Visualizing uncertainty

Daniel Anderson

Week 7, Class 2

Reviewing Lab 3

Data viz in the wild

Ann–Marie

Murat

Sarah Donaldson & Hyeonjin on deck

Agenda

- Framing uncertainty as relative frequencies
  - Discrete probabilities
  - Non–discrete probabilities
  - Understanding AUC calculations
- Understanding standard errors
  - Non–standard ways of visualizing SEs
- HOPs (briefly)
  - Also bootstrapping

Learning objectives

1. Understand there are lots of different ways to visualize uncertainty, and the best method may often be non–standard.

2. Understand how to implement basic methods, and the resources available to you to implement more advanced methods

Thinking about uncertainty

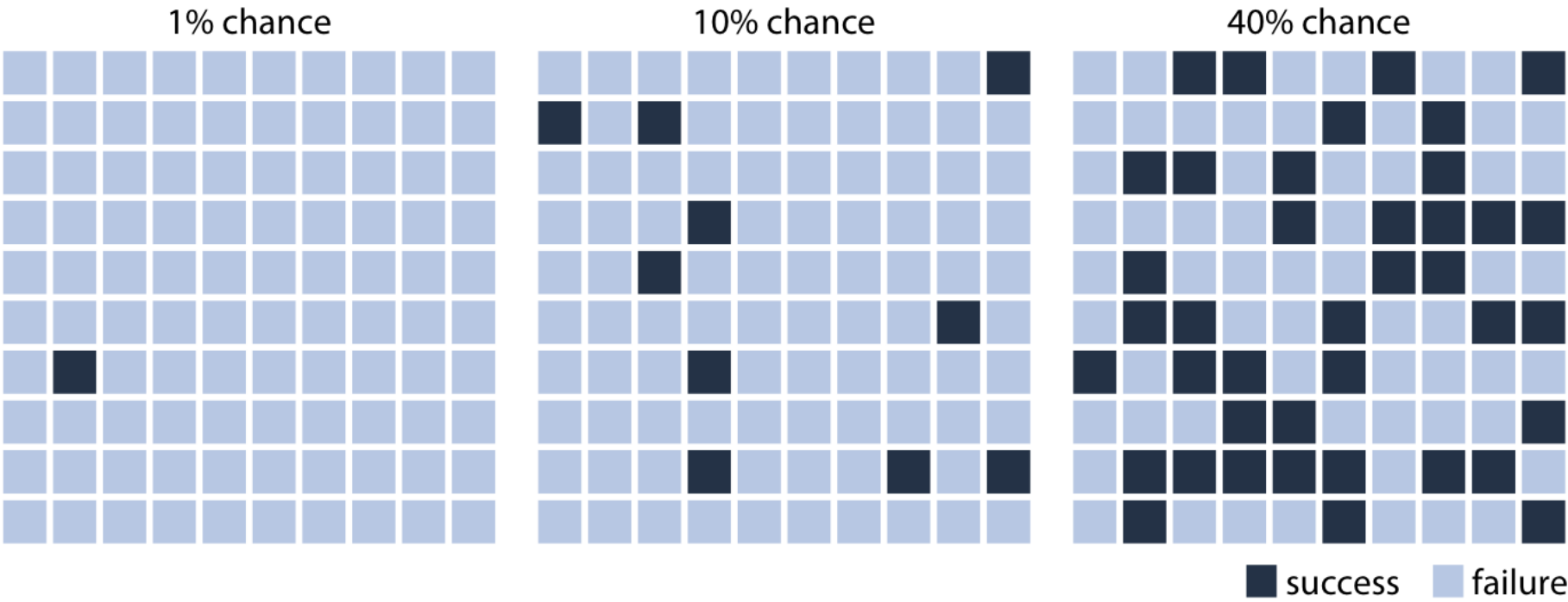
Uncertainty means exactly what it sounds like – we are not 100% sure.

- We are nearly always uncertain of future events (forecasting)
- We can also be uncertain about past events
  - I saw a parked car at 8 AM, but the next time I looked at 2PM it was gone. What time did it leave?

Quantifying uncertainty

- We quantify our uncertainty mathematically using probability
- Framing probabilities as frequencies is generally more intuitive

Framing a single uncertainty


1% chance
10% chance
40% chance
success
failure

How do we make these?

- Start out by making a grid

```
grid <- expand.grid(x = 1:20, y = 1:20)
head(grid)
```

```
##   x y
## 1 1 1
## 2 2 1
## 3 3 1
## 4 4 1
## 5 5 1
## 6 6 1
```

```
tail(grid)
```

```
##       x  y
## 395 15 20
## 396 16 20
## 397 17 20
## 398 18 20
## 399 19 20
## 400 20 20
```

Look at the grid

```
ggplot(grid, aes(x, y)) +
  geom_tile(color = "gray40",
            fill = "white") +
  theme_void()
```

Create occurrence rate

- For each sequence of x, create a variable that has the given occurence rate

How?

- Plenty of options, here's one

Consider 10%

```
nrow(grid)*.10 # n to sample
```

```
## [1] 40
```

```
set.seed(86753098)
samp <- sample(seq_len(nrow(grid)), nrow(grid)*.10)
head(samp)
```

```
## [1] 318 134 180 283 177 248
```

```
length(samp)
```

```
## [1] 40
```

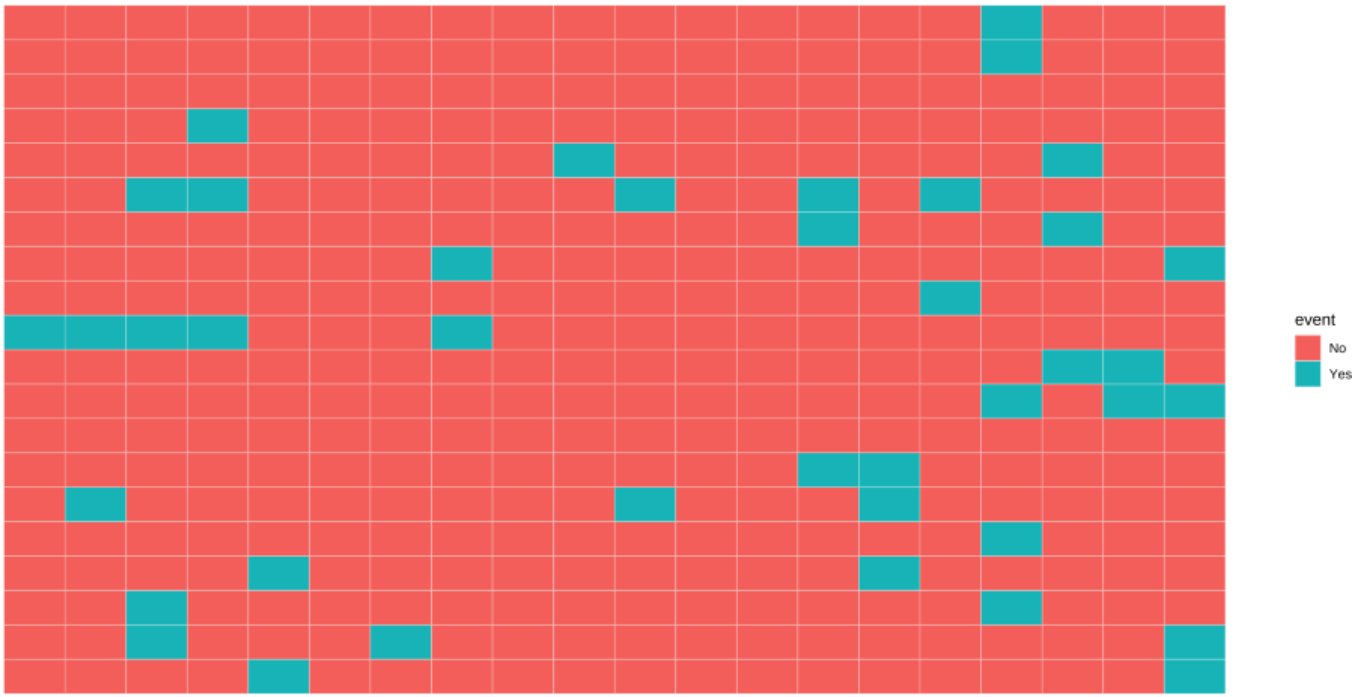
Create the variable

```
grid <- grid %>%
  rownames_to_column("row_id") %>%
  mutate(event = ifelse(row_id %in% samp, "Yes", "No"))
head(grid)
```

```
##   row_id x y event
## 1      1 1 1    No
## 2      2 2 1    No
## 3      3 3 1    No
## 4      4 4 1    No
## 5      5 5 1   Yes
## 6      6 6 1    No
```

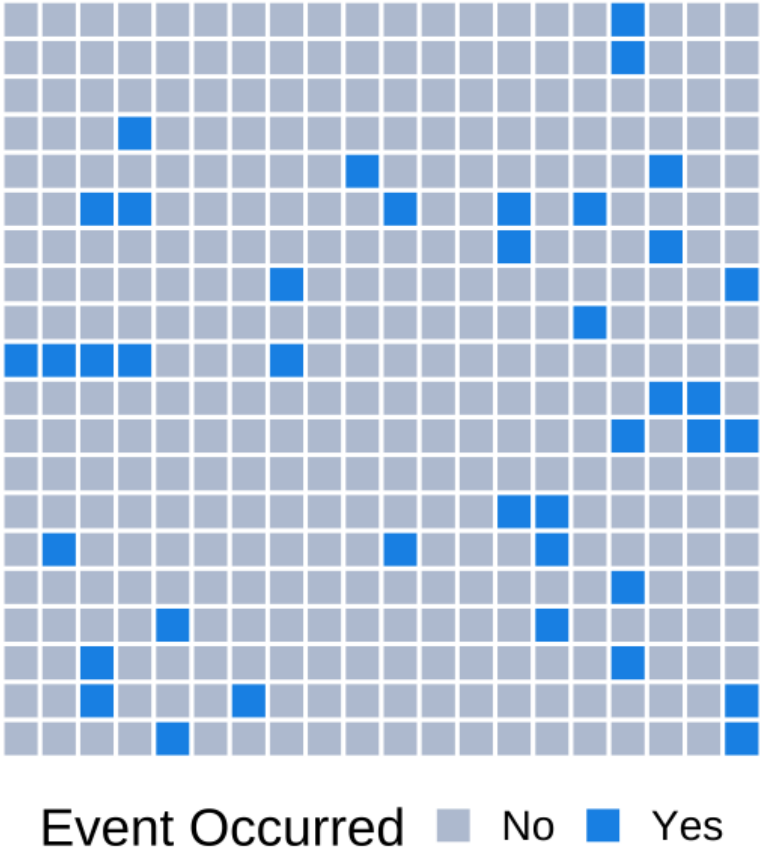
Fill in

```
ggplot(grid, aes(x, y)) +
  geom_tile(aes(fill = event), color = "white") +
  theme_void()
```



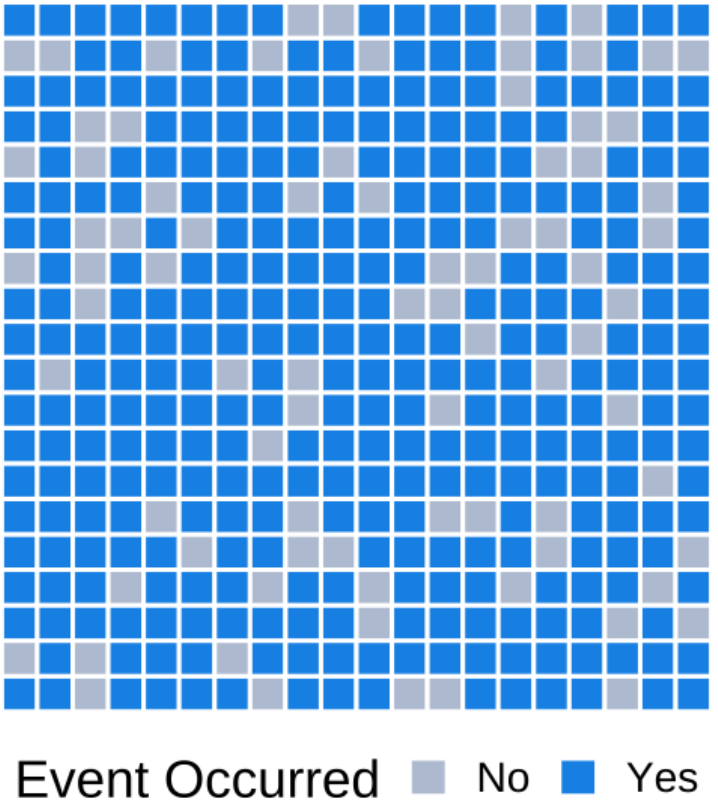
Customize

```
library(colorspace)
ggplot(grid, aes(x, y)) +
  geom_tile(aes(fill = event), color = "white", size = 1.4) +
  scale_fill_manual(
    name = "Event Occurred",
    values = c(
      desaturate(
        lighten("#1694E8", 0.5),
        0.7),
      "#1694E8"
      )
    ) +
  coord_fixed() +
  theme_void() +
  theme(legend.position = c(0.75, 0),
        legend.direction = "horizontal",
        plot.margin = margin(b = 1, unit = "cm"))
```


