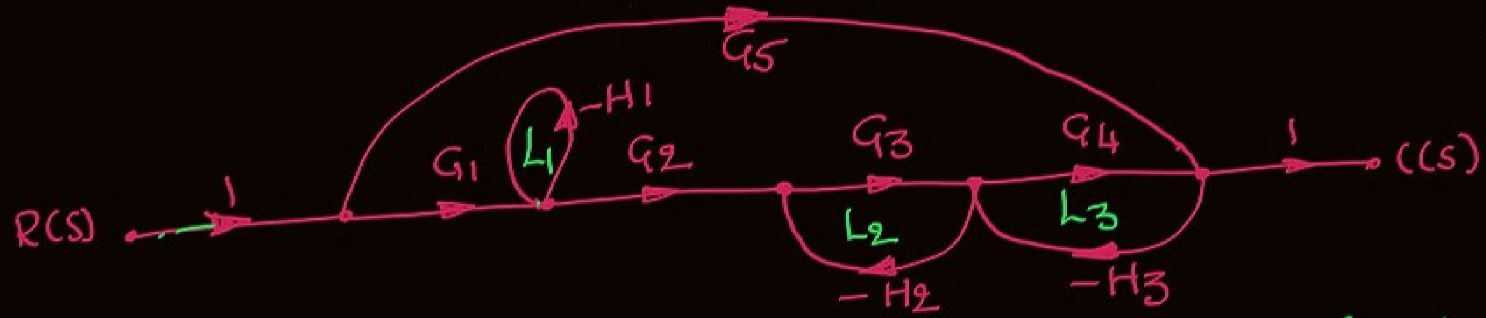
# Signal Flow Graphs (SFG)

Prob:- Find the overall TF to the given SFG




forward paths

$\boxed{P_1 \rightarrow G_1G_2G_3G_4}$

$P_2 \rightarrow G_5$

Loops

$L_1 = -H_1$

$L_2 = -G_3H_2$

$L_3 = -G_4H_3$

2-NTL

$L_1L_2 = +G_3H_1H_2$

$L_1L_3 = +G_4H_1H_3$

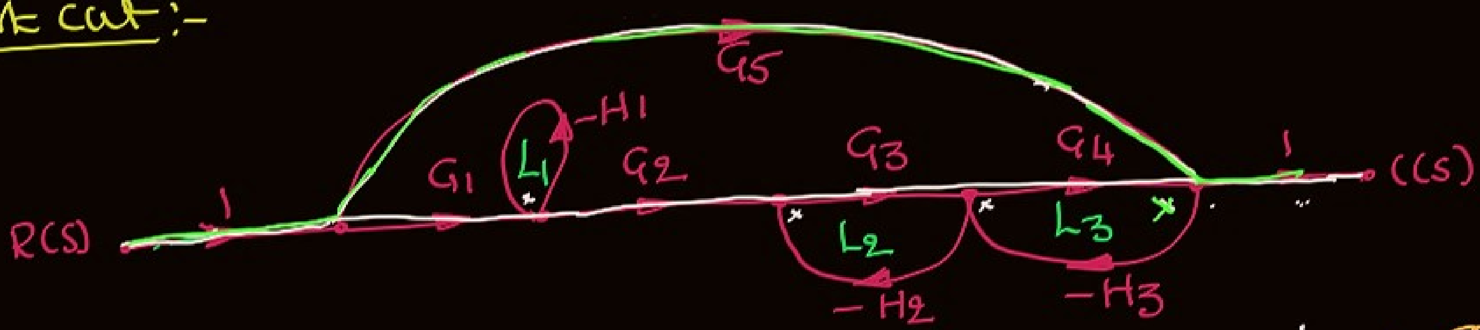
$\Delta = 1-(L_1+L_2+L_3)+L_1L_2+L_1L_3$

$\Delta_1 = 1$

$\Delta_2 = 1-(L_1+L_2)+L_1L_2$

$$\frac{C(s)}{R(s)} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta} = \frac{G_1G_2G_3G_4 + G_5(1+H_1+G_3H_2+G_3H_1H_2)}{1+H_1+G_3H_2+G_4H_3+G_3H_1H_2+G_4H_1H_3}$$

Short cut :-




$$\frac{C(s)}{R(s)} = \frac{G_1G_2G_3G_4 + G_5(1+H_1+G_3H_2+G_3H_1H_2)}{1+H_1+G_3H_2+G_4H_3+G_3H_1H_2+G_4H_1H_3}$$

$\Delta$ or $\Delta_K = 1 - 1L + 2NTL - 3NTL + 4NTL$

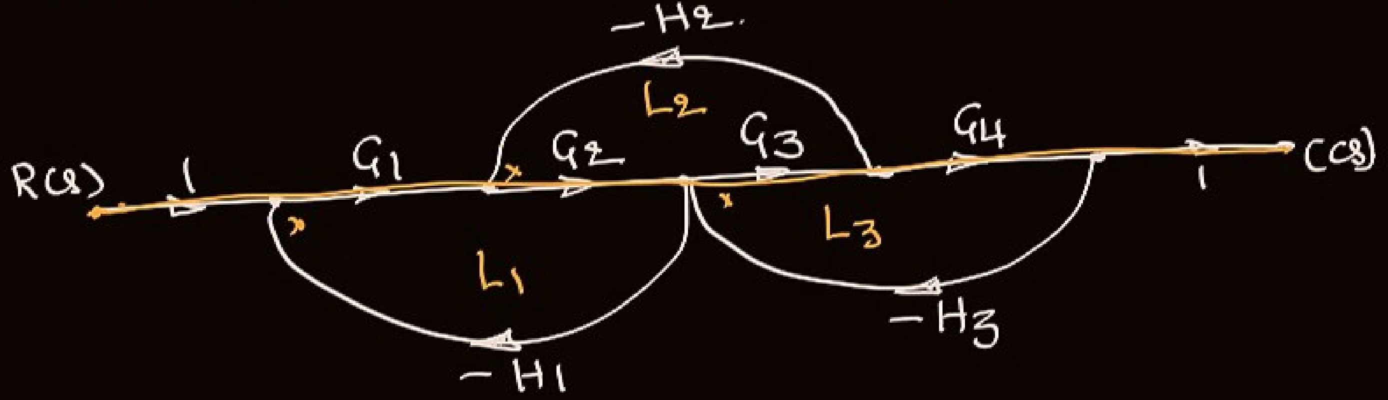
consider the Loops which are non-touching to the selected forward path

⇒ Take opposite sign for odd number of Non-touching loops

⇒ Take same sign for even number of Non-touching loops.

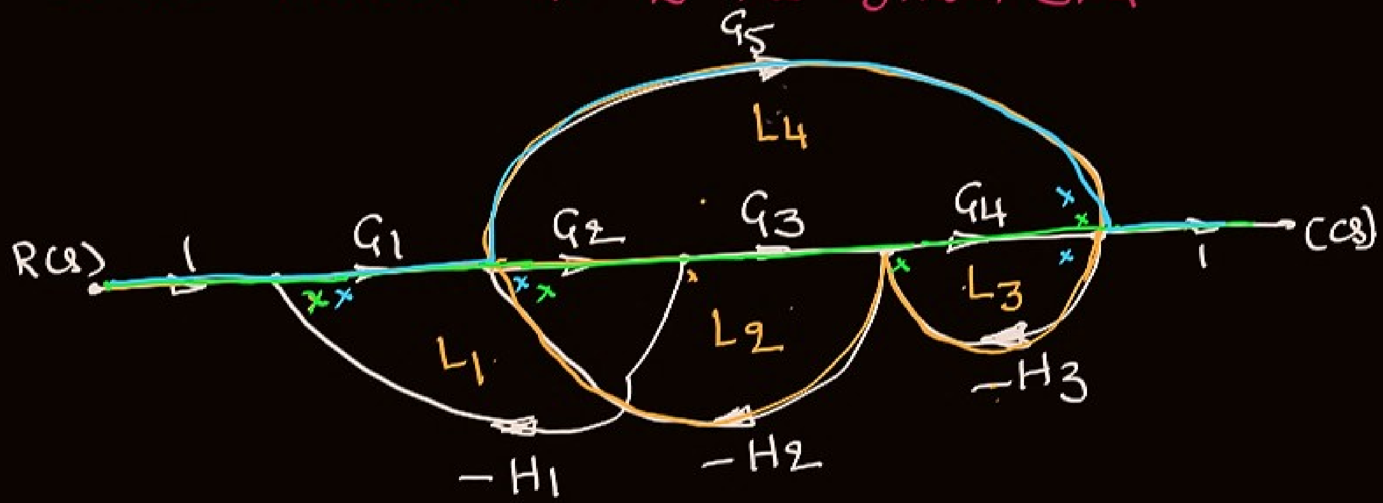
$$\frac{C(s)}{R(s)} = \frac{P_1\left[\begin{array}{l}\text{observe NTL to } P_1. \\ \text{If Non-touching Loops are exist, then} \\ (1 + \text{opposite sign of 1 Loop gains} \\ \quad + \text{Same sign of 2-NTL gain} \\ \quad + \text{opposite sign of 3-NTL gains} \\ \quad + \text{Same sign of 4-NTL gains} \\ \quad \vdots \qquad ) \end{array}\right] + P_2\left[\begin{array}{l}\text{observe NTL to } P_2 \\ \text{If NTL are exist, then} \\ (1 + \text{opposite sign of 1 Loop gains} \\ \quad + \text{Same sign of 2 NTL gains} \\ \quad + \text{opposite sign of 3-NTL gains} \\ \quad + \text{Same sign of 4-NTL gains} \\ \quad + \cdots \qquad ) \end{array}\right]}{\begin{array}{l}1 + \text{Opposite sign of 1 Loop gains} + \text{Same sign of 2 NTL gains} + \\ \text{opposite sign of 3-NTL gains} + \text{Same sign of 4 NTL gains} + \cdots \end{array}}$$

Prob :- find the TF to the given SFG.



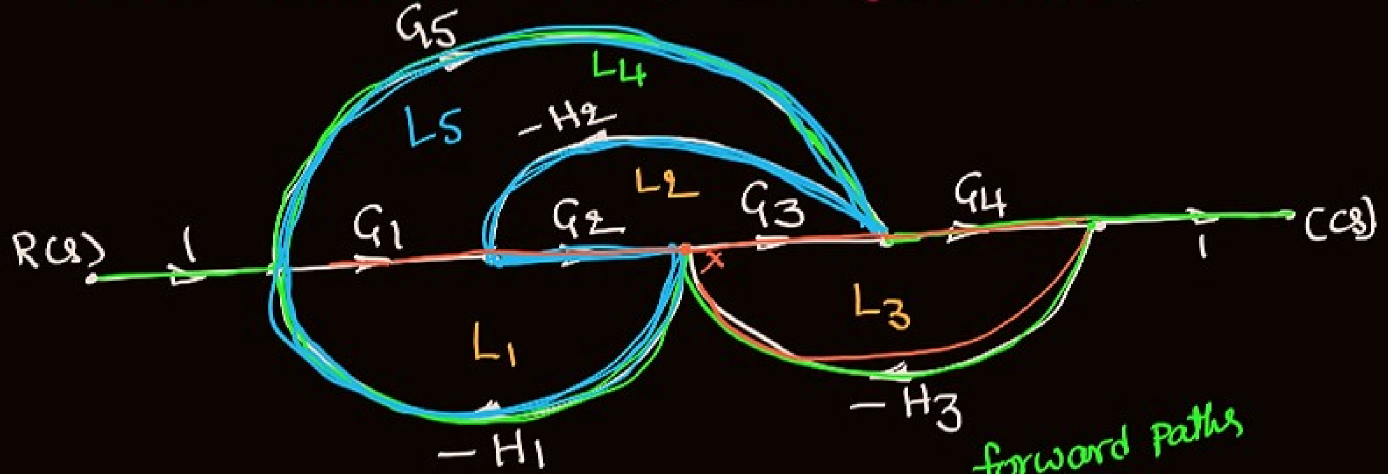

$$\frac{C(s)}{R(s)} = \frac{G_1 G_2 G_3 G_4}{1 + G_1 G_2 H_1 + G_2 G_3 H_2 + G_3 G_4 H_3}$$

