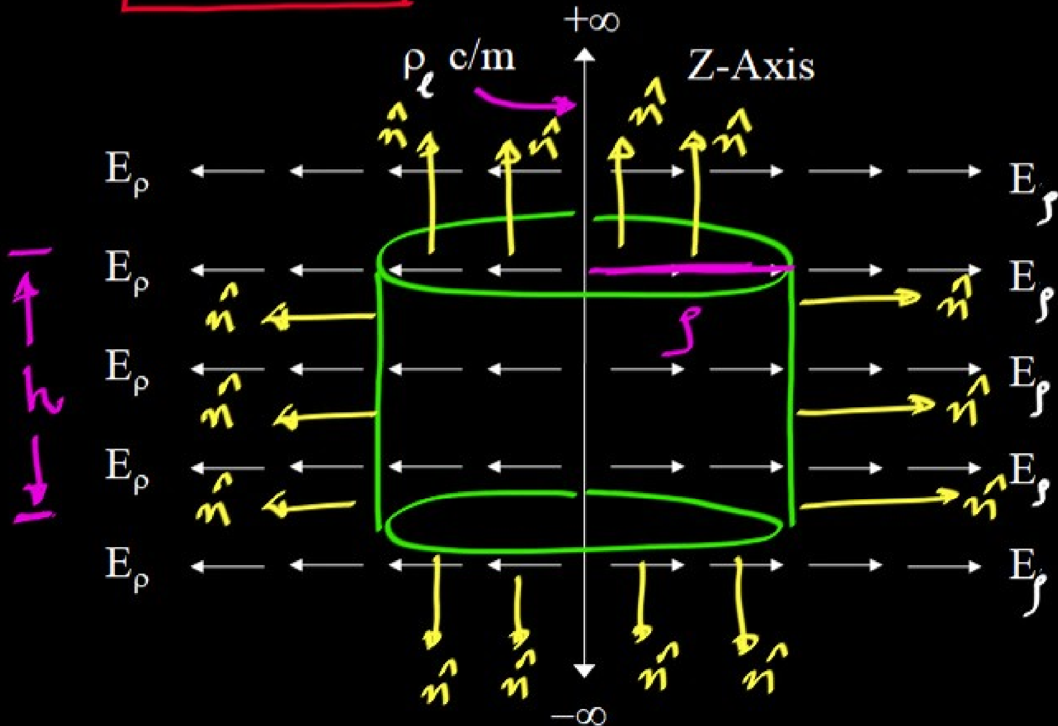
$\boxed{12-08-22}$

Ex-2 Infinite line charge ($\rho_l$ c/m)

$$\boxed{\bar{E} = \frac{\rho_l}{2\pi\varepsilon\rho}\hat{\rho}}$$




$$\oiint \bar{E}\cdot\bar{dA} = \frac{Q_{ENC}}{\epsilon} = \frac{\rho_l * h}{\epsilon}$$

$$\underbrace{\iint_{top} \bar{E}\cdot\bar{dA}}_{0} + \underbrace{\iint_{bottom} \bar{E}\cdot\bar{dA}}_{0} + \iint_{sides} \bar{E}\cdot\bar{dA} = \frac{\rho_l * h}{\epsilon}$$

$$\iint_{sides} E_\rho \, dA = \frac{\rho_l * h}{\epsilon}$$

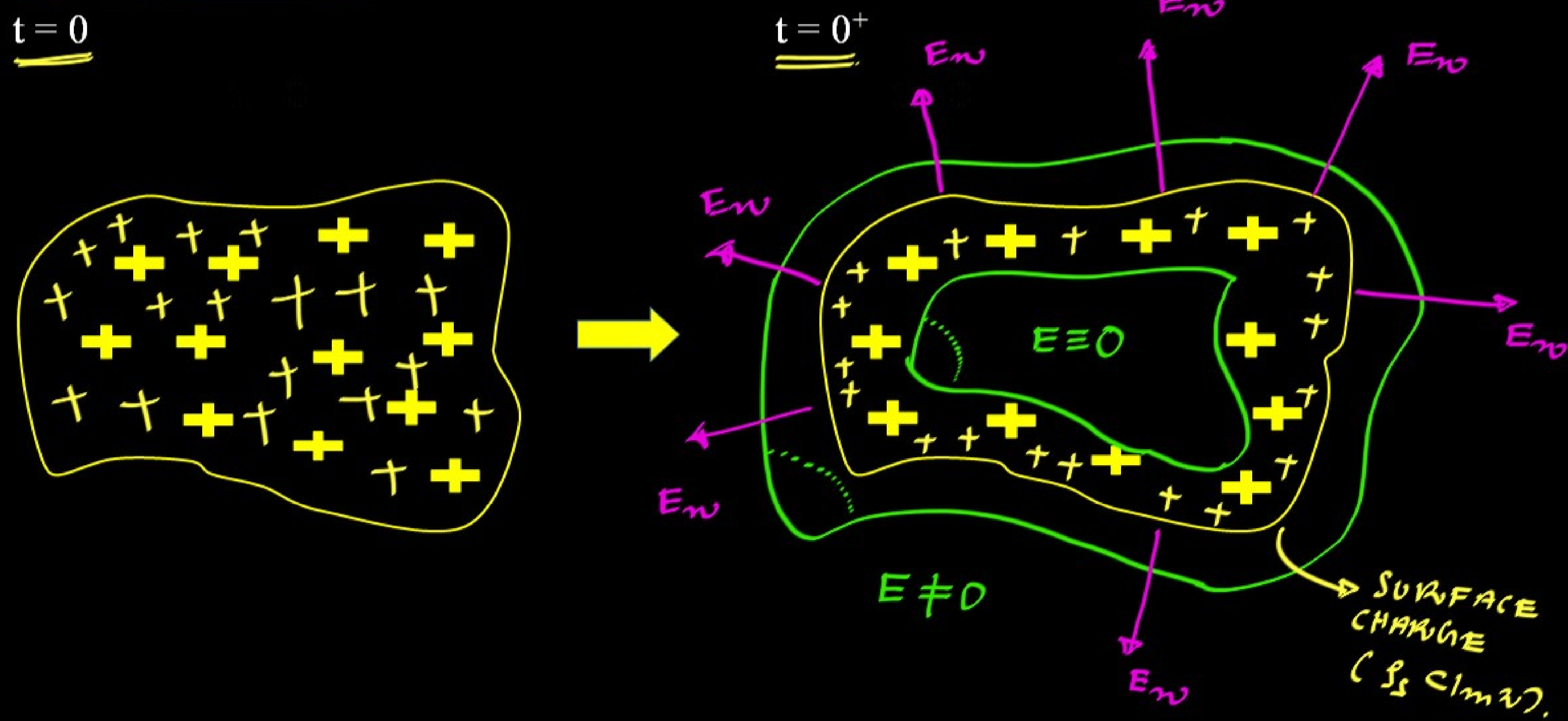
$$E_\rho \underbrace{\iint_{sides} dA}_{AREA} = \frac{\rho_l * h}{\epsilon}$$

$$E_\rho \, 2\pi\rho \, h = \frac{\rho_l h}{\epsilon}$$

$$E_\rho = \frac{\rho_l}{2\pi\epsilon\rho} \Rightarrow \boxed{\bar{E} = \frac{\rho_l}{2\pi\epsilon\rho}\hat{\rho}}$$

