BISECTION METHOD

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Bisection Method

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One of the simplest and most reliable iterative methods.

Based on Intermediate Value Theorem.

If f(x) is continuous on (a,b) and f(a)f(b)<0, then at least one real root lies in (a,b).

Basic Concept

Let
$$x_1=a$$
, $x_2=b$.

Midpoint
$$x_0=rac{x_1+x_2}{2}$$
 .

Three possible cases:

1.
$$f(x_0) = 0 \rightarrow \operatorname{root} \operatorname{at} x_0$$

2.
$$f(x_0)f(x_1) < 0 \rightarrow \text{root in } (x_1, x_0)$$

3.
$$f(x_0)f(x_2) < 0$$
 $ightarrow$ root in (x_0, x_2)

Iterative Approach

Test sign of $f(x_0)$ to determine which half contains the root.

Repeat the process to narrow down the interval.

Continue until the interval is sufficiently small.

Procedure

- 1. Choose two initial values of x such that $f(x_1)f(x_2)<0$.
- 2. Bisect the interval and choose the half with the sign change.
- 3. Each iteration improves accuracy by one binary digit.
- 4. Stop when interval size is less than required precision.

Largest Possible Roots

For polynomial $a_nx^n+a_{n-1}x^{n-1}+\cdots+a_1x+a_0=0$:

Largest possible root: $x_1=rac{a_{n-1}}{a_n}$.

Used as initial guess if no better estimate is available.

Search Bracket

Useful inequality:

$$|x| \leq \sqrt{\left(rac{a_{n-1}}{a_n}
ight)^2 - 2\left(rac{a_{n-2}}{a_n}
ight)}$$

Maximum absolute value of root:

$$|x_{ ext{max}}| = \sqrt{\left(rac{a_{n-1}}{a_n}
ight)^2 - 2\left(rac{a_{n-2}}{a_n}
ight)}$$

Ex-1: Estimate the possible initial guess value of the Polynomial.

$$2x^3 - 8x^2 + 2x + 12 = 0$$

Answer:

The largest possible root is:

$$x_1 = \frac{a_{n-1}}{a_n} = \frac{-8}{2} = +4$$

This implies that, no root can be larger than the value 9.

But all roots satisfy the relation.

$$|x| \le \sqrt{\left(rac{a_{n-1}}{a_n}
ight)^2 - 2\left(rac{a_{n-2}}{a_n}
ight)}$$

$$= \sqrt{\left(rac{-8}{2}
ight)^2 - 2\left(rac{2}{2}
ight)}$$

$$= \sqrt{16 - 2}$$

$$=\sqrt{14}$$

Therefore all real roots lie in the interval $(-\sqrt{14},\sqrt{14})$.

Ex-2:

Find the real of the equation $x^2 - 4x - 10 = 0$ using Bisection Method.

Sol:

$$|x|_{max} = \sqrt{\left(rac{-4}{1}
ight)^2 - 2\left(rac{-10}{1}
ight)}$$

$$= \sqrt{16 + 20}$$

$$=\sqrt{36}=6$$

Therefore, we have both the roots in the interval [-6,6].

The table gives the values of f(x) between -6 and 6.

Find initial root:

$$f(-2) = 2 > 0$$

 $f(-1) = -5 < 0$

f(-2) and f(-1) are opposite signs, so root lies between -2 and -1.

Initial root,
$$x_0 = rac{a+b}{2} = rac{-2+(-1)}{2} = rac{-3}{2} = -1.5$$

Root x_n	Signs of	Comments
	$f(x) = x^2 - 4x - 10$	
$x_0 = -1.5$	f(-1.5) = -1.75 < 0	Since $f(-2)>0$ and $f(-1.5)<0$, the root must in the interval
		the root must in the interval
		(-2,-1.5)

