

26. An input signal $x(t) = 2 + 5\sin(100\pi t)$ is sampled with a sampling frequency of 400 Hz and applied to the system whose transfer function is represented by $H(z) = \frac{1}{N} \left[\frac{1 - z^{-N}}{1 - z^{-1}} \right]$ where N represents the number of samples per cycle. The output $y(n)$ of the system under steady state is _____.

- (a) 0
 - (b) 1
 - (c) 2
 - (d) 5
- GATE 2m

$$x(t) = 2 + 5 \sin 100\pi t$$

$$f_s = 400 \text{ Hz} \Rightarrow T_s = \frac{1}{400}$$

$$x(nT_s) = 2 + 5 \sin 100\pi \frac{n}{400}$$

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$$x(n) = 2 + 5 \sin \frac{\pi}{4} n$$

$$N = \frac{2\pi}{\omega_0} \cdot m = \frac{2\pi}{\pi/4} \cdot m$$

$$= 8m$$

$$N = 8$$

$$H(z) = \frac{1}{8} \left[\frac{1 - z^{-8}}{1 - z^{-1}} \right]$$

$$H(e^{j\omega}) = \frac{1}{8} \left[\frac{1 - e^{-j8\omega}}{1 - e^{-j\omega}} \right]$$

$$H(e^{j0}) = \frac{1}{8} \left(\frac{1 - e^0}{1 - e^0} \right) = \frac{0}{0}$$

$$= \lim_{\omega \rightarrow 0} \frac{1}{8} \frac{0 - e^{-j8\omega} (-j8)}{0 - e^{-j\omega} (-j)}$$

$$= \frac{1}{8}$$

$$H(e^{j0}) = \frac{1}{8}$$

$$H(e^{j\pi/4}) = \frac{1}{8} \left(\frac{1 - e^{-j8 \cdot \frac{\pi}{4}}}{1 - e^{-j\pi/4}} \right)$$

$$= \frac{1}{8} \frac{(1 - 1)}{(\quad)} = 0$$

$$y[n] = 1 \cdot (2) + 0 \cdot \left(5 \sin \frac{\pi}{4} n \right)$$

$$y[n] = 2$$

$$\text{Ans} = 2$$



28. A ^{causal} LTI system is described by the D.E $2y(n) = \alpha y[n-2] - 2x[n] + \beta x[n-1]$
The system is stable only if

- (a) $|\alpha| = 2, |\beta| < 2$ (b) $|\alpha| > 2, |\beta| > 2$
(c) $|\alpha| < 2$, any β (d) $|\beta| < 2$, any α

$$2y(n) = \alpha y(n-2) - 2x(n) + \beta x(n-1)$$

$$2Y(z) = \alpha z^{-2} Y(z) - 2X(z) + \beta z^{-1} X(z)$$

$$Y(z) (2 - \alpha z^{-2}) = X(z) (-2 + \beta z^{-1})$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{-2 + \beta z^{-1}}{2 - \alpha z^{-2}}$$

For a causal and stable sys must lie inside unit circle.

$$2 - \alpha z^{-2} = 0$$

$$2z^2 - \alpha = 0$$

$$z = \pm \sqrt{\frac{\alpha}{2}}$$

$$\left| \sqrt{\frac{\alpha}{2}} \right| < 1$$

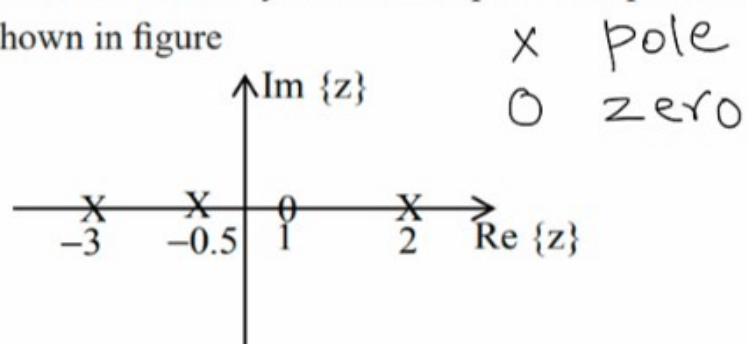
$$\left| \frac{\alpha}{2} \right| < 1$$

$$|\alpha| < 2$$

(c)

