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Problem Find Rank of the matrix $\begin{bmatrix} 6 & 0 & 4 & 4 \\ -2 & 14 & 8 & 18 \\ 14 & -14 & 0 & -10 \end{bmatrix}_{3\times 4}$ is ____

$$\begin{matrix} R_2 \to 3R_2 + R_1 \\ R_3 \to 3R_3 - 7R_1 \end{matrix} \begin{bmatrix} 6 & 0 & 4 & 4 \\ 0 & 42 & 28 & 58 \\ 0 & -42 & -28 & -58 \end{bmatrix}$$

$$R_3 \to R_3 + R_2 \begin{bmatrix} 6 & 0 & 4 & 4 \\ 0 & 42 & 28 & 58 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\rho(A) = 2$

3.4 Linearly independent and dependent vectors

01. An ordered 'n'-tuple $X = (x_1, x_2, \ldots x_n)$ is called an n- dimensional vector and $x_1, x_2, \ldots x_n$ are called components of 'X'.

R^2 – 2 dimensional $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$

R^3 – 3 dimensional $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$

R^4 – 4 dimensional $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$

02. A vector may be written as either a row matrix $X = [x_1 \ x_2 \ . . . \ x_n]$ which is called row vector (or)

a column matrix $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ | \\ x_n \end{bmatrix}$ which is called column vector.

3.4 Linearly independent and dependent vectors

4. Two vectors x_1, and x_2 are said to be linearly dependent if $X_1 = \alpha X_2$ or $X_2 = \alpha X_1$

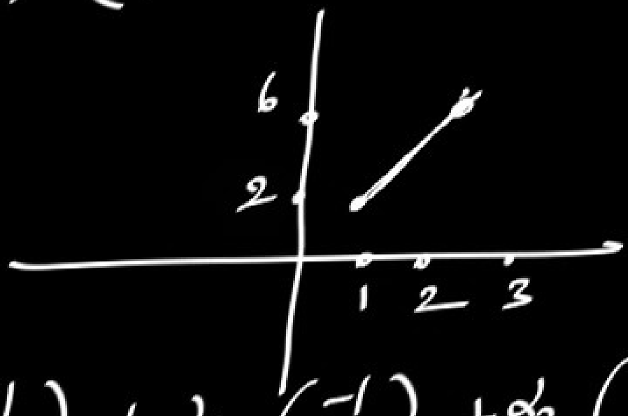
otherwise L.I

$\begin{pmatrix}1\\2\end{pmatrix} \begin{pmatrix}3\\6\end{pmatrix}$ $\quad \begin{pmatrix}3\\6\end{pmatrix} = 3\begin{pmatrix}1\\2\end{pmatrix}$ L.D

$\begin{pmatrix}-1\\3\end{pmatrix} \begin{pmatrix}3\\-9\end{pmatrix}$ $\quad \begin{pmatrix}3\\-9\end{pmatrix} = -3\begin{pmatrix}-1\\3\end{pmatrix}$ L.D

$\begin{pmatrix}1\\0\end{pmatrix} \begin{pmatrix}2\\4\end{pmatrix} \longrightarrow$ L.I

5.Linear combination of vectors:



If $X_1, X_2, \ldots X_r$ are 'r' vectors of order 'n' and $\alpha_1, \alpha_2, \ldots \alpha_r$ are 'r' scalars then the expression of the form $\alpha_1 X_1 + \alpha_2 X_2 + \alpha_3 X_3 \ldots\ldots + \alpha_r X_r$ is also a vector and it is called linear combination of the vectors $X_1, X_2, \ldots\ldots X_r$.

L.C $\leftarrow \alpha_1\begin{pmatrix}1\\2\end{pmatrix} + \alpha_2\begin{pmatrix}-1\\3\end{pmatrix} + \alpha_3\begin{pmatrix}3\\4\end{pmatrix}$

3.4 Linearly independent and dependent vectors

Linearly dependent vectors:

The vectors $X_1, X_2, \ldots X_r$ of same order 'n' are said to be linearly dependent if there exist scalars $\alpha_1, \alpha_2, \ldots \alpha_r$ not all zero such that $\alpha_1 X_1 + \alpha_2 X_2 + \alpha_3 X_3 \ldots\ldots + \alpha_r X_r = O$ where O denotes the zero vector of order n.

Linearly independent vectors:

The vectors $X_1, X_2, \ldots X_r$ of same order 'n' are said to be linearly dependent if there exist scalars $\alpha_1, \alpha_2, \ldots \alpha_r$ such that

$\alpha_1 X_1 + \alpha_2 X_2 + \alpha_3 X_3 \ldots\ldots + \alpha_r X_r = O \Rightarrow \alpha_1 = 0, \alpha_2 = 0, \ldots \alpha_r = 0$

where O denotes the zero vector of order n.

3.4 Linearly independent and dependent vectors

Define A, by considering given vectors as rows

verify $\begin{pmatrix}1\\2\\3\end{pmatrix}\begin{pmatrix}2\\4\\5\end{pmatrix}\begin{pmatrix}-1\\1\\3\end{pmatrix}$ are L.I/L.D

$$A = \begin{bmatrix}1 & 2 & 3\\ 2 & 4 & 5\\ -1 & 1 & 3\end{bmatrix}$$

$A_{m\times n}$

$m = n$

1. $|A| \neq 0$ (or) $e(A) = n$

Then the rows and columns of A are L.I

2. $|A| = 0$ (or) $e(A) < n$ then the rows and columns of A are L.D

$m \neq n$

$|A| = 1(7) - 2(11) + 3(8)$
$\neq 0$ L.I.

The vectors are said to be L.I if no row is zero row in row echelon form

3.4 Linearly independent and dependent vectors

Definition of rank of a matrix

Definition -3

Finally , rank of A = number of linearly independent rows(columns) of the given matrix

Ie. If rank of matrix is 3 means three linearly independent rows and three linearly independent columns will be there

i.e., In row echelon form — non zero rows are L.I
— zero rows are L.D

3.4 Linearly independent and dependent vectors

Important points

1.If X_1, X_2, …….., X_r are linearly dependent vectors then at least one of the vectors can be expressed as a linear combination of other vectors

2.Any subset of a linearly independent set is itself linearly independent set.

