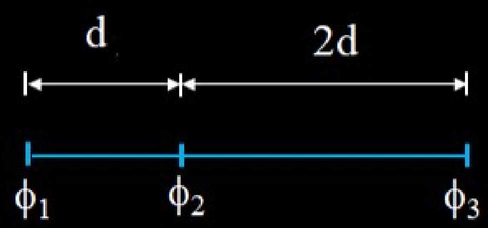
13 - 08 - 22

Q. The three values of a one-dimensional potential function ϕ shown in the given figure and satisfying laplace equation are related as



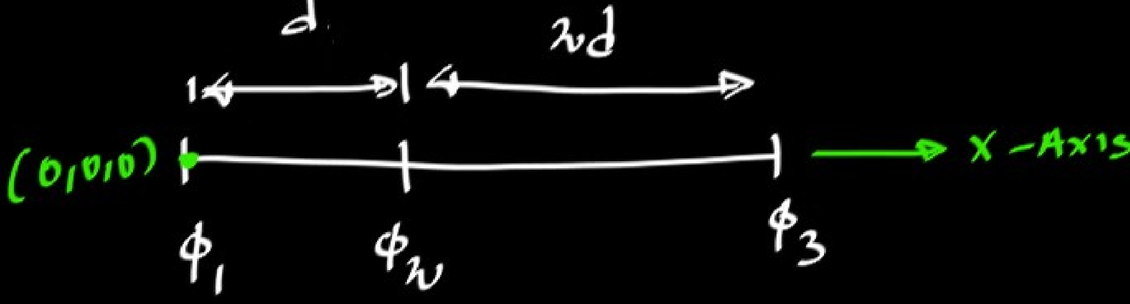
(a) $\phi_2 = \dfrac{2\phi_3 + \phi_1}{3}$

(b) $\phi_2 = \dfrac{2\phi_1 + \phi_3}{3}$

(c) $\phi_2 = \dfrac{2\phi_1 - \phi_3}{3}$

(d) $\phi_2 = \dfrac{\phi_1 + 3\phi_3}{2}$

Soln:



$$\nabla^2 V = 0$$

$$\frac{\partial^2 V}{\partial x^2} + \underbrace{\frac{\partial^2}{\partial y^2} V}_{0} + \underbrace{\frac{\partial^2}{\partial z^2} V}_{0} = 0$$

$$\frac{\partial^2 V}{\partial x^2} = 0$$

(I.O.B.S)

$$\frac{\partial V}{\partial x} = c_1$$

(I.O.B.S)

$$V = c_1 x + c_2$$

AT $x = 0$, $V(x=0) = \phi_1$

$$\phi_1 = c_1(0) + c_2$$

$$c_2 = \phi_1$$

AT $x = d$, $V(x=d) = \phi_2$

$$\phi_2 = c_1(d) + \phi_1$$

$$c_1 = \left[\frac{\phi_2 - \phi_1}{d}\right]$$

$$V = c_1 x + c_2$$

$$V = \left[\frac{\phi_2 - \phi_1}{d}\right] x + \phi_1$$

AT $x = 3d$, $V(x = 3d) = \phi_3$

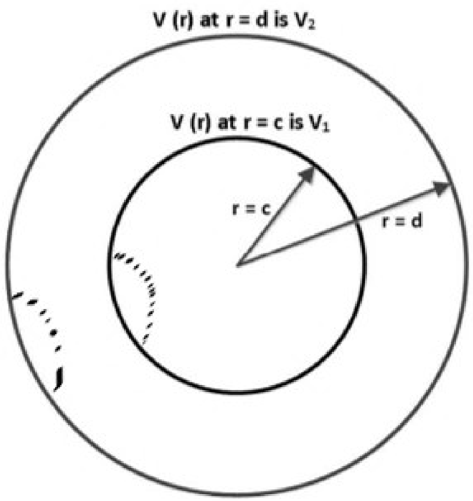
$$\phi_3 = \left[\frac{\phi_2 - \phi_1}{d}\right](3d) + \phi_1$$

$$\phi_3 = 3\phi_2 - 3\phi_1 + \phi_1$$

$$\phi_3 = 3\phi_2 - 2\phi_1$$

$$\phi_2 = \frac{\phi_3 + 2\phi_1}{3}$$

Q.53 As shown in the figure below, two concentric conducting spherical shells, centered at $r = 0$ and having radii $r = c$ and $r = d$ are maintained at potentials such that the potential $V(r)$ at $r = c$ is V_1 and $V(r)$ at $r = d$ is V_2. Assume that $V(r)$ depends only on r, where r is the radial distance. The expression for $V(r)$ in the region between $r = c$ and $r = d$ is



(A) $V(r) = \frac{c\,d\,(V_2 - V_1)}{(d-c)\,r} - \frac{V_1 c + V_2 d - 2 V_1 d}{d-c}$

(B) $V(r) = \frac{c\,d\,(V_1 - V_2)}{(d-c)\,r} + \frac{V_2 d - V_1 c}{d-c}$

(C) $V(r) = \frac{c\,d\,(V_1 - V_2)}{(d-c)\,r} - \frac{V_1 c - V_2 c}{d-c}$

(D) $V(r) = \frac{c\,d\,(V_2 - V_1)}{(d-c)\,r} - \frac{V_2 c - V_1 c}{d-c}$

Soln:

$$\nabla^2 V = -\frac{\cancel{\rho_v}}{\epsilon} = 0$$

$$\nabla^2 V = 0$$

$$\frac{1}{r^2\sin\theta}\left[\frac{\partial}{\partial r}\left(r^2\sin\theta\frac{\partial V}{\partial r}\right) + \underbrace{\frac{\partial}{\partial\theta}\left(\frac{r\sin\theta}{r}\frac{\partial V}{\partial\theta}\right)}_{0} + \underbrace{\frac{\partial}{\partial\phi}\left(\frac{r}{r\sin\theta}\frac{\partial V}{\partial\phi}\right)}_{0}\right] = 0$$

$$\frac{\sin\theta}{r^2\sin\theta}\frac{\partial}{\partial r}\left[r^2\frac{\partial V}{\partial r}\right] = 0$$

$$\Rightarrow \frac{\partial}{\partial r}\left[r^2\frac{\partial V}{\partial r}\right] = 0$$

(I.O.B.S)

$$r^2 \frac{\partial V}{\partial r} = c_1$$

$$\frac{\partial V}{\partial r} = \frac{c_1}{r^2}$$

(I.O.B.S)

$$V = -\frac{c_1}{r} + c_2$$

AT $r = c$, $V(r=c) = V_1$

$$V_1 = -\frac{c_1}{c} + c_2 \longrightarrow (1)$$

AT $r = d$, $V(r=d) = V_2$

$$V_2 = -\frac{c_1}{d} + c_2 \longrightarrow (2)$$

$$V_2 - V_1 = -\frac{c_1}{d} + \frac{c_1}{c}$$

$$V_2 - V_1 = c_1\left[\frac{1}{c} - \frac{1}{d}\right]$$

$$V_2 - V_1 = c_1\left[\frac{d-c}{dc}\right]$$

$$c_1 = \frac{dc[V_2 - V_1]}{[d-c]}$$

$$c_2 = V_1 + \frac{c_1}{c}$$

$$c_2 = V_1 + \frac{1}{c}\left\{\frac{dc(V_2 - V_1)}{d-c}\right\}$$

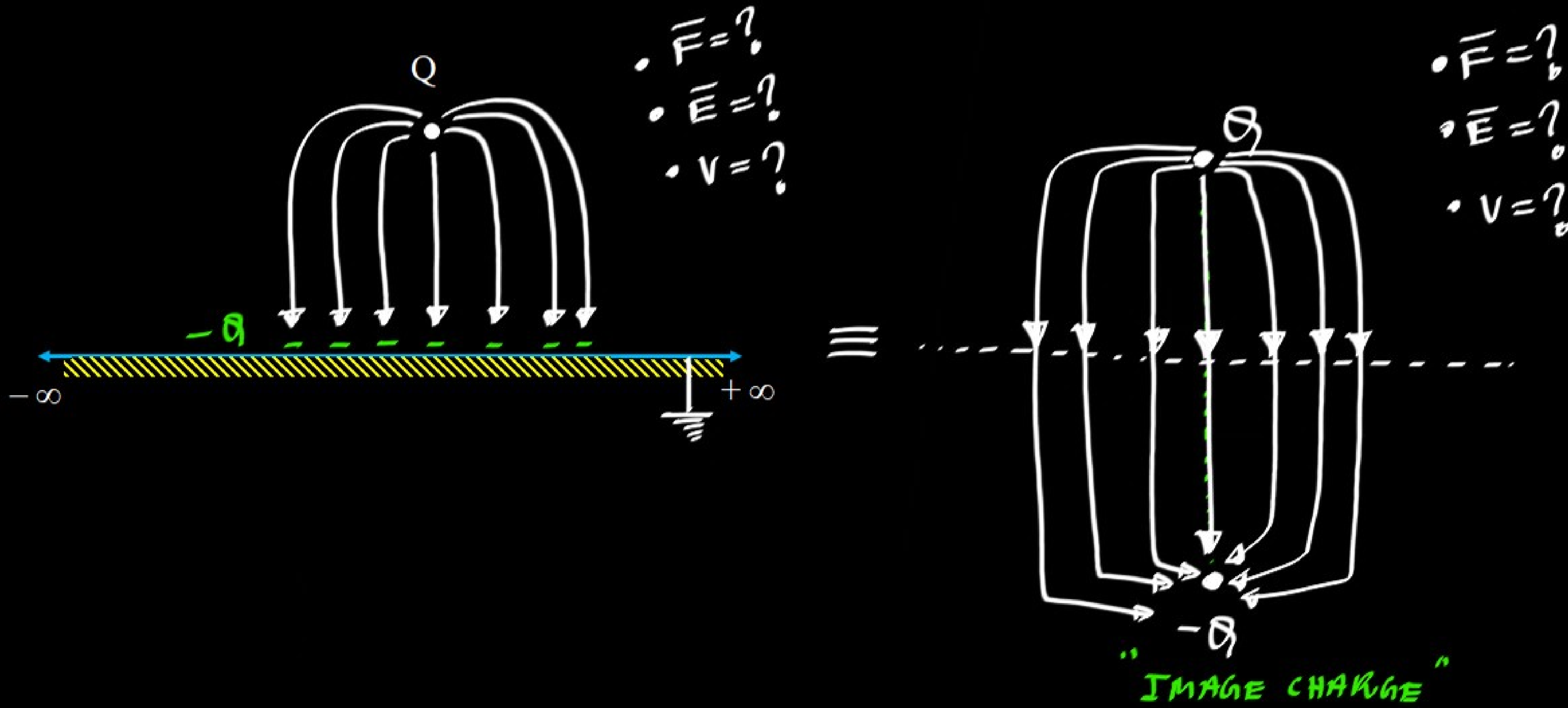
$$c_2 = V_1 + \frac{d(V_2 - V_1)}{(d-c)}$$

