M-Method

$$\text{Min } Z = 8x_1 + 4x_2$$

$$3x_1 + x_2 \geq 27$$
$$x_1 + x_2 = 21$$
$$x_1 + 2x_2 \leq 40$$
$$x_1, x_2 \geq 0$$

Standard Form

$$\text{Max } Z = -8x_1 - 4x_2 + 0S_1 + 0S_2 - MA_1 - MA_2$$

$$3x_1 + x_2 - S_1 + A_1 = 27$$
$$x_1 + x_2 + A_2 = 21$$
$$x_1 + 2x_2 + S_2 = 40$$
$$x_1, x_2, S_1, S_2, A_1, A_2 \geq 0$$

	C_j		-8	-4	0	0	$-M$	$-M$	
C_{Bi}	Basic	Sol	x_1	x_2	S_1	S_2	A_1	A_2	Ratio
$-M$	A_1	27	3	1	-1	0	1	0	$27/3 = 9$ ←
$-M$	A_2	21	1	1	0	0	0	1	$21/1 = 21$
0	S_2	40	1	2	0	1	0	0	$40/1 = 40$
		$-48M$	$-4M$	$-2M$	M	0	$-M$	$-M$	
	$C_j - Z_j$		$-8+4M$ ↑	$-4+2M$	$-M$	0	0	0	
-8	x_1	9	1	$1/3$	$-1/3$	0	$1/3$	0	$9/1/3 = 27$
$-M$	A_2	12	0	$2/3$	$1/3$	0	$-1/3$	1	$12/2/3 = 18$ ←
0	S_2	31	0	$5/3$	$1/3$	0	$-1/3$ ↓	0	$31 * 3/5 = 18.6$
	Z_j	$-72-12M$	-8	$-8/3 - 2/3M$ ↑	$8/3 - M/3$		$-8/3 + 1/3M$	$-M$	
	$C_j - M_j$		0	$-4/3 + 2/3M$	$-8/3 + M/3$		$8/3 - 2/3M$	0	
-8	x_1	3	1	0	$-1/2$	0	$1/2$	$-1/2$	
-4	x_2	18	0	1	$1/2$	0	$-1/2$	$3/2$	
0	S_2	1	0	0	$-1/2$	1	$1/2$	$-5/2$	
	Z_j	-96	-8	-4	2	0	-2	-2	
	$C_j - Z_j$		0	0	-2	0	$-M+2$	$-M+2$	$C_j - Z_j \leq 0$

Optimal Solution

Max $Z = -96$ Min $(-Z) = -(-96) = 96$

$Z = 96$ $x_1 = 3$

$x_2 = 18$