$h = x - x_0$

Taylor:

$$y_1 = y_0 + hy_0' + \frac{h^2}{2!} y_0'' + \frac{h^3}{3} y_0''' \dots$$

simultaneous 1st order eq:

$$y_1 = y_0 + hy_0' + \frac{h^2}{2!} y_0'' + \frac{h^3}{3} y_0''' + \dots$$

$$z_1 = z_0 + hz_0' + \frac{h^2}{2!} z_0'' + \frac{h^3}{3} z_0'''$$

Picard:

$$y_1^{(n)} = y_0 + \int_{x_0}^{x} f(x, y_1^{(n-1)})$$

Euler, simple: $y_{n+1} = y_n + hf(x_n, y_n)$

modified: $y_{n+1} = y_n + hf\left(x_n + \frac{h}{2}, y_n + \frac{h}{2} f(x_n, y_n)\right)$

improved: $y_{n+1} = y_n + \frac{h}{2}\left[f(x_n, y_n) + f(x_n + h, y_n + hf(x_n, y_n))\right]$

R.K 2nd order: $k_1 = hf(x, y)$, $k_2 = hf\left(x + \frac{h}{2}, y + \frac{k_1}{2}\right)$, $\Delta y = k_2$, $y_n = y_{n-1} + \Delta y$

3rd order: k_1 and k_2 are same as 2nd order;

$k_3 = hf(x + h, y + 2k_2 - k_1)$, $\Delta y = \frac{1}{6}(k_1 + 4k_2 + k_3)$, $y_n = y_{n-1} + \Delta y$,

4th order: k_1 and k_2 are same as 2nd order. $k_3 = hf(x + h/2, y + k_2/2)$

$k_4 = hf(x + h, y + k_3)$, $\Delta y = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$, $y_n = y_{n-1} + \Delta y$

R.K for simultaneous first order equation: $dy/dx = f(x, y, z)$, $dz/dx = g(x, y, z)$

$k_1 = hf(x, y, z)$, $k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}, z_0 + \frac{l_1}{2}\right)$, $k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}, z_0 + \frac{l_2}{2}\right)$

$k_4 = hf(x + h, y + k_3, z + k_3)$, $\Delta y = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$, $y_n = y_{n-1} + \Delta y$,

for l_1, l_2, l_3, l_4, use g instead of f used in k_1, k_2, k_3, k_4 respectively. Parameter and everything other thing is same.

$\Delta z = \frac{1}{6}(l_1 + 2l_2 + 2l_3 + l_4)$, $z_n = z_{n-1} + \Delta z$,

Milne: P: $y_{n+1} = y_{n-3} + \frac{4h}{3}(2y_{n-2}' - y_{n-1}' + 2y_n')$.

C: $y_{n+1} = y_{n-1} + \frac{h}{3}(y_{n-1}' + 4y_n' + y_{n+1}')$.

Adam's: P: $y_{n+1} = y_n + \frac{h}{24}\left[55y_n' - 59y_{n-1}' + 37y_{n-2}' - 9y_{n-3}'\right]$

C: $y_{n+1} = y_n + \frac{h}{24}\left[9y_{n+1}' + 19y_n' - 5y_{n-1}' + y_{n-2}'\right]$, Finite Diff. $y_i'' = \dfrac{y_{i+1} + y_{i-1} - 2y_i}{h^2}$, $y = y_i$,

Date: Replace y'', y with these values. Day: M T W T F S

Root finding:

Bisection: $x_1 = \cancel{\dfrac{x(a)+x(b)}{2}}$ ~~s.t f(x(a))f(x(b))~~

$x = \dfrac{a+b}{2}$, s.t $f(a) \times f(b) < 0$

$N \geq \dfrac{\log(b-a) - \log e}{\log 2}$.

