

Team Notebook

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Data Structures

2D BIT

```

1 template <typename T> class BIT2D {
2 private:
3     const int n, m;
4     vector<vector<T>> bit;
5 public:
6     BIT2D(int n, int m) : n(n), m(m), bit(n + 1, vector<T>(m + 1)) {}
7     void add(int r, int c, T val) {
8         r++, c++;
9         for (; r <= n; r += r & -r) {
10             for (int i = c; i <= m; i += i & -i) { bit[r][i] += val; }
11         }
12     }
13     T rect_sum(int r, int c) {
14         r++, c++;
15         T sum = 0;
16         for (; r > 0; r -= r & -r) {
17             for (int i = c; i > 0; i -= i & -i) { sum += bit[r][i]; }
18         }
19         return sum;
20     }
21     T rect_sum(int r1, int c1, int r2, int c2) {
22         return rect_sum(r2, c2) - rect_sum(r2, c1 - 1) - rect_sum(r1 - 1, c2) +
23             rect_sum(r1 - 1, c1 - 1);
24     }
25     vector<vector<int>> ar(n, vector<int>(m));
26     BIT2D<int> bit(n, m);
27     bit.add, bit.rect_sum

```

BIT

```

1  ll tree[N+5];
2  void update(int pos, ll val){
3      while (pos <= N)// must N = 4 * n
4      {
5          tree[pos] += val;
6          pos += (pos & -pos);
7      }
8  }
9  ll query(int pos){
10    ll sum = 0;
11    while (pos > 0)
12    {
13        sum += tree[pos];
14        pos -= (pos & -pos);
15    }
16    return sum;
17 }
18 ll sum(int l,int r){
19     return query(r) - query(l-1);

```

20 }

Sparse Table

```

1 int spt[N][22];
2 int n,ar[N];
3 int combine(int a,int b){
4     return min(a , b);
5 }
6 void buildST(){
7     for (int i = 0; i < n; i++) spt[i][0] = ar[i];
8
9     for (int j = 1; (1 << j) <= n; j++) {
10         for (int i = 0; (i + (1 << j) - 1) < n; i++) {
11             spt[i][j] = combine(spt[i + (1 << (j - 1))][j - 1] ,
12                                 spt[i][j - 1]);
13         }
14     }
15     int query(int l, int r){
16         if(l>r) return INT_MAX;
17         int j = (int)log2(r - l + 1);
18         return combine(spt[l][j], spt[r - (1 << j) + 1][j]);
19     }
20 //! 0 based index -> query O(1), pre compute nlog(n) and support
21 //! idempotent function f(a,a) = a
22 //! minimum, maximum, GCD, and XOR buildST();

```

Segment Tree

```

1 int ar[N],tree[4*N],lazy[4*N];// lazy seg start
2 int combine(int x,int y){return min(x , y);}
3 void build(int node,int l,int r){
4     lazy[node] = -1;//! change
5     if(l == r){tree[node] = ar[l];return;}
6     int mid = (l + r)/2;
7     build(node*2,l,mid);build(node*2+1,mid+1,r);
8     tree[node] = combine(tree[node*2],tree[node*2 + 1]);
9 }
10 void propagate(int node,int l,int r){
11     if(lazy[node] == -1) return;//! change
12     if(l != r){lazy[2*node] = lazy[node];//! change
13         lazy[2*node+1] = lazy[node];}
14     tree[node] = lazy[node];//! change
15     lazy[node] = -1; //!change
16 }
17 int query(int node,int l,int r,int a,int b){
18     propagate(node,l,r);
19     if(l > b || r < a){ return oo;}
20     if(a <= l && r <= b){return tree[node];}
21     int mid = (l + r)/2;
22     int x = query(node * 2,l,mid,a,b);
23     int y = query(node * 2 + 1,mid + 1,r,a,b);

```

```

24     return combine(x,y);
25 }
26 void update(int node,int l,int r,int a,int b,int val){
27     propagate(node,l,r); if(a > r || b < l) return;
28     if(a <= l && r <= b){
29         lazy[node] = val;//! change
30         propagate(node,l,r); return;
31     }
32     int mid = (l + r)/2; update(node*2,l,mid,a,b,val);
33     update(node*2+1,mid+1,r,a,b,val);
34     tree[node] = combine(tree[node*2],tree[node*2+1]);
35 }
36 ll tree[M],ar[M];// just seg Tree start
37 ll combine(int a,int b){return max(a,b);}
38 void init(int node,int b,int e){
39     if(b == e){tree[node] = ar[b];return;}
40     int Left = node * 2;int Right = node * 2 + 1;
41     int mid = (b+e)/2; init(Left,b,mid); init(Right,mid+1,e);
42     tree[node] = combine(tree[Left] , tree[Right]);
43 }
44 ll query(int node,int b,int e,int i,int j){
45     if(i>e || j<b) return 0;if(i<=b && j>=e){
46         return tree[node];
47     }
48     int Left = node * 2;int Right = node * 2 + 1;
49     int mid = (b+e)/2;ll p1 = query(Left,b,mid,i,j);
50     ll p2 = query(Right,mid+1,e,i,j);return combine(p1,p2);
51 }
52 void upgrade(int node,int b,int e,int i,int newValue){
53     if(b == e){tree[node] = newValue;ar[b] = newValue;
54     return;
55     }
56     int Left = node * 2;int Right = node * 2 + 1;int mid = (b+e)/2;
57     if(i <= mid)upgrade(Left,b,mid,i,newValue);
58     else upgrade(Right,mid+1,e,i,newValue);
59     tree[node] = combine(tree[Left] , tree[Right]);
60 }
61 // Iterative Segment Tree ->
62 int combine(int a,int b){return max(a,b);}
63 void build(int n){
64     for(int i=1;i<=n;i++) tree[n+i-1]=ar[i];
65     for(int i=n-1;i;i--) tree[i]=combine(tree[i<<1],tree[i<<1|1]);
66 }
67 void update(int p,int v,int n){
68     for(tree[p+=n-1]=v,p>=1;p;p>>=1){
69         tree[p]=combine(tree[p<<1],tree[p<<1|1]);
70     }
71 }
72 int query(int l,int r,int n){
73     int ans=NEUTRAL;
74     for(l+=n-1,r+=n-1;l<=r;l>>=1,r>>=1){
75         if(l&1) ans=combine(ans,tree[l++]);
76         if(!(r&1)) ans=combine(ans,tree[r--]);

```

```

77     }return ans;
78 }

```

Persistent Segment Tree

```

1 struct node{
2     int val;node* lc;node* rc;
3 };
4 int ar[N];
5 node* versions[N+5];
6 vector<node*> lagbe;
7 node* build(int l,int r){
8     if(l == r){
9         node* me = new node();lagbe.pb(me);
10        me->val = ar[l]; me->lc = NULL;
11        me->rc = NULL;return me;
12    }
13    node* my = new node();lagbe.pb(my);
14    int mid = (l + r)/2;
15    my->lc = build(l,mid);
16    my->rc = build(mid+1,r);
17    my->val = (my->lc->val + my->rc->val); return my;
18 int query(node* my,int l,int r,int L,int R){
19     if(R < l || r < L) return 0;
20     if(L <= l && r <= R){return my->val;}
21     int mid = (l + r)/2;
22     int p = query(my->lc,l,mid,L,R);
23     int q = query(my->rc,mid+1,r,L,R);
24     return p + q;
25 }
26 node* update(node* my,int l,int r,int ind,int val){
27     if(l == r){
28         node* new_my = new node(); lagbe.pb(new_my);
29         new_my->lc = NULL;new_my->rc = NULL;new_my->val = val;
30         return new_my;
31     }
32     node* new_me = new node();
33     lagbe.pb(new_me);
34     int mid = (l + r)/2;
35     if(ind <= mid){
36         node* x = update(my->lc,l,mid,ind,val);
37         new_me->lc = x;
38         new_me->rc = my->rc;
39     }else{
40         node* y = update(my->rc,mid+1,r,ind,val);
41         new_me->lc = my->lc;
42         new_me->rc = y;
43     }
44     new_me->val = new_me->lc->val + new_me->rc->val;
45     return new_me;
46 }
47 // after test case delete all node* ->for(auto u : lagbe){delete u;
48 }lagbe.clear();
49 int now = 1;versions[now] = build(1,n);versions[k] = update(
50 versions[k],1,n,a,x);

```

Merge Sort Tree

```

1 struct Merge_Sort_Tree {
2     int n;
3     vector<vector<int>> tree;
4     void build(vector<int> &a, int x, int l, int r) {
5         if (l + 1 == r) {
6             tree[x] = {a[l]};
7             return;
8         }
9
10        int m = (l + r) / 2;
11        build(a, 2 * x + 1, l, m);
12        build(a, 2 * x + 2, m, r);
13        merge(all(tree[2 * x + 1]), all(tree[2 * x + 2]),
14              back_inserter(tree[x]));
15    }
16    Merge_Sort_Tree(vector<int>& a) : n(a.size()) {
17        int SIZE = 1 << (lg(n) + bool(__builtin_popcount(n) - 1));
18        tree.resize(2 * SIZE - 1);
19        build(a, 0, 0, n);
20    }
21    int count(int lq, int rq, int mn, int mx, int x, int l, int r) {
22        if (rq <= l || r <= lq) return 0;
23        if (lq <= l && r <= rq) return lower_bound(all(tree[x]), mx)
24            - lower_bound(all(tree[x]), mn);
25
26        int m = (l + r) / 2;
27        int a = count(lq, rq, mn, mx, 2 * x + 1, l, m);
28        int b = count(lq, rq, mn, mx, 2 * x + 2, m, r);
29        return a + b;
30    }
31    int count(int lq, int rq, int mn, int mx) {
32        return count(lq - 1, rq, mn, mx + 1, 0, 0, n); // query 1
33        base call l to r how many mn to mx number
34    }
35 };

```

Trie

```

1 struct Node {
2     Node* links[26]; int cnt_pre = 0; int cnt_word = 0;
3     bool contains(char c) { return (links[c - 'a'] != NULL); }
4     void put(char c, Node* node) { links[c - 'a'] = node; }
5     Node* get(char c) { return links[c - 'a']; }
6     void increasePre() { cnt_pre++; }
7     void increaseWord() { cnt_word++; }
8     void decreasePre() { cnt_pre--; }
9     void decreaseWord() { cnt_word--; }
10    int getPre() { return cnt_pre; }
11    int getWord() { return cnt_word; }

```

```

12    void deleteNode() {
13        for(int i = 0; i < 26; ++i) {
14            if(links[i] != nullptr) { links[i]->deleteNode(); }
15        }
16    }
17 }
18 class Trie {
19 private:
20     Node *root;
21 public:
22     Trie() {
23         root = new Node();
24     }
25     void deleteNode() { root->deleteNode(); delete root; }
26     void insert(string s) {
27         Node* cur_node = root;
28         for(char c : s) {
29             if(!cur_node->contains(c)) {
30                 cur_node->put(c, new Node());
31             } // move reference node
32             cur_node = cur_node->get(c);
33             cur_node->increasePre();
34         }
35         cur_node->increaseWord();
36     }
37     int countWords(string s) {
38         Node* cur_node = root;
39         for(char c : s) {
40             if(!cur_node->contains(c)) {
41                 return 0;
42             }
43             cur_node = cur_node->get(c);
44         }
45         return cur_node->getWord();
46     }
47     int countPrefixes(string s) {
48         Node* cur_node = root;
49         for(char c : s) {
50             if(!cur_node->contains(c)) {
51                 return 0;
52             }
53             cur_node = cur_node->get(c);
54         }
55         return cur_node->getPre();
56     }
57     void erase(string s) {
58         Node* cur_node = root;
59         for(char c : s) {
60             if(!cur_node->contains(c)) {
61                 return;
62             }
63             cur_node = cur_node->get(c);
64         }
65     }

```

```

64     cur_node -> decreasePre();
65 }
66 cur_node -> decreaseWord();
67 }
68 int getMax(int num) {
69     Node* cur_node = root;
70     int mx = 0;
71     for(int i = 31; i >= 0; i--) {
72         int bit = (num >> i) & 1;
73         if(cur_node -> contains(bit ^ 1) && cur_node -> getCnt(bit ^ 1) > 0) {
74             mx = ((1LL << i) | mx);
75             cur_node = cur_node -> get(bit ^ 1);
76         } else {
77             cur_node = cur_node -> get(bit);
78         }
79     }
80     return mx;
81 }
82 };