3. Any super set of L. D set is L.D

4. If a set of vectors includes a zero vector then the set of vectors is linearly dependent set.

5.A set of (n+1) vectors in R^n is linearly dependent

3 vectors in R^2 are L.D $\left\{ \begin{pmatrix} 1 \\ 2 \end{pmatrix} \begin{pmatrix} 3 \\ 5 \end{pmatrix} \begin{pmatrix} 4 \\ 7 \end{pmatrix} \right\}$ are L.D

4 vectors in R^3 are L.D $\left\{ \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} \begin{pmatrix} 1 \\ 3 \\ 4 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} \right\}$ L.D

Problems on Rank of a Matrix

Q. A 5 × 7 matrix has all its entries equal to 1.

$\rho(A_{m\times n}) \le \min\{m,n\}$

Then the rank of a matrix is **[GATE – 94(CS)]**

(a) 7 ✗

(b) 5 ✗

(c) 1 ✓

(d) Zero ✗

$\rho(A_{5\times 7}) \le 5$

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ \cdot & & - & & & - & \\ \cdot & & - & & & - & \end{bmatrix}$$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - R_1$

$R_4 \to R_4 - R_1$

$R_5 \to R_5 - R_1$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ - & & - & & & - & \\ - & & - & & & - & \end{bmatrix}$$

Let $A = \begin{bmatrix} 3 & P & P \\ P & 3 & P \\ P & P & 3 \end{bmatrix}$. If rank of A is 1, then

P = 3.

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Sol: $A_{3\times3}$ and $\rho(A) = 1$

$\Rightarrow |A| = 0$ and all 2×2 minors are zero

Consider $\begin{vmatrix} 3 & P \\ P & 3 \end{vmatrix} = 0 \Rightarrow 9 - P^2 = 0$

$\Rightarrow P = \pm 3$

$P = 3$: $\begin{bmatrix} 3 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \end{bmatrix}$

$R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$\begin{pmatrix} 3 & 3 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ $\rho(A) = 1$

$P = -3$: $\begin{bmatrix} 3 & -3 & -3 \\ -3 & 3 & -3 \\ -3 & -3 & 3 \end{bmatrix}$

$R_2 \to R_2 + R_1$, $R_3 \to R_3 + R_1$

$\begin{pmatrix} 3 & -3 & -3 \\ 0 & 0 & -6 \\ 0 & -6 & 0 \end{pmatrix} \leftarrow$

$\rho(A) = 3$ $\begin{pmatrix} 3 & -3 & -3 \\ 0 & -6 & 0 \\ 0 & 0 & -6 \end{pmatrix}$

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Suppose that $A_{n\times n}$ is upper triangular matrix such that $a_{ii} = 0$, $i = 1, 2, \ldots, n$.

Then rank of A^n = ________.

(a) 0 (b) $n - 1$

(c) 1 (d) n

$a_{11} = 0$
$a_{22} = 0$

Sol: $n = 2$

$$A_{2\times 2} = \begin{bmatrix} a_{11} & a_{12} \\ 0 & a_{22} \end{bmatrix}$$

$$A_{2\times 2} = \begin{bmatrix} 0 & a_{12} \\ 0 & 0 \end{bmatrix}$$

$$A^2 = A \cdot A$$
$$= \begin{pmatrix} 0 & a_{12} \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & a_{12} \\ 0 & 0 \end{pmatrix}$$
$$= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$\rho(A^2) = 0$

similarly we can verify for $n = 3$

If A is a matrix of order 3 such that

$A.adj(A) = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ then the rank of A is

(a) 3 ✓

(b) 2

(c) 1

(d) 0

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Sol:

$A \cdot adj A = |A| \cdot I_3$

$\begin{pmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{pmatrix} = |A| \cdot I_3$

$5 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = |A| \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

clearly $|A| = 5 \neq 0$

$\rho(A) = 3$

Result $|A_{n \times n}| \neq 0$ then $\rho(A) = n$

$|A_{n \times n}| = 0$ $\rho(A) < n$

Problems on Rank of a Matrix

Q. The rank of the matrix

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix}$$ *is* ________ .[GATE – 17(EC)]

Sol: $R_4 \to R_4 + R_1$

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix}$$

$R_2 \leftrightarrow R_4$

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix}$$

$R_3 \to R_3 + R_2$

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix}$$

$R_4 \to R_4 + R_3$

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix}$$

$R_5 \to R_5 + R_4$

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$\rho(A) = 4$

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Problem Rank of the matrix $\begin{pmatrix} 1 & 1 & 0 & -2 \\ 2 & 0 & 2 & 2 \\ 4 & 1 & 3 & 1 \end{pmatrix}$ is ____

(a) 1 (b) 2 (c) 3 (d) 4

$R_2 \to R_2 - 2R_1$
$R_3 \to R_3 - 4R_1$

$$\begin{bmatrix} 1 & 1 & 0 & -2 \\ 0 & -2 & 2 & 6 \\ 0 & -3 & 3 & 9 \end{bmatrix}$$

$R_3 \to 2R_3 - 3R_2$

$$\begin{bmatrix} 1 & 1 & 0 & -2 \\ 0 & -2 & 2 & 6 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \rho(A) = 2$$

Problems on Rank of a Matrix

Q. The value of k, for which the vectors

(k, 2, 4) (1, 0, 2) (1, -1, -1)

are linearly independent is --------

$$A = \begin{bmatrix} k & 2 & 4 \\ 1 & 0 & 2 \\ 1 & -1 & -1 \end{bmatrix}$$

For L.I $|A| \neq 0$

$$\begin{vmatrix} k & 2 & 4 \\ 1 & 0 & 2 \\ 1 & -1 & -1 \end{vmatrix} \neq 0$$

$$k(2) - 2.(-3) + 4(-1) \neq 0$$

$$2k - 10 \neq 0 \Rightarrow k \neq 5$$

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Problems on Rank of a Matrix

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Q. If the vectors the vectors

(1.0, -1.0, 2.0) (7.0,3.0, x) and (2.0, 3.0, 1.0) in R^3

are linearly dependent, the value of x is 8--------

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 7 & 3 & x \\ 2 & 3 & 1 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & -1 & 2 \\ 7 & 3 & x \\ 2 & 3 & 1 \end{vmatrix} = 1(3-3x) + 1(7-2x) + 2(15)$$

$$= 40 - 5x$$

For L.D $|A| = 0$

$40 - 5x = 0$

$\Rightarrow x = 8$

Example verify $\{(1,2,1,-1)\ (3,2,1,-1)\ (1,1,1,4)\}$ are L.I/L.D

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$$A = \begin{bmatrix} 1 & 2 & 1 & -1 \\ 3 & 2 & 1 & -1 \\ 1 & 1 & 1 & 4 \end{bmatrix}$$

