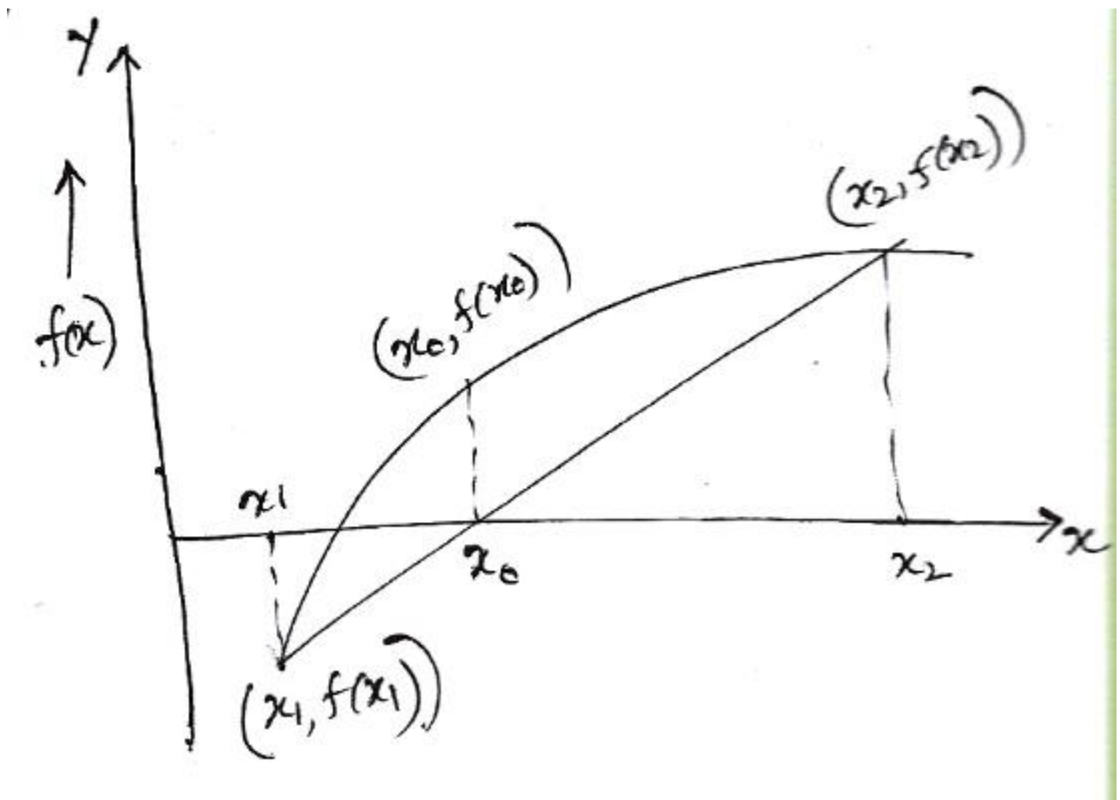
ICT 2105: Numerical Analysis

False position method/Method of false position

The Converge process in the bisection method is very slow. Because the interval between x_1 and x_2 is divided into two equal half and calculate the first approximate value[$x_0 = \frac{x_1+x_2}{2}$]. Actually it is used only to decide the next smaller interval. But a better approximation can be obtained by taking a straight line both the two end points. Thus the point of intersection of this line with x axis (x_0) gives an improved estimate of the root and is called the false position of the root. The value of this point (x_0) and its function is taking as the initial guesses.



False position formula:

We know that equation of the line joining the points (x_1, $f(x_1)$) and (x_2, $f(x_2)$) is given by:

$$\frac{f(x_2) - f(x_1)}{x_1 - x_2} = \frac{y - f(x_1)}{x - x_1}$$

$$\Rightarrow \frac{f(x_2) - f(x_1)}{x_1 - x_2} = \frac{0 - f(x_1)}{x_0 - x_1} \; [since\ the\ line\ intersects\ the\ x\ axis\ at\ x_0, when\ x = x_0; y = 0]$$

$$\Rightarrow x_0 - x_1 = -\frac{f(x_1)(x_2 - x_1)}{f(x_2) - f(x_1)}$$

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$$\Rightarrow\ x_0 = x_1 - \frac{f(x_1)(x_2 - x_1)}{f(x_2) - f(x_1)}$$

$$\Rightarrow\ x_0 = x_1 - \frac{f(x_1)(x_2 - x_1)}{f(x_2) - f(x_1)} \quad \textit{this equation is called the false position formula.}$$

$$\Rightarrow\ x_0 = \frac{x_1 f(x_2) - x_2 f(x_1)}{f(x_2) - f(x_1)}$$

$$\Rightarrow\ x_0 = \frac{af(b) - bf(a)}{f(b) - f(a)}\ [x_1 = a; x_2 = b]$$

Regula falsi method / Method of false position:

Ex-1: Find the positive root of $x^3 = 2x + 5$ by false position method.

