

# MID-1 - FORMULAE

$$p = \frac{x - x_0}{h}$$

$x$  = required value

$h$  = interval diff

## EQUAL-INTERVALS

NFO:

$$y_x = y_0 + p \Delta y_0 + \frac{p(p-1)}{2!} \Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!} \Delta^3 y_0$$

NBO:

$$y_x = y_0 + p \nabla y_0 + \frac{p(p+1)}{2!} \nabla^2 y_0 + \frac{p(p+1)(p+2)}{3!} \nabla^3 y_0$$

GAUSS-FORWARD:

$$y_x = y_0 + p \Delta y_0 + \frac{p(p-1)}{2!} \Delta^2 y_{-1} + \frac{p(p-1)(p+1)}{3!} \Delta^3 y_{-1} \\ + \frac{p(p-1)(p+1)(p-2)}{4!} \Delta^4 y_{-2}$$

∴

GAUSS-BACKWARD:

$$y_x = y_0 + p \Delta y_{-1} + \frac{p(p+1) \Delta^2 y_{-1}}{2!} + \frac{p(p+1)(p-1) \Delta^3 y_{-2}}{3!} + \frac{p(p+1)(p-1)(p+2) \Delta^4 y_{-2}}{4!}$$

BESSEL'S:-

$$y_x = y_0 + p \Delta y_0 + \frac{p(p-1)}{2!} \left( \frac{\Delta^2 y_{-1} + \Delta^2 y_0}{2} \right) + \frac{p(p-1)(p-0.5) \Delta^3 y_{-1}}{3!}$$

STIRLING'S:-

$$y_x = y_0 + p \left( \frac{\Delta y_0 + \Delta y_{-1}}{2} \right) + \frac{p^2}{2!} \Delta^2 y_{-1} + \frac{p(p-1)(p+1)}{3!} \left[ \frac{\Delta^3 y_{-1} + \Delta^3 y_{-2}}{2} \right] +$$

$$\frac{p^2(p-1)(p+1)}{4!} (\Delta^4 y_{-2})$$

LAPLACE-EVERETT'S:

$$y_x = \left[ p y_1 + \frac{p(p^2-1^2)}{3!} \Delta^2 y_0 + \frac{p(p^2-1^2)(p^2-2^2)}{5!} \Delta^4 y_{-1} \right] +$$

$$\left[ q y_0 + \frac{q(q^2-1^2)}{3!} \Delta^2 y_{-1} + \frac{q(q^2-1^2)(q^2-2^2)}{5!} \Delta^4 y_{-2} \right]$$

$$q = 1 - p$$

UNEQUAL INTERVALS

LANGRANGE'S:-

$$y_x = \frac{(x-x_1)(x-x_2)\dots\dots(x-x_n)}{(x_0-x_1)(x_0-x_2)\dots\dots(x_0-x_n)} y_0 +$$

(ignore 1)  $\frac{(x-x_0)(x-x_2)\dots\dots(x-x_n)}{(x_1-x_0)(x_1-x_2)\dots\dots(x_1-x_n)} y_1 +$

(ignore 2)  $\frac{(x-x_0)(x-x_1)\dots\dots(x-x_n)}{(x_2-x_0)(x_2-x_1)\dots\dots(x_2-x_n)} y_2 + \dots\dots$   
n-terms

NEWTON DIVIDED DIFFERENCE:-

$$y_x = y_0 + (x-x_0)\Delta y_0 + (x-x_0)(x-x_1)\Delta^2 y_0 + \\ (x-x_0)(x-x_1)(x-x_2)\Delta^3 y_0 \dots\dots\dots$$

NUMERICAL DIFFERENTIATION

EQUAL

NFD:-

$$\frac{dy}{dx} = \frac{1}{h} \left( \Delta y_0 + \frac{2p-1}{2!} \Delta^2 y_0 + \left( \frac{3p^2-6p+2}{3!} \right) \Delta^3 y_0 \right)$$

$$\frac{d^2 y}{dx^2} = \frac{1}{h^2} \left( \Delta^2 y_0 + (p-1) \Delta^3 y_0 \right)$$

NBD:-

$$\frac{dy}{dx} = \frac{1}{h} \left( \nabla y_0 + \frac{2p+1}{2!} \nabla^2 y_0 + \frac{(3p^2+6p+2)}{3!} \nabla^3 y_0 \right)$$

$$\frac{d^2 y}{dx^2} = \frac{1}{h^2} \left( \nabla^2 y_0 + (p+1) \nabla^3 y_0 \right)$$

STIRLINGS:-

$$\frac{dy}{dx} = \frac{1}{h} \left[ \left( \frac{\Delta y_0 + \Delta y_{-1}}{2} \right) + p \Delta^2 y_{-1} + \left( \frac{3p^2-1}{3!} \right) \left( \frac{\Delta^3 y_{-1} + \Delta^3 y_{-2}}{2} \right) \right]$$

$$\frac{d^2 y}{dx} = \frac{1}{h^2} \left[ \Delta^2 y_{-1} + p \left( \frac{\Delta^3 y_{-1} + \Delta^3 y_{-2}}{2} \right) \right]$$

UNEQUAL

NEWTON-DIVIDED DIFF:-

$$\frac{dy}{dx} = \Delta y_0 + (2x - x_0 - x_1) \Delta^2 y_0 \dots \dots \dots$$

NUMERICAL INTEGRATION

Error:

Simpsons:

$$\epsilon \leq \frac{M_4 (b-a) h^4}{180}$$

$M_4 = \text{max. value of } |f^{(4)}(x)| \text{ in interval } a, b$

TRAPEZOIDAL

$$\epsilon \leq \frac{M_2 (b-a) h^2}{12}$$

$M_2 = \text{max. value of } |f''(x)| \text{ in interval } a, b$

TRAPEZOIDAL:-

$$\int_0^n f(x) dx = \frac{h}{2} [y_0 + y_n + 2(y_1 + y_2 + y_3 + \dots + y_{n-1})]$$

$$h = \frac{b-a}{n}$$

(b = upper limit)  
(a = lower limit)

$$y_k = f(k)$$

SIMPSONS:-

$$\int_0^n y dx = \frac{h}{3} \left[ (y_0 + y_n) + 4(y_1 + y_3 + y_5 + \dots + y_{n-1}) + 2(y_2 + y_4 + y_6 + \dots + y_{n-2}) \right]$$

MAGIK!

$\Delta^2 y_0$ :

$$\Delta^2 y_0 = y_2 - 2y_1 + y_0$$

$$\Delta^2 y_{-1} = y_1 - 2y_0 + y_{-1}$$

$$\Delta^2 y_{-2} = y_0 - 2y_{-1} + y_{-2}$$

$$\Delta^2 y_1 = y_3 - 2y_2 + y_1$$

$$\Delta^2 y_2 = y_4 - 2y_3 + y_2$$

$\Delta^3 y_0$ :

$$\Delta^3 y_0 = y_3 - 3y_2 + 3y_1 - y_0$$

$$\Delta^3 y_{-1} = y_2 - 3y_1 + 3y_0 - y_{-1}$$



$$\Delta^3 y_{-2} = y_1 - 3y_0 + 3y_{-1} - y_{-2}$$

$$\Delta^3 y_1 = y_4 - 3y_3 + 3y_2 - y_1$$

$$\Delta^3 y_2 = y_5 - 3y_4 + 3y_3 - y_2$$