Question 1:

A patient is thought to have one of three diseases A_1, A_2 and A_3 whose probabilities under the given conditions are $\frac{1}{2}$, $\frac{1}{6}$ and $\frac{1}{3}$ respectively. A test is carried out to help the diagnosis and it yields a positive result with a probability of 0.1 for disease A_1, a probability of 0.2 for disease A_2 and a probability of 0.9 for disease A_3. What is the probability of A_1 disease after testing?

Question 2:

An overseas shipment of 6 foreign automobiles contains 3 that have slight paint blemishes. If an agency receives 3 of these automobiles at random, list the elements of the sample space S, using the letters B and N for blemished and non-blemished, respectively; then to each sample point assign a value x of the random variable X representing the number of automobiles with paint blemishes purchased by the agency. Find the probability distribution of X.

Question 3:

A continuous random variable X that can assume values between x = 2 and x = 5 has a density function given by

$$f(x) = \frac{2(1+x)}{27}.$$

Using the above pdf find:

(a) P(X<4)

(b) P(3≤X<4)

(c) Find cumulative distribution function and use it to evaluate P(3≤X<4).

Question 4:

The shelf life, in days, for bottles of a certain prescribed medicine is a random variable having the density function

$$f(x) = \begin{cases} \frac{20,000}{(x+100)^3} & x > 0 \end{cases}$$

Find the probability that a bottle of this medicine will have a shelf life of

(a) at most 200 days

(b) From 80 to 120 days

(c) Find Cumulative Distribution Function and use it to evaluate part(b).

Question 5:

Suppose that a Bayesian spam filter is trained on a set of 1000 spam messages and 250 messages that are not spam. The word "cruise" appears in 50 spam messages and in 2 messages that are not spam, while the word "urgent" appears in 100 spam messages and in 10 messages that are not spam. Would an incoming message be rejected as spam if it contains both words "cruise" and "urgent" and the threshold for rejecting spam is 0.9? (Assume, for simplicity, that the message is equally likely to be spam as it is not to be spam and that the two words are used independently.)

Question 6:

Obtain the probability distribution of a random variable X, where X denotes the number of aces in bridge hand.

Question 7:

Six boxes of components are ready to be shipped by a certain supplier. The number of defective components in each box is as follows:

Box	1	2	3	4	5	6
No of defectives	0	2	0	1	2	0

Obtain the probability distribution of number of defective components.

Question 8:

A store carries flash drives with either 1 GB, 2 GB, 4 GB, 8 GB, or 16 GB of memory. The accompanying table gives the distribution of Y 5 the amount of memory in a purchased drive:

Y	1	2	4	8	16
P(y)	0.05	0.10	0.35	0.40	0.10

Determine F(y) for each of the five possible values of Y

Question 9:

Let X be a continuous random variable with pdf

f(x) = $6x(1-x)$ $0 \leq x \leq 1$

   $= 0$ otherwise

   i)   Check that f(x) is a proper pdf.

ii)   Obtain an expression for distribution function of X.

iii)   Compute $P(\frac{1}{3} < X < \frac{2}{3})$

Question 10:

Three cooks A, B, and C, bake a special kind of cake, and the probabilities that it fails to rise are 0.02, 0.03 and 0.05 respectively. In the restaurant where they work, A bakes 50% of these cakes, B bakes 30% and C bakes 20%. What is the probability of failure caused by B?