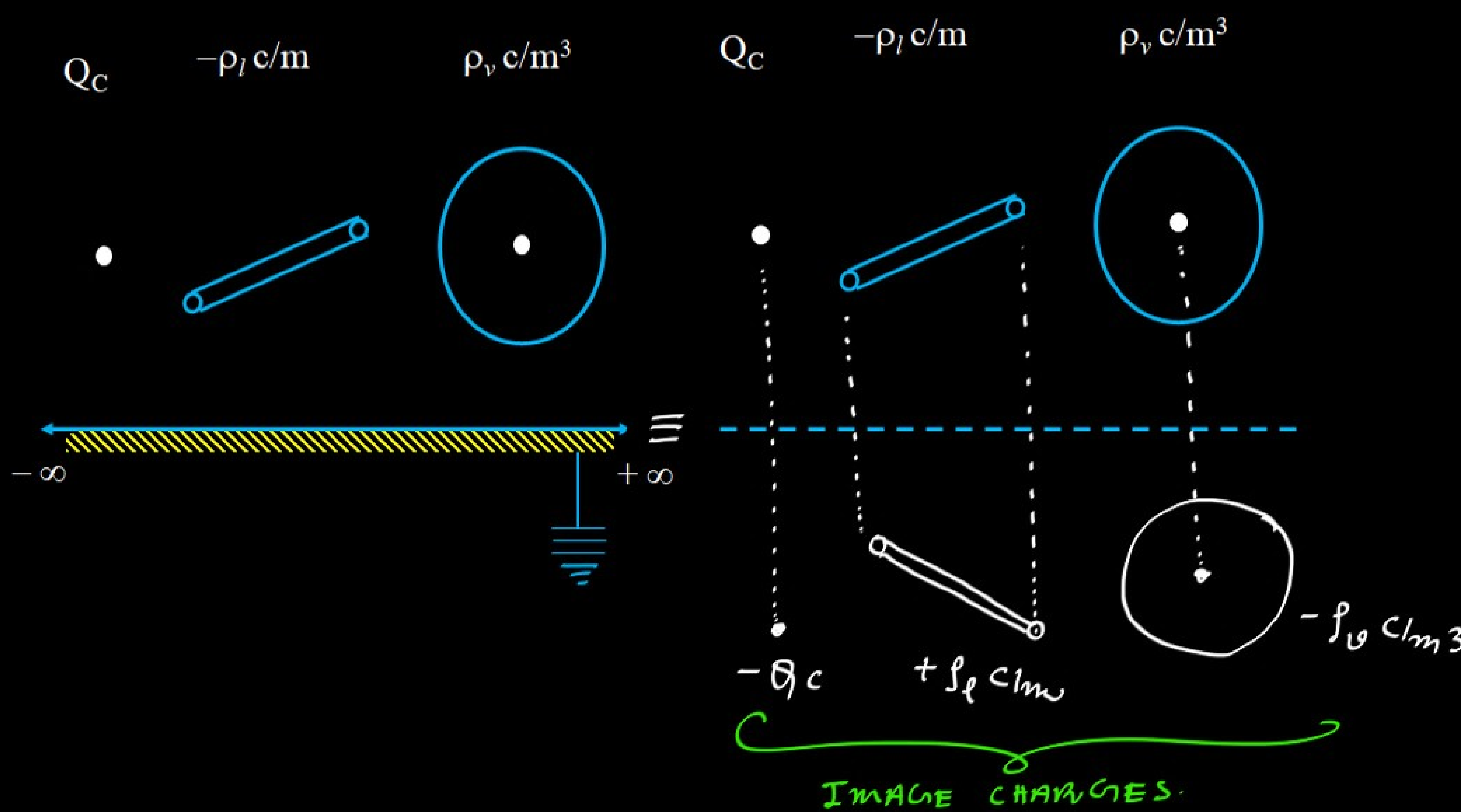
$$V = -\frac{1}{r}\left[\frac{dc(V_2 - V_1)}{d-c}\right] + V_1 + \frac{d(V_2 - V_1)}{(d-c)}$$

$$V = -\frac{1}{r}\left[\frac{dc(V_2 - V_1)}{d-c}\right] + \frac{V_1 d - V_1 c + dV_2 - dV_1}{(d-c)}$$

$$V = \frac{dc[V_1 - V_2]}{(d-c)r} + \frac{dV_2 - V_1 c}{(d-c)}$$

Method of Images





Q_C
$-\rho_l$ c/m
ρ_v c/m^3
$-\infty$
$+\infty$
$\equiv$
Q_C
$-\rho_l$ c/m
ρ_v c/m^3
$-Q$ c
$+\rho_l$ c/m
$-\rho_v$ c/m3
IMAGE CHARGES.

Charge distribution combined with infinite grounded sheet can be tackled by method of images.

1. Place image charge beneath the sheet

2. Remove the sheet and apply regular electrostatic techniques to find $(\bar{E}, \bar{F}, V)$

EC

Q. The force on a point charge + q kept at a distance d from the surface of an infinite grounded metal plate in a medium of permittivity $\in$ is **[GATE -14]**

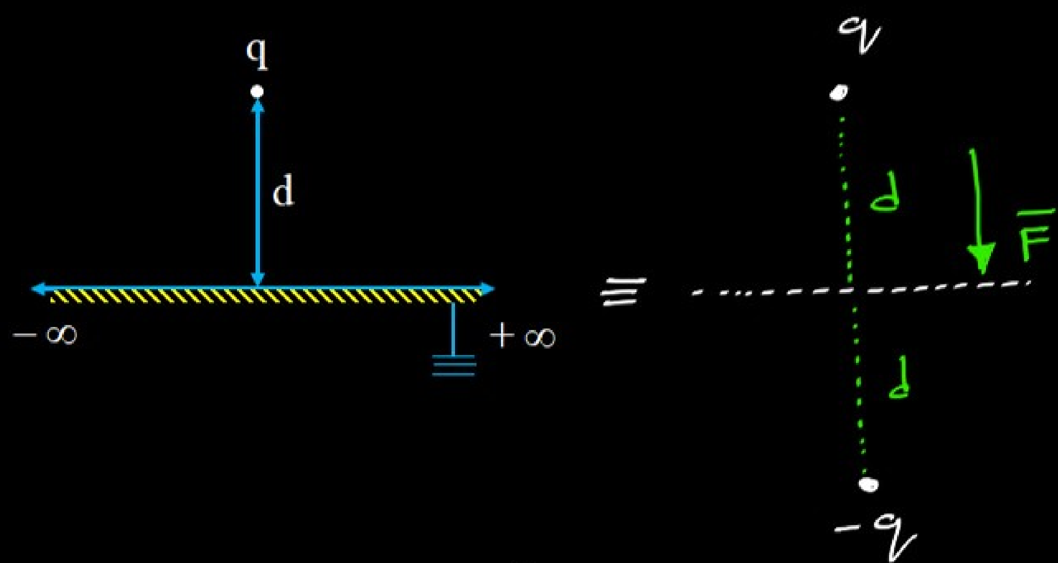
(a) 0

(b) $\dfrac{q^2}{16\pi\in d^2}$ Away from the plate

(c) $\dfrac{q^2}{16\pi\in d^2}$ Towards the plate

(d) $\dfrac{q^2}{4\pi\in d^2}$ Towards the plate

Soln:

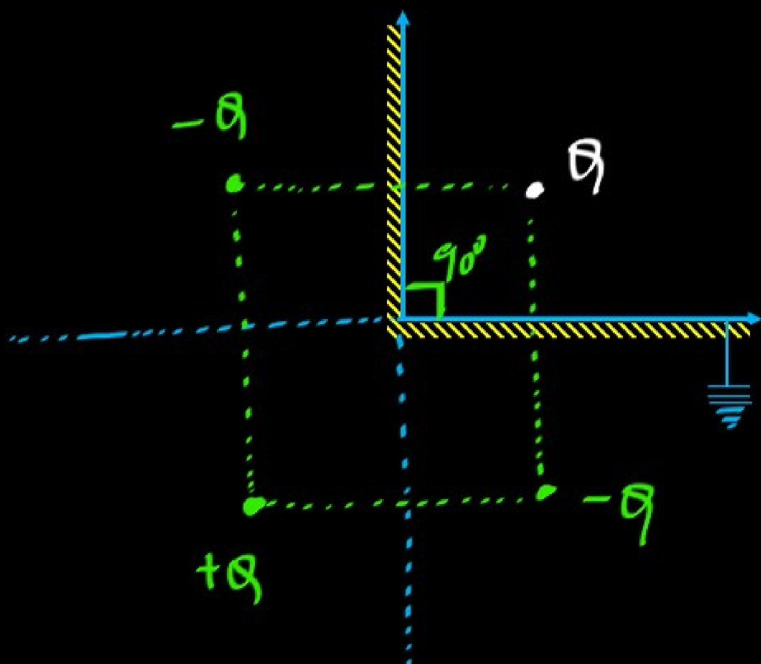


$$|\bar{F}| = \left| \frac{q(-q)}{4\pi\epsilon(2d)^2} \right|$$

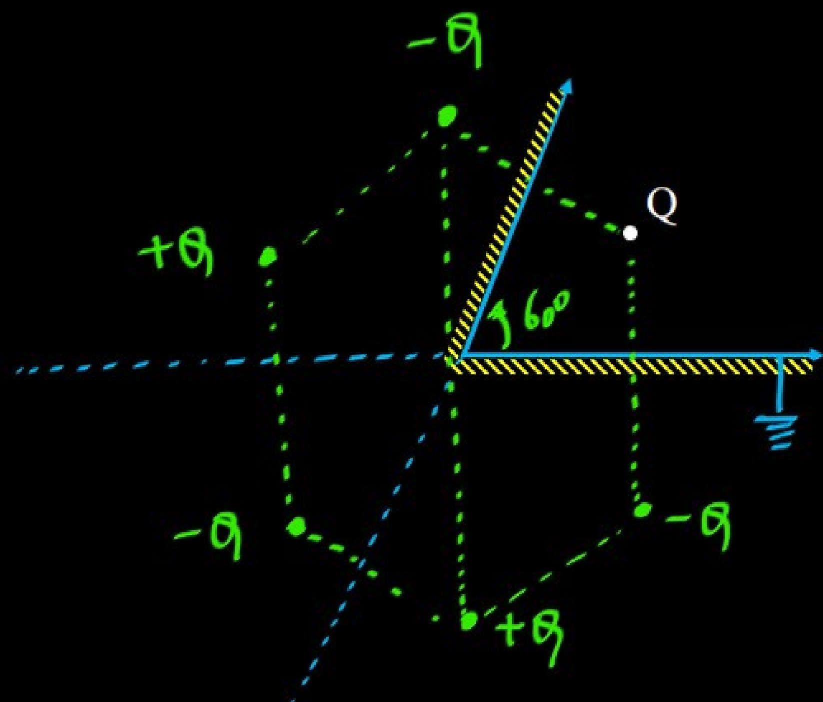
$$|\bar{F}| = \frac{q^2}{16\pi\epsilon d^2}$$: TOWARDS THE PLATE.

