

# HAPPY *Raksha* BANDHAN

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opportunity this Raksha Bandhan



#LetsACEit

11-08-22



Q. A Non-uniform surface charge density of  $\frac{1}{x^2+y^2+4} \frac{nc}{m^2}$  which is defined for  $\rho \leq 2.5 \text{ m}, z = 5 \text{ m}$ . Find total charge.

Soln:  $Q = \iint f_s dA$   $\frac{nc}{m^2}$

$(\rho, \phi, z)$   
1,  $\rho$ , 1

$\frac{nc}{m^2} \times m^2$

$$d\vec{A} = \rho d\rho d\phi \hat{z} + \underbrace{\rho d\phi dz}_{0} \hat{\rho} + \underbrace{dz d\rho}_{0} \hat{\phi}$$

$\hat{z} = \hat{z}, dz = 0, d\vec{A} = \rho d\rho d\phi \hat{z}$

$$dA = \rho d\rho d\phi$$

we know

$$x = \rho \cos \phi$$

$$y = \rho \sin \phi$$

$$x^2 + y^2 = \rho^2$$

$$f_s = \frac{1}{\rho^2 + 4} nc$$

$$Q = \iint \frac{1}{\rho^2 + 4} \rho d\rho d\phi$$

$$Q = \frac{1}{2} \int_0^{2.5} \frac{\rho}{\rho^2 + 4} d\rho \int_0^{2\pi} d\phi$$

$$Q = \frac{1}{2} \left[ \ln(\rho^2 + 4) \right]_0^{2.5} \left[ \phi \right]_0^{2\pi}$$

$$= \frac{1}{2} \left\{ \ln(2.5^2 + 4) - \ln(4) \right\} \times 2\pi$$

$$= \pi \left\{ \ln(2.5^2 + 4) - \ln(4) \right\}$$

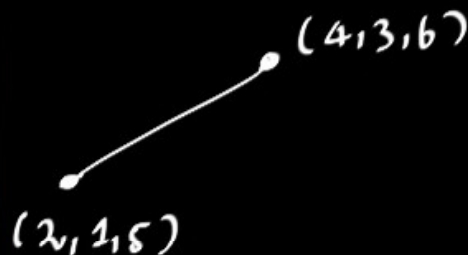
$$Q = \underline{2.95} nc$$

Q. Given  $\rho_l = 2x + 3y - 4z$  c/m the charge on the line segment extending from (2, 1, 5) to (4, 3, 6) is \_\_\_\_\_

- (a) 10 C
- (b) -10 C
- (c) 30
- (d) -30 C

Soln:

$$Q = \int \rho_l \, dl$$



$$\frac{x-2}{4-2} = \frac{y-1}{3-1} = \frac{z-5}{6-5} = t$$

$$0 \leq t \leq 1$$

$$x = 2t + 2 \Rightarrow \frac{dx}{dt} = 2$$

$$y = t + 1 \Rightarrow \frac{dy}{dt} = 1$$

$$z = t + 5 \Rightarrow \frac{dz}{dt} = 1$$

$$\rho_l = 2(2t + 2) + 3(t + 1) - 4(t + 5)$$

$$\rho_l = 6t - 13 \text{ c/m}$$

$$d\vec{l} = dx \hat{x} + dy \hat{y} + dz \hat{z}$$

$$d\vec{l} = \frac{dx}{dt} dt \hat{x} + \frac{dy}{dt} dt \hat{y} + \frac{dz}{dt} dt \hat{z}$$

$$d\vec{r} = x dt \hat{x} + y dt \hat{y} + dz \hat{z}$$

$$dl = \sqrt{(x dt)^2 + (y dt)^2 + (dz)^2}$$

$$\underline{dl = 3 dt}$$

$$Q = \int f_1 dl = \int (6t - 13) 3 dt$$

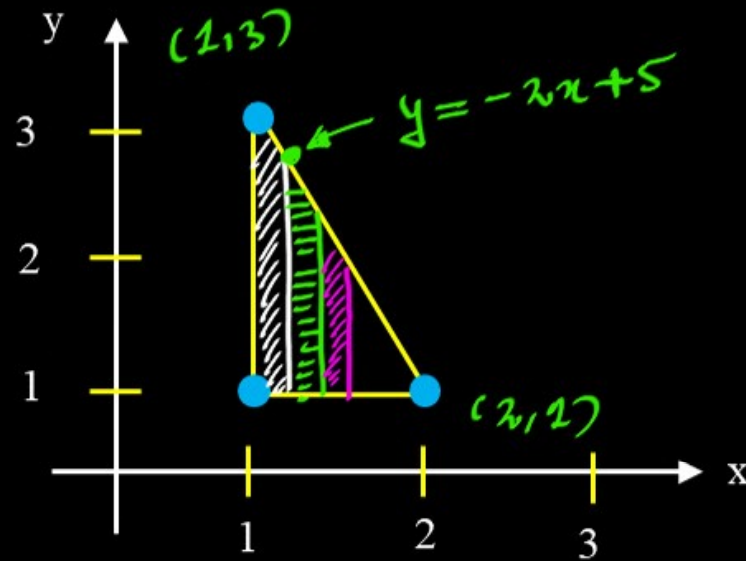
$$= 3 \left[ \frac{6t^2}{2} - 13t \right]_0^1$$

$$= 3 [ 3 \times 1^2 - 13 \times 1 ]$$

$$= 3 [ -10 ] = -30$$

$$\underline{Q = -30 C}$$

Q. The total charge on the triangle of figure, given surface charge density  $\rho_s = 6xy \text{ c/m}^2$  is \_\_\_\_\_



Soln:

$$Q = \iint \rho_s dA$$

$$Q = \iint 6xy \, dx \, dy$$

$$Q = 6 \int_{x=1}^2 \int_{y=1}^{-2x+5} xy \, dy \, dx$$

$$= 6 \int_{x=1}^2 x \left[ \frac{y^2}{2} \right]_1^{-2x+5} dx$$

$$\frac{x-1}{2-1} = \frac{y-3}{1-3}$$

$$(x-1)(-2) = y-3$$

$$-2x + 2 = y - 3$$

$$y = -2x + 5$$

$$= 3 \int x \{ (-2x+5)x - 1x \} dx$$

$$= 3 \int x [ 4x^2 + 25 - 20x - 1 ] dx$$

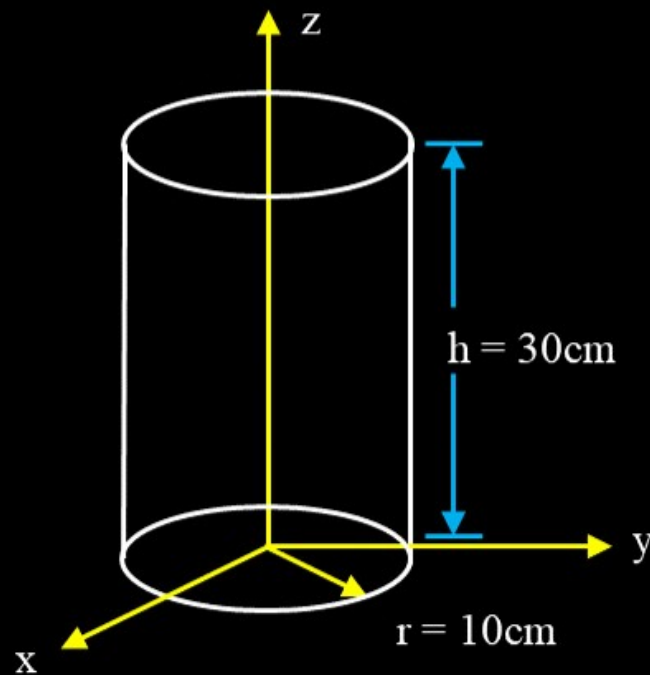
$$= 3 \int [ 4x^3 - 20x^2 + 24x ] dx$$

$$= 3 \left[ \cancel{4} \frac{x^4}{\cancel{4}} - 20 \frac{x^3}{3} + \cancel{20} \frac{x^2}{\cancel{2}} \right]_1^2$$

$$= 3 \left\{ \left( 2^4 - 20 \times \frac{2^3}{3} + 12 \times 2^2 \right) - \left( 1^4 - 20 \times \frac{1^3}{3} + 12 \times 1^2 \right) \right\}$$

$$= 3 \left[ 16 - \frac{160}{3} + 48 - 1 + \frac{20}{3} - 12 \right] = \underline{\underline{13}} \text{ C}$$

Q. If the volume charge density is  $\rho = 100 e^{-z} (x^2 + y^2)^{-1/4} \text{ C/m}^3$ , the total charge contained in the cylindrical shown in the figure is \_\_\_\_\_



