Lab 2: Sorting and Searching

ICT 2202: Algorithm Analysis Design and Lab

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Contents

1.1 Overview

The lab is designed to provide practical experience with four distinct algorithms, each demonstrating classic recursive divide and conquer techniques.

The first problem focuses on Merge Sort, where an array is recursively split into halves, sorted, and merged. The algorithm's performance is evaluated in three scenarios—best, average, and worst cases—with consistent time complexity of $O(nlog\ n)$, illustrating its efficiency and reliability.

The second problem involves Quick Sort, which sorts an array by partitioning it around a pivot and recursively sorting the partitions. The lab compares its performance in best, average, and worst cases, showing that although the best and average cases typically run in $O(nlog\ n)$, the worst case may degrade to $O(n^2)$. This highlights Quick Sort's sensitivity to input order.

The third problem implements a recursive Binary Search to locate a target element in a sorted array and further identifies all its multiple occurrences. This approach efficiently combines binary and linear search techniques to return every index where the target appears.

Lastly, the optional problem covers Strassen's matrix multiplication, a recursive method that divides matrices into sub-blocks and reduces the multiplication complexity from $O(n^3)$ to about $O(n^{2.81})$, demonstrating advanced algorithmic optimization.

Together, these four problems enrich understanding of recursive divide and conquer algorithms applied across sorting, searching, and matrix operations, offering both practical implementation skills and theoretical insights into their time complexities.

1.2 Problem 01: Merge Sort

1.2.1 Algorithms

Merge Sort Algorithm:

mergeSort(arr[], start, end)

This function is responsible for the divide and conquer steps.

1. Input: An array arr, the starting index start, and the ending index end.
2. Base Case:
    - If start >= end (meaning the current sub-array has one or zero elements), simply return. A single element is already sorted.
3. Divide:
    - Calculate the middle index: mid = start + (end - start) / 2.
4. Conquer (Recursive Calls):
    - Call mergeSort(arr, start, mid) to sort the left half.
    - Call mergeSort(arr, mid + 1, end) to sort the right half.
5. Combine:
    - Call the merge(arr, start, mid, end) function to merge the two now-sorted halves (start to mid and mid+1 to end).

merge(arr[], start, mid, end)

This function is responsible for the combine step, merging two sorted sub-arrays into one.

1. Input: The original array arr, and indices start, mid, and end that define two adjacent, sorted sub-arrays.
2. Create Temporary Arrays:
    - Calculate the sizes of the two sub-arrays:
        - len1 = mid - start + 1 (size of left sub-array)
        - len2 = end - mid (size of right sub-array)
    - Create two temporary arrays, left[len1] and right[len2].
3. Copy Data:
    - Copy elements from arr[start ... mid] into the left[] array.
    - Copy elements from arr[mid+1 ... end] into the right[] array.
4. Merge Back into Original Array:

- Initialize three pointers:
  - i = 0 (index for left[])
  - j = 0 (index for right[])
  - k = start (index for the original arr[] where the merged result will be placed)
- While both i < len1 and j < len2:
  - Compare the current element of left[i] with the current element of right[j].
  - Copy the smaller element into arr[k] and increment the respective pointers (i or j and k).
- Copy Remaining Elements: After one sub-array is exhausted, copy all remaining elements from the other sub-array into arr[].
  - While i < len1, copy left[i++] to arr[k++].
  - While j < len2, copy right[j++] to arr[k++].

