HW - 5 : W (10/04), in the conference

Quiz -5 : F (10/06)

Class -22

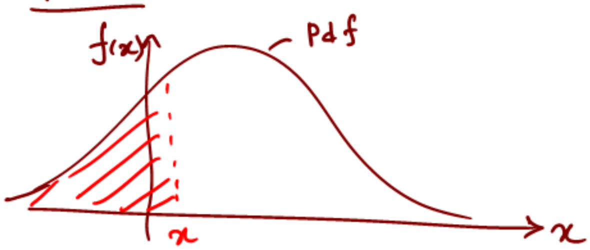
Rest of the term:
Sec 5.2 : Cumulative distribution function.
Sec 6.4 : Normal distribution
Sec 6.5 : Central Limit theorem
Sec 5.3 : Functions
Sec 5.4 : Joint distributions (continuous)
Sec 5.5 : Marginal distributions (continuous)

Note:

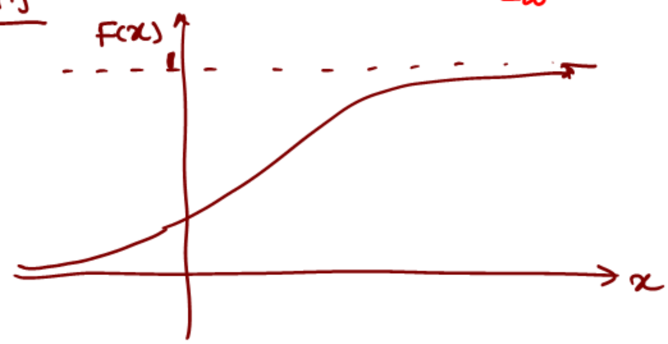
* For a discrete random variable, the cdf is always a step function.
* Usually for a continuous random variable,

$$F(x) = \int_{-\infty}^{x} f(t)\,dt$$

pdf



* cdf



Properties of cdfs

* $\lim_{x\to\infty} F(x) = 1$, $\lim_{x\to -\infty} F(x) = 0$

* $F(x)$ is non-decreasing (constant or increasing)
* $F(x)$ is right continuous, (ie $\lim_{x\to a^+} F(x) = F(a)$).

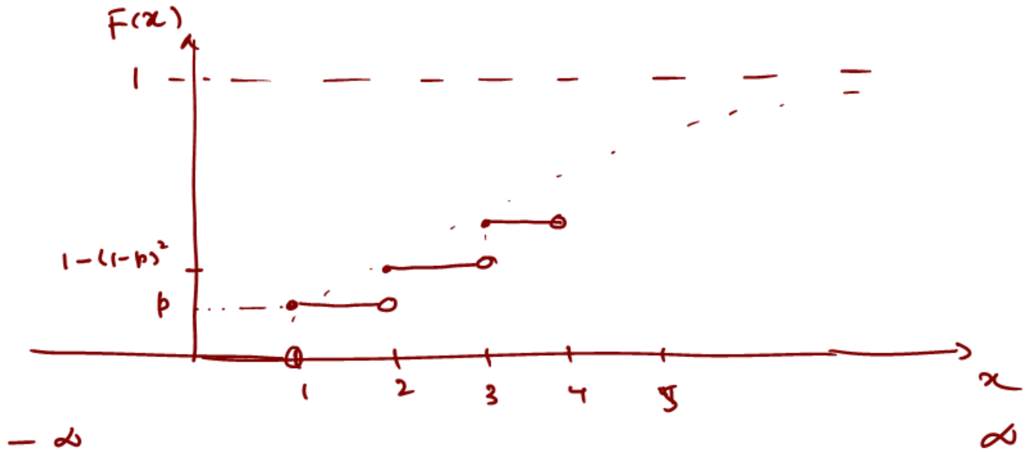
Eg:- Let $X \sim \text{Geometric}(p)$. Find the cdf of X.

Pmf: $p(x) = p(1-p)^{x-1}$, $x = 1, 2, 3, \ldots$

cdf:
$$F(x) = \sum_{k=1}^{x} p(k)$$
$$= \sum_{k=1}^{x} (1-p)^{k-1} p$$
$$= p + p(1-p) + p(1-p)^2 + \dots + p(1-p)^{x-1}$$
$$= \frac{p\{1-(1-p)^x]}{\cancel{1}-[\cancel{1}-p)}$$
$$= 1-(1-p)^x \quad : \quad x = 1, 2, 3, \dots$$

$$\sum_{i=1}^{n} ar^{i-1} = a + ar + ar^2 + \dots + ar^{n-1} = \frac{a(1-r^n)}{1-r}$$

$1-(1-p)^2$



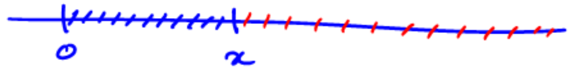
Eg:- Let $X \sim Exp(\lambda)$. Find cdf of X.

