**Problem 1**  
  
Show that as the sampling period approaches zero, the formula for the discrete-time Fourier transform given in approaches the formula for the Fourier transform.

Answer:

The sampling period . So, it can be rewritten.

As approaches zero, the discrete time , approaches continuous time . Moreover, the summation over approaches the equation below.

approaches the continuous FT . So, the formula for the DFT approaches the formula for the FT:

as the sampling period approaches zero.

**Problem 2**

the interpolation formula can be written like this

Prove the equation below.

Answer:

**Problem 3**

Specify the Nyquist rate and the Nyquist interval for each of the following signals:  
(a)   
(b)

Answer

Sinc is rectangular with 2B BW in FT domain. Sinc2 is the convolution of two rectangular signal that extends the BW to 4BW. Sinc3 extends the BW 3 times and Sinc4  extends the BW 4 times.

a)

Sinc BW is 100Hz.

Sinc2 BW is 200 Hz.

Sinc3 BW is 300 Hz. So, Nyquist rate is 600Hz and Nyquist interval is 1.66667ms

b)

Sinc4 BW is 400 Hz. And this is the highest signal frequency in (b). S0, Nyquist rate is 800 Hz, and Nyquist interval is 1.25 ms.

**Problem 4**

Consider uniform sampling of the sinusoidal wave

Determine the Fourier transform of the sampled waveform for the following sampling period:  
(a)   
(b)   
(c)

**Answer**

This is a cosine wave with angular frequency:

So, its Fourier Transform is:

If is sampled at period , the sampled signal is:

In the frequency domain, this gives:

So:  
The spectrum becomes periodic with spacing , and each replica is scaled by .

Case (a)

* No aliasing occurs, because

Replicas will be centered at:

So:

Plugging in :

Case (b)

* Critical Nyquist rate

Replicas are at:

No aliasing, but spectrum just touches at the boundary.

Case (c)

* Now: Aliasing occurs

Aliased frequency:

Try :

So the peaks are now incorrectly shifted due to aliasing.

**Problem 5**

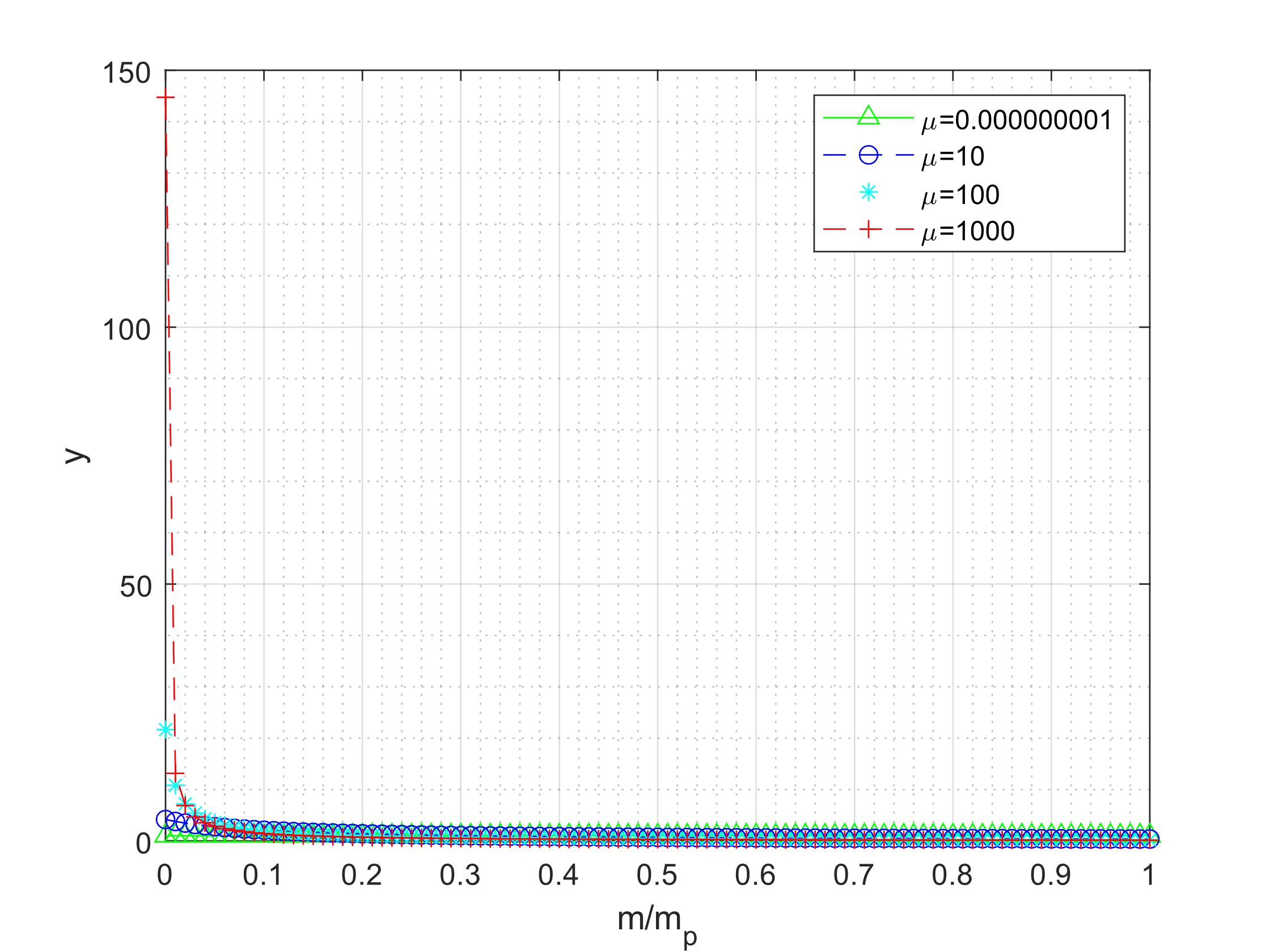
Derive the slope characteristics of -law and -law and plot for values of and

