Que:

Max $z = 2x_1 - 4x_2 + x_3$.

Subject to.

$x_1 + x_2 + 3x_3 + S_1 = 4$.

$x_1 + x_2 + S_2 = +8$

$x_1 + x_2 + x_3 = 10$

max

Total no. of coner points

$${}^{n}C_{m} = \frac{n!}{m!(n-m)!}$$

n = no. of variables.

m = no. of equations

x_1, x_2, x_3, S_1, S_2.

$n = 5$, $m = 3 \rightarrow 3$ eqs.

$${}^{5}C_{3} = \frac{5!}{3!\,2!} = \frac{20}{2} = 10$$

Optimal Sol is at corner points.
In this que, we have 10 corner points.
so, to find optimal sol we need to check on these 10 points.

Put

$(n-m)$ variables $= 0$
in first guess

Non-basic variables	Basic variable	Feasible condition	objective
$x_1 = 0, x_2 = 0$ ↓ with help of this we can get basic variables, by putting $x_1, x_2 = 0$ in eqs.	$x_3 = 10, S_2 = 8$, $S_1 = -26$ → infeasible ↓ with help of these variables we check if these are feasible if any variable is $-ve$, it will become infeasible.		

(Simplex Method)

PROBLEM 3-22 (C).

$$\max \; z = 3x_1 - x_2 + 3x_3 + 4x_4.$$

subject to,

$$x_1 + 2x_2 + 2x_3 + 4x_4 \leq 40$$

$$2x_1 - x_2 + x_3 + 2x_4 \leq 8$$

$$4x_1 - 2x_2 + x_3 - x_4 \leq 10$$

$$\underbrace{x_1, x_2, x_3, x_4}_{\text{Decision variables}} \geq 0.$$

- less than condition → we add variable (slack)
- greater than condition → we subtract " (surplus)

Step 1 → convert in eqs.

i) objective eqs.

$$z - 3x_1 + x_2 - 3x_3 - 4x_4 = 0. \quad \text{— (i)}$$

Subject to

Simplex method ... slack variables
not for surplus.

$$x_1 + 2x_2 + 2x_3 + 4x_4 + s_1 = 40.$$

$$2x_1 - x_2 + x_3 + 2x_4 + s_2 = 8.$$

$$4x_1 - 2x_2 + x_3 - x_4 + s_3 = 10.$$

Basics	x_1	x_2	x_3	x_4	s_1	s_2	s_3	Right hand values	
1) Z	-3	1	-3	-4	0	0	0	0 ← objective row	
2) S_1	1	2	2	4	1	0	0	40	$\frac{40}{4} =$
3) S_2	2	-1	1	2	0	1	0	8	$\frac{8}{2} = 4$
4) S_3	4	-2	1	-1	0	0	1	10	$\frac{10}{-1}$

pivot element

pivot row

pivot column

smallest ratio

→ Choose that variable from eq (1) which has highest coefficient, so we chose x_4 since it has -4.

→ If every variable on left side of eq 1) (x_1, x_2, x_3) had +ve coefficient sol. would simply be 0.

→ Find ratios, if ratio is -ve, ignore it. and choose smallest ratio from remaining.

we choose row 3.

- x_4 → incoming variable
- s_2 → leaving "

→ make pivot element 1 and other 0.

• Divide row 3 by 2.

→ $\frac{R_3}{2}$, $R_1 + 4\left(\frac{R_3}{2}\right)$, $R_2 - 4\left(\frac{R_3}{2}\right)$

$R_4 + 1\left(\frac{R_3}{2}\right)$

Basics	x_1	x_2	x_3	x_4	S_1	S_2	S_3	Right hand variables
z	1	-1	-1	0	0	1	0	16
S_1	-3	4	0	0	1	-2	0	24
x_4	1	$-\frac{1}{2}$	$\frac{1}{2}$	1	0	$\frac{1}{2}$	0	4
S_3	5	$-\frac{5}{2}$	$\frac{3}{2}$	0	0	$\frac{1}{2}$	1	14

After 1st iteration $S_1 = 24$ $x_4 = 4$ $S_3 = 14$

gives optimum value 16 ∴ its in eq of z row.

