## Q.2) Cutting plane Algorithm.

Special constraint (called Cuts) are Added to the solution space in a manner that Render an integ

The added cut do not eliminate any of the feasible integer point but must pass through at least one feasible or infeasible integer point.

These are basic requirment for any cut.

Q) Solve By fractional Cut

Ex (9.56) (Part (d))

$$\text{Min} \quad z = 5x_1 + 4x_2$$
$$3x_1 + 2x_2 \ge 5$$
$$2x_1 + 3x_2 \ge 7.$$
$$x_1, x_2 \ge 0, \quad \text{And integer}$$

soln

Already solve

The optimal tableau is

| | $x_1$ | $x_2$ | $x_3$ | $x_4$ | R.H.V |
|---|---|---|---|---|---|
| min z | 0 | 0 | $-7/5$ | $-2/5$ | $49/5$ |
| $x_1$ | 1 | 0 | $-3/5$ | $2/5$ | $1/5$ |
| $x_2$ | 0 | 1 | $2/5$ | $-3/5$ | $11/5$ |

The first cut is at $x_1$.

$$x_1 - \frac{3}{5}x_3 + \frac{2}{5}x_4 = \frac{1}{5}.$$

$$x_1 + \left(-1 + \frac{2}{5}\right)x_3 + \frac{2}{5}x_4 = \frac{1}{5}$$

$x_1 - x_3 = -\frac{2}{5}x_3 - \frac{2}{5}x_4 + \frac{1}{5} \le \frac{1}{5}$

And for integers

$-\frac{2}{5}x_3 - \frac{2}{5}x_4 + \frac{1}{5} \le 0$

$-\frac{2}{5}x_3 - \frac{2}{5}x_4 \le -\frac{1}{5}$

And cut is. $\frac{2}{5}x_3 + \frac{2}{5}x_4 \ge \frac{1}{5}$

$$-\frac{2}{5}x_3 - \frac{2}{5}x_4 + S_1 = -\frac{1}{5}$$

| | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $S_1$ | R.H.V |
|---|---|---|---|---|---|---|
| min z | 0 | 0 | $-7/5$ | $-2/5$ | 0 | $49/5$ |
| $x_1$ | 1 | 0 | $-3/5$ | $2/5$ | 0 | $1/5$ |
| $x_2$ | 0 | 1 | $2/5$ | $-3/5$ | 0 | $11/5$ |
| $S_1$ | 0 | 0 | $-2/5$ | $-2/5$ | 1 | $-1/5$ |
| | | | 3.5 | 1 | | |

$\min\left\{\left|\frac{-2/5}{-2/5}\right|, \left|\frac{-7/5}{-2/5}\right|\right\}$

$$-\frac{5}{2}R_4, \quad R_1 - R_4, \quad R_2 + R_4, \quad R_3 + \frac{3}{5}\left(-\frac{5}{2}R_4\right)$$

$-\frac{3}{2} = -1 + \frac{1}{2} = -\frac{1}{4} - \frac{3}{2}$ $\quad -1 + x = -\frac{3}{2} \quad 1 - \frac{5}{2} = -\frac{3}{2} \quad x = -\frac{1}{2}$ $\quad -x = -\frac{3}{2} - 1 \quad x = \frac{5}{2}$ $\quad -\frac{5}{2} + 1 = -\frac{3}{2} \quad = 1 - \frac{3}{2}$

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| | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $S_1$ | R.H.V |
|---|---|---|---|---|---|---|
| Min z | 0 | 0 | -1 | 0 | -1 | 10 |
| $x_1$ | 1 | 0 | -1 | 0 | 1 | 0 |
| $x_2$ | 0 | 1 | 1 | 0 | -3/2 | 5/2 |
| $x_4$ | 0 | 0 | 1 | 1 | -5/2 | 1/2 |

Made cut At $x_2$.

$$x_2 + x_3 - \frac{3}{2} S_1 = 5/2$$

$$\cancel{x_2 + x_3 + \left(-1 - \frac{1}{2}\right) S_1 = 2 + \frac{1}{2}}$$

~~The cut is~~

$$\cancel{\frac{1}{2} S_1 + S_2 = -\frac{1}{2}}$$

$\frac{1}{2} + \frac{1}{2}S_1 \leq 0$

$S_1 \leq -\frac{1}{2}$

$\frac{1}{2} S_1 + S_2 = -\frac{1}{2}$

$-2 + \frac{1}{2} = -\frac{3}{2}$

$$\cancel{x_2 + x_3 + \left(1 - \frac{5}{2}\right) S =}$$

$$x_2 + x_3 - \frac{3}{2} S_1 = 5/2$$

$$x_2 + x_3 + \left(-2 + \frac{1}{2}\right) S_1 = 2 + \frac{1}{2}$$

so

cut is $-\frac{1}{2} S_1 + \frac{1}{2} \leq 0$

$$-\frac{1}{2} S_1 + S_2 = -\frac{1}{2}$$

so Add this constrain

| | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $S_1$ | $S_2$ | R.H.V |
|---|---|---|---|---|---|---|---|
| Min z | 0 | 0 | -1 | 0 | -1 | 0 | 10 |
| $x_1$ | 1 | 0 | -1 | 0 | 1 | 0 | 0 |
| $x_2$ | 0 | 1 | 1 | 0 | -3/2 | 0 | 5/2 |
| $x_4$ | 0 | 0 | 1 | 1 | -5/2 | 0 | 1/2 |
| $S_2$ | 0 | 0 | 0 | 0 | -1/2 | 1 | -1/2 |

