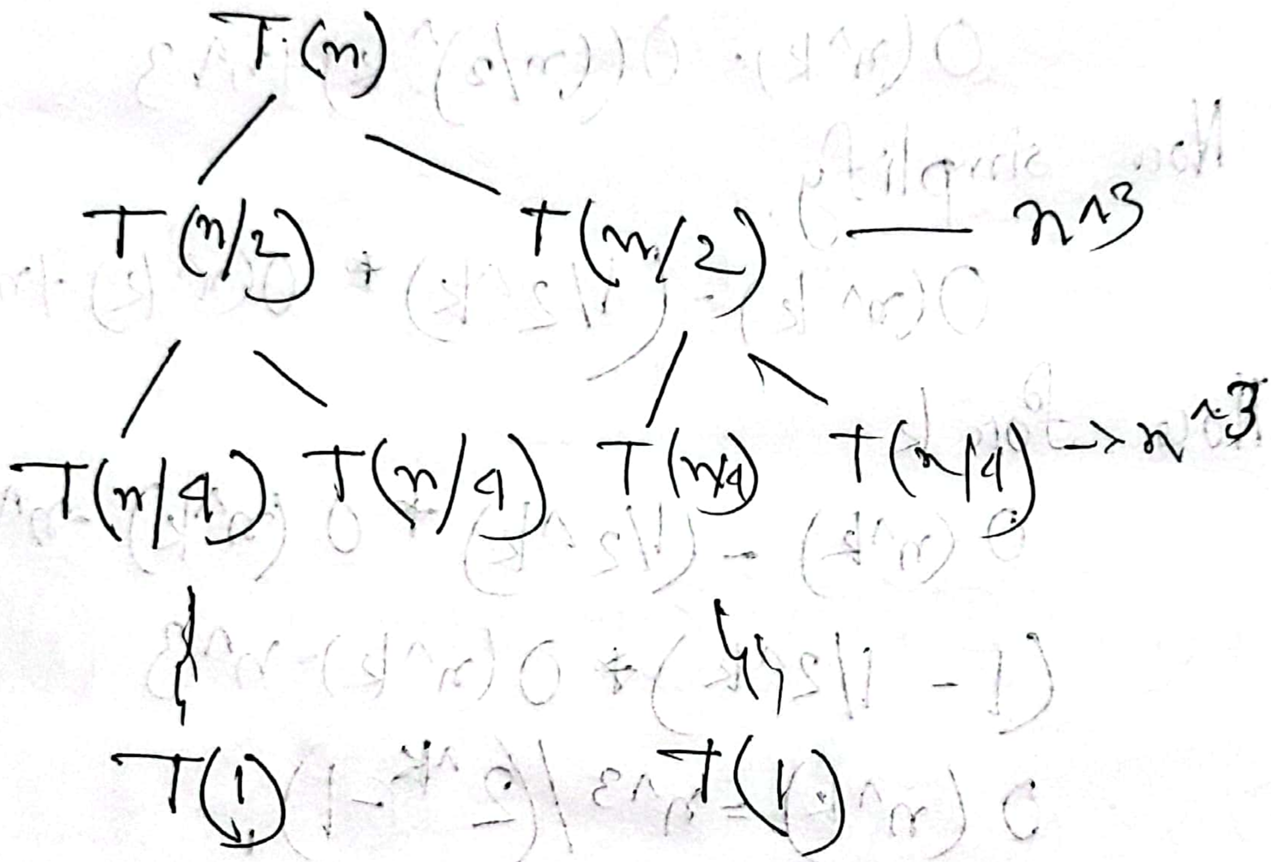
1 Given,

$$T(n) = T(n/2) + n^3$$



Subtitute method:

We will assume $T(n)$ is in $O(f(n))$ and ~~upper bound~~, Assume $T(n) = O(n^k)$ where k is constant to be determined