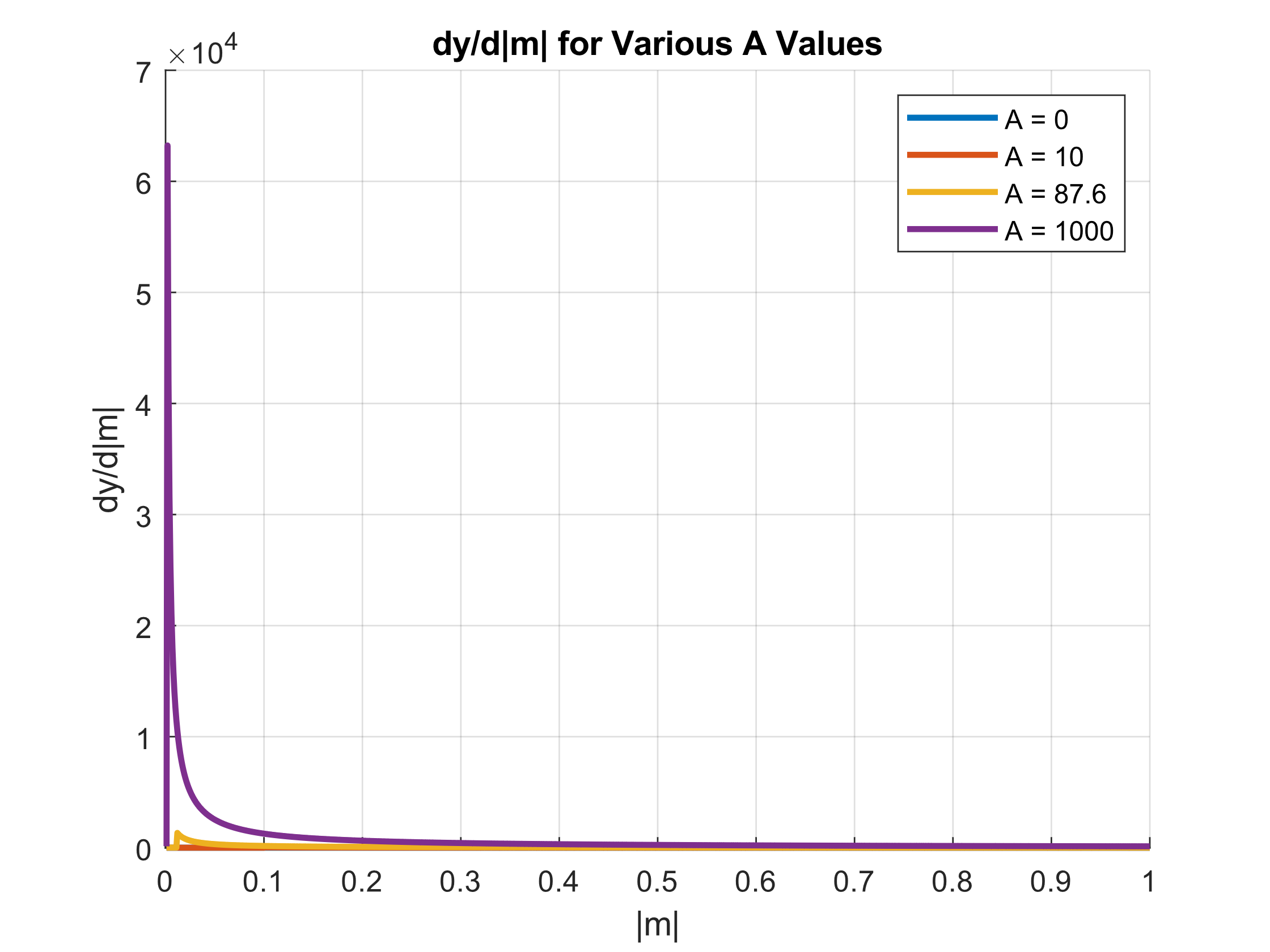
Solution  
(a) The logarithmic law is defined by