Soln:

$$Q = \iiint \rho_v dV$$

$$\rho^2 = x^2 + y^2$$

$$\rho_v = 100 e^{-z} (\rho^2)^{-1/4}$$

$$\rho_v = 100 e^{-z} \rho^{-1/2} \text{ C/m}^3$$

$$Q = \iiint 100 e^{-z} \rho^{-1/2} \cdot \rho d\rho d\phi dz$$



$$Q = 100 \iiint f^{1/2} e^{-z} dz d\phi dr$$

$$= 100 \int f^{1/2} dr \int d\phi \int e^{-z} dz$$

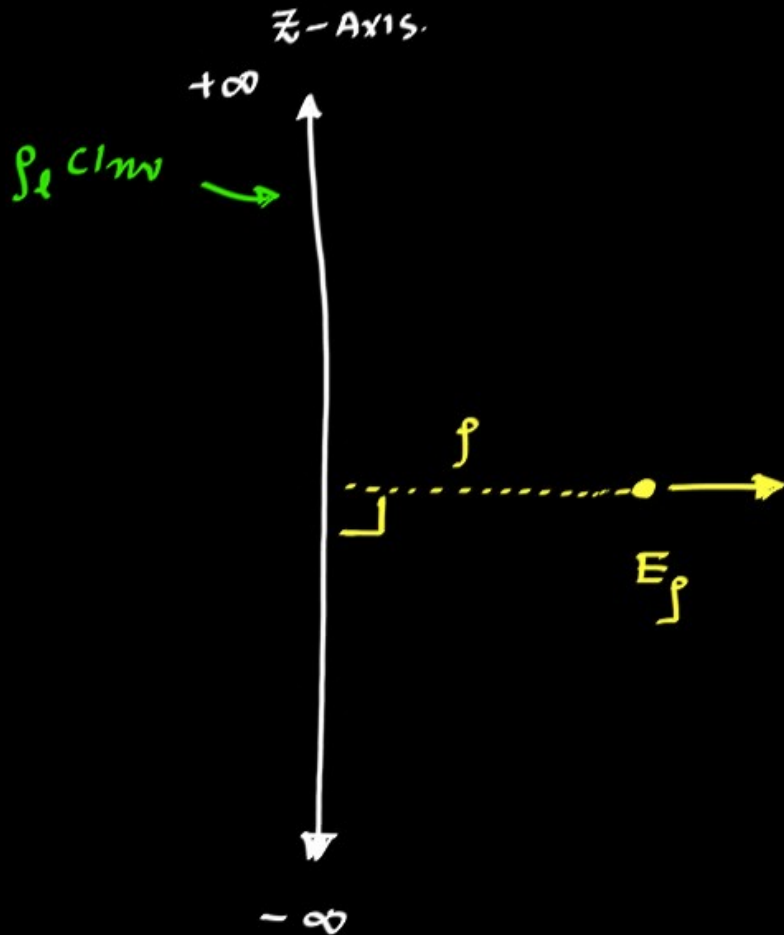
$$= 100 \left[ \frac{r^{3/2}}{3/2} \right]_0^{0.1m} \left[ \phi \right]_0^{2\pi} \left[ -e^{-z} \right]_0^{0.3m}$$

$$= \frac{100 \times 2}{3} \left[ (0.1)^{3/2} \right] \left[ 2\pi \right] \left\{ - (e^{-0.3} - 1) \right\}$$