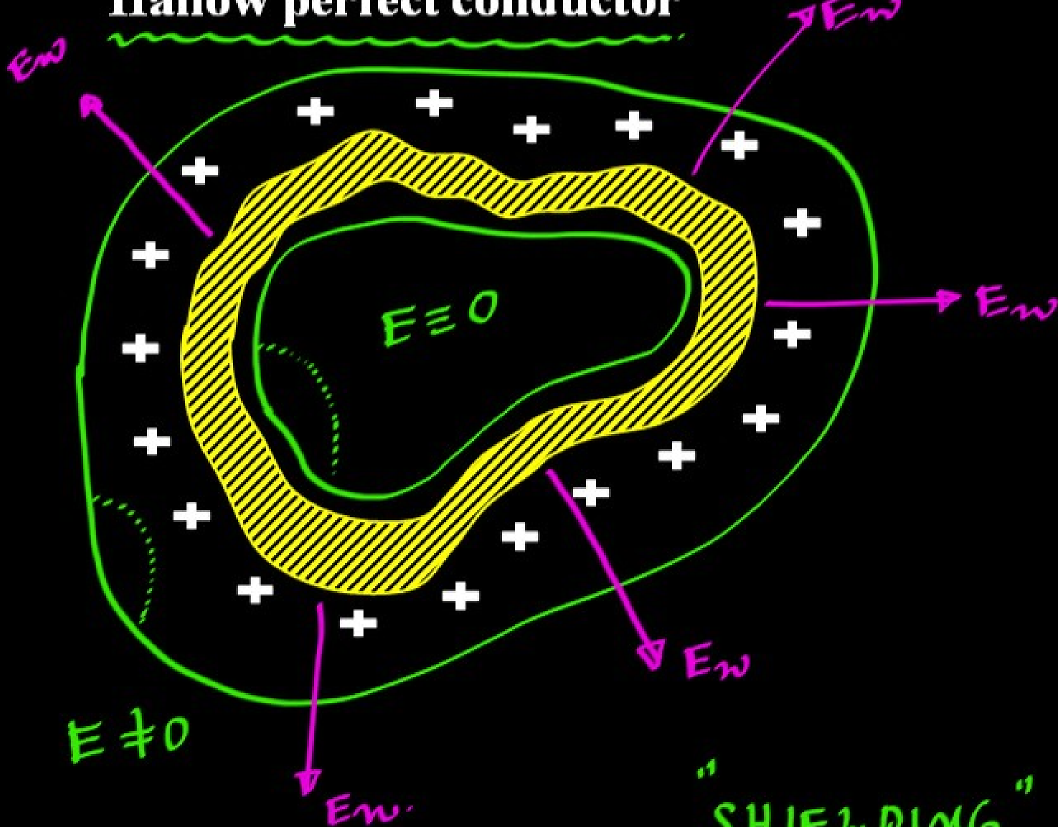
Ex: 3

**Solid perfect conductor**



Ex: 4

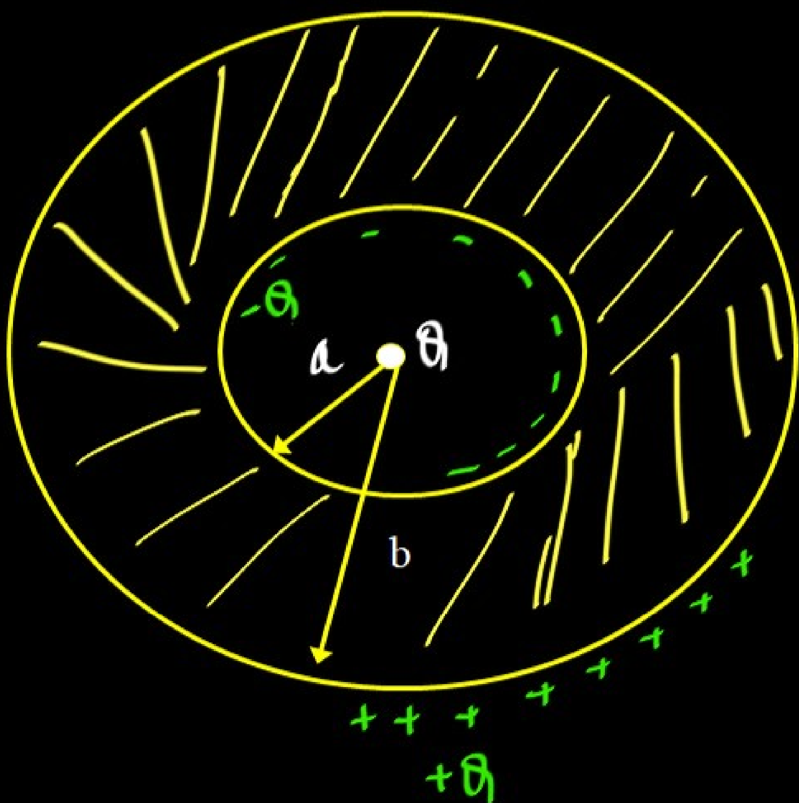
**Hallow closed perfect conductor**




"SHIELDING"

- Inside the perfect conductor, charges repel each other and tries to reside on surface of it (Which is characterized by surface charge $\rho_s$ $c/m^2$ ).
- Inside the perfect conductor the net electric field is zero every where.
- The electric field on surface on conductor is non zero and exists in normal direction.
- The electric field inside the hollow closed conducting surface is zero.

Ex: 5




$0 < r < a$

$E \neq 0$

$a < r < b$

$E = 0$

$r > b$

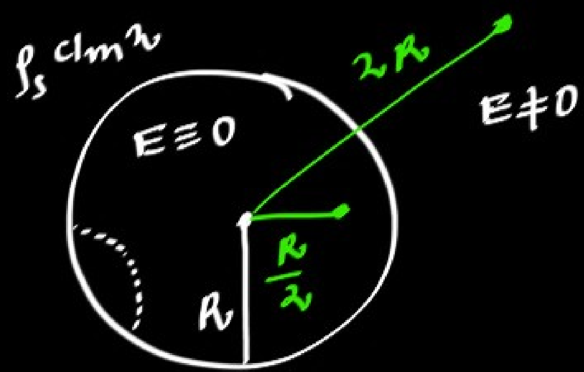
$E \neq 0$

Q) For a uniformly charged sphere of radius $R$ and

G) charge density $\rho_s\ C/m^2$, the ratio of magnitude of electric field at distance $R/2$ and $2R$ from the center i.e

$$\frac{E(r=R/2)}{E(r=2R)} \text{ is } \_\_\_\_\_.$$

Soln:




$$\frac{E(r=R/2)}{E(r=2R)} = \left[\frac{0}{\neq 0}\right] = 0$$

Q) Calculate the field at $E(R/2)$, $E(2R)$ due to the volume charge as sphere of $R$ radius and $\rho_v$ C/m3 charge density.

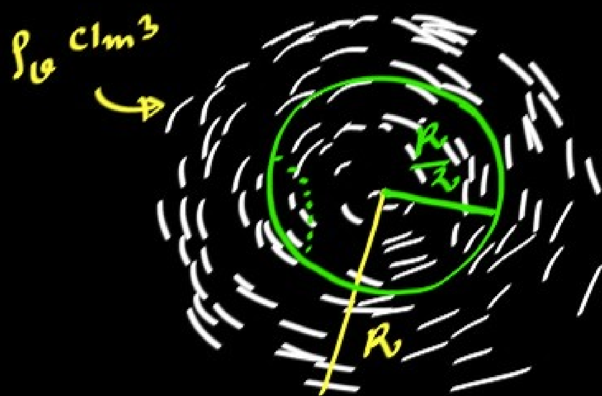
(a) $\frac{\rho_v R}{6\epsilon_0}$, $\frac{\rho_v R}{12\epsilon_0}$

(b) $\frac{\rho_v R}{12\epsilon_0}$, $\frac{\rho_v R}{6\epsilon_0}$

(c) $\frac{\rho_v R}{6\epsilon_0}$, $\frac{\rho_v R}{6\epsilon_0}$

(d) $\frac{\rho_v R}{12\epsilon_0}$, $\frac{\rho_v R}{12\epsilon_0}$.

