Name: ~~[illegible]~~ Talha Dogar

Roll No: 21L-7559

Section: 6K

Date: 20-2-24

ASSIGNMENT #1

QNo: 1 (a) Assume that x_1 denotes units of bookshelf and x_2 denotes units of coffee table.

$x_1 =$ units of bookshelf

$x_2 =$ units of coffee table.

$$2x_1 + x_2 \leq 6$$

$$7x_1 + 8x_2 \leq 28$$

$$x_1, x_2 \geq 0$$

Objective Equ:

Max: $z = 120x_1 + 80x_2$

Associate Equations:

$$2x_1 + x_2 = 6 \quad \text{— (i)}$$

$$7x_1 + 8x_2 = 28 \quad \text{— (ii)}$$

from (i) put $x_1 = 0$

$2(0) + x_2 = 6$

$x_2 = 6$ $(0,6)$

put $x_2 = 0$

$2x_1 + 0 = 6$

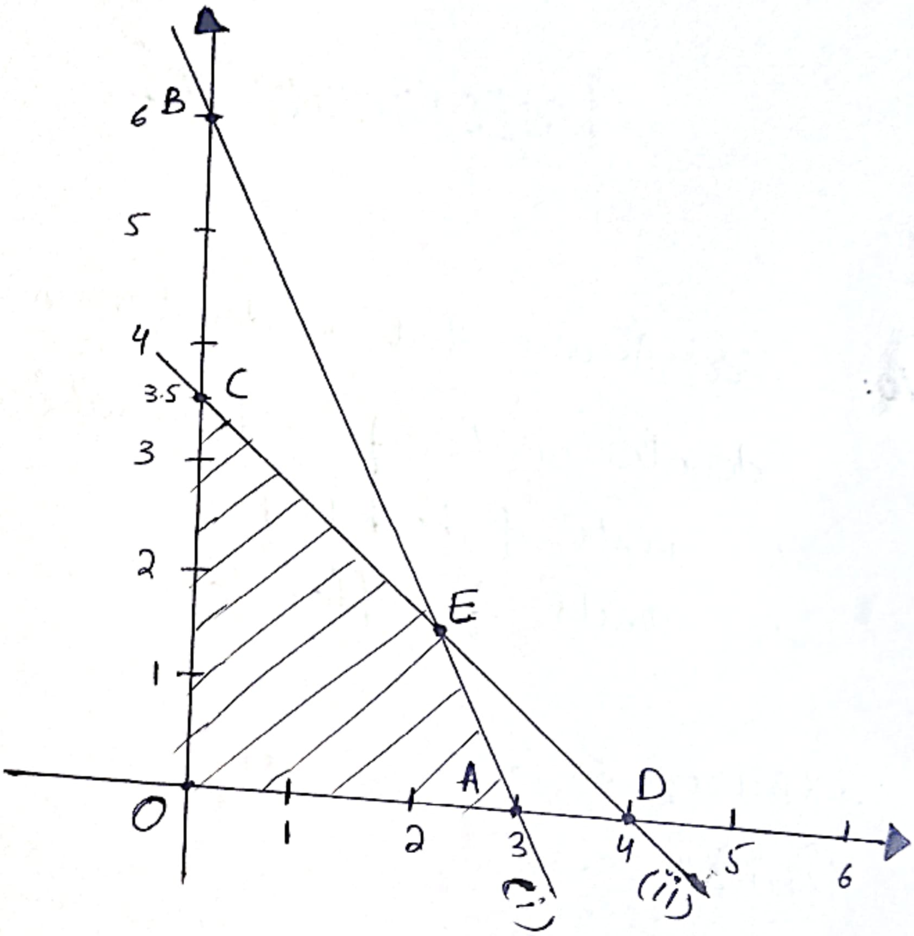
$2x_1 = 6$

$x_1 = 3$ $(3,0)$

from equ (ii)

Put $x_1 = 0$	Put $x_2 = 0$
$7(0) + 8x_2 = 28$	$7x_1 + 8(0) = 28$
$8x_2 = 28$	$7x_1 = 28$
$x_2 = 3.5$ $(0, 3.5)$	$x_1 = 4$ $(4, 0)$

Graph:



Testing At origin O(0,0):

~~for equ (i)~~ $2x_1 + x_2 \leq 6$

$2(0) + 0 \leq 6$

$0 \leq 6$

True

(Towards origin)

$7x_1 + 8x_2 \leq 28$

$7(0) + 8(0) \leq 28$

$0 \leq 28$

True

(Towards origin)

Shaded region in graph is feasible solution.

