1-3-23 Wednesday

Lecture # 11

Multiplication rule for dependent events

$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

If eik event se baki events affect ho rahay hon.

$$\boxed{P(A \cap B) = P(A) \cdot P(B/A)}$$

A and B

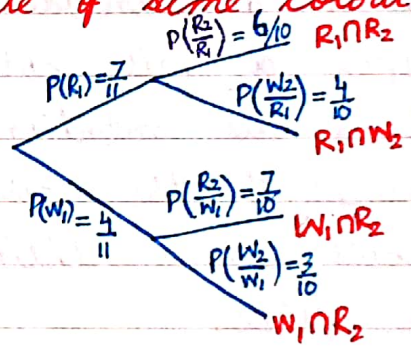
Q. Suppose a jar contains 7 red balls and 4 white balls. 2 balls are selected without replacement, what is the probability that i) both are red ii) first white and second red iii) both are of same colour?

R_1 = First red ball

R_2 = Second red ball

W_1 = First white ball

W_2 = Second white ball



i) $P(R_1 \cap R_2) = P(R_1) \cdot P(R_2/R_1)$

$$= \frac{7}{11} \cdot \frac{6}{10}$$

$$= \frac{42}{110}$$

ii) $P(W_1 \cap R_2) = P(W_1) \cdot P(R_2/W_1)$

$$= \frac{4}{11} \cdot \frac{7}{10} = \frac{28}{110}$$

iii) $P(R_1 \cap R_2)$ or $P(W_1 \cap W_2) = P(R_1) \cdot P\left(\frac{R_2}{R_1}\right) + P(W_1) \cdot P\left(\frac{W_2}{W_1}\right)$

$$= \frac{7}{11} \cdot \frac{6}{10} + \frac{4}{11} \cdot \frac{3}{10}$$

$$= \frac{54}{110}$$

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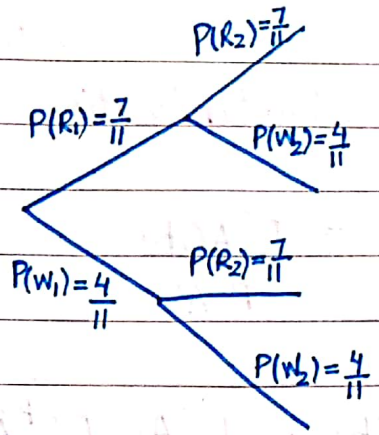
Date:

Multiplication rule for independent events :



$$\boxed{P(A \cap B) = P(A).P(B)}$$

Probability that second ball is red:

$$P(R_2) = P(W_1 \cap R_2) \text{ OR } P(R_1 \cap R_2)$$
$$= P(W_1)P(R_2) + P(R_1).P(R_2)$$
$$= \frac{4}{11}.\frac{7}{11} + \frac{7}{11}.\frac{7}{11} = \frac{7}{11}$$

Q. In a carnival game a contestent has to first spin a fair coin and then roll a dice whose faces are mentioned 1 to 6. The contestent wins a prize if the coin shows head and the dice shows score below 3. Find probability that contestent wins a prize.

$$P(A \cap B) = P(A).P(B)$$

A: coin shows head

B: dice shows below 3

$$P(A \cap B) = \frac{1}{2}.\frac{2}{6} = \frac{1}{6}$$

Bayes theorem OR Bayes rule

⇒ conditional probability ko fuse ke kay further conditional probability bnayeen gy

$$\boxed{P(A/B) = \frac{P(A \cap B)}{P(B)}}$$

sum of probabilities

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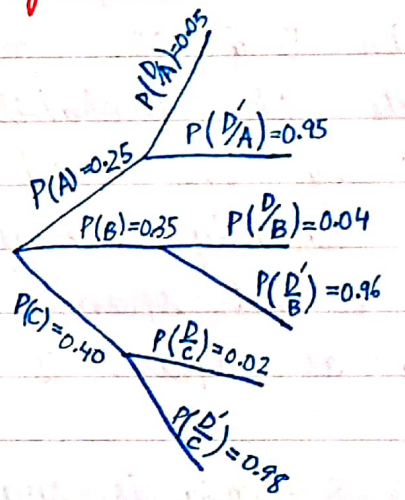
Q. In a bolt factory machines A, B and C manufactures 25%, 35% and 40% of total outcomes respectively. Of their outputs 5%, 4% and 2% respectively are defective. A bolt is selected at random and found to be defective. What's the probability that it came from C.



D: bolt is defective

$P(D/A) = 0.05$

$P(D/B) = 0.04$

$P(D/C) = 0.02$

$$P(C/D) = \frac{P(C \cap D)}{P(D)}$$

$$P(D) = P(D \cap A) + P(D \cap B) + P(D \cap C)$$

$$= P(A).P\left(\frac{D}{A}\right) + P(B).P\left(\frac{D}{B}\right) + P(C).P\left(\frac{D}{C}\right)$$

$$= 0.25 \times 0.05 + 0.35 \times 0.04 + 0.40 \times 0.02$$

$$P\left(\frac{C}{D}\right) = \frac{P(C \cap D)}{P(D)}$$

$$= \frac{P(C)\,P(D/C)}{P(D \cap A) + P(D \cap B) + P(D \cap C)}$$

$$= \frac{0.40 \times 0.02}{0.03}$$

$$= 0.26$$

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Date:

Application of Bays theorem:

Baysian span filter:

Q. Suppose that we have found that the word 'Rolex' occurs in 250 of 2000 msgs known to be spam and in 5 of 1000 msgs known not to be spam. Estimate the probability that an incoming msg containing the word 'Rolex' is spam assuming that it is equally likely that an incoming msg is spam or not spam. If our threshold for rejecting a msg as spam is 0.9, will we reject such msgs

S = msg is spam
S' = msg is not spam
r = rolex word occurs
r' = rolex word does not occur

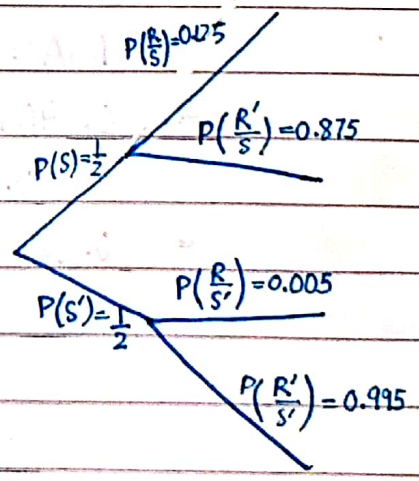
$$P(S/R) = \frac{P(S \cap R)}{P(R)}$$

$$P(R) = P(R \cap S) + P(R \cap S')$$

$$= P(S).P\left(\frac{R}{S}\right) + P(S').P\left(\frac{R}{S'}\right)$$

$$= \frac{1}{2}.0.125 + \frac{1}{2} \times 0.005$$

$$= 0.065$$



$$P(S/R) = \frac{P(S).P(R/S)}{P(S \cap R) + P(S' \cap R)}$$

$$= \frac{1/2 \times 0.125}{0.065} = 0.962$$ we reject such messages as spam

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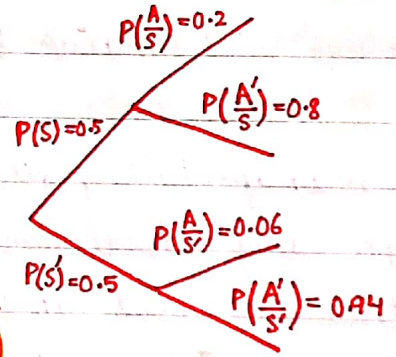
6-3-23 Monday

