Probability Distribution

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Probability Distribution

- Sample space: All possible outcomes of an experiment comprise a set of outcomes is called sample space

- Example: a coin toss 3 times and interest is number of heads then 0, 1, 2, 3 will of interest rather than sample space.

Probability Distribution

A dia tossed , our interest is even number

Experiment: dia toss
sample space : 1,2,3,4,5,6
Event: even , not even
probability: 1/2 , 1/2

X= even number, not even number (yes, no or (1,0))
variable: because value may change experiment to experiment
Random variable: yes because probability is associated for each value of x
descrete r.v. : yes because x vues are integer/descrete

Continuous r.v. : x =
wieight of coffee, x= .980 kg - 1.01 kg
Continuous r.v

A coin tossed 2 times, interest is same will come, or different will come

Experiment: Coin tossed two time s

Sample space: HH, HT, TH , TT
Event: Same or not same (HH, TT) or (HT, TH)

probability: 1/2, 1/2

Variable: x = same , not same (1,0)

Random variable? yes

Descrete r.v ? yes

Probability Distribution

- Random variable: variable whose value is determined by the outcome of a random experiment.
- Discrete random variable: random variable whose set of values assumed is countable.
- Continuous random variable: random variable whose set of values assumed is uncountable.

Discrete random variable

Two balls are drawn at random in succession without replacement from an urn containing 4 red balls and 6 black balls. Find the probabilities of all possible outcomes.

Solve:

Let X = # of red balls in the outcome

Possible outcomes:	RR	RB	BR	BB
X	2	1	1	0

Discrete random variable

$P(RR)=P(R) \,.\, P(R) = 4/10 \;.\; 3/9 = 2/15 = P(X=2)$

$P(X=1)=P(RB)= 4/10 \,.\, 6/9 = 4/15$

$P(X=1)=P(BR)= 6/10 \,.\, 4/9 = 4/15$

$P(X=0)=P(BB)= 6/10 \,.\, 5/9 = 1/3$

So probability distribution is

X	0	1	2
P(x)	1/3	8/15	2/15

Probability distribution: is a table listing all possible values together with the associated probabilities. The above is discrete probability distribution.

What do you mean by probability distribution?

Continuous random variable

A jar of coffee is picked at random from a filling process in which an automatic machine is filling coffee jars each with 1 kg of coffee. Due to some faults in the automatic process, the weight of a jar could vary from jar to jar in the range 0.9 kg to 1.05 kg.

Let X denote the weight of a jar of coffee selected. What is the range of X?

Ans: Possible outcomes: $0.9 <= X <= 1.05$

Continuous random variable

For continuous random variable the probability distribution is denoted by a function probability density function.

Instead of evaluating the probability of a value it expresses probability of a range of values.

$$\int_a^b f(x)dx = P(a \leq X \leq b)$$

Area under the curve from the range **a** to **b**

Expectation and variance of random variable

X	occurances	Probability
0	1	1/8
1	3	3/8
2	3	3/8
3	1	1/8

For example, a large number of time this experiment has been conducted. The results may be as foolows:

X= 0 , 1, 1 ,2 , 2, 1,3,1, 2, 1, 2,. 1, 3, 0,,,,,,,,,,, 2 .... (80 times)

0 -> 10
1 -> 30
2 -> 30
3 -> 10

Sum (x) = 0* 10 + 1* 30 + 2 * 30 + 3 * 10
E(x) = 0* 10/80 + 1* 30/80 + 2 * 30/80 + 3 * 10/80
= 0 * p(0) + 1* p(1) + 2* p(2) + 3* p3)
= sum[x * p(x)]

Bernoulli Distribution: Bernoulli (p)

- Used to model the distribution with two possible outcomes
  - E.g. coin flipping
  - A customer will click a login button or not
  - A server is up or down

- Parameter: p

$$p(x) = \begin{cases} 1-p & if \quad x = 0 \\ p & if \quad x = 1 \\ 0 & otherwise \end{cases}$$

Mean: *p*, Variance: *p*(1−*p*)

Binomial Distribution: bin(n,p)

- Used to model the number of *x* successes in *n* Bernoulli trials with probability *p* of success on each trial
  - e.g. number of defective items in a batch of size *n*
  - no. of packets that reach the destination without loss
- Constant probability for each observation

✓✓××✓×××✓✓✓××××.... ××

n trials, where x is the number of ✓

$$p(x) = \binom{n}{x} p^x (1-p)^{n-x} \quad \forall x \in \{0,1,\cdots\}$$

$$\textit{where} \binom{n}{x} = \text{combinations of selecting x items out of n objects}$$

$$= \frac{n!}{x!(n-x)!}$$

Mean: *np*, Variance: *np*(1−*p*)

Binomial Distribution: bin(n,p)

- If $Y_1, Y_2, \ldots, Y_n$ are independent *Bernoulli*(p) random variables, then $Y_1+Y_2+\ldots+Y_n \sim bin(n, p)$

- If $X_1, X_2, \ldots, X_n$ are independent random variables and $X_i \sim bin(t_i, p)$ random variables, then

$$X_1+X_2+\ldots+X_n \sim bin(t_1+t_2+\ldots+t_n, p)$$

✓✓××✓×××✓✓✓××××… ×× t_1 trials

××✓×✓✓✓✓✓×××××✓✓…✓✓ t_2 trials

…

…

××××✓✓×✓✓✓×××××…✓× t_n trials

Binomial Distribution

A binomial experiment is one that possesses the following properties:

- The experiment consists of **n** repeated trials
- Each trials results in an outcome that may be classified as a success or a failure.
- The probability of a success, denoted by **p**, remains constant from trial to trial.
- Repeated trials are independent

Binomial Distribution

- If X is a random variable that possesses values which are the number success in n trials of a binomial experiment, then X is called **binomial random variable**.
- Hence, the probability distribution of X is called binomial probability distribution.

$$P(X=x)=\binom{n}{x}\,p^x(1-p)^{(n-x)}$$

where n=number of trials;

x = # of success = 0,1,2, ….. n

P= probability of success

Binomial Distribution

- Mean $E(x)=np$
- Variance $Var(x)= np(1-p)$

Example: Binomial

A die is tossed 3 times, What is the probability of

a) No fives turning up?
b) 1 five?
c) 3 fives?

Example: Binomial

- This is binomial: 5 or not 5
- P(5)=1/6 ; p(not 5)= 5/6; n=3

a) x=0 $$P(X=0)=\binom{n}{0}(1/6)^x(1-1/6)^{(3-0)}=\frac{125}{216}$$

b) x=1 $$P(X=1)=\binom{n}{1}(1/6)^1(1-1/6)^{(3-1)}=\frac{75}{216}$$

c) x=2 $$P(X=2)=\binom{3}{2}(1/6)^2(1-1/6)^{(3-2)}=\frac{15}{216}$$

d) x=3 $$P(X=3)=\binom{3}{3}(1/6)^3(1-1/6)^{(3-3)}=\frac{1}{216}$$

Example: Binomial Distribution

- Hospital records shows that patient suffering from a diseases and 75% die of it. What is the probability that of 6 randomly selected patients, 4 will recover?

- Solve: Here, n=6; x=4 ; p=.25

$$P(X = 4) = \binom{6}{4} (.25)^4 (.75)^{(6-4)} = 0.03296$$

Example: Binomial Distribution

A manufacturer of meta pistons finds that on the average, 12% of his pistons are rejected because they are either oversize or undersize. What is the probability that a batch of 10 pistons will contain

a) No more than 2 rejects?

b) At least 2 reject?