CH #4.

Duality and Post-Optimal Analysis

The dual problem is defined systematically from the primal (or original) LP Model.

Rules for constructing the Dual Problem

primal Problem	Dual Problem		
objective	objective	constrains type	variable sign
Maximization	Minimization	$\geq$	unrestricted
Minimization	Maximization	$\leq$	unrestricted

Example 4.1-1

Primal LP Model.

$$\text{Max } z = 5x_1 + 12x_2 + 4x_3$$

s.t

$$x_1 + 2x_2 + x_3 \leq 10$$
$$2x_1 - x_2 + 3x_3 = 8$$
$$x_1, x_2, x_3 \geq 0.$$

Soln first we change into Eqn form: Dual variable

$$\text{Max } z = 5x_1 + 12x_2 + 4x_3$$

s.t

$$x_1 + 2x_2 + x_3 + x_4 = 10 \rightarrow y_1$$
$$2x_1 - x_2 + 3x_3 = 8 \rightarrow y_2$$

Dual Problem

$$\text{Min } z' = 10y_1 + 8y_2$$

subject to

$$y_1 + 2y_2 \geq 5$$
$$2y_1 + (-1)y_2 \geq 12$$
$$y_1 + 3y_2 \geq 4.$$
$$y_1 + 0y_2 \geq 0$$

y_1, y_2 unrestricted

$y_1 \geq 0$

So $y_2 \rightarrow$ unrestricted

$x_1 = 9$

$x_1 = 10 - 1$ (x_1^-, x_1^+)

$x_1 = (10) - (19)$ ($+ \text{nbo}$, $+ \text{nbs}$)

Ex 4.1-3 Primal LP Model

$$\text{Max } z = 5x_1 + 6x_2$$

s.t
$$x_1 + 2x_2 = 5$$
$$-x_1 + 5x_2 \geq 3$$
$$4x_1 + 7x_2 \leq 8$$

x_1 unrestricted, $x_2 \geq 0$.

Sol change into Eqn form

Due to x_1 is unrestricted we Replace it by $x_1 = x_1^- - x_1^+$

So

$$\text{Max } z = 5(x_1^- - x_1^+) + 6x_2$$

s.t
$$x_1^- - x_1^+ + 2x_2 = 5$$
$$-(x_1^- - x_1^+) + 5x_2 - x_3 = 3$$
$$4(x_1^- - x_1^+) + 7x_2 + x_4 = 8$$
$$x_1^-, x_1^+, x_2, x_3, x_4 \geq 0.$$

And.

$$\text{Max } z = 5x_1^- - 5x_1^+ + 6x_2$$

s.t
$$x_1^- - x_1^+ + 2x_2 = 5 \rightarrow y_1$$
$$-x_1^- + x_1^+ + 5x_2 - x_3 = 3 \rightarrow y_2$$
$$4x_1^- - 4x_1^+ + 7x_2 + x_4 = 8 \rightarrow y_3$$
$$x_1^-, x_1^+, x_2, x_3, x_4 \geq 0.$$

~~Primal~~ Dual Problem

$$\text{Min } z' = 5y_1 + 3y_2 + 8y_3$$

Subject to

$$y_1 - y_2 + 4y_3 \geq 5. \quad (1)$$
$$-y_1 + y_2 - 4y_3 \geq -5 \Rightarrow y_1 - y_2 + 4y_3 \leq 5 \quad (2)$$
$$2y_1 + 5y_2 + 7y_3 \geq 6$$
$$+(0)y_1 - y_2 + (0)y_3 \geq 0. \Rightarrow y_2 \leq 0$$
$$0 + 0 + y_3 \geq 0 \qquad y_3 \geq 0.$$

$y_1 \rightarrow$ unrestricted

$$\left.\begin{array}{l} x_1 + x_2 \geq 5 \\ x_1 + x_2 \leq 5 \end{array}\right\} \rightarrow x_1 + x_2 = 5$$

from Eq ① & ②. : And final Dual Problem.

$$\text{Min } z' = 5y_1 + 3y_2 + 8y_3.$$

subject to.

$$y_1 - y_2 + 4y_3 = 5.$$

$$2y_1 + 5y_2 + 7y_3 \geq 6$$

$$y_2 \leq 0, \quad y_3 \geq 0, \quad y_1 \rightarrow \text{unrestricted}.$$

Summary of the Rules for constructing the Dual.

