HW-4 Solutions

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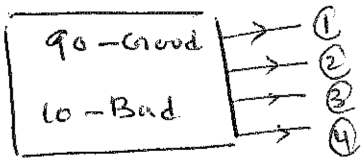


Let G_i – the i^{th} one is a good one.

$$P(\text{the batch is accepted}) = P(\text{All four are good})$$

$$= P(G_1 \cap G_2 \cap G_3 \cap G_4)$$

$$= P(G_1) \cdot P(G_2 | G_1) \cdot P(G_3 | G_1 \cap G_2) \cdot P(G_4 | G_1 \cap G_2 \cap G_3)$$

(multiplication rule)

$$= \frac{90}{100} \cdot \frac{89}{99} \cdot \frac{88}{98} \cdot \frac{87}{97}.$$

$$= \underline{\underline{\qquad\qquad}}$$

[redacted]

2] Let A – getting 2 spades and 3 – hearts.

B – the 1^{st} card is a spade.

2 Spades and 3 – hearts: $\underline{S}\ \underline{S}\ \underline{H}\ \underline{H}\ \underline{H}$

$$\frac{13}{52}\ \frac{12}{51}\ \frac{13}{50}\ \frac{12}{49}\ \frac{11}{48} = \frac{13^2 \cdot 12^2 \cdot 11}{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48}.$$

There are ${}^5C_2$ many ways this can happen.

$$\therefore P(A) = \frac{13^2 \cdot 12^2 \cdot 11}{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48} \times {}^5C_2 \quad \left(\text{Note that: } P(A) = \frac{{}^{13}C_3 \cdot {}^{13}C_2}{{}^{52}C_5} \text{ too}\right).$$

$A \cap B$: the 1^{st} card is a spade and having 2-spades and 3-hearts.

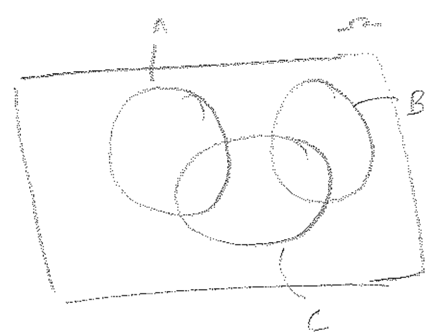
S _ _ _ _

There are ${}^4C_1$ (or ${}^4C_3$) many ways for this

$$\therefore P(A \cap B) = \frac{13^2 \cdot 12^2 \cdot 11}{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48} \times {}^4C_1$$

$$\therefore P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{{}^4C_1}{{}^5C_2} = \frac{4}{10} = \boxed{0.4}$$

3) $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$



Since A and B are disjoint, $\Rightarrow A \cap B = \Phi$

$P(A \cap B) = 0$ and

$A \cap B \cap C = \Phi \Rightarrow P(A \cap B \cap C) = 0.$

Further since A and C are independent,

$P(A \cap C) = P(A) \cdot P(C) = (0.3)(0.5) = 0.15$

~~$\therefore P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap C)$~~

Further $P(B \cap C) = P(B|C) \cdot P(C) = (0.1)(0.5) = 0.05$

$\therefore P(A \cup B \cup C) = 0.3 + 0.4 + 0.5 - 0 - 0.15 - 0.05 + 0$

$= \underline{1}$

#4)

Box-1	Box-2
4 − R	5 − R
8 − B	3 − B

Let H − toss is a head.

R − a red marble is chosen.