$R_2 \to R_2 - 3R_1$
$R_3 \to R_3 - R_1$
$$\begin{bmatrix} 1 & 2 & 1 & -1 \\ 0 & -4 & -2 & 2 \\ 0 & -1 & 0 & 5 \end{bmatrix}$$

$R_3 \to 4R_3 - R_2$

$$\begin{bmatrix} 1 & 2 & 1 & -1 \\ 0 & -4 & -2 & 2 \\ 0 & 0 & 2 & 18 \end{bmatrix}$$

all Rows are non zero rows

$\therefore$ the vectors are L.I.

Example verify $\{(1,2,1)\ (3,2,1)\ (1,1,3)\ (2,4,1)\}$ are

L.I | L.D

Soln L.D (∵ 4 vectors of R^3 are given)

Problems on Rank of a Matrix

Q. Consider the following statements

Statement -I if $\{X_1, X_2, X_3, X_4, X_5\}$ is linearly independent set of vectors then the set $\{X_1, X_2, X_3,\}$ is Linearly Independent set subset

Statement -II if $\{X_1, X_2, X_3, X_4,\}$ is linearly dependent set of vectors then the set $\{X_1, X_2, X_3, X_4, X_5\}$ is Linearly dependent set superset

Which of the following is true

(a) Only statement -I is true

(b) Only statement -II is true

(c) both statement -I and II are true

(d) both statement -I and II are wrong

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H.W (1) Rank of the matrix $\begin{bmatrix} 0 & 2 & 2 \\ 7 & 4 & 8 \\ -7 & 0 & -4 \end{bmatrix}$ is — (2)

(2) Rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 4 & 6 & 8 \end{bmatrix}$ is — Ans (2)

(3) Rank of the matrix $\begin{bmatrix} 5 & 0 & -5 & 0 \\ 0 & 2 & 0 & 1 \\ -5 & 0 & 5 & 0 \\ 0 & 1 & 0 & 2 \end{bmatrix}$ — Ans (3)

(4) Rank of the matrix $\begin{bmatrix} 1 & 2 & 2 & 3 \\ 3 & 4 & 2 & 5 \\ 5 & 6 & 2 & 7 \\ 7 & 8 & 2 & 9 \end{bmatrix}$ Ans (2)

4.System of Linear Equations

4.1 Terminology

4.2 Geometrical interpretation of system of linear equations

4.3 Matrix inverse method

4.4 Cramer's rule

4.5 Gauss- elimination method

4.6 Homogenous system of equations

4.7 Null space of homogenous system of linear equations

4.8 Problems on system of linear equations

4.1 Terminology

System of linear equations

The system of 'm' non-homogeneous linear equation in 'n' variables $x_1, x_2, \ldots x_n$ is given by

m = Number of equations
n = Number of variables

$$\left.\begin{array}{l} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2 \\ \ldots\ldots \quad \ldots\ldots \quad \ldots\ldots \quad \ldots\ldots \quad \ldots \\ a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n = b_m \end{array}\right\}$$

the set of these equations can be written in matrix form as *$AX = B$*

where *A is coefficient matrix, X is column vector of the variables and B is the column vector of constants $b_1, b_2, \ldots b_n$.* ✓

$$2x_1 + 3x_2 + 4x_3 = 5$$
$$x_1 - x_2 + x_3 = -1$$
$$x_1 + x_2 - x_4 = 0$$

$m = 3$

$n = 3$

$$\begin{bmatrix} 2 & 3 & 4 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5 \\ -1 \\ 0 \end{bmatrix}$$

$$AX = B$$

clearly X is always column vector

$$X = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$$

Note: $AX = B$ is called Non Homogenous system

$AX = 0$ is called Homogenous system

4.1 Terminology

1. Solution.

The value of the vector X, satisfying the system AX=B is called solution

$$(A|B) = \begin{bmatrix} 2 & 3 & 4 & 5 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & 0 \end{bmatrix}$$

2. Augmented matrix (A/B): The matrix containing A and B

3. Consistent (compatible): at least one solution exists

4. Inconsistent(incompatible) : the system has no solution

4.1Terminology

5.Over determined system:

If there are more equations than variables, that is, m > n, then the system is said to be over determined

$$\left.\begin{aligned} x + y &= 4 \\ 2x - 5y &= 7 \\ x + y &= -1 \end{aligned}\right\}$$

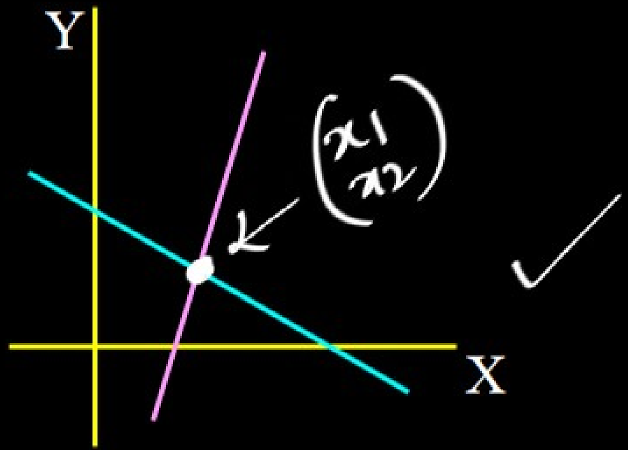
6. Under determined system:

If the system has fewer equations than variables, that is, m < n, then the system is called under determined and.