Corner points:

O(0,0)
A(3,0)
B(0,6)
C(0,3.5)
D(4,0)
E(x_1, x_2) = ?

To find E(a,b):

8(i) − (ii)

$$16x_1 + 8x_2 = 48$$
$$\pm 7x_1 \pm 8x_2 = \pm 28$$
$$9x_1 = 20$$
$$x_1 = 20/9$$

Put 'x_1' in (i)

$$2\left(\frac{20}{9}\right) + x_2 = 6$$
$$x_2 = 6 - \frac{40}{9}$$
$$x_2 = \frac{54-40}{9} = \frac{14}{9}$$
$$E(x_1, x_2) = \left(\frac{20}{9}, 14/9\right)$$

Now,

Corner Point	Objective Soln.
(0,0)	$z = 120(0) + 80(0) = 0$
(3,0)	$z = 120(3) + 80(0) = 360$
(0,3.5)	$z = 120(0) + 80(3.5) = 280$
(20/9, 14/9)	$z = 120(20/9) + 80\left(\frac{14}{9}\right) = 391.11$

Max. amount can be generated ~~of~~ of 391.11 \$ by producing $\frac{20}{9}$ units of bookshelves and $\frac{24}{9}$ units of coffee table.

QNO:1 (b)

$$9x + 5y \geq 500$$
$$7x + 9y \geq 300$$
$$5x + 3y \leq 1500$$
$$7x + 9y \leq 1900$$
$$2x + 4y \leq 1000$$
$$x, y \geq 0$$

Objective Equ:

Max: $Z = 1170x + 1110y$

$9x + 5y = 500$ — (i)

$7x + 9y = 300$ — (ii)

$5x + 3y = 1500$ — (iii)

$7x + 9y = 1900$ — (iv)

$2x + 4y = 1000$ — (v)

from (i)

Put $x = 0$

$9(0) + 5y = 500$

$5y = 500$

$y = 100$

$(0, 100)$

Put $y = 0$

$9x + 5(0) = 500$

$9x = 500$

$x = \frac{500}{9} = 55.5$

$(55.5, 0)$

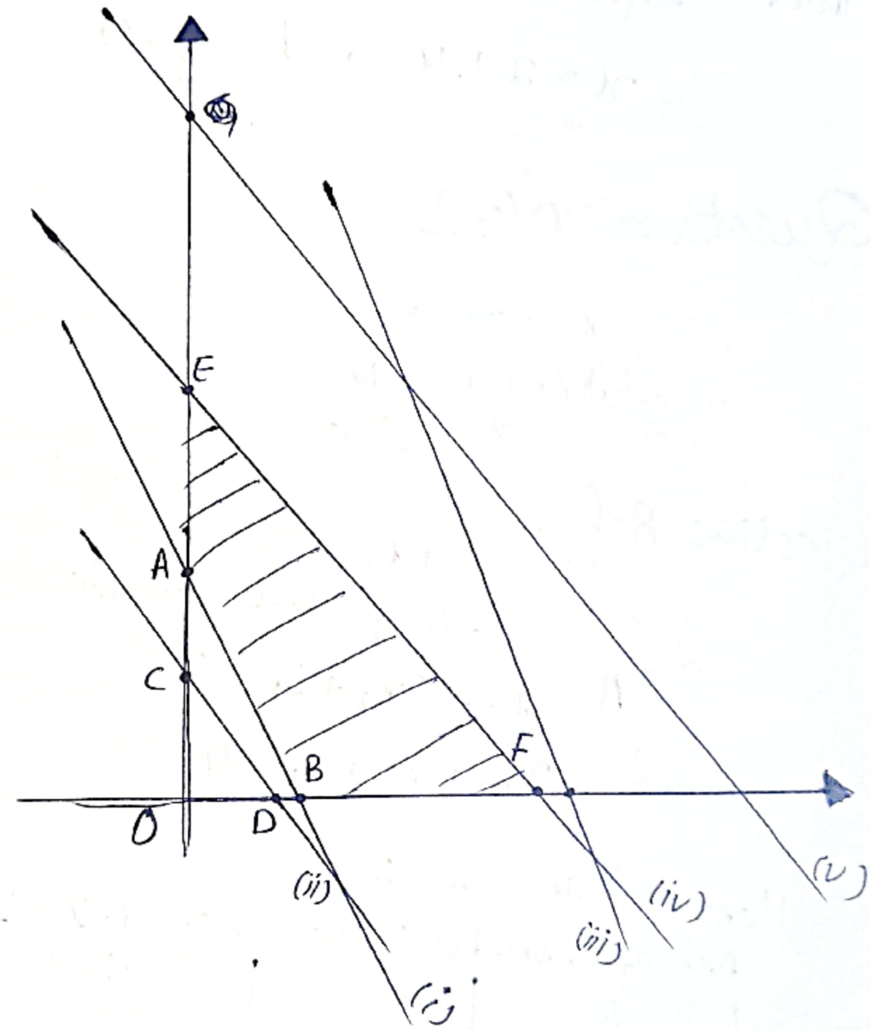
Similarly:

(ii): $7x+9y=300$ $(42.8, 0)$, $(0, 33.3)$

$5x+3y=1500$ $(300, 0)$, $(0, 500)$

$7x+9y=1900$ $(271.4, 0)$, $(0, 211.1)$

$2x+4y=1000$ $(500, 0)$, $(0, 250)$



Shaded region bounded by points:

B: $(55.5, 0)$

A: $(0, 100)$

F: $(271.4, 0)$

E: $(0, 211.1)$

Corner Points:	Objective Function:
$(55.5, 0)$	$z = 1170(55.5) + 1110(0) = 6493$
$(0, 100)$	$z = 1170(0) + 1110(100) = 111000$
$(271.4, 0)$	$z = 1170(271.4) + 1110(0) = 317538$
$(0, 211.1)$	$z = 1170(0) + 1110(211.1) = 234321$

Thus equ (z) maximize to value 317538 at $x = 271.4$ and $y = 0$.

Question No: 2

$$x_1 + 5x_2 \leq 5$$
$$2x_1 + x_2 \leq 4$$
$$x_1, x_2 \geq 0$$

Objective Soln:

$$Z = x_1 + x_2$$

Adding slack variables:

$$x_1 + 5x_2 + S_1 = 5$$
$$2x_1 + x_2 + S_1 = 4$$

Here $n = 4$, $m = 2$

No. of non-basic variables $= 4 - 2 = 2$

Non-basic variables	Basic Variables	Basic Soln.	Feasible	Objective Soln.
(x_1, x_2)	(S_1, S_2)	$(5, 4)$	Yes	0
(x_1, S_1)	(x_2, S_2)	$(1, 3)$	Yes	1
(x_2, S_1)	(x_1, S_2)	$(5, -6)$	No	—
(x_1, S_2)	(x_2, S_1)	$(4, -15)$	No	—
(x_2, S_2)	(x_1, S_1)	$(2, 3)$	Yes	2
(S_1, S_2)	(x_1, x_2)	$(5/3, 2/3)$	Yes	2.33

Optimal soln is 2.33 when $x = 5/3$, $y = 2/3$

Question No: 3
(Excel Solver)

$$45A + 60B + 40C + 40D + 45E + 30F \geq 70$$

$$20A + 30B + 40C + 40D + 45E + 30F \geq 100$$

$$50A + 30B + 20C + 25D + 50E + 20F \geq 20$$

$$4A + 9B + 11C + 10D + 9E + 10F \geq 20$$

$$A, B, C, D, E, F \geq 0$$

Min: $z = 2A + 3B + 5C + 6D + 8E + 8F$

(Solve in excel solver file)

Question No: 4 (Graphically)

$$6x_1 + 9x_2 \leq 100$$
$$2x_1 + x_2 \leq 20$$
$$x_1, x_2 \geq 0$$

Objective Equ:

$z = 2x_1 + 3x_2$

Associated Equations:

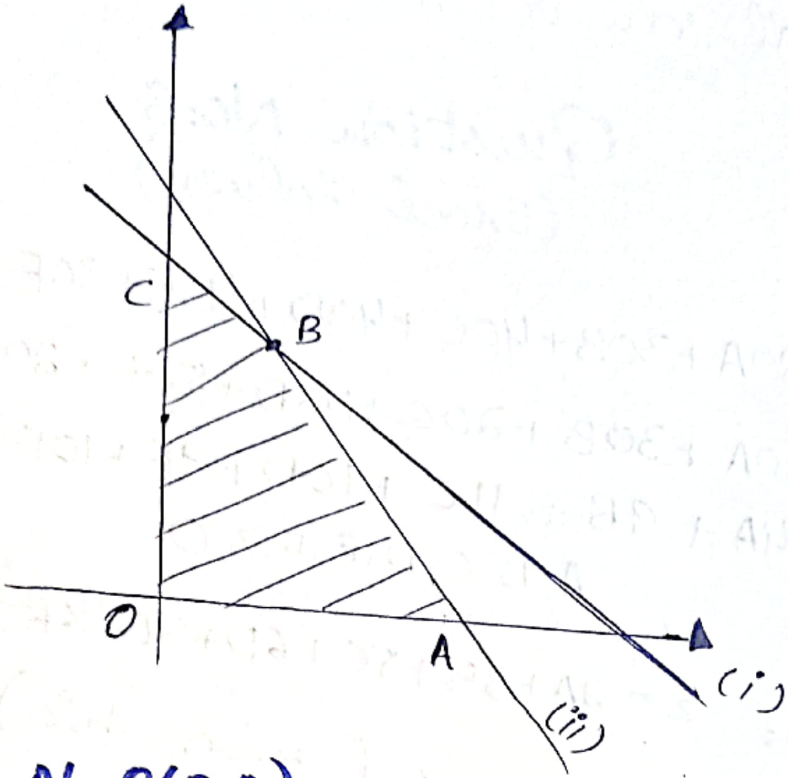
$6x_1 + 9x_2 = 100$ — (i)

$2x_1 + x_2 = 20$ — (ii)

(i): $6x_1 + 9x_2 = 100$ $(16.6, 0)$, $(0, 11.1)$

(ii): $2x_1 + x_2 = 20$ $(10, 0)$, $(0, 20)$

Graph:



Testing At O(0,0)

$6(0)+9(0) \leq 100$

$0 \leq 100$

True

Towards origin

$2(0)+0 \leq 20$

$0 \leq 20$

True

Towards origin

Corner Points:

O: (0,0)

A(10,0)

C(0,11.1)

B(x,y)

(i) − 9(ii)

$$6x_1 + 9x_2 = 100$$
$$\pm 18x_1 \pm 9x_2 = \pm 180$$
$$-12x_1 = -80$$
$$x_1 = \frac{80}{12} = \frac{20}{3}$$

Put in (iii)

$2(\frac{20}{3}) + x_2 = 20$

$x_2 = 20 - \frac{40}{3}$

$x_2 = \frac{60-40}{3} = 20/3$

Corner Points	Objective function:
(0,0)	$z = 2(0) + 3(0) = 0$
(10,0)	$z = 2(10) + 3(0) = 20$
(0,11.1)	$z = 2(0) + 3(11.1) = 33.3$
(20/3, 20/3)	$z = 2(20/3) + 3(20/3) = 33.3$

Hence, optimal solution = 33.3 can be achieved in two ways.
(0,11.1) and (20/3, 20/3)

QNO:4 (Algebraically)

By simplex Method:

Adding slack variables:

$6x_1 + 9x_2 + S_1 = 100$
$2x_1 + x_2 + S_2 = 20$
$x_1, x_2, S_1, S_2 \geq 0$

$z = 2x_1 + 3x_2$
$z - 2x_1 - 3x_2 = 0$

$\downarrow$ incoming

Basic	x_1	x_2	s_1	s_2	Solution	
Z	-2	-3	0	0	0	
exiting $\rightarrow$ s_1	6	9	1	0	100	$100/9 = 11.1$
s_2	2	1	0	1	20	$20/1 = 20$

$$\frac{1}{9}R_2 \;;\quad R_1 + 3\left(\frac{1}{9}R_2\right) \;;\quad R_3 - \left(\frac{1}{9}R_2\right)$$

Basic	x_1	x_2	s_1	s_2	Solution	
Z	0	0	$1/3$	0	$100/3$	
x_2	$2/3$	1	$1/9$	0	$100/9$	$= \frac{50}{3} = 16.6$
s_2	$4/3$	0	$-1/9$	1	$80/9$	$= \frac{20}{3}$

. Max. value $100/3 = 33.3$ is acquired at $x_2 = 100/9$ and $S_2 = 80/9$.

