we ... to make final ... in halt state before ma...

Day.

Date

$= 0 + \frac{x^2}{2} + \frac{x^5}{20}.$

$\boxed{\text{Put } n=2}$

$y_3 = y_0 + \int_{x_0}^{x} f(x, y_2)\,dx.$

$= y_0 + \int_{x_0}^{x} x + \left[\frac{x^2}{2} + \frac{x^5}{20}\right]^2 dx$

$= y_0 + \int_0^x x + \frac{x^4}{4} + \frac{2x^2}{(2)}\left(\frac{x^5}{20}\right) + \frac{x^{10}}{400}\,dx$

$= y_0 + \int_0^x \left(x + \frac{x^4}{4} + \frac{x^7}{20} + \frac{x^{10}}{400}\right)dx$

$y_3 = y_0 + \frac{x^2}{2} + \frac{x^5}{20} + \frac{x^8}{160} + \frac{x^{11}}{4400}$

third approximation

$y_3 = \frac{(0.1)^2}{2} + \frac{(0.1)^5}{20} + \frac{(0.1)^8}{160} + \frac{(0.1)^{11}}{4400}$

$y_3 = 5\times10^{-3} = 0.005$

Q: Solve, find successive approximation upto 4 th order, $y' + y = e^x$, $y(0) = 0$

$\frac{dy}{dx} + y = e^x.$

$\frac{dy}{dx} = e^x - y.$

Let $n = 5$

$h = \frac{x_n - x_0}{n} = \frac{0.1 - 0}{5} = 0.02.$

$y_1 = y_0 + h\,(f(x_0, y_0)) = 1 + 0.02\left(\frac{1-0}{1+0}\right)$

$= 1.02$

$x_1 = x_0 + h = 0 + 0.02 = 0.02$

$y_2 = y_1 + h\,(f(x_1, y_1)) = 1.02 + 0.02\left(\frac{1.02 - 0.02}{1.02 + 0.02}\right)$

$= 1.0392.$

$x_2 = x_1 + h = 0.04.$

$y_3 = y_2 + h\,f(x_2, y_2) = 1.0392 + 0.02\left(\frac{1.0392 - 0.0}{1.0392 + 0.0}\right)$

$y_3 = 1.0577$

$x_3 = x_2 + h = 0.06.$

$y_4 = y_3 + h\,f(x_3, y_3) = 1.0577 + 0.02\left(\frac{1.0577 - 0.0}{1.0577 + 0.0}\right)$

$y_4 = 1.0756.$

$x_4 = 0.08$

$y_5 = 1.0756 + 0.02\left(\frac{1.0756 - 0.08}{1.0756 + 0.08}\right)$

$y_5 = 1.0928 \longrightarrow y_5 = 1.0928$

$x_5 = 0.1$

$y(0.1) = 1.09$

~~$y_6 = 1.0928 + 0.02\left(\frac{1.0928 - 0.1}{1.0928 + 0.1}\right)$~~

$y(0.1) = 1.0928$ upto 4 places

Date:

Day:

$f(x_1, y_1^{(2)}) = (0.2)^2 + (1.2597)^2$

$= 1.8268$

$y_1^{(3)} = 1 + \frac{0.2}{2}(1 + 1.6265) = 1.2626.$

$f(x_1, y_1^{(3)}) = (0.2)^2 + (1.2626)^2$

$= 1.6344.$

$y_1^{(4)} = 1 + \frac{0.2}{2}(1 + 1.6344) = 1.2634.$

Very long Method.

use Book's formula instead

Single step method

Euler Modified.

$$y_{n+1} = y_n + hf\left[x_n + \frac{h}{2}, y_n + \frac{h}{2} f(x_n, y_n)\right]$$

$n = 0, 1, 2,$

$x_n = x_0 + nh, \quad y_n = y(x_n).$

(Notes page 570)

High convergence rate $\longrightarrow$ error reduction faster

Bisection 1/2
regula 1 , iteration 1
Secant 1.62 , raphson 2

Single Step Method

Taylor Series Method

Consider given differential eq.

$$\frac{dy}{dx} = f(x, y) \quad \text{—} (1)$$

with initial condition $\boxed{y(x_0) = y_0}$

If $y(x)$ is exact sol. of eq (1) subject to given condition, Then Taylor's series for $y(x)$ around $x = x_0$ is given by,

$$y_1 = y(x) = y_0 + (x - x_0)\, y_0' + \frac{(x - x_0)^2}{2!} y_0'' + \dots \quad \text{—} (2)$$

Put $\boxed{x - x_0 = h}$ in (2)

Day:

Date:

$f(x, y) = e^x - y, \quad x_0 = 0, \quad y_0 = 0$

$y_{n+1} = y_0 + \int_{x_0}^{x} f(x, y_n)\, dx.$

$y_{n+1} = y_0 + \int_{x_0}^{x} e^x - y_n \, dx.$

$\boxed{\text{Put } n=0}$.

$y_1 = y_0 + \int_{x_0}^{x} e^x - y_0 \, dx. = e^x \Big]_0^x$

$= e^x - 1$

$y_1 = e^x - 1$

$\boxed{\text{Put } n=1.}$

$y_2 = y_0 + \int_{x_0}^{x} (e^x - y_1)\, dx,$

$= y_0 + \int_{x_0}^{x} (e^x - [e^x - 1])\, dx.$

$y_2 = 0 + \int_{0}^{x} 1 \, dx = x.$

$y_2 = x$

$\boxed{\text{Put } n=2}$

$y_3 = y_0 + \int_{x_0}^{x} (e^x - y_2)\, dx.$

$y_3 = e^x - \frac{x^2}{2} - 1$

By Taylor's series

$y_1 = y(x_1) = y_0 + h y_0' + \frac{h^2}{2!} y_0'' + \frac{h^3}{3!} y_0'''$

$y_1 = y(1.1) = 0 + (0.1)(1) + \frac{(0.1)^2}{2} 2 + \frac{(0.1)^3}{3!} 2 + \frac{(0.1)^4}{4!}(2) + \ldots$

$y_1 = 0.11034 \approx 0.1103$ upto 4 places.

