Complex Bode Plots

n- complex poles

$$\boxed{G(s)H(s) = \frac{1}{\left(1+\frac{2\xi}{\omega_n}\cdot s+\frac{s^2}{\omega_n^2}\right)^n}}$$

or

⇒ Std form of 2nd order sys:

$$G(s)H(s) = \left(\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}\right)$$

$\boxed{\div\,\omega_n^2}$ $$GH(s) = \frac{1}{\left(1+\frac{2\xi}{\omega_n}\cdot s+\frac{s^2}{\omega_n^2}\right)^n}$$

n- complex zeros

$$\boxed{GH(s) = \left(1+\frac{2\xi}{\omega_n}\cdot s+\frac{s^2}{\omega_n^2}\right)^n}$$

⇒ or

$$GH(s) = \left(\frac{s^2+2\xi\omega_n s+\omega_n^2}{\omega_n^2}\right)$$

$\boxed{\div\,\omega_n^2}$

$$GH(s) = \left(1+\frac{2\xi}{\omega_n}\cdot s+\frac{s^2}{\omega_n^2}\right)^n$$

$s \to j\omega$

$$GH(j\omega) = \frac{1}{\left[1 + \frac{2\xi}{\omega_n}(j\omega) + \left(\frac{-\omega^2}{\omega_n^2}\right)\right]^n}$$

$$= \frac{1}{\left[\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right] + j\,2\xi\left(\frac{\omega}{\omega_n}\right)\right]^n}$$

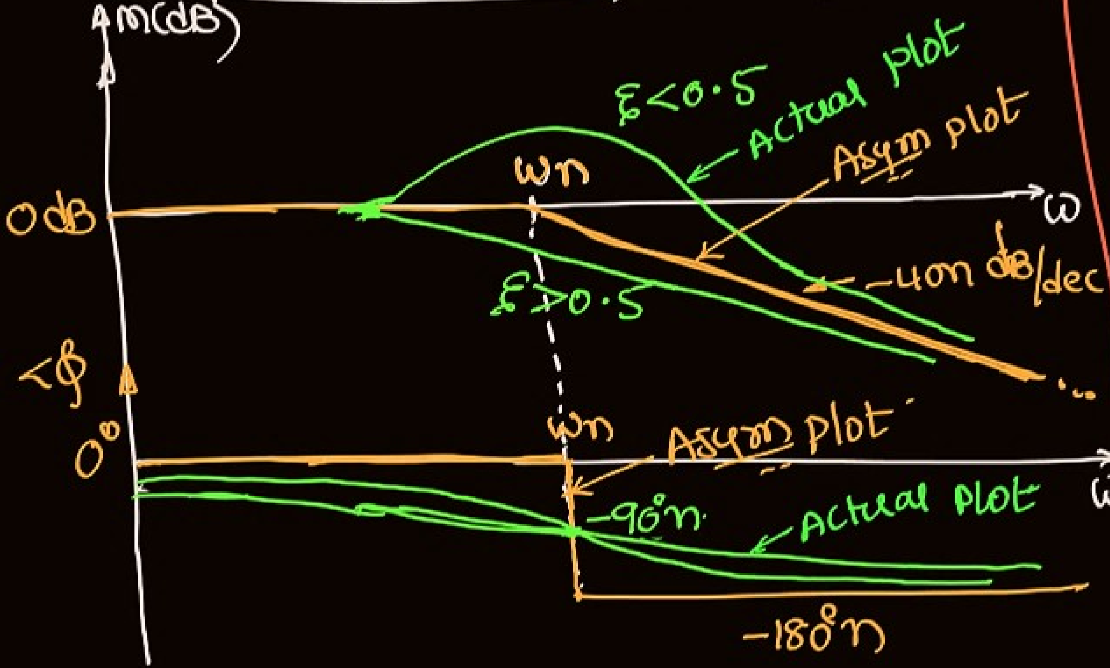
$$M = \frac{1}{\left(\sqrt{\left(1 - \left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2\xi\left(\frac{\omega}{\omega_n}\right)\right)^2}\right)^n}$$

$$\boxed{M_{(dB)\,\text{Actual}} = -20n \log \sqrt{\left(1 - \left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2\xi\left(\frac{\omega}{\omega_n}\right)\right)^2}}$$

$$\boxed{\phi_{\text{actual}} = -n \cdot \tan^{-1}\left(\frac{2\xi(\omega/\omega_n)}{1 - \left(\frac{\omega}{\omega_n}\right)^2}\right)}$$

ω_n is called corner frequency.

xxxx	slope	$\angle\phi$
< CF	0 dB/dec	0°
> CF	$-40n$ dB/dec	$-180^\circ n$



✓	S	$\angle\phi$
< CF	0	0°
> CF	$+40n$	$+180^\circ n$

$$\boxed{M_{\text{Actual}}(\text{dB}) = -20n \log \sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2\xi\left(\frac{\omega}{\omega_n}\right)\right)^2}}$$

$$\boxed{\phi_{\text{actual}} = -n \cdot \tan^{-1}\left(\frac{2\xi(\omega/\omega_n)}{1-\left(\frac{\omega}{\omega_n}\right)^2}\right)}$$

$$\left\{\begin{matrix} M_{\text{correction}} \\ \text{at } \omega=\omega_n \end{matrix}\right\} = -20n \log \sqrt{(2\xi)^2}$$

$$= \underline{\underline{-20n \log(2\xi)}}$$

$$\left\{\begin{matrix} \phi_{\text{correction}} \\ \text{at } \omega=\omega_n \end{matrix}\right\} = \underline{\underline{-90^\circ n}}$$

*⇒ The correction at CF is depends on ξ in the magni- -tude plot, where as in the phase plot, the correction at CF is independent of ξ & ω.

*⇒ Other than corner frequencies, the correction depends on ξ & ω in both the plots.

Prob :- Draw the Bode plot to the given TF of a MPS.

$$G(s)H(s) = \frac{s^2\left(1+s/20+s^2/100\right)^4}{\left(1+s/3+s^2/9\right)^3\left(1+s/20\right)^4}$$

$$G(s)H(s) = \frac{1 \cdot s^2\left(1+\frac{s}{20}+\frac{s^2}{100}\right)^4}{\left(1+\frac{s}{3}+\frac{s^2}{9}\right)^3\left(1+\frac{s}{20}\right)^4}$$

$\omega_n^2 = 100$

$\boxed{\omega_n = 10 \text{ rad/sec}}$

$\frac{2\xi}{\omega_n} = \frac{1}{20}$, with $\omega_n = 10$

$\Rightarrow \boxed{\xi = \frac{1}{4} = 0.25}$

$\Rightarrow \omega_n^2 = 9$

$\boxed{\omega_n = 3 \text{ rad/sec}}$

$\Rightarrow \frac{2\xi}{\cancel{\omega_n}\,3} = \frac{1}{\cancel{3}} \Rightarrow \boxed{\xi = 0.5}$

CF $\Rightarrow$ $(3, 10, 20)$ rad/sec

$$M_{\omega=0.1} = 20\log 1 + 20\log|\omega^2|$$

$$= 20\log 1 + 40\log 0.1$$

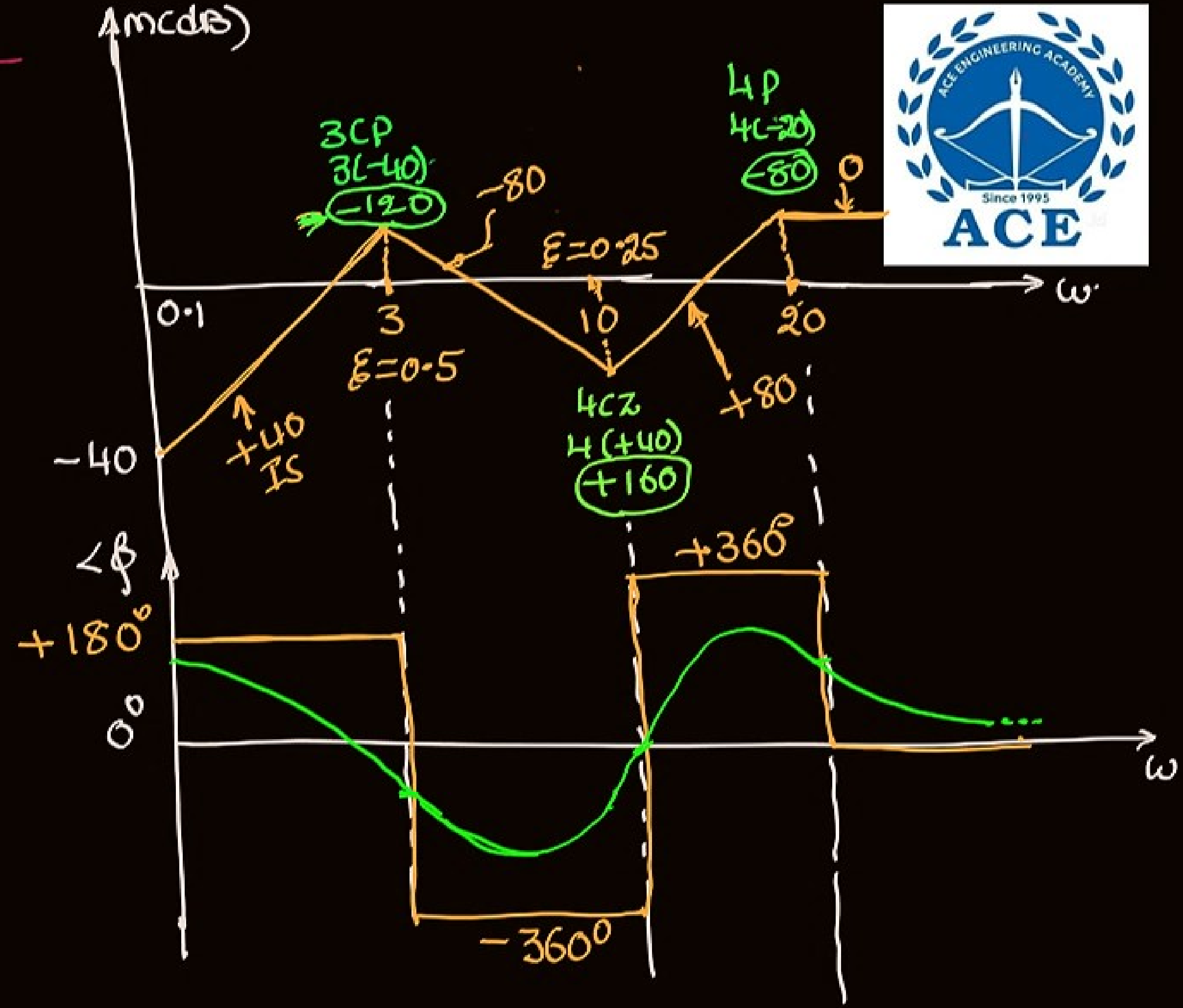
$$\boxed{M_{/\omega=0.1} = -40\text{ dB}}$$

