

2 . 2

21L5251 CLO - 02

$$Q8 \quad e^x y \quad y'(x) = e^{-y} + e^{-2x-y}$$

$$\int \left( \frac{1}{e^x} + \frac{1}{e^{3x}} \right) dx = \int \frac{y}{e^y} dy$$

$$c - e^{-x} - e^{-3x} = e^y (y^{-1})$$

$$c - \frac{1}{e^x} - \frac{1}{3e^{3x}} = e^y y - e^y$$

$$c - \frac{3e^{2x} + 1}{3e^{3x}} = e^y y - e^y$$

$$OR \quad \ln \left( c - \frac{3e^{2x} + 1}{3e^{3x}} \right) = y^2 - y$$

$$Q12 \quad \sin 3x dx + 2y \cos^3 3x dy = 0$$

$$\frac{\sin 3x}{\cos^3 3x} dx = -2y dy$$

$$\int \tan 3x \sec^2 3x dx = \int -2y dy$$

$$\left( \frac{\tan^2 3x}{8} \right) + c = -y^2$$

$$c - \frac{\tan^2 3x}{8} = y^2$$

$$y = \pm \sqrt{c - \frac{\tan^2 3x}{8}}$$

$$Q13 \quad (e^y + 1)^2 e^{-y} dx + (e^x + 1)^3 e^{-x} dy = 0$$

$$(e^y + 1)^2 e^{-y} dx = - (e^x + 1)^3 e^{-x} dy$$
$$-\int \frac{1}{(e^x + 1)^3 e^{-x}} dx = \int \frac{1}{(e^y + 1)^2 e^{-y}} dy$$

$$\int \frac{-e^x}{(e^x + 1)^3} dx = \int \frac{e^y}{(e^y + 1)^2} dy$$

Integrating

$$-(e^x + 1)^{-2} + C = (e^y + 1)^{-1}$$
~~$$-(e^x + 1)^{-2} + C - 1 = e^y$$~~
$$\ln(-(e^x + 1)^{-2} + C) = y$$

$$Q22 \quad (e^x + e^{-x}) \frac{dy}{dx} = y^2$$

$$\int \frac{1}{e^x + e^{-x}} dx = \int \frac{1}{y^2} dy$$
$$\int \frac{e^x + e^{-x}}{(e^{2x} + 1)^2} dx = \frac{-1}{y}$$

$$\int \frac{e^x}{e^{2x} + 1} dx = \int \frac{1}{y^2} dy$$

$$c + \tan^{-1}(e^x) = \frac{1}{y}$$

$$c + \frac{1}{\tan^{-1}(e^x)} = y$$



$$Q28 \quad (1+x^4)dy + x(1+4y^2)dx = 0 \quad y(1) = 0$$

$$(1+x^4)dy = -x(1+4y^2)dx$$

$$\frac{1+x^4}{x} dy = (1+4y^2) dx$$

$$\int \frac{1}{1+4y^2} dy = \int \frac{x}{1+x^4} dx$$

$$\int \frac{1}{1+(2y)^2} dy = \int \frac{x}{1+x^4} dx$$

let  $u = 2y$  then  $2dy = du$

$$\text{and } \frac{1}{2} \int \frac{1}{1+u^2} du = \frac{1}{2} \tan^{-1}(u) = \frac{1}{2} \tan^{-1}(2y)$$

let  $v = x^2$  then  $2x dx = dv$

$$\text{and } \frac{1}{2} \int \frac{1}{1+v^2} dv = \frac{1}{2} \tan^{-1}(v) = \frac{1}{2} \tan^{-1}(x^2)$$