Soln:



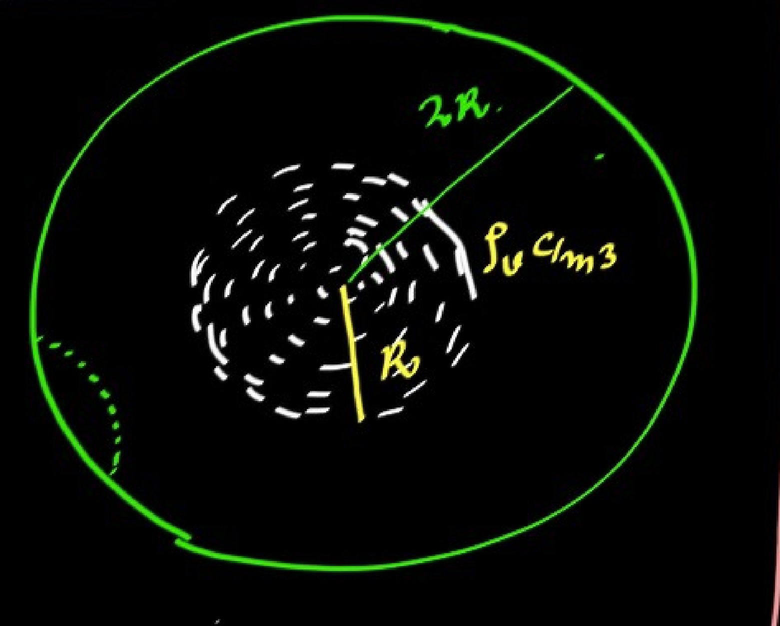

$$\oint\!\!\oint \bar{E}\cdot\bar{dA} = \frac{Q_{ENC}}{\epsilon} = \frac{1}{\epsilon_0}\cdot\rho_v\,\frac{4}{3}\pi\left(\frac{R}{2}\right)^3$$

$$\oint\!\!\oint E_r\,dA = \frac{4\pi R^3}{3\times8\times\epsilon_0}\rho_v$$

$$E_r \oint\!\!\oint dA = \frac{4\pi R^3}{3\times 8\times \epsilon_0}\rho_v$$

$$E_r\, 4\pi\left(\frac{R}{2}\right)^2 = \frac{4\pi R^3}{3\times 8\times \epsilon_0}\rho_v$$

$$E_r = \frac{\rho_v R}{6\epsilon_0}$$




$$\oint\!\!\oint \bar{E}\cdot\bar{dA} = \frac{Q_{enc}}{\epsilon_0} = \frac{1}{\epsilon_0}\rho_v \frac{4}{3}\pi R^3$$

$$\oint\!\!\oint E_r\, dA = \frac{\rho_v}{\epsilon_0}\cdot\frac{4\pi}{3}R^3$$

$$E_r \oint\!\!\oint dA = \frac{\rho_v}{\epsilon_0}\cdot\frac{4\pi}{3}R^3$$

$$E_r\, 4\pi(2R)^2 = \frac{\rho_v}{\epsilon_0}\cdot\frac{4\pi R^3}{3}$$

$$E_r = \frac{\rho_v R}{12\epsilon_0}$$

Q A solid sphere made of insulating material has a radius $R$ and has a total charge $Q$ distributed uniformly in its volume. What is the magnitude of electric field intensity $E$ at distance $r$ $(0 < r < R)$ inside the sphere?

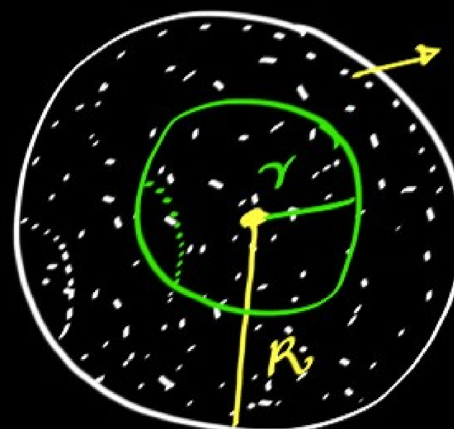
(a) $\frac{1}{4\pi\epsilon_0} \frac{Qr}{R^3}$

(b) $\frac{3}{4\pi\epsilon_0} \frac{Qr}{R^3}$

(c) $\frac{Q}{4\pi\epsilon_0 r}$

(d) $\frac{1}{4\pi\epsilon_0} \frac{QR}{r^3}$

Sol'n:




$\rho_v = \frac{Q}{\frac{4}{3}\pi R^3} \ \frac{C}{m^3}$

$0 \leq r \leq R$

$$\oiint \bar{E} \cdot \bar{dA} = \frac{Q_{enc}}{\epsilon} = \frac{1}{\epsilon} \cdot \rho_v \cdot \frac{4}{3}\pi r^3$$

$$\oiint E_r \, dA = \frac{1}{\epsilon} \cdot \frac{Q}{\frac{4}{3}\pi R^3} \cdot \frac{4}{3}\pi r^3$$

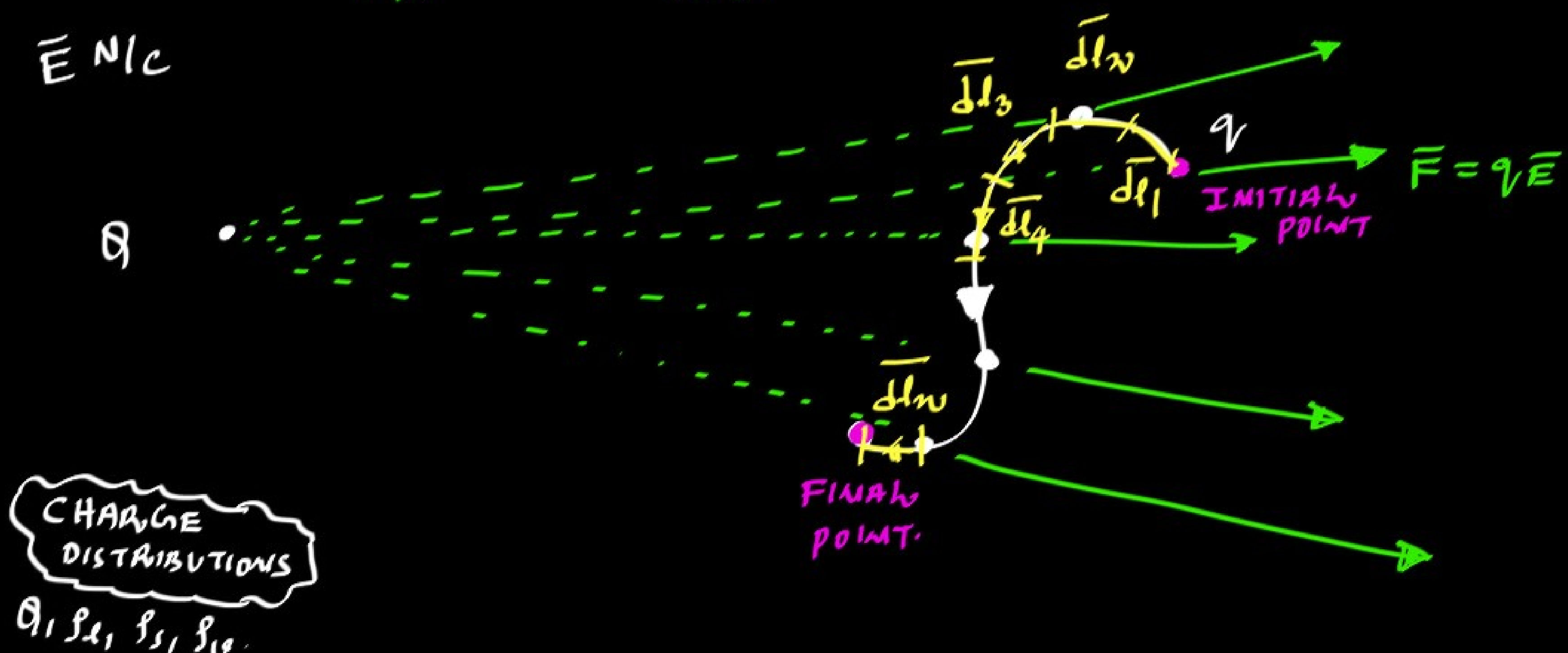
$$E_r \oiint dA = \frac{Q r^3}{\epsilon R^3}$$