$$G(s)H(s) = \frac{s^2\left(1+\frac{s}{20}+\frac{s^2}{100}\right)^4}{\left(1+\frac{s}{3}+\frac{s^2}{9}\right)^3\left(1+\frac{s}{20}\right)^4}$$

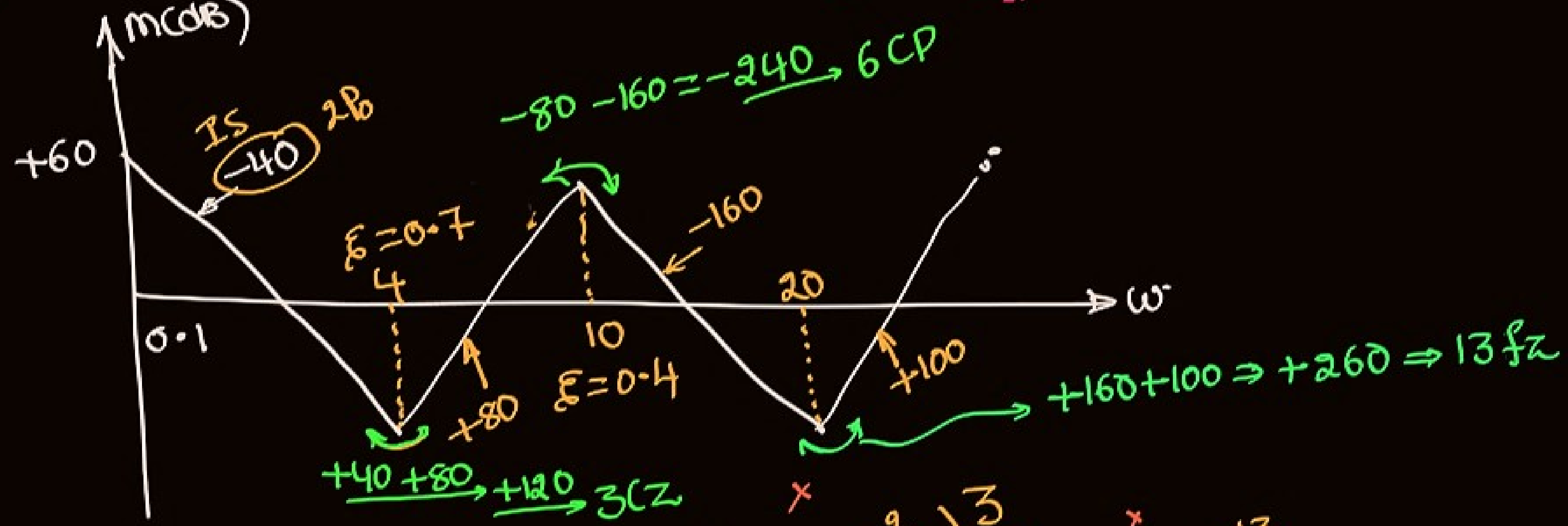
IS:-

$2Z_0 \Rightarrow +40$

CF $\boxed{3, 10, 20}$



Prob:- find TF to the given asymp mag plot of a MPS



$$G(s)H(s) = \frac{K\left(1+\frac{2\times0.7}{4}\cdot s+\frac{s^2}{4^2}\right)^3\left(1+s/20\right)^{13}}{s^2\left(1+\frac{2\times0.4}{10}\cdot s+\frac{s^2}{10^2}\right)^6}$$

$$\Rightarrow 60\Big|_{\omega=0.1} = 20\log K - 40\log 0.1$$

$$60 = 20\log K + 40$$

$$20 = 20\log K$$

$$1 = \log K \Rightarrow \boxed{K = 10}$$

POLAR PLOTS

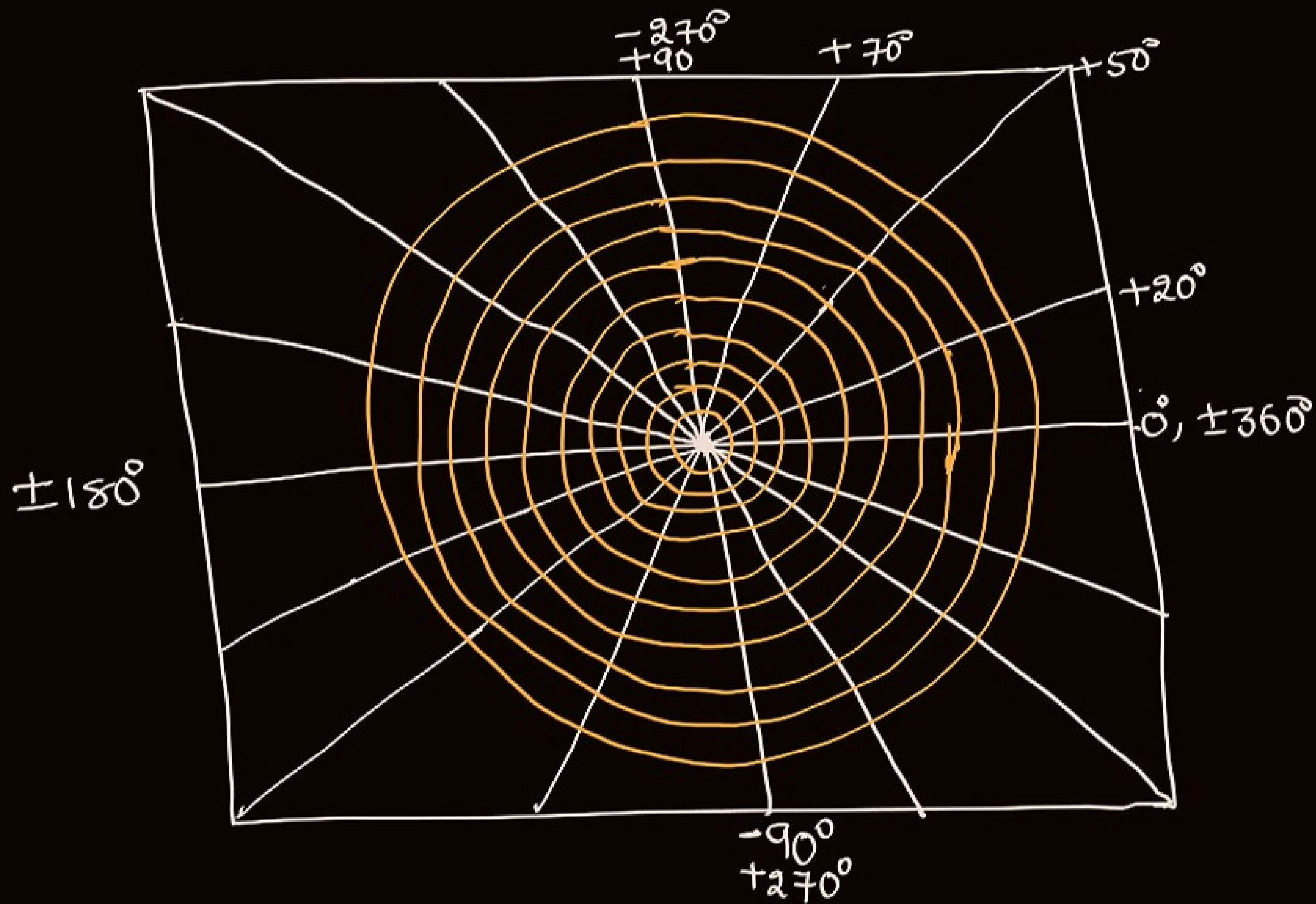
Purpose :-

1. To draw the frequency response of OLTF $G(s)H(s)$.
2. To find the closed loop system stability.
3. To find the range of K value for CL stability.
4. To find the Gain Margin & phase margin, gain cross-over freq, phase cross-over freq.
5. To find Relative stability by using GM & PM.

Polar Plot :- It is a frequency response, it consist magnitude verses phase plot.