Q4-2 Problem set 4.1A.

Q No 2/ So new ~~Optimal~~ Primal Problem Becomes

$$\text{Min } z = 15x_1 + 12x_2$$
$$x_1 + 2x_2 \geq 3$$
$$2x_1 - 4x_2 \leq 5$$
$$3x_1 + x_2 = 4$$
$$x_1, x_2 \geq 0.$$

Soln change into eqn form.

	Dual variable
$\text{Min } z = 15x_1 + 12x_2$	
$x_1 + 2x_2 - x_3 = 3.$	$\rightarrow y_1$
$2x_1 - 4x_2 + x_4 = 5$	$\rightarrow y_2$
$3x_1 + x_2 = 4.$	$\rightarrow y_3$
$x_1, x_2, x_3 \geq 0.$	

Dual Problem

$$\text{Max } z' = 3y_1 + 5y_2 + 4y_3.$$

s.t
$$y_1 + 2y_2 + 3y_3 \leq 15.$$
$$2y_1 - 4y_2 + y_3 \leq 12.$$
$$-y_1 + 0 + 0 \leq 0. \Rightarrow y_1 \geq 0.$$
$$0 + y_2 + 0 \leq 0 \Rightarrow y_2 \leq 0.$$

And y_3 is unrestricted.

4-4(b)

Q No 4/ b/ Primal Problem is

$$\text{Min } z = 6x_1 + 3x_2$$

s.t
$$6x_1 - 3x_2 + x_3 \geq 25$$
$$3x_1 + 4x_2 + x_3 \geq 55$$
$$x_1, x_2, x_3 \geq 0$$

Soln. change into eqn

$$\text{Min } z = 6x_1 + 3x_2.$$
$$6x_1 - 3x_2 + x_3 - x_4 = 25 \rightarrow y_1$$
$$3x_1 + 4x_2 + x_3 - x_5 = 55. \rightarrow y_2$$
$$x_i \geq 0 \quad i = 1, 2, 3, 4$$

Dual Problem

$$\text{Max } z' = 25y_1 + 55y_2$$

s.t $6y_1 + 3y_2 \le 6$

$-3y_1 + 4y_2 \le 3$

$y_1 + y_2 \le 0$

$-y_1 \le 0 \Rightarrow y_1 \ge 0$

4(c). $-y_2 \le 0 \Rightarrow y_2 \ge 0$

c) $\text{Max } z = x_1 + x_2$

s.t $2x_1 + x_2 = 5$

$3x_1 - x_2 = 6$

x_1, x_2 unrestricted.

Sol: Due to x_1, x_2 unrestricted we change them into $x_1 = x_1^- - x_1^+$, $x_2 = x_2^- - x_2^+$. where $x_1^-, x_1^+, x_2^-, x_2^+ \ge 0$.

So

$$\text{Max } z = x_1^- - x_1^+ + x_2^- - x_2^+$$

Dual variable

$2x_1^- - 2x_1^+ + x_2^- - x_2^+ = 5 \rightarrow y_1$

$3x_1^- - 3x_1^+ - x_2^- + x_2^+ = 6 \rightarrow y_2$

Dual Problem

~~24~~ $\text{Min } z' = 5y_1 + 6y_2$

s.t $2y_1 + 3y_2 \ge 1$ — (1)

$-2y_1 - 3y_2 \ge -1 \Rightarrow 2y_1 + 3y_2 \le 1$ — (2)

$y_1 - y_2 \ge 1$ — (3)

$-y_1 + y_2 \ge -1 \Rightarrow y_1 - y_2 \le 1$ — (4)

Eq (1), (2) And Eq (3), (4) show that.

(1)(2) $\Rightarrow$ $2y_1 + 3y_2 = 1$

(3)(4) $\Rightarrow$ $y_1 - y_2 = 1$.

y_1, y_2 unrestricted.

Optimal Dual Solution

The Primal and dual solutions are closely related, in the sense that the optimal solution of either Problem yields the optimal solution to the other.

