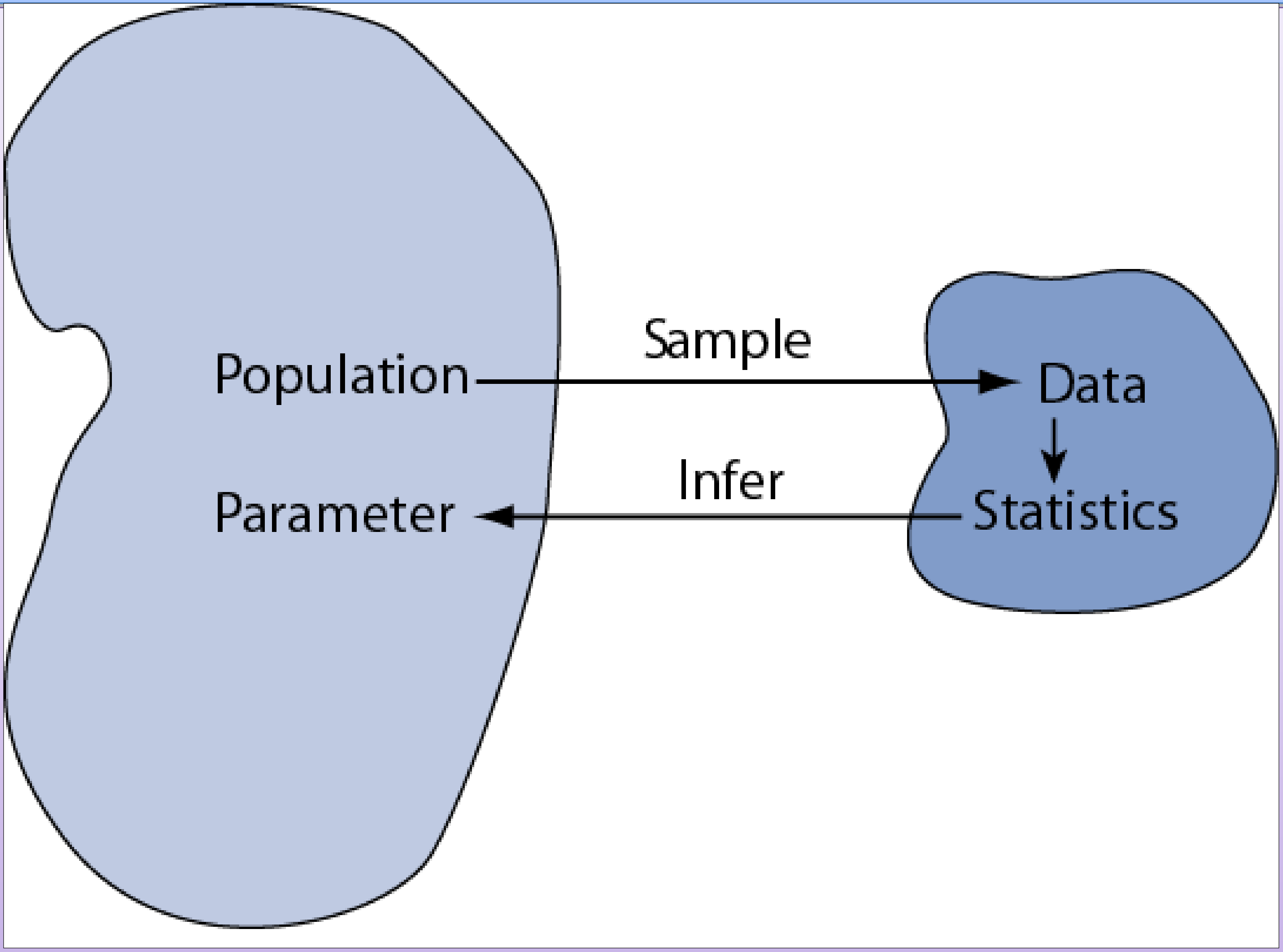
Hypothesis Testing

Md. Fazlul Karim Patwary, Professor, IIT, JU

Some Terms in Prior

- **Population** ≡ all possible values
- **Sample** ≡ a portion of the population
- **Statistical inference** ≡ generalizing from a sample to a population with calculated degree of certainty
- Two forms of statistical inference
  - **Hypothesis testing**
  - **Estimation**
- **Parameter** ≡ a characteristic of population, e.g., population mean μ
- **Statistic** ≡ calculated from data in the sample, e.g., sample mean ($\bar{x}$)

Some Terms in Prior



Some Terms in Prior

- Hypothesis
  - An educated guess
  - A claim or statement about a property of a Population

Goal: The goal in Hypothesis Testing is to analyze a sample in an attempt to distinguish between

population characteristics that are likely to occur

and

$\bar{x}$

population characteristics that are *unlikely* to occur.

