Assume each team has 0.5 probability to win a game.

* Four games : ($X = 4$)

* 1^{st} team win all 4 games.

$$P(1^{st} \text{ team win all } 4) = \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2} = \frac{1}{16}$$

Similarly

$$P(2^{nd} \text{ team win all } 4) = \frac{1}{16}$$

$$P(4) = P(X=4) = \frac{1}{16} + \frac{1}{16} = 2/16.$$

* Five games ($X = 5$)

1^{st} team

BAAAA, ABAAA, AABAA, AAABA, ~~AAAAB~~

$$P(1^{st} \text{ team}) = \left(\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\right) \times 4$$

$$P(X=5) = 2 \times 4 \times \left(\frac{1}{2}\right)^5 = 1/4$$

* Six games ($X = 6$)

BBAAAA, ABBAAA, AABBAA, AAABBA, ~~AAAABB~~

BABAAA, ABABAA, AABABA, AAABA~~B~~

BAABAA, ABAABA

BAAABA

$$P(X=6) = 2 \times 10 \times \frac{1}{2^6} = 5/16$$

* Seven Games ($X=7$)

$$P(X=7) = 1 - P(4) - P(5) - P(6)$$
$$= 1 - 2/16 - 4/16 - 5/16 = 5/16$$

[$\because$ all the possible values of X are : 4, 5, 6, 7]

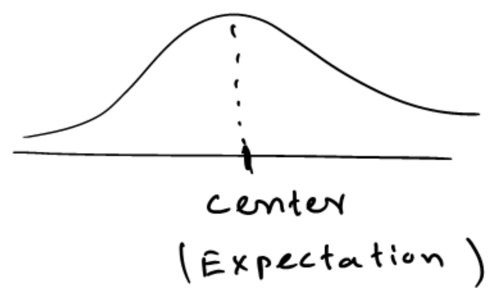
$\therefore$ Probability distribution of X :

$X=x$	4	5	6	7
$P(X=x)$	$\frac{2}{16}$	$\frac{4}{16}$	$\frac{5}{16}$	$\frac{5}{16}$

Alternative way :

$$P(X=x) = p(x) = \begin{cases} \frac{2}{16} & : x=4 \\ \frac{4}{16} & : x=5 \\ \frac{5}{16} & : x=6,7 \\ 0 & : \text{otherwise.} \end{cases}$$

Sec 1.5 : Expected value



Defn $E(X)$

The expected value of random variable X (or mean of X, or Average of X) is defined as

$$E(X) = \sum_{x} x \cdot p(X=x)$$

Eg: consider the distribution $p(x) = \begin{cases} \frac{1}{16} & : x=0 \\ 6/16 & : x=1 \\ 9/16 & : x=2 \end{cases}$

then

$$E(X) = \sum_{x=0}^{2} x \cdot p(X=x) = 0 \cdot \frac{1}{16} + 1 \cdot \frac{6}{16} + 2 \cdot \frac{9}{16} = \frac{24}{16} = 3/2$$

Eg:-

If you play roulette and bet \$1 on black then you win \$1 with probability 18/38 and you lose \$1 with probability 20/38. What is the expected win?

x	1	-1
$p(x)$	$18/38$	$20/38$

X - winning amount

$$E(X) = 1 \cdot \frac{18}{38} + (-1) \cdot \frac{20}{38} = \frac{-2}{38} = -0.0526$$

Eg-3)

Expectation of Geometric distribution.

Probability distribution: $p(x) = \underbrace{p(1-p)}^{x-1}$: $x = 1, 2, 3, \ldots$

So

$$E(X) = \sum_{x=1}^{\infty} x \cdot \underbrace{p(x)}$$

$$= \sum_{x=1}^{\infty} x \cdot p(1-p)^{x-1}$$

$$= p\Big[\underbrace{1 \cdot (1-p)^0 + 2(1-p)^1 + 3(1-p)^2 + \ldots}_{S_\infty}\Big] \longrightarrow \text{(A)}$$

Let $S_\infty = 1 + 2(1-p) + 3(1-p)^2 + \ldots \longrightarrow (1)$

$(1) \times (1-p)$

$(1-p) S_\infty = \quad 1-p + 2(1-p)^2 + 3(1-p)^3 + \ldots \longrightarrow (2)$

$(1) - (2)$:

$$\underbrace{S_\infty - (1-p) S_\infty}_{p S_\infty} = 1 + 1-p + (1-p)^2 + \ldots$$

$$p S_\infty = \frac{1}{1-(1-p)} = \frac{1}{p}$$

$$\Rightarrow S_\infty = \frac{1}{p^2}.$$

$S_\infty = a + ar + ar^2 + \ldots$

$$\boxed{S_\infty = \frac{a}{1-r}}$$

So from (A)

$$E(X) = p \cdot \frac{1}{p^2} = 1/p$$

$\boxed{\begin{array}{l} * \text{ If } X \text{ follows a Geometric distribution with } p(\text{Success}) = p, \\ \qquad E(X) = \dfrac{1}{p} \end{array}}$