$$= \frac{n!}{n_1! \cdot n_2! \cdot \ldots n_r!}$$

$$= \binom{n}{n_1, n_2, n_3 \ldots, n_r}$$

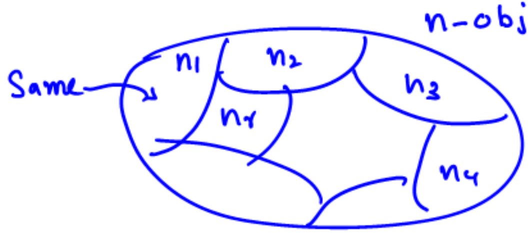
HW - 3 : W (9/13)

Class -10

Note:

* $^nC_r = \binom{n}{r}$ is called the Binomial coefficient.
* $\binom{n}{n_1, n_2, \ldots n_r}$ is called the Multinomial coefficeint.
* Partition is also used when all objects are not different.

n-object, r-groups.



$$\# \text{ of arrangements} = \frac{n!}{n_1! \cdot n_2! \cdot \ldots n_r!} = \binom{n}{n_1, n_2, \ldots n_r}.$$

Eg:-

A house has 12 rooms. We want to paint 4 yellow, 3 purple, and 5 red. How many ways can this be done?

$n = 12, \quad n_1 = 4, \quad n_2 = 3, \quad n_3 = 5$

$$\# \text{ of ways} = \frac{12!}{4! \cdot 3! \cdot 5!} = 27720.$$

There are 39 students in a class. In how many ways can a professor give out 9 A’s, 13 B’s, 12 C’s and 5 F’s?

$n = 39, \quad n_1 = 9, \quad n_2 = 13, \quad n_3 = 12, \quad n_4 = 5$

$$\# \text{ of ways} = \frac{39!}{9! \cdot 13! \cdot 12! \cdot 5!} = 1.57 \times 10^{22}.$$

How many different words can be obtained by rearranging the word “TATOO”.

~~n=5~~

T-2 , O-2 , A-1

$n=5$, $n_1=2$, $n_2=2$, $n_3=1$

$$\#\text{ of words} = \frac{5!}{2!\cdot 2!\cdot 1!} = 30$$

A class consisting of 4 graduate students and 12 undergraduate students randomly divide into four groups of 4. What is the probability that each group includes a graduate student?

$$P(\text{each group has a grad}) = \frac{\#\text{ of ways to have a grad in each group}}{\text{Total number of choices.}}$$

* without any restrictions

$$\#\text{ of ways to chose 4 groups} = \frac{16!}{4!\cdot 4!\cdot 4!\cdot 4!}$$

* Eeach group has a grad Student

$$\#\text{ of ways to distribute 4 grads} = 4!$$

After that,

$$\#\text{ of ways to distribute 12 under grads} = \frac{12!}{3!\cdot 3!\cdot 3!\cdot 3!}$$

$$\#\text{ of ways to have a grad in each group} = 4! * \frac{12!}{3!\cdot 3!\cdot 3!\cdot 3!}$$

$$P(\text{each group has a grad}) = \frac{4!\cdot \frac{12!}{3!\cdot 3!\cdot 3!\cdot 3!}}{\frac{16!}{4!\cdot 4!\cdot 4!\cdot 4!}} = \underline{\qquad}$$

Sec 2.2 : Binomial and Multinomial Distributions

* Binomial Distribution

Consider n identical and independent trials such that each trial has only two outcomes (Success and failure) with $P(\text{Success}) = p$.

Let X – # of Successes out of n-trials, then X follows a Binomial distribution with parameter n and p.

Probability distribution:

$$P(X=x) = {}^{n}C_{x}\, p^{x}(1-p)^{n-x}, \quad x = 0, 1, 2, \dots n.$$

$$\left(\text{ie } X \sim \text{Binomial}(n, p)\right).$$

Note:

If $X \sim \text{Binomial}(\overset{\text{\# of trials}}{n}, \overset{\text{Probability of Success on each trial.}}{p})$ then,

* $\sum_{x} p(x) = \sum_{x=0}^{n} {}^{n}C_{x}\, p^{x}(1-p)^{n-x} = 1.$

Proof: $\underbrace{\sum_{x=0}^{n} {}^{n}C_{x}\, p^{x}(1-p)^{n-x}}$ $\overset{\text{binomial theorem.}}{=} (p + (1-p))^{n} = 1^{n} = 1.$

* $E(X) = np$ and $Var(X) = np(1-p).$

Proof - HW.

Suppose a die is rolled 5 times.
a) What is the probability of getting exactly 2 fours?
b) What is the probability of getting at least a four?
c) What is the probability of getting at most 4 fours?
d) Find the expected value and the variance of the number of fours out of five trials.

$n=5, \quad p=\frac{1}{6} = P(\text{a 4 is obtained}).$

Let X – # of 4s out of 5 – trials.

a) $P(X=2) = {}^5C_2 (1/6)^2 (1-1/6)^{5-2}$

check.

$= 10 \cdot \frac{1}{6^2} \cdot \frac{5^3}{6^3} = \boxed{0.0804.}$

b) Possible values

$0, \underbrace{1, 2, 3, 4, 5}$

$P(X \geq 1) = P(1) + P(2) + P(3) + P(4) + P(5)$

$= 1 - P(0)$

$= 1 - {}^5C_0 (1/6)^0 (5/6)^5 = 0.665$

c) $\underbrace{0, 1, 2, 3, 4}, 5$

$P(X \leq 4) = 1 - P(5)$

$= 1 - {}^5C_5 (1/6)^5 (5/6)^0 = 0.999.$

d) $E(X) = np$

$= 5 \cdot \frac{1}{6} = 5/6$

$Var(X) = np(1-p)$

$= 5 \frac{1}{6} \cdot \frac{5}{6}$

$= 25/36$