* General Union

For any events $A_1, A_2, \dots, A_n$

$$\underbrace{P\left(\bigcup_{i=1}^{n} A_i\right)}_{P(A_1 \cup A_2 \cup \dots \cup A_n)} = \sum_{i=1}^{n} P(A_i) - \sum_{i<j} \underbrace{P(A_i \cap A_j)} + \sum_{i<j<k} P(A_i \cap A_j \cap A_k)$$

$$+ \dots + (-1)^{n+1} P(A_1 \cap A_2 \cap \dots \cap A_n).$$

Quiz -3 : Today (9/15)

Class -13

Exam -1 : T (9-19) : Covers First Two chapters

Practice Exam : M (9-18, discuss in the class)

Eg:-

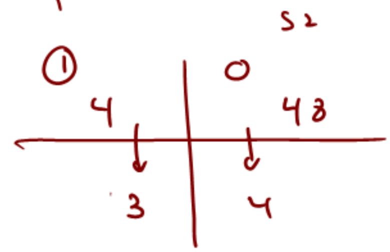
You pick seven cards out of deck of 52.
What is the probability that you have at least one 3 of a kind?

Let A_i – having 3 cards of type i, $i = 1, 2, 3, 4, \dots 13$.

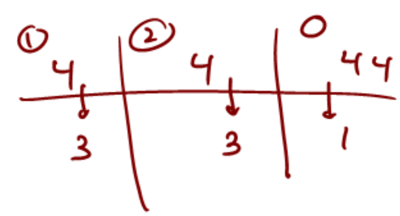
[1 – A, 11 – jack, 12 – queen, 13 – king].

$$P\left(\bigcup_{i=1}^{13} A_i\right) = \sum_{i=1}^{13} P(A_i) - \sum_{i<j} \underbrace{P(A_i \cap A_j)} + \cancel{\sum_{i<j<k} P(A_i \cap A_j \cap A_k)}$$

$$P(A_1) = \frac{{}^{4}C_{3} * {}^{48}C_{4}}{{}^{52}C_{7}} = P(A_i) \text{ for } i = 2, \dots 13.$$



$$P(A_1 \cap A_2) = \frac{{}^{4}C_{3} * {}^{4}C_{3} * {}^{44}C_{1}}{{}^{52}C_{7}} = P(A_i \cap A_j) \text{ for all } i, j = 2, \dots 13, \; i<j$$



\# of terms

$$P\left(\bigcup_{i=1}^{13} A_i\right) = {}^{13}C_{1} * \frac{{}^{4}C_{3} * {}^{48}C_{4}}{{}^{52}C_{7}} - {}^{13}C_{2} * \frac{{}^{4}C_{3} * {}^{4}C_{3} * {}^{44}C_{1}}{{}^{52}C_{7}}$$

$$= \underline{\underline{\qquad}}$$

Eg:- 2)

Suppose we roll a die 15 times. What is the probability that we do not see each of the 6 numbers at least once?

Let A_i – we never see i, $i = 1, 2, 3, \dots 6$. $P\left(\bigcup_{i=1}^{6} A_i\right) = ?$

$$P\left(\bigcup_{i=1}^{6} A_i\right) = \sum_{i=1}^{n} P(A_i) - \sum_{i<j} P(A_i \cap A_j) + \sum_{i<j<k} P(A_i \cap A_j \cap A_k) - \dots - \underbrace{\cancel{P(A_1 \cap A_2 \dots A_6)}}.$$

$\frac{4}{4}$ — — — — —

$P(A_i) = \left(\frac{5}{6}\right)^{15}$, $i = 1, 2, \dots 6$ # of times $= {}^6C_1 = 6$

$P(A_i \cap A_j) = \left(\frac{4}{6}\right)^{15}$, $i = j = 1, 2, \dots 6$, $i < j$ # of times $= {}^6C_2$

$P(A_i \cap A_j \cap A_k) = \left(\frac{3}{6}\right)^{15}$ # of times $= {}^6C_3$

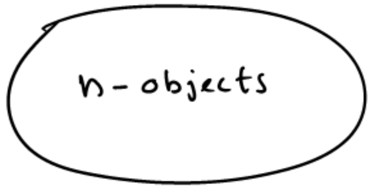
using the pattern,

$$P\left(\bigcup_{i=1}^{6} A_i\right) = {}^6C_1\left(\frac{5}{6}\right)^{15} - {}^6C_2\left(\frac{4}{6}\right)^{15} + {}^6C_3\left(\frac{3}{6}\right)^{15} - {}^6C_4\left(\frac{2}{6}\right)^{15} + {}^6C_5\left(\frac{1}{6}\right)^{15}$$

$$= 0.3558$$

Summary [combinatorics]

① All are different (n-objects)



* Order doesn't matter

* Take all of them (do not arrange)

  # of ways = 1

* Pick $k\ (\le n)$ of them (do not arrange)

  # of ways $= {}^{n}C_{k} = \binom{n}{k} = \dfrac{n!}{(n-k)!\cdot k!}$

  (combinations)

* Order matters

* Take all of them and arrange

  # of ways $= n!$

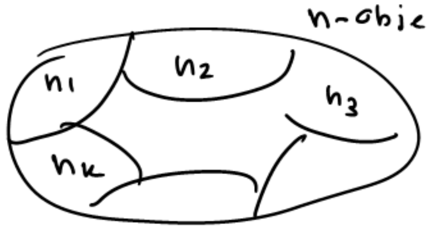
* pick $k\ (\le n)$ of them and arrange

  # of ways $= {}^{n}P_{k} = \dfrac{n!}{(n-k)!}$

  (Permutations)

② All are not different.

n-object, k-groups



* Order doesn't matter

* Take all of them

  # of ways = 1

* Pick $k\ (\le n)$ of them

  did not cover in the class.

* Order matters

* Take all of them and arrange

  # of ways $= \dfrac{n!}{n_1!\cdot n_2!\dots n_k!}$ (Partition)

* Pick $k\ (\le n)$ of them and arrange

  did not cover in the class.