Prerequisites:

At first we must recall the following summation formulas:

- $\sum_{i=1}^{n}(i) = \frac{n(n+1)}{2}$

Solution:

Things we need to understand,

- There is no repetition of a single number. Every number in the series is unique.
- The numbers are incrementing by 1.
- The i-th parenthesized term has exactly i- numbers within the parentheses.

Now to find out the last integer of the i-th parenthesized term, it is actually the $\sum_{i=1}^{n}(i) = \frac{n(n+1)}{2}$. For example:

i	i-th term	Last number within the i-th parenthesized term	$\sum_{i=1}^{n}(i) = \frac{n(n+1)}{2}$
1	1	1	1
2	(2+3)	3	3
3	(4+5+6)	6	6
4	(7+8+9+10)	10	10
. . .	. . .	. . .	. .
n	X	X_z	$\sum_{i=1}^{n}(i)$

So, basically we are just summing from 1 to $\frac{n(n+1)}{2}$. So, the expression should be:

$$\sum_{i=1}^{\frac{n(n+1)}{2}} (i) = \frac{\frac{n(n+1)}{2}\left(\frac{n(n+1)}{2}+1\right)}{2}$$

$$= \frac{n(n^3+2n^2+3n+2)}{8} \Rightarrow \frac{n(n+1)(n^2+n+2)}{8}.$$

Here,

$\frac{n(n+1)}{2}$ is the total number of elements not terms.

$\therefore 1+2+3+\ldots+\frac{n(n+1)}{2}$

Contributor:

A. A. Noman Ansary

GitHub : [showrav-ansary]

LinkedIn: [showrav-ansary]

Edited : Md-Minhajul-Islam