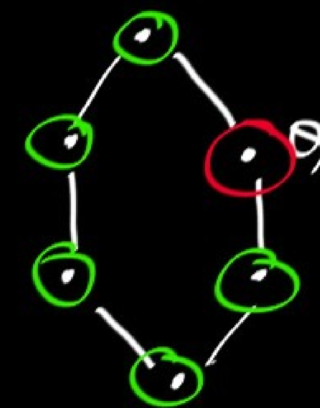
Ex: (1)

$$\left[\frac{360^\circ}{90^\circ} - 1\right] = \underline{\underline{3}}$$



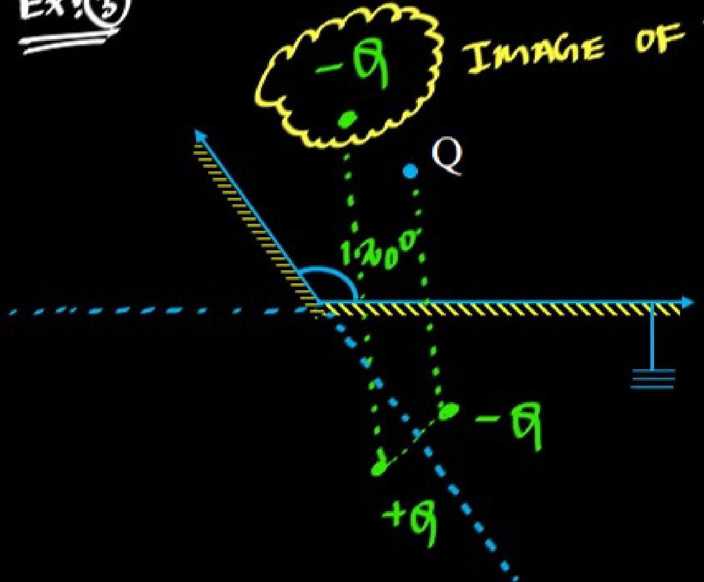
Ex: 2 $\left[\frac{360^\circ}{60^\circ} - 1\right] = 5$





Ex: 3

IMAGE OF IMAGE ....



$$\left[\frac{360^\circ}{120^\circ} - 1\right] = 2$$

* If the image of image charge exists in region of interest then, method of image charge technique can't be used

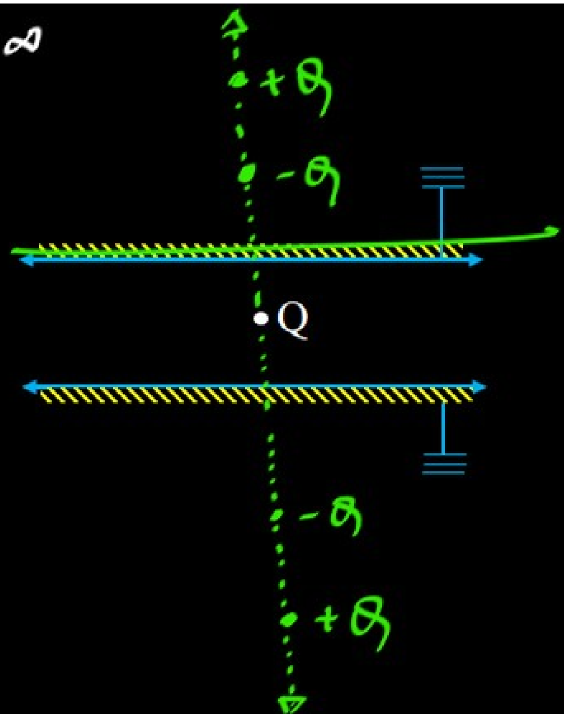
* $$[\text{Number of image Changes}] = \left[\frac{360^\circ}{\phi} - 1\right]$$

SEMI

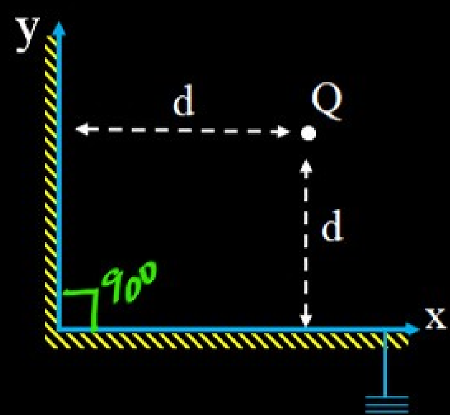
Where ϕ is angle of inclination between two infinite grounded sheets

Ex: $\phi = 30^\circ$, $\# IC = \left[\frac{360^\circ}{30^\circ} - 1\right] = \underline{\underline{11}}$

Ex: $\left[\frac{360^\circ}{0^\circ} - 1\right] \to \infty$



Q Two semi-infinite conducting sheet are placed at right angle to each other as shown in figure. A point charge of Q is placed at a distance of d from both sheets. The net force on the charge is $\frac{Q^2}{4\pi\epsilon_0}\frac{\overline{K}}{d^2}$ Where $\overline{K}$ is given by



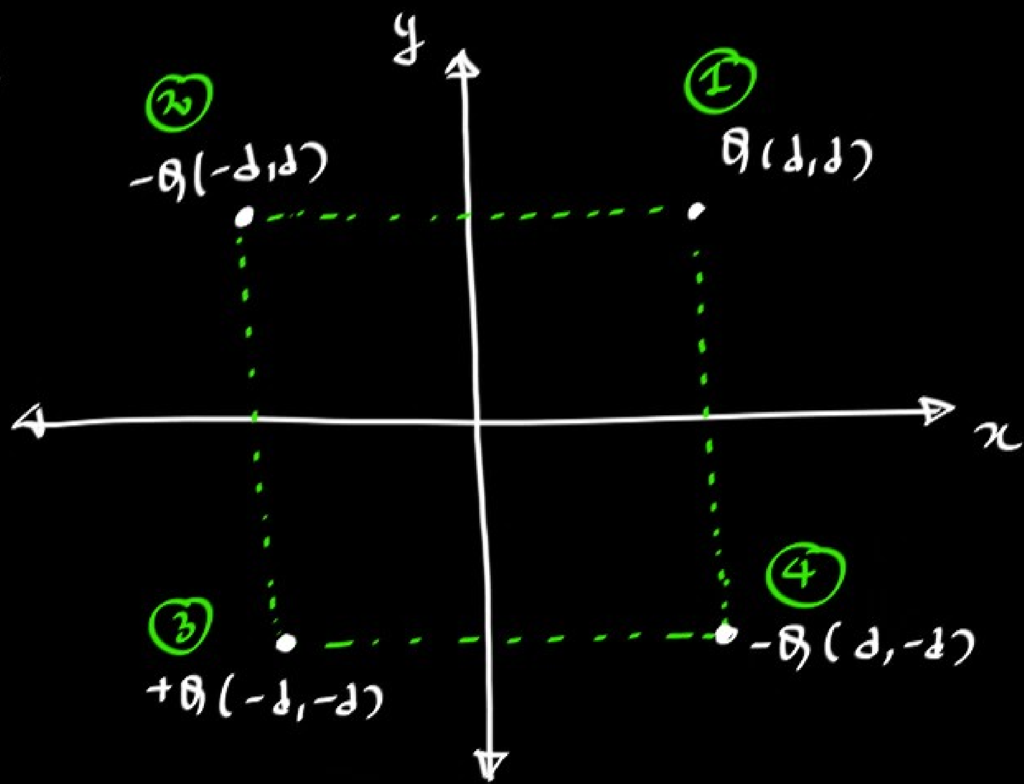
(a) 0

(b) $-\frac{1}{4}\hat{x}-\frac{1}{4}\hat{y}$

(c) $-\frac{1}{8}\hat{x}-\frac{1}{8}\hat{y}$

(d) $\frac{1-2\sqrt{2}}{8\sqrt{2}}\hat{x}+\frac{1-2\sqrt{2}}{8\sqrt{2}}\hat{y}$

Soln:



$$\bar{F}_T = \bar{F}_{21} + \bar{F}_{31} + \bar{F}_{41}$$

$$\bar{F} = \frac{Q_1 Q_2}{4\pi\epsilon}\left[\frac{\bar{R}}{R^3}\right]$$

$$\bar{F}_T = \frac{-Q^2}{4\pi\epsilon}\left[\frac{2d\hat{x}}{(2d)^3}\right] + \frac{Q^2}{4\pi\epsilon}\left[\frac{2d\hat{x}+2d\hat{y}}{(\sqrt{4d^2+4d^2})^3}\right] + \frac{(-Q^2)}{4\pi\epsilon}\left[\frac{2d\hat{y}}{(2d)^3}\right]$$

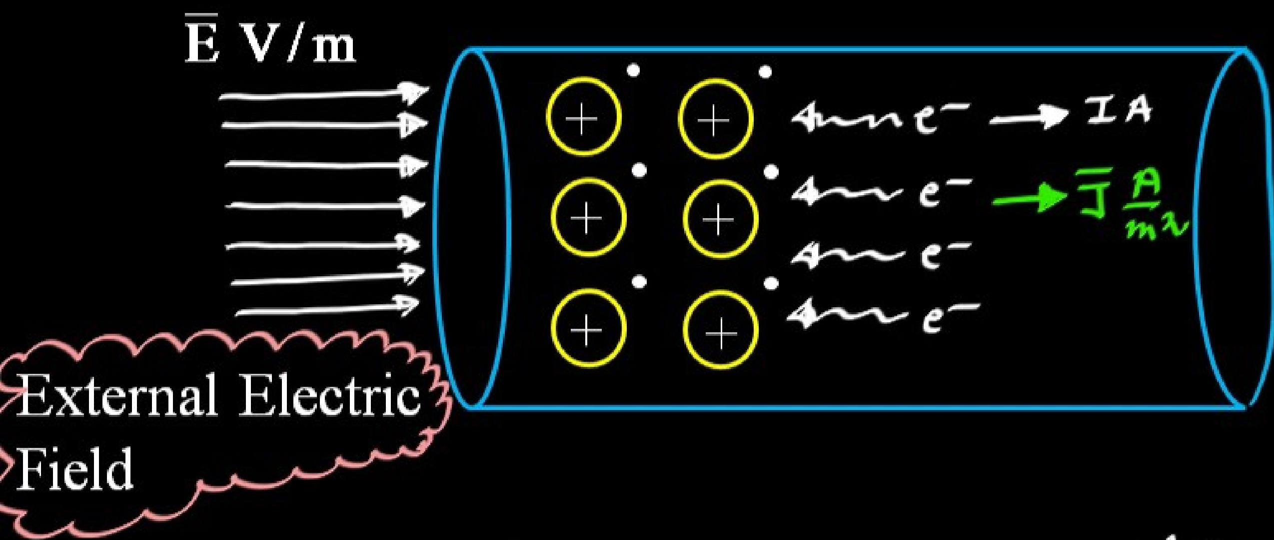
$$\bar{F}_T = \frac{-Q^2}{4\pi\epsilon}\left[\frac{\hat{x}}{4d^2}\right] + \frac{Q^2}{4\pi\epsilon}\left[\frac{2d\hat{x}+2d\hat{y}}{16\sqrt{2}d^3}\right] + \frac{(-Q^2)}{4\pi\epsilon}\left[\frac{\hat{y}}{4d^2}\right]$$

