Consider chess tournament problem:

(Ai- had a type i opponent (i=1,2,3), B- won the game)

P(A1)=0.5, p(A2)=.25, P(A3)=0.25, P(B|A1)=0.3, P(B|A2)=0.4, P(B|A3)=0.5.

P(B)= 0.375 (previous section).

Suppose you won the game, what is the probability that you had a type 1 opponent? P(A+|B)=?

By Bayes' theorem

$$P(A \mid B) = \frac{P(B \mid A \mid) \cdot P(A \mid)}{P(B)}$$

$$= \frac{(0.3)(0.5)}{0.375}$$

Eg: >1

Consider four sided die problem:

(Ai- The first roll is i (i=1,2,3,4), B- the sum is at least four)

P(Ai)=1/4 (i=1,2,3,4),

P(B|A1)=1/2, P(B|A2)=1/3, P(B|A3)=0, P(B|A4)=1.

P(B)= 9/16 (previous section).

a) Suppose the sum is at least four, what is the probability that the first roll is 2? $|A_1|B = 2$

b) Suppose the sum is less than four, what is the probability that the first roll is 1?

a)
$$P(A_{2}|B) = P(B|A_{2}) \cdot P(A_{2})$$

$$= \frac{(3/3)(1/4)}{9/11} = \frac{1}{9/11}$$

b)
$$P(A_1|B^C) = P(B^C|A_1) \cdot P(A_1)$$

$$= \underbrace{(1 - P(B|A_1)) \cdot P(A_1)}_{1 - P(B)}$$

$$= \underbrace{(1 - \frac{1}{2}) \cdot \frac{1}{4}}_{1 - \frac{9}{16}} = \underbrace{(\frac{2}{4})}_{1 - \frac{9}{16}}$$

29 -

Three factories make 20, 30, and 50% of the computer chips for a company. The probability of a defective chip is 0.04, 0.03, and 0.02 for three factories. We have a defective chip. What is the probability that it came from factory 1?

Let
$$Ai-$$
 chip came from factory i , $i=1,2,3$.
B — the chip is defective.

$$p(A_1) = 0.2$$
 , $p(A_2) = 0.3$, $p(A_3) = 0.5$.

By Bayes theorem

$$P(A_1|B) = \frac{P(B|A_1) \cdot P(A_1)}{P(B|A_1) \cdot P(A_2) + P(B|A_3) \cdot P(A_3)}$$

$$= \frac{(0.04)(0.2)}{(0.04)(0.2) + (0.03)(0.3) + (0.02)(0.5)}$$

$$= \frac{8}{3}_{2} = \frac{1}{4} = 0.15$$

Let X and Y are two random variables associated

with Same random experiment. Then the probability (it the joint distribution of x and Y)

distribution of X and Y is given by

$$P(X=x,Y=y) = P(x,y) = P\left(\frac{\{X=x\} \cap \{Y=y\}}{\text{event}} \right)$$

Note:

$$\frac{5}{2} p(x,y) = 1.$$

Eg: Roll two four Sided fair dice.

Let X - the maximum of the two numbers Y - the Sum

- a) Find the joint distribution of X and Y
 - b) Find P(X = 2, Y = 3)

Possible values of X: 1, 2, 3, 4 possible values of Y: 2, 3, 4, 5, 6, 7, 8

Joint distribution of x and Y.

			·				_		
	X	2	3	4	2	6	ት 	8	
- h		1	0	0	٥	0	0	0	< (1)()
1	2	٥	2/6	% <u>.</u>	0	0	0	0	(1,2),(3,1), (2,2)
	3	0	0	2/16	2/16	1/6	0	6	\leftarrow $(1,3),(3,1),(2,3),(3,2),(3,3)$
	۾ ا	0	0	0	2/1	6 2/	L 2/16	/15	€ (1,3),(3,1), (2,3),(3,2), (3,3)
		1							

$$P(X=1, Y=1) = P(\{(i_1,i)\}) = \frac{1}{14}$$

b)
$$P(X \le 1, Y \le 3) = P(1,2) + P(1,3) + P(2,2) + P(2,3)$$

= $\frac{1}{16} + 0 + 0 + \frac{2}{16} = \frac{3}{16}$

Marginal Distributions

suppose p(x,y) is the joint distribution of X and Y.

Then the marginal distribution of X:

$$p(x=x) = \sum_{y} p(x_i,y),$$

The marginal distribution of Y:

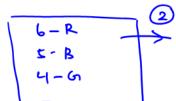
$$P(Y=Y) = \sum_{x} P(x,y)$$

Eg: Suppose we draw 2 balls out of an unn with 6-red, 5-blue omd 4 green balls.

Let X-# of red balls, Y-# of blue balls.

- a) Find the joint distribution function of x and Y
- b) Find the marginal distributions of X and Y?

Possible values of K: 0,1,2
possible value of Y: 0,1,2



Joint distribution:

X		t ,	2						
0	1/105	20/105	10/105) (X=0) =	6+20+16 = 36/05				
ι	24/105	30/105	0	P(x=1) =	105				
2	15/205	0	0	P(X=2)	= 15/105				
$P(Y=0) = \frac{45}{105}$ $P(Y=3) = \frac{10}{105}$									
p(x=0,	$ =0\rangle = P$	(2-97em) = -	= SC2 =	105				
P(X=0,	Y=1)= P(156	$=\frac{20}{105}$				
P (012)	- P((2-blue)	= _;	15C7 =	10/05				

Marginal distribution of X:

X=x	0	1	2
p (X = x	36	<u>54</u> 105	201

OR
$$P(X=x) = \begin{cases} \frac{3}{p}/105 : x = 0 \\ \frac{54}{105} : x = 1 \\ \frac{15}{105} : x = 2 \end{cases}$$