$$E_r \, 4\pi r^2 = \frac{Q r^3}{\epsilon R^3}$$

$$\boxed{E_r = \frac{Q r}{4\pi\epsilon R^3}}$$

# Work done in order to move charge / electrostatic potential (V volts)

Work : Motion of objects against applied field
$(q)$ $(\bar{E})$



## WORK DONE BY THE CHARGE.

$$dW = \bar{F}' \cdot \overline{dl} = -\bar{F} \cdot \overline{dl}$$

$$dW = -q\,\bar{E} \cdot \overline{dl}$$

$$W = -\int_{iw}^{fi} q\,\bar{E} \cdot \overline{dl}$$

$$\boxed{W = -q \int_{iw}^{fi} \bar{E} \cdot \overline{dl}} \quad J$$

$$\frac{W}{q} = -\int_{iw}^{fi} \bar{E} \cdot \overline{dl} \quad \frac{J}{C}$$

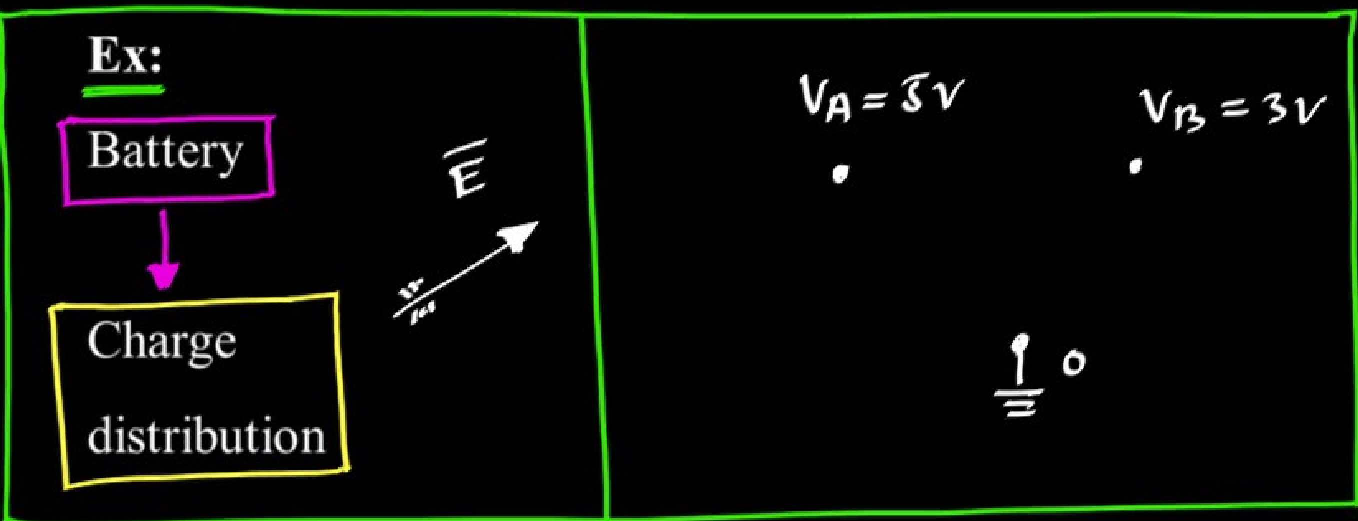
$$\boxed{V = -\int_{iw}^{fi} \bar{E} \cdot \overline{dl}}$$

volts

[V/m] *

m

$$V = -\int_{in}^{fi} \overline{E} \cdot \overline{dl}$$




$V_{AB} = V_A - V_B = 2V$

FINAL POINT.

INITIAL POINT

**Absolute Potential:** It is the potential defined with respect to zero reference point.

**Ex:** $V_A$, $V_B$.

Q8 An electric field intensity is given by

G $\bar{E} = x\hat{x} + y\hat{y} + z\hat{z}$ V/m. find potential at $X(0,0,1)$ with respect to $Y(2,3,4)$

Soln:

$$V_{XY} = V_X - V_Y = -\int_Y^X \bar{E}\cdot\bar{dl} = -\int_Y^X (x\hat{x} + y\hat{y} + z\hat{z})\cdot(dx\,\hat{x} + dy\,\hat{y} + dz\,\hat{z})$$

$V_X$: F.P $\quad$ $V_Y$: I.P

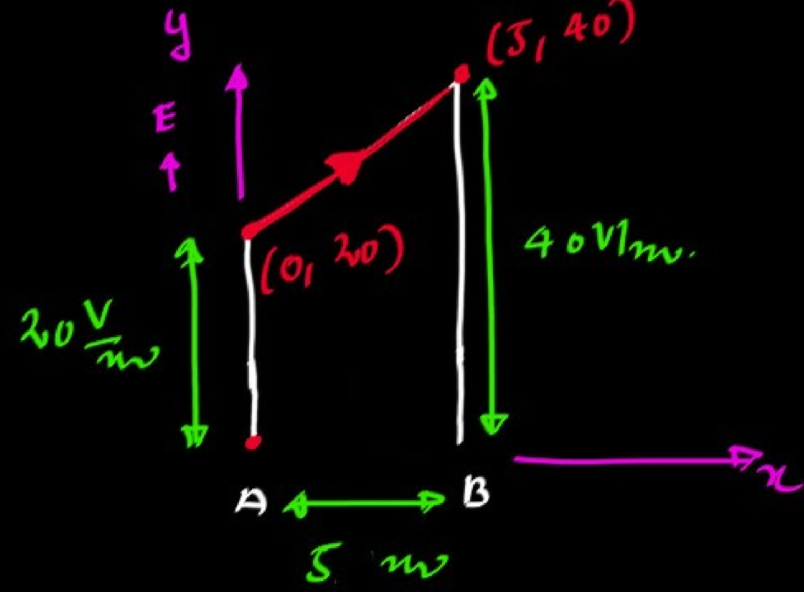
$$= -\int_Y^X (x\,dx + y\,dy + z\,dz) = -\left[\frac{x^2}{2}\Big|_2^0 + \frac{y^2}{2}\Big|_3^0 + \frac{z^2}{2}\Big|_4^1\right]$$

$$= -\frac{1}{2}\left[(0-4) + (0-9) + (1-16)\right] = -\frac{1}{2}\left[-28\right]$$

$$V_{XY} = 14 \text{ Volt}$$

(G)

(8) The electric field between two points A and B is shown, let $\psi_A$ and $\psi_B$ be the electrostatic potentials at A and B respectively. The value of $\psi_B - \psi_A$ is ____.




$$\frac{x-0}{5-0} = \frac{E-20}{40-20}$$

$$\frac{E-20}{20} = \frac{x}{5}$$

$$E = 4x + 20$$

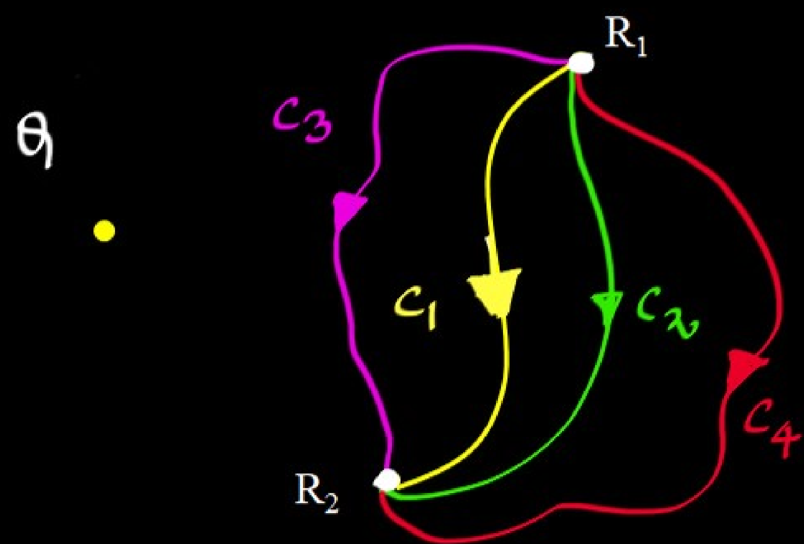
$$V_{BA} = \underset{\text{F.p}}{V_B} - \underset{\text{I.p}}{V_A} = -\int_0^{5m} E\,dx = -\int_0^5 (4x+20)\,dx$$

$$= -\left[4\frac{x^2}{2} + 20x\right]_0^5 = -\left[2\times 5^2 + 20\times 5\right]$$

$$= -\left[2\times 25 + 100\right] = -150 \text{ Volt.}$$

## Potential due to point charge (Q)

$$\overline{E} = \frac{Q}{4\pi\varepsilon R^2}\,\hat{r}$$




$$V = -\int_{R_1}^{R_2} \overline{E}\cdot\overline{dl}$$

$$= -\int_{R_1}^{R_2} \frac{Q}{4\pi\epsilon R^2}\,\hat{r}\cdot\left[dr\,\hat{r} + \underbrace{r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi}}_{0}\right]$$

$$= -\frac{Q}{4\pi\epsilon}\int_{R_1}^{R_2} \frac{1}{R^2}\,dr = \frac{Q}{4\pi\epsilon}\left[\frac{1}{R}\right]_{R_1}^{R_2}$$

$$V = \frac{Q}{4\pi\epsilon}\left[\frac{1}{R_2} - \frac{1}{R_1}\right]$$

FINAL POINT.

