Blur Filters

Blur Filters - Box Filters

1. It can be used to reduce noise,
2. to reveal image structures at different scales, or
3. for upsampling and downsampling images
4. Blurring is implemented by computing local averages over small neighborhoods of input pixel values. This can be done by a convolution
5. There are also non-linear approaches for blurring

Down-Sampling (Reducing Image Size)

1. A pre-blur filter is applied before resizing to smooth out high-frequency components and prevent artifacts.
   a. Reduces aliasing by removing high-frequency details.
   b. Ensures a more uniform representation of the image at a smaller scale.
   c. Prevents jagged edges

Up-Sampling (Increasing Image Size)

1. A post-blur filter is used after interpolation to smooth out pixel transitions.
   a. Reduces blocky artifacts and jagged edges from interpolation.
   b. Makes the up-scaled image appear more natural and less pixelated.
   c. Helps blend interpolated pixels smoothly with existing pixels.

(a) (b) (c) (d)

Figure 17.2: (a) Input image. (b) Blurring with a square, (c) a horizontal, and (d) a vertical line. Each color channel is filtered independently.

Properties of Box Filter

1. The box filter is a low-pass filter. That is, it attenuates the high spatial-frequency content of the input image.
2. 2D box filter is separable
3. The box filter is not a perfect blurring filter. A blur filter should attenuate high spatial frequencies with stronger attenuation for higher spatial frequencies.
    a. Box = [1, 1, 1]
    b. “[…, 1, –1, 1, –1, 1, –1, …]”
    c. “[…, 0.5, 0.5, –1, 0.5, 0.5, –1, …]”
4. It can cause artifacts to appear

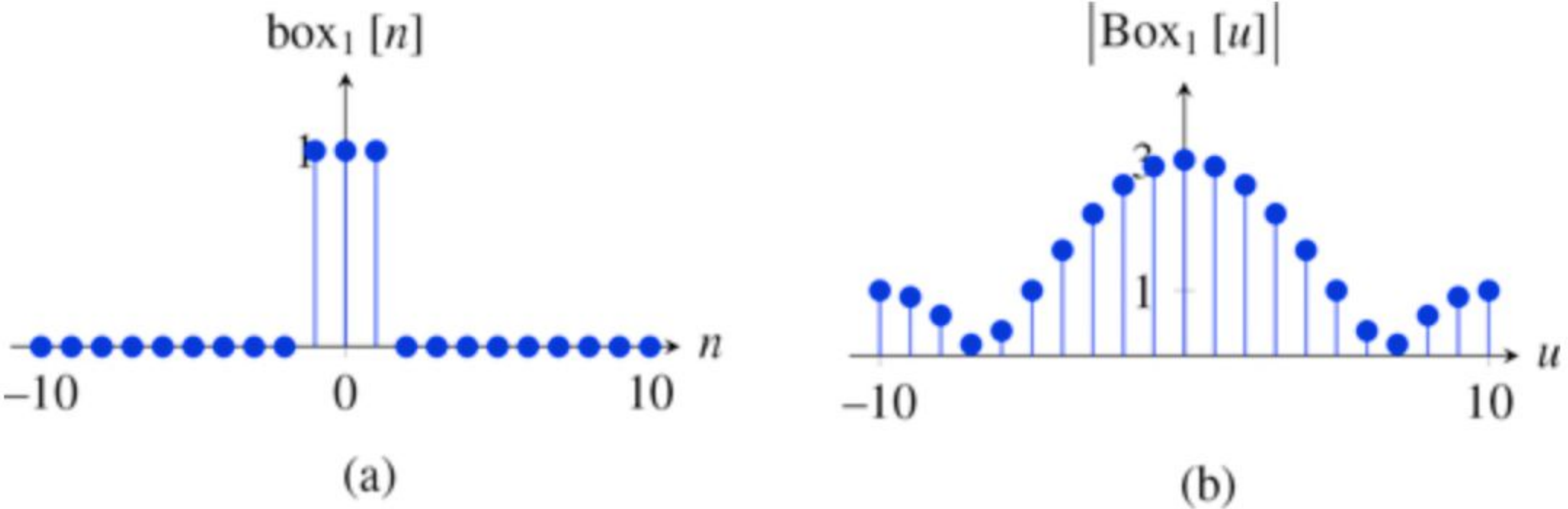

Figure 17.3: (a) A one-dimensional (1D) box filter ([1, 1, 1]), and (b) its Fourier transform over 20 samples. Note that the frequency gain is not monotonically decreasing with spatial frequency.

