STANDARD FORMS

10 - 08 - 22

1. $\nabla \cdot (\ )$
2. $\nabla \times (\ )$
3. $\nabla (\ )$
4. $\nabla^2 (\ )$
5. $\overline{dl}$
6. $\overline{dA} / ds$
7. dv

DIFFERENTIAL LENGTH ($\overline{d\ell}$)

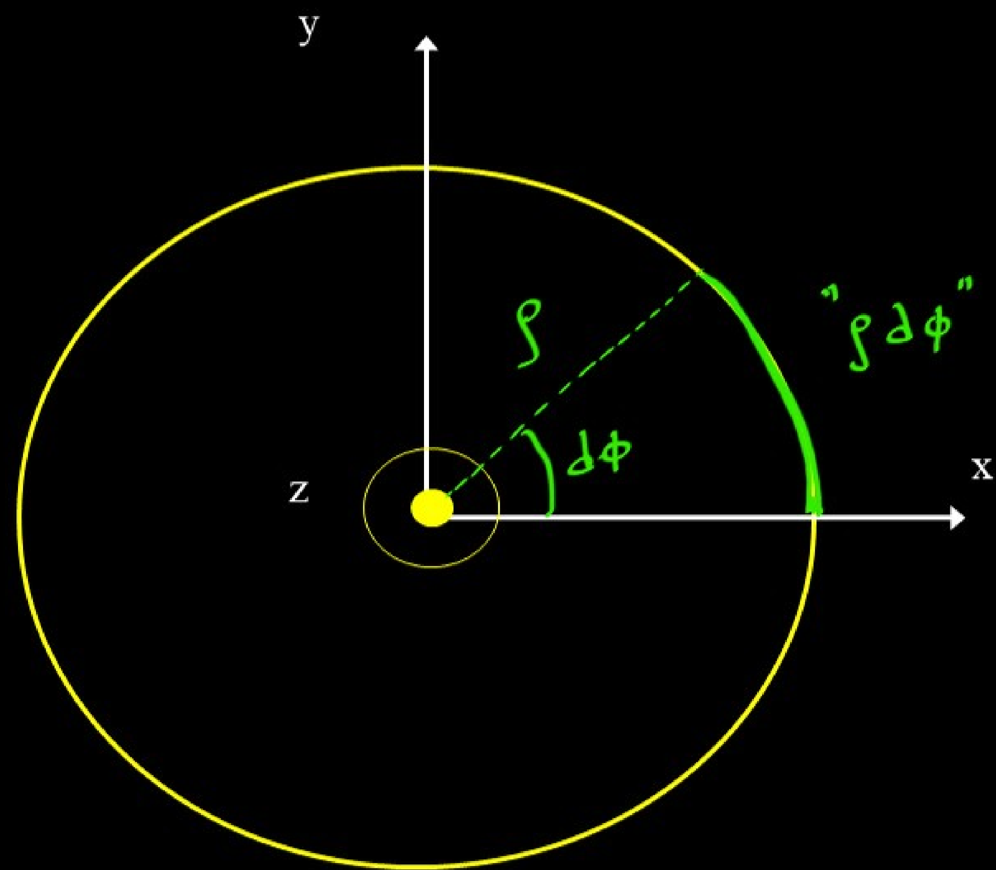
(x, y, z): $(\overline{d\ell}) = (1)\ dx\hat{x} + (1)\ dy\hat{y} + (1)\ dz\hat{z}$

(ρ, ϕ, z): $(\overline{d\ell}) = (1)\ d\rho\hat{\rho} + (\rho)\ d\phi\hat{\phi} + (1)\ dz\hat{z}$

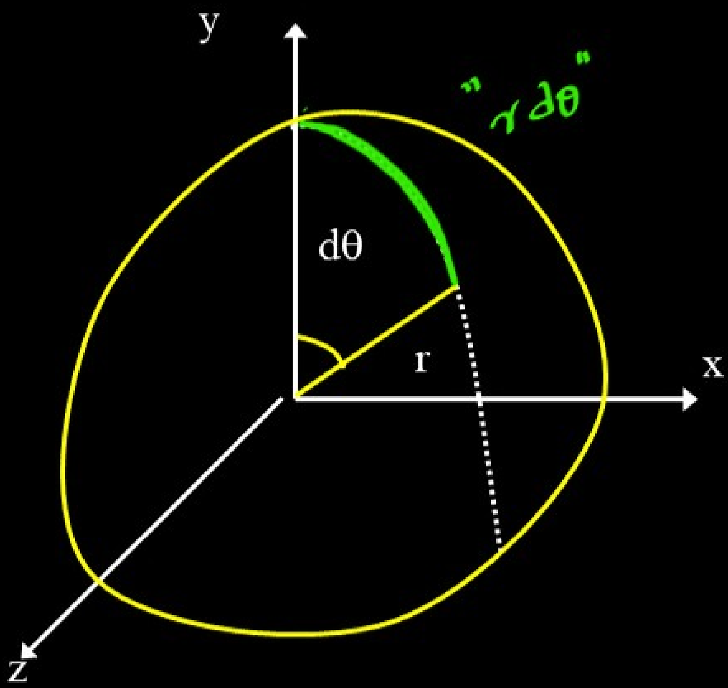
(r, θ, ϕ): $(\overline{d\ell}) = (1)\ dr\hat{r} + (r)\ d\theta\hat{\theta} + (r\sin\theta)\ d\phi\hat{\phi}$

(u, v, w) $\boxed{\overline{d\ell} = h_1\, du\,\hat{u} + h_2\, dv\,\hat{v} + h_3\, dw\,\hat{w}}$

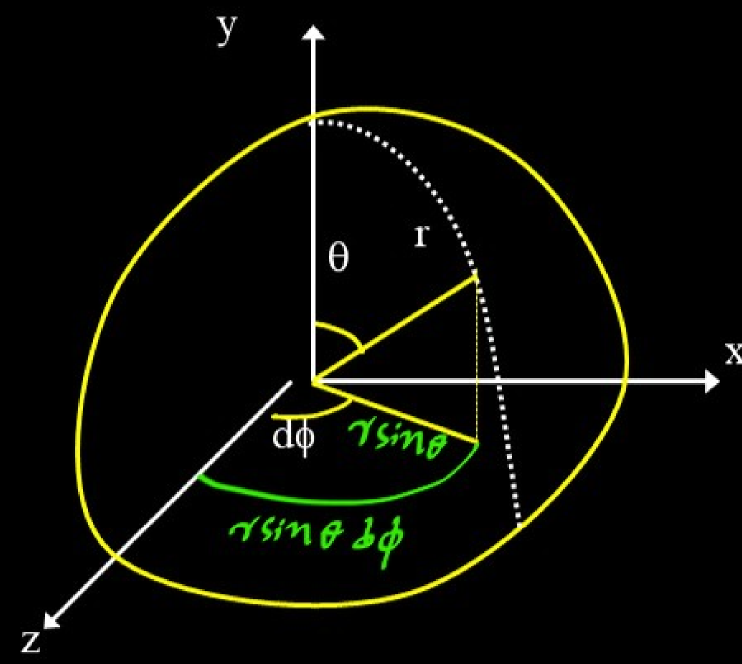
(I) TOP-VIEW OF CYLINDER



(II) SPHERICAL CO-ORD



(III) SPHERICAL CO-ORD



CO-ORDINATES **SCALING FACTORS**

u v w	h_1	h_2	h_3
x y z	1	1	1
ρ ϕ z	1	ρ	1
r θ ϕ	1	r	$r\sin\theta$

DIFFERENTIAL SURFACE ($\overline{ds}/\overline{dA}$)

$$\boxed{\overline{dA} = h_1 h_2 du dv \hat{w} + h_2 h_3 dv dw \hat{u} + h_3 h_1 dw du \hat{v}}$$

(x, y, z) : $\overline{dA} = dxdy\hat{z} + dydz\hat{x} + dzdx\hat{y}$

1, 1, 1

(ρ, ϕ, z) : $\overline{dA} = \rho d\rho d\phi\hat{z} + \rho d\phi dz\hat{\rho} + dzd\rho\hat{\phi}$