⇒ The frequency range for polar plot is $(0 \text{ to } \infty)$. where for nyquist plot is $(-\infty \text{ to } +\infty)$

⇒ Polar plots are not a complete frequency response plot. The complete frequency response plot is Nyquist plot.


-270°
+90
+70°
+50°
+20°
0°, ±360°
±180°
-90°
+270°

Prob :- Draw the polar plot for $G(s)H(s) = 1/s$

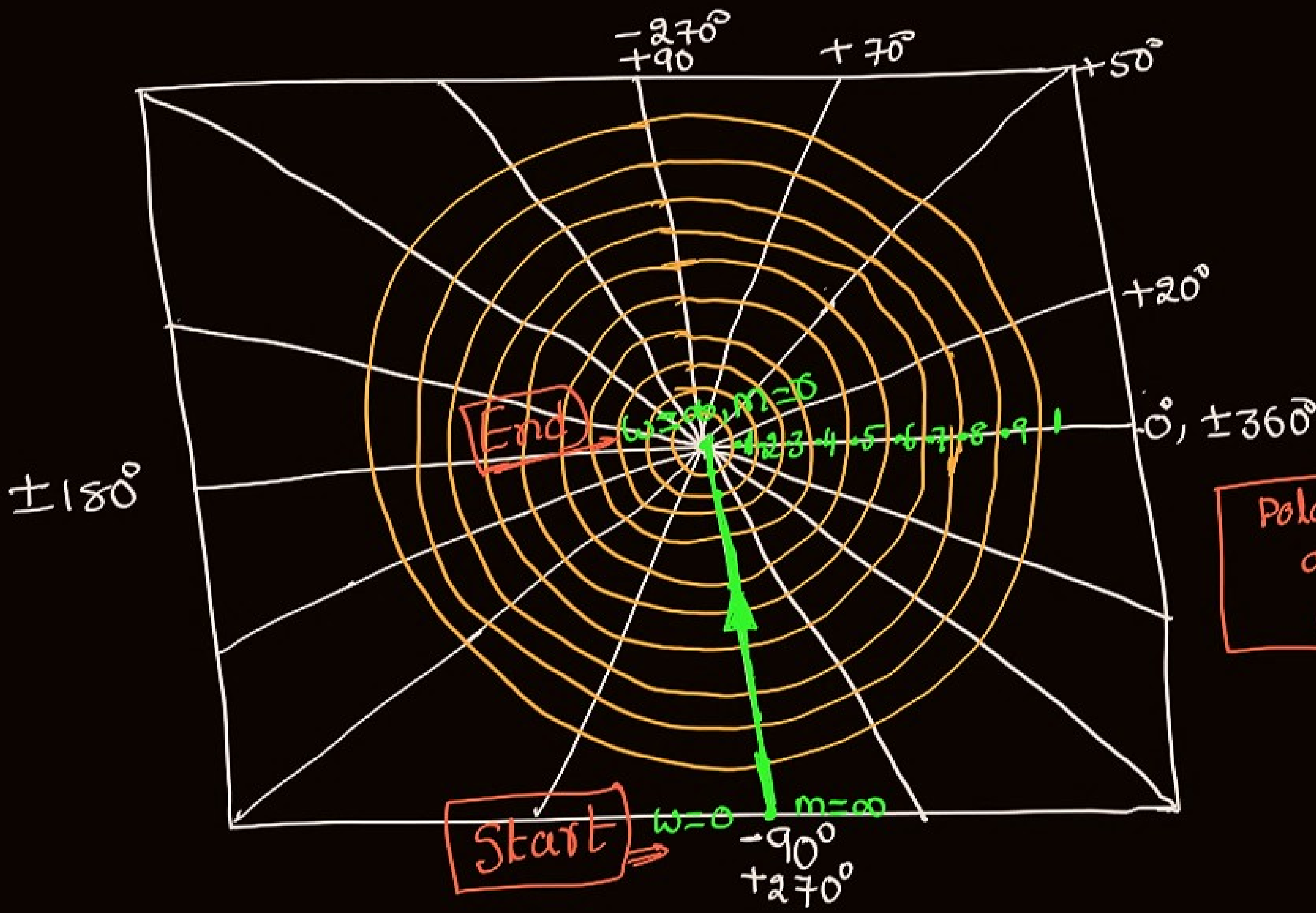
Sol :- $GH(s) = 1/s$

$s \to j\omega \Rightarrow GH(j\omega) = \frac{1}{j\omega}$

$\Rightarrow M = \left(\frac{1}{\omega}\right)$

$\angle\phi = \angle GH(j\omega) = \frac{\angle 1}{\angle j\omega} = \frac{0^\circ}{90^\circ} = -90^\circ$

ω	M	$\angle\phi$
0 →	∞ →	-90°
1 →	1 →	-90°
2 →	0.5 →	-90°
5 →	0.2 →	-90°
10 →	0.1 →	-90°
⋮	⋮	
∞ →	0 →	-90°



Polar Plot starts at $\omega=0$ and ends at $\omega=\infty$

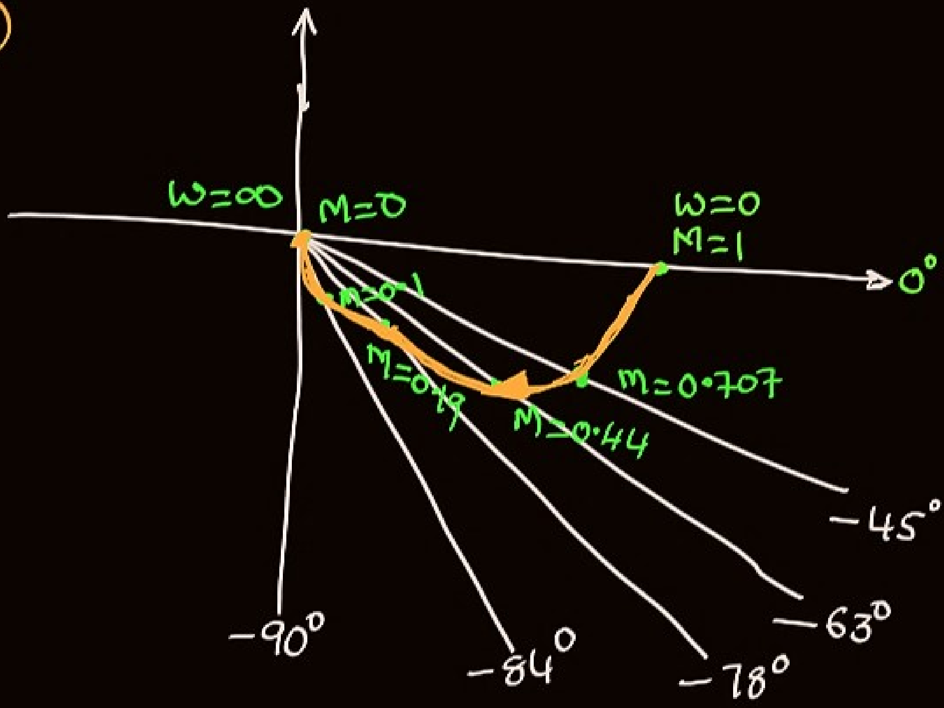
Prob:- Draw the Polar plot for $G(s)H(s) = \frac{1}{(sT+1)}$

Sol:- $s \rightarrow j\omega$ $\quad GH(j\omega) = \frac{1}{(j\omega T+1)}$

$$M = \frac{1}{\sqrt{(\omega T)^2+1}}, \quad \angle\phi = \frac{\angle 1}{\angle(j\omega T+1)}$$

$$\angle\phi = \frac{0^\circ}{\tan^{-1}\left(\frac{\omega T}{1}\right)} = -\tan^{-1}(\omega T)$$

ω	M	$\angle\phi$
0 →	1 →	0°
$1/T$ →	0.707 →	-45°
$2/T$ →	0.44 →	-63°
$5/T$ →	0.19 →	-78°
$10/T$ →	0.1 →	-84°
⋮	⋮	⋮
∞ →	0 →	-90°



⇒ Draw Polar Plots

(i) $G(s)H(s) = \left(\frac{s+1}{s+10}\right)$ [High pass filter
Lead compensator.]

