ELECTROMAGNETICS

08-08-22

ELECTROMAGNETICS

SYLLABUS:

1. VECTOR CALCULAS AND CO-ORDINATE SYSTEM (1M - 2M)
2. ELECTROSTATICS
3. MAGNETOSTATICS
4. TIME VARYING FIELDS

GATE: 4M - 6M

ESE: 8% - 10%

STATE SERVICE: (3Q - 4Q)

$$\begin{tikzcd}
V \arrow[d] & I \arrow[d] \arrow[r] & \text{KVL, KCL, OHM'S LAW.} \\
\bar{E} \arrow[r, bend left] & \bar{H} \arrow[l, bend left] & : \text{EM-PHEMOMENON}
\end{tikzcd}$$

(MAXWELL'S EQUATIONS)

MAXWELL'S EQUATIONS

1. $\nabla \times \overline{H} = \overline{J} + \frac{\partial \overline{D}}{\partial t}$

2. $\nabla \times \overline{E} = -\frac{\partial \overline{B}}{\partial t}$

3. $\nabla \cdot \overline{D} = \rho_v$

4. $\nabla \cdot \overline{B} = 0$

where

$\overline{D} = \epsilon \overline{E}$

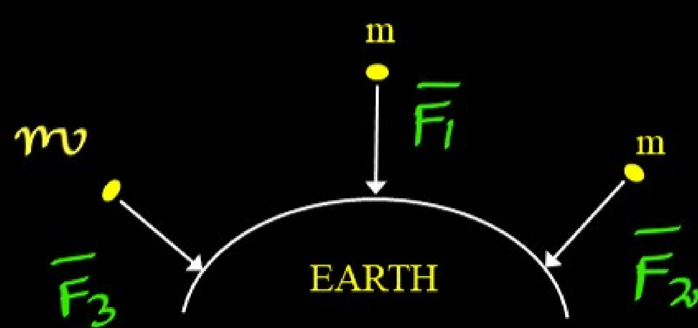
$\overline{B} = \mu \overline{H}$

- Only the concept of voltage, current can't explain different aspects of Electrical Engineering. (transformer action, motor action,capacitance,inductance,LLT…..ect).
- EM - Theory will give flexible solutions and explanations to the
  ELECTRICAL.
  different aspects of Engineering.
- EM - Theory Explains the Interplay or Inter Dependence or Co-existing mechanism between both electric and magnetic fields which is explained with the help of Four Maxwell's Equations and script is written Interms of Mathematics (Vector Calculas).

Topic: 1. VECTOR CALCULAS AND CO-ORDINATE SYSTEM

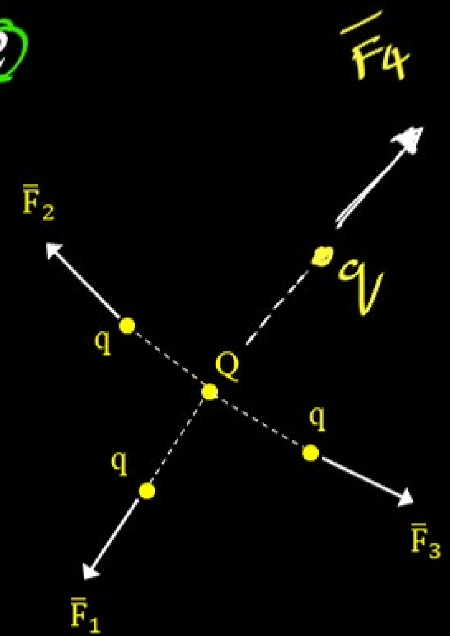
FIELD: Field is the physical quantity which takes different values at different locations.

Ex: 1



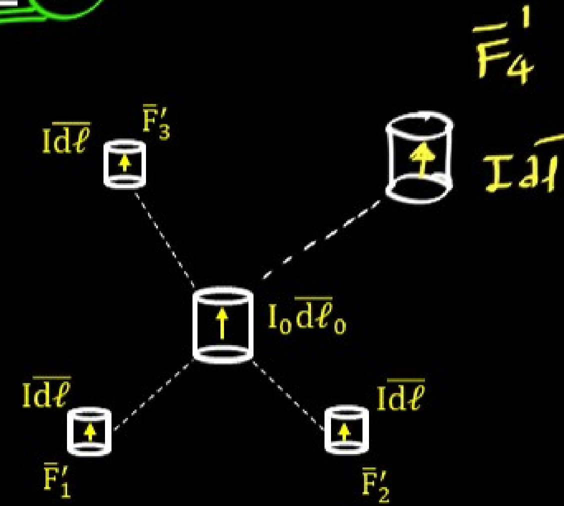
GRAVITATIONAL FIELD.

Ex: 2



ELECTRIC FIELD.

Ex: 3



MAGNETIC FIELD.