1, ρ, 1

(r, θ, ϕ) : $\widehat{dA} = r dr d\theta\hat{\phi} + r^2 sin\theta d\theta d\phi\hat{r} + rsin\theta d\phi dr\hat{\theta}$

1, r, rsinθ

DIFFERENTIAL VOLUME (dv)

$$\boxed{dv = h_1 h_2 h_3\, du\, dv\, dw}$$

(x, y, z) : $dv = dx\,dy\,dz$

$1, 1, 1$

(ρ, ϕ, z) : $dv = \rho\, d\rho\, d\phi\, dz$

$1, \rho, 1$

(r, θ, ϕ) : $dv = r^2 \sin\theta\, dr\, d\theta\, d\phi$

$1, r, r\sin\theta$

GRADIENT: SCALAR FIELD: f (u, v, w) → f

$$\boxed{\nabla f = \frac{1}{h_1}\frac{\partial f}{\partial u}\hat{u} + \frac{1}{h_2}\frac{\partial f}{\partial v}\hat{v} + \frac{1}{h_3}\frac{\partial f}{\partial w}\hat{w}}$$

(x, y, z) : f(x, y, z) → f

1, 1, 1

$$\nabla f = \frac{\partial f}{\partial x}\hat{x} + \frac{\partial f}{\partial y}\hat{y} + \frac{\partial f}{\partial z}\hat{z}$$

(ρ, ϕ, z) : f (ρ, ϕ, z) → f

1, ρ, 1

$$\nabla f = \frac{\partial f}{\partial \rho}\hat{\rho} + \frac{1}{\rho}\frac{\partial f}{\partial \phi}\hat{\phi} + \frac{\partial f}{\partial z}\hat{z}$$

(r, θ, ϕ) : f (r, θ, ϕ) → f

1, r, r sinθ

$$\nabla f = \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}\hat{\phi}$$

DIVERGENE AND CURL

Vector Field : $\bar{F} = F_u \hat{u} + F_v \hat{v} + F_w \hat{w}$

$$\boxed{\nabla . \bar{F} = \frac{1}{h_1 h_2 h_3}\left[\frac{\partial}{\partial u}(h_2 h_3 F_u) + \frac{\partial}{\partial v}(h_1 h_3 F_v) + \frac{\partial}{\partial w}(h_1 h_2 F_w)\right]}$$

$$\boxed{\nabla \; X \; \bar{F} = \frac{1}{h_1 h_2 h_3}\begin{vmatrix} h_1 \hat{u} & h_2 \hat{v} & h_3 \hat{w} \\ \dfrac{\partial}{\partial u} & \dfrac{\partial}{\partial v} & \dfrac{\partial}{\partial w} \\ h_1 F_u & h_2 F_v & h_3 F_w \end{vmatrix}}$$

(x, y, z) : $\bar{F} = F_x\hat{x} + F_y\hat{y} + F_z\hat{z}$

1, 1, 1

$$\nabla.\bar{F} = \frac{\partial}{\partial x}F_x + \frac{\partial}{\partial y}F_y + \frac{\partial}{\partial z}F_z$$

$$\nabla \times \bar{F} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix}$$

$(\rho, \phi, z) \quad : \quad \bar{F} = F_\rho \hat{\rho} + F_\phi \hat{\phi} + F_z \hat{z}$

$1, \rho, 1$

$$\nabla . \bar{F} = \frac{1}{\rho}\left[\frac{\partial}{\partial \rho}(\rho F_\rho) + \frac{\partial}{\partial \phi}(F_\phi) + \frac{\partial}{\partial z}(\rho F_z)\right]$$

$$\nabla \times \bar{F} = \frac{1}{\rho}\begin{vmatrix} \hat{\rho} & \rho\hat{\phi} & \hat{z} \\ \dfrac{\partial}{\partial \rho} & \dfrac{\partial}{\partial \phi} & \dfrac{\partial}{\partial z} \\ F_\rho & \rho F_\phi & F_z \end{vmatrix}$$

(r, θ, ϕ) : $\bar{F} = F_r\hat{r} + F_\theta\hat{\theta} + F_\phi\hat{\phi}$

$1, r, r\sin\theta$

$$\nabla.\bar{F} = \frac{1}{r^2 sin\theta}\left[\frac{\partial}{\partial r}(r^2 sin\theta F_r) + \frac{\partial}{\partial \theta}(r sin\theta F_\theta) + \frac{\partial}{\partial \phi}(r F_\phi)\right]$$

$$\nabla \times \bar{F} = \frac{1}{r^2 sin\theta}\begin{vmatrix} \hat{r} & r\hat{\theta} & r sin\theta\,\hat{\phi} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ F_r & rF_\theta & r sin\theta F_\phi \end{vmatrix}$$

LAPLACIAN OPERATOR

Scalar Field: f(u, v, w) → f

$$\boxed{\nabla^2 f = \frac{1}{h_1 h_2 h_3}\left[\frac{\partial}{\partial u}\left(\frac{h_2 h_3}{h_1}\frac{\partial f}{\partial u}\right) + \frac{\partial}{\partial v}\left(\frac{h_1 h_3}{h_2}\frac{\partial f}{\partial v}\right) + \frac{\partial}{\partial w}\left(\frac{h_1 h_2}{h_3}\frac{\partial f}{\partial w}\right)\right]}$$

(x, y, z) : f(x, y, z) → f

1, 1, 1

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

$(\rho, \phi, z)\ :\ \ f(\rho, \phi, z) \rightarrow f$

$1,\ \rho,\ 1$

$$\nabla^2 f = \frac{1}{\rho}\left[\frac{\partial}{\partial \rho}\left(\rho \frac{\partial f}{\partial \rho}\right) + \frac{\partial}{\partial \phi}\left(\frac{1}{\rho}\frac{\partial f}{\partial \phi}\right) + \frac{\partial}{\partial z}\left(\rho \frac{\partial f}{\partial z}\right)\right]$$

(r, θ, ϕ) : f(r, θ, ϕ) → f

$1, r, r sin\theta$

$$\nabla^2 f = \frac{1}{r^2 sin\theta}\left[\frac{\partial}{\partial r}\left(r^2 sin\theta \frac{\partial f}{\partial r}\right) + \frac{\partial}{\partial \theta}\left(\frac{r sin\theta}{r}\frac{\partial f}{\partial \theta}\right) + \frac{\partial}{\partial \phi}\left(\frac{r}{r sin\theta}\frac{\partial f}{\partial \phi}\right)\right]$$

Q The scalar field in certain region is defined as $r^2 \sin\theta \cos\phi$. Find its gradient at point $(1, \pi/4, \pi/4)$

Soln: $f = r^2 \sin\theta \cos\phi$

$$\nabla f = \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}\hat{\phi}$$

$$\nabla f = \frac{\partial}{\partial r} r^2 \sin\theta \cos\phi \,\hat{r} + \frac{1}{r}\frac{\partial}{\partial \theta} r^2 \sin\theta \cos\phi \,\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi} r^2 \sin\theta \cos\phi \,\hat{\phi}$$

$$\nabla f = 2r\sin\theta \cos\phi \,\hat{r} + r\cos\theta \cos\phi \,\hat{\theta} - r\sin\phi \,\hat{\phi}$$

AT $(r, \theta, \phi) = (1, \pi/4, \pi/4)$

$$\nabla f = 2 \times 1 \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}\hat{r} + 1 \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}\hat{\theta} - 1 \times \frac{1}{\sqrt{2}}\hat{\phi} = \hat{r} + \frac{1}{2}\hat{\theta} - \frac{1}{\sqrt{2}}\hat{\phi}$$

Q. If $\bar{F}(\rho, \phi, z) = \rho\hat{\rho} + \rho sin^2\phi\hat{\phi} - z\hat{z}$, which one of the following is TRUE?