Event Occurred
No
Yes

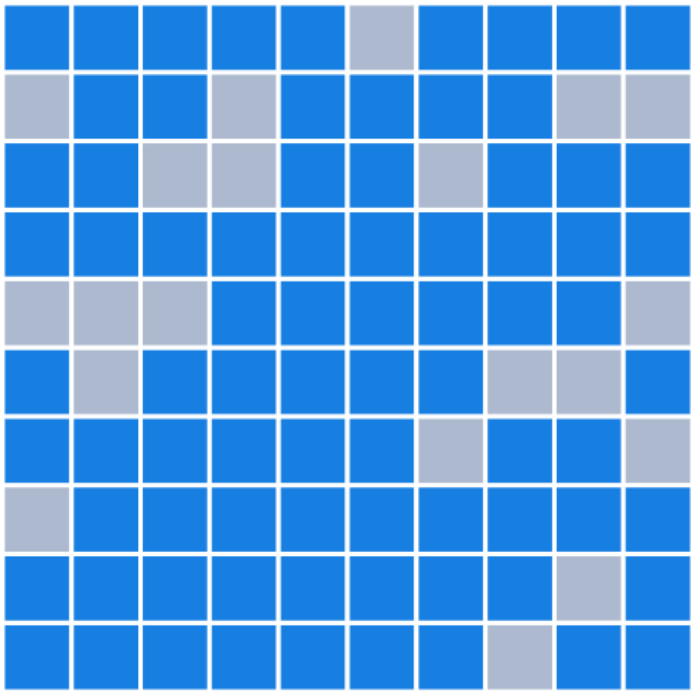
Chance of rain

80%



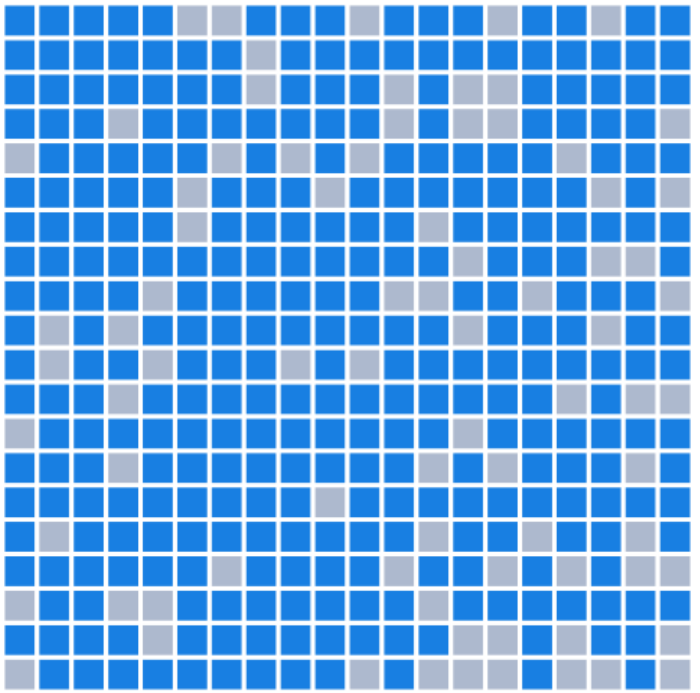
Vary grid size

10 x 10



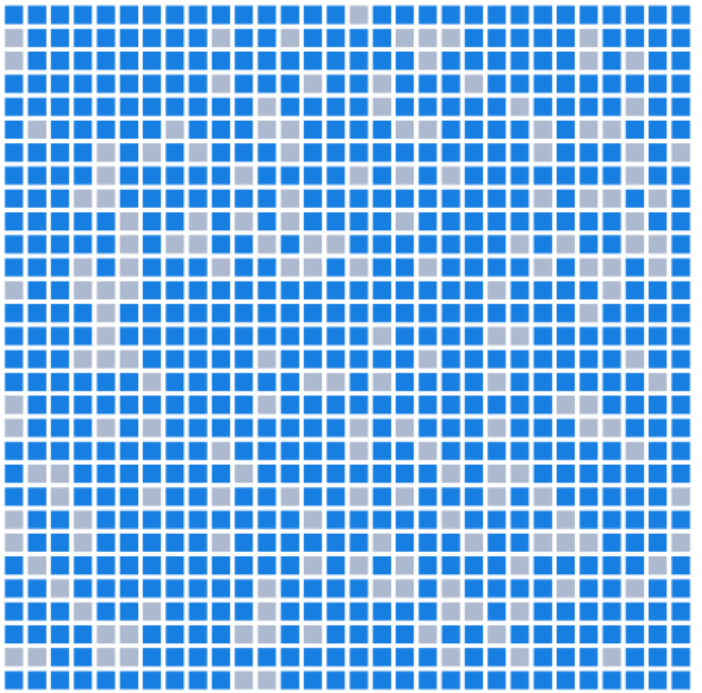
Vary grid size

20 x 20



Vary grid size

30 x 30



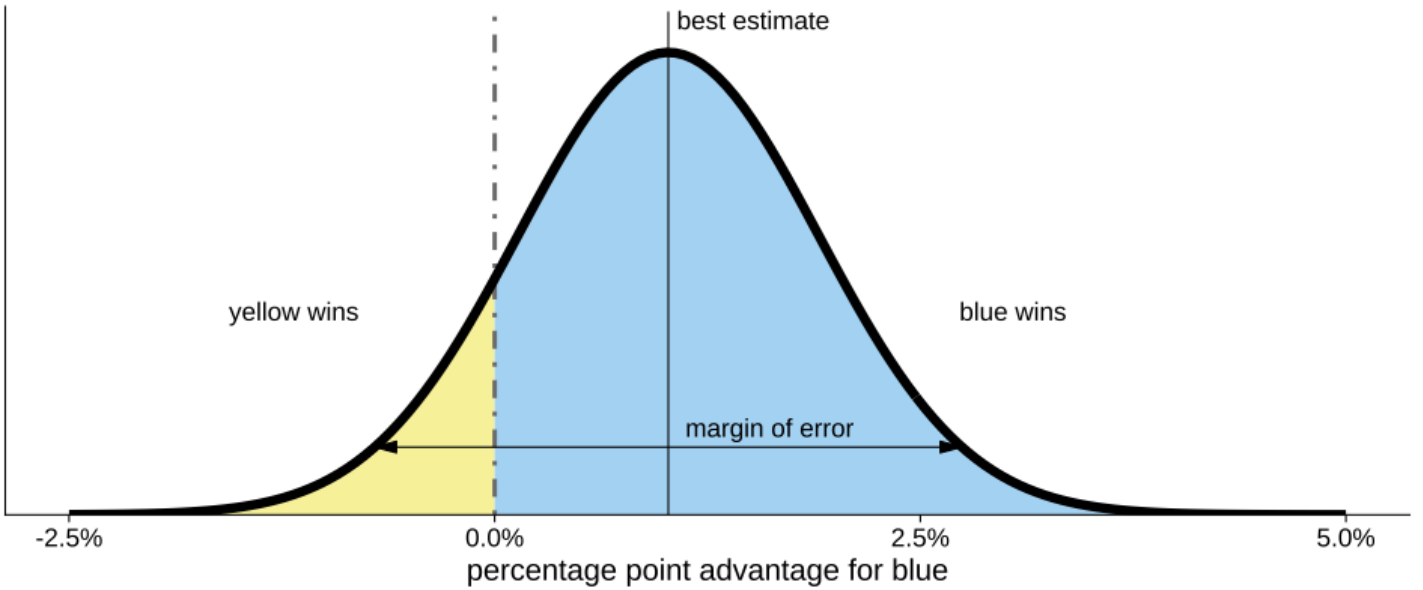
(probs too many)

Non–discrete probabilities

Hypothetical

Blue party has 1% advantage (technically 1.02%) w/ margin of error of 1.76 points

Who will win?



A bit of math

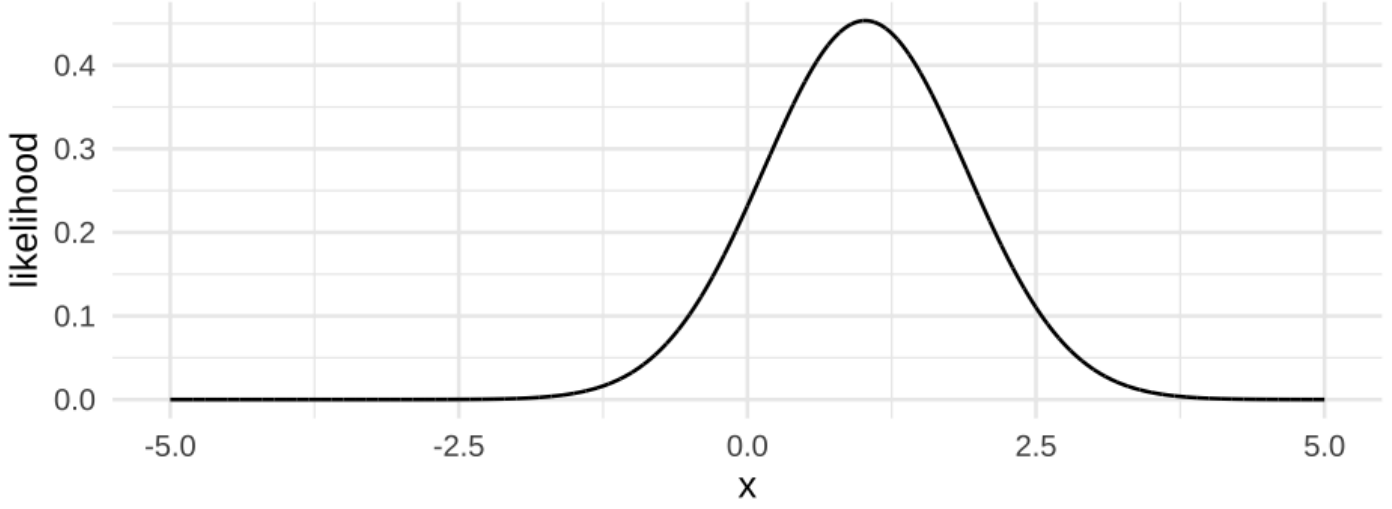
Our prior distribution was defined by $\mu = 1.02$ and $sd = 0.88$ (margin of error divided by two).

- What's the chance the end result is below zero?

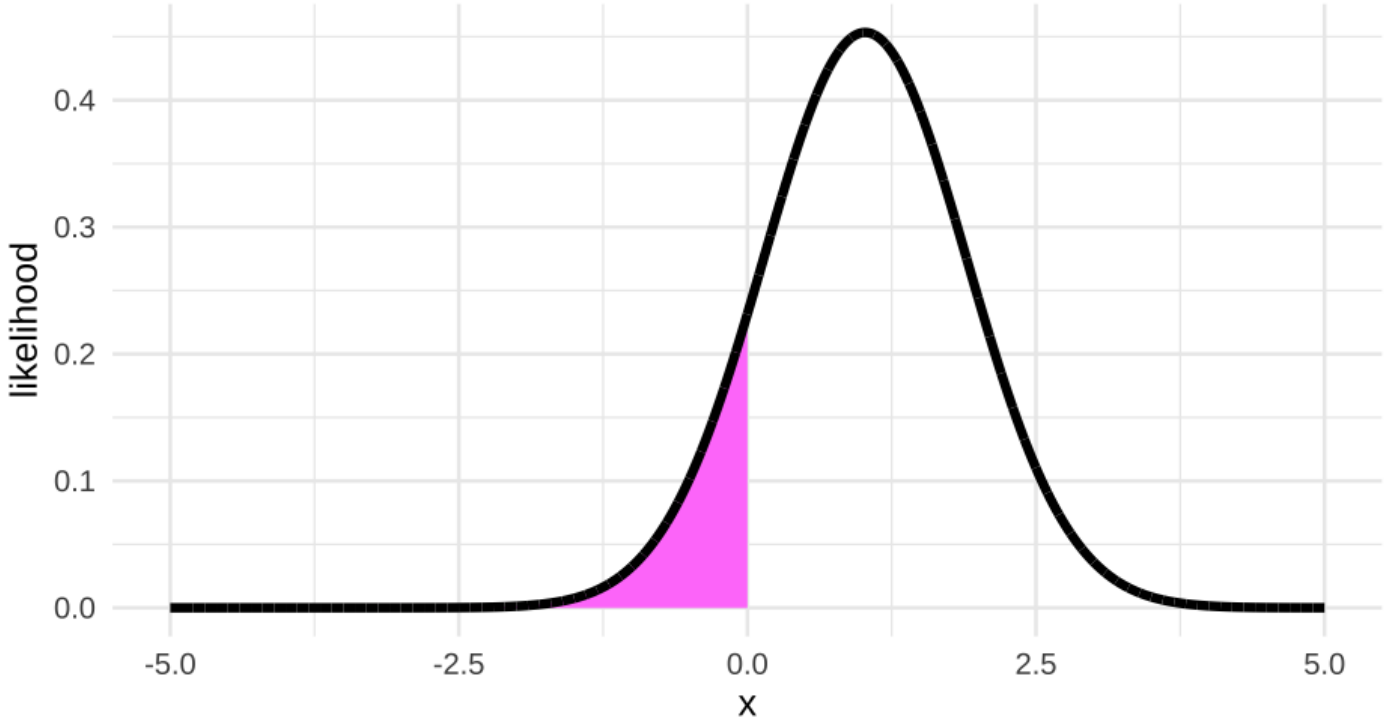
The hard way

Calculate the exact probability of data below zero under this distribution

```
x <- seq(-5, 5, 0.001)
likelihood <- dnorm(x, 1.02, 0.88)
sim <- data.frame(x, likelihood)
ggplot(sim, aes(x, likelihood)) +
  geom_line(size = 1.2)
```



How do we calculate this portion?