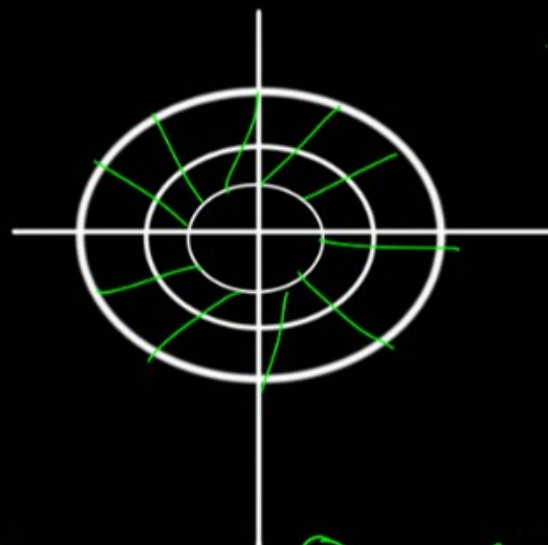
all the poles

29. Consider an LTI system whose pole-zero pattern is shown in figure



- (a) Find the ROC of system function, if it is known to be stable?
- (b) Is it possible for the given pole-zero plot to be a causal & stable system?
- (c) How many possible ROC's are there?

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(a) $0.5 < |z| < 2 \Rightarrow \text{ROC}$
includes unit circle

(b) not possible to have both causal and stable

(c) $|z| > 3$, $|z| < 0.5$
 $0.5 < |z| < 2$, $2 < |z| < 3$

$0.5 < |z| < 3$ is invalid as it's including pole at $z=2$



30. The impulse response $h(n)$ of a LTI system is real. The transfer function $H(z)$ of the system has only one pole and it is at $z = 4/3$. The zeros of $H(z)$ are non-real and located at $|z| = 3/4$. The system is
- (a) stable & causal (b) unstable & anticausal
(c) unstable & causal (d) stable & anticausal

If $h(n)$ is real and if at all $H(z)$ has complex zeros and complex poles then they always occur in conjugate pairs.

$$H(z) = \frac{(z - \frac{3}{4}e^{j\theta})(z - \frac{3}{4}e^{-j\theta})}{z - 4/3} = \frac{z^2 + \dots}{z - 4/3}$$

$$= z + \dots$$

$$h(n) = f(n+1) + \dots \quad h(n) \neq 0 \quad n < 0$$

System is Anticausal $\Rightarrow |z| < \frac{4}{3}$ ✓

System is stable

$$|z| < 1.33$$

Roc includes unit circle



32. Consider the system with transfer function, $H(z) = \frac{z^{-1}}{1 - 2z^{-1}}$. Then the corresponding stable impulse response is

(a) $-0.5 \delta(n) - 0.5 (2)^n u[-n - 1]$

(b) $2^{n-1} u[n - 1]$

(c) $0.5 \delta(n) + 0.5 (2)^{n-1} u[n - 1]$

(d) $0.5 \delta(n) + 2^n u[-n - 1]$

$$-a^n u(-n-1) \rightarrow \frac{z}{z-a} \quad |2| < |a|$$

$$H(z) = \frac{1}{z-2} \quad |z| > 2, |z| < 2 \checkmark \rightarrow \text{Roc is including unit circle}$$

$$= \frac{1}{2} \frac{2}{z-2}$$

$$= \frac{1}{2} \left(\frac{2-2+z}{z-2} \right)$$

$$= \frac{1}{2} \left(-1 + \frac{z}{z-2} \right)$$

$$h(n) = -0.5 \delta(n) - 0.5 2^n u(-n-1)$$

Ans (a)



33. Suppose $x[n]$ is an absolutely summable discrete-time signal. Its z -transform is a rational function with two poles and two zeros. The poles are at $z = \pm 2j$. Which one of the following statements is TRUE for the signal $x[n]$?