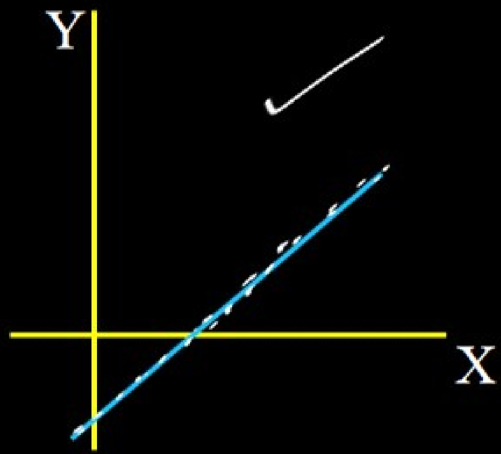
$$\boxed{\begin{aligned} x + y + 2z &= 1 \\ 2x - y + 7z &= 5 \end{aligned}}$$

(7) determined system $m = n$

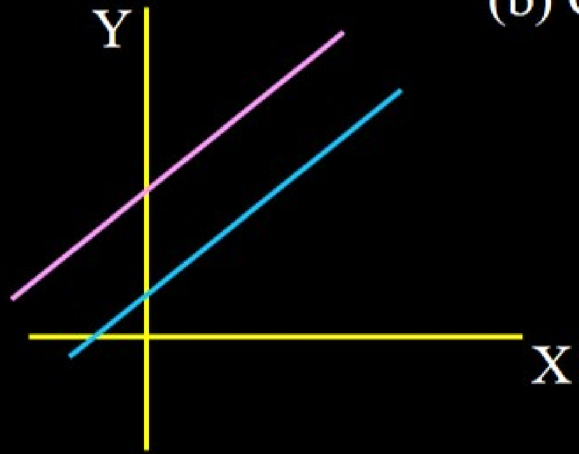
4.2Geometrical representation of system of linear equations



(a) Consistent



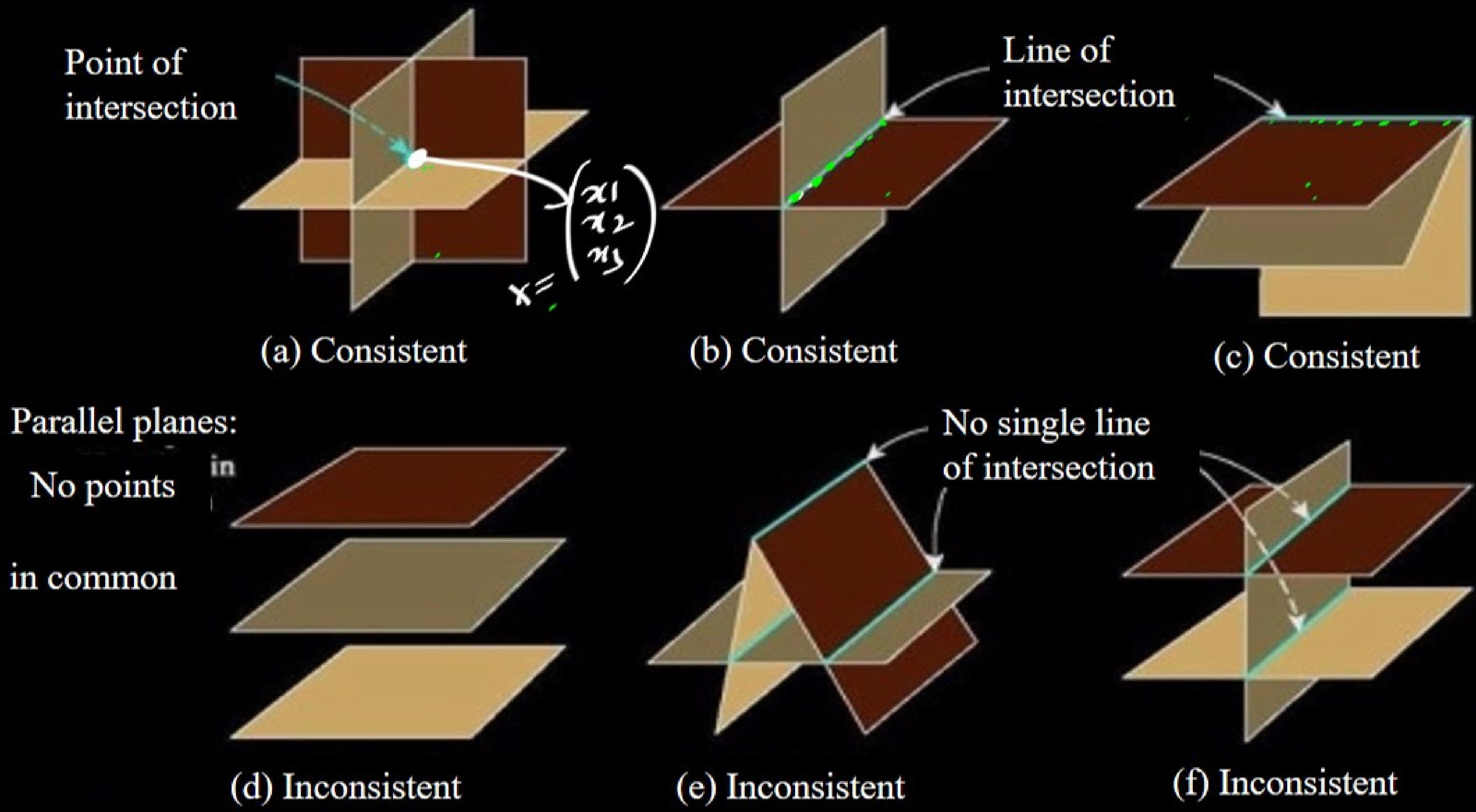
(b) Consistent



(c) Inconsistent

$$a_{11}x_1 + a_{12}x_2 = b_1$$
$$a_{21}x_1 + a_{22}x_2 = b_2$$

A linear system of two equations in two variables interpreted as lines in 2 - space



A linear system of three equations in three variables interpreted as planes in 3 - space

4.2Geometrical representation of system of linear equations

Finally the possible solutions
For $AX = B$ are

(a) unique solution ✓

(or)

(b) Infinitely many solutions

(or)

(c) No solution

The following statements will not be correct.