$$\bar{F}_T = \frac{-Q^2}{4\pi\epsilon}\left[\frac{\hat{x}}{4d^2}\right] + \frac{Q^2}{4\pi\epsilon}\left[\frac{\hat{x}+\hat{y}}{8\sqrt{2}d^2}\right] + \frac{(-Q^2)}{4\pi\epsilon}\left[\frac{\hat{y}}{4d^2}\right]$$

$$\bar{F}_T = \frac{Q^2}{4\pi\epsilon d^2}\left\{\hat{x}\left[\frac{1}{8\sqrt{2}} - \frac{1}{4}\right] + \hat{y}\left[\frac{1}{8\sqrt{2}} - \frac{1}{4}\right]\right\}$$

$$\bar{F}_T = \frac{Q^2}{4\pi\epsilon d^2}\left[\hat{x}\left(\frac{1-2\sqrt{2}}{8\sqrt{2}}\right) + \hat{y}\left(\frac{1-2\sqrt{2}}{8\sqrt{2}}\right)\right] = \frac{Q^2}{4\pi\epsilon d^2}\bar{K}$$

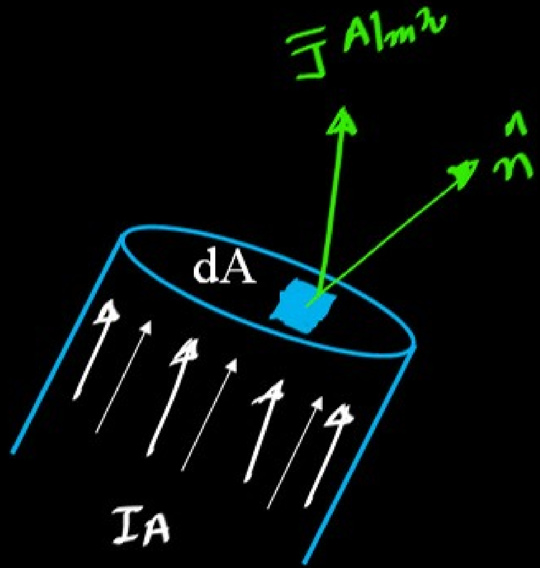
Current (IA) / Current Density ($\bar{J}$ A/m^2)



* $\bar{F} = q\bar{E}$

* $I = \frac{dq}{dt}$ A

* $q = n_e(-e)$



$\bar{J}$: Current density (A/m^2)

$$\bar{J} = \frac{dI}{d\bar{A}} \ A/m^2$$

$$dI = \bar{J} \cdot d\bar{A}$$

$$\boxed{I = \iint \bar{J} \cdot d\bar{A}}$$

$A \qquad \frac{A}{m^2} \times m^2$

OHM'S LAW:

$$\bar{J} \propto \bar{E}$$

$$\boxed{\bar{J} = \sigma \bar{E}}$$

→ OHM'S LAW IN FIELD FORM. (OR) POINT FORM

σ : CONDUCTIVITY

(σ ℧/m)

In Circuit Theory

$$E = \frac{V}{\ell}, J = \frac{I}{A}, \sigma = \frac{1}{\rho}$$

$$\bar{J} = \sigma \bar{E} \Rightarrow J = \sigma E \Rightarrow \frac{I}{A} = \frac{1}{\rho}\frac{V}{\ell}$$

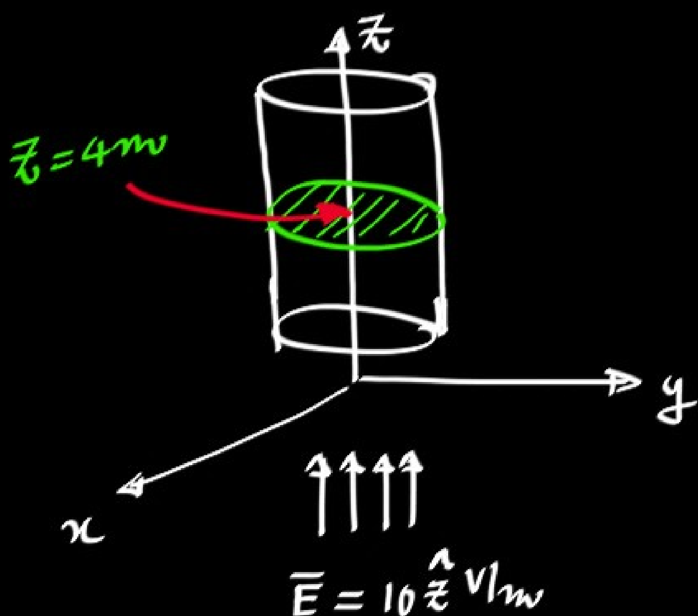
$$V = \left(\frac{\rho \ell}{A}\right) I$$

$$\boxed{V = RI}$$

Q. An imperfect conducting ROD of circular cross-section has radius of 1 m. The conductivity of the ROD varies as $\sigma = 10^5 (\rho - 10^{-2})$ ℧/m is subjected to external axial electric field $\bar{E} = 10\hat{z}$ V/m

Find current crossing through the surface z = 4 m in $\hat{z}$ direction.

Soln:



$$\bar{J} = \sigma\bar{E} = 10^5(\rho - 10^{-2}) \times 10\,\hat{z}$$

$$\bar{J} = 10^6(\rho - 10^{-2})\,\hat{z} \quad A/m^2$$

$$I = \iint \bar{J}\cdot\bar{dA}$$

$$(\rho, \phi, z):\ \bar{dA} = \rho\, d\rho\, d\phi\, \hat{z} + \underbrace{\rho\, d\phi\, dz}_{0}\, \hat{\rho} + \underbrace{dz\, d\rho}_{0}\, \hat{\phi}$$

1, ρ, 1

$$z = 4m,\quad dz = 0,\quad \bar{dA} = \rho\, d\rho\, d\phi\, \hat{z}$$

$$I = \iint 10^6(\rho - 10^{-2})\hat{z} \cdot \rho \, d\rho \, d\phi \, \hat{z}$$

$$= 10^6 \iint (\rho^2 - 10^{-2}\rho) \, d\rho \, d\phi$$

$$= 10^6 \int (\rho^2 - 10^{-2}\rho) \, d\rho \int d\phi$$

$$= 10^6 \left[\frac{\rho^3}{3} - \frac{10^{-2}\rho^2}{2} \right]_0^1 \left[\phi \right]_0^{2\pi}$$

$$= 10^6 \left[\frac{1}{3} - 10^{-2} \times \frac{1}{2} \right] \left[2\pi \right]$$

$$= 10^6 \left[\frac{1}{3} - \frac{1}{200} \right] \left[2\pi \right]$$

$$I = \underline{2.06 * 10^6 \, A.}$$

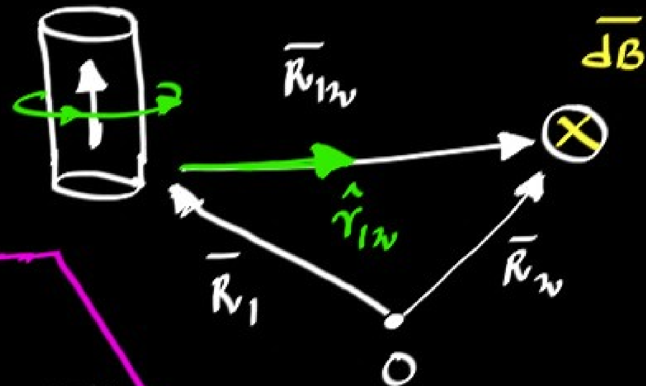
Topic - 3

MAGNETOSTATICS

Magnetostatics defines the magnetic field and develops the different techniques of finding magnetic field due to various current distributions

BIOT-SAVARTS LAW

"$I\,\bar{dl}$"



$$\bar{R}_{12} = \bar{R}_2 - \bar{R}_1$$

$$\hat{r}_{12} = \frac{\bar{R}_{12}}{R_{12}} = \frac{\bar{R}_{12}}{|\bar{R}_{12}|}$$

CURRENT ELEMENTS

$I\,\bar{dl}$ A-m

$$|\bar{dB}| \propto \frac{I\,dl}{R_{12}^2}$$

$$|\bar{dB}| = \frac{\mu}{4\pi}\,\frac{I\,dl}{R_{12}^2}$$

$$\boxed{\bar{dB} = \frac{\mu}{4\pi}\,\frac{I\,\bar{dl} \times \hat{r}_{12}}{R_{12}^2}}$$

where

$$\hat{r}_{12} = \frac{\bar{R}_{12}}{R_{12}}$$

$$\bar{dB} = \frac{\mu}{4\pi}\,\frac{I\,\bar{dl} \times \bar{R}_{12}}{R_{12}^3}$$

$\mu = \mu_o \, \mu_r$

μ = Permeability

μ_r = Relative Permeability

$\mu_r = 1$ (AIR)

$\mu_o = 4\pi \times 10^{-7}$ H/m

Absolute permeability

NOTE:

$$\bar{B} = \mu \bar{H}$$

MAGNETIC FLUX DENSITY (Wb/m^2)

