BONUS:
In how many different ways can the letters A, A, B, B, B, C, D, E be arranged if the letter C must be to the right of the letter D?

1st Method (Partitions)

Without any restrictions:

# of ways $= \dfrac{8!}{2! \cdot 3!} = 3360$

In half of those arrangements letter C is right to D.

$\therefore$ # of arrangements with C is right of D $= \dfrac{1}{2} * 3360 = \boxed{1680}$

2nd Method (Multiplication Rule * Partitions)

# of ways to have C right of D $= 7+6+5+4+3+2+1 = 28$

# of ways the rest can be distributed $= \dfrac{6!}{3! \cdot 2!}$

$\therefore$ # of arrangements with C is right of D $= 28 * \dfrac{6!}{3! \cdot 2!} = \boxed{1680}$

HW - 4 : W (9/27), in the conference

Class -16

Chapter-3: Conditional Probability

Defn

For any events A and B with $P(A) > 0$, the probability B will occur given that A has occurred is

$$P(B|A) = \frac{P(A \cap B)}{P(A)} \; ; \; P(A) > 0.$$

Note:

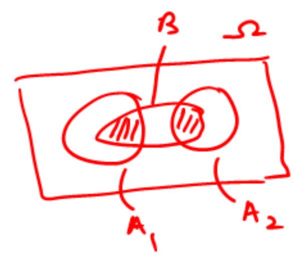
* Conditional Probability follows probability axioms.

* $P(\Omega/B) = 1$.

Proof: $P(\Omega|B) = \frac{P(\Omega \cap B)}{P(B)} = \frac{P(B)}{P(B)} = 1.$

* If A_1 and A_2 are disjoint, (ie $A_1 \cap A_2 = \Phi$)

$$P(A_1 \cup A_2 / B) = P(A_1|B) + P(A_2|B).$$



Proof:

Since A_1 and A_2 are disjoint, $A_1 \cap B$ and $A_2 \cap B$ are also disjoint

$$P(A_1 \cup A_2 | B) = \frac{P((A_1 \cup A_2) \cap B)}{P(B)}$$

$$= \frac{P((A_1 \cap B) \cup (A_2 \cap B))}{P(B)}$$

$$= \frac{P(A_1 \cap B)}{P(B)} + \frac{P(A_2 \cap B)}{P(B)}$$

$$= P(A_1|B) + P(A_2|B).$$

Eg-1) We toss a coin 3 times. If fist toss is a head, what is the probability ~~that~~ of having more heads than tails.

$$\Omega = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}.$$

Let A — having more heads than tails. $P(A/B) = ?$

B — first toss is a head.

$$P(B) = 4/8 = 1/2 \quad , \quad P(A \cap B) = 3/8$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{3/8}{4/8} = 3/4$$

2) Suppose that 5 good fuses and two defective ones have been mixed up. To find the defective fuces, we test them one by one at random and without replacement.

What is the probability that we ~~at~~ find both of the defectives at the first two test.

7

5-G → ①

2-D → ②

Let D_1 — 1^{st} draw is a defective

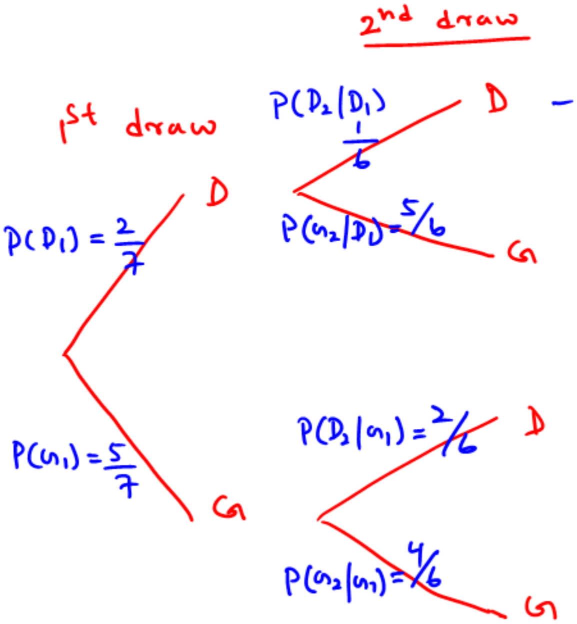
D_2 — 2^{nd} draw is a defective, $P(A_1 \cap A_2) = ?$

$$\boxed{P(A \cap B) = P(B/A) \cdot P(A)}$$

$$P(D_1) = 2/7 \quad , \quad P(D_2/D_1) = \frac{1}{6}$$

$$P(D_1 \cap D_2) = P(D_1) \cdot P(D_2|D_1) = \frac{2}{7} \cdot \frac{1}{6} = \frac{2}{42} = 1/21.$$