(1) more than one solution but finite

(2) two solutions

(3) three solutions

(4) finite number of solutions

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4.3Matrix inverse method

$m = n$

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$A_{n\times n}X = B$

- $|A| \neq 0$

  Unique solution

  $X = A^{-1} \cdot B = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$

- $|A| = 0$

  - $(\text{adj}A)\cdot B = 0$

    Infinitely many solutions

  - $(\text{adj}A)\cdot B \neq 0$

    No solution

Note $|A| \neq 0$ unique solution exists

$|A| = 0$ NO unique solution

4.4 Cramer's Rule for $A_{n\times n}X = B$

$m = n$

$\Delta \neq 0$

unique solution

$x_1 = \frac{\Delta_1}{\Delta}$, $x_2 = \frac{\Delta_2}{\Delta}$, $x_3 = \frac{\Delta_3}{\Delta}$

$\Delta = 0$ and all $\Delta_i = 0$
Then infinitely many solutions

$\Delta = 0$ and one or more
$\Delta_i \neq 0$ then no solution

Let A be a square matrix of order n

$\Delta = |A|$

And let Δ_i be the determinant of the matrix A_i , obtained by replacing the ith column of A by the right hand side column vector B

Consider 3x3 System

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$$A \quad X \quad = B$$

$$\Delta = |A|$$

$$\Delta_1 = \begin{vmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{vmatrix}$$

$$\Delta_2 =$$

$$\Delta_3 =$$

4.5 Gauss – Elimination method

m = Number of equations
n = Number of variables

$A_{m\times n}X = B$

$\rho(A/B) \neq \rho(A)$

Inconsistent

$\rho(A/B) = \rho(A)$

Consistent

$\rho(A/B) = \rho(A) = r = n$

Unique solution

$\rho(A/B) = \rho(A) = r < n$

Infinitely many solutions

Method

let $AX = B$

$$\Rightarrow (A|B) = \left[\begin{array}{c|c} \quad & - \\ & - \\ & - \end{array}\right]$$

$\downarrow$ Reduce into Row echelon form.

suppose Final form is

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 3 \\ 0 & 2 & 4 & 4 \\ 0 & 0 & 2 & 6 \end{array}\right]$$

$\rho(A) = 3$

$\rho(A|B) = 3$

$\rho(A|B) = \rho(A) = 3 =$ variables

unique solution exists

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 6 \end{pmatrix}$$

$$x + 2y + 3z = 3 \Rightarrow x = 2$$
$$2y + 4z = 4 \Rightarrow y = -4$$
$$2z = 6 \Rightarrow z = 3$$

The solution is $X = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ -4 \\ 3 \end{pmatrix}$ ✓

Scheme: Forward elimination and Backward substitution suppose final form is

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & 2 & 4 & 8 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$\rho(A|B) = 2$

$\rho(A) = 2$

$\rho(A) = \rho(A|B) = 2 <$ variables $\Rightarrow$ Infinitely many solutions

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 8 \\ 0 \end{pmatrix} \Rightarrow \left.\begin{array}{r} x + 2y + 3z = 1 \\ 2y + 4z = 8 \end{array}\right\}$$

let $z = k$ be parameter

$\Rightarrow \quad y = \dfrac{8-4k}{2}$

$x =$ interms of k

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$$X = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} - \\ - \\ k \end{pmatrix}$$

$k = 0$

$k = 1$

$k = -1$

$k = 2$ - - - .

Infinitely many solutions

Suppose

Final form is

$$\underbrace{\begin{pmatrix} 1 & -1 & 2 & | & 1 \\ 0 & 3 & 6 & | & 2 \\ 0 & 0 & 0 & | & 5 \end{pmatrix}}$$

$\rho(A) = 2$ $\qquad$ $\rho(A|B) = 3$

$\rho(A) \neq \rho(A|B)$

NO solution

$$\begin{pmatrix} 1 & -1 & 2 \\ 0 & 3 & 6 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 5 \end{pmatrix}$$

$x - y + 2z = 1$

$3y + 6z = 2$

$0 = 5$ ✗

No solution

Imp

Result: A system of linear equations AX= B is consistent, if the vector B can be written as a linear combination of the columns of A.

ie.., B is linearly dependent on columns of A $\quad \alpha_1(\cdot)+\alpha_2(\cdot)+\alpha_3(\cdot)=B$

$\Rightarrow \rho(A)=\rho(A|B)$

If B is not a linear combination of columns of A, then the system AX=B is inconsistent .

ie B is linearly independent of columns of A $\quad \Rightarrow \rho(A) \neq \rho(A|B)$

4.6 Homogenous system of linear equationsAX=O

Trivial solution: the solution x= 0 is called trivial solution

(unique solution) (zero solution)

Non trivial solution: the solution x≠ 0 is called non trivial solution

(non zero solution)

$$x + 2y = 0$$
$$3x + 5y = 0$$
$$\begin{pmatrix}0\\0\end{pmatrix}$$

$$2x + y = 0$$
$$4x + 2y = 0$$
$$\begin{pmatrix}0\\0\end{pmatrix}\begin{pmatrix}2\\-2\end{pmatrix}\begin{pmatrix}-2\\2\end{pmatrix}\begin{pmatrix}6\\-6\end{pmatrix}\cdots$$
$$x = \alpha\begin{pmatrix}2\\-2\end{pmatrix}$$

$$x - y + z = 0$$
$$x + y + z = 0$$
$$\begin{pmatrix}0\\0\\0\end{pmatrix}\begin{pmatrix}1\\0\\-1\end{pmatrix}\begin{pmatrix}-1\\0\\1\end{pmatrix}\begin{pmatrix}2\\0\\-2\end{pmatrix}----$$
$$x = \alpha\begin{pmatrix}1\\0\\-1\end{pmatrix}.$$

Nullspace $X = \left\{\begin{pmatrix}0\\0\end{pmatrix}\begin{pmatrix}2\\-2\end{pmatrix}\begin{pmatrix}-2\\2\end{pmatrix}\cdots\right\}$

4.6 Homogenous system of linear equations AX=O

Note

The possibilities for AX=O

1. Only trivial solution

(or)

2. Infinite number of nontrivial solutions

i.e., Every Homogenous system is always consistent

The following statements are never correct for AX=0

1) No Solution
2) Nonzero unique solution
3) Finite number of nonzero solution

Note out of Infinitely many solutions few solutions are L.I.

i.e solutions are like

1) $X = k_1 (\ ) \leftarrow$ one independent solution (Linearly)

(or)

2) $X = k_1 (\ ) + k_2 (\ ) \leftarrow$ Two L.I solutions

(or)

3) $X = k_1 (\ ) + k_2 (\ ) + k_3 (\ ) \leftarrow$ 3 L.I solutions

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$A_{m\times n}X = 0$

$m = n$

1. $|A| \neq 0$
   Then only trivial solution → unique solution
2. $|A| = 0$
   Then infinitely many solutions

