Image Sampling and Aliasing

Introduction

1. “Sampling is the process of transforming a continuous signal into a discrete one”
2. Questions related to sampling
    a. What are the possible sampling patterns to sample a signal?
    b. How can we characterize the loss of information?
    c. And how do we reduce artifacts?

Sampling

1. We often have to optimize two competing factors:
    a. Maximize the amount of available information. This requires as many samples as possible.
    b. Minimize the computational cost and memory requirements. This implies that we want to minimize the number of samples we collect.

Aliasing

1. Consider the continuous signal $\ell(t) = \cos(wt)$ with
   a. frequency $w = 18\pi$
   b. period $T = 2\pi/w = 1/9$

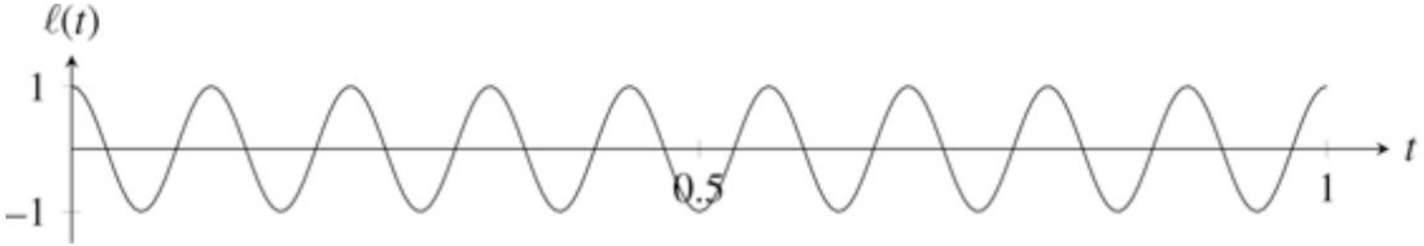


Figure 20.1: Continuous cosine wave, $\ell(t) = \cos(wt)$, with frequency $w = 18\pi$.

Aliasing

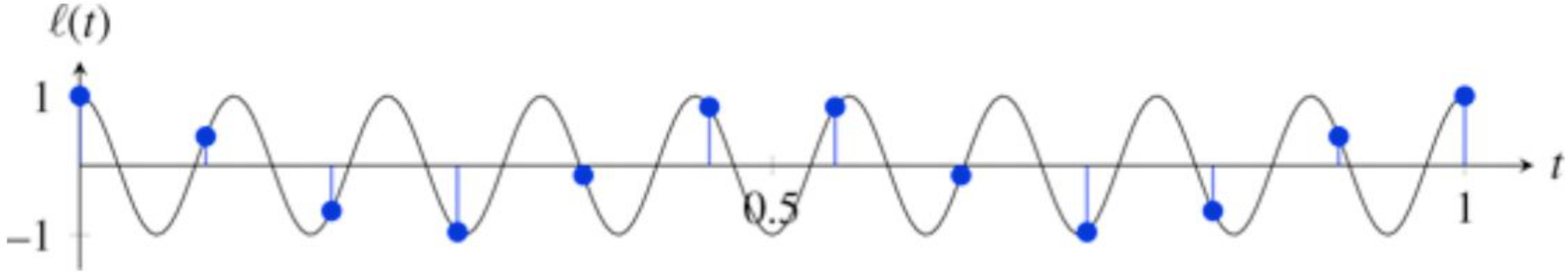


Figure 20.2: Sampling the cosine wave with a sampling period of $T_s = 1/11$.

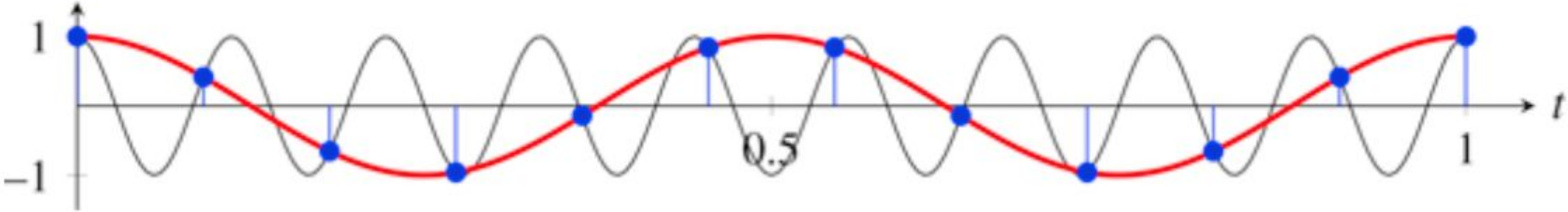


Figure 20.3: There are infinite waves (only two shown) that perfectly pass by all the samples.

