

Primary Test of $O(\sqrt{N})$

* All divisors of a Number N occur in pairs of (a, b) s.t. $a * b = N$

Ex: 12 divisors are 1, 2, 3, 4, 6, 12
pairs are $(1, 12), (2, 6), (3, 4)$

* For a divisor pair (a, b) , one of them lies below \sqrt{N} and other lies above \sqrt{N}

(i) "Both a and b are ~~not~~ below \sqrt{N} "
→ False because it contradicts $a * b = N$. But the statement states $a * b < N$

(ii) "Both a and b are above \sqrt{N} "
False because it shows $a * b > N$
which contradicts $a * b = N$

(iii) One is below \sqrt{N} and above \sqrt{N} because if $b < \sqrt{N}$

then $1 < \sqrt{N}/b$, where $a = \sqrt{N} * (1+2)$. Hence $a > \sqrt{N}$, $] \sqrt{N}$ to \sqrt{N}
then $1 > \sqrt{N}/b$. Hence $a < \sqrt{N}$

Fact: (i) if we find a pair, then
we don't have to find another.
(ii) One of them lies above
or below \sqrt{N} .

So, Time complexity is $O(\sqrt{N})$
and main code is

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for (int i = 2; i * i <= n; i++)
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