Quiz -4 : Today (9/27)

HW - 5 : W (10/04), in the conference

Class -21

Rest of the term:
Sec 5.1 : Continuous Random Variables
Sec 5.2 : Cumulative distribution function.
Sec 6.4 : Normal distribution
Sec 6.5 : Central Limit theorem
Sec 5.3 : Functions
Sec 5.4 : Joint distributions (continuous)
Sec 5.5 : Marginal distributions (continuous)

If $X \sim Exp(\lambda)$,

② $E(X) = \frac{1}{\lambda}$

$$E(X) = \int_{-\infty}^{\infty} x f(x) dx = \int_0^\infty x \cdot \lambda e^{-\lambda x} dx$$

$$= \int_0^\infty \underbrace{\lambda x}_{u} \cdot \underbrace{e^{-\lambda x} dx}_{dv}$$

$$\boxed{\int_a^b u\,dv = uv\Big|_a^b - \int_a^b v\,du}$$

$u = \lambda x \quad\quad dv = e^{-\lambda x} dx$

$\downarrow \quad\quad \downarrow$

$du = \lambda dx \quad\quad v = \frac{e^{-\lambda x}}{-\lambda}$

$$= \lambda x \cdot \frac{e^{-\lambda x}}{-\lambda}\Big|_0^\infty - \int_0^\infty \frac{e^{-\lambda x}}{-\lambda} \cdot \lambda dx$$

$$= -\Big[\underbrace{\lim_{a\to\infty} (a \cdot e^{-\lambda a})}_{=0} - \underbrace{0 \cdot e^{-\lambda(0)}}_{=0}\Big] + \Big[\frac{e^{-\lambda x}}{-\lambda}\Big]_0^\infty$$

$$= -\frac{1}{\lambda}[0 - 1] = \frac{1}{\lambda} \;\square.$$

③ $Var(X) = \frac{1}{\lambda^2}$ (Proof HW)

$E(X^2) = ?$

$$Var(X) = \underbrace{E(X^2)}_{} - (\underbrace{E(X)}_{})^2$$

$$= \frac{2}{\lambda^2} - \frac{1}{\lambda^2}$$

④ $P(X \geq a) = e^{-\lambda a}$, a – constant. (>0)

Proof:

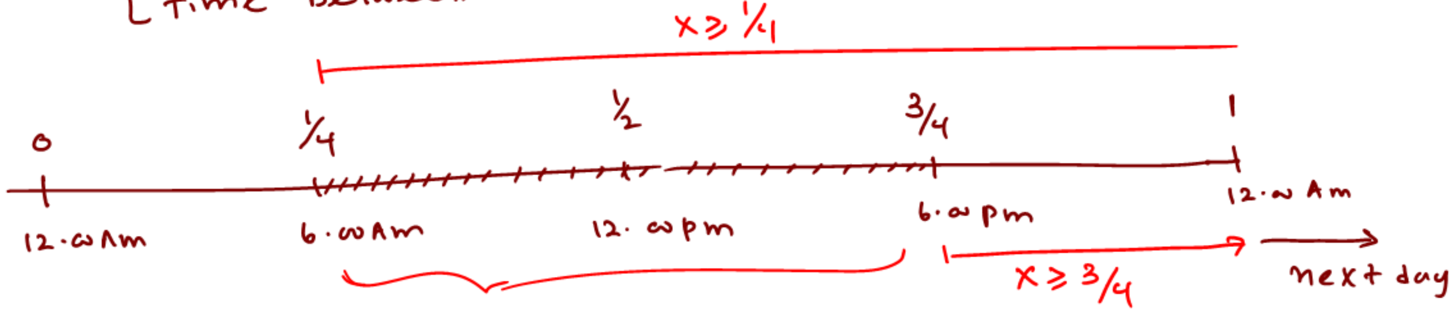
$$P(X \geq a) = \int_a^\infty \lambda e^{-\lambda x} dx.$$

Eg:-

The time until a small meteorite first lands anywhere in the Sahara desert is modeled as an exponential random variable with a mean of 10 days. The time is currently midnight. What is the probability that a meteorite first lands sometime between 6 a.m. and 6 p.m. of the first day?