```

Graph Algorithms

DSU

```

1 class DSU{
2     vector<int> parent,Size;
3 public:
4     DSU(int n){
5         for(int i = 0; i <= n; i++){
6             parent.push_back(i);
7             Size.push_back(1);
8         }
9     }
10    int find_par(int node){
11        if(parent[node] == node){
12            return node;
13        }
14        return parent[node] = find_par(parent[node]);
15    }
16    void union_size(int u, int v) {
17        int pu = find_par(u);
18        int pv = find_par(v);
19        if(pu == pv) return;
20
21        if(Size[pu] < Size[pv]){
22            parent[pu] = pv;
23            Size[pv] += Size[pu];
24        }
25        else{
26            parent[pv] = pu;
27            Size[pu] += Size[pv];

```

```

28        }
29    }
30    int size(int node){
31        return Size[find_par(node)];
32    };

```

Dijkstra

```

1 void dijkstra(int s,int n,vi &dis,vector<vector<pii>> &graph){
2     priority_queue<pii,vector<pii>,greater<pii> pq;
3     for(int i = 1; i <= n; i++){
4         dis[i]=oo;
5     }
6     dis[s]=0;
7     pq.push({0,s});
8     while(!pq.empty()){
9         int a = pq.top().S,co = pq.top().F;
10    pq.pop();
11    if(co > dis[a]) continue;
12    for (auto u: graph[a]){
13        int b = u.F,w = u.S;
14        if (dis[a]+w < dis[b]){
15            dis[b] = dis[a]+w;
16            pq.push({dis[b],b});
17        }
18    }
19 }
20 }

```

Bellman Ford

```

1 struct edge {
2     int u,v,w,x;
3 };
4 vector<edge> e;
5 int dist[N],par[N],neg[N];
6 void BellmenFord(int s,int n,int m) {
7     for(int i = 0; i <= n; i++) {
8         dist[i] = oo,neg[i] = 0;
9     }
10    dist[s] = 0;
11    for(int i = 1; i <= n; i++) {
12        for(int j = 0; j < m; j++) {
13            int u = e[j].u,v = e[j].v,w = e[j].w;
14            if(dist[u] + w < dist[v]) {
15                dist[v] = dist[u] + w;
16                par[v] = u;
17                if(i == n) {
18                    neg[v] = 1;
19                }
20            }
21        }
22    }

```

```
23 } // neg[i] tells it is a part of negative cycle or not
```

Floyd Warshall

```
1 for(int i = 1; i <= n; i++) {
2     for(int j = 1; j <= n; j++) {
3         nxt[i][j] = j;
4     }
5 }
6 for(int k = 1; k <= n; k++) {
7     for(int i = 1; i <= n; i++) {
8         for(int j = 1; j <= n; j++) {
9             if(dist[i][k] + dist[k][j] < dist[i][j]) {
10                dist[i][j] = dist[i][k] + dist[k][j];
11                nxt[i][j] = nxt[i][k];
12            }
13        }
14    }
15 }
16 int s = 1, t = 4;
17 cout << s << " ";
18 int cur = s;
19 while(cur != t) {
20     cur = nxt[cur][t];
21     cout << cur << " ";
22 }
```

Articulation Point

```
1 vi graph[N]; set<int> point;
2 int low[N], tin[N], vis[N], timer;
3 void dfs(int node, int par){
4     vis[node] = 1;
5     low[node] = tin[node] = ++timer;
6     int child = 0;
7     for(auto u : graph[node]) {
8         if(u == par) continue;
9         if(vis[u]) low[node] = min(low[node], tin[u]);
10        else {
11            child++;
12            dfs(u, node);
13            low[node] = min(low[node], low[u]);
14            if(low[u] >= tin[node] && par != -1) point.insert(node);
15        }
16    }
17    if(child > 1 && par == -1) point.insert(node);
18 }
```

Bridge

```
1 set<pii> bridge;
2 int low[N], tin[N], vis[N], timer = 0;
3 void dfs(int node, int par){
```

```
4     vis[node] = 1;
5     low[node] = tin[node] = ++timer;
6     for(auto child : graph[node]) {
7         if(child == par) continue;
8         if(vis[child]) {
9             low[node] = min(low[node], tin[child]);
10        }
11        else {
12            dfs(child, node);
13            low[node] = min(low[node], low[child]);
14            if(low[child] > tin[node]) {
15                int x = min(node, child), y = max(node, child);
16                bridge.insert({x, y});
17            }
18        }
19    }
20 }
```

SCC

```
1 // given a directed graph return the minimum number of edges to be
2 // added so that the whole graph become an SCC
3 bool vis[N];
4 vector<int> g[N], r[N], G[N], vec; //g is the condensed graph
5 void dfs1(int u) {
6     vis[u] = 1;
7     for(auto v: g[u]) if(!vis[v]) dfs1(v);
8     vec.push_back(u);
9 }
10 vector<int> comp;
11 void dfs2(int u) {
12     comp.push_back(u);
13     vis[u] = 1;
14     for(auto v: r[u]) if(!vis[v]) dfs2(v);
15 }
16 int idx[N], in[N], out[N];
17 int main() {
18     int n, m;
19     cin >> n >> m;
20     for(int i = 1; i <= m; i++) {
21         int u, v;
22         cin >> u >> v;
23         g[u].push_back(v);
24         r[v].push_back(u);
25     }
26     for(int i = 1; i <= n; i++) if(!vis[i]) dfs1(i);
27     reverse(vec.begin(), vec.end());
28     memset(vis, 0, sizeof vis);
29     int scc = 0;
30     for(auto u: vec) {
31         if(!vis[u]) {
32             comp.clear();
33             dfs2(u);
```

```

33     scc++;
34     for(auto x: comp) idx[x]=scc;
35   }
36 }
37 for(int u = 1; u <= n; u++) {
38   for(auto v: g[u]) {
39     if(idx[u] != idx[v]) {
40       in[idx[v]]++, out[idx[u]]++;
41       G[idx[u]].push_back(idx[v]);
42     }
43   }
44 }
45 int needed_in=0, needed_out=0;
46 for(int i = 1; i <= scc; i++) {
47   if(!in[i]) needed_in++;
48   if(!out[i]) needed_out++;
49 }
50 int ans = max(needed_in, needed_out);
51 if(scc == 1) ans = 0;
52 cout << ans << '\n';
53 }

```

BCC

```

1 //bcc[i] is which num Biconnected element it
2 set<int> g[N];//use set avoid multiple same edges
3 vector<int> bcc[N], st;
4 int low[N], dis[N], T = 0, sz = 0;
5 vector<vector<int>> allBCC;
6 void dfs(int u, int pre = -1) {
7   low[u] = dis[u] = ++T;
8   st.push_back(u);
9   for(auto v : g[u]) {
10     if(!dis[v]) {
11       dfs(v, u);
12       low[u] = min(low[u], low[v]);
13       if(low[v] >= dis[u]) {
14         sz++; vector<int> comp;
15         int x;
16         do{
17           x = st.back();
18           st.pop_back();
19           bcc[x].push_back(sz);
20           comp.push_back(x);
21         } while (x != v);
22         bcc[u].push_back(sz);
23         comp.push_back(u);
24         sort(comp.begin(), comp.end());
25         comp.erase(unique(comp.begin(), comp.end()), comp.end());
26         allBCC.push_back(comp);
27       }
28     }

```

```

29     else if(v != pre)
30       low[u] = min(low[u], dis[v]);
31   }
32 }
33 void Goriber_solve(){
34   int n,m;cin >> n >> m;
35   for (int i = 0; i < m; i++) {
36     int u, v;cin >> u >> v;
37     if (u == v) continue;
38     g[u].insert(v);g[v].insert(u);
39   }
40   for(int i = 0; i < n; i++)if(!dis[i])dfs(i);
41   for (int i = 0; i < n; i++) {
42     if(g[i].empty()) {sz++;
43       allBCC.push_back({i});
44     }
45   }
46   cout << allBCC.size() << "\n";
47   for(auto &comp : allBCC) {
48     cout << comp.size();
49     for(int x : comp) cout << " " << x;
50     cout << "\n";
51   }
52 }//if bcc[u].size() > 1 it is an AP

```

Topological Sort

```

1 void dfs(int node) {
2   vis[node] = 1;
3   for(auto u : graph[node]) {
4     if(!vis[u])dfs(u);
5   } ans.pb(node);
6 }
7 void Goriber_solve() {
8   int n,m;cin >> n >> m;bool ok = 0;
9   for(int i = 0; i < m; i++) {
10     int u,v;cin >> u >> v;
11     if(u == v)ok = 1;graph[u].pb(v);
12   }
13   if(ok){cout << "IMPOSSIBLE\n";return;}
14   for(int i = 1; i <= n; i++) {
15     if(!vis[i]) {dfs(i);}
16   }
17   reverse(all(ans));vi pos(n+1);
18   for(int i = 0; i < sz(ans); i++) {
19     pos[ans[i]] = i+1;
20   }
21   for(int i = 1; i <= n; i++) {
22     for(auto u : graph[i]) {
23       if(pos[i] > pos[u]) {cout << "IMPOSSIBLE\n";return;}
24     }
25   }
26 }

```

```

27     for(auto u : ans) {cout << u << " ";}cout << endl;
28 }



## Cycle Detection


1 vi graph[N];// for directed graph
2 int vis[N],start = -1,finish,par[N];
3 void dfs(int node){
4     vis[node] = 1;
5     for(auto child : graph[node]){
6         if(vis[child] == 0){
7             par[child] = node;
8             dfs(child);
9             if(start != -1) return;
10        }
11        else if(vis[child] == 1){
12            start = child;
13            finish = node;
14            return;
15        }
16    }
17    vis[node] = 2;
18 }
19 bool is_cycle(int node,int par) {// for undirected graph
20     vis[node] = true;
21     ans.pb(node);
22     for(int child : graph[node]) {
23         if(child == par) continue;
24         if(vis[child] == 0) {
25             if(is_cycle(child,node) == 1) return 1;
26         }
27         else {
28             ans.pb(child);
29             return 1;
30         }
31     }
32     ans.pop_back();
33     return 0;
34 }
35 //main function->
36 for(int i = 1; i <= n; i++){
37     if(vis[i] == 0){dfs(i);if(start != -1)break;}
38     int x = ans.back();vi path;path.pb(x);ans.pop_back();int node = ans
39     .back();
40     path.pb(node);ans.pop_back();node = ans.back();
41 }path.pb(x);

```

LCA

```

1 vector<int> graph[N];
2 int Level[N];int dp[N][22];
3 void dfs(int node,int par,int lev){

```

```

4     dp[node][0] = par;
5     Level[node] = lev;
6     for(int i = 1; i <= 20; i++){
7         dp[node][i] = dp[dp[node][i-1]][i-1];
8     }
9     for(auto child : graph[node]){
10        if(child != par){
11            dfs(child,node,lev+1);
12        }
13    }
14 }
15 int get_kth_par(int node,int k){
16     for(int i = 20; i >= 0; i--){
17         if((k >> i) & 1){
18             node = dp[node][i];
19         }
20     }
21     return node;
22 }
23 int get_lca(int a,int b){
24     if(Level[a] > Level[b]){
25         swap(a,b);
26     }
27     int k = Level[b] - Level[a];
28     b = get_kth_par(b,k);
29     if(a == b){ return a;}
30     for(int i = 20; i >= 0; i--){
31         if(dp[a][i] != dp[b][i]){
32             a = dp[a][i];
33             b = dp[b][i];
34         }
35     }
36 }
37 } //dfs(1,0,0);

```

Bipartite Matching

```

1 const int N = 3e5 + 9;// Time complexity O(E*sqrt(V))
2 struct HopcroftKarp {
3     static const int inf = 1e9;int n;
4     vector<int> l, r, d;vector<vector<int>> g;
5     HopcroftKarp(int _n, int _m) {
6         n = _n;int p = _n + _m + 1;g.resize(p);
7         l.resize(p, 0);r.resize(p, 0);d.resize(p, 0);
8     }
9     void add_edge(int u, int v) {
10        g[u].push_back(v + n); //right id is increased by n, so is l[u]
11    }
12    bool bfs() {
13        queue<int> q;
14        for (int u = 1; u <= n; u++) {
15            if (!l[u]) {d[u] = 0; q.push(u);}
16            else d[u] = inf;
17        } d[0] = inf;
18        while (!q.empty()) {