Method 1

$$\begin{pmatrix}\text{optimal value of} \\ \text{dual variable } y_i\end{pmatrix} = \begin{pmatrix}\text{optimal primal z-coefficient of starting} \\ \text{variable } x_i \\ + \\ \text{orignal objective coefficient of } x_i\end{pmatrix}$$

Method 2

$$\begin{pmatrix}\text{optimal value of} \\ \text{dual variable } y_i\end{pmatrix} = \begin{pmatrix}\text{Row vectors of} \\ \text{orignal objective coefficient} \\ \text{of optimal primal basic variable}\end{pmatrix} \times \begin{pmatrix}\text{optimal} \\ \text{primal} \\ \text{inverse}\end{pmatrix}$$

~~Ex 4.2~~

4-10 Problem set 4.2.C

Q No 1) $\max z = 10x_1 + 4x_2 + 5x_3$

s.t. $5x_1 - 7x_2 + 3x_3 \geq 25$ → Here is 20

$x_1, x_2, x_3 \geq 0$

find the dual problem soln

Sol: change into eqn form

$\max z = 10x_1 + 4x_2 + 5x_3$

$5x_1 - 7x_2 + 3x_3 - x_4 = 25$ → y_1

$x_i \geq 0. \; i = 1, 2, 3, 4.$

Then we solve this primal problem.

Dual Problem $\min z' = 25y_1$

$5y_1 \geq 10 \Rightarrow y_1 \Rightarrow y_1 \geq 2$

$-7y_1 \geq 4. \qquad y_1 \leq \frac{-4}{7} = -0.57$

$3y_1 \geq 5. \qquad y_1 \geq \frac{5}{3} = 1.667$

$-y_1 \geq 0. \Rightarrow y_1 \leq 0 \qquad y_1 \leq 0.$

so

$\cancel{5y_1 \Rightarrow} \quad -\frac{4}{7} \leq y_1 \leq 0$ so $\min z' = 0.$

Q No 2) $\min z = 10x_1 + 4x_2 + 5x_3$

$5x_1 - 7x_2 + 3x_3 \geq 25$

$x_1, x_2, x_3 \geq 0$

Sol: change into eqn form.

$\min z = 10x_1 + 4x_2 + 5x_3$

$5x_1 - 7x_2 + 3x_3 - x_4 = 25$ → y_1

$x_i \geq 0 \; i = 1, 2, 3, 4,$

Dual Problem $\max z' = 25y_1$

$5y_1 \leq 10 \Rightarrow y_1 \leq 2.$

$-7y_1 \leq 4 \Rightarrow y_1 \geq \frac{-4}{7}$

$3y_1 \leq 5 \Rightarrow y_1 \leq \frac{5}{3}$

$-y_1 \le 0 \Rightarrow y_1 \ge 0.$

so Range of y_1 is $0 \le y_1 \le \frac{5}{3}$

At range of

$0 \le y_1 \le \frac{5}{3}$ satisfied All constrain

so Max $z' = 25\left(\frac{5}{3}\right) = \frac{125}{3}$

Q No 2) Soln $\boxed{4-11}$

convert the given problem into eqn form.

