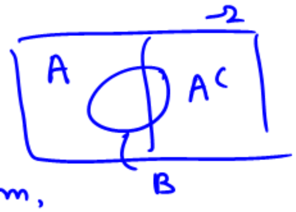
Note:

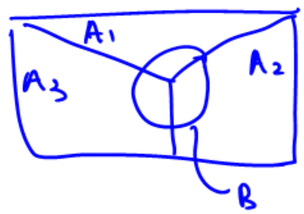
* when $n = 2$



Total probability theorem,

$$P(B) = P(B|A) \cdot P(A) + P(B|A^c) \cdot P(A^c).$$

* when $n = 3$



Total probability theorem,

$$P(B) = P(B|A_1) \cdot P(A_1) + P(B|A_2) \cdot P(A_2) + P(B|A_3) \cdot P(A_3) \square.$$

HW - 4 : W (9/27), in the conference

Class -17

Eg:-

You enter to a chess tournament where your probability of winning a game is 0.3 against half of the players (call them type 1), 04 against the quarter of the players (call them type 2), and 0.5 against the remaining quarter of the players (call them type 3). You play a game against a randomly chosen opponent. What is the probability of winning?

Let A_i – Playing against type i opponent, $i = 1, 2, 3$.

B – winning the game $\qquad P(B) = ?$

$P(A_1) = 0.5$, $\quad P(A_2) = 0.25$, $\quad P(A_3) = 0.25$

$P(B|A_1) = 0.3$ $\quad P(B|A_2) = 0.4$ $\quad P(B|A_3) = 0.5$

$P(A_1) + P(A_2) + P(A_3) = 1 = P(\Omega).$

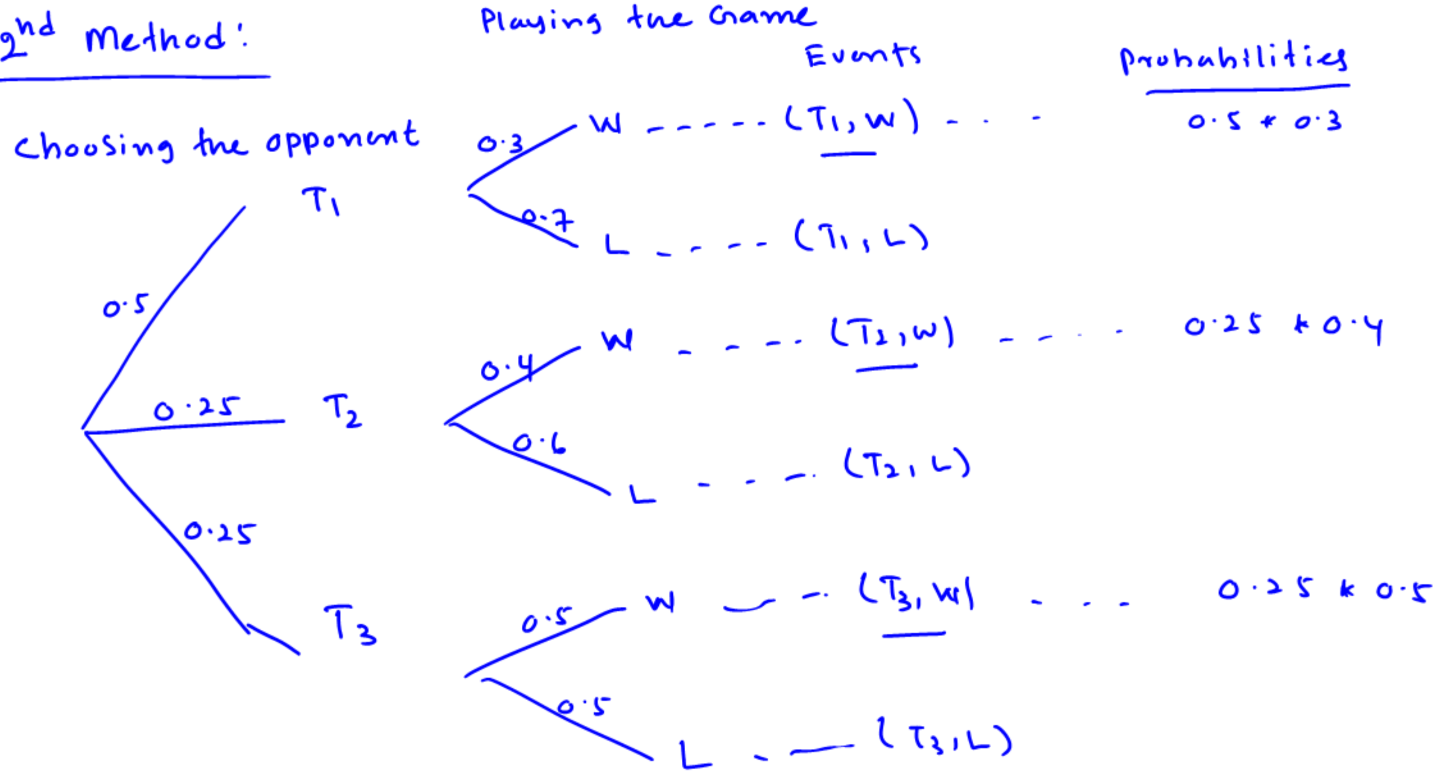
By Total Probability theorem,

$$P(B) = P(B|A_1) \cdot P(A_1) + P(B|A_2) \cdot P(A_2) + P(B|A_3) \cdot P(A_3)$$

$$= (0.3)(0.5) + (0.4)(0.25) + (0.5)(0.25)$$

$$= 0.375$$

2^{nd} Method:



$$P(\text{winning}) = P(T_1, W) + P(T_2, W) + P(T_3, W)$$
$$= 0.375$$

Eg:-

We roll a four sided fair die. If the result is 1 or 2, we roll once more but otherwise, we stop. What is the probability that the sum of the rolls is at least four?

1^{st} roll :	1		2		3	4	
→ 2^{nd} roll	1	2	3	4	0	0	$P(B) = ?$

Let A_i – getting "i" for the first roll, $i = 1, 2, 3, 4$.

B – Sum is at least 4.

$P(A_i) = 1/4$, $i = 1, 2, 3, 4$.

$P(B/A_1) = 2/4$, $P(B/A_2) = 3/4$,

$P(B/A_3) = 0$, $P(B/A_4) = 1$.