$m \neq n$

$m < n$
always infinitely many solutions

$m > n$
1. $\rho(A) = r = n$
   only trivial solutions
2. $\rho(A) = r < n$
   infinitely many solutions

Finally

1) $\rho(A) = r = n$
   only trivial solution
2) $\rho(A) = \underline{r < n}$
   Infinitely many solution

4.7 Null space of homogenous system

1. Null space : set of all solutions of the homogenous system AX=O

2. Nullity(dimension of Null space)

= the number of linearly independent solutions

= n-r

here *n is the number of variables and r is the rank of the matrix A*

Imp

3. Rank + nullity = n

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Break

20 mins

Problems on System of Linear Equations

Q. Consider the system

$$\begin{cases} kx + y + z = 1 \\ x + ky + z = 1 \\ x + y + kz = 1 \end{cases}$$

If the system has a unique solution then which of the following is true?

(a) k=1 and k = - 2

(b) k≠1 and k ≠ - 2 ✓

(c) k=1 and k ≠ - 2

(d) k≠1 and k= - 2

For unique solution $|A| \neq 0$

$$\begin{vmatrix} k & 1 & 1 \\ 1 & k & 1 \\ 1 & 1 & k \end{vmatrix} \neq 0$$

$C_1 \to C_1 + C_2 + C_3$

$$\begin{vmatrix} k+2 & 1 & 1 \\ k+2 & k & 1 \\ k+2 & 1 & k \end{vmatrix} \neq 0$$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - R_1$

$$\begin{vmatrix} k+2 & 1 & 1 \\ 0 & k-1 & 0 \\ 0 & 0 & k-1 \end{vmatrix} \neq 0$$

$(k+2)(k-1)^2 \neq 0 \Rightarrow k \neq -2$ and $k \neq 1$

Problems on System of Linear Equations

Q. Consider the following system of equations:

$3x + 2y = 1$

$4x + 7z = 1$

$x + y + z = 3$

$x - 2y + 7z = 0$

The number of solutions for this system is ……….. 1

[GATE-14-CS-Set1]

m = equations $= 4$

n = variables $= 3$

$$(A|B) = \begin{bmatrix} 3 & 2 & 0 & 1 \\ 4 & 0 & 7 & 1 \\ 1 & 1 & 1 & 3 \\ 1 & -2 & 7 & 0 \end{bmatrix}$$

$R_2 \to 3R_2 - 4R_1$

$R_3 \to 3R_3 - R_1$

$R_4 \to 3R_4 - R_1$

$$\begin{bmatrix} 3 & 2 & 0 & 1 \\ 0 & -8 & 21 & -1 \\ 0 & 1 & 3 & 8 \\ 0 & -8 & 21 & -1 \end{bmatrix}$$

$R_3 \to 8R_3 + R_2$ $\quad$ $R_4 \to R_4 - R_2$

$$\begin{bmatrix} 3 & 2 & 0 & 1 \\ 0 & -8 & 21 & -1 \\ 0 & 0 & 45 & 63 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\rho(A) = \rho(A|B) = 3$

unique solution

Problems on System of Linear Equations

Q. Consider the system of linear equations:

$x - 2y + 3z = -1$

$x - 3y + 4z = 1$ and

$-2x + 4y - 6z = k,$

The value of 'k' for which the system has infinitely many solution is _____. **[GATE – 15 – EC – SET1]**

$$(A|B) = \begin{bmatrix} 1 & -2 & 3 & -1 \\ 1 & -3 & 4 & 1 \\ -2 & 4 & -6 & k \end{bmatrix}$$

$$\begin{array}{l} R_2 \to R_2 - R_1 \\ R_3 \to R_3 + 2R_1 \end{array} \begin{bmatrix} 1 & -2 & 3 & -1 \\ 0 & -1 & 1 & 2 \\ 0 & 0 & 0 & k-2 \end{bmatrix}$$

If $k = 2 \Rightarrow \left[\begin{array}{ccc|c} 1 & -2 & 3 & -1 \\ 0 & -1 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$\rho(A) = 2 = \rho(A|B) <$ variables

infinately many solution

Consider the linear equations

$$x - 2y + z = 3,$$

$$2x + \alpha z = -2,$$

$$-2x + 2y + \alpha z = 1.$$

In order to have unique solution to this linear system of equations the value of α should not be equal to

(a) $\frac{-2}{3}$ ✓ (b) $\frac{2}{3}$

(c) $\frac{4}{3}$ (d) $\frac{-4}{3}$

$$|A| \neq 0$$

$$\begin{vmatrix} 1 & -2 & 1 \\ 2 & 0 & \alpha \\ -2 & 2 & \alpha \end{vmatrix} \neq 0$$

$$1(-2\alpha) + 2(4\alpha) + 1(4) \neq 0$$

$$4 + 6\alpha \neq 0$$

$$\alpha \neq -\frac{2}{3}$$

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Problems on System of Linear Equations

Q. Let AX=B be a system of three equations in three variables x, y and z. If A has three linearly independent columns and B is a linear combination of the columns of A, then which of the following is true?

(a) The system has unique solution

(b) The system has infinitely many solutions

(c) The system has no solution

(d) The system AX=O has non-zero solution

$m=3 \quad n=3$

$A_{3\times3} X_{3\times1} = B_{3\times1}$

$\rho(A)=3$ & $|A| \neq 0$

B is l.c of colums of A

$\Rightarrow$ $\boxed{AX=B \text{ is consistent}}$

Problems on System of Linear Equations

Q. Consider the matrix

$$A = \begin{bmatrix} k & k & k \\ 0 & k-1 & k-1 \\ 0 & 0 & k^2-1 \end{bmatrix}$$

If the system AX=O has only one independent solution then k = ______.