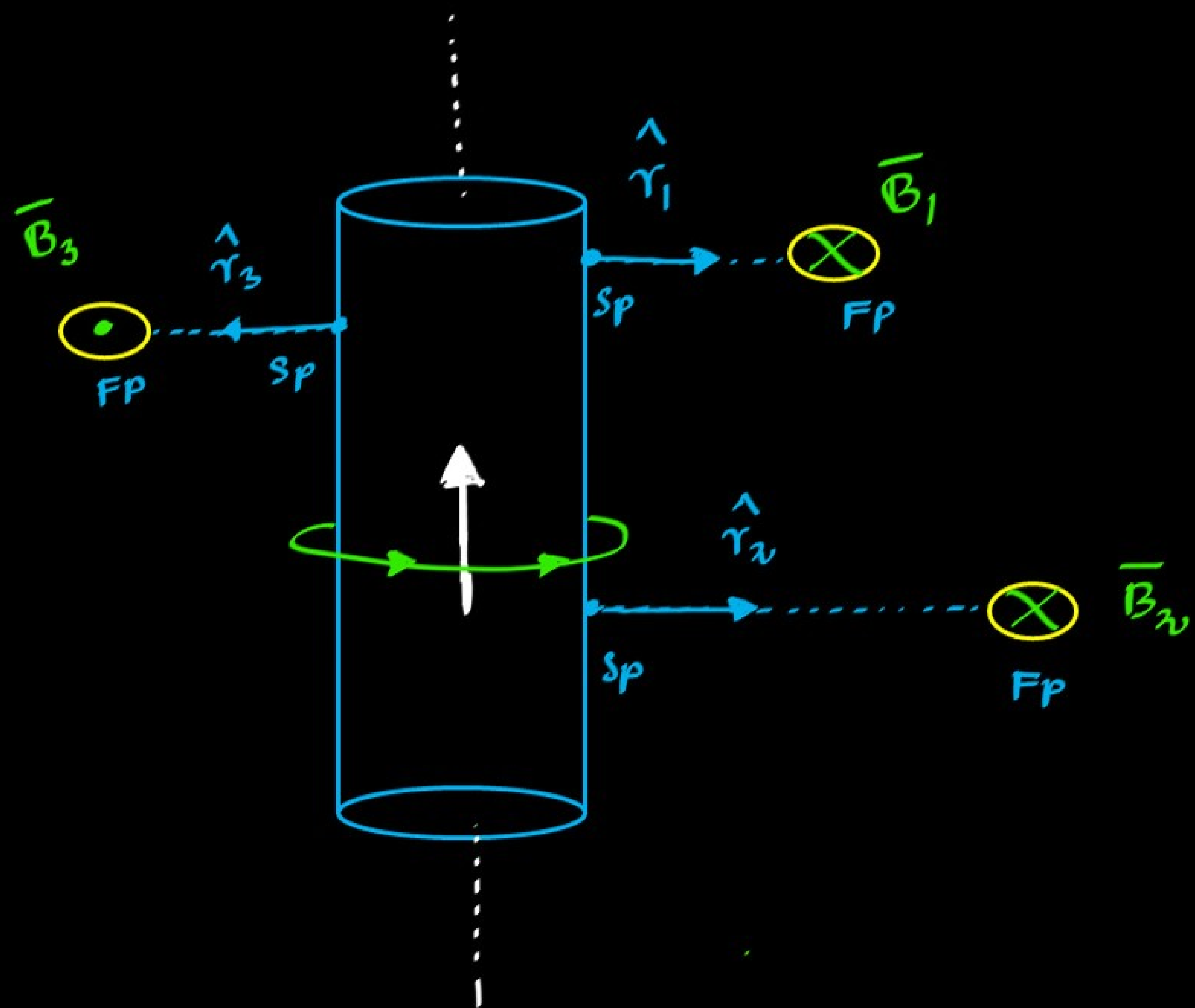
MAGNETIC FIELD INTENSITY (A/m).

$$d\bar{B} = \frac{\mu}{4\pi} \frac{I\,d\bar{l} \times \hat{r}_{12}}{R_{12}^2} \quad \frac{Wb}{m^2}$$

$$d\bar{H} = \frac{1}{4\pi} \frac{I\,d\bar{l} \times \hat{r}_{12}}{R_{12}^2} \quad \frac{A}{m}$$

$$d\bar{H} = \frac{1}{4\pi} \frac{I\,d\bar{l} \times \bar{R}_{12}}{R_{12}^3} \quad \frac{A}{m}$$

Ex:



[Magnetic field direction] = [Current direction] x [Unit Vector]

" $\hat{M}$ " " $\hat{C}$ " " $\hat{u}$ "

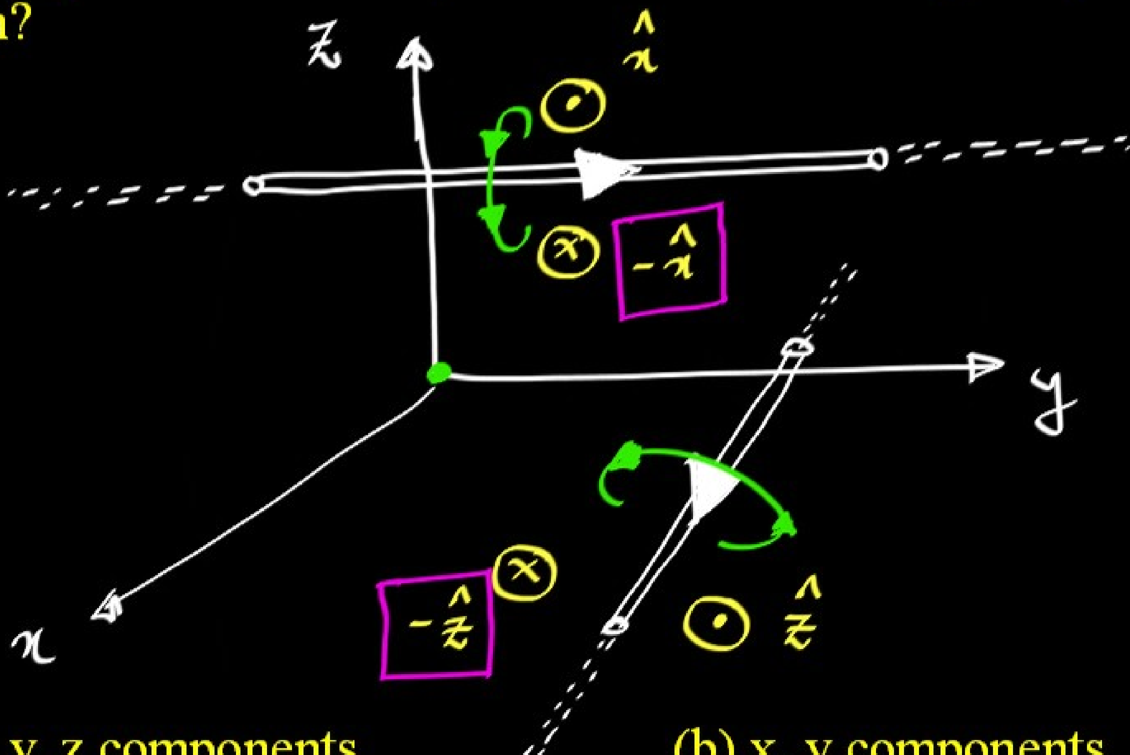
****

$$\boxed{\hat{M} = \hat{C} \times \hat{u}}$$

* Where unit vector is shortest distance vector from source point to field point.

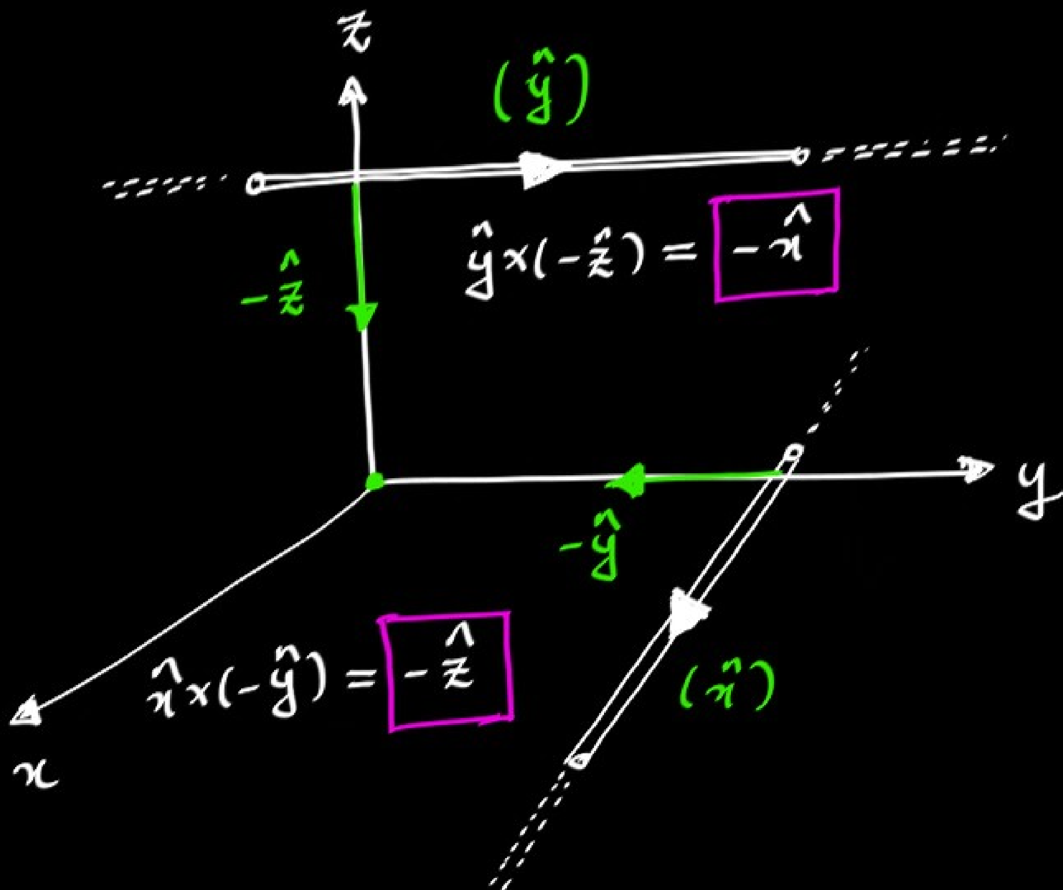
$$\text{i.e } \boxed{Sp : \xrightarrow{\hat{u}} : FP}$$

Q. Two infinitely long wires carrying current are as shown in fig below. One wire is in the yz-plane and parallel to the y-axis. The other wire is in the xy-plane and parallel to the x-axis which components of the resulting magnetic field are non-zero at the origin? (GATE-2009)



(a) x, y, z components
(b) x, y components
(c) y, z components
(d) x, z components

$$\boxed{\hat{n} = \hat{c} \times \hat{u}}$$

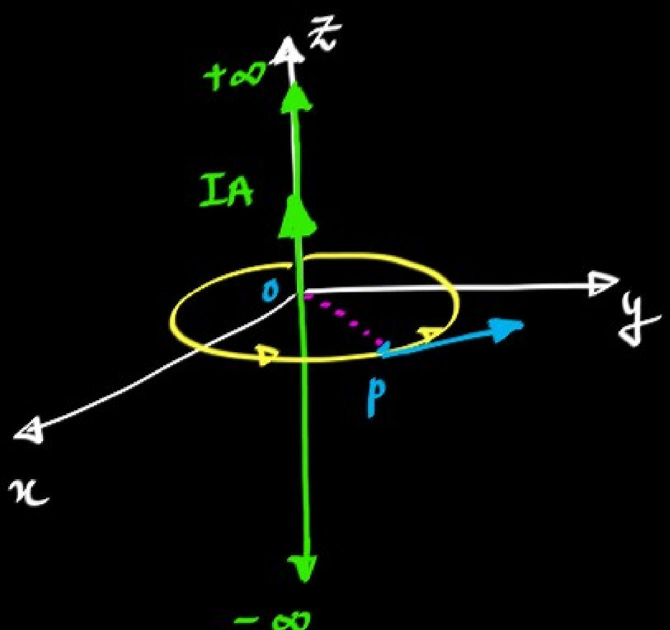


Q. An infinity long straight conductor located along z-axis carries a current I in the +ve z-direction. The magnetic field at any point P in xy-plane is in which direction?