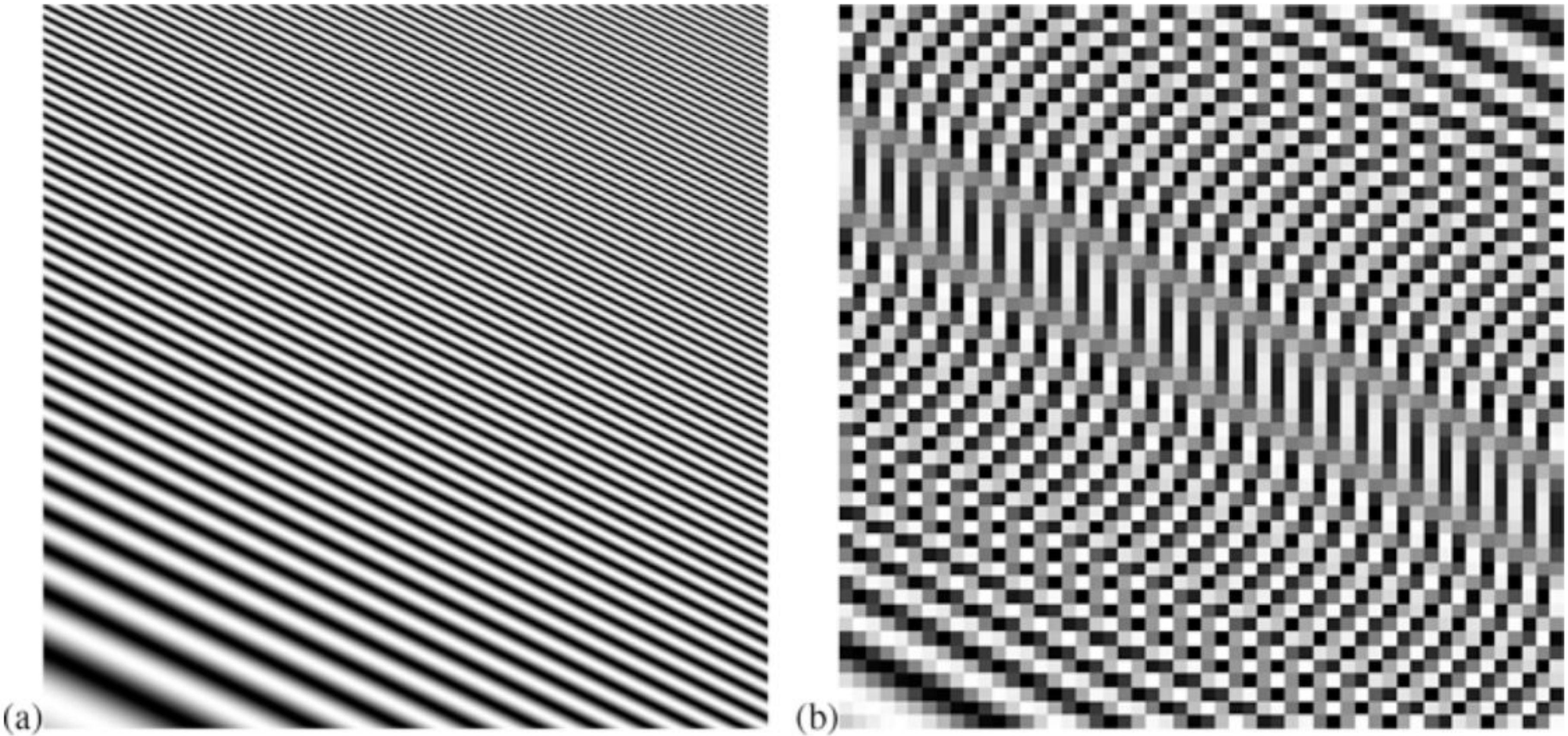