Lecture # 12

Example :

Suppose that we train a Bayesian spam filter on a set of 2000 spam messages and 1000 messages that are not spam. The word "stock" appears in 400 spam messages and 60 messages that are not spam. And the word "undervalued" appears in 200 spam messages and 25 messages that are not spam. Estimate the probability that an incoming message containing both the words "stock" and "undervalued" is spam, assuming that we have no prior knowledge about whether it is spam. Will we reject such messages as spam when we set the threshold at 0.9?



A = word is stock

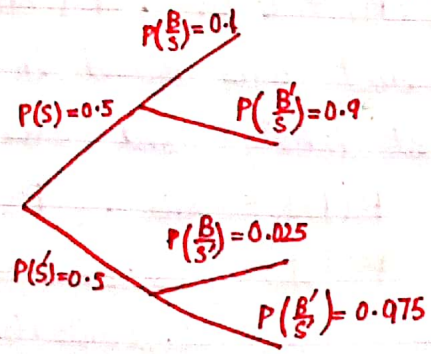
B = word is undervalued

we have to find

$$P\left(\frac{S}{A\cap B}\right) = \frac{P(S).P\left(\frac{A}{S}\right).P\left(\frac{B}{S}\right)}{P(S).P\left(\frac{A}{S}\right).P\left(\frac{B}{S}\right)+P(S')P\left(\frac{B}{S'}\right)P\left(\frac{B}{S'}\right)}$$

$$= \frac{0.5\times0.2\times0.1}{(0.5\times0.2\times0.1)+(0.5\times0.06\times0.025)}$$

$$= 0.93 > 0.9$$



we reject such messages as spam.

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Date:

Q. Two cards are drawn in succession without replacement from an ordinary deck of cards. Find probability that an event $A_1 \cap A_2 \cap A_3$ occurs where A_1 = first card is ace red, A_2 = second card is 10 or Jack, A_3 = third card is >3 but <7.

$$P(A_1 \cap A_2 \cap A_3) = P(A_1)\, P(A_2/A_1)\, P(A_3/A_1 \cap A_2)$$

$$= \frac{2}{52} \times \frac{8}{51} \times \frac{12}{50} = \frac{8}{5525}$$

Q. The prob that a man will be alive in 25 year is 3/5 and the probability that his wife will be alive in 25 years is 2/3. Find the probability that:

i) Both will be alive $P(A \cap B) = P(A)\,P(B) = 3/5 \times 2/3 = 2/5$

ii) Only man will be alive $P(A \cap \bar{B}) = 3/5 \times (1 - 2/3) = 1/5$

iii) Only wife will be alive $P(\bar{A} \cap B) = (1 - 3/5)(2/3) = 4/15$

iv) Atleast 1 will be alive $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 3/5 + 2/3 - 2/5 = 13/15$

v) No one will be alive $P(\bar{A} \cup \bar{B}) = 1 - P(A \cup B) = 1 - 13/15 = 2/15$

Q. A manufacturing firm employs three analytical plans for the design and development of a particular product. For cost reasons, all three are used at varying times. In fact, plan 1, 2 and 3 are used for 30%, 20% and 50% of the products respectively. The defect rate is different for the 3 plans and are as follows:

$P(D/P_1) = 0.01$, $P(D/P_2) = 0.03$, $P(D/P_3) = 0.02$. If a random product was found to be defected, which one of the plans is most likely used and responsible?

$P(P_1) = 3/10$, $P(P_2) = 2/10$, $P(P_3) = 5/10$

$$P(P_1/D) = \frac{0.3 \times 0.01}{0.2 \times 0.03 + 0.3 \times 0.01 + 0.5 \times 0.02} = 0.158$$

$$P(P_2/D) = \frac{0.3 \times 0.01}{0.3 \times 0.01 + 0.3 \times 0.01 + 0.5 \times 0.02} = 0.316$$

$$P(P_3/D) = \frac{0.5 \times 0.02}{0.3 \times 0.01 + 0.3 \times 0.01 + 0.5 \times 0.02} = 0.526$$

Practise: 2.81, 2.83, 2.87, 2.75, 2.76, 2.94

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8-3-23

Lecture # 13
Chapter # 3

Wednesday

Random variable

$SS = \{ \underset{2}{HH}, \underset{1}{HT}, \underset{1}{TH}, \underset{0}{TT} \}$

x : no of heads
x : 0, 1, 2

x	f(x)
0	1/4
1	2/4
2	1/4
	1

$f(x) \geq 0$
$\Sigma f(x) = 1$
$P(x = x) = f(x)$

Eg 3.8 A shipment of 20 similar lappies to a retail outlet contains 3 that are defective. If a school makes a random purchase of 2 of these lappies, find the probability distribution for the number of defectives.

Total	Good	Defective
20	17	3

x : No of defective
x : 0, 1, 2

$$P(x=0) = \frac{\binom{3}{0}\binom{17}{2}}{\binom{20}{2}}$$

Probability function ← $$P(x=x) = \frac{\binom{3}{x}\binom{17}{2-x}}{\binom{20}{2}} = f(x)$$

x	f(x) or P(x)	
0	68/95	P(x=0)
1	51/190	P(x=1)
2	3/190	P(x=2)
	1	

$f(x) \geq 0$
$\Sigma f(x) = 1$
$P(x = x) = f(x)$

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Date:

S.S $= \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

X : No of tails

X : 0, 1, 2, 3

X	f(x)
0	1/8
1	3/8
2	3/8
3	1/8
	1

$$\frac{\binom{3}{x}}{8}$$

Discrete probability function

OR

Probability mass function

$f(x) = P(X = x)$

(exact wohi probability)

Commulative distribution function

OR

Distribution function

$\Rightarrow$ $F(x) = P(x \leq x)$

(uhan tk ki probability ka sum)

$f(x) = \frac{1}{16}\binom{4}{x}$ for $x = 0, 1, 2, 3, 4$

x	$f(x) = 1/16 \binom{4}{x}$	F(x)
0	1/16	1/16
1	4/16	1/16 + 4/16 = 5/16
2	6/16	5/16 + 6/16 = 11/16
3	4/16	11/16 + 4/16 = 15/16
4	1/16	15/16 + 1/16 = 1