(IES-2008)

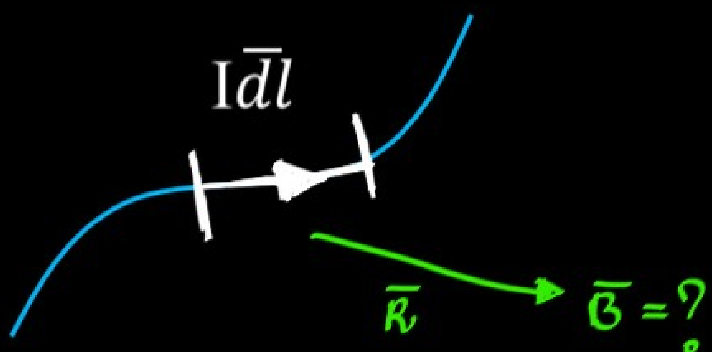
(a) In the +ve z-direction
(b) In the −ve z-direction
(c) In the direction perpendicular to the radial line OP (in xy-plane)
(d) Along the radial line OP

Soln:



Different Current Distributions and Magnetic field Expression

I. Line Current (IA):



$$d\bar{B} = \frac{\mu}{4\pi} \frac{I\bar{dl} \times \hat{r}}{R^2}$$

$$\bar{B} = \int \frac{\mu}{4\pi} \frac{I\bar{dl} \times \hat{r}}{R^2}$$

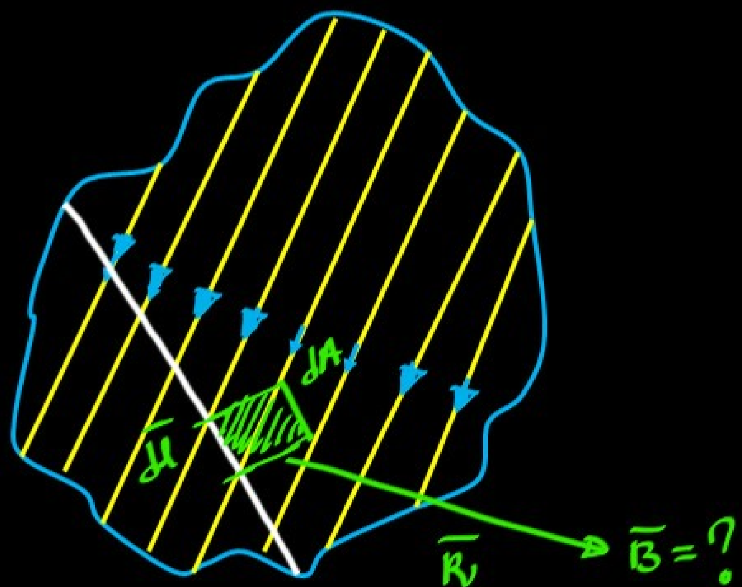
$$\boxed{\bar{B} = \frac{\mu}{4\pi} \int \frac{I\bar{dl} \times \hat{r}}{R^2}}$$

CURRENT ELEMENT

"$I\bar{dl}$" A-m

II. Sheet current ($\bar{K}$ A/m)

IA: Total Current



$$\bar{K} = \frac{dI}{d\bar{l}} \quad A/m$$

$$dI = \bar{K}\cdot d\bar{l}$$

$$\boxed{I = \int \bar{K}\cdot d\bar{l}}$$

$A \qquad \frac{A}{m}\cdot m$

CURRENT ELEMENT

$$I d\bar{l} = \bar{K}\, dA \quad (A\text{-}m).$$

$A\text{-}m \qquad \frac{A}{m}\text{-}m^2$

$$d\bar{B} = \frac{\mu}{4\pi}\frac{I d\bar{l}\times\hat{r}}{R^2}$$

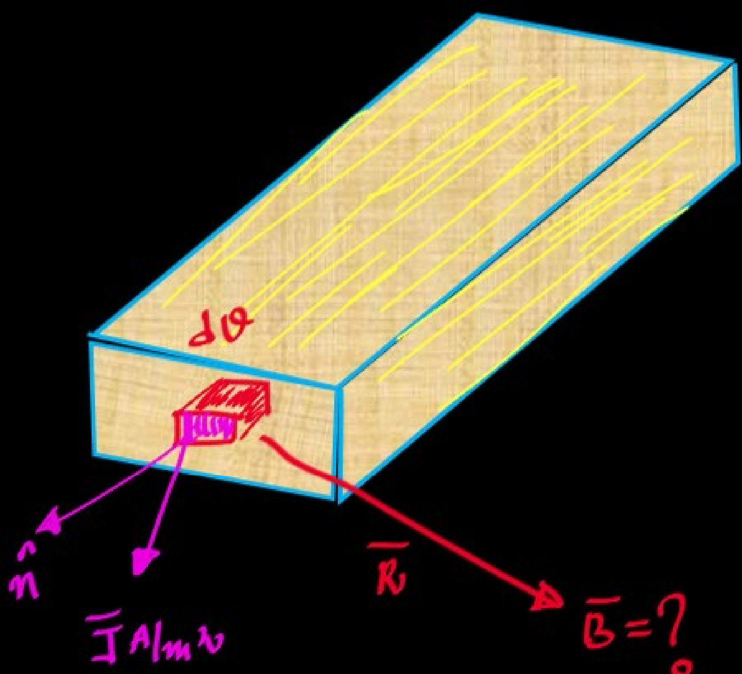
$$d\bar{B} = \frac{\mu}{4\pi}\frac{\bar{K}\, dA\times\hat{r}}{R^2}$$

$$\bar{B} = \iint \frac{\mu}{4\pi}\frac{\bar{K}\, dA\times\hat{r}}{R^2}$$

$$\boxed{\bar{B} = \frac{\mu}{4\pi}\iint \frac{\bar{K}\, dA\times\hat{r}}{R^2}}$$

III. Volume current ($\bar{J}$ A/m^2)

IA: Total current



$$\bar{J} = \frac{dI}{d\bar{A}} \quad \frac{A}{m^2}$$

$$dI = \bar{J}\cdot d\bar{A}$$

$$\boxed{I = \iint \bar{J}\cdot d\bar{A}}$$

CURRENT ELEMENT

$$I\,d\bar{l} = \bar{J}\,dv \quad \text{(A-m)}$$

A-m $\qquad \frac{A}{m^2}$ - m^3

$$d\bar{B} = \frac{\mu}{4\pi}\,\frac{I\,d\bar{l}\times\hat{r}}{R^2}$$

$$d\bar{B} = \frac{\mu}{4\pi}\,\frac{\bar{J}\,dv\times\hat{r}}{R^2}$$

$$\bar{B} = \iiint \frac{\mu}{4\pi}\,\frac{\bar{J}\,dv\times\hat{r}}{R^2}$$

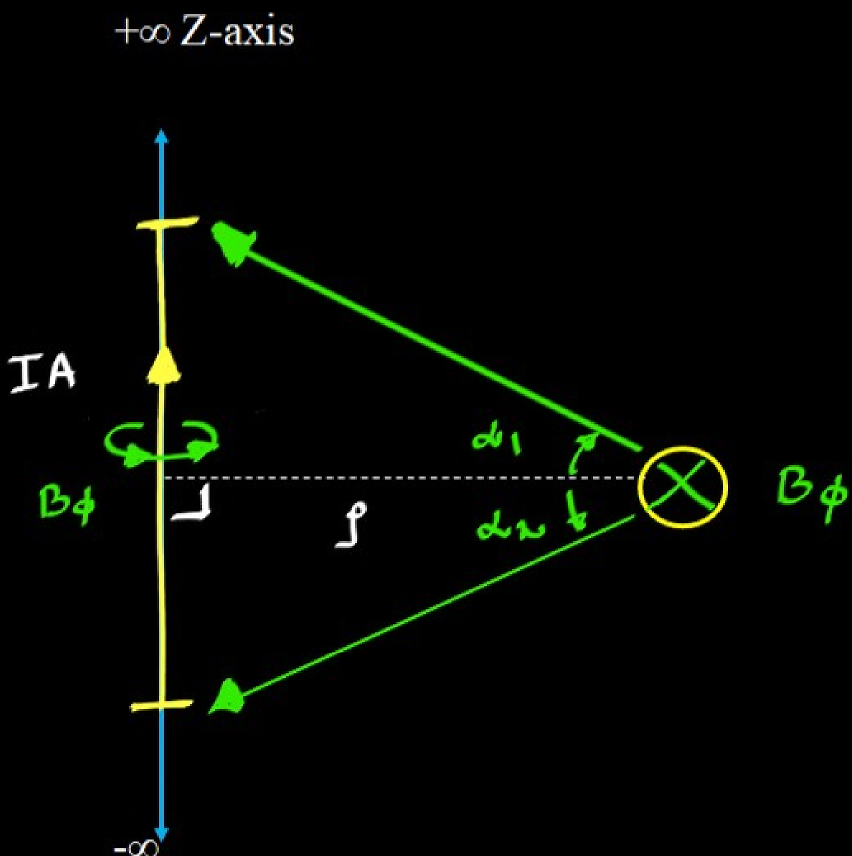
$$\boxed{\bar{B} = \frac{\mu}{4\pi}\iiint \frac{\bar{J}\,dv\times\hat{r}}{R^2}}$$

Note: **Current Element (A-m)**

$$I\overline{d\ell} = \overline{K}dA = \overline{J}dv \quad (A-m)$$

$A - m \qquad \frac{A}{m} - m^2 \qquad \frac{A}{m^2} - m^3$

Magnetic field due to finite length conductor (IA)



$$\bar{B} = \frac{\mu}{4\pi}\int \frac{I\,\bar{dl}\times\hat{r}}{R^2} \quad wb/m^2$$

$$\boxed{\bar{B} = \frac{\mu I}{4\pi\rho}\left[\sin\alpha_1 + \sin\alpha_2\right]\hat{\phi}} \quad \frac{wb}{m^2}$$