Now we can substitute this into recurrence rela

$T(n) = T(n/2) + n^3$

$O(n^k) = O((n/2)^k) + n^3$

Now simplify,

$O(n^k) = (1/2^k) * O(n^k) + n^3,$

Now for k,

$O(n^k) - (1/2^k) * O(n^k) = n^3$

$(1 - 1/2^k) * O(n^k) = n^3$

$O(n^k) = n^3/(2^k - 1),$

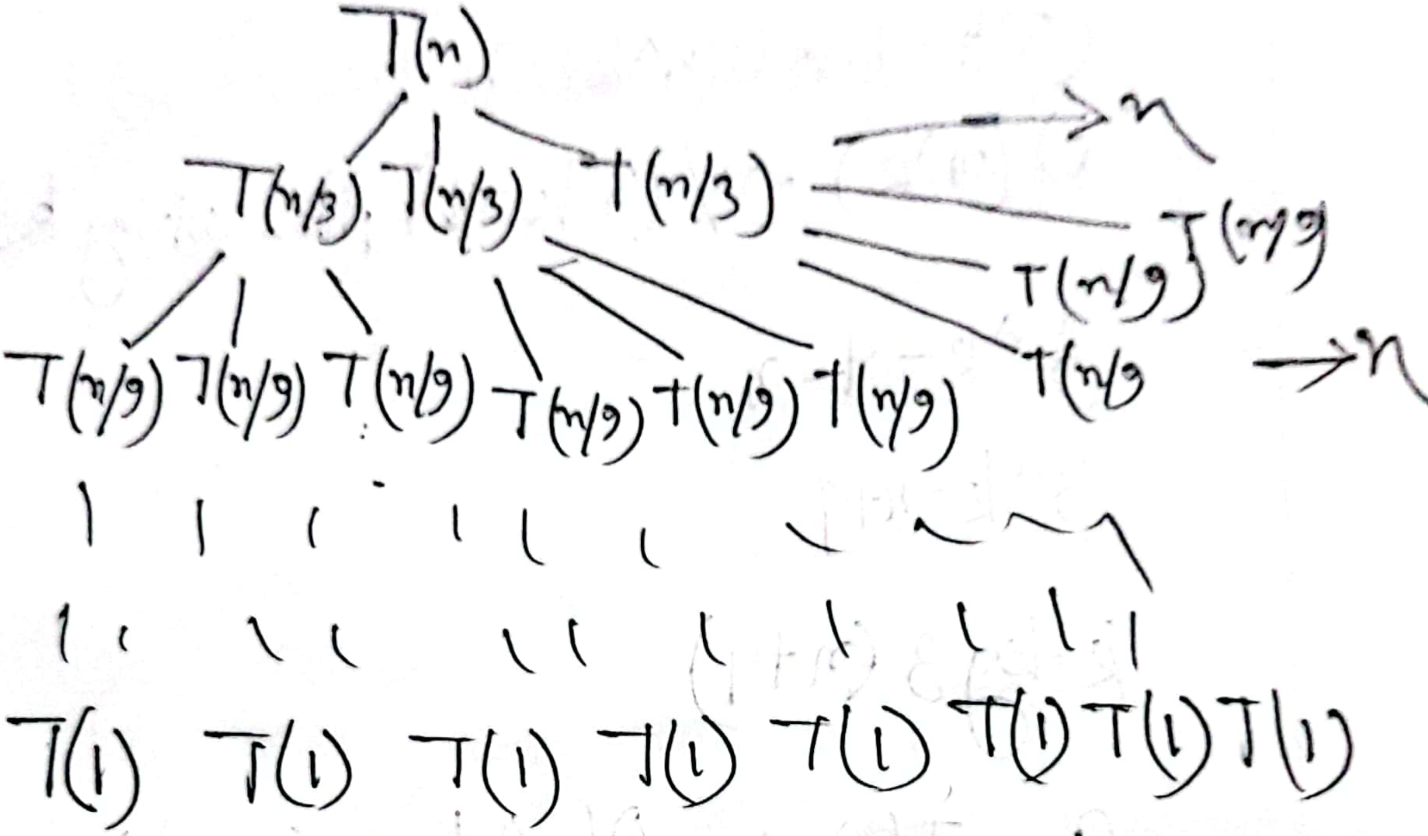
Since k is constant,

$2^k - 1 = n$

$k = \log 2(n+1)$, So, $T(n)$ is in

$O(n^\wedge \log 2(n+1))$.

(2) $T(n) = 4T(n/3) + n$, Recursion Tree



Substitute method: Assume $T(n)$ is in $O(f(n))$, and $T(n) = O(n^k)$ is constant. So,

