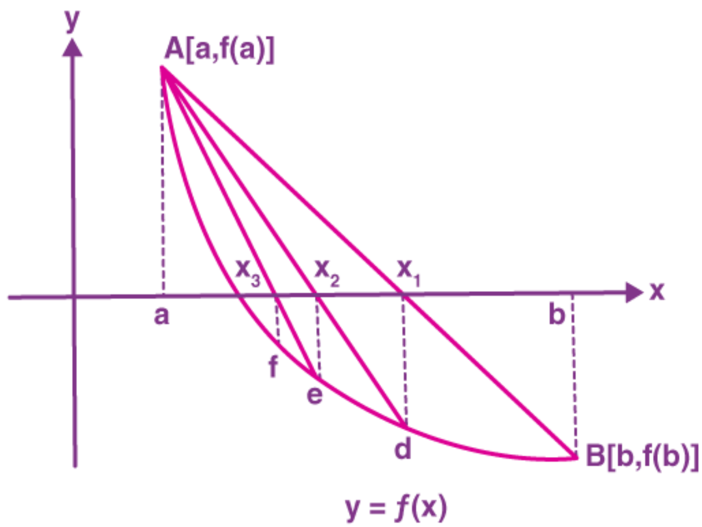
REGULA FALSI METHOD

HP

False Position Method



False Position Method (Regula Falsi) - Derivation

1. Given a function $f(x)$, we want to find root x such that $f(x) = 0$.
2. Choose two points a and b such that $f(a)$ and $f(b)$ have opposite signs. This guarantees a root between a and b.
3. Consider the straight line (chord) connecting the points $A(a, f(a))$ and $B(b, f(b))$.
4. The equation of this line is:

$$\frac{y - f(a)}{x - a} = \frac{f(b) - f(a)}{b - a}$$

5. To find the root approximation, set $y = 0$ (where the line crosses the x-axis), then solve for x:

$$0 - f(a) = \frac{f(b) - f(a)}{b - a}(x - a)$$

6. Rearranged, the formula for x becomes:

$$x = \frac{af(b) - bf(a)}{f(b) - f(a)}$$

7. If $f(x) = 0$, then x is the root. Otherwise, replace a or b by x depending on the sign of $f(x)$ and repeat.

Ex-1: Find the positive root of $x^3 - 2x - 5 = 0$ by false position method.

Sol:

let, $f(x) = x^3 - 2x - 5$

$f(2) = -1 < 0$

$f(3) = 16 > 0$

So one root lies between 2 and 3.

First iteration

Calculate x_2 using the false position formula:

- $x_2 = \dfrac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$
- $x_2 = \dfrac{2(16) - 3(-1)}{16 - (-1)} = \dfrac{32 + 3}{17} = \dfrac{35}{17} \approx 2.0588$

Evaluate $f(x_2)$:

- $f(x_2) = f(2.0588) = (2.0588)^3 - 2(2.0588) - 5 \approx -0.3908$

Since $f(x_2) < 0$ and $f(x_1) > 0$, the root lies between x_2 and x_1.

Second iteration

Let $x_0 = x_2 = 2.0588$ and $x_1 = 3$.

Calculate x_3:

- $x_3 = \dfrac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$
- $x_3 = \dfrac{2.0588(16) - 3(-0.3908)}{16 - (-0.3908)} = \dfrac{32.9408 + 1.1724}{16.3908} \approx 2.0813$

Evaluate $f(x_3)$:

- $f(x_3) = f(2.0813) = (2.0813)^3 - 2(2.0813) - 5 \approx -0.1014$

Third iteration

Let $x_0 = x_3 = 2.0813$ and $x_1 = 3$.

Calculate x_4:

- $x_4 = \dfrac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$
- $x_4 = \dfrac{2.0813(16) - 3(-0.1014)}{16 - (-0.1014)} = \dfrac{33.2998 + 0.3042}{16.1014} \approx 2.0906$

Evaluate $f(x_4)$:

- $f(2.0906) = (2.0906)^3 - 2(2.0906) - 5 \approx -0.0259$

Fourth iteration

Let $x_0 = x_4 = 2.0906$ and $x_1 = 3$.

