

Prerequisites:

At first we must recall the following summation formulas/rules:

- $\sum_{i=1}^n (c) = nc$
- $\sum_{i=1}^n (i) = \frac{n(n+1)}{2}$
- $\sum_{i=1}^n (x_i \pm y_i) = \sum_{i=1}^n (x_i) \pm \sum_{i=1}^n (y_i)$
- $\sum_{i=1}^n (ci) = c \sum_{i=1}^n (i)$
- $\sum_{i=1}^n (i^2) = \frac{n(n+1)(2n+1)}{6}$

Solution:

Now let us evaluate the expression:

$$\begin{aligned} & 1n + 2(n-1) + 3(n-2) + 4(n-3) + \dots + n \\ &= n + 2n - 2 + 3n - 6 + 4n - 12 + \dots + n(n-(n-1)) \\ &= (1n + 2n + 3n + \dots + n^2) \\ & \quad - (2 + 6 + 12 + \dots + n(n-1)) \\ &= n(1 + 2 + 3 + \dots + n) \\ & \quad - 2(1 + 3 + 6 + \dots + \frac{n(n-1)}{2}) \\ &= n \sum_{i=1}^n (i) - 2 \left(\sum_{i=1}^1 (i) + \sum_{i=1}^2 (i) + \sum_{i=1}^3 (i) + \dots + \sum_{i=1}^{n-1} (i) \right) \\ &= n \sum_{i=1}^n (i) - 2 \sum_{j=1}^{n-1} \left(\sum_{i=1}^j (i) \right) \\ &= n \sum_{i=1}^n (i) - 2 \sum_{j=1}^{n-1} \left(\frac{j(j+1)}{2} \right) \\ &= n \sum_{i=1}^n (i) - 2 \sum_{j=1}^{n-1} \left(\frac{1}{2}j + \frac{1}{2}j^2 \right) \end{aligned}$$

$$\begin{aligned}
&= n \sum_{i=1}^n (i) - 2 \cdot \frac{1}{2} \sum_{j=1}^{n-1} (j) - 2 \cdot \frac{1}{2} \sum_{j=1}^{n-1} (j^2) \\
&= n \frac{n(n+1)}{2} - \frac{n(n-1)}{2} - \frac{n(n-1)(2n-1)}{6} \\
&= \frac{n^3 + 3n^2 + 2n}{6}
\end{aligned}$$

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