Put these values in (ii)

$$2x_1 + \frac{100}{9} + 80/9 = 20$$

$$2x_1 = 0$$
$$x_1 = 0 \quad \text{satisfy equ.}$$

So, Max. value of $z = 100/3$ can also achieved for points $x_1 = 0$, $x_2 = \frac{100}{9}$ which is shown below:

$$3/4 R_3 \;;\quad R_2 - \frac{2}{3}\left(3/4 R_2\right)$$

Basic	x_1	x_2	S_1	S_2	solution
z	0	0	$1/3$	0	$100/3$
x_2	0	1	$1/6$	$-1/2$	$20/3$
x_1	1	0	$-1/12$	$3/4$	$10/3$

Question No: 5
(a)
M-technique:

Objective:
Max: $z = 6x_1 + 4x_2$

$$x_1 + x_2 \leq 5$$
$$x_2 \geq 8$$
$$x_1, x_2 \geq 0$$

$$x_1 + x_2 + S_1 = 5$$
$$x_2 - S_2 = 8$$
$$x_1, x_2, S_1, S_2 \geq 0$$

Artificial variables:
Max $z = 6x_1 + 4x_2 - MR_1$
Subject to:
$$x_1 + x_2 + S_1 = 5$$
$$x_2 - S_2 + R_1 = 8$$
$$x_1, x_2, S_1, S_2, R_1 \geq 0$$

Non-basic variables $= n - m = 5 - 2 = 3$

Let $M = 100$

Basic	x_1	x_2 (incoming)	S_1	S_2	R_1	Solution
z	-6	-4	0	0	100	0
exit $\leftarrow S_1$	1	1	1	0	0	5
R_1	0	1	0	-1	1	8

Row_1 $100Row_3$

-6	-104	0	100	0	-800	
1	1	1	0	0	5	$5/1 = 5$
0	1	0	-1	1	8	$8/1 = 8$

$R_1 + 104R_2$; $R_3 - 104R_2$

z	98	0	104	100	0	-280
x_2	1	1	1	0	0	5
R_1	-1	0	-1	-1	1	3

As there is no negative value in z row but value of R_1 (artificial value) is still +ve so it is not feasible soln.

QNO: 5 (b)

$$\text{Max}: \; z = 5x_1 + 12x_2 + 4x_3$$

$$x_1 + 2x_2 + x_3 = 10$$

$$2x_1 - x_2 + x_4 = 8$$

$$x_1, x_2, x_3, x_4 \geq 0$$

$$z - 5x_1 - 12x_2 - 4x_3 = 0$$

Basic	x_1	x_2	x_3	x_4	Soln.
z	-5	-12	-4	0	0
x_3	1	2	1	0	10
x_4	2	-1	0	1	8

$R_1 + 4R_2$

x_2 (incoming)

z	-1	-4	0	0	40	
exit x_3	1	2	1	0	10	$10/2 = 5$
x_4	2	-1	0	1	8	$8/-1 = -8$

$R_2/2$; $R_1 + 4\left(\frac{R_2}{2}\right)$; $R_3 + \frac{R_2}{2}$

z	1	0	2	0	60
x_2	$1/2$	1	$1/2$	0	5
x_4	$5/2$	0	$1/2$	1	13

Max solution $z = 60$ at $x_2 = 5$, $x_4 = 13$, $x_1 = 0$, $x_3 = 0$.

Two Phase Method:

Max: $z = 6x_1 + 4x_2$

$x_1 + x_2 \leq 5$

$x_2 \geq 8$

$x_1, x_2 \geq 0$

$x_1 + x_2 + S_1 = 5$

$x_2 - S_2 + R_1 = 8$

$x_1, x_2, S_1, S_2, R_1 \geq 0$

Minimize z'. $z' - R_1 = 0$

Basic	x_1	x_2	S_1	S_2	A_1	Sol.
z	0	[illegible]	0	0	-1	0
S_1	1	1	1	0	0	5
R_1	0	1	0	-1	1	8

R_{out} Row$_3$

incoming x_2

z	0	1	0	-1	0	8	
exit ← S_1	1	1	1	0	0	5	$5/1 = 5$
R_1	0	1	0	-1	1	8	$8/1 = 8$

$R_1 - R_2$; $R_3 - R_2$

z	-1	0	-1	-1	0	3
x_2	1	1	1	0	0	5
R_1	-1	0	-1	-1	1	3

As value of z is +ve the solution is infeasible.