Simplex method

Consider the following set of constraints:

$$x_1 + 2x_2 + 2x_3 + 4x_4 \le 40$$
$$2x_1 - x_2 + x_3 + 2x_4 \le 8$$
$$4x_1 - 2x_2 + x_3 - x_4 \le 10$$
$$x_1, x_2, x_3, x_4 \ge 0$$

Minimize $z = 5x_1 - 4x_2 + 6x_3 - 8x_4$.

Simplex method

$$x_1 + 2x_2 + 2x_3 + 4x_4 + s_1 = 40$$
$$2x_1 - x_2 + x_3 + 2x_4 + s_2 = 8$$
$$4x_1 - 2x_2 + x_3 - x_4 + s_3 = 10$$
$$x_1, x_2, x_3, x_4, s_1, s_2, s_3 \ge 0$$

$$z - 5x_1 + 4x_2 - 6x_3 + 8x_4 + 0s_1 + 0s_2 + 0s_3 = 0$$

Simplex method

Consider the following set of constraints:

$$\begin{aligned} x_1 + 2x_2 + 2x_3 + 4x_4 &\le 40 \\ 2x_1 - x_2 + x_3 + 2x_4 &\le 8 \\ 4x_1 - 2x_2 + x_3 - x_4 &\le 10 \\ x_1, x_2, x_3, x_4 &\ge 0 \end{aligned}$$

$$\begin{aligned} x_1 + 2x_2 + 2x_3 + 4x_4 + s_1 &= 40 \\ 2x_1 - x_2 + x_3 + 2x_4 + s_2 &= 8 \\ 4x_1 - 2x_2 + x_3 - x_4 + s_3 &= 10 \\ x_1, x_2, x_3, x_4, s_1, s_2, s_3 &\ge 0 \end{aligned}$$

Minimize $z = 5x_1 - 4x_2 + 6x_3 - 8x_4$.

$$z - 5x_1 + 4x_2 - 6x_3 + 8x_4 + 0s_1 + 0s_2 + 0s_3 = 0$$

Basic	x1	x2	x3	x4	S1	S2	S3	solution
z	-5	4	-6	8	0	0	0	0
S1 row	1	2	2	4	1	0	0	40
S2 row	2	-1	1	2	0	1	0	8
S3row	4	-2	1	-1	0	0	1	10

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$$4x_1 - 2x_2 + x_3 - x_4 \le 10$$
$$x_1, x_2, x_3, x_4 \ge 0$$

Simplex method

Minimize $z = 5x_1 - 4x_2 + 6x_3 - 8x_4$.

$$z - 5x_1 + 4x_2 - 6x_3 + 8x_4 + 0s_1 + 0s_2 + 0s_3 = 0$$

Basic	x1	x2	x3	x4	S1	S2	S3	solution
z	-5	4	-6	8	0	0	0	0
S1 row	1	2	2	4	1	0	0	40
S2 row	2	-1	1	2	0	1	0	8
S3row	4	-2	1	-1	0	0	1	10
z	-13	8	-10	0	0	-4	0	-32
S1	-3	4	0	0	1	-2	0	24
x4	1	$-\frac{1}{2}$	$\frac{1}{2}$	1	0	$\frac{1}{2}$	0	4
S3	5	$-\frac{5}{2}$	$\frac{3}{2}$	0	0	$\frac{1}{2}$	1	14
z	-7	0	-10	0	-2	0	0	-80
x2	$-\frac{3}{4}$	1	0	0	$\frac{1}{4}$	$-\frac{1}{2}$	0	6
x4	$\frac{5}{8}$	0	$\frac{1}{2}$	1	$\frac{1}{8}$	$\frac{1}{4}$	0	7
S3	$\frac{25}{8}$	0	$\frac{3}{2}$	0	$\frac{5}{8}$	$-\frac{3}{4}$	1	29

Summery of Simplex method

- **Optimality condition.** The entering variable in a maximization (minimization) problem is the *nonbasic* variable with the most negative (positive) coefficient in the z-row.
- **Feasibility condition.** For both the maximization and the minimization problems, the leaving variable is the *basic* variable associated with the smallest nonnegative ratio with *strictly positive* denominator. Ties are broken arbitrarily.
- ***Gauss-Jordan row operations.***
- **1.** *Pivot row*
  - **a.** Replace the leaving variable in the *Basic* column with the entering variable.
  - **b.** New pivot row = Current pivot row $\div$ Pivot element
- **2.** *All other rows, including z*
  - New row = (Current row) $-$ (Its pivot column coefficient) $*$ (New pivot row).
- The optimum is reached at the iteration where all the z-row coefficients are nonnegative (nonpositive).