$s \to j\omega$

$\longrightarrow GH(j\omega) = \left(\frac{j\omega+1}{j\omega+10}\right)$

$$M = \sqrt{\frac{\omega^2+1}{\omega^2+100}}$$

$$\angle\phi = \frac{\angle(j\omega+1)}{\angle(j\omega+10)} = +\tan^{-1}(\omega) - \tan^{-1}\left(\frac{\omega}{10}\right)$$

(ii) $G(s)H(s) = \left(\frac{s+10}{s+1}\right)$ [Low pass filter
Lag compensator]

$s \to j\omega$

$\longrightarrow GH(j\omega) = \left(\frac{j\omega+10}{j\omega+1}\right)$

$$M = \sqrt{\frac{\omega^2+100}{\omega^2+1}}$$

$$\angle\phi = \frac{\angle(j\omega+10)}{\angle(j\omega+1)} = -\tan^{-1}(\omega) + \tan^{-1}\left(\frac{\omega}{10}\right)$$

$$M = \sqrt{\frac{\omega^2+1}{\omega^2+100}}$$

$$\angle\phi = \frac{\angle(j\omega+1)}{\angle(j\omega+10)} = +\tan^{-1}(\omega) - \tan^{-1}\left(\frac{\omega}{10}\right)$$

ω	M	$\angle\phi$
0	$\rightarrow$ 0.1	$\rightarrow$ 0°
1	$\rightarrow$ 0.14	$\rightarrow$ $+39^\circ$
2	$\rightarrow$ 0.22	$\rightarrow$ $+52^\circ$
5	$\rightarrow$ 0.44	$\rightarrow$ $+52^\circ$
10	$\rightarrow$ 0.71	$\rightarrow$ $+39^\circ$
⋮	⋮	⋮
∞	$\rightarrow$ 1	$\rightarrow$ 0°

$M/_{\omega=0} \Rightarrow \underline{\underline{0.1}}$ & $M/_{\omega=\infty} \Rightarrow 1$

$$M = \sqrt{\frac{\omega^2+100}{\omega^2+1}}$$

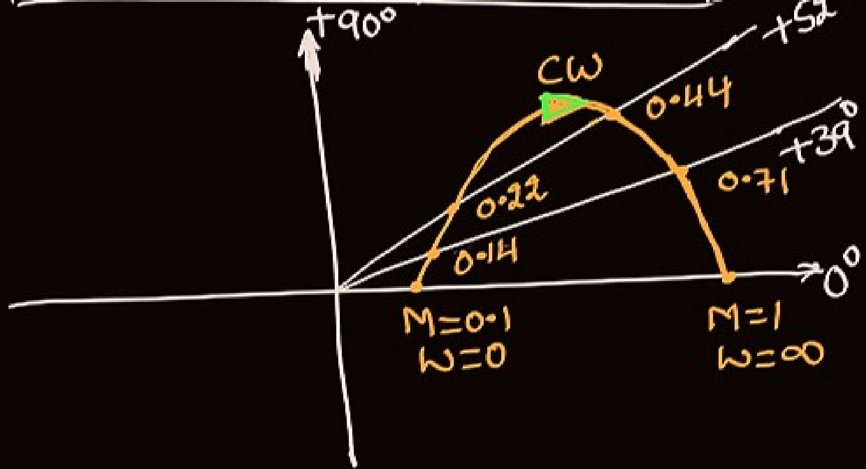
$$\angle\phi = \frac{\angle(j\omega+10)}{\angle(j\omega+1)} = -\tan^{-1}(\omega) + \tan^{-1}\left(\frac{\omega}{10}\right)$$

ω	M	$\angle\phi$
0	$\rightarrow$ 10	$\rightarrow$ 0°
1	$\rightarrow$ 7.1	$\rightarrow$ -39°
2	$\rightarrow$ 4.4	$\rightarrow$ -52°
5	$\rightarrow$ 2.2	$\rightarrow$ -52°
10	$\rightarrow$ 1.4	$\rightarrow$ -39°
⋮	⋮	⋮
∞	1	$\rightarrow$ 0°

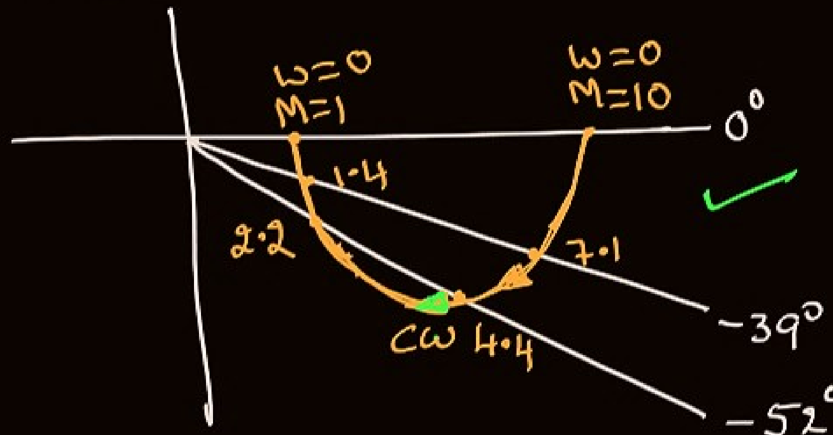
$M/_{\omega=0} \Rightarrow 10$ ✓

$M/_{\omega=\infty} \Rightarrow \underline{1}$

ω	M	∠φ
0 →	0.1 →	0°
1 →	0.14 →	+39°
2 →	0.22 →	+52°
5 →	0.44 →	+52°
10 →	0.71 →	+39°
⋮	⋮	⋮
∞ →	1 →	0°



ω	M	∠φ
0 →	10 →	0°
1 →	7.1 →	−39°
2 →	4.4 →	−52°
5 →	2.2 →	−52°
10 →	1.4 →	−39°
⋮	⋮	⋮
∞	1 →	0°



Short cut procedure to draw polar plot :→

⇒ This procedure is valid only when $\boxed{M|_{\omega=0} \geq M|_{\omega=\infty}}$

→ If $\boxed{M|_{\omega=0} < M|_{\omega=\infty}}$, then draw the plot by stadard procedure.

⇒ This case $[M|_{\omega=0} < M|_{\omega=\infty}]$ may occurs if TF consists of

(i) only zeros [No poles]

(ii) Any zero at origin

(iii) If the number of poles ≤ number of zeros, then verify magnitude at $\omega=0$ & $\omega=\infty$.

⇒ If $M|_{\omega=0} < M|_{\omega=\infty}$, then draw the plot by using std procedure

Draw plot by std procedure.

Short cut procedere :-

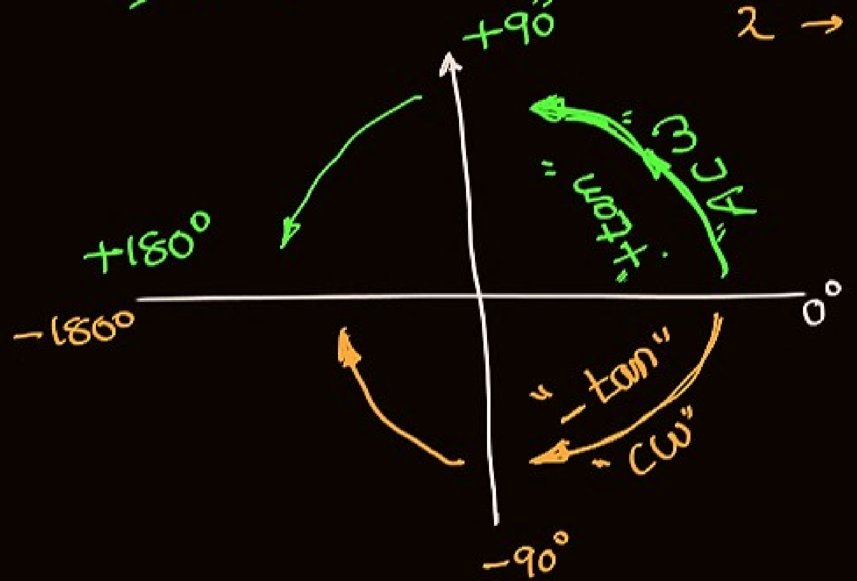
(S1) Find Magnitude & Phase at $\underline{\omega=0}$ & $\underline{\omega=\infty}$.

(S2) $\phi = \pm \cancel{90n} + \tan^{-1}(\omega)$

$\omega=0 \longrightarrow 0$

$\omega=1 \rightarrow \pm 45^{\circ}$

$2 \rightarrow \mp 63^{\circ}$



⇒ The Direction of Polar plot is decided by "tan" terms in $\angle\phi$.

⇒ "−tan" push the plot in clock wise direction.

→ "+tan" push the plot in anticlock wise direction.

53 Starting direction:-

$$\phi = \cancel{\pm 180n} - \tan^{-1}(\omega) + \tan^{-1}\left(\frac{\omega}{2}\right) + \tan^{-1}\left(\frac{\omega}{3}\right)$$

"$-\tan$" is large $\longrightarrow$ CW

"$+\tan$" is large $\longrightarrow$ ACW

⇒ For starting direction, observe the large "tan" term in the phase angle ϕ.

⇒ If "$+\tan$" value large, then direction is ACW.

⇒ If "$-\tan$" value large, then direction is CW.

(S4) Ending direction:-

$$\phi = \cancel{\pm 90^\circ n)} - \tan^{-1}(\omega) + \tan^{-1}\left(\frac{\omega}{2}\right) + \tan^{-1}\left(\frac{\omega}{3}\right)$$

$\underline{2(+) \,\&\, 1(-)} \rightarrow$ "+tan" terms more $\longrightarrow$ ACW

If "−tan" terms are more $\longrightarrow$ CW.

⇒ For ending direction, count the number of "+ve" & "−ve" tan terms. If "−tan" terms are more, then direction is CW.

⇒ If "+ tan" terms are more, then direction is ACW.

<u>NOTE</u>:- If the number of + tan terms = number of − tan terms, then draw the plot by using starting direction. In this case ending direction is not consider.