$E_a = |x - x'|$ $E_p = 100 E_r$,

$E_r = \left|\dfrac{x - x'}{x}\right|$, a = abs, r = rel. P = perc.

false: $\dfrac{a f(b) - b f(a)}{f(b) - f(a)}$ $\boxed{\text{conv.} = 1}$ $f(a) f(b) < 0$

iterative: Rearrange fn. s.t $f'(a) < 1$, $f'(b) < 1$ $f(a) f(b) < 0$

$x_k = f(x_{k-1})$ ~~take x_0~~ initially $a < x_0 < b$

Newton: $\boxed{\text{convergen} = 2}$ $x_k = x_{k-1} - \dfrac{f(x_{k-1})}{f'(x_{k-1})}$ $f(a) f(b) < 0$

initially $a < x_0 < b$

with multiplicity P, $x_k = x_{k-1} - \dfrac{P f(x_{k-1})}{f'(x_{k-1})}$

Secant: $x_{k+1} = \cancel{x_k + \dfrac{x_{k-1} f(x_k) - x_k f(x_{k-1})}{f(x_k) - f(x_{k-1})}}$,

$\boxed{\text{conv.} : 1.68}$ $x_{k+1} = x_k - \dfrac{(x_k - x_{k-1}) f(x_k)}{f(x_k) - f(x_{k-1})}$

$$P = \frac{x - x_0}{h}$$

Newton Forward:

$$y(x) = y_0 + P\Delta y_0 + \frac{P(P-1)}{2!}\Delta^2 y_0 + \frac{P(P-1)(P-2)}{3!}\Delta^3 y_0 + \dots$$

Date: ______

Newton Backward:

$$y(x) = y_n + P\nabla y_n + \frac{P(P+1)}{2!}\nabla^2 y_n + \frac{P(P+1)(P+2)}{3!}\nabla^3 y_n + \dots$$

Gauss forward:

$$y(x) = y_0 + P\Delta y_0 + \frac{P(P-1)}{2!}\Delta^2 y_{-1} + \frac{(P+1)P(P-1)}{3!}\Delta^3 y_{-1} + \frac{(P+1)P(P-1)(P-2)}{4!}\Delta^4 y_{-2} + \dots$$

$(0 < P < 1)$

Gauss Backward:

$$y(x) = y_0 + P\Delta y_{-1} + \frac{P(P+1)}{2!}\Delta^2 y_{-1} + \frac{(P-1)P(P+1)}{3!}\Delta^3 y_{-2} + \frac{(P-1)P(P+1)(P+2)}{4!}\Delta^4 y_{-2}$$

$\hookrightarrow (-1 < P < 0)$

Stirling (average of g. bk and g. forward)

$$y(x) = y_0 + P\left(\frac{\Delta y_0 + \Delta y_{-1}}{2}\right) + \frac{P^2}{2!}\Delta^2 y_{-1} + \frac{P(P^2-1)}{3!}\,\frac{\Delta^3 y_{-1} + \Delta^3 y_{-2}}{2} + \frac{P^2(P^2-1)}{4!}\Delta^4 y_{-2}$$

$\hookrightarrow \left(-\tfrac{1}{2} \le P \le \tfrac{1}{2}\right)$

Bessel:

$$y(x) = \frac{y_0 + y_1}{2} + (P - \tfrac{1}{2})\Delta y_0 + \frac{P(P-1)}{2!}\left(\frac{\Delta^2 y_{-1} + \Delta^2 y_0}{2}\right) + \frac{(P-\tfrac{1}{2})P(P-1)}{3!}\cdot\Delta^3 y_{-1} + \frac{(P+1)P(P-1)(P-2)}{4!}\cdot\frac{\Delta^4 y_{-2} + \Delta^4 y_{-1}}{2}$$

$$+ \frac{(P-\tfrac{1}{2})(P+1)P(P-1)(P-2)}{5!}\Delta^5 y_{-2}$$

$\left(\tfrac{1}{4} \le P \le \tfrac{3}{4}\right)$

Lagrange: suppose we have

x_0	x_1	---	x_n
y_0	y_1	---	y_n

$$y(x) = \frac{(x-x_1)(x-x_2)\cdots(x-x_n)}{(x_0-x_1)(x_0-x_2)\cdots(x_0-x_n)} \times y_0 + \frac{(x-x_0)(x-x_2)\cdots(x-x_n)}{(x_1-x_0)(x_1-x_2)\cdots(x_1-x_n)} \times y_1 \cdots \frac{(x-x_0)(x-x_1)\cdots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\cdots(x_n-x_{n-1})} \times y_n$$

