Calculus I

Useful Symbols:-

$\equiv$ $\equiv$ Identical to $\equiv$ مشابہ ہے

$=$ $\equiv$ Equation $\equiv$ مساوات

e.g. $\equiv$ For example $\equiv$ مثلاً $\equiv$ exemple gratia

s.t $\equiv$ $\ni$ $\equiv$ Such That $\equiv$ جبکہ

i.e. $\equiv$ That is $\equiv$ یعنی $\equiv$ id est

viz. $\equiv$ namely $\equiv$ یعنی $\equiv$ videlicet

No. $\equiv$ Number $\equiv$ عدد $\equiv$ nombre

w.r.t. $\equiv$ with respect to $\equiv$ بلحاظ

$\therefore$ $\equiv$ Therefore $\equiv$ اسلئے

$\because$ $\equiv$ Because $\equiv$ چونکہ

$\Longrightarrow$ $\equiv$ Implies That $\equiv$ سے اخذ ہوتا ہے

$$\left[\left\{\left(\overline{}\right)\right\}\right]$$

Bar

Square bracket   Parentheses   Braces

$>$ $\equiv$ is greater Than $\equiv$ بڑا ہے

$<$ $\equiv$ is less Than $\equiv$ چھوٹا ہے

N.B. $\equiv$ Note well $\equiv$ قابل غور $\equiv$ Nota bene

$\forall$ $\equiv$ For all $\equiv$ عام

Paper's Pattern:- (Expected) 10% Similar to Exs; 20% Similar to Assignments
20% Class work (with some changes); 50% unseen, Model questions etc.

بسم اللہ الرحمن الرحیم

Calculus

Books Recommended. (1) Thomas' Calculus
(Calculus and Analytic Geometry)
by George B. Thomas

4. Calculus
by E. Boyce & Richard DiPrima

5. Calculus
by Tom. M. Apostol

2. Calculus & " "
by Howard Antum

3. " " "
by Sherman K. Stien

Thomas ch 1. Preliminaries:-
Def. :- Def :- Def is an agreement. - -
Def. Arithmatic (حساب):-
Def. Mathematics (Maths ریاضی):-
Def. Mathematician:- A - - -

واللہ جمیل و یحب الجمال

God is beautiful and He loves beautiful persons.

Def. Calculus:-

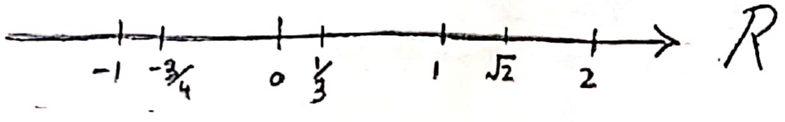
Def. Real Nos.:- These are the Nos. that can be expressed as decimals such as

$$-\frac{3}{4} = -0.75000\ldots$$

$$\frac{1}{3} = 0.3333\ldots$$

$$\sqrt{2} = 1.4142\ldots$$

Def The real Nos. can be expressed as pts. on a No. line called the real line.



Rules for inequalities :-

If $a, b, c \in R$, Then

1. $a < b \implies a+c < b+c$

e.g. $3 < 10$

$\implies 3+2 < 10+2$

2. $a < b \implies a-c < b-c$

e.g. $3 < 10$

$\implies 3-2 < 10-2$

3. $a < b$ & $c > 0$

$\implies ac < bc$

e.g. For $3, 8 \in R$

$4 > 0$

Then $3 < 8 \implies 3 \cdot 4 < 8 \cdot 4$

4. $a < b$ & $c < 0 \implies bc < ac$

Also $a < b \implies -a > -b$

e.g. $3 < 9 \implies -3 > -9$

5. $a > 0 \implies \frac{1}{a} > 0$

e.g. $3 > 0 \implies \frac{1}{3} > 0$

6. If a & b are both +ve or −ve.

Then $a < b \implies \frac{1}{b} < \frac{1}{a}$

Def. Set of Natural Nos. $= \{1, 2, 3, \ldots\}$

" " Integers $= \mathbb{Z} = \{0, \pm1, \pm2, \ldots\}$

" " Rational Nos $= \mathbb{Q} = \{ a/b \mid a, b \in \mathbb{Z}, b \neq 0\}$

Def An Irrational No. is a No. which has non terminating & non repeating decimal representation :-

e.g. π, $\sqrt{2}$, $\sqrt[3]{5}$ & $\log_{10} 3$ ...

