National University of Computer and Emerging Sciences, Lahore Campus

Course Name:	**Operations Research**	**Course Code:**	**MT 4031**
Degree Program:	**BCS**	**Semester:**	**Spring 2024**
Exam Duration:	**60 min.**	**Total Marks:**	**30**
Paper Date:		**Weight**	**15 %**
Sections:	**All**	**Page(s):**	
Exam Type:	**Sessional-I**		

Instruction/ Notes:

i. **Clearly write your name, roll no and section on the first page of answer book.**
ii. **Attempt all questions neatly.**
iii. **Exchange of calculators is not allowed.**
iv. **Read the questions carefully for clarity of context and understanding of meaning and make assumptions wherever required, for neither the invigilator will address your queries, nor the teacher/examiner will come to the examination hall for any assistance.**

<u>Question 1</u>: (marks: 8+4)

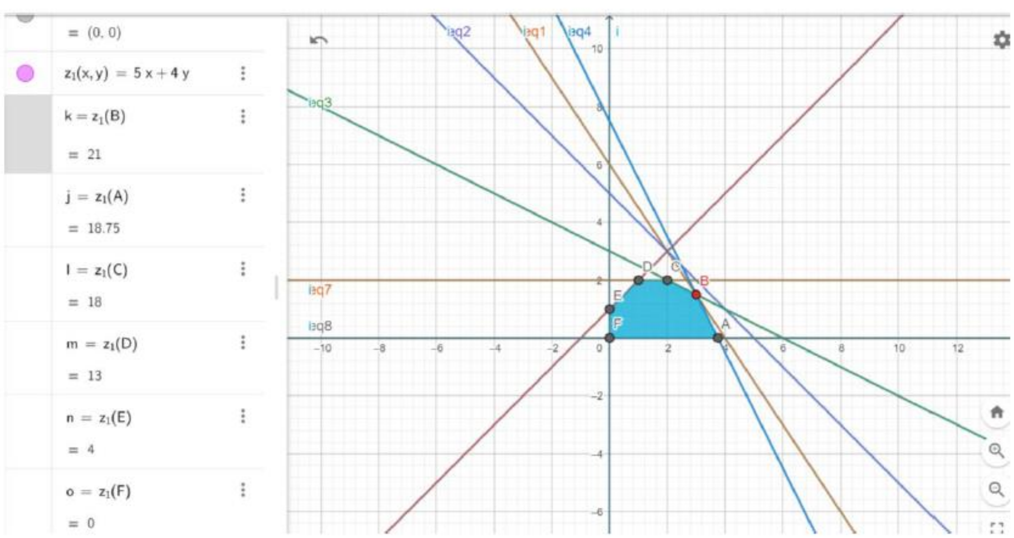
a. Solve the following linear programming model graphically.

$$\text{Max} \quad z = 5x + 4y$$

subject to

$$\begin{aligned} 6x + 4y &\le 24 \\ 6x + 3y &\le 22.5 \\ x + y &\le 5 \\ x + 2y &\le 6 \\ -x + y &\le 1 \\ y &\le 2 \\ x, y &\ge 0 \end{aligned}$$

b. Identify the redundant constraints and show that their removal does not affect the solution space or the optimal solution.



First and third are redundant constraints.

Drawing all lines---3 marks.
Shading the feasible sol. space--- 2 marks.
Corner points-----2 marks.
Optimal value--- 1 mark

Question 2: **(marks: 3+7)**

A compony produces 3 types of toys. The maximum production limit of the three types per month is 7 toys in total. Production time of a type 1, type 2 and type 3 toy is 2 hours, 5 hours and 3 hours respectively. The minimum work hours available in a month are 10 hours. The profit of a type 1, type 2 and type 3 toy is \$1, \$2 and \$3 respectively.

i. formulate a linear programming model for the given scenario.
ii. Use any appropriate technique to find the number of each type of toys to be produced to maximize the profit.

