Sum of Odd Numbers

Odd numbers are defined as any number that is not a multiple of 2 or has its units place digit to be odd such as 3, 45, 67, etc. The sum of odd numbers is the total summation of the odd numbers taken together for any specific range given. We will be learning about the sum of odd numbers and the sum of first n odd numbers using formulas and examples in this article.

Sum of Odd Numbers Definition

Sum of odd numbers is defined as the summation of odd numbers taken together and added up to calculate the result. The sum of odd numbers starts from 1 and goes up to infinity. We can find the sum of odd numbers for any range such as 1 to 100, 1 to 50, and so on by using the sum of n odd numbers formula involving the concept of arithmetic progression discussed in the next section.

Sum of n Odd Numbers Formula

We know that the general form of an odd number is (2n - 1) where n is an integer. Also, consecutive odd numbers have a common difference of 2. Therefore, the series of odd numbers form an arithmetic progression. The sum of n odd numbers formula is described as follows,

Sum of n odd numbers = n^2 where n is a natural number and represents the number of terms.

Sum of n Odd Numbers Formula

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Sum of 'n' odd numbers = S_n

$S_n = 1 + 3 + 5 + \dots\dots\dots + (2n - 1)$

$S_n = n^2$ where, n = number of terms

Thus, to calculate the sum of first n odd numbers together without actually adding them individually, we can use the sum of n odd numbers formula i.e., 1 + 3 + 5 +..........n terms = n^2.

Sum of First n Odd Numbers Proof

Let us now derive the sum of n odd natural numbers formula. We know that the series of odd numbers is given as 1, 3, 5,...(2n - 1) which forms an arithmetic progression with a common difference of 2. Let the sum of first n odd numbers be represented as S_n = 1 + 3 + 5 +...+ (2n - 1). Here 1 represents the first odd number and (2n - 1) represents the last odd number.

Thus, the first term (a) = 1, last term (l) = 2n - 1 and common difference (d) = 2.

The sum of n terms of an AP is given by the formula S_n= n/2 × [a + l].

By substituting the values of 'a' and 'l' in the above formula we get,

S_n = n/2 × [1 + (2n - 1)]
S_n= n/2 × [2n]
S_n= n × n
S_n= n^2

Thus, when n = 1, $S_1 = 1^2 = 1$

when n = 2, $S_2 = 2^2 = 4$

when n = 3, $S_3 = 3^2 = 9$

Therefore, we have proved that the sum of first n odd numbers is equal to n^2. Let's take an example to understand this.

Example: Find the sum of odd numbers 1 to 50.

We know that there are 25 odd numbers between 1 to 50. Thus, by using the sum of n odd numbers formula which is n^2, we get, $S_{25} = 25^2 = 625$.

We can alternatively show this using the formula S_n = n/2 × [a + l]. We know that the sum of odd numbers 1 to 50 is represented as S_n = 1 + 3 + ... + 49.
Thus, a = 1, l = 49, and n = 25.
S_{25} = (25/2) × [1 + 49]
= (25/2) × 50
= 25 × 25 = 625

Thus, the sum of odd numbers 1 to 50 is equal to 625.