Name : Muhammad Laraib Akhtar
Roll no : 21L-5294
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Question 1

$P(A_1) = \frac{1}{2}$, $P(A_2) = 1/6$, $P(A_3) = 1/3$,

$P(+ve/A_1) = 0.1$, $P(+ve/A_2) = 0.2$, $P(+ve/A_3) = 0.9$

$$P(A_1/+ve) = \frac{P(+ve/A_1) * P(A_1)}{P(+ve)}$$

$$P(+ve) = P(A_1)P(+ve/A_1) + P(A_2)P(+ve/A_2) + P(A_3)P(+ve/A_3)$$
$$= (0.5)(0.1) + (1/6)(0.2) + (1/3)(0.9)$$
$$= 0.05 + 0.0333 + 0.3$$
$$P(+ve) = 0.383$$

$$P(A_1/+ve) = \frac{(0.1)(0.5)}{0.383}$$
$$= \frac{0.05}{0.383}$$
$$= 0.13$$

Question 2

Sample Space(s)	x
N N N	0
N N B	1
N B N	1
N B B	2
B N N	1
B N B	2
B B N	2
B B B	3

$$P(X=3) = \frac{\binom{3}{3}\binom{3}{0}}{\binom{6}{3}} = \frac{1}{20} = 0.05$$

$$P(X=2) = \frac{\binom{3}{2}\binom{3}{1}}{\binom{6}{3}} = \frac{3\times3}{20} = \frac{9}{20} = 0.45$$

$$P(X=1) = \frac{\binom{3}{1}\binom{3}{2}}{\binom{6}{3}} = \frac{9}{20} = 0.45$$

$$P(X=0) = \frac{\binom{3}{0}\binom{3}{3}}{\binom{6}{3}} = \frac{1}{20} = 0.05$$

$$P(X=x) = \frac{\binom{3}{x}\binom{3}{3-x}}{\binom{6}{3}}$$

Question 3

$$f(x) = \begin{cases} \frac{2(1+x)}{27} & 2 \le x \le 5 \\ 0 & \text{elsewhere} \end{cases}$$

a) $P(X<4) = \int_{-\infty}^{4} f(x)\,dx$

$$= \int_{-\infty}^{2} f(x)dx + \int_{2}^{4} f(x)\,dx$$

$$= 0 + \int_{2}^{4} \frac{2(1+x)}{27}\,dx$$

$$= \int_{2}^{4} \frac{2}{27} + \frac{2x}{27}\,dx$$

$$= \left[\frac{2x}{27} + \frac{x^2}{27}\right]_{2}^{4}$$

$$= \left[\frac{2(4)}{27} + \frac{4^2}{27}\right] - \left[\frac{2(2)}{27} + \frac{2^2}{27}\right]$$

$$= \frac{8}{9} - \frac{8}{27}$$

$$= \frac{16}{27}$$

b) $P(3 \leq x < 4) = \int_3^4 f(x)\,dx$

$$= \int_3^4 \frac{2(1+x)}{27}\,dx$$

$$= \left[\frac{2x}{27} + \frac{x^2}{27}\right]_3^4$$

$$= \frac{8}{9} - \left[\frac{2(3)}{27} + \frac{3^2}{27}\right]$$

$$= \frac{8}{9} - \frac{5}{9}$$

$$= \frac{3}{9}$$

$$= \frac{1}{3}$$

c) $\int_2^x \frac{2(1+x)}{27}\,dx$

$$= \frac{2x}{27} + \frac{x^2}{27} - \frac{8}{27}$$

$$= \frac{2x + x^2 - 8}{27}$$

$$F(3) = \frac{2(3) + 3^2 - 8}{27}$$

$$= \frac{7}{27}$$

$$F(4) = \frac{2(4) + (4)^2 - 8}{27}$$

$$= \frac{8 + 16 - 8}{27}$$

$$= \frac{16}{27}$$

$$F(4) - F(3) = \frac{16}{27} - \frac{7}{27} = \frac{9}{27}$$

$$= \frac{1}{3}$$

Question 4

a) $\int_{0}^{200} \frac{20{,}000}{(x+100)^3}\,dx$

let $x+100 = u$

$dx = du$

$u_1 = 200+100$

$u_1 = 300$

$u_2 = 0+100$

$u_2 = 100$

$$\int_{100}^{300} \frac{20\,000}{u^3}\,du$$

$$\left[\frac{20\,000\,u^{-2}}{-2}\right]_{100}^{300}$$

$$= \left[-\frac{10000}{u^2}\right]_{100}^{300}$$

$$= \left[\frac{-10000}{300^2}\right] - \left[\frac{-10000}{100^2}\right] = -\frac{1}{9} + 1$$

$$= \frac{8}{9}$$

(b) $P(80 \le x \le 120) = \int_{80}^{120} \frac{2000}{(x+100)^3}\,dx$

let $u = x+100$

$du = dx$

$$\left[\frac{-10000}{u^2}\right]_{180}^{220}$$

$$= \frac{-25}{121} + \frac{25}{81}$$

$$= \frac{1000}{9801}$$

$$= 0.1020$$

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c) $\int_0^x \frac{20000}{(x+100)^3}\,dx$