min $z = 50x_1 + 60x_2 + 30x_3$

s.t

$5x_1 + 5x_2 + 3x_3 - x_4 = 50 \rightarrow y_1$

$x_1 + x_2 - x_3 - x_5 = 20 \rightarrow y_2$

$7x_1 + 6x_2 - 9x_3 - x_6 = 30 \rightarrow y_3$

$5x_1 + 5x_2 + 5x_3 - x_7 = 35 \rightarrow y_4$

$2x_1 + 4x_2 - 15x_3 - x_8 = 10 \rightarrow y_5$

$12x_1 + 10x_2 - x_9 = 90 \rightarrow y_6$

$x_2 - 10x_3 - x_{10} = 20 \rightarrow y_7$

$x_1, x_2, x_3, \ldots x_{10} \ge 0.$

The dual problem

Max $z' = 50y_1 + 20y_2 + 30y_3 + 35y_4 + 10y_5 + 90y_6 + 20y_7.$

s.t $5y_1 + y_2 + 7y_3 + 5y_4 + 2y_5 + 12y_6 \le 50$

$5y_1 + y_2 + 6y_3 + 5y_4 + 4y_5 + 10y_6 + y_7 \le 60.$

$3y_1 - y_2 - 9y_3 + 5y_4 - 15y_5 - 10y_7 \le 30.$

$-y_1 \le 0 \Rightarrow y_1 \ge 0.$

similarly $y_1, y_2, \ldots y_7 \ge 0$

Q 4-13

Sol/ Soln : The Dual Problem is

$$\text{Max } z' = 30y_1 + 60y_2 + 40y_3$$

s.t

$$3y_1 + 4y_2 + y_2 \le 4$$
$$y_1 + 3y_2 + 2y_3 \le 1$$
$$y_1 \text{ is unrestricted}$$
$$-y_2 \le 0 \Rightarrow y_2 \ge 0$$
$$y_3 \le 0.$$

Soln of this dual problem is

Method 1

Starting Basic variable	x_4	x_5	x_6.
coefficient in optimal	-98.6.	-100	-0.2
coefficient in original	100.	100	0
Dual variable	y_1	y_2	y_3

Soln of Dual variable.

$$y_1 = -98.6 + 100 \Rightarrow y_1 = 1.4.$$
$$y_2 = -100 + 100 \Rightarrow y_2 = 0.$$
$$y_3 = -0.2 + 0 \Rightarrow y_3 = -0.2.$$
$$(y_1, y_2, y_3) = (1.4, 0, -0.2).$$

$$\text{max } z' = 30(1.4) + 60(0) + 40(-0.2).$$

$$\boxed{\text{max } z' = 34}$$

Method 2.

$$y_1, y_2, y_3 = (\text{coefi. of } x_1, \text{ coff of } x_2, \text{ coff of } x_3) \times (\text{inverse matrix.}$$

$$(y_1, y_2, y_3) = [4, 1, 0] \begin{bmatrix} 0.4 & 0 & -0.2 \\ -0.2 & 0 & 0.6 \\ 1 & -1 & 1 \end{bmatrix}$$

$$= [1.6 - 0.2 + 0 \quad 0 + 0 + 0, \quad -0.2 \times 4 + 0.6 + 0]$$

$$(y_1, y_2, y_3) = (1.4, 0, -0.2).$$

4—15

Q No 6) Soln The Associated Dual Problem is.

$$\text{Min } z' = 3y_1 + 4y_2$$

s.t
$$y_1 + 2y_2 \geq 1$$
$$2y_1 - y_2 \geq 5$$
$$y_1 + 0y_2 \geq 0 \Rightarrow y_1 \geq 0.$$
$$y_2 \text{ is unrestricted.}$$

Soln of this problem from optimal tableau of given

Method 1 (Direct).

$$y_1 = 0 + 3 \Rightarrow y_1 = 3.$$
$$y_2 = 99 - 100 \Rightarrow \cancel{y_2 = 99}.\ y_2 = -1$$
$$\text{Min } z' = 3(3) + 4(-1) \Rightarrow \boxed{z' = 5}$$

Method 2. Coef in starting z-eqn.

$$(y_1, y_2) = (x_3, x_1) \times \text{inverse Matrix.}$$

$$= (3, 1)\begin{bmatrix} 1 & -0.5 \\ 0 & 0.5 \end{bmatrix}$$

$$= [3+0 \quad -1.5+0.5] = [3 \quad -1]$$

$\boxed{y_1 = 3}$, $\boxed{y_2 = -1}$

$$z' = 3(3) + 4(-1)$$
$$= 5$$

Q No 7/ 4-16

$$\text{Max } z = x_1 + x_2$$

s.t

$$2x_1 + 3x_2 \le 12 \longrightarrow y_1$$
$$-3x_1 + 2x_2 \le -4 \longrightarrow y_2$$
$$3x_1 - 5x_2 \le 2 \longrightarrow y_3$$
$$x_1 \text{ unrestricted}, \quad x_2 \ge 0$$

Soln change into eqn form.

$$\text{Max } z = x_1 + x_2$$
$$2x_1 + 3x_2 + x_3 = 12$$
$$-3x_1 + 2x_2 + x_4 = -4$$
$$3x_1 - 5x_2 + x_5 = 2$$

into Matrix form.

