Quiz -5 : F (10/06)

HW - 6 : T (10/10)

Class -24

Rest of the term:
Sec 6.5 : Central Limit theorem
Sec 5.3 : Functions

Final Exam : R (10/12)

Cover materials after Exam -1
One double sided handwritten sheet is allowed
Bring your calculator

Practice Exam : Discuss in the class T (10/10)

Promlem: What if the distribution of the population is unknown?

Answer: use the central Limit Theorem.

Central Limit Theorem

Let $X_1, X_2, \dots, X_n$ be a random Sample with common mean μ and variance σ^2. Assume n is large ($n \geq 35$).

1) If $S_n = X_1 + X_2 + X_3 + \dots + X_n$ is the Sample total then,

$$\underbrace{\frac{S_n - n\mu}{\sqrt{n}\,\sigma}} \overset{app}{\sim} N(0,1).$$

2) If $M_n = \dfrac{X_1 + X_2 + X_3 + \dots + X_n}{n}$ is the Sample mean

$$\frac{M_n - \mu}{\sigma/\sqrt{n}} \overset{app}{\sim} N(0,1).$$

$\sqrt{\sigma^2/n}$

Eg:-

n

We load on a plane 100 packages whose weights are independent random variables that are uniformly distributed between 5 and 50 pounds. What is the probability that the total weight exceeds 3000 pounds?

Let X_i – the weight of the i^{th} package, $i = 1, 2, \dots, 100$.

then $X_i \sim$ uniform $(5, 50)$

If $X \sim$ uniform(a,b)

$E(X) = \frac{a+b}{2}$

$var(X) = \frac{(b-a)^2}{12}$.

$$\mu = E(X_i) = \frac{5+50}{2} = 27.5$$

$$\sigma^2 = var(X_i) = \frac{(50-5)^2}{12} = 168.75, \quad i = 1, 2, \dots 100.$$

Let S_{100} = total weight

$= X_1 + X_2 + \dots + X_{100}$.

$$E(S_{100}) = n\mu = 100 \cdot (27.5) = 2750$$

$$var(S_{100}) = n\sigma^2 = 100 \cdot (168.75) = 16875$$

$\sqrt{n}\, var(X_i)$

$\therefore$ Standard deviation of $S_{100} = \sqrt{16875}$

$$P(S_{100} \geq 3000) = P\left(\underbrace{\frac{S_{100} - 2750}{\sqrt{16875}}} \geq \frac{3000 - 2750}{\sqrt{16875}} \right)$$

$= P(Z \geq 1.92)$ [where $Z \sim N(0,1)$ By C.L.T]

$= 1 - P(Z < 1.92)$

$\approx 1 - 0.9726$

$= 0.0274$

Eg–2)

M_{50}

The income of college students is distributed with a mean income per year is \$12,000 and a standard deviation of \$6,000. If we randomly sample 50 college students,

a) What is the expected average income of our sample?

b) What is the variance of the average income of our sample?

c) What is the probability that the average income of our sample is less than \$10,000?

$\mu = 12000$, $\sigma^2 = 6000^2$

Let X_i – the income of the i^{th} Student, $i = 1, 2, \dots 50$.

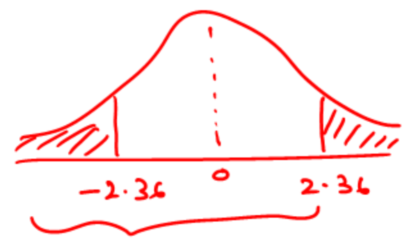
$\mu = E(X_i) = 12000$, $\sigma^2 = var(X_i) = 6000^2$

Let $M_{50} = \dfrac{X_1 + X_2 + \cdots + X_{50}}{50}$ — Sample average.

a) $E(M_{50}) = \mu = 12000.$

b) $\operatorname{var}(M_{50}) = \sigma^2/n = \dfrac{6000^2}{50} = 720{,}000.$

c)
$$P(M_{50} < 10000) = P\left(\frac{M_{50} - 12000}{\sqrt{720{,}000}} < \frac{10000 - 12000}{\sqrt{720{,}000}} \right)$$
$$= P(Z < -2.36)$$
$$= P(Z > 2.36)$$
$$= 1 - P(Z < 2.36)$$
$$= 1 - 0.9909$$
$$= 0.0091$$



Sec 5.3 : Functions of Random Variable

Recall: Discrete case:

Eg:- Let the pmf of X is

$$p(x) = \begin{cases} 1/5 & : x = -2, -1, 0, 1, 2 \\ 0 & : \text{otherwise} \end{cases}$$

Let $Y = g(X) = X^2$. Find the pmf of Y

Possible values of Y : 0, 1, 4

$\{x = 0\}$ $\{x = -1, 1\}$ $\{x = -2, 2\}$

$$P(Y=0) = P(X=0) = 1/5$$

$$P(Y=1) = P(X=-1) + P(X=1) = 2/5$$

$$P(Y=4) = P(X=-2) + P(X=2) = 2/5$$

So the pmf of Y:

$$p(y) = \begin{cases} 1/5 & : y=0 \\ 2/5 & : y=1,4 \\ 0 & : \text{otherwise.} \end{cases}$$