$$Q = \underline{\underline{3.433}} \text{ C}$$



## Electric Field Due to Infinite Uniform line Charge ( $\rho_l$ C/m)



$$\vec{E} = \int \frac{\rho_l dl}{4\pi\epsilon_0 r^2} \hat{r}$$

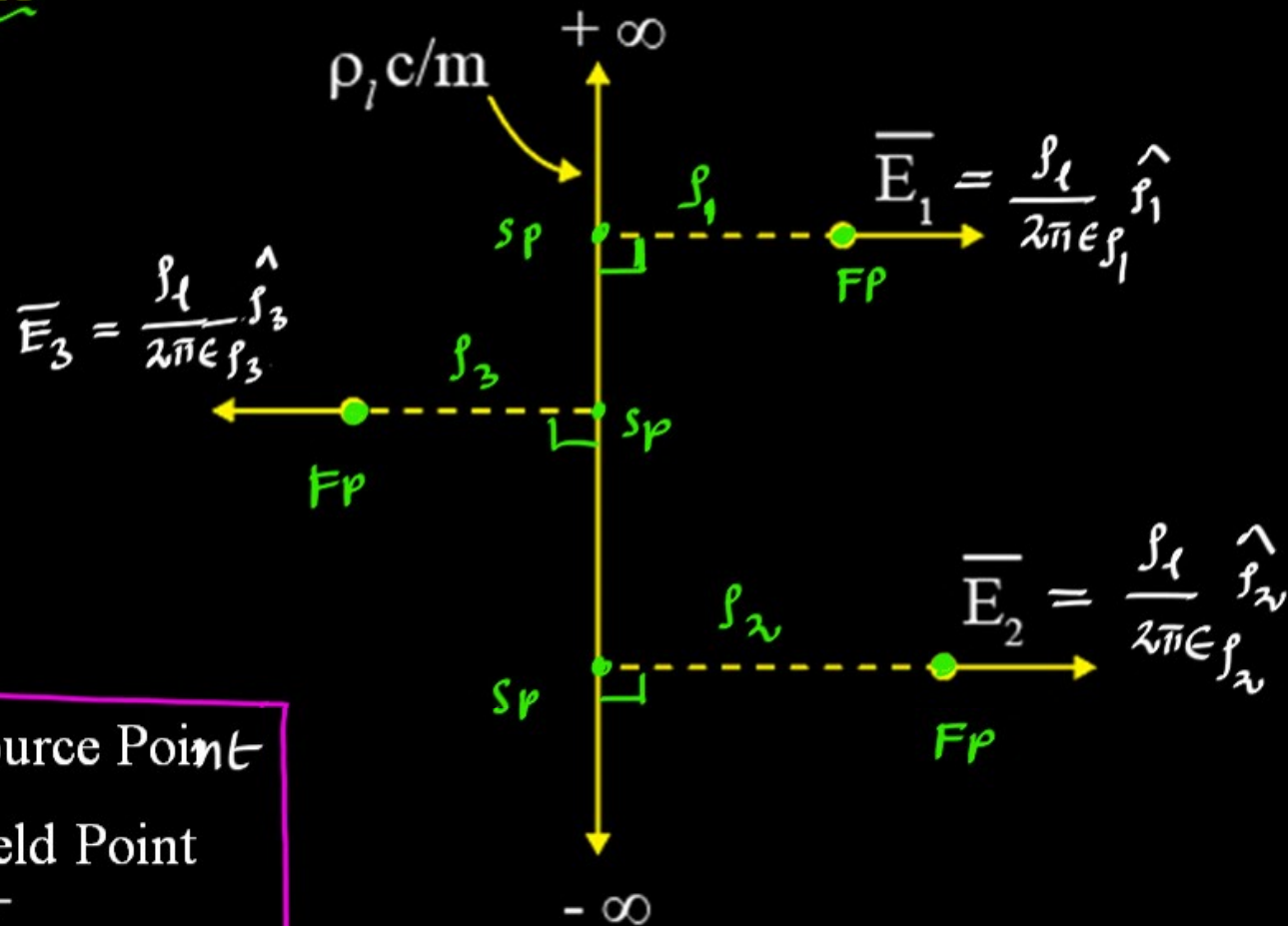


$$\vec{E} = \frac{\rho_l}{2\pi\epsilon_0 \rho} \hat{\rho}$$

$\perp$  DISTANCE

RADIAL DIRECTION

## Example

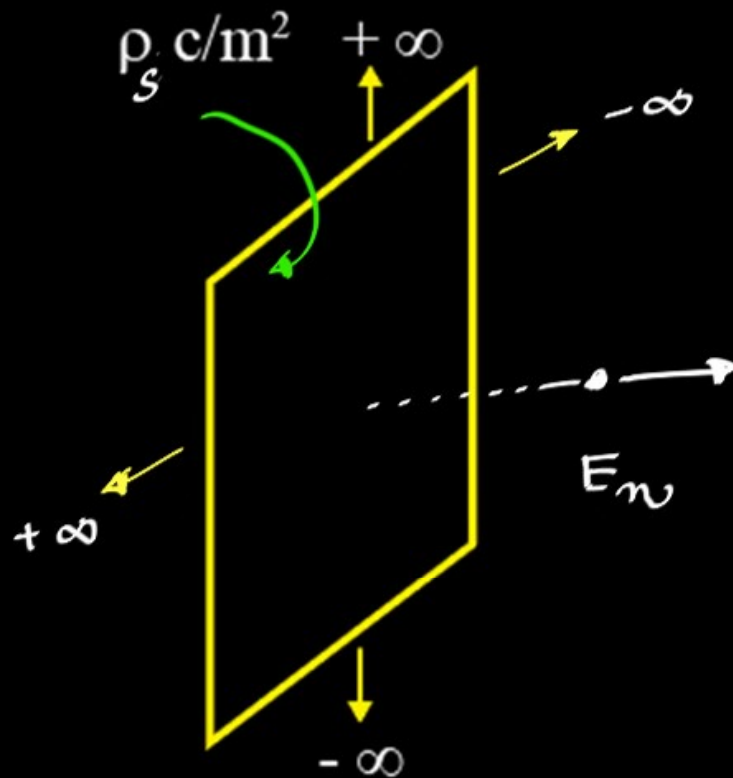


SP: Source Point

FP: Field Point

$$\text{SP} : \frac{\vec{E}}{\rho} : \text{FP}$$

## Electric field due to Infinite Uniform Sheet Charge ( $\rho_s \text{ C/m}^2$ )



$$\vec{E} = \iint \frac{\rho_s dA}{4\pi\epsilon R^2} \hat{r}$$

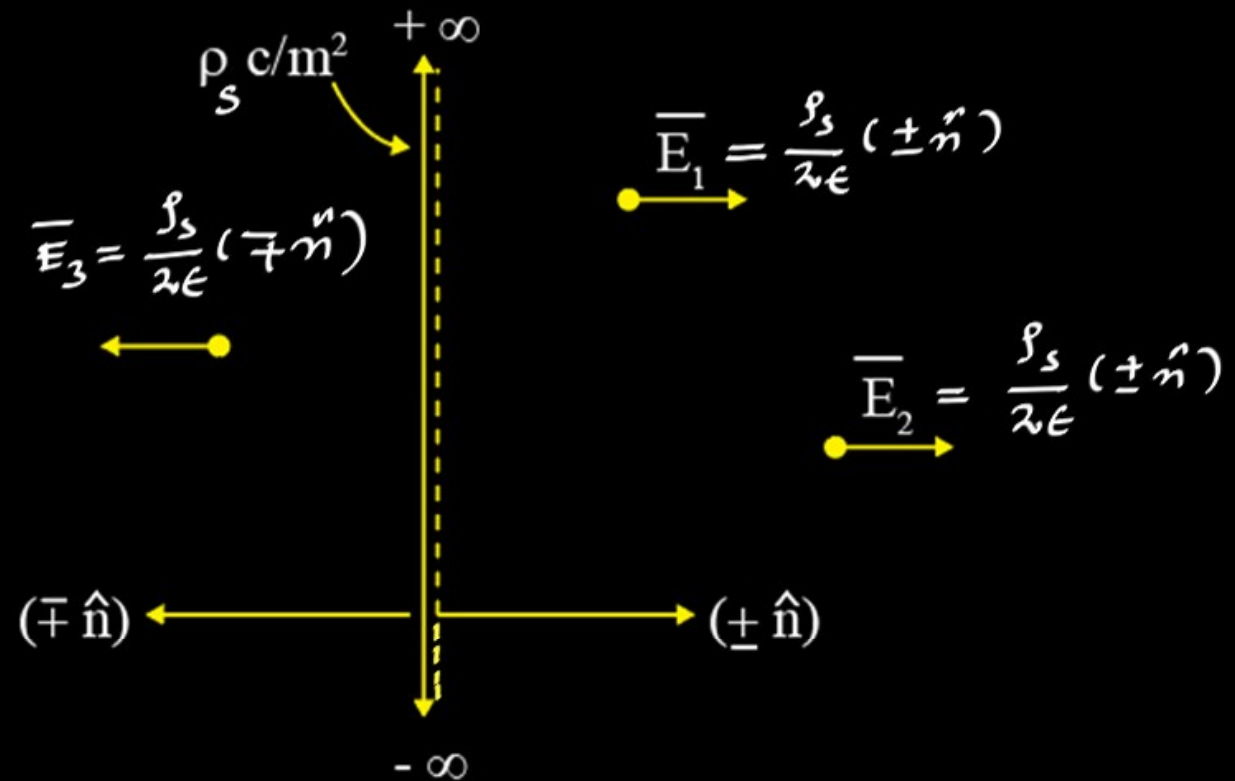


$$\vec{E} = \frac{\rho_s}{2\epsilon} (\pm \hat{n})$$

INDEPENDENT  
OF DISTANCE

NORMAL  
DIRECTION.

## Example



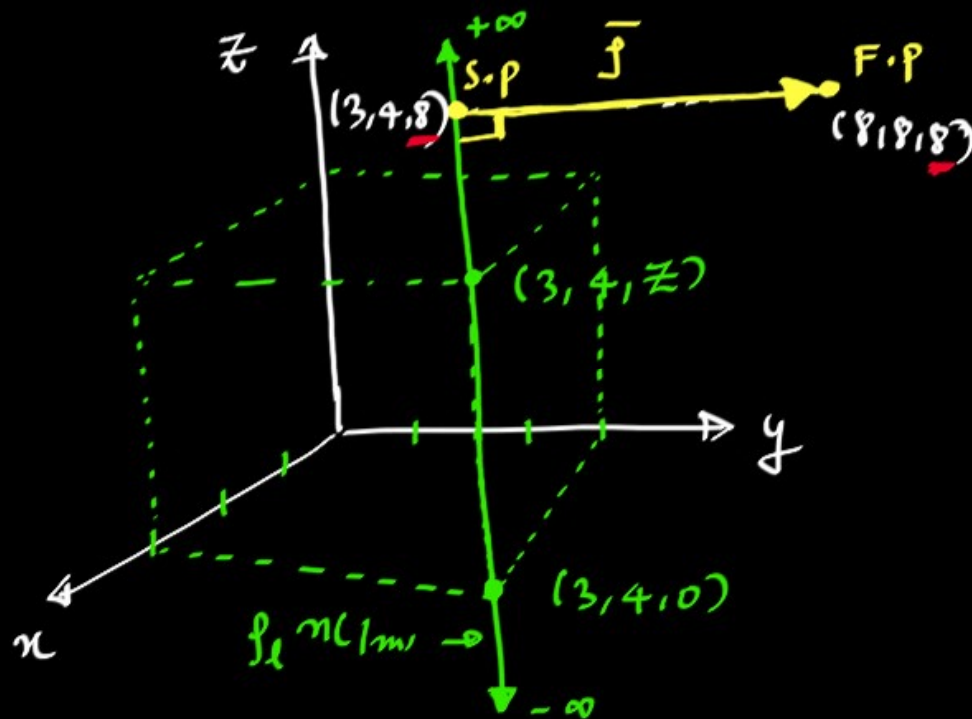
Q. Infinite uniform line of <sup>charge</sup>  $10 \text{ nc/m}$  is located at  $x=3\text{m}$ ,  $y=4\text{m}$ , and parallel to z-axis. Find electric field due to this infinite line at following points

(a)  $(0,0,0)$

(b)  $(4,5,6)$

(c)  $(8,8,8)$

Soln:



$$\vec{E} = \frac{\rho_l}{2\pi\epsilon_0} \hat{j}, \quad \hat{j} = \frac{\vec{j}}{j}$$

$$\vec{E} = \frac{\rho_l}{2\pi\epsilon_0} \left[ \frac{\vec{j}}{j^2} \right]$$

$$\textcircled{a} \text{ } sp(3, 4, \hat{z}) \xrightarrow[\bar{f}_1]{\substack{4 \\ || \\ 0}} FP(0, 0, 0)$$

$$\begin{aligned} \bar{E}_1 &= \frac{f_1}{2\pi\epsilon_0} \left[ \frac{\bar{f}_1}{f_1 z} \right] \\ &= \frac{10 \times 10^{-9}}{2\pi \times \frac{1}{36\pi \times 10^9}} \left[ \frac{-3\hat{x} - 4\hat{y}}{(\sqrt{9+16})^2} z \right] \\ &= \frac{180}{25} [-3\hat{x} - 4\hat{y}] \\ \bar{E}_1 &= -21.6\hat{x} - 28.8\hat{y} \end{aligned}$$

$$\textcircled{b} \text{ } sp(3, 4, \hat{z}) \xrightarrow[\bar{f}_2]{\substack{4 \\ || \\ 6}} FP(4, 5, 6)$$

$$\begin{aligned} \bar{E}_2 &= \frac{f_2}{2\pi\epsilon_0} \left[ \frac{\bar{f}_2}{f_2 z} \right] \\ &= \frac{10 \times 10^{-9}}{2\pi \times \frac{1}{36\pi \times 10^9}} \left[ \frac{\hat{x} + \hat{y}}{(\sqrt{1^2+1^2})^2} z \right] \\ \bar{E}_2 &= \frac{180}{25} [\hat{x} + \hat{y}] \\ \bar{E}_2 &= 90\hat{x} + 90\hat{y} \end{aligned}$$

$$\textcircled{c} \quad S_p(3, 4, \overline{8}) \xrightarrow[\overline{f_3}]{} F.p(8, 8, 8)$$

$\parallel$   
 $\overline{8}$

$$\overline{E}_3 = \frac{f_1}{2\pi\epsilon_0} \left[ \frac{\overline{f_3}}{f_3^2} \right]$$



Q An Infinity long uniform charge of density  $30 \text{ nC/m}$  is located at  $y=3\text{m}$ ,  $z=5\text{m}$ . The field intensity at  $(0,6,2)$  is  $\vec{E} = 64.7\hat{y} - 86.3\hat{z} \text{ V/m}$ . What is the field intensity at  $(5,6,2)$ ?