Prob :- find overall TF to the given SFG




$$\frac{C(s)}{R(s)} = \frac{G_1G_2G_3G_4 + G_1G_5}{\underset{-L_1}{1+G_1G_2H_1} \underset{-L_2}{+G_2G_3H_2} \underset{-L_3}{+G_4H_3} \underset{-L_4}{-G_5H_2H_3} \underset{+\;L_1\;L_3}{+G_1G_2H_1G_4H_3}}$$

Prob:- find overall TF to the given SFG




forward paths

$G_1G_2G_3G_4$ $+G_4G_5$

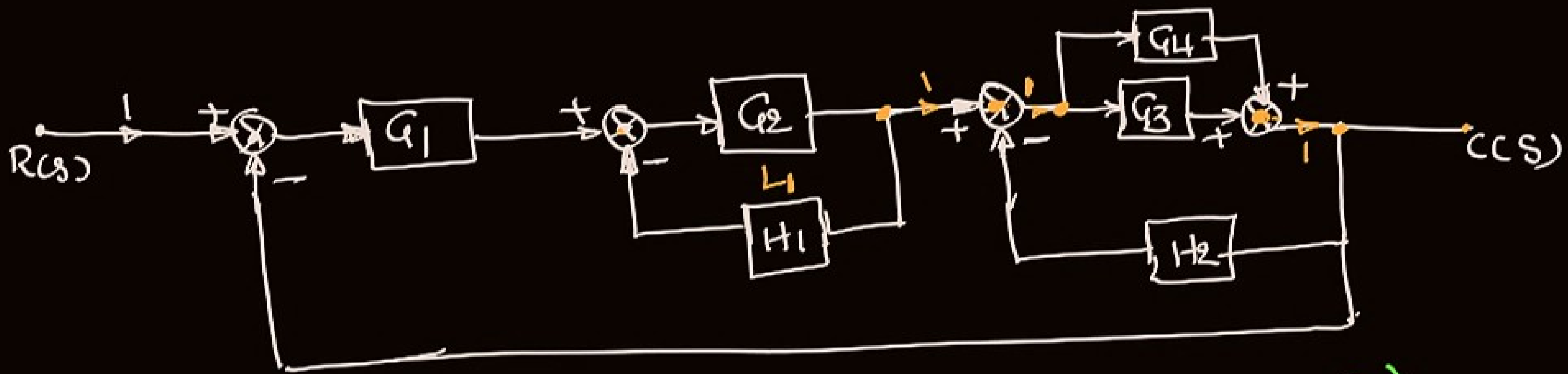
$1+G_1G_2H_1+G_2G_3H_2+G_3G_4H_3$ $-G_4G_5H_1H_3$ $-G_2G_5H_1H_2$ $\cancel{G_1G_2G_3G_4H_1H_3}$

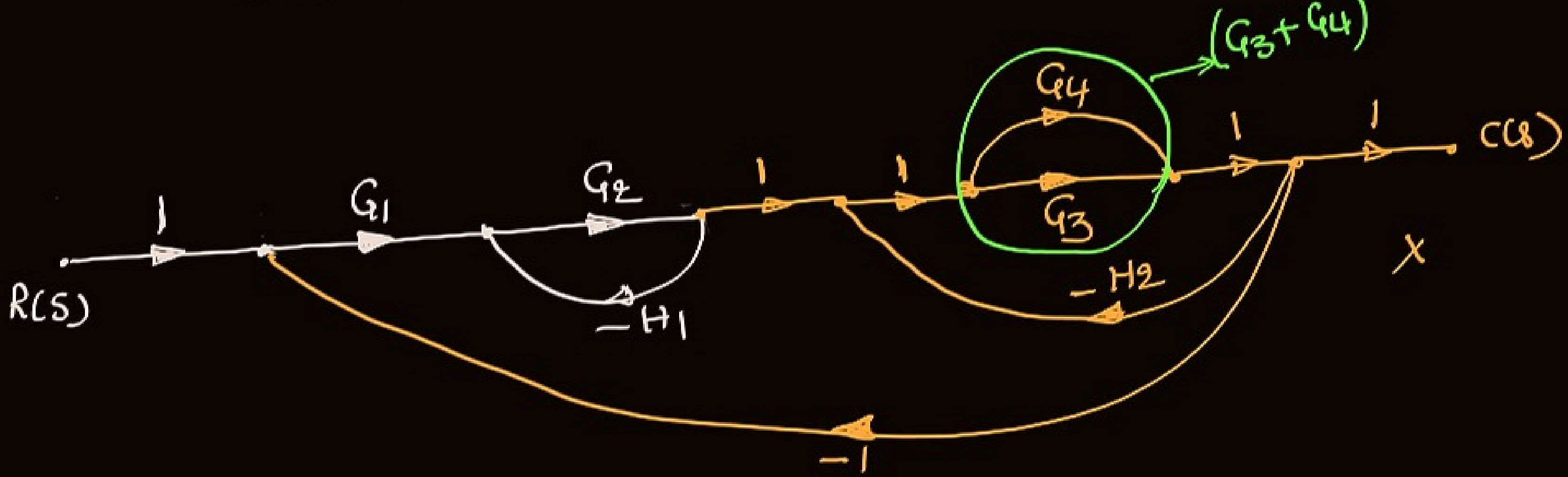
Loops

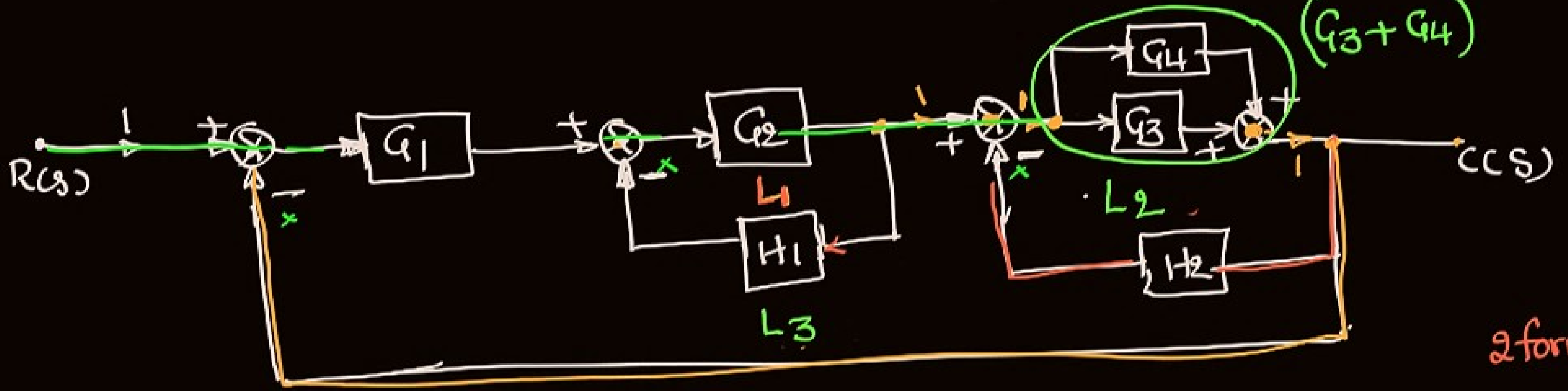
Loop

Ans $\Rightarrow$ $$\frac{C}{R}=\frac{G_1G_2G_3G_4+G_4G_5}{1+G_1G_2H_1+G_2G_3H_2+G_3G_4H_3-G_4G_5H_1H_3-G_2G_5H_1H_2}$$

Prob :- find TF to the given BD by using Mason's gain formula.



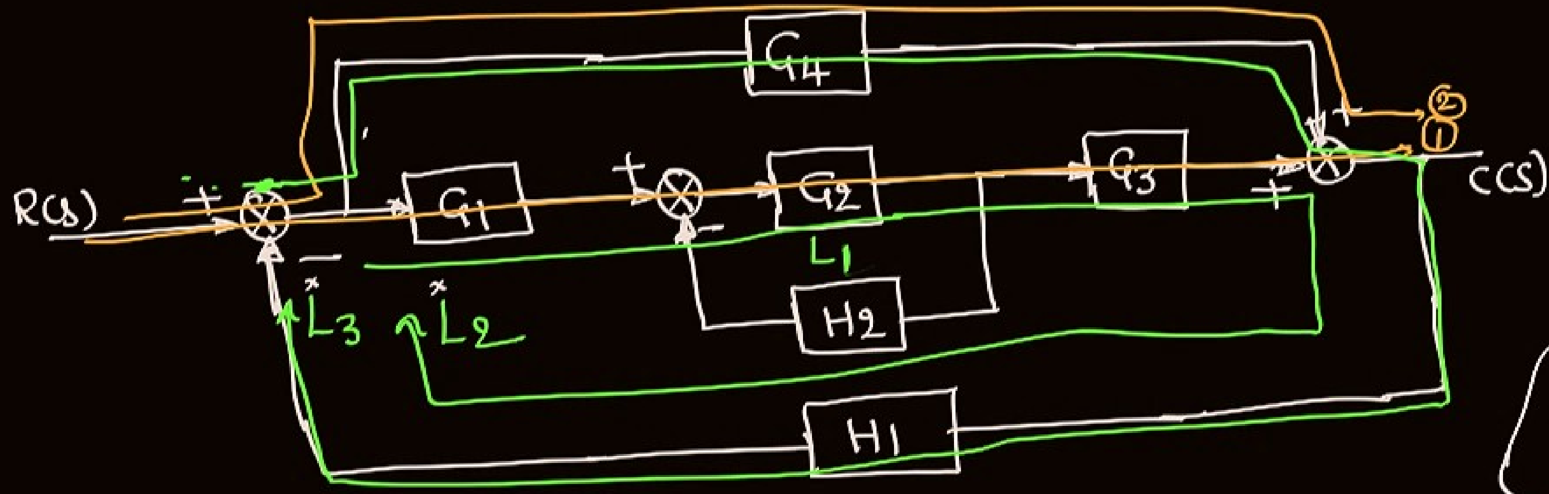



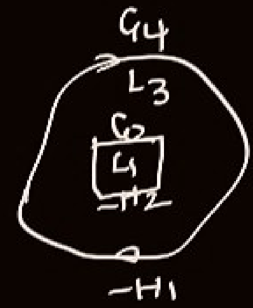



2 forwar paths.

$$\frac{C(s)}{R(s)} = \frac{G_1G_2(G_3+G_4)}{\underset{-L_1}{1+G_2H_1} + \underset{-L_2}{(G_3+G_4)H_2} + \underset{-L_3}{G_1G_2(G_3+G_4)\cdot 1} + \underset{L_1\ L_2}{G_2H_1(G_3+G_4)H_2}}$$

Prob:- find the TF. to the given BD.