$T(n) = 4T(n/3) + n$

$O(n^k) = 4(O(n/3)^k) + n$. Now simplify,

$O(n^k) = (4/3^k) * O(n^k) + n$, Now,

$O(n^k) - (4/3^k) * O(n^k) = n$

$(1-4/3 \wedge k) * O(n \wedge 1/2) - n$

$(3 \wedge k - 4) * O(n \wedge 1/2) - n$

$O(n \wedge 1/2) - n / (3 \wedge k - 4)$, Now solving k

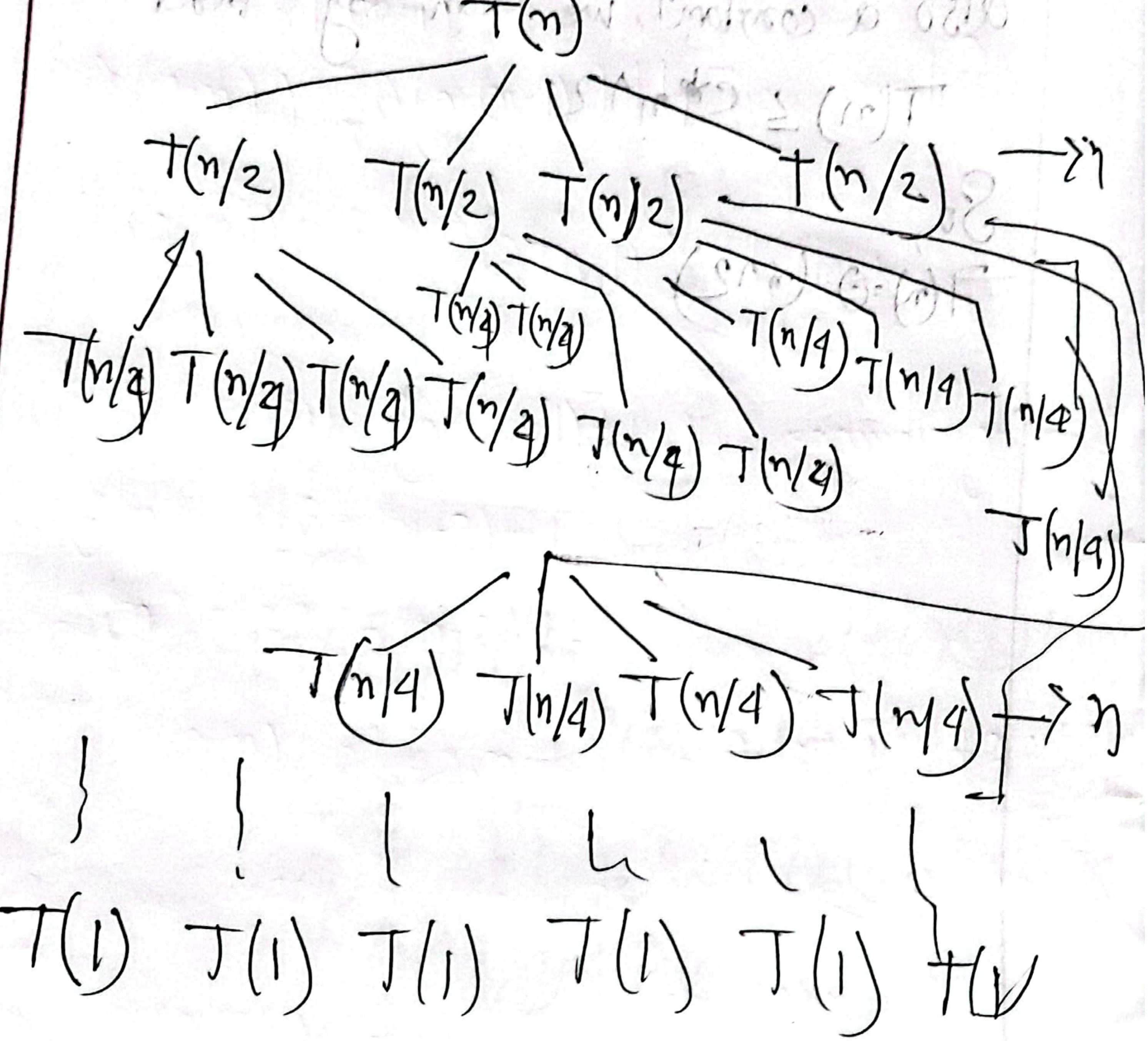
$3 \wedge k - 4 = n$

$3 \wedge k = n + 4$

$k = \log 3 (n+4)$

So, $T(n)$ is $O(n \wedge \log 3 (n+4))$

③ $T(n) = 4T(n/2) + n$, Given recursion tree.



