Exercise 1.1 (Howard Anton)

~~Q1~~ See Q2

© ~~Q2~~ $\frac{2}{11} = 0.181818\ldots$

$= 0.\overline{18}$

$$\begin{array}{r} 0.181818 \\ 11 \overline{)\,2.0} \\ \underline{11} \\ 90 \\ \underline{88} \\ 20 \end{array}$$

It is also known as recurring decimal.

If $2<x<6$, Then which of the following is true

© Q3 (e) $-6 < -x < -2$

$\Rightarrow (-1)(-6) > (-1)(-x) > -1(-2)$

or $6 > x > 2$ or $2<x<6$

$\therefore$ It is true

i.e. The Condition is also sufficient which is not reqd in this question

$\therefore$ If $2<x<6$

Then $-2>-x>-6$

$\Rightarrow -6<-x<-2$

i.e. The Condition is necessary.

(g) $-6<-x<2$

It is necessary but not sufficient

(f) $|x-4| < 2$ $\forall x \in R$

$\Rightarrow \pm(x-4) < 2$

$\Rightarrow x-4 < 2$ OR $-(x-4) < 2$

$\Rightarrow x < 6$ ⟩(i) $\Rightarrow x-4 > -2$

$\Rightarrow x > 2$ ⟩(ii)

$\therefore 2 < x < 6$

$\therefore$ It is true

a) $0<x<4$

It is not necessarily true:

~~$\because$ x maybe $=5$~~

~~& $5>4$~~.

(b) $0 < x-2 < 4 \Rightarrow --- \Rightarrow 2<x<6$

$\therefore$ It is true

Q4) If $-1 < y-5 < 1$

(a) $4 < y < 6 \Rightarrow 4-5 < y-5 < 6-5$

$\Rightarrow -1 < y-5 < 1$

$\therefore$ It is true.

(g) $\frac{1}{6} < \frac{1}{y} < \frac{1}{4}$

$\Rightarrow \quad 6 > y > 4 \quad$ or $\quad 4 < y < 6$

$\Rightarrow$ Sol: (a)

