HW - 5 : W (10/04), in the conference

Quiz -5 : F (10/06)

Rest of the term:
Sec 6.4 : Normal distribution
Sec 6.5 : Central Limit theorem
Sec 5.3 : Functions

Final Exam : R (10/12)

Cover materials after Exam -1
One double sided handwritten sheet is allowed
Bring your calculator

Practice Exam : Discuss in the class T (10/10)

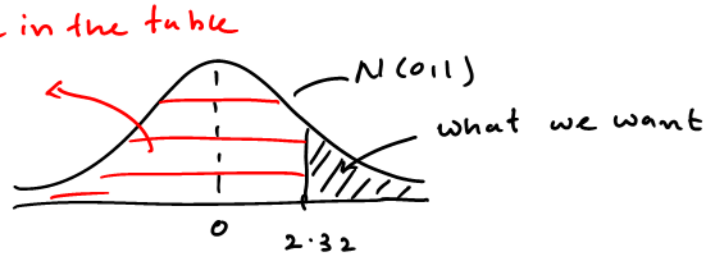
If $Z \sim N(0,1)$, find

3) $P(Z > 2.32)$

$= 1 - \underbrace{P(Z \le 2.32)}$

$= 1 - 0.9898$

$= 0.0102.$



4) $P(Z < -2.56)$

$= P(Z > 2.56)$

$= 1 - P(Z < 2.56)$

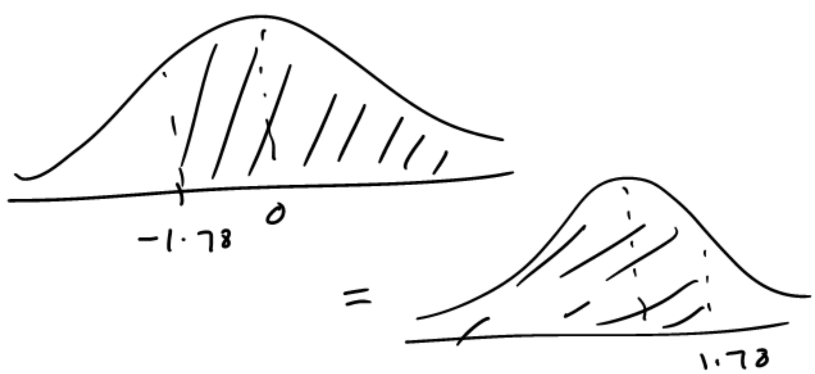
$= 1 - 0.9948$

$= 0.0052$

5) $P(Z > -1.78)$

$= P(Z < 1.78)$

$= 0.9625.$



Standardize

Let $X \sim N(\mu, \sigma^2)$ then $Z = \frac{X-\mu}{\sigma} \sim N(0, 1)$

$E(Z) = E\left(\frac{X-\mu}{\sigma}\right) = \frac{1}{\sigma}(E(X) - \mu) = \frac{1}{\sigma}(\mu - \mu) = 0$

$\mathrm{var}(Z) = \mathrm{var}\left(\frac{X-\mu}{\sigma}\right) = \frac{1}{\sigma^2}\mathrm{var}(X) = \frac{1}{\sigma^2} \cdot \sigma^2 = 1$

Now,

$$P(X > a) = P\left(\underbrace{\frac{X-\mu}{\sigma}}_{\parallel} > \frac{a-\mu}{\sigma}\right)$$ constant

$$= P\left(Z > \frac{a-\mu}{\sigma}\right)$$

$$= 1 - \Phi\left(\frac{a-\mu}{\sigma}\right)$$

$P\left(Z < \frac{a-\mu}{\sigma}\right)$

from the table

Eg:-

σ μ

An average light bulb manufactured by the Acme Corporation lasts 300 days with a standard deviation of 50 days. Assuming that bulb life is normally distributed, what is the probability that an Acme light bulb will last at most 365 days?