2nd method (Tree diagram method)



Events ($D_1 \cap D_2$)	probabilities
(D, D)	$2/7 * 1/6$
(D, G)	$2/7 * 5/6$
(G, D)	$\frac{5}{7} * 2/6$
(G, G)	$\frac{5}{7} * 4/6$

$$P(\text{both defective}) = P(D, D) = 2/7 \cdot 1/6 = 1/21$$

* Multiplication Rule (General)

For any events $A_1, A_2, \ldots, A_n$.

$$P\left(\bigcap_{i=1}^{n} A_i\right) = P(A_1) \cdot P(A_2|A_1) \cdot P(A_3|A_1 \cap A_2) \cdots\cdots P\left(A_n \middle| \bigcap_{i=1}^{n-1} A_i\right).$$

Note:

* when $n = 2$

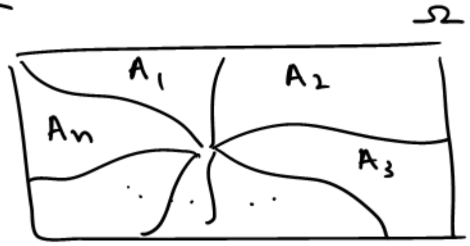
$$P(A_1 \cap A_2) = P(A_1) \cdot P(A_2|A_1).$$

* when $n = 3$

$$P(A_1 \cap A_2 \cap A_3) = P(A_1) \cdot P(A_2|A_1) \cdot P(A_3|A_1 \cap A_2).$$

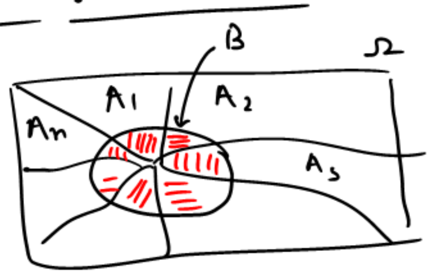
Sec 3.2 : Two Stage Experiments

Defn Partition



A collection of events $A_1, A_2, \ldots A_n$ is called a partion of Ω if $\bigcup_{i=1}^{n} A_i = \Omega$ and $A_i \cap A_j = \phi$ for all $i, j = 1, 2, \ldots n$ and $i \neq j$.

Total Probability theorem



Let $A_1, A_2, \ldots, A_n$ be a partition of Ω. Then for any event B,

$$P(B) = \sum_{i=1}^{n} P(B/A_i) \cdot P(A_i).$$

$$P(B) = P(B/A_1) \cdot P(A_1) + P(B|A_2) \cdot P(A_2) + \cdots + P(B|A_n) \cdot P(A_n).$$

Proof:

$$P(B) = P\left(\bigcup_{i=1}^{n} \underline{A_i \cap B}\right)$$

Since $A_1, A_2, \ldots A_n$ are pairwise disjoint, $A_1 \cap B, A_2 \cap B, \ldots A_n \cap B$ are also pairwise disjoint.

$$\therefore P(B) = \sum_{i=1}^{n} \underbrace{P(A_i \cap B)}$$

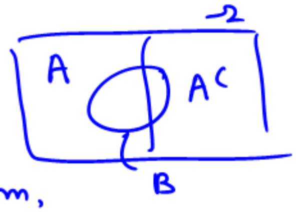
$P(A \cap B) = P(B/A) \cdot P(A).$

$$= \sum_{i=1}^{n} P(B/A_i) \cdot P(A_i) \quad \text{(Multiplication Rule)}$$

$\square$.

Note:

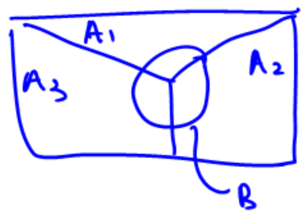
* when $n=2$



Total probability theorem,

$$P(B) = P(B|A)\cdot P(A) + P(B|A^c)\cdot P(A^c).$$

* when $n=3$



Total probability theorem,

$$P(B) = P(B|A_1)\cdot P(A_1) + P(B|A_2)\cdot P(A_2) + P(B|A_3)\cdot P(A_3)\,\square.$$