Practice Exam : Today (M ,9-18), discuss in the class

Exam -1 : T (9-19) :
Cover first two chapters
allowed one double sided handwritten sheet
Bring your calculator
(cell phone is not allowed as a calculator)

Practice Exam (for Exam-1)

MA 2621 – A 17

1. Insurance Underwriters have established that the probability of city experiencing disasters in the next five years is 0.3 for a Tornado, 0.4 for Hurricane, and 0.1 for both Tornado and Hurricane.

   a) What is the probability of city experiencing only a Tornado in the next five years? $P(T \cap H^c) = ?$

Let T – experiencing a tornado, H – " " Hurricane

$P(T) = 0.3$, $P(H) = 0.4$

$P(T \cap H) = 0.1$

$$P(T) = P(T \cap H^c) + P(T \cap H)$$

$$P(T \cap H^c) = P(T) - P(T \cap H)$$

$$= 0.3 - 0.1 = \boxed{0.2}$$

   b) What is the probability of city experiencing neither a Tornado nor Hurricane in the next five years?

$$P(T^c \cap H^c) = P((T \cup H)^c)$$

De Morgan's Law: $(T \cup H)^c = T^c \cap H^c$

$$P((T \cup H)^c) = 1 - P(T \cup H)$$

$$= 1 - [P(T) + P(H) - P(T \cap H)]$$

$$=$$

$$= \boxed{0.4}$$

2. A math teacher gave her class two tests. 25% of the class passed both tests and 42% of the class passed the first test.

   a) What percent of those who passed the first test also passed the second test?

Let A_1 – Passing the first test, A_2 – " " Second test.

$P(A_1) = 0.42$

$P(A_1 \cap A_2) = 0.25$

$$P(A_2 | A_1) = \frac{P(A_1 \cap A_2)}{P(A_1)} = \frac{0.25}{0.42} = 25/42.$$

   b) What percent of those who passed the first test failed the second test?

$P(A) + P(A^c) = 1$

1st method

$$P(A_2^c | A_1) = 1 - P(A_2 | A_1) = 1 - 25/42 = 17/42$$

$P(A|B) + P(A^c|B) = 1$

2nd method

$$P(A_2^c | A_1) = \frac{P(A_2^c \cap A_1)}{P(A_1)} = \frac{P(A_1) - P(A_1 \cap A_2)}{P(A_1)} = \frac{.42 - 0.25}{0.42}$$

$$= 17/42.$$

3. a) Say a yogurt shop has three flavors (C, V and S), two sizes, (L and M) and 5 different toppings. How many different yogurts can be ordered? (Multiplication Rule)

$\underset{3}{\underset{\uparrow}{\_\_\_}}$ $\underset{2}{\underset{\uparrow}{\_\_\_}}$ $\underset{5}{\underset{\uparrow}{\_\_\_}}$

$\#$ of yogurts $= 3 * 2 * 5 = 30$

b) There are 3 Science books 5 Math books ~~are~~ on a shelf. In how many different ways could you arrange them such that all 3 Science books are together? (Permutation).

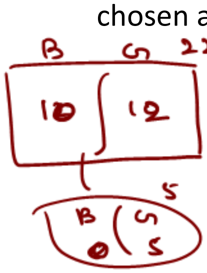
$M_1 \; M_2 \; \boxed{S_1 \; S_2 \; S_3} \; M_3 \; M_4 \; M_5$

Fist bind 3 - Science books together. (now you have a big Science book)

$\#$ of ways to arrange Science books $= 3!$

Total $\#$ of way $= 6! * 3! = 4320$

c) Given a class of 12 girls and 10 boys. What is the probability that a committee of five, chosen at random from the class, consists only of girls? (Combinations) $P(A) = \frac{n(A)}{n(\Omega)}$



$$P(5\text{-girls}) = \frac{{}^{12}C_5 * \boxed{{}^{10}C_0}}{{}^{22}C_5} = 0.0301$$

d) How many different words (letter sequences) can be obtained by rearranging the letters in the word "**STATISTICS**"? $n = 10$ (Partitions).