(DRDO-2009)

(a) $\nabla.\bar{F}\Big|_{\phi=0°} < \nabla.\bar{F}\Big|_{\phi=\pi/2}$

(b) $\nabla.\bar{F}\Big|_{\phi=\pi/4} = \nabla.\bar{F}\Big|_{\phi=0°}$

(c) $\nabla.\bar{F}\Big|_{\phi=0°} > \nabla.\bar{F}\Big|_{\phi=\pi/2}$

(d) $\nabla.\bar{F}\Big|_{\phi=\pi/4} = 2\nabla.\bar{F}\Big|_{\phi=0°}$

Soln: $\bar{F} = \rho\hat{\rho} + \rho\sin 2\phi\,\hat{\phi} - z\hat{z}$

$$\nabla\cdot\bar{F} = \frac{1}{\rho}\left[\frac{\partial}{\partial\rho}(\rho F_\rho) + \frac{\partial}{\partial\phi}(F_\phi) + \frac{\partial}{\partial z}(\rho F_z)\right]$$

$$= \frac{1}{\rho}\left[\frac{\partial}{\partial\rho}(\rho\cdot\rho) + \frac{\partial}{\partial\phi}(\rho\sin 2\phi) + \frac{\partial}{\partial z}[\rho(-z)]\right]$$

$$= \frac{1}{\rho}\left[2\rho + \rho\, 2\sin\phi\cos\phi - \rho\right]$$

$$= \frac{1}{\rho}\left[\rho + \rho\sin 2\phi\right]$$

$$\nabla\cdot\bar{F} = 1 + \sin 2\phi.$$

AT $\phi = 0^\circ$

$$\nabla\cdot\bar{F} = 1 + \sin 2\times 0^\circ$$
$$= 1 + 0 = 1$$

AT $\phi = \frac{\pi}{4}$

$$\nabla\cdot\bar{F} = 1 + \sin 2\times\frac{\pi}{4}$$
$$= 1 + 1 = 2$$

AT $\phi = \frac{\pi}{2}$

$$\nabla\cdot\bar{F} = 1 + \sin 2\times\frac{\pi}{2}$$
$$= 1 + 0$$
$$= 1.$$

Q. Find the CURL of following

(a) $\overline{F_1}(\rho,\phi,z) = \hat{\phi} = 1\hat{\phi}$

(b) $\overline{F_2}(\rho,\phi,z) = \frac{\hat{\phi}}{\rho} = \frac{1}{\rho}\hat{\phi}$

(c) $\overline{F_3}(r,\theta,\phi) = \hat{r} + \frac{\hat{\theta}}{r} + \frac{\hat{\phi}}{r\sin\theta}$

(a)

$$\nabla\times\overline{F_1} = \frac{1}{\rho}\begin{vmatrix} \hat{\rho} & \rho\hat{\phi} & \hat{z} \\ \frac{\partial}{\partial\rho} & \frac{\partial}{\partial\phi} & \frac{\partial}{\partial z} \\ F_\rho & \rho F_\phi & F_z \end{vmatrix}$$

$$= \frac{1}{\rho}\begin{vmatrix} \hat{\rho} & \rho\hat{\phi} & \hat{z} \\ \frac{\partial}{\partial\rho} & \frac{\partial}{\partial\phi} & \frac{\partial}{\partial z} \\ 0 & \rho & 0 \end{vmatrix}$$

$$= \frac{1}{\rho}\left\{\hat{\rho}(0-0) - \rho\hat{\phi}(0-0) + \hat{z}(1-0)\right\}$$

$$\nabla\times\overline{F_1} = \frac{\hat{z}}{\rho}$$

(b) $$\nabla \times \bar{F}_2 = \frac{1}{\rho}\begin{vmatrix} \hat{\rho} & \rho\hat{\phi} & \hat{z} \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ 0 & \rho \times \frac{1}{\rho} & 0 \end{vmatrix} = \frac{1}{\rho}\begin{vmatrix} \hat{\rho} & \rho\hat{\phi} & \hat{z} \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ 0 & 1 & 0 \end{vmatrix}$$

$$= 0$$

(c)

$$\nabla \times \bar{F}_3 = \frac{1}{r^2 \sin\theta} \begin{vmatrix} \hat{r} & r\hat{\theta} & r\sin\theta\,\hat{\phi} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi} \\ F_r & rF_\theta & r\sin\theta\, F_\phi \end{vmatrix}$$

$$= \frac{1}{r^2 \sin\theta} \begin{vmatrix} \hat{r} & r\hat{\theta} & r\sin\theta\,\hat{\phi} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi} \\ 1 & r\times\frac{1}{r} & r\sin\theta\times\frac{1}{r\sin\theta} \end{vmatrix}$$

$1 \qquad 1 \qquad 1$

$$= 0$$

Q. Find the value of "n" for solenoidal field $\bar{A}(r,\theta,\phi) = 4r^n\hat{r}$

Soln:

$$\nabla \cdot \bar{A} = 0$$

$$\frac{1}{r^2 \sin\theta}\left[\frac{\partial}{\partial r}(r^2 \sin\theta\, A_r) + \frac{\partial}{\partial \theta}(r \sin\theta\, \underset{0}{A_\theta}) + \frac{\partial}{\partial \phi}(r\, \underset{0}{A_\phi})\right] = 0$$

$$\Rightarrow \frac{\partial}{\partial r}(r^2 \sin\theta\, A_r) = 0$$

$$\sin\theta \frac{\partial}{\partial r}(r^2\, 4r^n) = 0$$

$$4\sin\theta \frac{\partial}{\partial r}(r^{n+2}) = 0$$

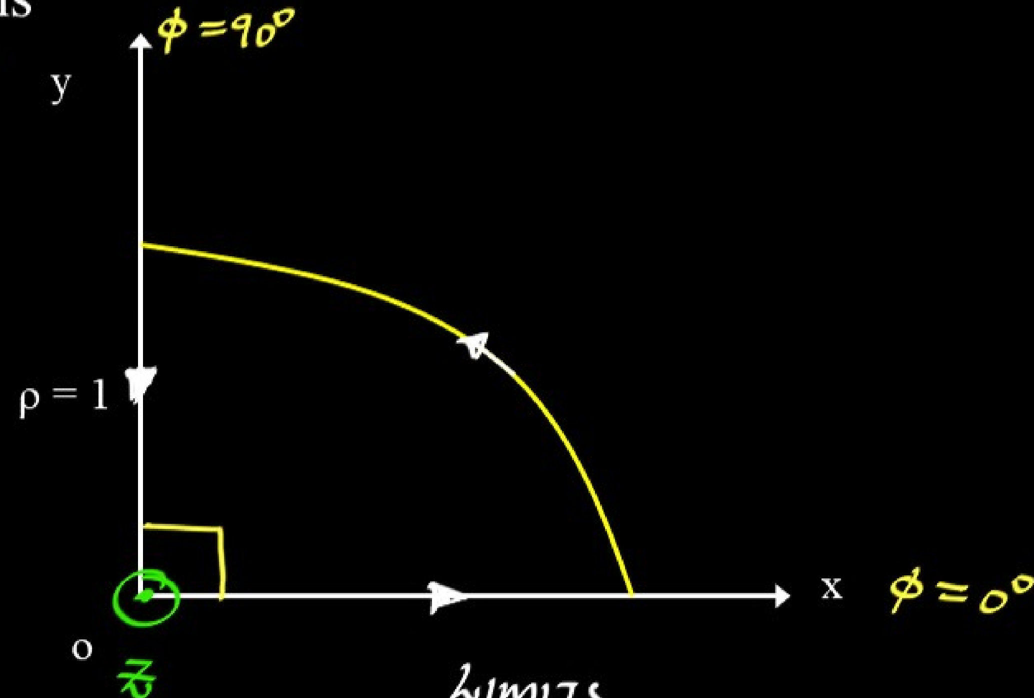
$$4\sin\theta\, (n+2)\, r^{n+1} = 0$$

$$\Rightarrow n+2 = 0 \Rightarrow n = -2$$

Q. Given a vector field $\bar{A} = 2\rho cos\phi\hat{\rho}$. In cylindrical co-ordinates for the contour as shown below $\oint \bar{A}.\overline{d\ell}$ is