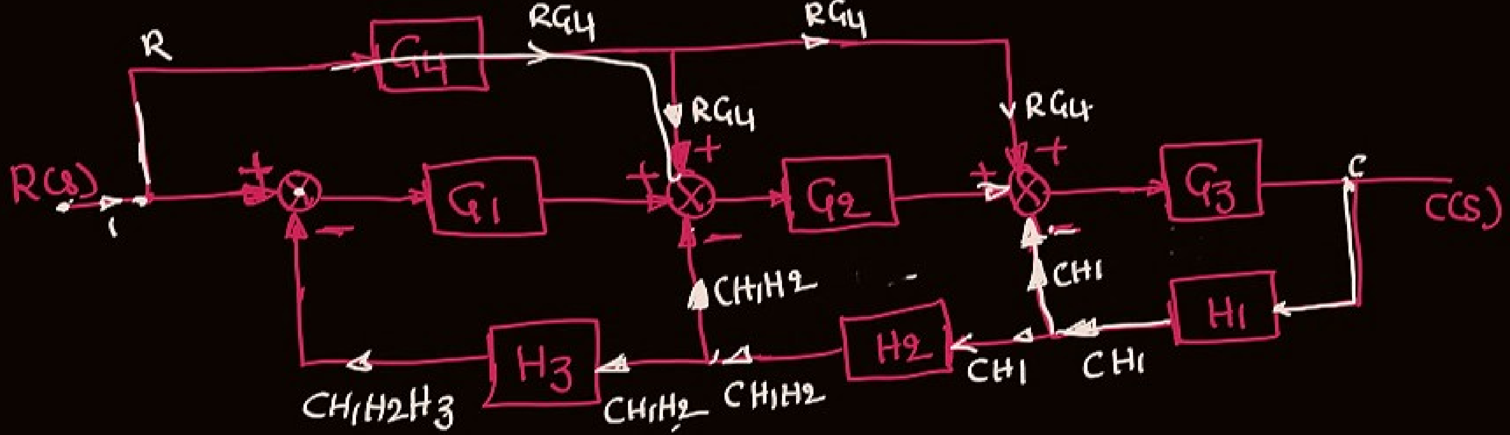


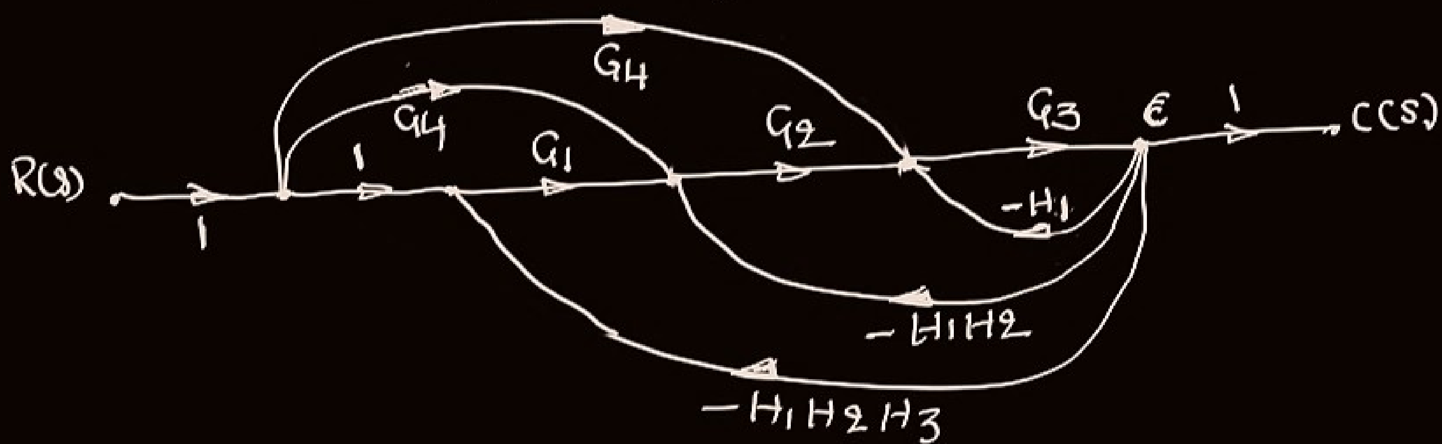



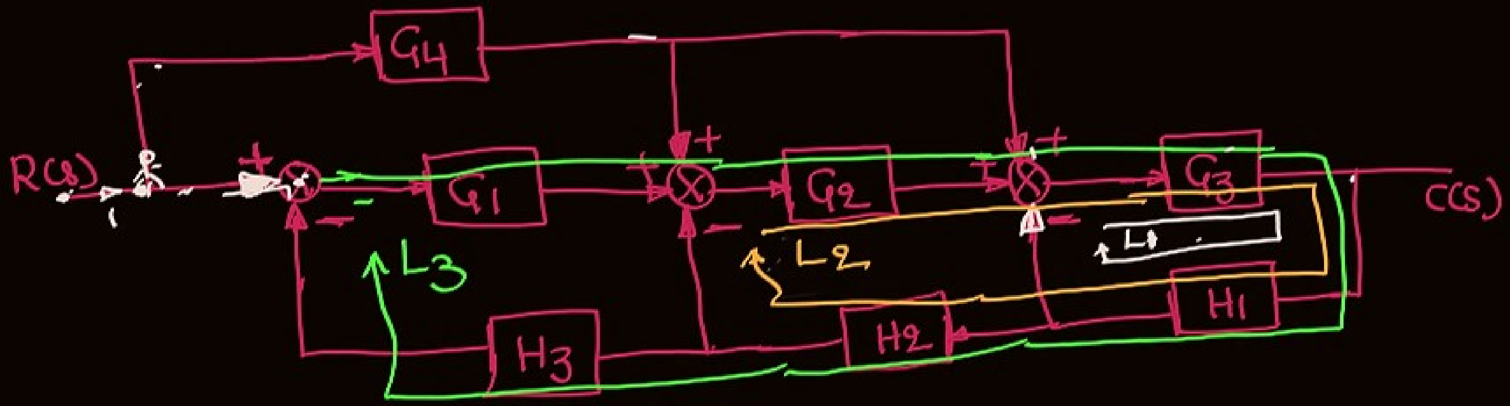

$$\frac{C(s)}{R(s)} = \frac{G_1G_2G_3 + G_4(1+G_2H_2)}{\underset{-L_1}{1+G_2H_2} + \underset{-L_2}{G_1G_2G_3H_1} + \underset{-L_3}{G_4H_1} + \underset{L_1L_3}{G_2H_2 \cdot G_4H_1}}$$

Prob :- find TF to the given BD.

ESE/Gate

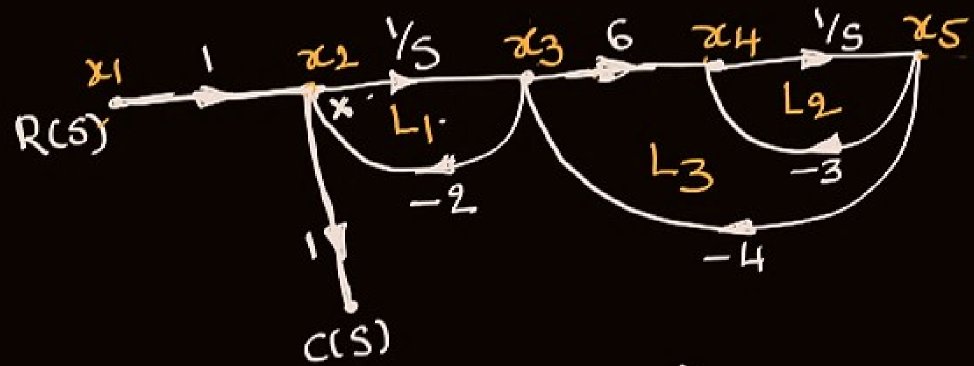





$$\frac{C(s)}{R(s)} = \frac{G_1G_2G_3 + G_4G_2G_3 + G_4G_3}{1 + G_3H_1 + G_2G_3H_1H_2 + G_1G_2G_3H_1H_2H_3}$$

Prob :- Find the C/R to the given system.

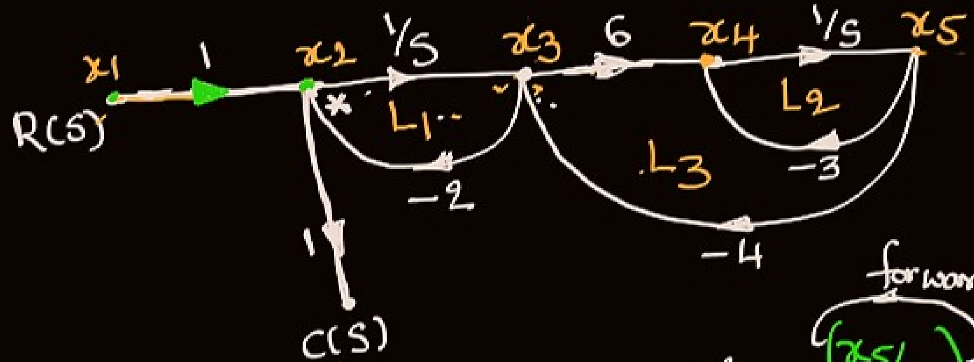



$$\Rightarrow \frac{X_2}{X_1} = \frac{C(s)}{R(s)} = \frac{1\left(1+3/s+24/s\right)}{\left(1+2/s+3/s+24/s+6/s^2\right)} = \Delta$$

$$= \frac{\left(1+27/s\right)}{\left(1+29/s+6/s^2\right)} = \frac{s(s+27)}{s^2+29s+6}$$

⇒ If I/P signal is not reaches the loops, then that loops are not consider

⇒ In the above problem, the I/P signal reaches to all the loops. Hence all the loops are consider.




⇒ Mason's gain formula gives ratio w.r.to i/p node only. But not with Link nodes.

$$\frac{x_3}{x_1} = \frac{\frac{1}{s}\left(1+\frac{3}{s}\right)}{\Delta}$$

$$\frac{x_4}{x_1} = \frac{6/s}{\Delta}$$

$$\frac{x_5}{x_1} = \frac{6/s^2}{\Delta} = \frac{6}{s^2+29s+6}$$

forward path gain

$$\frac{x_5}{x_2} = \frac{(x_5/x_1)}{(x_2/x_1)} = \frac{6/s^2}{1\left(1+3/s+24/s\right)} = \frac{6/s^2}{(1+27/s)} = \frac{6}{s(s+27)}$$

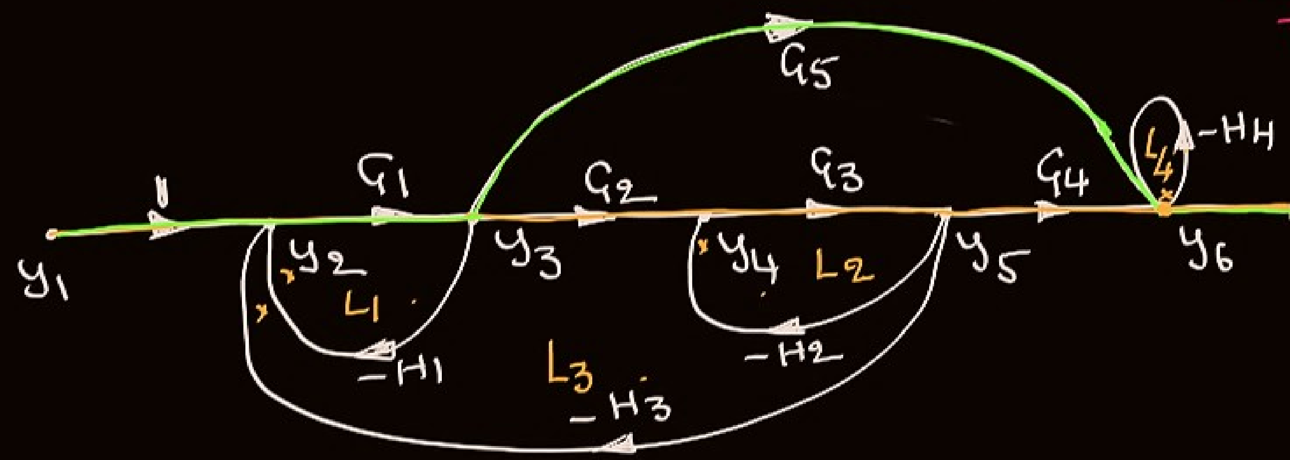
forward path Gain

$$\frac{x_4}{x_3} = \frac{(x_4/x_1)}{(x_3/x_1)} = \frac{6/s}{\frac{1}{s}\left(1+3/s\right)} = \frac{6s}{s+3}$$

forward path gain

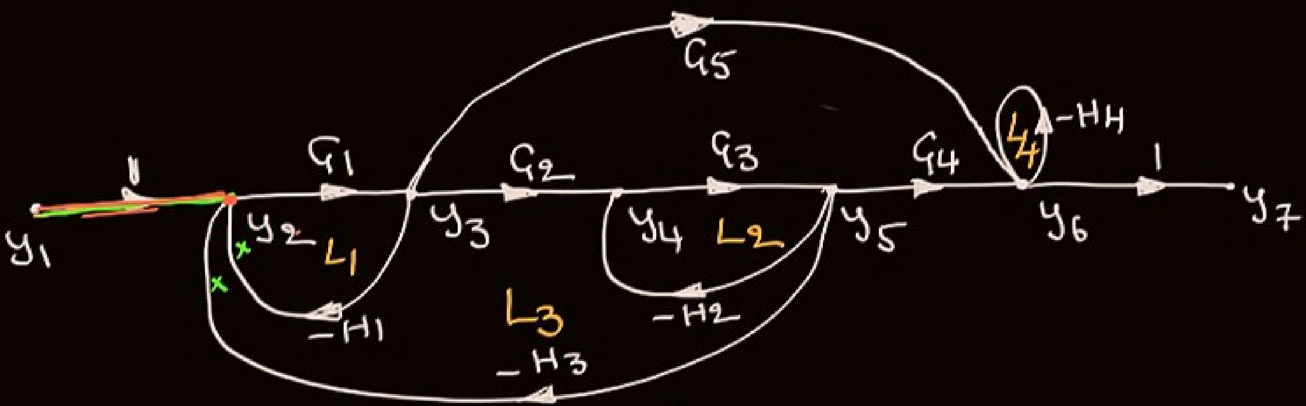
$$\frac{x_3}{x_2} = \frac{x_3/x_1}{x_2/x_1} = \frac{\frac{1}{s}\left(1+3/s\right)}{1\left(1+3/s+24/s\right)} = \frac{(s+3)/s^2}{(s+27)/s} = \frac{(s+3)}{s(s+27)}$$

Prob:- Find $\frac{y_6}{y_1}$, $\frac{y_7}{y_1}$, $\frac{y_5}{y_1}$, $\frac{y_2}{y_1}$, $\frac{y_7}{y_2}$, $\frac{y_5}{y_3}$, $\frac{y_4}{y_3}$ ---- Ratio of any two Nodes.




