

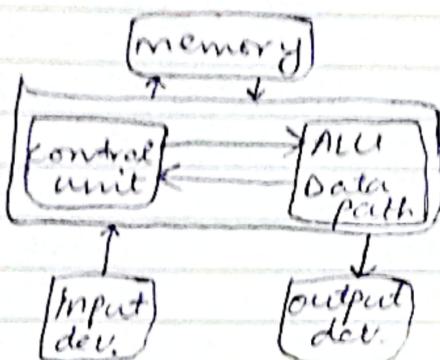
PLD - Rec 1 - 14 Feb 21

Ch 1, 2, 3, 4

Assignments  
from 4<sup>th</sup> ed.

logic and computer  
design fundamentals.  
(5<sup>th</sup> ed)

Block diagram of digital computer.



- Arrows show flow of data.
- Diff. comp. of comp. system are shown in diagram.
- Memory save data and program.
- Which are diff. input dev?
- " " " " output dev?

Data Path : Arithmetic and logic operations

Control unit : co controls everything

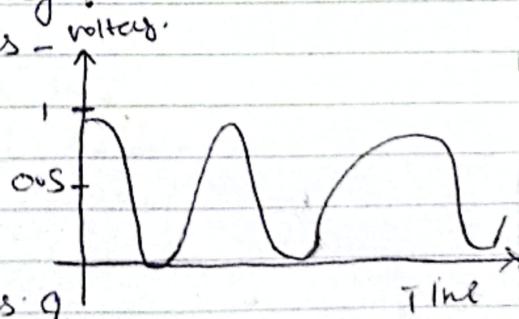
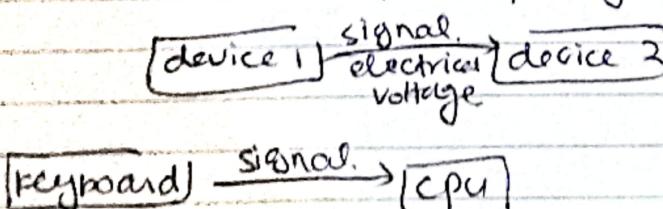
sample Prog: integer a,b,c: e.g. • flow of prog. • Running the sys.  
input a int; • Running programs fetching  
input b int; code line one by  
 $c = a + b$  one.  
print c;

If you are a control unit

- How control unit running the sample prog.
- data is traveling from one to other component.
- data come from keyboard.
- Addition in data path.
- Storing in memory.
- display on screen LCD.

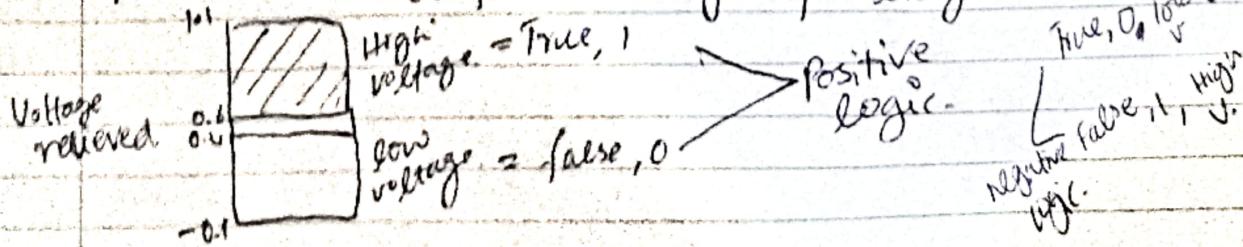
How is data traveling?

Data travels in form of signals - voltage.

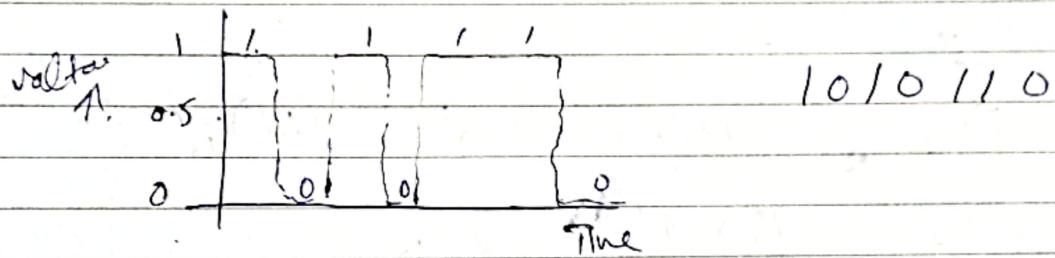
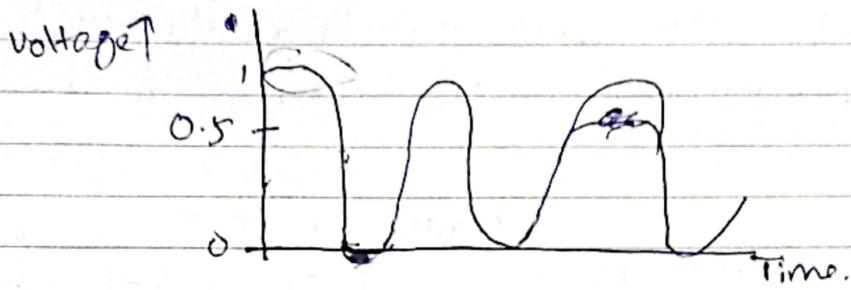


How will we save this signal as 9.

For this we define a range of voltage.



Now voltage has been converted into 0/1 binary numbers.



If dotted line indicates start of a binary number which binary no. have we received in above signal.  
Answer? 1010110

This is how data is received into/from a device. This 0/1 is called binary digit or bit.

Binary to decimal conv.

Q. (Number) Base

$$(11010)_2 = (?)_{10} \\ 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 \rightarrow = (26)_{10}$$

$$(0.6875)_{10} = (011)_2$$

$$(625.6875)_{10} = (100110001101)_2$$

$$(11010.11)_2 = (?)_{10}$$

$$1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 + 1 \times 2^{-1} + 1 \times 2^{-2}$$

$$(625)_{10} = (1001110001) \quad 26 + 0.5 + 0.25$$

$$(11010.11) = (26.75)_{10}$$

keep mult. until we get 000 in fraction part.

$$\begin{array}{r} 625 \\ 2 | 312 \\ 2 | 156 \\ 2 | 78 \\ 2 | 39 \\ 2 | 19 \\ 2 | 9 \\ 2 | 4 \\ 2 | 2 \\ \hline & 0 \end{array}$$

$$(625)_{10} = (1001110001) \\ \text{integer} \quad \text{fraction}$$

$$0.6875 \times 2 = 1.375 \rightarrow (1) \\ 1.375 - 1 = 0.375 \rightarrow (0) \\ 0.375 \times 2 = 0.75 \rightarrow (0) \\ 0.75 \times 2 = 1.5 \rightarrow (1)$$

$$0.6875 = 0.1010110001 \\ \text{in fraction part.}$$

4 types of number system.

① (Decimal)<sub>10</sub> ② (Binary)<sub>2</sub> ③ (Octal)<sub>8</sub> ④ (Hexadecimal)<sub>16</sub>

$$(724.542)_{10} = 7 \times 10^2 + 2 \times 10^1 + 4 \times 10^0 + 5 \times 10^{-1} + 4 \times 10^{-2} + 2 \times 10^{-3}$$

↓                    ↓  
most.                least.  
significant        significant  
digit                digit

Digits in base 10 are

0, 1, 2, ..., 8, 9

10 A  
11 B  
12 C  
13 D  
14 E  
15 F  
→ we use A, B, C, after 9

Digits in base 8.