(ESE - 04)

(a)  $\vec{E}$

(b)  $\left[ \frac{6^2+1^2}{6^2+5^2+1^2} \right]^{1/2} \vec{E}$

(c)  $\left[ \frac{6^2+1^2}{6^2+5^2+1^2} \right] \vec{E}$

(d)  $\left[ \frac{6^2+5^2+1^2}{6^2+1^2} \right]^{1/2} \vec{E}$

ESE - EC  
PRELIMS

\* ELECTROSTATICS  
\* MAGNETOSTATICS

(COLLECT FOR PRACTICE)

Soln:

$$\begin{array}{ccc} \text{sp}(\underbrace{\lambda}_{\substack{4 \\ || \\ 0}}, 3, 5) & \xrightarrow{\bar{f}_1} & \text{FP}(0, 6, \lambda) \end{array}$$

$$\bar{E}_1 = \left[ \frac{f_1}{2\pi\epsilon_0} \right] \left[ \begin{array}{c} \bar{f}_1 \\ \bar{f}_1 \lambda \end{array} \right] = \bar{E}$$

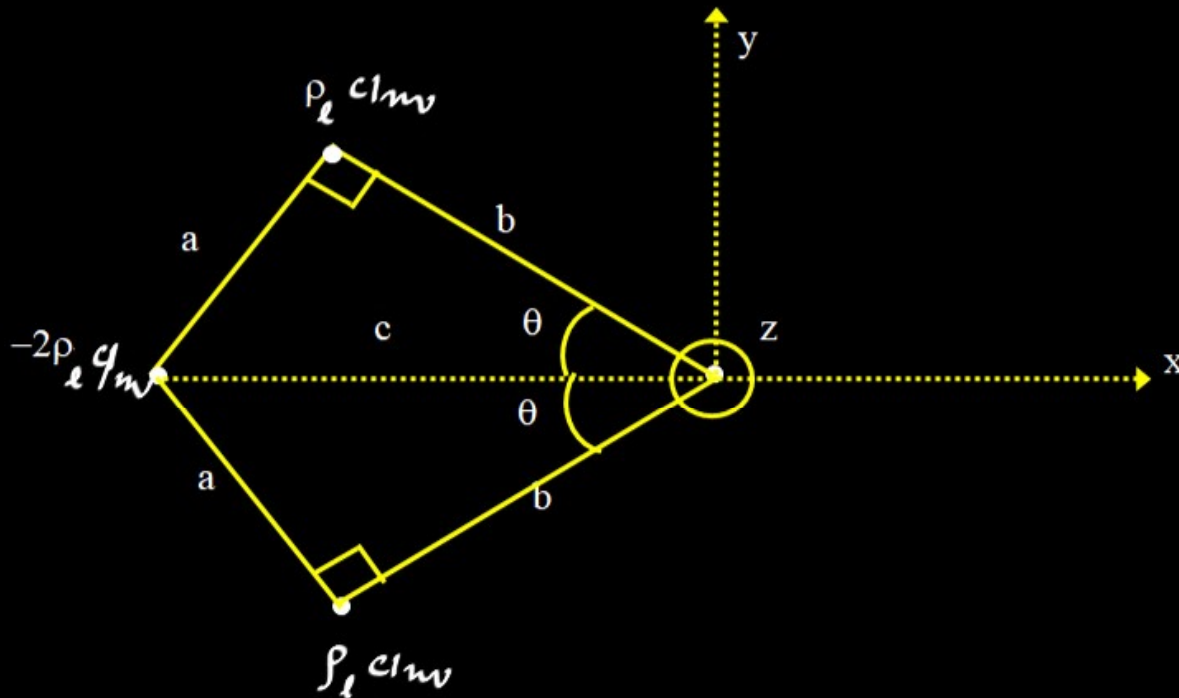
$$\bar{f}_1 = \underline{\underline{3\hat{y} - 3\hat{z}}}$$

$$\begin{array}{ccc} \text{sp}(\underbrace{\lambda}_{\substack{4 \\ || \\ 5}}, 3, 5) & \xrightarrow{\bar{f}_\lambda} & \text{FP}(5, 6, \lambda) \end{array}$$

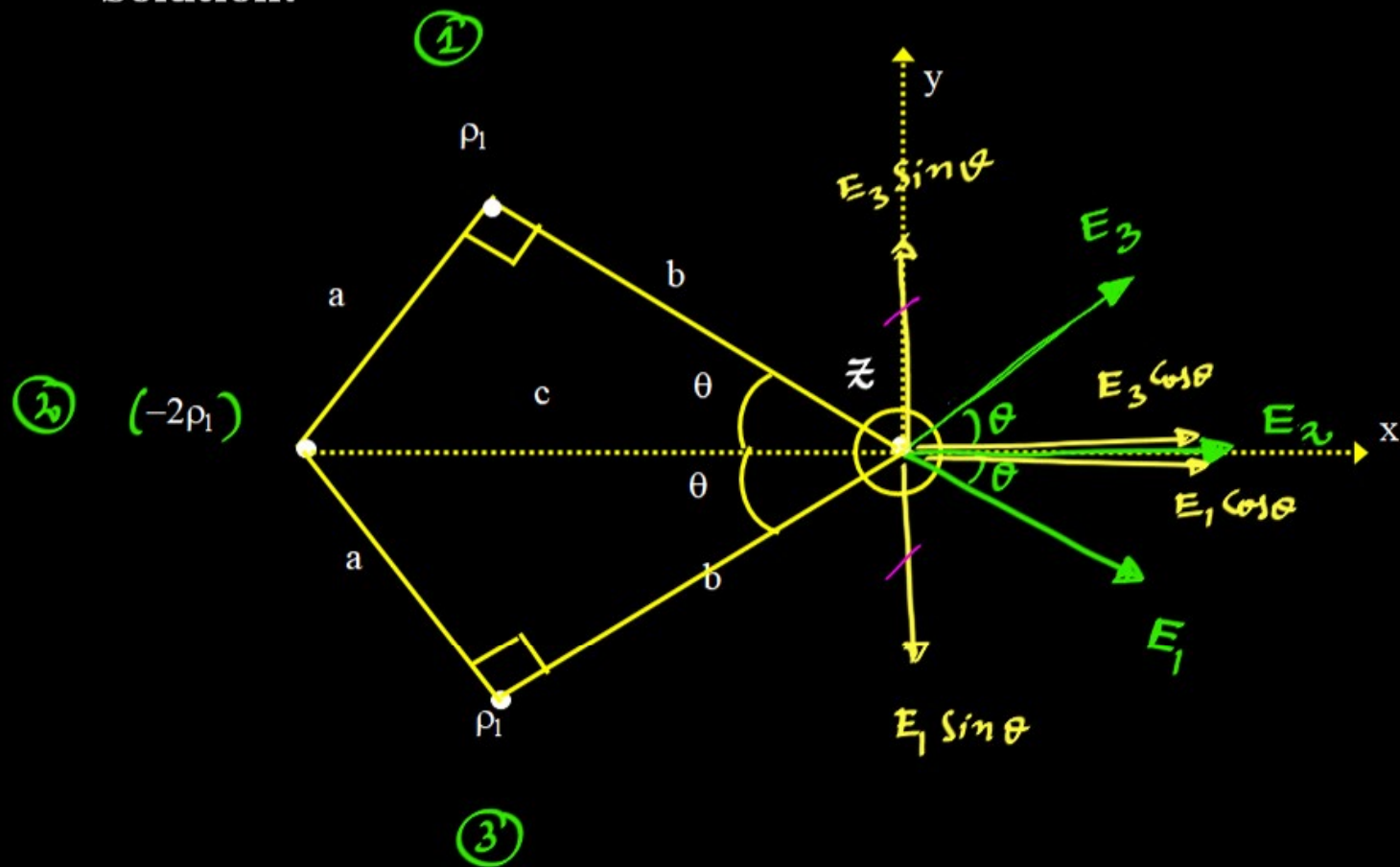
$$\bar{E}_\lambda = \left[ \frac{f_\lambda}{2\pi\epsilon_0} \right] \left[ \begin{array}{c} \bar{f}_\lambda \\ \bar{f}_\lambda \lambda \end{array} \right] = \bar{E}_1 = \bar{E}$$

$$\bar{f}_\lambda = \underline{\underline{3\hat{y} - 3\hat{z} = \bar{f}_1}}$$

Q. Figure shows “KITE” shape in the xy-plane having line charges parallel to the z-axis through three of its corners. Find electric field at the fourth corner



**Solution:**



$$\bar{E}_T = E_1 \cos \theta \hat{r} + E_3 \cos \theta \hat{r} + E_2 \hat{r} = 2E_1 \cos \theta \hat{r} + E_2 \hat{r}$$

$$E_1 = E_3 = \frac{f_1}{2\pi\epsilon(b)}$$

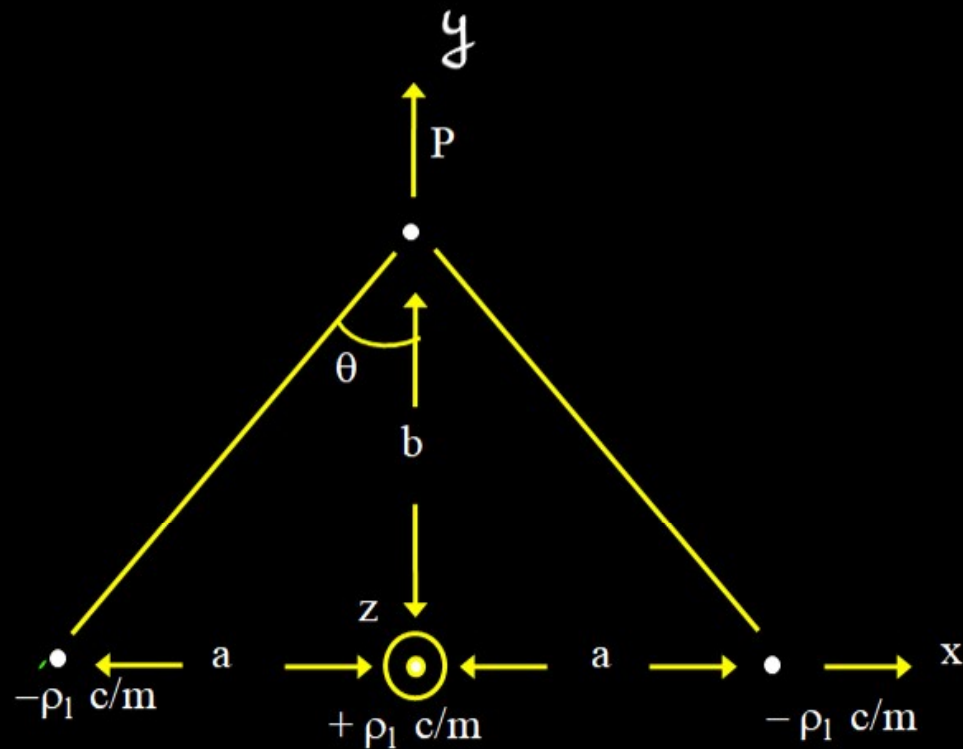
$$E_2 = \frac{(-2f_1)}{2\pi\epsilon(c)}$$