Prob :- Draw the Polar Plot $G(s)H(s) = \left(\frac{1}{S+1}\right)$

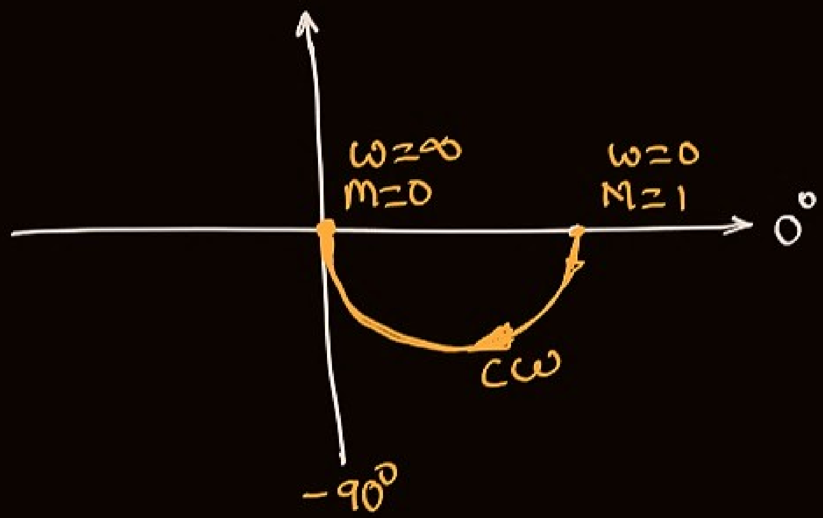
Sol → $GH(S) = \frac{1}{(S+1)}$

$S \to j\omega \Rightarrow GH(j\omega) = \frac{1}{(j\omega+1)}$

$M = \frac{1}{\sqrt{\omega^2+1}}$, $\angle\phi = \frac{\angle 1}{\angle(j\omega+1)} = -\tan^{-1}(\omega)$ → CW

$\omega = 0 \quad 1\angle 0^0$

$\omega = \infty \quad 0\angle -90^0$



Prob :- Draw the polar plot to the following TF.

$$\Rightarrow G(s)H(s) = \frac{1}{(s+1)(s+2)}$$

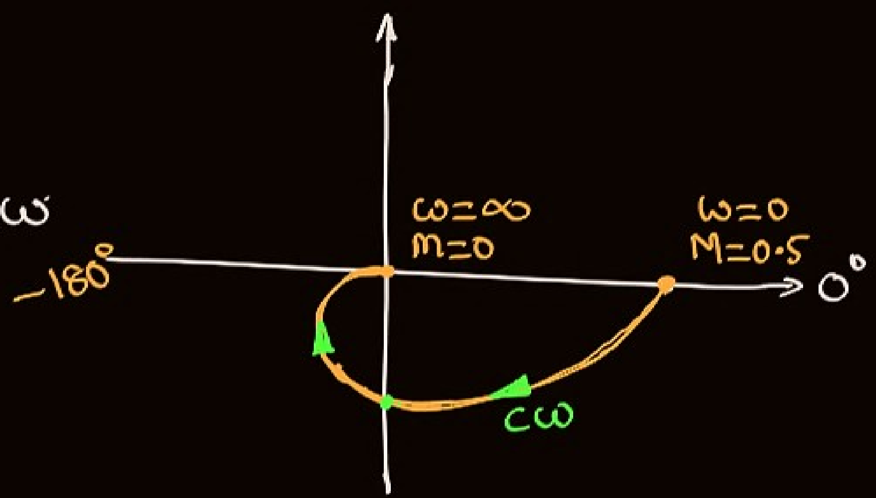
Sol $s \rightarrow j\omega \Rightarrow GH(j\omega) = \frac{1}{(j\omega+1)(j\omega+2)}$

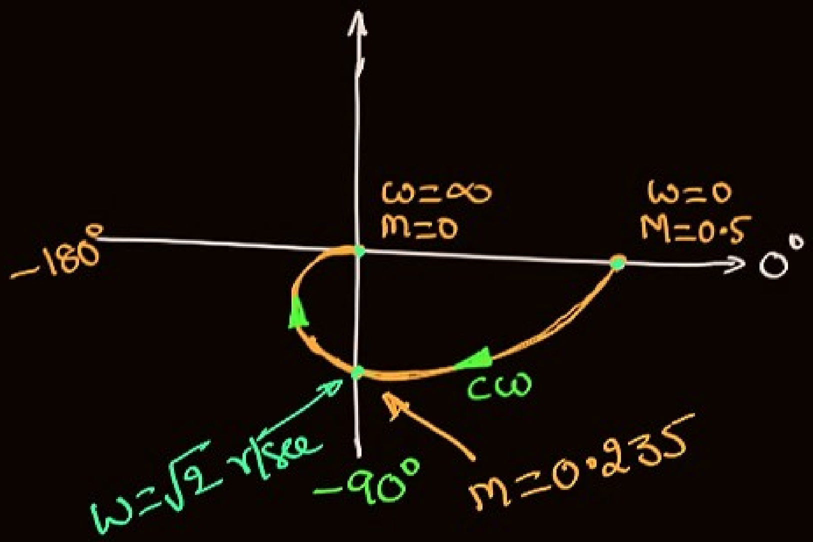
$$M = \frac{1}{\sqrt{\omega^2+1}\sqrt{\omega^2+4}}$$

$$\angle\phi = -\tan^{-1}(\omega) - \tan^{-1}\left(\frac{\omega}{2}\right) \longrightarrow \text{CW}$$

$\omega = 0 \quad 0.5\angle 0^\circ$

$\omega = \infty \quad 0\angle -180^\circ$





IP with $-90^\circ \Rightarrow 0.235\angle -90^\circ$ (Polar)

Rectangular $M(\cos\theta, j\sin\theta)$

$\Rightarrow 0.235(\cos(-90^\circ), j\sin(-90^\circ))$

$\Rightarrow 0.235(0, -j1)$

$\Rightarrow (0, -j0.235)$ ✓

Intersection point with -90°

$$\angle G(j\omega)H(j\omega) = -90^\circ$$

$$+90^\circ = +\tan^{-1}(\omega) + \tan^{-1}\left(\frac{\omega}{2}\right)$$

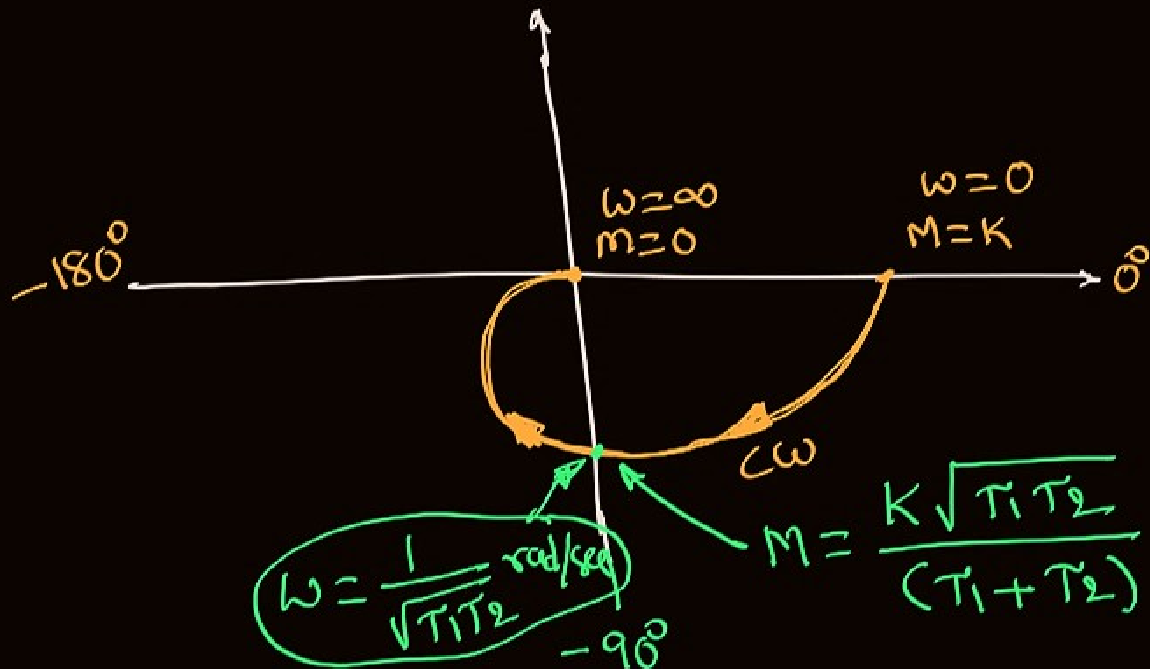
$$90^\circ = \tan^{-1}\left[\frac{\omega + \omega/2}{1-\omega^2/2}\right]$$

$$\infty = \left(\frac{\omega+\omega/2}{1-\omega^2/2}\right)_{=0} \Rightarrow \left(1-\frac{\omega^2}{2}\right) = 0$$

$$\Rightarrow \omega^2 = 2 \Rightarrow \boxed{\omega = \sqrt{2} \text{ rad/sec}}$$

$$M\Big|_{\omega=\sqrt{2}} = \frac{1}{\sqrt{2+1}\sqrt{2+4}} = \frac{1}{\sqrt{18}} = 0.235$$