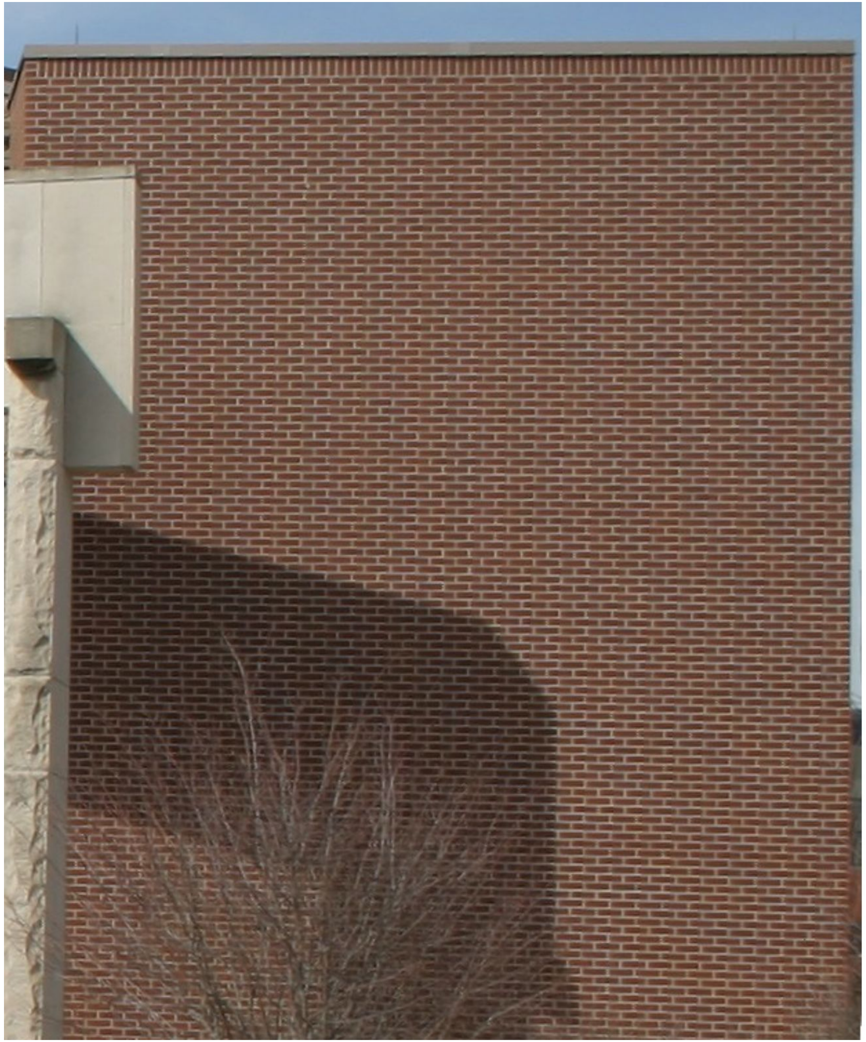