(a) 1 (b) $1 - \frac{\pi}{2}$

(c) $1 + \frac{\pi}{2}$ (d) -2



Limits

$$\left\{ \begin{array}{c} 0 \le \rho \le 1 \\ 0 \le \phi \le \pi/2 \\ z = 0 \end{array} \right\}$$

$$I = \oint \bar{A} \cdot \bar{dl} = \underline{\iint \nabla \times \bar{A} \cdot \bar{ds}}$$

(ρ, ϕ, z) : $\bar{dA} = \rho\, d\rho\, d\phi\, \hat{z} + \underbrace{\rho\, d\phi\, dz\, \hat{\rho} + dz\, d\rho\, \hat{\phi}}_{0}$

$2, \rho, 1$

$$\bar{dA} = \rho\, d\rho\, d\phi\, \hat{z}$$

$$\nabla \times \bar{A} = \frac{1}{\rho} \begin{vmatrix} \hat{\rho} & \rho\hat{\phi} & \hat{z} \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_\rho & \rho A_\phi & A_z \end{vmatrix} = \frac{1}{\rho} \begin{vmatrix} \hat{\rho} & \rho\hat{\phi} & \hat{z} \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ 2\rho\cos\phi & 0 & 0 \end{vmatrix}$$

$$= \frac{1}{\rho} \left\{ \hat{\rho}(0-0) - \rho\hat{\phi}(0-0) + \hat{z}\left(0 + 2\rho \sin\phi \right) \right\}$$

$$\nabla \times \bar{A} = 2 \sin\phi\, \hat{z}$$

$$I = \iint \nabla \times \bar{A} \cdot \bar{ds} = \iint 2\sin\phi\, \hat{z} \cdot \rho\, d\rho\, d\phi\, \hat{z}$$

$$= 2 \iint \sin\phi\, \rho\, d\rho\, d\phi$$

$$= 2 \int \sin\phi\, d\phi \int \rho\, d\rho$$

$$= 2 \left[-\cos\phi\right]_0^{\pi/2} \left[\frac{\rho^2}{2}\right]_0^1$$

$$= \{-(0-1)\}(1) = \underline{\underline{1}}.$$

Q. Determine the net flux of the vector field $\bar{F}(\rho, \phi, z) = \rho\hat{\rho} + \hat{\phi} + z\hat{z}$ leaving the closed half cylinder $\rho \leq 1, 0 \leq \phi \leq \pi, 0 \leq z \leq 1$.

Soln:

NET FLUX $= \oiint \bar{F} \cdot \overline{dA} = \iiint \nabla \cdot \bar{F} \, dv$

$$\nabla \cdot \bar{F} = \frac{1}{\rho}\left[\frac{\partial}{\partial \rho}(\rho F_\rho) + \frac{\partial}{\partial \phi}(F_\phi) + \frac{\partial}{\partial z}(\rho F_z)\right]$$

$$= \frac{1}{\rho}\left[\frac{\partial}{\partial \rho}(\rho \cdot \rho) + \frac{\partial}{\partial \phi}(1) + \frac{\partial}{\partial z}(\rho \cdot z)\right]$$

$$= \frac{1}{\rho}\left[2\rho + 0 + \rho\right] = 3$$

$$dv = \rho \, d\rho \, d\phi \, dz$$

NET FLUX $= \iiint (3) \, \rho \, d\rho \, d\phi \, dz$

$$= 3 \int \rho \, d\rho \int d\phi \int dz$$

$$= 3 \left[\frac{\rho^2}{2}\right]_0^1 \left[\phi\right]_0^\pi \left[z\right]_0^1$$

$$= 3 \left[\frac{1}{2}\right][\pi][1]$$

NET FLUX $= 1.5\pi$

Q. The region specified by

$$\left\{(\rho, \phi, z): 3 \leq \rho \leq 5, \frac{\pi}{8} \leq \phi \leq \frac{\pi}{4}, 3 \leq z \leq 4.5\right\}$$

in cylindrical co-ordinates has volume of ______ m³.

(GATE-16-Set-1)

Soln Volume $= \iiint dv = \iiint \rho \, d\rho \, d\phi \, dz$

$$= \int \rho \, d\rho \int d\phi \int dz = \left[\frac{\rho^2}{2}\right]_3^5 \left[\phi\right]_{\pi/8}^{\pi/4} \left[z\right]_3^{4.5}$$

$$= \frac{1}{2}[25-9]\left[\pi/4 - \pi/8\right][4.5-3]$$

Volume $= \underline{4.71}$ m3.

Q. Determine the volume of region defined in spherical co-ordinate system as

$$r \le 1,\ 0 \le \theta \le \frac{\pi}{2},\ 0 \le \phi \le \frac{\pi}{2}$$

Soln: Volume $= \iiint dv = \iiint r^2 \sin\theta \, dr\, d\theta\, d\phi = \int r^2 dr \int \sin\theta \, d\theta \int d\phi$

$$= \left[\frac{r^3}{3}\right]_0^1 \left[-\cos\theta\right]_0^{\pi/2} \left[\phi\right]_0^{\pi/2}$$

$$= \frac{1}{3}\left\{-(0-1)\right\}(\pi/2) = \frac{\pi}{6}$$

Volume $= \underline{0.523}\ m^3$.

Q. Determine the area of surface defined in spherical co-ordinate system as

$r = 2\text{m}$ and $\frac{\pi}{4} \leq \theta \leq \frac{\pi}{3}$

Soln: $\text{AREA} = \iint dA$

(r, θ, ϕ)

$1, r, r\sin\theta$

$\overline{dA} = \underbrace{r\,dr\,d\theta\,\hat{\phi}}_{0} + r^2\sin\theta\,d\theta\,d\phi\,\hat{r} + \underbrace{r\sin\theta\,d\phi\,dr\,\hat{\theta}}_{0}$

$r = 2m,\ dr = 0,\ \overline{dA} = r^2\sin\theta\,d\theta\,d\phi\,\hat{r}$

$dA = r^2\sin\theta\,d\theta\,d\phi$

$\text{AREA} = \iint r^2\sin\theta\,d\theta\,d\phi$

$= 2^2\int\sin\theta\,d\theta\int d\phi$

$= 4\,[-\cos\theta]_{\pi/4}^{\pi/3}\,[\phi]_0^{2\pi}$

$= 4\left\{-\left(\frac{1}{2} - \frac{1}{\sqrt{2}}\right)\right\}(2\pi)$

$\text{AREA} = \underline{5.2}\ m^2.$

Q. Determine the area of surface defined in cylindrical co-ordinate system as

$\rho \leq 2$ m, $z = 5$ m

Soln: $\text{AREA} = \iint dA$

(ρ, ϕ, z)

$1, \rho, 1$

$\overline{dA} = \rho\, d\rho\, d\phi\, \hat{z} + \rho\, d\phi\, \underset{0}{dz}\, \hat{\rho} + \underset{0}{dz}\, d\rho\, \hat{\phi}$

$z = 5m, \; dz = 0, \; \overline{dA} = \rho\, d\rho\, d\phi\, \hat{z}$

$dA = \rho\, d\rho\, d\phi$

$\text{AREA} = \iint \rho\, d\rho\, d\phi = \int \rho\, d\rho \int d\phi$

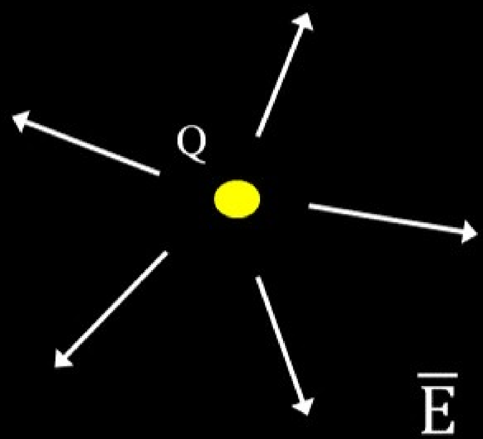
$= \left[\frac{\rho^2}{2}\right]_0^2 \left[\phi\right]_0^{2\pi}$

