Example: Muller Method

Find a root of an equation $f(x)=2x^3-2x-5$ using Muller Method

Solution:
Here $2x^3-2x-5=0$

Let $f(x)=2x^3-2x-5$

Here

x	0	1	2
$f(x)$	-5	-5	7

$x_0=1$, $x_1=2$, $x_2=1.5$

1^{st} iteration :

$$f(x_0)=f(1)=2\cdot 1^3-2\cdot 1-5=-5$$

$$f(x_1)=f(2)=2(2)^3-2(2)-5=7$$

$$f(x_2)=f(1.5)=2(1.5)^3-2(1.5)-5=-1.25$$

$$h_1=x_1-x_0=2-1=1$$

$$h_2=x_2-x_1=1.5-2=-0.5$$

$$\delta_1=\frac{f(x_1)-f(x_0)}{h_1}=\frac{7--5}{1}=12$$

$$\delta_2=\frac{f(x_2)-f(x_1)}{h_2}=\frac{-1.25-7}{-0.5}=16.5$$

$$a=\frac{\delta_2-\delta_1}{h_2+h_1}=\frac{16.5-12}{-0.5+1}=9$$

$$b=a\times h_2+d_2=9\times -0.5+16.5=12$$

$$c=f(x_2)=-1.25$$

$$x_3=x_2+\frac{-2c}{b\pm\sqrt{b^2-4ac}}$$

$$x_3=x_2+\frac{-2c}{b+sign(b)\sqrt{b^2-4ac}}$$

$$= 1.5 + \frac{-2 \times -1.25}{12 + \sqrt{12^2 - 4 \times 9 \times -1.25}}$$

$$= 1.5 + \frac{2.5}{12 + \sqrt{189}}$$

$$= 1.5 + \frac{2.5}{12 + 13.74773}$$

$$= 1.5971$$

Relative percent error

$$\varepsilon_a 1 = \left| \frac{x_3 - x_2}{x_3} \right| \times 100\ \% = \left| \frac{1.5971 - 1.5}{1.5971} \right| \times 100\ \% = 6.07953\ \%$$

Now,

$x_0 = x_1 = 2$

$x_1 = x_2 = 1.5$

$x_2 = x_3 = 1.5971$

2^{nd} iteration :

$$f(x_0) = f(2) = 2(2)^3 - 2(2) - 5 = 7$$

$$f(x_1) = f(1.5) = 2(1.5)^3 - 2(1.5) - 5 = -1.25$$

$$f(x_2) = f(1.5971) = 2(1.5971)^3 - 2(1.5971) - 5 = -0.04672$$

$$h_1 = x_1 - x_0 = 1.5 - 2 = -0.5$$

$$h_2 = x_2 - x_1 = 1.5971 - 1.5 = 0.0971$$

$$\delta_1 = \frac{f(x_1) - f(x_0)}{h_1} = \frac{-1.25 - 7}{-0.5} = 16.5$$

$$\delta_2 = \frac{f(x_2) - f(x_1)}{h_2} = \frac{-0.04672 - -1.25}{0.0971} = 12.39272$$

$$a = \frac{\delta_2 - \delta_1}{h_2 + h_1} = \frac{12.39272 - 16.5}{0.0971 \pm 0.5} = 10.19419$$

$$b = a \times h_2 + d_2 = 10.19419 \times 0.0971 + 12.39272 = 13.38253$$

$$c = f(x_2) = -0.04672$$

$$x_3 = x_2 + \frac{-2c}{b \pm \sqrt{b^2 - 4ac}}$$

$$x_3 = x_2 + \frac{-2c}{b + sign(b)\sqrt{b^2 - 4ac}}$$

$$= 1.5971 + \frac{-2 \times -0.04672}{13.38253 + \sqrt{13.38253^2 - 4 \times 10.19419 \times -0.04672}}$$

$$= 1.5971 + \frac{0.09343}{13.38253 + \sqrt{180.99718}}$$

$$= 1.5971 + \frac{0.09343}{13.38253 + 13.45352}$$

$$= 1.60058$$

Relative percent error

$$\varepsilon_a 2 = \left|\frac{x_3 - x_2}{x_3}\right| \times 100\ \% = \left|\frac{1.60058 - 1.5971}{1.60058}\right| \times 100\ \% = 0.21753\ \%$$

Now,

$x_0 = x_1 = 1.5$

$x_1 = x_2 = 1.5971$

$x_2 = x_3 = 1.60058$

3^{rd} <u>iteration :</u>

$$f(x_0) = f(1.5) = 2(1.5)^3 - 2(1.5) - 5 = -1.25$$

$$f(x_1) = f(1.5971) = 2(1.5971)^3 - 2(1.5971) - 5 = -0.04672$$

$$f(x_2) = f(1.60058) = 2(1.60058)^3 - 2(1.60058) - 5 = -0.00028$$

$$h_1 = x_1 - x_0 = 1.5971 - 1.5 = 0.0971$$

$$h_2 = x_2 - x_1 = 1.60058 - 1.5971 = 0.00348$$

$$\delta_1 = \frac{f(x_1) - f(x_0)}{h_1} = \frac{-0.04672 - -1.25}{0.0971} = 12.39272$$

$$\delta_2 = \frac{f(x_2) - f(x_1)}{h_2} = \frac{-0.00028 - -0.04672}{0.00348} = 13.33768$$

$$a = \frac{\delta_2 - \delta_1}{h_2 + h_1} = \frac{13.33768 - 12.39272}{0.00348 + 0.0971} = 9.39535$$

$$b = a \times h_2 + d_2 = 9.39535 \times 0.00348 + 13.33768 = 13.37039$$

$$c = f(x_2) = -0.00028$$

$$x_3 = x_2 + \frac{-2c}{b \pm \sqrt{b^2 - 4ac}}$$

$$x_3 = x_2 + \frac{-2c}{b + sign(b)\sqrt{b^2 - 4ac}}$$

$$= 1.60058 + \frac{-2 \times -0.00028}{13.37039 + \sqrt{13.37039^2 - 4 \times 9.39535 \times -0.00028}}$$

$$= 1.60058 + \frac{0.00056}{13.37039 + \sqrt{178.7779}}$$

$$= 1.60058 + \frac{0.00056}{13.37039 + 13.37079}$$

$$= 1.6006$$

Relative percent error

$$\varepsilon_{a^3} = \left| \frac{x_3 - x_2}{x_3} \right| \times 100\,\% = \left| \frac{1.6006 - 1.60058}{1.6006} \right| \times 100\,\% = 0.00131\,\%$$

Approximate root of the equation $2x^3 - 2x - 5 = 0$ using Muller method is 1.6006

n	x_0	x_1	x_2	$f(x_0)$	$f(x_1)$	$f(x_2)$	a	b	c	x_3	ε_{a^n}
1	1	2	1.5	-5	7	-1.25	9	12	-1.25	1.5971	6.07953
2	2	1.5	1.5971	7	-1.25	-0.04672	10.19419	13.38253	-0.04672	1.60058	0.21753
3	1.5	1.5971	1.60058	-1.25	-0.04672	-0.00028	9.39535	13.37039	-0.00028	1.6006	0.00131