(a) 0, -1

(b) -1, 1

(c) 0, 1

(d) 0, 1, -1

$n = 3$

Nullity $= n - r = 1$

$\Rightarrow 3 - r = 1 \Rightarrow r = 2$

$k = 1$: $\begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ $r = 1$

$\therefore$ k is not equal 1

$k = 0$: $\begin{pmatrix} 0 & 0 & 0 \\ 0 & -1 & -1 \\ 0 & 0 & -1 \end{pmatrix} \simeq \begin{pmatrix} 0 & -1 & -1 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{pmatrix} \rightarrow r = 2$

$k = -1$: $\begin{pmatrix} -1 & -1 & -1 \\ 0 & -2 & -2 \\ 0 & 0 & 0 \end{pmatrix}$ $r = 2$

$\therefore k = 0, -1$

Problems on System of Linear Equations

$|A| = \begin{vmatrix} 2 & -2 \\ 1 & -1 \end{vmatrix} = 0$

Infinitely many solutions

Q. The equation

$$\begin{bmatrix} 2 & -2 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \text{has}$$

[GATE-13[EE]]

(a) no solution ✗

(b) only one solution ✗

(c) non-zero unique solution ✗

(d) multiple solutions ✓

Problems on System of Linear Equations

Q. Let A be 3×3 matrix with rank 2.

Then AX = O has [GATE – 05(IN)]

(a) only the trivial solution X = O ✗

(b) one independent solution ✓

(c) two independent solutions

(d) three independent solutions

2 + (Validity) =

$m = 3 \quad n = 3$

$r = \rho(A) = 2 \Rightarrow r < n$

Infinitely many soln

$n - r$ solutions are L.I

$3 - 2 = 1$ L.I solution exists

$$\begin{pmatrix} 2 & 5 & 4 \\ 0 & 2 & 4 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \begin{array}{l} 2x + 5y + 4z = 0 \\ 2y + 4z = 0 \end{array}$$

let $z = k \Rightarrow y = -2k$

$\Rightarrow x = 3k$

$$X = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3k \\ -2k \\ k \end{pmatrix} = k\begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}$$

L.I

4.8 Problems on System of Linear Equations

Q. Suppose that P is a 4 x 5 matrix such that every solution of the equation Px = 0 is a scalar multiple of $[2\ 5\ 4\ 3\ 1]^T$.

The rank of P is 4. **(GATE-21-CE-SET2)**

$X = k\underbrace{\begin{bmatrix} 2 \\ 5 \\ 4 \\ 3 \\ 1 \end{bmatrix}}_{L.I}$ $n = 5$

NO of L.I solution is 1

ie $n - r = 1$

$5 - r = 1$

$r = 4$

Problems on System of Linear Equations

Q. The dimension of null space of the matrix

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & -1 & 0 \\ -1 & 0 & -1 \end{bmatrix}$$

$n = 3$

Is ……………………… **(GATE-13-IN)**

$R_2 \leftrightarrow R_1 \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \\ -1 & 0 & -1 \end{pmatrix}$

$R_3 \rightarrow R_3 + R_1 \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \\ 0 & -1 & -1 \end{pmatrix}$

$R_3 \rightarrow R_3 + R_2 \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix} \rightarrow r = 2$

Sol: dimension of Nullspace $= n - r$

$= 3 - r$

$= 3 - 2$

$= 1$

$r = \rho(A) = 2 \Rightarrow$

Problems on System of Linear Equations

Q. For the matrix

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 4 & -3 & 3 \\ 0 & 2 & -1 \end{bmatrix}$$

$|A| = 2(-3) - 0 + 1(8) = 2 \neq 0$

1) A is non singular

2) $\rho(A) = 3$

3) $AX = 0$ has unique solution

Which of the following is /are true ?

(a) The matrix A is non singular ✓

(b) The rank of A is 3 ✓

(c) The system AX=O has unique solution ✓

(d) Det A= 6 ✗

MSQ

5.Eigen Values and Eigen Vectors

5.1 Definition of Eigen value and Eigenvector

5.2 Algebraic multiplicity and geometric multiplicity

5.3 Properties of eigenvalues and eigenvectors

5.4 Diagonalization

5.5 Cayley- Hamilton theorem

5.6 Problems

5.1Definition of eigen values and eigen vectors

Let A be a square matrix of order ‘n’ and ‘λ’ be a scalar.

1.$|A - \lambda I| = 0$ is called the *characteristic equation* of A.

2.The roots of characteristic equation are called *Eigen values* (*characteristic roots / latent roots*) of A.

3.Corresponding to each Eigen value λ, there exists a non-zero vector X such that $AX = \lambda X$ or $(A - \lambda I)X = O$.

Here, X is called an *Eigen vector* (*characteristic vector or latent vector*) of A.

Note 1 Eigen vector is never equals to zero vector

Note2 we can have zero as eigen value

5.1 Definition of eigen values and eigen vectors

Example. $A = \begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix}$

Ch eauation $|A - \lambda I| = 0$

$$\left| \begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right| = 0$$

$$\Rightarrow \begin{vmatrix} 2-\lambda & -1 \\ 1 & 3-\lambda \end{vmatrix} = 0$$

$$(2-\lambda)(3-\lambda) + 1 = 0$$

$$\lambda^2 - 5\lambda + 7 = 0$$

Roots are eigen values.

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$$A = \begin{pmatrix} 4 & 2 \\ -1 & 1 \end{pmatrix}$$

$$\left| \begin{pmatrix} 4 & 2 \\ -1 & 1 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right| = 0$$

$$\begin{vmatrix} 4-\lambda & 2 \\ -1 & 1-\lambda \end{vmatrix} = 0$$

$$(4-\lambda)(1-\lambda) + 2 = 0$$

$$\lambda^2 - 5\lambda + 6 = 0$$

$$(\lambda-2)(\lambda-3) = 0$$

$$\lambda = 2, 3$$

eignvalues are $\underline{2, 3}$.

<u>eigenvectors corresponding to $\lambda = 2$</u>

$$(A - 2I)X = 0$$

$$\left(\begin{pmatrix} 4 & 2 \\ -1 & 1 \end{pmatrix} - 2 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right) \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 2 \\ -1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \longrightarrow \text{Homogeneous system}$$

$$R_2 \to 2R_2 + R_1 \quad \begin{pmatrix} 2 & 2 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$2x + 2y = 0$$

$$\Rightarrow x + y = 0$$

let $y = k$

$\Rightarrow x = -k$

eigenvector $X = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -k \\ k \end{pmatrix}$

$= k \begin{pmatrix} -1 \\ 1 \end{pmatrix}$ ✓

$\underline{k=1}$ $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$

$\underline{k=2}$ $\begin{pmatrix} -2 \\ 2 \end{pmatrix}$

$\underline{k=-1}$ $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ - - - -

Infinitely many eigenvectors are there

Hence one L.I eigenvector exists corresponding to $\lambda = 2$

Similarly for $\lambda = 3$ also we can find eigenvectors

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5.1 Definition of eigen values and eigen vectors

observation: (1) For $A_{n \times n}$ matrix n eigenvalues exists

(2) For each eigenvalue Infinitely many eigenvectors exists

(3) out of these infinitely many eigen vectors few are L.I.

