

PROBABILITY FOR APPLICATIONS

HW #1 (Problems 1-8)

Due: T (8/29)

#1. A box contains a red marble, a blue marble and a yellow marble. An experiment consist of drawing two marbles at random out of the box, one after the other without replacement.

- (a) List the sample space.
- (b) How many outcomes are in the event “A- exactly one marble is red”? Calculate $P(A)$.
- (c) Repeat parts (a) and (b) for an experiment that draws two marbles at random from the box, one after the other with replacement.

#2. In automotive repair, experience has shown that a rough-running engine can be attributed to bad ignition wires 35% of the time, bad spark plugs 80% of the time, and both 20 % of the time.

- (a) Suppose a mechanic begins to diagnose the problem by checking the spark plugs first and find them to be bad. With this information, what is the probability that the wires are also bad?
- (b) If the sparks plugs are found to be good, what is the probability that wires are bad?

#3. The experience of a car dealer shows that 60% of the customers purchase a stereo as an option with their cars, 50% purchase tinted windows, and 35% purchase both. Find the probability that a customer purchase at least one of these two options.

#4. A 6 sided die is thrown repeatedly, until the first time (if ever) that a ‘1’ is tossed.

- a. What is the sample space for this experiment?
- b. Let A be the event that the first ‘1’ occurs at the 4th throw or later. If you wanted to calculate $P(A)$ what would be the most efficient way to calculate this? (*Refer to a non-‘1’ generically as an “f”-failure.*)
- c. **(optional)** Generalize a formula for the probability of the first ‘1’ occurring at the n -th toss or later?

(For final answer make use of finite geometric series; which equals two series: an infinite series starting at 0 minus an infinite series from a later start point.)

#5. Let A and B be events where $P(A) = 1/3$ and $P(B) = 1/4$. Show bounds for $P(A \cup B)$ and $P(A \cap B)$.

#6. In Galileo's time people thought that when three dice were rolled, a sum of 9 and sum of 10 had same probability since each could be obtained in 6 ways:

9: $1+2+6, 1+3+5, 1+4+4, 2+2+5, 2+3+4, 3+3+3$ 10: $1+3+6, 1+4+5, 2+4+4, 2+3+5, 2+2+6, 3+3+4$. Compute the probabilities of these sums and show that 10 is a more likely total than 9.

#7. Suppose A and B are connected in parallel as shown in the figure.

In this case, at least one component must work for the system to work. Again, suppose that components function independently, with probabilities 0.9 and 0.8 for A and B respectively. Calculate the probability that the system works.

#8. Let $P(A \cap B) = 0.5$, $P(B) = 0.25$ and $P(A \cup B) = 0.75$.

(a) Find $P(A \cap B)$, $P(A)$ and $P(B^c \cap A^c)$.

(b) Are A and B independent?

Hint: You may use $B^c \cap A^c = (A \cup B)^c$ (De Morgan's Law)

Extra Suggested Problems: Durrett : p26: 1,3,6,19,21,25,31,33.