INITIAL POINT

**NOTE:** The potential difference between two points is only function of location of initial point and location of final points, but not on the shape of the contour choosen from initial to final point.

CONSIDER

$$V = \frac{q}{4\pi\epsilon}\left[\frac{1}{R_2} - \frac{1}{R_1}\right]$$

LET $R_2 = R$

$$K = \frac{-q}{4\pi\epsilon R_1}$$

$$\boxed{V = \frac{q}{4\pi\epsilon R} + K}$$

FOR POTENTIAL AT POINT

FOR ABSOLUTE POTENTIAL:

$$R_1 \longrightarrow \infty$$

$$K = \frac{-q}{4\pi\epsilon R_1} = 0$$

$$\boxed{V = \frac{q}{4\pi\epsilon R}}$$

FOR POTENTIAL DIFFERENCE.

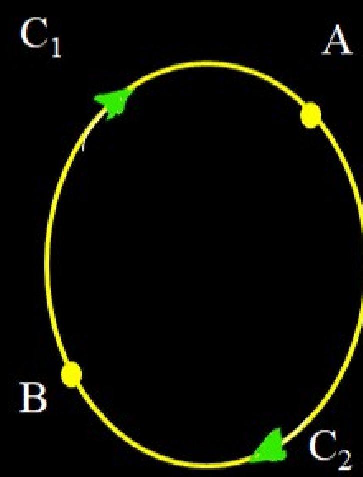
## Conservative Property of Electrostatic Field ($\bar{E}$ v/m)

$\bar{E}$ V/m

CHARGE

DISTRIBUTION

$Q$, $\rho_l$, $\rho_s$, $\rho_v$



$$\oint \bar{E}\cdot d\bar{l} = \underset{(C_2)}{\int_A^B} \bar{E}\cdot d\bar{l} + \underset{(C_1)}{\int_B^A} \bar{E}\cdot d\bar{l} = \underset{(C_2)}{\int_A^B} \bar{E}\cdot d\bar{l} - \underset{(C_1)}{\int_A^B} \bar{E}\cdot d\bar{l} = 0$$

$$\boxed{\oint \bar{E}\cdot d\bar{l} = 0}$$

FROM STOKE'S THEOREM:

$$\oint \bar{E}\cdot d\bar{l} = \iint \nabla\times\bar{E}\cdot d\bar{A} = 0$$

$$\Rightarrow \boxed{\nabla\times\bar{E} = 0}$$

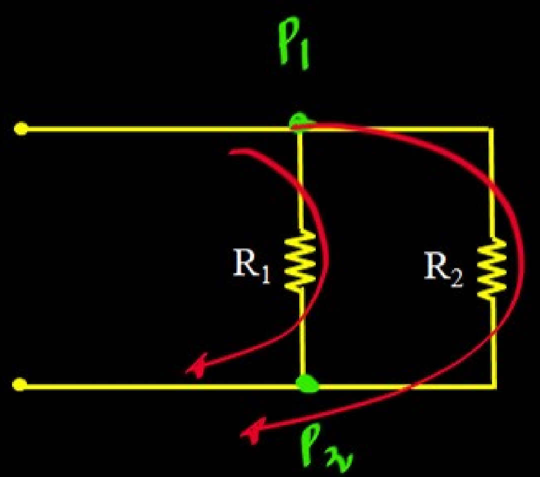
CONSERVATIVE PROPERTY.

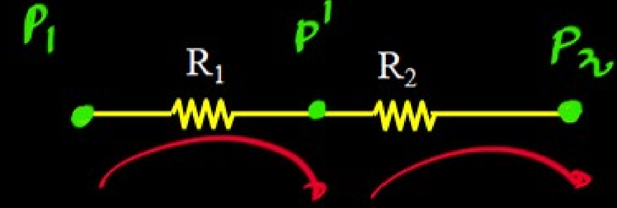
$$\boxed{\oint \bar{E}\cdot d\bar{l} = 0}$$

INTEGRAL FORM.

$$\boxed{\nabla\times\bar{E} = 0}$$

DIFFERENTIAL FORM.

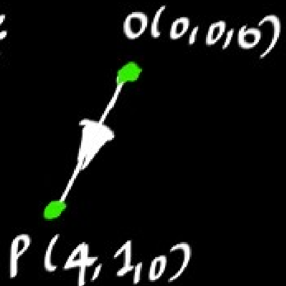

P1
R1
R2
P2


P1
R1
P'
R2
P2

Q. Find the potential at (4, 1, 0) due to the field $\bar{E} = y\hat{x} + x\hat{y}$ V/m, with respect to the origin.

(a) 4V (b) −4V (c) 0 (d) None.

Soln:




$$V_{PO} = \underset{\text{F.P}}{V_P} - \underset{\text{I.P}}{V_O} = -\int_O^P \bar{E}\cdot d\bar{l} = -\int_O^P (y\hat{x} + x\hat{y})\cdot(dx\hat{x} + dy\hat{y} + dz\hat{z})$$

$$\frac{x-0}{4-0} = \frac{y-0}{1-0}$$

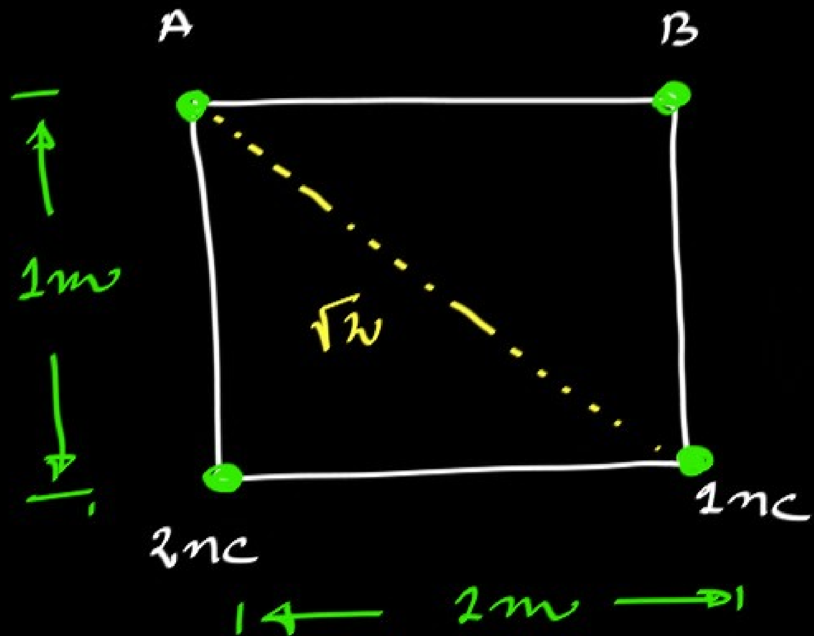
$$y = \frac{x}{4}$$

$$dy = \frac{dx}{4}$$

$$= -\int_0^P (y\,dx + x\,dy) = -\int_0^P \frac{x}{4}dx + x\frac{dx}{4} = -\frac{1}{2}\int x\,dx$$

$$= -\frac{1}{2}\left[\frac{x^2}{2}\right]_0^4 = -\frac{1}{4}\cdot 4^2 = -4 \text{ volts.}$$

Q) Find potential at point A w.r.t point B for below charge configuration.