Algebraic multiplicity of λ : Number of times λ Repeated.

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$A_{3\times3} \longrightarrow$ Its eigen values are $-1, -1, 4$

A.M of -1 — 2

A.M of 4 — 1.

$A_{4\times4} \rightarrow 0, 0, 0, 1$

A.M of 0 — 3

A.M of 1 — 1

Geometric multiplicity of λ

$= \text{Number of L.I eigenvectors of } \lambda$

$= n - r$

$\cancel{(A-\lambda I)X = 0}$

$\cancel{PX = 0}$

here n = order of A

r = Rank of $(A-\lambda I)$

Result For any eigenvalue λ, $1 \le G.M \le A.M$

5.1Definition of eigen values and eigen vectors

Shortcut method for 2x2 matrix

$\lambda^2 - S_1\lambda + S_2 = 0$

$S_1 = Trace\ of\ the\ matrix$

$S_2 = Det\ of\ \ the\ matrix$

$A = \begin{pmatrix} 5 & -4 \\ 1 & 1 \end{pmatrix}$

$\lambda^2 - 6\lambda + 9 = 0$

5.1Definition of eigen values and eigen vectors

Shortcut method for 3×3

$\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$

$S_1 = Trace\ of\ the\ matrix$

$S_2 = Sum\ of\ the\ principal\ minors\ of\ \ the\ matrix$

$S_3 = Det\ of\ the\ matrix$

$$\lambda^3 - 8\lambda^2 + 10\lambda - 17 = 0$$

$$A = \begin{bmatrix} 2 & 1 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 5 \end{bmatrix}$$

$S_1 = \text{trace} = 2+1+5 = 8$

$S_2 = \begin{vmatrix} 1 & 2 \\ 0 & 5 \end{vmatrix} + \begin{vmatrix} 2 & 3 \\ 0 & 5 \end{vmatrix} + \begin{vmatrix} 2 & 1 \\ 0 & 1 \end{vmatrix}$

$= 5 + 10 + 2$

$= 17$

$S_3 = 10$

Detailed method

$$\begin{vmatrix} 2-\lambda & 1 & 3 \\ 0 & 1-\lambda & 2 \\ 0 & 0 & 5-\lambda \end{vmatrix} = 0$$

to be simplified

5.1 Definition of eigen values and eigen vectors

Is there any short cut method to find characteristic equation of higher order matrices?

NO

Just simplify $|A-\lambda I| = 0$

5.3 Properties of eigen values and eigenvectors

Imp

1. Eigen values of lower triangular matrix ,upper triangular matrix and diagonal matrix are just the diagonal elements only.

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 7 \end{pmatrix}$$

eigenvalues are 1, 5, 7

2. If $\lambda_1, \lambda_2, \ldots\ldots \lambda_n$ are Eigen values of matrix A of order n, then

   (a) $\lambda_1 + \lambda_2 + \ldots. + \lambda_n =$ trace of A

   ie, sum of eigen values = Tr(A)

   (b) $\lambda_1 \cdot \lambda_2 \ldots\ldots \cdot \lambda_n = |A|$

   product of eigen values = det A

3. 0 is an Eigen value of matrix A if and only if A is singular.

4. If all the Eigen values of A are non-zero then A is non-singular

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eigenvalues of $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$ are

(a) 0, 0, 3 ✓

(b) 1, 1, 1 ✗

(c) 1, 2, 1 ✗

(d) 1, -1, 4 ✗

10 marks

Detailed method

$$\begin{vmatrix} 1-\lambda & 1 & 1 \\ 1 & 1-\lambda & 1 \\ 1 & 1 & 1-\lambda \end{vmatrix} = 0$$

4 marks

$$\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$$

trace = 3 ✓

det = 0 ✓

1 mark

trace ✓

det ✓

5.3 Properties of eigen values and eigenvectors

5. If λ is an eigen value of a matrix A and k is a scalar then

(a) λ^m is eigen value of matrix A^m $(\because m \in N)$

(b). $k\lambda$ is an eigen value of matrix kA.

Note: Eigen vectors remains same only

(c). $\lambda+k$ is an eigen value of matrix $A+kI$

(d). $\lambda-k$ is an eigen value of matrix $A-kI$

(e) $a_0+a_1\lambda+a_2\lambda^2+\ldots\ldots+a_n\lambda^n$ is an eigen value of matrix

$a_0I+a_1A+a_2A^2+\ldots\ldots\ldots+a_nA^n$

5.3Properties of eigen values and eigenvectors

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6. If λ is an Eigen value of a non-singular matrix A then

(i) $\frac{1}{\lambda}$ is an Eigen value of A^{-1} and

(ii) $\frac{|A|}{\lambda}$ is an Eigen value of adj (A) (here $\lambda \neq 0$)

The Eigen values of A and A^T are same

.(but The Eigen vectors of A and A^T are not same)

8. Corresponding to each Eigen value of a matrix there exist infinitely many eigen vectors.

5.3 Properties of eigen values and eigenvectors

09.If $\lambda_1,\lambda_2,\ldots.\lambda_n$ are distinct eigen values of a square matrix A of order 'n' then the corresponding eigen vectors $X_1,X_2,\ldots,X_n$ of matrix A are linearly independent.

Note: But in the case of repeated eigen values eigen vectors are may or may not be linearly independent

10.The Eigen vectors of A and A^{-1} same

11. Eigen vectors corresponding to distinct Eigen values of symmetric matrix are orthogonal.

Matrix	Eigen values
Hermitian matrix(symmetric matrix)	Always real
Skew-Hermitian matrix (Skew-Symmetric matrix)	Purely imaginary or zero
Unitary matrix(Orthogonal matrix)	having absolute value one
Idempotent matrix	Zero , one only
Involutary matrix	1 , -1 only
Nilpotent matrix	zero only