GATE - 2015

- (a) It is a finite duration signal.
- (b) It is a causal signal.
- (c) It is a non-causal signal.
- (d) It is a periodic signal.

$$X(z) = \frac{1}{(z + 2j)(z - 2j)}$$

Roc should include unit circle

✗ $|z| > 2$, $|z| < 2$ ✓ Non-causal signal.



34. $y(n) - 0.8y(n-1) = x(n) + 1.25x(n+1)$. Its right sided impulse response is _____

- (a) Causal (b) Unbounded
(c) Periodic (d) Non-negative

$$Y(z) - 0.8z^{-1}Y(z) = X(z) + 1.25z^1X(z)$$

$$Y(z)(1 - 0.8z^{-1}) = X(z)(1 + 1.25z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 + 1.25z}{1 - 0.8z^{-1}}$$

$$= \frac{1}{1 - 0.8z^{-1}} + 1.25 \frac{z}{1 - 0.8z^{-1}}$$

$$= \frac{z}{z - 0.8} + 1.25z \frac{z}{z - 0.8}$$

$$h(n) = (0.8)^n u(n) + 1.25(0.8)^{n+1} u(n+1)$$

$$h(n) \neq 0 \quad n < 0$$

\Rightarrow Not causal

Ans (d)

35. Consider the following statements regarding a linear discrete - time system

$$H(z) = \frac{z^2 + 1}{(z + 0.5)(z - 0.5)}$$

1. The system is stable.
2. The initial value $h(0)$ of the impulse response is - 4.
3. The steady-state output is zero for a sinusoidal discrete time input of frequency equal to one-fourth the sampling frequency.

Which of these statements are correct?

- | | |
|----------------|-------------|
| (a) 1, 2 and 3 | (b) 1 and 2 |
| (c) 1 and 3 | (d) 2 and 3 |

$$\text{poles} = \pm 0.5$$

poles are inside
unit circle \Rightarrow stable

$$h(0) = \lim_{z \rightarrow \infty} H(z)$$

$$= \lim_{z \rightarrow \infty} \frac{z^2 (1 + 1/z^2)}{z^2 (1 + 0.5/z)(1 - 0.5/z)}$$

$$= \frac{1}{(1+0)(1-0)}$$

$$= 1$$

(2) is false

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35. Consider the following statements regarding a linear discrete - time system

$$H(z) = \frac{z^2 + 1}{(z+0.5)(z-0.5)}$$

1. The system is stable.
2. The initial value $h(0)$ of the impulse response is -4.
3. The steady-state output is zero for a sinusoidal discrete time input of frequency equal to one-fourth the sampling frequency.

Which of these statements are correct?

- | | |
|----------------|-------------|
| (a) 1, 2 and 3 | (b) 1 and 2 |
| (c) 1 and 3 | (d) 2 and 3 |

$$H(e^{j\omega}) = \frac{e^{j2\omega} + 1}{(e^{j\omega} + 0.5)(e^{j\omega} - 0.5)}$$

$$x(t) = A \sin \omega_0 t$$

$$\omega_0 = \frac{\omega_s}{4}$$

$$x(nT_s) = A \sin \frac{\omega_s}{4} \cdot nT_s$$

$$= A \sin \frac{2\pi}{4} n$$

$$x(n) = A \sin \frac{\pi}{2} n$$

$$\Rightarrow H(e^{j\pi/2}) = \frac{e^{j2 \cdot \frac{\pi}{2}} + 1}{() ()}$$

$$H(e^{j\pi/2}) = 0$$

$$y(n) = 0 (A \sin \frac{\pi}{2} n) = 0$$

(3)

is correct.