Some Terms in Prior

- **Null hypothesis (H_0)** is a claim of "no difference in the population"
- **Alternative hypothesis (H_a)** claims "H_0 is false"

We collect data and seek evidence against H_0 as a way of bolstering H_a

$\bar{x}$

Some Terms in Prior

Null Hypothesis

- Statement about the value of a population parameter
- Represented by H_0
- Always stated as an Equality

Alternative Hypothesis

- Statement about the value of a population parameter that must be true if the null hypothesis is false
- Represented by H_1
- Stated in on of three forms
  - $>$
  - $<$
  - $\neq$

Example: Hypothesis

- **The problem:** In the 1970s, 20–29 year old men in the U.S. had a mean μ body weight of 170 pounds. Standard deviation σ was 40 pounds. We test whether mean body weight in the population now differs.
- **Null hypothesis** $H_{0:}\ \mu = 170$ ("no difference")
- The **alternative hypothesis** can be either

  $H_{a:}\ \mu > 170$ (**one-sided test**) or
  $H_{a:}\ \mu \neq 170$ (**two-sided test**)

Example: Hypothesis

- In a trial a jury must decide between two hypotheses. The null hypothesis is

  H_0: The defendant is innocent

- The alternative hypothesis or research hypothesis is

  H_1: The defendant is guilty

- The jury does not know which hypothesis is true. They must make a decision on the basis of evidence presented.

Some Terms in Prior

- **Test statistic -** Quantity based on sample data and null hypothesis used to test between null and alternative hypotheses

- **Rejection region -** Values of the test statistic for which we reject the null in favor of the alternative hypothesis

Type I vs. Type II Error

Test Result – / True State	H_0 True	H_0 False
H_0 True	Correct Decision	Type I Error
H_0 False	Type II Error	Correct Decision

$$\alpha = P(Type\ I\ Error) \quad \beta = P(Type\ II\ Error)$$

- Goal: Keep α, β reasonably small

Alpha vs. Beta

- a is the probability of Type I error
- b is the probability of Type II error
- The experimenters have the freedom to set the α-level for a particular hypothesis test. That level is called the level of significance for the test.
- Changing a can (and often does) affect the results of the test—whether you reject or fail to reject H_0.

Alpha vs. Beta

- It would be wonderful if we could force both α and β to equal zero.
-
- Unfortunately, these quantities have an inverse relationship. As α increases, β decreases and vice versa.

- The only way to decrease both α and β is to increase the sample size.

- To make both quantities equal zero, the sample size would have to be infinite—you would have to sample the entire population.

Some Terms in Prior

- Every hypothesis test ends with the experimenters either
  - Rejecting the Null Hypothesis, or
  - Failing to Reject the Null Hypothesis

- If one never able to ***accept*** the Null Hypothesis, the best comment about the Null Hypothesis is that there not enough evidence, based on a sample, to reject it!

Brief Steps in Hypothesis Testing

The critical concepts are theses:

1. There are two hypotheses, the null and the alternative hypotheses.
2. The procedure begins with the assumption that the null hypothesis is true.
3. The goal is to determine whether there is enough evidence to infer that the alternative hypothesis is true, or the null is not likely to be true.
4. There are two possible decisions:

– Conclude that there is enough evidence to support the alternative hypothesis. Reject the null.

– Conclude that there is *not* enough evidence to support the alternative hypothesis. Fail to reject the null.

Hypothesis Testing Steps

A. Null and alternative hypotheses
B. Test statistic
C. P-value and interpretation
D. Significance level (optional)

Example: Hypothesis

- **The problem:** In the 1970s, 20–29 year old men in the U.S. had a mean μ body weight of 170 pounds. Standard deviation σ was 40 pounds. We test whether mean body weight in the population now differs.
- **Null hypothesis** $H_{0:}$ μ = 170 ("no difference")
- The **alternative hypothesis** can be either

  $H_{a:}$ μ > 170 (**one-sided test**) or
  $H_{a:}$ μ ≠ 170 (**two-sided test**)

Test Statistic

This is an example of a one-sample test of a mean when σ is known. Use this statistic to test the problem:

$$z_{\text{stat}} = \frac{\bar{x} - \mu_0}{SE_{\bar{x}}}$$

$$\text{where } \mu_0 \equiv \text{population mean assuming } H_0 \text{ is true}$$

$$\text{and } SE_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

Example: *z* statistic

- For the illustrative example, $\mu_0 = 170$
- We know $\sigma = 40$
- Take an SRS of $n = 64$. Therefore

$$SE_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{40}{\sqrt{64}} = 5$$

- If we found a sample mean of 173, then

$$z_{\text{stat}} = \frac{\bar{x} - \mu_0}{SE_{\bar{x}}} = \frac{173 - 170}{5} = 0.60$$