```

```

19     int u = q.front(); q.pop();
20     for (auto v : g[u]) {
21         if (d[r[v]] == inf) {
22             d[r[v]] = d[u] + 1; q.push(r[v]);
23         }
24     }
25     return d[0] != inf;
26 }
27 bool dfs(int u) {
28     if (!u) return true;
29     for (auto v : g[u]) {
30         if (d[r[v]] == d[u] + 1 && dfs(r[v])) {
31             l[u] = v; r[v] = u; return true;
32         }
33     }
34     d[u] = inf;
35     return false;
36 }
37 int maximum_matching() {
38     int ans = 0;
39     while (bfs()) {
40         for (int u = 1; u <= n; u++) if (!l[u] && dfs(u)) { ans++; }
41     }
42     return ans;
43 }
44 vector<pii> get_pairs() {
45     vector<pii> pairs;
46     for (int u = 1; u <= n; u++) {
47         if (l[u]) {
48             pairs.emplace_back(u, l[u] - n);
49         }
50     }
51 }

```

Max Flow (Dinic)

```

1 const int N = 5010; const long long inf = 1LL << 61;
2 struct Dinic {
3     struct edge {
4         int to, rev; long long flow, w; int id;
5     };
6     int n, s, t, mxid; vector<int> d, flow_through;
7     vector<int> done; vector<vector<edge>> g;
8     Dinic() {}
9     Dinic(int _n) {
10         n = _n + 10; mxid = 0; g.resize(n);
11     }
12     void add_edge(int u, int v, long long w, int id = -1) {
13         edge a = {v, (int)g[v].size(), 0, w, id};
14         edge b = {u, (int)g[u].size(), 0, 0, -2}; //for bidirectional
15         edges cap(b) = w
16         g[u].emplace_back(a); g[v].emplace_back(b); mxid = max(mxid, id);
}

```

```

17     bool bfs() {
18         d.assign(n, -1); d[s] = 0; queue<int> q; q.push(s);
19         while (!q.empty()) {
20             int u = q.front(); q.pop();
21             for (auto &e : g[u]) {
22                 int v = e.to;
23                 if (d[v] == -1 && e.flow < e.w) d[v] = d[u] + 1, q.push(v);
24             }
25         }
26         return d[t] != -1;
27     }
28     long long dfs(int u, long long flow) {
29         if (u == t) return flow;
30         for (int &i = done[u]; i < (int)g[u].size(); i++) {
31             edge &e = g[u][i];
32             if (e.w <= e.flow) continue;
33             int v = e.to;
34             if (d[v] == d[u] + 1) {
35                 long long nw = dfs(v, min(flow, e.w - e.flow));
36                 if (nw > 0) {
37                     e.flow += nw; g[v][e.rev].flow -= nw;
38                     return nw;
39                 }
40             }
41         }
42         return 0;
43     }
44     long long max_flow(int _s, int _t) {
45         s = _s; t = _t; long long flow = 0;
46         while (bfs()) {
47             done.assign(n, 0);
48             while (long long nw = dfs(s, inf)) flow += nw;
49         }
50         flow_through.assign(mxid + 10, 0);
51         for (int i = 0; i < n; i++) for (auto e : g[i]) {
52             if (e.id >= 0) flow_through[e.id] = e.flow;
53         }
54         return flow;
55     }
56     void reset_flows() {
57         for (int i = 0; i < n; ++i) {
58             for (auto &e : g[i]) {
59                 e.flow = 0;
60             }
61         }
62     }
63 }

```

Mathematics

Sieve

```

1 // use bitset<N> is_prime; to have O(N/64) memory complexity
2 // using bitset you can solve upto around N = 10^8 in 1s
3 const int N = 1e8; vector<int> primes; bitset<N>is_prime;
4 void sieve_v0() {

```

```

5   for (int i = 3; i < N; i += 2) {
6     is_prime[i] = 1;
7   }
8   for (int i = 3; i * i < N; i += 2) {
9     if (is_prime[i]) {
10       for (int j = i * i; j < N; j += i) {
11         is_prime[j] = 0;
12       }
13     }
14   }
15   is_prime[2] = 1;
16   for (int i = 2; i < N; i++) {
17     if(is_prime[i]){ primes.push_back(i); }
18   }
19 } // sieve with smallest prime factors (spf)
20 int spf[N];
21 void sieve() {
22   for (int i = 2; i < N; i++) { spf[i] = i; }
23   for (int i = 2; i * i < N; i++) {
24     if (spf[i] == i) {
25       for (int j = i * i; j < N; j += i) {
26         spf[j] = min(spf[j], i);
27       }
28     }
29   }
30   for (int i = 2; i < N; i++) {
31     if (spf[i] == i) {
32       primes.push_back(i);
33     }
34   }
35 }
36 sum of divisor = ((x^(a-1)/(x - 1)) * (y ^ (b-1)/ (y-1)));

```

Euler Totient

```

1 int phi[N];bool mark[N];
2 void sieve(){
3   for(int i = 1; i < N; i++){ phi[i] = i; }
4   for(int i = 2; i < N; i++){
5     if(mark[i]) continue; phi[i] = i - 1;
6     for(int j = i + i; j < N; j += i){
7       mark[j] = true; phi[j] -= phi[j] / i;
8     }
9   }
10 }
11 long long euler_phi(long long x){
12   long long ans = x;
13   for (long long i = 2; i * i <= x; i++){
14     if (x % i == 0){
15       ans /= i; ans *= i - 1;
16       while (x % i == 0){ x /= i; }
17     }
18   }

```

```

19   if (x > 1){
20     ans /= x;ans *= x - 1;
21   }
22   return ans;
23 }
24 1-> f(n) = gcd(1,n)+gcd(2,n)+...+gcd(n,n)
25 ans,f(n) = sum{divisor of n as d} d * phi(n/d)

```

Extended Euclid

```

1 struct triplate { int x,y,gcd;};
2 //! ax + by = gcd(a,b)
3 triplate extendedEuclid(int a, int b) {
4   if(b == 0) return {1, 0, a};
5   triplate smallans = extendedEuclid(b, a % b);
6   triplate ans; ans.x = smallans.y; ans.gcd = smallans.gcd;
7   ans.y = (smallans.x - (a / b) * smallans.y);
8   return ans;
9 }// x^-1 = power(x,phi(mod)-1,mod) power(a,b,mod) = (a ^ (b%phi(mod)))

```

Mobius

```

1 void mobius() { mu[1] = 1;
2   for(int i = 2; i <= N; i++) {
3     if(!isComposite[i]) {
4       primes.push_back(i);
5       mu[i] = -1;
6     }
7     for(int p : primes) {
8       if(1LL * i * p > N)break;
9       isComposite[i * p] = 1;
10      if(i % p == 0) {mu[i * p] = 0;break;
11      }
12      else {mu[i * p] = -mu[i];}
13    }
14  }
15 }
16 1-> Number of unordered pairs(i, j) with gcd(a_i, a_j) = 1 freq[x]
17   = how many array elements equal x
18 cnt[d] = number of array elements divisible by d,pair count = sum{1
19   to (mx of ar) as d} mu[d] * nCr(cnt[d],2)
20 subsequence count = sum{1 to mx} mu[d] * (pow(2,cnt[d])-1)
21 2->How many pair range[1..N] gcd(x,y) = 1 ,ans = sum{1 to N as d}
22   mu[d]* (N/d)^2
23 3-> Count Numbers <= N that are divisable by none of primes
24 L = lcm of all prime.. divs = all divisor of L,ans = sum{divs as d}
25   mu[d]*(N/d)
26 4-> How many square free num in [1..N] count = sum{1 to sqrt(N) as d
27   } mu[d]*(N/(d*d))
28 5-> how many Lattice point with gcd = 1 or 1 <= x <= n and 1 <= y
29   <= m and gcd(x,y) = 1
30   ,ans = sum{1 to min(x,y) as d} mu[d]*(n/d)*(m/d)

```

Inclusion-Exclusion

```

1 int ans = n,la = (1 << k);
2 for(int i = 1; i < la; i++) { int lc = 1;
3   for(int j = 0; j < k; j++) {
4     if(getbit(i,j)) { lc = lcm(lc,ar[j]); if(lc > n)break;
5   }
6   } int cnt = __builtin_popcountll(i);
7   if(cnt & 1)ans -= (n / lc);
8   else ans += (n / lc);
9 }
```

Derangements find n

```

1 int Derangements(int n) { // 0(n)
2   vector<int> dp(n + 1); dp[1] = 0; dp[2] = 1;
3   for (int i = 3; i <= n; i++) {
4     dp[i] = (i - 1) * (dp[i - 1] + dp[i - 2]);
5   } return dp[n];
6 }
7 int find_n(int x) { // n*(n+1)/2 = x
8   int n = (-1 + sqrt(1 + 8 * x)) / 2;
9   if((n * (n + 1)) / 2 > x) { n--; }
10 } return n;
11 }
```

Baby-step Giant-step

```

1 // Returns minimum x for which a ^ x % m = b % m
2 int baby_step(int a,int b,int m) {
3   a%m,b%m; int k=1,add=0,g;
4   while((g=__gcd(a,m))>1) {
5     if(b==k) return add;if(b%g) return -1;
6     b/=g,m/=g,add++;k=(k*a/g)%m;
7   }
8   int n=sqrt(m)+1,int mul=1;
9   for(int i=0; i<n; i++)mul=(mul*a)%m;
10  unordered_map<int,int>val;
11  for(int q=0,cur=b; q<=n; q++) {
12    val[cur]=q;cur=(cur*a)%m;
13  }
14  for(int p=1,cur=k; p<=n; p++) {
15    cur=(cur*mul)%m;
16    if(val.count(cur)) {return p*n-val[cur]+add;}
17  } return -1;
18 }
```

BigMod/nCr

```

1 int power(int a,int b,int mod) {
2   ll ans = 1;
3   while(b>0) {
4     if(b&1) { ans = (ans*a)%mod;}
```

```

5       a = (a*a)%mod;b >>= 1;
6   } return ans;
7 }
8 int fact[N];
9 void Pre() {
10   fact[0] = 1;
11   for(int i = 1; i < N; i++) { fact[i] = (fact[i-1] * i) % mod; }
12 }
13 int nCr(int n,int r) {
14   if(n < r) return 0; ll up = fact[n];
15   int down = (fact[r]*fact[n-r]) % mod;
16   down = power(down,mod-2,mod);ll res = (up * down) % mod;
17   return res;
18 }
19 Catalan Number Cn = (1/(n+1)*nCr(2n,n); Or Cn = nCr(2*n,n) - nCr(2*n,n-1);
```

Chinese Remainder Theorem

```

1 int ex_gcd(int a,int b,int &x,int &y) {
2   if(b == 0) { x = 1; y = 0; return a;}
3   int x1,y1; int ans = ex_gcd(b,a % b,x1,y1);
4   x = y1; y = x1 - y1 * (a / b); return ans;
5 }
6 int chinese_rem_theorem(int a,int m1,int b,int m2) {
7   int p,q; int gc = ex_gcd(m1,m2,p,q);
8   return (a*m2*q % (m1*m2) + b*m1*p % (m1*m2)) % (m1*m2);
9 }
10 void Goriber_solve() {
11   int n; cin >> n; vii a(n);
12   for(int i = 0; i < n; i++) { cin >> a[i].F >> a[i].S;}
13   int x = chinese_rem_theorem(a[0].F,a[0].S,a[1].F,a[1].S);
14   int mm = lcm(a[0].S,a[1].S);
15   for(int i = 2; i < n; i++) {
16     x = chinese_rem_theorem(x,mm,a[i].F,a[i].S); mm = lcm(mm,a[i].S);
17   }
18 }
```

Dynamic Programming SOS DP

```

1 const ll N = 20; const ll M = (1 << N);
2 void add_super(int dp[]){
3   for(int bit = 0; bit < N; bit++){
4     for(int i = 0; i < M; i++){
5       if(i & (1 << bit)){ dp[i] += dp[i^(1<<bit)];}
6     }
7   }
8 }
9 void remove_super(int dp[]){

