$P(Y=0) = P(X=0) = 1/5$

$P(Y=1) = P(X=-1) + P(X=1) = 2/5$

$P(Y=4) = P(X=-2) + P(X=2) = 2/5$

So the pmf of Y:

$$p(y) = \begin{cases} 1/5 & : y=0 \\ 2/5 & : y = 1, 4 \\ 0 & : \text{otherwise.} \end{cases}$$

Quiz -5 : Today (10/06)

HW - 6 : T (10/10)

Class -25

Final Exam : R (10/12)

Practice Exam : Discuss in the class T (10/10)

Cover materials after Exam -1
One double sided handwritten sheet is allowed
Bring your calculator

Note:

When X and Y are continuous, this method can not be used.

In the continuous case, there are two methods

1. cdf method
2. Transformation method.

1. cdf method

Let X be a continuous random variable and $Y = g(X)$.

There are two steps

1) Find the cdf of Y ($F_Y(y)$) (in terms of $F_X(x)$).
2) Differentiate to obtain the pdf of Y ($f_Y(y)$)

$F_X(x) = \int_{-\infty}^{x} f(t)\,dt$

$\Rightarrow f(x) = \frac{d}{dx}[F_X(x)]$

Eg:- Let $\underline{X} \sim$ uniform $[0,1]$ and $Y = \sqrt{X}$. Find the pdf of Y.

$X \sim$ uniform $[a,b]$

Pdf:

$$f(x) = \frac{1}{b-a} \quad : a \le x \le b$$

Pdf of X:

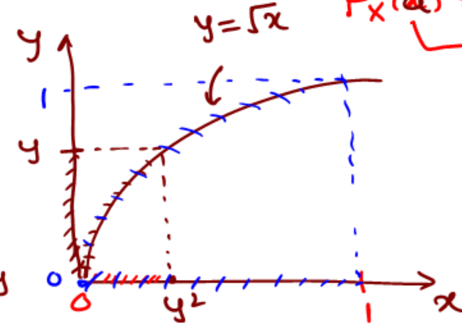
$$\underline{f_X(x) = 1} \quad ; \quad 0 \le x \le 1$$

1) $F_Y(y) = P(Y \le y) = P(\sqrt{X} \le y)$

$$= P(X \le y^2)$$

$$= \underbrace{F_X(y^2)}$$

$$F_X(a) = \underline{P(X \le a)}$$



2) Take the derivative with respect to y (d.w.r.t. y)

$$f_Y(y) = \underbrace{f_X(y^2)} \cdot \underbrace{\frac{d(y^2)}{dy}}$$

$$\frac{d}{dx}(f(g(x))) = \underline{f'(g(x))} \cdot \underline{g'(x)}$$

$$= \underline{1 \cdot 2y} \quad ; \quad 0 \le y \le 1$$

Pdf of Y:

$$\underline{f_Y(y) = 2y \quad ; \quad 0 \le y \le 1}$$

OR

$$f_Y(y) = \begin{cases} 2y & : 0 \le y \le 1 \\ 0 & : \text{otherwise} \end{cases}$$

Eg-2)

John Slow is driving from Boston to New York area a distance of 180 miles. His average speed is uniformly distributed between 30 and 60 miles per hour. What is the pdf of duration of the trip?

Let X – Speed and Y – duration then,

$$X = \frac{180}{Y} \Rightarrow Y = \frac{180}{X} = g(X)$$

Pdf of X: $\quad f_X(x) = \frac{1}{60-30} = \frac{1}{30} \quad ; \quad 30 \le x \le 60.$

1) cdf of Y:

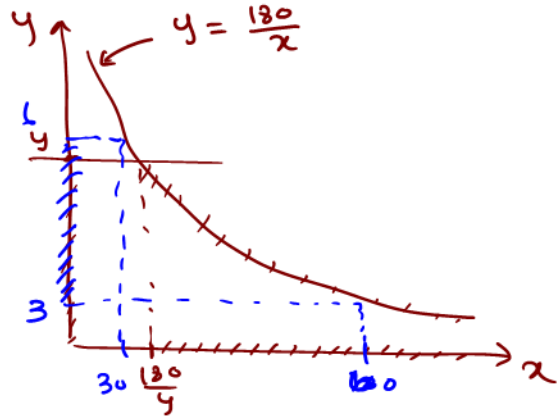
$$F_Y(y) = P(Y \le y) = P\left(\frac{180}{X} \le y\right)$$

$$= P\left(\frac{X}{180} \ge \frac{1}{y}\right)$$

$$= P\left(X \ge \frac{180}{y}\right)$$

$$= 1 - \underbrace{P\left(X < \frac{180}{y}\right)}$$

$$= 1 - F_X\left(\frac{180}{y}\right)$$



$$F_X(x) = P(X \le x)$$

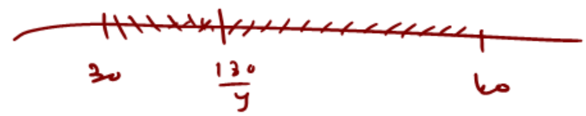
2) d.w.r.t. y

$$f_Y(y) = -f_X\left(\frac{180}{y}\right) \cdot \frac{d}{dy}\left(\frac{180}{y}\right)$$

$$= -\underbrace{f_X\left(\frac{180}{y}\right)}\left(\frac{-180}{y^2}\right)$$

$$= \frac{1}{30} \cdot \frac{180}{y^2}$$

$$= \frac{6}{y^2} \; ; \; 3 \le y \le 6$$



$$\frac{d}{dy}(1/y) = \frac{d}{dy}(y^{-1}) = \frac{-1}{y^2}$$