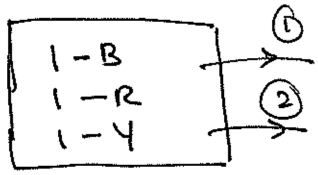
MA 2621 – HW – I – Solution Key

1)

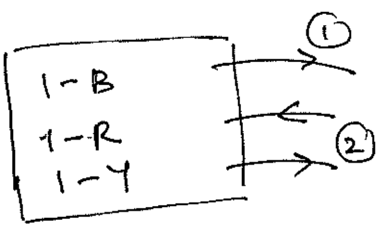


a) Sample Space $= \Omega = \{BR, BY, RB, RY, YR, YB\}$.

b) $A = \{BR, RB, RY, YR\}$.

$$P(A) = \frac{n(A)}{n(\Omega)} = \frac{4}{6} = 2/3$$

c)



Sample Space $= \Omega = \{BB, BR, BY, RR, RB, RY, YY, YR, YB\}$.

$A = \{BR, RB, RY, YR\}$

$$P(A) = \frac{n(A)}{n(\Omega)} = 4/9$$

2) Let W – expeariancing bad ignition wires and
S' – " " spark plugs.

then $P(W) = 0.35$, $P(S') = 0.8$ and $P(W \cap S') = 0.2$

a)

$$P(\text{bad wires} \mid \text{bad spark plugs}) = P(W \mid S')$$

$$= P\frac{(W \cap S')}{P(S')}$$

$$= \frac{0.2}{0.8} = 0.25$$

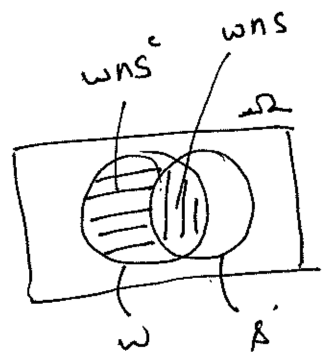
b) $P(\text{bad wires} \mid \text{good Spark plugs}) = P(W \mid S^c)$

$$= \frac{P(W \cap S')}{P(S^c)}$$

But $P(S') = 1 - P(S') = 0.2$ and

$P(W \cap S^c) + P(W \cap S) = P(W)$

$\therefore P(W \cap S^c) = 0.35 - 0.2 = 0.15$



$$\therefore P(W \mid S') = \frac{0.15}{0.2} = 3/4 = 0.75$$

3 Let S - Purchasing a car with Stereo

T - " a car with tinted windows.

Then $P(S) = 0.6$, $P(T) = 0.5$, and $P(S \cap T) = 0.35$

$P(\text{At leas one of the two options}) = P(S \cup T)$

$= P(S) + P(T) - P(S \cap T)$

$= 0.6 + 0.5 - 0.35$

$= 0.75$

4 a) Let $f - 2, 3, 4, 5, 6$

Sample Space :

$\Omega = \{1, f1, ff1, fff1, ffff1, ---- \}$.

Note: Here note that outcomes are not equally likely

b) $A = \{fff1, ffff1, ---- \}$.

So $A^c = \{1, f1, ff1\}$

$\therefore P(A^c) = \frac{1}{6} + \frac{5}{6} \cdot \frac{1}{6} + \left(\frac{5}{6}\right)^2 \cdot \frac{1}{6}$

$= 0.4213$

$\therefore P(A) = 1 - P(A^c)$

$= 1 - 0.4213 = 0.5787$

c) Let B – the first "1" occurs at the n^{th} ~~th~~row or later.

$$B = \{ \underbrace{ff\cdots f}_{n-1 \text{ times}}1 , \underbrace{ff\cdots f}_{n\text{-times}}1 , \cdots \}.$$

Now $$B^c = \{ 1, f1, ff1, \cdots \underbrace{fff\cdots f}_{n-2 \text{ times}}1 \}.$$

$$\therefore P(B^c) = \frac{1}{6} + \frac{1}{6}\left(\frac{5}{6}\right) + \frac{1}{6}\left(\frac{5}{6}\right)^2 + \cdots + \frac{1}{6}\left(\frac{5}{6}\right)^{n-2}$$

$$= \frac{1}{6}\left[1 + \frac{5}{6} + \left(\frac{5}{6}\right)^2 + \cdots + \left(\frac{5}{6}\right)^{n-2}\right] \quad (\text{only } n-1 \text{ terms})$$

$$= \frac{1}{6}\left[\frac{1-(5/6)^{n-1}}{1-5/6}\right]$$

$$= 1 - \left(\frac{5}{6}\right)^{n-1}$$

$$\therefore P(B) = 1 - P(B^c) = 1 - \left(1 - (5/6)^{n-1}\right) = (5/6)^{n-1} \text{ //.}$$