$$\cos \theta = \frac{b}{c}$$

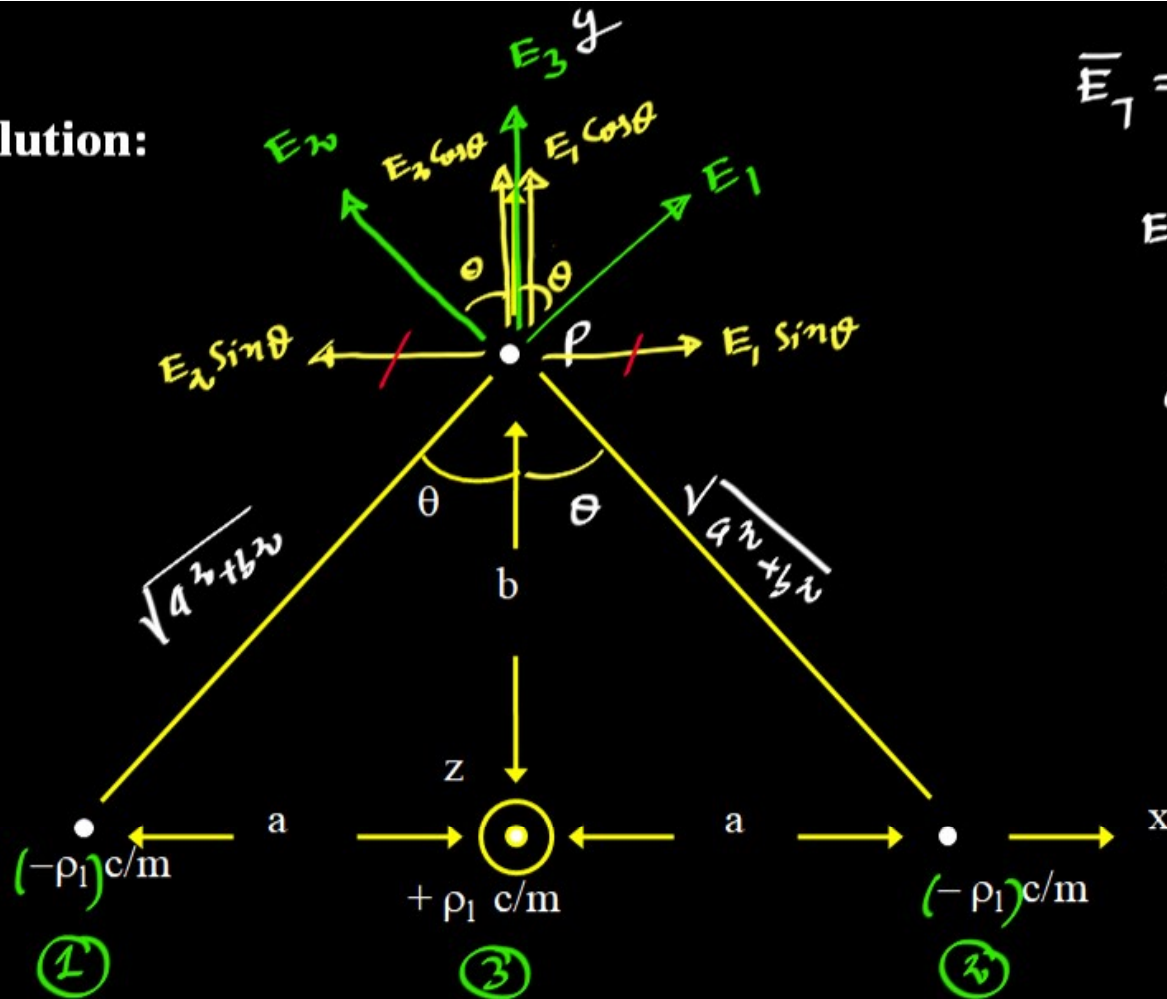
$$\bar{E}_T = 2 \times \frac{f_1}{2\pi\epsilon(b)} \cdot \frac{b}{c} \hat{r} + \frac{(-2f_1)}{2\pi\epsilon(c)} \hat{r}$$

$$= \frac{2f_1}{2\pi\epsilon(c)} \hat{r} + \frac{(-2f_1)}{2\pi\epsilon(c)} \hat{r} = \underline{\underline{0}}$$

Q. Three infinitely long charged lines run parallel to the z-axis, as shown in figure the lines have uniform charge densities as shown, determine the electric field at P



**Solution:**



$$\vec{E}_T = 2 E_1 \cos \theta \hat{y} + E_3 \hat{y}$$

$$E_1 = \frac{(-\rho_l)}{2\pi\epsilon(\sqrt{a^2 + b^2})}$$

$$\cos \theta = \frac{b}{\sqrt{a^2 + b^2}}$$

$$E_3 = \frac{\rho_l}{2\pi\epsilon(b)}$$

$$\vec{E}_T = \frac{2(-\rho_l)}{2\pi\epsilon(\sqrt{a^2 + b^2})} \cdot \frac{b}{\sqrt{a^2 + b^2}} \hat{y} + \frac{\rho_l}{2\pi\epsilon(b)} \hat{y}$$



$$\bar{E}_T = \frac{-2f_1 \hat{y}}{2\pi \epsilon b} \left[ \frac{b}{a^2 + b^2} \right] + \frac{f_1}{2\pi \epsilon b} \hat{y}$$

$$\bar{E}_T = \frac{f_1}{2\pi \epsilon b} \hat{y} \left[ 1 - \frac{2b^2}{a^2 + b^2} \right]$$

$$\bar{E}_T = \frac{f_1 \hat{y}}{2\pi \epsilon b} \left[ \frac{a^2 + b^2 - 2b^2}{a^2 + b^2} \right]$$

$$\bar{E}_T = \frac{f_1 \hat{y}}{2\pi \epsilon b} \left[ \frac{a^2 - b^2}{a^2 + b^2} \right]$$

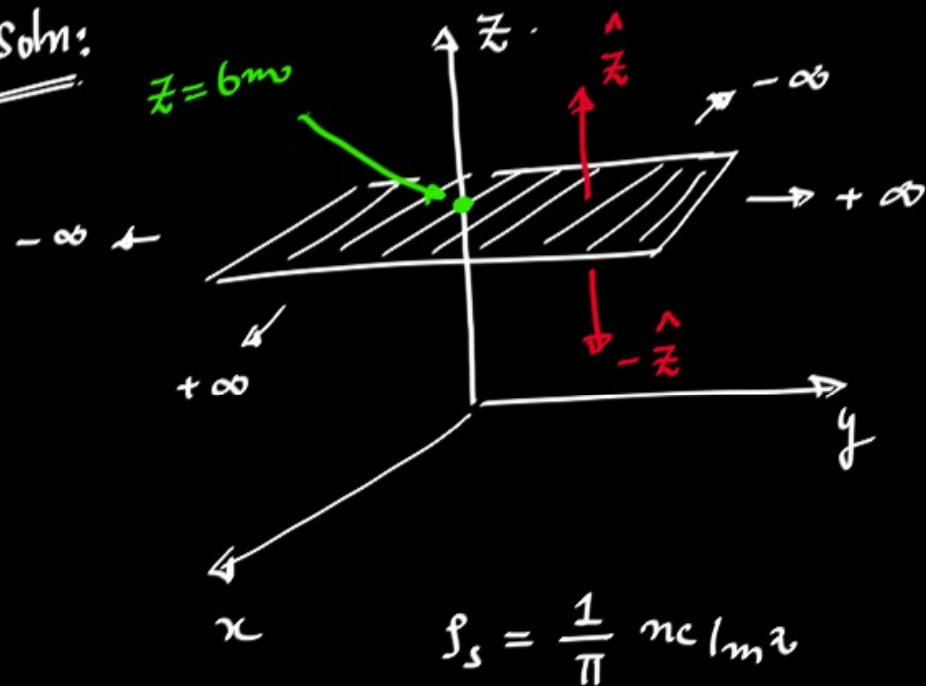
Q. Infinite uniform surface charge of  $\frac{1}{\pi} \text{ nc/m}^2$  is located on  $z = 6$  m plane, find electric field at following points.