Examples of Vector Fields

1. Water Flow in River
2. Air Distribution Around the Fan
3. Gravitational Field
4. Electric Field
5. Magnetic Field

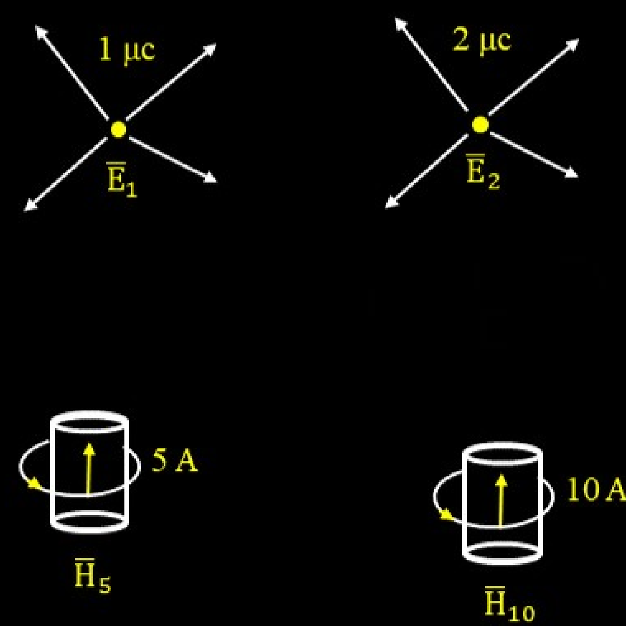
Examples of Scalar Fields

1. Temperature
2. Electrostatic Potential (Voltage)

ATTRIBUTES OF FIELDS

1. Magnitude
2. Nature
3. Uniqueness

Ex:



VECTOR CALCULAS

- The Vector Calculas Operators Measures Fields Quantitatively Captures their physical Nature and defines them mathematically Unique.

Differential Operator (∇)

$$\boxed{\nabla = \frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z}}$$

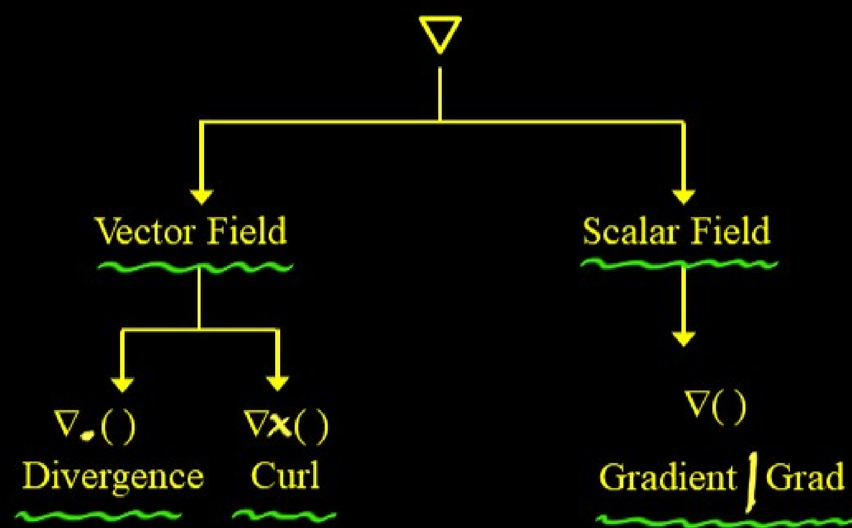
3D – SPATIAL DERIVATIVE.

$$\boxed{(\hat{x}, \hat{y}, \hat{z})}$$

$$(\hat{i}, \hat{j}, \hat{k})$$

$$(\hat{a}_x, \hat{a}_y, \hat{a}_z)$$

$$(\hat{u}_x, \hat{u}_y, \hat{u}_z)$$


∇
Vector Field
Scalar Field
$\nabla \cdot (\,)$
Divergence
$\nabla \times (\,)$
Curl
$\nabla (\,)$
Gradient | Grad

THE DIVERGENCE OF VECTOR FIELD

Let $\bar{F} = F_x\hat{x} + F_y\hat{y} + F_z\hat{z}$ be A

Vector Field

$F_x(x, y, z) \,/\, F_y(x, y, z) \,/\, F_z(x, y, z)$

$$\nabla \cdot \bar{F} = \left[\frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z}\right] \cdot \left[F_x\hat{x} + F_y\hat{y} + F_z\hat{z}\right]$$

$\hat{x} \cdot \hat{x} = 1$	$\hat{x} \cdot \hat{y} = 0$	$\hat{y} \cdot \hat{x} = 0$
$\hat{y} \cdot \hat{y} = 1$	$\hat{y} \cdot \hat{z} = 0$	$\hat{z} \cdot \hat{y} = 0$
$\hat{z} \cdot \hat{z} = 1$	$\hat{z} \cdot \hat{x} = 0$	$\hat{x} \cdot \hat{z} = 0$

$$\boxed{\nabla \cdot F = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}}$$

O/p: SCALAR FIELD.

- $\nabla \cdot (\,) = +ve$
  - Source Field
- $\nabla \cdot (\,) = -ve$
  - Sink Field
- $\nabla \cdot (\,) = 0$
  - Divergenceless
  - Solenoidal

ACE

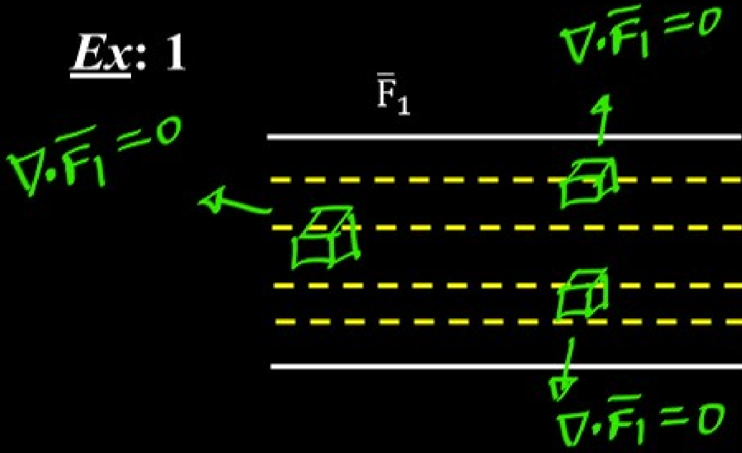
***

$$\boxed{\nabla \cdot (\,) = \left[\frac{\text{NET FLOW}}{\text{SMALL VOLUME}}\right]}$$