```

```

10    for(int bit = 0; bit < N; bit++){
11        for(int i = M-1; i >= 0; i--){
12            if(i & (1 << bit)){ dp[i] -= dp[i^(1<<bit)];}
13        }
14    }
15 }
16 void add_sub(int dp[]){
17    for(int bit = 0; bit < N; bit++){
18        for(int i = M - 1; i >= 0; i--){
19            if(i & (1 << bit)){dp[i^(1<<bit)] += dp[i];}
20        }
21    }
22 }
23 void remove_sub(int dp[]){
24    for(int bit = 0; bit < N; bit++){
25        for(int i = 0; i < M; i++){
26            if(i & (1 << bit)){dp[i^(1<<bit)] -= dp[i];}
27        }
28    }
29 }
30 1.number of element y such that x|y=x ->add_super
31 2.number of element y such that x&y=x ->add_sub
32 3.number of element y such that x&y=0 ->add_super,cnt(2^bit-1)^x
33 4.(a1&a2&a4&a5...&ax) = k -> add_sub(cnt[]), cnt[i]=pow(2,cnt[i])
   -1, remove_sub(cnt[]);

```

Digit DP

```

1 vi digit;int dp[11][100][100][3];
2 int sol(int i,int m1,int m2,int f) {
3     if(i == n) { return m1 == 0 && m2 == 0; }
4     int &ret = dp[i][m1][m2][f]; if(ret != -1) return ret;
5     int res = 0; int lim = (f == 1 ? 9 : digit[i]);
6     for(int j = 0; j <= lim; j++) {
7         res += sol(i+1,(m1*10 + j) % k, (m2 + j) % k,(f || j < digit[i]
8             ));
8     }return ret = res;
9 }
10 int cal(int val) {
11     digit.clear();
12     while(val > 0) {
13         digit.push_back(val % 10);val /= 10;
14     }
15     reverse(all(digit));n = sz(digit);mem(dp,-1);
16     return sol(0,0,0,0);
17 }
18 void solve() {
19     cin >> l >> r >> k;if(k > 90)cout << 0 << endl;
20     else cout << cal(r) - cal(l-1) << endl;
21 }

```

Bit Mask DP

```

1 int f(int mask){
2     if(mask == ((1 << n) - 1))return 0; int &ret = dp[mask];
3     if(ret != -1) return ret;int res = INF;
4     for(int i = 0; i < n; i++) {
5         if((mask & (1 << i)) == 0) { int c = cost[i][i];
6             for(int j = 0; j < n; j++) {
7                 if(mask & (1 << j)) { c += cost[i][j];}
8             }
8             res = min(res,c + f(mask | (1 << i)));
10        }
11    }ret = res;
12 }

```

DP Path Print

```

1 int f(int i,int rem) {
2     if(i == n + 1) {if(rem == 0) return 0;return -n;}
3     int &ret = dp[i][rem];
4     if(ret != -1) return ret;
5     return ret = max(1 + f(i+1,(ar[i]+rem) % k),f(i+1,rem));
6 }
7 void print(int i,int rem) {
8     if(i == n + 1 || ans == 0) { return; }
9     if(1 + f(i+1,(ar[i]+rem)%k) == ans) {
10        cout << i << " ";ans--;
11        print(i+1,(rem + ar[i])%k);
12    }
13    else print(i+1,rem);
14 }

```

String Algorithms

Hashing

```

1 struct rH{
2     const int k = 239,mod = 1000000207;string s;int n;vi pw,vl;
3     rH(string s): s(s),n(sz(s)),pw(n+1),vl(n+1){ pw[0] = 1,vl[0] =
0;
4         for(int i = 0; i < n; i++){
5             vl[i+1] = (1LL *vl[i] * k % mod + s[i]) % mod; pw[i+1]
= (1LL*pw[i] * k) % mod;
6         }
7     }
8     int h(int l){ return vl[l]; }
9     int h(int l,int r){ return (h(r+1) - (1LL*h(l) * pw[r-l+1]) %
mod + mod) % mod; }
10 }
11 int k = 1201 ,mod = 1000003891;k = 31,mod = 1000003247;rH hs1(s);
12 cin >> l >> r;
12 l--,r--;int val = hs.h(l,r);int x = (n - l - 1);int y = x - (r - 1)
;int val2 = hs2.h(y,x);

```

KMP

```

1 vi build_lps(string p) {
2     int sz = p.size();vi lps;lps.assign(sz + 1, 0);int j = 0;lps[0] =
3         0;
4     for(int i = 1; i < sz; i++) {
5         while(j >= 0 && p[i] != p[j]) {
6             if(j >= 1) j = lps[j - 1]; else j = -1;
7             }j++;lps[i] = j;
8     }return lps;
9 }
10 void kmp(vi lps, string s, string p) {
11     int psz = p.size(), sz = s.size(), j = 0;
12     for(int i = 0; i < sz; i++) {
13         while(j >= 0 && p[j] != s[i])
14             if(j >= 1) j = lps[j - 1];
15             else j = -1;
16             j++;
17         if(j == psz) {j = lps[j - 1];ans.pb(i - psz + 1); }
18     }
}

```

Z Algorithm

```

1 // An element Z[i] of Z array stores length of the longest
2 // substring
3 // starting from str[i] which is also a prefix of str[0..n-1].
4 // The first entry of Z array is meaning less as complete string is
5 // always prefix of itself. Here Z[0]=0.
6 vector<int> z_function(string s) {
7     int n = (int)s.length();vector<int> z(n);
8     for(int i = 1, l = 0, r = 0; i < n; ++i){
9         if(i <= r){z[i] = min(r - i + 1, z[i - 1]);}
10        while (i + z[i] < n && s[z[i]] == s[i + z[i]]){++z[i];}
11        if(i + z[i] - 1 > r){l = i, r = i + z[i] - 1; }
12    }return z;
}

```

Manacher

```

1 struct manacher { vector<int> p;
2     void run_manacher(string s) {
3         int n = sz(s);p.assign(n, 1);int l = 1,r = 1;
4         for(int i = 1; i < n; i++) {
5             p[i] = max(0LL,min(r - i,p[l + r - i]));
6             while(i + p[i] < n && i - p[i] >= 0 && s[i + p[i]] == s[i - p[i]]){
7                 p[i]++;
8             }
9             if(i + p[i] > r) {
10                 l = i - p[i];r = i + p[i];
11             }
12         }
}

```

```

13     }
14     void build(string s) {
15         string t = "#";for (char c : s) t += c, t += "#";run_manacher(t
16             );
17     }
18     int get_longest(int cen,bool odd) {
19         int pos = 2 * cen + 1 + (!odd);return p[pos] - 1;
20     }
21     bool is_palindrome(int l,int r) {
22         if((r - 1 + 1) <= get_longest((l + r) / 2,1 % 2 == r % 2))
23             return 1;
24         else return 0;
25     }
26 };manacher m;m.build(s);

```

Suffix Array

```

1 #define MAX_N 500010
2 string T; int RA[MAX_N],tempRA[MAX_N],n;
3 int SA[MAX_N],tempSA[MAX_N],c[MAX_N];int Phi[MAX_N],PLCP[MAX_N],LCP
4 [MAX_N];
5 void countingSort(int k){
6     int i, sum, maxi = max(300, n);memset(c, 0, sizeof c);
7     for(i=0;i<n;i++)c[i+k<n?RA[i+k]:0]++;
8     for(i = sum = 0; i < maxi; i++){
9         int t = c[i];c[i] = sum;sum += t;
10    }
11    for(i = 0; i < n; i++){tempSA[c[SA[i]+k < n ? RA[SA[i]+k] :
12        0]++] = SA[i];}
13    for(i = 0; i < n; i++)SA[i]=tempSA[i];
14    void constructSA(){
15        int i, k, r;for(i = 0; i < n; i++)RA[i] = T[i];
16        for(i = 0; i < n; i++)SA[i] = i;
17        for(k = 1; k < n; k <= 1){
18            countingSort(k);countingSort(0);tempRA[SA[0]] = r = 0;
19            for(i = 1; i < n; i++){tempRA[SA[i]] =(RA[SA[i]]==RA[SA[i
20                -1]] && RA[SA[i]+k]==RA[SA[i-1]+k])?r:+r;}
21            for(i = 0; i < n; i++)RA[i] = tempRA[i]; if(RA[SA[n-1]] ==
22                n-1)break;
23        }
24        void computeLCP() {
25            int i, L;Phi[SA[0]] = -1;for(i = 1; i < n; i++)Phi[SA[i]] = SA[
26                i-1];
27            for(i = L = 0; i < n; i++){
28                if(Phi[i] == -1) {PLCP[i] = 0;continue;}while(T[i + L] == T
29                    [Phi[i] + L])L++;
30                PLCP[i] = L;L = max(L-1, 0);
31            }
32        }
33    }for(i = 0; i < n; i++)LCP[i] = PLCP[SA[i]];
34 } // main function code -
35 cin >> T;T.pb('$');n = sz(T);constructSA();computeLCP();
36 1.Substring Search->Binary Search on Suffix array

```

```

31 2.Counting Substrings->Binary serach SA Upper-Lowerbound
32 3.Distinct Substrings->(n*(n+1))/2 - sumOf(LCP)
33 4.Longest Common Substring->T+'#'+Pat+'$',SAi < n1 && SAj > n1
   maxOf LCP
34 5.Finding Longest Substring that occur >= K time-> LCP K-1
   consecutive = k time max is length of Substring
35 6.Kth smallest substring->suflen = n-SA[i]-1,i=1 to n-1,new_substr=
   suflen-LCP[i],if new<=k,len=LCP[i]+k,start=SA[i]

```

Geometry

Basic Geometry

```

1 struct pt {
2     dl x, y;pt() : x(0), y(0) {} pt(dl a, dl b) : x(a), y(b) {}
3 };// Function to calculate the cross product (direction) of vectors
   (p1p2) and (p1p3)
4 int direction(const pt &p1, const pt &p2, const pt &p3) {
5     return (p2.x - p1.x) * (p3.y - p1.y) - (p2.y - p1.y) * (p3.x -
   p1.x);
6 }// Function to check if pt p3 lies on the segment p1p2
7 bool onsegment(const pt &p1, const pt &p2, const pt &p3) {
8     return min(p1.x, p2.x) <= p3.x && p3.x <= max(p1.x, p2.x) &&
9         min(p1.y, p2.y) <= p3.y && p3.y <= max(p1.y, p2.y);
10 }
11 pt rotateCCW(pt a,dl ang){
12     pt v = {a.x,a.y};dl c = cos(ang),s = sin(ang);
13     pt res = {v.x*c-v.y*s,v.x*s+v.y*c};
14     return res;}
15 pt rotateCCW(pt a,pt b,dl ang){
16     pt v = {b.x-a.x,b.y-a.y};// vec from a to b
17     dl c = cos(ang),s = sin(ang); // ang is radian
18     pt res = {v.x*c-v.y*s,v.x*s+v.y*c};
19     return res; //rotateCW is (-s put); }
20 pt unit(pt a){
21     double val = sqrt(a.x*a.x+a.y*a.y); return pt(a.x/val,a.y/val);
22 }
22 pt vec(pt a, pt b){
23     return {b.x - a.x, b.y - a.y};}
24 dl sqnorm(pt a){
25     return (a.x*a.x + a.y*a.y);}
26 //given point a,b and angle abc and len of bc then find point c
27 pt find_point(pt a,pt b,dl ang,dl len){
28     pt c = vec(b,a); c = unit(c);
29     c.x *= len, c.y *= len;
30     c = rotateCCW(c,ang);
31     c.x += b.x, c.y += b.y;
32     return c;}
33 pt find_point(pt b,dl ang,dl len){//same as up
34     pt c = {b.x + len*cos(ang),b.y + len*sin(ang)};
35     return c; // here ang is CCW , if CW it PI - ang}
36 dl dot(pt a, pt b){