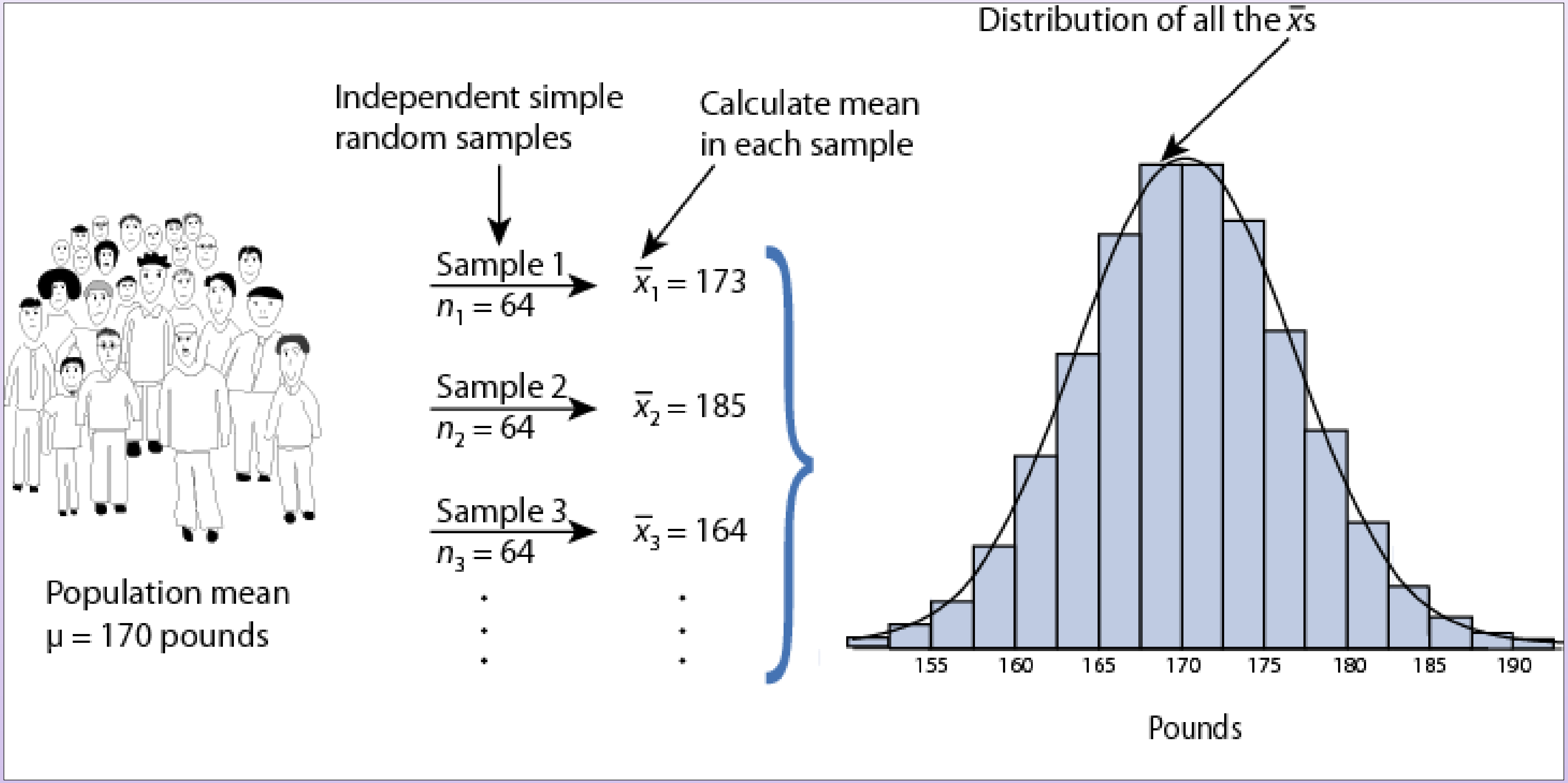
Example: *z* statistic

If we found a sample mean of 185, then

$$z_{\text{stat}} = \frac{\bar{x} - \mu_0}{SE_{\bar{x}}} = \frac{185 - 170}{5} = 3.00$$

- $\mu_{H0} + z\ \sigma_x = 8000 + 1.96(40) = 8078.4$
- 185 + 1.96 x 40 =263.4
- $\mu_{H0} - z\ \sigma_x = 8000 + 1.96(40) = 8078.4$
- 185 - 1.96 x 40 =106.6
- 106.4 <= pop mean <= 263.4