$$\begin{array}{c} \\ z \\ x_3 \\ x_4 \\ x_5 \end{array} \begin{array}{c} \begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & x_5 & \text{R.H.V} \end{array} \\ \begin{bmatrix} -1 & -1 & 0 & 0 & 0 & 0 \\ 2 & 3 & 1 & 0 & 0 & 12 \\ -3 & 2 & 0 & 1 & 0 & -4 \\ 3 & -5 & 0 & 0 & 1 & 2 \end{bmatrix} \end{array}$$

$$R_1 + \frac{R_2}{3}, \quad \frac{R_2}{3}, \quad R_3 - 2\frac{R_2}{3}, \quad R_4 + 5\left(\frac{R_2}{3}\right)$$

$$\begin{array}{c} \\ z \\ x_2 \\ x_4 \\ x_5 \end{array} \begin{array}{c} \begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & x_5 & \text{R.H.V} \end{array} \\ \begin{bmatrix} -1/3 & 0 & 1/3 & 0 & 0 & 4 \\ 2/3 & 1 & 1/3 & 0 & 0 & 4 \\ -13/3 & 0 & -2/3 & 1 & 0 & -12 \\ 19/3 & 0 & 5/3 & 0 & 1 & 22 \end{bmatrix} \end{array}$$

$$R_1 + \frac{1}{3}\left(\frac{3}{19}R_4\right), \quad R_2 - \frac{2}{3}\left(\frac{3}{19}R_4\right), \quad R_3 + \frac{13}{3}\left(\frac{3}{19}R_4\right), \quad \frac{3}{19}R_4$$

optimal

x_1	x_2	x_3	x_4	x_5	R.H.V
0	0	8/19	0	1/19	98/19
0	1	3/19	0	-2/19	32/19
0	0	9/19	1	13/19	58/19
1	0	5/19	0	3/19	66/19

This is the optimal tableau of primal problem.

Now find the optimal soln of dual problem.

The dual problem is.

$$\text{Min } z' = 12y_1 - 4y_2 + 2y_3$$

s.t

$$2y_1 - 3y_2 + 3y_3 \geq 1$$
$$3y_1 + 2y_2 - 5y_3 \geq 1.$$
$$y_1 \geq 0.$$
$$y_2 \geq 0 \quad , \quad y_3 \geq 0.$$

soln from optimal tableau of primal problem

Method 1.

y_1 — Starting basic variable

coeff in optimal table of z-row

coeff in original table in z-row

Dual variable

x_3	x_4	x_5
8/19	0	1/19
0	0	0.
y_1	y_2	y_3.

so $y_1 = 8/19 + 0 \Rightarrow \boxed{y_1 = 8/19}$

$y_2 = 0 + 0 \Rightarrow \boxed{y_2 = 0}$

$\boxed{y_3 = 1/19}$

$$\text{Min } z' = 12\left(\frac{8}{19}\right) - 4(0) + 2\left(\frac{1}{19}\right).$$

$$\boxed{\text{Min } z' = \frac{98}{19}} \Rightarrow \text{optimal.}$$

in original z-eqn.

method 2

(y_1, y_2, y_3) = coeff of x_2, coeff of x_4, coeff of x_4, x_1) × inverse matrix

$$= [1, 0, 1] \begin{bmatrix} 3/19 & 0 & -2/19 \\ 9/19 & 1 & 13/19 \\ 5/19 & 0 & 3/19 \end{bmatrix}$$

$$= \left[\frac{3}{19} + 0 + \frac{5}{19}, \; 0 + 0 + 0, \; \frac{-2}{19} + 0 + \frac{3}{19}\right].$$

$$(y_1, y_2, y_3) = \left(\frac{8}{19}, \; 0, \; \frac{1}{19}\right)$$

4.2.4 Simplex tableau computations

Formula 1: constraint column computations.

in Any simplex iteration, a left hand or Right hand side column is computed as follow.

$$\begin{pmatrix}\text{Constraint column}\\ \text{in iteration } i\end{pmatrix} = \begin{pmatrix}\text{inverse in}\\ \text{iteration } i\end{pmatrix} \times \begin{pmatrix}\text{original}\\ \text{constraint column}\end{pmatrix}$$

Formula 2. objective z-row computations:

The objective eqn coefficient (reduced cost) of x_j is computed as.

$$\begin{pmatrix}\text{Primal z-eqn}\\ \text{coeff of variable } x_j\end{pmatrix} = \begin{pmatrix}\text{L.H.side of } j^{th}\\ \text{dual constraint}\end{pmatrix} - \begin{pmatrix}\text{R.H. side of}\\ j^{th} \text{ dual constraint}\end{pmatrix}$$

Q4-20 Ex 4.2 D

Q No 2) The dual Problem is

$$\text{Min } z' = 21y_1 + 21y_2$$

s.t $2y_1 + 7y_2 \geq 4$ — ①

$7y_1 + 2y_2 \geq 14$ — ②

$y_1 \geq 0$, — ③ $y_2 \geq 0$. — ④

4-20(a)

a) Basic variable $= (x_2, x_4)$, inverse $= \begin{pmatrix}\frac{1}{7} & 0\\ -\frac{2}{7} & 1\end{pmatrix}$

means R.H.side of that's constraint

$$(x_2, x_4) = (\text{inverse}) \times (\text{original constraint calum})$$

$$\begin{bmatrix}x_2\\ x_4\end{bmatrix} = \begin{bmatrix}\frac{1}{7} & 0\\ -\frac{2}{7} & 1\end{bmatrix}\begin{bmatrix}21\\ 21\end{bmatrix}$$

$$\begin{bmatrix}x_2\\ x_4\end{bmatrix} = \begin{bmatrix}3+0\\ -6+21\end{bmatrix} \Rightarrow \begin{bmatrix}x_2\\ x_4\end{bmatrix} = \begin{bmatrix}3\\ 15\end{bmatrix}$$

CamScanner

Both are ≥ 0 so they are feasible.

For optimal.

if the optimal value of primal and dual are same. then the soln is optimal.

Primal. [redacted]

$$\text{Max } z = 4x_1 + 14x_2$$

Basis. (given). $(x_2, x_4) = (3, 15)$

$$\text{max } z = 4(0) + 14(3) \Rightarrow \boxed{z = 42}$$

Dual.

First compute y_1, y_2.

$(y_1, y_2) = ($Coefficient of (x_2, x_4) in orignal ~~of~~ z-eqn$)$ $\times$ inverse.

$$= (14, 0) \times \begin{bmatrix} 1/7 & 0 \\ -2/7 & 1 \end{bmatrix}$$

$$(y_1, y_2) = (2+0 \quad 0+0) \Rightarrow y_1 = 2, \; y_2 = 0$$

so

$$\text{Min } z' = 21(2) + 21(0) \Rightarrow \boxed{z' = 42}$$

$z = z'$ so the given value are optimal tableau value.

Basic variable $= (x_2, x_1)$, inverse $= \begin{pmatrix} \frac{7}{45} & -\frac{2}{45} \\ -\frac{2}{45} & \frac{7}{45} \end{pmatrix}$.

$$\begin{bmatrix} x_2 \\ x_1 \end{bmatrix} = \begin{bmatrix} 7/45 & -2/45 \\ -2/45 & 7/45 \end{bmatrix} \begin{bmatrix} 21 \\ 21 \end{bmatrix}.$$

$$= \begin{bmatrix} \frac{49}{15} - \frac{14}{15} \\ -\frac{14}{15} + \frac{49}{15} \end{bmatrix} = \begin{bmatrix} 7/3 \\ 7/3 \end{bmatrix} \begin{matrix} > 0 \\ > 0. \end{matrix}$$

The

Basic variable $(x_2, x_1) = \left(\frac{7}{3}, \frac{7}{3}\right)$ are ≥ 0 so soln is feasible.

For optimality.

Because problem is to Maximize.

optimality mean there is no non-negative coefficient of non-basic variable in z-eqn.

"First we compute value of dual variables".