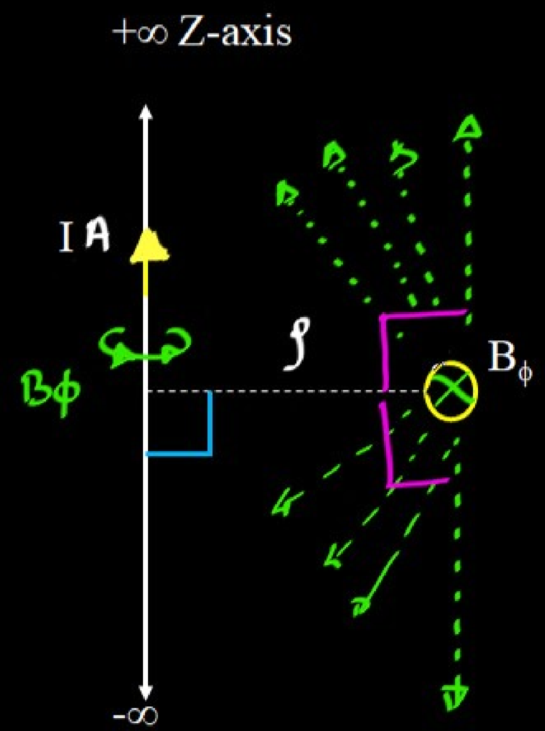
$\perp^{le}$ DISTANCE

$\hat{m} = \hat{c}\times\hat{y}$

$$\bar{B} = \mu\bar{H}$$

$$\boxed{\bar{H} = \frac{I}{4\pi\rho}\left[\sin\alpha_1 + \sin\alpha_2\right]\hat{\phi}} \quad A/m$$

For Infinite Line:



$$\alpha_1 = \alpha_2 = \pi/2$$

$$\bar{B} = \frac{\mu I}{4\pi\rho}[1+1]\,\hat{\phi}$$

$$\boxed{\bar{B} = \frac{\mu I}{2\pi\rho}\,\hat{\phi}} \quad \text{Wb/m}^2$$

⊥ʳ DISTANCE

$$\hat{m} = \hat{c} \times \hat{n}$$

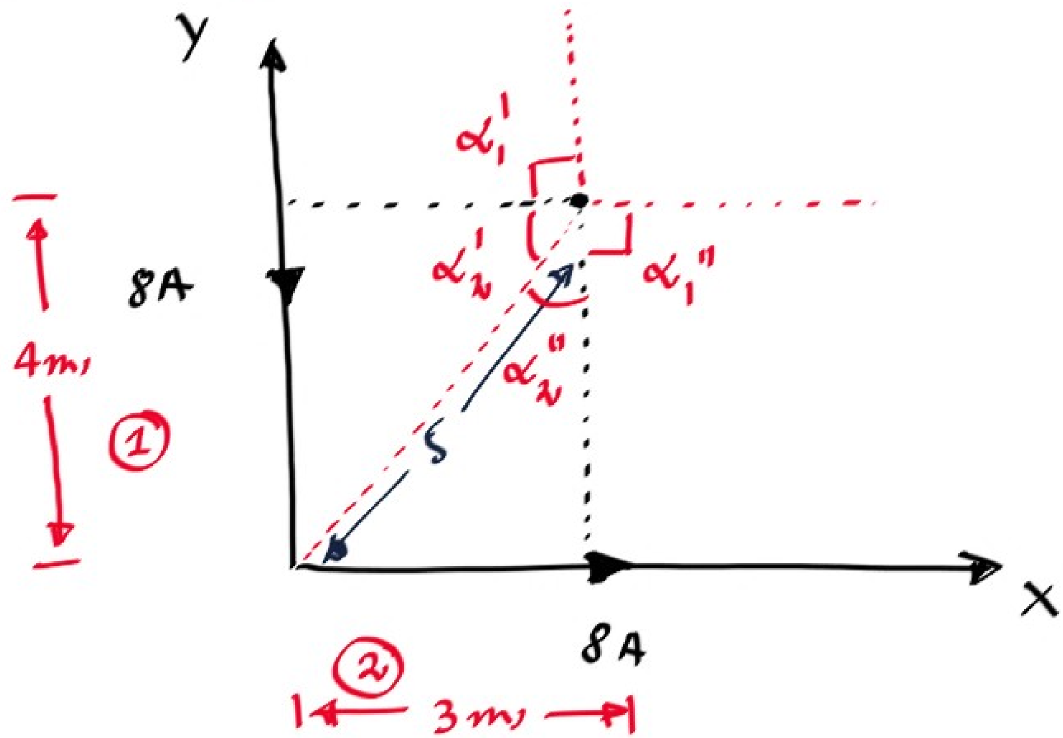
$$\bar{B} = \mu\bar{H}$$

$$\boxed{\bar{H} = \frac{I}{2\pi\rho}\,\hat{\phi}} \quad \text{A/m}$$

Q. An 8A current carrying wire flows along the y +ve axis and also along +x-axis with the current from y to x-axis. Calculate the $\bar{H}$ at (3, 4, 0)

(a) $\frac{2}{\pi}\hat{a}_z$ (b) $\frac{4}{\pi}\hat{a}_z$ (c) $\frac{-2}{\pi}\hat{a}_z$ (d) $\frac{-4}{\pi}\hat{a}_z$

Soln:



$$\bar{H}_1 = \frac{I_1}{4\pi\rho_1}\left[\sin\alpha_1' + \sin\alpha_2'\right](\hat{c}_1 \times \hat{u}_1)$$

$$\bar{H}_1 = \frac{8}{4\pi(3)}\left[1 + \frac{4}{5}\right](-\hat{y} \times \hat{x})$$

$$\bar{H}_1 = \frac{2}{3\pi}\left[\frac{9}{5}\right]\hat{z}$$

$$\bar{H}_1 = \frac{6}{5\pi}\hat{z} \text{ A/m.}$$

$$\bar{H}_2 = \frac{I_2}{4\pi\rho_2}\left[\sin\alpha_1'' + \sin\alpha_2''\right](\hat{c}_2 \times \hat{u}_2)$$

$$\bar{H}_2 = \frac{8}{4\pi(4)}\left[1 + \frac{3}{5}\right](\hat{x} \times \hat{y})$$

$$\bar{H}_2 = \frac{2}{4\pi}\left[\frac{8}{5}\right]\hat{z}$$

$$\bar{H}_2 = \frac{4}{5\pi}\hat{z}$$

$$\bar{H}_T = \bar{H}_1 + \bar{H}_2 = \frac{6}{5\pi}\hat{z} + \frac{4}{5\pi}\hat{z}$$

$$\bar{H}_T = \frac{10}{5\pi}\hat{z}$$

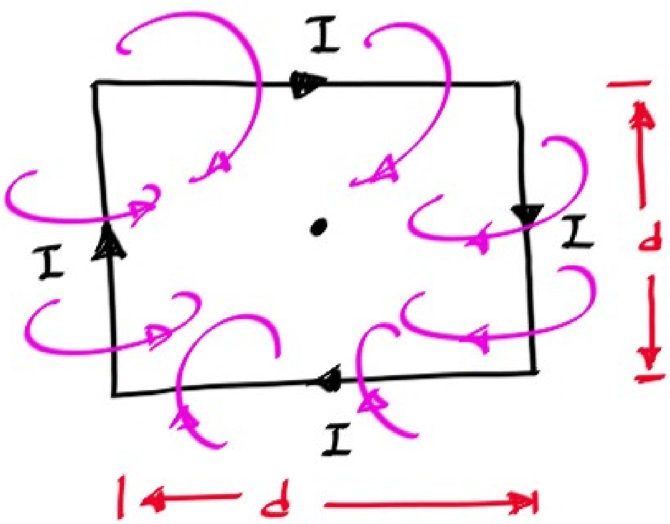
$$\bar{H}_T = \frac{2}{\pi}\hat{z} \text{ A/m.}$$

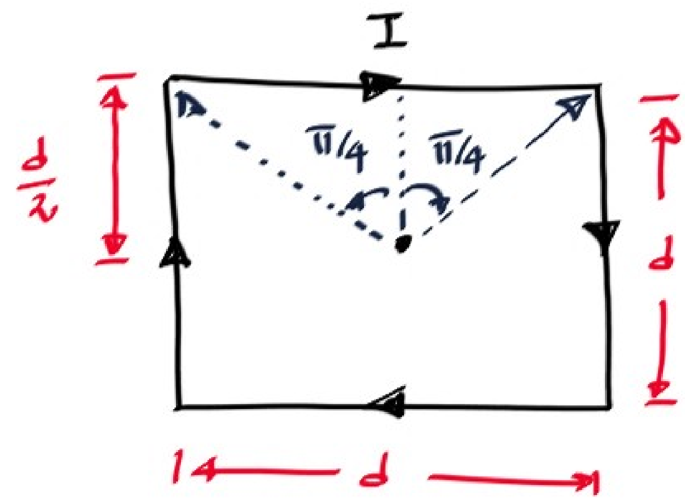
Q. Find the H at the center of square loop of current of I amps flowing clockwise in z = 0 plane with side d.

(a) $\frac{2\sqrt{2}\,I}{\pi d}$ (b) $\frac{\sqrt{2}\,I}{\pi d}$

(c) $\frac{4\sqrt{2}\,I}{\pi d}$ (d) $\frac{2I}{\pi d}$

Soln:





$$H_s = \frac{I}{4\pi\rho}\left[\sin\alpha_1 + \sin\alpha_2\right]$$

$$H_s = \frac{I}{4\pi(d/2)}\left[\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right]$$

$$H_s = \frac{I}{2\pi d}\left[\frac{2}{\sqrt{2}}\right]$$

$$H_s = \frac{I}{\pi d\sqrt{2}}$$

$$H_T = 4\,H_s = \frac{4I}{\pi d\sqrt{2}}$$

$$\boxed{H_T = \frac{2\sqrt{2}\,I}{\pi d}} \quad \frac{A}{m}.$$

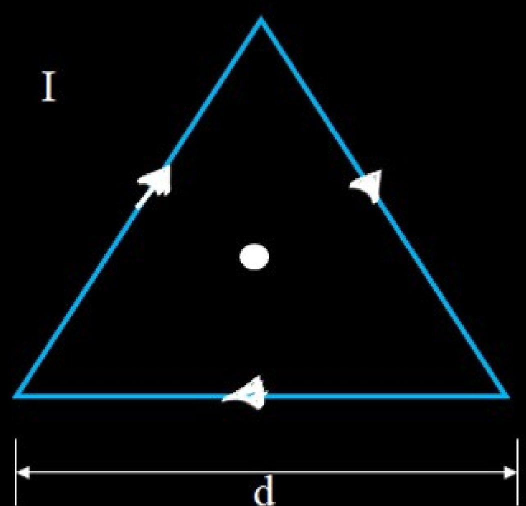
NOTE

* For 'N' equal sided polygon with side 'd' and carrying current IA Then the magnitude of magnetic field at it's geometric center is

****

$$\boxed{H = \frac{NI}{\pi d}\tan\left(\frac{180^0}{N}\right)\sin\left(\frac{180^o}{N}\right)}$$