0, 1, 2, ..., 7-1

(7) is 5 to  
(8) is 10 so  
base of radix

$$(15)_{16} = 1 \times 16^1 + 5 \times 16^0 = (21)_{10}$$

$$(F)_{16} = 15 \times 16^0 = (15)_{10}$$

(18) is not in base-8.

bcg of 8

up to

1010

4.0

## COPS

14 Feb

- ① Pointers have data type  $\rightarrow$  integers.
- ② This is type of variable, which is pointed.  
 $\text{char} \rightarrow p$ .

$\text{char } a = 'A'$

$0x2A$  [A] a.

$\text{char } *p \quad (\text{char}) * p = \&a;$

$0x2E \quad | 0x2A | p$

$\text{cout} \ll a; // A.$

passing array  
as a function.

$\text{cout} \ll p; // 0x2A$

$\text{cout} \ll \&p; // A$

③ identifiers

{ same type }

$\rightarrow$  Pointing a pointer copies.

$\text{char } *q = p.$

$0x3F \quad | 0x2A | q$

Now q is pointing towards a.

$\text{cout} \ll \&q; // A.$

$\text{cout} \ll q; // 0x2A$

$0x40 \quad | 0x2A | b$

~~char~~ b = 0x2A;

$\cdot p = &b;$

int \*r = 0;  $\rightarrow$  good idea to  
initiate to 0.

$\text{cout} \ll r \text{ if } 0.$

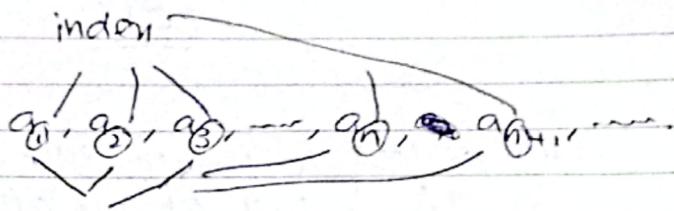
$0x4A \quad | 0x00 | P$

, its a null pointer.

May 2011

Thomas  
Cal. → Ch 10

Sequence:



→ order is important. terms

→ There is integer corresponding to each term.

$$1 \rightarrow a_1 = 3$$

$$2 \rightarrow a_2 = 6$$

$$n \rightarrow a_n = 3n$$

→ An infinite seqn is a fn. whose domain is set of integers.

$$f(n) = a_n$$

other notations:

$\{a_n\}$  or  $\{a_1, a_2, a_3, \dots, a_n, \dots\}$  or  $\{a_n\}_{n=1}^{\infty}$   
Ex 10.1 Q15

$$a_n = (-1)^{n+1} n^2$$

$$\text{Q18} \quad -\frac{3}{2}, -\frac{1}{6}, \frac{1}{12}, \frac{3}{20}, \frac{5}{20}, \dots$$

$$\begin{aligned} & + \frac{2}{4+n} \underset{n=0}{\sim} & & + \frac{2}{4+2(n)} \underset{n=0}{\sim} \\ & 4+n & & \end{aligned}$$

$$a_n = a_1 + (n-1)d$$

$$\begin{aligned} & +(-3 + (n-1)2) \underset{n=0}{\sim} & & n(n+1) \underset{n=0}{\sim} \\ & -3 + 2n - 2 & & \\ & -2n - 5. & & \end{aligned}$$

$$\text{Q25} \quad 1, 0, 1, 0, 1, \dots$$

$$(-1)^{\frac{n+1}{2}}$$

2

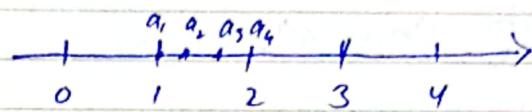
$$Q24$$

$$\frac{(n)^3}{(5)^{n+1}}$$

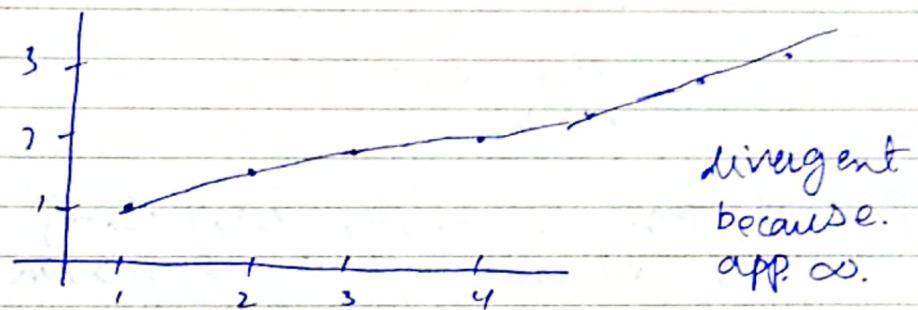
Graphical represent.

$$a_n = \sqrt[n]{n} = \sqrt[1]{1}, \sqrt[2]{2}, \sqrt[3]{3}, \sqrt[4]{4}, \dots$$

Real axis



① As a fn.

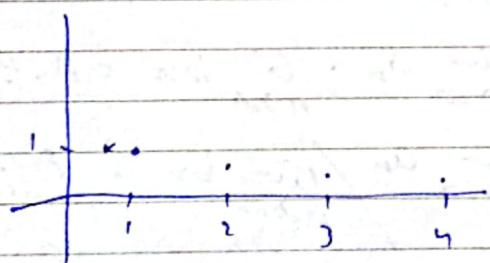


$$a_n = \frac{1}{n}$$

Real axis

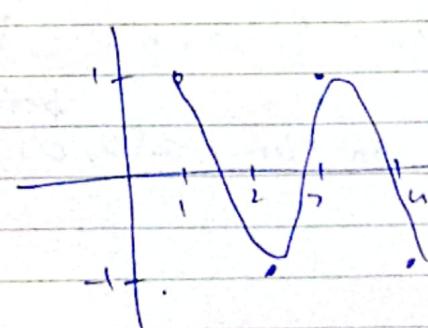
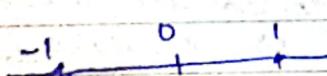
$$= \left\{ 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots \right\}$$

②



convergent.  
because  
app. 0.

$$a_n = (-1)^{n+1}$$



divergent  
bcz does not  
app. single pt.

## Convergent & divergent seq.

- If  $\{a_n\}$  converges to 'L'

$$\lim_{n \rightarrow \infty} a_n = L.$$

- If no such L exist, seq. is divergent.

- The seq. diverges to infinity if:

$$\lim_{n \rightarrow \infty} a_n = \infty.$$

- Similarly  $\infty$ ,  $a_n$  diverges to -ve infinity if.

$$\lim_{n \rightarrow \infty} a_n = -\infty.$$

- The seq.  $\{1, -2, 3, -4, 5, 7, -8, \dots\}$   $\{1, 0, 2, 0, 3, 0, \dots\}$  are divergent without approaching boundary or  $\infty$ .

