HW - 2 : W (9/06) , in the conference

Quiz -2 : F (9/08) : Sections 1.4 to 1.6

Class -7

Extra Office Hours: Today (T, 9/5) from 3.10 pm to 5 pm

Note:

1) Expectation of a function of a random variable.

Let $g(X)$ be a function of random variable X, then

$$E(g(X)) = \sum_x g(x) \cdot p(x).$$

$$E(X) = \sum_x x \cdot p(x)$$

Eg: consider the distribution $p(x) = \begin{cases} 1/16 & : x = 0 \\ 6/16 & : x = 1 \\ 9/16 & : x = 2 \end{cases}$

Find $E(X^2)$. $\quad g(X) = X^2$.

$$E(X^2) = 0^2 \cdot \frac{1}{16} + 1^2 \cdot \frac{6}{16} + 2^2 \cdot \frac{9}{16}$$

$$= 21/8$$

2) $E(\underbrace{aX+b}_{\text{New}}) = a\,E(X) + b$, a, b — constants. $\quad g(X) = aX + b$

Proof:

$$E(aX+b) = \sum_x (ax+b) \cdot p(x)$$

$$= \sum_x a x\, p(x) + \sum_x b\, p(x)$$

$$= a \underbrace{\sum_x x\, p(x)} + b \underbrace{\sum_x p(x)}_{=1}$$

$$= a\,E(X) + b \quad \square.$$

3) If $X_1, X_2, \ldots, X_n$ are r.v.s, then

$$E(X_1 + X_2 + \cdots + X_n) = E(X_1) + E(X_2) + \cdots + E(X_n)$$

ie $$E\left(\sum_{i=1}^{n} X_i\right) = \sum_{i=1}^{n} E(X_i)$$

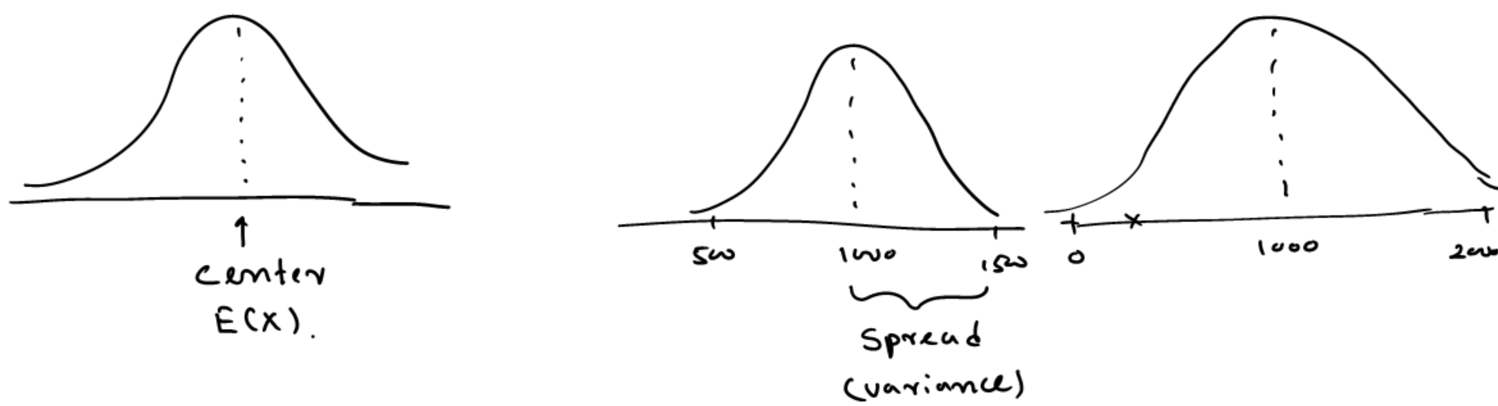
Sec 1.6 : Variance

Defn k^{th} moment $\qquad g(x) = x^k$

k^{th} moment of random variable X is defined as.

$$E(X^k) = \sum_{x} x^k \cdot p(x)$$

Variance



Defn Variance (Var(X))

Variance of r.v. X is defined as

$$Var(X) = E\left([X - E(X)]^2\right). \qquad g(x) = (x - E(x))^2$$

Note:

1) $\text{var}(X) = E(X^2) - [E(X)]^2$ — (more applicable)

Proof:

$E(aX+b) = aE(X)+b$

$$\text{Var}(X) = E[\underbrace{(X-E(X))^2}]$$

$$= E[X^2 - 2X\cdot E(X) + (E(X))^2]$$

$$= E(X^2) - 2E(X)\cdot E(X) + (E(X))^2$$

$$= E(X^2) - (E(X))^2 \quad \square.$$

2) $\text{var}(aX+b) = a^2\,\text{var}(X)$

Proof:

$$\text{Var}(aX+b) = E(\underbrace{(ax+b)^2}) - [\underbrace{E(ax+b)}]^2$$

$$= E[a^2x^2 + 2abx + b^2] - [aE(X)+b]^2$$

$$= a^2E(x^2) + 2ab\cancel{E(X)} + \cancel{b^2} - (a^2(E(X))^2 + 2ab\cancel{E(X)} + \cancel{b^2})$$

$$= a^2 \underbrace{[E(x^2) - (E(X))^2]}_{\text{var}(X)}$$

$$= a^2\,\text{var}(X) \quad \square.$$

Defn) Standard deviation of X.

Standard deviation of random variable X is

$$\sigma(X) = \sqrt{\text{var}(X)}$$

Sigma

Note:

If X has a Geometric distribution with $P(\text{Success}) = p$.

* probability dis^n : $P(X=x) = p(1-p)^{x-1}$; $x = 1, 2, 3, \ldots$

* Expectation : $E(X) = \frac{1}{p}$

* variance : $var(X) = \frac{1-p}{p^2}$

$var(X) = E(X^2) - (E(X))^2$

Proof: HW.

Eg:- Find the mean and the Standard deviation of the following distribution.

$X=x$	0	1	2	3	4	5	8	10
$P(X=x)$	0.02	0.68	0.07	0.08	0.10	0.01	0.02	0.02

$E(X) = 0\cdot(0.02) + 1\cdot(0.68) + 2(0.07) + \cdots + 10(0.02)$

$= 1.87$

$E(X^2) = 0^2(0.02) + 1^2(0.68) + 2^2(0.07) + \cdots + 10^2(0.02)$

$= 9.06$

$\therefore var(X) = E(X^2) - (E(X))^2$

$= 9.06 - (1.87)^2$

$= 5.5631$

$\therefore$ Standandard deviation $= \sigma(X) = \sqrt{var(X)} = \sqrt{5.5631} = 2.35$