$\int_{100}^{x+100} \frac{20000}{u^3}\,du$

$\left[\frac{-10000}{u^2}\right]_{100}^{x+100}$

$\frac{-10000}{x^2+200x+10000} - \left[\frac{-10000}{10000}\right]$

$\frac{-10000}{x^2+200x+10000} + 1$

$\frac{-\cancel{10000} + x^2 + 200x + \cancel{10000}}{(x+100)^2}$

$\frac{x^2+200x}{(x+100)^2}$

$F(80) = \frac{56}{81}$

$F(120) = \frac{96}{121}$

$F(120)-F(80) = \frac{96}{121} - \frac{56}{81}$

$= \frac{1000}{9801}$

$= 0.1020$

Question 5

$S = 1000$ cruise $= C$ &

$N = 250$ urgent $= u$

$P(C/S) = 50/1000 = 0.05$

$P(C/N) = 2/250 = 0.08$

$P(u/S) = 100/1000 = 0.1$

$P(u/N) = 10/250 = 0.04$

$P(S/C \cap u) = \frac{P(S)\,P(C/S)\,P(u/S)}{P(S)\,P(C/S)\,P(u/S) + P(N)\,P(C/N)\,P(u/N)}$

$$= \frac{(0.5)(0.05)(0.1)}{(0.5)(0.05)(0.1) + (0.5)(0.008)(0.04)}$$

$$= \frac{0.0025}{0.0025 + 0.00016} = \frac{0.0025}{0.00266}$$

$= 0.9398$

$=$ Yes (since $0.9398 > 0.9$)

Question 6

$$P(X=0) = \frac{\binom{4}{0}\binom{48}{13}}{\binom{52}{13}} = 0.3038$$

$$P(X=1) = \frac{\binom{4}{1}\binom{48}{12}}{\binom{52}{13}} = 0.4388$$

$$P(X=2) = \frac{\binom{4}{2}\binom{48}{11}}{\binom{52}{13}} = 0.2134$$

$$P(X=3) = \frac{\binom{4}{3}\binom{48}{10}}{\binom{52}{13}} = 0.0412$$

$$P(X=4) = \frac{\binom{4}{4}\binom{48}{9}}{\binom{52}{13}} = 0.0026$$

$$P(X=x) = \frac{\binom{4}{x}\binom{48}{13-x}}{\binom{52}{13}}$$

Question 7

No. of defective components	$P(x)$	~~$P(x)$~~ x
0	3/6	3
1	1/6	1
2	2/6	2

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Question8

Y	1	2	4	8	16
P(y)	0.05	0.10	0.35	0.40	0.10
F(y)	0.05	0.15	0.5	0.9	1

$$F(y) = \begin{cases} 0.05 & y \le 1 \\ 0.15 & 1 < y \le 2 \\ 0.5 & 2 < y \le 4 \\ 0.9 & 4 < y \le 8 \\ 1 & 8 < y \le 16 \end{cases}$$

$F(1) = f(1)$
$F(2) = f(1) + f(2)$
$F(4) = f(1) + f(2) + f(4)$
$F(8) = f(1) + f(2) + f(4) + f(8)$
$F(16) = f(1) + f(2) + f(4) + f(8) + f(16)$

Question 9

i) (i) $f(x) \geq 0$

$f(0) = 0$

$f(1) = 0$

hence proven

(ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$= \int_{-\infty}^{0} fx\,dx + \int_{0}^{1} fx\,dx + \int_{1}^{\infty} fx\,dx$

$= 0 + 0 + \int_{0}^{1} 6x - 6x^2\,dx$

$= 6\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1$

$= 6\left[\frac{1}{2} - \frac{1}{3}\right]$

$= 1$ proven.

(iii) $P(a < x < b) = F(b) - F(a) =$ ~~E(x~~

$\int_{0}^{X} fx\,dx$

$6\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^X$

$= \frac{6x^2}{2} - \frac{6x^3}{3}$

$= 3x^2 - 2x^3$

$F(1) = 1$

$F(0) = 0$

$F(1) - F(0) = 1$

hence proven.

b) $F(x) = P(X \leq x) = \int_{-\infty}^{x} f(x)\,dx$

$= 6\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^x$

$= 3x^2 - 2x^3$

c) $P\left(\frac{1}{3} < x < \frac{2}{3}\right)$

$6\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_{1/3}^{2/3}$

$6\left[\frac{4}{9 \times 2} - \frac{8}{27 \times 3}\right] - 6\left[\frac{1}{9 \times 2} - \frac{8}{27 \times 3}\right]$

$6\left[\frac{2}{9} - \frac{8}{81}\right] - 6\left[\frac{1}{18} - \frac{8}{81}\right]$

$6\left[\frac{10}{81}\right] - 6\left[-\frac{7}{162}\right]$

$\frac{20}{27} - -\frac{7}{27}$

$= \frac{13}{27}$

Question 10

$P(A) = 0.5$, $P(B) = 0.3$, $P(C) = 0.2$

$P(F/A) = 0.02$, $P(F/B) = 0.03$, $P(F/C) = 0.05$

$$P(B/F) = \frac{P(F/B) * P(B)}{P(F)}$$

$$P(F) = P(A)\,P(F/A) + P(B)P(F/B) + P(C)P(F/C)$$

$$= (0.5)(0.02) + (0.3)(0.03) + (0.2)(0.05)$$

$$= 0.029$$

$$P(B/F) = \frac{0.03 \times 0.3}{0.029}$$

$$= 0.31$$

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