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36. **Assertion (A):** The stability of the system is assured if the Region of Convergence (ROC) includes the unit circle in the Z-plane.

Reason (R): For a causal stable system all the poles should be outside the unit circle in the Z-plane.

A : True

R : False

37. **Statement (I):** The system function

$$H(z) = \frac{z^3 - 2z^2 + z}{z^2 + \frac{1}{4}z + \frac{1}{8}} \text{ is not causal.}$$

Statement (II): If the numerator of $H(z)$ is of lower order than the denominator, the system may be causal.

$$H(z) = \frac{-1}{2} + \frac{1}{2} \frac{z}{z-2}$$

$$h(n) = -\frac{1}{2} \delta(n) - \frac{1}{2} 2^n u(-n-1)$$

$$h(n) \neq 0 \quad n < 0$$

$$\begin{aligned} \text{I} \quad H(z) &= z + \left(\right) \\ h(n) &= \delta(n+1) + \left(\right) \neq 0 \quad n < 0 \Rightarrow \text{Not causal} \\ &\Rightarrow \text{Not causal} \end{aligned}$$

$$\begin{aligned} \text{II} \quad H(z) &= \frac{1}{z-2} \quad |z| < 2 \\ &= \frac{2}{2(z-2)} = \frac{1}{2} \frac{(2-2+2)}{z-2} \end{aligned}$$

whenever the numerator of $H(z)$ is of lower order than the denominator, the sys may or may be causal.



38. Statement (I):

Z-transform approach is used to analyze the discrete time systems and is also called as pulse transfer function approach.

Statement (II):

The sampled signal is assumed to be a train of impulses whose strengths, or areas, are equal to the continuous time signal at the sampling instants.

$H(z)$ is the ratio of Z-Transform of sampled output to the Z-Transform of sampled input

$$H(z) = \frac{Y(z)}{X(z)} = \frac{Z.T\{y(n)\}}{Z.T\{x(n)\}}$$

Both I and II are true and II is the correct explanation for I



$$z = e^{j\omega}$$

Continuous	$\omega \rightarrow 0$ $s \rightarrow 0$	$\omega \rightarrow \infty$ $s \rightarrow \infty$	$\omega = \omega_c$
Discrete	$\omega \rightarrow 0$ $z \rightarrow 1$	$\omega \rightarrow \pi$ $z \rightarrow -1$	
LPF	1	0	$1/\sqrt{2}$
HPF	0	1	$1/\sqrt{2}$
BPF	0	0	1
BRF	1	1	0
APF	1	1	1

Identify the nature of the filter $H(s) = \frac{s}{s^2+3s+3}$

$$H(0) = \frac{0}{0+0+3} = 0$$

$$H(\infty) = \lim_{s \rightarrow \infty} \frac{s}{s^2 \left(1 + \frac{3}{s} + \frac{3}{s^2}\right)}$$

$$= \frac{1}{s \left(1 + \frac{3}{s} + \frac{3}{s^2}\right)}$$

$$= \frac{1}{\infty(1+0+0)} = 0$$

Ans BPF

$$H(s) = \frac{s^2}{s^2+3s+3}$$

$$H(0) = \frac{0}{3} = 0$$

$$H(\infty) = \lim_{s \rightarrow \infty} \frac{s^2}{s^2 \left(1 + \frac{3}{s} + \frac{3}{s^2}\right)}$$

$$= \lim_{s \rightarrow \infty} \frac{1}{1+0+0}$$

$$= 1$$

Ans : HPF

Identify the nature of the filter with Impulse response $h(n) = (-0.8)^n u(n)$

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$$H(z) = \frac{z}{z + 0.8}$$

$$\omega = 0$$
$$H(1) = \frac{1}{1 + 0.8} = \frac{1}{1.8} = 0.555$$

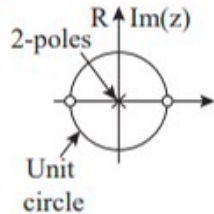
$$\omega = \pi$$
$$H(-1) = \frac{-1}{-1 + 0.8} = \frac{-1}{-0.2} = 5$$

HPF

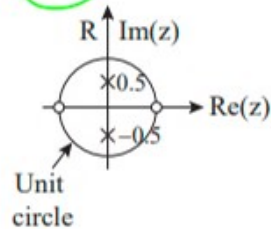
The pole-zero plots of three discrete-time systems P, Q and R on the z-plane are shown below.