1.2.2 Code

```
#include <bits/stdc++.h>
using namespace std;
using namespace std::chrono;

void merge(int arr[], int start, int mid, int end) {
    int len1 = mid - start + 1;
    int len2 = end - mid;

    int left[len1], right[len2];

    for (int i = 0; i < len1; i++)
        left[i] = arr[start + i];

    for (int j = 0; j < len2; j++)
        right[j] = arr[mid + 1 + j];

    int i = 0, j = 0, k = start;

    while (i < len1 && j < len2) {
        if (left[i] <= right[j])
            arr[k++] = left[i++];
        else
            arr[k++] = right[j++];
    }

    while (i < len1)
        arr[k++] = left[i++];

    while (j < len2)
        arr[k++] = right[j++];
}
void mergeSort(int arr[], int start, int end) {
    if (start >= end)
        return;

    int mid = start + (end - start) / 2;

    mergeSort(arr, start, mid);
    mergeSort(arr, mid + 1, end);
    merge(arr, start, mid, end);
}

void display(int arr[], int n) {
    for (int i = 0; i < n; i++)
        cout << arr[i] << " ";

    cout << endl;
}
```

```
int main() {
    int arr1[] = {1, 5, 8, 12, 15, 18, 21, 25, 28, 30,
33, 35, 38, 40, 42, 45, 47, 49, 52, 55};
    int arr2[] = {12, 31, 35, 8, 32, 17, 5, 44, 9, 28,
40, 21, 30, 15, 7, 50, 19, 11, 46, 3};
    int arr3[] = {60, 58, 55, 52, 49, 45, 42, 38, 35,
33, 30, 28, 25, 21, 18, 15, 12, 8, 5, 1};

    int n = 20;


    auto start1 = high_resolution_clock::now();
    mergeSort(arr1, 0, n - 1);
    auto end1 = high_resolution_clock::now();
    auto duration1 =
duration_cast<microseconds>(end1 - start1);
    cout << "Best Case (Already Sorted):" << endl;
    display(arr1, n);
    cout << "Time taken: " << duration1.count() <<
" microseconds\n\n";


    auto start2 = high_resolution_clock::now();
    mergeSort(arr2, 0, n - 1);
    auto end2 = high_resolution_clock::now();
    auto duration2 =
duration_cast<microseconds>(end2 - start2);
    cout << "Average Case (Random Order):" <<
endl;
    display(arr2, n);
    cout << "Time taken: " << duration2.count() <<
" microseconds\n\n";


    auto start3 = high_resolution_clock::now();
    mergeSort(arr3, 0, n - 1);
    auto end3 = high_resolution_clock::now();
    auto duration3 =
duration_cast<microseconds>(end3 - start3);
    cout << "Worst Case (Reverse Sorted):" <<
endl;
    display(arr3, n);
    cout << "Time taken: " << duration3.count() <<
" microseconds\n";

    return 0;
}
```

1.2.3 Output

Best Case (Already Sorted):

1 5 8 12 15 18 21 25 28 30 33 35 38 40 42 45 47 49 52 55

Time taken: 1 microseconds

Average Case (Random Order):

3 5 7 8 9 11 12 15 17 19 21 28 30 31 32 35 40 44 46 50

Time taken: 1 microseconds

Worst Case (Reverse Sorted):

1 5 8 12 15 18 21 25 28 30 33 35 38 42 45 49 52 55 58 60

Time taken: 2 microseconds

1.2.4 Explanation

The implemented algorithm is Merge Sort, which uses a recursive divide-and-conquer approach to sort arrays. The array is repeatedly divided into smaller subarrays until each subarray contains a single element. These subarrays are then merged in a sorted manner to reconstruct the fully sorted array. The merging process compares elements from the left and right subarrays and places the smaller element into the result array. This process ensures that at each recursive step, the subarrays remain sorted.

Time Complexity Analysis

Merge Sort consistently performs at O(n log n) time complexity for all input cases. This includes the best case, where the array is already sorted, the average case, where elements are in random order, and the worst case, where the array is in reverse order. Unlike some other algorithms, such as Quick Sort, Merge Sort's execution does not depend on the initial ordering of the array because it always divides and merges the array completely.

Space Complexity

The algorithm requires additional memory of O(n) due to temporary arrays used during the merging process. Each merge operation uses a temporary left and right array to combine elements, and this auxiliary space grows linearly with the size of the input array.

Analysis

For a 20-element array, the best case scenario (already sorted array) was sorted in 1 microsecond. The average case, consisting of a randomly ordered array, was also sorted in 1 microsecond. The worst case scenario (reverse sorted array) required 2 microseconds. All cases produced the correct sorted output.

Merge Sort demonstrates stable and predictable performance across different input scenarios. The slight difference in time between the worst case and other cases is negligible, caused by minor variations in memory operations during merging. Its consistent runtime and reliable sorting behavior make Merge Sort particularly suitable for applications where predictable performance is important.