$f(2) = F(2) - F(1)$

$= \frac{11}{16} - \frac{5}{16}$

$= \frac{6}{16}$

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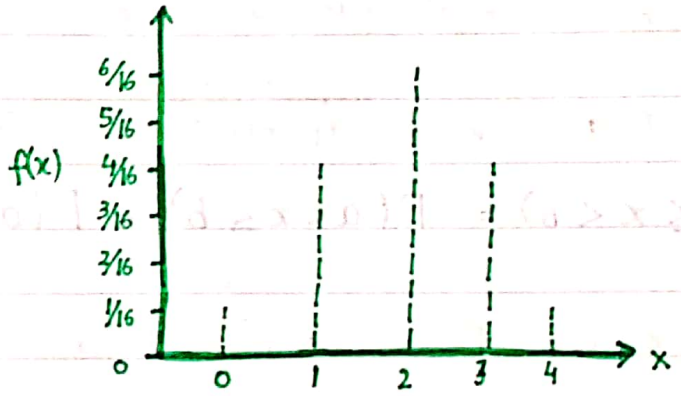
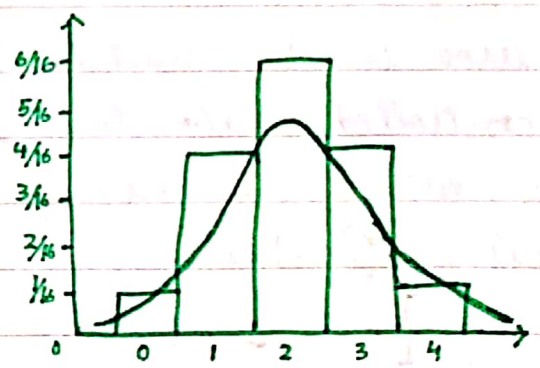
$$F(x) = \begin{cases} 0 & \text{for } x<0 \\ 1/16 & \text{for } 0 \le x < 1 \\ 5/16 & \text{for } 1 \le x < 2 \\ 11/16 & \text{for } 2 \le x < 3 \\ 15/16 & \text{for } 3 \le x < 4 \\ 1 & \text{for } x \geq 4 \end{cases}$$



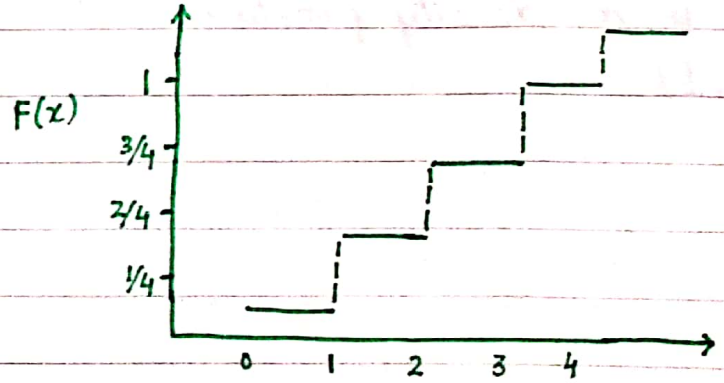
Probability mass function

Practise Qs:

3.1, 3.2, 3.22, 3.10



Probability histogram



Discrete commulative distribution function

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Date: 13-3-23 Monday

Lecture # 14

Density probability function :

The function $f(x)$ is a probability density function for continuous random variable x if

1. $f(x) \geq 0$
2. $\int_{-\infty}^{\infty} f(x)\,dx = 1$
3. • $P(a \leq x \leq b) = \int_a^b f(x)dx$ * jab pdf (probability density function) kay through probability nikalein

   • $P(a \leq x \leq b) = F(b) - F(a)$ * jab cdf (cumulative distributive function) kay through probability nikalein

* $P(a < x < b) = P(a \leq x \leq b) = P(a < x \leq b) = P(a \leq x \leq b)$

Pg 89 Exp 3.11

Q. Suppose that the error in the reaction temperature in Centigrade for a controlled laboratory experiment a continuous random variable X having the probability density function $f(x) = \begin{cases} \frac{x^2}{3} & -1 < x < 2 \\ 0 & \text{elsewhere} \end{cases}$

a) verify that $f(x)$ is a density function * integral should be = 1

b) find $P(0 \leq x \leq 1)$

a) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$= \int_{-1}^{2} \frac{x^2}{3}\,dx = 1$

$= \frac{1}{3}\left|\frac{x^3}{3}\right|_{-1}^{2}$

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$$= \frac{1}{3}\left[\frac{2^3}{3} + \frac{1}{3}\right]$$

$$= \frac{9}{9} = 1$$

b) $\int_0^1 \frac{x^2}{3}\,dx$

$$= \frac{1}{9}\left|x^3\right|_0^1$$

$$= \frac{1}{9}[1-0]$$

$$= \frac{1}{9}$$

Commulative distributive function :

$$F(x) = P(x \le x) = \int_{-\infty}^{x} f(x)\,dx$$ lower limit to x

* For previous question

$$= \int_{-1}^{x} \frac{x^2}{3}\,dx = \frac{1}{3}\left[\frac{x^3}{3}\right]_{-1}^{x}$$

$$= \left|\frac{x^3}{9}\right|_{-1}^{x}$$

$$= \frac{1}{9}[x^3+1]$$

$$F(x) = \frac{x^3+1}{9}$$

$P(0 \le x \le 1)$ using $F(x)$:

$$P(a \le x \le b) = F(b) - F(a)$$

$$P(0 \le x \le 1) = \frac{1+1}{9} - \frac{1}{9}$$

$$= \frac{1}{9}$$

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Date:

Question 3.30 :

$$f(x) = \begin{cases} k(3-x^2) & -1<x<1 \\ 0 & \text{elsewhere} \end{cases}$$

a) determine k

b) find F(x)

c) find $p(x < \frac{1}{2})$ using F(x)

a) $\int_{-1}^{1} k(3-x^2)\, dx = 1$

$k\int_{-1}^{1} 3\, dx - k\int_{-1}^{1} x^2\, dx = 1$

$3k\,|x|_{-1}^{1} - k\left|\frac{x^3}{3}\right|_{-1}^{1} = 1$

$6k - \frac{2}{3}k = 1$

$\frac{16}{3}k = 1$

$k = \frac{3}{16}$

c) using p.d.f

$p(x < \frac{1}{2})$

$\int_{-1}^{1/2} \frac{3}{16}(3-x^2)\,dx$

b) $F(x) = \int_{-1}^{x} \frac{3}{16}(3-x^2)\,dx$

$= \frac{3}{16}\int_{-1}^{x} (3-x^2)\,dx$

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$$= \frac{3}{16}\left[3x - \frac{x^3}{3}\right]_{-1}^{x}$$

$$= \frac{3}{16}\left[\frac{9x - x^3}{3} - \left(-3 - \left(\frac{-1}{3}\right)\right)\right]$$

$$= \frac{1}{16}\left(-x^3 + 9x + 8\right)$$

$$= \frac{-x^3}{16} + \frac{9x}{16} + \frac{1}{2}$$

Q 3.36 On a laboratory assignment if the equipment is working, the density function of the observed outcome X is $f(x) = 2(1-x) \quad 0 < x < 1$