$$\frac{1}{2} \tan^{-1}(2y) = \frac{1}{2} \tan^{-1}(x^2) + C$$

Taking off arctan from both sides

$$2y = x^2 + \cancel{\tan(C)}$$

$$y = \frac{x^2}{2} + \cancel{\frac{2\tan C}{2}}$$

$$0 = \frac{1}{2} + \frac{2\tan C}{2} \rightarrow C = +\frac{\pi}{4}$$

$$y = \frac{x^2}{2} - \frac{\pi}{4}$$

Q29  $\frac{dy}{dx} = ye^{-x^2}$   $y(4) = 1$

$$\frac{1}{y} dy = e^{-x^2} dx$$

$$4 \int \frac{1}{y} dy = \int e^{-x^2} dx$$

$$\ln(y) = f$$

Q29  $\frac{dy}{dx} = ye^{-x^2}$   $y(4) = 1$

$$4 \int \frac{1}{y} \frac{dy}{dt} dt = \int_4^x e^{-t^2} dt$$

$$\ln y(x) - \ln y(4) = \int_4^x e^{-t^2} dt$$

$$\ln y(x) = \ln y(4) + \int_4^x e^{-t^2} dt \quad (\ln y(4) = \ln 1 = 0)$$

$$\ln y(x) = \int_4^x e^{-t^2} dt$$

$$y(x) = e^{\int_4^x e^{-t^2} dt}$$



Q37

$$1) \frac{dy}{dx} = x\sqrt{1-y^2}$$

BS are 0 for  $y=\pm 1$

Solving DE,

$$\sin^{-1} y = \frac{x^2}{2} + c$$

$$y = \sin \frac{x^2}{2} + \sin c$$

No value of  $c$  gives neither  $y=1$  nor  $y=-1$   
so, both  $y=1, y=-1$  are singular solutions.

$$2) (e^x + e^{-x}) \frac{dy}{dx} = y^2$$

BS of DE are zero for  $y=0$

After solving,

$$c + \frac{1}{\tan(e^x)} = y$$

No value of  $c$  gives  $y=0$ , so it is a  
singular solution.

Q46

$$\frac{dy}{dx} = \frac{\sin \sqrt{x}}{\sqrt{y}}$$

$$\int \sqrt{y} dy = \int \sin \sqrt{x} dx$$

$$\frac{2y^{3/2}}{3} = 2\sin(\sqrt{x}) - 2\sqrt{x}\cos(\sqrt{x}) + C$$

$$y = \left( \frac{3}{2} \left( \sin \sqrt{x} - \sqrt{x} \cos \sqrt{x} + C \right) \right)^{2/3}$$

$$\int \sin \sqrt{x} dx$$

$$\text{let } \sqrt{x} = u$$

$$\frac{1}{2\sqrt{x}} dx = du$$

$$dx = 2u du$$

$$2 \int u \sin u du$$

$$= 2 \sin(u) - 2\cos(u)u + C$$

Q49  $\frac{dy}{dx} = \frac{e^{\sqrt{x}}}{\sqrt{y}}$   $y(1) = 4$

$$\frac{dy}{dx} = \frac{e^{\sqrt{x}}}{\sqrt{y}}$$

$$\text{let } \sqrt{x} = u$$

$$\frac{1}{2\sqrt{x}} = du \Rightarrow dx = 2u du$$

$$\int y dy = \int e^{\sqrt{x}} dx$$

$$\frac{y^2}{2} = 2(\sqrt{x}e^{\sqrt{x}} - e^{\sqrt{x}} + C)$$

$$2 \int ue^u du$$

$$= 2(ue^u - e^u + C)$$

$$\text{put } y = 4 \text{ and } x = 1$$

$$21 = C$$

$$\frac{y^2}{2} = 2(\sqrt{x}e^{\sqrt{x}} - e^{\sqrt{x}} + 8)$$

$$y = \sqrt{4\sqrt{x}e^{\sqrt{x}} - 4e^{\sqrt{x}} + 16}$$

$$y = 2 \sqrt{\sqrt{x}e^{\sqrt{x}} - e^{\sqrt{x}} + 4}$$



2 - 3

21L5251 CL0-3

Q3  $\frac{dy}{dx} + y = e^{3x}$

$$I = e^{\int P(x) dx}$$

$$= e^x$$

$P(x) = 1$

$(-\infty, +\infty)$

$e^x \frac{dy}{dx} + e^x y = e^{4x}$

$\frac{d}{dx}(e^x y) = e^{4x}$

$e^x y = \int e^{4x}$

$e^x y = \frac{e^{4x}}{4} + C$

$y = \frac{e^{4x} + C}{4e^x}$

$y = \frac{1}{4} e^{3x} + e^{-x} C \quad | \quad Ce^{-x} \text{ is transient}$

Q12

$(1+x) \frac{dy}{dx} - xy = x + x^2$

$\frac{dy}{dx} - \frac{x}{1+x} y = \frac{x+x^2}{1+x} = \frac{x(1+x)}{1+x} = x$

$\int \frac{x}{1+x} dx = x - \ln(1+x) + C$

$e^{-(x - \ln(1+x))} = \frac{e^{-x}}{(1+x)^{-1}} = \frac{e^{-x}}{e^x} = \frac{1}{e^x}$

$$\int \frac{dy}{dx} \left( \frac{-xe^x}{(x+1)^2} y \right) = \int \frac{xe^x}{x+1} dx$$