```

```

37     return (a.x*b.x + a.y*b.y);}
38 dl angle(pt a,pt o,pt b){
39     pt oa = vec(o , a), ob = vec(o , b);
40     if(sqnorm(oa) == 0 || sqnorm(ob) == 0){
41         return 0;
42     }
43     double alpha = acos(dot(oa, ob) / sqrt(sqnorm(oa) * sqnorm(ob)))
44         );
45     alpha = alpha * 180 / PI;//angle in degree
46     return alpha;// angle of AOB}
47 double abs(pt a) {return sqrt(sqnorm(a));}
48 int triArea2(const pt &a, const pt &b, const pt &c) {
49     return (a.x*(b.y-c.y) + b.x*(c.y-a.y) + c.x*(a.y-b.y));
50 }
51 dl sqDist(const pt &a, const pt &b) {
52     return ((a.x-b.x)*(a.x-b.x) + (a.y-b.y)*(a.y-b.y));
53 }
54 dl angle(dl a, dl b,dl c){
55     dl alpha = acos((b*b-c*c-a*a)/(-2*a*c)); //alpha = alpha*180.0/
   PI;//angle (ABC) in degree
56     return alpha;}
57 dl Slope(pt a, pt b){
58     if(a.x == b.x && a.y == b.y) return oo;
59     double dx = b.x - a.x;
60     double dy = b.y - a.y;
61     if(dx == 0) return oo;
62     return (double)dy / dx;}
63 Basic Law > circle arc area = 0.5*r*r*theta ,radius of incircle r
   = A/s
64 radius of circumcircle R=(a*b*c)/(4*A),a spherical cap with height
   H and R = H/2
65 dl volume = (PI * h * h/ 3.0) * (3.0 * R - h);Polygone n point and
   same distance s
66 dl area_polygone=(n*s*s)/(4*(tan(PI/n)));dl redius_polygone = s /
   (2*sin(PI / n));
67 glass area = ((pi*h)/3) *((r1*r1)+(r2*r2)+(r1*r2)) ;N inner circle
   in One circle R
68 dl theta = PI / n;dl r = (R * sin(theta)) / (1 + sin(theta));dl
   area = PI*r*r;
69 dl E = (R - r);dl AreaNotcoverInner = ((n * r * r) / tan(theta)) -
   (area * (n - 2)) / 2.0;
70 dl AreaNotcoverOuter = Area - n * area - AreaNotcoverInner;
71 Tseq = (2^N - 1) = Even Subsequence + OddSubsequence ;Eseq = 2^Even
   ;
72 Oseq = 2^(Odd - 1);OddSumSeq = (2^Even * (2^(Odd - 1)));EvenSumSeq
   = ((2^N) - 1 - OddSumSeq);
73 Odd= Total number of odd in the array.;Oseq=Total number of
   subsequences with only odd;
Num ;OddSumSeq = Total number of subsequences with odd sum

```

Convex Hull

¹ ConvexHull : Grahams Scan O(n lg n), integer implementation ;P[]:

```

    holds all the points, C[]: holds points on the hull
2 np: number of points in P[], nc: number of points in C[]
3 to handle duplicate, call makeUnique() before calling convexHull()
    call convexHull() if you have np >= 3
4 to remove co-linear points on hull, call compress() after
    convexHull()
5 struct point{ int x, y; };
6 point P[N], C[N], P0;
7 int triArea2(const point &a, const point &b, const point &c) {
8     return (a.x*(b.y-c.y) + b.x*(c.y-a.y) + c.x*(a.y-b.y));
9 }
10 int sqDist(const point &a, const point &b) {
11     return ((a.x-b.x)*(a.x-b.x) + (a.y-b.y)*(a.y-b.y));
12 }
13 bool comp(const point &a, const point &b) {
14     int d = triArea2(P0, a, b); if(d < 0) return false;
15     if(!d && sqDist(P0, a) > sqDist(P0, b)) return false;
16     return true;
17 }
18 bool normal(const point &a, const point &b) {
19     return ((a.x==b.x) ? a.y < b.y : a.x < b.x);
20 }
21 bool issame(const point &a, const point &b) {
22     return (a.x == b.x && a.y == b.y);
23 }
24 void makeUnique(int &np) {
25     sort(&P[0], &P[np], normal);
26     np = unique(&P[0], &P[np], issame) - P;
27 }
28 void convexHull(int &np, int &nc) {
29     int i, j, pos = 0;
30     for(i = 1; i < np; i++){
31         if(P[i].y<P[pos].y || (P[i].y==P[pos].y && P[i].x<P[pos].x))
32             pos = i;
33     }
34     swap(P[0], P[pos]); P0 = P[0]; sort(&P[1], &P[np], comp);
35     j = np - 1, i = np - 1;
36     while(j > 0 && triArea2(P0, P[j], P[j-1]) == 0)j--;
37     while(i > j){
38         swap(P[i], P[j]); i--, j++;
39     }
40     for(i = 0; i < 3; i++) C[i] = P[i];
41     for(i = j = 3; i < np; i++) {
42         while(triArea2(C[j-2], C[j-1], P[i]) < 0) j--;
43         C[j++] = P[i];
44     }
45     nc = j;
46 }
47 void compress(int &nc) {
48     int i, j, d;
49     C[nc] = C[0];
50     for(i=j=1; i < nc; i++) {
            d = triArea2(C[j-1], C[i], C[i+1]);

```

```

51         if(d || (!d && issame(C[j-1], C[i+1]))) C[j++] = C[i];
52     }
53     nc = j;
54 }

```

Circle Intersection

```

1 dl commonArea(const Circle &a, const Circle &b) {
2     int dsq = sqDist(a, b); double d = sqrt((double)dsq);
3     if(sq(a.r + b.r) <= dsq) return 0;
4     if(a.r >= b.r && sq(a.r-b.r) >= dsq) return PI * b.r * b.r;
5     if(a.r <= b.r && sq(b.r-a.r) >= dsq) return PI * a.r * a.r;
6     double angleA = 2.0 * acos((a.r * a.r + dsq - b.r * b.r) / (2.0
7         * a.r * d));
8     double angleB = 2.0 * acos((b.r * b.r + dsq - a.r * a.r) / (2.0
9         * b.r * d));
10    return 0.5 * (a.r * a.r * (angleA - sin(angleA)) + b.r * b.r *
11        (angleB - sin(angleB)));
12 }

```

Point in Polygon

```

1 bool inConvexPoly(int nc, const pt &p) {
2     int st = 1, en = nc - 1, mid;
3     while(en - st > 1) {
4         mid = (st + en)>>1;
5         if(direction(C[0], C[mid], p) < 0) en = mid;
6         else st = mid;
7     }
8     if(direction(C[0], C[st], p) < 0) return false;
9     if(direction(C[st], C[en], p) < 0) return false;
10    if(direction(C[en], C[0], p) < 0) return false;
11    return true;
12 } //Ray casting algo to check point in convex -> start here
13 for(int i = 0; i < n; ++i){ // Check if the pt is on any boundary
14     edge
15     pt a = polygon[i];
16     pt b = polygon[(i + 1) % n];
17     if(onSegment(a, b, p)) {return "BOUNDARY";}
18 } // Ray casting algorithm ; int count = 0;
19 for(int i = 0; i < n; ++i) {
20     pt a = polygon[i]; pt b = polygon[(i + 1) % n];
21     if ((a.y > p.y) != (b.y > p.y)) { // Check if the pt is between
22         the y-coordinates of the edge
23         // Calculate the x-intersection of the edge with the horizontal
24         ray
25         double xIntersect = (double)(b.x - a.x) * (p.y - a.y) / (b.y -
a.y) + a.x;
26         if(p.x <= xIntersect) {count++;}
27     }
28 } if(count % 2 == 1){return "INSIDE";} else {return "OUTSIDE";}

```

Segment Intersection

```

1 struct pt { // Segment Intersection check
2     int x, y; pt() : x(0), y(0) {} pt(int a, int b) : x(a), y(b) {}  

3 };
4 // Function to calculate the cross product (direction) of vectors (p1p2)
5 // and (p1p3)
6 int direction(const pt &p1, const pt &p2, const pt &p3) {
7     return (p2.x - p1.x) * (p3.y - p1.y) - (p2.y - p1.y) * (p3.x -
8         p1.x);
9 }
10 // Function to check if pt p3 lies on the segment p1p2
11 bool onsegment(const pt &p1, const pt &p2, const pt &p3) {
12     return min(p1.x, p2.x) <= p3.x && p3.x <= max(p1.x, p2.x) &&
13         min(p1.y, p2.y) <= p3.y && p3.y <= max(p1.y, p2.y);
14 }
15 // Function to check if two segments (p1p2) and (p3p4) intersect
16 inline bool intersect(const pt &p1, const pt &p2, const pt &p3,
17     const pt &p4) {
18     int d1, d2, d3, d4;
19     d1 = direction(p3, p4, p1); // Direction of (p3, p4, p1)
20     d2 = direction(p3, p4, p2); // Direction of (p3, p4, p2)
21     d3 = direction(p1, p2, p3); // Direction of (p1, p2, p3)
22     d4 = direction(p1, p2, p4); // Direction of (p1, p2, p4)
23     // General case: Segments (p1p2) and (p3p4) intersect if
24     // directions differ
25     if (((d1 < 0 && d2 > 0) || (d1 > 0 && d2 < 0)) && ((d3 < 0 &&
26         d4 > 0) || (d3 > 0 && d4 < 0))) return true;
27     // Special cases: Check if the pts are collinear and lie on the
28     // segment
29     if (!d3 && onsegment(p1, p2, p3)) return true; // p3 is on
30     segment (p1p2)
31     if (!d4 && onsegment(p1, p2, p4)) return true; // p4 is on
32     segment (p1p2)
33     if (!d1 && onsegment(p3, p4, p1)) return true; // p1 is on
34     segment (p3p4)
35     if (!d2 && onsegment(p3, p4, p2)) return true; // p2 is on
36     segment (p3p4)
37     return false;
38 } // Segment intersection point start here
39 pair<bool, pt> doSegmentsIntersect(const pt &p1, const pt &p2, const
40 pt &p3, const pt &p4) {
41     int d1 = direction(p3, p4, p1); d1 d2 = direction(p3, p4, p2);
42     d1 d3 = direction(p1, p2, p3); d1 d4 = direction(p1, p2, p4);
43     if (d1 * d2 < 0 && d3 * d4 < 0) { // Cramer's Rule.
44         d1 det = (p1.x - p2.x) * (p3.y - p4.y) - (p1.y - p2.y) * (
45             p3.x - p4.x);
46         d1 ix = ((p1.x*p2.y - p1.y*p2.x)*(p3.x - p4.x) - (p1.x -
47             p2.x)*(p3.x*p4.y - p3.y*p4.x)) / det;
48         d1 iy = ((p1.x*p2.y - p1.y*p2.x)*(p3.y - p4.y) - (p1.y -
49             p2.y)*(p3.x*p4.y - p3.y*p4.x)) / det;
50         return {true, pt(ix, iy)};
51 } // Special cases - endpts on other segment

```

```

37     if (d1 == 0 && onSegment(p3, p4, p1)) return {true, p1};
38     if (d2 == 0 && onSegment(p3, p4, p2)) return {true, p2};
39     if (d3 == 0 && onSegment(p1, p2, p3)) return {true, p3};
40     if (d4 == 0 && onSegment(p1, p2, p4)) return {true, p4};
41     return {false, pt(0,0)};
42 }

```

Polygon Area

```

1 int area = 0, b = 0;
2 for(int i = 0; i < n; i++){
3     Point l = p[i], r = p[i+1]; area += (l.x * r.y - r.x * l.y); l.x
4         -= r.x, l.y -= r.y;
5     int g = gcd(l.x, l.y); b += abs(g);
6     peaks theorem a = abs(area) - b + 2; area = abs(area) / 2; // same
7     same but
8     A=I+2 B 1 , A = area of polygone, I = inside point, B = Boundary
9     point; p[n] = p[0]; int area = 0, b = 0;
10    for(int i = 0; i < n; i++){
11        Point l = p[i], r = p[i+1]; area += (l.x * r.y - r.x * l.y);
12        l.x -= r.x, l.y -= r.y; int g = gcd(l.x, l.y); b += abs(g);
13    } int a = abs(area) - b + 2;