$x_1 = \frac{-2 - 1.5}{2} = -1.75$ $x_2 = \frac{-1.75 - 1.5}{2} = -1.625$ $x_3 = \frac{-1.75 - 1.625}{2}$ $x_4 = \frac{-1.75 - 1.6875}{2}$ $x_5 = \frac{-1.75 - 1.7188}{2}$ $x_6 = \frac{-1.75 - 1.7344}{2}$ $x_7 = \frac{-1.7422 - 1.7402}{2}$ $x_8 = \frac{-1.7422 - 1.7412}{2}$ $x_9 = \frac{-1.7422}{2}$ $x_{1} = \frac{-1.7422}{2}$ $x_{2} = -1.7412$ $x_{3} = \frac{-1.7422}{2}$ $x_{4} = \frac{-1.7422}{2}$ $x_{5} = \frac{-1.7422 - 1.7402}{2}$ $x_{6} = \frac{-1.7422 - 1.7412}{2}$ $x_{7} = \frac{-1.7422 - 1.7412}{2}$ $x_{1} = \frac{-1.7422 - 1.7412}{2}$ $x_{1} = \frac{-1.7422 - 1.7412}{2}$ $x_{1} = \frac{-1.7422 - 1.7412}{2}$ $x_{2} = -1.7412$ $x_{3} = \frac{-1.7422 - 1.7412}{2}$ $x_{4} = \frac{-1.7422 - 1.7412}{2}$ $x_{5} = \frac{-1.7422 - 1.7412}{2}$ $x_{7} = \frac{-1.7422 - 1.7402}{2}$ $x_{1} = \frac{-1.7422 - 1.7402}{2}$ $x_{2} = -1.7412$ $x_{3} = \frac{-1.7422 - 1.7412}{2}$ $x_{4} = \frac{-1.7422 - 1.7402}{2}$ $x_{5} = \frac{-1.7422 - 1.7402}{2}$ $x_{7} = \frac{-1.7422 - 1.7402}{2}$ $x_{8} = \frac{-1.7422 - 1.7402}{2}$ $x_{9} = \frac{-1.7422 - 1.7402}{2}$ $x_{1} = -1.7412$ $x_{2} = -1.7412$ $x_{1} = -1.7412$ $x_{1} = -1.7412$ $x_{2} = -1.7412$ $x_{3} = -1.7412$ $x_{4} = -1.75 - 1.625$ $x_{1} = -1.7412$ $x_{5} = $					
$x_2 = \frac{-1.75 - 1.5}{2} = -1.625 \qquad f(-1.625) = -0.859 \\ < 0 \qquad (-1.75, -1.625) \qquad The root must in the interval \\ (-1.75, -1.625) \qquad f(-1.6875) = -0.4 < 0 \qquad (-1.75, -1.625) \qquad The root must in the interval \\ (-1.75, -1.625) \qquad f(-1.6875) = -0.4 < 0 \qquad The root must in the interval \\ (-1.75, -1.6875) \qquad f(-1.7188) = -0.1705 \\ < 0 \qquad (-1.75, -1.7188) \qquad (-1.75, -1.7188) \qquad (-1.75, -1.7188) \qquad (-1.75, -1.7188) \qquad The root must in the interval \\ < 0 \qquad (-1.75, -1.7188) \qquad f(-1.7344) = -0.054 \\ < 0 \qquad (-1.75, -1.7344) \qquad (-1.75, -1.7344) \qquad (-1.75, -1.7344) \qquad (-1.75, -1.7344) \qquad The root must in the interval \\ < 0 \qquad (-1.734, -1.7422) \qquad f(-1.7344, -1.7422) \qquad The root must in the interval \\ < 0 \qquad (-1.7422, -1.7383) \qquad f(-1.7402) = -0.0109 \\ < 0 \qquad (-1.7422, -1.7383) \qquad f(-1.7422, -1.7402) \qquad The root must in the interval \\ < 0 \qquad (-1.7422, -1.7402) \qquad (-1.7422, -1.7402) \qquad The root must in the interval \\ < 0 \qquad (-1.7422, -1.7402) \qquad (-1.7422, -1.7402) \qquad The root must in the interval \\ < 0 \qquad (-1.7422, -1.7402) \qquad (-1.7422, -1.7402) \qquad The root must in the interval \\ < 0 \qquad (-1.7422, -1.7402) \qquad (-1.7422, -1.7402) \qquad The root must in the interval \\ < 0 \qquad (-1.7422, -1.7402) \qquad (-1.7422, -1.7402) \qquad The root must in the interval \\ < 0 \qquad (-1.7422, -1.7402) \qquad (-1.7422, -1.7402) \qquad (-1.7422, -1.7402) \qquad (-1.7422, -1.7402) \qquad The root must in the interval \\ < 0 \qquad (-1.7422, -1.7402) \qquad (-1.7422, -1.7402$	-2 - 1.5	f(-1.75) = 0.0625 > 0	Since $f(-1.5) < 0$ and $f(-1.5) < 0$		
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$x_{5} = \frac{2}{2}$ $= -1.7344$ $x_{6} = \frac{-1.75 - 1.7344}{2}$ $x_{7} = \frac{-1.7422}{2}$ $= -1.7383$ $x_{8} = \frac{-1.7422 - 1.7383}{2}$ $x_{9} = \frac{-1.7422 - 1.7402}{2}$ $x_{1} = \frac{-1.7422 - 1.7402}{2}$ $x_{2} = -1.7422 - 1.7402$ $x_{3} = \frac{-1.7422 - 1.7402}{2}$ $x_{4} = \frac{-1.7422 - 1.7402}{2}$ $x_{5} = \frac{-1.7422 - 1.7402}{2}$ $x_{6} = \frac{-1.7422 - 1.7402}{2}$ $x_{7} = \frac{-1.7422 - 1.7402}{2}$ $x_{8} = \frac{-1.7422 - 1.7402}{2}$ $x_{9} = \frac{-1.7422 - 1.7402}{2}$ $x_{1} = \frac{-1.7422 - 1.7402}{2}$ $x_{2} = -1.7412$ $x_{3} = \frac{-1.7422 - 1.7402}{2}$ $x_{4} = \frac{-1.7422 - 1.7402}{2}$ $x_{5} = \frac{-1.7422 - 1.7402}{2}$ $x_{7} = \frac{-1.7422 - 1.7402}{2}$ $x_{8} = \frac{-1.7422 - 1.7402}{2}$ $x_{9} = \frac{-1.7422 - 1.7402}{2}$ $x_{1} = \frac{-1.7422 - 1.7402}{2}$ $x_{2} = \frac{-1.7422 - 1.7402}{2}$ $x_{3} = \frac{-1.7422 - 1.7402}{2}$ $x_{4} = \frac{-1.7422 - 1.7402}{2}$ $x_{5} = \frac{-1.7422 - 1.7402}{2}$ $x_{7} = \frac{-1.7422 - 1.7402}{2}$ $x_{8} = \frac{-1.7422 - 1.7402}{2}$ $x_{9} = \frac{-1.7422 - 1.7402}{2}$ $x_{1} = \frac{-1.7422 - 1.7402}{2}$ $x_{2} = \frac{-1.7422 - 1.7402}{2}$ $x_{3} = \frac{-1.7422 - 1.7402}{2}$ $x_{4} = \frac{-1.7422 - 1.7402}{2}$ $x_{5} = \frac{-1.7422 - 1.7402}{2}$ $x_{7} = \frac{-1.7422 - 1.7402}{2}$ $x_{8} = \frac{-1.7422 - 1.7402}{2}$ $x_{1} = \frac{-1.7422 - 1.7402}{2}$ $x_{2} = \frac{-1.7422 - 1.7402}{2}$ $x_{3} = \frac{-1.7422 - 1.7402}{2}$ $x_{4} = \frac{-1.7422 - 1.7402}{2}$ $x_{5} = \frac{-1.7422 - 1.7402}{2}$ $x_{7} = \frac{-1.7422 - 1.7402}{2}$ $x_{7} = \frac{-1.7422 - 1.7402}{2}$ $x_{7} = \frac{-1.7422 - 1.7402}{2}$ $x_{8} = \frac{-1.7422 - 1.7402}{2}$ $x_{1} = \frac{-1.7422 - 1.7402}{2}$ $x_{2} = \frac{-1.7422 - 1.7402}{2}$ $x_{3} = \frac{-1.7422 - 1.7402}{2}$ $x_{4} = \frac{-1.7422 - 1.7402}{2}$ $x_{5} = \frac{-1.7422 - 1.7402}{2}$ $x_{7} = \frac{-1.7422 - 1.7402}{2}$ $x_{8} = \frac{-1.7422 - 1.7402}{2}$ $x_{1} = \frac{-1.7422 - 1.7402}{2}$ $x_{2} = \frac{-1.7422 - 1.7402}{2}$ $x_{3} = \frac{-1.7422 - 1.7402}{2}$ $x_{4} = \frac{-1.7422 - 1.7402}{2}$ $x_{5} = \frac{1.7422 - 1.7402}{2}$ $x_{7} = \frac{1.7422 - 1.7402}{2}$ $x_{7} = 1.7422 - 1.74$					
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$x_6 = \frac{-1.75 - 1.7344}{2}$ $= -1.7422$ $x_7 = \frac{-1.7344 - 1.7422}{2}$ $= -1.7383$ $x_8 = \frac{-1.7422 - 1.7383}{2}$ $x_9 = \frac{-1.7422 - 1.7402}{2}$	$x_5 = {2}$	< 0	(-1.75, -1.7344)		
$x_{6} = \frac{2}{2}$ $= -1.7422$ $x_{7} = \frac{-1.7344 - 1.7422}{2}$ $= -1.7383$ $x_{8} = \frac{-1.7422 - 1.7383}{2}$ $x_{9} = \frac{-1.7422 - 1.7402}{2}$ $x_{9} = \frac{-1.7422 - 1.7402}{2}$ $x_{1} = \frac{-1.7422 - 1.7402}{2}$ $x_{1} = \frac{-1.7422 - 1.7402}{2}$ $x_{2} = -1.7412$ $x_{3} = \frac{-1.7422 - 1.7402}{2}$ $x_{4} = \frac{-1.7422 - 1.7402}{2}$ $x_{5} = \frac{-1.7422 - 1.7402}{2}$ $x_{6} = \frac{-1.7422 - 1.7402}{2}$ $x_{7} = \frac{-1.7422 - 1.7402}{2}$ $x_{8} = \frac{-1.7422 - 1.7402}{2}$ $x_{9} = \frac{-1.7422 - 1.7402}{2}$ $x_{1} = \frac{-1.7422 - 1.7402}{2}$ $x_{2} = \frac{-1.7422 - 1.7402}{2}$ $x_{3} = \frac{-1.7422 - 1.7402}{2}$ $x_{4} = \frac{-1.7422 - 1.7402}{2}$ $x_{5} = \frac{-1.7422 - 1.7402}{2}$ $x_{7} = \frac{-1.7422 - 1.7402}{2}$ $x_{8} = \frac{-1.7422 - 1.7402}{2}$ $x_{9} = \frac{-1.7422 - 1.7402}{2}$ $x_{1} = \frac{-1.7422 - 1.7402}{2}$ $x_{2} = \frac{-1.7422 - 1.7402}{2}$ $x_{3} = \frac{-1.7422 - 1.7402}{2}$ $x_{4} = \frac{-1.7422 - 1.7402}{2}$ $x_{5} = \frac{-1.7422 - 1.7402}{2}$ $x_{7} = \frac{-1.7422 - 1.7402}{2}$ $x_{8} = \frac{-1.7422 - 1.7402}{2}$ $x_{1} = \frac{-1.7422 - 1.7402}{2}$ $x_{1} = \frac{-1.7422 - 1.7402}{2}$ $x_{2} = \frac{-1.7422 - 1.7402}{2}$ $x_{3} = \frac{-1.7422 - 1.7402}{2}$ $x_{4} = \frac{-1.7422 - 