Reasoning Behind μ and z_{stat}



Sampling distribution of xbar under H_0: μ = 170 for n = 64 $\Rightarrow$

$$\bar{x} \sim N(170,5)$$

P-value

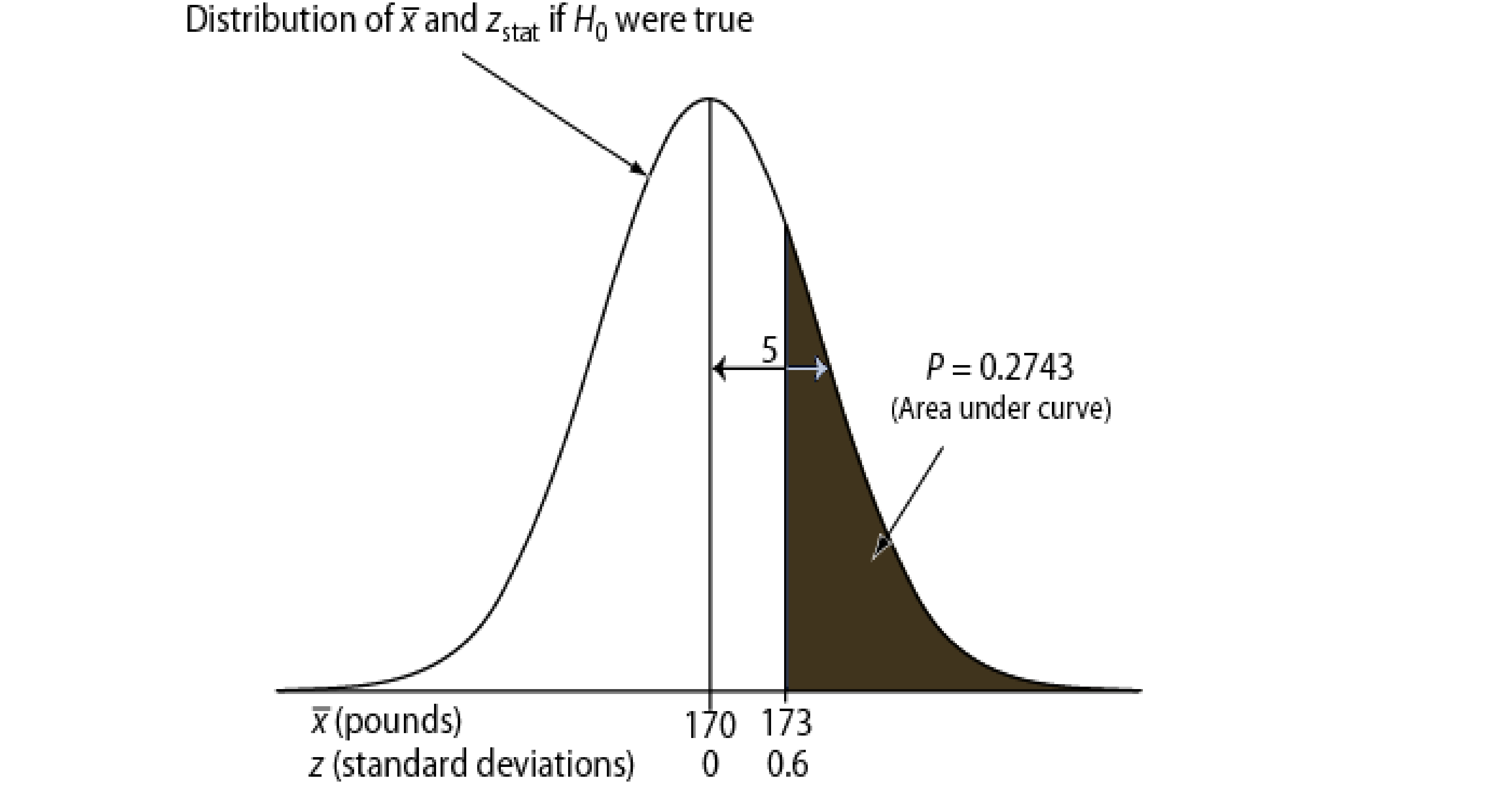
- The *P*-value answer the question: What is the probability of the observed test statistic or one more extreme **when H_0 *is true***?
- This corresponds to the AUC in the tail of the Standard Normal distribution beyond the $z_{stat.}$
- Convert *z* statistics to *P*-value :

  For H_a: $\mu > \mu_0 \Rightarrow P = \Pr(Z > z_{stat})$ = right-tail beyond z_{stat}

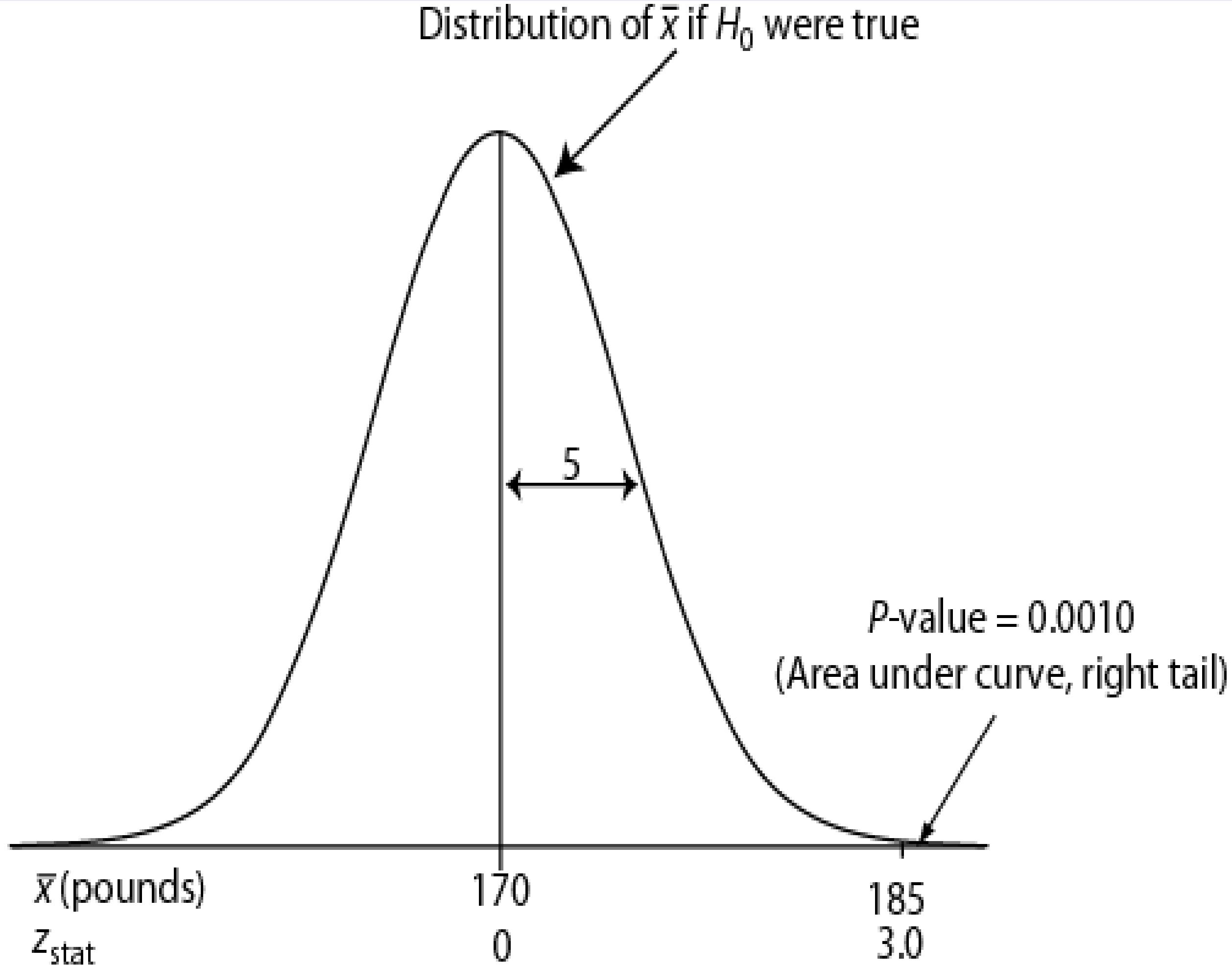
  For H_a: $\mu < \mu_0 \Rightarrow P = \Pr(Z < z_{stat})$ = left tail beyond z_{stat}

