ICT 2105: Numerical Analysis

Secant Method

The **secant method** is a root-finding procedure in numerical analysis that uses a series of roots of secant lines to better approximate a root of a function f. Let us learn more about the second method, its formula, advantages and limitations, and secant method solved example with detailed explanations in this article.

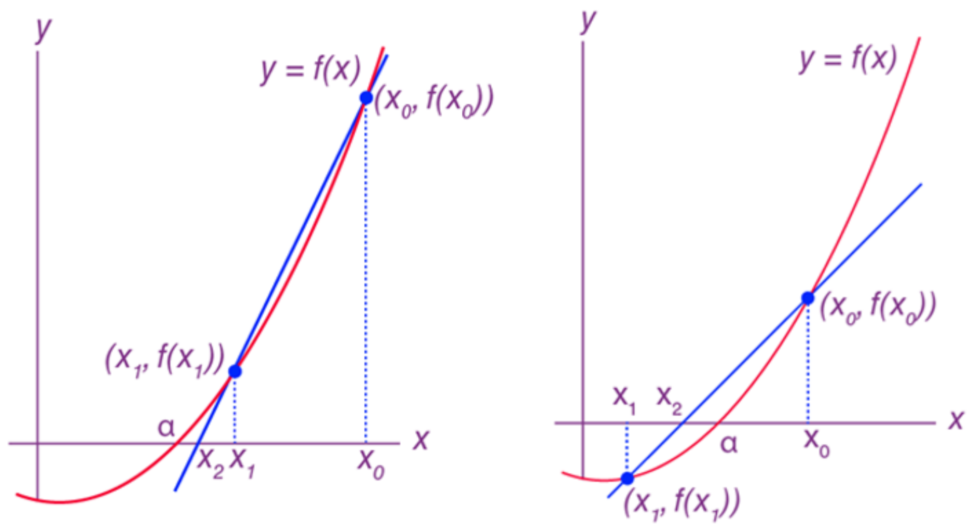
The tangent line to the curve of $y = f(x)$ with the point of tangency $(x_0, f(x_0)$ was used in Newton's approach. The graph of the tangent line about $x = \alpha$ is essentially the same as the graph of $y = f(x)$ when $x_0 \approx \alpha$. The root of the tangent line was used to approximate α.

Let's pretend we have two root estimations of root α, say, x_0 and x_1. Then, we have a linear function $q(x) = a_0 + a_1x$

using $q(x_0) = f(x_0)$, $q(x_1) = f(x_1)$.

This line is also known as a secant line. Its formula is as follows:

$$q(x) = \frac{(x_1-x)f(x_0)+(x-x_0)f(x_1)}{x_1-x_0}$$



The linear equation $q(x) = 0$ is now solved, with the root denoted by x_2. This results in

$$x_2 = x_1 - f(x_1).\ \frac{x_1-x_0}{f(x_1)-f(x_0)}$$

Let the above form be equation (1)

The procedure can now be repeated. Employ x_1 and x_2 to create a new secant line, and then use the root of that line to approximate α;…

Secant Method Steps

The secant method procedures are given below using equation (1).

Step 1: Initialization
x_0 and x_1 of α are taken as initial guesses.

Step 2: Iteration
In the case of n = 1, 2, 3, …,

$$x_{n+1} = x_n - f(x_n).\frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})}$$

until a specific criterion for termination has been met (i.e., The desired accuracy of the answer or the maximum number of iterations has been attained).

Example-1
Compute two iterations for the function $f(x) = x^3 – 5x + 1 = 0$ using the secant method, in which the real roots of the equation f(x) lies in the interval (0, 1).

Solution:
Using the given data, we have,
$x_0 = 0$, $x_1 = 1$, and
$f(x_0) = 1$, $f(x_1) = -3$

Using the secant method formula, we can write
$x_2 = x_1 – [(x_0 – x_1) / (f(x_0) – f(x_1))]f(x_1)$

Now, substitute the known values in the formula,
$= 1 – [(0 – 1) / ((1-(-3))](-3)$
$= 0.25.$
Therefore, $f(x_2) = – 0.234375$

Performing the second approximation, ,
$x_3 = x_2 – [(x_1 – x_2) / (f(x_1) – f(x_2))]f(x_2)$
$=(- 0.234375) – [(1 – 0.25)/(-3 – (- 0.234375))](- 0.234375)$
$= 0.186441$
Hence, $f(x_3) = 0.074276$

Example-2
Use the secant method to estimate the root of $f(x) = e^{-x} - x$.

Solution:

Start with initial estimates of $x_{-1} = 0$ and $x_0 = 1.0$. . .