$0 \quad$ otherwise

a) $P(x \leq 1/3)$

b) $P(x > 0.5)$

c) Find distribution function and use it to evaluate part 3

a) $P(x \leq 0.33) = \int_{-\infty}^{+\infty} fx\,dx$

$$= \int_{0}^{0.33} fx\,dx$$

$$= \int_{0}^{0.33} 2(1-x)\,dx$$

$$= 2\left[x - \frac{x^2}{2}\right]_{0}^{0.33}$$

$$= 2\left[0.33 - \frac{(0.33)^2}{2}\right]$$

$$= 0.55 = \frac{5}{9}$$

b) $P(x > 0.5) = \int_{-\infty}^{+\infty} fx\,dx$

$$= \int_{0.5}^{1} 2(1-x)\,dx$$

$$= \frac{1}{4} = 0.25$$

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Probability using cdf

- $P(a<x<b) = F(b)-F(a)$
- $P(x>b) = 1-F(b)$
- $P(x \le b) = F(b)$

Date:

c) $F(x) = \int_{-\infty}^{\infty} f(x)\,dx$

$= \int_0^x 2(1-x)\,dx$

$= 2\left[x - \frac{x^2}{2}\right]_0^x$

$= \cancel{2}\left(\frac{2x-x^2}{\cancel{2}}\right)$

$= 2x - x^2$

$F(0.5) = 2(0.5) - (0.5)^2 = 0.75$

$1 - F(0.5) = 1 - 0.75 = 0.25$

Q. Consider the density function $f(x) = K\sqrt{x}$ $0<x<1$
0 otherwise

a) Evaluate 'k'

b) Find $F(x)$ & use it to evaluate $P(0.3 < x < 0.6)$

a)

$f(x) = k\sqrt{x}$

$k\int_0^1 \sqrt{x}\,dx = 1 = \frac{2k}{3}\left[x^{3/2}\right]_0^1$

$= \frac{2k}{3} = 1$

$\Rightarrow k = \frac{3}{2}$

b)

For $0 \le x < 1$, $F(x) = \frac{3}{2}\int_0^x \sqrt{t}\,dt$

$= \left[t^{3/2}\right]_0^x$

$= x^{3/2}$

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Now,

$$F(x) = \begin{cases} 0 & x<0 \\ x^{3/2} & 0 \le x < 1 \\ 1 & x \ge 1 \end{cases}$$

$$P(0.3 < X < 0.6) = F(0.6) - F(0.3)$$
$$= (0.6)^{3/2} - (0.3)^{3/2}$$
$$= 0.3004$$

Q. An investment firm offers its customers muncipal bonds that mature after varying no. of offers given that the cdf of T, the no. of years of maturity for a randomly selected bond

$$F(t) = \begin{cases} 0 & t<1 \\ 1/4 & 1 \le t < 3 \\ 1/2 & 3 \le t < 5 \\ 3/4 & 5 \le t < 7 \\ 1 & t \ge 7 \end{cases}$$

Find

a) $P(T=5)$ *hint $F(5)-F(4)$

b) $P(T>3)$ *hint $1-F(3)$

c) $P(1.4 < T < 6)$

d) $P(T \le 5 / T \ge 2)$

a) $P(T=5) = F(5) - F(4)$
$= \frac{3}{4} - \frac{1}{2}$
$= \frac{1}{4}$

b) $P(T>3) = 1 - F(3)$
$= 1 - \frac{1}{2} = \frac{1}{2}$

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Date:

c) $P(1.4 < T < 6) = F(6) - F(1.4)$

$= \frac{3}{4} - \frac{1}{4}$

$= \frac{1}{2}$

d) $\frac{P(2 \leq T \leq 5)}{P(T \geq 2)} = \frac{F(5) - F(2)}{1 - F(2)}$

$= \frac{3/4 - 1/4}{1 - 1/4}$

$= \frac{2}{3}$

Q 3.27 The time of failure in hrs of an imp piece of equipment used in a manufacturer DVD player has the density function

$$f(x) = \begin{cases} \frac{1}{2000} \exp\left(\frac{-x}{2000}\right) & x \geq 0 \\ 0 & x < 0 \end{cases}$$

a) Find $F(x)$ (cdf)

b) determine the probability that the component last more than 1000 hrs before the component needs to be replaced.

c) determine the probability that the component fails before 2000 hrs.

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20-3-23 Monday

P.d.f

Q. $f(x) = \begin{cases} 2x & 0<x<1 \\ 0 & \text{otherwise} \end{cases}$

$P\left[x \le \frac{1}{2} / \frac{1}{3} \le x \le \frac{2}{3}\right]$

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{P[x \le \frac{1}{2} \cap \frac{1}{3} \le x \le \frac{2}{3}]}{P(\frac{1}{3} \le x \le \frac{2}{3})}$$

$$= \frac{P(\frac{1}{3} \le x \le \frac{1}{2})}{P(\frac{1}{3} \le x \le \frac{2}{3})}$$

$$= \frac{\int_{1/3}^{1/2} 2x \, dx}{\int_{1/3}^{2/3} 2x \, dx}$$

$$= \frac{5/36}{1/3}$$

$$= \frac{5}{12}$$

Q. $F(x) = \begin{cases} 0 & \text{for } x<0 \\ \frac{2x^2}{5} & \text{for } 0<x\le 1 \\ -\frac{3}{5} + \frac{2}{5}\left(3x - \frac{x^2}{2}\right) & 1<x\le 2 \\ & x>2 \end{cases}$

Find $\overset{f(x)}{\text{pdf}}$ and $P(|x| < 1.5)$

$$f(x) = \frac{d}{dx} F(x)$$

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Q.

$$f(x) = \begin{cases} 0 & \text{for } x<0 \\ \frac{4x}{5} & \text{for } 0<x\leq 1 \\ \frac{2}{5}(3-x) & \text{for } 1<x\leq 2 \\ 0 & \text{for } x>2 \end{cases}$$

Solution:

$$P(-1.5 \leq x \leq 1.5) = \int_{-1.5}^{0} 0\,dx + \int_{0}^{1} \frac{4x}{5}\,dx + \int_{1}^{1.5} \frac{2}{5}(3-x)\,dx$$

$$= \frac{4}{5}\left|\frac{x^2}{2}\right|_0^1 + \left|\frac{2}{5}\left(3x-\frac{x^2}{2}\right)\right|_1^{1.5}$$

$$= 0.75$$

Joint Probability Distribution

- $f(x, y) = P(x = x \text{ and } y = y)$ ← Joint probability function
- $F(x, y) = P(x \leq x \text{ and } y \leq y)$ ← Joint commulative distribution function

- $f(x_i, y_j) \geq 0$

$i = 1, 2, \ldots, m$
$j = 1, 2, \ldots, n$

- $\sum_i \sum_j f(x_i, y_j) = 1$

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Joint probability distribution of x and y