Solution: let $f(x) = \ x^3 - 2x + 5 \dots\dots\dots\dots\dots\dots\dots\dots..(1)$

Here, $f(2) = 8 - 9 = -1 < 0;\quad f(3) = 27 - 6 - 5 = 16 > 0$

So the root lies between 2 and 3. So, **a=2; b=3**

Now Regula falsi method:

$$x_0 = \frac{af(b) - bf(a)}{f(b) - f(a)} = \frac{2f(3) - 3f(2)}{f(3) - f(2)} = \frac{\left(2 * 16 - 3 * (-1)\right)}{16 - (-1)} = 2.058824$$

Now from (1) we can write:

$$f(x_0) = (x_0)^3 - 2x_0 + 5 = (2.058824)^3 - 2 * 2.058824 + 5 = \ -0.390795 < 0$$

So the root lies between 2.058824 and 3 $\Rightarrow [a = 2.058824, b = 3]$

$$x_1 = \frac{af(b) - bf(a)}{f(b) - f(a)} = \frac{2.058824 * f(3) - 3f(2.058824)}{f(3) - f(2.058824)} = 2.081264$$

From (1) we can write:

$$f(x_1) = (x_1)^3 - 2x_1 + 5 = (2.081264)^3 - 2 * 2.081264 + 5 = \ -0.147200 < 0$$

So the root lies between 2.081264 and 3 $\Rightarrow [a = 2.081264, b = 3]$

$$x_2 = \frac{af(b) - bf(a)}{f(b) - f(a)} = \frac{2.081264 * f(3) - 3f(2.081264)}{f(3) - f(2.081264)} = 2.089639$$

From (1) we can write:

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$$f(x_2) = (x_2)^3 - 2x_2 + 5 = (2.089639)^3 - 2 * 2.089639 + 5 = -0.054679 < 0$$

So the root lies between 2.089639 and 3 $\Rightarrow [a = 2.089639, b = 3]$

$$x_3 = \frac{af(b) - bf(a)}{f(b) - f(a)} = \frac{2.089639 * f(3) - 3 * f(2.089639)}{f(3) - f(2.089639)} = 2.09274$$

From (1) we can write:

$$f(x_3) = (x_3)^3 - 2x_3 + 5 = (2.09274)^3 - 2 * 2.09274 + 5 = -0.020198 < 0$$

So the root lies between 2.09274 and 3 $\Rightarrow [a = 2.09274, b = 3]$

$$x_4 = \frac{af(b) - bf(a)}{f(b) - f(a)} = \frac{2.09274 * f(3) - 3 * f(2.09274)}{f(3) - f(2.09274)} = 2.093884$$

From (1) we can write:

$$f(x_4) = (x_4)^3 - 2x_4 + 5 = (2.093884)^3 - 2 * 2.093884 + 5 = -0.007448 < 0$$

So the root lies between 2.093884 and 3 $\Rightarrow [a = 2.093884, b = 3]$

$$x_5 = \frac{af(b) - bf(a)}{f(b) - f(a)} = \frac{2.093884 * f(3) - 3 * f(2.093884)}{f(3) - f(2.093884)} = 2.094306$$

From (1) we can write:

$$f(x_5) = (x_5)^3 - 2x_5 + 5 = (2.094306)^3 - 2 * 2.094306 + 5 = -0.002740 < 0$$

So the root lies between 2.094306 and 3 $\Rightarrow [a = 2.094306, b = 3]$

$$x_6 = \frac{af(b) - bf(a)}{f(b) - f(a)} = \frac{2.094306 * f(3) - 3 * f(2.094306)}{f(3) - f(2.094306)} = 2.094461$$

From (1) we can write:

$$f(x_6) = (x_6)^3 - 2x_6 + 5 = (2.094461)^3 - 2 * 2.094461 + 5 = -0.0010098 < 0$$

So the root lies between 2.094461 and 3 $\Rightarrow [a = 2.094461, b = 3]$

$$x_7 = \frac{af(b) - bf(a)}{f(b) - f(a)} = \frac{2.094461 * f(3) - 3 * f(2.094461)}{f(3) - f(2.094461)} = 2.09451$$

From (1) we can write:

$$f(x_7) = (x_7)^3 - 2x_7 + 5 = (2.09451)^3 - 2 * 2.09451 + 5 = -0.0003914 < 0$$

So the root lies between 2.09451 and 3 $\Rightarrow [a = 2.09451, b = 3]$

$$x_8 = \frac{af(b) - bf(a)}{f(b) - f(a)} = \frac{2.09451 * f(3) - 3 * f(2.09451)}{f(3) - f(2.09451)} = 2.09453$$

$$\therefore x_7 = x_8 = 2.0945$$

Hence the required root is : 2.0945

Exercise:

Solve for a positive root of the following equation by using Regula Falsi Method.

