**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

**Solution:**

Let X be the time required for servicing the transmissions, which is normally distributed with mean (μ) = 45 minutes and standard deviation (σ) = 8 minutes

We want to find P(X > 60), which means the probability that the service time is greater than 60 minutes.

Using the Z-score formula, we can standardize the value 60:

Z = (X - μ) / σ

= (60 - 45) / 8

= 15 / 8

= 1.875

Now, we need to find the probability corresponding to Z = 1.875 in the standard normal distribution table or using a calculator.

Using a standard normal distribution table or calculator, the probability for Z = 1.875 is approximately 0.9693.

Now, we know that P(X > 60) = 0.9693.

However, we need to consider that the work begins 10 minutes after the car is dropped off, so we need to find the probability that the service time (X) exceeds 50 minutes (60 - 10 = 50).

Now, we need to find P(X > 50):

Z = (X - μ) / σ

= (50 - 45) / 8

= 5 / 8

= 0.625

Using a standard normal distribution table or calculator, the probability for Z = 0.625 is approximately 0.2676.

Therefore, the probability that the service manager cannot meet his commitment is approximately 0.2676.

The correct answer is option B. 0.2676.

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

**Solution:** The Statement given above is false and explained in the below solution

To determine the number of employees that are older than 44 and between 38 and 44, we need to calculate the corresponding probabilities using the normal distribution.

Let X be the age of a clerical employee, which is normally distributed with mean (μ) = 38 and standard deviation (σ) = 6.

We want to find P(X > 44), which is the probability that an employee is older than 44.

Z = (X - μ) / σ Z = (44 - 38) / 6 Z = 1

Using a standard normal distribution table or calculator, the probability for Z = 1 is approximately 0.8413.

Next, we want to find P(38 < X < 44), which is the probability that an employee's age is between 38 and 44.

Z1 = (38 - 38) / 6 Z1 = 0

Z2 = (44 - 38) / 6 Z2 = 1

Using the standard normal distribution table or calculator, the probability for Z1 = 0 and Z2 = 1 is approximately 0.3413.

So, the proportion of employees older than 44 is 0.8413, and the proportion between 38 and 44 is 0.3413. Since 0.8413 is greater than 0.3413, there are more employees older than 44 than between 38 and 44.

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**Solution :**

To find the number of employees under the age of 30 expected to be attracted to the training program, we need to calculate the probability of an employee being under 30 using the normal distribution.

We want to find P(X < 30), which is the probability that an employee is younger than 30.

Z = (X - μ) / σ

= (30 - 38) / 6

= -1.3333

Using a standard normal distribution table or calculator, the probability for Z = -1.3333 is approximately 0.0912.

Now, to find the number of employees expected to be attracted to the training program, we will multiply the probability by the total number of employees:

Number of employees expected to be attracted = 0.0912 \* 400 ≈ 36.48

Since the number of employees cannot be in fractions, we would round it to the nearest whole number, which means about 36 employees are expected to be under the age of 30 at the centre.

Therefore, the statement B is true.

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Answer:

2X1: If X1 follows a normal distribution with mean μ and variance σ^2, then 2X1 would also follow a normal distribution. When we multiply a random variable by a constant (in this case, 2), it affects the mean and the variance.

The mean of 2X1 would be: E(2X1) = 2 \* E(X1) = 2 \* μ

The variance of 2X1 would be: Var(2X1) = (2^2) \* Var(X1) = 4 \* σ^2

So, the distribution of 2X1 is also normal, with mean 2μ and variance 4σ^2.

X1 + X2: If X1 and X2 are independent and identically distributed (iid) normal random variables, then their sum (X1 + X2) would also follow a normal distribution. When we add two independent random variables, the mean of the resulting distribution would be the sum of the individual means, and the variance of the resulting distribution would be the sum of the individual variances.

The mean of X1 + X2 would be: E(X1 + X2) = E(X1) + E(X2) = μ + μ = 2μ

The variance of X1 + X2 would be: Var(X1 + X2) = Var(X1) + Var(X2) = σ^2 + σ^2 = 2σ^2

So, the distribution of X1 + X2 is also normal, with mean 2μ and variance 2σ^2.

In summary:

· 2X1 follows a normal distribution with mean 2μ and variance 4σ^2.

· X1 + X2 follows a normal distribution with mean 2μ and variance 2σ^2.

Both distributions have the same mean but different variances. The variance of 2X1 is larger than the variance of X1 + X2. This is because when we multiply a random variable by a constant, the variance increases by the square of that constant. On the other hand, when we add two independent random variables, the variance of the sum is simply the sum of their individual variances.

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

**Solution:**

To find the values a and b, we can use the standard normal distribution (Z-score) and then convert back to the original normal distribution using the mean and standard deviation.

Step 1: Find the Z-scores corresponding to the 0.005 and 0.995 probabilities (since the distribution is symmetric):

For 0.005 probability (lower tail), Z1 = -2.576 For 0.995 probability (upper tail), Z2 = 2.576

Step 2: Convert the Z-scores back to the original values using the mean (μ = 100) and standard deviation (σ = 20):

a = μ + Z1 \* σ = 100 + (-2.576) \* 20 ≈ 48.48 b = μ + Z2 \* σ = 100 + 2.576 \* 20 ≈ 151.52

Therefore, the two values, a and b, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99, are approximately 48.5 and 151.5.

The correct answer is option D. 48.5, 151.5.

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

**Answer: The rupee range ( centered on the mean ) with 95% probability for the annual profit is approximately 4.39M rupees to 28.39M rupees.**

1. Specify the 5th percentile of profit (in Rupees) for the company

**Answer: The 5th percentile of profit for the company is approximately 0.855M rupees.**

1. Which of the two divisions has a larger probability of making a loss in a given year?

**Answer: Division 2 has higher probability of making a loss for a given year.**