Therefore, differentiation with respect to yields

(b) The A-law is defined by (see Eq. (5.25):

Hence, differentiation with respect to yields

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**Problem 6**

Consider a sinusoidal wave of frequency and amplitude , applied to a delta modulator of step size . Show that slope-overload distortion will occur if

where is the sampling period. What is the maximum power that may be transmitted without slope-overload distortion?

Answer:

The modulating wave is

The slope of is given by

The maximum slope of is therefore equal to .  
The maximum average slope of the approximating signal produced by the delta modulator is , where is the step size and is the sampling period. The limiting value of is therefore given by

Assuming a load of 1 ohm , the transmitted Average power is . Therefore, the maximum power that nay be transmitted without slope-overload distortion is equal to .

**Problem 7**

Consider a delta modulation (DM) system used to transmit a voice signal, which is uniformly sampled at the rate of 64 kHz . Assume the following specifications:

(a) To avoid slope overload distortion, what is the minimum permissible value of the step size used in the system?  
(b) Determine the average power of granular noise.  
(c) Determine the minimum-channel bandwidth needed to transmit the DM encoded data.

**Answer:**

Sampling rate   
Voice signal bandwidth   
Maximum signal amplitude volts  
(a) To avoid slope overload, we must satisfy the following requirement

Solving for the step size , we write

Substituting the given values into yields

or

volts  
Provided that the step size is 0.33 volt, then slope-overload distortion is avoided.

(b)

Let denote the granular noise, viewed as a function of time . The average power of granular noise (analogous to quantization noise in PCM), is defined by

With set at 0.33 volt, the average power of granular noise is therefore 0.03 watts (assuming that the power is calculated for a load of 1 ohm ).

(c) The minimum channel bandwidth needed to transmit the DM encoded signal is the inverse of the sampling rate 64 kHz if RZ.  
  
For NRZ it’s 32KHz.

**Problem 8**

Consider a compact disc that uses pulse-code modulation to record audio signals whose bandwidth . Specifications of the modulator include the following:  
Quantization: uniform with 512 levels  
Encoding: binary  
Determine (a) the Nyquist rate, and (b) the minimum permissible bit rate.

**Answer**

(a) The Nyquist rate is .  
(b) To accommodate 512 quantization levels, we require a binary code with bits, which would have to satisfy the following requirement:

Hence, . The sampling period milliseconds must be divided into 9 bits. The minimum sampling rate is therefore

**Problem 9**

This problem addresses the digitization of a television signal using pulse-code modulation. The signal bandwidth is 4.5 MHz . Specifications of the modulator include the following:  
Sampling: in excess of the Nyquist rate  
Quantization: uniform with 1024 levels  
Encoding: binary  
Determine (a) the Nyquist rate, and (b) the minimum permissible bit rate.