$$\Rightarrow G(s)H(s) = \frac{K}{(sT_1+1)(sT_2+1)}$$



$$\omega = \frac{1}{\sqrt{T_1T_2}}\ \text{rad/sec}$$

$$M = \frac{K\sqrt{T_1T_2}}{(T_1+T_2)}$$

$$GH(s) = \frac{1}{(s+1)(s+2)}$$

$$= \frac{0.5}{(1+s)(1+0.5s)}$$

$K = 0.5,\ T_1 = 1,\ T_2 = 0.5$ sec

$$\omega = \frac{1}{\sqrt{1\times 0.5}} = \sqrt{2}\ \text{r/sec}$$

$$M = \frac{0.5\sqrt{1\times 0.5}}{(1+0.5)}$$

$$= 0.235.$$

$$\Rightarrow G(s)H(s) = \frac{1}{(s+1)(s+2)(s+3)}$$

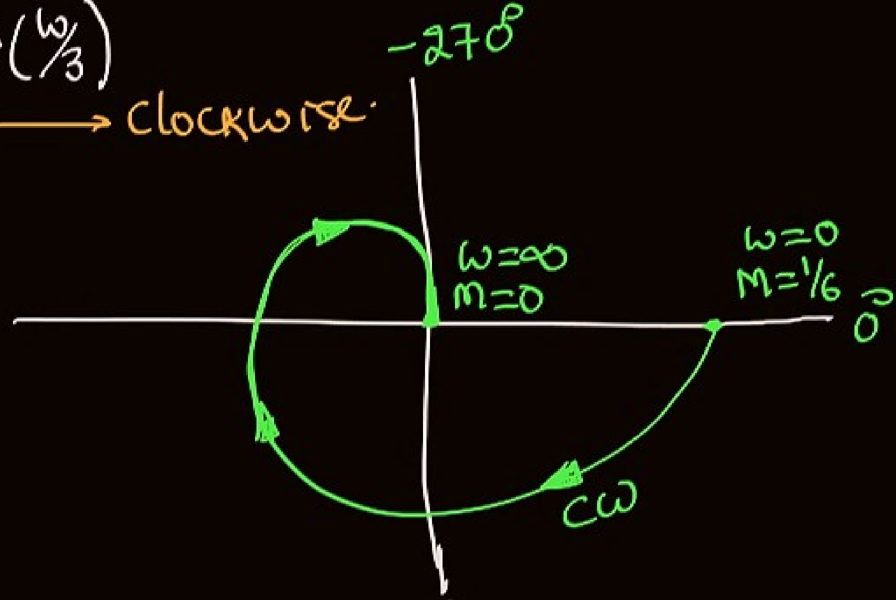
$$s \to j\omega \Rightarrow GH(j\omega) = \frac{1}{(j\omega+1)(j\omega+2)(j\omega+3)}$$

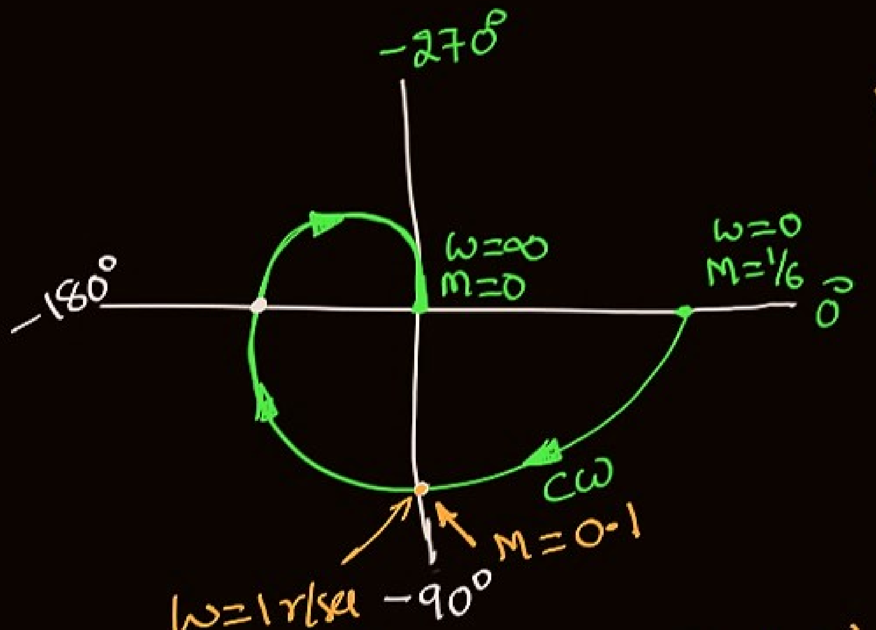
$$M = \frac{1}{\sqrt{(\omega^2+1)(\omega^2+4)(\omega^2+9)}}$$

$$\angle\phi = -\tan^{-1}(\omega) - \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}\left(\frac{\omega}{3}\right) \longrightarrow \text{Clockwise}$$

$\omega = 0 \quad \frac{1}{6}\angle 0^\circ$

$\omega = \infty \quad \underline{\underline{0\angle -270^\circ}}$ ✓





IP with -90° $\Rightarrow$ $0.1\angle -90^\circ$ (Polar)

Rect $\Rightarrow$ $0.1(0, -j1) \Rightarrow$

$\Rightarrow (0, -j0.1)$ Rectangular.

Intersection point with -90°

$$\angle GH(j\omega) = -90^\circ$$

$$+90^\circ = +\tan^{-1}(\omega) + \tan^{-1}\left(\frac{\omega}{2}\right) + \tan^{-1}\left(\frac{\omega}{3}\right)$$

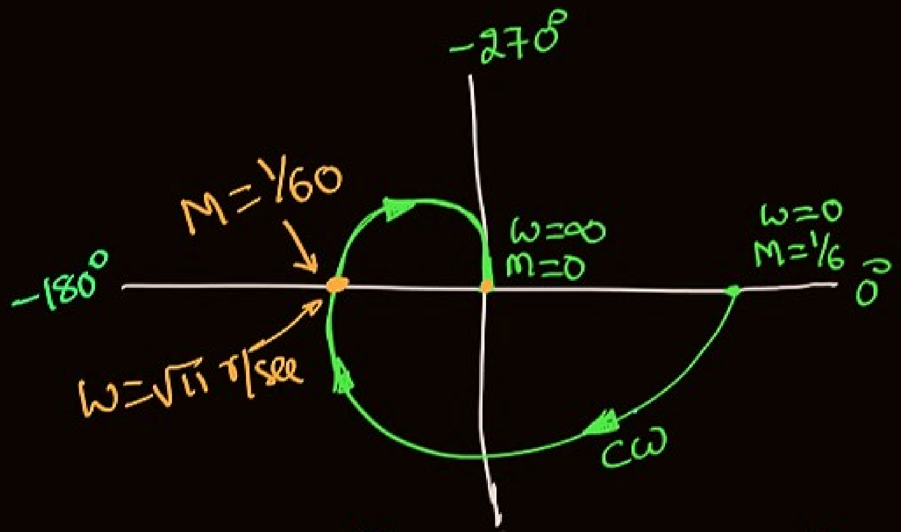
$$90^\circ = \tan^{-1}\left[\frac{\left(\omega + \frac{\omega}{2} + \frac{\omega}{3}\right) - \left(\omega \cdot \frac{\omega}{2} \cdot \frac{\omega}{3}\right)}{1 - \left[\omega\frac{\omega}{2} + \frac{\omega}{2}\frac{\omega}{3} + \frac{\omega}{3}\omega\right]}\right]$$

$$\infty = \left[\frac{\omega + \omega/2 + \omega/3 - \omega^3/6}{1 - \left[\omega^2/2 + \omega^2/6 + \frac{\omega^2}{3}\right]}\right] = 0$$

$$1 - \left[\frac{\omega^2}{2} + \frac{\omega^2}{6} + \frac{\omega^2}{3}\right] = 0$$

$$1 = \frac{3\omega^2 + 1\omega^2 + 2\omega^2}{6} \Rightarrow 1 = \frac{6\omega^2}{6} \Rightarrow \boxed{\omega = 1} \text{ r/sec}$$

$$M_{|\omega=1} = \frac{1}{\sqrt{(1+1)(1+4)(1+9)}} = 0.1$$



IP with -180° → $\frac{1}{60}\angle -180^\circ$ (Polar)

Rectangular → $\frac{1}{60}(-1, j0)$

$= \left(-\frac{1}{60}, j0\right)$

Intersection point with -180°

$$\angle GH(j\omega) = -180^\circ$$

$$+180^\circ = +\tan^{-1}(\omega) + \tan^{-1}\left(\frac{\omega}{2}\right) + \tan^{-1}\left(\frac{\omega}{3}\right)$$

$$180^\circ = \tan^{-1}\left[\frac{\left(\omega + \frac{\omega}{2} + \frac{\omega}{3}\right) - \left(\omega \cdot \frac{\omega}{2} \cdot \frac{\omega}{3}\right)}{1 - \left[\omega\frac{\omega}{2} + \frac{\omega}{2}\frac{\omega}{3} + \frac{\omega}{3}\omega\right]}\right]$$