- If  $\{a_n\}, \{b_n\}$  are sequences s.t.  $\lim_{n \rightarrow \infty} a_n = A, \lim_{n \rightarrow \infty} b_n = B$ .

$$① \lim_{n \rightarrow \infty} (a_n \pm b_n) = \lim_{n \rightarrow \infty} a_n \pm \lim_{n \rightarrow \infty} b_n = A \pm B.$$

$$② \lim_{n \rightarrow \infty} (k \cdot a_n) = k \lim_{n \rightarrow \infty} a_n = kA.$$

$$③ \lim_{n \rightarrow \infty} (a_n \cdot b_n) = \lim_{n \rightarrow \infty} a_n \times \lim_{n \rightarrow \infty} b_n = A \cdot B.$$

$$\lim_{n \rightarrow \infty} \left( \frac{a_n}{b_n} \right) = \frac{\lim_{n \rightarrow \infty} a_n}{\lim_{n \rightarrow \infty} b_n} = A/B.$$

$$a_n = \{1, 2, 3, \dots\} \quad b_n = \{-1, -2, -3, \dots\}$$

both  $a_n, b_n$  are divergent.

but  $a_n + b_n = \{0, 0, 0, \dots\}$  is converges on 0.

reflected  
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ation

if  $a_n$  converges and  $b_n$  diverges then  $a_n + b_n$  diverges and also  $a_n - b_n$  also diverges

If  $a_n = \text{conv.}$ ,  $b_n = \text{conv.}$ ,  $(a_n + b_n) = \text{conv.}$

If  $(a_n \pm b_n) = \text{conv}$  then may or may not  $a_n = \text{conv}$   $b_n = \text{conv}$ .

If  $a_n = \text{div.}$ ,  $b_n = \text{div.}$  then  $(a_n \pm b_n) = \text{may or may not conv or div.}$

← →  
Eng

## Foundations of communication

follows  
so  
→ Com. Context: The phys., social, cult., histrol., psycholog. situations, that surrounds communication event

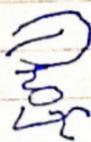
last down upon your self was situation  
→ Phys. sit → geogr. - location, environment  
social → nature of relationship b/w participants.

(Prep. Practice. Perform.) 3P.

Conv. Proc. Pg 11.

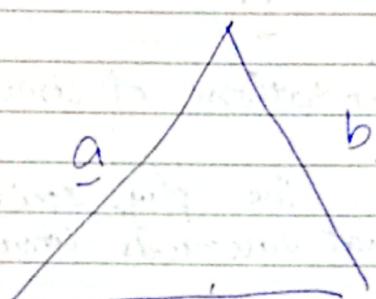
context, participants, channels message, interference.

Maria and danien are meandering through the park - taking and drinking bottled water. Danien finishes his bottle, replace the lid and loses the bottle into the bushes at the side of the path. Maria who has been listening to danien talk, comes to a stop, puts her hands on her hip, stares at danien and says angrily "I can't believe what you just did."



- 1) Content (a) physical (b) social (c) historical (d) symbolic
- 2) Participant
- 3) Message
- 4) Channel
- 5) Feedback
- 6) Interference

272-



$(a = \exists b \text{ s.t. } a \neq b)$  ||

$(a = \exists b \text{ s.t. } a \neq c)$  ||

$(b = \exists c \text{ s.t. } b \neq a)$

Interesting.

BCD - lec 2 - 16 Feb

Data ranges : Let 1 bit represent 1 box   
- minimum value of it can have.  0  
maximum " " " " "  1

Range  $0 \rightarrow 1$   
 $\rightarrow 2$  bits  $\boxed{\square \square}$

min. value.  0, max value  1  
all possible values  $00, 01, 10, 11$   
0 1 2 3

Range  $0 \rightarrow 3$ .  
 $\rightarrow$  Let  $n$  be no. of bits.

Range  $0 \rightarrow 2^n - 1$ .

$\rightarrow$  Range of fraction.

Range:  $0 - \frac{2^n - 1}{2^n}$

Conventions: we say 1 kilo =  $10^3$ , 4 kg = 4000g.  
But in dealing with data:

$2^{10}$  = 1K (kilo).  $1\text{KB} = 1024$  byte.

$2^{20}$  = Mega (M).  $1\text{GM} = 16 \times 2^{20}$  bytes.

$2^{30}$  = Giga (G).

$2^{40}$  = Tera (T).

Arithmetic operations.

Addition

bto  $\rightarrow$

$$\begin{array}{r} 1010 \\ + 0101 \\ \hline 1010 \end{array}$$

If ans  $\geq$  base then  $\rightarrow$  factorize.

b2  $\rightarrow$

✓

$$\begin{array}{r} 0110001 \\ 0010001 \\ \hline 1000100 \end{array} \rightarrow 2 \cdot \frac{12}{1-0}$$

10  
11  
12  
13  
14  
15

$$b46 \rightarrow \begin{array}{r} 111 \\ 9CA17F \\ 325E81 \\ \hline CFO000 \end{array} \rightarrow 16 \begin{array}{r} 16 \\ 16 \\ \hline 1-0. \end{array}$$

$$\begin{array}{r} 59F \\ E46 \\ \hline \cancel{100} \\ -13E5 \end{array} \quad 21 \quad \begin{array}{r} 1621 \\ 1-5. \end{array} \quad \begin{array}{r} 1619 \\ 163 \end{array} \quad \boxed{\text{basis } 16}$$

754

$$\begin{array}{r} 469 \\ \hline 285 \end{array}$$

✓

Subtraction.

$$\begin{array}{r} 10110 \\ -10010 \\ \hline 00100 \end{array}$$

1=2.

$$\begin{array}{r} 10110100 \\ -11001101 \\ \hline \end{array}$$

?

?

$$\begin{array}{r} 10011 \rightarrow 19.52 \\ 111010 \rightarrow 30.52 \end{array}$$

as  $10011 < 11101$

so  $10011 < 11101$ .

10+0

1000+

11110

10011

598010  
7993

$$\begin{array}{r} 0.2 \rightarrow 22.0 \rightarrow 270 \\ 100010 \\ \hline 0.01101 \\ \hline 0.00101 \end{array}$$

## Binary mult.

$$\begin{array}{r} 1011 \\ \times 101 \\ \hline \end{array}$$

$$\begin{array}{r} 31 \\ 762 \\ \times 45 \\ \hline 14672 \\ 3710 \\ \hline 43772 \end{array}$$

? H.W.

$$(673 \cdot 12)_8 = (?)_2$$

$$(3A6 \cdot C)_{16} = (?)_2$$

$$(E9C \cdot 04B)_{16} = (?)_8$$

Binary  $\rightarrow$  octal/Hexade.

$$\begin{array}{r} 101100 \\ \hline b_2 \rightarrow b_n \end{array}$$

for base  $= 2^n$  we need min. n bits to represent any digit of that base.

Bas.  $8 = 2^3$  Method is valid for  $2^n$  only.

octal	0	1	2	3	4	5	6	7
binary	0	01	10	11	100	101	110	111
	000	001	010	011	100	101	110	111

$$\text{consider } (789.526)_{10}$$

if had to add 0  $\rightarrow$

left in 789  
right in .526

$$\begin{array}{r} 00789 \\ \hline \cdot 52600 \end{array} \quad \begin{array}{r} 78900 \\ \hline \cdot 00526 \end{array}$$

for base  $n$   
 $\rightarrow$  more of  $n$ . as  $\sqrt[n]{n}$ .