$$\frac{-xe^x}{(x+1)^2} y =$$

$$\int \frac{dy}{dx} \frac{-x}{e^x} y dx = \int \frac{(x+1)x}{e^x} dx$$

$$\frac{f(x)}{e^x} y = f(x^2 + 3x + 3) e^{-3x} + C$$

$$y = e^{-2x} (x^3 + 3x^2 + 3x) + xC$$

$$\int \frac{dy}{dx} \frac{x+1}{e^x} y dx = -(x^2 + 3x + 3) e^{-3x} + C$$

$$\frac{x+1}{e^x} y = -(x^2 + 3x + 3) e^{-3x} + C$$

$$y = (-x^2 e^{-2x} + 3x e^{-2x} + 3 e^{-2x} + C)(x+1)$$

No transient term

$$-1 < x < \infty$$



$$Q14 \quad xy' + (1+x)y = e^{-x} \sin 2x$$

$$y' + \frac{1+x}{x} y = \frac{e^{-x} \sin 2x}{x}$$

$$\int \frac{1+x}{x} = \ln x + x$$

$$e^{\ln x + x} = xe^x$$

$$\int \frac{d}{dx} (xe^x y) dx = \int e^x \sin 2x dx$$

$$xe^x y = -\frac{\cos 2x}{2} + C$$

$$y = \frac{-\cos 2x}{2xe^x} + \frac{ce^{-x}}{x}$$

$$0 < x < \infty$$

$$Q27 \quad xy' + y = e^x \quad y(1) = 2$$

$$y' + \frac{1}{x} y = \frac{e^x}{x}$$

$$\int \frac{1}{x} dx = \ln x$$

$$e^{\ln x} = x \quad (-\infty, \infty)$$

$$\int \frac{d}{dx} (xy) dx = \int e^x dx$$

$$xy = e^x + C$$

$$y = \frac{e^x}{x} + \frac{c}{x} \quad \leftarrow y(1) = 2 \quad \left| \begin{array}{l} \frac{2}{e} = c \\ y = \frac{e^x}{x} + \frac{2}{ex} \\ 0 < x < \infty \end{array} \right.$$

$$Q34 \quad x(x+1) \frac{dy}{dx} + xy = 1$$

$$\frac{dy}{dx} + \frac{1}{x+1} y = \frac{1}{x(x+1)}$$

$$e^{\int \frac{1}{x+1} dx} = x+1 \quad (-\infty, +\infty)$$

$$\int \frac{d}{dx} ((x+1)y)^{dx} = \int \frac{1}{x} dx$$

$$(x+1)y = \ln x + c$$

$$y = \frac{\ln x}{x+1} + \frac{c}{x+1}$$

$$-1 < x < \infty$$

$$y(e) = 1$$

$$1 = \frac{1}{e+1} + \frac{c}{e+1}$$

$$e+1 = c+1$$

$$c = e$$

$$y = \frac{\ln x}{x+1} \frac{e}{x+1}$$

$$Q40 \quad (1+x^2) \frac{dy}{dx} + 2xy = f(x) \quad y(0) = 2$$

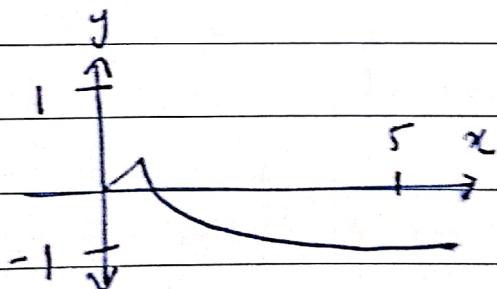
$$f(x) = \begin{cases} x, & 0 \leq x \leq 1 \\ -x, & x \geq 1 \end{cases}$$

$$y' + \frac{2x}{1+x^2} y = \begin{cases} \frac{x}{1+x^2}, & 0 \leq x \leq 1 \\ \frac{-x}{1+x^2}, & x \geq 1 \end{cases}$$

$$I = 1+x^2$$

$$y = \begin{cases} \frac{1}{2} - \frac{1}{2(1+x^2)}, & 0 \leq x < 1 \end{cases}$$

$$\begin{cases} \frac{3}{2} - \frac{1}{2}, & x \geq 1 \end{cases}$$



2.4

$$Q6 \quad \left( 2y - \frac{1}{x} + \cos 3x \frac{dy}{dx} + \frac{y}{x^2} - 4x^3 + 3y \sin 3x \right) = 0$$

