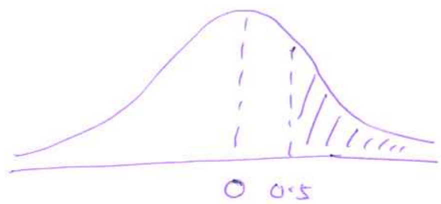
# HW-6 Solution Key

#1) Let $X$ – the temperature of the day. Then $X \sim N(10, 10^2)$

$$P(X \leq 15) = P\left(\frac{X-10}{10} \leq \frac{15-10}{10}\right)$$



$$= P(Z \leq 0.5)$$

$$= \cancel{P(Z \leq 0.5)}$$

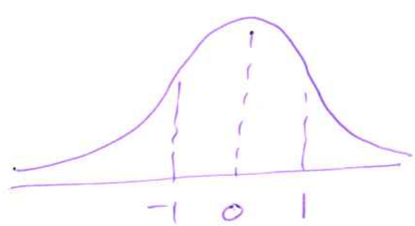
(10 pts)

$$= 0.6915.$$

Q2) Let $X$ – the score of the test. Then $X \sim N(62, 12^2)$.

a) $$P(50 \leq X \leq 74) = P\left(\frac{50-62}{12} \leq \frac{X-62}{12} \leq \frac{74-62}{12}\right)$$

$$= P(-1 \leq Z \leq 1)$$



$$= P(Z \leq 1) - P(Z \leq -1)$$

$$= P(Z \leq 1) - [1 - P(Z \leq 1)]$$

(10 pts)

$$= 2P(Z \leq 1) - 1$$

$$= 2(0.8413) - 1$$

$$= 0.6826$$

Q3) Let $X_i$ – the weight of the $i^{th}$ widsets, $i = 1, 2, \ldots, 36$.

Then $X_1, X_2, \ldots, X_{36} \sim N(200, 10^2)$

$\therefore \mu = E(X_i) = 200$ and $\sigma^2 = Var(X_i) = 10^2$

a) Let $M_{36} = \dfrac{X_1 + X_2 + \cdots + X_{36}}{36}$ = mean weight then,

(15 pts)

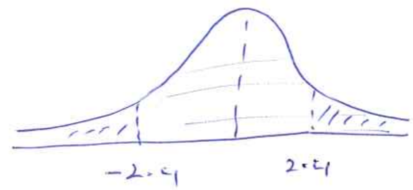
$E(M_{36}) = \mu = 200$, $Var(M_n) = \sigma^2/n = 10^2/36 = (10/6)^2$

$$P(M_{36} \geq 196) = P\left(\frac{M_{36}-200}{10/6} \geq \frac{196-200}{10/6}\right)$$

$$= P(Z \geq -2.4) \quad (\text{By C.L.T})$$

$$= P(Z \leq 2.4)$$

$$= 0.9918$$




b) Let $S_{36} = X_1 + X_2 + \cdots + X_{36}$ = total weight, then

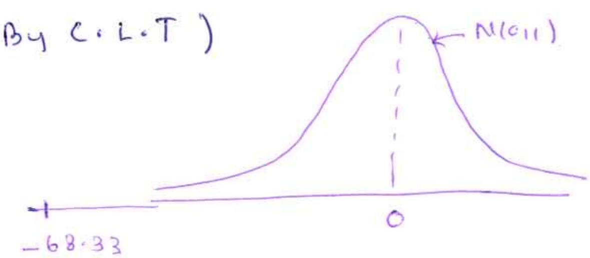
$E(S_{36}) = n\mu = 36(200) = 7200$ , $var(S_n) = n\sigma^2 = 36 \cdot 10^2 = (60)^2$

$$P(S_{36} \leq 3100) = P\left(\frac{S_{36}-7200}{60} \leq \frac{3100-7200}{60}\right)$$

$$= P(Z \leq -68.33) \quad (\text{By C.L.T})$$

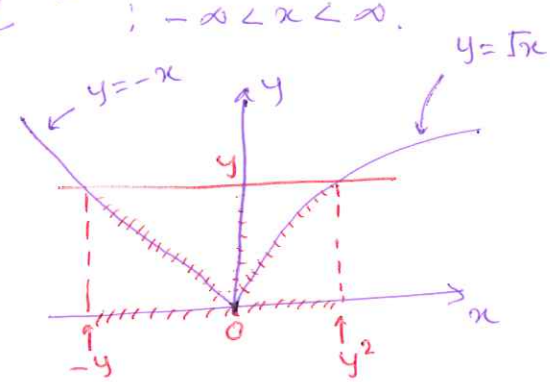
(10 pts)

$$\simeq 0$$




Q4) Let $X \sim N(0,1)$ then $f_X(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$ ; $-\infty < x < \infty$.