[GATE-17 S2]

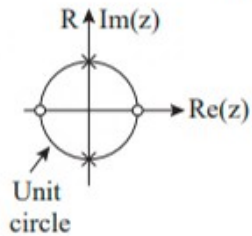
①



②



③



Which one of the following is TRUE about the frequency selectivity of these systems?

- (a) All three are high-pass filters
- (b) All three are band-pass filters ✓
- (c) All three are low-pass filters
- (d) P is a low-pass filter, Q is a band-pass filter and R is a high-pass filter.

①

$$H(z) = \frac{(z-1)(z+1)}{z^2}$$

$$H(1) = 0$$

$$H(-1) = 0$$

BPF

②

$$H(z) = \frac{z^2 - 1}{(z - j0.5)(z + j0.5)}$$

$$= \frac{z^2 - 1}{z^2 + 0.25} \quad \text{BPF}$$

③

$$H(z) = \frac{z^2 - 1}{(z+j)(z-j)} = \frac{z^2 - 1}{z^2 + 1} \quad \text{BPF}$$

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A discrete real allpass system has a pole at $z = 2 \angle 30^\circ$: it, therefore, **[GATE-06]**

- (a) also has a pole at $1/2 \angle 30^\circ$
- (b) has a constant phase response over the z-plane:
 $\arg|H(z)| = \text{const}$
- (c) is stable only if it is anticausal
- (d) has a constant phase response over the unit circle: $\arg|H(e^{j\Omega})| = \text{const}$

For real all pass system, poles occur in conjugate pairs

$$z = 2 \angle 30^\circ = 2e^{j\pi/6}$$

the other pole is $2e^{-j\pi/6}$

$|z| < 1 \leftarrow |2e^{j\pi/6}|$
 $|z| < 2$ or $|z| > 2$
 Anticausal $|z| < 2$
 \Rightarrow ROC is including unit circle. \Rightarrow stable
 ans (c)



A discrete-time all-pass system has two of its poles at $0.25\angle 0^\circ$ and $2\angle 30^\circ$. Which one of the following statements about the system is TRUE?

(GATE -18)

- (a) It has two more poles at $0.5\angle 30^\circ$ and $4\angle 0^\circ$.
- (b) It is stable only when the impulse response is two-sided.
- (c) It has constant phase response over all frequencies.
- (d) It has constant phase response over the entire z-plane.

(b) $0.25 < |z| < 2$
ROC includes unit circle
 \Rightarrow System is stable.
Ans (b)

$$H(z) = \frac{d + z^{-1}}{1 + d z^{-1}}, \text{ find } |H(e^{j\omega})|, \angle H(e^{j\omega})$$

$$H(e^{j\omega}) = \frac{d + e^{-j\omega}}{1 + d e^{-j\omega}} = \frac{d + \cos\omega - j\sin\omega}{1 + d\cos\omega - j d\sin\omega}$$

$$|H(e^{j\omega})| = \sqrt{\frac{(d + \cos\omega)^2 + (\sin\omega)^2}{(1 + d\cos\omega)^2 + (d\sin\omega)^2}} = \sqrt{\frac{d^2 + \cos^2\omega + 2d\cos\omega + \sin^2\omega}{1 + d^2\cos^2\omega + 2d\cos\omega + d^2\sin^2\omega}}$$

$$= \sqrt{\frac{d^2 + 1 + 2d\cos\omega}{1 + d^2 + 2d\cos\omega}}$$

$$|H(e^{j\omega})| = 1 \quad \forall \omega \quad \left\{ \begin{array}{l} \angle H(e^{j\omega}) = \tan^{-1}\left(\frac{-\sin\omega}{d + \cos\omega}\right) \\ \quad - \tan^{-1}\left(\frac{-d\sin\omega}{1 + d\cos\omega}\right) \end{array} \right.$$