1.7402}{2}$ $x_{5} = \frac{-1.7422 - 1.7402}{2}$ $x_{7} = \frac{-1.7422 - 1.7402}{2}$ $x_{7} = \frac{-1.7422 - 1.7402}{2}$ $x_{8} = \frac{-1.7422 - 1.7402}{2}$ $x_{1} = \frac{-1.7422 - 1.7402}{2}$ $x_{2} = \frac{-1.7422 - 1.7402}{2}$ $x_{3} = \frac{-1.7422 - 1.7402}{2}$ $x_{4} = \frac{-1.7422 - 1.7402}{2}$ $x_{5} = \frac{-1.7422 - 1.7402}{2}$ $x_{7} = \frac{-1.7422 - 1.7402}{2}$ $x_{7} = \frac{-1.7422 - 1.7402}{2}$ $x_{7} = \frac{-1.7422 - 1.7402}{2}$	=-1.7344				
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$x_7 = \frac{-1.7344 - 1.7422}{2}$ $x_8 = \frac{-1.7422 - 1.7383}{2}$ $x_9 = \frac{-1.7422 - 1.7402}{2}$	$x_6 = {2}$		(-1.7344, -1.7422)		
$x_7 = \frac{2}{2}$ $= -1.7383$ $x_8 = \frac{-1.7422 - 1.7383}{2}$ $= -1.7402$ $x_9 = \frac{-1.7422 - 1.7402}{2}$	=-1.7422				
$x_7 = \frac{2}{2}$ $= -1.7383$ $x_8 = \frac{-1.7422 - 1.7383}{2}$ $= -1.7402$ $x_9 = \frac{-1.7422 - 1.7402}{2}$	-1.7344 - 1.7422	f(-1.7383) = -0.025	The root must in the interval		
$x_{8} = \frac{-1.7422 - 1.7383}{2}$ $= -1.7402$ $x_{9} = \frac{-1.7422 - 1.7402}{2}$ $= -1.7402$ $f(-1.7402) = -0.0109$ < 0 $f(-1.7422, -1.7402)$ $= -0.003$ ≈ 0 $\therefore The appropriate root is$ -1.7412	Y_ =	• •	(-1.7422, -1.7383)		
$x_{9} = \frac{2}{2}$ $= -1.7402$ $x_{9} = \frac{-1.7422 - 1.7402}{2}$ $x_{9} = \frac{-1.7422 - 1.7402}{2}$ $x_{9} = \frac{-1.7422 - 1.7402}{2}$ $x_{1} = -0.003$ $x_{1} = 0.003$ $x_{2} = 0.003$ $x_{3} = 0.003$ $x_{4} = 0.003$ $x_{5} = 0.003$ $x_{7} = 0.003$ $x_{7} = 0.003$ $x_{8} = 0.003$	= -1.7383				
$x_{9} = \frac{2}{2}$ $= -1.7402$ $x_{9} = \frac{-1.7422 - 1.7402}{2}$ $x_{9} = \frac{-1.7422 - 1.7402}{2}$ $x_{9} = \frac{-1.7422 - 1.7402}{2}$ $x_{1} = -0.003$ $x_{1} = 0.003$ $x_{2} = 0.003$ $x_{3} = 0.003$ $x_{4} = 0.003$ $x_{5} = 0.003$ $x_{7} = 0.003$ $x_{7} = 0.003$ $x_{8} = 0.003$	-1.7422 - 1.7383	f(-1.7402) = -0.0109	The root must in the interval		
$x_9 = \frac{-1.7422 - 1.7402}{2}$ $f(-1.7412) = -0.003$ \therefore The appropriate root is ≈ 0 -1.7412	$\chi_{g} = {2}$	< 0	(-1.7422, -1.7402)		
$x_9 = \phantom{aaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaa$	=-1.7402				
≈ 0 -1.7412	-1.7422 - 1.7402	f(-1.7412) = -0.003	∴ The appropriate root is		
=-1.7412	$x_9 = {2}$	≈ 0	- 1.7412		
2.7.122	=-1.7412				