Properties of Box Filter

1. If you convolve two boxes you do not get another box. Instead you get a triangle.
2. It means that if you blur an image twice with box filters, what you get is not the equivalent to blurring only once with a larger box filter

Gaussian Filter

1. Addresses problems of box filter
2. 2D Gaussian filter is separable (2 1D filters)

Blur Filters

1. “Another application of blurring is to remove distracting high-resolution image details.”
2.

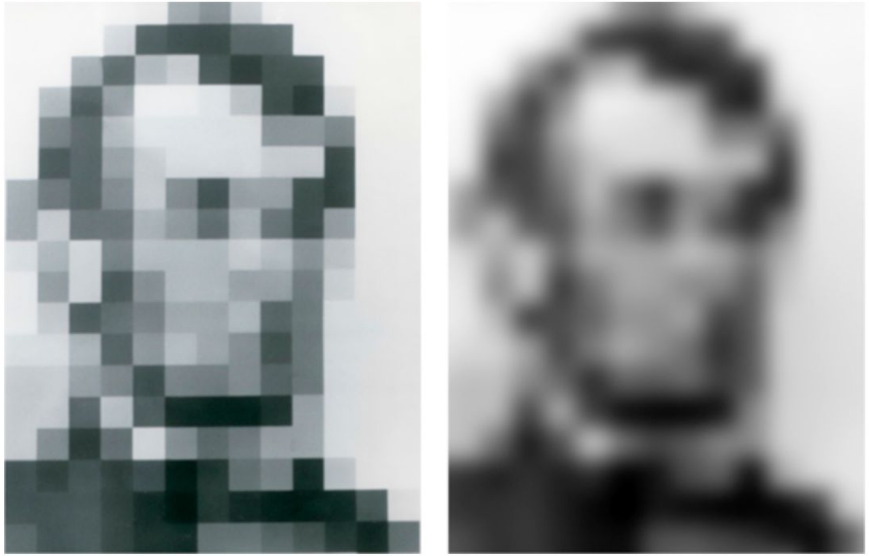

Figure 17.5: (Left) Input image. (Right) Blurred version. The left version has many spurious details introduced by the blocky style of the image. The right image has been blurred by a large Gaussian filter. *Source*: Image by Bela Julesz and Leon Harmon, 1971 [185].

Properties of Continuous Gaussian

1. “The n-dimensional Gaussian is the only completely circularly symmetric operator that is separable.”
2. “The continuous Fourier transform (FT) of a Gaussian is also a Gaussian.”
3. “The width of the Gaussian Fourier transform decreases with σ”
4. “The convolution of two n-dimensional Gaussians is an n-dimensional Gaussian”

Properties of Continuous Gaussian

1. Due to discretization error, the convolution of discretized Gaussians is not a Gaussian”
2. “Note that the convolution of Gaussian with the wave [1, –1, 1, –1, …] is not zero.”

Binomial Filters

1. “In practice, there are very efficient discrete approximations to the Gaussian filter that, for certain σ values, have nicer properties than when working with discretized Gaussians.”
2. “One common approximation of the Gaussian filter is to use binomial coefficients. Binomial filters are obtained by successive convolutions of the box filter [1, 1]”
3.

b_0									1										$\sigma_0^2 = 0$
b_1								1		1									$\sigma_1^2 = 1/4$
b_2							1		2		1								$\sigma_2^2 = 1/2$
b_3						1		3		3		1							$\sigma_3^2 = 3/4$
b_4					1		4		6		4		1						$\sigma_4^2 = 1$
b_5				1		5		10		10		5		1					$\sigma_5^2 = 5/4$
b_6			1		6		15		20		15		6		1				$\sigma_6^2 = 3/2$
b_7		1		7		21		35		35		21		7		1			$\sigma_7^2 = 7/4$
b_8	1		8		28		56		70		56		28		8		1		$\sigma_8^2 = 2$

Figure 17.6: Binomial coefficients. To build the Pascal's triangle, each number is the sum of the number above to the left and the one above to the right.