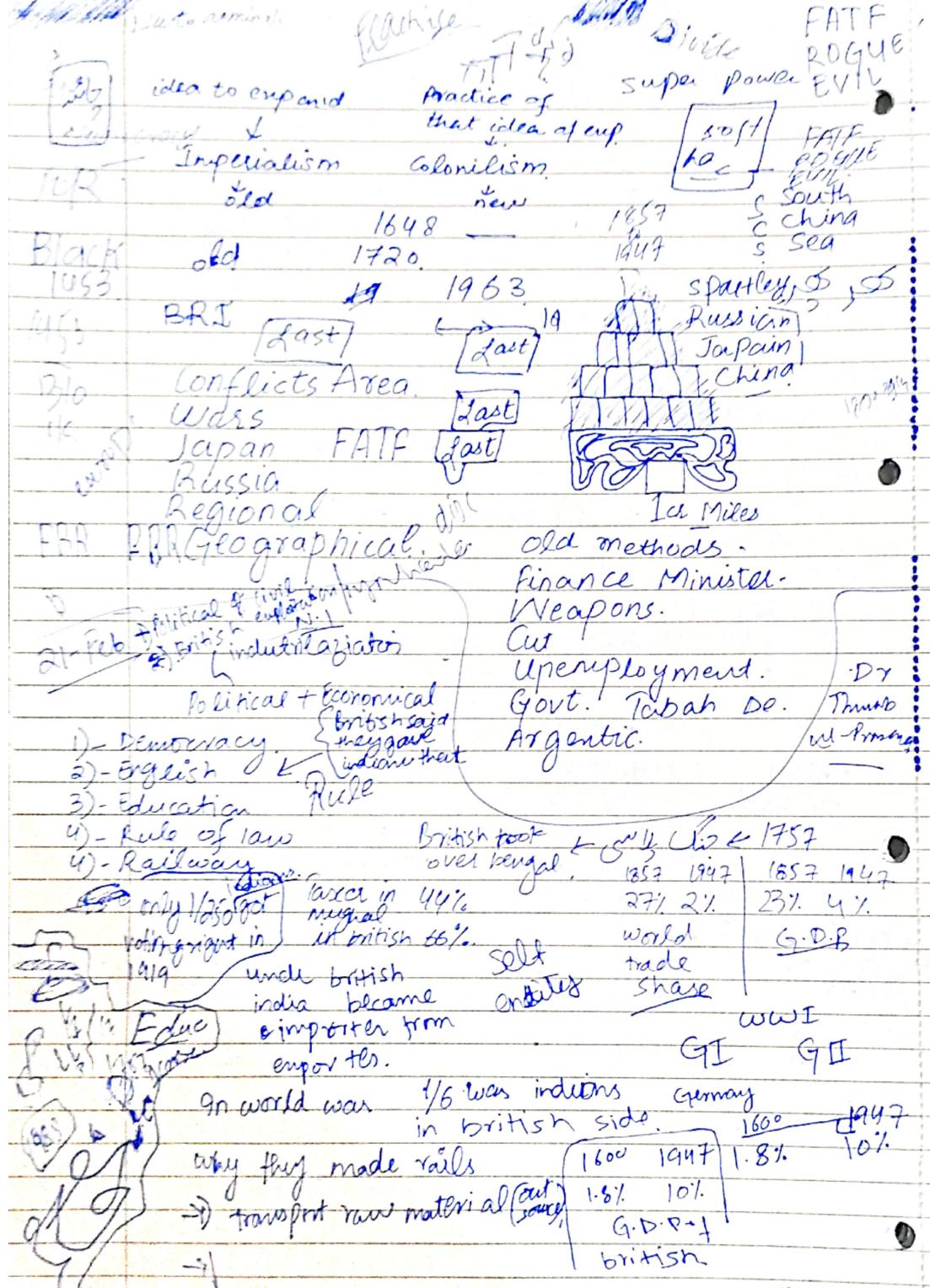
similarly  
for  $b^n$ .  
 $\rightarrow$  group of  $n$ .

$$(26153.7406)_8$$

$$\begin{array}{r} 1011000011010111110000011 \\ \hline \downarrow \end{array}$$

$$010\ 110\ 001\ 101\ 011\ 111\ 100\ 000\ 110$$

$$(2\ 6\ 1\ 5\ 3\ 7\ 4\ 0\ 6)_8$$



4P

OO PS.

(22) 201

(24)<sub>10</sub>

( )<sub>10</sub>

$$2 \times 16^1 + 8 \times 16^0$$

$$32 + 8$$

$$32$$

int a=10, b=20, c=30;

int \*p = &a;

\*p = \*p+1; → ordinary arithmetic.

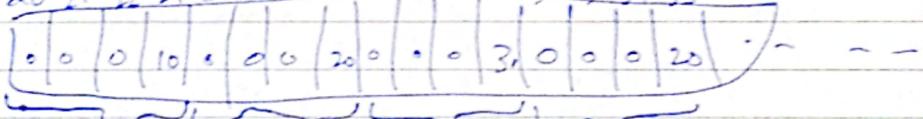
cout << p; // 11, cout << p // 10, 20.

p = p+1;

cout << p; // 0, 24.

cout << p; // 20.

on On  
20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35



a      b      c      p.

const int a=10;

const int \*p = &a; if a is const then p must  
 $\star p = 11$ . Error

•  $p = p + 1$ ; → will change address and logical error  
to avoid this we use this: int a=10.

• int \* const p = &a;  
 $p = p + 1$  Now will give error at

No syntax or conflict error.

const int a = 10;  
 const int \* const p = &a;  
 ↑                      ↑  
 This const will prevent change of value of a or \*p.  
 This const will prevent changing of address of p.

→ array is const. pointer to a non-const. value.

for (int i=0; i<3, i++) {  
 int arr[3] = {10, 20, 30};  
 int \*ptr = arr;

for (int i=0; i<3, i++) {  
 arr++;  
 const int \*ptr = arr; → error

1st  
 On 203  
 (ptr + i)  
 2nd  
 3rd  
 int \* ptr = arr;  
 for (int i=0; i<3, i++) {  
 cout << \*ptr;

ptr++; ↑ will work

int a = 10, b = 20;  
 int \*ptr = &a;  
 cout << \*ptr; → 10  
 cout << b; → 20  
 cout << \*ptr; → 10  
 cout << b; → 20  
 cout << \*ptr; → 10  
 cout << b; → 20

```

int a=10, b=20;
int &r=a;
cout << a; // 10
cout << r; // 10
a=b;
cout << a; // 20
cout << r; //

```

Cal-II      Ket2

17-Feb.

\* Sandwich Th. for seq.

$$\text{if } a_n \leq b_n \leq c_n.$$

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} c_n = L. \quad [\lim_{n \rightarrow \infty} b_n]$$

$$\Rightarrow \text{then. } \lim_{n \rightarrow \infty} b_n = L.$$

Example. (1).  $a_n = \frac{\cos n}{n}$

$$-1 \leq \cos n \leq 1$$

$$\Rightarrow -\frac{1}{n} \leq \frac{\cos n}{n} \leq \frac{1}{n}$$

As  $\lim_{n \rightarrow \infty} \frac{1}{n} = 0.$

$$\Rightarrow 0 \leq \lim_{n \rightarrow \infty} \frac{\cos n}{n} \leq 0.$$

$$\Rightarrow \lim_{n \rightarrow \infty} \frac{\cos n}{n} = 0.$$

(2).  $a_n = \frac{(-1)^n}{n}$        $-1, \frac{1}{2}, -\frac{1}{3}, \frac{1}{4}, -\frac{1}{5}.$

$$-\frac{1}{n} \leq \frac{(-1)^n}{n} \leq \frac{1}{n}$$

as  $\frac{1}{n} \rightarrow 0,$  as  $n \rightarrow \infty.$

$$-\frac{1}{2} \leq \frac{1}{2} \leq \frac{1}{2}.$$

$$\Rightarrow \frac{(-1)^n}{n} \rightarrow 0$$

$$-\frac{1}{3} \leq -\frac{1}{3} = \frac{1}{3}.$$

Cont. fr. th.