$$\cos 3x \frac{dy}{dx} = \frac{1}{x} - 2y + \frac{y}{x^2} - 4x^3 + 3y \sin 3x$$

$$\cos 3x dy = \left( \frac{1}{x} - 2y + \frac{y}{x^2} - 4x^3 + 3y \sin 3x \right) dx$$

$$N(y) dy = M(x) dx$$

$$M(x) dx + (N(y)) dy = 0$$

$$\frac{dN(y)}{dx} = -3 \sin 3x$$

$$\frac{dM(x)}{dy} = -2 + \frac{1}{x^2} + 3 \sin 3x$$

Equation not exact

$$Q18 \quad (2y \sin x \cos x - y + 2y^2 e^{xy^2}) dx = (x - \sin^2 x - 4xy e^{xy^2}) dy$$

$$(2y \sin x \cos x - y + 2y^2 e^{xy^2}) dx + (-x + \sin^2 x + 4xy e^{xy^2}) dy = 0$$

$$\frac{dM}{dy} = 2 \sin x \cos x - 1 + 2y e^{xy^2} + 4y^3 e^{xy^2} \cdot x$$

$$\frac{dN}{dx} = -1 + 2 \sin x \cos x + 4y e^{xy^2} + 4xy^3 e^{xy^2}$$

Equation is exact

Integrating  $m(x, y) dx$

$$\int (2y \sin x \cos x - y + 2y^2 e^{xy^2}) dx$$

$$= 2y \int \sin x \cos x dx - \int y dx + 2y^2 \int e^{xy^2} dx$$

$$= y \sin^2 x - yx + 2e^{xy^2} + g(y)$$

$$\int 2y \sin x \cos x dx$$

~~cancel from LHS~~

$$\frac{\sin^2 x}{2}$$

$$e^{xy^2}$$

$$\frac{e^{xy^2}}{y^2}$$

Derivating wrt y

$$= \sin^2 x - x + 4e^{xy^2} \cdot xy + g'(y)$$

~~$\sin^2 x - x + 4e^{xy^2} \cdot xy + g'(y) = -x + \sin^2 x + 4xye^{xy^2}$~~

$$g'(y) = 0$$

$$g(y) = 0$$

$$C = y \sin^2 x - yx + 2e^{xy^2} + \cancel{c}$$

$$Q. 25 \quad (y^2 \cos x - 3x^2 y - 2x) dx + (2y \sin x - x^3 + \ln y) dy = 0$$

$$y(0) = e$$

$$\frac{dM}{dy} = 2y \cos x - 3x^2$$

$$\frac{dN}{dx} = 2y \cos x - 3x^2$$

Exact

$$\begin{aligned} & \int (y^2 \cos x - 3x^2 y - 2x) dx \\ &= \int y^2 \cos x dx - 3y \int x^2 dx - 2 \int x dx \\ &= y^2 \sin x - yx^3 - x^2 + g(y) \end{aligned}$$

Differentiating wrt y

$$= 2y \sin x - x^3 + g'(y)$$

$$\begin{aligned} & \int \ln y \\ & y \ln y - \int y \frac{1}{y} dy \\ & y \ln y - y \end{aligned}$$