$x \backslash y$	y_1	y_2	$\dots$	y_j	$\dots$	y_n	$P(x=x_i)$
x_1	$f(x_1,y_1)$	$f(x_1,y_2)$	$\dots$	$f(x_1,y_j)$	$\dots$	$f(x_1,y_n)$	$g(x_1)$
x_2	$f(x_2,y_1)$	$f(x_2,y_2)$	$\dots$	$f(x_2,y_j)$	$\dots$	$f(x_2,y_n)$	$g(x_2)$
$\vdots$	$\vdots$	$\vdots$		$\vdots$		$\vdots$	$\vdots$
x_i	$f(x_i,y_1)$	$f(x_i,y_2)$	$\dots$	$f(x_i,y_j)$	$\dots$	$f(x_i,y_n)$	$g(x_i)$
$\vdots$	$\vdots$	$\vdots$		$\vdots$		$\vdots$	$\vdots$
x_m	$f(x_m,y_1)$	$f(x_m,y_2)$	$\dots$	$f(x_m,y_j)$	$\dots$	$f(x_m,y_n)$	$g(x_m)$
$P(y=y_i)$	$h(y_1)$	$h(y_2)$	$\dots$	$h(y_j)$	$\dots$	$h(y_n)$	1

Marginal function of x:

$$g(x_i) = \sum_{j=1}^{n} f(x_i, y_j)$$

$$= f(x_i, y_1) + f(x_i, y_2) + \dots + f(x_i, y_n)$$

$$g(x_i) = f(x = x_i)$$

Marginal function of y:

$$h(y_j) = \sum_{i=1}^{m} f(x_i, y_j)$$

$$= f(x_1, y_j) + f(x_2, y_j) + \dots + f(x_m, y_j)$$

$$h(y_j) = f(y = y_j)$$

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Date:

Exp 3.14: 2 ball pens are selected at random from a box that contains 3 blue pens, 2 red pens and 3 green pens. If X is a number of blue pens and Y is number of red pens selected, find a) joint probability function $f(x,y)$, b) $P(x+y \le 1)$, c) marginal probability distribution of x and y, d) The conditional probability distribution of $f(x/1)$, e) verify whether x and y are independent or not.

a)

B	R	G		$SS = \binom{8}{2}$
3	2	3	= 8	

x = no of blue pens

y = no of red pens

x : 0, 1, 2

y : 0, 1, 2

$$f(x,y) = \frac{\binom{3}{x}\binom{2}{y}\binom{3}{2-x-y}}{\binom{8}{2}}$$

x \ y	0	1	2	g(x)
0	3/28	6/28	1/28	10/28
1	9/28	6/28	0	15/28
2	3/28	0	0	3/28
h(y)	15/28	12/28	1/28	

b) $P(x+y \le 1) = f(0,0) + f(0,1) + f(1,0)$

$$= \frac{3}{28} + \frac{6}{28} + \frac{9}{28} = \frac{18}{28}$$

c)

x	g(x)
0	10/28
1	15/28
2	3/28
	1

y	h(y)
0	15/28
1	12/28
2	1/28

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d)

conditional probability distribution:

$$f(x_i/y_j) = P\left(\frac{x=x_i}{y=y_j}\right)$$

$$= \frac{P(x=x_i \text{ and } y=y_j)}{P(y=y_j)}$$

$$= \frac{f(x_i, y_j)}{h(y_j)}$$

$$f(y_j/x_i) = P\left(\frac{y=y_j}{x=x_i}\right)$$

$$= \frac{P(y=y_j \text{ and } x=x_i)}{P(x=x_i)}$$

$$= \frac{f(x_i, y_j)}{g(x_i)}$$

we have to find:

$$f(x/1) = P\left(\frac{x=x}{y=1}\right)$$

$$= \frac{P(x=x \text{ and } y=1)}{P(y=1)}$$

$$= \frac{f(x,1)}{h(1)}$$

$$h(1) = f(0,1) + f(1,1) + f(2,1)$$

$$= 6/28 + 6/28 + 0 = 12/28$$

$$f(0/1) = \frac{f(0,1)}{h(1)} = \frac{6/28}{12/28} = \frac{1}{2}$$

$$f(1/1) = \frac{f(1,1)}{h(1)} = \frac{6/28}{12/28} = \frac{1}{2}$$

$$f(2/1) = \frac{f(2,1)}{h(1)} = 0$$

X	$f(x/1)$
0	1/2
1	1/2
2	0
	1

e) as $f(0,1) \neq g(0)h(1) \Rightarrow \frac{3}{14} \neq \frac{5}{14} \cdot \frac{3}{7}$ so events are dependent

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Date: 22-3-23 Wednesday

Q. $P(x > x) = e^{-\lambda x}$ $x > 0$

i) Find cdf of x

ii) Find $P(T < x < 2T)$ where $T = \frac{1}{\lambda}$

Sol:

i) $F(x) = P(x \le x) = 1 - P(x > x)$

$= 1 - e^{-\lambda x}$

$P(a \le x \le b) = F(b) - F(a)$

$P(T \le x \le 2T) = 1 - e^{-\lambda(2T)} - [1 - e^{-\lambda(T)}]$

$= 1 - e^{-\lambda(2\frac{1}{\lambda})} - (1 - e^{-\lambda(1/\lambda)})$

$= 0.233$

Continuous bivariate probability density function

$f(x, y) \ge 0$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\, dy\, dx = 1$$

$$P(a \le x \le b,\ c \le y \le d) = \int_a^b\int_c^d f(x,y)\, dy\, dx$$

$$F(x,y) = \int_{-\infty}^{x}\int_{-\infty}^{y} f(x,y)\, dy\, dx$$

Margina function of x

$$g(x) = \int_{-\infty}^{\infty} f(x,y)\, dy$$

$$h(y) = \int_{-\infty}^{\infty} f(x,y)\, dx$$

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Conditional pdf of x given y

$$f\left(\frac{x}{y}\right) = \frac{f(x,y)}{h(y)}$$

$$f\left(\frac{y}{x}\right) = \frac{f(x,y)}{g(x)}$$

- if $f(x,y) = g(x) \cdot h(y)$ then x and y are independent
- if $f(x,y) \neq g(x) \cdot h(y)$ then x and y are dependent