Let $X(z) = \frac{1}{1-z^{-3}}$ be the Z - transform of a causal signal $x[n]$. Then, the value of $x[2]$ and $x[3]$ are

[GATE-14-S1]

- (a) 0 and 0 (b) 0 and 1
(c) 1 and 0 (d) 1 and 1

$$X(z) = \frac{z^3}{z^3 - 1}$$

$$x(n) = u(n/3) = \{ \underset{A}{1}, 0, 0, 1, 0, 0, 1, 0, 0, \dots \}$$

$$x(2) = 0, \quad x(3) = 1 \quad (b)$$

$$\frac{1}{1-z} = 1 + z + z^2 + z^3 + \dots \quad |z| < 1$$

$$\frac{1}{1-z^{-3}} = 1 + z^{-3} + z^{-6} + z^{-9} + \dots$$

$|z^{-3}| < 1 \quad \frac{1}{|z|^3} < 1 \Rightarrow |z| > 1$



The z-Transform of a sequence $x[n]$ is given as $X(z) = 2z + 4 - 4/z + 3/z^2$. If $y[n]$ is the first difference of $x[n]$, then $Y(z)$ is given by

[GATE-15 S2]

(a) $2z + 2 - \frac{8}{z} + \frac{7}{z^2} - \frac{3}{z^3}$

(b) $-2z + 2 - \frac{6}{z} + \frac{1}{z^2} - \frac{3}{z^3}$

(c) $-2z - 2 + \frac{8}{z} - \frac{7}{z^2} + \frac{3}{z^3}$

(d) $4z - 2 - \frac{8}{z} - \frac{1}{z^2} + \frac{3}{z^3}$

$$y[n] = x[n] - x[n-1]$$

$$Y(z) = X(z) - z^{-1}X(z)$$

$$Y(z) = 2z + 4 - 4z^{-1} + 3z^{-2} - 2 - 4z^{-1} + 4z^{-2} - 3z^{-3}$$

$$Y(z) = 2z + 2 - 8z^{-1} + 7z^{-2} - 3z^{-3}$$

(a)



The z -transform of a signal $x[n]$ is given by $4z^{-3} + 3z^{-1} + 2 - 6z^2 + 2z^3$. It is applied to a system, with a transfer function $H(z) = 3z^{-1} - 2$. Let the output be $y(n)$. Which of the following is true?

[GATE-09]

- (a) $y(n)$ is non causal with finite support
- (b) $y(n)$ is causal with infinite support
- (c) $y(n) = 0; |n| > 3$
- (d) $\text{Re}[Y(z)]_{z=e^{j\theta}} = -\text{Re}[Y(z)]_{z=e^{-j\theta}}$

$$\text{Im}[Y(z)]_{z=e^{j\theta}} = \text{Im}[y(z)]_{z=e^{-j\theta}}; -\pi \leq \theta < \pi$$

$$Y(z) = X(z) \cdot H(z)$$

$$= 12z^{-4} + 9z^{-2} + \cancel{6z^{-1}} - 18z^{-1} + 6z^2 - 8z^3 - \cancel{6z^{-1}} - 4 + 12z^2 - 4z^3$$

$$= -4z^3 + 18z^2 - 18z - 4 + 9z^{-2} - 8z^{-3} + 12z^{-4}$$

$$y(n) = \{-4, 18, -18, -4, 0, 9, -8, 12\}$$

\uparrow
 $n=0$

Ans (a)

