HW - 1 : W (8/30) in the conference   Quiz -1 : F (9/01)   Class-4

2) For any events A and B, $P(A \cap B) = P(A) \cdot P(B|A)$

$[\text{or } P(A \cap B) = P(B) \cdot P(A|B)]$.

Eg:

Draw two cards from a deck:
Let A= the first card is a spade and B= the second card is a spade.
What is the probability that both cards are spades?
(Regular card deck (52 cards) : 13- spades, 13-clubs, 13-hearts, 13-diamonds)

$P(A \cap B) = ?$

$P(A) = \frac{13}{52}$, $P(B|A) = \frac{12}{51}$

$S' - 13 \to ①$
$O - 39 \to ②$

$$P(A \cap B) = P(A) \cdot P(B|A)$$
$$= \frac{13}{52} \cdot \frac{12}{51} = \boxed{\quad}.$$

3. If A and B are disjoint events that have positive probabilities then they are not independent.

Since $\underset{>0}{P(A)} \cdot \underset{>0}{P(B)} \neq P(A \cap B) = 0$.

See EX. 1.8

4) General result:

* Events $A_1, A_2, A_3, \ldots, A_n$ are said to be pairwise independent if

$$P(A_i \cap A_j) = P(A_i) \cdot P(A_j) \text{ for all } i, j \in \{1, 2, \ldots n\} \text{ and } i \neq j.$$

* Events $A_1, A_2, \ldots, A_n$ are said to be independent if

$$P(A_1 \cap A_2 \cap \cdots A_n) = P(A_1) \cdot P(A_2) \cdots P(A_n)$$
$$\left[\text{ie } P\left(\bigcap_{i=1}^{n} A_i\right) = \prod_{i=1}^{n} P(A_i)\right].$$

Let A = Alice and Betty have the same birthday.
B = Betty and Carol have the same birthday.
C = Carol and Alice have the same birthday.
Are A, B, C pairwise independent? Are A, B, C independent?

There are 365 posibilities for Alice's birthday.
" " 365 " for Betty's birthday.

There are 365^2 possibilities for Alice's and Betty's birthdays.

There are 365 possibilities for same birthday.

$$P(A) = \frac{365}{365^2} = \frac{1}{365}.$$

$$P(B) = P(C) = \frac{1}{365}.$$

$A \cap B$ = all three have the same birthday.

$$P(A \cap B) = \frac{365}{365^3} = \frac{1}{365^2}$$

$$\therefore \quad P(A) \cdot P(B) = \frac{1}{365} \cdot \frac{1}{365} = \frac{1}{365^2} = P(A \cap B)$$

$\therefore$ A and B are independent

Similarly, we can show that A and C and B and C are independent.

$\therefore$ A, B and C are pairwise independent.

$$A \cap B \cap C = A \cap B$$ (not true in general).

$$\Rightarrow \quad P(A \cap B \cap C) = P(A \cap B) = \frac{1}{365^2}$$

But
$$P(A) \cdot P(B) \cdot P(C) = \frac{1}{365^3} \neq \frac{1}{365^2} = P(A \cap B \cap C)$$

$\therefore$ A, B and C are not independent.

Note:

Pairwise independence doesn't imply independence.

Sec 1.4 : Random Variable

Defn

A random variable (r.v.) is a function that assigns a real number to each outcome in the sample space Ω of a random experiment.

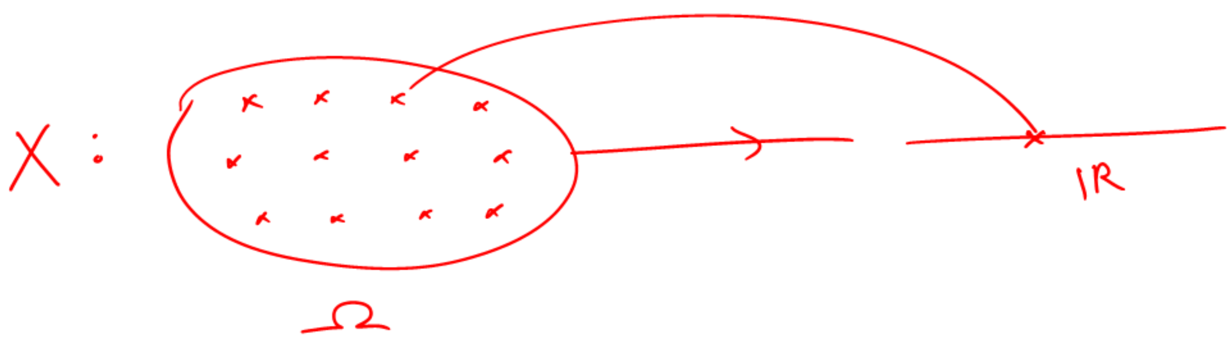
Eg:- Tossing a coin three times.

Let X - # of heads out of 3 tosses.

$$\Omega = \{ \underset{\downarrow \atop 3}{HHH},\ \underset{\downarrow \atop 2}{HHT},\ \underset{\downarrow \atop 2}{HTH},\ \underset{\downarrow \atop 2}{THH},\ \underset{\downarrow \atop 1}{HTT},\ \underset{\downarrow \atop 1}{THT},\ \underset{\downarrow \atop 1}{TTH},\ \underset{\downarrow \atop 0}{TTT} \}.$$

Let A - getting three heads then $A = \{HHH\}$

(not a random variable, just an event).



Note:

* A random variable is called discrete if it can take only discrete numbers.
* A random variable is called continuous if it can take any value from an interval (a subset of the real number set).

Eg: 1. Roll two dice and X – the Sum of the two numbers.

Possible values : 2, 3, 4, ... , 12 $\quad \Omega = \{(1,1), (1,2), \dots (6,6)\}$.

finite and discrete.

2. Roll a die until a 4 appears and let X – # of rolls.

$\Omega = \{4, f4, ff4, fff4, \dots\}$ $\quad f : 1, 2, 3, 5, 6$

infinite, discrete

3. Height of a Student Selected randomly from MA 2621 class.

continuous.