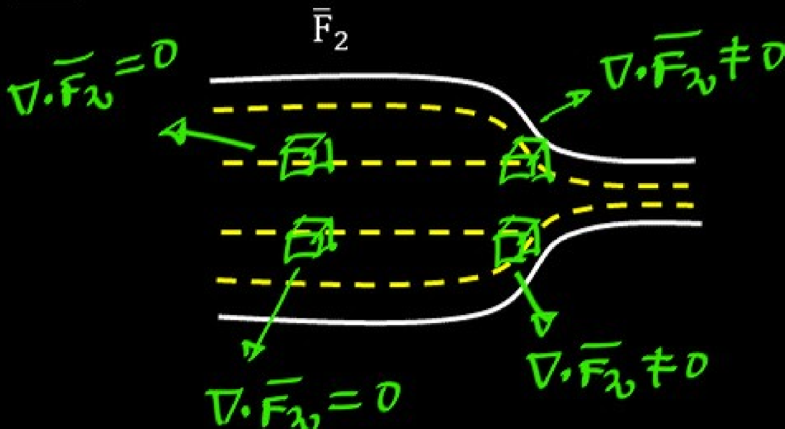
* CLOSED SURFACE → NET FLOW (IN AND OUT)

* OPEN SURFACE → FLOW (IN OR OUT).

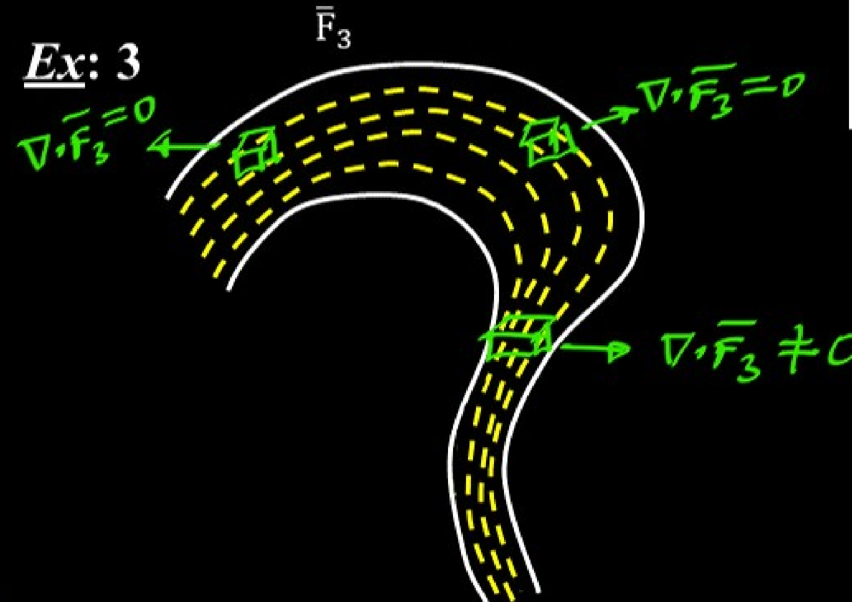
***Ex*: 1**



***Ex*: 2**



***Ex*: 3**



CIRCULATING FIELDS MAY ALSO HAVE DIVERGENCE.

THE CURL OF VECTOR

Let $\bar{F} = F_x\hat{x} + F_y\hat{y} + F_z\hat{z}$ be

A Vector Field

$$\nabla \times \bar{F} = \left[\frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z}\right] \times \left(F_x\hat{x} + F_y\hat{y} + F_z\hat{z}\right)$$

$\hat{x} \times \hat{y} = \hat{z}$	$\hat{y} \times \hat{x} = -\hat{z}$	$\hat{x} \times \hat{x} = 0$
$\hat{y} \times \hat{z} = \hat{x}$	$\hat{z} \times \hat{y} = -\hat{x}$	$\hat{y} \times \hat{y} = 0$
$\hat{z} \times \hat{x} = \hat{y}$	$\hat{x} \times \hat{z} = -\hat{y}$	$\hat{z} \times \hat{z} = 0$

$$\nabla \times \bar{F} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix}$$

O/p: VECTOR FIELD.