$y_7 = y_6 \times 1 = y_6$

$$\frac{y_7}{y_1} = \frac{y_6}{y_1} = \frac{G_1G_2G_3G_4 + G_1G_5(1+G_3H_2)}{\left[1 + \underbrace{G_1H_1}_{L_1} + \underbrace{G_3H_2}_{L_2} + \underbrace{G_1G_2G_3H_3}_{L_3} + \underbrace{H_4}_{L_4} + \underbrace{G_1H_1G_3H_2}_{L_1L_2} + \underbrace{G_1H_1H_4}_{L_1L_4} + \underbrace{G_3H_2 \cdot H_4}_{L_2L_4} + \underbrace{G_1G_2G_3H_3 \cdot H_4}_{L_3L_4} + \underbrace{G_1H_1 \cdot G_3H_2 \cdot H_4}_{L_1L_2L_4}\right]} \Rightarrow \Delta$$



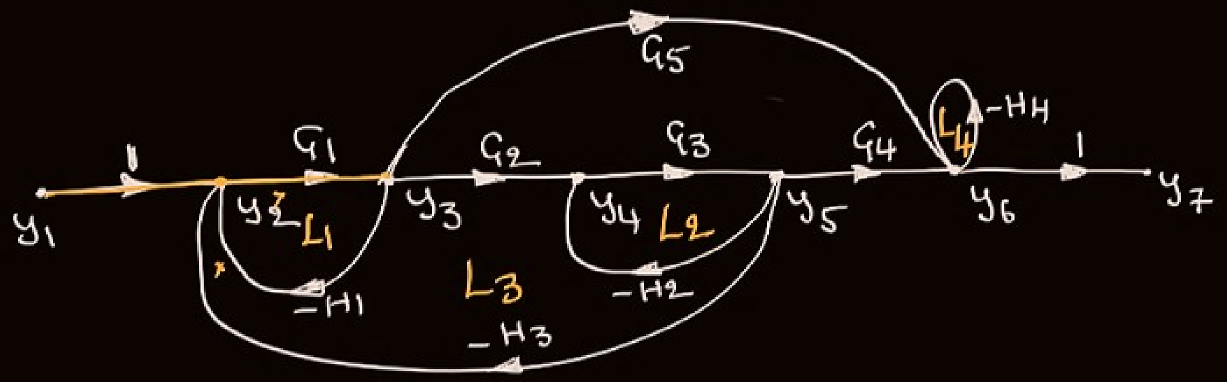

$$\frac{y_5}{y_1} = \frac{G_1 G_2 G_3 (1 + H_4)}{\Delta}$$

$$\frac{y_2}{y_1} = \frac{1 (1 + G_3 H_2 + H_4 + G_3 H_2 H_4)}{\Delta}$$

forward path gains

$$\frac{y_7}{y_2} = \frac{y_7 / y_1}{y_2 / y_1} = \frac{G_1 G_2 G_3 G_4 + G_1 G_5 (1 + G_3 H_2)}{1 (1 + G_3 H_2 + H_4 + G_3 H_2 H_4)}$$

forward path gains

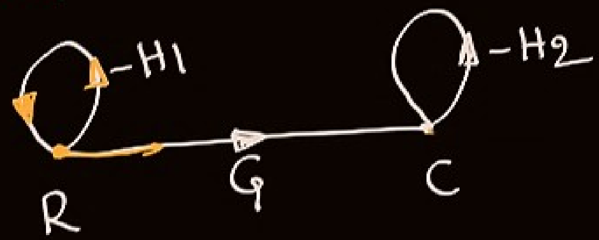



$$\rightarrow \frac{y_5}{y_3} = \frac{y_5/y_1}{y_3/y_1} = \frac{\cancel{G_1} G_2 G_3 (1+H_4)}{\cancel{G_1}\,(1+G_3H_2+H_4+G_3H_2H_4)} = \frac{G_2G_3(1+H_4)}{(1+G_3H_2)+H_4(1+G_3H_2)}$$

$$= \frac{G_2G_3\cancel{(1+H_4)}}{(1+G_3H_2)\cancel{(1+H_4)}} = \left(\frac{G_2G_3}{1+G_3H_2}\right)$$

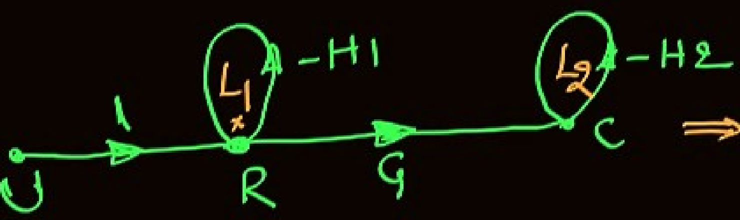
$$\Rightarrow \frac{y_4}{y_3} = \frac{y_4/y_1}{y_3/y_1} = \frac{\cancel{G_1}G_2(1+H_4)}{\cancel{G_1}(1+G_3H_2+H_4+G_2H_2H_4)} = \frac{G_2\cancel{(1+H_4)}}{(1+G_3H_2)\cancel{(1+H_4)}} = \left(\frac{G_2}{1+G_3H_2}\right)$$

Prob :- find $C/R$




(1) ⇒ In the above prob, R is not a Input node. Input node is the one, it should have only outgoing branches.

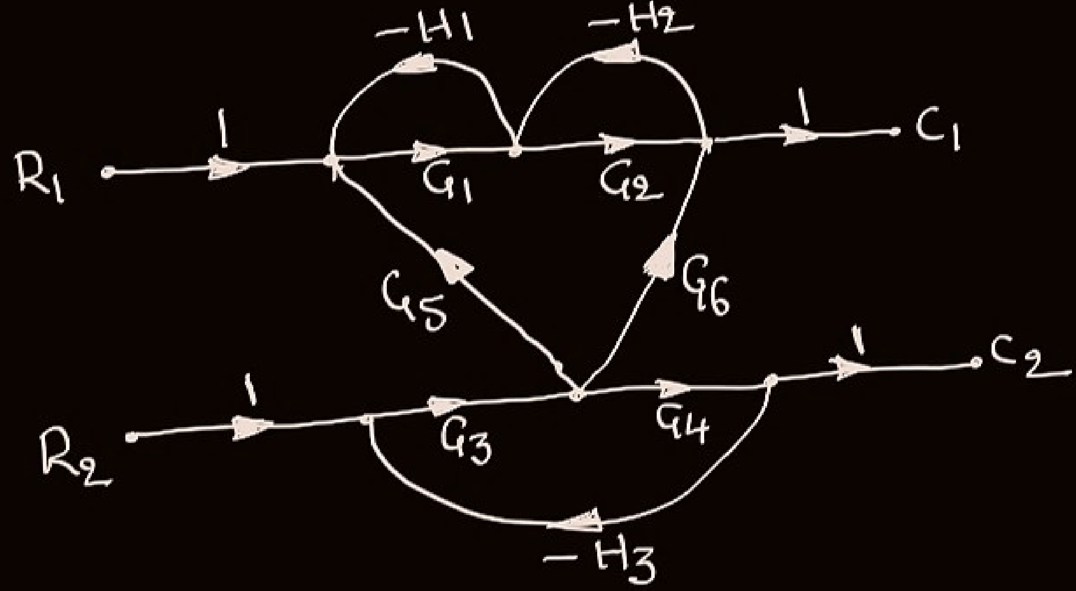
(2) ⇒ In the above prob, we require to create, a dummy i/p node with a path gain of 1, as shown in figure.

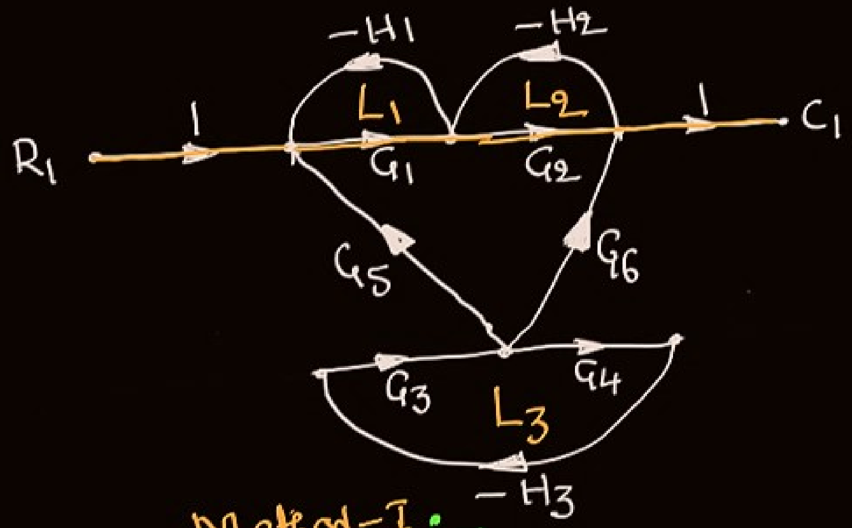



(Dummy i/p Node)

$$\Rightarrow \frac{C}{R} = \frac{C/U}{R/U} = \frac{G}{1(1+H_2)} = \frac{G}{1+H_2}$$

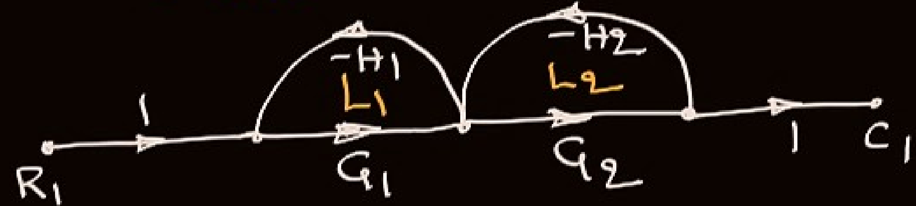
Prob :- find $C_1/R_1$, $C_1/R_2$, $C_1$, $\frac{C_2}{R_1}$, $\frac{C_2}{R_2}$, $C_2$ to the given Multi-Input & Multi-o/p System.