Integrate

```
zab <- filter(sim, x <= 0)
pracma::trapz(zab$x, zab$likelihood)
```

```
## [1] 0.123
```

Easier: Simulate

```
random_draws <- rnorm(1e5, 1.02, 0.9)
table(random_draws > 0) / 1e5
```

```
##
## FALSE  TRUE
##  0.13  0.87
```

Discretized plot

```
ppoints(50)
```

```
##  [1] 0.01 0.03 0.05 0.07 0.09 0.11 0.13 0.15 0.17 0.19 0.21 0.23 0.25 0.
## [15] 0.29 0.31 0.33 0.35 0.37 0.39 0.41 0.43 0.45 0.47 0.49 0.51 0.53 0.
## [29] 0.57 0.59 0.61 0.63 0.65 0.67 0.69 0.71 0.73 0.75 0.77 0.79 0.81 0.
## [43] 0.85 0.87 0.89 0.91 0.93 0.95 0.97 0.99
```

```
qnorm(ppoints(50), 1.02, 0.9)
```

```
##  [1] -1.07371 -0.67271 -0.46037 -0.30821 -0.18668 -0.08388  0.00625  0.0
##  [9]  0.16125  0.22989  0.29422  0.35504  0.41296  0.46847  0.52195  0.5
## [17]  0.62408  0.67321  0.72133  0.76861  0.81521  0.86126  0.90690  0.9
## [25]  0.99744  1.04256  1.08774  1.13310  1.17874  1.22479  1.27139  1.3
## [33]  1.36679  1.41592  1.46627  1.51805  1.57153  1.62704  1.68496  1.7
## [41]  1.81011  1.87875  1.95279  2.03375  2.12388  2.22668  2.34821  2.5
## [49]  2.71271  3.11371
```

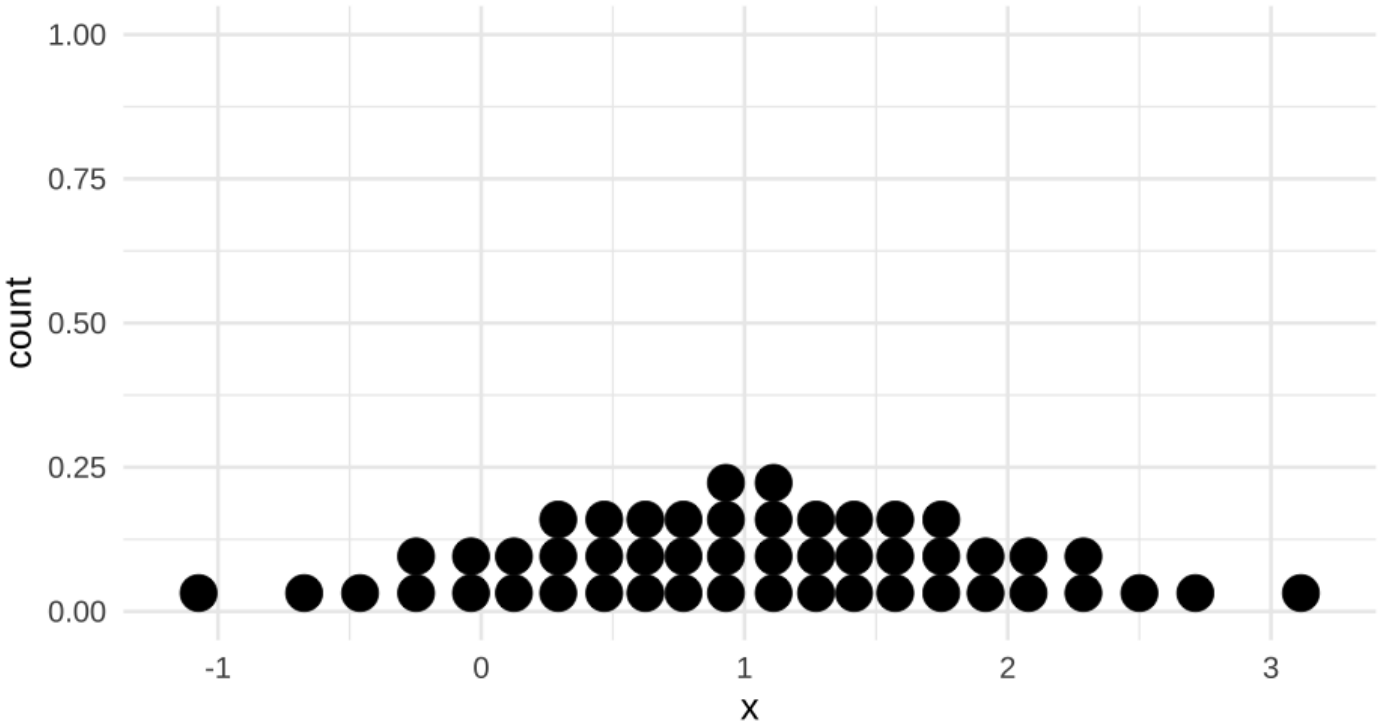
```r
discretized <- data.frame(
  x = qnorm(ppoints(50), 1.02, 0.9)
  ) %>%
  mutate(winner = ifelse(x <= 0, "#b1daf4", "#f8f1a9"))

head(discretized)
```

```
##         x  winner
## 1 -1.0737 #b1daf4
## 2 -0.6727 #b1daf4
## 3 -0.4604 #b1daf4
## 4 -0.3082 #b1daf4
## 5 -0.1867 #b1daf4
## 6 -0.0839 #b1daf4
```

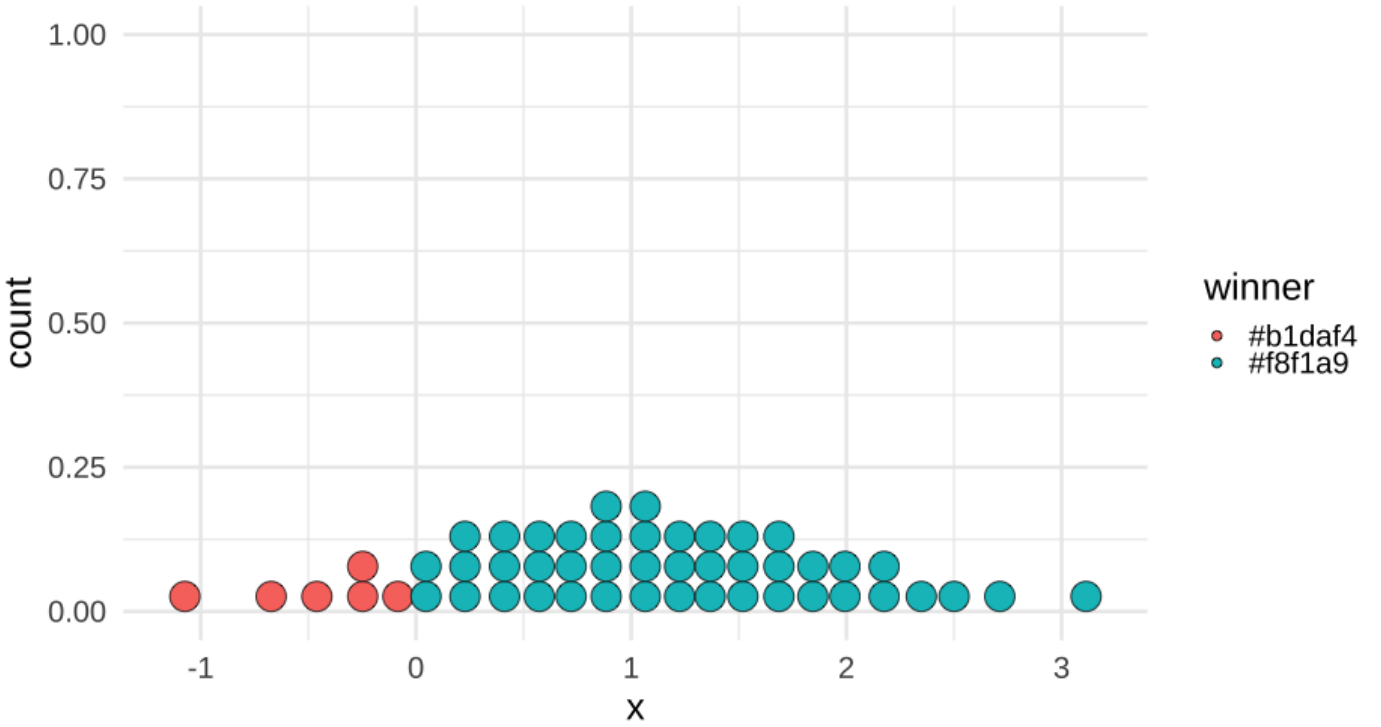
```r
tail(discretized)
```

```
##       x  winner
## 45 2.12 #f8f1a9
## 46 2.23 #f8f1a9
## 47 2.35 #f8f1a9
## 48 2.50 #f8f1a9
## 49 2.71 #f8f1a9
## 50 3.11 #f8f1a9
```

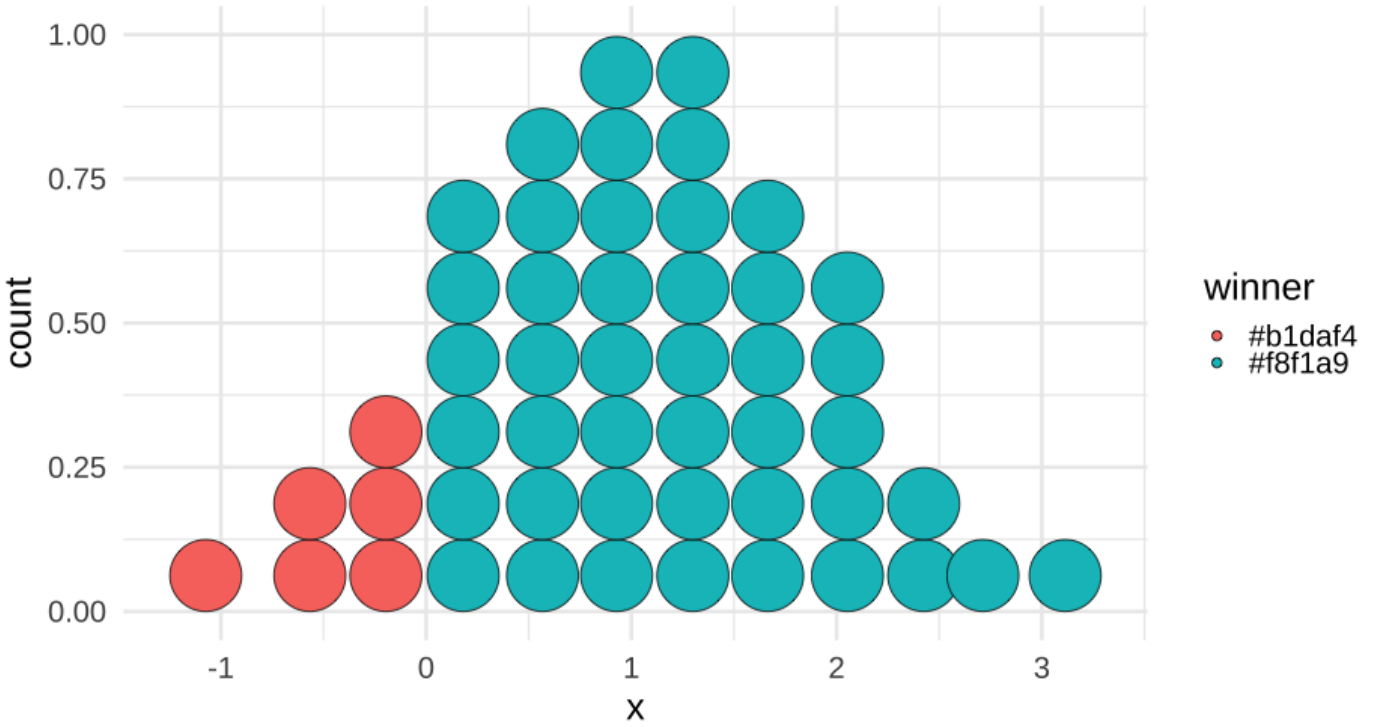
```
ggplot(discretized, aes(x)) +
  geom_dotplot()
```