Now, we can select both x_2, x_3, we

select x_3.

Check ratios to find pivot col.

ratio of row 3 is lowest $4/\frac{1}{2} = 8$

so, it is pivot row

$x_3 \to$ incoming

$x_4 \to$ leaving

optimality finishes when every value gets +ve in row of 2.

→ $2R_3$, $R_1 + 2R_3$, $R_4 - \frac{3}{2}\cdot(2R_3)$.

Basic	x_1	x_2	x_3	x_4	S_1	S_2	S_3	Right hand
2	3	-2	0	2	0	3	0	24
S_1	-3	4	0	0	1	-2	0	24
x_3	2	-1	1	2	0	1	0	8
S_3	2	-1	0	-3	0	-1	1	2

After 2nd,
↳ optimal value 24.

pivot row 2, pivot col x_2

↓ ration $\frac{24}{4} = 6$. incoming → x_2, leaving S_1

→ $\frac{R_2}{4}$, $R_1 + 2(\frac{R_2}{4})$, $R_3 + \frac{R_2}{4}$, $R_4 + \frac{R_2}{4}$.

Basics	x_1	x_2	x_3	x_4	S_1	S_2	S_3	right hand
2	$\frac{3}{2}$	0	0	2	$\frac{1}{2}$	2	0	36
x_2	$-\frac{3}{4}$	1	6	0	$\frac{1}{4}$	$-\frac{1}{2}$	0	8
x_3	$5/4$	0	1	2	$\frac{1}{4}$	$\frac{1}{2}$	0	14.
S_3	$5/4$	0	0	-3	$\frac{1}{4}$	$-\frac{3}{2}$	1	8

After 3rd optimal value 36.

→ Not possible further, all values are +ve

Results:

$x_2 = 6$, $x_3 = 14$, $S_3 = 8$.
↓
Basis

$x_1 = 0$, $x_4 = 0$
$S_1 = 0$, $S_2 = 0$
↓
Non basic

optimal = 36

Waleed Ishtiaq

Minimize

Choose col which has highest positive value of ~~eq~~ coefficient in eq.

Keep doing until all -ve values come in table.

LEC 5

Two phase method

- M-method

Embolced grading

LEC6

Special CASES IN SIMPLEX METHOD.

1) Degenracy.

→ Tie in choice of Entering variable

Basic	x_1	x_2
Z	-4	-4
S_1		
S_2		

tie

→ Tie in choice of leaving variables

Basic	x_1	x_2	S_1	S_2	RHS	Ratios
Z	-4	-4				
S_1	2				4	2
S_2	4				8	2

tie

→ RHS value contain zero

3-52.

max $z = 3x_1 + 2x_2$.

st. $4x_1 - x_2 \overset{+S_1}{\leq} 4$.

$4x_1 + 3x_2 \overset{+S_2}{\leq} 6$.

$4x_1 + x_2 \overset{+S_3}{\leq} 4$

$x_1, x_2 \geq 0$

$z - 3x_1 - 2x_2 - s_1 - s_2 - s_3$

Basic	x_1	x_2	s_1	s_2	s_3	RHV	ratio
z	-3	-2	0	0	0	0	0
s_1	4	-1	1	0	0	4	1
s_2	4	3	0	1	0	6	3/2
s_3	4	1	0	0	1	4	1

tie

→ Imp.