1.3 Problem 02: Quick Sort

1.3.1 Algorithms

quickSort(arr[], start, end)

This function handles the recursive division of the array.

1. Input: An array arr, starting index start, ending index end
2. Base Case:
    - If start < end, proceed (array has more than one element)
    - Otherwise, return (single element is already sorted)
3. Partition:
    - Call partition(arr, start, end) to rearrange array and get pivot index
4. Recursive Calls:
    - Call quickSort(arr, start, pivotIndex - 1) for left partition
    - Call quickSort(arr, pivotIndex + 1, end) for right partition

partition(arr[], start, end)

This function implements the core partitioning logic using the Lomuto partition scheme.

1. Input: Array arr, indices start and end
2. Choose Pivot:
    - Select last element as pivot: pivot = arr[end]
3. Initialize Pointer:
    - index = start - 1 (tracks position where pivot will be placed)
4. Iterate and Partition:
    - For each element arr[j] from start to end-1:
        - If arr[j] <= pivot:
            - Increment index
            - Swap arr[index] and arr[j]
5. Place Pivot:
    - Swap arr[index + 1] with arr[end]
    - Return index + 1 (final pivot position)

1.3.2 Code

```
#include <bits/stdc++.h>
using namespace std;
using namespace std::chrono;

int partition(int arr[], int start, int end) {
    int pivot = arr[end];
    int index = start - 1;

    for (int j = start; j < end; j++) {
        if (arr[j] <= pivot) {
            index++;
            swap(arr[index], arr[j]);
        }
    }
    swap(arr[index + 1], arr[end]);
    return index + 1;
}

void quickSort(int arr[], int start, int end) {
    if (start < end) {
        int pivotIndex = partition(arr, start, end);
        quickSort(arr, start, pivotIndex - 1);
        quickSort(arr, pivotIndex + 1, end);
    }
}

void display(int arr[], int n) {
    for (int i = 0; i < n; i++) cout << arr[i] << " ";
    cout << endl;
}

int main() {
    int best[]    = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10,11, 12,
13, 14, 15, 16, 17, 18, 19, 20};
    int average[] = {12, 5, 18, 7, 3, 16, 1, 10, 20,
9,14, 2, 19, 6, 11, 4, 13, 8, 17, 15};
    int worst[]   = {20, 19, 18, 17, 16, 15, 14, 13,
12, 11,10, 9, 8, 7, 6, 5, 4, 3, 2, 1};

    int n = sizeof(best) / sizeof(best[0]);

    auto start1 = high_resolution_clock::now();
    quickSort(best, 0, n - 1);
    auto end1 = high_resolution_clock::now();
    auto duration1 =
duration_cast<microseconds>(end1 - start1);
    cout << "Best Case (Already Sorted):" << endl;
    display(best, n);
    cout << "Time taken: " << duration1.count() <<
" microseconds\n\n";


    auto start2 = high_resolution_clock::now();
    quickSort(average, 0, n - 1);
    auto end2 = high_resolution_clock::now();
    auto duration2 =
duration_cast<microseconds>(end2 - start2);
    cout << "Average Case (Random Order):" <<
endl;
    display(average, n);
    cout << "Time taken: " << duration2.count() <<
" microseconds\n\n";


    auto start3 = high_resolution_clock::now();
    quickSort(worst, 0, n - 1);
    auto end3 = high_resolution_clock::now();
    auto duration3 =
duration_cast<microseconds>(end3 - start3);
    cout << "Worst Case (Reverse Sorted):" <<
endl;
    display(worst, n);
    cout << "Time taken: " << duration3.count() <<
" microseconds\n";

    return 0;
}
```

1.3.4 Output

Best Case (Already Sorted):

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

Time taken: 1 microseconds

Average Case (Random Order):

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

Time taken: 1 microseconds

Worst Case (Reverse Sorted):

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

Time taken: 1 microseconds

1.3.5 Explanation

The implemented algorithm is Quick Sort, which is a divide-and-conquer sorting algorithm. It works by selecting a pivot element from the array and partitioning the array such that all elements smaller than the pivot are placed to its left and all elements greater than the pivot are placed to its right. The process is then recursively applied to the left and right subarrays until the entire array is sorted.

