Ex 3.1

Q6 $t = 0$ 100 mg

$t = 6$ $100 - 3\%$ of $100 = 97$

$t = 24$?

$y = Ce^{kt}$

$97 = 100e^{6k}$

$\frac{97}{100} = e^{6k}$

$\ln\left(\frac{97}{100}\right) = 6k$

$\frac{1}{6}\ln\left(\frac{97}{100}\right) = k$

$k = -0.0051$

$y \approx 100e^{(-0.0051)24}$

$y \approx 88.49$ mg → Answer

Q7 from Q6 $k = -0.0051$ and $C = 100$mg

$y = 50$, $y = Ce^{kt}$

$50 = 100e^{-0.0051t}$

$\ln(0.5) = -0.005t$

$-0.69 = -0.005t$

$t = 138.6$ hours

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Q19

$T_m = 70$

$\frac{dT}{dt} = k(T - 70)$ $\quad T(0) = 98.6$

$\int \frac{dT}{T-70} = \int k\, dt$

$\ln|T-70| = kt + c$

$T - 70 = ce^{kt}$

$T = 70 + ce^{kt}$

for $t = 0$, $T = 98.6$

$98.6 = 70 + c$

$c = 28.6$

$T - 70 = 28.6\, e^{kt}$

$T = 70 + 28.6 e^{kt}$

$T(t) = 85 = 70 + 28.6 e^{kt}$, $\quad T(t+1) = 80 = 70 + 28.6\, e^{k(t+1)}$

$\frac{15}{28.6} = e^{kt}$, $\quad \frac{10}{28.6} = e^{kt} \cdot e^{k}$

$\frac{10}{28.6} = \frac{15}{28.6} e^{k}$

$\frac{2}{3} = e^{k}$

$k = -0.405$

$e^k \cdot e^t = 15$

28.6

$e^t = 15$

28.6×0.666

$t =$

$$t = \frac{1}{k} \ln \frac{15}{28.6} \approx 1.6$$

Q25 $R' = R_{in} - R_{out}$

$R_{in} = 2 \times 5 = 10$

$$R_{out} = \frac{10 \times R(t)}{500 - 5t}$$

$$= \frac{2R(t)}{100 - t}$$

$$R' = 10 - \frac{2R(t)}{100 - t}$$

$R = 1000 - 10t + c(100 - t)^2$

$R(0) = 0, \quad c = -\frac{1}{10}$

tank empties in 100 min

Example 4

$$\frac{dT}{dt} = k(T - 70), \quad T(0) = 300$$

$$\frac{dT}{T-70} = k\,dt$$

$$\ln|T-70| = kt + c_1$$

$$T = 70 + c_2 e^{kt}$$

$$300 = 70 + c_2$$

$$c_2 = 230$$

$$T = 70 + 230e^{kt}$$

$$k = -0.19018$$

$$T(t) = 70 + 230\,e^{-0.19018t}$$