$(y_1, y_2) = (\text{Coefficient of } (x_2, x_1) \text{ in original z-eqn}) \times \text{inverse}$.

$$= [14, 4] \begin{bmatrix} 7/45 & -2/45 \\ -2/45 & 7/45 \end{bmatrix}$$

$$= \left[\frac{98}{45} - \frac{8}{45} \quad -\frac{28}{45} + \frac{28}{45}\right]$$

$[y_1, y_2] = [2 \quad 0] \Rightarrow \boxed{y_1 = 2}, \boxed{y_2 = 0}$

Now compute coefficient of non basic coefficient (x_3, x_4)

Coeff of x_3 in z-eqn = L.H.S of 3rd constraint of dual - R.H.S

$= y_1 - 0$

Cofficient of $x_3 = 2 - 0 = 2 > 0$.

similarly

coefficient of $x_4 = y_2 - 0$

$= 0 - 0 = 0 \geq 0$.

"Both non-basic $(x_3, x_4) = (2, 0)$ have non-negative coefficient in z-eqn."

So the soln is optimal

$\text{Max } z = 4x_1 + 14x_2$

$= 4\left(\frac{7}{3}\right) + 14\left(\frac{7}{3}\right)$

$\boxed{z = 42}$

$\text{Min } z' = 21y_1 + 21y_2$

$z' = 21(2) + 21(0)$

$\boxed{z' = 42}$

optimal soln = 42.

Q4-21

Q No 3 Max $z = 3x_1 + 2x_2 + 5x_3$ — Dual variable

subject to $x_1 + 2x_2 + x_3 + x_4 = 30$ — y_1

$3x_1 + 2x_3 + x_5 = 60$ — y_2

$x_1 + 4x_2 + x_6 = 20$ — y_3

$x_1, x_2, x_3 \ldots x_6 \geq 0$

check the optimality and feasibility of following basic solutions

a) Basic variable = x_4, x_3, x_6

$$\text{inverse} = \begin{pmatrix} 1 & -1/2 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

if the basic variable x_4, x_3, x_6 have ≥ 0 they are feasible

constraint column

original constraint column.

$[x_4, x_3, x_6]$ = inverse × R.H.S of original problem.

$$\begin{bmatrix} x_4 \\ x_3 \\ x_6 \end{bmatrix} = \begin{bmatrix} 1 & -1/2 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 30 \\ 60 \\ 20 \end{bmatrix}$$

$$\begin{bmatrix} x_4 \\ x_3 \\ x_6 \end{bmatrix} = \begin{bmatrix} 30 - 30 + 0 \\ 0 + 30 + 0 \\ 0 + 0 + 20 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 \\ 30 \\ 20 \end{bmatrix}$$

All basic variable $(x_4, x_3, x_6) = (0, 30, 20)$ have non-negative variable ≥ 0. so it's feasible soln.

For optimality.

First of All we compute dual variable.

Dual Problem.

$$\text{Min } z' = 30y_1 + 60y_2 + 20y_3.$$

s.t

$$y_1 + 3y_2 + y_3 \geq 3 \quad \text{—— (1)}$$
$$2y_1 + 0y_2 + 4y_3 \geq 2 \quad \text{—— (2)}$$
$$y_1 + 2y_2 + y_3 \geq 5 \quad \text{—— (3)}$$
$$y_1 \geq 0 \quad \text{—— (4)}$$
$$y_2 \geq 0 \quad \text{—— (5)}$$
$$y_3 \geq 0 \quad \text{—— (6)}$$

For dual variable calculation.

(y_1, y_2, y_3) = Coef of (x_4, x_3, x_6) in original z-eqn × inverse.

$$= [0, 5, 0] \times \begin{bmatrix} 1 & -1/2 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \left[0 + 0 + 0 \quad 0 + \frac{5}{2} + 0 \quad 0 + 0 + 0\right]$$

$(y_1, y_2, y_3) = [2 \quad \frac{5}{2} \quad 0]$.

Now compute non basic (x_1, x_4, x_5) of primal Problem to check the optimality.

if non-basic (x_1, x_2, x_5) have non-negative (due to Maximization) value in z-row then given soln values provide the optimal soln.

Coeff of x_1 = L.H.Side of dual 1st constrain − R.H.S of dual 1st constrain

$= y_1 + 3y_2 + y_3 - 3$

coeff of $x_1 = 2 + 3(\frac{5}{2}) + 0 - 3 = \frac{9}{2} > 0.$

similarly

~~coeff of $x_2 = 2y_1 + 2y_2 + 4y_3 - 2$~~

~~$= 2(2) + 2(\frac{5}{2}) + 4(0) - 2 = 7 - 2 = 5 > 0.$~~

~~coeff of x~~

coeff of $x_2 = 2y_1 + 4y_3 - 2$

$= 2(0) + 4(0) - 2 = -2 < 0.$

so the soln is ~~infeasible~~ not optimal.