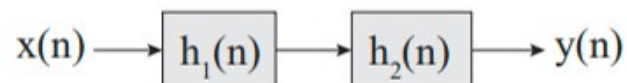


A cascade system having the impulse responses

$$h_1(n) = \{1, -1\} \text{ and } h_2(n) = \{1, 1\}$$

\uparrow \uparrow
 time origin time origin

is shown in the figure below, where symbol \uparrow denotes the time origin. [GATE-17-S2]



The input sequence $x(n)$ for which the cascade system produces an output sequence

$$y(n) = \{1, 2, 1, -1, -1\}$$

\uparrow
time origin

(a) $x(n) = \{1, 2, 1, 1\}$

\uparrow
time origin

(b) $x(n) = \{1, 1, 2, 2\}$

\uparrow
time origin

(c) $x(n) = \{1, 1, 1\}$

\uparrow
time origin

(d) $x(n) = \{1, 2, 2, 1\}$

\uparrow
time origin

$$\text{Overall I.R} = h_1(n) * h_2(n)$$

$$\begin{array}{r}
 \begin{array}{cc} 1 & -1 \\ 1 & 1 \end{array} \\
 \hline
 \begin{array}{cc} 1 & -1 \\ \times & 1 \end{array}
 \end{array}$$

$$h(n) = \{1, 0, -1\}$$

$$y(n) = x(n) * h(n)$$

$$y(2) = x(2) + h(2) \Rightarrow x(2) = \frac{y(2)}{h(2)}$$



$$X(z) = \frac{1 + 2z^{-1} + z^{-2} - z^{-3} - 2z^{-4} - z^{-5}}{1 - z^{-2}}$$

$$= \frac{(1 + 2z^{-1} + z^{-2}) - z^{-3}(1 + 2z^{-1} + z^{-2})}{1 - z^{-2}}$$

$$= \frac{(1 + 2z^{-1} + z^{-2})(1 - z^{-3})}{1 - z^{-2}}$$

$$= \frac{(1 + z^{-1})^2 (1 - z^{-1}) (1 + z^{-1} + z^{-2})}{(1 - z^{-1})(1 + z^{-1})} = (1 + z^{-1})(1 + z^{-1} + z^{-2})$$

$$= 1 + z^{-1} + z^{-2} + z^{-1} + z^{-2} + z^{-3}$$

$$= 1 + 2z^{-1} + 2z^{-2} + z^{-3}$$

$$x(n) = \left\{ \underset{q}{1}, 2, 2, \underset{(d)}{1} \right\}$$



The causal signal with z-transform $z^2(z-a)^{-2}$ is
($u[n]$ is the unit step signal) **[GATE-21]**

