(a) $T(n) = T(n-1) + n$ — ①

$T(n-1) = T(n-1-1) + n-1 = T(n-2) + n-1$ — ②

$T(n-2) = T(n-2-1) + n-2 = T(n-3) + n-2$ — ③

So, we can take a guess from here that for $n-k$,

$$T(n-k) = T(n-k-1) + n-k$$

Putting value of ② in ①,

$$T(n) = T(n-2) + n-1 + n = T(n-2) + 2n-1.$$

Putting value of ③ in here:

$$T(n) = T(n-3) + n-2 + 2n-1 = T(n-3) + 3n-3$$

So, for every value we can take a guess. $(n-k) + kn - k$, here $k = (n-1)$ in

$k = n^2 - n$ So $O(n^2)$.

(b) $T(n) = T(n/2) + O(1)$ — (1)

$T(n/2) = T(n/4) + O(1)$ — (2)

$T(n/4) = T(n/8) + O(1)$ — (3)

Putting value of (2) in (1).

$T(n) = T(n/4) + O(1) + O(1)$

Putting value of (3) in here

$T(n) = T(n/8) + 3O(1) = T(n/2^3) + 3O(1)$

so for every k we get

$T(n/k) = T(n/2^k) + kO(1)$

Here assuming

$k = 2^k$

$\log k = k \log k$

Replacing ~~with k~~ k with n

$\log n = k \log 2$ $\therefore O(\log n)$

(c) $T(n) = 2T(n/2) + n$ —— (1)

$T(n/2) = 2T(n/4) + n + n = 2T(n/4) + 2n$ —— (2)

$T(n/4) = 2T(n/8) + 3n$ —— (3)

Putting value of (2) in (1),

$T(n) = 2T(n/4) + 2n$

Putting value of (3) in (1) here,

$T(n) = \cancel{2T(n/(n/8))} + 2T(n/8) + 3n$

$T(n^k) = 2T(n/2^k) + kn$ [if $\boxed{k=3}$]

assuming

$2 \cdot 2^k = n^{1/2}$

$k \log_2 k = n \log 1/2$

So, Replacing k with n,

$\therefore O(n \log n)$

(d) $T(n) = 2T(n/2+17)+n$

Following the same method of

we get

$2 \cdot 2^{n}k = n^{nk}$ [17 is constant]

$k\log_2 k = n\log k$

Replacing k with n we get,

$O(n\log n)$

(e) Given,

$T(n)\ 2T(n/3) + \Theta(n)$

By assumption, we have,

$T(n/3) \leq c*(n/3)$

Now, substitute this into the origina

recurrence:

$T(n) = 2T(n/3) + \Theta(n) \leq 2 * (c * n/3) + \Theta(n)$

[Substituting $T(n/3)$]

$= (2/3) * c * n + \Theta(n)$

Analyzing the inequality:

$(2/3) * c * n + \Theta(n) \leq c * n$ (if we choose $c \geq 3$)

Since, $(2/3) * c$ is a constant and $\Theta(n)$ is also a constant, we can say that:

$T(n) \leq c * n$

So, This establish $T(n) = \Theta(n)$

(f) Given,

$T(n) = 4T(n/2) + \Theta(n)$

Assuming that $T(k) \leq c * k^2$ for some positive constant c and all values of k less than n,

Now, substitute this assumption into original recurrence:

$$T(n) = 4T(n/2) + \Theta(n)$$

By assumption, we have,

$$T(n/2) \leq c * (n/2)^{\wedge}2$$

Now,

$$T(n) = 4\,T(n/2) + \Theta(n) \leq 4 * (c * (n/2)^{\wedge}2) + \Theta(n)$$

(Substituting $T(n/2)$)

$$= (4/4) * c * (n/2)^{\wedge}2 + \Theta(n)$$

$$= c * (n^{\wedge}2/4) + \Theta(n)$$

Analyizing this inequality:

$$c * (n^{\wedge}2/4) + \Theta(n) \leq c * n^{\wedge}2$$

Since 8/4 is a constant and $\Theta(n)$ is also a constant, we can say that,

$$T(n) \leq c * n^2$$

So,

$$T(n) = \Theta(n^2)$$