4.4. ADDITIONAL SIMPLEX ALGORITHM

4.4.1 Dual simplex Algorithm

The dual simplex Algorithm starts with a better than optimal and infeasible basic soln.

Dual feasibility condition

The leaving variable "x_γ" is the basic variable having the most negative value [ties and broken Arbitrary]. if all the basic variable is non-negative the Algorithm is end.

Dual optimality condition

$$\min \left\{ \left| \frac{\bar{c}_j}{\alpha_{\gamma j}} \right| , \alpha_{\gamma j} < 0 \right\}.$$

Read out from book.

"To start LP optimal and infeasible, two op condition must be met:

1) The objective function must satisfy the optimality condition of the regular simplex method.
2) All the constrain must be of type ($\leq$)

"inequalities of the type ($\geq$) are converted to ($\leq$) by multiplying both sides of the inequality by (-1). if LP includes ($=$) constraints, the eqn can be replaced by two inequalities."

For example $x_1 + x_2 = 1$

then $x_1 + x_2 \geq 1$, $x_1 + x_2 \leq 1$

and $-x_1 - x_2 \leq -1$, $x_1 + x_2 \leq 1$.

then solve the problem.

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DNO 2 b Solve by Dual simplex iteration

$$\text{Min } z = 5x_1 + 6x_2$$

s.t

$$x_1 + x_2 \geq 20$$
$$4x_1 + x_2 \geq 40$$
$$x_1, x_2 \geq 0$$

Soln The inequalities must be ($\leq$) in this form so multiply each constraint by -1.

$$\text{Min } z = 5x_1 + 6x_2$$
$$-x_1 - x_2 \leq -20$$
$$-4x_1 - x_2 \leq -40$$
$$x_1, x_2 \geq 0 \quad \cancel{-x_1, -x_2 \leq 0.} \text{ Not this}$$

then change into eqn form.

$$\text{Min } z = 5x_1 + 6x_2$$
$$-x_1 - x_2 + x_3 = -20$$
$$-4x_1 - x_2 + x_4 = -40$$
$$x_3, x_4 \geq 0, \quad -x_1, -x_2 \leq 0.$$

into matrix form.

$$\begin{array}{c|ccccc} & x_1 & x_2 & x_3 & x_4 & R.H.V \\ \hline z & -5 & -6 & 0 & 0 & 0 \\ x_3 & -1 & -1 & 1 & 0 & -20 \\ x_4 & -4 & -1 & 0 & 1 & -40 \end{array}$$

The soln is optimal but infeasible so apply dual simplex method.

So x_1 is the entering variable

And x_4 is Leaving variable

$$\frac{R_3}{-4}, \quad R_2 + \left(\frac{R_3}{-4}\right), \quad R_1 + 5\left(\frac{R_3}{-4}\right)$$

1.

	x_1	x_2	x_3	x_4	R.H.V
z	0	$-19/4$	0	$-5/4$	50
x_3	0	$-3/4$	1	$-1/4$	-10
x_1	1	$1/4$	0	$-1/4$	10
Ratio		$\left\lvert\frac{-19/4}{-3/4}\right\rvert = \frac{19}{3}$		$\left\lvert\frac{-5/4}{-1/4}\right\rvert = 5$	

Soln is still optimal and infeasible.

So $x_4 \to$ Entering variable, $x_3 \to$ Leaving.

$\frac{-4}{1}(R_2)$, $R_1 + \frac{5}{4}(-4R_2)$, $R_3 + \frac{1}{4}(-4R_2) \Rightarrow R_3 - R_2$.

	x_1	x_2	x_3	x_4	R.H.V
z	0	-1	-5	0	100
x_4	0	3	-4	1	40
x_1	1	1	-1	0	20

so this tablau is optimal and also feasible

Hence The optimal value is

$\boxed{x_1 = 20,\ x_2 = 0}$ $\boxed{z = 100}$