  For H_a: $\mu \neq \mu_0 \Rightarrow P = 2 \times$ one-tailed *P*-value
- Use Table B or software to find these probabilities

One-sided P-value for z_{stat} of 0.6

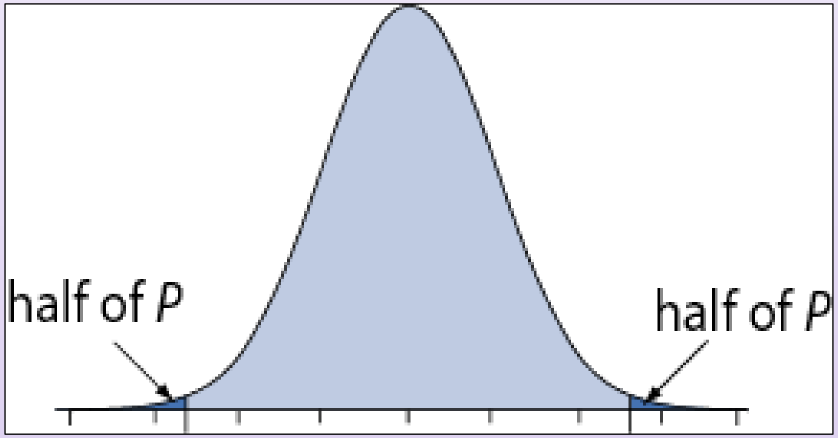


One-sided P-value for z_{stat} of 3.0



Two-Sided *P*-Value

- One-sided $H_a \Rightarrow$ AUC in tail beyond z_{stat}
- Two-sided $H_a \Rightarrow$ consider potential deviations in both directions $\Rightarrow$ double the one-sided *P*-value



Examples: If one-sided *P* = 0.0010, then two-sided *P* = 2 × 0.0010 = 0.0020. If one-sided *P* = 0.2743, then two-sided *P* = 2 × 0.2743 = 0.5486.

Interpretation

- *P*-value answer the question: What is the probability of the observed test statistic … **when H_0 *is true*?**
- Thus, smaller and smaller *P*-values provide stronger and stronger evidence against H_0
- Small *P*-value $\Rightarrow$ strong evidence

Interpretation

Conventions*

$P > 0.10 \Rightarrow$ non-significant evidence against H_0

$0.05 < P \leq 0.10 \Rightarrow$ marginally significant evidence

$0.01 < P \leq 0.05 \Rightarrow$ significant evidence against H_0

$P \leq 0.01 \Rightarrow$ highly significant evidence against H_0

Examples

$P = .27 \Rightarrow$ non-significant evidence against H_0

$P = .01 \Rightarrow$ highly significant evidence against H_0

*** It is *unwise* to draw firm borders for "significance"**

(Summary) One-Sample z Test

A. Hypothesis statements

H_0: $\mu = \mu_0$ vs.

H_a: $\mu \neq \mu_0$ (two-sided) or

H_a: $\mu < \mu_0$ (left-sided) or

H_a: $\mu > \mu_0$ (right-sided)

B. Test statistic

$$\mathrm{z}_{\mathrm{stat}} = \frac{\bar{x} - \mu_0}{SE_{\bar{x}}} \text{ where } SE_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

C. P-value: convert z_{stat} to P value

D. Significance statement (usually not necessary)

Two-Sample t-tests

Clearly, we are often faced with making judgments for circumstances that involve more than one population and sample. For the moment, we will focus on the so-called *two sample situation.* That is, we consider two populations.