```

Distance from Point to line Segment

```

1 // Function to calculate the distance from point C to line segment
2 // AB
3 double linePointDist(int A[], int B[], int C[], bool isSegment) {
4     int ABx = B[0] - A[0], ABy = B[1] - A[1]; int ACx = C[0] - A[0],
5         ACy = C[1] - A[1];
6     int BCx = C[0] - B[0], BCy = C[1] - B[1]; double crossProd = ABx
7         * ACy - ABy * ACx;
8     double ABLength = sqrt(ABx * ABx + ABy * ABy); double dist =
9     fabs(crossProd) / ABLength;
10    if (isSegment) {
11        if ((ABx * BCx + ABy * BCy) > 0) return sqrt(BCx * BCx +
12            BCy * BCy); // Distance to B
13        if ((-ABx * ACx - ABy * ACy) > 0) return sqrt(ACx * ACx +
14            ACy * ACy); // Distance to A
15    } return abs(dist); }

```

Circle from 3 Points

```

1 // Function to find the center of the circle given 3 points
2 pair<dl, dl> findCircleCenter(dl x1, dl y1, dl x2, dl y2, dl x3, dl
3 y3) {
4     dl mx1 = (x1 + x2) / 2, my1 = (y1 + y2) / 2, mx2 = (x2 + x3) /
5         2, my2 = (y2 + y3) / 2;
6     dl A1 = y2 - y1, B1 = x1 - x2, A2 = y3 - y2, B2 = x2 - x3; dl
7     det = A1 * B2 - A2 * B1;
8     dl cx = (B2 * (A1 * mx1 - A2 * mx2) - B1 * (A1 * mx2 - A2 * mx1
9         )) / det;

```

```

6     dl cy = (A1 * (B2 * my2 - B1 * my1) - A2 * (B1 * my1 - B2 * my2)
7         ) / det;
8     return {cx, cy};
}

```

Advanced Algorithms

Heavy-Light Decomposition

```

1 const ll N = 1e5 + 123; const ll M = 4*N + 5; vi graph[N]; ll tree[M];
2 int Sz[N],heavy[N],Par[N],lev[N]; int Head[N],pos[N],idx = 1,ar[N],
3 val[N];
4 void dfs(int node,int pa){
5     Sz[node] = 1; Par[node] = pa;
6     for(auto u : graph[node]){
7         if(u == pa) continue;
8         lev[u] = lev[node] + 1;dfs(u,node);Sz[node] += Sz[u];
9         if(Sz[u] > Sz[heavy[node]] || heavy[node] == 0){
10            heavy[node] = u;
11        }
12    }
13 void dfsHLD(int node,int chain){
14     Head[node] = chain;ar[idx] = val[node];pos[node] = idx;idx++;
15     if(heavy[node] != 0){
16         dfsHLD(heavy[node],chain);
17     }
18     for(auto u : graph[node]){
19         if(u == Par[node]) continue;
20         if(heavy[node] != u){dfsHLD(u,u);}
21     }
22 }
23 int Query(int a,int b){
24     int mx = 0;
25     while(Head[a] != Head[b]){
26         if(lev[Head[a]] < lev[Head[b]]){ swap(a,b);}
27         mx = max(mx,query(1,1,idx,pos[Head[a]],pos[a]));
28         a = Par[Head[a]];
29     }
30     if(lev[a] < lev[b])swap(a,b);int l = pos[b]+1,r = pos[a];
31     if(l <= r)mx = max(mx,query(1,1,idx,l,r));//edge base b+1 to a,
32     node base b to a
33     return mx;
34 } // main function start->
35 int n,q;cin >> n;
36 for(int i = 1; i <= n; i++){
37     graph[i].clear();Sz[i] = 0;lev[i] = 0;heavy[i] = 0;pos[i] = 0,
38     val[i] = 0;Head[i] = 0,Par[i] = 0;
39 }
40 map<int,pair<int,int>> bd;vector<tu> edge;
41 for(int i = 1; i < n; i++){
42     int u,v,w;cin >> u >> v >> w;graph[u].push_back(v);graph[v].

```

```

        push_back(u);
43     bd[i] = {u,v};edge.push_back({u,v,w});}
44     idx = 1;dfs(1,0);val[1] = 0;
45     for(auto u : edge){int a = u[0];if(lev[u[0]] < lev[u[1]])a = u[1];
46         val[a] = u[2];}
47     dfsHLD(1,1);init(1,1,idx);
48     while(1){string s;cin >> s;if(s == "DONE")break;
49         if(s == "CHANGE"){
50             int i,v;cin >> i >> v;pair<int,int> x = bd[i];int a = x.F;
51             if(lev[x.F] < lev[x.S])a = x.S;int del = pos[a];upgrade
52                 (1,1,idx,del,v);}
53         else{int a,b;cin >> a >> b;cout << Query(a,b) << endl;}
54     }
}

```

Mo's Algorithm

```

1 int ar[N],ans,n,answers[N],cnt[N];int block_size;
2 void remove(int idx){ cnt[ar[idx]]--; if(cnt[ar[idx]] == 0)ans--;}
3 void add(int idx){ cnt[ar[idx]]++; if(cnt[ar[idx]] == 1)ans++;}
4 struct Query{
5     int l, r, idx;
6     bool operator < (const Query &x) const {
7         if(l / block_size == x.l / block_size) return r > x.r;
8         return l / block_size < x.l / block_size;}
9 };// main function
10 void Goriber_solve(){cin >> n; for(int i = 0; i < n; i++){cin >> ar
11 [i];}int q;cin >> q;
12 block_size = (int) sqrt (n + .0) + 1;vector<Query> queries;
13 for(int i = 0; i < q; i++){int l,r;cin >> l >> r;queries.pb({l,
14 r,i});}
15 sort(all(queries));int cur_l = 0,int cur_r = -1;
16 for (Query &q : queries) {q.l--, q.r--;
17     while (cur_l > q.l) {cur_l--;add(cur_l);}
18     while (cur_r < q.r) {cur_r++;add(cur_r);}
19     while (cur_l < q.l) {remove(cur_l);cur_l++;}
20     while (cur_r > q.r) {remove(cur_r);cur_r--;}
21     answers[q.idx] = ans;}
22 for(int i = 0; i < q; i++){
23     cout << answers[i] << endl;}
}

```

Square Root Decomposition

```

1 int n;vector<int> a(n);int len = (int) sqrt (n + .0) + 1;vector<
2     int> b(len);
3 for(int i=0; i<n; ++i) b[i / len] += a[i];
4 while(q--) {
5     int l, r;int sum = 0;
6     for(int i=l; i<=r; ){
7         if(i % len == 0 && i + len - 1 <= r) {
8             sum += b[i / len];i += len;}
9         else {sum += a[i];++i;}
10    }
}

```

Euler Tour

```

1 vector<int> graph[N]; vector<int> flat_tree; map<int, pair<int, int>>
    mp; int cnt = 0;
2 void dfs(int node, int par) {
3     flat_tree.push_back(node); mp[node].first = cnt; cnt++;
4     for(auto child : graph[node]) {
5         if(child == par) continue; dfs(child, node);
6     }
7     mp[node].second = cnt; flat_tree.push_back(node); cnt++;
}

```

Tree Diameter

```

1 vi graph[N];
2 void dfs(int node, int par, int lev, vi &d){ d[node] = lev;
3     for(auto u : graph[node]){
4         if(u != par){ dfs(u,node,lev+1,d);}
5     }
6 }
7 void Goriber_solve(){
8     int n; cin >> n; for(int i = 0; i <= n; i++) {graph[i].clear();}
9     for(int i = 1; i < n; i++){
10         int u,v; cin >> u >> v; u--, v--; graph[u].pb(v); graph[v].pb(u)
11         ;
12     }
13     vi d1(n,0),d2(n,0); dfs(0,-1,0,d1); int a = max_element(all(d1))
14         - d1.begin(); dfs(a,-1,0,d1);
15     int b = max_element(all(d1)) - d1.begin(); dfs(b,-1,0,d2); }

```

2D Algorithms

2D Pre Sum + mat Rotation + grid 8 direction move

```

1 pre[i][j] = ar[i][j] + pre[i-1][j] + pre[i][j-1] - pre[i-1][j-1];
2 cout << pre[c][d] - pre[a-1][d] - pre[c][b-1] + pre[a-1][b-1];
3 for(int i = 0; i < n; i++) {
4     for(int j = 0; j < n; j++) {res[j][n - i - 1] = mat[i][j];}
5 }
6 int fx[] = {+0,+0,+1,-1,-1,+1,-1,+1}; int fy[] =
7 {-1,+1,+0,+0,+1,+1,-1,-1};

```

Spiral Matrix

```

1 int x = (n + 1) / 2, y = (n + 1) / 2; int num = 0; ans[x][y] = 0; // based matrix
2 for (int i = 2; i <= n; ++i) {
3     if (i % 2 == 0) { ans[x][++y] = ++num;
4         for (int j = 1; j < i; ++j) ans[++x][y] = ++num;
5         for (int j = 1; j < i; ++j) ans[x][--y] = ++num;
6     }
7     else {ans[x][--y] = ++num;
8         for (int j = 1; j < i; ++j) ans[--x][y] = ++num;
}

```

```

9         for (int j = 1; j < i; ++j) ans[x][++y] = ++num;
10    }
}

```

Matrix Exponentiation

```

1 vector<vi> matMul(vector<vi>&mat1, vector<vi>&mat2, int n){ vector<vi>
    res(n, vi(n,0));
2     for(int i = 0; i < n; i++){
3         for(int j = 0; j < n; j++){
4             for(int k = 0; k < n; k++){res[i][j]=(res[i][j]+((ll)mat1[i]
5                 ][k]*mat2[k][j])%mod)%mod;}
6         }
7     }
8     return res;
9 }
10 vector<vi> matExpo(vector<vi>& mat, int k, int n){
11     if(k == 1) return mat; vector<vi> res=matExpo(mat,k/2,n); res =
12         matMul(res,res,n);
13     if(k & 1) res=matMul(res,mat,n);
14     return res;
15 }
16 F(k+1) = | mat^k | F(k) |
17 F(k) = | | F( k 1 ) |
18 S(k) = | | S( k 1 ) |
19 S( k 1 ) = | | S( k 2 ) |
20 vi mulBase(vector<vi>&mat, vi &base, int n){ vi res(n,0);
21     for(int i = 0; i < n; i++){
22         for(int j = 0; j < n; j++){
23             res[i] += (ll)mat[i][j] * base[j] % mod; res[i] %= mod;
}
24     }
25 }
26 } return res;
27 }

```

Utilities

Path Compression

```

1 vi a(n), b(n); sort(all(b)); b.erase(unique(all(b)), b.end());
2 for(int i = 0; i < n; i++) {a[i] = lower_bound(all(b), a[i]) - b.
begin() + 1;}

```

Histogram

```

1 int getMaxArea(vector<int>&arr, int n) {
2     stack<int> s; s.push(-1); int area = arr[0]; int i = 0; vector<int>
        left(n, -1), right(n, n);
3     while(i < n) {
4         while(!s.empty() && s.top() != -1 && arr[s.top()] > arr[i]) {
5             right[s.top()] = i; s.pop();
6         }
7         if(i > 0 && arr[i] == arr[i - 1]) {
8             left[i] = left[i - 1];
9         } else { lefr[i] = s.top(); }
}

```

```

10     s.push(i);i++;
11 }
12 for (int j = 0; j < n; j++) {
13     area = max(area, arr[j] * (right[j] - left[j] - 1));
14 } return area;
15 }
```

Is Intersect All Points

```

1 auto f = [&] (dl m) -> bool {vector<pair<dl,dl>> p;
2     for(int i = 0; i < n; i++) {
3         dl r = x[i] + m*v[i];dl l = x[i] - m*v[i];p.push_back({l,r});
4     }
5     dl l = -INF,r = INF;
6     for(int i = 0; i < n; i++) {
7         dl left = p[i].F;dl right = p[i].S;
8         if(l > right || r < left) {return 0;} l = max(l,left); r = min(25
9             r,right);
10    } return 1;
11};
```

Unique Line Equations

```

1 tuple<int,int,int> norm(int a, int b, int c) {
2     int d = (_gcd(a,_gcd(b,c))); a/=d; b/=d; c/=d;int s = a?a:b?b:c
3     ;
4     if(s < 0) a=-a, b=-b, c=-c;
5     return {a,b,c};
6 }
7 int countLines(vector<pair<int,int>>& p) {
8     set<tuple<int,int,int>> s;
9     for(int i=0; i<p.size(); i++) {
10        for(int j=i+1; j<p.size(); j++) {
11            auto [x1,y1]=p[i], [x2,y2]=p[j]; int a=y2-y1, b=x1-x2, c=x2*
12                y1-x1*y2; s.insert(norm(a,b,c));
13        }
14    }return s.size();
15 }
```