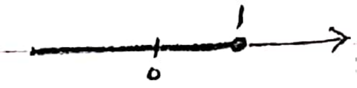
Q6) $\quad 8 - 3x \geq 5$

$\Rightarrow 8 - 3x - 8 \geq 5 - 8$

$\Rightarrow -3x \geq -3$

$\Rightarrow (-\frac{1}{3})(-3x) \leq (-\frac{1}{3})(-3)$

$\Rightarrow x \leq 1 \quad \therefore S.S. = (-\infty, 1]$



© Q12) $\quad -\frac{x+5}{2} \leq \frac{12+3x}{4}$

$\Rightarrow -(\frac{x+5}{2}) \times 4 \leq \frac{(12+3x)}{4} \times 4$

$\Rightarrow -2x - 10 \leq 12 + 3x$

$\Rightarrow -2x - 10 + 2x \leq 12 + 3x + 2x$

or $\quad -10 \leq 12 + 5x$

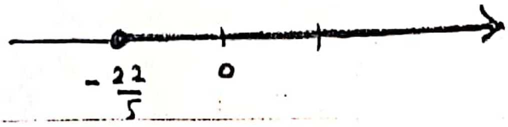
or $\quad -10 - 12 \leq 12 + 5x - 12$

$\Rightarrow -22 \leq 5x$

$\Rightarrow \frac{1}{5}(-22) \leq \frac{1}{5}(5x)$

$\Rightarrow x \geq -\frac{22}{5}$

$\therefore S.S. = [-\frac{22}{5}, \infty)$



Q18) $\left|\frac{s}{2}-1\right| = 1$

$\Rightarrow \pm\left(\frac{s}{2}-1\right) = 1$

$\Rightarrow \frac{s}{2}-1 = 1$ OR $-\left(\frac{s}{2}-1\right) = 1$

$\Rightarrow \frac{s}{2} = 2$

$\Rightarrow s = 4 \quad \langle(i)$

or $\frac{s}{2}-1 = -1$

$\Rightarrow \frac{s}{2} = 0$

$\Rightarrow s = 0 \quad \langle(ii)$

$\therefore$ S.S. $= \{0, 4\}$

Q17) $|8-3s| = 9/2$

$\Rightarrow \pm(8-3s) = 9/2$

$\Rightarrow 8-3s = 9/2$ OR $-(8-3s) = 9/2$

$\Rightarrow -3s = 9/2 - 8$

$= \frac{9-16}{2} = -7/2$

$\Rightarrow s = \frac{7}{6} \quad \langle(i)$

$\Rightarrow -8+3s = +9/2$

$\Rightarrow 3s = +9/2 + 8$

$= \frac{+9+16}{2}$

$= \frac{25}{2}$

$\therefore s = 25/6 \quad \langle(ii)$

$\therefore$ S.S. $= \{7/6, 25/6\}$

Q 25) $\left|\frac{z}{5}-1\right| \leq 1$

$\Rightarrow \pm(z/5 - 1) \leq 1$

$\Rightarrow z/5 - 1 \leq 1$ OR $-(z/5 - 1) \leq 1$

$\Rightarrow z/5 \leq 2$

$\Rightarrow z \leq 10 \quad \langle(i)$

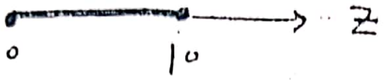
$\Rightarrow z/5 - 1 \geq -1$

$\Rightarrow z/5 \geq 0$

$\Rightarrow z \geq 0$ or $0 \leq z \quad \langle(ii)$

$\therefore$ S.S. $= 0 \leq z \leq 10$

$= [0, 10]$



Q33) $\frac{|r+1|}{2} \geq 1$

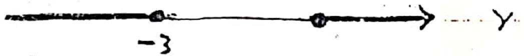
$\Rightarrow \pm\left(\frac{r+1}{2}\right) \geq 1$

$\Rightarrow \frac{r+1}{2} \geq 1$ or $-\left(\frac{r+1}{2}\right) \geq 1$

Q37) $4 < x^2 < 9$
See P-14

$\Rightarrow r+1 \geq 2$	$\Rightarrow -(r+1) \geq 2$
$\Rightarrow r \geq 1$	$\Rightarrow r+1 \leq -2$
$\therefore S_1 = [1, \infty)$ (i)	$\Rightarrow r \leq -3$
	$\therefore S_2 = (-\infty, -3]$

$\therefore S.S. = (-\infty, -3] \cup [1, \infty)$



Ⓒ Q39) $(x-1)^2 < 4$ N.B. $\sqrt{a^2} = |a|$

$\Rightarrow |x-1| < 2$ $\Rightarrow \pm(x-1) < 2$

$\Rightarrow x-1 < 2$ or $-(x-1) < 2$

$\Rightarrow x < 3$ (i)   or $x-1 > -2$

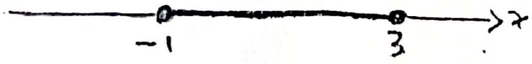
$\Rightarrow x > -1$ (ii)

N.B.
rels ≡ relations

$\therefore$ From rels (i) & (ii), we get

$-1 < x < 3$

$\therefore S.S = (-1, 3)$



Ⓒ Q41) $x^2 - x < 0 \Rightarrow x(x-1) < 0$ which is −ve.

To get whole statement −ve

one of two must be −ve

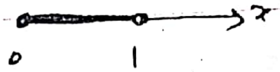
$\Rightarrow x < 0$ & $x-1 > 0$
(ii) $\Rightarrow x > 1$ (iii)

Rels. (i) & (iii) can not hold simultaneously

ii Let $x > 0$, (iii) & $x-1 < 0$
$\Rightarrow x < 1$ (iv)

From rels (iii) & (iv), we get $0 < x < 1$

$\therefore$ S.S. = $(0, 1)$



Q.43) $|-a| = a$

For $a = 0$ —— (i)

$|-a| = |-0| = 0 = a$ (See eqn. (i))

For $a = 2$, $|-a| = |-2| = 2 = a$

$\therefore |-a| = a$ holds $\forall\, a \geq 0$

now for $a = -3$ —— (iii)

$|-a| = |-(-3)| = |3| = 3$

$\neq a$ (See eqn (iii))

Similarly it is false $\forall$ -ve real nos.

Alternate method
Q44) By Q43

$|-(1-x)| = (1-x)$

$\Rightarrow 1-x \geq 0 \Rightarrow x \leq 1$

$\therefore$ S.S. $= (-\infty, 1]$

ⓒ ✓ Q44) $|x-1| = 1-x$

$\Rightarrow \pm(x-1) = 1-x \Rightarrow x-1 = 1-x \Rightarrow 2x = 2$
$\Rightarrow x = 1$ —(i)

or $-(x-1) = 1-x \Rightarrow -x+1 = 1-x \Rightarrow 0=0$ which does not give any sol.

$\therefore$ S.S. $= \{1\}$

See alternate method. In This case alternate method is better.