Let $y = \begin{cases} -x & ; x < 0 \\ \sqrt{x} & ; x \geq 0 \end{cases} = g(x)$




(10 pts)

a) Since the function is not monotonic over the range of the random variable $X$, $(-\infty, \infty)$, the transformation method can not be used.

b) cdf method!

1) Pdf of $Y$:

$F_Y(y) = P(Y \le y)$

$= P(-y \le X \le 0) + P(0 \le X \le y^2)$ (from the graph).

$= \cancel{F_X(0)} - F_X(-y) + F_X(y^2) - \cancel{F_X(0)}$

(15 pts) $= F_X(y^2) - F_X(-y)$

2) d.w.r.t. $y$,

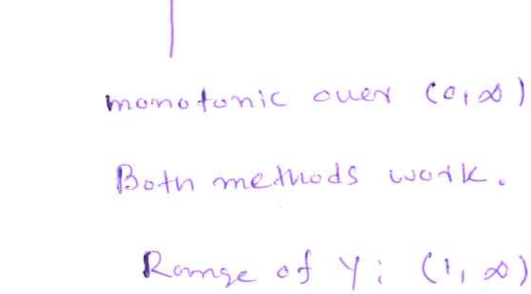
$f_Y(y) = f_X(y^2)\cdot(2y) - f_X(-y)\cdot(-1)$

$= \frac{1}{\sqrt{2\pi}} e^{-(y^2)^2} \cdot 2y + \frac{1}{\sqrt{2\pi}} e^{-(-y)^2}$

$\therefore f_Y(y) = \frac{1}{\sqrt{2\pi}} \cdot 2y e^{-y^4} + \frac{1}{\sqrt{2\pi}} e^{-y^2}$ : $\underbrace{0 < y < \infty}$ from the graph.

Q5) Let $X \sim EXP(\lambda)$ then, $f_X(x) = \lambda e^{-\lambda x}$ ! $x > 0$

Let $g(x) = y = e^x$




a) Yes

$y = e^x = g(x) \Rightarrow x = \ln y = g^{-1}(y)$

$\frac{d(g^{-1}(y))}{dy} = \frac{d}{dy}(\ln y) = \frac{1}{y}$

monotonic over $(0, \infty)$.

Both methods work.

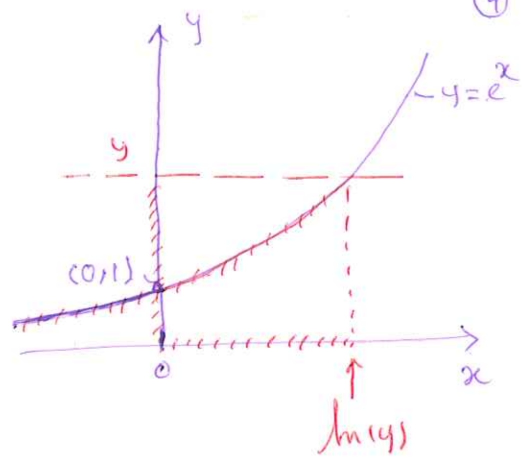
(15 pts) $\Rightarrow \left|\frac{d(g^{-1}(y))}{dy}\right| = \frac{1}{y}$ ($\because y > 0$)

Range of $Y$: $(1, \infty)$.

$\therefore f_Y(y) = f_X(g^{-1}(y)) \cdot \left|\frac{d(g^{-1}(y))}{dy}\right|$

$= \lambda e^{-\lambda \ln y} \cdot \frac{1}{y} \Rightarrow$ $\boxed{f_Y(y) = \lambda \cdot \frac{1}{y} \cdot e^{-\lambda \ln y} : 1 < y < \infty.}$

b) cdf method:




1) $F_Y(y) = P(Y \le y) = P(e^X \le y)$

$= P(X \le \ln(y))$

$= F_X(\ln y)$

2) d.w.r.t. y,

$f_Y(y) = f_X(\ln y) \cdot \frac{d(\ln(y))}{dy}$

(15 pts) $= \lambda e^{-\lambda \ln y} \cdot \frac{1}{y}$

$\therefore\ f_Y(y) = \lambda e^{-\lambda \ln y} \cdot \frac{1}{y}\ ;\ 1 < y < \infty.$

$$\boxed{f_Y(y) = \lambda \cdot \frac{1}{y} e^{-\lambda \ln y}\ ;\ 1 < y < \infty}$$