Binomial Filters - Properties

1. “Binomial coefficients provide a compact approximation of the Gaussian coefficients using only integers.”
2. “One remarkable property of the binomial filters is that bn ◦ bm = bn+m, and, therefore, , which is the analogous to the Gaussian property in the continuous domain. That is, the convolution of two binomial filters is another binomial filter.”
3. “The simplest approximation to the Gaussian filter is the 3-tap binomial kernel: b_2 = [1, 2, 1]

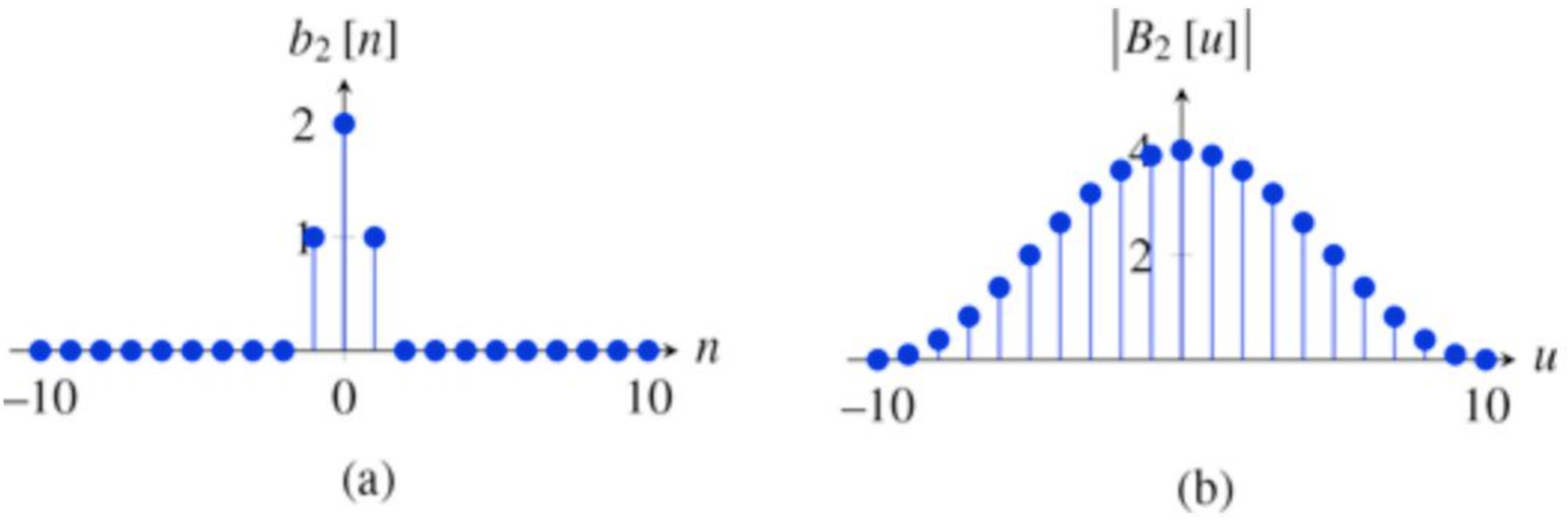

Figure 17.7: (a) A one-dimensional three-tap approximation to the Gaussian filter ([1, 2, 1]) and, (b) its Fourier transform for $N = 20$ samples.

$$B_2[u] = 2 + 2\cos(2\pi u/N)$$

Binomial Filters - Properties

1. “For all the binomial filters bn, when they are convolved with the wave [1, –1, 1, –1, …], the result is the zero signal [0, 0, 0, 0, …]. This is a very nice property of binomial filters and will become very useful later when talking about downsampling an image”

2D Binomial Filters

1. "The Gaussian in 2D can be approximated, using separability, as the convolution of two binomial filters one vertical and another horizontal."
2.

$$b_{2,2} = b_{2,0} \circ b_{0,2} = \begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \circ \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} \tag{17.16}$$

2D binomial filter:

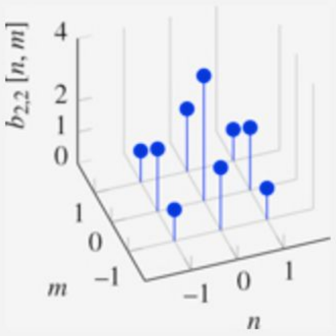

Figure 17.8: Image corrupted by a checkerboard-pattern noise (Left) and its output to two different blur kernels: (middle) 3×3 box filter. (right) Binomial filter $b_{2,2}$.

2D Binomial Filters

1. “Blur kernels are useful when
2. **building image pyramids**, or
3. **in neural networks when performing different pooling operations** or when **resizing feature maps**.”

References

1. Foundations of Computer Vision - Chapter 17