Special Problems

Kth Number L to R Using Persistent Seg Tree

```

1 //number of elements greater than k in the subsequence ai, ai+1 ...
2     aj
3 struct node{
4     int val;
5     node* lc;
6     node* rc;
7 };
8 int ar[N];
node* versions[N+5];
```

```

9     vector<node*> lagbe;
10
11 node* build(int l,int r){
12     if(l == r){
13         node* me = new node();
14         lagbe.pb(me);
15         me->val = 0;
16         me->lc = NULL;
17         me->rc = NULL;
18         return me;
19     }
20     node* my = new node();
21     lagbe.pb(my);
22     int mid = (l + r)/2;
23     my->lc = build(l,mid);
24     my->rc = build(mid+1,r);
25     my->val = (my->lc->val + my->rc->val);
26     return my;
27 }
28 int query(node* n1,node* n2,int l,int r,int L,int R){
29     if(n1 == NULL || n2 == NULL) return 0;
30     if(R < l || r < L) return 0;
31     if(L <= l && r <= R){
32         return n1->val - n2->val;
33     }
34     int mid = (l + r)/2;
35     int p = query(n1->lc,n2->lc,l,mid,L,R);
36     int q = query(n1->rc,n2->rc,mid+1,r,L,R);
37     return p + q;
38 }
39 node* update(node* my,int l,int r,int ind,int val){
40     if(l == r){
41         node* new_my = new node();
42         lagbe.pb(new_my);
43         new_my->lc = NULL;
44         new_my->rc = NULL;
45         new_my->val = my->val + val;
46         return new_my;
47     }
48     node* new_me = new node();
49     lagbe.pb(new_me);
50     int mid = (l + r)/2;
51     if(ind <= mid){
52         node* x = update(my->lc,l,mid,ind,val);
53         new_me->lc = x;
54         new_me->rc = my->rc;
55     }
56     else{
57         node* y = update(my->rc,mid+1,r,ind,val);
58         new_me->lc = my->lc;
59         new_me->rc = y;
60     }
61     new_me->val = new_me->lc->val + new_me->rc->val;
```

```

62     return new_me;
63 }
64 vi com;
65 void Goriber_solve()
66 {
67     int n,q;
68     cin >> n;
69     for(int i = 1; i <= n; i++){
70         cin >> ar[i];
71         com.pb(ar[i]);
72     }
73     sort(all(com));
74     com.resize(unique(all(com))-com.begin());
75     map<int,int> pos;
76     int x = 0;
77     for(auto u : com)pos[u] = x++;
78     int si = sz(com)+1;
79     versions[0] = build(0,si);
80     for(int i = 1; i <= n; i++){
81         versions[i] = update(versions[i-1],0,si,pos[ar[i]],1);
82     }
83     int ans = 0;
84     cin >> q;
85     while(q--){
86         int a,b,c;
87         cin >> a >> b >> c;
88         a = (a ^ ans);
89         b = (b ^ ans);
90         c = (c ^ ans);
91         int val = upper_bound(all(com),c)-com.begin();
92         if(a <= 0)a = 1;
93         if(b > n)b = n;
94         if(a > b){
95             cout << 0 << endl;
96             ans = 0;
97             continue;
98         }
99         ans = query(versions[b],versions[a-1],0,si,val,si);
100        cout << ans << endl;
101    }
102    for(auto u : lagbe){
103        delete u;
104    }
105 }

```

How Many Subarrays XOR < k

```

1 class Trie{
2     int cnt(int num, int k){
3         Node* cur = root;int ret = 0;
4         for(int i = 31; i >= 0; i--){
5             int num1 = (num >> i) & 1;int num2 = (k >> i) & 1;
6             if(num2){

```

```

7                 if(cur -> contains(num1)){ ret += cur -> get(num1)
8                     -> getPre();}
9                 if(cur -> contains(num1 ^ 1))cur = cur -> get(num1
10                    ^ 1);
11                 else return ret;
12             }
13             else{
14                 if(cur -> contains(num1))cur = cur -> get(num1);
15                 else return ret;
16             }
17         }
18     }
19 };// main function trie.insert(0); xr ^= ar[i]; ans += trie.cnt(xr,
20 k);trie.insert(xr);

```

Min Length with Max XOR $\geq k$

```

1 class Trie {
2     int getMax(int num,int lo) {
3         Node* cur_node = root;int mx = 0;
4         for(int i = 31; i >= 0; i--) {
5             int bit = (num >> i) & 1;int opo = bit ^ 1;
6             if(cur_node -> contains(opo) && cur_node -> get(opo) ->
7                 mx_index >= lo) {
8                 mx = ((1LL << i) | mx);cur_node = cur_node -> get(opo);
9             }
10            else {cur_node = cur_node -> get(bit);}
11        }
12        return mx;
13    }
14 };// main function -> int n,k; cin >> n >> k; int ans = n + 1; Trie
15     trie;
16     for(int i = 1; i <= n; i++) { int x; cin >> x; trie.insert(x,i);
17         int lo = 1, hi = i,cnt = -1;
18         while(lo <= hi) { int mid = (lo + hi) / 2;
19             if((trie.getMax(x,mid)) >= k) { cnt = mid; lo = mid + 1;}
20             else hi = mid - 1;
21         } if(cnt != -1)ans = min(ans,i + 1 - cnt);
22     } if(ans == n + 1)cout << -1 << endl; else cout << ans << endl;

```

$$I < J < K \text{ and } a_i > a_j > a_k$$

```

1 vi a(n), b(n); for(int i = 0; i < n; i++) { cin >> a[i]; b[i] = a[i]
2     ]
3 sort(all(b)); b.erase(unique(all(b)), b.end());
4 for(int i = 0; i < n; i++) { a[i] = lower_bound(all(b), a[i]) - b.
5     begin() + 1; } vi L(n), R(n);
6 for(int i = 0; i < n; i++) {
7     L[i] = sum(a[i]+1,n+1); update(a[i],1); mem(tree,0); // use BIT}
8     for(int i = n-1; i >= 0; i--) {
9         R[i] = sum(1,a[i]-1); update(a[i],1);
10    } int ans = 0;

```

```

9   for(int i = 0; i < n; i++) {
10    ans += L[i] * R[i];
11 } cout << ans << endl;

```

Tetrahedron Formulas

```

1 glass area = ((pi*h)/3) *((r1*r1)+(r2*r2)+(r1*r2)) //Some
   tetrahedron formulas
2 inline double volume(double u, double v, double w, double U, double
   V, double W) {
3   double u1,v1,w1; u1 = v * v + w * w - U * U;
4   v1 = w * w + u * u - V * V; w1 = u * u + v * v - W * W;
5   return sqrt(4.0*u*u*v*v*w*w - u*u*u1*u1 - v*v*v1*v1 - w*w*w1*w1
   + u1*v1*w1) / 12.0; }
6 double surface(double a, double b, double c) {
7   return sqrt((a + b + c) * (-a + b + c) * (a - b + c) * (a + b -
   c)) / 4.0}
8 double insphere(double WX, double WY, double WZ, double XY, double
   XZ, double YZ) {
9   double sur, rad;
10  sur = surface(WX, WY, XY) + surface(WX, XZ, WZ) + surface(WY,
   YZ, WZ) + surface(XY, XZ, YZ);
11  rad = volume(WX, WY, WZ, YZ, XZ, XY) * 3.0 / sur;   return rad
   ;}

```

Templates

Template with PBDS

```

1 #include <ext/pb_ds/assoc_container.hpp>
2 #include <ext/pb_ds/tree_policy.hpp>
3 using namespace __gnu_pbds;template <class T>
4 using ordered_set = tree<T,null_type,less<T>, rb_tree_tag,
   tree_order_statistics_node_update>;
5 ios_base::sync_with_stdio(0);cin.tie(0);cout.tie(0);
6 #define endl '\n' #define F first #define S second
7 #define all(s) (s).begin(), (s).end()
8 #define rall(s) s.rbegin(), s.rend()
9 #define sz(x) (int)x.size()
10 #define mem(a, b) memset(a, b, sizeof(a))
11 #define cnt_bit(x) __builtin_popcountll(x)
12 #define getbit(n, i) (((n) & (1LL << (i))) != 0)
13 #define msb(x) 31 - __builtin_clz(x)
14 typedef array<int,3> tu;
15 mt19937_64 gen(random_device{}());int r_num =
   uniform_int_distribution<long long>{0, 1e18}(gen);
16 view->layout 3 column then group max 2 column
17 Sublime text setup ->tool->build system->newbuild
18 {
19   "shell_cmd": "g++ ${file} -o a.exe && a.exe < input.in >
   output.in"
20 }

```

```

21 {
22   "cmd": [
23     "g++ -std=c++20 \"${file_name}\" -o \"${file_base_name}\" &&
       timeout 4s ./\"${file_base_name}\" < input.txt > output.
       txt"
24   ],
25   "selector": "source.c++",
26   "shell": true,
27   "working_dir": "${file_path}"
28 }

```

GP Hash Table

```

1 struct custom_hash {
2   static uint64_t splitmix64(uint64_t x) {
3     x += 0x9e3779b97f4a7c15;
4     x = (x ^ (x >> 30)) * 0xb5f58476d1ce4e5b9;
5     x = (x ^ (x >> 27)) * 0x94d049bb13311eb;
6     return x ^ (x >> 31);
7   }
8   size_t operator()(uint64_t x) const {
9     static const uint64_t FIXED_RANDOM = chrono::steady_clock::now
       ().time_since_epoch().count();
10    return splitmix64(x + FIXED_RANDOM);
11  }
12  size_t operator()(const pair<int, int>&x) const{
13    return ((long long)x.first) ^ (((long long)x.second) << 32);
14  }
15}; unordered_map<pair<ll, ll>, vector<ll>, custom_hash> bd;
16 unordered_map<pair<ll, ll>, ll, custom_hash> dp;

```

Saad Segment Tree

```

1 struct ST {
2   struct Node { // change here
3     long long sum = 0; Node () {}
4   };
5   vector<long long> a;
6   vector<Node> t;
7   Node op (Node x, Node y) { // change here
8     x.sum += y.sum;
9     return x;
10  }
11  ST (vector<long long> &A) { // change here
12    a = A;
13    t.resize (4 * a.size());
14    bld (1, 0, a.size() - 1);
15  }
16  void bld (int i, int l, int r) {
17    if (l == r) {
18      t[i].sum = a[l];
19      return;
20    }

```

```

21     int m = (l + r) / 2;
22     bld (i * 2, l, m);
23     bld (i * 2 + 1, m + 1, r);
24     t[i] = op (t[i * 2], t[i * 2 + 1]);
25 }
26 void upd (int i, int l, int r, int p, int v) {
27     if (l == r) {
28         a[p] += v; // change here
29         t[i].sum = a[p];
30         return;
31     }
32     int m = (l + r) / 2;
33     if (p <= m) upd (i * 2, l, m, p, v);
34     else upd (i * 2 + 1, m + 1, r, p, v);
35     t[i] = op (t[i * 2], t[i * 2 + 1]);
36 }
37 Node qry (int i, int l, int r, int L, int R) {
38     if (R < l || r < L) return Node();
39     if (L <= l && r <= R) return t[i];
40     int m = (l + r) / 2;
41     return op (qry (i * 2, l, m, L, R),
42                 qry (i * 2 + 1, m + 1, r, L, R));
43 }
44 };
45 struct ST {
46     struct N {
47         long long s = 0, m = 1e18;
48     };
49     vector<N> t;
50     vector<long long> a, lz;
51     int n;
52     N op (N x, N y) {
53         x.s += y.s;
54         x.m = min (x.m, y.m);
55         return x;
56     }
57     void push (int i, int l, int r, long long v) {
58         t[i].s = v * (r - l + 1);
59         t[i].m = v;
60         if (l != r) lz[i * 2] = lz[i * 2 + 1] = v;
61         lz[i] = 0;
62     }
63     void build (int i, int l, int r) {
64         if (l == r) {
65             t[i].s = a[l];
66             t[i].m = a[l];
67             return;
68         }
69         int m = (l + r) / 2;
70         build (i * 2, l, m);
71         build (i * 2 + 1, m + 1, r);
72         t[i] = op (t[i * 2], t[i * 2 + 1]);
73     }

