

# Defn Probability Distribution

Let  $X$  - be a discrete random variable,  $x$  - any real number then the probability distribution function "p" is the probability of the event  $\{X=x\}$ .

we use

$$P(X=x) = p(x) = p(\underbrace{\{X=x\}}_{\text{event}}).$$

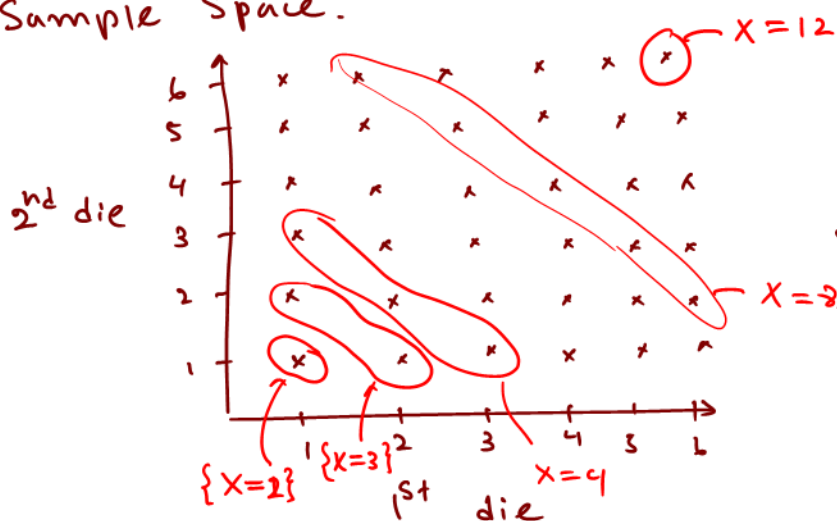
Note:

- \* R.V. - upper case letters
- \* values - lower case letters.

Eg:- Rolling two fair dice.

Let  $X$  - the Sum of two numbers.

Sample Space:



Possible values:

$x = 2, 3, 4, \dots, 12$

$$\{X=2\} = \{(1,1)\}.$$

$$P(X=2) = P(\{X=2\}) = P(\{(1,1)\}) = 1/36$$

$$P(X=3) = P(\{(1,2), (2,1)\}) = 2/36$$

$$P(X=4) = 3/36$$

$$P(X=5) = 4/36$$

$$P(X=6) = 5/36$$

$$P(X=7) = 6/36$$

$$P(X=8) = 5/36$$

$$P(X=9) = 4/36$$

$$P(X=10) = 3/36$$

$$P(X=11) = 2/36$$

$$P(X=12) = 1/36$$

∴ Probability distribution of  $X$ :

$X=x$	2	3	4	5	6	7	8	9	10	11	12
$P(X=x)$	$1/36$	$2/36$	$3/36$	$4/36$	$5/36$	$6/36$	$5/36$	$4/36$	$3/36$	$2/36$	$1/36$

OR

$$P(X=x) = \begin{cases} \frac{1}{36} & : x = 2, 12 \\ \frac{2}{36} & : x = 3, 11 \\ \frac{3}{36} & : x = 4, 10 \\ \frac{4}{36} & : x = 5, 9 \\ \frac{5}{36} & : x = 6, 8 \\ \frac{6}{36} & : x = 7 \\ 0 & : \text{otherwise} \end{cases}$$

## Geometric Distribution

Consider a sequence of independent and identical trials, such that each has only two outcomes (Success and failure) with  $p(\text{Success}) = p$  on each trial (Binary trials).

Let  $X$  - the # of trials to come up the 1<sup>st</sup> Success.

Then  $X$  has a geometric distribution and the probability distribution is

$$p(X=x) = (1-p)^{(x-1)} \cdot p, \quad x=1, 2, 3, \dots$$

Eg:-

$$p = 0.03$$

Product produced by a machine has a 3% defective rate.

a) What is the probability that the first defective occurs at the fifth item inspected.

$$p(X=5) = ?$$

b) What is the probability that first defective occurs in the first five inspections.

Let  $X$  - # of trials to get the first defective,

$$a) p(X=5) = (1-0.03)^{5-1} (0.03)$$

$$= (0.97)^4 (0.03)$$

$$= 0.02656$$

$$p(X=x) = (1-0.03)^{x-1} (0.03) \\ x=1, 2, 3, \dots$$

$$b) p(X \leq 5) = p(1) + p(2) + p(3) + p(4) + p(5)$$

$$= (1-0.03)^0 (0.03) + \dots + (1-0.03)^4 (0.03)$$

$$= 0.14127$$

$$* p(X \leq 5) = 1 - p(X > 5)$$

1, 2, 3, 4, 5, 6, 7, ...  
 $X > 5$

Eg:-

In a baseball event the first team to win four games wins the championship.

What is the probability distribution of number of games ( $X$ ).

Assume each team has 0.5 probability to win a game.

\* Four games: ( $X=4$ )

\* 1<sup>st</sup> team win all 4 games.

$$P(1^{\text{st}} \text{ team win all 4}) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{16}$$

Similarly.

$$P(2^{\text{nd}} \text{ team win all 4}) = \frac{1}{16}$$

$$P(4) = P(X=4) = \frac{1}{16} + \frac{1}{16} = \frac{2}{16}.$$

\* Five games ( $X=5$ )

1<sup>st</sup> team

B A A A A, A B A A A, A A B A A, A A A B A, ~~A A A A B~~

$$P(1^{\text{st}} \text{ team}) = \left(\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}\right) \times 4$$

$$P(X=5) = 2 \times 4 \times \left(\frac{1}{2}\right)^5$$

If you play roulette and bet \$1 on black then you win \$1 with probability 18/38 and you lose \$1 with probability 20/38. What is the expected win?