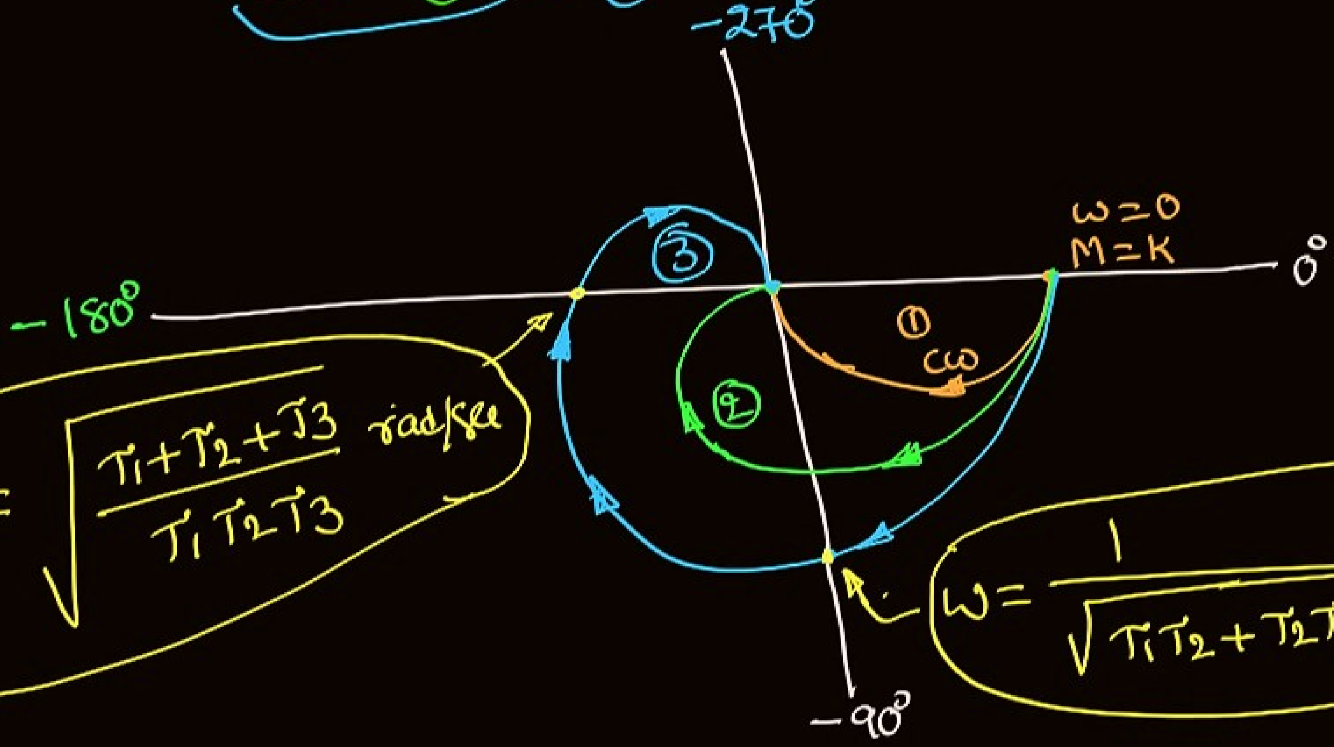
$$0 = \left[\frac{\omega + \omega/2 + \omega/3 - \omega^3/6}{1 - \left[\omega^2/2 + \omega^2/6 + \frac{\omega^2}{3}\right]}\right] = 0$$

$$\Rightarrow \left(\omega + \omega/2 + \omega/3 - \omega^3/6\right) = 0$$

$$\frac{6\omega + 3\omega + 2\omega}{6} = \omega^3/6 \Rightarrow 11\omega = \omega^3 \Rightarrow \boxed{\omega = \sqrt{11}\ r/sec}$$

$$M_{/\omega=\sqrt{11}} = \frac{1}{\sqrt{11+1}\sqrt{11+4}\sqrt{11+9}} = \frac{1}{60}$$

$$\Rightarrow \quad GH(\omega) = \frac{K}{(sT_1+1)(sT_2+1)(sT_3+1)}$$



$$\omega = \sqrt{\frac{T_1+T_2+T_3}{T_1T_2T_3}} \text{ rad/sec}$$

$$\omega = \frac{1}{\sqrt{T_1T_2+T_2T_3+T_3T_1}} \text{ rad/sec}$$

Conclusion:- The addition of each finite pole, shift the ending angle by -90° in the clockwise direction.

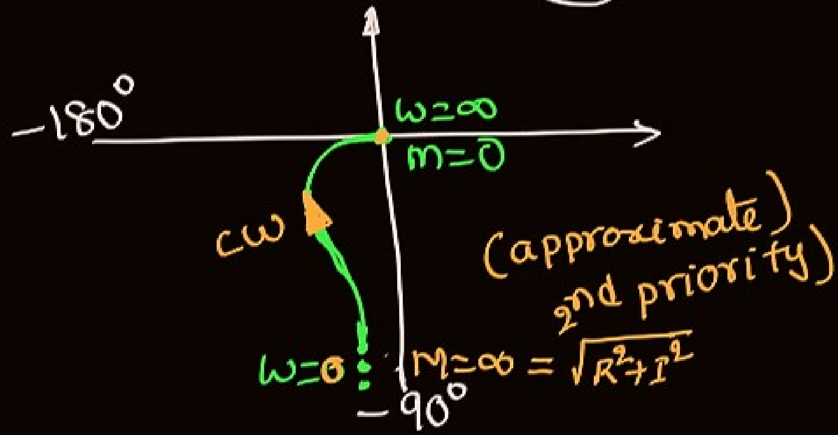
Prob:- Draw the polar plot. $\Rightarrow G(s)H(s) = \dfrac{1}{\underline{s}(s+1)}$

Sol:- $s \to j\omega \Rightarrow GH(j\omega) = \dfrac{1}{j\omega(j\omega+1)}$

$$\Rightarrow M = \frac{1}{\omega\sqrt{\omega^2+1}}, \quad \angle\phi = -90^\circ - \tan^{-1}(\omega) \longrightarrow \text{CW}$$

$\omega = 0 \quad \infty\angle -90^\circ$

$\omega = \infty \quad 0\angle -180^\circ$



Polar $\rightarrow$ with $-90°$ $\rightarrow$ $\boxed{\infty\angle -90°}$

Rationalization :-

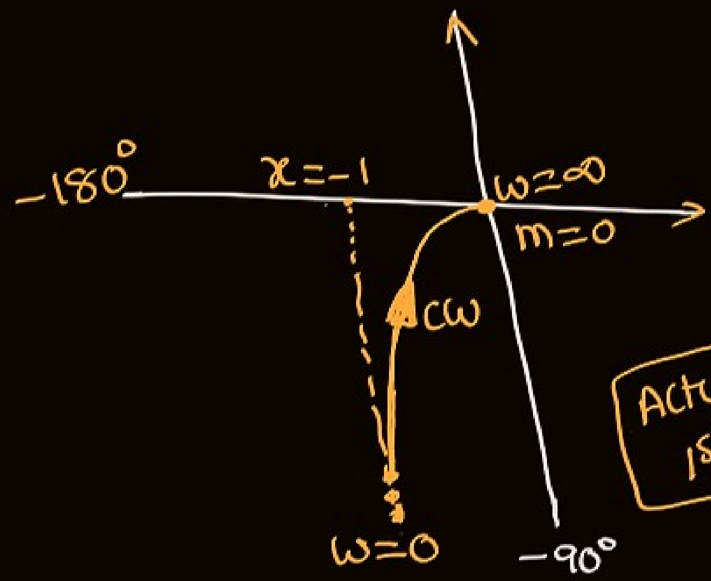
$$\Rightarrow \frac{(1-j\omega)}{j\omega(1+j\omega)(1-j\omega)}$$

$$\Rightarrow \frac{(1-j\omega)}{j\omega(1+\omega^2)} \Rightarrow \frac{-j(1-j\omega)}{\omega(1+\omega^2)}$$

$$\Rightarrow \frac{-\cancel{\omega}}{\cancel{\omega}(1+\omega^2)} - \frac{j}{\omega(1+\omega^2)}$$

$$\Rightarrow \frac{-1}{1+\omega^2} - \frac{j}{\omega(1+\omega^2)}$$

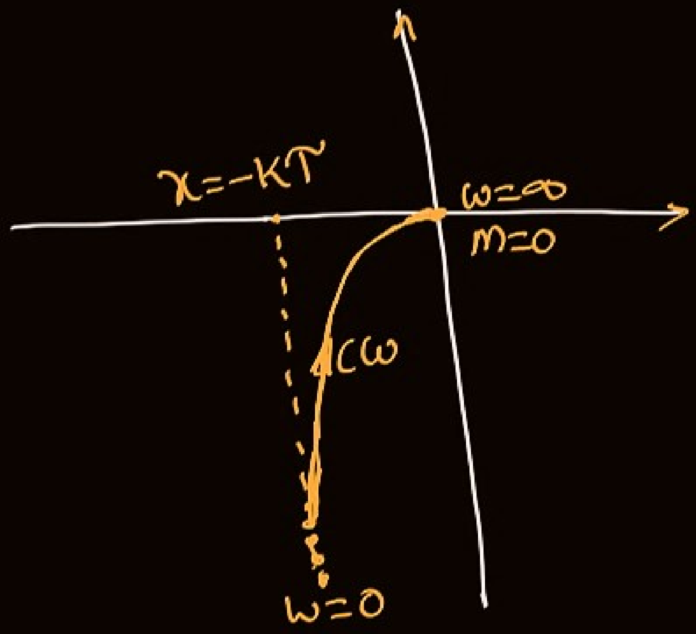
$\Rightarrow$ $\omega = 0 \Rightarrow (-1 - j\infty)$ (Rectangular)