	City A	City B
Mean	μ_A	μ_B
SD	σ_A	σ_B

Question:

Do you believe the two populations have the same mean?

Two Sample Hypotheses

H_0: $\mu_A = \mu_B$ versus H_1: $\mu_A \neq \mu_B$

or equivalently

H_0: $\Delta = \mu_A - \mu_B = 0$ versus H_1: $\Delta = \mu_A - \mu_B \neq 0$.

Parameters vs. Estimates

Population 1		Population 2	
Parameter	Estimate	Parameter	Estimate
Populations of individual values			
μ_1	$\bar{x}_1$	μ_2	$\bar{x}_2$
σ_1^2	s_1^2	σ_2^2	s_2^2
σ_1	s_1	σ_2	s_2
Populations of means, samples of size n_1 and n_2			
μ_1	$\bar{x}_1$	μ_2	$\bar{x}_2$
σ_1^2/n_1	s_1^2/n_1	σ_2^2/n_2	s_2^2/n_2
$\sigma_1/\sqrt{n_1}$	$s_1/\sqrt{n_1}$	$\sigma_2/\sqrt{n_2}$	$s_2/\sqrt{n_2}$

Parameters vs. Estimates

We are interested in

$$d = \bar{x}_1 - \bar{x}_2$$

$\Delta = \mu_1 - \mu_2$

If the samples are independent, then

$$Var(\bar{x}_1 - \bar{x}_2) = Var(\bar{x}_1) + Var(\bar{x}_2)$$

$$Var(\bar{x}_1 - \bar{x}_2) = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}$$

When

$$\sigma_1^2 = \sigma_2^2, \quad Var(\bar{x}_1 - \bar{x}_2) = \sigma^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)$$

Pooled Average

Hence, we use the weighted average of the two sample variances, with the weighting done according to sample size. This weighted average is called the *pooled estimate:*

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{(n_1 - 1) + (n_2 - 1)}$$

To estimate $\text{Var}(\bar{x}_1 - \bar{x}_2)$, we can use

$$s_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)$$

Example 1: Blood Pressures of Children

To investigate the question of whether the children of city A and city B have the same systolic blood pressure, a random sample of n = 10 children was selected from each city and their blood pressures measured. These samples provided the following data:

Statistic	City A	City B
N	10	10
$\bar{x}$ (mmHg)	105.8	97.2
s^2 $(\text{mmHg})^2$	78.62	22.40
s (mmHg)	8.87	4.73

Blood Pressures of Children

We are interested in the difference:

$$\Delta = \mu_A - \mu_B$$

and we have $\overline{x}_A$ as an estimate of μ_A and $\overline{x}_B$ as an estimate of μ_B; hence it is reasonable to use:

$$d = \overline{x}_A - \overline{x}_B = 105.8 - 97.2 = 8.6 \text{ (mm Hg)}$$

as an estimate of $\Delta = \mu_A - \mu_B$.

Confidence Interval for μ$_A$- μ$_B$, using s_p

$$C\left[(\bar{x}_A - \bar{x}_B) - t_{0.975} s_p \sqrt{\frac{1}{n_A} + \frac{1}{n_B}} \le \mu_A - \mu_B \le (\bar{x}_A - \bar{x}_B) + t_{0.975} s_p \sqrt{\frac{1}{n_A} + \frac{1}{n_B}} \right] = 0.95$$

$$df = (n_A - 1) + (n_B - 1)$$

$$\bar{x}_A - \bar{x}_B = 8.6 \qquad t_{0.975} = 2.1009 \qquad df = 18$$

$$s_p^2 = \frac{(n_A - 1)s_A^2 + (n_B - 1)s_B^2}{(n_A - 1) + (n_B - 1)} = \frac{9(78.62) + 9(22.4)}{18} = 50.51$$

Confidence Interval for $\mu_A - \mu_B$, using s_p

$$S_p = \sqrt{50.51} = 7.11$$

$$C\left[\ 8.6 - 2.1009(7.11)\sqrt{\frac{1}{10}+\frac{1}{10}} \le \mu_A - \mu_B \le 8.6 + 2.1009(7.11)\sqrt{\frac{1}{10}+\frac{1}{10}}\ \right] = 0.95$$

$$C\left[\ 1.92 \le \mu_A - \mu_B \le 15.27\right] = 0.95$$

Example 2: *AJPH, April 1994*

TABLE 2—Nurse Practitioners' Responses to 10 Clinical Case Scenarios on Occupational Health Test

Case Scenario Type	Occupational Program Graduates' No. of Correct Responses, Mean ± SD $n = 31$	Nonoccupational Program Graduates' No. of Correct Responses, Mean ± SD $n = 223$
Occupational disease identification*	3.9 ± 1.0	2.2 ± 1.2
Occupational case management*	4.1 ± 1.2	3.4 ± 1.5
Both case scenario types*	8.0 ± 1.7	5.6 ± 2.4

Note. Five clinical case scenarios were included in each category.
**P* < .05 (occupational as compared with nonoccupational program graduates).