$P(H) = 0.4$, $P(R|H) = 4/12$, $P(H^c) = 0.6$, $P(R|H^c) = 5/8$

a) By total probability theorem:

$$P(R) = P(R|H)\cdot P(H) + P(R|H^c)\cdot P(H^c)$$
$$= 4/12 \cdot 4/10 + 5/8 \cdot 6/10$$
$$= \underline{\underline{0.5083}}$$

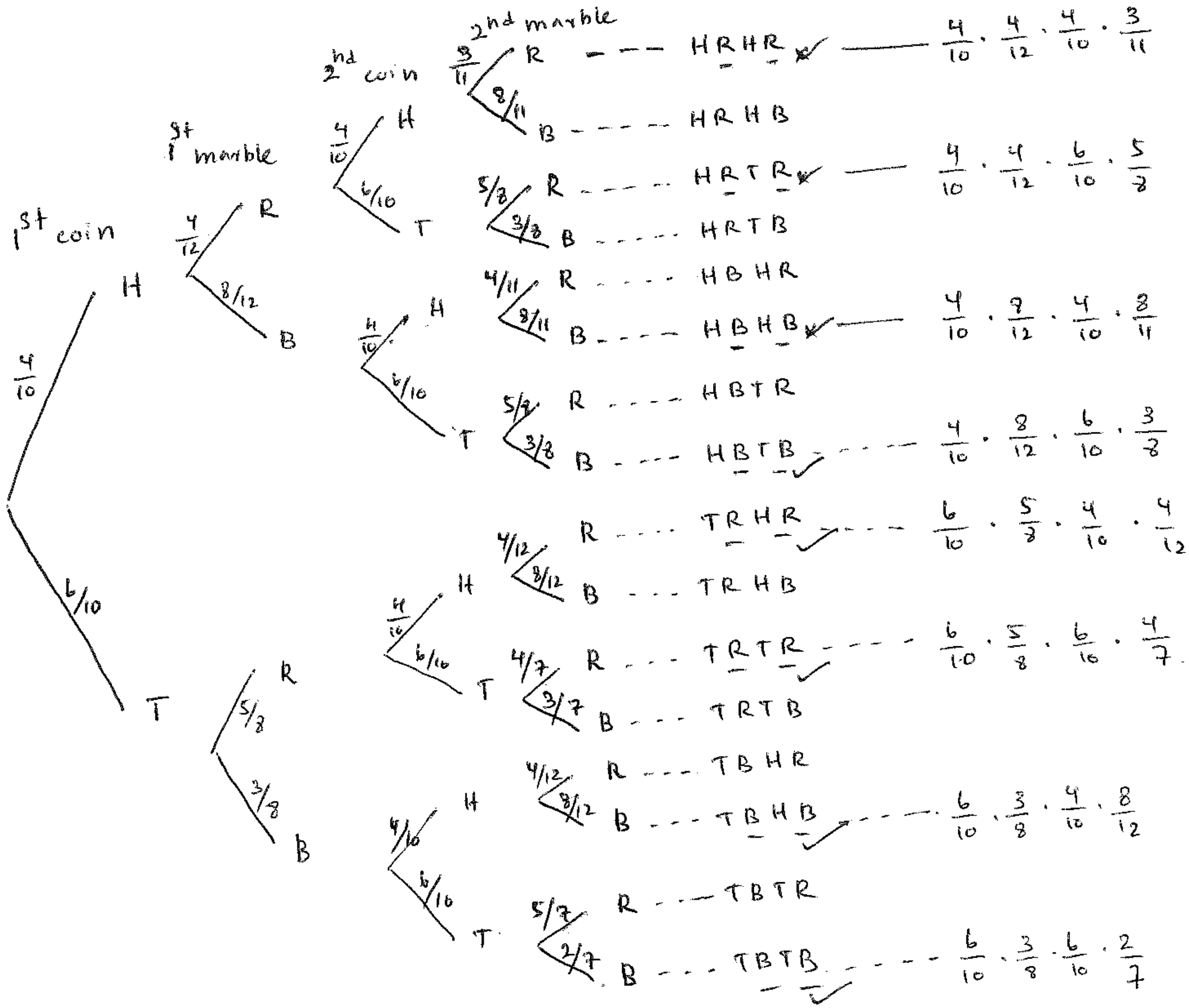
b) By Baye's theorem,

$$P(H|R) = \frac{P(R|H)\cdot P(R)}{P(R|H)\cdot P(R) + P(R|H^c)\cdot P(H^c)}$$
$$= \frac{4/12 \cdot 4/10}{0.5083}$$
$$= \underline{\underline{0.2623}}$$

