$$\frac{dy}{dx} = f(x) = (x-1.65)^2 + 5.25$$

$h = 1.0$

$y(1.0) = 2.5 \quad \therefore x = 1.0$

$y(2.0) = ?$

$f(x_1, y_1) = 5.6725$

(a) Midpoint method

$$y_{i+\frac{1}{2}} = y_i + f(x_i, y_i) \cdot \frac{h}{2}$$

$$\Rightarrow y_{1.5} = y_1 + f(x_1) \cdot \frac{h}{2}$$

$$\Rightarrow y_{1.5} = 2.5 + \left[(1-1.65)^2 + 5.25\right] \times \frac{1.0}{2} = 2.5 + 2.83625$$

$$= 5.33625$$

$$y_2 = y_{1.5} + \left[(1.5-1.65)^2 + 5.25\right] \times \frac{1.0}{2}$$

$$\therefore y_2 = 7.9725$$

% of error $= \left[\frac{7.9725 - 6.5}{6.5}\right]\% = 22.65\%$

(b) Ralston's method.

$$y_{i+1} = y_i + \left(\frac{1}{3}k_1 + \frac{2}{3}k_2\right)h$$

$k_1 = f(x_i, y_i)$ for $i=1$, $k_1 = 5.6725$

$k_2 = f(x_i + \frac{3}{4}h, y_i + \frac{3}{4}h)$ for $i=1$, $k_2 = f(1.75) =$

$= (1.75-1.65)^2 + 5.2$

$= 5.26.$

$k_2 = f(x_i + \frac{3}{4}h, y_i + \frac{3}{4}h)$ for $i=1$, $k_2 = f(1.75) =$

$= (1.75 - 1.65)^2 + 5.$

$= 5.26.$

$y_2 = y_1 + \left(\frac{1}{3} \times 5.6725 + \frac{2}{3} \times 5.26\right) \times 1.0$

$= 2.5 + 5.3975 = 7.8975$

% of error $= \left| \frac{7.8975 - 6.5}{6.5} \right| \% = 21.5\%$.

4th order RK method,

$$y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)h.$$

$k_1 = f(x_i, y_i)$ for $x=1$, $k_1 = 5.6725$

$k_2 = f(x_i + \frac{h}{2}, y_i + \frac{h}{2})$. for $x=1$, $k_2 = k_3 = f(1.5)$

$k_3 = f(x_i + \frac{h}{2}, y_i + \frac{h}{2})$. $= (1.5 - 1.65)^2$

$= 5.2725$

$k_4 = f(x_i + h, y_i + h)$ for $x=1$, $k_4 = f(2)$

$= (2 - 1.65)^2 + 5.25$

$= 5.3725.$

$y_2 = y_1 + \frac{1}{6}(5.6725 + 4 \times 5.2725 + 5.3725) \times 1.0$

$= 7.85$

% of error $= \left| \frac{7.85 - 6.5}{6.5} \right| \% = 20.7\%.$