$= \frac{2^2}{2} \times 2\pi = 4\pi$

$\text{AREA} = \underline{12.56}\ m^2.$

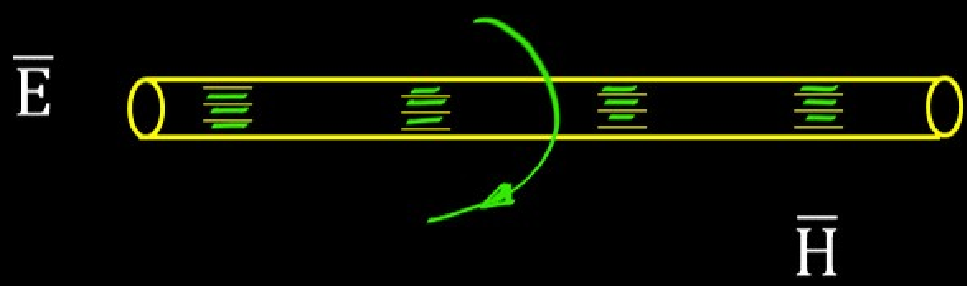
STORY OF EM-THEORY

(I) ELECTROSTATICS



- Charge is at rest
- No supply
- Only $\bar{E}$ – Field Exists

(II) MAGNETOSTATICS



- Charges are moving with uniform velocity.
- Constant supply
- Both $\overline{\mathrm{E}}$ and $\overline{H}$ fields exists but there is no em-phenomenon.

(III) TIME-VARYING FIELDS

$\bar{E} \rightleftarrows \bar{H}$: EM-PHENOMON

- Charges are accelerated and de-accelerated.
- Time-varying supply
- Both $\overline{\mathrm{E}}$ and $\overline{H}$ fields exists and there is em-phenomenon.

TOPIC : 2

ELECTROSTATICS

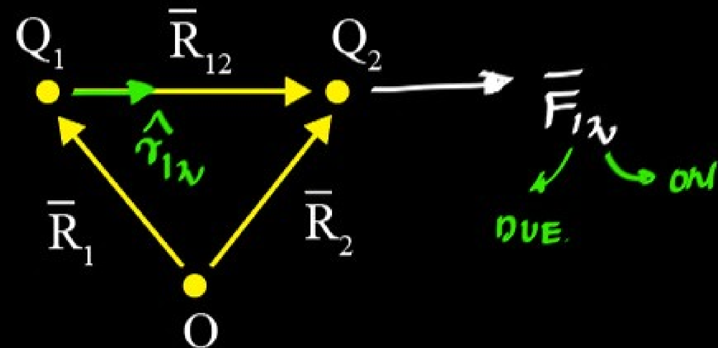
ELECTROSTATICS

- Electrostatics Defines the electric field and develops the different techniques of finding electric field due to various charge distributions

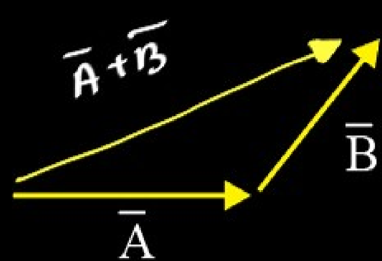
Coulombs Law

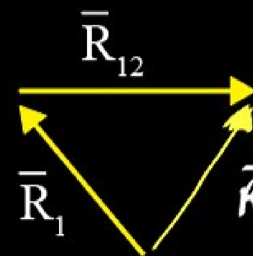
$$\bar{R}_{12} = \bar{R}_2 - \bar{R}_1$$

$$\hat{r}_{12} = \frac{\bar{R}_{12}}{R_{12}} = \frac{\bar{R}_{12}}{|\bar{R}_{12}|}$$



Ex:





$$\bar{R}_2 = \bar{R}_1 + \bar{R}_{12}$$

$$\bar{R}_{12} = \bar{R}_2 - \bar{R}_1$$

$$|\bar{F}_{12}| \propto \frac{Q_1 Q_2}{R_{12}^2}$$

$$|\bar{F}_{12}| = \frac{1}{4\pi\epsilon} \frac{Q_1 Q_2}{R_{12}^2}$$

***

$$\boxed{\bar{F}_{12} = \frac{Q_1 Q_2}{4\pi\epsilon R_{12}^2} \hat{r}_{12}} \ (N)$$

where

$$\hat{r}_{12} = \frac{\bar{R}_{12}}{R_{12}}$$

$$\boxed{\bar{F}_{12} = \frac{Q_1 Q_2}{4\pi\epsilon} \left[\frac{\bar{R}_{12}}{R_{12}^3}\right]} \ (N)$$

$\in = \in_0 \in_r$: Permittivity

$\epsilon_0 = \frac{1}{36\pi \times 10^9}$ F/m : Absolute Permittivity

$\in_r$: Relative Permittivity

$\in_r = 1$ (Air)

$\in_\gamma > 1$ (Other than air)

Q. Two point charges Q_1 and Q_2 are located at (3, 0, 0) and (1, 2, 0) find the relation between Q_1 and Q_2 such that x and y components of total force on test charge q_o placed at (1, -1, 0) are equal.

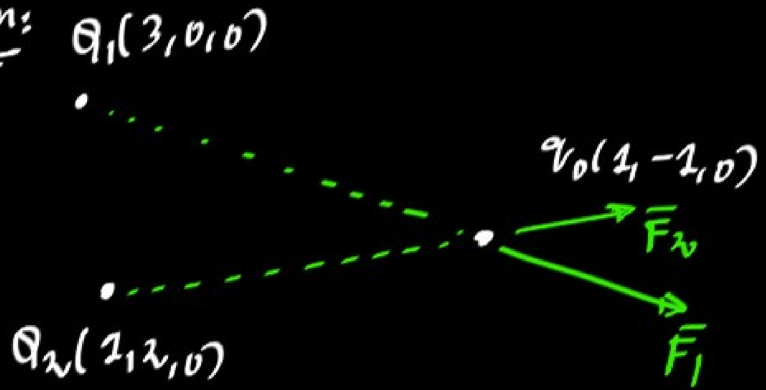
(a) $\frac{Q_1}{Q_2} = \frac{5\sqrt{5}}{9}$

(b) $\frac{Q_1}{Q_2} = \frac{9}{5\sqrt{5}}$

(c) $\frac{Q_1}{Q_2} = 5\sqrt{5}$

(d) $\frac{Q_1}{Q_2} = 9$

Soln:



$$\bar{F}_T = \bar{F}_1 + \bar{F}_2$$

$$\bar{F} = \frac{q_1 q_2}{4\pi\epsilon}\left[\frac{\bar{R}}{R^3}\right]$$

$$\bar{F}_T = \frac{Q_1 q_0}{4\pi\epsilon}\left[\frac{-2\hat{x} - \hat{y}}{(\sqrt{4+1})^3}\right] + \frac{Q_2 q_0}{4\pi\epsilon}\left[\frac{-3\hat{y}}{(3)^3}\right]$$

$$\bar{F}_T = \frac{Q_1 q_0}{4\pi\epsilon}\left[\frac{-2\overset{\#}{\hat{x}} - \overset{*}{\hat{y}}}{5\sqrt{5}}\right] + \frac{Q_2 q_0}{4\pi\epsilon}\left[-\frac{\overset{*}{\hat{y}}}{9}\right]$$

$$x_{comp} = y_{comp.}$$

$$\frac{Q_1 q_0}{4\pi\epsilon}\left[\frac{-2}{5\sqrt{5}}\right] = \frac{Q_1 q_0}{4\pi\epsilon}\left[\frac{-1}{5\sqrt{5}}\right] + \frac{Q_2 q_0}{4\pi\epsilon}\left[-\frac{1}{9}\right]$$

$$\frac{2Q_1}{5\sqrt{5}} = \frac{Q_1}{5\sqrt{5}} + \frac{Q_2}{9}$$

$$\frac{Q_1}{5\sqrt{5}} = \frac{Q_2}{9}$$

$$\frac{Q_1}{Q_2} = \frac{5\sqrt{5}}{9}.$$

Q. A charge 'Q' is divided between two points charges, what should be the values of this charge on the objects so that the force between them is maximum