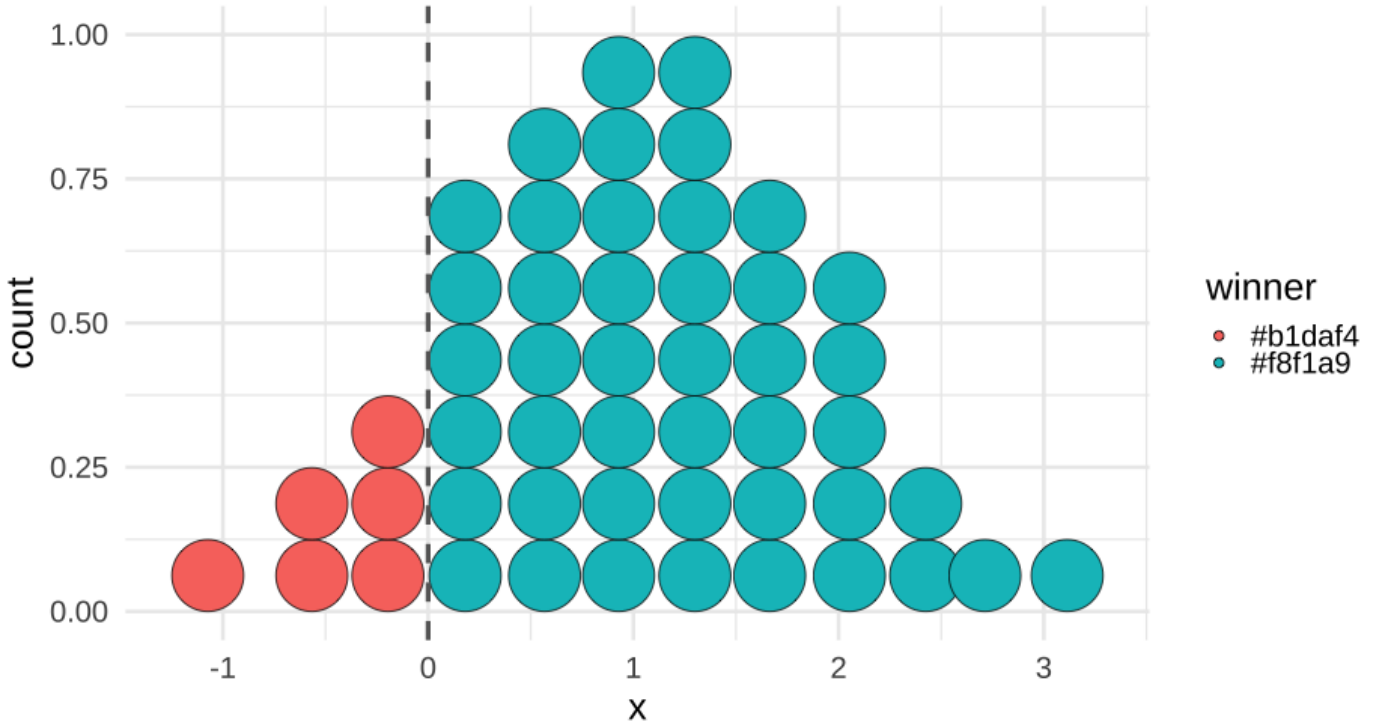
```
ggplot(discretized, aes(x)) +
  geom_dotplot(aes(fill = winner))
```



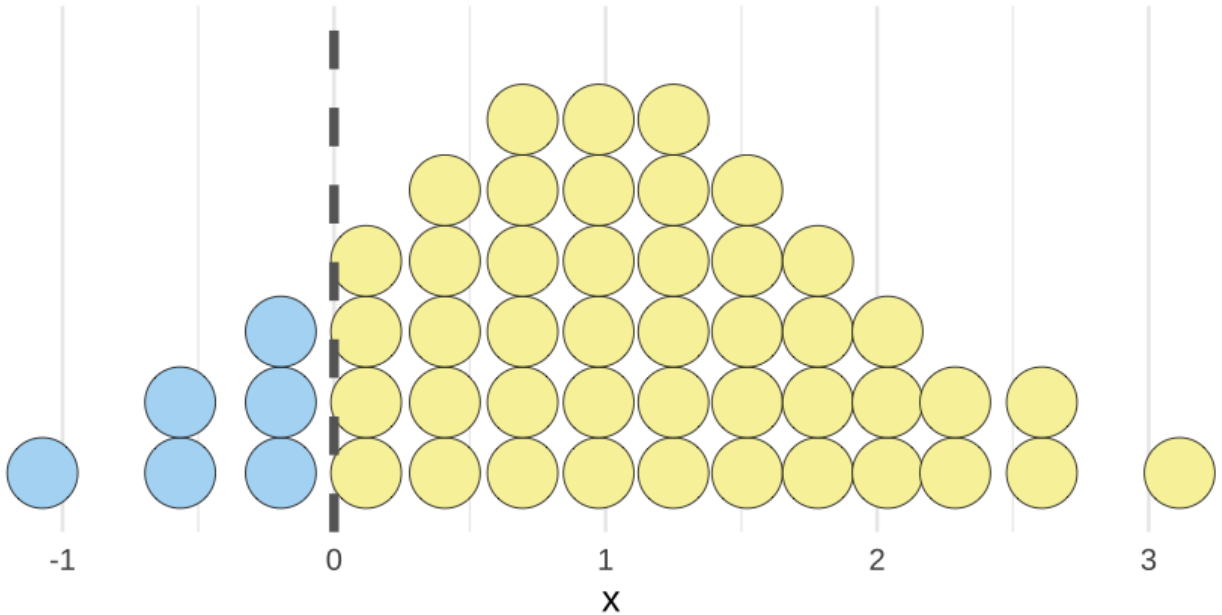
```
ggplot(discretized, aes(x)) +
  geom_dotplot(aes(fill = winner), binwidth = 0.35)
```



```
ggplot(discretized, aes(x)) +
  geom_dotplot(aes(fill = winner), binwidth = 0.35) +
  geom_vline(xintercept = 0,
             color = "gray40",
             linetype = "dashed",
             size = 1.5)
```

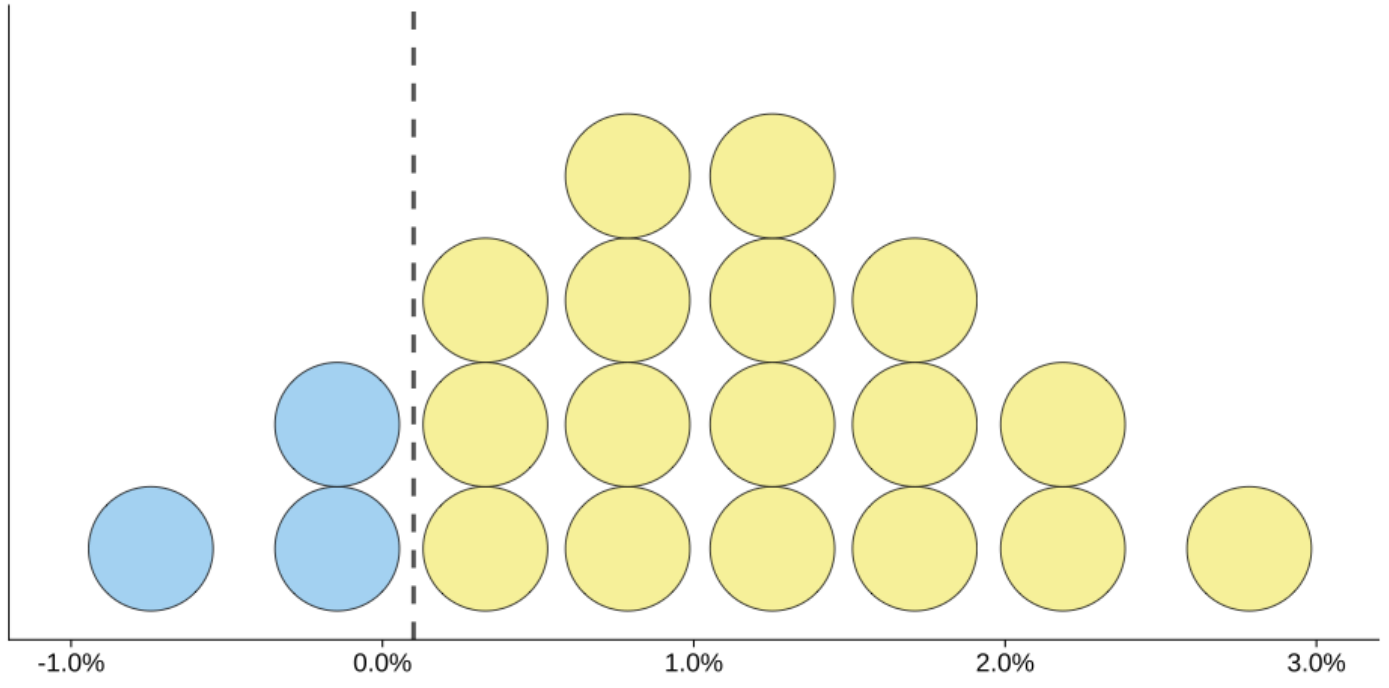


```
ggplot(discretized, aes(x)) +
  geom_dotplot(aes(fill = winner), binwidth = 0.26) +
  geom_vline(xintercept = 0,
             color = "gray40",
             linetype = 2,
             size = 3) +
  scale_fill_identity(guide = "none") +
  scale_y_continuous(name = "",
                     breaks = NULL)
```



Probs too many though

```
discretized2 <- data.frame(
  x = qnorm(ppoints(20), 1.02, 0.9)
  ) %>%
  mutate(winner = ifelse(x <= 0, "#b1daf4", "#f8f1a9"))

ggplot(discretized2, aes(x)) +
  geom_dotplot(aes(fill = winner), binwidth = 0.4) +
  geom_vline(
    xintercept = 0.1,
    color = "gray40",
    linetype = 2,
    size = 1.4) +
  scale_fill_identity(guide = "none") +
  scale_x_continuous(
    name = "",
    limits = c(-1, 3),
    labels = scales::percent_format(scale = 1)
  ) +
  theme_dviz_open(20, font_family = "Roboto Light") +
  scale_y_continuous(breaks = NULL,
                     name = "") +
  labs(caption = "Each ball represents 5% probability")
```


-1.0%
0.0%
1.0%
2.0%
3.0%
Each ball represents 5% probability

Uncertainty of point estimates

Quick review (hopefully a review)

- What is a standard error?
- Standard deviation of the sampling distribution
- What is the sampling distribution?
- Samples from the underlying, population–based, generative distribution
- What does this mean, exactly?
- Let's simulate to explore

Simulation

- Imagine the "real" distribution has $\mu = 100$ and $\sigma = 10$.
- Let's draw a sample of 10 from this distribution

```
set.seed(123)
samp10a <- rnorm(n = 10, mean = 100, sd = 10)
samp10a
```

```
##  [1]  94.4  97.7 115.6 100.7 101.3 117.2 104.6  87.3  93.1  95.5
```

- Calculate the mean

```
mean(samp10a)
```

```
## [1] 101
```

Do it a second time

```
samp10b <- rnorm(n = 10, mean = 100, sd = 10)
samp10b
```

```
##  [1] 112.2 103.6 104.0 101.1  94.4 117.9 105.0  80.3 107.0  95.3
```

```
mean(samp10b)
```

```
## [1] 102
```

Do it a bunch of times

```
samples <- replicate(1000, rnorm(10, mean = 100, sd = 10),
                     simplify = FALSE)
samples
```

```
## [[1]]
##  [1]  89.3  97.8  89.7  92.7  93.7  83.1 108.4 101.5  88.6 112.5
##
## [[2]]
##  [1] 104.3  97.0 109.0 108.8 108.2 106.9 105.5  99.4  96.9  96.2
##
## [[3]]
##  [1]  93.1  97.9  87.3 121.7 112.1  88.8  96.0  95.3 107.8  99.2
##
## [[4]]
##  [1] 102.5  99.7  99.6 113.7  97.7 115.2  84.5 105.8 101.2 102.2
##
## [[5]]
##  [1] 103.8  95.0  96.7  89.8  89.3 103.0 104.5 100.5 109.2 120.5
##
## [[6]]
##  [1]  95.1  76.9 110.1  92.9  93.1 110.3  97.2  87.8 101.8  98.6
##
## [[7]]
##  [1] 100.1 103.9  96.3 106.4  97.8 103.3 111.0 104.4  96.7 111.5
##
## [[8]]
```

Calculate all means

```
map_dbl(samples, mean) %>%
  head()
```

```
## [1]  95.8 103.2  99.9 102.2 101.2  96.4
```

- What's the $\boldsymbol{sd}$ of these means? That's the standard error.

```
map_dbl(samples, mean) %>%
  sd()
```

```
## [1] 3.14
```

Sample size

Let's re–do this, pulling a sample of 100 each time.

```
samples2 <- replicate(1000, rnorm(100, mean = 100, sd = 10),
                      simplify = FALSE)
map_dbl(samples2, mean) %>%
  sd()
```