c) By Bayes' Theorem, (R^c − a blue marble is chosen)

$$P(H|R^c) = \frac{P(R^c|H)\cdot P(H)}{P(R^c|H)\cdot P(H) + P(R^c|H^c)\cdot P(H^c)}$$
$$= \frac{(8/12)(4/10)}{(8/12)(4/10) + (3/8)(6/10)} = \underline{\underline{0.5424}}$$

#4) d) easy to use a tree diagram:



∴ P(getting same color marbles) = P(HRHR) + P(HRTR) + P(HBHB) + P(HBTB)

P(TRHR) + P(TRTR) + P(TBHB) + P(TBTB)

=

#5) Let A_i – the i^{th} coin is chosen.

H – toss is a head

$P(A_i) = \frac{1}{3}$, $i = 1, 2, 3$. $P(H/A_1) = 1$, $P(H/A_2) = 0$, $P(H/A_3) = 0.5$

$P(\text{opposite is a tail} / \text{toss is a head}) = P(A_3/H)$

$$= \frac{P(H/A_3) \cdot P(A_3)}{P(H/A_1) \cdot P(A_1) + P(H/A_2) \cdot P(H_2) + P(H/A_3) \cdot P(H_3)}$$

$$= \frac{(0.5)\frac{1}{3}}{1 \cdot \frac{1}{3} + 0 \cdot \frac{1}{3} + (0.5)\frac{1}{3}}$$

$$= \frac{0.5}{1.5} = \frac{1}{3}$$

#6) Let H – high risk, M – medium risk L – low risk

F – filling a claim.

$P(H) = 0.15$ $P(M) = 0.25$ $P(L) = 0.6$

$P(F/H) = 0.04$ $P(F/M) = 0.02$ $P(F/L) = 0.01$

a) Total Probability theorem,

$$P(F) = P(F/H) \cdot P(H) + P(F/M) \cdot P(M) + P(F/L) \cdot P(L)$$

$$= (0.04)(0.15) + (0.02)(0.25) + (0.01)(0.60)$$

$$= 0.017$$

$\therefore P(F^c) = 1 - P(F) = 1 - 0.017 = \underline{\underline{0.983}}$

b) Bayes' Theorem,

$$P(H|F) = \frac{P(F|H)\cdot P(H)}{P(F)} = \frac{(0.04)(0.15)}{0.017} = 0.3530$$

c) Bayes' Theorem,

$$P(L|F^c) = \frac{\overset{?}{P(F^c/L)}\cdot \overset{\checkmark}{P(L)}}{\underset{\checkmark}{P(F^c)}}$$

But $P(F^c/L) = 1 - P(F/L) = 1 - 0.01 = 0.99$

$$\therefore P(L/F^c) = \frac{(0.99)(0.6)}{0.983} = 0.6043$$

#7)

Box 1	Box-2	Box-3
5-R	7-R	6-R
5-G	3-G	4-G

Let A_i - i^{th} box is chosen.

G - green ball is chosen.

$P(A_1) = 1/4$, $P(A_2) = 1/2$ $P(A_3) = 1/4$

a) Total Probability theorem,

$$P(G) = P(G/A_1)\cdot P(A_1) + P(G/A_2)\cdot P(A_2) + P(G/A_3)\cdot P(A_3)$$

$$= (5/10)\cdot(1/4) + (3/10)(2/4) + (4/10)(1/4) = 15/40 = 3/8$$

b) Bayes' Theorem: (B^c – a red ball is chosen)

$$P(A_2 | B^c) = \frac{P(B^c / A_2) \cdot P(A_2)}{P(B^c/A_1) \cdot P(A_1) + P(B^c/A_2) \cdot P(A_2) + P(B^c/A_3) \cdot P(A_3)}$$