(ESE-15)

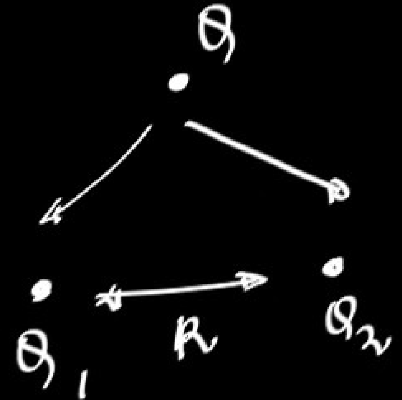
(a) $\frac{Q}{3}$

(b) $\frac{Q}{2}$

(c) $(Q - 2)$

(c) $2Q$

Soln:



$$Q = Q_1 + Q_2$$

$$F_{12} = \frac{Q_1 Q_2}{4\pi\epsilon R^2} = \frac{Q_1}{4\pi\epsilon R^2}[Q - Q_1]$$

$$F_{12} = \frac{1}{4\pi\epsilon R^2}[Q_1 Q - Q_1^2]$$

$$\frac{dF_{12}}{dQ_1} = 0$$

$$\frac{1}{4\pi\epsilon R^2}[Q - 2Q_1] = 0$$

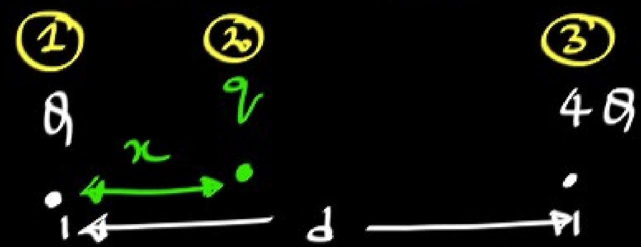
$$\Rightarrow Q = 2Q_1$$

$$Q_1 = \frac{Q}{2}$$

$$Q_2 = \frac{Q}{2}.$$

Q. Two positive charges Q and 4Q are separated by a distance d, in AIR, A third charge is so placed that the entire system is in equilibrium determine the location, the magnitude and the sign of the third charge

Soln:



$$|\bar{F}_{12}| = |\bar{F}_{23}| = |\bar{F}_{13}|$$

$$\frac{Qq}{4\pi\epsilon x^2} = \frac{q4Q}{4\pi\epsilon(d-x)^2} = \frac{Q4Q}{4\pi\epsilon d^2}$$

$$\Rightarrow \frac{Qq}{4\pi\epsilon x^2} = \frac{4Qq}{4\pi\epsilon (d-x)^2}$$

$$\frac{1}{x^2} = \frac{4}{(d-x)^2}$$

$$\frac{1}{x} = \frac{\pm 2}{(d-x)}$$

$$d-x = \pm 2x$$

$$d = \pm 2x + x$$

$$d \begin{cases} 2x + x = 3x \\ -2x + x = -x \end{cases}$$

$$x \begin{cases} \dfrac{d}{3} \\ -d \end{cases}$$

$$\boxed{x = \frac{d}{3}}$$

$$\frac{Qq}{4\pi\epsilon x^2} = \frac{Q(4Q)}{4\pi\epsilon d^2}$$

$$\frac{q}{x^2} = \frac{4Q}{d^2}$$

$$q = \frac{4Q}{d^2} x^2$$

$$q = \frac{4Q}{d^2} \cdot \left(\frac{d}{3}\right)^2$$

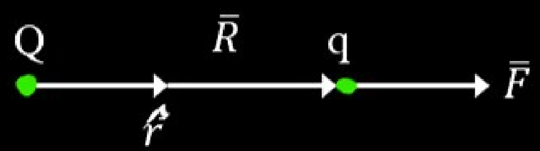
$$q = \frac{4Q}{9} \, C$$

FOR EQUILIBRIUM "q" MUST BE

-ve CHARGE

i.e "$-\dfrac{4Q}{9}$ C"

ELECTIRC FIELD (OR) ELECTRIC FIELD INTENSITY ($\bar{E}$ $\frac{N}{C}$ or $\frac{V}{m}$)



$$\boxed{\bar{E} = \frac{Q}{4\pi \in R^2}\hat{r} \quad \frac{N}{C}}$$

Q: Field producing charge

q: Test charge

$$\bar{F} = \frac{Qq}{4\pi \in R^2}\hat{r}$$

$$\boxed{\bar{F} = \mathrm{q}\bar{E}}$$

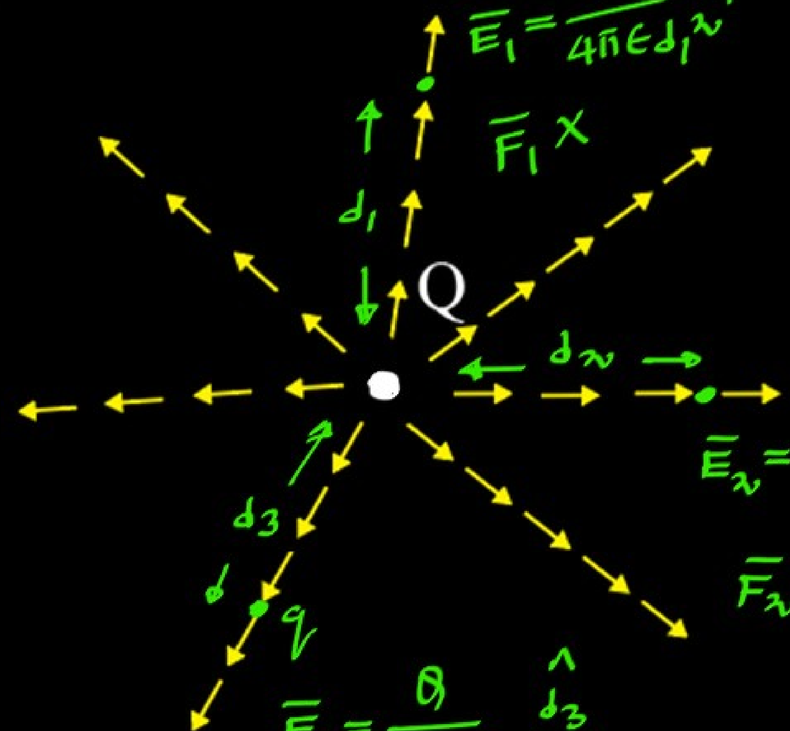
$$\frac{\bar{F}}{q} = \frac{Q}{4\pi \in R^2}\hat{r}$$

Q: Field Producing Charge

q: Test Charge

$$\boxed{\bar{E} = \frac{Q}{4\pi\epsilon R^2}\hat{r}}$$

$$\bar{F} = q\bar{E}$$



$$\bar{E}_1 = \frac{Q}{4\pi\epsilon d_1^2}\hat{d}_1$$

$\bar{F}_1$ X

$$\bar{E}_2 = \frac{Q}{4\pi\epsilon d_2^2}\hat{d}_2$$

$\bar{F}_2$ X

$$\bar{E}_3 = \frac{Q}{4\pi\epsilon d_3^2}\hat{d}_3$$

$$\bar{F}_3 = q\bar{E}_3$$

Q. A positive charge of QC is located at point A(0, 0, 3) and negative charge of magnitude QC is located at point B(0, 0, -3). The electric filed intensity at point C(4,0,0) is in the