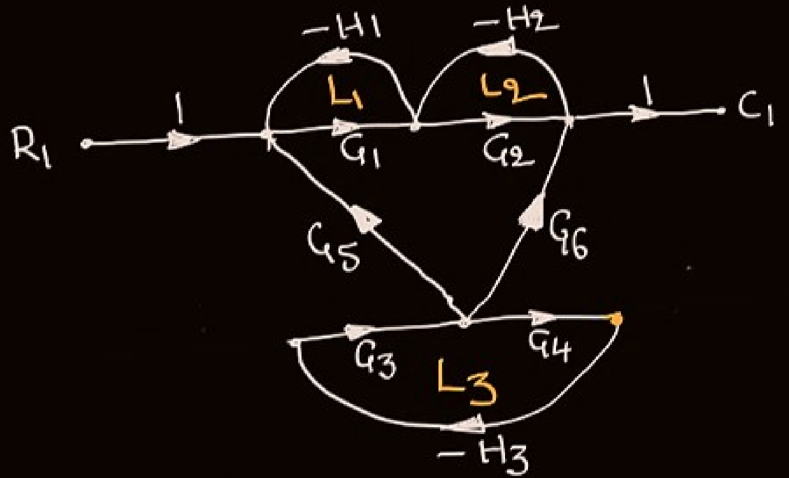

**Method-I** :-

$\Rightarrow \left.\dfrac{C_1}{R_1}\right|_{R_2=C_2=0}$ :→ $\dfrac{C_1}{R_1}$ ⇒ From input $R_1$, there is no path to Loop $L_3$. Hence Loop $L_3$ is not consider.




$$\frac{C_1}{R_1} = \left(\frac{G_1G_2}{1+G_1H_1+G_2H_2}\right)$$

$$C_1 \text{ due to } R_1 = \left(\frac{G_1G_2R_1}{1+G_1H_1+G_2H_2}\right)$$



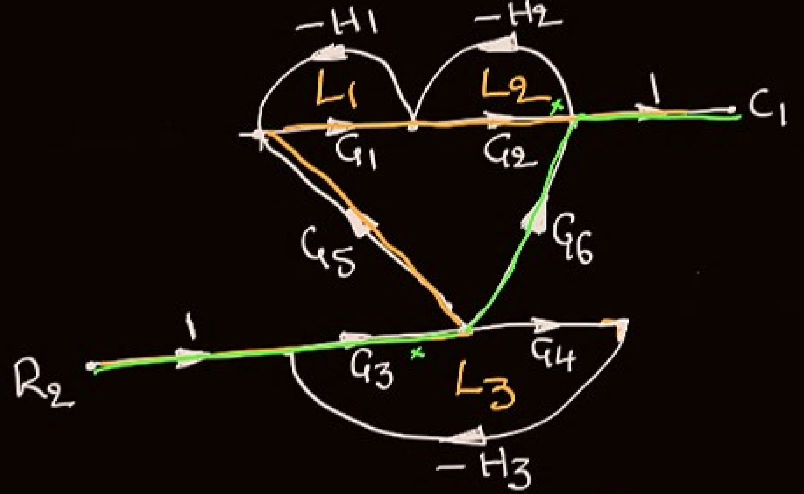

Method – II :–

If $L_3$ consider

$$\frac{C_1}{R_1} = \frac{G_1G_2(1+G_3G_4H_3)}{1+G_1H_1+G_2H_2+\underbrace{G_3G_4H_3}+\underbrace{G_1H_1G_3G_4H_3}+\underbrace{G_2H_2G_3G_4H_3}}$$

$$\frac{C_1}{R_1} = \frac{G_1G_2(1+G_3G_4H_3)}{(1+G_1H_1+G_2H_2)+G_3G_4H_3(1+G_1H_1+G_2H_2)} = \frac{G_1G_2\cancel{(1+G_3G_4H_3)}}{(1+G_1H_1+G_2H_2)\cancel{(1+G_3G_4H_3)}}$$

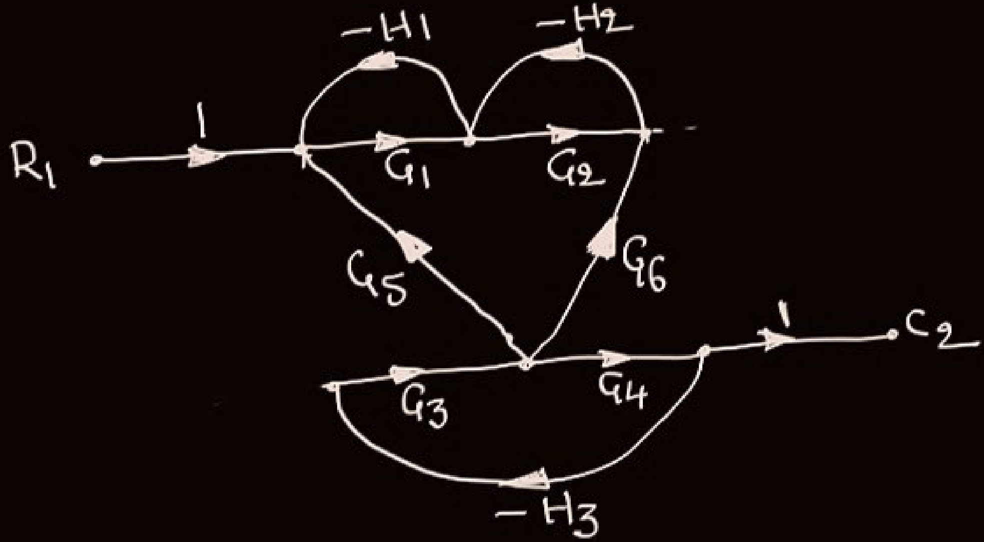
$$\frac{C_1}{R_1} = \left(\frac{G_1G_2}{1+G_1H_1+G_2H_2}\right) \Rightarrow C_1 \text{ due to } R_1 = \left(\frac{G_1G_2R_1}{1+G_1H_1+G_2H_2}\right)$$



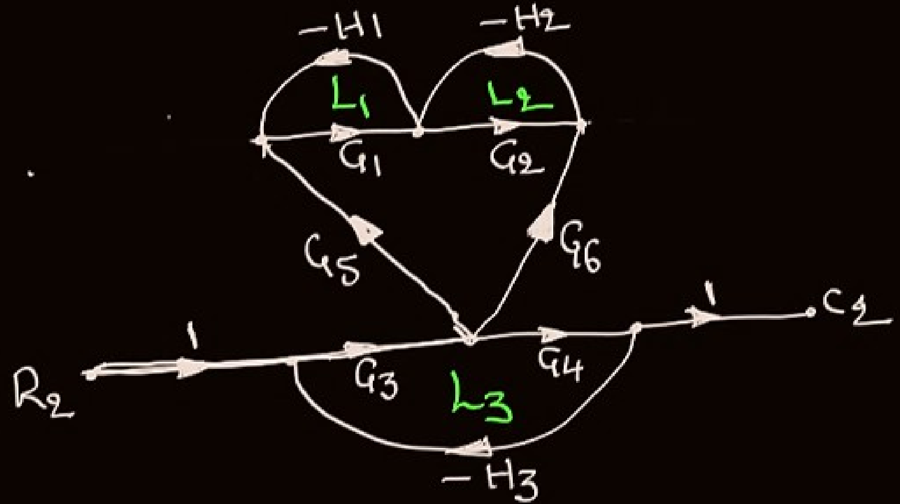

$$\frac{C_1}{R_2} \Big/ R_1 = C_2 = 0 \Rightarrow$$

$$\frac{C_1}{R_2} = \frac{G_3G_5G_1G_2 + G_3G_6(1+G_1H_1)}{1+G_1H_1+G_2H_2+G_3G_4H_3+G_1H_1G_3G_4H_3+G_2H_2G_3G_4H_3}$$

$$C_1 = C_1 \text{ due to } R_1 + C_1 \text{ due } R_2 = \frac{G_1G_2R_1}{1+G_1H_1+G_2H_2} + \frac{[G_3G_5G_1G_2 + G_3G_6(1+G_1H_1)]R_2}{1+G_1H_1+G_2H_2+G_3G_4H_3+G_1H_1G_3G_4H_3+G_2H_2G_3G_4H_3}$$




$$\Rightarrow \left.\frac{C_2}{R_1}\right|_{R_2=C_1=0} = 0 \text{ (No forward path)}$$




$$\Rightarrow \left.\frac{C_2}{R_2}\right|_{R_1 = C_1 = 0} \Rightarrow \frac{C_2}{R_2} = \frac{G_3G_4(1+G_1H_1+G_2H_2)}{1+G_1H_1+G_2H_2+G_3G_4H_3+G_1H_1G_3G_4H_3+G_2H_2G_3G_4H_3}$$

$$= \frac{G_3G_4(1+G_1H_1+G_2H_2)}{(1+G_1H_1+G_2H_2)+G_3G_4H_3(1+G_1H_1+G_2H_2)} = \frac{G_3G_4\cancel{(1+G_1H_1+G_2H_2)}}{\cancel{(1+G_1H_1+G_2H_2)}(1+G_3G_4H_3)}$$

$$\frac{C_2}{R_2} = \frac{G_3G_4}{1+G_3G_4H_3}$$

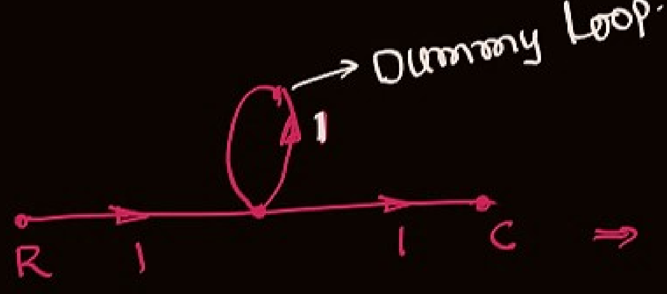
$$C_2 \text{ due to } R_2 = \frac{(G_3G_4)\,R_2}{1+G_3G_4H_3}$$

$$C_2 = C_2 \text{ due to } R_1 + C_2 \text{ due to } R_2$$

$$= 0 + \frac{G_3G_4R_2}{(1+G_3G_4H_3)}$$

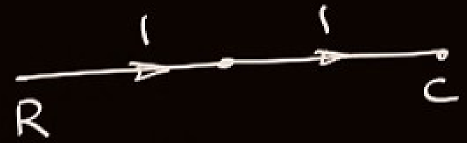
$$C_2 = \left(\frac{G_3G_4R_2}{1+G_3G_4H_3}\right)$$

Prob:- find C/R.




$\Rightarrow$ A self loop with path gain of 1 is called Dummy Loop. Which is not consider

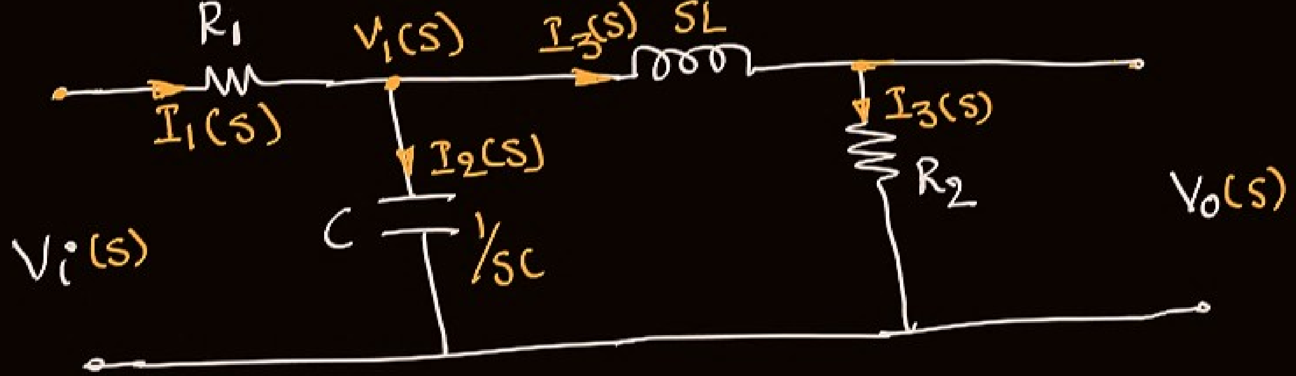
$\Rightarrow$




$$\frac{C}{R} = 1$$

## ⇒ Construction of SFG to Electrical network :-

Prob :- Construct SFG to E-N/W.