Figure 20.4: (a) A continuous image (here shown by sampling very finely). (b) Its heavily sampled version resulting in an image with size 52×52.


Aliasing

1. Our perception seems to follow the slow and smooth assumption
2. We have prior towards smooth and slow trajectories and textures.
3. In the presence of noise, or missing information, our visual system will tend to follow the prior.

Sampling Theorem

1. The sampling theorem (also known as Nyquist theorem) states that for a signal to be perfectly reconstructed from a set of samples (*under the slow and smooth prior*),
   a. the sampling frequency $\mathbf{w_s = 2\pi/T_s}$ has to be greater than $\mathbf{2w_{max}}$
   b. OR the sampling period, $\mathbf{T_s}$, has to be $< \mathbf{T_{min}/2}$, $\mathbf{T_{min} = 2\pi/w_{max}}$

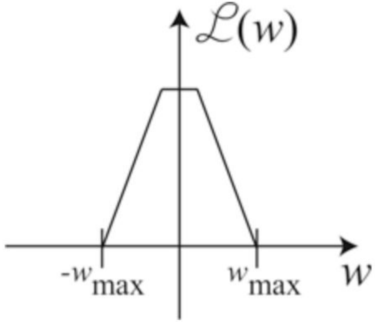
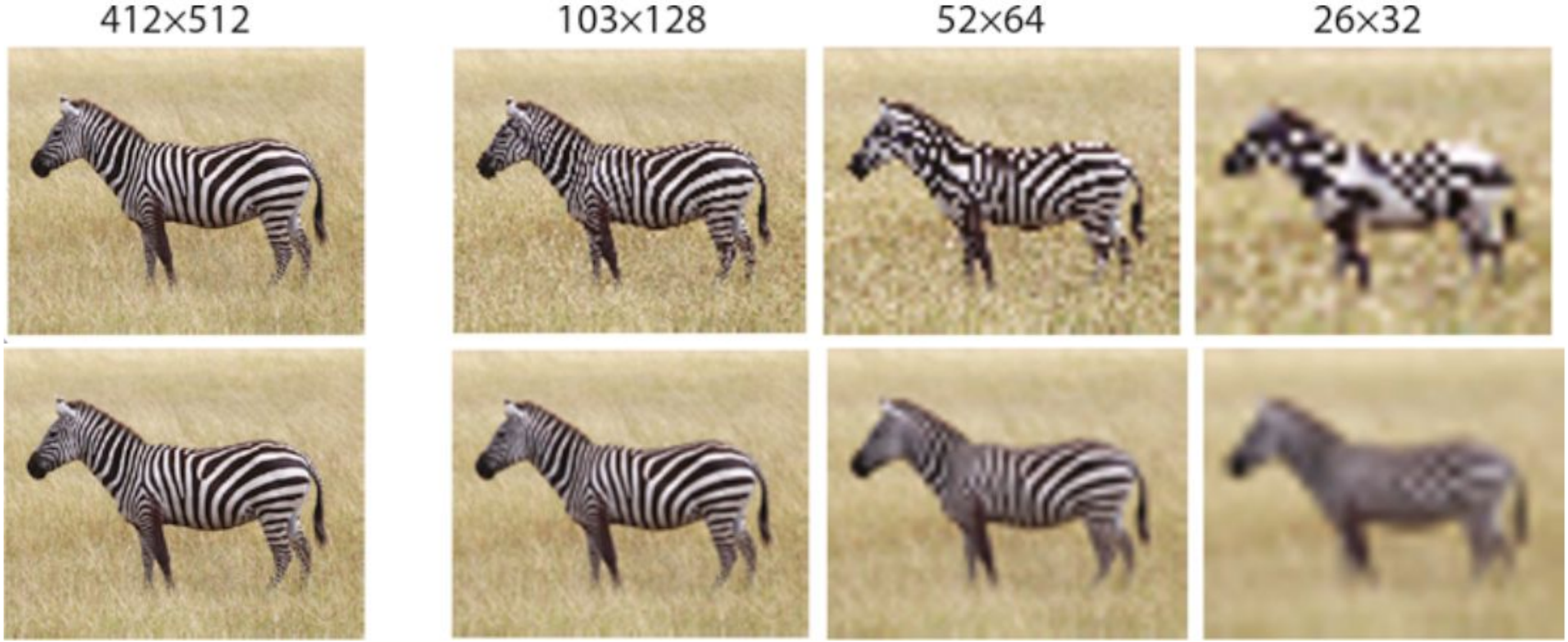


Figure 20.5: A band-limited signal with maximum frequency w_{max}.

Anti-Aliasing

1. A picture downsampled at different resolutions (412×512, 103×128, 52×64, and 26×32) and then reconstructed to the original resolution (412×512 pixels)



Conclusion

1. Aliasing is not always bad and it might be possible to use aliasing to recover high-frequency content that would be otherwise lost. Superresolution algorithms can learn to extract, from the aliasing pattern, fine image details
2. When an image is encoded by multiple channels, aliasing in one channel does not mean that the information is lost because multiple channels could contain complementary information that allows the recovery of high resolution details

References

1. Foundations of Computer Vision - Chapter 20