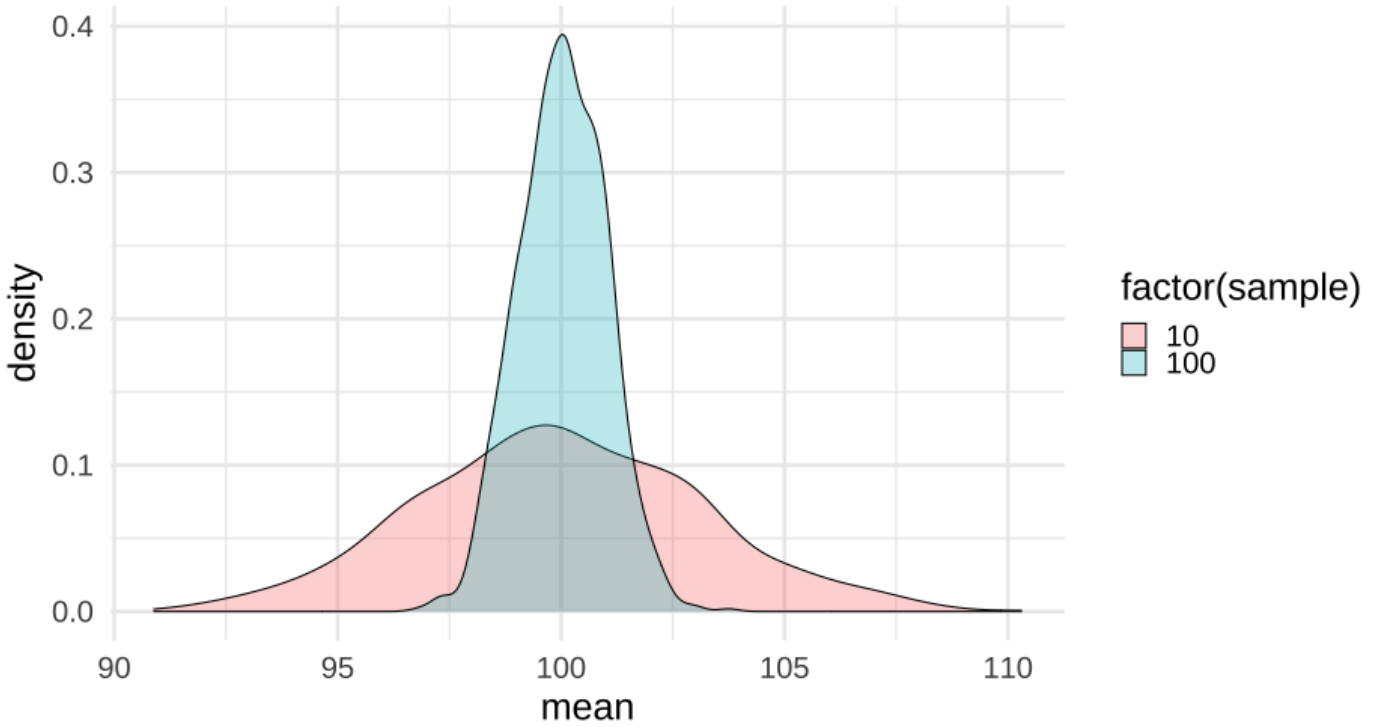
```
## [1] 0.973
```

Visualize the sampling distributions

```
sample_means <- tibble(
  iter = rep(1:1000, 2),
  sample = rep(c(10, 100), each = 1000),
  mean = c(map_dbl(samples, mean),
           map_dbl(samples2, mean))
)
sample_means
```

```
## # A tibble: 2,000 x 3
##    iter sample      mean
##   <int>  <dbl>     <dbl>
## 1     1     10  95.75441
## 2     2     10 103.2204
## 3     3     10  99.91284
## 4     4     10 102.2169
## 5     5     10 101.2308
## 6     6     10  96.37082
## # … with 1,994 more rows
```

```
ggplot(sample_means, aes(mean)) +
  geom_density(aes(fill = factor(sample)), alpha = 0.3)
```



Fit a model

```
m <- lm(cty ~ displ + class, mpg)
summary(m)
```

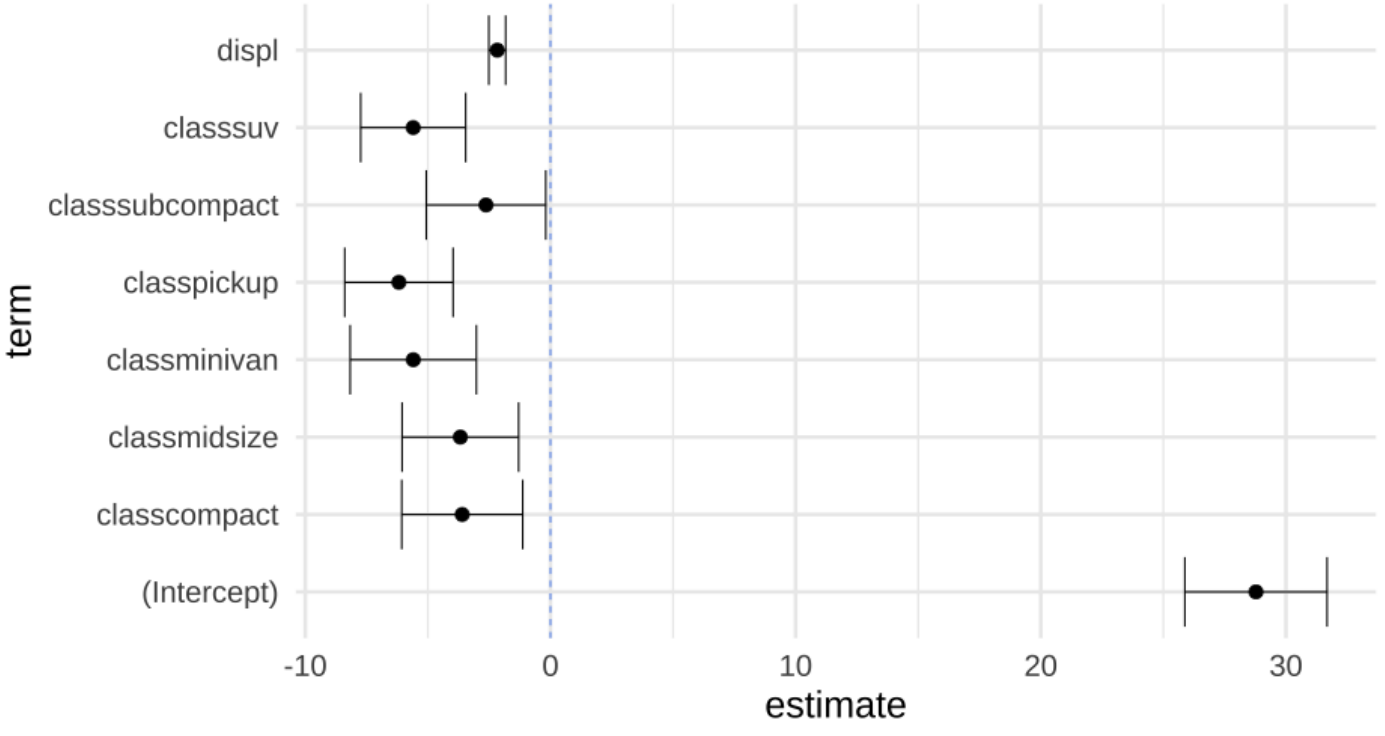
```
##
## Call:
## lm(formula = cty ~ displ + class, data = mpg)
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -5.269 -1.150 -0.016  1.034 12.978
##
## Coefficients:
##                 Estimate Std. Error t value Pr(>|t|)
## (Intercept)       28.777      1.473   19.54  < 2e-16 ***
## displ             -2.172      0.175  -12.43  < 2e-16 ***
## classcompact      -3.599      1.252   -2.87   0.0044 **
## classmidsize      -3.676      1.206   -3.05   0.0026 **
## classminivan      -5.595      1.306   -4.28  2.7e-05 ***
## classpickup       -6.182      1.121   -5.51  9.6e-08 ***
## classsubcompact   -2.629      1.237   -2.13   0.0346 *
## classsuv          -5.599      1.087   -5.15  5.7e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.25 on 226 degrees of freedom
## Multiple R-squared:  0.729,    Adjusted R-squared:  0.721
```

Visualize with standard errors

```
tidied_m <- broom::tidy(m, conf.int = TRUE)

tidied_m
```

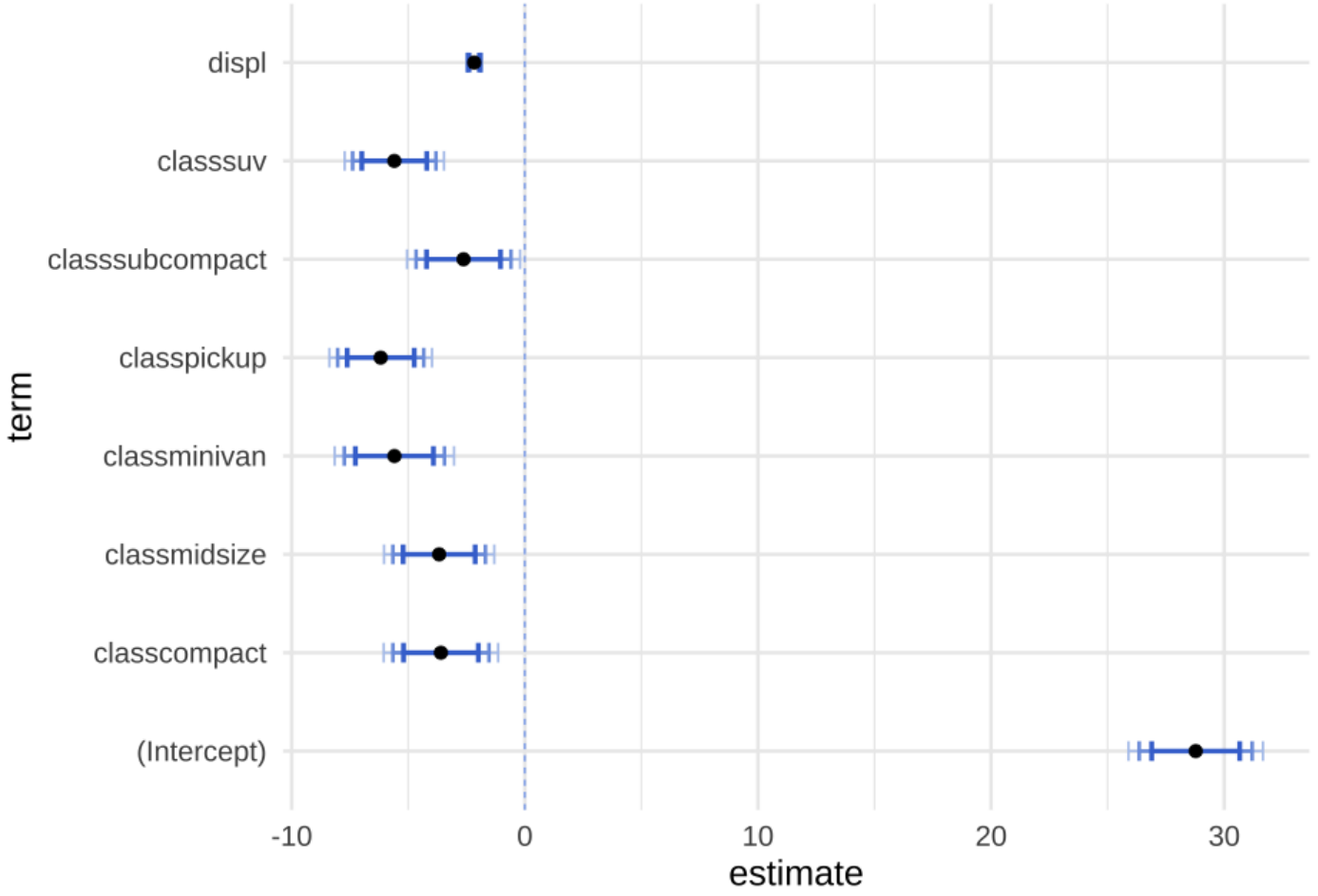
```
## # A tibble: 8 x 7
##   term          estimate std.error  statistic      p.value  conf.low
##   <chr>            <dbl>     <dbl>      <dbl>        <dbl>     <dbl>
## 1 (Intercept)  28.77682  1.472892   19.53763  1.905873e-50 25.87446
## 2 displ        -2.171562 0.1746638 -12.43281  2.197130e-27 -2.515740
## 3 classcompact -3.599125 1.252190   -2.874265 4.436052e- 3 -6.066585
## 4 classmidsize -3.675526 1.206253   -3.047061 2.585762e- 3 -6.052466
## 5 classminivan -5.595070 1.305993   -4.284151 2.714490e- 5 -8.168550
## 6 classpickup  -6.182466 1.121448   -5.512931 9.600087e- 8 -8.392297
## # … with 2 more rows, and 1 more variable: conf.high <dbl>
```

```
ggplot(tidied_m, aes(term, estimate)) +
  geom_hline(yintercept = 0,
             color = "cornflowerblue",
             linetype = 2) +
  geom_errorbar(aes(ymin = conf.low, ymax = conf.high)) +
  geom_point() +
  coord_flip()
```



Multiple error bars

```
library(colorspace)
ggplot(tidied_m, aes(term, estimate)) +
  geom_hline(yintercept = 0,
             color = "cornflowerblue",
             linetype = 2) +
  geom_errorbar(aes(ymin = estimate + qnorm(.025)*std.error,
                    ymax = estimate + qnorm(.975)*std.error),
                color = lighten("#4375D3", .6),
                width = 0.2,
                size = 0.8) + # 95% CI
  geom_errorbar(aes(ymin = estimate + qnorm(.05)*std.error,
                    ymax = estimate + qnorm(.95)*std.error),
                color = lighten("#4375D3", .3),
                width = 0.2,
                size = 1.2) + # 90% CI
  geom_errorbar(aes(ymin = estimate + qnorm(.1)*std.error,
                    ymax = estimate + qnorm(.9)*std.error),
                color = "#4375D3",
                width = 0.2,
                size = 1.6) + # 80% CI
  geom_point() +
  coord_flip()
```

