MA 2621 - A 17

HW #5 (Problems 1-8) **Due W 10/04**

#1. Consider the following joint probability distribution of X and Y

Y \ X	1	2	3	4
4	**0**	**1/20**	**1/20**	**1/20**
3	**1/20**	**2/20**	**3/20**	**1/20**
2	**1/20**	**2/20**	**3/20**	**1/20**
1	**1/20**	**1/20**	**1/20**	**0**

a) Find marginal distributions $P(x)$ and $P(y)$.

b) If $Z = X + 2Y$, find $P(z)$.

c) Using b), find $E(Z)$.

d) Find $E(X)$ and $E(Y)$ then find $E(Z)$ using the expectation of X and expectation of Y

e) Are X and Y independent?

#2. Let joint Probability distribution function of random variables X and Y be

$$P(x,y) = \begin{cases} 1/3 & \text{if } (x,\ y) = (1,\ 1) \\ 1/3 & \text{if } (x,\ y) = (2,\ 0) \\ 1/3 & \text{if } (x,\ y) = (0,\ 0) \\ 0 & \text{otherwise } . \end{cases}$$

Are X and Y independent?

#3. Each morning John eats some eggs. On any given morning, the number of eggs he eats is **equally likely** to 1, 2, 3, 4, or 5 independent of what he has done in the past. Let X be the number of eggs that John eats in 10 days. Find the mean and the variance of X.

#4. The time till failure of an electronic component has an Exponential distribution and it is known that 10% of components have failed by 1000 hours.

(a) What is the probability that a component is still working after 5000 hours?

(b) Find the mean and standard deviation of the time till failure.

#5. Let X be a continuous random variable with PDF,

$$f_X(x) = \begin{cases} ax^2 & : 0 < x < 2. \\ 0 & : otherwise \end{cases}$$

a) Find a.
b) Find variance of X.
c) Find Cumulative Distribution Function (CDF) of X.

#6. Let X be a continuous random variable with PDF,

$$f_X(x) = \begin{cases} ke^{-2x} & : \quad x > 0. \\ 0 & : otherwise \end{cases}$$

a) Find k.
b) Find mean of X.
c) Find variance of X.
d) Find CDF of X.

#7. Let X be a **discrete** random variable with probability distribution (probability mass function),

$$P(x) = c\,(1/3)^x, \quad x = 0, 1, 2, \ldots$$

a) Find c such that $P(x)$ is a legitimate PMF.
b) Find Cumulative Distribution Function (cdf) of X, $F(x)$, $x \in \{0, 1, 2, \ldots\}$

#8. Suppose X has probability density function $f_X(x) = \begin{cases} 4x^3 & : \quad 0 < x < 1. \\ 0 & : otherwise \end{cases}$

a) Find cdf of X.
b) **Using the answer in art a)**, find $P(X < 1/2)$.
c) **Using the answer in art a)**, find $P(1/3 < X < 2/3)$.

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Suggested Problems

Chapter-3: 60-66, Chapter-5: 2,4,5,6,8,9,10,11.