Example 2: *AJPH, April 1994*

	1 OCCP Prog	2 Non OCCP Prog
n	31	223
mean	4.1	3.4
SD	1.2	1.5
S^2	1.44	2.25

Example 2: *AJPH, April 1994*

1. **The hypothesis:** $H_0: \mu_1 = \mu_2$ *vs.* $H_1: \mu_1 \neq \mu_2$
2. **The assumptions:** Independent random samples from normal distributions,
3. **The α level:** $\alpha = 0.05$
4. **The test statistic:**
5. **The critical region:** Reject H_0 if t is not between

Example 2: *AJPH, April 1994*

6. Test result:

$$s_p^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{(n_1-1)+(n_2-1)} \qquad s_p^2 = \frac{30(1.44)+222(2.25)}{30+222}$$

$$s_p^2 = \frac{542.7}{252} = 2.154 \qquad s_p = \sqrt{2.154} = 1.47$$

$$\sqrt{\frac{1}{n_1}+\frac{1}{n_2}} = \sqrt{\frac{1}{31}+\frac{1}{223}} = 0.19$$

$$t = \frac{4.1-3.4}{1.47(0.19)} = 2.5$$

7. The Conclusion: Reject H_0 since t = 2.5 is not between ± 1.97; 0.01 < p < 0.02

Independent Random Samples from Two Populations of Serum Uric Acid Values

	Sample 1	Sample 2
	1.2	1.7
	0.8	1.5
	1.1	2.0
	0.7	2.1
	0.9	1.1
	1.1	0.9
	1.5	2.2
	0.8	1.8
	1.6	1.3
	0.9	1.5
Sum	10.6	16.1
Mean	1.06	1.61

Independent Random Samples from Two Populations of Serum Uric Acid Values

1. The hypothesis: $H_0: \mu_1 = \mu_2$ vs. $H_1: \mu_1 \neq \mu_2$

2. The α-level: $\alpha = 0.05$

3. The assumptions: Independent Random Samples
Normal Distribution, $\sigma_1^2 = \sigma_2^2$

4. The test statistic:

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

Confidence Interval for $\mu_A - \mu_B$, using s_p

5. The reject region: Reject $H_0: \mu_1 = \mu_2$ if t is not between $\pm\, t_{0.975}(18) = 2.1009$

6. The result:

$$t = \frac{1.06 - 1.61}{0.37\sqrt{\frac{1}{10} + \frac{1}{10}}} = \frac{-0.55}{0.37(0.45)} = -3.3$$

7. The conclusion: Reject $H_0: \mu_1 = \mu_2$, since t is not between $\pm\, 2.1009$

Serum Uric Acid Values Before And After a Special Meal

	Before	After
	1.2	1.7
	0.8	1.5
	1.1	2.0
	0.7	2.1
	0.9	1.1
	1.1	0.9
	1.5	2.2
	0.8	1.8
	1.6	1.3
	0.9	1.5
Sum	10.6	6.1
Mean	1.06	1.61

Serum Uric Acid Values Before And After a Special Meal

Serum Uric Acid Values Before And After A Special Meal

Worksheet

Person	Before	After	d[a]	d^2
1	1.2	1.7	0.5	0.25
2	0.8	1.5	0.7	0.49
3	1.1	2.0	0.9	0.81
4	0.7	2.1	1.4	1.96
5	0.9	1.1	0.2	0.04
6	1.1	0.9	-0.2	0.04
7	1.5	2.2	0.7	0.49
8	0.8	1.8	1.0	1.00
9	1.6	1.3	-0.3	0.09
10	0.9	1.5	0.6	0.36
Sum	10.6	16.1	5.5	5.53
Mean	1.06	1.61	0.55	
Sum^2/n			3.025	
SS			2.505	
Variance			0.278	
SD			0.528	

d[a] = After - Before

Serum Uric Acid Values Before And After a Special Meal

1. The hypothesis: $H_0: \Delta = 0$ vs. $H_1: \Delta \neq 0$,
where $\Delta = \mu_{After} - \mu_{Before}$

2. The α-level: $\alpha = 0.05$

3. The assumptions: Random Sample of Differences,
Normal Distribution

4. The test statistic:

$$t = \frac{\bar{d}}{s_{\bar{d}}} = \frac{\bar{x}_{After} - \bar{x}_{Before}}{s_d / \sqrt{n}}$$

Serum Uric Acid Values Before And After a Special Meal

5. The rejection region: Reject H_0: $\Delta = 0$, if t is not between $\pm\, t_{0.975}(9) = 2.26$

6. The result:

$$t = \frac{0.55}{0.528/\sqrt{10}} = \frac{0.55}{0.528/(3.16)} = 3.29$$

7. The conclusion: Reject H_0: $\Delta = 0$ since t is not between $\pm\, 2.26$

Chi-Square as a Statistical Test

- ***Chi-square test:*** an **inferential statistics** technique designed to test for **significant relationships** between two variables organized in a bivariate table.

- Chi-square requires **no assumptions** about the shape of the population distribution from which a sample is drawn.