```

```

74     void upd (int i, int l, int r, int L, int R, long long v) {
75         if (lz[i]) push (i, l, r, lz[i]);
76         if (r < L || R < l) return;
77
78         if (L <= l && r <= R) {
79             push (i, l, r, v);
80             return;
81         }
82
83         int m = (l + r) / 2;
84         upd (i * 2, l, m, L, R, v);
85         upd (i * 2 + 1, m + 1, r, L, R, v);
86         t[i] = op (t[i * 2], t[i * 2 + 1]);
87     }
88     N qry (int i, int l, int r, int L, int R) {
89         if (r < L || R < l) return N();
90         if (lz[i]) push (i, l, r, lz[i]);
91         if (L <= l && r <= R) return t[i];
92
93         int m = (l + r) / 2;
94         return op (qry (i * 2, l, m, L, R), qry (i * 2 + 1, m + 1,
95                                         r, L, R));
96     }
97     ST (vector<long long> &A) {
98         a = A;
99         n = A.size();
100        t.resize (4 * n);
101        lz.assign (4 * n, 0);
102        build (1, 0, n - 1);
103    }
104    void InUpdate (int i, long long v) {
105        upd (1, 0, n - 1, i, i, v);
106    }
107    void R_Update (int L, int R, long long v) {
108        upd (1, 0, n - 1, L, R, v);
109    }
110    N query (int L, int R) {
111        return qry (1, 0, n - 1, L, R);
112    }
113 }
114 };

```

Kaium Segment Tree

```

1 struct Node { // Maximum Subarray Sum
2     int sum;
3     int sub; // Value of interest
4     int pre;
5     int suf;
6
7     Node(): sum(0), sub(-1e18), pre(sub), suf(sub) {}
8     Node(int x): sum(x), sub(x), pre(x), suf(x) {}

```

```

9  operator int() { return sub; }
10 Node operator+(const Node& b) {
11     Node c;
12     c.sum = sum + b.sum;
13     c.sub = max({ sub, b.sub, suf+b.pre });
14     c.pre = max(pre, sum+b.pre);
15     c.suf = max(b.suf, suf+b.sum);
16
17     return c;
18 }
19 };
20
21 struct SegTree {
22     int n;
23     vector<Node> Tree;
24     SegTree(int size) {
25         // Round up to closest power of 2
26         n = 1 << (32 - __builtin_clz(--size));
27         Tree.resize(2*n);
28     }
29
30     // Constructor to initialize by array
31     SegTree(const VI& A): SegTree(A.size()) {
32         for (int i = 0; i < A.size(); i++)
33             Tree[n + i] = Node(A[i]);
34         // Build the segment tree
35         ROF (i, 1, n-1)
36         Tree[i] = Tree[2*i] + Tree[2*i+1];
37     }
38     // Update value at position p
39     void update(int p, int value) {
40         p += n;
41         Tree[p] = Node(value);
42         for (p /= 2; p > 0; p /= 2)
43             Tree[p] = Tree[2*p] + Tree[2*p+1];
44     }
45     // Query on interval [l, r)
46     Node query(int l, int r) {
47         Node resL, resR;
48         for (l += n, r += n; l < r; l /= 2, r /= 2) {
49             if (l % 2) resL = resL + Tree[l++];
50             if (r % 2) resR = Tree[--r] + resR;
51         } // Non-commutative merge
52         return resL + resR;
53     }
54 };

```

Kaium I/O

```

1 #define DEBUG
2 #define LOCAL
3 // #define TCASE
4 // #define PB_DS

```

```

5
6 #include <bits/stdc++.h>
7 using namespace std;
8
9 #ifndef LOCAL
10 #define _LB_ cerr << "{ "
11 #define _RB_ cerr << " }"
12 #else
13 #define endl '\n'
14 #define _LB_
15 #define _RB_
16 #endif
17
18 #define _SP_ << &" "[!i++]
19 #define _FN_OUT_ { _LB_; int i = 0; for (auto& a : A) out _SP_ a;
20         _RB_; return out; }
21 #define _FN_IN_ { for (auto& a : A) in >> a; return in; }
22 #define _OP_OUT_ ostream& operator<<(ostream& out, const
23 #define _OP_IN_ istream& operator>>(istream& in,
24
25 #define temT template <class T>
26 #define temTs template <class... Ts>
27 #define temTN template <class T, size_t N>
28
29 temTs _OP_OUT_ pair<Ts...>& p) { _LB_; out << p.first << " " << p.
30         second; _RB_; return out; }
31 temTs _OP_IN_ pair<Ts...>& p) { return in >> p.first >> p.second; }
32 temTN _OP_OUT_ array<T, N>& A) _FN_OUT_
33 temTN _OP_IN_ array<T, N>& A) _FN_IN_
34 #define def_out(cont) temT _OP_OUT_ cont<T>& A) _FN_OUT_
35 #define def_in(cont) temT _OP_IN_ cont<T>& A) _FN_IN_
36 def_in(vector) def_in(deque) def_out(vector) def_out(deque) def_out
37         (set) def_out(multiset)
38 temTs _OP_OUT_ map<Ts...>& A) _FN_OUT_
39
40 #ifdef PB_DS
41 #include <ext/pb_ds/assoc_container.hpp>
42 #include <ext/pb_ds/tree_policy.hpp>
43 using namespace __gnu_pbds;
44 #define _RBT_ rb_tree_tag, tree_order_statistics_node_update
45 temT using ordered_set = tree<T, null_type, less<T>, _RBT_>;
46 temT using ordered_multiset = tree<T, null_type, less_equal<T>,
47         _RBT_>;
48 def_out(ordered_set) def_out(ordered_multiset)
49 #endif
50
51 temTs istream& c_in(Ts... args) { return ((cin >> args), ...); }
52 temTs ostream& c_out(const Ts... args) { int i = 0; return ((cout
53         _SP_ args), ...) << endl; }
54 ostream& c_out(bool b) { return c_out(b ? "YES" : "NO"); }
55
56 #ifdef LOCAL

```

```

53 temTs ostream& c_err(const Ts&... args) { int i = 0; return ((cerr
54 _SP_ args), ...) << endl; }
55 #else
56 #define c_err(...)
57 #endif
58 #define d_bug(args...) c_err(#args, '=', args)
59
60 #define all(A) A.begin(), A.end()
61 #define rall(A) A.rbegin(), A.rend()
62 #define sum_of(A) accumulate(all(A), 0ll)
63 #define min_of(A) *min_element(all(A))
64 #define max_of(A) *max_element(all(A))
65
66 #define int long long
67 #define uint unsigned int
68 using VI = vector<int>;
69 using VC = vector<char>;
70 using DI = deque<int>;
71 using QI = queue<int>;
72 using SI = set<int>;
73 using MII = map<int, int>;
74 using VVI = vector<VI>;
75 template<size_t N> using AI = array<int, N>;
76 using II = AI<2>; // instead of pair<int, int>
77 using III = AI<3>; // ... tuple<int, int, int>
78 using VII = vector<II>;
79 using VIII = vector<III>;
80 using VVII = vector<VII>;
81 template<class T> using PQ = priority_queue<T, vector<T>, greater<T
82 >>;
83 #define VMAT(n, m) VVI(n, VI(m))
84 #define VMATA(n, m, a) VVI(n, VI(m, a))
85 #define AMAT(n, m) vector<AI<m>>(n)
86 #define FOR(i, l, r) for (int i = l; i <= r; i++)
87 #define ROF(i, l, r) for (int i = r; i >= l; i--)

II pow = { 1, 1 };
for (int i = 0; i < n; i++) {
    int c = s[i] - 'a' + 1;
    sum = (sum + II{ c, c } * pow % mod) % mod;
    pref[i+1] = sum;
}
pow = pow * base % m;
}

II substr_hash(int i, int j) { // [i, j)
    i = max(0ll, min(n, i));
    j = max(0ll, min(n, j));
    II hash = mod + pref[j] - pref[i];
    hash = hash % mod * inv[i] % mod;
    return hash;
}
};

Min Cut

```

Kaium Hashing

```

1 II operator+(II a, II b) { return { a[0] + b[0], a[1] + b[1] }; }
2 II operator-(II a, II b) { return { a[0] - b[0], a[1] - b[1] }; }
3 II operator*(II a, II b) { return { a[0] * b[0], a[1] * b[1] }; }
4 II operator%(II a, II b) { return { a[0] % b[0], a[1] % b[1] }; }
5
6 struct StrHash {
7     int n;
8     II base, mod;
9     vector<II> pref, inv;
10
11     StrHash(string s, II b = { 31, 37 }, \
12             II m = { 1000000007, 1000000009 }): n(s.size()), \
13             base(b), mod(m), pref(n+1), inv(n+1) {
14         II sum = { 0, 0 };

```

```

1 int n, m; c_in(n, m);
2 VII E(m); c_in(E);
3
4 VVI G(n+1);
5 auto M = VMAT(n+1, n+1);
6
7 for (auto [u, v]: E) {
8     G[u].push_back(v);
9     G[v].push_back(u);
10    M[u][v] = 1;
11    M[v][u] = 1;
12 }
13
14 auto max_flow = [&](int s, int t) {
15     VC Vis(n+1);
16     int delta = 1ll << 30;
17
18     auto dfs = [&](int u, int fu, auto& self) -> int {
19         if (u == t) return fu;
20         Vis[u] = true;
21
22         for (auto v : G[u]) if (!Vis[v] && M[u][v] >= delta) {
23             int pushed = self(v, min(fu, M[u][v]), self);
24             if (pushed) {
25                 M[u][v] -= pushed;
26                 M[v][u] += pushed;
27                 return pushed;
28             }
29         }
30     };
31
32     dfs(s, 0);
33
34     return M[s][t];
35 };

```

```

29     }
30     return 0;
31 };
32
33 int flow = 0;
34
35 while (delta) {
36     int new_flow;
37     do {
38         fill(all(Vis), false);
39         new_flow = dfs(s, INF, dfs);
40         flow += new_flow;
41     } while (new_flow);
42
43     delta >= 1;
44 }
45 return flow;
46 };
47 c_out(max_flow(1, n));
48 VII Cuts;
49 VC Vis(n+1);
50
51 auto dfs_cuts = [&](int u, auto& self) -> void {
52     Vis[u] = true;
53     for (auto v : G[u])
54         if (!Vis[v] && M[u][v])
55             self(v, self);
56 };
57 dfs_cuts(1, dfs_cuts);

```

Convex Hull (kaium)

```

1 II operator-(II p1, II p2) \
2 { return { p1[0] - p2[0], p1[1] - p2[1] }; }
3 int orientation(II p1, II p2, II p) {
4     II v1 = p2 - p1, v2 = p - p1;
5     int cross = v1[0] * v2[1] - v1[1] * v2[0];
6     return (cross > 0) - (cross < 0);
7 }
8 deque<II> convex_hull(vector<II>& V, \
9 bool collinear = false) {
10    II lp = V[0]; // find the lowest point
11    for (auto& p : V) if (p[1] < lp[1] || \
12        (p[1] == lp[1] && p[0] < lp[0])) lp = p;
13
14    deque<array<double, 3>> D;
15    int n = V.size();
16    // generate angle, distance, index for sorting
17    FOR (i, 0, n-1) {
18        II v = V[i] - lp;
19        double angle = atan2(v[1], v[0]);
20        double dist2 = v[0] * v[0] + v[1] * v[1];
21        D.push_back({ angle, dist2, (double)i });

```

```

22    }
23    sort(all(D));
24    // reverse collinear points at the back
25    if (collinear) {
26        double angle = D.back()[0];
27        auto it = D.rbegin();
28        while (it < D.rend() && (*it)[0] == angle) it++;
29        reverse(D.rbegin(), it);
30    }
31    deque<II> H;
32    int i; // add first two points
33    i = D[0][2]; H.push_front(V[i]); D.pop_front();
34    i = D[0][2]; H.push_front(V[i]); D.pop_front();
35    for (auto& d : D) {
36        i = d[2];
37        II p = V[i];
38        // < for keeping collinear points, <= otherwise
39        while (orientation(H[1], H[0], p) < 0)
40            H.pop_front();
41        H.push_front(p);
42    }
43    reverse(all(H));
44    return H;
45 }

```