$S - \overset{n_1}{3}, \; T - \overset{n_2}{3}, \; A - \overset{n_3}{1}, \; I - \overset{n_4}{2}, \; C - \overset{n_5}{1}$

$$\# \text{ of words} = \frac{10!}{3! \cdot 3! \cdot 1! \cdot 2! \cdot 1!} = 50400$$

4. a) In each of 4 races, the Democrats have a 60% chance of winning. Assuming that the races are independent of each other. What is the probability that the Democrats will win a majority of the races? $n=4$, $p=0.6$ (Binomial)

$P(x) = {}^nC_x p^x (1-p)^{n-x}$

$x = 0, 1, \ldots n$

Let X − # of races that democrats will win,

$0, 1, 2, \underline{3, 4}$

$P(X>2) = P(3) + P(4)$

$= {}^4C_3 (0.6)^3 (0.4)^1 + {}^4C_4 (0.6)^4 (0.4)^0$

$= 0.4752$

b) Suppose parts are of two varieties: good (with probability 90/92) and slightly defective (with probability 2/92). Parts are produced one after the other. What is the probability that at least 3 parts must be produced ~~until~~ there is a slightly defective part produced?

$P(x) = p(1-p)^{x-1}$

$x = 1, 2, 3 \ldots$

(Geometric), $p = 2/92$

Let X − # of parts produced till a defective is observed.

$X \sim \text{Geometric}(p = 2/92)$.

$\underbrace{1, 2,}\ \underline{3, 4, 5, \ldots\ldots}$

$P(X \geq 3) = 1 - [P(1) + P(2)]$

$= \frac{2}{92} + \frac{2}{92}\left(\frac{90}{92}\right)$

$=$

λ

c) If electricity power failures occur with an average of 0.15 failures per week, calculate the probability that there will not be more than one failure during a particular week.

$\lambda = 0.15$ (Poisson).

$p(x) = \frac{e^{-\lambda}\lambda^x}{x!}$

Let X − # failures during a week.

$x = 0, 1, 2, \ldots$

Then $X \sim \text{Poisson}(\lambda = 0.15)$

$P(X \leq 1) = P(0) + P(1) = \frac{e^{-0.15}(0.15)^0}{0!} + \frac{e^{-0.15}(0.15)^1}{1!}$

$= e^{-0.15}[1.15] = 0.9898$

5. Consider the Probability mass function of random variable X,

$$P_X(x) = \begin{cases} 0.2 & : x = 0, 10 \\ 0.1 & : x = 5 \\ 0.5 & : x = 20 \end{cases}$$

a) Find expectation of 2X +1

* Expectation

$E(X) = \sum_x x\,p(x)$

$E(g(X)) = \sum_x g(x)\,p(x)$

* $E(aX+b) = aE(X) + b$

a, b – constant

1st method:

$E(X) = 0\cdot(0.2) + 10(0.2) + 5(0.1) + 20(0.5) = 12.5$

Then $E(2X+1) = 2E(X) + 1 = 2(12.5) + 1 = 26$

2nd method:

$E(2X+1) = (2(0)+1)(0.2) + (2(10)+1)(0.2) + (2(5)+1)(0.1) + (2(20)+1)(0.5)$

$= 26$

* variance

* $Var(X) = E((X - E(X))^2)$

$= \sum_x (x - E(X))^2\,p(x)$

* $Var(X) = E(X^2) - (E(X))^2$

* $Var(aX+b) = a^2\,Var(X)$

a, b – constants.

b) Variance of 3X-2.

$Var(3X-2) = 9\cdot Var(X)$

$Var(X) = E(X^2) - (E(X))^2$

$E(X^2) = 0^2(0.2) + 10^2(0.2) + 5^2(0.1) + 20^2(0.5)$

$= 222.5$

$Var(X) = 222.5 - (12.5)^2 = 66.25$

$Var(3X-2) = 9(66.25) = 596.25$