If  $a_n \rightarrow l$  and  $f$  is cont.  
 $\rightarrow f(a_n) \rightarrow f(l)$

$$\frac{1+a_n}{n} \rightarrow 1 \text{ as } n \rightarrow \infty$$

$$\sqrt{\frac{1+a_n}{n}} \rightarrow 1 \text{ as } n \rightarrow \infty$$

$$e^{a_n} \rightarrow e^l \text{ as } n \rightarrow \infty$$

Q41

$$a_n = \int_{n+1}^{2n}$$

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \int_{n+1}^{2n} \frac{1}{x} dx = \int_{n \rightarrow \infty} \lim_{n \rightarrow \infty} \frac{2n}{n+1}$$
$$= \int \lim_{n \rightarrow \infty} \frac{2}{1 + \frac{1}{n}}$$

Q42  $a_n = \frac{1}{(0.9)^n}$

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{1}{(0.9)^n}$$

Let  $0.9 < 1$ .

$$\lim_{n \rightarrow \infty} (0.9)^n = 0$$

$$a_n = \frac{1}{(0.9)^n} \rightarrow \begin{array}{l} \text{diverge to} \\ \text{infinity} \end{array}$$

| If  $x \leq 1$

$$\lim_{x \rightarrow 0} x^n = 0$$

| If  $x > 1$ , then  $x^n$  diverges

Q49:  $a_n = \frac{\ln(n+1)}{\sqrt{n}}$

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \left( \frac{\ln(n+1)}{\sqrt{n}} \right) \rightarrow \left( \frac{\infty}{\infty} \text{ form} \right)$$

using L'opital rule.

$$\lim_{n \rightarrow \infty} \left( \frac{\frac{1}{n+1}}{\frac{1}{2\sqrt{n}}} \right) \rightarrow \left( \frac{\infty}{\infty} \text{ form} \right) \frac{2\sqrt{n}}{n+1}$$

again,

$$\lim_{n \rightarrow \infty} \left( \frac{\frac{1}{2\sqrt{n}}}{\frac{1}{4n+2}} \right) \lim_{n \rightarrow \infty} \frac{1}{n+2\sqrt{n}} = 0 \rightarrow \text{converges}$$

$$51) \quad a_n = (3/n)^{1/n}$$

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} (3/n)^{1/n} \quad (0^\circ \text{ form})$$

$$\ln a_n = \ln (3/n)^{1/n} \quad \ln(0) \rightarrow \infty.$$

$$\ln a_n = \frac{\ln 3 - \ln n}{n}.$$

$$= \lim_{n \rightarrow \infty} \frac{0 - 1/n}{1} \quad (\text{L'Hopital rule.})$$

$$\therefore \lim_{n \rightarrow \infty} -\frac{1}{n} = 0.$$

$$\lim_{n \rightarrow \infty} \ln a_n = 0.$$

(an is convergent)

$$\Rightarrow a^n \Rightarrow e^0 = 1.$$

Ex

$$a_n = \left(1 + \frac{x}{n}\right)^n$$

$$\ln a_n = n \ln \left(1 + \frac{x}{n}\right) \quad (0.0 \text{ form}).$$

$$\ln a_n = \ln \left(1 + \frac{x}{n}\right)/1/n.$$

$$\lim_{n \rightarrow \infty} \ln a_n = \lim_{n \rightarrow \infty} \left( \frac{\ln \left(1 + \frac{x}{n}\right)}{1/n} \right) \quad \% \text{ form}$$

$$= \lim_{n \rightarrow \infty} \frac{\left(\frac{1}{1+n/n}\right)\left(\frac{-x}{n^2}\right)}{-1/n^2}$$

$$= \lim_{n \rightarrow \infty} \frac{x}{1+x^2/n} = x.$$

$$\lim_{n \rightarrow \infty} a_n \approx e^x$$

### Rules.

$$1) \lim_{n \rightarrow \infty} \sqrt[n]{n} = 1 \text{ its equal to 1 acc to book pg 578}$$

$$2) \lim_{n \rightarrow \infty} n^n = 1.$$

$$3) \lim_{n \rightarrow \infty} \frac{n^n}{n!} = 0.$$

$$(3) \frac{1}{3} < 0.$$

1  
27

Q64  $a_n = (-4)^n / n!$   
as  $n^n / n! \rightarrow 0. \Rightarrow (-4)^n / n! \rightarrow 0.$

Recursive definition

1) Values of initial term(s).

2) Rule (Recursive formula).

$$1, 2, 3, \dots, a_1 = 1, a_n = a_{n-1} + 1,$$

$$1, 2, 6, 24, \dots, a_1 = 1, a_n = n(a_{n-1})$$

3) Fibonacci numbers:

$$1, 1, 2, 3, 5, \dots.$$

$\downarrow$   $\uparrow$   $\uparrow$

$$a_1 = 1, a_2 = 1, a_n = a_n, a_{n+1} = a_n + a_{n-1}$$

$$2, 2 + 1/2, 2 + \frac{1}{2 + \frac{1}{2}}, 2 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2}}}, \dots$$

$(a_n) = a_{n-1} + a_{n-2} \quad (n > 2)$

$$a_1 = 2, a_n = a_1 + \frac{1}{a_{n-1}}$$

## Bounded monoatomic seq.

→  $2, 4, 6, 8, \dots$

bounded from below.  $a_n \geq a_1$

→  $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots$   $a_n \leq M$

bounded from above. all are smaller than  $M$ .

If  $M$  is largest such no. then  $M$  is greatest lower bound.

If  $m$  is smallest such no. then  $m$  is least upper bound.

$\dots, -2, -1, 0, 1, 2, \dots$

unbounded..

①

Non-decreasing sequence:

If  $a_n \leq a_{n+1}$ .

$1, 1, 2, 3, 5, \dots$

→ If  $a_n \geq a_{n+1}$  → non-increasing seq.

→ If seq. is non-dec or non-incr. then seq. is monotonic seq.

$a_n = \{1, 0, 2, 0, 3, 0, \dots\}$

not monotonic.

If a seq. is bounded and monotonic then the seq. converges.

Ex 10<sup>1</sup>      09, 26, 34, 47, 58, 69, 74

Binary coded decimal (BCD).

0 0000  
1 0001  
2 0010

largest number is 9 so we need 4 bits.

2 | 396

2 | 198 - 6

99 - 0

1 | 49 - 1

24 - 1

12 - 0

6 - 0

3 - 0

1 - 1

$$(396)_{10} = (0011 \ 1001 \ 0110)_\text{BCD}$$

This is BCD number not binary number.

$$(396)_{10} = (1100 \ 01100)_2$$

$$(1100 \ 01100)_2 = (0011 \ 1001 \ 0110)_\text{BCD}$$

$$(X)_2 \neq (X)_\text{BCD}$$

$$(011001)_2, \quad (1011001)_\text{BCD}$$

$$(0001 \ 1000 \ 0101)_\text{B.C.D.} \neq (131)_2$$

even  $\rightarrow$  1s.

$$(0001 \ 1000 \ 0101)_\text{BCD} \xrightarrow{\text{bit of P.B.}} \text{even.}$$

$$(0001 \ 1000 \ 0101)_\text{BCD}$$

P.B for odd.

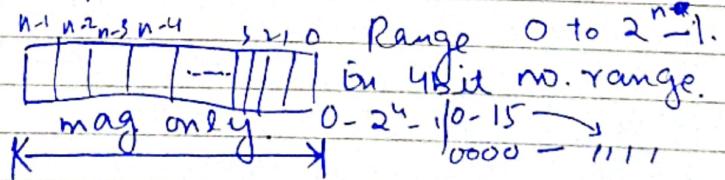
Binary no. (s).

unsigned signed B.N

Signed mag.  
repres.