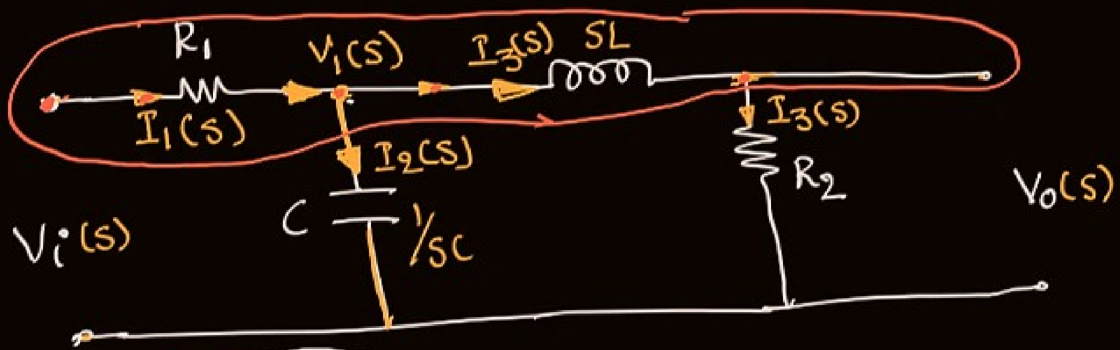

procedure :-

(S1) Select the branch currents & Node voltages.

(S2) Apply Laplace Transform to the N/W Variables & elements.

(S3) write the equations for unknown currents & unknown voltages.

(S4) Draw the SFG to the above equations.

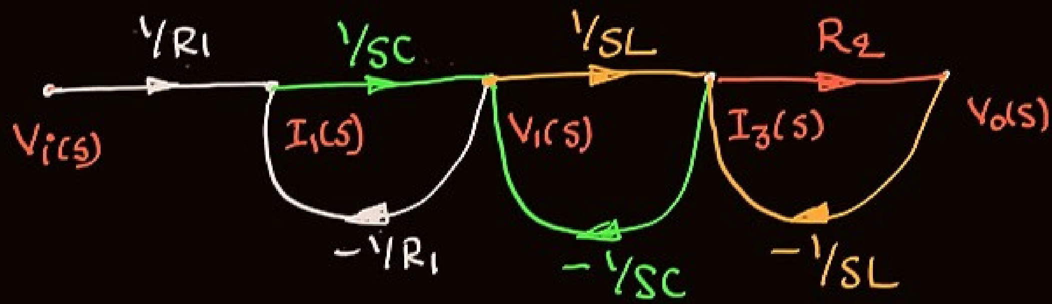



$$\Rightarrow I_1(s) = \frac{V_i(s) - V_1(s)}{R_1} \quad \text{— (1)}$$

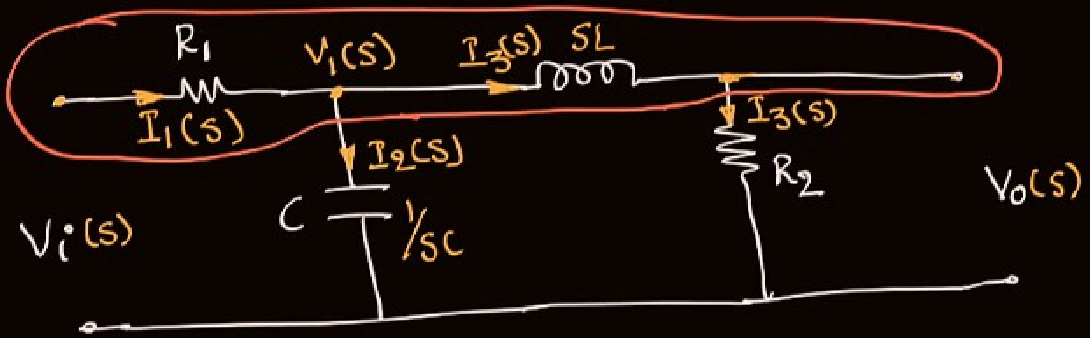
$$\Rightarrow V_1(s) = I_2(s)\left(\frac{1}{sC}\right) = \left[\frac{I_1(s) - I_3(s)}{sC}\right] \quad \text{— (2)}$$

$$\Rightarrow I_3(s) = \left(\frac{V_1(s) - V_o(s)}{sL}\right) \quad \text{— (3)}$$

$$\Rightarrow V_o(s) = I_3(s) \cdot R_2 \quad \text{— (4)}$$

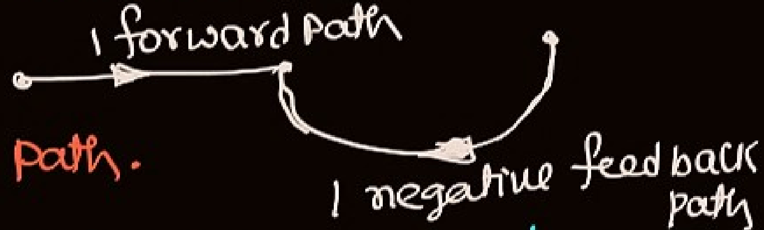



$$\boxed{TF_{E-N/W} = TF_{BD \text{ (or) } SFG}}$$



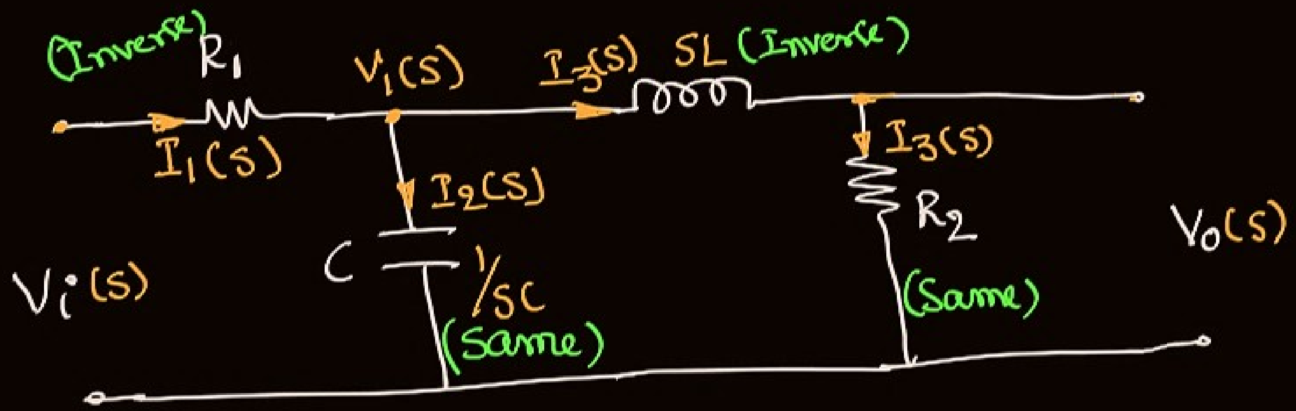

<u>Short cut</u> :- ① The nodes in the SFG are nothing but the variable along series branch.

② Each element in the electrical n/w gives <u>one forward path</u> & <u>one negative feedback path</u> except Last element ($R_2$).

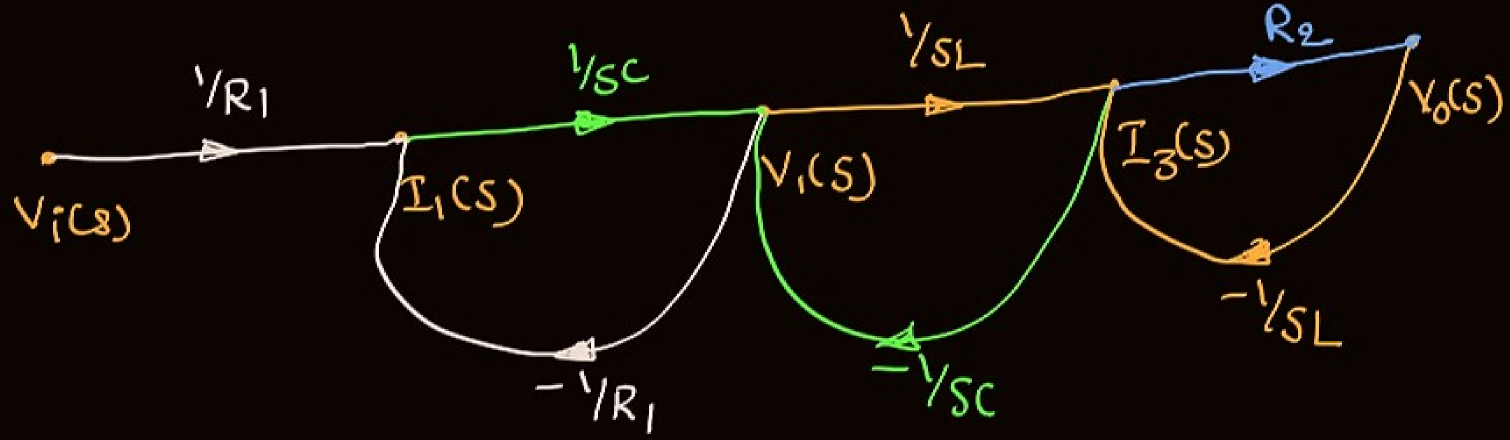
1 element ⇒ 


That Last element gives only forward path.

③ Take the inverse of Impedance for series branch elements as a path gain and take same impedance for shunt branch elements as a path gain.

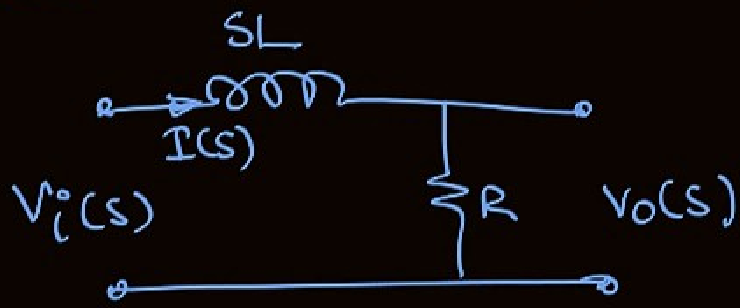


(Inverse) $R_1$
$V_1(S)$
$I_3(S)$
SL (Inverse)
$I_1(S)$
$I_2(S)$
$I_3(S)$
$R_2$
$V_0(S)$
C
$1/SC$
(Same)
$V_i(S)$
(Same)

