HW - 3 : W (9/13) | Quiz -3 : F (9/15) | Class -11

Exam -1 : T (9-19) : Covers First Two chapters | Practice Exam : M (9-18, discuss in the class)

Eg:-

A student takes a test with 16 multiple choice questions. Assuming each question has four choices and she chooses answers at random, what is the probability that she will get exactly 3 rights?

$n = 16$, $p = P(\underbrace{\text{choosing the correct answer}}_{\text{success}}) = 1/4$.

X - # of correct answers.

$$P(X=3) = \underbrace{{}^{16}C_3}\,(1/4)^3\,(3/4)^{16-3} = 0.2079.$$

$$\underbrace{\_\ \_\ \_\ \_\ \_\ \_\ \_}_{{}^{16}C_3}$$

Multinomial Distribution

Consider n identical and independent trials such that each trial has "k" (>2) outcomes with probabilities $p_1, p_2, \ldots, p_k$, with probability of getting n_i outcomes of type i then probability is

$n = n_1 + n_2 + \cdots + n_k$

$$P(n_1, n_2, \ldots, n_k) = \frac{n!}{n_1! \cdot n_2! \cdots n_k!}\, p_1^{n_1}\, p_2^{n_2}\, p_3^{n_3} \cdots p_k^{n_k}.$$

Note:

* Binomial (only two outcomes for each trial)

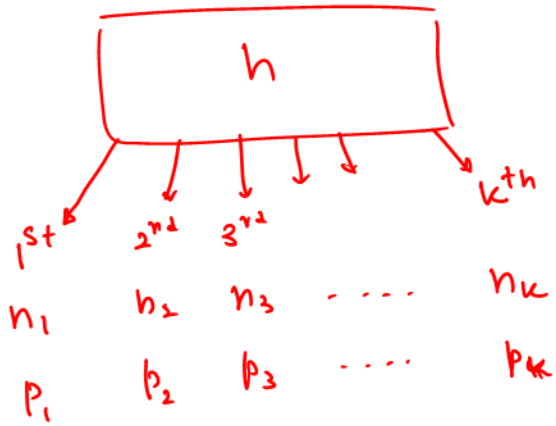
$\boxed{n}$

x - successes: p | $n-x$ - failures: $1-p$

Prob: distribution

$$P(X=x) = {}^{n}C_x\, p^x (1-p)^{n-x}$$
$$= \frac{n!}{x! \cdot (n-x)!}\, p^x (1-p)^{n-x}.$$

* Multinomial (k out comes for each trial)



Prob. distribution :

$$P(n_1, n_2, \dots, n_k) = \frac{n!}{n_1! \cdot n_2! \dots n_k!} p_1^{n_1} p_2^{n_2} \cdots p_k^{n_k}$$

$$n_1 + n_2 + \dots + n_k = n$$

$$p_1 + p_2 + \dots + p_k = 1$$

* Binomial is a Special case of multinomial.

$k = 2$, $n_1 = x$, $n_2 = n - x$ $\Rightarrow$ $n_1 + n_2 = x + n - x = n$

$p_1 = p$, $p_2 = 1 - p$ $\Rightarrow$ $p_1 + p_2 = p + 1 - p = 1$.

A baseball player gets a hit with probability 0.3, a walk with probability 0.1 and an out with probability 0.6. If he bats four times during a game, what is the probability that he will get 1 hit, 1 walk and 2 outs?

$n = 4$, $k = 3$, $p_1 = 0.3$, $p_2 = 0.1$, $p_3 = 0.6$

$n_1 = 1$, $n_2 = 1$, $n_3 = 2$

$$P(1\text{-hit}, 1\text{-walk}, 2\text{-outs}) = \frac{4!}{1! \cdot 1! \cdot 2!} (0.3)^1 (0.1)^1 (0.6)^2$$

$$= 0.1296$$

The output of a machine is graded excellent 70% of the time, good 20% of the time, defective 10% of the time. What is the probability that a sample of size 15 has 10 excellent, 3 good, 2 defective items?

$n = 15$, $k = 3$, $p_1 = 0.7$, $p_2 = 0.2$, $p_3 = 0.1$ | 1

$n_1 = 10$, $n_2 = 3$, $n_3 = 2$ | 15

$$P(10\text{-exce}, 3\text{-good}, 2\text{-def}) = \frac{15!}{10! \cdot 3! \cdot 2!} \cdot (0.7)^{10} (0.2)^3 (0.1)^2$$

$$=$$

See EX. 2.15, 2.16, 2.17, 2.18.

Sec 2.3 Poisson Approximation to the Binomial

Binomial distribution:

$$P(X=x) = \underbrace{{}^nC_x}_{\frac{n!}{(n-x)!\,x!}} p^x (1-p)^{n-x} \quad , \quad x = 0, 1, 2, \cdots n.$$

$P(X \le a) = ?$

Defn Poisson Distribution

Random variable X is said to have a poisson Distribution with parameter λ (lamda) (or $X \sim \text{Poisson}(\lambda)$) if

$$P(X=x) = \frac{e^{-\lambda}\lambda^x}{x!} \; ; \; x = 0, 1, 2, \ldots\ldots$$

Note:

* Poisson distribution is used to find # of Successes when the average number of Successes is given.
* λ - average number of Successes
* X - Actual number of Successes. (total # of trials is not fixed).
* $$\sum_{x=0}^{\infty} p(x) = \sum_{x=0}^{\infty} \frac{e^{-\lambda}\lambda^x}{x!}$$
  $$= e^{-\lambda}\underbrace{\left[\frac{\lambda^0}{0!} + \frac{\lambda^1}{1!} + \frac{\lambda^2}{2!} + \cdots\right]}_{e^{\lambda}}$$
  $$= 1 \quad \square$$

$Var(X) = E(X^2) - [E(X)]^2$

* $E(X) = \lambda$, $Var(X) = \lambda$.

Proof: Hw.

Eg:- Suppose the average number of lions seen on 1-day Safari is 5.

a) what is the probability that a tourist will see exactly 4 lions on the next day.

Let X - # of lions will be seen.

$\lambda = 5$. then, $P(X=x) = \frac{e^{-\lambda}\lambda^{x}}{x!}, x=0,1,2,\ldots$

$X \sim \text{Poisson}(\lambda=5)$.

$$P(\underset{x}{X=4}) = \frac{e^{-5}5^{4}}{4!} = 0.17547.$$

b) what is the probability that the tourist will see fewer than 4 lions?

$P(X<4) =$