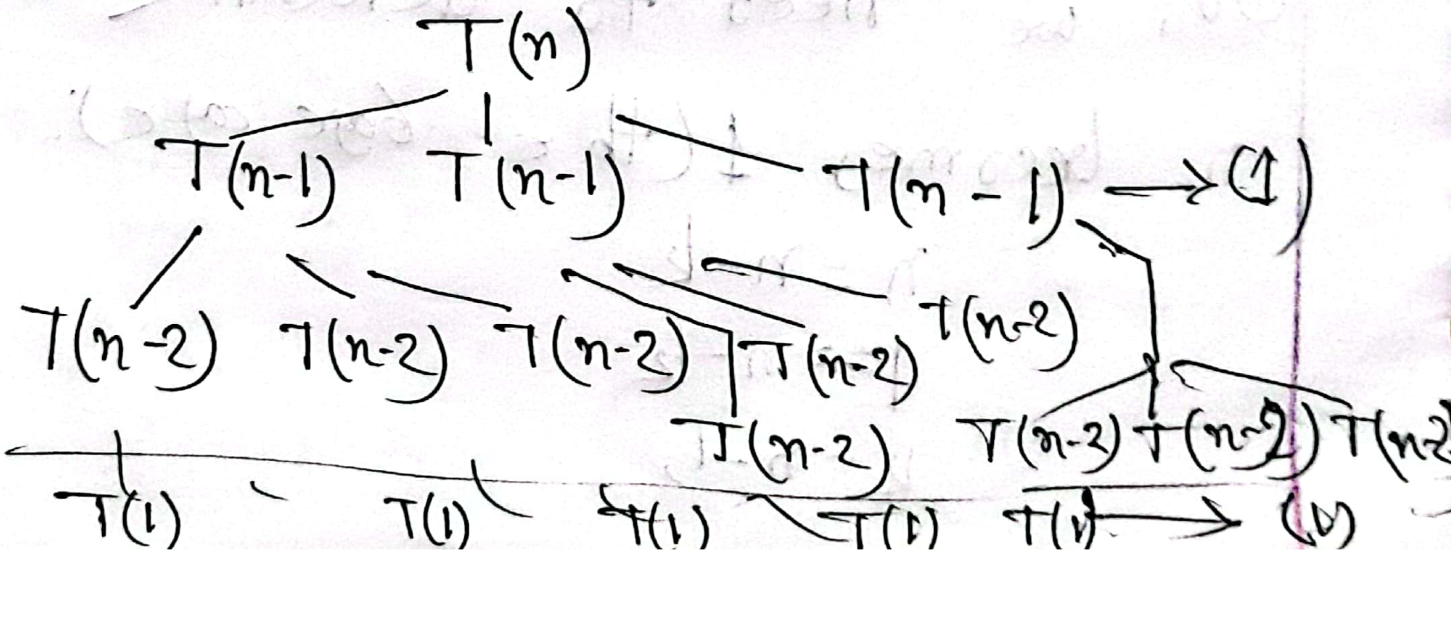
We can guess $T(n) = O(n^2)$

using sustitution method, we can assume that $T(n/2) <= c*(n/2)^2$ for some constant $c>0$. So,

$$T(n) = 4c*(n/2)^2 + n = c*n^2 + n$$

if we choose c to be large enough compared to n, then $cn^2 >= n$. So, $T(n) <= cn^2$ which verifies

(4) $T(n) = 3T(n-1) + 1$, its recurrence tree



Assuming $T(n)$ is in $O(f(n))$, So,

$T(n) = 3T(n-1) + 1$

$O(g(n)) = 3O(g(n-1)) + 1$, Now simplify

$O(g(n)) = 3 * O(g(n-1)) + 1$, Now

$g(n) = 3g(n-1) + 1$, Now expanding

$g(n) = 3[3g(n-2) + 1] + 1$

$g(n) = 3^2 g(n-2) + 3 + 1$, so

$g(n) = 3^k g(n-k) + 3^0 + 3^1 + \ldots + 3^{(k-1)}$

So, we need to determine k whe

n becomes 1 (the base case):

$n = n-k$

$1 = n-k$

$k = n-1$,

So, $g(n)$ can be expressed as:

$g(n) = 3^{(n-1)} + 3^0 + 3^1 + .. + 3^{(n-2)}$.

Now simplify and find the upper bound,

$g(n) = 3^{(n-1)} + 1 + 3 + 3^2 + - + 3^{(n-2)}$

$g(n) = 3^{(n-1)} + (1 + 3 + 3^2 + - + 3^{(n-2)})$,

The sum in the parentheses is a geometric series.

$1 + 3 + 3^2 + ... + 3^{(n-2)} - (3^{(n-1)} - 1)/(3-1) =$

$(3^{(n-1)} - 1)/2$

So, $g(n) = 3^{(n-1)} + (3^{(n-1)} - 1)/2$

$g(n) = (3^{(n-1)} + 3^{(n-1)} - 1)/2$

$g(n) = (3 * 3^{(n-1)} - 1)/2$

$g(n) = 3^{(n-1)} - 1/2$, So, $T(n)$ is $O(3^n)$