Newton divided formulae: (by ex)

$$f(x) = y_0 + (x-x_0)\Delta y_0 + (x-x_0)(x-x_1)\Delta^2 y_0 + (x-x_0)(x-x_1)(x-x_2)\Delta^3 y_0$$

x_0 5 y_0 12 $\quad \Delta y_0 = \frac{13-12}{6-5} = 1 \quad \Delta^2 y_0 = \frac{1/3 - 1}{9-5} = -1/6 \quad \Delta^3 y_0 = \frac{2/15 + 1/6}{11-5} = \frac{1}{20}$

x_1 6 y_1 13 $\quad \Delta y_1 = \frac{14-13}{9-6} = \frac{1}{3} \quad \Delta^2 y_1 = \frac{1 - 1/3}{11-6} = 2/15$

x_2 9 y_2 14 $\quad \Delta y_2 = \frac{16-14}{11-9} = 1$

x_3 11 y_3 16

inverse lagrange: (y is given, to find x)

$$x = \frac{(y-y_1)(y-y_2)\cdots(y-y_n)}{(y_0-y_1)(y_0-y_2)\cdots(y_0-y_n)}(x_0) + \frac{(y-y_0)(y-y_2)\cdots(y-y_n)}{(y_1-y_0)(y_1-y_2)\cdots(y_1-y_n)}(x_1) + \dots + \frac{(y-y_0)(y-y_1)\cdots(y-y_{n-1})}{(y_n-y_0)(y_n-y_1)\cdots(y_n-y_{n-1})}(x_n)$$

- Unsupervised learning: (no labels).
  - ↳ clustering: grouping similar data points
  - ↳ association: finding relationship b/w data points.

Trapezoidal: any no. of interval.

$$\int_a^b f(x)\,dx = \frac{h}{2}\left(y_0 + y_n + 2(y_1 + y_2 + \cdots y_{n-1})\right)$$

simpson (1/3) (even no of intervals).

$$\int_a^b f(x)\,dx = \frac{h}{3}\left(y_0 + y_n + 4(y_1 + y_3 + \cdots y_{2n-1}) + 2(y_2 + y_4 + \cdots y_{2n-2})\right)$$

simpson (3/8) (multiple of 3.

$$\int_a^b f(x)\,dx = \frac{3h}{8}\left(y_0 + y_n + 3(y_1 + y_2 + y_4 + y_5 + y_7 + \cdots) + 2(y_3 + y_6 + y_9 + \cdots)\right)$$

Inverse Interpolation: (Newton divided successive approx.) we will be given y, we will calculate "x"

$$P_1 = \frac{1}{\Delta y_0}(\underset{\text{given}}{y} - y_0)$$

$$P_2 = \frac{1}{\Delta y_0}\left(y - y_0 - \frac{P_1(P_1 - 1)}{2!}\Delta^2 y_0\right)$$

$$P_3 = \frac{1}{\Delta y_0}\left(y - y_0 - \frac{P_2(P_2 - 1)}{2!}\Delta^2 y_0 - \frac{P_2(P_2-1)(P_2-2)}{3!}\Delta^3 y_0\right)$$

$$\vdots$$

$$P_n = \frac{1}{\Delta y_0}\left(y - y_0 - \frac{P_{n-1}(P_{n-1} - 1)}{2!}\Delta^n y_0 + \cdots\right)$$

we will go untill we have two equal successive p's upto 3 defined place → now put that p in $x = Ph + x_0$