Ex: Find the root of the equation $x^3-x-1=0$, correct to two decimal places.

Solution:

Let
$$f(x) = x^3 - x - 1$$

 $f(1) = 1 - 1 - 1 = -1 < 0$
 $f(2) = 8 - 2 - 1 = 5 > 0$

So the root lies between 1 and 2.

	-				<u> </u>
a	X	b	f(a)	f(x)	f(b)
1	1.5	2	-1	0.875	5
1	1.25	1.5	-1	-0.2968	0.875
1.25	1.375	1.5	-0.2968	0.2246	0.875
1.25	1.3125	1.375	-0.2968	-0.0515	0.2246
1.3125	1.3438	1.375	-0.0515	0.0828	0.2246
1.3125	1.3282	1.3438	-0.0515	0.0149	0.0828
1.3125	1.3204	1.3282	-0.0515	-0.0183	0.0149
1.3204	1.3243	1.3282	-0.0515	-0.0018	0.0149
1.3243	1.3262	1.3282	-0.0018	0.0063	0.0149

So up to two decimal places the root is 1.32

Ex: $f(x) = x^3 + 3x - 5$, where [a = 1, b = 2], DOA = 0.0001

a	X	b	f(a)	f(x)	f(b)
1	1.5	2	-1	2.875	9
1	1.25	1.5	-1	0.7031	2.875
1	1.125	1.25	-1	-0.2012	0.7031
1.125	1.1875	1.25	-0.2012	0.2370	0.7031
1.125	1.15625	1.1875	-0.2012	0.0145	0.2370
1.125	1.140625	1.15625	-0.2012	-0.0941	0.0145
1.140625	1.1484375	1.15625	-0.0941	-0.0400	0.0145
1.1484375	1.15234375	1.15625	-0.0400	-0.0128	0.0145
1.1484375	1.154296	1.15625	-0.0128	0.0008	0.0145

Answer is: 1.154296

Ex: Find the root $f(x)=3x+\sin(x)-e^x=0$

a	Х	b	f(a)	f(x)	f(b)
1	1.5	2	0.2991	0.0444	-1.3541
1.5	1.75	2	0.0444	-0.4740	-1.3541
1.5	1.625	1.75	0.0444	-0.1750	-0.4740
1.5	1.5625	1.625	0.0444	-0.0559	-0.1750
1.5	1.53125	1.5625	0.0444	-0.0035	-0.0559
1.5	1.515625	1.53125	0.0444	0.0210	-0.0035
1.515625	1.5234375	1.53125	0.0210	0.0089	-0.0035
1.5234375	1.52734375	1.53125	0.0089	0.0027	-0.0035
1.52734375	1.529296875	1.53125	0.0027	-0.0003	-0.0035
1.52734375	1.528320313	1.529296875	0.0027	0.0012	-0.0003
1.528320313	1.528808594	1.529296875	0.0012	0.0004	-0.0003
1.528808594	1.529052734	1.529296875	0.0004	0.0000	-0.0003

The root is 1.5292966875

Ex: $X^3 - 7X^2 + 14X - 6 = 0$ on [0,1]

a	Х	b	f(a)	f(x)	f(b)
0	0.5	1	-6	-0.625	2
0.5	0.75	1	-0.625	0.9843	2
0.5	0.625	0.75	-0.625	0.2597	0.9843
0.5	0.5625	0.625	-0.625	-0.1619	0.2597
0.5625	0.59375	0.625	-0.1619	0.0540	0.2597
0.5625	0.578125	0.59375	-0.1619	-0.0526	0.0540
0.578125	0.5859375	0.59375	-0.0526	0.00103	0.0540

Root is approximately: x = 0.5859375