Q. $f(x,y) = \begin{cases} \frac{1}{8}(6-x-y) & 0 \le x \le 2,\ 2 \le y \le y \\ 0 & \text{otherwise} \end{cases}$

a) verify that $f(x,y)$ is a joint density function

b) $P(x \le 3/2,\ y \le 5/2)$

c) Find marginal pdf of x and y i.e., $g(x)$ and $h(y)$

d) Find the conditional p.d.f $f(x/y)$ and $f(y/x)$

a) $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\, dy\, dx = 1$

$$\int_0^2 \int_2^4 \frac{1}{8}(6-x-y)\, dy\, dx = 1$$

$$= \frac{1}{8}\int_0^2 \left[6y - xy - \frac{y^2}{2}\right]_2^4 dx$$

$$= \frac{1}{8}\int_0^2 [(24-4x-8)-(12-2x-2)]\, dx$$

$$= \frac{1}{8}\int_0^2 (6-2x)\, dx$$

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Date:

$$= \frac{1}{8}\left[6x - x^2\right]_0^2$$

$$= \frac{1}{8}\left[(12-4)-0\right]$$

$$= \frac{1}{8}(8)$$

$$= 1$$

b) $P(x \leq 3/2,\ y \leq 5/2)$

$$\int_0^{3/2}\int_2^{5/2} \frac{1}{8}(6-x-y)\,dy\,dx$$

$$= 9/32$$

c) Marginal pdf of x and y

$$g(x) = \int_{-\infty}^{\infty} f(x,y)\,dy$$

$$= \int_2^4 \frac{1}{8}(6-x-y)\,dy$$

$$= \frac{1}{8}\int_2^4 (6-x-y)\,dy$$

$$= \frac{1}{8}\left[6y - xy - \frac{y^2}{2}\right]_2^4$$

$$= \frac{1}{8}\left[(24-4x-8)-(12-2x-2)\right]$$

$$= \frac{1}{8}\left[6-2x\right]$$

$$= \frac{1}{4}(3-x) \quad 0 \leq x \leq 2$$

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$$h(y) = \int_{-\infty}^{\infty} f(x,y)\,dx$$

$$= \int_{0}^{2} \frac{1}{8}(6-x-y)\,dx$$

$$= \frac{1}{8}\left[6x - \frac{x^2}{2} - xy\right]_0^2$$

$$= \frac{1}{8}\left[(12-2-2y) - 0\right]$$

$$= \frac{1}{4}(5-y) \qquad 2 \le y \le 4$$

d)

$$f(x/y) = \frac{f(x,y)}{h(y)}$$

$$= \frac{1/8\,(6-x-y)}{1/4\,(5-y)}$$

$$= \frac{6-x-y}{2(5-y)}$$

$$f(y/x) = \frac{f(x,y)}{g(x)}$$

$$= \frac{1/8\,(6-x-y)}{1/4\,(3-x)}$$

$$= \frac{(6-x-y)}{2(3-x)}$$

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Date: 27-3-23 Monday

Mathematical expectation ⇒ mean

$$\mu_x = E(x) = \sum_{i=1}^{n} x\, f(x) \quad \text{(Discrete variable)}$$

$$\mu_x = E(x) = \int_{-\infty}^{\infty} x\, f(x) \quad \text{(continuous variable)}$$

variance :

$$\text{variance} = E(x^2) - [E(x)]^2$$

$$E(x^2) = \sum x^2 f(x) \quad \text{(Discrete)}$$

$$E(x^2) = \int_{-\infty}^{\infty} x^2 f(x)\, dx \quad \text{(continuous)}$$

Alternative :

$$\sigma^2 = var(x) = E(x-\mu)^2 = \sum (x-\mu)^2 f(x) \quad \text{(Discrete)}$$

$$\sigma^2 = \text{"} var(x) = E(x-\mu)^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\, dx \quad \text{(continuous)}$$

Q. what is the mathematical expectation for the no. of heads when 3 fair coins are tossed.

S.S = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

x : no of heads

x : 0, 1, 2

x	$f(x)$	$x\, f(x)$	$x^2 f(x)$
0	1/8	0	0
1	3/8	3/8	3/8
2	3/8	6/8	12/8
3	1/8	3/8	9/8
	1	12/8	24/8

$$E(x) = \sum x\, f(x)$$

$$E(x) = \frac{12}{8} = 1.5$$

1.5 heads per toss

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for variance :

$$var(x) = E(x^2) - [E(x)]^2$$

$$E(x^2) = \Sigma x^2 f(x)$$

$$= 24/8$$

$$var(x) = \frac{24}{8} - (1.5)^2$$

$$= 0.75$$

Q. If the continuous r.v (random variable) x has p.d.f

$$f(x) = \begin{cases} \frac{3}{4}(3-x)(x-5) & 3 \le x \le 5 \\ 0 & \text{otherwise} \end{cases}$$

Find mean, variance and SD of x

$$E(x) = \int_{-\infty}^{\infty} x\, f(x)\, dx$$

$$= \int_{3}^{5} x\left[\tfrac{3}{4}(3-x)(x-5)\right] dx$$

$$= \frac{3}{4}\int_{3}^{5} (-x^3 + 8x^2 - 15x)\, dx$$

$$= \frac{3}{4}\left[\frac{-x^4}{4} + \frac{8x^3}{3} - \frac{15x^2}{2}\right]_3^5$$

$$= \frac{3}{4}\left[\left(-\frac{125}{12} + \frac{64}{4}\right)\right]$$

$$= 4$$

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Date:

$$E(x^2) = \int_{-\infty}^{\infty} x^2 f(x)\, dx$$

$$= \int_3^5 x^2 \left[\frac{3}{4}(3-x)(x-5)\right] dx$$

$$= \frac{3}{4}\int_3^5 (-x^4 + 8x^3 - 15x^2)\, dx$$

$$= \frac{3}{4}\left[-\frac{x^5}{5} + \frac{8x^4}{4} - \frac{15x^3}{3}\right]_3^5$$

$$= \frac{81}{5}$$

$$var(x) = E(x^2) - [E(x)]^2$$

$$= \frac{81}{5} - (4)^2$$

$$var(x) = 0.2$$

standard deviation $= \sqrt{var(x)}$

$$= \sqrt{0.2}$$

$$= 0.447$$

Q.

$$f(x) = \begin{cases} x^2/3 & -1<x<2 \\ 0 & \text{otherwise} \end{cases}$$

Find $E(x)$ and $var(x)$

$$E(x) = \int_{-1}^{2} x \cdot f(x)\, dx$$

$$= \int_{-1}^{2} x \cdot \frac{x^2}{3}\, dx$$

$$= \frac{5}{4}$$

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$$E(x^2) = \int_{-1}^{2} x^2 \cdot \frac{x^2}{3}\, dx$$

$$= \frac{11}{5}$$

$$var(x) = E(x^2) - [E(x)]^2$$

$$var(x) = \frac{11}{5} - \left(\frac{5}{4}\right)^2$$

$$var(x) = 0.64$$

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Chapter 4: Def 4.1, Theorem 4.1, Def 4.3, Theorem 4.2

day / date: 29-3-23 Wednesday

Expectation of a function of random variable

$g(x)$ or $h(x)$

$E[g(x)] = \sum g(x) f(x)$ (discrete)

$E[g(x)] = \int_{-\infty}^{\infty} g(x) f(x)\, dx$ (continuous)

where $g(x)$ is any function of x

Q. suppose that the no. of cars X that pass through a car wash b/w 4-5pm on any sunny friday has a following probability distribution

X	$P(x=x) = f(x)$	$2x-1$	$(2x-1)(f(x))$
4	1/12	$2(4)-1=7$	7/12
5	1/12	9	9/12
6	1/4	11	11/4
7	1/4	13	13/4
8	1/6	15	15/6
9	1/6	17	17/6
			12.67

let

let $g(x) = 2x-1$ represent the amount of money in dollars paid to the attendant by the manager. Find the attendants expected earning for the particular time period.