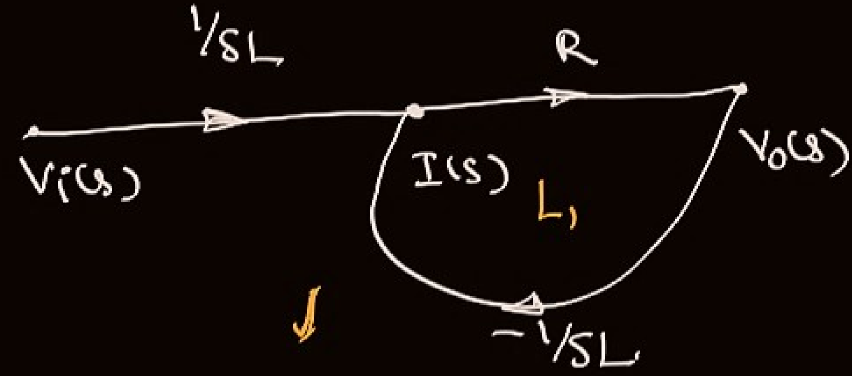



$1/R_1$
$1/SC$
$1/SL$
$R_2$
$V_0(S)$
$V_i(S)$
$I_1(S)$
$V_1(S)$
$I_3(S)$
$-1/SL$
$-1/SC$
$-1/R_1$

⇒ Prob :- Draw SFG

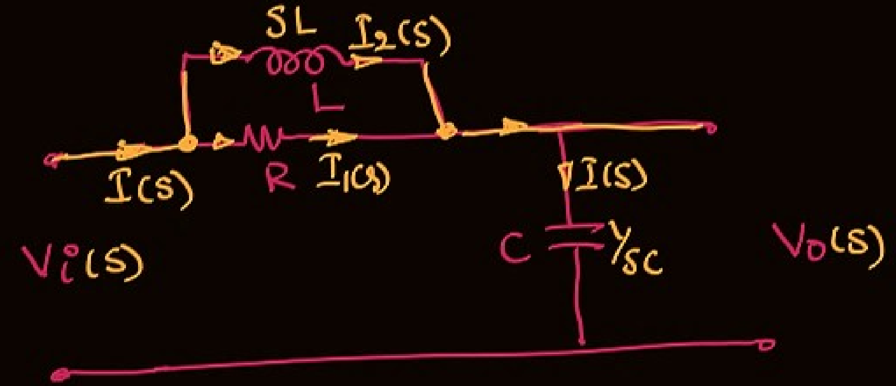



$$\frac{V_o(s)}{V_i(s)} = \left(\frac{R}{SL+R}\right)$$




$$\frac{V_o(s)}{V_i(s)} = \frac{R/SL}{1+R/SL} = \left(\frac{R}{SL+R}\right)$$

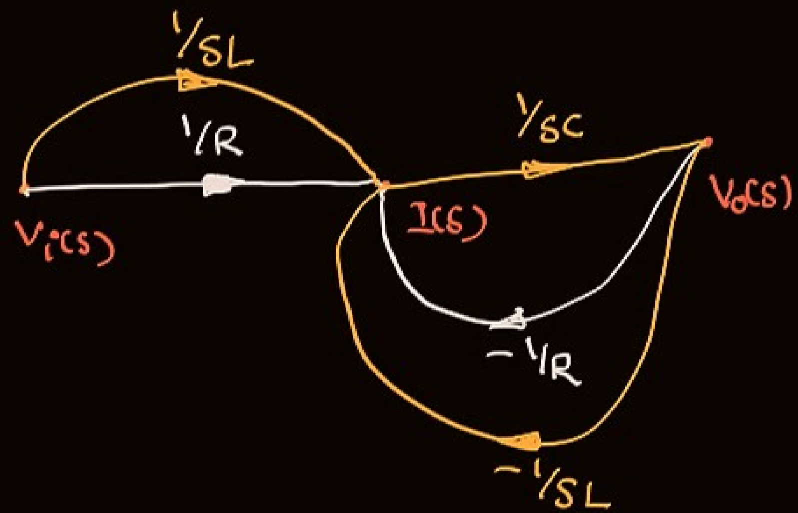
Prob:- Draw SFG.




$$I(s) = I_1(s) + I_2(s)$$

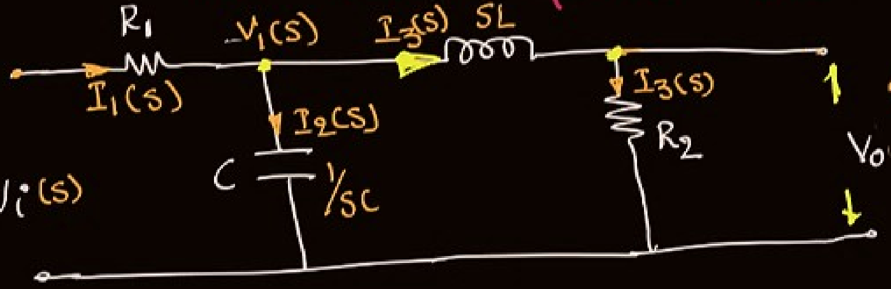
$$\Rightarrow I(s) = \frac{V_i(s) - V_o(s)}{R} + \frac{V_i(s) - V_o(s)}{sL}$$

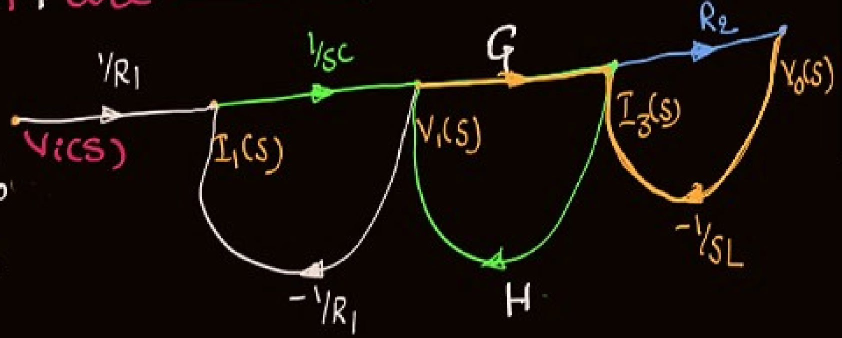
$$V_o(s) = I(s) \cdot \left(\frac{1}{sC}\right)$$



Prob:- An electrical system & SFG are shown in figure.

The Path Gains G & H in SFG are ______




Sol :- G $\Rightarrow$ start from SFG:-

$\Rightarrow$ G is incomming branch to $I_3(S)$

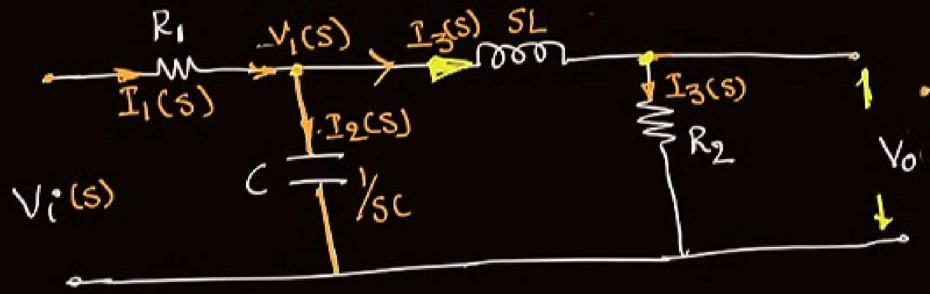
$$I_3(S) = G_1 V_1(S) - V_0(S)\left(\frac{1}{SL}\right) \quad \text{--- (1)}$$

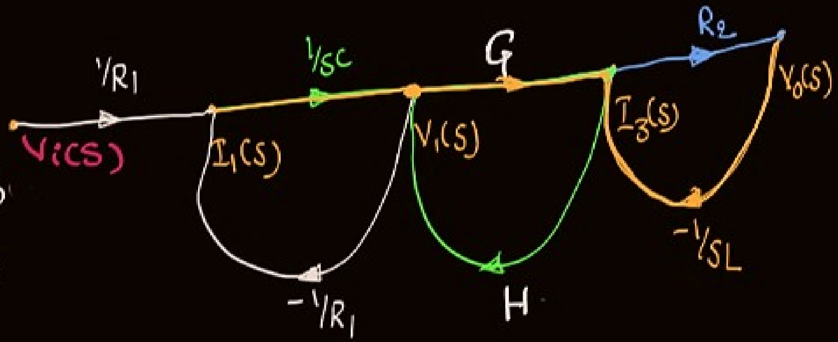
from N/W:-

$$I_3(S) = \frac{V_1(S) - V_0(S)}{SL} = \left[\frac{1}{SL}V_1(S) - \frac{1}{SL}V_0(S)\right] \quad \text{--- (2)}$$

compare

$$\Rightarrow \boxed{G_1 = \frac{1}{SL}}$$



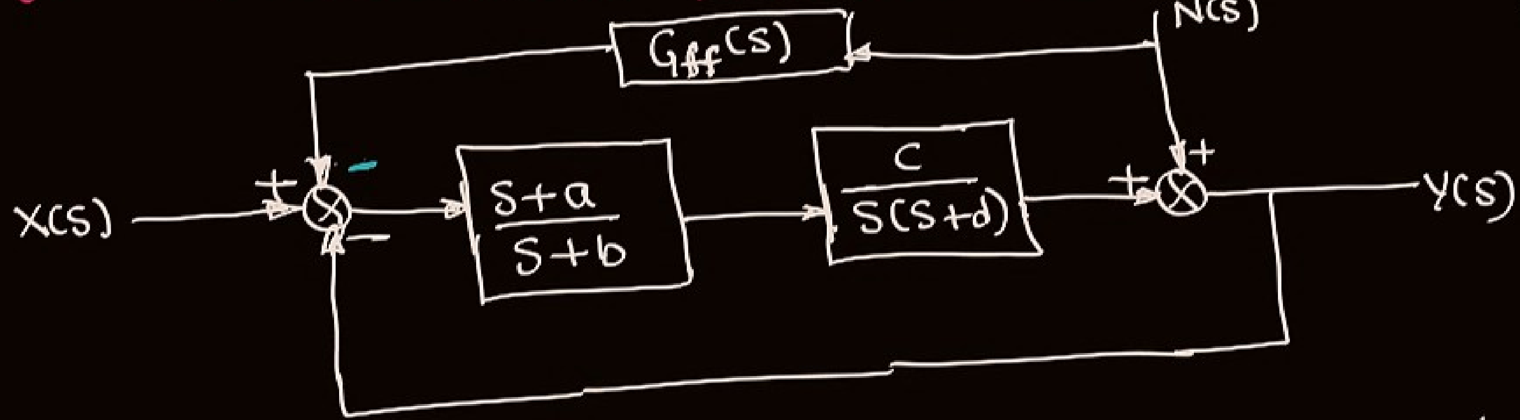

H ⇒ SFG :- H is incomming branch to $V_1(S)$

$$V_1(S) = \left[\frac{1}{SC} I_1(S) + H\cdot I_3(S)\right] \quad — ①$$

from N/W $V_1(S) = I_2(s)\cdot\frac{1}{SC} = \frac{I_1(s) - I_3(s)}{SC} = \left[\frac{1}{SC} I_1(S) - \frac{1}{SC} I_3(s)\right] \quad — ②$ compare

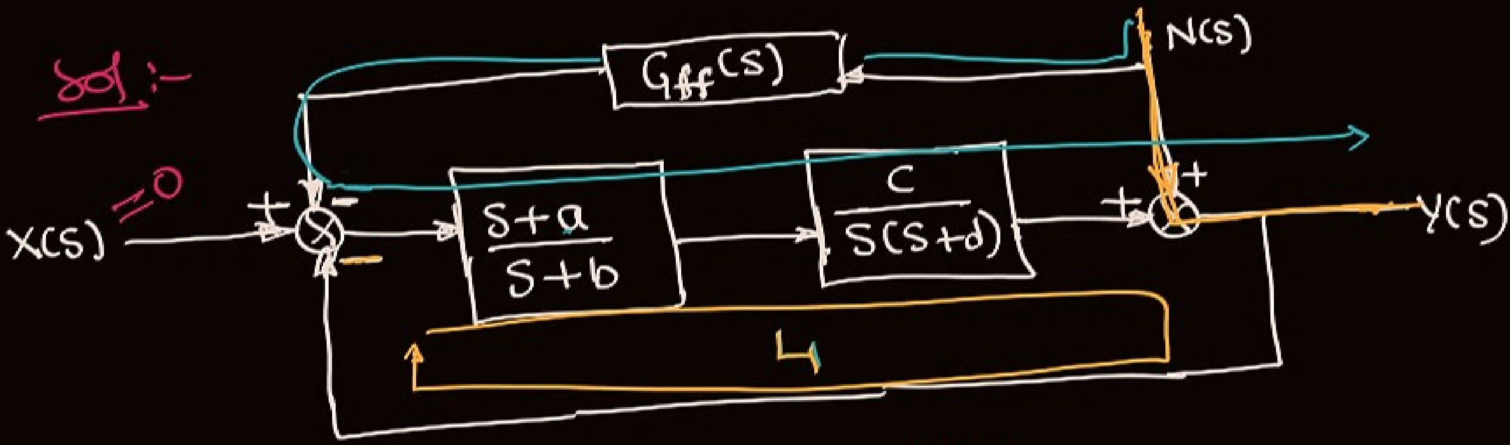
$$\boxed{H = -1/SC}$$

Prob :- For LTI System shown in figure. X(s) is i/p & Y(s) is o/p. In order to nullify the effect of noise in the o/p, the gain of feed forward path is ______




⇒ Feed forward paths are introduced intentionally, to reduce the effect of noise (or) Disturbance in the o/p.