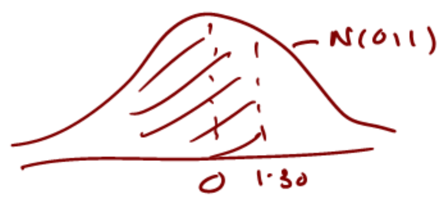
$\mu = 300, \ \sigma = 50$

Let X - lifetime of a bulb, then $X \sim N(300, 50^2)$ variance.

$$P(X \leq 365) = P\left(\frac{X-300}{50} \leq \frac{365-300}{50}\right)$$

$$= P(Z \leq 1.30)$$

$$= 0.9032$$



Eg:- The annual Snowfal at a particular geographic location is modeled as a normal random variable with mean 60 inches and, a Standard deviation 20 inches. What is the Probability that this year's Snowfall will be at least 80 inches?

$\mu = 60, \quad \sigma = 20$

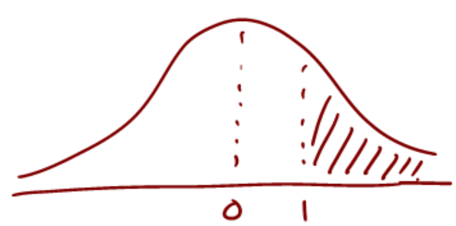
Let X - this year's Snowfall, then $X \sim N(60, 20^2)$.

$$P(X \geq 80) = P\left(\frac{X-60}{20} \geq \frac{80-60}{20}\right)$$

$$= P(Z \geq 1)$$

$$= 1 - P(Z < 1)$$

$$= 1 - 0.8413 = 0.1587$$



Sec 6.5 : Central Limit Theorem

$\binom{100}{10}$

* Random Sample

Defn

A sequence of independent and identically distributed (iid) random variables $X_1, X_2, X_3, \ldots, X_n$ is called a random sample. Let $E(X_i) = \mu$ and $var(X_i) = \sigma^2$, $i = 1, 2, 3, \ldots n$.

$\boxed{X_1 \quad X_2 \quad X_3 \quad \quad X_{10}}$ — random sample.

Sample-I —	x_{11}	x_{12}	x_{13}	....	x_{110}
-2	x_{21}	x_{22}	x_{23}	...	x_{210}
⋮					
$\binom{100}{10}$		—	—		—

* Sample total — random variable.

Let $S_n = X_1 + X_2 + \cdots + X_n$

$$
\begin{aligned}
E(S_n) &= E(X_1 + X_2 + \cdots + X_n) \\
&= E(X_1) + E(X_2) + \cdots + E(X_n) \\
&= \mu + \mu + \cdots + \mu \quad (n\text{-times}) \\
&= n\mu .
\end{aligned}
$$

$$
\begin{aligned}
Var(S_n) &= var(X_1 + X_2 + \cdots + X_n) \\
&= var(X_1) + var(X_2) + \cdots + var(X_n) \quad [\because X_1, \ldots X_n \text{ are independent}] \\
&= \sigma^2 + \sigma^2 + \cdots + \sigma^2 \quad (n\text{-times}) \\
&= n\sigma^2 .
\end{aligned}
$$

* Sample mean — random variable.

Let $M_n = \dfrac{X_1 + X_2 + \cdots + X_n}{n} = \dfrac{S_n}{n}$

$$E(M_n) = E\left(\frac{S_n}{n}\right) = \frac{1}{n} \cdot E(S_n) = \frac{1}{n} \cdot (n\mu) = \mu.$$

$$\mathrm{var}(M_n) = \mathrm{var}\left(\frac{S_n}{n}\right) = \frac{1}{n^2}\,\mathrm{var}(S_n) = \frac{1}{n^{\cancel{2}}} \cdot \cancel{n}\sigma^2 = \sigma^2/n.$$

Note:

Let $X_1, X_2, \ldots, X_n \overset{iid}{\sim} N(\mu, \sigma^2)$, then

a) $S_n \sim N(n\mu, n\sigma^2)$.

$$\Rightarrow \frac{S_n - n\mu}{\sqrt{n\sigma^2}} \sim N(0,1)$$

b) $M_n \sim N(\mu, \sigma^2/n)$

$$\Rightarrow \frac{M_n - \mu}{\sqrt{\sigma^2/n}} \sim N(0,1).$$