Soln: $V = \dfrac{Q}{4\pi\epsilon_0 R}$

$$V_A = \frac{2\times10^{-9}}{4\pi\epsilon_0 \times 1} + \frac{1\times10^{-9}}{4\pi\epsilon_0 \times \sqrt{2}}$$

$$V_B = \frac{1\times10^{-9}}{4\pi\epsilon_0 \times 1} + \frac{2\times10^{-9}}{4\pi\epsilon_0 \sqrt{2}}$$

$$V_{AB} = V_A - V_B$$

$$V_{AB} = \frac{1\times10^{-9}}{4\pi\epsilon_0}\left[2 + \frac{1}{\sqrt{2}} - 1 - \frac{2}{\sqrt{2}}\right]$$

$$V_{AB} = \frac{1\times10^{-9}}{4\pi\epsilon_0}\left[1 - \frac{1}{\sqrt{2}}\right]$$

$$V_{AB} = \frac{1\times10^{-9}}{4\pi \times \frac{1}{36\pi\times10^{9}}} \left[1 - 0.707\right]$$

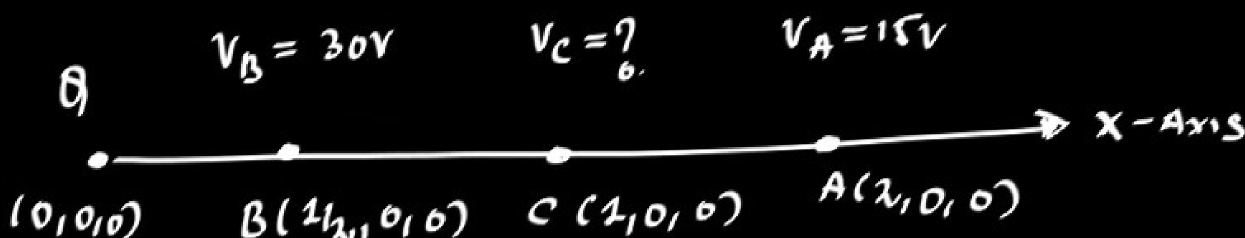
$$V_{AB} = 9\left[1 - 0.707\right]$$

$$V_{AB} = 2.63 \text{ volts.}$$

Q) In the field of a charge Q at the origin, the potentials at A(2,0,0) and B(1/2,0,0) are $V_A = 15V$ and $V_B = 30V$ respectively what will be the potential at C(1,0,0) ?

(a) 25V (b) 22.5V (c) 20V (d) 17.5V.

Soln:



$$V = \frac{Q}{4\pi\epsilon R} + K$$

$$V_A = \frac{Q}{4\pi\epsilon(2)} + K = 15 \longrightarrow (1)$$

$$V_B = \frac{Q}{4\pi\epsilon(1/2)} + K = 30$$

$$\frac{2Q}{4\pi\epsilon} + K = 30 \longrightarrow (2)$$

$(2) = 2 \times (1)$

$$\frac{2Q}{4\pi\epsilon} + K = \frac{2Q}{4\pi\epsilon(2)} + 2K$$

$$\frac{2Q}{4\pi\epsilon} - \frac{Q}{4\pi\epsilon} = K.$$

$$K = \frac{Q}{4\pi\epsilon}$$

sub K in (1)

$$\frac{Q}{4\pi\epsilon(2)} + \frac{Q}{4\pi\epsilon} = 15$$

$$\frac{Q}{4\pi\epsilon}\left[\frac{3}{2}\right] = 15$$

$$Q = 40\pi\epsilon.$$

$$V_C = \frac{Q}{4\pi\epsilon(1)} + K$$

$$V_C = \frac{40\pi\epsilon}{4\pi\epsilon} + \frac{Q}{4\pi\epsilon}$$

$$V_C = 10 + \frac{40\pi\epsilon}{4\pi\epsilon}$$

$$V_C = 10 + 10$$

$V_C = 20$ volts.

# The gradient relation.

Consider

$$V = -\int_{in}^{fi} \overline{E}.\,\overline{dl}$$

$$\boxed{\begin{array}{l} V(x, y, z) \\ V(\rho, \phi, z) \\ V(r, \theta, \phi) \end{array}}$$

$$V = -\int_{in}^{fi} \overline{E} \cdot \overline{dl}$$

$$\Delta V = -\overline{E} \cdot \overline{dl}$$

$$\left[\frac{\partial V}{\partial x} dx + \frac{\partial V}{\partial y} dy + \frac{\partial V}{\partial z} dz\right] = -\overline{E} \cdot \overline{dl}$$

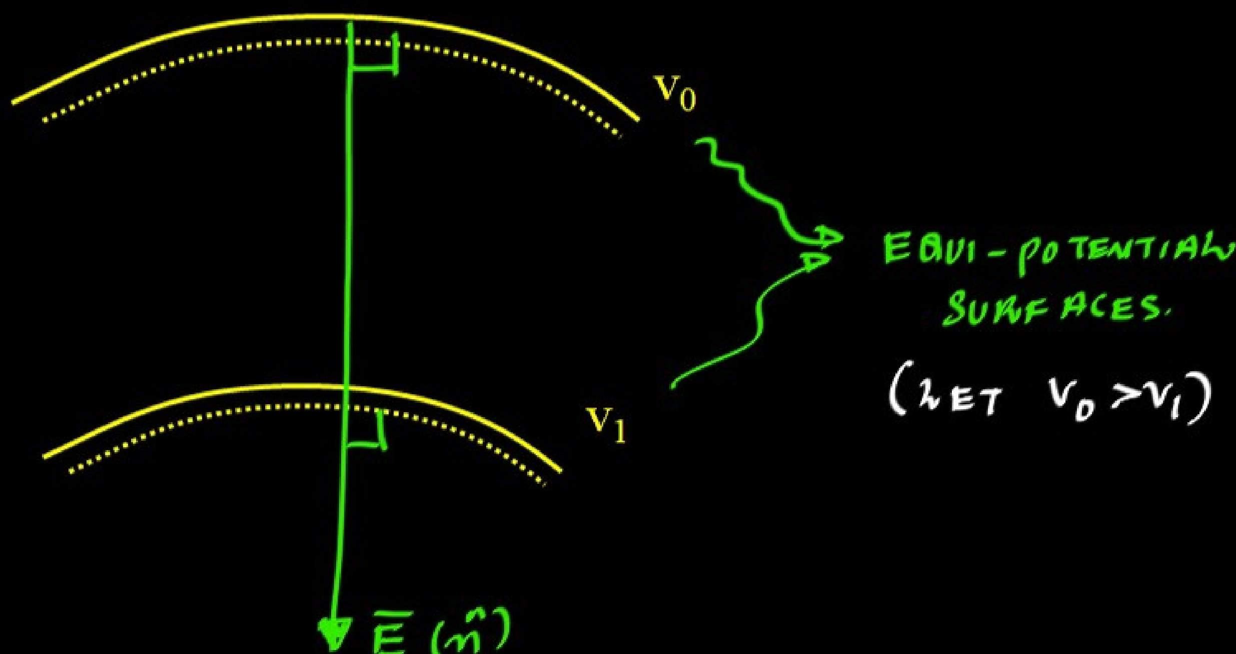
$$\left[\frac{\partial V}{\partial x} \hat{x} + \frac{\partial V}{\partial y} \hat{y} + \frac{\partial V}{\partial z} \hat{z}\right] \cdot \cancel{\underbrace{\left[dx\,\hat{x} + dy\,\hat{y} + dz\,\hat{z}\right]}_{\overline{dl}}} = -\overline{E} \cdot \cancel{\overline{dl}}$$

$$\overline{E} = -\left[\frac{\partial V}{\partial x} \hat{x} + \frac{\partial V}{\partial y} \hat{y} + \frac{\partial V}{\partial z} \hat{z}\right] = -\underbrace{\left[\frac{\partial}{\partial x} \hat{x} + \frac{\partial}{\partial y} \hat{y} + \frac{\partial}{\partial z} \hat{z}\right]}_{\nabla} V$$

****

$$\boxed{\overline{E} = -\nabla V}$$

→ ELECTRIC FIELD PROJECTS NORMALLY FROM HIGHER EQUI-POTENTIAL SURFACE TO LOWER EQUI-POTENTIAL SURFACE.



$V_0$
$V_1$
EQUI-POTENTIAL SURFACES.
(LET $V_0 > V_1$)
$\bar{E}\ (\hat{n})$

Q. An electrostatic potential is given by $\phi = 2x\sqrt{y}$ volt in the rectangular co-ordinate system. The magnitude of electric field at x = 1 m, y = 1 m is ____ V/m.