Note:

* Here we use:

$$S_n = a + ar + ar^2 + \cdots + ar^{n-1} \quad (n\text{-terms})$$

$$= \frac{a(1-r^n)}{1-r}$$

* You may need:

$$S_\infty = a + ar + ar^2 + \cdots \quad (\text{infinite Geometric})$$

$$= \frac{a}{1-r}.$$

#5

$P(A) = \frac{1}{3}, \quad P(B) = \frac{1}{4}$

Since $\Phi \subseteq A \cap B \subseteq B$

$\Rightarrow p(\Phi) \leq p(A \cap B) \leq p(B)$

$\Rightarrow \boxed{0 \leq p(A \cap B) \leq \frac{1}{4}}$

* Since $A \subseteq A \cup B$

$\Rightarrow p(A) = \frac{1}{3} \leq p(A \cup B)$

Further, $p(A \cup B) \leq p(A) + p(B)$

$\Rightarrow p(A \cup B \leq \frac{1}{3} + \frac{1}{4} = \frac{7}{12}$

$\therefore \boxed{\frac{1}{3} \leq p(A \cup B) \leq \frac{7}{12}}$

#6

Sample Space $= \Omega = \{ (1,1,1), (1,1,2), \ldots (6,6,6) \}$

- 216 outcomes. $= 6^3$

\# of ways to get combination $\overbrace{1+2+6}^{\text{all different}} = 6$

$1+2+6 \rightarrow$ 1,2,6
1,6,2
2,1,6
2,6,1
6,1,2
6,2,1

$\therefore p(\text{sum } 1+2+6) = \frac{6}{6^3} = \frac{1}{6^2}$

Similarlly,

$$P(\text{Sum } 1+3+5) = P(\text{Sum } 2+3+4) = 1/6^2$$

$\#$ of ways to get combination $1+\underbrace{4+4}^{\text{Same}} = 3$

$$\therefore P(\text{Sum } 1+4+4) = \frac{3}{6^3}$$

Similarlly,

$$P(\text{Sum } 2+2+5) = 3/6^3$$

$\#$ of ways to get $3+3+3 = 1$

$$\therefore P(\text{Sum } 3+3+3) = 1/6^3$$

$$\therefore P(\text{Sum} = 9) = \frac{6}{6^3} + \frac{6}{6^3} + \frac{3}{6^3} + \frac{3}{6^3} + \frac{6}{6^3} + \frac{1}{6^3}$$

$$= \frac{25}{6^3}$$

Following a Similar argument,

$$P(1+3+6) = P(1+4+5) = P(2+3+5) = 6/6^3$$

$$P(2+4+4) = P(2+2+6) = P(3+3+4) = 3/6^3$$

$$\therefore P(\text{Sum} = 10) = 3 \cdot \frac{6}{6^3} + 3 \cdot \frac{3}{6^3} = 27/6^3$$

$\therefore$ Sum is equal to 10 is more likely than the Sum is equal to 9.

#7 $P(A) = 0.9$, $P(B) = 0.8$.

A or B (or both)

$$P(\text{System works}) = P(A \cup B)$$
$$= P(A) + P(B) - \underbrace{P(A \cap B)}_{P(A)\cdot P(B)}$$
$$= P(A) + P(B) - P(A)\cdot P(B) \quad \left[\because \text{A and B are independent}\right]$$
$$= 0.9 + 0.8 - (0.9)(0.8)$$
$$= \boxed{0.98}$$

#8 $P(A|B) = 0.5$, $P(B) = 0.25$, $P(A \cup B) = 0.75$

a) $$P(A|B) = \frac{P(A \cap B)}{P(B)} \quad (\text{Defn of conditional Prob})$$

$$\Rightarrow P(A \cap B) = P(A|B)\cdot P(B) = (0.5)(0.25) = 0.125$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$\Rightarrow P(A) = P(A \cup B) - P(B) + P(A \cap B)$$
$$= 0.75 - 0.25 + 0.125$$
$$= 0.625$$

$$P(B^c | A^c) = \frac{P(B^c \cap A^c)}{P(A^c)} = \frac{P[(A \cup B)^c]}{P(A^c)} = \frac{0.25}{0.375} = 0.6667$$

b) Since $P(A|B) = 0.5 \neq P(A) = 0.67$,

A and B are not independent.