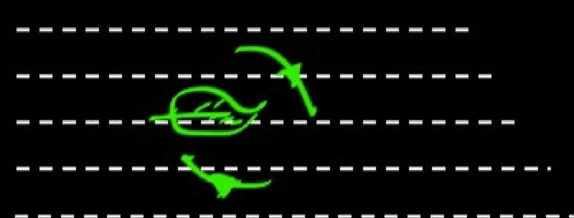
$\nabla \times (\ ) = 0$

- Curl Free
- Conservative
- Irrotational

$$\boxed{\nabla \times (\ ) = \left[\frac{\text{NET ROTATION}}{\text{SMALL AREA}}\right]}$$

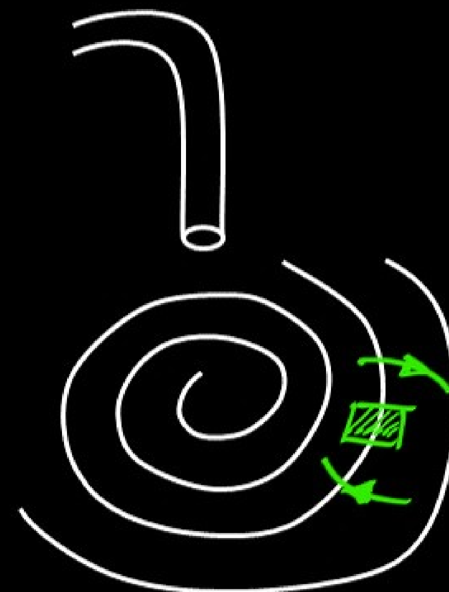
$$\boxed{\nabla \times (\,) = \left[\frac{\text{Net Rotation}}{\text{Small Area}}\right]}$$

Ex: 1. Water Flow In River



STRAIGHT FORWARD NATURED FIELDS MAY ALSO HAVE CURL.

Ex: 2. Wash Basin



(8) CHECK WHETHER THE VECTOR FIELD

$\bar{E} = yz\,\hat{x} + xz\,\hat{y} + xy\,\hat{z}$ IS SOLENOIDAL

OR CONSERVATIVE?

Soln:

$$\nabla\cdot\bar{E} = \frac{\partial}{\partial x}(yz) + \frac{\partial}{\partial y}(xz) + \frac{\partial}{\partial z}(xy)$$

$$= 0 + 0 + 0$$

$$\nabla\cdot\bar{E} = 0$$

→ SOLENOIDAL.

$$\nabla\times\bar{E} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz & xz & xy \end{vmatrix}$$

$$= \hat{x}\left[\frac{\partial}{\partial y}xy - \frac{\partial}{\partial z}xz\right]$$

$$-\hat{y}\left[\frac{\partial}{\partial x}xy - \frac{\partial}{\partial z}yz\right]$$

$$\hat{z}\left[\frac{\partial}{\partial x}xz - \frac{\partial}{\partial y}yz\right].$$

$$\nabla \times \bar{E} = \hat{x}[x - x] - \hat{y}[y - y] + \hat{z}[z - z]$$

$$\nabla \times \bar{E} = 0\hat{x} - 0\hat{y} + 0\hat{z} = 0$$

CONSERVATIVE.

ESE.

Q. THE NATURE OF VECTOR FIELD

$\bar{F} = 3x^2yz\,\hat{x} + x^3z\,\hat{y} + (x^3y - 2z)\,\hat{z}$ IS

(a) ROTATIONAL $\}$ $\nabla \times \bar{F} \neq 0$

(b) DIVERGENCE LESS

(c) SOLENOIDAL.

$\left.\begin{array}{l}\text{(b)}\\\text{(c)}\end{array}\right\} \nabla \cdot \bar{F} = 0$

(d) CONSERVATIVE. $\}$ $\nabla \times \bar{F} = 0$

Soln:

$$\nabla \times \bar{F} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 3x^2yz & x^3z & x^3y - 2z \end{vmatrix}$$

$$= \hat{x}\left[x^3 - x^3\right] - \hat{y}\left[3x^2y - 3x^2y\right] + \hat{z}\left[3x^2z - 3x^2z\right]$$

$$\nabla \times \bar{F} = 0\hat{x} - 0\hat{y} + 0\hat{z}$$

$$\nabla \times \bar{F} = 0$$

G-2M

Q) GIVEN AN IRROTATIONAL VECTOR FIELD

$\bar{F} = (K_1 xy + K_2 z^3)\hat{x} + (3x^2 - K_2 z)\hat{y} + (3xz^2 - y)\hat{z}$

THEN THE VALUE OF $\nabla\cdot\bar{F}$ AT $(1, 1, -2)$ IS

(a) 6

(b) −6

(c) 12

(d) −12.

soln:

$$\nabla\cdot\bar{F} = \frac{\partial}{\partial x}(K_1 xy + K_2 z^3) + \frac{\partial}{\partial y}(3x^2 - K_2 z) + \frac{\partial}{\partial z}(3xz^2 - y)$$

$$\nabla\cdot\bar{F} = (K_1 y + 0) + (0) + (6xz - 0)$$

$$\nabla\cdot\bar{F} = K_1 y + 6xz$$

AT $(1, 1, -2)$

$$\nabla\cdot\bar{F} = K_1(1) + 6(1)(-2) = K_1 - 12 = 6 - 12 = -6$$

$$\nabla \times \bar{F} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ (k_1 xy + k_2 z^3) & (3x^2 - k_2 z) & (3xz^2 - y) \end{vmatrix} = 0 = 0\hat{x} + 0\hat{y} + 0\hat{z}$$

$\hat{z}$

$$\Rightarrow \frac{\partial}{\partial x}(3x^2 - k_2 z) - \frac{\partial}{\partial y}(k_1 xy + k_2 z^3) = 0$$

$$6x - k_1 x = 0$$

$$(6 - k_1)x = 0$$

$$\Rightarrow k_1 = 6$$

Q) A VECTOR FIELD IS GIVEN AS $\bar{F} = x^2y\,\hat{x} + yz^2\,\hat{y} + f(x,y,z)\,\hat{z}$, WHAT SHOULD BE THE FUNCTION $f(x,y,z)$, SO THAT THE VECTOR $\bar{F}$ IS SOLENOIDAL.