$$2y \sin x - x^3 + g'(y) = 2y \sin x - x^3 + \ln y$$

$$g'(y) = \ln y$$

$$g(y) = y \ln y - y$$

$$C = 2y^2 \sin x - x^3 y + y \ln y - y - x^2$$

$$C = e \ln e - e$$

$$C = 0$$



$$y^2 \sin x - x^3 y - x^2 + y \ln y - y = 0$$

Q32

$$y(x+y+1) dx + (x+2y) dy = 0$$

$$\frac{dM}{dy} = x+2y+1$$

$$\frac{dN}{dx} = 1$$

Finding integrating factor

$$= \frac{x+2y+x-1}{x+2y}$$

$$e^{\int dx}$$

$$= e^x$$

$$ye^x(x+y+1) dx + e^x(x+2y) dy = 0$$

$$\frac{dM}{dy} = xe^x + 2e^x y + e^x$$

$$\frac{dN}{dx} = xe^x + 2e^x y + e^x$$

Exact

$$\int e^x (x + 2y) dy$$

$$= e^x \int (x + 2y) dy$$

$$= xe^x y + y^2 e^x + f(x)$$

Differentiating wrt x

$$xye^x + ye^x + y^2 e^x + f'(x) = xye^x + y^2 e^x + ye^x$$

$$f'(x) = 0$$

$$f(x) = 0$$

$$c = xe^x y + y^2 e^x$$

$$Q33 \quad 6xy dx + (4y + 9x^2) dy = 0$$

$$\frac{dM}{dy} = 6x$$

$$\frac{dN}{dx}$$

$$\frac{dN}{dx} = 18x$$

$$= \frac{+18x - 6x}{6xy}$$

$$= +2$$

$$\frac{1}{y}$$

$$= e^{\int \frac{2}{y} dy} = y^2$$



$$6x^3y^3 dx + (4y^3 + 9x^2y^2) dy = 0$$

$$\frac{dM}{dy} = \cancel{18x^2y^2} - \cancel{18x^2y^2}$$

$$\frac{dN}{dx} = 18xy^2$$

Exact

$$\begin{aligned} & \int 6x^3y^3 dx \\ &= 3x^2y^3 + g(y) \\ & \cancel{3x^2y^3} + g(y) = \cancel{1}y^3 \\ &= 9x^2y^2 + g'(y) \\ & 9x^2y^2 + g'(y) = 4y^3 + 9x^2y^2 \end{aligned}$$

$$\begin{aligned} g'(y) &= 4y^3 \\ g(y) &= y^4 \end{aligned}$$

$$C = 3x^2y^3 + y^4$$



$$\varphi 37 \quad x dx + (x^2 y + 4y) dy = 0 \quad y(4) = 0$$

$$\frac{dx}{dy} = 0$$

dy

$$\frac{dN}{dx} = 2xy$$

$\frac{dy}{dx}$

$$= \frac{2xy - 0}{x}$$

$$= 2y$$

$$e^{\int 2y dy}$$

$$= e^{y^2}$$

$$xe^{y^2} dx + e^{y^2} (x^2 y + 4y) dy = 0$$

$$\frac{dx}{dy} = xe^{y^2} \cdot 2y$$

$$\frac{dN}{dx} = 2xye^{y^2}$$

$$\int xe^{y^2} dx$$

$$= \frac{x^2 e^{y^2}}{2} + g(y)$$

Differentiating wrt y

$$= x^2 e^{y^2} \cdot f' y + g'(y) = x^2 e^{y^2} y + e^{y^2} y$$

$$e^{y^2} y = g'(y)$$

$$g(y) = \frac{e^{y^2}}{2} - 2e^{y^2}$$

$$C = \frac{x^2 e^{y^2}}{2} + \frac{e^{y^2}}{2} - 2e^{y^2}$$

$$C = 10$$

$$10 = \frac{x^2 e^{y^2}}{2} + \frac{e^{y^2}}{2} - 2e^{y^2}$$

$$20 = (x^2 + 4) e^{y^2}$$

2.5

Q9

$$-ydx + (x + \sqrt{xy})dy = 0$$

Dividing by  $x$

$$-\left(\frac{y}{x}\right)dx + \left(1 + \sqrt{\frac{y}{x}}\right)dy = 0 \quad \left(v = \frac{y}{x}\right)$$