For $x=0$
$|0-1| = (1-0)$
For $x=-3$
$|-3-1| = 1-(-3)$
For $x = 5 > 1$, $|5-1| \neq 1-5$

Triangle Inequality:-

Q 45] $|a+b|^2 = (a+b)^2$

$= a^2 + 2ab + b^2$

$\le |a|^2 + 2|a||b| + |b|^2$ $\left\{ \because a^2 = |a|^2 \ \& \ ab \le |a||b| \right\}$

$\le (|a| + |b|)^2$

$\Rightarrow |a+b| \le |a| + |b|$

46] $|ab| = |a||b|$

Case I: Let $a>0, b>0$ Then $|ab| = ab = |a||b|$

Case II: Let $a<0$, $b>0$, Then $|ab| = -ab$

$= |a||b|$

$\{ \because a<0 \ \therefore |a| = -a \}$

Case III Let $a>0, b<0$, Then $|ab| = -ab$

$= a(-b) = |a||b|$

Case III $a<0, b<0$, $|ab| = ab$

$= (-a)(-b) = |a||b|$

CaseV when $a=0$

Then $|ab| = |0| = 0 = 0 \cdot |b| = |a||b|$

similarly when $b=0$ or both are '0'

Then $|ab| = |a||b|$

Hence $|ab| = |a||b| \ \forall \ a,b \in R$

Q 47] If $|x| \le 3$ & $x > -\frac{1}{2}$ or $-\frac{1}{2} < x$ —(i)

$\Rightarrow \pm x \le 3 \Rightarrow x \le 3$ —(ii)

or $-x \le 3 \Rightarrow x \ge -3$

or $-3 \le x$ —(iii)

Reals (i) & (iii) hold

4)

Simultaneously if $x > -\frac{1}{2}$ or $-\frac{1}{2} < x$ —— (iv)

$\therefore -\frac{1}{2} < x \leq 3$. (using rels. (iii) & (iv))

ⓒ Q37)

$$4 < x^2 < 9$$

$$\Rightarrow 2 < |x| < 3 \qquad \{\because \sqrt{x^2} = |x|\}$$

$$\Rightarrow 2 < \pm x < 3$$

$$\Rightarrow 2 < x < 3 \quad \Rightarrow S_1 = (2, 3) \text{ —(I)}$$

or $2 < -x < 3$

$$\Rightarrow (-1)2 > (-1)(-x) > (-1)3$$

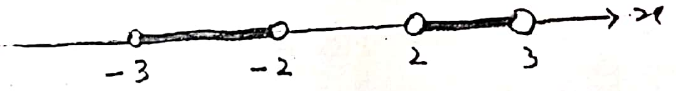
$$\Rightarrow -2 > x > -3$$

or $-3 < x < -2$

$$\therefore S_2 = (-3, -2) \text{ —— (II)}$$

Hence from eqn ① + ②, we get

S.S. = $(-3, -2) \cup (2, 3)$



ASSIGNMENT Exercise 1.1. P – 7 (15)

Q 1 Express 1/9 as a repeating decimal, using a bar to indicate the repeating digits. What are the decimal representations of 2/9, 3/9, 8/9 ?

Q 9. $2x - \frac{1}{2} > 7x + \frac{7}{6}$

Q 27. $\left|3 - 1/x\right| < 1/2$

Q 11. $\frac{4}{5}(x-2) < \frac{1}{3}(x-6)$

Q 38 $1/9 < x^2 < 1/4$

Q $x^2 - 3x > 10$

Ans S.S. $(-\infty, -2) \cup (5, +\infty)$

Q 34 $\left|\frac{3x}{5} - 1\right| > \frac{2}{5}$

Page # 23 | James Stewart (EX: 1.1)

Q # 34 $F(x) = |2x+1|$

Q # 35 $G(x) = \frac{3x + |x|}{x}$

Q # 36 $G(x) = \frac{|x|}{x^2}$

Q: 42 $x^2 - x - 2 \geq 0$, Q 43

Plus 3 model Questions

EX 7
P-9
Howard Anton

$\frac{2x-5}{x-2} < 1 \Rightarrow \frac{2x-5}{x-2} - 1 < 0 \Rightarrow \frac{2x-5-x+2}{x-2} < 0$

$\Rightarrow \frac{x-3}{x-2} < 0 \Rightarrow x - 3 \neq 0$ {∵ In This Case L.H.S = 0 & $0 \nless 0$}

i.e. $x \neq 3$

Also $x - 2 \neq 0$ i.e. $x \neq 2$

now L.H.S < 0 i.e. –ve

$\Rightarrow x - 3 > 0$ & $x - 2 < 0 \Rightarrow x > 3$ & $x < 2$

which is not possible

OR $x - 3 < 0$ & $x - 2 > 0 \Rightarrow x < 3$ & $x > 2$

i.e. $2 < x < 3$ which is true

∴ S.S = (2, 3)

E. Boyce book

Example $\frac{(x-3)(x+1)}{(x+2)} > 0$ $x \neq -2$

S.S = $(-2, -1) \cup (3, \infty)$