(a) $-2xyz - \frac{z^3}{3}$

(b) $2xyz - \frac{z^3}{3}$

(c) $2xyz + \frac{z^3}{3}$

(d) $xyz + z^3$

Soln:

$$\nabla\cdot\bar{F} = \frac{\partial}{\partial x}x^2y + \frac{\partial}{\partial y}yz^2 + \frac{\partial}{\partial z}f(x,y,z) = 0$$

$$2xy + z^2 + \frac{\partial}{\partial z}f(x,y,z) = 0$$

$$\partial f(x,y,z) = (-2xy - z^2)\,\partial z.$$

$$f(x,y,z) = \int(-2xy - z^2)\,dz = -2xyz - \frac{z^3}{3}$$

(G-1)

(Q) GIVEN $\bar{A} = (x + az)\hat{x} + (y + bx)\hat{y} + (x + cz)\hat{z}$

THEN THE VALUE OF "C" SUCH THAT VECTOR IS SOLENOIDAL.

(a) -1

(b) -2

(c) -3

(d) 0

Soln: $\nabla \cdot \bar{A} = 0$

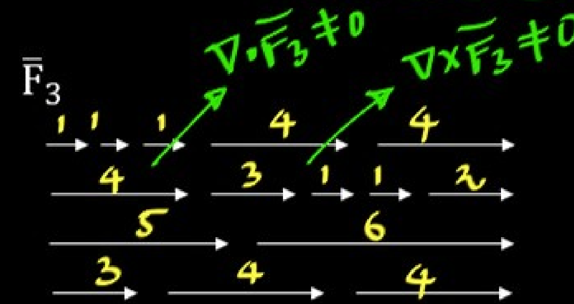
$$\frac{\partial}{\partial x}(x + az) + \frac{\partial}{\partial y}(y + bx) + \frac{\partial}{\partial z}(x + cz) = 0$$

$$1 + 1 + c = 0$$

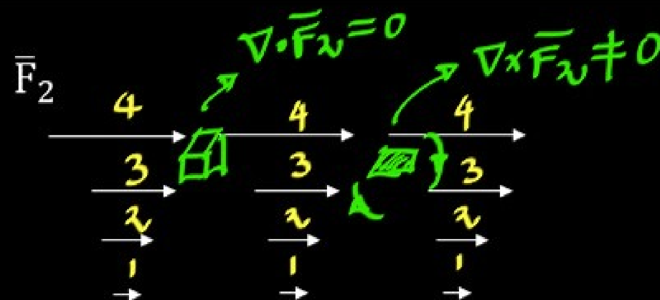
$$c = -2$$

Examples of Vector Fields Responsible for Curl and Divergence

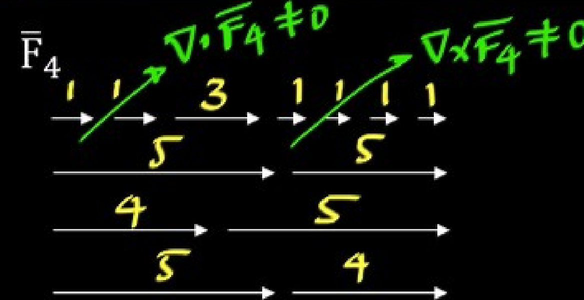
Magnitude of vector field is changing along the direction of orientation



Magnitude is changing along and $\perp^{le}$ direction



Magnitude of vector field is changing $\perp^{le}$ to the direction of orientation



Magnitude is changing along and $\perp^{le}$ direction

MAXWELL'S EQUATIONS:

① $\nabla \times \overline{H} = \overline{J} + \frac{\partial \overline{D}}{\partial t}$

② $\nabla \times \overline{E} = -\frac{\partial \overline{B}}{\partial t}$

③ $\nabla \cdot \overline{D} = \nabla \cdot \epsilon \overline{E} = \rho_v$

④ $\nabla \cdot \overline{B} = \nabla \cdot \mu \overline{H} = 0$

SOURCES

(CHARGES, CURRENTS)

- Any arbitrary vector field can be uniquely defined by it's both curl and divergence.
- The description of vector field in 3D-space is possible by defining both it's curl and divergence.

Note : Maxwell's Equations are Defining curl and Divergence for both electric and magnetic fields and establishes relation with their respective sources (charges and currents)

THE GRADIENT OF SCALAR FIELD

Let f(x, y, z) be a scalar field

$$f(x, y, z) \rightarrow f$$

$$\nabla f = \left(\frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z}\right) f$$

$$\boxed{\nabla f = \frac{\partial f}{\partial x}\hat{x} + \frac{\partial f}{\partial y}\hat{y} + \frac{\partial f}{\partial z}\hat{z}}$$

O/p: VECTOR FIELD.