Now, $x_1 = 1.1$, $y_1 = 0.1103$, $h = 0.1$.

$x_2 = x_1 + h = 1.2$

$y_1' = x_1 + y_1$, $= 1.1 + 0.1103 = 1.2103$

$y_1'' = 1 + y_1'$, $= 1 + 1.2103 = 2.2103$

$y_1''' = y_1''$, $= 2.2103$

$y^{IV} = y_1'''$, $= 2.2103$

$y_2 = y(1.2) = y_1 + h y_1' + \frac{h^2}{2!} y_1'' + \frac{h^3}{3!} y_1'''$

$= 0.1103 + (0.1)(1.2103) + \frac{(0.1)^2}{2}(2.2103) + \frac{(0.1)^3}{6}(2.2103) + \frac{(0.1)^4}{24}(2.2103) + \ldots$

$\boxed{y_2 = 0.24276 \approx 0.2428.}$

Date:

Day:

Picard Method:

$$y_{n+1} = y_0 + \int_{x_0}^{x} f(x, y_n)\,dx.$$

Q: Solve by Piccard upto third approximation.

$$\frac{dy}{dx} = x + y^2, \quad y(0) = 0 \qquad y(0.1) = ?$$

$$x_0 = 0, \quad y_0 = 0.$$

$$f(x, y) = x + y^2.$$

$\boxed{\text{Put } n = 0.}$

$$y_1 = y_0 + \int_{x_0}^{x} f(x, y_0)\,dx$$

$$= y_0 + \int_{x_0}^{x} (x + y_0^2\,dx).$$

$$y_1 = 0 + \int_{0}^{x} x + 0\,dx = \frac{x^2}{2}\Big]_0^x = \frac{x^2}{2}$$

$\boxed{\text{Put } n = 1.}$

$$y_2 = y_0 + \int_{x_0}^{x} f(x, y_1)$$

$$y_2 = 0 + \int_0^x (x + y_1^2)\,dx.$$

$$= 0 + \int_0^x x + \left(\frac{x^2}{2}\right)^2 dx$$

$$= 0 + \int_0^x x + \frac{x^4}{4}\,dx$$

Date:

Single step

EULER'S METHOD

In this method, we get approximate value of ordinary differential eq.

$\frac{dy}{dx} = f(x,y)$ with initial condition $y(x_0) = y_0$

$\boxed{y_{n+1} = y_n + h\, f(x_n, y_n)}$ $\quad n = 0, 1, 2 \ldots$

$y_n = y_{n-1} + h\, f(x_{n-1}, y_{n-1})$, $\quad n = 1, 2, 3, \ldots$

where $h = \frac{x_n - x_0}{n}$
interval size, $n \rightarrow$ no. of intervals.

* If we increase n, y value reaches more accuracy, but it will still be approximate.

$\boxed{x_n = x_0 + nh}$

Put $n = 1$

$y_1 = y_0 + h\, f(x_0, y_0)$

Put $n = 2$

$y_2 = y_1 + h(f(x_1, y_1))$

Que, $\frac{dy}{dx} = \frac{y - x}{y + x}$, $y(0) = 1$

Find y corresponding to $x = 0.1$

$x_0 = 0$, $y_0 = 1$, $f(x,y) = \frac{y - x}{y + x}$.

$x_n = 0.1$

L = {x | x has substring aba} Σ{a, b}

we cannot make partial decision, we have

to make final an...

$0 + \frac{0.2}{2}$, 1+

Date

Day

Runge Kutta method of 2nd order (Modified Euler)

R.K 1st order (Simple Euler)

~~RKS 2nd order~~

&

Euler Improved

$$y_{n+1} = y_n + \frac{h}{2}\{f(x_n, y_n) + f[x_n+h, y_n + hf(x_n, y_n)]\}$$

$$y_{n+1}^* = y_n + \frac{h}{2}\{f(x_n, y_n) + f[x_n+h, y_n + hf(x_n, y_n)]\}$$

For y_1 we use y_0 & x_0

Day.

Date.

$y_1 = y_0 + h y_0' + \frac{h^2}{2!} y_0'' + \ldots$

similarly Taylor series for $y(x)$ around $x = x_1$ is given by

For y_2 we use y_1 and x_1

$y_2 = y_1 + h y_1' + \frac{h^2}{2!} y_1'' + \ldots$

$\rightarrow \boxed{y_{n+1} = y_n + h y_n' + \frac{h^2}{2} y_n'' + \frac{h^3}{3!} y_n''' + \ldots}$

Solve $\frac{dy}{dx} = x + y$ by Taylor's series method, start from $\boxed{x = 1, y = 0}$ and carry to $x = 1.2$ with $h = 0.1$

need to find y_2

Given $\frac{dy}{dx} = x + y = f(x, y)$

$y(1) = 0$

x_0 y_0

initial values can be given in form of eq. we need to extract them.

$\boxed{x_0 = 1, \ y_0 = 0, \quad h = 0.1.}$

$\boxed{x_1 = x_0 + h} = 1 + 0.1 = 1.1.$

enough a y^{iv}

$y' = x + y$, $y_0' = 1 + 0 = 1$

$y'' = 1 + y'$, $y_0'' = 1 + 1 = 2.$

$y''' = y''$, $y_0''' = 2$

$y^{iv} = y'''$, $y_0^{iv} = 2$