Muller's Method

$P(x) = a_0 + a_1(x-c) + a_2(x-c)^2$ - - - (1) polynomial

If we choose $c = x_3$ then

$$P(x) = a_0 + a_1(x - x_3) + a_2(x - x_3)^2 \quad \text{- - - (11)}$$

since x_4 is a root of $P(x)$, at $x = x_4$, $P(x) = 0$ and

therefore, equtn (2) becomes

$$a_2(x_4 - x_3)^2 + a_1(x_4 - x_3) + a_0 = 0$$

solving ~~eqn~~ for $(x_1 - x_3)$ we get

$$x_1 - x_3 = \frac{-2a_0}{a_1 \pm \sqrt{a_1^2 - 4a_2a_0}} \quad \text{- - (3)}$$

~~Since x_4 is a root of $P(x)$, at $x = x_4$, $P(x) = 0$ and~~

~~therefore, eqn (2)~~

The constants a_0, a_1, a_2 can be defined in terms of Known function values $f(x_1)$, $f(x_2)$ and $f(x_3)$ as follows

At $x = x_1$, x_2 and x_3 we have

$$a_2(x_1 - x_3)^2 + a_1(x_1 - x_3) + a_0 = P(x_1) = f(x_1)$$

$$a_2(x_2 - x_3)^2 + a_1(x_2 - x_3) + a_0 = P(x_2) = f(x_2)$$

$$a_2(x_3 - x_3) + a_1(x_3 - x_3) + a_0 = P(x_3) = f(x_3)$$

Letting $h_1 = x_1 - x_2$, $h_2 = x_2 - x_3$ and denoting $f_1 = f(x_1)$, we get

$$a_2 h_1^2 + a_1 h_1 + a_0 = f_1$$
$$a_2 h_2^2 + a_1 h_2 + a_0 = f_2$$
$$\cancel{0} + 0 + a_0 = f_3$$

Since $a_0 = f_3$, we can obtain a_1 and a_2 by solving the equations

$$\begin{cases} a_2 h_1^2 + a_1 h_1 = f_1 - f_3 = d_1 \\ a_2 h_2^2 + a_1 h_2 = f_2 - f_3 = d_2 \end{cases}$$

$$\to a_1 = \frac{d_2 h_1^2 - d_1 h_2^2}{h_1 h_2 (h_1 - h_2)}$$

$$a_2 = \frac{d_1 h_2 - d_2 h_1}{h_1 h_2 (h_1 - h_2)}$$

from (3)

$$x_4 = x_3 + h_4$$

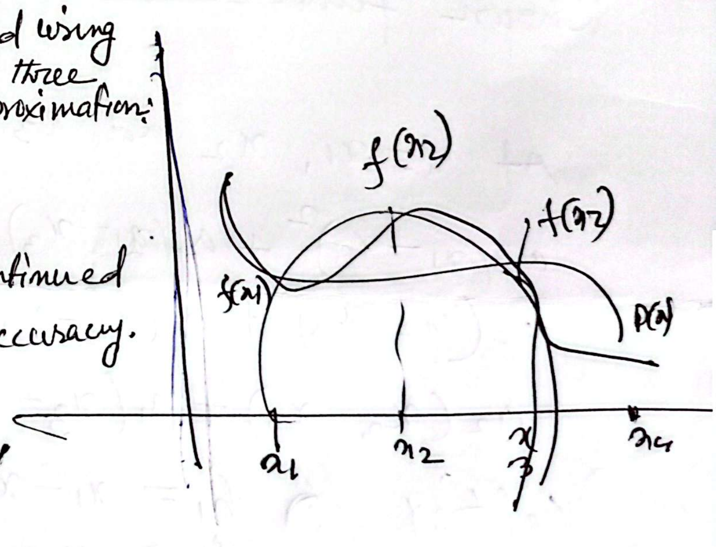
where $$h_4 = \frac{-2a_0}{a_1 \pm \sqrt{a_1^2 - 4a_2 a_0}}$$

h_4's chosen such that h_4 is as small in magnitude as possible so that x_3's close to x_4. That is, the magnitude of $(a_1 \pm \sqrt{a_1^2 - 4a_2a_0})$ should be large.

→ This process is then repeated using x_2, x_3 and x_4 as the initial three points to obtain the next approximation:

$$x_5 = x_4 + h_5$$

This process is ~~then repeated~~ continued till $f(x_i)$ is within the specified accuracy.



Muller's Method Example

Ex. Solve the equation:

$$f(x) = x^3 + 2x^2 + 10x - 20 = 0$$

Sol^n: Let us assume the three starting points as

Iteration I

$$x_1 = 0, \ x_2 = 1, \ x_3 = 2$$

$f_1 = -20$
$f_2 = -7$
$f_3 = 16$

$h_1 = x_1 - x_3 = -2$
$h_2 = x_2 - x_3 = -1$
$d_1 = f_1 - f_3 = -36$
$d_2 = f_2 - f_3 = -23$
$D = h_1 h_2 (h_1 - h_2)$
$= 2(-2+1) = -2$

$$\therefore a_1 = \frac{(-23)(-2)^2 - (-36)(-1)^2}{-2} = 28$$

$$a_2 = \frac{(-36)(-1) - (-23)(-2)}{-2} = 5$$

$$h = \frac{-2 \times 16}{28 \pm \sqrt{28^2 - 4(5)(16)}} = -\frac{32}{49.540659} \quad \text{(Choosing + sign)}$$

$$= -0.645934$$

$$x_4 = x_3 + h = 1.3540659$$

Iteration 2

$$x_1 = 1, \ x_2 = 2, \ x_3 = 1.3540659$$

$h_1 = x_1 - x_3 = -0.3540659$
$h_2 = x_2 - x_3 = 0.645934$
$f_1 = -7$
$f_2 = 16$
$f_3 = f(1.3540659) = -0.3096797$

$d_1 = f_1 - f_3 = -6.6903202$
$d_2 = f_2 - f_3 = 16.3096797$
$D = h_1 h_2 (h_1 - h_2) = 0.2287031$
$a_1 = \dfrac{d_2 h_1^2 - d_1 h_2^2}{D} = 21.145459$
$a_2 = \dfrac{d_1 h_2 - d_2 h_1}{D} = 6.3540717$
$a_0 = f_3 = -0.3096797$

$$\therefore h = \frac{-2a_0}{a_1 \pm \sqrt{a_1^2 - 4a_2 a_0}} = \frac{0.6193594}{42.47622} = 0.0145813$$

$$\therefore x_4 = x_3 + h = 1.3686472$$

This process can be continued to obtain better accuracy.
The correct answer is 1.368808107

<u>Ex.</u> (a) $x^3 - x - 2$, $x_1 = 1$, $x_2 = 1.2$ and $x_3 = 1.4$

(b) $1 + 2x - \tan x$, $x_1 = 1.5$, $x_2 = 1.4$ and $x_3 = 1.3$

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