MAGNITUDE:

$$|\nabla f| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2 + \left(\frac{\partial f}{\partial z}\right)^2}$$

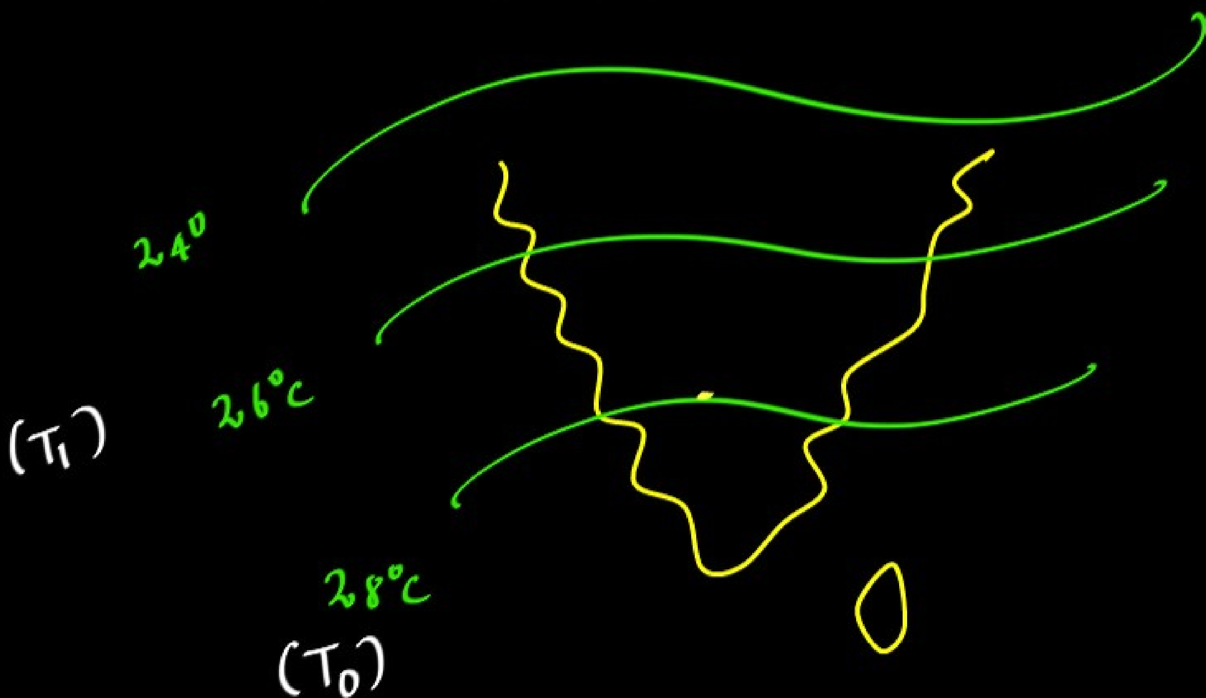
- Maximum rate of change of scalar field in 3D – space

DIRECTION:

$$\widehat{\nabla f} = \frac{\nabla f}{|\nabla f|} = \hat{n}$$

- Normal to the level surfaces or equi-value surfaces

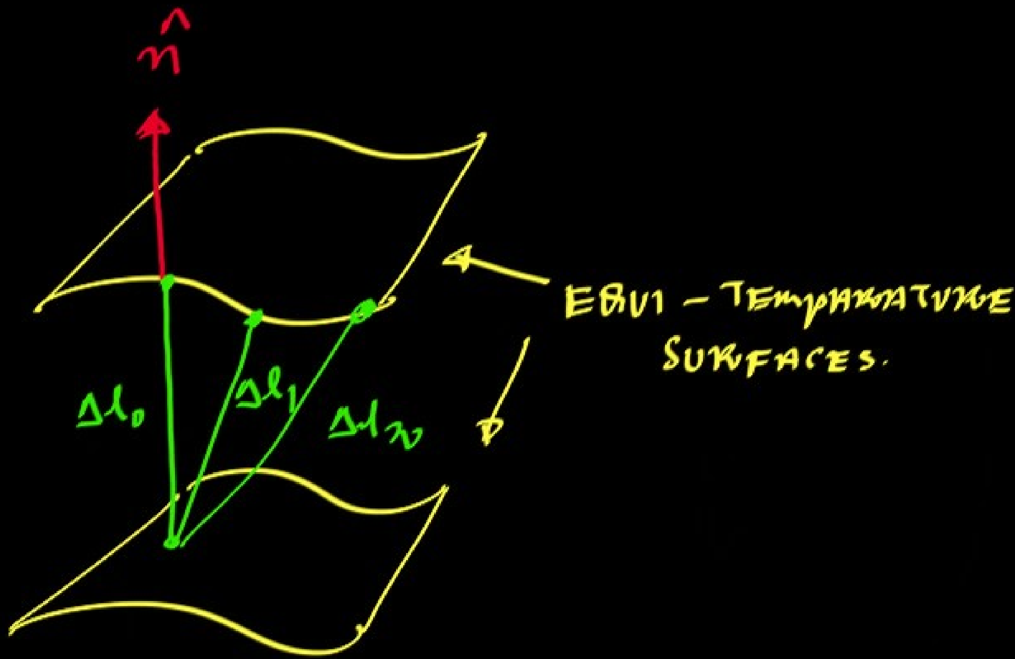
ISOTHERMS:



T_1

$(\Delta T, \Delta l)$

T_0

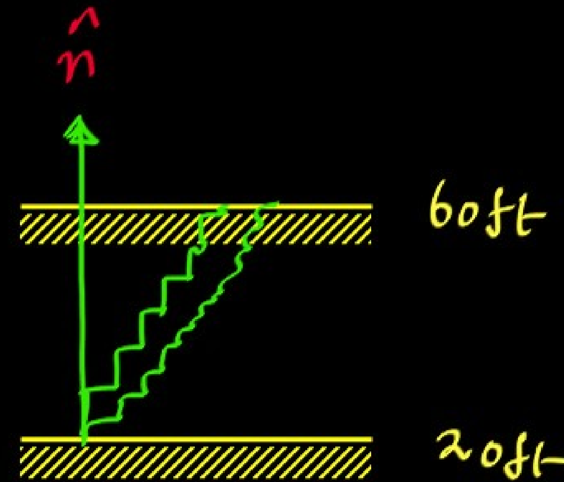


$$\frac{\Delta T}{\Delta l_0} > \frac{\Delta T}{\Delta l_1} > \frac{\Delta T}{\Delta l_2}$$

MAX

$$\boxed{\nabla T = \frac{dT}{dl}\,\hat{n} \Big/ \text{MAX}}$$

STEEP.