(a) (0,0,0)

(b) (10,10,10)

(c) (20, 20, -20)

Soln:



$$\vec{E} = \frac{\rho_s}{2\epsilon_0} (\pm \hat{n}) = \frac{1/\pi \times 10^{-9}}{2 \times \frac{1}{36\pi \times 10^9}} (\pm \hat{z})$$

$$\vec{E} = 18 (\pm \hat{z})$$

AT (0,0,0)

$$\vec{E} = 18 (-\hat{z})$$

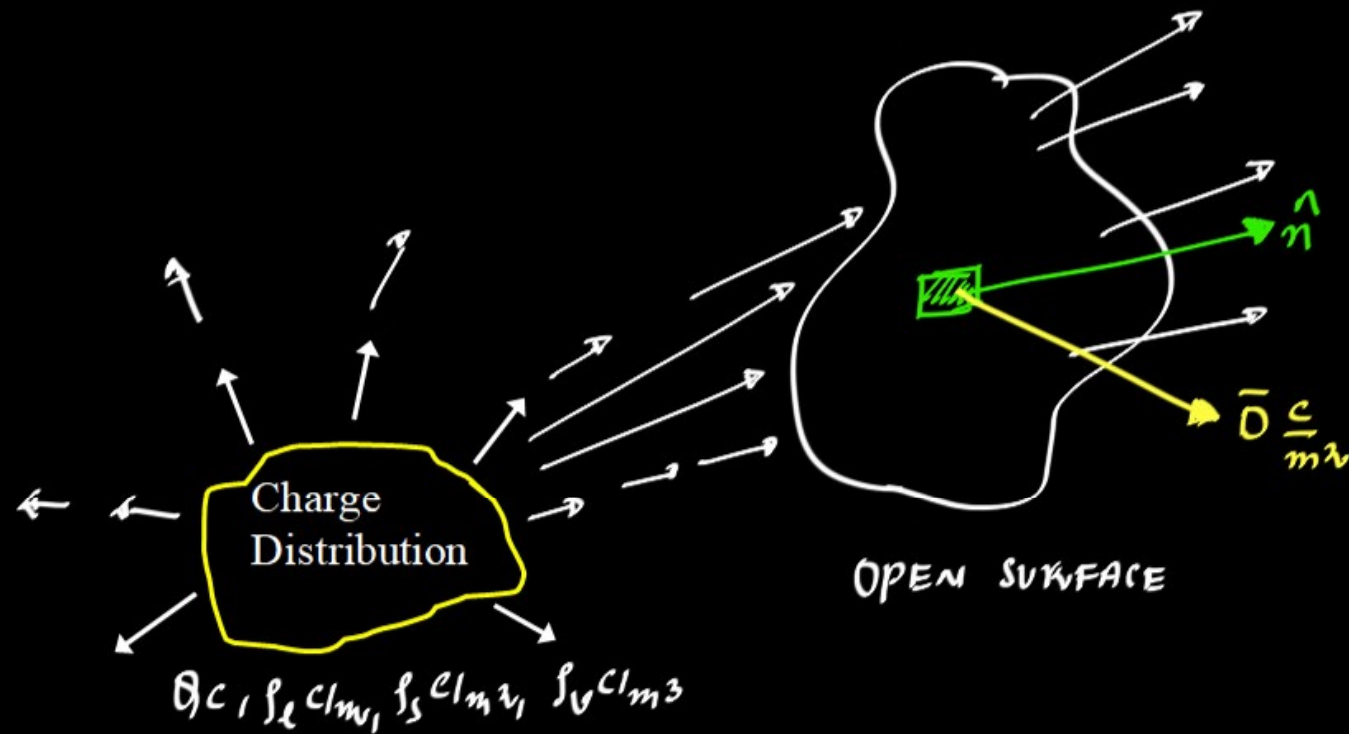
AT (10,10,10)

$$\vec{E} = 18 (\hat{z})$$

AT (20, 20, -20)

$$\vec{E} = 18 (-\hat{z})$$

Electric flux ( $\Psi_e$  C) / Electric flux density ( $\bar{D}$  C/m<sup>2</sup>)



$$\bar{D} = \frac{d\Psi_e}{dA} \text{ C/m}^2$$

$$d\Psi_e = \bar{D} \cdot d\bar{A}$$

$$\Psi_e = \iint \bar{D} \cdot d\bar{A}$$

C.

$\frac{C}{m^2} \times m^2$

NOTE:

$$\vec{D} = \epsilon \vec{E}$$

ELECTRIC  
FLUX DENSITY  
(# OF LINES)  
 $\left(\frac{C}{m^2}\right)$

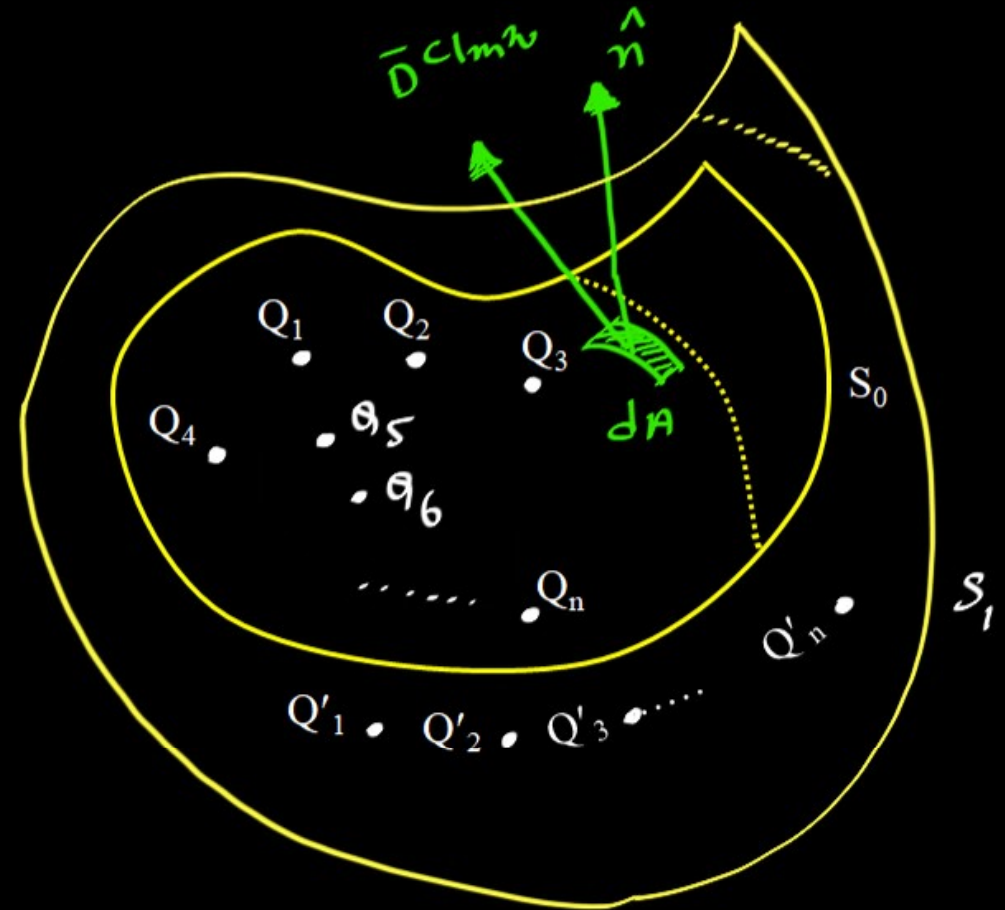
ELECTRIC FIELD  
INTENSITY  
(\* MAGNITUDE)  
(\* STRENGTH)  
(V/m)

## Gauss Law

The net electric flux coming out of any closed surface is equal to total charge enclosed by that surface.

i.e.  $[\Psi_e]_{\text{NET}} = Q_{\text{ENCLOSED}}$

$$\oint \vec{D} \cdot d\vec{A} = Q_{\text{ENC}}.$$



$$[\psi_e]_{\text{net}_{S_0}} = \oiint_{S_0} \vec{D} \cdot \vec{dA} = Q_{ENC} = Q_1 + Q_2 + Q_3 + \dots + Q_n$$

$$[\psi_e]_{\text{net}_{S_1}} = \oiint_{S_1} \vec{D} \cdot \vec{dA} = Q_{ENC} = (Q_1 + Q_2 + Q_3 + \dots + Q_n) + (Q'_1 + Q'_2 + Q'_3 + \dots + Q'_n)$$



## Gauss Law for volume charge ( $\rho_v$ c/m<sup>3</sup>)

$$[\psi_e]_{net} = Q_{enc}$$

$$\oiint \vec{D} \cdot d\vec{A} = \iiint \rho_v dV$$

→ GAUSS LAW IN INTEGRAL FORM.

## FROM DIVERGENCE THEOREM.

$$\begin{aligned} \oiint \vec{D} \cdot d\vec{A} &= \iiint \underline{\nabla \cdot \vec{D}} dV \\ &= \iiint \underline{\rho_v} dV \end{aligned}$$

$$\nabla \cdot \vec{D} = \rho_v$$

→ GAUSS LAW IN DIFFERENTIAL FORM.