$E[(g(x)] = \sum (2x-1)(f(x))$

$E(2x-1) = 12.67$

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Q. Let x be a random variable with pdf

$$f(x) = \begin{cases} 2(x-1) & 1<x<2 \\ 0 & \text{otherwise} \end{cases}$$

find the expected values of $g(x) = 2x-1$ and $g(x) = x^2$

$$E(2x-1) = \int_{-\infty}^{\infty} (2x-1)\, f(x)\, dx$$

$$= \int_1^2 (2x-1)\, 2(x-1)\, dx$$

$$= 2\int_1^2 (2x^2-3x+1)\, dx$$

$$= 2\left| \frac{2x^3}{3} - \frac{3x^2}{2} + x \right|_1^2$$

$$= 2\left[\left(\frac{16}{3} - 6 + 2\right) - \left(\frac{2}{3} - \frac{3}{2} + 1\right)\right]$$

$$= \frac{7}{3}$$

$$E[g(x)] = E(x^2)$$

$$= \int_1^2 x^2 f(x)\, dx$$

$$= \int_1^2 x^2 .\, 2(x-1)\, dx$$

$$= 2\int_1^2 (x^3 - x^2)\, dx$$

$$= 2\left| \frac{x^4}{4} - \frac{x^3}{3} \right|_1^2$$

$$= 2\left[\left(4 - \frac{8}{3}\right) - \left(\frac{1}{4} - \frac{1}{3}\right)\right]$$

$$= \frac{17}{6}$$

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variance:

this formula is easy

day / date:

$$\sigma_x^2 = var(x) = E(x-\mu)^2 \quad OR \quad E(x^2) - [E(x)]^2$$

$$\sigma_{g(x)}^2 = var[g(x)] = E[g(x) - \mu_{g(x)}]^2$$

$$var[g(x)] = E[g(x)^2] - [Eg(x)]^2$$

Q. Find variance of $g(x) = 2x+3$

x	$f(x)$	$2x+3$	$(2x+3)f(x)$	$4x^2-12x+9$	$(4x^2-12x+9)f(x)$
0	1/4	3	3/4	9	9/4
1	1/8	5	5/8	1	1/8
2	1/2	7	7/2	1	1/2
3	1/8	9	9/8	9	9/8
			6		4

$$var[g(x)] = E[g(x) - \mu_{g(x)}]^2$$

$$\mu_{g(x)} = E(2x+3) = 6$$

$$= E[2x+3-6]^2$$

$$\sigma^2 = E(2x-3)^2$$

$$= E(4x^2 - 12x + 9)$$

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Q. Let x be a random variable with pdf

$$f(x) = \begin{cases} x^2/3 & -1<x<2 \\ 0 & \text{otherwise} \end{cases}$$

Find variance of $g(x) = 4x+3$

$$\sigma^2_{g(x)} = E\left[g(x) - \mu_{g(x)}\right]^2$$

$$\mu_{g(x)} = \int_{-1}^{2} (4x+3)\, f(x)\, dx$$

$$E(4x+3) = \int_{-1}^{2} (4x+3) \cdot \frac{x^2}{3}\, dx$$

$$= 8$$

$$\sigma^2_{g(x)} = E\left[4x+3-8\right]^2$$

$$= E(4x-5)^2$$

$$= \int_{-1}^{2} (4x-5)^2 \frac{x^2}{3}\, dx$$

$$= \frac{1}{3}\int_{-1}^{2} (16x^4 - 40x^3 + 25x^2)\, dx$$

$$= \frac{1}{3}\left| \frac{x^5}{5} - \frac{40x^4}{4} + \frac{25x^3}{3} \right|_{-1}^{2}$$

$$= \frac{51}{5}$$

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Mathematical expectation Q's

day / date:

inspector

Q. A lot containing 9 components is sampled by a quality. The lot contains 4 good components and 3 defective components. A sample of 3 is taken by inspector. Find the expected value of the no of good components in this sample.

$E(x) = \Sigma x P(x)$ $\qquad x = 0, 1, 2, 3$

$$P(x = x) = \frac{\binom{4}{x}\binom{3}{3-x}}{\binom{7}{3}}$$

$$E(x) = 0\left(\frac{1}{35}\right) + 1\left(\frac{12}{35}\right) + 2\left(\frac{18}{35}\right) + 3\left(\frac{4}{35}\right)$$

$$= \frac{12}{7} = 1.7$$

Q4.3 Let x be random variable that denotes the life in hrs of a certain electronic device. the prob density function is

$$f(x) = \begin{cases} \frac{20,000}{x^3} & x > 100 \\ 0 & \text{otherwise} \end{cases}$$

find expected life of device?

$$E(x) = \int_{-\infty}^{\infty} x f(x)\, dx$$

$$= \int_{100}^{\infty} x \frac{20,000}{x^3}\, dx$$

$$= 20,000 \left| \frac{x^{-1}}{x^{-1}} \right|_{100}^{\infty}$$

$$= -20,000 \left[-\frac{1}{100} \right] = 200$$

on avg component's life is 200

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day / date:

Q4.8. Let the random variable x represents the no of vehicles that are used for official business purposes on any working day. The prob distribution for company A is

X	f(x)
1	0.3
2	0.4
3	0.3

and that for company B is

X	f(x)
0	0.2
1	0.1
2	0.3
3	0.3
4	0.1

show that the variance of the prob distribution for comp B is greater than that company A.

Company A:

$var(x) = E(x-\mu)^2 p(x)$

$= \sum (x-\mu)^2 p(x)$

$\mu = E(x) = \sum x P(x)$

$= 1\times0.3 + 2\times0.4 + 3\times0.3 = 2$

$var(A) = 0.3 + 0 + 0.3 = 0.6$

Company B:

$var(x) = E(x-\mu)^2 p(x)$

$= \sum (x-\mu)^2 P(x)$

$\mu = E(x) = \sum x P(x)$

$= 0 + 1\times0.1 + 2\times0.3 + 3\times0.3 + 4\times0.1 = 2$

$var(x) = 4(0.2) + 1(0.1) + 0 + 0.3 + 4(0.1)$

$= 1.6$

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Practise Q's : 4.49 , 4.38 , 4.34

day / date:

4.9

Q. Let the random variable x represents the no of defective parts of a machine when 3 parts are sampled from a production time. The following is probability distribution of x

x	$P(x)$	$x\,P(x)$	$x^2 P(x)$
0	0.51	0	0
1	0.38	0.38	0.38
2	0.10	0.2	0.4
3	0.01	0.03	0.09
		0.61	0.87

$$var(x) = E(x^2) - (E(x))^2$$
$$= 0.87 - (0.61)^2$$
$$= 0.5$$

4.50 For a laboratory assignment if an equipment is working the density function of the observed outcome x is $f(x) = \begin{cases} 2(1-x) & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$

find variance and standard deviation.

a) $$E(x^2) = \int_{-\infty}^{\infty} x^2 f(x)\,dx$$
$$= \int_0^1 x^2\, 2(1-x)\,dx$$
$$= \int_0^1 2x^2 - 2x^3\,dx$$
$$= 2\left[\frac{x^3}{3} - \frac{x^4}{4}\right]_0^1$$
$$= 2\left(\frac{1}{3} - \frac{1}{4}\right)$$
$$= 0.167$$

b) $$\sqrt{0.167}$$
$$= 0.4$$

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Chapter # 5 day / date: 3-4-23 Monday

Binomial Distribution

1. The outcome of each trial may be classified into one of the two categories (success or failure).
2. The repeated trials are all independent.
3. The probability of success remains constant throughout the trial.
4. The experiment is repeated a fixed number of times say n.

