



Shahjalal University of Science and Technology (SUST)

Department of Electrical and Electronic Engineering (EEE)

Experiment name: [Jacobian matrix and power-flow solution
by Newton–Raphson](#)

[Experiment No: 04](#)

[Course Title: Power System -I](#)
[Course Code: EEE -326](#)

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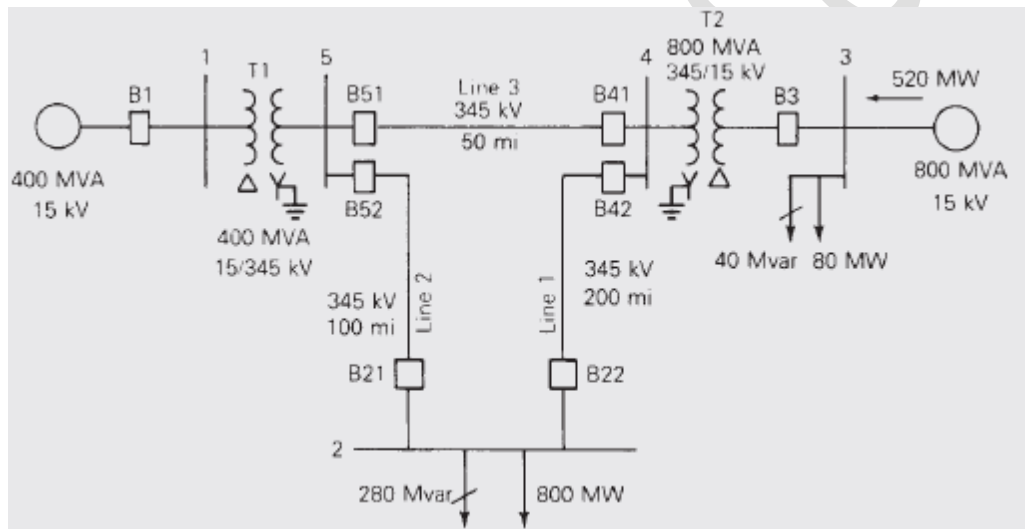


Objective: Determine the dimension of the Jacobian matrix for the power system in Example 6.9. Also calculate ΔP in Step 1 and ΔQ in Step 2 of the first Newton–Raphson iteration. Assume zero initial phase angles and 1.0 perunit initial voltage magnitudes (except $V_3 = 1.05$).

Equipment: Power World Simulator v17

Procedure:

Following figure shows a single-line diagram of a five-bus power system. Input data are given in Tables 1, 2, and 3. As shown in Table 1, bus 1, to which a generator is connected, is the swing bus. Bus 3, to which a generator and a load are connected, is a voltage-controlled bus. Buses 2, 4, and 5 are load buses.



The input data and unknowns are listed in Table 4. For bus 1, the swing bus, P_1 and Q_1 are unknowns. For bus 3, a voltage-controlled bus, Q_3 and d_3 are unknowns. For buses 2, 4, and 5, load buses, V_2 , V_4 , V_5 and d_2 , d_4 , d_5 are unknowns.

The elements of Y_{bus} are computed from the equation described in class. Since buses 1 and 3 are not directly connected to bus 2,

$$Y_{21} = Y_{23} = 0$$

Where, half of the shunt admittance of each line connected to bus 2 is included in Y_{22} (the other half is located at the other ends of these lines).



| Bus | Type | V per unit | δ degrees | P_G per unit | Q_G per unit | P_L per unit | Q_L per unit | Q_{Gmax} per unit | Q_{Gmin} per unit |
|-----|---------------------|------------------|---------------------|----------------------|----------------------|----------------------|----------------------|---------------------------|---------------------------|
| 1 | Swing | 1.0 | 0 | — | — | 0 | 0 | — | — |
| 2 | Load | — | — | 0 | 0 | 8.0 | 2.8 | — | — |
| 3 | Constant voltage | 1.05 | — | 5.2 | — | 0.8 | 0.4 | 4.0 | -2.8 |
| 4 | Load | — | — | 0 | 0 | 0 | 0 | — | — |
| 5 | Load | — | — | 0 | 0 | 0 | 0 | — | — |

* $S_{base} = 100$ MVA, $V_{base} = 15$ kV at buses 1, 3, and 345 kV at buses 2, 4, 5

Table:1 (Bus input data)

| Bus-to-Bus | R' per unit | X' per unit | G' per unit | B' per unit | Maximum MVA per unit |
|------------|------------------|------------------|------------------|------------------|----------------------------|
| 2-4 | 0.0090 | 0.100 | 0 | 1.72 | 12.0 |
| 2-5 | 0.0045 | 0.050 | 0 | 0.88 | 12.0 |
| 4-5 | 0.00225 | 0.025 | 0 | 0.44 | 12.0 |