Solution

Objective Function:

Maximize: $Z = 1X_1 + 2X_2 + 3X_3$

Subject to:

$1X_1 + 1X_2 + 1X_3 \le 7$

$2X_1 + 5X_2 + 3X_3 \ge 10$

$X_1, X_2, X_3 \ge 0$

3 marks

Objective Function:

Maximize: $Z = 1X_1 + 2X_2 + 3X_3 + 0S_1 + 0S_2 - MA_1$

Subject to:

$1X_1 + 1X_2 + 1X_3 + 1S_1 + 0S_2 + 0A_1 = 7$

$2X_1 + 5X_2 + 3X_3 + 0S_1 - 1S_2 + 1A_1 = 10$

$X_1, X_2, X_3, S_1, S_2, A_1 \ge 0$

Initial Table

Table 1	C_j	1	2	3	0	0	-M	
C_b	Base	X_1	X_2	X_3	S_1	S_2	A_1	R
0	S_1	1	1	1	1	0	0	7
-M	A_1	2	5	3	0	-1	1	10
	Z	-2M-1	-5M-2	-3M-3	0	M	0	-10M

standard form and initial table---2 marks

Enter the variable X_2 and the variable A_1 leaves the base. The pivot element is **5**

Iteration 1

Table 2	C_j	1	2	3	0	0	-M	
C_b	Base	X_1	X_2	X_3	S_1	S_2	A_1	R
0	S_1	3/5	0	2/5	1	1/5	-1/5	5
2	X_2	2/5	1	3/5	0	-1/5	1/5	2
	Z	-1/5	0	-9/5	0	-2/5	M+2/5	4

Enter the variable X_3 and the variable X_2 leaves the base. The pivot element is **3/5**

Iteration 2

Table 3	C_j	1	2	3	0	0	-M	
C_b	Base	X_1	X_2	X_3	S_1	S_2	A_1	R
0	S_1	1/3	-2/3	0	1	1/3	-1/3	11/3
3	X_3	2/3	5/3	1	0	-1/3	1/3	10/3
	Z	1	3	0	0	-1	M+1	10

Enter the variable S_2 and the variable S_1 leaves the base. The pivot element is **1/3**

1.5 marks for each iteration

Iteration 3

Table 4	C_j	1	2	3	0	0	-M	
C_b	Base	X_1	X_2	X_3	S_1	S_2	A_1	R
0	S_2	1	-2	0	3	1	-1	11
3	X_3	1	1	1	1	0	0	7
	Z	2	1	0	3	0	M	21

The optimal solution is Z = 21

$X_1 = 0, X_2 = 0, X_3 = 7, S_1 = 0, S_2 = 11, A_1 = 0$

0.5 marks

<u>Question 3</u>: (marks: 8)

Determine dual price (the value of objective function) and the feasibility range of the variables from the given optimal tableau:

							Solution			
Basic	x_1	x_2	x_3	x_4	x_5	x_6	RHS	D_1	D_2	D_3
z	4	0	0	1	2	0	1350	1	2	0
x_2	$-\frac{1}{4}$	1	0	$\frac{1}{2}$	$-\frac{1}{4}$	0	100	$\frac{1}{2}$	$-\frac{1}{4}$	0
x_3	$\frac{3}{2}$	0	1	0	$\frac{1}{2}$	0	230	0	$\frac{1}{2}$	0
x_6	2	0	0	-2	1	1	20	-2	1	1

Solution

Dual prices: The value of the objective function can be written as

$$z = 1350 + \mathbf{1}D_1 + \mathbf{2}D_2 + \mathbf{0}D_3$$

1 mark

Feasibility range: The current solution remains feasible if all the basic variables remain nonnegative—that is,

$$x_2 = 100 + \tfrac{1}{2}D_1 - \tfrac{1}{4}D_2 \geq 0$$
$$x_3 = 230 + \tfrac{1}{2}D_2 \geq 0$$
$$x_6 = 20 - 2D_1 + D_2 + D_3 \geq 0$$

1 mark

The given conditions can produce the individual *feasibility ranges* associated with changing the resources *one at a time* (as defined in Section 3.6.1). For example, a change in operation 1 time only means that $D_2 = D_3 = 0$. The simultaneous conditions thus reduce to

$$\left.\begin{array}{l} x_2 = 100 + \frac{1}{2}D_1 \geq 0 \Rightarrow D_1 \geq -200 \\ x_3 = 230 > 0 \\ x_6 = 20 - 2D_1 \geq 0 \Rightarrow D_1 \leq 10 \end{array}\right\} \Rightarrow -200 \leq D_1 \leq 10$$

This means that the dual price for operation 1 is valid in the feasibility range $-200 \leq D_1 \leq 10$.

We can show in a similar manner that the feasibility ranges for operations 2 and 3 are $-20 \leq D_2 \leq 400$ and $-20 \leq D_3 \leq \infty$. respectively (verify!).

2 marks for finding feasibility range for each D.