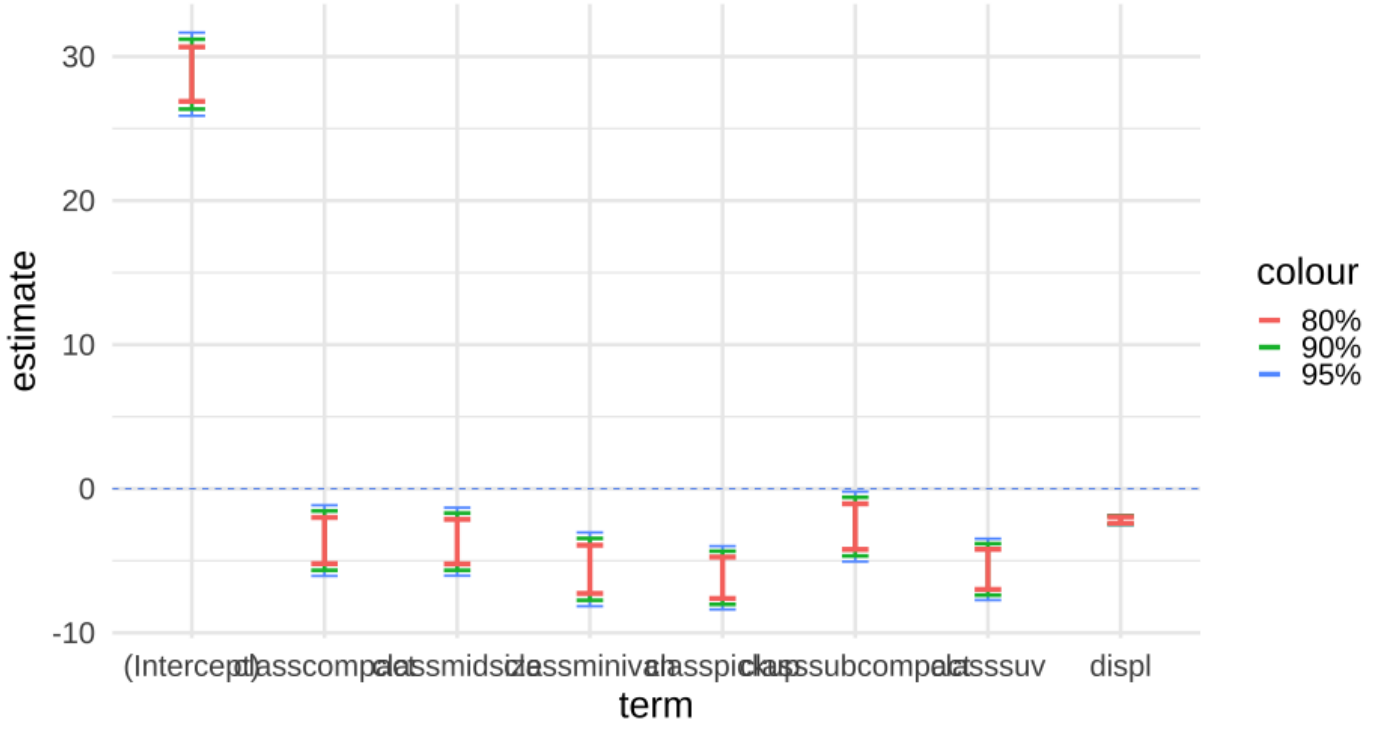

displ
classsuv
classsubcompact
classpickup
classminivan
classmidsize
classcompact
(Intercept)
term
-10
0
10
20
30
estimate

Add levels to legend

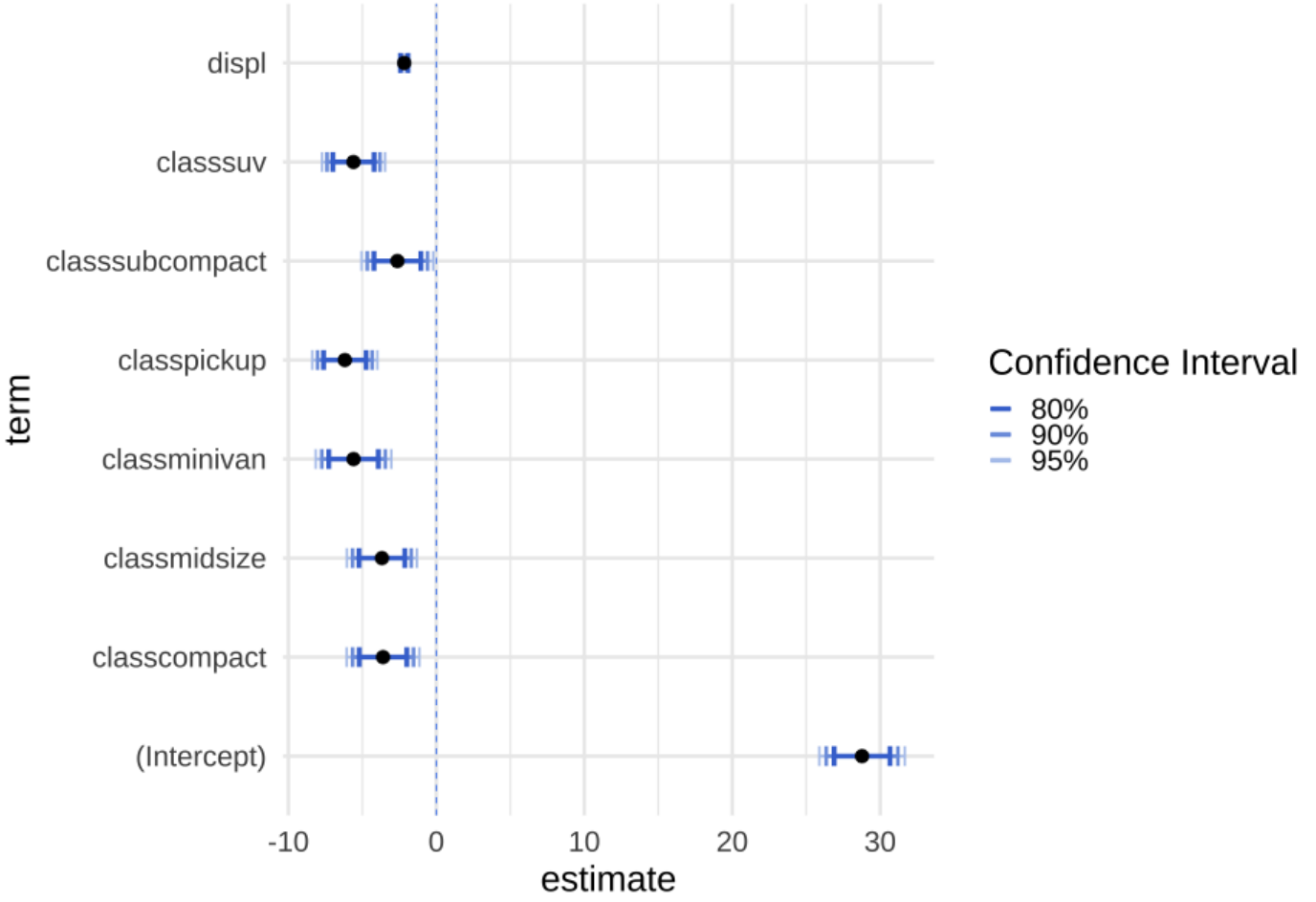
Include the color spec in `aes()`

```
p <- ggplot(tidied_m, aes(term, estimate)) +
  geom_hline(yintercept = 0,
             color = "cornflowerblue",
             linetype = 2) +
  geom_errorbar(aes(ymin = estimate + qnorm(.025)*std.error,
                    ymax = estimate + qnorm(.975)*std.error,
                    color = "95%"),
                width = 0.2,
                size = 0.8) +
  geom_errorbar(aes(ymin = estimate + qnorm(.05)*std.error,
                    ymax = estimate + qnorm(.95)*std.error,
                    color = "90%"),
                width = 0.2,
                size = 1.2) +
  geom_errorbar(aes(ymin = estimate + qnorm(.1)*std.error,
                    ymax = estimate + qnorm(.9)*std.error,
                    color = "80%"),
                width = 0.2,
                size = 1.6)
```

```
p
```

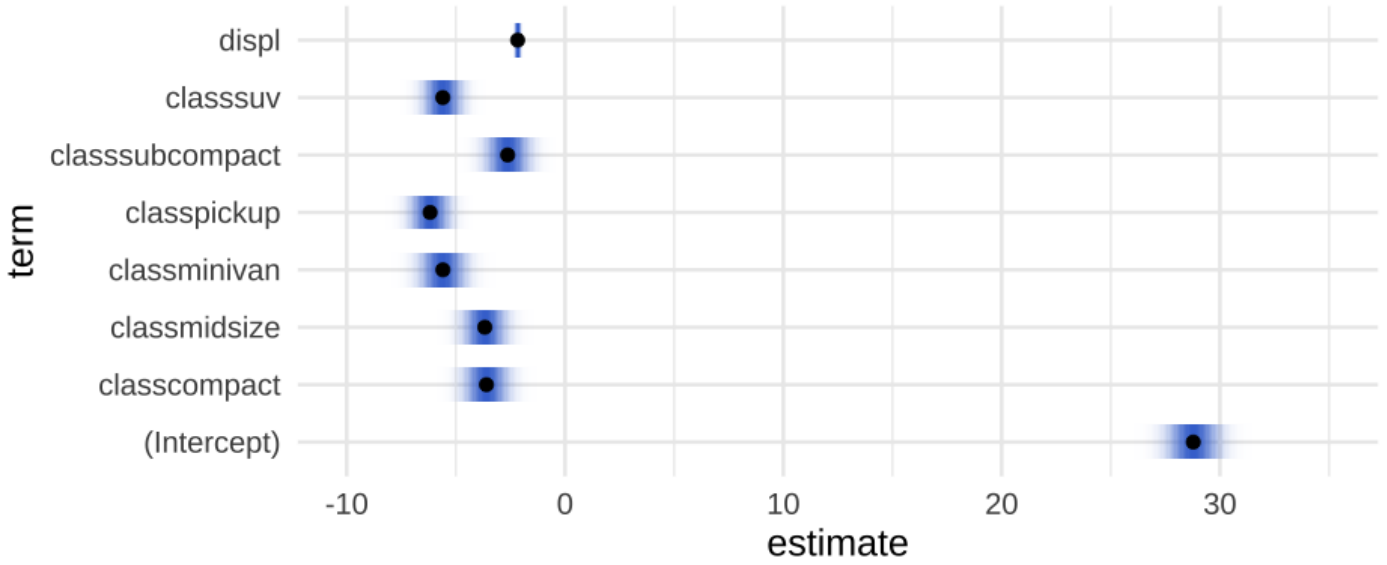


```
p +
  scale_color_manual("Confidence Interval",
                     values = c("#4375D3",
                                lighten("#4375D3", .3),
                                lighten("#4375D3", .6))) +
  geom_point() +
  coord_flip()
```


displ
classsuv
classsubcompact
classpickup
classminivan
classmidsize
classcompact
(Intercept)
term
-10
0
10
20
30
estimate
Confidence Interval
80%
90%
95%

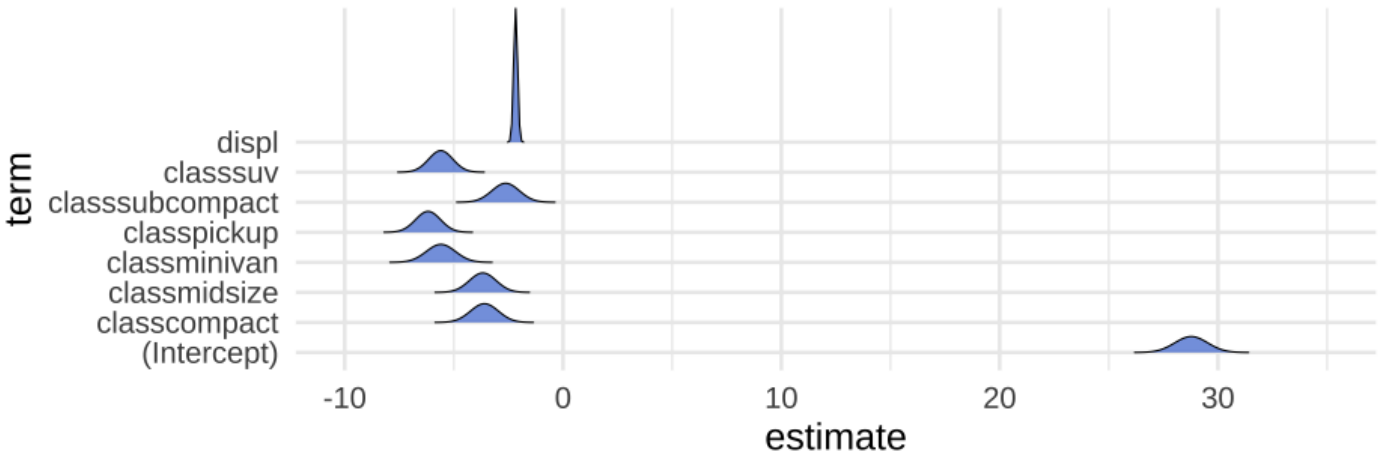
Density stripes

```
#devtools::install_github("wilkelab/ungeviz")
library(ungeviz)
ggplot(tidied_m, aes(estimate, term)) +
  stat_confidence_density(aes(moe = std.error),
                          fill = "#4375D3",
                          height = 0.6) +
  xlim(-10, 35) +
  geom_point()
```



Actual densities

```
library(ggridges)
ggplot(tidied_m, aes(estimate, term)) +
  stat_confidence_density(aes(moe = std.error,
                              height = stat(density)),
                          geom = "ridgeline",
                          confidence = 0.95,
                          min_height = 0.001,
                          alpha = 0.7,
                          fill = "#4375D3") +
  xlim(-10, 35)
```