**[GATE-92-EE]**

Soln:

$\bar{E} = -\nabla V$

$\nabla V = \frac{\partial V}{\partial x}\hat{x} + \frac{\partial V}{\partial y}\hat{y} + \underbrace{\frac{\partial V}{\partial z}\hat{z}}_{0}$

$\nabla V = 2\sqrt{y}\,\hat{x} + 2x\frac{1}{2\sqrt{y}}\hat{y}$

$\nabla V = 2\sqrt{y}\,\hat{x} + \frac{x}{\sqrt{y}}\hat{y}$

At x = 1 m, y = 1 m

$\nabla V = 2\hat{x} + \hat{y}$

$\bar{E} = -\nabla V = -2\hat{x} - \hat{y}$

$|\bar{E}| = \sqrt{1^2 + 2^2} = \sqrt{5}$

$|\bar{E}| = 2.236$ V/m

Q) The electrostatic potential is described as $\rho z \cos^2\phi$ volts find electric field at $(1, \pi/4, 1)$.

Soln:
$$\nabla V = \frac{\partial V}{\partial \rho}\hat{\rho} + \frac{1}{\rho}\frac{\partial V}{\partial \phi}\hat{\phi} + \frac{\partial V}{\partial z}\hat{z}$$

$$\nabla V = z\cos^2\phi\,\hat{\rho} + \frac{1}{\rho}\rho z\, 2\cos\phi(-\sin\phi)\,\hat{\phi} + \rho\cos^2\phi\,\hat{z}$$

$$\nabla V = z\cos^2\phi\,\hat{\rho} - z\sin 2\phi\,\hat{\phi} + \rho\cos^2\phi\,\hat{z}$$

$$\bar{E} = -\nabla V = -z\cos^2\phi\,\hat{\rho} + z\sin 2\phi\,\hat{\phi} - \rho\cos^2\phi\,\hat{z}$$

AT $(1, \pi/4, 1) = (\rho, \phi, z)$

$$\bar{E} = -1\times\left(\frac{1}{\sqrt{2}}\right)^2\hat{\rho} + (1)(1)\,\hat{\phi} - 1\times\left(\frac{1}{\sqrt{2}}\right)^2\hat{z} \Rightarrow \bar{E} = -\frac{1}{2}\hat{\rho} + \hat{\phi} - \frac{1}{2}\hat{z}$$

Q. Two electric charges q and -2q are placed at (0,0) and (6,0) on the xy – plane. The equation of the zero equipotential curve in the xy-plane is

**[GATE-16-EE]**

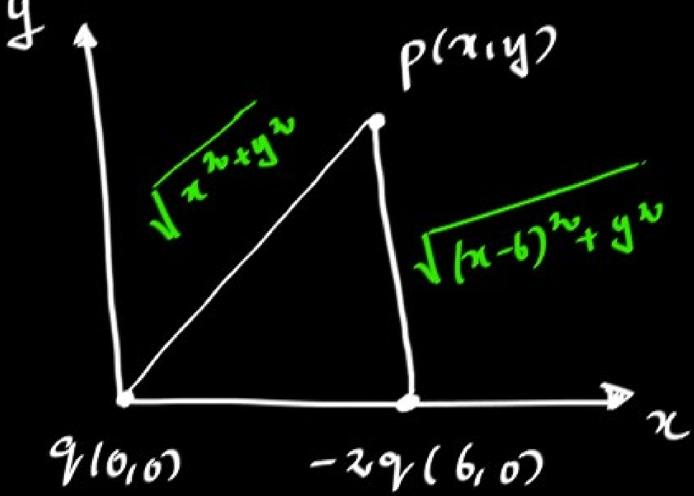
(a) $x = -2$

(b) $y = 2$

(c) $x^2 + y^2 = 2$

(d) $(x + 2)^2 + y^2 = 16$

Soln:




$$V = \frac{Q}{4\pi\epsilon R} + \cancel{K}^{0}$$

$$V = \frac{Q}{4\pi\epsilon R}$$

$$V_P = \frac{q}{4\pi\epsilon\sqrt{x^2+y^2}} + \frac{(-2q)}{4\pi\epsilon\sqrt{(x-6)^2+y^2}} = 0$$

$$\frac{q}{4\pi\epsilon\sqrt{x^2+y^2}} = \frac{2q}{4\pi\epsilon\sqrt{(x-6)^2+y^2}}$$

$$\frac{1}{\sqrt{x^2+y^2}} = \frac{2}{\sqrt{(x-6)^2+y^2}}$$

$$(x-6)^2 + y^2 = 4x^2 + 4y^2$$

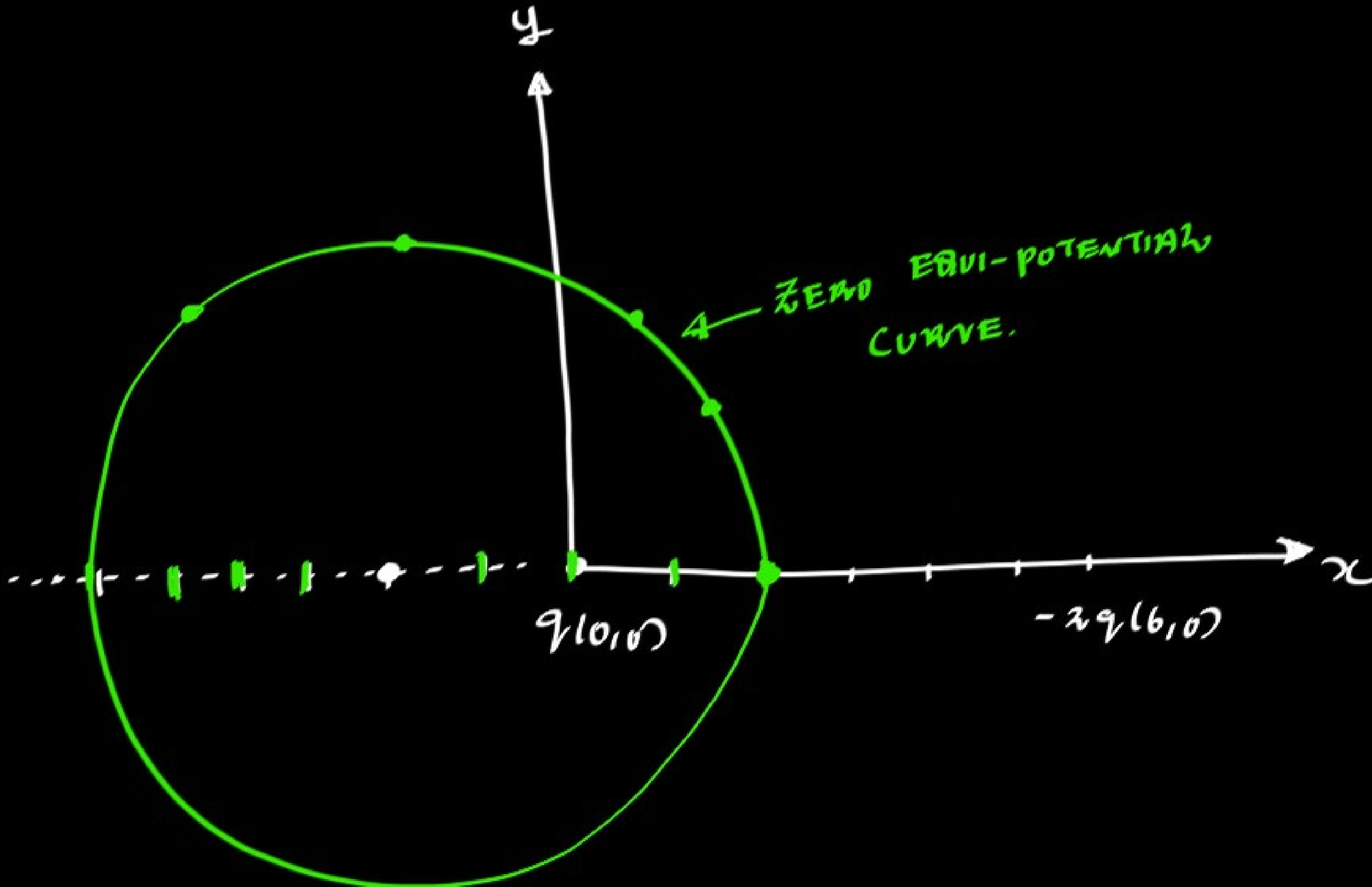
$$x^2 + 36 - 12x + y^2 = 4x^2 + 4y^2$$

$$3x^2 + 3y^2 + 12x = 36$$

$$x^2 + 4x + y^2 = 12$$

$$x^2 + 4x + 4 + y^2 = 12 + 4$$

$$(x+2)^2 + y^2 = 16.$$



y
x
ZERO EQUI-POTENTIAL CURVE.
q(0,0)
-2q(6,0)

# Poisson's Equation

CONSIDER

$$\nabla \cdot \bar{D} = \rho_v .$$

$$\bar{D} = \epsilon \bar{E}$$

FOR HOMOGENEOUS MEDIUM

($\epsilon$ : CONSTANT)

$$\nabla \cdot \epsilon \bar{E} = \rho_v$$

$$\nabla \cdot \bar{E} = \frac{\rho_v}{\epsilon} \longrightarrow (1)$$

CONSIDER

$$\bar{E} = -\nabla V \longrightarrow (2)$$

$$\nabla \cdot (-\nabla V) = \frac{\rho_v}{\epsilon}$$

$$\boxed{\nabla^2 V = -\frac{\rho_v}{\epsilon}}$$

→ POISSON'S EQUATION.