Q. The scalar field in certain region is described as $2xy^2z$

(a) Find it's gradient

(b) Find direction of unit normal acting on level surface at $(1,1,-1)$

Soln: $f = 2xy^2z$

(a) $\nabla f = \frac{\partial f}{\partial x}\hat{x} + \frac{\partial f}{\partial y}\hat{y} + \frac{\partial f}{\partial z}\hat{z}$

$\nabla f = 2y^2z\,\hat{x} + 4xyz\,\hat{y} + 2xy^2\,\hat{z}$

(b) $\hat{n} = \hat{\nabla f} = \dfrac{\nabla f}{|\nabla f|}\Big/_{(1,1,-1)}^{AT} = \dfrac{-2\hat{x} - 4\hat{y} + 2\hat{z}}{\sqrt{4+16+4}}$

$\sqrt{4 \times 3 \times 2} = 2\sqrt{6}$

$$\hat{n} = \frac{-2\hat{x} - 4\hat{y} + 2\hat{z}}{\sqrt{24}}$$

$$\hat{n} = \left[\frac{-\hat{x} - 2\hat{y} + \hat{z}}{\sqrt{6}} \right]$$

THE LAPLACIAN OPERATOR (∇^2)

$$\nabla . \nabla = \left(\frac{\partial}{\partial x}\hat{x}\,\frac{\partial}{\partial y}\hat{y}+\frac{\partial}{\partial z}\hat{z}\right).\left(\frac{\partial}{\partial x}\hat{x}\,\frac{\partial}{\partial y}\hat{y}+\frac{\partial}{\partial z}\hat{z}\right)$$

$$\boxed{\nabla^2 = \frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}}$$

<u>NOTE:</u> $\nabla^2(\quad) = 0$

$\hookrightarrow$ HARMONIC FUNCTION.

<u>USEFULL:</u> •. Poisson's Equation

Q. Find the value of K for harmonic function

"sinhxcosKye^{PZ}"

Soln: $f = \sinh x \cos Ky\, e^{PZ}$

$\nabla^2 f = 0$

$$\frac{\partial^2}{\partial x^2} f + \frac{\partial^2}{\partial y^2} f + \frac{\partial^2}{\partial z^2} f = 0$$

$$\frac{\partial^2}{\partial x^2} \sinh x \cos Ky\, e^{PZ} + \frac{\partial^2}{\partial y^2} \sinh x \cos Ky\, e^{PZ} + \frac{\partial^2}{\partial z^2} \sinh x \cos Ky\, e^{PZ} = 0$$

$$\begin{array}{ccc} \sinh x & \cos ky & e^{pz} \\ \downarrow & \downarrow & \downarrow \\ \cosh x & -k\sin ky & pe^{pz} \\ \downarrow & \downarrow & \downarrow \\ \sinh x & -k^2\cos ky & p^2 e^{pz} \end{array}$$

$$\sinh x \cos ky\, e^{pz} + \sinh x(-k^2 \cos ky)\, e^{pz} + \sinh x \cos ky\,(p^2 e^{pz}) = 0$$

$$\sinh x \cos ky\, e^{pz}\,[\underbrace{1 - k^2 + p^2}_{=0}] = 0$$

$$\Longrightarrow \quad 1 - k^2 + p^2 = 0 \quad \Rightarrow \quad k^2 = 1 + p^2$$

$$k = \pm\sqrt{1+p^2}$$

VECTOR IDENTITIES

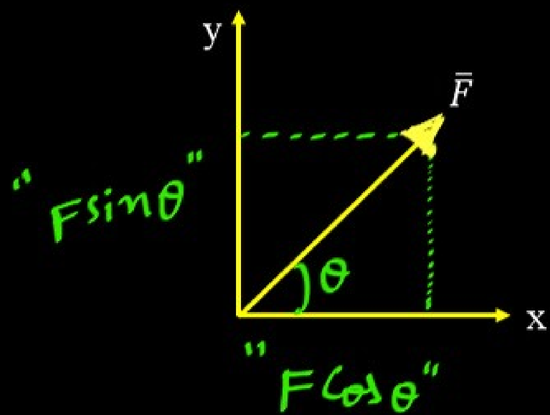
NULL IDENTITIES

I $\nabla \cdot \nabla \times (\text{VECTOR}) \equiv 0$

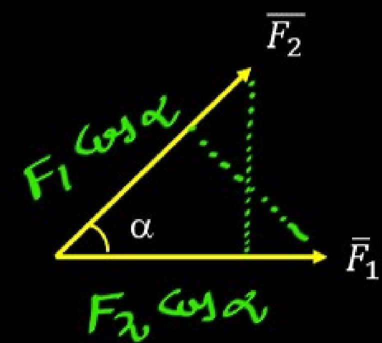
II $\nabla \times \nabla (\text{SCALAR}) \equiv 0$