$$-vdx + (1 + \sqrt{v})(vdx + xdv)$$

$$-vdx + vdx + \sqrt{v}dx + xdv + x\sqrt{v}dv = 0$$

~~$\frac{d}{dx}v^2$~~

$$\sqrt{v}dx = -dv(x(1+\sqrt{v}))$$

$$\frac{\sqrt{v}}{1+\sqrt{v}} \frac{1}{dv} = \frac{1}{dx}(-x)$$

$$\int \frac{1+\sqrt{v}}{\sqrt{v}} dv = - \int \frac{1}{x} dx$$

$$-2v^{-\frac{1}{2}} + \ln v = -\ln x + C$$

$$\ln y = 2\sqrt{xy} + C$$

$$y(\ln y - C)^2 = 4x$$



$$P.13 \quad (x + ye^{y/x})dx - xe^{y/x}dy = 0$$

Dividing by  $x$

$$(1 + ye^v)dx - e^v(xdv + vdx) = 0 \quad (y = vx)$$

~~$\frac{dx}{dv}$   $\frac{dv}{dx}$~~

$$dx + ve^v dx - e^v x dv - e^v v dx = 0$$

$$dx = e^v x dv$$

$$\int \frac{1}{x} dx = \int e^v dv$$

$$\ln x + c = e^v$$

$$\ln x + c = e^{y/x}$$

$$y(1) = 0$$

$$c = e^0$$

$$= 1$$

$$\ln x \cdot e^{y/x} = 0 - 1$$

$$17 \quad \frac{dy}{dx} = y(xy^3 - 1)$$

$$\frac{dy}{dx} = xy^4 + y$$

$$\frac{dy}{dx} + y = xy^4$$

$$y^{-4} \frac{dy}{dx} + y^{-3} = x$$

$$v = y^{-3}$$

$$v' = -3y^{-4}y'$$

$$\frac{v'}{-3} = y^{-4}y'$$

$$\frac{v'}{-3} + v = x$$

$$v' + 3v = -3x$$

$$e^{\int 3dx}$$

$$= e^{-3x}$$

$$\int \frac{d}{dx} (e^{-3x} v) dx = \int -3x e^{-3x} dx$$

$$e^{-3x} v = \cancel{C} + \frac{(3x+1)e^{3x}}{3} + C$$

$$v = \frac{+3x+1}{3} + \frac{C}{e^{-3x}}$$

$$y^{-3} = \frac{+3x+1}{3} + \frac{C}{e^{3x}}$$



$$Q21 \quad x^2 y' - 2xy = 3y^4 \quad y(1) = 1/2$$

$$y' - \frac{2}{x}y = \frac{3}{x^2}y^4$$

8

$$y^{-4} y' - \frac{2}{x} y^{-3} = \frac{3}{x^2}$$

$$v = y^{-3}$$

$$\frac{v'}{-3} = y^{-4} y'$$

$$\frac{v'}{-3} - \frac{2}{x} v = \frac{3}{x^2}$$

$$\frac{v'}{-3} + \frac{6}{x} v = -\frac{9}{x^2}$$

$$e^{\int \frac{6}{x} dx} = x^6$$

$$\int \frac{d}{dx} (x^6 v) dx = \int -\frac{9}{x^2} \cdot x^6 dx$$

$$x^6 v = -\frac{9}{5} x^5 + C$$

$$v = -\frac{9}{5} x^{-1} + \frac{C}{x^6}$$

$$y^{-3} = -\frac{9}{5x} + \frac{C}{x^6}$$

$$C = \frac{49}{5}$$

$$y^{-3} = -\frac{9}{5} x^{-1} + \frac{49}{5} x^{-6}$$

$$24 \quad \frac{dy}{dx} = \frac{1-x-y}{x+y}$$

~~$$\frac{dy}{dx} = \frac{1}{x+y}$$~~

$$\frac{dy}{dx} = \frac{1-x-v}{1+v} \quad (y=v^x)$$

~~$$(xdv + vdx)(1+v) = \frac{1}{x} - 1 - v$$~~

~~$$xdv + vxdv + vdx + v^2dx = \frac{1}{x} - 1 - v$$~~

~~$$xdv(v+1) + dx(v^2+v) = \frac{1}{x} - 1 - v$$~~

~~$$+ v + xdv(v+1) = -dx(v^2+v)$$~~

Q 24  $\frac{dy}{dx} = \frac{1-x-y}{x+y}$  let  $v = x+y$

$$\frac{dv}{dx} - 1 = \frac{1-v}{v} \quad \frac{dv}{dx} = 1 + \frac{dy}{dx}$$

~~$$vdv - v^2 = (1-v)dx$$~~

$$\int vdv = \int dx$$

$$v^2 = x + C$$

$$(x+y)^2 = 2x + C_1$$



$$\frac{dy}{dx} = \frac{3x+2y}{3x+2y+2}$$

let  $v = 3x+2y$

$$\frac{dv}{dx} = 3 + 2 \frac{dy}{dx}$$

$$\frac{1}{2} \frac{dv}{dx} - \frac{3}{2} = \frac{v}{v+2}$$

$$\frac{1}{2} \left( \frac{dv}{dx} - 3 \right) = \frac{v}{v+2}$$

$$\frac{dv}{dx} - 3 = \frac{2v}{v+2}$$

$$\frac{dv}{dx} = \frac{5v+6}{v+2}$$

$$\int \frac{v+2}{5v+6} dv = \int dx$$

~~$\int \frac{dv}{5v+6} = \ln(v+6)$~~

$$\frac{1}{5} (3x+2y) + \frac{4}{25} \ln |75x+50y+30| = x + C$$

$$C = \frac{4}{25} \ln 95$$

$$\frac{1}{5} (3x+2y) + \frac{4}{25} \ln |75x+50y+30| = x + \frac{4}{25} \ln 95$$

$$5y - 5x + 2\ln|75x+50y+30| = 2\ln 95$$