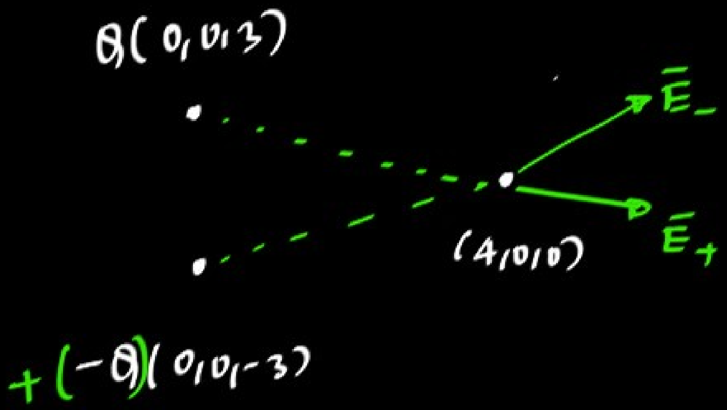
(a) Negative x - direction

(b) Negative z - direction

(c) POSITIVE x - direction

(d) positive z - direction

$$\bar{E} = \frac{Q}{4\pi\epsilon R^2}\hat{r} = \frac{Q}{4\pi\epsilon}\left[\frac{\bar{R}}{R^3}\right]$$



$$\bar{E}_T = \frac{Q}{4\pi\epsilon}\left[\frac{4\hat{x}-3\hat{z}}{(\sqrt{16+9})^3}\right] + \frac{(-Q)}{4\pi\epsilon}\left[\frac{4\hat{x}+3\hat{z}}{(\sqrt{16+9})^3}\right]$$

$$\bar{E}_T = \frac{Q}{4\pi\epsilon}\left[\frac{4\hat{x}-3\hat{z}-4\hat{x}-3\hat{z}}{125}\right]$$

$$\bar{E}_T = \frac{6Q}{4\pi\epsilon \times 125}(-\hat{z})$$

Q. 4 charges lies along the x-axis at (n, 0, 0) with n = 1, 2, 3, 4. If the charges are n!Q, the total field strength at the origin is maximum contributed by ___ and minimum by ___ charge.

(a) 1^{st}, 4^{th}

(b) 4^{th}, 1^{st}

(c) 2^{nd}, 4^{th}

(d) 4^{th}, 2^{nd}

$1! = 1$

$2! = 2 \times 1 = 2$

$3! = 3 \times 2 \times 1 = 6$

$4! = 4 \times 3 \times 2 \times 1 = 24$

Sol:

$$\bar{E} = \frac{Q}{4\pi\epsilon R^2}\hat{r} \propto \frac{Q}{R^2}$$

$\underline{\underline{1^{st}}}$: $\frac{Q}{1^2} = Q$

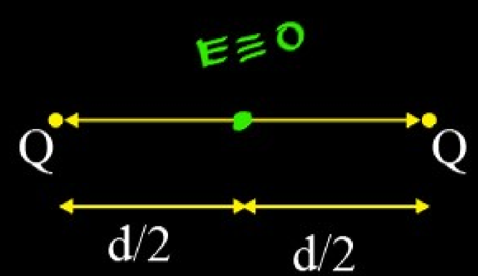
$\underline{\underline{2^{nd}}}$: $\frac{2Q}{2^2} = \frac{Q}{2}$ (min)

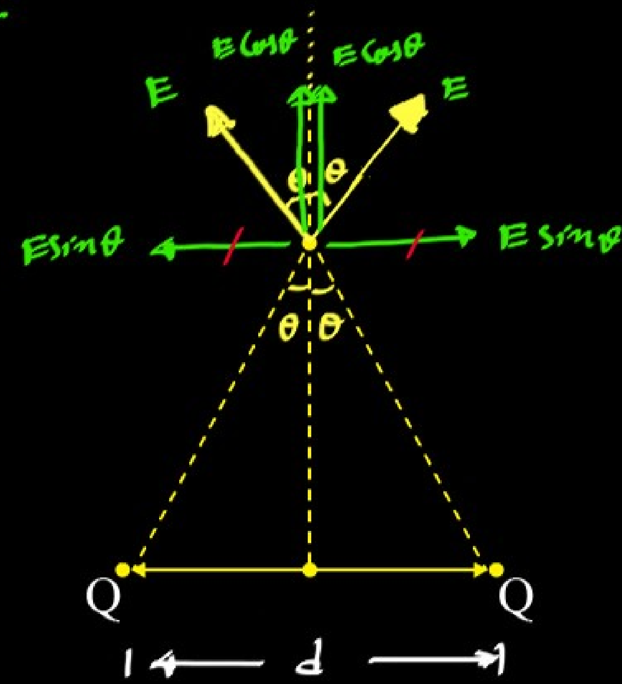
$\underline{\underline{3^{rd}}}$: $\frac{6Q}{3^2} = \frac{2Q}{3}$

$\underline{\underline{4^{th}}}$: $\frac{24Q}{4^2} = \frac{3Q}{2}$ (max)

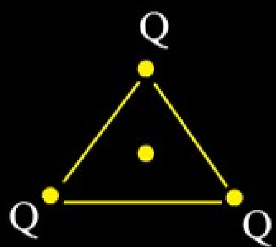
Symmetrical Charge Distribution

Example 1

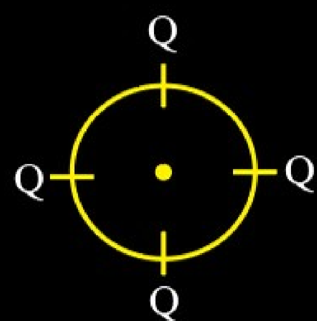




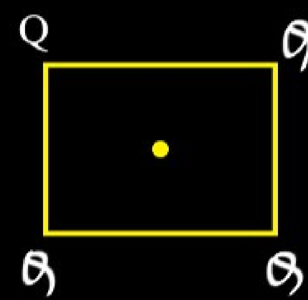
Example 2



Example 3



Example 4

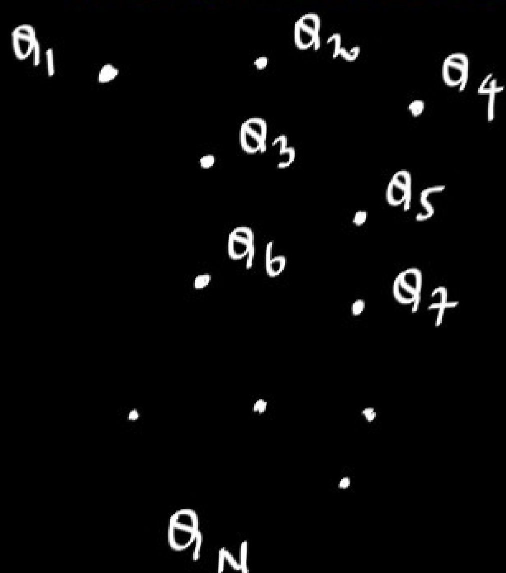


- For symmetrical charge distribution the net electric field (Electric force) is zero at its geometric centre.