INTEGRALS

Basic (1): Vector Resolving

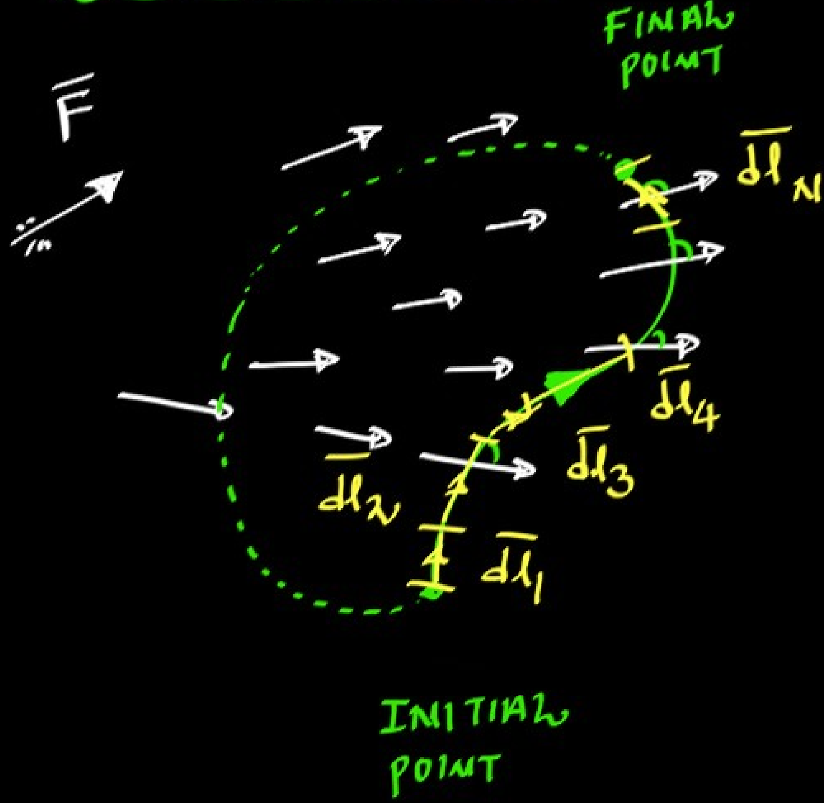


Basic (2): Dot Product



$$\overline{F_1}\cdot\overline{F_2} = F_1(F_2\cos\alpha)$$
$$= F_2(F_1\cos\alpha).$$

LINE INTEGRAL:



① $\bar{F}\cdot\bar{dl}$

② $\bar{F}\cdot\bar{dl}_1 + \bar{F}\cdot\bar{dl}_2 + \bar{F}\cdot\bar{dl}_3 + \dots + \bar{F}\cdot\bar{dl}_n$

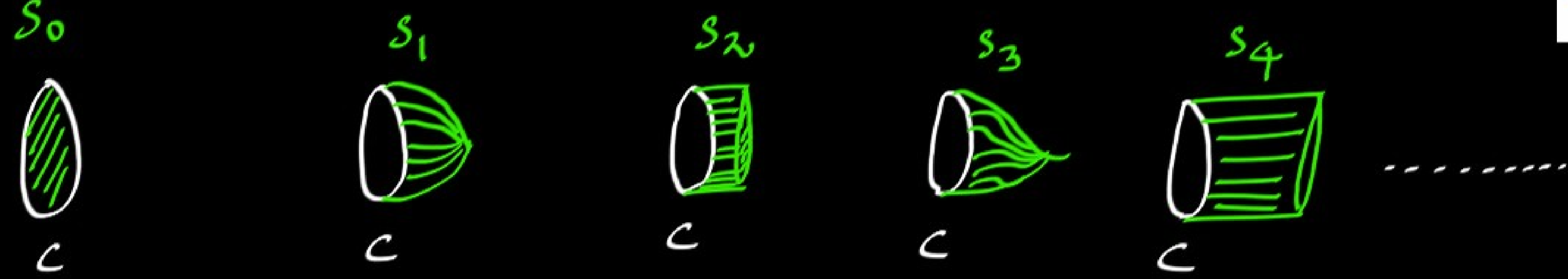
$$= \int_{in}^{fi} \bar{F}\cdot\bar{dl}$$

- LINE INTEGRAL
- CONTOUR INTEGRAL
- ROTATION
- CIRCULATION.

③ $\oint \bar{F}\cdot\bar{dl}$

- CLOSED LINE INTEGRAL
- CLOSED CONTOUR INTEGRAL
- NET ROTATION.
- NET CIRCULATION.

Ex:



* CLOSED CONTOUR (C) ENCLOSES OPEN SURFACES (S_0, S_1, S_2, S_3, S_4 -----) OF INFINITE SHAPES.

* CLOSED LINE INTEGRAL NEED NOT BE ZERO ALWAYS.

* APPLICATION: "EVALUATION OF WORK DONE."

Ex:

① FORCE FIELD $\xrightarrow{\int(\ )\cdot\overline{dl}}$ WORK DONE.

* ② ELECTRIC FIELD $\xrightarrow{\int(\ )\cdot\overline{dl}}$ E.M.F

* ③ MAGNETIC FIELD $\xrightarrow{\int(\ )\cdot\overline{dl}}$ M.M.F.