Sol:-

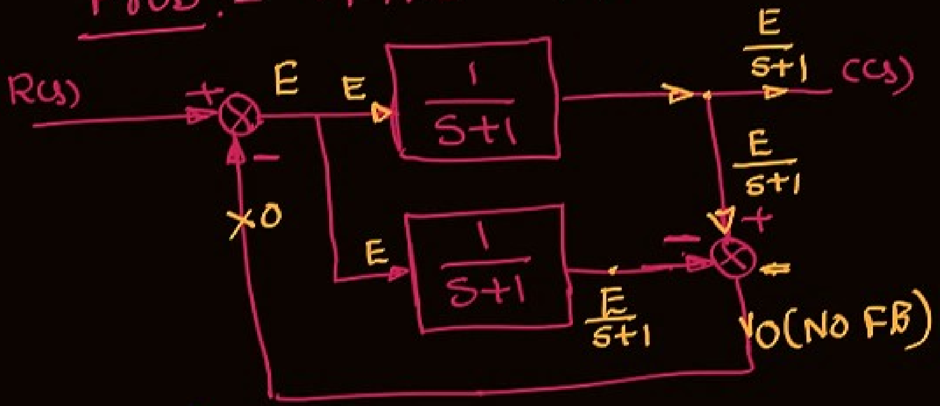



$\Rightarrow$ O/p due to Noise $= 0 \Rightarrow \frac{Y(s)}{N(s)} = 0$

$$\frac{Y(s)}{N(s)} = \frac{1 - G_{ff}(s)\left(\frac{c(s+a)}{s(s+b)(s+d)}\right)}{1 + \frac{c(s+a)}{s(s+b)(s+d)}} = 0 \Rightarrow 1 - G_{ff}(s)\left[\frac{c(s+a)}{s(s+b)(s+d)}\right] = 0$$

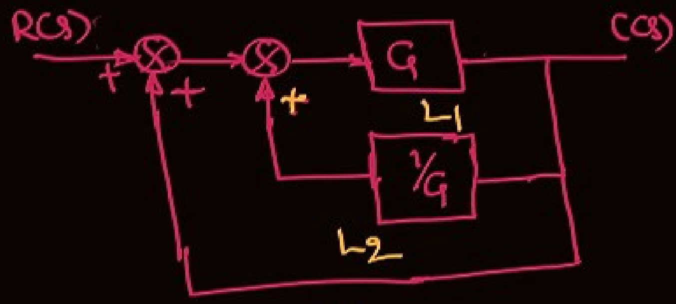
$$\Rightarrow \boxed{G_{ff}(s) = \frac{s(s+b)(s+d)}{c(s+a)}}$$

Prob:- find TF




⇓

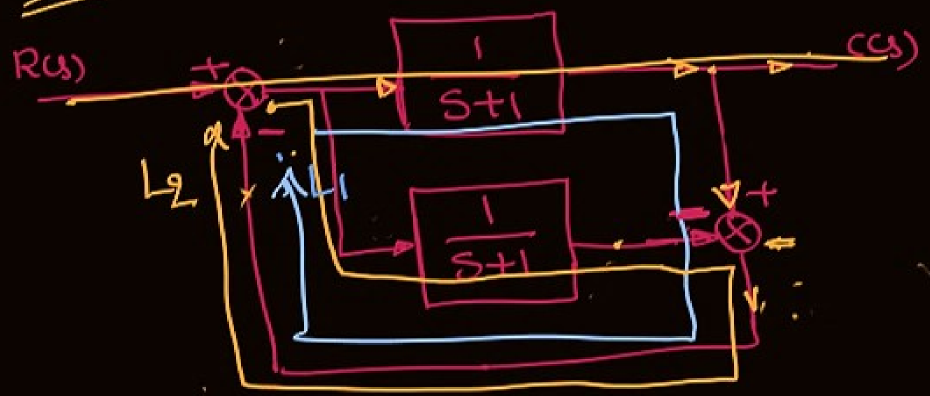

$$\frac{C(s)}{R(s)} = \left(\frac{1}{s+1}\right)$$




$$\frac{C}{R} = \frac{G}{1 - G\left(\frac{1}{G}\right) - G}$$

$$= \frac{G}{1 - 1 - G} = \frac{G}{-G} = -1$$

SFG



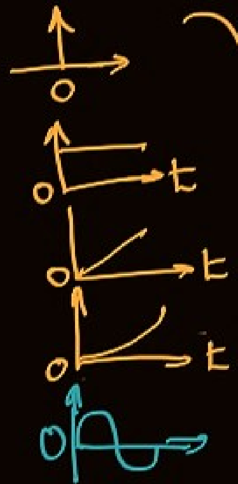

SFG :-

$$\frac{C}{R} = \frac{1/s+1}{1 + \cancel{\frac{1}{s+1}} - \cancel{\frac{1}{s+1}}} = \frac{1}{s+1}$$

⇒ Chapter 3 TIME DOMAIN ANALYSIS

Purpose :- To evaluate performance of the system with respect to time.

$r(t)$ → TF (or) MEM SYSTEM → $c(t)$



TDA

fDA

LTI

$c(t)$

$t_d, t_r, t_p, t_s, t_{ss}$ ↑ – Slow Res

$\% M_p$ ↑ , $e_{ss}$ ↑ → Less Accurate

→ Less Relative Stable

$0$ → $t$

⇒ **Time Response:** → Observing the variations in the o/p with respect to time is called Time Response.


→ $\boxed{\text{Time Response } c(t) = C_{tr}(t) + C_{ss}(t)}$

Time response is nothing but sum of transient & steady state response.

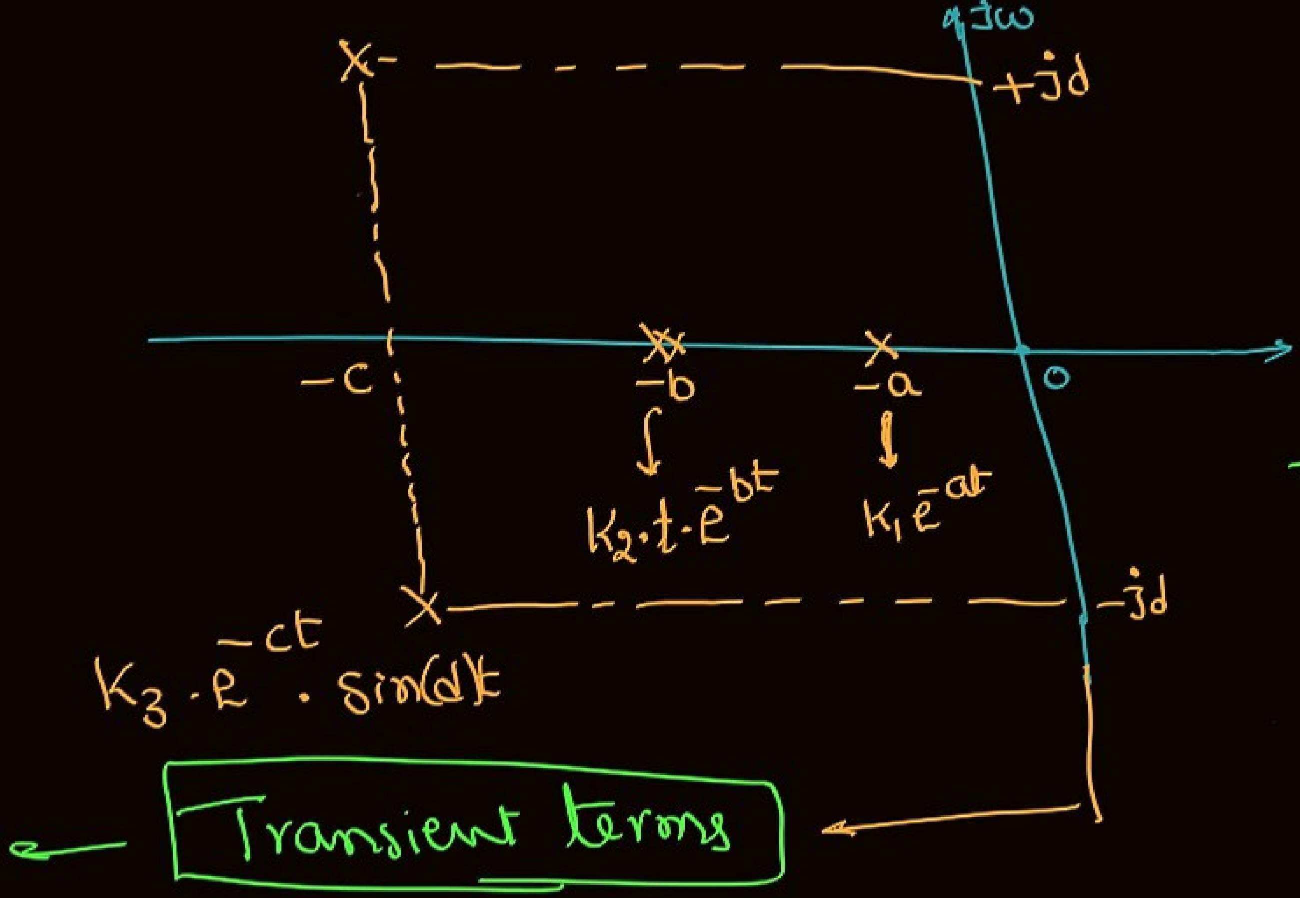
**Prob:-** Identify transient and steady state terms in the given time response

$$c(t) = \Big[\underbrace{10 + 2\sin 2t + 3\cos 3t}_{\text{(SS terms)}} + \underbrace{4t\cdot\cancel{e^{-4t}}^{\,0} + 5\cancel{e^{-5t}}^{\,0}\sin 5t + 6t\cdot\cancel{e^{-6t}}^{\,0}\cos 6t}_{\text{(tr terms)}}\Big]\,u(t)$$

$\Rightarrow$ Transient term is the one, which becomes zero as time becomes very large.

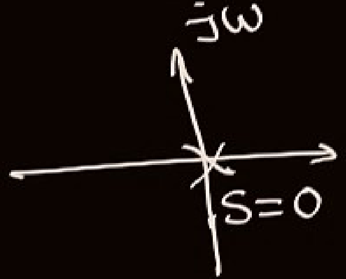
$$\boxed{\lim_{t\to\infty} c_{tr}(t) = 0}$$

(exponential decay) $\longrightarrow$ Transient)

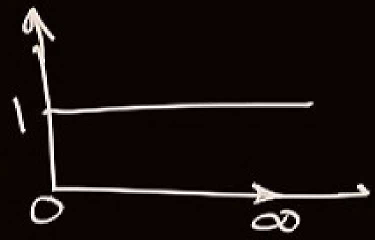


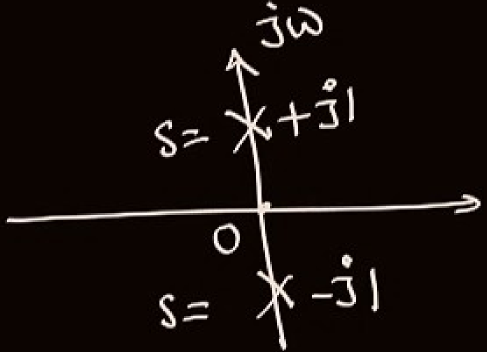

$\Rightarrow$ The poles which lies in the Left half of S-plane gives Transient term.

⇒ The Poles which lies on $j\omega$-axis gives steady state term.

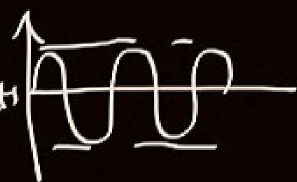



$\frac{1}{s} \Rightarrow 1u(t) \rightarrow$




$$\frac{1}{(s-j1)(s+j1)} = \frac{1}{s^2+1} = \sin(t)u(t)$$



SS

**Transient Response:-** It is a part of the time response, that becomes zero as time becomes $\infty$.

i.e $\boxed{\lim_{t \to \infty} C_{tr}(t) = 0}$

⇒ The term which consists exponential decay is called transient term.

⇒ **Steady state Response:-** It is the part of the time response, that remains, after transients becomes zero.