2 / 1	2	3	4
Sum 3	4	5	6

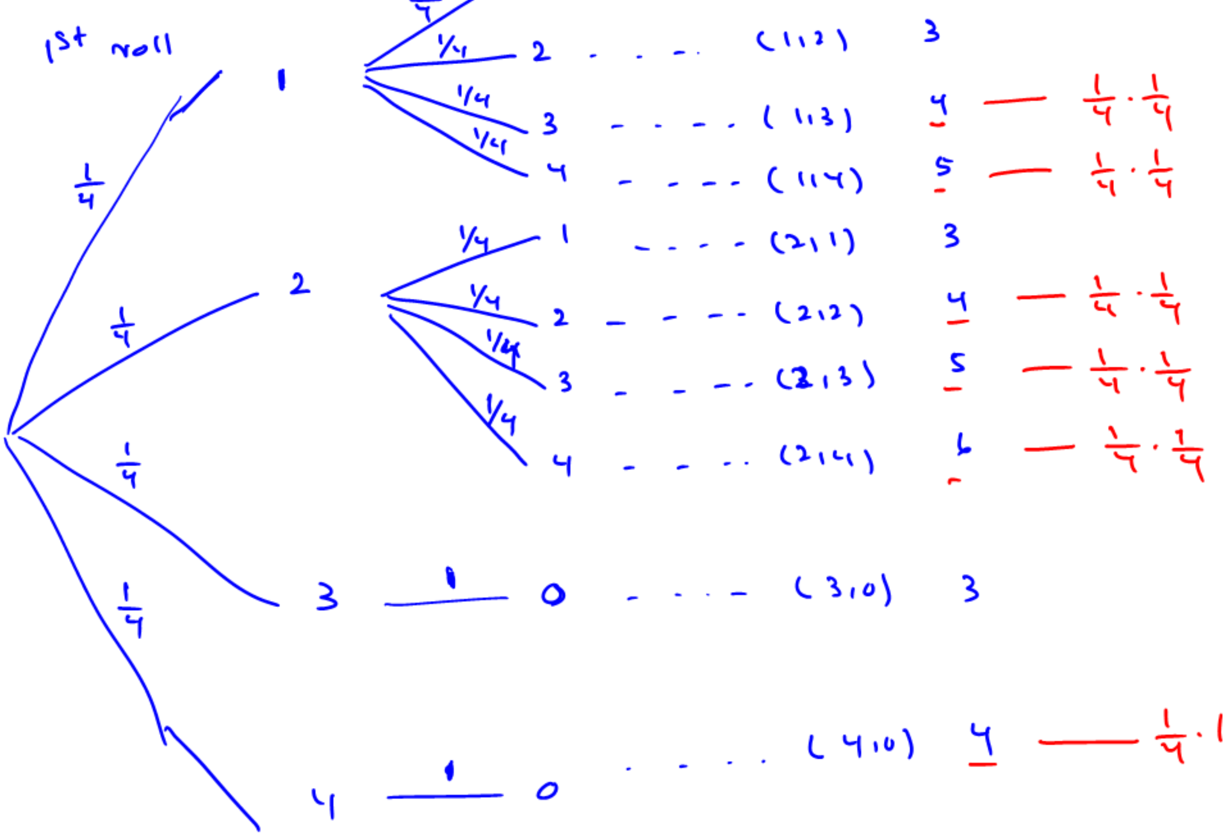
By Total probability theorem,

$$P(B) = \sum_{i=1}^{4} P(B|A_i)\cdot P(A_i)$$

$$= \left(\tfrac{1}{2}\right)\left(\tfrac{1}{4}\right) + \left(\tfrac{3}{4}\right)\left(\tfrac{1}{4}\right) + 0\cdot\tfrac{1}{4} + 1\cdot\tfrac{1}{4}$$

$$= \frac{9}{16}$$

Tree diagram Method



$$P(\text{Sum is at least } 4) = 5\cdot\frac{1}{16} + \frac{1}{4}$$

$$= \frac{9}{16}.$$

Sec 3.3 Bayes' Theorem

$0 \le P(A) \le 1$

Theorem

Let $A_1, A_2, \ldots, A_n$ be a partition of Ω such that $P(A_i) > 0$ for all $i = 1, 2, \ldots n$. Then for any event B,

$$P(A_i | B) = \frac{P(B|A_i) \cdot P(A_i)}{\sum_{j=1}^{n} P(B|A_j) \cdot P(A_j)}, \quad i = 1, 2, \ldots n.$$

$$= \frac{P(B|A_i)\, P(A_i)}{P(B)}, \quad i = 1, 2, \ldots n.$$

Proof: $P(A_i|B) = \dfrac{P(B \cap A_i)}{P(B)}$ (defn of con prob).

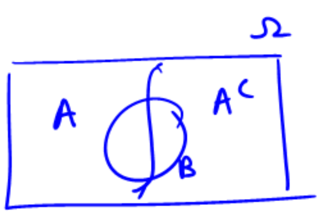
$$= \frac{P(B|A_i) \cdot P(A_i)}{\sum_{j=1}^{n} P(B|A_j) \cdot P(A_j)}$$

(Multiplication rule and Total probability theorem)

$\square$

Note:

* when $n = 2$



Bayes' Theorem

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B|A) \cdot P(A) + P(B|A^c) \cdot P(A^c)}$$

$$P(A^c|B) = \frac{P(B|A^c) \cdot P(A^c)}{P(B|A) \cdot P(A) + P(B|A^c) \cdot P(A^c)}.$$

Consider chess tournament problem:
(A_i- had a type i opponent (i=1,2,3), B- won the game)
$P(A1)=0.5$, $p(A2)=.25$, $P(A3)=0.25$, $P(B|A1)=0.3$, $P(B|A2)=0.4$, $P(B|A3)=0.5$.
$P(B)= 0.375$ (previous section).
Suppose you won the game, what is the probability that you had a type 1 opponent? $P(A_1|B)=?$

By Bayes' theorem.

$$P(A_1|B) = \frac{P(B|A_1)\cdot P(A_1)}{P(B)}$$

$$= \frac{(0\cdot 3)(0\cdot 5)}{0\cdot 375}$$

$$=$$

Consider four sided die problem:
(A_i- The first roll is i (i=1,2,3,4), B- the sum is at least four)
$P(A_i)=1/4$ (i=1,2,3,4),
$P(B|A1)=1/2$, $P(B|A2)=1/3$, $P(B|A3)=0$, $P(B|A4)=1$.
$P(B)= 9/16$ (previous section).

a) Suppose the sum is at least four, what is the probability that the first roll is 2?

b) Suppose the sum is less than four, what is the probability that the first roll is 1?