Ex: 1 $N = 3$

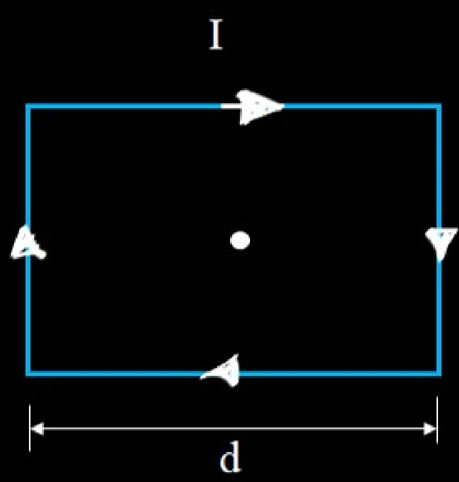


$$H = \frac{3I}{\pi d} \cdot \tan 60^\circ \cdot \sin 60^\circ$$

$$H = \frac{3I}{\pi d} \cdot \sqrt{3} \cdot \frac{\sqrt{3}}{2}$$

$$\boxed{H = \frac{9I}{2\pi d}} \quad A/m.$$

Ex: 2 N = 4

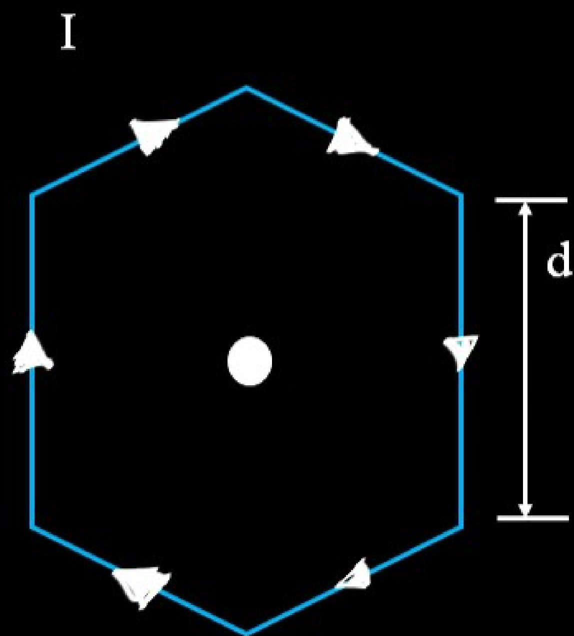


$$H = \frac{4I}{\pi d} \cdot \tan 45^\circ \cdot \sin 45^\circ$$

$$H = \frac{4I}{\pi d} \cdot 1 \cdot \frac{1}{\sqrt{2}}$$

$$H = \frac{2\sqrt{2}\, I}{\pi d}$$

Ex: 3 N = 6



$$H = \frac{6I}{\pi d} \cdot \tan 30^\circ \cdot \sin 30^\circ$$

$$H = \frac{6I}{\pi d} \cdot \frac{1}{\sqrt{3}} \cdot \frac{1}{2}$$

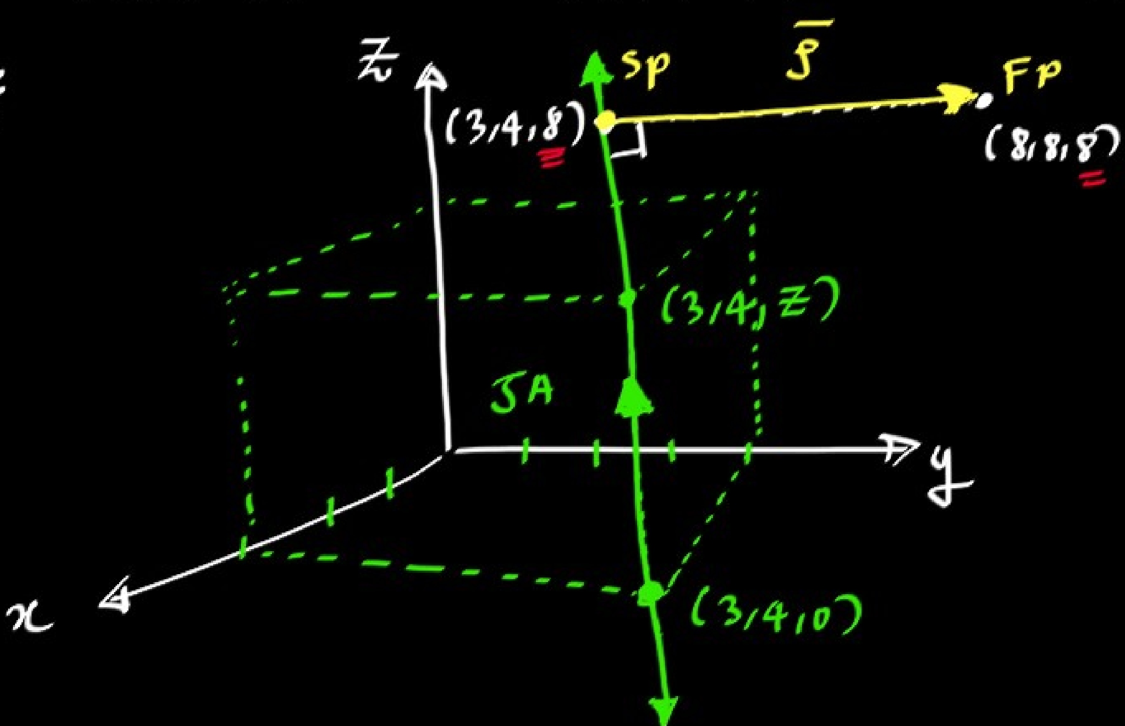
$$H = \frac{\sqrt{3}\, I}{\pi d}.$$

Q. A long conductor carrying A. D. C current of 5A in positive z direction and located at x = 3m, y = 4 m. Find $\overline{H}$ A/m at following points.

H.w   H.w

(a) (0, 0 ,0)   (b) (4, 5 ,6)   (c) (8, 8, 8)

Soln:



$$\overline{H} = \frac{I}{2\pi\rho}(\hat{c} \times \hat{u})$$

$$\overline{H} = \frac{I}{2\pi\rho}(\hat{z} \times \hat{\rho}) \quad , \quad \hat{\rho} = \frac{\overline{\rho}}{\rho}$$

$$\overline{H} = \frac{I}{2\pi\rho}\left\{\hat{z} \times \frac{\overline{\rho}}{\rho}\right\}$$

$$\overline{H} = \frac{I}{2\pi}\left\{\frac{\hat{z} \times \overline{\rho}}{\rho^2}\right\}$$

(a) $SP(3,4,z) \xrightarrow{\overline{\rho_1}} FP(0,0,0)$

$z = 0$

$\hat{x} \to \hat{y} \to \hat{z} \to \hat{x}$

$$\overline{H_1} = \frac{I}{2\pi}\left\{\frac{\hat{z}\times\overline{\rho_1}}{\rho_1^2}\right\}$$

$$\overline{H_1} = \frac{5}{2\pi}\left\{\frac{\hat{z}\times(-3\hat{x}-4\hat{y})}{\left(\sqrt{3^2+4^2}\right)^2}\right\}$$

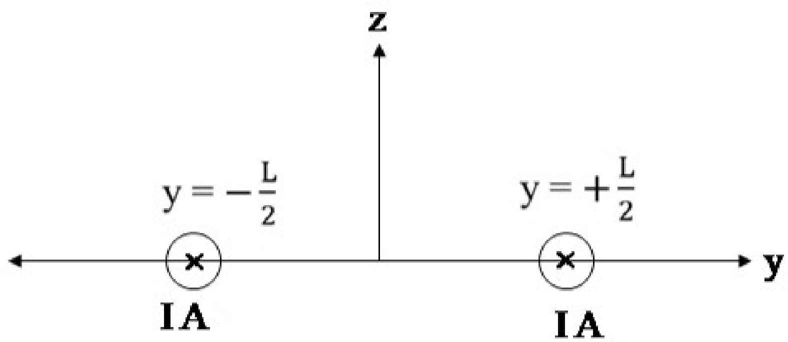
$$\overline{H_1} = \frac{5}{2\pi}\left[\frac{-3\hat{y}+4\hat{x}}{25}\right]$$

$$\overline{H_1} = \frac{1}{10\pi}\left[4\hat{x}-3\hat{y}\right] \text{ A/m.}$$

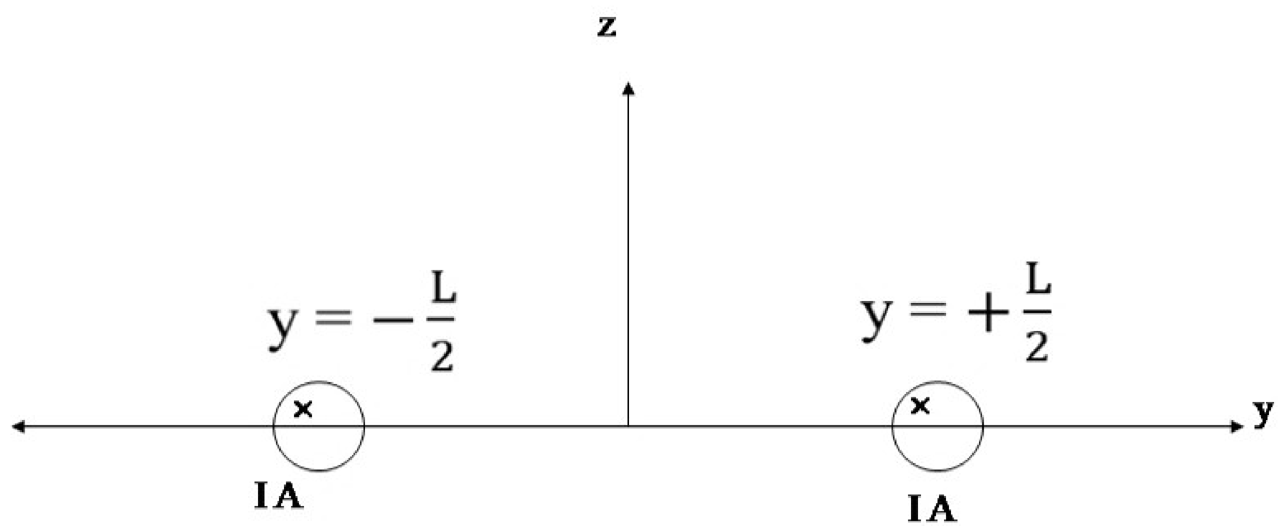
Q. A steady current I is flowing in the direction –ve x direction through each of two infinitely long wires at $y = \pm\frac{L}{2}$ as shown in figure. The $\overline{B}$ field at (0, L, 0) is

(GATE-15)

H.W



(a) $-\frac{4\mu_0 I}{3\pi L}\hat{z}$ (b) $\frac{4\mu_0 I}{3\pi L}\hat{z}$ (c) 0 (d) $\frac{4\mu_0 I}{3\pi L}\hat{z}$



z
$y = -\frac{L}{2}$
$y = +\frac{L}{2}$
y
IA
IA