Table:2 (Line input data)

| Bus-to-Bus | R per unit | X per unit | G_c per unit | B_m per unit | Maximum MVA per unit | Maximum TAP Setting per unit |
|------------|--------------------|--------------------|----------------------|----------------------|----------------------------|---------------------------------------|
| 1-5 | 0.00150 | 0.02 | 0 | 0 | 6.0 | — |
| 3-4 | 0.00075 | 0.01 | 0 | 0 | 10.0 | — |

Table:3 (Transformer input data)

| Bus | Input Data | Unknowns |
|-----|--|-----------------|
| 1 | $V_1 = 1.0, \delta_1 = 0$ | P_1, Q_1 |
| 2 | $P_2 = P_{G2} - P_{L2} = -8$ $Q_2 = Q_{G2} - Q_{L2} = -2.8$ | V_2, δ_2 |
| 3 | $V_3 = 1.05$ $P_3 = P_{G3} - P_{L3} = 4.4$ | Q_3, δ_3 |
| 4 | $P_4 = 0, Q_4 = 0$ | V_4, δ_4 |
| 5 | $P_5 = 0, Q_5 = 0$ | V_5, δ_5 |

Table:4 (Input data and unknowns)



Since there are $N = 5$ buses, (6.6.2) and (6.6.3) constitute $2(N - 1) = 8$ equations, for which $J(i)$ has dimension 8×8 .

However, there is one voltage-controlled bus, bus 3. Therefore, V_3 and the equation for $Q_3(x)$ could be eliminated, with $J(i)$ reduced to a 7×7 matrix.

From Step 1 and (6.6.2),

$$\begin{aligned}\Delta P_2(0) &= P_2 - P_2(x) = P_2 - V_2(0) \{ Y_{21} V_1 \cos[\delta_2(0) - \delta_1(0) - \theta_{21}] \\ &\quad + Y_{22} V_2 \cos[-\theta_{22}] + Y_{23} V_3 \cos[\delta_2(0) - \delta_3(0) - \theta_{23}] \\ &\quad + Y_{24} V_4 \cos[\delta_2(0) - \delta_4(0) - \theta_{24}] \\ &\quad + Y_{25} V_5 \cos[\delta_2(0) - \delta_5(0) - \theta_{25}] \} \\ \Delta P_2(0) &= -8.0 - 1.0 \{ 28.5847(1.0) \cos(84.624^\circ) \\ &\quad + 9.95972(1.0) \cos(-95.143^\circ) \\ &\quad + 19.9159(1.0) \cos(-95.143^\circ) \} \\ &= -8.0 - (-2.89 \times 10^{-4}) = -7.99972 \text{ per unit}\end{aligned}$$

From Step 2 and J_1 given in Table 6.5

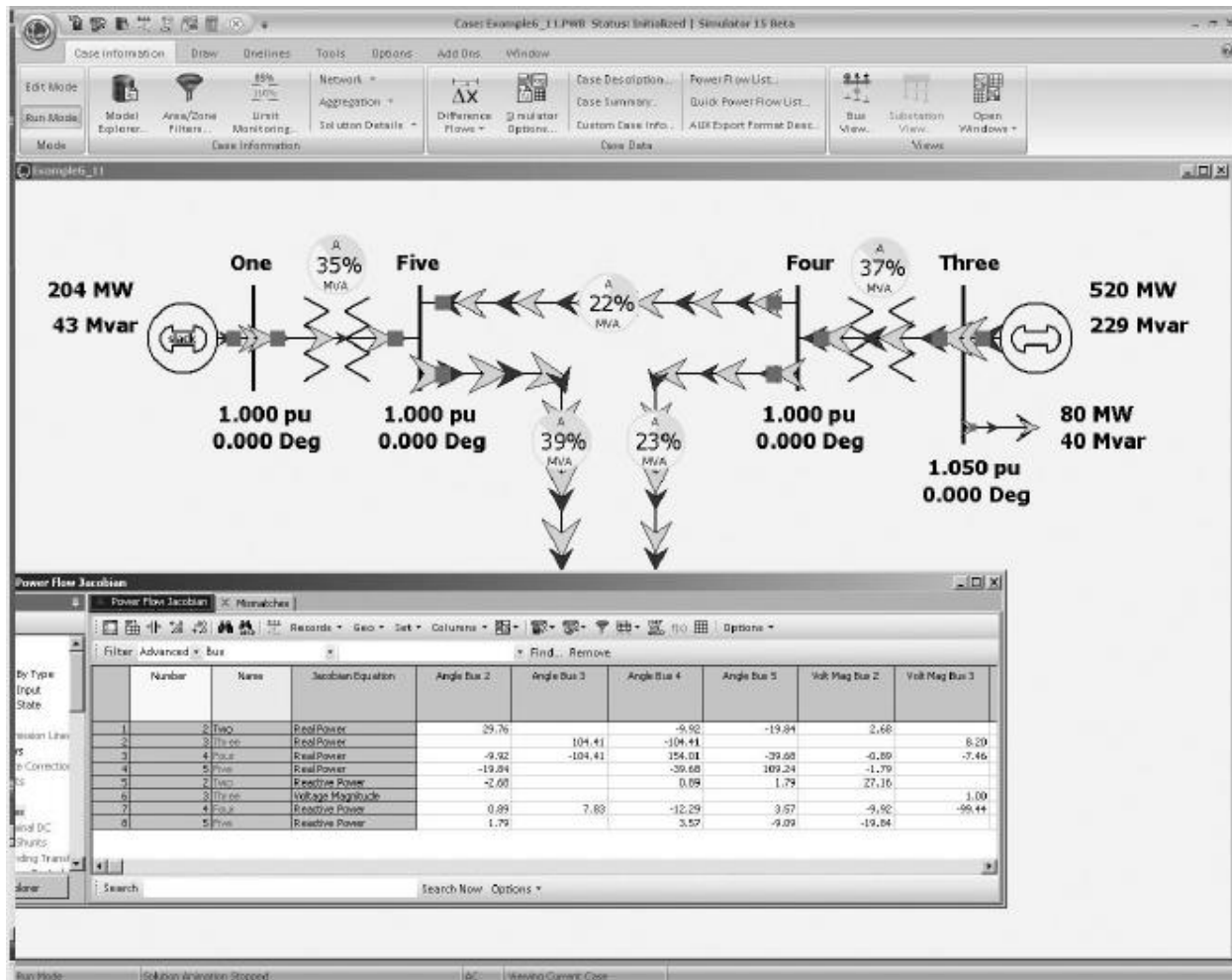
$$\begin{aligned}J_{124}(0) &= V_2(0) Y_{24} V_4(0) \sin[\delta_2(0) - \delta_4(0) - \theta_{24}] \\ &= (1.0)(9.95972)(1.0) \sin[-95.143^\circ] \\ &= -9.91964 \text{ per unit}\end{aligned}$$