- At point away from the centre of the charge distribution, the resultant electric field exists in vertical direction
  (Cancellation of horizontal components takes place due to symmetry)

Types of Charge Distributions and Electric Field

1 Point / Discrete Charge (QC)

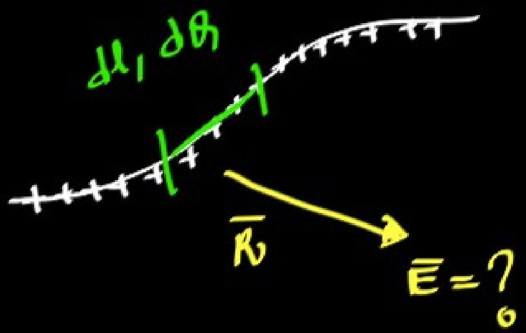


$$Q_T = Q_1 + Q_2 + Q_3 + \cdots + Q_N.$$

$$\boxed{\bar{E}_T = \sum_{i=1}^{N} \frac{Q_i}{4\pi\epsilon R_i^2}\hat{r}_i}$$

II Line Charge (ρ_l c/m)

Total charge : Q c



$$\rho_l = \frac{dQ}{dl} \ c/m$$

$$dQ = \rho_l \, dl$$

$$\boxed{Q = \int \rho_l \, dl}$$

$Q \rightarrow C$, $\rho_l \rightarrow \frac{C}{m}$, $dl \rightarrow m$: $\frac{C}{m} \times m$

$$d\bar{E} = \frac{dQ}{4\pi\epsilon R^2}\hat{r}$$

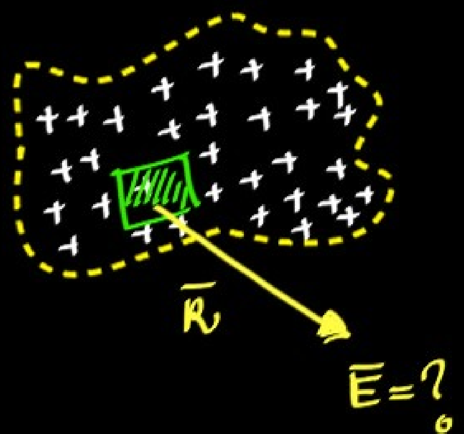
$$d\bar{E} = \frac{\rho_l \, dl}{4\pi\epsilon R^2}\hat{r}$$

$$\boxed{\bar{E} = \int \frac{\rho_l \, dl}{4\pi\epsilon R^2}\hat{r}}$$

III surface Charge (ρ_s c/m^2)

Total charge : QC

(dA, dQ)



$$\rho_s = \frac{dQ}{dA} \quad \frac{c}{m^2}$$

$$dQ = \rho_s\, dA$$

$$\boxed{Q = \iint \rho_s\, dA}$$

$C \qquad \frac{C}{m^2} \times m^2$

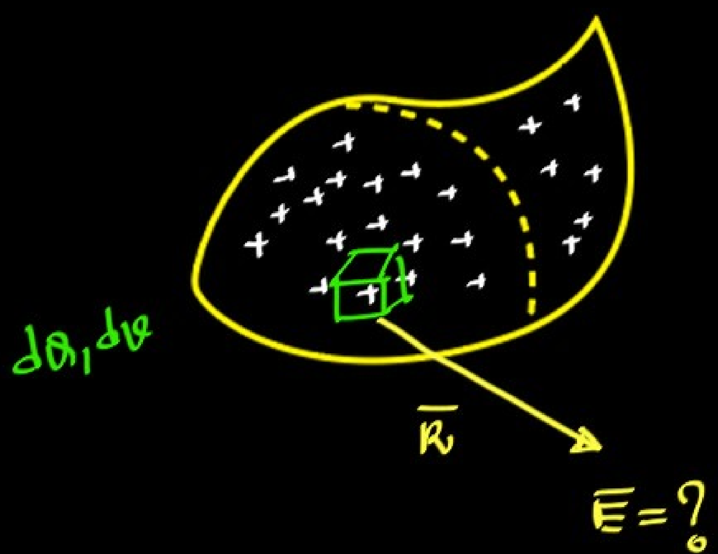
$$d\bar{E} = \frac{dQ}{4\pi\epsilon R^2}\hat{r}$$

$$d\bar{E} = \frac{\rho_s\, dA}{4\pi\epsilon R^2}\hat{r}$$

$$\boxed{\bar{E} = \iint \frac{\rho_s\, dA}{4\pi\epsilon R^2}\hat{r}}$$

(IV) **Volume Charge** $\left(\rho_v \; c/m^3\right)$

Total charge : QC



$$\rho_v = \frac{dQ}{dv} \; \frac{c}{m^3}$$

$$dQ = \rho_v \, dv$$

$$\boxed{Q = \iiint \rho_v \, dv}$$

$Q \to C$, $\rho_v \to \frac{c}{m^3}$, $dv \to m^3$: $\frac{c}{m^3} \times m^3$

$$d\bar{E} = \frac{dQ}{4\pi\epsilon R^2} \hat{r}$$

$$d\bar{E} = \frac{\rho_v \, dv}{4\pi\epsilon R^2} \hat{r}$$

$$\boxed{\bar{E} = \iiint \frac{\rho_v \, dv}{4\pi\epsilon R^2} \hat{r}}$$

Q. A sphere of radius 0.2 m centered at the origin contains electrical charge of density $\left[\frac{2}{\sqrt{x^2+y^2}}\right]\frac{c}{m^3}$. Find the total charge contained within the sphere.

Soln:

Limits

$$\left\{\begin{array}{l} 0 \le r \le 0.2\,m \\ 0 \le \theta \le \pi \\ 0 \le \phi \le 2\pi \end{array}\right\}$$

$$Q = \iiint \rho_v \, dv$$

$$\rho_v = \frac{2}{\sqrt{x^2+y^2}}$$

$$x = (r\sin\theta)\cos\phi$$

$$y = (r\sin\theta)\sin\phi$$

$$x^2+y^2 = r^2\sin^2\theta$$

$$\sqrt{x^2+y^2} = r\sin\theta$$

$$\rho_v = \frac{2}{r\sin\theta}$$

$$dv = r^2\sin\theta\, dr\, d\theta\, d\phi$$

$$Q = \iiint \frac{2}{r\sin\theta} r^2 \sin\theta\, dr\, d\theta\, d\phi$$

$$Q = 2\iiint r\, dr\, d\theta\, d\phi$$

$$Q = 2\int r\,dr \int d\theta \int d\phi$$

$$= 2\left[\frac{r^2}{2}\right]_0^{0.2} \left[\theta\right]_0^{\pi} \left[\phi\right]_0^{2\pi}$$

$$= (0.2)^2 \times \pi \times 2\pi$$

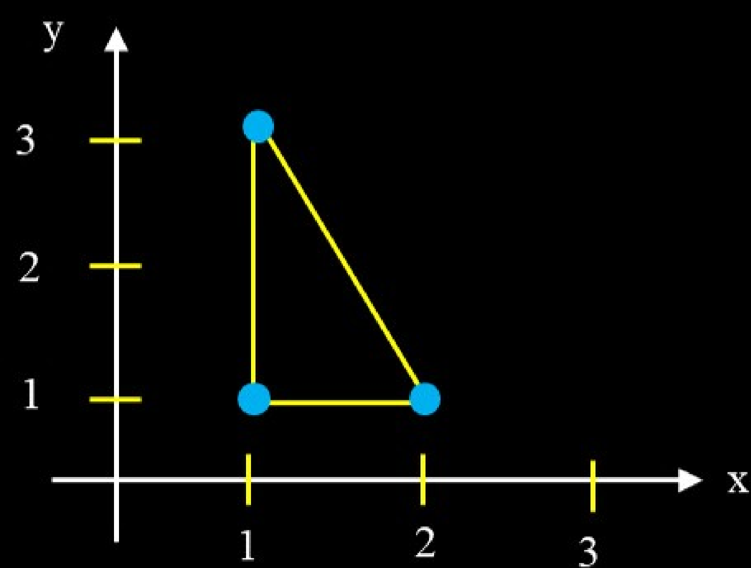
$$Q = 0.789\ C$$

Q. A Non-uniform surface charge density of $\frac{1}{x^2+y^2+4}\frac{nc}{m^2}$ which is defined for $\rho \leq 2.5$ m, $z = 5$ m. Find total charge.

H.w

Q. The total charge on the triangle of figure, given surface charge density $\rho_s = 6xy\ c/m^2$ is ___________

H.w



Q. Given $\rho_l = 2x + 3y - 4z$ c/m the charge on the line segment extending from (2, 1, 5) to (4, 3, 6) is ________

(a) 10 C

(b) –10 C

(c) 30

(d) –30 C

H.W

Q. If the volume charge density is $\rho = 100\ e^{-z}\ (x^2 + y^2)^{-1/4}\ c/m^3$, the total charge contained in the cylindrical shown in the figure is ______

H.10