NOTE:

$$\textcircled{1} \quad [\psi_e]_{\text{net}} = \theta_{\text{ENC.}}$$

$$\textcircled{2} \quad \oiint \vec{D} \cdot d\vec{A} = \iiint \rho_v dv$$

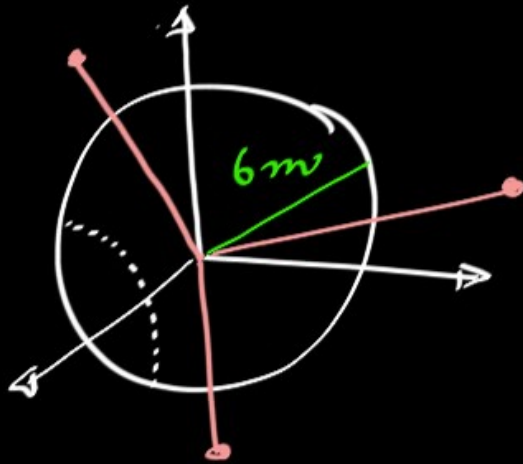


$$\textcircled{3} \quad \nabla \cdot \vec{D} = \rho_v$$



Q. Three charges of  $2\mu\text{C}$ ,  $8\mu\text{C}$ ,  $-4\mu\text{C}$  at  $(4, 8, 3)$ ,  $(2, -2, -3)$  and  $(-4, 0, 1)$  flux leaving the sphere of  $6\text{m}$  radius with the centre at origin.

Soln:



$$R_1 = \sqrt{4^2 + 8^2 + 3^2} > 6$$

$$R_2 = \sqrt{2^2 + (-2)^2 + (-3)^2} < 6$$

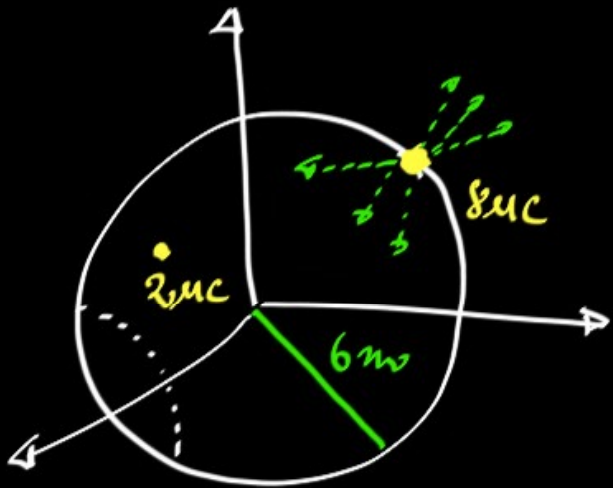
$$R_3 = \sqrt{(-4)^2 + 0^2 + 1^2} < 6$$

$$\begin{aligned} [\Psi_e]_{\text{net}} &= Q_{\text{enc}} = 8\mu\text{C} - 4\mu\text{C} \\ &= \underline{\underline{4\mu\text{C}}} \end{aligned}$$

$$\mathbf{E}_x: 2\mu c (2, 1, 0) \\ 8\mu c (0, 6, 0)$$

$$R_1 = \sqrt{2^2 + 1^2 + 0^2} < 6$$

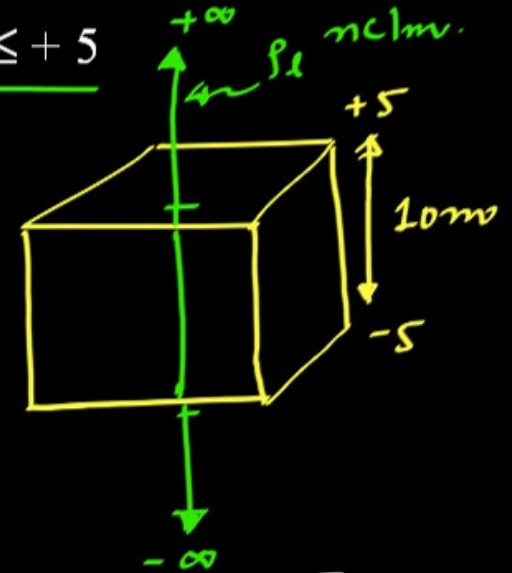
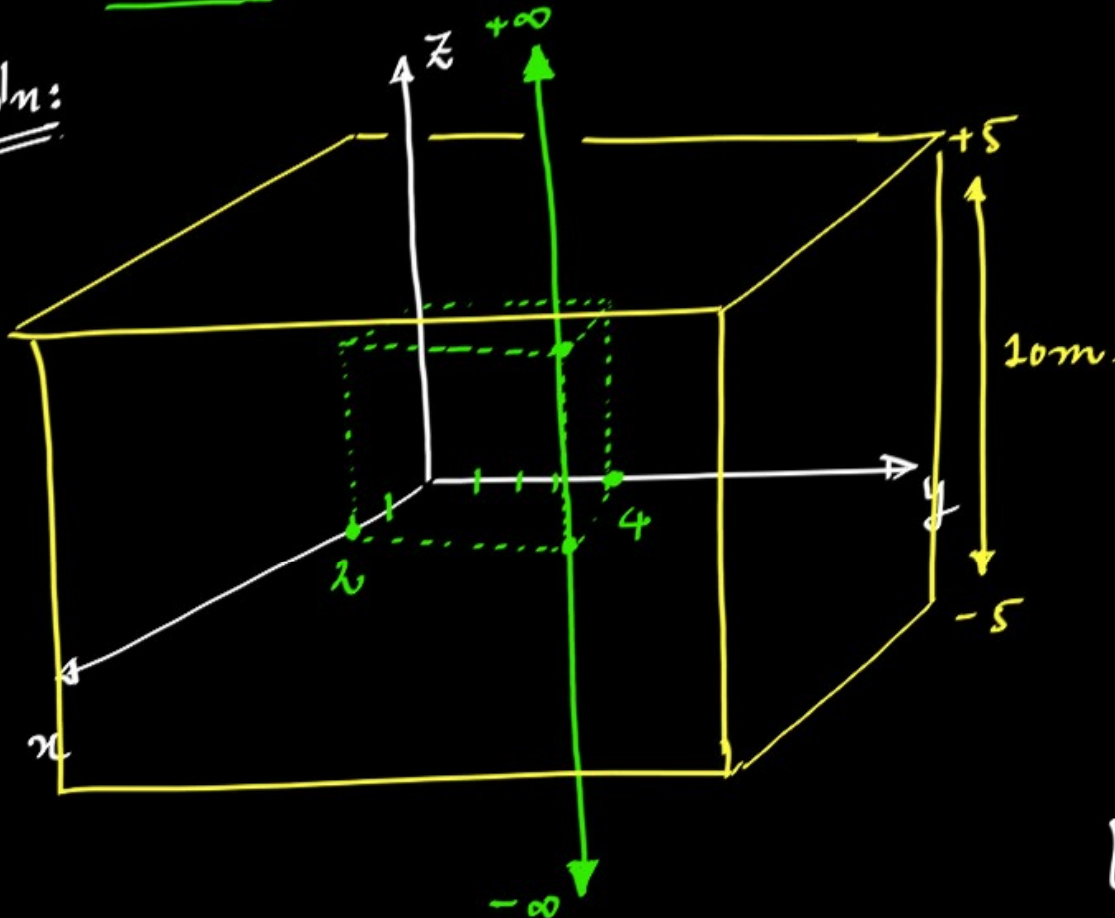
$$R_2 = \sqrt{0^2 + 6^2 + 0^2} = 6$$



$$\begin{aligned} [\psi_e]_{\text{net}} &= Q_{\text{enc}} \\ &= 2\mu c + \left[ \frac{8\mu c}{2} \right] \\ &= \underline{\underline{6\mu c}} \end{aligned}$$

Q If a line charge of density  $\rho_l = 15 \text{ nc/m}$  exists at  $x = 2 \text{ m}$ ,  $y = 4 \text{ m}$  for all  $z$ . Calculate the net flux leaving the cube  $-5 \leq x, y, z \leq +5$

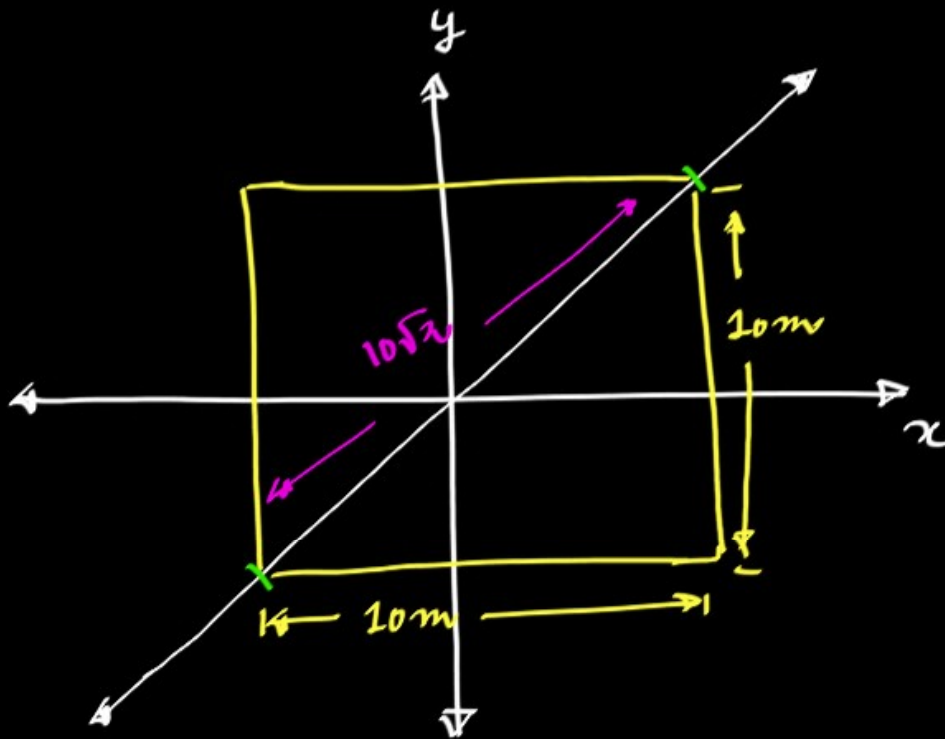
Soln:



$$\begin{aligned}
 [\psi_e]_{\text{net}} &= Q_{\text{enc}} = \int_{-5}^{+5} \rho_l dl \\
 &= [15][l]_{-5}^{+5} = (15 \times 10) \\
 [\psi_e]_{\text{net}} &= \underline{150 \text{ nc}}
 \end{aligned}$$