To see the complete convergence of this case, open PowerWorld Simulator. **Select Case Information, Network, Mismatches** to see the **initial mismatches**, and **Case Information, Solution Details, Power Flow Jacobian** to view the **initial Jacobian matrix**. As is common in commercial power flows, PowerWorld Simulator actually includes rows in the Jacobian for voltage-controlled buses. When a generator is regulating its terminal voltage, this row corresponds to the equation setting the bus voltage magnitude equal to the generator voltage setpoint. However, if the generator hits a reactive power limit, the bus type is switched to a load bus.

To step through the New-Raphson solution, from the **Tools Ribbon** select **Solve, Single Solution—Full Newton**. Ordinarily this selection would perform a complete Newton–Raphson iteration, stopping only when all the mismatches are less than the desired tolerance. However, for this case, in order to allow you to see the solution process, the maximum number of iterations has been set to 1, allowing the voltages, mismatches and the



Jacobian to be viewed after each iteration. To complete the solution, continue to select Single Solution— Full Newton until the solution convergence to the values shown in Tables 6.6, 6.7 and 6.8 (in about three iterations).



| Bus # | Voltage Magnitude (per unit) | Phase Angle (degrees) | Generation | | Load | |
|-------|------------------------------|-----------------------|---------------|---------------|---------------|---------------|
| | | | PG (per unit) | QG (per unit) | PL (per unit) | QL (per unit) |
| 1 | 1.000 | 0.000 | 3.948 | 1.144 | 0.000 | 0.000 |
| 2 | 0.834 | -22.407 | 0.000 | 0.000 | 8.000 | 2.800 |
| 3 | 1.050 | -0.597 | 5.200 | 3.376 | 0.800 | 0.400 |
| 4 | 1.019 | -2.834 | 0.000 | 0.000 | 0.000 | 0.000 |
| 5 | 0.974 | -4.548 | 0.000 | 0.000 | 0.000 | 0.000 |
| TOTAL | | | 9.148 | 4.516 | 8.800 | 3.200 |



Table: Bus output data for the power system given in

| Line # | Bus to Bus | | P | Q | S |
|--------|------------|---|--------|--------|-------|
| 1 | 2 | 4 | -2.920 | -1.392 | 3.232 |
| | 4 | 2 | 3.036 | 1.216 | 3.272 |
| 2 | 2 | 5 | -5.080 | -1.408 | 5.272 |
| | 5 | 2 | 5.256 | 2.632 | 5.876 |
| 3 | 4 | 5 | 1.344 | 1.504 | 2.016 |
| | 5 | 4 | -1.332 | -1.824 | 2.260 |

Table: Line output data for the power system

| Tran. # | Bus to Bus | | P | Q | S |
|---------|------------|---|--------|--------|-------|
| 1 | 1 | 5 | 3.948 | 1.144 | 4.112 |
| | 5 | 1 | -3.924 | -0.804 | 4.004 |
| 2 | 3 | 4 | 4.400 | 2.976 | 5.312 |
| | 4 | 3 | -4.380 | -2.720 | 5.156 |

Table: Transformer output data for the power system