Probability mass function: $\boxed{P(x=x) = \binom{n}{x} p^x q^{n-x}}$ $0 \le x \le n$

used to define binomial distribution
parameter: n, p
mean $= np$
variance $= npq$

x = no. of success
p = probability of success
q = probability of failure
n = no of trials

Q. A and B play a gam in which A has the probability of winning is 2/3 in a series of 8 games. a) what's the probability that a will win exactly 4 games? b) what's the probability that A will win at least 4 games?

a) $P(x=x) = \binom{n}{x} p^x q^{n-x}$

$P(x=4) = \binom{8}{4}\left(\frac{2}{3}\right)^4\left(\frac{1}{3}\right)^{8-4}$

$p(\text{winning}) = 2/3$
$p(\text{not winning}) = q$
$q = 1-p = 1-2/3 = 1/3$

$P(x=4) = 0.1707$

b) $P(x \ge 4) = P(x=4) + P(x=5) + P(x=6) + P(x=7) + P(x=8)$

$= \binom{8}{4}\left(\frac{2}{3}\right)^4\left(\frac{1}{3}\right)^4 + \binom{8}{5}\left(\frac{2}{3}\right)^5\left(\frac{1}{3}\right)^3 + \binom{8}{6}\left(\frac{2}{3}\right)^6\left(\frac{1}{3}\right)^2 + \binom{8}{7}\left(\frac{2}{3}\right)^7\left(\frac{1}{3}\right)^1 + \binom{8}{8}\left(\frac{2}{3}\right)^8\left(\frac{1}{3}\right)^0$

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e.g 5.3

day / date:

Q. A large chain retailer purchases a certain kind of electronic device from a manufacturer. The manufacturer indicates that defective rate of device is 3%

a) The inspector randomly picks 20 items from a shipment. what's the prob that there will be atleast 1 defective item among these 20?

b) Suppose that the retailer receives 10 shipments in a month and the inspector randomly tests 20 devices per shipment. what's the prob that there will be exactly 3 shipments each containing at least 1 defective device among the 20 that are defective and tested from the shipment?

a) $P(x \geq 1) = 1 - P(x < 1)$

$= 1 - P(x = 0)$

$= 1 - \left[\binom{20}{0}(0.03)^0(0.97)^{20}\right]$

$= 0.4562$

$n = 20$
$p = 0.03$
$q = 0.97$

b) $P(x = 3) = \binom{10}{3}(0.4562)^2(1-0.4562)^7$

$= 0.1602$

$n = 10$
$p = 0.4562$
$q = 1 - 0.4562$

Mathematical Distribution

1. The outcome of each trial may be classified into one of the k mutually exclusive categories $C_1, C_2, \ldots, C_k$.
2. The repeated trials are all independent.
3. The probability of ith outcome is P_i, which remains constant and $\Sigma P_i = 1$.
4. The experiment is repeated a fixed number of times.

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$$P(x_1 = x_1, x_2 = x_2, \dots, x_k = x_k) = \binom{n}{x_1, x_2, \dots, x_k} P_1^{x_1} P_2^{x_2} \dots P_k^{x_k}$$

$$\binom{n}{x_1, x_2, \dots, x_k} = \frac{n!}{x_1!\, x_2! \dots x_k!}$$

Parameters : $n, P_1, P_2, \dots, P_k$

Mean = nP_i

Variance = $nP_i q_i$

Q. The complexity of arrivals and departures of planes at an airport is such that computed simulation is often use to model the ideal conditions. For a certain airport with 3 runways, it is known that in the ideal setting the following are the probabilities that the individual runways are excessed by a randomly arriving commercial jet.

Runaway 1 : $P_1 = 2/9$

Runaway 2 : $P_2 = 1/6$

Runaway 3 : $P = 11/18$

what's the prob that 6 randomly arriving airplanes are distributed in the following fashion.

Runaway 1 : 2 Airplanes

Runaway 2 : 1 Airplanes

Runaway 3 : 3 Airplanes

$$P(x_1 = 2, x_2 = 1, x_3 = 3) = \binom{6}{2 \quad 1 \quad 3} \left(\frac{2}{9}\right)^2 \left(\frac{1}{6}\right)^1 \left(\frac{11}{18}\right)^3$$

$$= \frac{6!}{2!\ 1!\ 3!} \left(\frac{2}{9}\right)^2 \left(\frac{1}{6}\right)^1 \left(\frac{11}{18}\right)^3$$

$$= 0.1127$$

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Binomial Distribution

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5.1

Q. The prob that a certain kind of component will survive a shock test is 3/4 Find the prob that exactly 2 of the next 4 components tested survived.

$$P(x=2) = \binom{4}{2}\left(\frac{3}{4}\right)^2\left(\frac{1}{4}\right)^2$$

$$= \frac{27}{128}$$

5.2

Q. The prob that a patient recovers from a rare blood disease is 0.4. If 15 patients are known to have this disease, what's the prob that

a) at least 10 survive

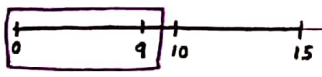
b) from 3 to 8 survive

c) exactly 5 survive

a) $P(x \geq 10) = P(x=10) + P(x=11) + P(x=12) + P(x=13)$
$+ P(x=14) + P(x=15)$

$1 - \sum_{x=0}^{9} b(x,n,p)$

$= 0.0338$



1 - this gives answer

b) $P(3 \leq x \leq 8) = \sum_{x=0}^{8} b(x,n,p) - \sum_{x=0}^{2} b(x,n,p)$

$= 0.905 - 0.0271$

$= 0.8779$

c) $P(x=5) = \binom{15}{5}(0.4)^5(0.6)^{10}$

$= 0.186$

variance = 3.6

mean = 6

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Q. A nationwide survey of clg seniors in a uni revealed that almost 70% disapproved smoking If 12 seniors are selected at random and asked their opinions, find the prob that the number who disapprove smoking is

a) anywhere from 7 to 9

b) atmost 5

a) $P(7 \le x \le 9) = \binom{12}{7}(0.7)^7(0.3)^5 + \binom{12}{8}(0.7)^8(0.3)^4 + \binom{12}{9}(0.7)^9(0.3)^3$

$= 0.6294$

b) $P(x \le 5) = \binom{12}{0}(0.7)^0(0.3)^{12} + \dots + \binom{12}{5}(0.7)^5(0.3)^7$

$= 0.0386$

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