Eg:- Suppose the average number of lions seen on 1-day Safari is 5.

a) What is the probability that a tourist will see exactly 4 lions on the next day.

Let X - # of lions will be seen.

$\lambda = 5$.

then, $p(X=x) = \frac{e^{-\lambda}\lambda^{x}}{x!}, x = 0, 1, 2 \ldots$

$X \sim$ Poisson $(\lambda = 5)$.

$p(X = 4) = \frac{e^{-5} 5^{4}}{4!} = 0.17547.$

x

Quiz -3 : F (9/15)

Class -12

Exam -1 : T (9-19) : Covers First Two chapters

Practice Exam : M (9-18, discuss in the class)

b) What is the probability that the tourist will see fewer than 4 lions?

Possible values: 0, 1, 2, 3, ...

$$P(X<4) = P(0) + P(1) + P(2) + P(3)$$

$$= \frac{e^{-5}5^{0}}{0!} + \frac{e^{-5}5^{1}}{1!} + \frac{e^{-5}5^{2}}{2!} + \frac{e^{-5}5^{3}}{3!}$$

$$= e^{-5}\left[1 + 5 + \frac{25}{2} + \frac{125}{6}\right]$$

$$= 0.26503$$

Poisson Approximation to the Binomial

Theorem

Suppose S_n has a Binomial distribution with parameters n and p_n. If $p_n \to 0$ and $n \cdot p_n \to \lambda$ as $n \to \infty$, then

$$P(S_n = x) \longrightarrow \frac{e^{-\lambda}\lambda^x}{x!}.$$

– Probability calculated using Poisson (λ).

Note:

For large n and small p, Binomial probabilities can be approximated by Poisson distribution, with $\lambda = n \cdot p$.

Eg:-

Suppose we roll two dies 12 times. Let D be the number of times a double six appears. Find exact and approximation values of P(D=k) for k=0,1,2.

$n = 12$, $p = P(\{6,6\}) = 1/36$ $\qquad \{(6,6)\}$.

$\lambda = n \cdot p = 12 \cdot \frac{1}{36} = 1/3$.

* **Exact value:** (Binomial)

$$P(X=0) = {}^{12}C_0 (1/36)^0 (35/36)^{12} = 0.7132$$

* **Appro: value:** (Poisson)

$$P(X=0) \simeq \frac{e^{-1/3}(1/3)^0}{0!} = 0.7165$$

* **Exact value:**

$$P(X=1) = {}^{12}C_1 (1/36)^1 (35/36)^{11} = 0.2445$$

* **Appro: value:**

$$P(X=1) \simeq \frac{e^{-1/3}(1/3)^1}{1!} = 0.2388$$

* Exact value:

$$P(X=2) = {}^{12}C_2 \left(\tfrac{1}{36}\right)^2 \left(\tfrac{35}{36}\right)^{10} = 0.0384$$

* Appro: value:

$$P(X=2) \simeq \frac{e^{-1/3}(1/3)^2}{2!} = 0.0398$$

Eg:

If we are in a group of 183 individuals. What is the probability that no one else has my birthday?

$n = 182$, $p = \frac{1}{365}$

Let X – # of people who have the same birthday.

Then, $X \sim \text{Binomial}\left(182, \frac{1}{365}\right)$

Exact:

$$P(X=0) = {}^{182}C_0 \left(\tfrac{1}{365}\right)^0 \left(\tfrac{364}{365}\right)^{182} = 0.6069$$

App: value:

$$\lambda = n \cdot p = \frac{182}{365}$$

$$P(X=0) \simeq \frac{e^{-\frac{182}{365}}\left(\frac{182}{365}\right)^0}{0!} = 0.6073$$

Sec 2.5 : Probabilities of Unions

Recall:

For any events A and B,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

$P(A \cap B) \neq P(A) \cdot P(B)$.

* For any events A, B and C,

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C).$$

Proof:

$$P(A\cup B\cup C) = P(A) + \underbrace{P(B\cup C)} - P(A\cap \underbrace{(B\cup C)})$$

$$= P(A) + P(B) + P(C) - P(B\cap C) - P[(A\cap B)\cup(A\cap C)].$$

$$= P(A) + P(B) + P(C) - P(B\cap C) - \{P(A\cap B) + P(A\cap C) - P(A\cap B\cap A\cap C)\}$$

$$= P(A) + P(B) + P(C) - P(A\cap B) - P(A\cap C) - P(B\cap C) + P(A\cap B\cap C).$$

Eg:-

Suppose we roll 3 dice. What is the probability that we get at least one 6?

Let A_i - we get 6 on the "i^{th}" die, $i = 1, 2, 3$. $P(A_1\cup A_2\cup A_3) = ?$

then $P(A_1) = \frac{1}{6} = P(A_2) = P(A_3)$

$$P(A_1\cap A_2) = \frac{1}{36} = P(A_2\cap A_3) = P(A_1\cap A_3)$$

$$P(A_1\cap A_2\cap A_3) = \frac{1}{216}$$

$$P(A_1\cup A_2\cup A_3) = P(A_1) + P(A_2) + P(A_3) - P(A_1\cap A_2) - P(A_2\cap A_3) - P(A_1\cap A_3) + P(A_1\cap A_2\cap A_3)$$

$$= 3\cdot\frac{1}{6} - 3\cdot\frac{1}{36} + \frac{1}{216}$$

$$= \frac{91}{216}$$

2nd method: Using Binomial Distribution.

$n = 3$, $p = 1/6$, X - # of 6s.

$$P(X\geq 1) = P(1) + P(2) + P(3)$$

$$= {}^3C_1 (1/6)^1 (5/6)^2 + {}^3C_2 (1/6)^2 (5/6)^1 + {}^3C_3 (1/6)^3 (5/6)^0$$

$$= 91/216$$

$$= 1 - P(0) = 1 - {}^3C_0 (1/6)^0 (5/6)^3 = 91/216$$

* General Union

For any events $A_1, A_2, \dots, A_n$

$$P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} P(A_i) - \sum_{i<j} P(A_i \cap A_j) + \sum_{i<j<k} P(A_i \cap A_j \cap A_k) + \dots + (-1)^{n+1} P(A_1 \cap A_2 \cap \dots \cap A_n).$$

Eg:-

You pick seven cards out of deck of 52. What is the probability that you have three of a kind?