- Since there is tie and both variables tieing are slack, so we can select either, if there was one decision variable we would select slack variable to leave

-ve ratio

$R_1 + 3\left(\frac{R_2}{4}\right)$, $\frac{R_2}{4}$, $R_3 - R_2$, $R_4 - R_2$

	x_1	x_2	s_1	s_2	s_3	RHV	Ratio
z	0	$-\frac{11}{4}$	3/4	0	0	3	–
x_1	1	$-1/4$	1/4	0	0	1	–
s_2	0	4	-1	1	0	2	1/2
s_3	0	2	-1	0	1	0	0

leaving

min ratio

$R_1 + \frac{11}{4}\left(\frac{R_4}{2}\right), R_3 + \frac{1}{4}\left(\frac{R_4}{2}\right), R_3 - 4\left(\frac{R_4}{2}\right), \frac{R_4}{2}$

Basic	x_1	x_2	S_1	S_2	S_3	RHV	Ratio
Z	0	0	$-5/8$	0	$11/8$	3	invalid
x_1	1	0	$1/8$	0	$1/8$	1	8
S_2	0	0	1	1	-2	2	2
x_2	0	1	$-1/2$	0	$1/2$	0	0

again 0 we will be stuck in cycle.

"we use Purturbation method"

So, we choose value of min. ratio, above 0 if we had decision variable in that row

$R_1 + 5/8 R_3, R_2 - \frac{1}{8} R_3, R_4 + \frac{1}{2} R_3$

Basic	x_1	x_2	S_1	S_2	S_3	
Z	0	0	0	$5/8$	$1/8$	$17/4$
x_1	1	0	0	$-1/8$	$3/8$	$3/4$
S_1	0	0	1	1	-2	2
x_2	0	1	0	$1/2$	$-1/2$	1

Basic

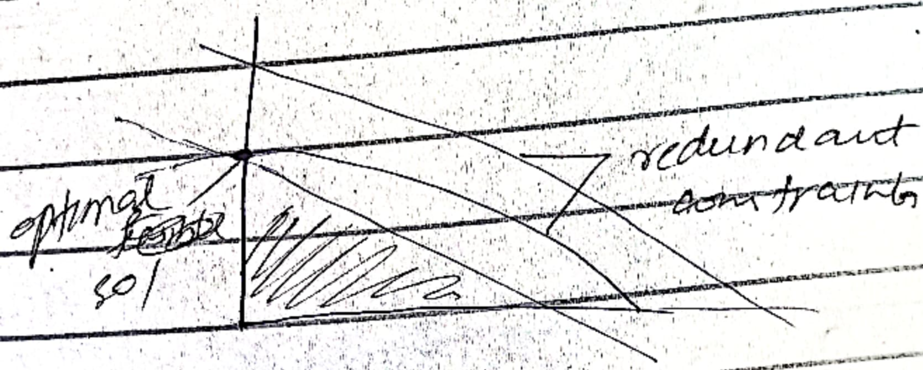
optimal $17/4$

Alternative Optima:

→ when objective fun is parallel to non-redundant binding constraint

optimal sol remains same, but corresponding value alternates

Constraints, which don't bound feasible region → redundant constraints



$$MAX \quad z = x_1 + 2x_2 + 3x_3.$$

s.t

$x_1 + 2x_2 + 3x_3 + s_1 \leq 10$

$x_1 + x_2 + s_2 \leq 5$

$x_1 + s_3 \leq 1$

Identify three basic sol.

	x_1	x_2	x_3	s_1	s_2	s_3	RHV	
Z	-1	-2	-3	0	0	0	0	
S_1	1	2	3	1	0	0	10	10/3
S_2	1	1	0	0	1	0	5	∞
S_3	1	0	0	0	0	1	1	∞

$R_1 + R_2$, $R_2/3$, $x_3 \to$ incoming
$S_1 \to$ outgoing

	x_1	x_2	x_3	s_1	s_2	s_3	
Z	0	0	0	1	0	0	10
X_3	1/3	2/3	1	1/3	0	0	10/3
S_2	1	1	0	0	1	0	5
S_3	1	0	0	0	0	1	1

Sol 1 $x_3 = 10/3$, $X_1 = 0$, $X_2 = 0$
$S_1 = 0$, $S_2 = 5$, $S_3 = 1$