Time Complexity Analysis:

- Best Case: $O(nlog\ n)$— occurs when the pivot divides the array into nearly equal halves at each step. This ensures minimal recursion depth.
- Average Case: $O(nlog\ n)$— occurs for a random order of elements, where the partitioning generally results in balanced subarrays.
- Worst Case: $O(n^2)$— occurs when the pivot consistently divides the array into highly unbalanced partitions (e.g., already sorted or reverse sorted arrays in classical Quick Sort).

Space Complexity:
Quick Sort has O(log n) space complexity due to recursion stack space. Unlike Merge Sort, it does not require additional arrays for merging.

Analysis:

- Best Case (Already Sorted): The array [1, 2, 3, …, 20] was sorted in 1 microsecond.
- Average Case (Random Order): The array [12, 5, 18, …, 15] was sorted in 1 microsecond.
- Worst Case (Reverse Sorted): The array [20, 19, 18, …, 1] was sorted in 1 microsecond.

All three cases produced the correct sorted array [1, 2, 3, …, 20].

For small arrays, Quick Sort performs extremely efficiently, and the execution times for best, average, and worst cases are almost identical. In this implementation, the pivot selection strategy (choosing the last element) did not significantly impact performance for small datasets. However, for larger arrays, the worst-case scenario could lead to deeper recursion and longer execution times, highlighting the classical disadvantage of Quick Sort compared to algorithms like Merge Sort.

1.4 Problem 03: Binary Search Algorithm

1.4.1 Algorithms

binarySearch(int arr[], int left, int right, int target)

Input:
An array arr[] of *n* sorted elements, two indices left and right, and a target value to search.

Output:
The index of target in arr[] if found, otherwise -1.

Steps:

1. **Base Case:**
    - If left > right, return -1.
2. **Divide:**
    - Compute mid = left + (right - left) / 2.
3. **Conquer:**
    - If arr[mid] == target, return mid.
    - Else if arr[mid] > target, recursively search the left half:
        - binarySearch(arr, left, mid - 1, target)
    - Else, recursively search the right half:
        - binarySearch(arr, mid + 1, right, target)
4. **Return:**
    - Return the value obtained from the recursive call

findAllOccurrences(int arr[], int n, int target)

Steps:

1. **Initialize:** Create empty list indices.
2. **Search:** Call binarySearch(arr, 0, n-1, target) → index.
3. **Check:** If index == -1, return indices.
4. **Left Scan:**
    1. left = index - 1
    2. While left >= 0 and arr[left] == target, add left to indices and decrement left.
5. **Middle:** Add index to indices.
6. **Right Scan:**
    1. right = index + 1
    2. While right < n and arr[right] == target, add right to indices and increment right.
7. **Return:** Sort indices and return.
8. Return indices.

1.4.2 Code

```
#include <bits/stdc++.h>
using namespace std;

int binarySearch(int arr[], int left, int right, int target)
{
    if (left > right)
        return -1;

    int mid = left + (right - left) / 2;

    if (arr[mid] == target)
        return mid;
    else if (arr[mid] > target)
        return binarySearch(arr, left, mid - 1, target);
    else
        return binarySearch(arr, mid + 1, right, target);


}

vector<int> findAllOccurrences(int arr[], int n, int
target) {

    vector<int> indices;
    int index = binarySearch(arr, 0, n - 1, target);

    if (index == -1)
        return indices;

    int left = index - 1;

    while (left >= 0 && arr[left] == target) {
        indices.push_back(left);
        left--;
    }

    indices.push_back(index);

    int right = index + 1;


    while (right < n && arr[right] == target) {

        indices.push_back(right);
        right++;
    }
sort(indices.begin(), indices.end());

    return indices;
}


void displayArray(int arr[], int n) {
    for (int i = 0; i < n; i++) cout << arr[i] << " ";
    cout << endl;
}

void displayIndices(vector<int> indices) {
    if (indices.empty())
        cout << "Element not found." << endl;
    else {
        cout << "Element found at indices: ";

        for (int idx : indices)
            cout << idx << " ";

        cout << endl;

    }
}

int main() {
    int arr[] = {9, 3, 2, 1, 9, 3, 4, 2};
    int n = sizeof(arr) / sizeof(arr[0]);

    cout << "Original Array: ";
    displayArray(arr, n);

    sort(arr, arr + n);

    cout << "Sorted Array: ";
    displayArray(arr, n);

    int target = 2;
    cout << "Searching for element: " << target << endl;


    vector<int> occurrences = findAllOccurrences(arr, n,
target);
    displayIndices(occurrences);

    return 0;
```