Def The set of irrational nos. $= Q' = \{\pi, \sqrt{2}, \sqrt[3]{5}, \ldots\}$

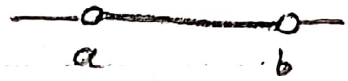
Def **Interval**: It is a subset of the real line which contain at least two nos. and all the nos. lying between any two of its elements.

The interval corresponding to line segment are called **Finite Interval** (See fig(i)) whereas the interval corresponding to ray of real line are called **Infinite Interval**. (See fig (ii))

fig (i)

Finite intervals

$(a, b) \equiv \{x \mid a < x < b\}$

$\equiv$ 

$[a, b] \equiv$ 

$[a, b) \equiv$ 

$(a, b] \equiv$ 

fig (ii)

Infinite intervals

Half open

open interval

$(a, \infty) \equiv \{x \mid x > a\}$ 

$[a, \infty)$

$(-\infty, b) \equiv \{x \mid x < b\}$

$(-\infty, b] \equiv \{x \mid x \le b\}$ 

$(-\infty, \infty) = R$ 

In extended domain

These are known as closed interval

Ex 1 (a) P. 4

$2x-1 < x+3$

$\Rightarrow 2x-1+1 < x+3+1$

or $2x < x+4$

$\Rightarrow 2x-x < x+4-x$

$\Rightarrow x < 4$

$\therefore$ S.S $= (-\infty, 4)$

Ex. 1 (b)

$-\frac{x}{3} < 2x+1$

$\Rightarrow 3(-x/3) < 3(2x+1)$

$\Rightarrow -x < 6x+3$

$\Rightarrow -x+x < 6x+3+x$

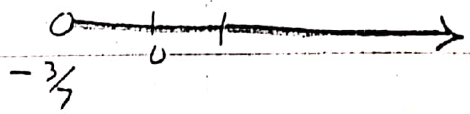
$\Rightarrow 0 < 7x+3$

$\Rightarrow 0-3 < 7x+3-3$

or $-3 < 7x \Rightarrow \frac{1}{7}(-3) < \frac{1}{7}(7x)$

$\Rightarrow -\frac{3}{7} < x$

$\therefore$ S. S. $= (-3/7, \infty)$



(c) Ex. 1 (c) $\frac{6}{x-1} \geq 5$

$\left\{ \begin{array}{l} x-1 \neq 0 \\ \text{i.e. } x \neq 1 \end{array} \right.$

obviously it holds only if

$x-1 > 0$ i.e. $x-1$ is +ve

$\Rightarrow x > 1$ ———— (i)

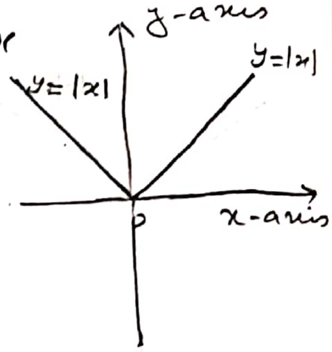
Now $\frac{6}{x-1} \geq 5$

$\Rightarrow \ldots \Rightarrow x \le \frac{11}{5}$

$\therefore$ S.S. $= (1, 11/5]$

(c) Def Absolute value of a No. x

(James Stewart Page # 19 Example # 8) $= |x| = \begin{cases} x & x \ge 0 \\ -x & x < 0 \end{cases}$



e.g. Let $x = 3$ — (i)

Then $|x| = |3| = 3 = x$

If $x = -5$ — (ii)

Then $|x| = 5 = -(-5)$

$= -x$ (using $\equiv$ (ii))

N.B. (i) $|-a| = |a|$, ~~for a > 0~~

(ii) $|ab| = |a|\,|b|$

(iii) $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$

(iv) $|a+b| \le |a| + |b|$ — Triangle Inequality

Exs ---, ---, ---, ---

Ex 4 P6 Sol. $|2x-3| = 7$

$\Rightarrow \pm(2x-3) = 7 \Rightarrow (2x-3) = \pm 7$

$\Rightarrow 2x = \pm 7 + 3$

$= 7+3, -7+3$

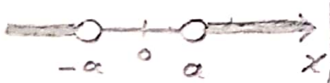
$= 10, -4$

$\Rightarrow x = 5, -2$, as reqd.

