Problem 1.4

$$6x - 5y = 10$$
$$-13x + 10y = -20$$

$$\left[\begin{array}{cc|c} 6 & -5 & 10 \\ -13 & 10 & -20 \end{array}\right] \cdot 2 \sim \left[\begin{array}{cc|c} 6 & -5 & 10 \\ -1 & 0 & 0 \end{array}\right] \cdot(-1) \sim \left[\begin{array}{cc|c} 6 & -5 & 10 \\ 1 & 0 & 0 \end{array}\right] \cdot -6 \sim \left[\begin{array}{cc|c} 0 & -5 & 10 \\ 1 & 0 & 0 \end{array}\right] \cdot \frac{1}{-5} \sim \left[\begin{array}{cc|c} 0 & 1 & -2 \\ 1 & 0 & 0 \end{array}\right]$$

$\frac{1}{-5}(0x - 5y) = (10)\frac{1}{-5}$

$0x - \frac{5y}{-5} = \frac{10}{-5} \rightarrow 0x + y = -2$

Problem 3.4

$$\left[\begin{array}{ccc|c} 2 & 6 & 15 & -12 \\ 4 & 7 & 13 & -10 \\ 3 & 6 & 12 & -9 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 2 & 6 & 15 & -12 \\ 4 & 7 & 13 & -10 \\ 3 & 6 & 12 & -9 \end{array}\right] \cdot \frac{1}{3} \sim \left[\begin{array}{ccc|c} 2 & 6 & 15 & -12 \\ 4 & 7 & 13 & -10 \\ 1 & 2 & 4 & -3 \end{array}\right] \cdot -4 \cdot -2 \sim \left[\begin{array}{ccc|c} 0 & 2 & 7 & -6 \\ 0 & -1 & -3 & 2 \\ 1 & 2 & 4 & -3 \end{array}\right] \cdot 2 \sim \left[\begin{array}{ccc|c} 0 & 0 & 1 & -2 \\ 0 & -1 & -3 & 2 \\ 1 & 0 & -2 & 1 \end{array}\right] \cdot 2 \sim \left[\begin{array}{ccc|c} 0 & 0 & 1 & -2 \\ 0 & -1 & 0 & -4 \\ 1 & 0 & 0 & -3 \end{array}\right] \cdot(-1) \left[\begin{array}{ccc|c} 0 & 0 & 1 & -2 \\ 0 & 1 & 0 & 4 \\ 1 & 0 & 0 & -3 \end{array}\right]$$

Entydig lösning

$x = 0$
$y = -2$

Problem 3.5

$$\left[\begin{array}{ccc|c} 2 & 3 & 5 & 21 \\ 1 & -1 & -5 & -2 \\ 2 & 1 & -1 & 11 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 2 & 3 & 5 & 21 \\ 1 & -1 & -5 & -2 \\ 2 & 1 & -1 & 11 \end{array}\right] \cdot -2 \cdot -2 \sim \left[\begin{array}{ccc|c} 0 & 5 & 15 & 25 \\ 1 & -1 & -5 & -2 \\ 0 & 3 & 9 & 15 \end{array}\right] \cdot \frac{1}{5} \cdot \frac{1}{3} \sim \left[\begin{array}{ccc|c} 0 & 1 & 3 & 5 \\ 1 & -1 & -5 & -2 \\ 0 & 1 & 3 & 5 \end{array}\right] \cdot -1 \sim \left[\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 1 & -1 & -5 & -2 \\ 0 & 1 & 3 & 5 \end{array}\right] \cdot 1 \sim \left[\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 1 & 0 & -2 & 3 \\ 0 & 1 & 3 & 5 \end{array}\right]$$

Entydig lösning

$x = -3$
$y = 4$
$z = -2$

$x = 9$
$y = -4$
$z = 3$

$2x + 3y + 5z = 21$
$x - y - 5z = -2$
$2x + y - z = 11$

$0 = 0$
$x - y - 5z = -2$
$y + 3z = 5$

$0 = 0$
$x - 2z = 3 \rightarrow x = 3 + 2z$
$y + 3z = 5 \quad y = 5 - 3z$

$x = 3 + 2t$
$y = 5 - 3t$
$z = t \quad , t \in \mathbb{R}$

$z = 3$
$x = 9$
$y = -4$

Oändligt många lösningar

Problem 3.6

$$\left[\begin{array}{ccc|c} 4 & -2 & 2 & 5 \\ -6 & 3 & -3 & -2 \\ 10 & -5 & 9 & 4 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 4 & -2 & 2 & 5 \\ -6 & 3 & -3 & -2 \\ 10 & -5 & 9 & 4 \end{array}\right] \cdot 2 \sim \left[\begin{array}{ccc|c} 4 & -2 & 2 & 5 \\ -6 & 3 & -3 & -2 \\ 4 & -2 & 6 & 2 \end{array}\right] \cdot -1 \sim \left[\begin{array}{ccc|c} 4 & -2 & 2 & 5 \\ -6 & 3 & -3 & -2 \\ 0 & 0 & 4 & -3 \end{array}\right] \cdot 2 \sim \left[\begin{array}{ccc|c} 4 & -2 & 2 & 5 \\ -2 & 1 & -1 & 3 \\ 0 & 0 & 4 & -3 \end{array}\right] \cdot 2 \sim \left[\begin{array}{ccc|c} 0 & 0 & 0 & 11 \\ -2 & 1 & -1 & 3 \\ 0 & 0 & 4 & -3 \end{array}\right]$$

STOP

Contradiction

$0 = 11 \rightarrow$ Inga lösningar

$$\left[\begin{array}{ccc|c} 4 & -2 & 2 & 5 \\ -6 & 3 & -3 & -2 \\ 10 & -5 & 4 & 4 \end{array}\right] \cdot \frac{-3}{2} \sim \left[\begin{array}{ccc|c} 4 & -2 & 2 & 5 \\ 4 & -2 & 2 & -3 \\ 10 & -5 & 4 & 4 \end{array}\right] \cdot -2 \sim \left[\begin{array}{ccc|c} 4 & -2 & 2 & 5 \\ 0 & 0 & 0 & -8 \\ 10 & -5 & 4 & 4 \end{array}\right]$$

$4x - 2y + 2z = 5$
$4x - 2y + 2z = -3$
$10x - 5y + 4z = 4$