2s complement  
representation

① unsigned no's.



→ Bit extension: e.g. making 6 bit no. from original 4 bits w.r.t.  
e.g. 976.  $\rightarrow$  [6-digit] 000976

$$(9)_{10} = (1001)_2 \Rightarrow = (0000 \ 1001)_2$$

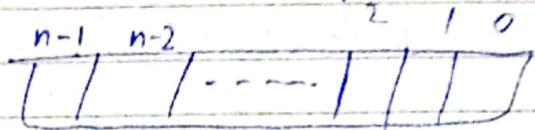
make in 8 digit

sign in binary  
'-' → 1  
'+' → 0

Q3. signed numbers? for n-bit rep.

e.g. 4-bit no.

+ve range.



K \* magnitude →

0 000 → 0 111.  
+0 → +7      sign bit

→ -ve. no. range from 1 000 to 1 111

→ for n bits (range). -0 to -7

- $(2^{n-1}-1)$  to  $+ (2^{n-1}-1)$  ( $+7$ ) to ( $-7$ ) for 4-bit → range

Bit extension 4-bit to 8-bit.

$$(+9)_{10} = (01001)_2 \rightarrow (0000\ 1001)_2 = (+9)_{10}$$

$$(-9)_{10} = (11001)_2 \rightarrow (\underline{1}\ 0000\ 1001)_2 = (-9)_{10}$$

signed mag. rep.

Decimal Mag. rep. (signed mag.) 2's complement

+7 0111 0111

+6 0110 0110

+5 0101 0101

+4 0100 0100

+3 0011 0011

+2 0010 0010

+1 0001 0001

+0 0000 0000

-1 1000 0000

-2 1010 1111

-3 1011 1110

-4 1100 1101

-5 1101 1100

-6 1110 1011

-7 1111 1010

(-8) 1000 1001

can't be rep. in 4 bits

3). Signed 2's complement representation.

→ 2's complement of  $N$  : (Algorithm)

→ i). start traversing bits from right to left.

2) - leave or copy all the data

3) - copy first 1-

4) - flip rest of the bits i.e. make 0 to 1 and 1 to 0.

2's comp. of  $N = 01011100$

→ Positive no. are identical representation.

In both sys. left most bit represents sign.

→ 2's comp. in table (Back side).

→ signed 2's complement

Range e.g. 4 bit no.

+ve nos. range.

0 000

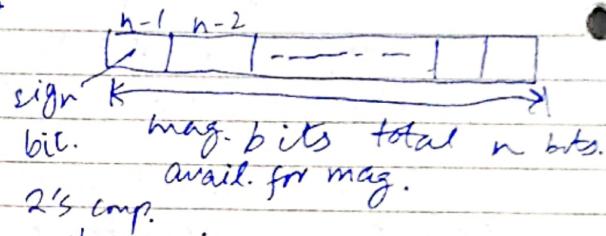
to

0 111

-ve no. range.

n-bit range.

$-(2^{n-1})$  to  $(2^{n-1} - 1)$



2's comp.  
mag.

$1000$  to  $1011 \rightarrow$   
2's comp.  $\rightarrow$  1's comp.  
 $-(1000) \rightarrow -(000)$   
 $(-8)_{10} \rightarrow (-1)_{10}$

What is

$$\begin{array}{c} \text{sign bit extension.} \\ (0111)_2 = +7. \\ \downarrow \\ (00000111) \end{array}$$

2's comp

$(00001000)_2$ .

$$\begin{array}{c} (000)_2 = -8 \\ 4 \text{ bit} \rightarrow 8 \text{ bit} \\ (0111) \quad (000) \end{array}$$

in 2's comp. rep. '0' is used for bit extension

21 Feb

Q: Divide and

int a = 20

int b = a.

cout << a ; // 10

b = 20

cout << a // 20.

int a = 10

int \* p = &a.

cout << a // 10

\*p = 20

cout << a // 20.

4- outfit

shalwar

→ hook

Once I went out of city in my village. supposed for 2-3 hrs stay but due to some personal situations it became longer including a night stay. So I was wearing shalwar kameez, It helped me in such a way I can wear it during sleep time.

① → 51  
visit.

quality/ clarity/ relevance/ connection

of sentence.

Content points

3

Msg : *L2 + L3*

G

Uttar

Ur

→

→

$$S_1 = a_1$$

$$S_2 = a_1 + a_2$$

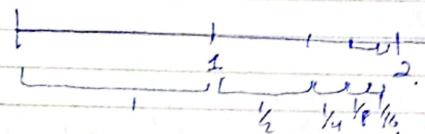
$$S_3 = a_1 + a_2 + a_3$$

$n^{\text{th}}$  partial sum  $S_n = a_1 + a_2 + a_3 + \dots + a_n = \sum_{k=1}^n a_k$

$\{S_n\} \rightarrow$  partial sum of series.

$|S_n \rightarrow L$  (series converges.  
and sum of series is  $L$ )

$\sum_{n=1}^{\infty} a_n = L$ , if  $S_n$  doesn't converge,  
so series diverges.



Geometric series.

-  $a + ar + ar^2 + \dots + ar^{n-1}$   
 $a \neq 0$ ,  $r$  fixed.  
 $r$  ratio.

$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots$$

$$a = 1, r = \frac{1}{2}, \dots$$

$$+ \frac{1}{32}$$

$$S_1 = a, S_2 = a + ar, S_n = a + ar + ar^2 + \dots + ar^{n-1} \rightarrow S_n = ar + ar^2 + ar^3 + \dots + ar^n$$

$$\textcircled{1}, \textcircled{2} \quad S_n - rS_n = a - ar^n$$

$$S_n = \frac{a(1-r^n)}{(1-r)}$$

if  $r = 1 \rightarrow S_n \rightarrow$  diverges.  $\rightarrow S_n = n a = a + a + a + a + \dots$

if  $r \neq 1 \rightarrow S_n = a - a + a - a + a - \dots (-1)^n a \rightarrow$  series oscillates

$$\text{e.g. } S_0 = 0$$

$\Rightarrow$  if  $|r| < 1$  then  $S_n = a \frac{(1-r^n)}{1-r} \rightarrow$  diverges.

$\Rightarrow$  if  $|r| > 1 \rightarrow$  series converges to  $\frac{a}{1-r}$ .