First iteration:

$$x_{-1} = 0 \qquad f(x_{-1}) = 1.00000$$

$$x_0 = 1 \qquad f(x_0) = -0.63212$$

$$x_1 = 1 - \frac{-0.63212(0 - 1)}{1 - (-0.63212)} = 0.61270 \qquad \varepsilon_t = 8.0\%$$

Second iteration:

$$x_0 = 1 \qquad f(x_0) = -0.63212$$
$$x_1 = 0.61270 \qquad f(x_1) = -0.07081$$

(Note that both estimates are now on the same side of the root.)

$$x_2 = 0.61270 - \frac{-0.07081(1 - 0.61270)}{-0.63212 - (-0.07081)} = 0.56384 \qquad \varepsilon_t = 0.58\%$$

Third iteration:

$$x_1 = 0.61270 \qquad f(x_1) = -0.07081$$

$$x_2 = 0.56384 \qquad f(x_2) = 0.00518$$

$$x_3 = 0.56384 - \frac{0.00518(0.61270 - 0.56384)}{-0.07081 - (-0.00518)} = 0.56717 \qquad \varepsilon_t = 0.0048\%$$

Example-3

Find a root of an equation $\int(x) = x^3 - x - 1$ using Secant method.

Solution:

Here $x^3 - x - 1 = 0$

Let $f(x) = x^3 - x - 1$

Here

x	0	1	2
$f(x)$	-1	-1	5

1^{st} iteration :

$x_0 = 1$ and $x_1 = 2$

$f(x_0) = f(1) = -1$ and $f(x_1) = f(2) = 5$

$$\therefore x_2 = x_0 - f(x_0) \cdot \frac{x_1 - x_0}{f(x_1) - f(x_0)}$$

$$x_2 = 1 - (-1) \times \frac{2 - 1}{5 - (-1)}$$

$x_2 = 1.16667$

$\therefore f(x_2) = f(1.16667) = -0.5787$

2^{nd} iteration :

$x_1 = 2$ and $x_2 = 1.16667$

$f(x_1) = f(2) = 5$ and $f(x_2) = f(1.16667) = -0.5787$

$$\therefore x_3 = x_1 - f(x_1) \cdot \frac{x_2 - x_1}{f(x_2) - f(x_1)}$$

$$x_3 = 2 - 5 \times \frac{1.16667 - 2}{-0.5787 - 5}$$

$x_3 = 1.25311$

$\therefore f(x_3) = f(1.25311) = -0.28536$

3^{rd} iteration :

$x_2 = 1.16667$ and $x_3 = 1.25311$

$f\left(x_2\right) = f(1.16667) = -0.5787$ and $f\left(x_3\right) = f(1.25311) = -0.28536$

$$\therefore x_4 = x_2 - f\left(x_2\right) \cdot \frac{x_3 - x_2}{f\left(x_3\right) - f\left(x_2\right)}$$

$$x_4 = 1.16667 - (-0.5787) \times \frac{1.25311 - 1.16667}{-0.28536 - (-0.5787)}$$

$x_4 = 1.33721$

$\therefore f\left(x_4\right) = f(1.33721) = 0.05388$

4^{th} iteration :

$x_3 = 1.25311$ and $x_4 = 1.33721$

$f\left(x_3\right) = f(1.25311) = -0.28536$ and $f\left(x_4\right) = f(1.33721) = 0.05388$

$$\therefore x_5 = x_3 - f\left(x_3\right) \cdot \frac{x_4 - x_3}{f\left(x_4\right) - f\left(x_3\right)}$$

$$x_5 = 1.25311 - (-0.28536) \times \frac{1.33721 - 1.25311}{0.05388 - (-0.28536)}$$

$x_5 = 1.32385$

$\therefore f\left(x_5\right) = f(1.32385) = -0.0037$

5^{th} iteration :

$x_4 = 1.33721$ and $x_5 = 1.32385$

$f(x_4) = f(1.33721) = 0.05388$ and $f(x_5) = f(1.32385) = -0.0037$

$$\therefore x_6 = x_4 - f(x_4) \cdot \frac{x_5 - x_4}{f(x_5) - f(x_4)}$$

$$x_6 = 1.33721 - 0.05388 \times \frac{1.32385 - 1.33721}{-0.0037 - 0.05388}$$

$x_6 = 1.32471$

$\therefore f(x_6) = f(1.32471) = -0.00004$

Approximate root of the equation $x^3 - x - 1 = 0$ using Secant method is 1.32471

n	x_0	$f(x_0)$	x_1	$f(x_1)$	x_2	$f(x_2)$	Update
1	1	-1	2	5	1.16667	-0.5787	$x_0 = x_1$ $x_1 = x_2$
2	2	5	1.16667	-0.5787	1.25311	-0.28536	$x_0 = x_1$ $x_1 = x_2$
3	1.16667	-0.5787	1.25311	-0.28536	1.33721	0.05388	$x_0 = x_1$ $x_1 = x_2$
4	1.25311	-0.28536	1.33721	0.05388	1.32385	-0.0037	$x_0 = x_1$ $x_1 = x_2$
5	1.33721	0.05388	1.32385	-0.0037	1.32471	-0.00004	$x_0 = x_1$ $x_1 = x_2$