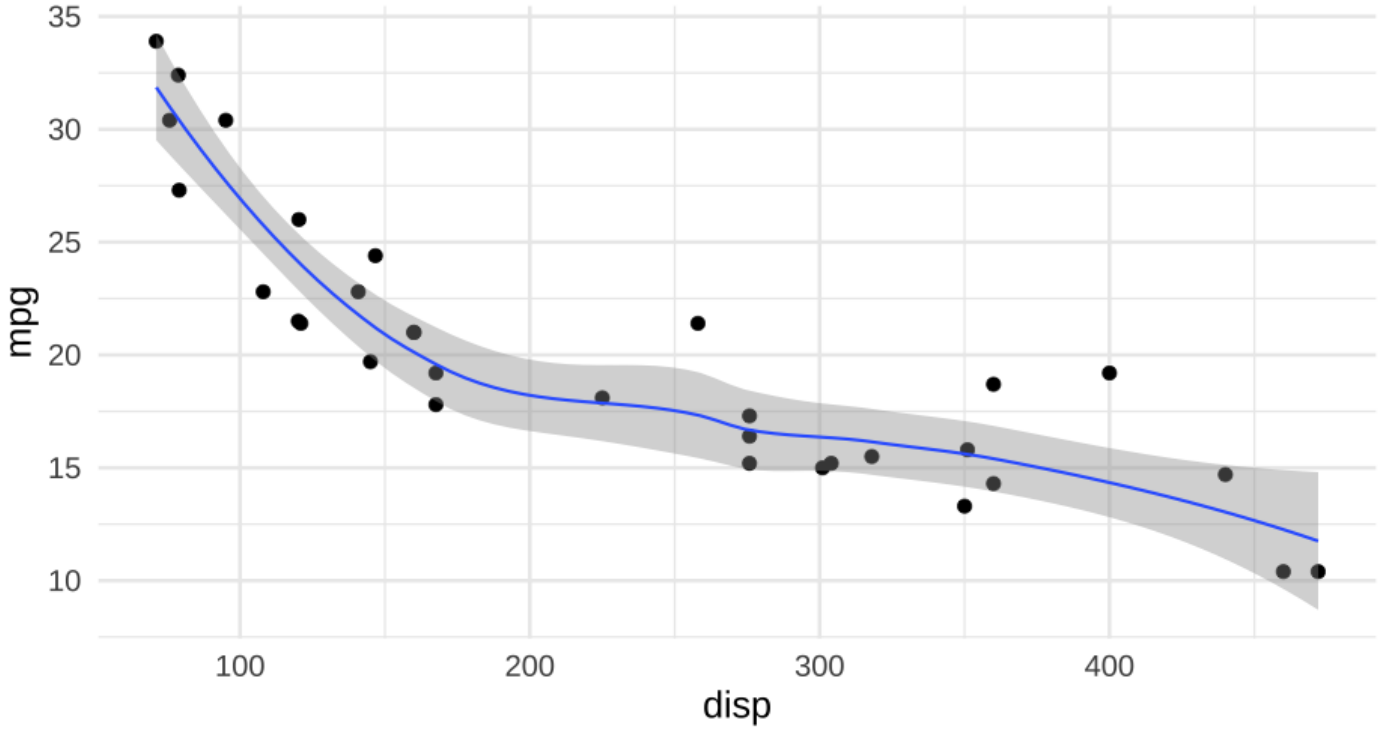


HOPs

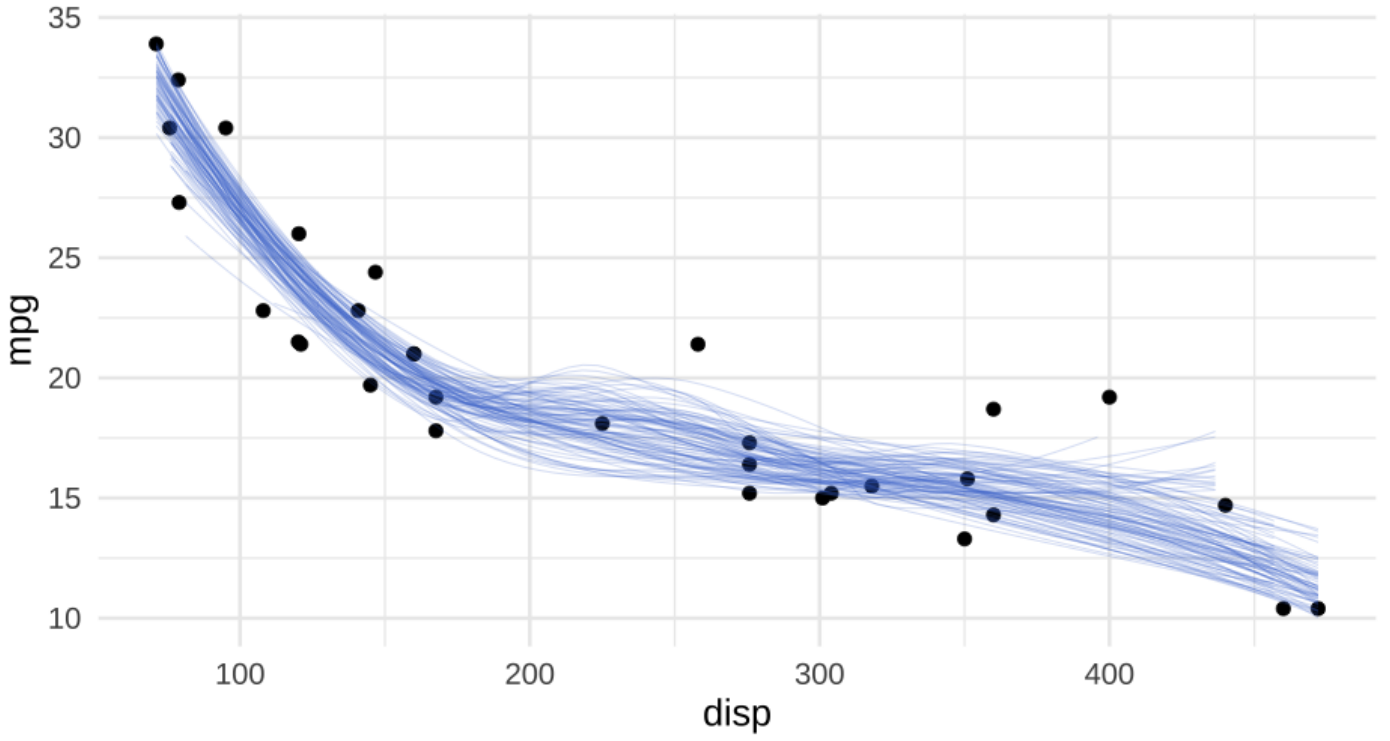
Hypothetical Outcome Plots (and related plots)

Standard regression plot

```
ggplot(mtcars, aes(disp, mpg)) +
  geom_point() +
  geom_smooth()
```



Alternative



How?

Bootstrapping

```
row_samps <- replicate(100,
                       sample(seq_len(nrow(mtcars)),
                              nrow(mtcars),
                              replace = TRUE),
                       simplify = FALSE)
row_samps
```

```
## [[1]]
##  [1] 31  6 32 12  1 14  2 11 20 10 26 10 22 30 25 25  5 31 19 13 19 20 1
## [25]  4 12 12 25 20 21 16 23
##
## [[2]]
##  [1] 11 23 14  1 24 20 10 30 27 24 22 23 25  1 18 18 25  8  8 16 25 19 3
## [25] 11 10 21  6 14 14 12 24
##
## [[3]]
##  [1] 27 29 22  5  6  8 14 16  7 13 17 13 21 10  7 21  7 20 30 30  5 10
## [25]  4 15 16 21 27 23 19  7
##
## [[4]]
##  [1] 16  7  8 28  3 17 13 26  8 30  3 32 20 10  2  6 19 21 11  6 16  9 1
## [25] 25  4 27 29 19 21  1 16
```

Extract samples

```
d_samps <- map_df(row_samps, ~mtcars[.x, ], .id = "sample")
head(d_samps)
```

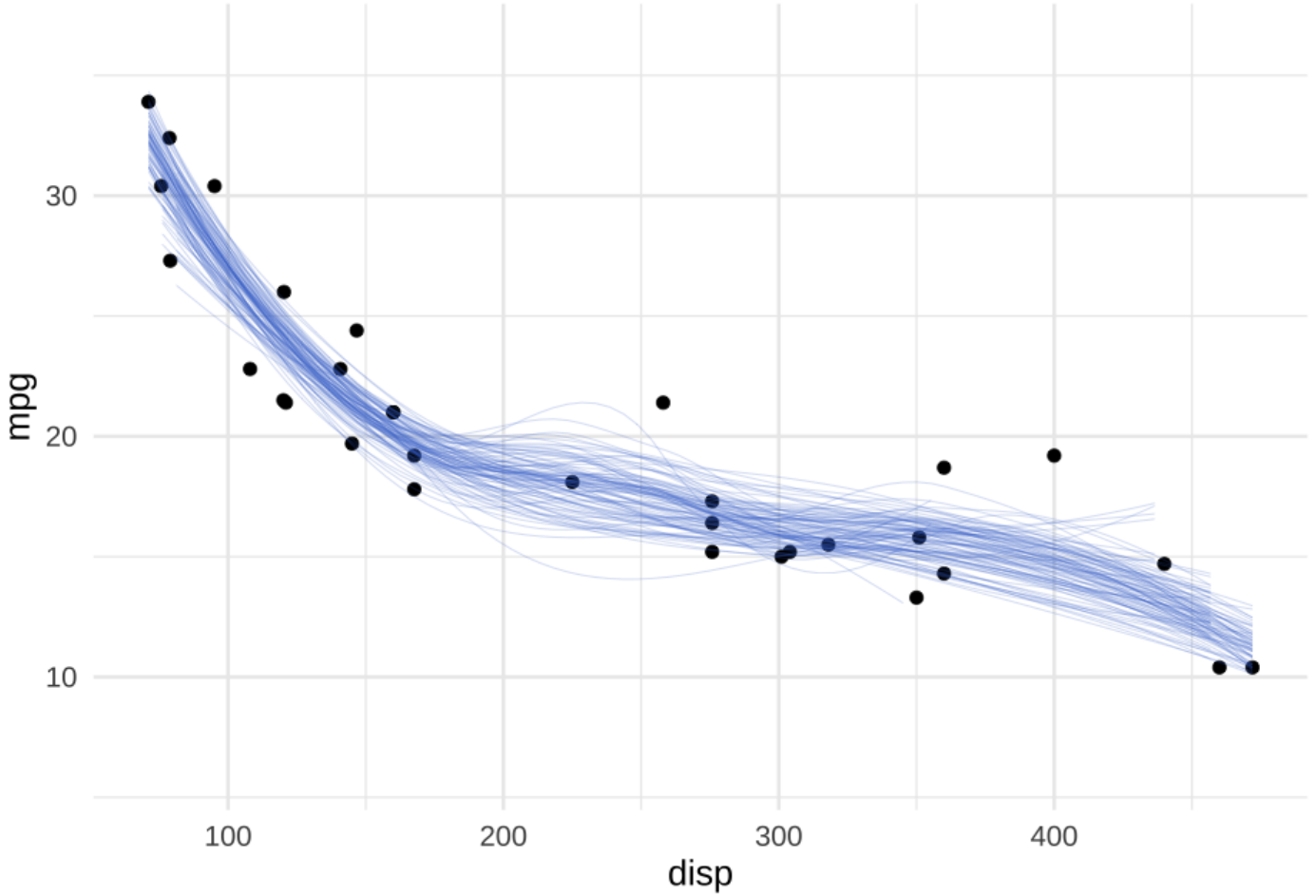
```
##                   sample  mpg cyl disp  hp drat   wt qsec vs am gear car
## Maserati Bora...1      1 15.0   8  301 335 3.54 3.57 14.6  0  1    5
## Valiant...2            1 18.1   6  225 105 2.76 3.46 20.2  1  0    3
## Volvo 142E...3         1 21.4   4  121 109 4.11 2.78 18.6  1  1    4
## Merc 450SE...4         1 16.4   8  276 180 3.07 4.07 17.4  0  0    3
## Mazda RX4...5          1 21.0   6  160 110 3.90 2.62 16.5  0  1    4
## Merc 450SLC...6        1 15.2   8  276 180 3.07 3.78 18.0  0  0    3
```

```
tail(d_samps)
```

```
##                             sample  mpg cyl  disp  hp drat   wt qsec vs
## Lincoln Continental.1...3195    100 10.4   8 460.0 215 3.00 5.42 17.8  0
## Pontiac Firebird...3196         100 19.2   8 400.0 175 3.08 3.85 17.1  0
## Fiat X1-9.1...3197              100 27.3   4  79.0  66 4.08 1.94 18.9  1
## Mazda RX4 Wag.3...3198          100 21.0   6 160.0 110 3.90 2.88 17.0  0
## Hornet Sportabout...3199        100 18.7   8 360.0 175 3.15 3.44 17.0  0
## Lotus Europa...3200             100 30.4   4  95.1 113 3.77 1.51 16.9  1
##                             gear carb
## Lincoln Continental.1...3195    3    4
## Pontiac Firebird...3196         3    2
## Fiat X1-9.1...3197              4    1
## Mazda RX4 Wag.3...3198          4    4
```