FOR CHARGE FREE REGION ( $\rho_v = 0$ )

$$\boxed{\nabla^2 V = 0}$$

→ LAPLACIAN EQUATION.

Q. The potential (scalar) distribution is given as $V = 10y^3 + 20x^3$. If $\epsilon_0$ is permittivity of free space. What is the charge density $\rho$ at the point (2,0)? **[IES 2005]**

(a) $-200\,\epsilon_0$ (b) $-200/\epsilon_0$

(c) $200\,\epsilon_0$ (d) $-240\,\epsilon_0$

Soln.:

$$\nabla^2 V = -\frac{\rho_v}{\epsilon_0}$$

$$\rho_v = -(\nabla^2 V)\,\epsilon_0$$

$$\nabla^2 V = \frac{\partial^2}{\partial x^2}V + \frac{\partial^2}{\partial y^2}V + \underbrace{\frac{\partial^2}{\partial z^2}V}_{0}$$

$$\nabla^2 V = 20\cdot 3\cdot 2x + 10\cdot 3\cdot 2\cdot y$$

AT (2,0)

$$\nabla^2 V = 120(2) + 60(0) = 240$$

$$\rho_v = -240\,\epsilon_0$$

Q. In a source free region in vacuum if the electrostatic potential $\phi = 2x^2 + 2y^2 + Cz^2$. The value of constant C must be ________.

**[IES 2005]**

Soln: $\nabla^2 V = -\cancel{\frac{\rho_v}{\epsilon}} = 0$

$\frac{\partial^2}{\partial x^2} V + \frac{\partial^2}{\partial y^2} V + \frac{\partial^2}{\partial z^2} V = 0$

$2 \cdot 2 \cdot 1 + 2 \cdot 2 \cdot 1 + C \cdot 2 \cdot 1 = 0$

$8 + 2C = 0$

$C = -4$

Q Charge needed with a unit sphere centered at the origin for production a potential field $V = \frac{-6r^5}{\epsilon_0}$. For $r \le 1$ is

**[IES 1999]**

(a) 12 πC

(b) 60 πC

(c) 120 πC

(d) 150 πC

Soln: $\rho_v = -\epsilon_0 [\nabla^2 V]$

$$\nabla^2 V = \frac{1}{r^2 \sin\theta}\left[\frac{\partial}{\partial r}\left(r^2 \sin\theta \frac{\partial V}{\partial r}\right) + \underbrace{\frac{\partial}{\partial \theta}\left(\frac{r\sin\theta}{r}\frac{\partial V}{\partial \theta}\right) + \frac{\partial}{\partial \phi}\left(\frac{r}{r\sin\theta}\frac{\partial V}{\partial \phi}\right)}_{0}\right]$$

$$\nabla^2 V = \frac{\sin\theta}{r^2 \sin\theta} \frac{\partial}{\partial r}\left[r^2 \frac{\partial V}{\partial r}\right] = \frac{1}{r^2} \frac{\partial}{\partial r}\left[r^2 \frac{\partial}{\partial r}\left(-\frac{6r^5}{\epsilon_0}\right)\right]$$

$$= \frac{1}{r^2} \frac{\partial}{\partial r}\left[r^2 \left(-\frac{6 \cdot 5 \cdot r^4}{\epsilon_0}\right)\right]$$

$$= -\frac{30}{r^2 \epsilon_0} \frac{\partial}{\partial r}(r6)$$

$$= -\frac{30 \times 6 \times r^5}{r^2 \epsilon_0}$$

$$\nabla^2 V = -\frac{180 r^3}{\epsilon_0}$$

$$\rho_v = -\epsilon_0 \nabla^2 V = -\epsilon_0 \left[\frac{-180 r^3}{\epsilon_0}\right]$$

$\rho_v = 180 r^3 \ C/m^3$

$Q = \iiint \rho_v \, dv$

$= \iiint 180 r^3 \, r^2 \sin\theta \, dr \, d\theta \, d\phi$

$= 180 \int r^5 \, dr \int \sin\theta \, d\theta \int d\phi$

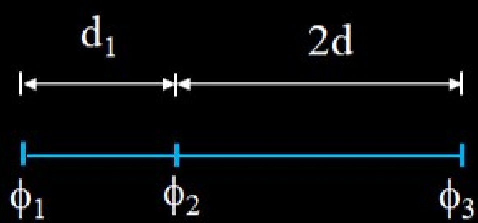
$= 180 \left[\frac{r^6}{6}\right]_0^1 \left[-\cos\theta\right]_0^{\pi} \left[\phi\right]_0^{2\pi}$

$= 30 \left[1^6\right] \{-(-1-1)\} (2\pi)$

$Q = 120\pi \ C$

Q. The three values of a one-dimensional potential function $\phi$ shown in the given figure and satisfying laplace equation are related as

H-W



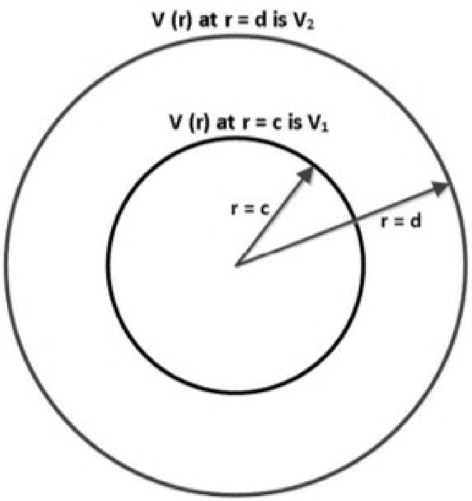

(a) $\phi_2 = \dfrac{2\phi_3 + \phi_1}{3}$

(b) $\phi_2 = \dfrac{2\phi_1 + \phi_3}{3}$

(c) $\phi_2 = \dfrac{2\phi_1 - \phi_3}{3}$

(d) $\phi_2 = \dfrac{\phi_1 + 3\phi_3}{2}$

Q.53 As shown in the figure below, two concentric conducting spherical shells, centered at $r = 0$ and having radii $r = c$ and $r = d$ are maintained at potentials such that the potential $V(r)$ at $r = c$ is $V_1$ and $V(r)$ at $r = d$ is $V_2$. Assume that $V(r)$ depends only on r, where r is the radial distance. The expression for $V(r)$ in the region between $r = c$ and $r = d$ is




(A)
$$V(r) = \frac{c\,d\,(V_2 - V_1)}{(d - c)\,r} - \frac{V_1\,c + V_2\,d - 2\,V_1\,d}{d - c}$$

(B) $V(r) = \dfrac{c\,d\,(V_1 - V_2)}{(d - c)\,r} + \dfrac{V_2\,d - V_1\,c}{d - c}$

(C) $V(r) = \dfrac{c\,d\,(V_1 - V_2)}{(d - c)\,r} - \dfrac{V_1\,c - V_2\,c}{d - c}$

(D) $V(r) = \dfrac{c\,d\,(V_2 - V_1)}{(d - c)\,r} - \dfrac{V_2\,c - V_1\,c}{d - c}$

H.W