(a) We are given

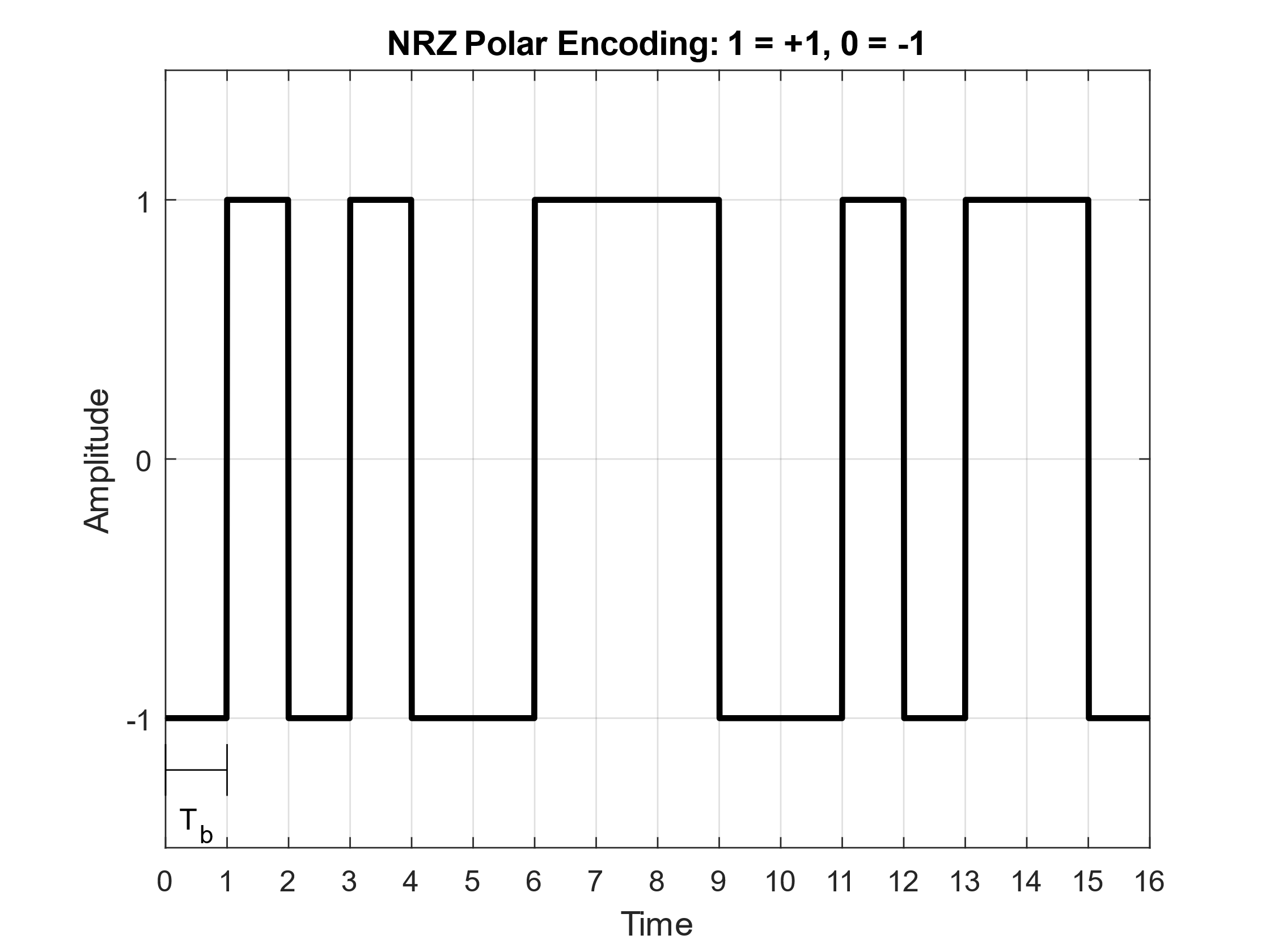
* Video bandwidth
* Sampling rate in excess of the Nyquist rate
* Uniform quantization using 1024 levels
* Binary encoding  
  (b) The Nyquist rate is . Actual sampling rate   
  The sampling period is therefore

This sampling rate must be divided into bits, where

Hence, . The bit duration is therefore

The permissible bit rate is therefore 103.5 megabits/s.

**Problem 10**



This shows a PCM signal in which the amplitude levels of +1 volt and -1 volt are used to represent binary symbols 1 and 0 , respectively. The code word used consists of three bits. Find the sampled version of an analog signal from which this PCM signal is derived.

**Answer**

The transmitted code words, representing the PCM waveform

|  |  |
| --- | --- |
|  | code |
| 1 | 001 |
| 2 | 010 |
| 3 | 011 |
| 4 | 100 |
| 5 | 101 |
| 6 | 110 |

Accordingly, the sampled analog signal from which these code words are derived is shown below