Calculate x_5:

- $x_5 = \dfrac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$
- $x_5 = \dfrac{2.0906(16) - 3(-0.0259)}{16 - (-0.0259)} = \dfrac{33.4496 + 0.0777}{16.0259} \approx 2.0935$

Evaluate $f(x_5)$:

- $f(2.0935) = (2.0935)^3 - 2(2.0935) - 5 \approx -0.0066$

Fifth iteration

Let $x_0 = x_5 = 2.0935$ and $x_1 = 3$.

Calculate x_6:

- $x_6 = \dfrac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - fastep(x_0)}$
- $x_6 = \dfrac{2.0935(16) - 3(-0.0066)}{16 - (-0.0066)} = \dfrac{33.496 + 0.0198}{16.0066} \approx 2.0944$

Solution

The positive root of the equation $x^3 - 2x - 5 = 0$ is approximately 2.094.

Example: Solve $3x - \cos(x) - 1 = 0$ using False Position Method

Interval: $[a = 0, b = 1]$

Let
$f(x) = 3x - \cos(x) - 1$

Calculate
$f(0) = 3(0) - \cos(0) - 1 = 0 - 1 - 1 = -2$ (which is less than 0)
$f(1) = 3(1) - \cos(1) - 1 \approx 3 - 0.5403 - 1 = 1.4597$ (which is greater than 0)

Since $f(0) < 0$ and $f(1) > 0$, there is at least one root between 0 and 1.

The formula to find the next approximation x_n is:

$$x_n = b - f(b) \times \frac{b - a}{f(b) - f(a)}$$

where a and b are the current interval endpoints, and $f(a)$, $f(b)$ are the function values at those points.

a	b	x_n (approx.)	$f(a)$	$f(b)$	$f(x_n)$	Update
0	1	0.5781	-2	1.4597	-0.1032	$a = x_n$
1	0.5781	0.6060	1.4597	-0.1032	-0.0039	$b = x_n$
1	0.6060	0.6070	1.4597	-0.0039	-0.0004	$b = x_n$
1	0.6070	0.6071	1.4597	-0.0004	-0.00008	$b = x_n$
1	0.6071	0.6071	1.4597	-0.00008	-0.00006	Converged

So the root is 0.6071

Problem 1 : Find the root of the equation $f(x) = x^3 - x - 2$ in the interval [1, 2].

Solution:

Lets assume Initial Points i.e., $a = 1$, $b = 2$

Now,

$f(1) = 1^3 - 1 - 2 = -2$

$f(2) = 2^3 - 2 - 2 = 4$

Since $f(1) \cdot f(2) < 0$, the root lies between 1 and 2.

Iteration 1:

$c = a - [f(a) \cdot (b - a) / f(b) - f(a)]$

$\Rightarrow c = 1 - [\,(-2) \cdot (2 - 1) / 4 - (-2)\,]$

$\Rightarrow c = 1 - (-2 / 6) = 4 / 3$

$\Rightarrow c = 1.3333$

and $f(1.3333) = 1.3333^3 - 1.3333 - 2 = -0.1481$

Since $f(2) \cdot f(1.3333) < 0$, update the interval to [4/3, 2].

Iteration 2:

$c = 1.3333 - (-0.1481) \cdot (2 - 1.3333) / 4 - (-0.1481)$

$\Rightarrow c = 1.3333 - (-0.1481 \cdot 0.6667 / 4.1481)$

$\Rightarrow c = 1.3672$

and $f(1.3672) = 1.3672^3 - 1.3672 - 2 = 0.1197$

Since $f(1.3333) \cdot f(1.3672) < 0$, update the interval to [1.3333, 1.3672].

Iteration 3:

$c = 1.3333 - (-0.1481) \cdot (1.3672 - 1.3333) / 0.1197 - (-0.1481)$

$\Rightarrow c = 1.3513$

and $f(1.3513) = 1.3513^3 - 1.3513 - 2 = -0.0061$

Since $f(1.3333) \cdot f(1.3513) < 0$, update the interval to [1.3513, 1.3672].