Chi-Square as a Statistical Test

- A statistical method used to determine goodness of fit
  - Goodness of fit refers to how close the observed data are to those predicted from a hypothesis

- Note:
  - The chi square test does not prove that a hypothesis is correct
    - It evaluates to what extent the data and the hypothesis have a good fit

Limitations of the Chi-Square Test

- The chi-square test does **not** give us much information about the ***strength*** of the relationship or its ***substantive significance*** in the population.
- The chi-square test is **sensitive** to ***sample size***. The size of the calculated chi-square is **directly proportional** to the size of the sample, independent of the strength of the relationship between the variables.
- The chi-square test is also **sensitive** to **small expected frequencies** in one or more of the cells in the table.

Statistical Independence

- ***Independence (statistical):*** the **absence of association** between two cross-tabulated variables. The percentage distributions of the dependent variable within each category of the independent variable are **identical**.

  - The **research hypothesis** (H_1) proposes that the two variables are **related** in the population.

  - The **null hypothesis** (H_0) states that **no association exists** between the two cross-tabulated variables in the population, and therefore the variables are **statistically independent**.

Chi-Square as a Statistical Test

H_1: The two variables are **related** in the population.

Gender and fear of walking alone at night are ***statistically dependent***.

Afraid	Men	Women	Total
No	83.3%	57.2%	**71.1%**
Yes	16.7%	42.8%	**28.9%**
Total	**100%**	**100%**	**100%**

Chi-Square as a Statistical Test

H_0: There is **no association** between the two variables.

Gender and fear of walking alone at night are ***statistically independent***.

Afraid	Men	Women	Total
No	71.1%	71.1%	**71.1%**
Yes	28.9%	28.9%	**28.9%**
Total	**100%**	**100%**	**100%**

Concept of Expected Frequencies

Expected frequencies f_e : the cell frequencies that would be **expected** in a bivariate table **if** the two tables were **statistically independent**.

Observed frequencies f_o: the cell frequencies **actually observed** in a bivariate table.

$$f_e = \frac{(\text{column marginal})(\text{row marginal})}{N}$$

To obtain the expected frequencies for any cell in any cross-tabulation in which the two variables are assumed independent, **multiply** the row and column totals for that cell and **divide** the product by the total number of cases in the table.

Calculating the Obtained Chi-Square

- The test statistic that **summarizes** the differences between the **observed** (*fo*) and the **expected** (*fe*) frequencies in a bivariate table.

$$\chi^2 = \sum \frac{(f_e - f_o)^2}{f_e}$$

f_e = expected frequencies
f_o = observed frequencies

df = (r – 1)(c – 1)

where

r = the number of rows

c = the number of columns

Chi-Square as a Statistical Test

- The outcome
  - F_1 generation
    - All offspring have straight wings and gray bodies
  - F_2 generation
    - 193 straight wings, gray bodies
    - 69 straight wings, ebony bodies
    - 64 curved wings, gray bodies
    - 26 curved wings, ebony bodies
    - 352 total flies

- Applying the chi square test
  - Step 1: Propose a null hypothesis (H_0) that allows us to calculate the expected values based on Mendel's laws
    - The two traits are independently assorting

Chi-Square as a Statistical Test

– Step 2: Calculate the expected values of the four phenotypes, based on the hypothesis

- According to our hypothesis, there should be a 9:3:3:1 ratio on the F_2 generation

Phenotype	Expected probability	Expected number	Observed number
straight wings, gray bodies	9/16	9/16 X 352 = 198	193
straight wings, ebony bodies	3/16	3/16 X 352 = 66	64
curved wings, gray bodies	3/16	3/16 X 352 = 66	62
curved wings, ebony bodies	1/16	1/16 X 352 = 22	24

Chi-Square as a Statistical Test

– Step 3: Apply the chi square formula

$$\chi^2 = \frac{(O_1 - E_1)^2}{E_1} + \frac{(O_2 - E_2)^2}{E_2} + \frac{(O_3 - E_3)^2}{E_3} + \frac{(O_4 - E_4)^2}{E_4}$$

$$\chi^2 = \frac{(193 - 198)^2}{198} + \frac{(69 - 66)^2}{66} + \frac{(64 - 66)^2}{66} +$$

$$\chi^2 = 0.13 + 0.14 + 0.06 + 0.73$$

$$\chi^2 = 1.06$$

Expected number	Observed number
198	193
66	64
66	62
22	24

Chi-Square as a Statistical Test

- Step 4: Interpret the chi square value
  - The calculated chi square value can be used to obtain probabilities, or P values, from a chi square table
    - These probabilities allow us to determine the likelihood that the observed deviations are due to random chance alone
  - Low chi square values indicate a high probability that the observed deviations could be due to random chance alone
  - High chi square values indicate a low probability that the observed deviations are due to random chance alone
  - If the chi square value results in a probability that is less than 0.05 (ie: less than 5%) it is considered ***statistically significant***
    - The hypothesis is rejected

Chi-Square as a Statistical Test

- Step 4: Interpret the chi square value
  - Before we can use the chi square table, we have to determine the degrees of freedom (*df*)
    - The *df* is a measure of the number of categories that are independent of each other
    - If you know the 3 of the 4 categories you can deduce the 4th (total number of progeny – categories 1-3)
    - *df* = *n* – 1
      - where *n* = total number of categories
    - In our experiment, there are four phenotypes/categories
      - Therefore, df = 4 – 1 = 3
  - Refer to Table