Q If a line charge density  $\rho_l = 15 \text{ nc/m}$  exists at  $y = x$ . In the  $z = 0$  plane. Calculate the net flux leaving the cube  $-5 \leq x, y, z \leq +5$

Soln:



$$\begin{aligned}
 [\psi_e]_{\text{net}} &= Q_{\text{enc}} \\
 &= 15 \times 10^{-9} \times 10\sqrt{2} \\
 &= 150\sqrt{2} \text{ nC}
 \end{aligned}$$

$$[\psi_e]_{\text{net}} = \underline{\underline{150\sqrt{2} \text{ nC}}}$$

Q. Let  $\vec{D} = 2\rho^2 \hat{\rho} + z\hat{z}$  nc/m<sup>2</sup> exists inside the a cylindrical region enclosed by the surface  $\rho \leq 1$  m,  $z = 0$  and  $z = 5$  m. Find total charge enclosed by the surface.

Soln:  $Q_{enc} = [\Psi_e]_{net} = \oint \vec{D} \cdot \vec{dA} = \iiint \nabla \cdot \vec{D} \, dv$  (GATE)

$$\nabla \cdot \vec{D} = \frac{1}{\rho} \left[ \frac{\partial}{\partial \rho} (\rho D_\rho) + \frac{\partial}{\partial \phi} (D_\phi) + \frac{\partial}{\partial z} (\rho D_z) \right]$$

$$= \frac{1}{\rho} \left[ \frac{\partial}{\partial \rho} (\rho \cdot 2\rho^2) + \frac{\partial}{\partial \phi} (0) + \frac{\partial}{\partial z} (\rho \cdot z) \right]$$

$$\nabla \cdot \vec{D} = \frac{1}{\rho} [2 \cdot 3 \cdot \rho^2 + \rho] = (2 + 6\rho)$$

$$Q_{enc} = \iiint (1+6f) f \, d\phi \, dz$$

$$= \int (6f^2 + f) \, d\phi \, dz$$

$$= \left[ \frac{6f^3}{3} + \frac{f^2}{2} \right]_0^1 [\phi]_0^{2\pi} [z]_0^5$$

$$= \left[ 2 \times 1^3 + \frac{1^2}{2} \right] [2\pi] [5]$$

$$= \frac{5}{2} \times 2\pi \times 5$$

$$\underline{\underline{Q_{enc} = 25\pi \, nC}}$$



Q) The electric flux density in certain region is given as  $\vec{D} = 4r^2 \cos \theta \hat{\theta}$  nC/m<sup>2</sup>, find its charge density at  $(1, \pi/4, \pi/3)$

Soln:  $\rho_v = \nabla \cdot \vec{D}$

$$\nabla \cdot \vec{D} = \frac{1}{r^2 \sin \theta} \left[ \frac{\partial}{\partial r} (r^2 \sin \theta \underline{D_r}) + \frac{\partial}{\partial \theta} (r \sin \theta \underline{D_\theta}) + \frac{\partial}{\partial \phi} (r \underline{D_\phi}) \right]$$

$$= \frac{r}{r^2 \sin \theta} \frac{\partial}{\partial \theta} [\sin \theta \cdot 4r^2 \cos \theta]$$

$$= \frac{2r^2 \cdot r}{r^2 \sin \theta} \frac{\partial}{\partial \theta} [\sin 2\theta]$$

$$\rho_v = \frac{2r}{\sin \theta} (\cos 2\theta) 2 = \left[ \frac{4r \cos 2\theta}{\sin \theta} \right]$$

AT  $(r, \theta, \phi) = (1, \pi/4, \pi/3)$

$$\rho_v = \frac{4 \times 1 \times \cos 2\pi/4}{\sin \pi/4}$$

$\rho_v = 0$

## Finding electric field using Gauss Law


Consider  $\oiint \vec{D} \cdot d\vec{A} = Q_{ENC}$

$$\vec{D} = \epsilon \vec{E}$$

FOR HOMOGENEOUS MEDIUM  
( $\epsilon$ : CONSTANT)

$$\oiint \epsilon \vec{E} \cdot d\vec{A} = Q_{ENC}$$

$$\oiint \vec{E} \cdot d\vec{A} = \frac{Q_{ENC}}{\epsilon}$$



A green circle with a green arrow pointing to it from the text 'GAUSSIAN SURFACE'.

$$\oiint \vec{E} \cdot d\vec{A} = \frac{Q_{ENC}}{\epsilon}$$

GAUSSIAN SURFACE.

GAUSSIAN SURFACE

$$\oint \vec{E} \cdot \vec{dA} = \frac{Q_{\text{ENC}}}{\epsilon}$$

Step 1

Choose Gaussian surface such that

$$\vec{E} \cdot \vec{dA} = \vec{E} \cdot dA \hat{n} = E dA \cos\theta$$

$$\begin{array}{l} \rightarrow E dA (\theta = 0^\circ) \\ \rightarrow 0 (\theta = 90^\circ) \end{array}$$

Step 2

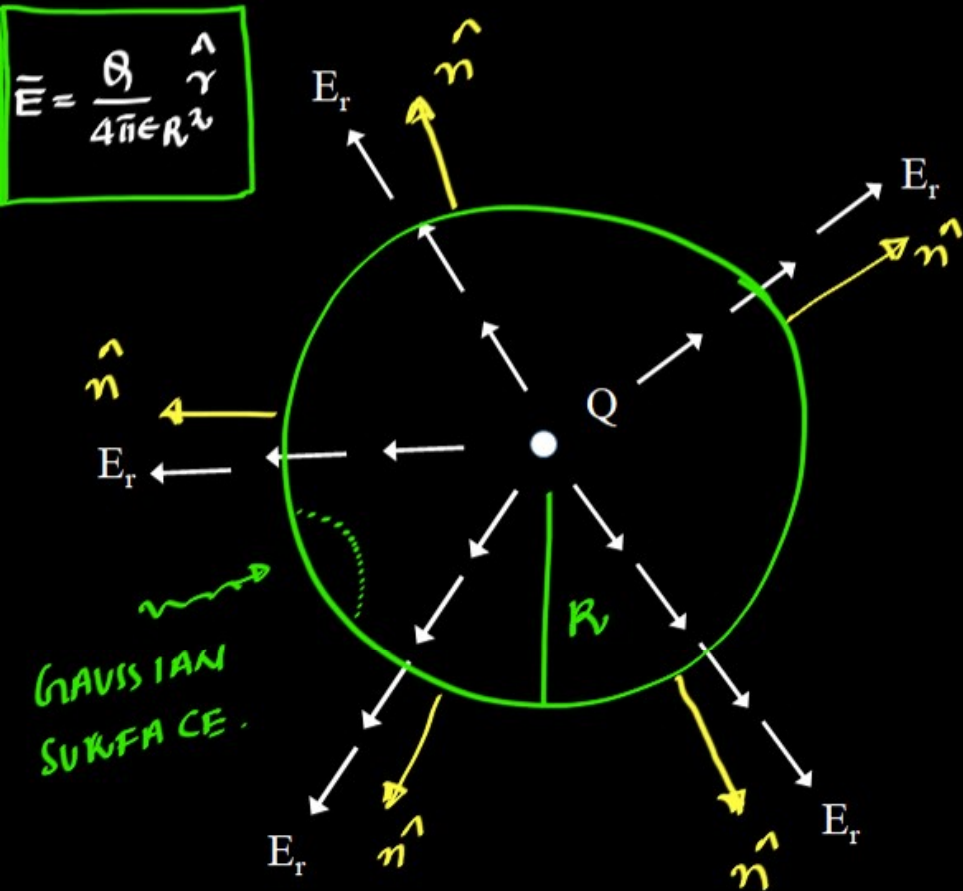
If  $\vec{E} \cdot \vec{dA} = E dA$  then E must be constant on Gaussian surface

$$\text{i.e., } \oint E dA = \frac{Q_{\text{ENC}}}{\epsilon}$$

$$E \oint dA = \frac{Q_{\text{ENC}}}{\epsilon}$$

### Ex-1 Point Charge (QC)

$$\vec{E} = \frac{Q}{4\pi\epsilon R^2} \hat{r}$$



$$\oiint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon} = \frac{Q}{\epsilon}$$

$$\oiint E_r dA = \frac{Q}{\epsilon}$$

$$E_r \oiint dA = \frac{Q}{\epsilon}$$

$$E_r 4\pi R^2 = \frac{Q}{\epsilon}$$

$$E_r = \frac{Q}{4\pi\epsilon R^2}$$

$$\vec{E} = \frac{Q}{4\pi\epsilon R^2} \hat{r}$$

Ex-2 Infinite line charge ( $\rho_l$  c/m)

$$\vec{E} = \frac{\rho_l}{2\pi\epsilon\rho} \hat{\rho}$$

H-w