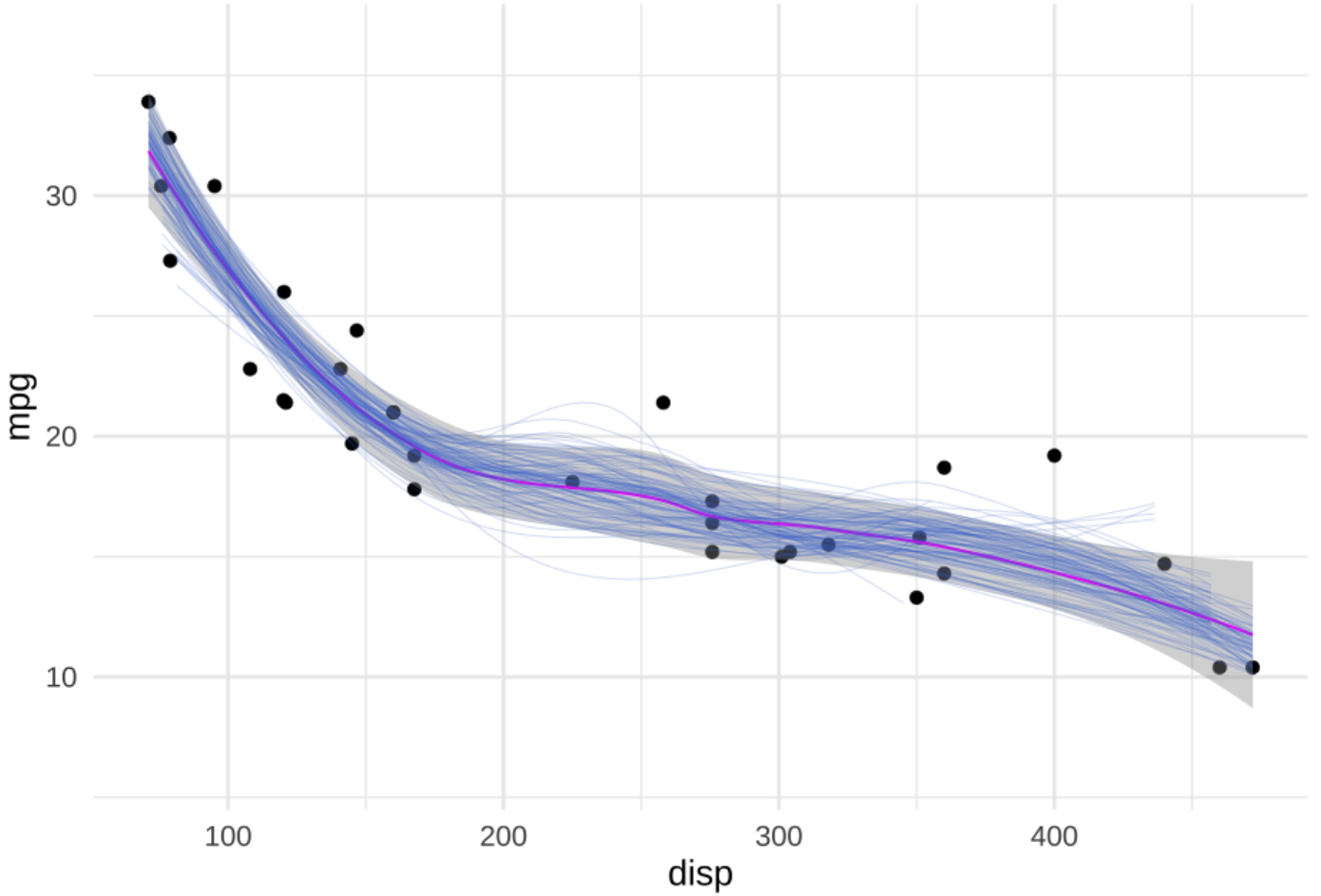
Plot both data sources

```
ggplot(mtcars, aes(disp, mpg)) +
  geom_point() +
  stat_smooth(aes(group = sample),
              data = d_samps,
              geom = "line",
              color = "#4375D3",
              fullrange = TRUE,
              size = 0.1)
```


mpg
30
20
10
100
200
300
400
disp

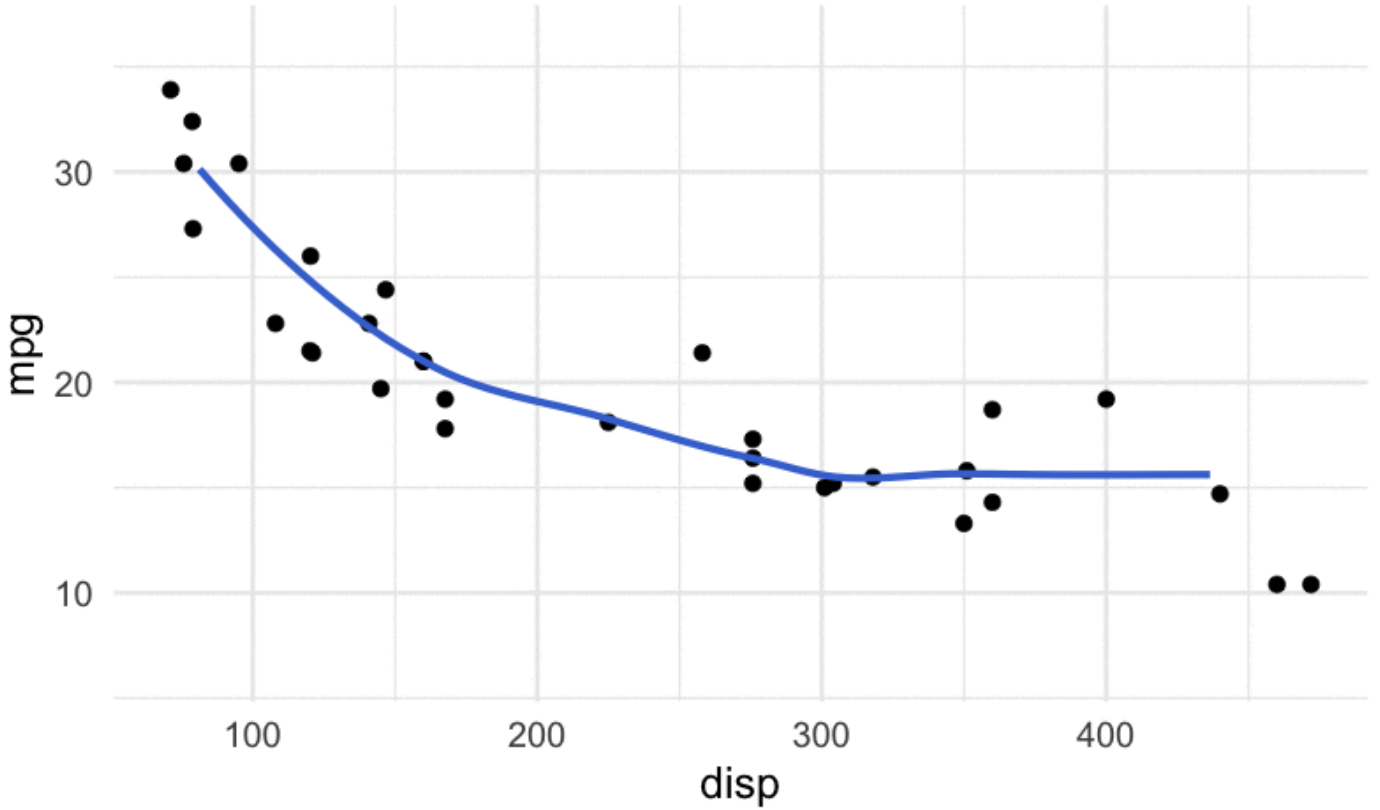
Note, they match up

```
ggplot(mtcars, aes(disp, mpg)) +
  geom_point() +
  geom_smooth(color = "magenta") +
  stat_smooth(aes(group = sample),
              data = d_samps,
              geom = "line",
              color = "#4375D3",
              fullrange = TRUE,
              size = 0.1)
```


30
20
10
mpg
100
200
300
400
disp

HOPs

Hops animate the process, so you can't settle on one "truth"



How?

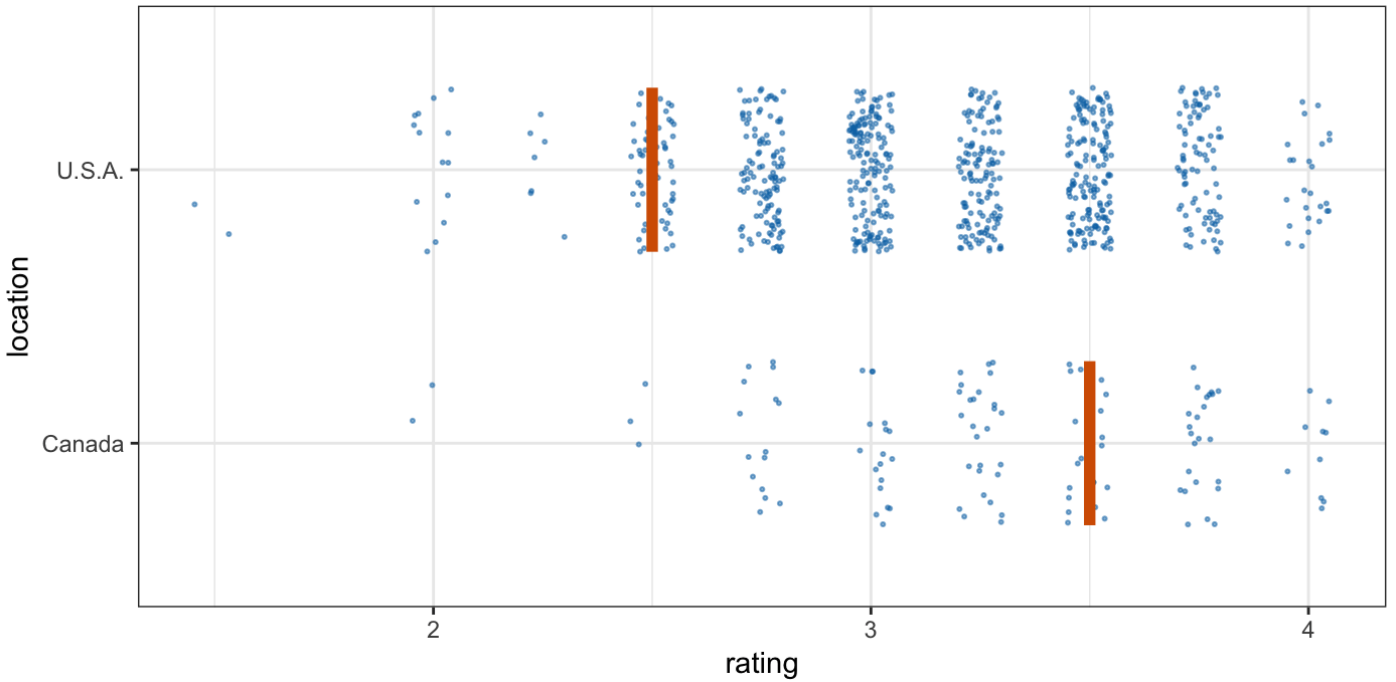
gganimate::transition_states

```
library(gganimate)
ggplot(mtcars, aes(disp, mpg)) +
  geom_point() +
  stat_smooth(data = d_samps,
              geom = "line",
              size = 2,
              color = "#4375D3",
              fullrange = TRUE) +
  transition_states(sample,
                    transition_length = 0.5,
                    state_length = 0.5) +
   ease_aes('linear') # Smoother transitions
```

Another example

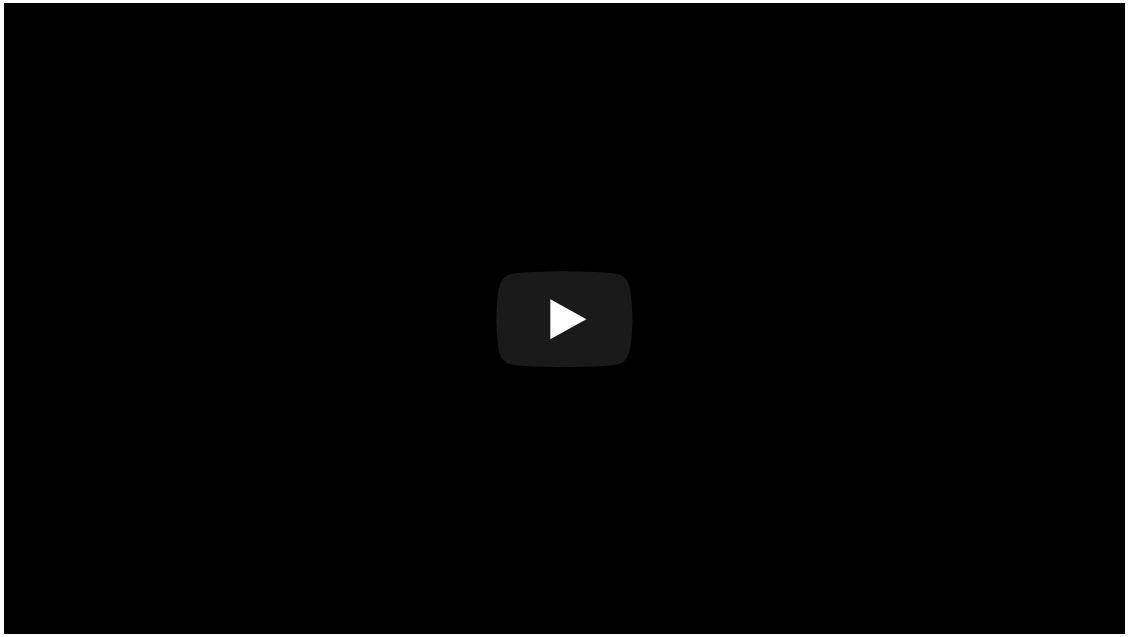
Canada has the highest average rating for their chocolate bars in the world, according to a recent study. But the sample size is also smaller than other countries, like the US.

How certain are we that Canada's chocolate is better? Compare the ratings of a randomly chosen Canadian chocolate bar to a randomly chosen US Chocolate bar.


location
U.S.A.
Canada
2
3
4
rating

Another example

From Dr. Kay again



Conclusions

- Lots of tools at your disposal (perhaps so many it can be difficult to choose)
- Do try to communicate uncertainty whenever possible
- I'd recommend checking out Clause Wilke's talk from `rstudio::conf(2019L)`, where he talks about the ungeviz package.

Next time

Tables and Fonts