HW-3 (Solutions)

#1 Multiplication Rule:

$$\underset{5}{\uparrow} \quad \underset{4}{\uparrow} \quad \underset{3}{\uparrow} \quad \underset{2}{\uparrow} \quad \underset{1}{\uparrow} \quad \underset{1}{\uparrow}$$

$\#$ of ways $= 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 1 = 120$

#2

$n = 6$

a) $\#$ of arrangements $= 6! = 720$

b) $\underset{3}{\_} \quad \underset{5}{\_} \quad \underset{4}{\_} \quad \underset{3}{\_} \quad \underset{2}{\_} \quad \underset{1}{\_}$

$\#$ of arrangements $= 3 \cdot 5! = 360$

c) $\underset{3}{\_} \quad \underset{3}{\_} \quad \underset{2}{\_} \quad \underset{2}{\_} \quad \underset{1}{\_} \quad \underset{1}{\_}$

$\#$ of arrangements $= 3 \cdot 3 \cdot 2 \cdot 2 \cdot 1 \cdot 1 = (3!)^2 = 36$

#3

a) Total $\#$ of ways of drawing 8 cards out of 52 cards $= {}^{52}C_8$

$\#$ of ways to chose 8 cards with exactly 3 aces $= \begin{pmatrix}\text{\# of ways to} \\ \text{choose 3-Aces}\end{pmatrix} * \begin{pmatrix}\text{\# of ways to} \\ \text{chose 5} \\ \text{other cards}\end{pmatrix}$

$$= \frac{{}^{4}C_3 * {}^{48}C_5}{{}^{52}C_8}$$

$$= 0.00910$$

b) $p(\text{8 cards with exactly 2 kings}) = \dfrac{{}^{4}C_2 * {}^{48}C_6}{{}^{52}C_8}$

$= 0.0978.$

c) # of ways to pick 8 cards with exactly 3-aces and exactly 2-kings $\Big\} = {}^{4}C_3 * {}^{4}C_2 * {}^{44}C_3$

$\therefore p\left(\begin{matrix}\text{8 cards with exactly 3-aces}\\ \text{and exactly 2-kings}\end{matrix}\right) = \dfrac{{}^{4}C_3 \cdot {}^{4}C_2 \cdot {}^{44}C_3}{{}^{52}C_8}$

$\underbrace{p(\text{3-aces or 2-kings})}$ $= 0.000422$

$\therefore$ Probability $= 0.00910 + 0.0978 - 0.000422 =$ ______

#4) MASSACHUSETTS

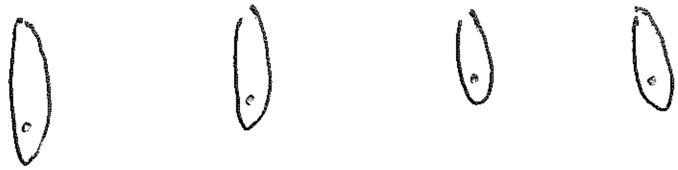
M=1, A-2, S-4, C-1, H-1, U-1, E-1, T-2

$n = 13,\ n_1 = 2,\ n_2 = 4,\ n_3 = 2, \ldots$

# of different words $= \dfrac{13!}{2! \cdot 4! \cdot 2!} = \boxed{\dfrac{13!}{96}}$ – (do not Simplify)

#5) # of ways to distribute 52 cards in 4 groups such that each group has same # of cards $\Big\} = \begin{pmatrix} & & 52 & \\ 13 & 13 & 13 & 13 \end{pmatrix}$

To find the ~~each~~ # of ways with each player (group of cards) has an ace, First divide 4-aces among 4-players (to 4-groups)



# of ways to distribute 4-aces among 4-players = 4!

Then,

# of ways to distribute the other 48 cards $= \frac{48!}{12!\cdot 12!\cdot 12!\cdot 12!}$

$$\therefore P(\text{each player got an ace}) = \frac{4!\cdot \frac{48!}{12!\cdot 12!\cdot 12!\cdot 12!}}{\frac{52!}{13!\cdot 13!\cdot 13!\cdot 13!}}$$

$$= 0.105498$$

#6 Binomial distribution.

a) $n = 999$ (except you), $p = \frac{1}{365}$

Let X - # of guests who have the same birthday.

$$P(X=1) = {}^{999}C_1 \left(\frac{1}{365}\right)^1 \left(\frac{364}{365}\right)^{998} = \underline{\underline{0.01137}}$$

b) $P(X \geq 2) = 1 - P(0) - P(1)$

$$= 1 - {}^{999}C_0 \left(\frac{1}{365}\right)^0 \left(\frac{364}{365}\right)^{999} - 0.01137$$

$$= \underline{\underline{0.924106}}$$

(did not grade)

#7 Binomial, $n = 20$, $p = P(\text{being a left handed}) = 0.20$

Let X - # of left handed students in the class

[Note: 20% of all students are left handed (not for this class).]

Here note that if $X \geq 2$ (# of right handed ≤ 18) and $X \leq 5$,

Students will have chairs to their needs.

$$P(2 \leq X \leq 5) = P(2) + P(3) + P(4) + P(5)$$

$$= {}^{20}C_2 (0.2)^2 (0.8)^{18} + {}^{20}C_3 (0.2)^3 (0.8)^{17} + {}^{20}C_4 (0.2)^4 (0.8)^{16} + {}^{20}C_5 (0.2)^5 (0.8)^{15}$$

$$= \underline{\hspace{3cm}}$$

#8 Multinomial distribution

$n = 12$, $k = 3$, $P_1 = 0.4$, $P_2 = 0.35$, $P_3 = 0.25$

$n_1 = 7$, $n_2 = 2$, $n_3 = 3$

$$P(7, 2, 5) = \frac{12!}{7! \cdot 2! \cdot 3!} (0.4)^7 (0.35)^2 (0.25)^3$$

$$= \underline{0.0000776}$$