Pdf: $$f(x) = \lambda e^{-\lambda x} \quad : \quad x \geq 0, \quad \lambda > 0$$

cdf:
$$F(x) = \int_{-\infty}^{x} f(t)\,dt$$
$$= \int_{0}^{x} \lambda e^{-\lambda t}\,dt$$
$$= \cancel{\lambda} \cdot \frac{e^{-\lambda t}}{-\cancel{\lambda}} \Big|_{0}^{x}$$
$$= -[e^{-\lambda x} - 1] = 1 - e^{-\lambda x}$$

$$F(x) = \underline{1 - e^{-\lambda x}} \quad : \quad \underline{x \geq 0}$$ OR $$F(x) = \begin{cases} 1 - e^{-\lambda x} & : x \geq 0 \\ 0 & \text{Otherwise} \end{cases}$$

2nd method: (using $P(X \geq a) = e^{-\lambda a}$)



$$F(x) = P(X \leq x)$$
$$= 1 - P(X > x)$$
$$= 1 - e^{-\lambda a} \quad ; \quad x \geq 0.$$

Sec 6.4 Normal Distribution

A continuous random variable X is said to have a normal distribution (ie Gausian distribution) with parameters μ and σ^2 (ie $X \sim N(\underset{\text{mean}}{\mu}, \underset{\text{variance}}{\sigma^2})$), if

$$f(x) = \underbrace{\frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}} \quad ; \quad -\infty < x < \infty, \ -\infty < \mu < \infty$$
$$\sigma > 0.$$

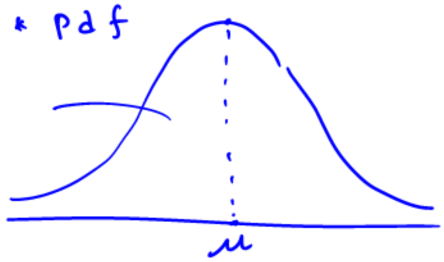
Note:

* $\displaystyle\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \, dx = 1$

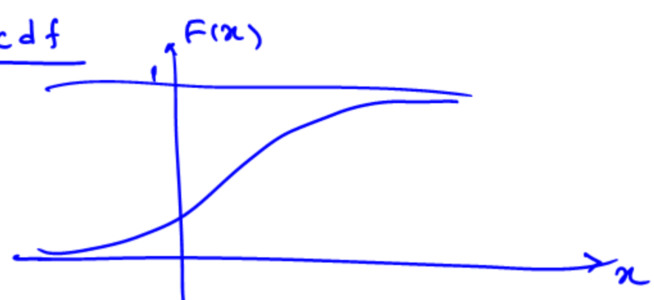
* $E(X) = \mu$, $var(X) = \sigma^2$.
  [Standard deviation $= \sigma$].

* Graphs

* pdf



* cdf



* If $Y = aX + b$ and $X \sim N(\mu, \sigma^2)$, then $Y \sim \underline{N}(a\mu+b, a^2\sigma^2)$

$$E(Y) = E(aX+b) = a\underbrace{E(X)}_{=\mu}+b = a\mu+b$$

$$Var(Y) = Var(aX+b) = a^2 Var(X) = a^2\sigma^2$$

The Standard Normal Distribution

A normal random variable Z with mean 0 (ie $E(Z)=0$) and variance 1 (ie $Var(Z)=1$) is called a Standard normal random variable (ie $Z \sim \underbrace{N(0,1)}$)

Standard normal.

Note:

* pdf of Z : $P(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}$: $-\infty < z < \infty$

* cdf of Standard normal distribution is

$$\Phi(z) = P(Z \le z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-t^2/2}\,dt$$

* Values of $\Phi(z)$ are tabulated for different values of z.

Eg:-

$1.76 \to 1.7 + 0.06$

1. $P(Z \le 1.76) = 0.9608$

2. $P(Z \le 1.06) = 0.8554$

3. $P(Z > 2.32) =$

4. $P(Z < -2.56) =$