1.4.3 Output

Original Array: 9 3 2 1 9 3 4 2

Sorted Array: 1 2 2 3 3 4 9 9

Searching for element: 2

Element found at indices: 1 2

1.4.4 Explanation

The implemented algorithm is designed to find all occurrences of a target value in a sorted array. It works by first applying a recursive binary search to locate any one occurrence of the target. Once an occurrence is found, the algorithm scans to the left and right of that index to collect all consecutive elements equal to the target. The indices of all occurrences are stored in a list, which is then sorted in ascending order before being returned. This approach efficiently combines the speed of binary search with simple linear scanning to ensure that all positions of the target are captured.

Time Complexity Analysis:

- Best Case: O(log n + k) — occurs when the target is present only a few times, where k is the number of occurrences. Binary search quickly finds one occurrence, and the linear scan is minimal.
- Average Case: O(log n + k) — occurs when the target occurs a moderate number of times. Binary search finds one occurrence efficiently, and scanning covers all occurrences.
- Worst Case: O(n log n) — occurs when the target appears in all positions, requiring the scan to cover the entire array and sorting the list of n indices.

Space Complexity:
The space complexity is O(k + log n), where k is the number of occurrences stored in the list, and O(log n) comes from the recursion stack used by binary search. In the worst case, when the target occurs at all positions, the space requirement becomes O(n).

Analysis:

- Few Occurrences: The array [1, 2, 3, 4, 5, 6, 7, 8] with target 5 returned [4] almost instantly.
- Multiple Occurrences: The array [2, 4, 4, 4, 6, 8] with target 4 returned [1, 2, 3] efficiently.
- All Elements Same: The array [7, 7, 7, 7, 7] with target 7 returned [0, 1, 2, 3, 4] in slightly higher time due to linear scanning and sorting.

This implementation performs efficiently for small and medium-sized arrays. For large arrays where the target occurs frequently, the linear scan and sorting step can increase execution time, but overall, it is faster than searching each element individually without binary search.

1.5 Problem 04: Strassen Matrix Multiplication

1.5.1 Algorithms

add(X, Y, n, sign)
Input: Matrices X and Y of size n×n, sign (1 for addition, -1 for subtraction)
Output: Matrix Z = X ± Y

Steps:

1. Initialize Z as an n×n matrix.
2. For i = 0 to n-1:
   For j = 0 to n-1:
   Z[i][j] = X[i][j] + sign × Y[i][j]
3. Return Z

Strassen(A, B)
Input: Matrices A and B of size n×n (n is power of 2)
Output: Matrix C = A × B

Steps:

1. If n = 1:
   C[0][0] = A[0][0] × B[0][0]
   Return C
2. Compute newSize = n / 2
3. Divide A into submatrices: A11, A12, A21, A22
   Divide B into submatrices: B11, B12, B21, B22
4. Compute the 7 products recursively:
   M1 = Strassen(add(A11, A22, newSize, 1), add(B11, B22, newSize, 1))
   M2 = Strassen(add(A21, A22, newSize, 1), B11)
   M3 = Strassen(A11, add(B12, B22, newSize, -1))
   M4 = Strassen(A22, add(B21, B11, newSize, -1))
   M5 = Strassen(add(A11, A12, newSize, 1), B22)
   M6 = Strassen(add(A21, A11, newSize, -1), add(B11, B12, newSize, 1))
   M7 = Strassen(add(A12, A22, newSize, -1), add(B21, B22, newSize, 1))
5. Compute result submatrices:
   C11 = M1 + M4 - M5 + M7
   C12 = M3 + M5
   C21 = M2 + M4
   C22 = M1 - M2 + M3 + M6
6. Combine C11, C12, C21, C22 into C
7. Return C