A

10/83

OB

$S_0 = 1 +$

Tues

=

=

line  
ratio

80 =

$S_1 =$

$S_2 =$

$S_3 =$

$S_4 =$

$S_5 =$

$S_6 =$

$\frac{1}{1-r}$

$$(-1)^{n-1} 2^{n-1}$$

10.2 Q3.  $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \dots + \frac{(-1)^n}{2^n} + \dots$

Q3.  $\sum_{n=0}^{\infty} \left( \frac{1}{2^n} + \frac{(-1)^n}{5^n} \right)$

$$S_0 = 1 + 1, S_1 = (1 + 1) + \left( \frac{1}{2} - \frac{1}{5} \right), S_2 = (1 + 1) \left( \frac{1}{2} - \frac{1}{5} \right) \left( \frac{1}{2} + \frac{1}{5} \right)$$

There are eight terms:

$$= (1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \dots + \frac{1}{2^n}) + (1 - \frac{1}{5} + \frac{1}{5^2} - \frac{1}{5^3} + \dots - \frac{1}{5^n})$$

$$= \frac{(1)(1 - (\frac{1}{2})^n)}{1 - \frac{1}{2}} + \frac{(1)(1 - (\frac{1}{5})^n)}{1 - \frac{1}{5}}$$

if  $n > 0$ .

$$\lim_{n \rightarrow \infty} S_n = (1) \frac{1 - (\frac{1}{2})^\infty}{1 - \frac{1}{2}} + (1) \frac{1 - (\frac{1}{5})^\infty}{1 - \frac{1}{5}} \quad \lim_{n \rightarrow \infty} S_n = 0.$$

$$= \frac{1}{\frac{1}{2}} + \frac{1}{\frac{1}{5}} = 1/\frac{1}{6}$$

$$S_0 = \sum_{n=0}^{\infty} \left( \frac{2^{n+1}}{5^n} \right).$$

$$\frac{1}{2} \cdot \frac{1}{5} = \frac{1}{10}$$

$$S_0 = \frac{1}{2} / \frac{1}{2} = \frac{2^{n+1}}{5^n}$$

$$S_1 = \frac{1}{5}$$

$$\frac{2 \cdot 5}{15} = \frac{2}{3}$$

$$S_2 = \frac{2}{5^2}$$

$$\frac{2^2 \cdot 5^2}{25} = \frac{4}{25}$$

$$S_3 = \frac{2}{5^2} + \frac{1}{5} + \frac{2}{5^2} + \frac{2^2}{5^3}$$

$$S_4 = \frac{2}{5^2} + \frac{1}{5} + \frac{2}{5^2} + \frac{2^2}{5^3} + \frac{2^3}{5^4}$$

$$S_5 = \frac{2}{5^2} + \frac{1}{5} + \frac{2}{5^2} + \frac{2^2}{5^3} + \frac{2^3}{5^4} + \frac{2^4}{5^5} + \frac{2^5}{5^6} + \frac{2^6}{5^7} + \frac{2^7}{5^8}$$

$$= \frac{(\frac{2}{5})(1 - (\frac{2}{5})^5)}{(1 - \frac{2}{5})} = \frac{1/2}{3/5} = \frac{5}{6}$$

$$= 2 \left( 1 - \left( \frac{2}{5} \right)^5 \right)$$

$$= \frac{2}{1 - 2/5} = \frac{10}{3}$$

Geometric

$$(-2) + (-2)^2 + \dots + (-2)^n + \dots$$

Geo. series with  $r > 1 \rightarrow \text{diverges}$

→ Repeating decimals

(24)

$$1.\overline{414} = 1 + \frac{414}{1000} + \frac{414}{(1000)^2} + \frac{414}{(1000)^3} + \dots$$

When  
 $|r| < 1$

$$\frac{a}{1-r} = \text{sum of series}$$

$$1.\overline{414} = 1 + \frac{414}{1000} + \frac{414}{(1000)^2} + \frac{414}{(1000)^3} + \dots - \frac{414}{(1000)^n}$$

$$a = 1 + \frac{414}{1000} \left[ 1 + \frac{414}{1000} + \frac{414}{(1000)^2} + \dots + \frac{414}{(1000)^{n-1}} \right]$$

$$S_n = \left( 1 + \frac{414}{1000} \right) \left( \frac{1}{1 - \frac{414}{1000}} \right) \approx 1413/999$$

(36)

Telescoping series:

$$\sum_{n=1}^{\infty} \left( \frac{3}{n^2} - \frac{3}{(n+1)^2} \right).$$

$$S_1 = 3 - \frac{3}{2^2}$$

$$S_2 = 3 - \underbrace{\frac{3}{2^2} + \frac{3}{2^2}}_{\cancel{+ 3/2^2}} + \frac{3}{3^2}$$

$$S_3 = 3 - \frac{3}{2^2} + \frac{3}{2^2} + \frac{3}{3^2} - \frac{3}{3^2} + \frac{3}{4^2} + \dots - \frac{3}{n^2} + \frac{3}{(n+1)^2}$$

$$3 - \frac{3}{(n+1)^2}$$

$$\text{lim } S_n = 3 - \cancel{\frac{3}{(n+1)^2}} \left( 1 - \frac{3}{2} \right). (3)$$

→ nth term test for divergence series:

The if  $\lim_{n \rightarrow \infty} a_n$  converges then  $a_n \rightarrow 0$ .

\* if  $a_n \rightarrow 0$  then series may diverge.

if  $\lim_{n \rightarrow \infty} a_n$  fails to exist or is different from 0, then  $\sum a_n$  diverges.

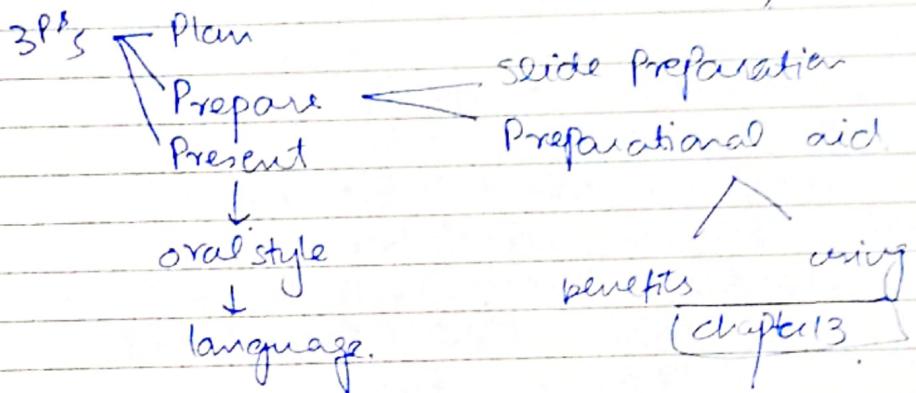
eg. ①  $\sum_{n=1}^{\infty} n^2 \rightarrow$  diverge as  $n^2 \rightarrow \infty$ .

②  $\sum_{n=1}^{\infty} \frac{n+1}{n} = \lim_{n \rightarrow \infty} 1 + \frac{1}{n} = 1 + 0.$

(5, 10, 25, 32) 10.2

← English.

- 1) Sub.
- 2) Ali
- 3) Ab
- 4) Bush
- 5) Hanga
- 6)



CH14 → speak clearly

speaking appropriate

Structure of PPTA.

- 1) Intro
- 2) Middle
- 3) Cond.

DLD

lect 4  
23 Feb

2<sup>n</sup> 2's complement.  
formula  $2^n - N$

student 2<sup>y</sup>

Tent 28.

- reading 36-40

5x bullet p (slide)

tiny new Roman

calibri (slide)

⇒ Algo of sub. 1) Borrow method., 2) Using 2's comp.

Ex:  $6 - 13 = ?$

we will do  $6 + (-13) \rightarrow$  add. will be performed.

→ char 2 - 110 1101

2) signed @ 110 01101

3) Bit extension  $+6 = 0110$

4) No. of bits same.  $+6 = 00110$   $\rightarrow$  2's comp.