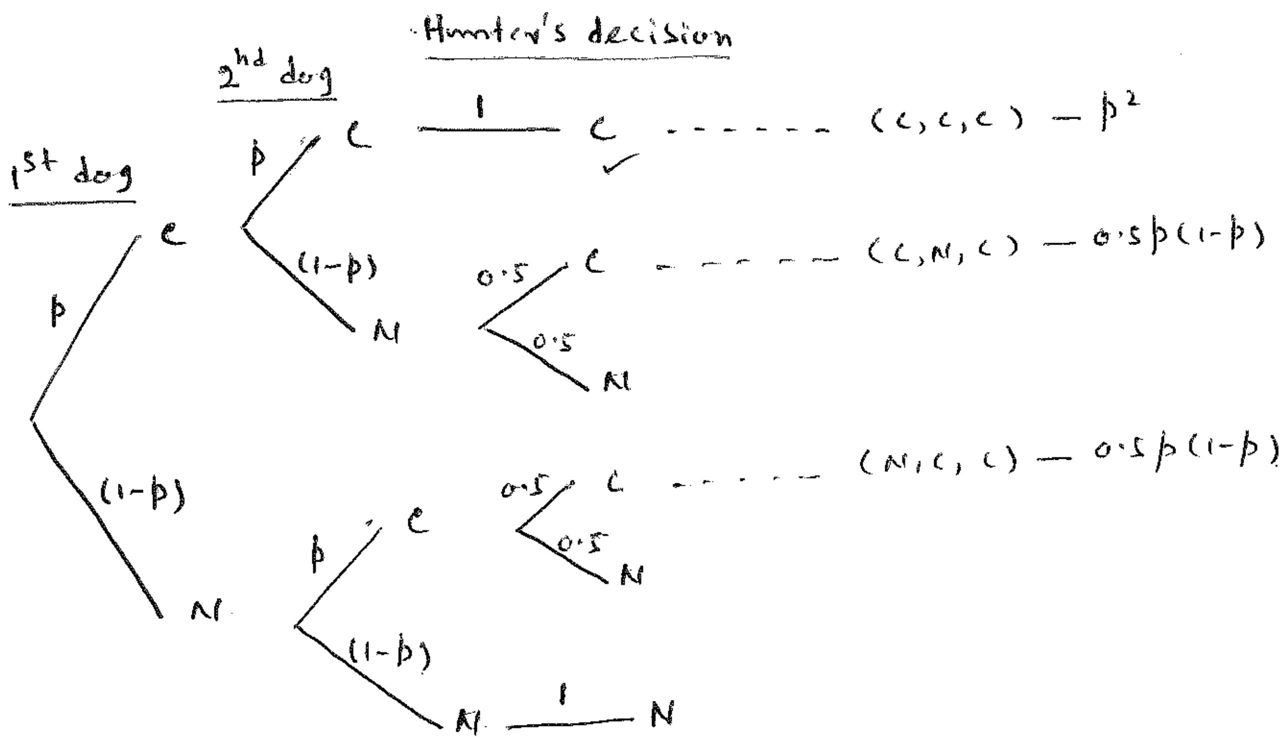
$$= \frac{(7/10)(2/4)}{(5/10)(1/4) + (7/10)(2/4) + (6/10)(1/4)}$$

$$= \frac{14/40}{25/40} = 14/25$$

OR

$$P(A_2 / B^c) = \frac{P(B^c / A_2) \cdot P(A_2)}{P(B^c)} \longleftarrow P(B^c) = 1 - P(B)$$

#8) For the hunter's strategy:



p(Hunter is finding the correct path) $= p^2 + 0.5p(1-p) + 0.5p(1-p)$
$= p.$

* If the hunter lets one dog to choose the path : (2^{nd} Strategy):

p(choosing the correct path) $= p.$

Thus, both methods have same Probability of choosing the correct path.

So both Strategies have Same effect.