5. If $A \subseteq B \Rightarrow P(A) \leq P(B)$

Proof – H:W:

HW - 1 : W (8/30) in the conference

Quiz -1 : F (9/01)

Class-3

Sec 1.3 Conditional Probability

Eg– Roll a six sided fair die.

$\Omega = \{1, 2, 3, 4, 5, 6\}$.

Define A – Even number is obtained

B – number 6 is obtained

What is the probability of getting 6 given that the outcome is even.

condition

New Sample space $= \Omega_1 = A = \{2, 4, 6\}$

$P(B|A) = 1/3$

Defn Conditional Probability

Conditional probability of an event A, given an event B is defined by

condition

$$P(A/B) = \frac{P(A \cap B)}{P(B)} \quad : P(B) \neq 0$$

Eg: $A = \{2, 4, 6\}$, $B = \{6\}$, $A \cap B = \{6\}$

$$\cancel{P(A/B) = \frac{P(A \cap B)}{P(B)} = 1/6}$$

$$P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{1/6}{1/2} = 1/3$$

(more in chapter – 3).

Independence

Two events A and B are independent if the occurence of A has no any influence of occurrence of B.

Def$^{\underline{n}}$ Events A and B are independent if and only if (iff)

$$P(A|B) = P(A)$$

Note: Events A and B are independent iff $P(A \cap B) = P(A) \cdot P(B)$.

1) ~~If~~ A and B are independent (not true in general)

Proof:

$$P(A) \cdot P(B) = P(A|B) \cdot P(B) \quad [\because A \text{ and } B \text{ are independent}]$$

$$= \frac{P(A \cap B)}{\cancel{P(B)}} \cdot \cancel{P(B)}$$

$$= P(A \cap B) \quad \square.$$

Eg:- Roll two fair dice.

Let A – the first die shows 5

B – the second die shows 2

Are A and B independent?

$$\Omega = \{(1,1), (1,2), \ldots, (6,6)\}$$

$A = \{(5,1), (5,2) \ldots (5,6)\}$ $\quad A \cap B = \{(5,2)\}$

$B = \{(1,2), \ldots (6,2)\}$

$$P(A) = \frac{6}{36} = 1/6, \quad P(B) = 1/6, \quad P(A \cap B) = 1/36.$$

$$P(A) \cdot P(B) = \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36} = P(A \cap B) \checkmark$$

$\therefore$ A and B are independent.