1. $x^3 - 4x - 1 = 0 \rightarrow f(1) = -2 < 0; f(0) = 1 > 0\ [a = 0; b = 1]$
2. $3x - cos\ cos\ x\ - 1 = 0 \rightarrow f(0.6) = -0.0253 < 0; f(0.61) = 0.0104 > 0$
   $[a = 0.6; b = 0.61]$
3. $x^3 - 2x - 5 = 0 \rightarrow f(2) = -3 < 0; f(3) = 13 > 0\ [a = 2; b = 3]$
4. $x^3 - 5x + 3 = 0 \rightarrow f(1) = -1 < 0; f(2) = 1 > 0\ [a = 1; b = 2]$
5. $xx\ - 1.2 = 0 \rightarrow f(2) = -0.597940 < 0; f(3) = 0.23136 > 0\ [a = 2; b = 3]$
6. $x\ - 12 = 0 \rightarrow f(3) = -4.0986 < 0; f(4) = 2.6137 > 0\ [a = 3; b = 4]$

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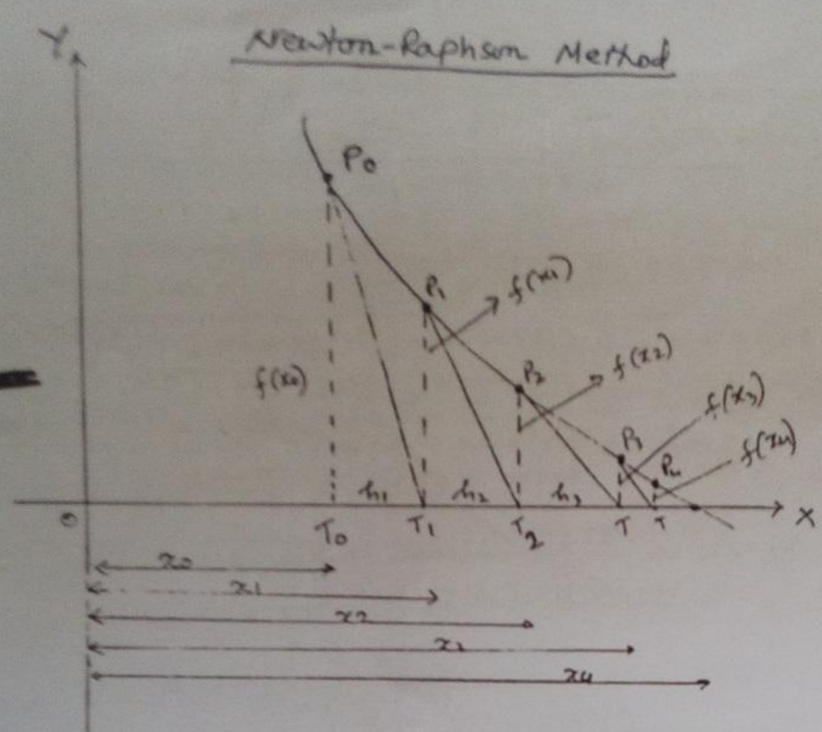
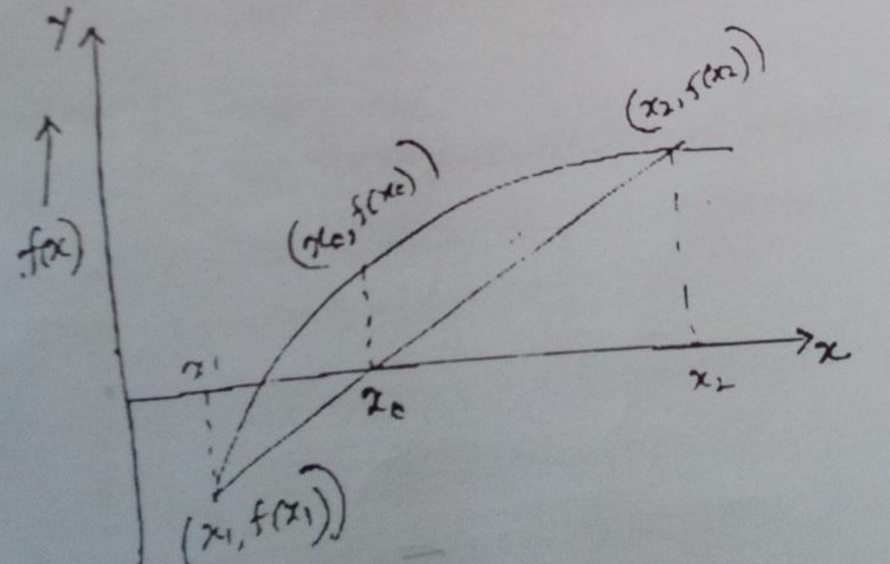
Newton-Raphson Method





False Position Method