- $\lambda = \frac{1}{10}$ (in days).

Let $\underline{X}$ - time for the 1^{st} land (in days)
[time between 0 land and the 1^{st} land].



$$P\left(\begin{array}{l}\text{Fist land is in}\\ \text{between 6 am and 6 pm}\end{array}\right) = P\left(\tfrac{1}{4} \le X \le 3/4\right)$$

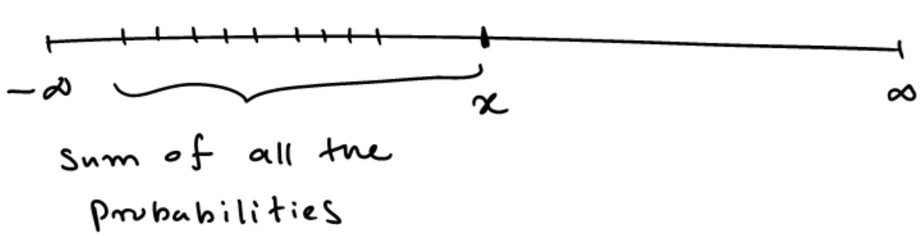
$$= \int_{1/4}^{3/4} \frac{1}{10} e^{-\frac{1}{10}x}\, dx \qquad P(X \underline{\geq} a) = e^{-\lambda a}$$

$$= P(X > 1/4) - P(X > 3/4)$$

$$= e^{-\frac{1}{10}(1/4)} - e^{-\frac{1}{10}(3/4)}$$

$$= 0.0476$$

Sec 5.2 Cumulative Distribution Functions



$F(-\infty) = 0$

$F(\infty) = 1$

Defn

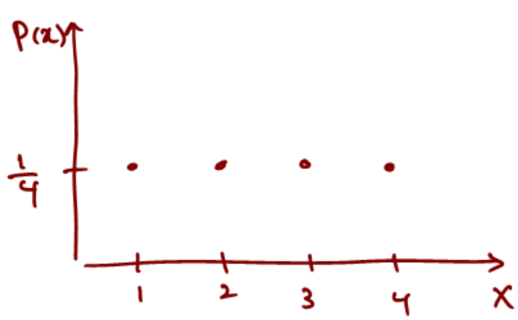
The cumulative distribution function (cdf) of a random variable X is given by

$$F(x) = \begin{cases} \sum_{k \le x} p(k) & : X - \text{discrete} \\ \int_{-\infty}^{x} f(t)\,dt & : X - \text{continuous.} \end{cases}$$

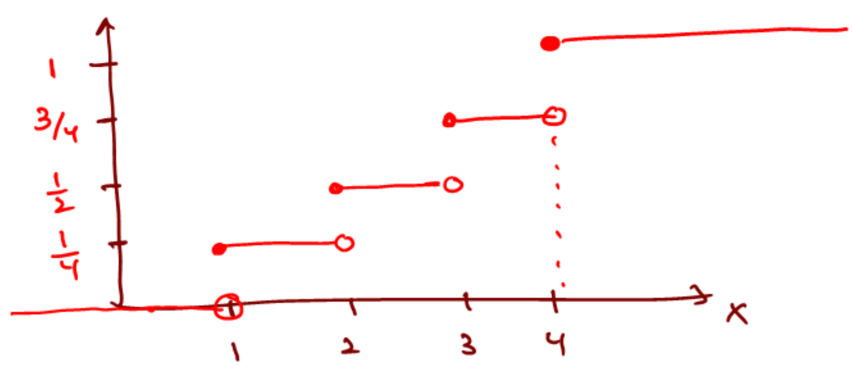
Eg: Let $p(x) = \begin{cases} 1/4 & : x = 1, 2, 3, 4 \\ 0 & : \text{otherwise} \end{cases}$. Find the cdf of X.

Graphs

* Pmf



* cdf



cumulative distribution function of X:

$$F(x) = \begin{cases} 0 & : x < 1 \\ 1/4 & : 1 \le x < 2 \\ 1/2 & : 2 \le x < 3 \\ 3/4 & : 3 \le x < 4 \\ 1 & : x \ge 4 \end{cases}$$