1.5.2 Code

```
#include <bits/stdc++.h>
using namespace std;

vector<vector<int>> add(const vector<vector<int>> &a, const vector<vector<int>> &b, int n, int sign = 1) {
    vector<vector<int>> res(n, vector<int>(n));
    for (int i = 0; i < n; i++)
        for (int j = 0; j < n; j++)
            res[i][j] = a[i][j] + sign * b[i][j];
    return res;
}

vector<vector<int>> strassen(const vector<vector<int>> &mat1, const vector<vector<int>> &mat2) {
    int n = mat1.size();
    vector<vector<int>> res(n, vector<int>(n, 0));
    if (n == 1) {
        res[0][0] = mat1[0][0] * mat2[0][0];
        return res;
    }
    int newSize = n / 2;
    vector<vector<int>> a11(newSize, vector<int>(newSize)), a12(newSize, vector<int>(newSize)), a21(newSize, vector<int>(newSize)), a22(newSize, vector<int>(newSize));
    vector<vector<int>> b11(newSize, vector<int>(newSize)), b12(newSize, vector<int>(newSize)), b21(newSize, vector<int>(newSize)), b22(newSize, vector<int>(newSize));
    for (int i = 0; i < newSize; i++)
        for (int j = 0; j < newSize; j++) {
            a11[i][j] = mat1[i][j]; a12[i][j] = mat1[i][j + newSize]; a21[i][j] = mat1[i + newSize][j]; a22[i][j] = mat1[i + newSize][j + newSize];
            b11[i][j] = mat2[i][j]; b12[i][j] = mat2[i][j + newSize]; b21[i][j] = mat2[i + newSize][j]; b22[i][j] = mat2[i + newSize][j + newSize];
        }
    auto m1 = strassen(add(a11, a22, newSize), add(b11, b22, newSize));
    auto m2 = strassen(add(a21, a22, newSize), b11);
    auto m3 = strassen(a11, add(b12, b22, newSize, -1));
    auto m4 = strassen(a22, add(b21, b11, newSize, -1));
    auto m5 = strassen(add(a11, a12, newSize), b22);
    auto m6 = strassen(add(a21, a11, newSize, -1), add(b11, b12, newSize));
    auto m7 = strassen(add(a12, a22, newSize, -1), add(b21, b22, newSize));
    auto c11 = add(add(m1, m4, newSize), add(m7, m5, newSize, -1), newSize);
    auto c12 = add(m3, m5, newSize);
    auto c21 = add(m2, m4, newSize);
    auto c22 = add(add(m1, m3, newSize), add(m6, m2, newSize, -1), newSize);
    for (int i = 0; i < newSize; i++)
        for (int j = 0; j < newSize; j++) {
            res[i][j] = c11[i][j]; res[i][j + newSize] = c12[i][j]; res[i + newSize][j] = c21[i][j]; res[i + newSize][j + newSize] = c22[i][j];
        }
    return res;
}

int main() {
    int n;
    cout << "Enter the size of the square matrices (power of 2):  ";
    cin >> n;
    vector<vector<int>> mat1(n, vector<int>(n)), mat2(n, vector<int>(n));
cout << "Enter elements of the first matrix row by row:\n";

    for (int i = 0; i < n; i++)
        for (int j = 0; j < n; j++)
            cin >> mat1[i][j];
cout << "Enter elements of the second matrix row by row:\n";
    for (int i = 0; i < n; i++)
        for (int j = 0; j < n; j++)
            cin >> mat2[i][j];
    vector<vector<int>> res = strassen(mat1, mat2);
cout << "Result of Strassen's matrix multiplication:\n";
    for (int i = 0; i < n; i++) {
        for (int j = 0; j < n; j++)
            cout << res[i][j] << " ";
        cout << endl;
    }
    return 0;
}
```

1.5.3 Output

Enter the size of the square matrices (power of 2): 2

Enter elements of the first matrix row by row:

1 2

3 4

Enter elements of the second matrix row by row:

5 6

7 8

Result of Strassen's matrix multiplication:

19 22

43 50

1.5.4 Explanation

The implemented algorithm performs matrix multiplication using the Strassen algorithm ,which is an optimized divide-and-conquer method. Unlike the conventional approach, which multiplies matrices in $O(n^3)$ time, Strassen reduces the number of recursive multiplications, achieving a time complexity of approximately $O(n^{2.81})$The algorithm works by first checking for the base case, where the matrix size is 1; in that case, it simply multiplies the single elements. For larger matrices, each input matrix is divided into four submatrices of equal size. Seven intermediate matrices, M1 through M7, are then computed recursively using specific combinations of these submatrices, involving both additions and subtractions. These seven products are then combined in a particular way to form the four submatrices of the final result. Finally, these four submatrices are merged to reconstruct the complete matrix. This approach reduces the number of multiplications from eight (in standard divide-and-conquer) to seven at each recursion level, which improves efficiency for large matrices. The space complexity is higher than standard multiplication due to the storage of temporary submatrices and recursion stack, and the algorithm works most naturally when the matrix size is a power of two, though padding can be used otherwise. Overall, this implementation demonstrates an efficient and elegant application of divide-and-conquer optimization for matrix multiplication, making it suitable for medium to large matrices.

Time Complexity Analysis:

- Best Case / Average Case / Worst Case $O(n\log_2 7) \approx O(n^{2.81})$
    - The time complexity arises from recursively multiplying 7 submatrices instead of 8, reducing the exponent from 3 to ~2.81.
- Base Case: $O(1)$ when $n = 1$.

Space Complexity:

- Recursive Calls: $O(n^2)$ for storing intermediate matrices (M1 to M7, C11 to C22) and recursion stack.
- Overall, space is higher than standard multiplication due to temporary matrices.

Analysis:

- Small Matrices: Works efficiently but overhead from recursion may not always outperform classical multiplication for very small matrices.
- Medium / Large Matrices: Provides a significant speedup compared to naive $O(n^3)$ multiplication.
- Limitations:
    - Requires n to be a power of 2. Padding may be needed for other sizes.
    - More complex to implement and higher memory usage than standard multiplication.

This implementation effectively demonstrates divide-and-conquer with optimization, reducing the number of multiplications while keeping additions manageable. It is particularly suitable for large square matrices.

1.6 Discussion

The lab exercises explored divide-and-conquer strategies for sorting, searching, and matrix multiplication.

Merge Sort

In the merge sort implementation, the recursive splitting and merging of arrays consistently produced correct sorted sequences across already sorted, random, and reverse-sorted inputs. The algorithm demonstrated stable performance, with the output arrays correctly arranged in ascending order for all cases, and the time measurements reflected minimal variation, confirming its reliability and consistency.

Quick Sort

In the quick sort experiments, the recursive partitioning around pivot elements successfully sorted arrays of different initial orders. The outputs verified that the algorithm efficiently rearranged elements to achieve complete order, and the time measurements indicated that the algorithm handled all cases effectively within a very short duration. Observations confirmed that quick sort performed well even for worst-case inputs in this small dataset, while preserving in-place sorting characteristics.

Binary Search

For binary search with multiple occurrences, sorting the array prior to searching allowed efficient location of a target element. The algorithm correctly identified all indices where the target appeared, demonstrating its effectiveness in handling repeated elements. The approach ensured that no occurrence was missed, and the results validated the correctness of combining recursive search with linear checks for neighboring duplicates.

Strassen Matrix Multiplication

The Strassen matrix multiplication implementation successfully computed the product of two square matrices. By recursively dividing matrices into submatrices and combining the intermediate results, the algorithm produced the correct final matrix, even for small inputs. Observations indicated that the recursive approach accurately executed the multiplication logic, preserving the structural integrity of the matrices and confirming the correctness of the divide-and-conquer method.

Overall, the lab reinforced the practical application of recursive divide-and-conquer techniques, showing that these algorithms reliably solve problems in sorting, searching, and matrix multiplication, producing correct results efficiently and consistently.