- (a) $a^{2n}u[n]$ (b) $(n+1)a^n u[n]$
(c) $n^{-1}a^n u[n]$ (d) $n^2 a^n u[n]$

$$a^n u(n) \rightarrow \frac{z}{z-a}$$

$$n u(n) \rightarrow \frac{z}{(z-1)^2}$$

$$a^n n u(n) \rightarrow \frac{z/a}{(\frac{z}{a}-1)^2} = \frac{a z}{(z-a)^2}$$

$$a^n x(n) \rightarrow X(z/a)$$

$$X(z) = \frac{z^2}{(z-a)^2} = \frac{z^2 - az + az}{(z-a)^2}$$

$$= \frac{z(z-a)}{(z-a)^2} + \frac{az}{(z-a)^2}$$

$$= \frac{z}{z-a} + \frac{az}{(z-a)^2}$$

$$a^n u(n) + a^n n u(n) \\ (n+1)a^n u(n) //$$

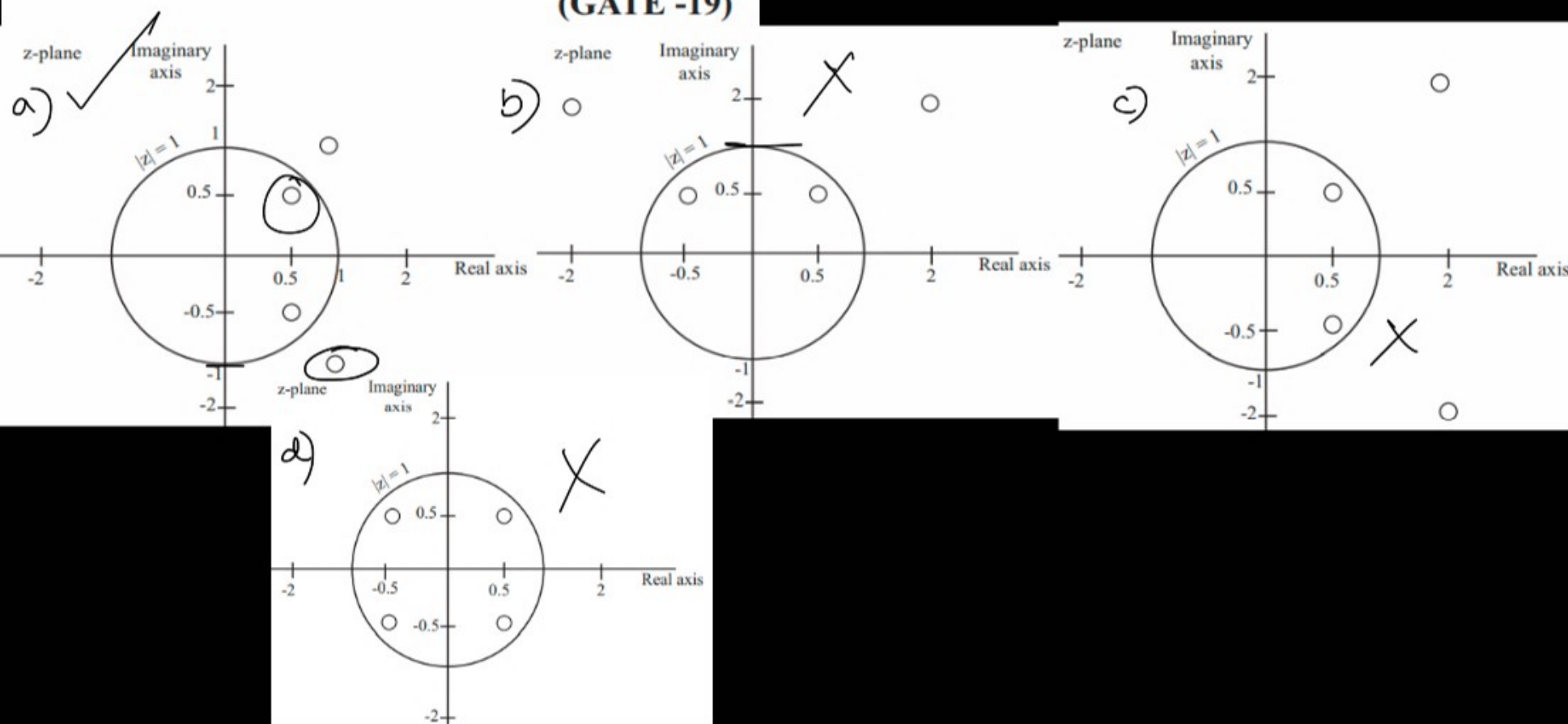


$$z = \frac{1}{2} + \frac{1}{2}j$$

$$z = \frac{1}{\frac{1}{2} + \frac{1}{2}j} = \frac{2}{1+j} = \frac{2(1-j)}{(1+j)(1-j)} = 1-j$$

$$= 1-j$$

(GATE -19)



Which one of the following pole-zero plots corresponds to the transfer function of an LTI system characterized by the input-output difference equation given below?

$$y[n] = \sum_{k=0}^3 (-1)^k x[n-k]$$

(GATE -20)