N.B. Generally $|x| < a$ represents interior of the interval & it has a common sol. without any gap set, whereas $|x| > a$ gives exterior of the interval & in general its sol. consists of union of intervals with a certain gap between them.

$|x| < a$

& $|x| > a$



Def. For a +ve No. D

$$|a| < D \iff -D < a < D.$$

$$\cancel{|a| \leq D \iff -D \leq a \leq D.}$$

e.g. Let $|a| \leq 3$

$\Rightarrow \pm a \leq 3$

$\Rightarrow a \leq 3 \quad \text{(i)} \quad \& \quad -a \leq 3$

$\Rightarrow a \geq -3$

or $-3 \leq a \quad \text{(ii)}$

$\therefore$ From rels. (i) & (ii), we get

$$-3 \leq a \leq 3$$

i.e. $|a| \leq 3 \Rightarrow -3 \leq a \leq 3$ — (I)

Conversely let $-3 \leq a \leq 3$

$\Rightarrow -3 \leq a \Rightarrow 3 \geq -a$ or $-a \leq 3$ (iii)

& $a \leq 3$ —— (iv)

$\therefore$ From rels (iii) & (iv), we get

$\pm a \leq 3$ i.e. $|\alpha| \leq 3$

i.e $-3 \leq a \leq 3 \Rightarrow |a| \leq 3$ (II)

$\therefore$ From rels. (I) & (II), we get

$$|a| \leq 3 \iff -3 \leq a \leq 3$$

Ex 5 / P-6

$$\left|5 - \frac{2}{x}\right| < 1 \iff -1 < 5 - \frac{2}{x} < 1$$

$$\Rightarrow -1 - 5 < 5 - \frac{2}{x} - 5 < 1 - 5$$

$$\text{or} \quad -6 < -\frac{2}{x} < -4$$

$$\Rightarrow (-\tfrac{1}{2})(-6) > (-\tfrac{1}{2})(-\tfrac{2}{x}) > (-\tfrac{1}{2})(-4)$$

$$\Rightarrow 3 > \frac{1}{x} > 2$$

$$\Rightarrow \frac{1}{3} < x < \frac{1}{2}$$

$$\therefore \; S.S. = (\tfrac{1}{3}, \tfrac{1}{2})$$

Ex 6 (a) $|2x-3| \le 1$

P 7

$$\Rightarrow -1 \le 2x-3 \le 1$$

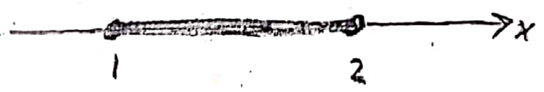
$$\Rightarrow -1+3 \le 2x-3+3 \le 1+3$$

$$\text{or} \quad 2 \le 2x \le 4$$

$$\Rightarrow \frac{1}{2}\cdot 2 \le \tfrac{1}{2}\, 2x \le \tfrac{1}{2}\cdot 4$$

$$\Rightarrow 1 \le x \le 2 \qquad \therefore S.S. = [1, 2]$$

See fig



(b) $|2x-3| \ge 1$

$$\Rightarrow \pm(2x-3) \ge 1$$

$$\Rightarrow 2x-3 \ge 1 \quad \text{or} \quad -(2x-3) \ge 1$$

$$\Rightarrow 2x-3 \le -1$$

$$- - - - \quad x \ge 2 \qquad \Rightarrow \cdots \quad x \le 1$$

$$\therefore \; S.S. = (-\infty, 1] \cup [2, \infty)$$

See fig

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