Iteration 4:

$c = 1.3513 - (-0.0061) \cdot (1.3672 - 1.3513) / 0.1197 - (-0.0061)$

$\Rightarrow c = 1.3535$

and $f(1.3535) = 1.3535^3 - 1.3535 - 2 = 0.0003$

Since $f(1.3513) \cdot f(1.3535) < 0$, update the interval to $[1.3513, 1.3535]$.

Iteration 5:

$c = 1.3513 - (-0.0061) \cdot (1.3535 - 1.3513) / 0.0003 - (-0.0061)$

$\Rightarrow c = 1.3520$

and $f(1.3520) = 1.3520^3 - 1.3520 - 2 = -0.0001$

Since, there are no significant changes in the value of approximate root.

Thus, the root is approximately $x = 1.3522$.

Ex: $x^3-4x-1=0$ [where, $a=-1$, $b=0$]

Sol:

$f(x)=x^3-4x-1$

$f(0)=-1<0$

$f(-1)=2>0$

a	b	$x_n=\frac{af(b)-bf(a)}{f(b)-f(a)}$	$f(a)$	$f(b)$	$f(x_n)$	Update
-1	0	$x_1=-0.3333$	2	-1	$.2961$	$a=b$, $b=x_1$
0	-0.3333	$x_2=-0.2572$	-1	0.2961	0.0118	$a=a$, $b=x_2$
0	-0.2572	$x_3=-0.2542$	-1	0.0118	-0.0004	$a=b$, $b=x_3$
-0.2572	-0.2542	$x_4=-0.2543$	0.0118	-0.0004	0.0007	$a=b$, $b=x_4$
-0.2542	-0.2543	$x_5=-0.2543$	-0.004	$.0007$		

So the root is -0.2543

Finding a positive root of the equation $x \log_{10} x - 1.2 = 0$ using the Regula Falsi method.

Solution:

Regula Falsi formula: $x_{n+1} = \dfrac{a_n f(b_n) - b_n f(a_n)}{f(b_n) - f(a_n)}$

If $f(x_{n+1})$ has the same sign as $f(a_n)$, then set $a_{n+1} = x_{n+1}$ and $b_{n+1} = b_n$.

If $f(x_{n+1})$ has the same sign as $f(b_n)$, then set $b_{n+1} = x_{n+1}$ and $a_{n+1} = a_n$.

Calculate $f(a)$ and $f(b)$

$$f(a) = f(2) = 2\log_{10} 2 - 1.2$$

$$f(a) \approx -0.5979$$

$$f(b) = f(3) = 3\log_{10} 3 - 1.2$$

$$f(b) \approx 0.2314$$

Calculate $f(x_1)$

$$f(x_1) = f(2.7081) = 2.7081 \log_{10} 2.7081 - 1.2$$

$$f(x_1) \approx -0.0220$$

Update the interval

Since $f(x_1)$ has the same sign as $f(a)$, set $a = x_1 = 2.7081$ and $b = 3$.

Calculate x_2

$$x_2 = \frac{a f(b) - b f(a)}{f(b) - f(a)}$$
$$x_2 = \frac{2.7081(0.2314) - 3(-0.0220)}{0.2314 - (-0.0220)}$$
$$x_2 \approx 2.7402$$

Calculate $f(x_2)$

$$f(x_2) = f(2.7402) = 2.7402 \log_{10} 2.7402 - 1.2$$
$$f(x_2) \approx -0.0002$$

Update the interval

Since $f(x_2)$ has the same sign as $f(a)$, set $a = x_2 = 2.7402$ and $b = 3$.

Calculate x_3

$$x_3 = \frac{a f(b) - b f(a)}{f(b) - f(a)}$$
$$x_3 = \frac{2.7402(0.2314) - 3(-0.0002)}{0.2314 - (-0.0002)}$$
$$x_3 \approx 2.7406$$

Calculate $f(x_3)$

$$f(x_3) = f(2.7406) = 2.7406 \log_{10} 2.7406 - 1.2$$
$$f(x_3) \approx 0$$

Solution

The positive root of the equation $x \log_{10} x - 1.2 = 0$ is approximately 2.7406.