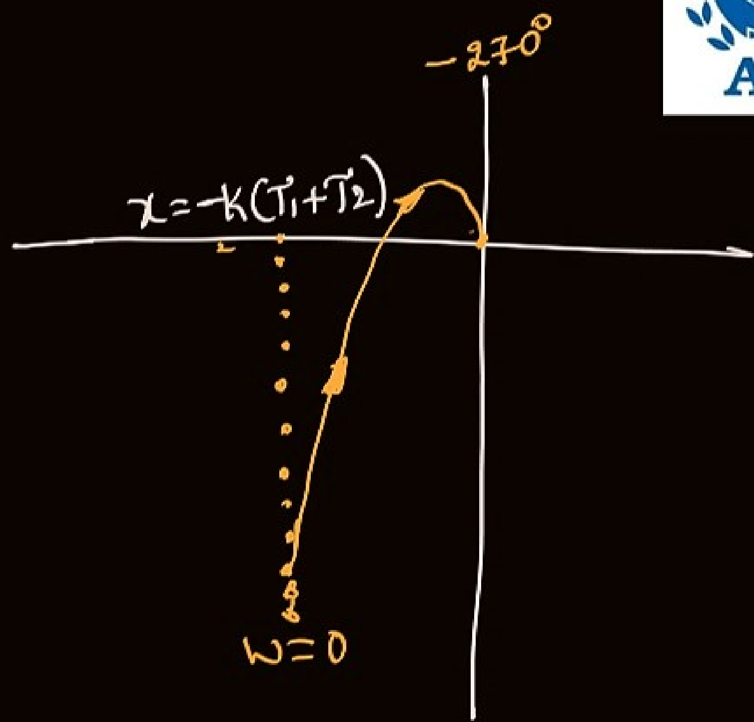
Actual plot 1st priority

$\Rightarrow$ At $\omega=0$, the polar plot is asymptotic to the straight line of $x=-1$

$$GH(s) = \frac{K}{s(sT+1)}$$



$$GH(s) = \frac{K}{s(sT_1+1)(sT_2+1)}$$



Prob :- $GH(s) = \frac{1}{s^2(s+1)}$

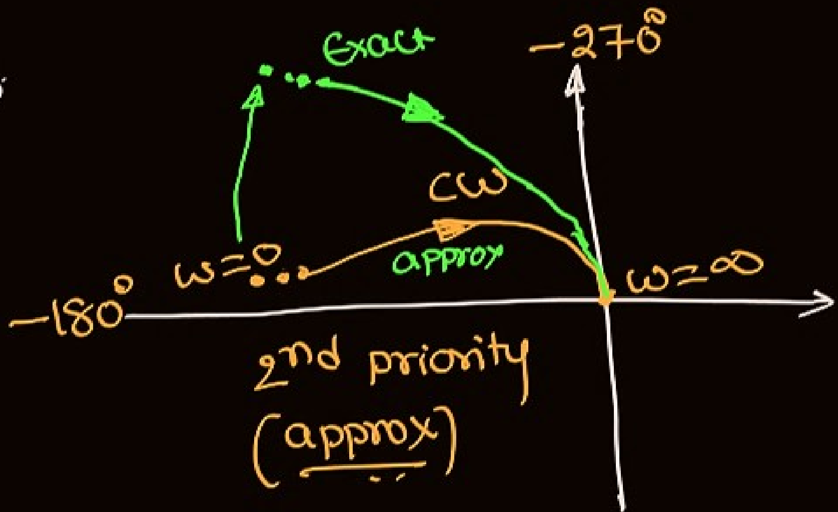
$s \to j\omega \quad GH(j\omega) = \frac{1}{(j\omega)^2(j\omega+1)}$

$M = \frac{1}{\omega^2\sqrt{\omega^2+1}}$

$\angle\phi = -180^\circ - \underrightarrow{\tan^{-1}(\omega)}\ \text{cw}$

$\omega = 0 \quad \infty\angle{-\underline{180}^\circ}$

$\omega = \infty \quad 0\angle{-270^\circ}$



Rationalization :-

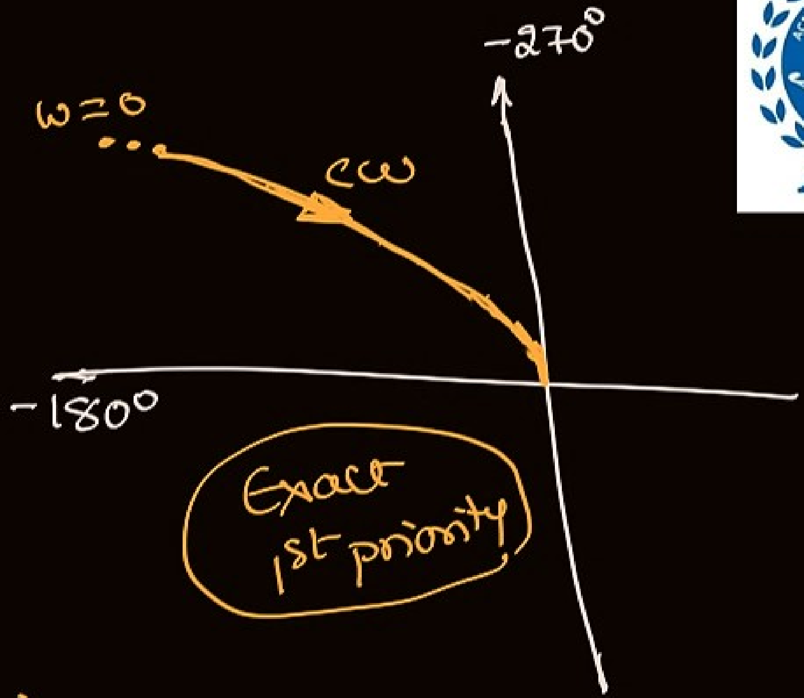
$$\frac{(1-j\omega)}{(j\omega)^2(1+j\omega)(1-j\omega)}$$

$$\frac{(1-j\omega)}{-\omega^2(1+\omega^2)}$$

$$-\frac{1}{\omega^2(1+\omega^2)}+\frac{j\cancel{\omega}}{\omega^{\cancel{2}}(1+\omega^2)}$$

$$-\frac{1}{\omega^2(1+\omega^2)}+\frac{j}{\omega(1+\omega^2)}$$

$\omega = 0 \Rightarrow (-\infty + j\infty)$ (Rectangular)



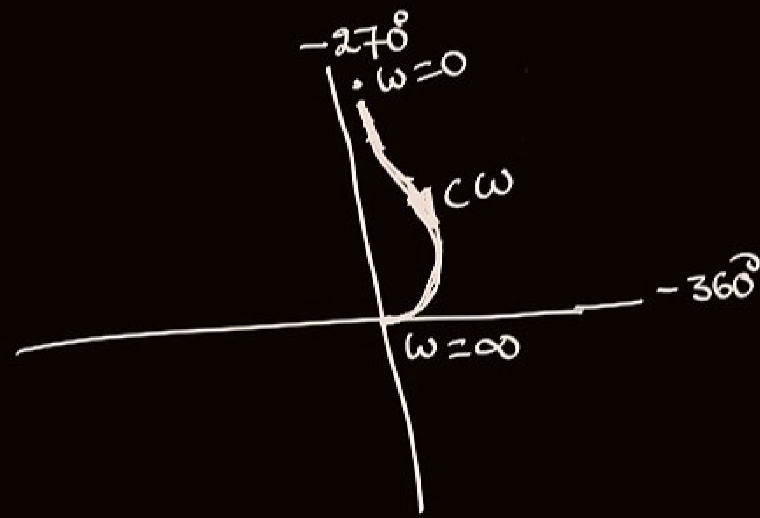
Prob $GH(s) = \dfrac{1}{s^3(s+1)}$

$s \to j\omega \Rightarrow GH(j\omega) = \dfrac{1}{(j\omega)^3\,(j\omega+1)}$

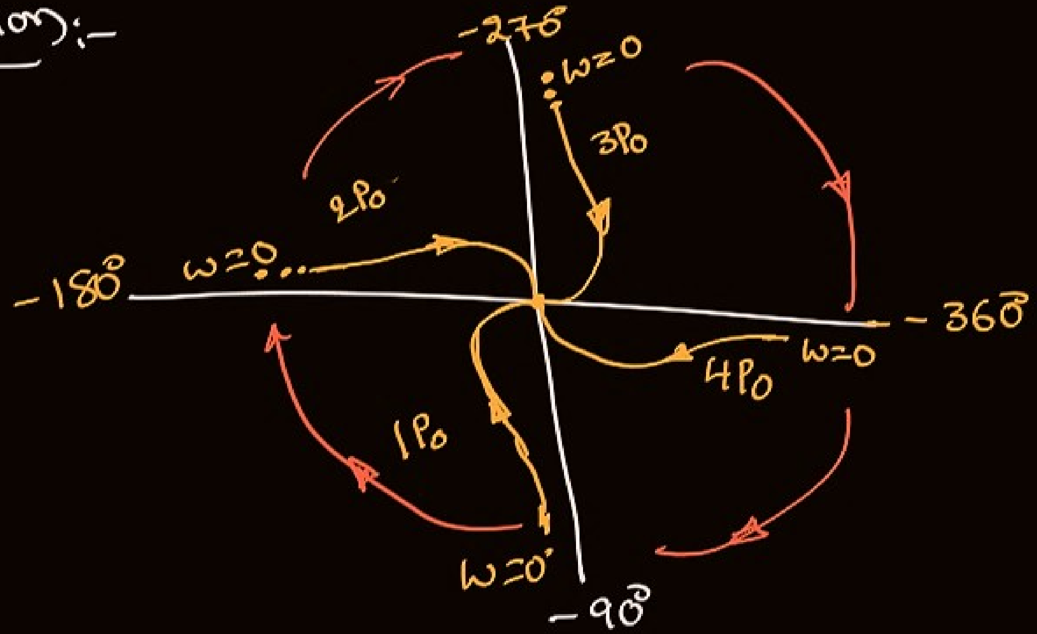
$M = \dfrac{1}{\omega^3\sqrt{\omega^2+1}}$ $\quad \angle\phi = -270^\circ - \tan^{-1}(\omega) \longrightarrow cw$

$\omega = 0 \quad \infty\angle -270^\circ$

$\omega = \infty \quad 0\angle -360^\circ$



Conclusion:-



⇒ The addition each pole at origin, shift total plot by -90° in the clockwise direction.