5)  $A - B = A + (-B)$ .

$$(+A) - (+B) = (\pm A) + (-B)$$

$$(+A) - (-B) = (+A) \oplus (+B)$$

Reason: In our processor hardware sub is not supported - we use add. operation.

hardware less hardware performed.

fast process for addition taking 2's comp. before that if necessary

we are done here let's check.

$$\begin{array}{r} 00110 \\ +11001 \\ \hline 1011001 \end{array}$$

$$\begin{array}{r} 000111 \\ +11001 \\ \hline 1011001 \end{array}$$

$\Rightarrow -6 - 13 = ?$

1)  $6 = 110 \quad 13 = 1101 \quad -6 + (-13)$

2)  $+6 = 0110 \quad +13 = 01101$

3)  $+6 = 00110 \quad +13 = 001101$

4)  $+6 = 000110 \quad +13 = 001101$

$(-A + -B) \rightarrow 2's \text{ comp}$

$\begin{array}{r} 11010 \\ 11001 \\ \hline 1001101 \end{array}$

$\rightarrow$  Why we do bit extension -

We have some no. of bits available in 3 nos. A, B, C.  
for eg. without bit extension.  $8 - 8 = 8 + (-8) = A + B - C$

$8 = 1000 \quad -8 + (-8)$

$+8 = 01000$

10000

$\rightarrow$  Why we discard last carry.

Ex.

$13 \quad 1101 \quad 01101$

$0110 \quad 00110$

$M-N \quad 01101$

$11010$

$\downarrow \text{discard} \quad 00011$

$2's \text{ comp of } N = 2^n - N$

$M-N, M=2^m M + (-N)$

$M + (2^n - N)$

$(M - N) + (2^n)$

discard

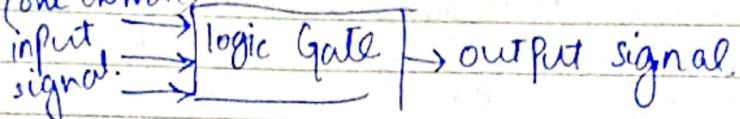
$00011 + \frac{2^5}{100000} + 7$

discarded!

→ Hardware components that manipulate binary info. is called "digital circuit".

→ Digital circuits are perf. diff. operations.

→ Basic circuit is called logic gate → Base of logic gates make complex circuits. all circuits - (one or more)



output of gates are applied to inputs of other gates to form a digital circuit.

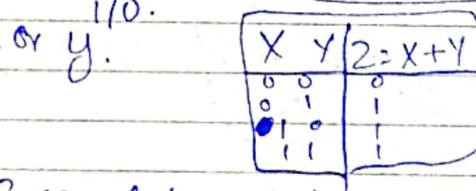
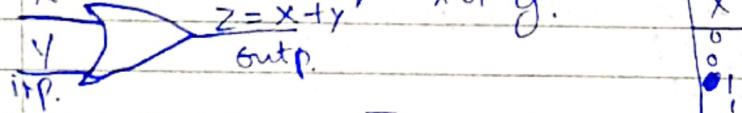
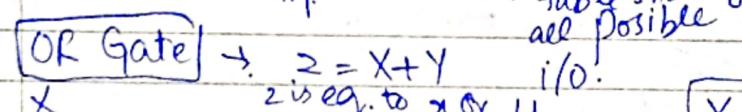
Electrical signal i.e. voltage/current exist throughout a digit system. in either of two recognizable values.

→ Each gate performs a specific logical operation - 3 basic logic operations associated with binary variables

- AND | AND Gate: - Perf. AND oper.

- OR | Rep.:  $\bar{z} = \bar{x} \cdot \bar{y}$  OR  $z = xy$   $z$  is equal to  $x$  and  $y$

- NOT | Group X → Y → Outp.  $z = \bar{x}$  Truth table showing all possible i/o.

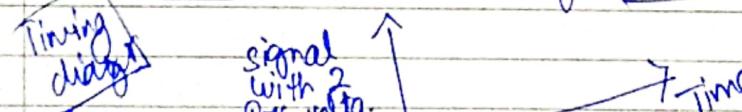


NOT Gate:  $z = \bar{x}$   $z$  equal to not  $x$ .

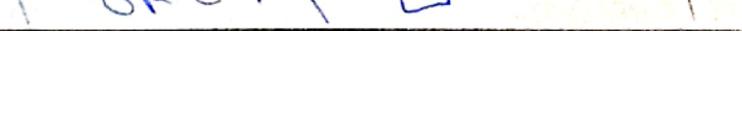
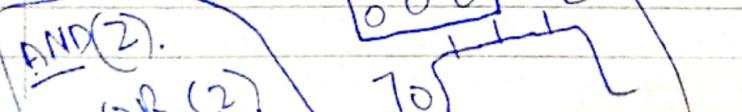
$$x \rightarrow \text{AND} \rightarrow z = \bar{x}$$

X	$z = \bar{x}$
0	1
1	0

Timing diagram of AND, OR, NOT



Timing diagram of AND, OR, NOT



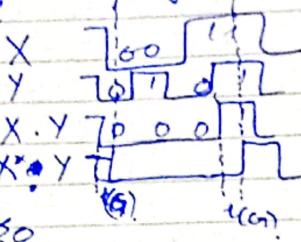
1-Abdul  
2-Sift  
3-Zesh.  
4-Saad.  
5-Suleiman

Gate Delay: → Property of gate.

- ① The length of time it take for an input change to result in the corresponding output change denoted by  $t_{\text{gate}}$ .

Depends upon:

→ Gate Type → No. of inputs.  
→ circuit design → underlying tech.



Multiple inputs in gate.

$A \rightarrow D$ ,  $B \rightarrow D$ ,  $C \rightarrow D$   $F = ABC$ . F will be one ( $A, B, C = (1, 1, 1)$ )  
and 0 otherwise (for rest of comb.).

$A \rightarrow D$ ,  $B \rightarrow D$ ,  $C \rightarrow D$ ,  $G = A + B + C + D$ . G will be zero (0) only when  $(A, B, C, D) = (0, 0, 0, 0)$ .  
for rest of combination it is 1.

United Indians say → War of independence. 3

1843 → Sindh, British → Mutiny. } 1857

1848 → Punjab, MEC. } 1886

1857, 1858, 1859, 1860, 1861, 1862, 1863, 1864, 1865, 1866, 1867, 1868, 1869, 1870, 1871, 1872, 1873, 1874, 1875, 1876, 1877, 1878, 1879, 1880, 1881, 1882, 1883, 1884, 1885, 1886, 1887, 1888, 1889, 1890, 1891, 1892, 1893, 1894, 1895, 1896, 1897, 1898, 1899, 1900, 1901, 1902, 1903, 1904, 1905, 1906, 1907, 1908, 1909, 1910, 1911, 1912, 1913, 1914, 1915, 1916, 1917, 1918, 1919, 1920, 1921, 1922, 1923, 1924, 1925, 1926, 1927, 1928, 1929, 1930, 1931, 1932, 1933, 1934, 1935, 1936, 1937, 1938, 1939, 1940, 1941, 1942, 1943, 1944, 1945, 1946, 1947, 1948, 1949, 1950, 1951, 1952, 1953, 1954, 1955, 1956, 1957, 1958, 1959, 1960, 1961, 1962, 1963, 1964, 1965, 1966, 1967, 1968, 1969, 1970, 1971, 1972, 1973, 1974, 1975, 1976, 1977, 1978, 1979, 1980, 1981, 1982, 1983, 1984, 1985, 1986, 1987, 1988, 1989, 1990, 1991, 1992, 1993, 1994, 1995, 1996, 1997, 1998, 1999, 2000, 2001, 2002, 2003, 2004, 2005, 2006, 2007, 2008, 2009, 2010, 2011, 2012, 2013, 2014, 2015, 2016, 2017, 2018, 2019, 2020, 2021, 2022, 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enough knowledge

- 2- good thing → best control.
- 3- wanted to engage  
include.
- 4- hand gesture
- 5- posture.