$-2R_5$, $R_1 + (-2R_5)$, $R_2 - (-2R_5)$, $R_3 + \frac{3}{2}(-2R_5)$

$R_4 + \frac{5}{2}(-2R_5)$

2

$$
\begin{array}{c|ccccccc}
 & x_1 & x_2 & x_3 & x_4 & S_1 & S_2 & R.H.V \\
\hline
\min z & 0 & 0 & -1 & 0 & 0 & -2 & 11 \\
x_1 & 1 & 0 & -1 & 0 & 0 & 2 & -7 \\
x_2 & 0 & 1 & 1 & 0 & 0 & -3 & 4 \\
x_4 & 0 & 0 & 1 & 1 & 0 & -5 & 3 \\
S_1 & 0 & 0 & 0 & 0 & 1 & -2 & 1
\end{array}
$$

$-R_2,\ R_1 - R_2,\ R_3 + R_2,\ R_4 + R_2.$

$$
\begin{array}{c|ccccccc}
 & x_1 & x_2 & x_3 & x_4 & S_1 & S_2 & R.H.V \\
\hline
\min z & -\phi & 0 & 0 & 0 & 0 & -4 & 12 \\
x_3 & -1 & 0 & 1 & 0 & 0 & -2 & 1 \\
x_2 & 1 & 1 & 0 & 0 & 0 & -1 & 3 \\
x_4 & 1 & 0 & 0 & 1 & 0 & -3 & 2 \\
S_1 & 0 & 0 & 0 & 0 & 1 & -2 & 1
\end{array}
$$

This is the optimal integer tableau where

$z = 12,\ x_1 = 0,\ x_2 = 3.$

## Ex 9.2 B

**Question No 6.** Problem 9–70 (a)

Solve the following problem by fractional cut.

a) Max $Z = 4x_1 + 6x_2 + 2x_3$

s.t

$$4x_1 - 4x_2 \le 5$$
$$-x_1 + 6x_2 \le 5$$
$$-x_1 + x_2 + x_3 \le 5$$
$$\cancel{x_1, x_2, x_3 \le 5}$$
$$x_1, x_2, x_3 \ge 0 \text{ and integers}$$

**soln**

First find the optimum of this tableau

| | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | R.H.V | |
|---|---|---|---|---|---|---|---|---|
| max z | −4 | −6 | −2 | 0 | 0 | 0 | 0 | |
| $x_4$ | 4 | −4 | 0 | 1 | 0 | 0 | 5 | ∞ |
| $x_5$ | −1 | 6 | 0 | 0 | 1 | 0 | 5 | 0.83 |
| $x_6$ | −1 | 1 | 1 | 0 | 0 | 1 | 5 | 5 |

$\frac{R_3}{6}$, $R_1 + R_3$, $R_2 + 4\left(\frac{R_3}{6}\right)$; $R_4 - \frac{R_3}{6}$

| | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | R.H.V | |
|---|---|---|---|---|---|---|---|---|
| max z | −5 | 0 | −2 | 0 | 1 | 0 | 5 | |
| $x_4$ | 10/3 | 0 | 0 | 1 | 2/3 | 0 | 25/3 | 5/2 |
| $x_2$ | −1/6 | 1 | 0 | 0 | 1/6 | 0 | 5/6 | ∞ |
| $x_6$ | −5/6 | 0 | 1 | 0 | −1/6 | 1 | 25/6 | ∞ |

$\frac{3}{10}R_2$, $R_1 + 5\left(\frac{3}{10}R_2\right)$, $R_3 + \frac{1}{6}\left(\frac{3}{10}R_2\right)$, $R_4 + \frac{5}{6}\left(\frac{3}{10}R_2\right)$

| | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | R.H.V |
|---|---|---|---|---|---|---|---|
| max z | 0 | 0 | −2 | 3/2 | 2 | 0 | 35/2 |
| $x_1$ | 1 | 0 | 0 | 3/10 | 1/5 | 0 | 5/2 |
| $x_2$ | 0 | 1 | 0 | 1/20 | 1/5 | 0 | 5/4 |
| $x_6$ | 0 | 0 | 1 | 1/4 | ~~7/5~~ 0 | 1 | 25/4 ~~35/12~~ |

$R_1 + 2R_4$

| | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | R.H.V |
|---|---|---|---|---|---|---|---|
| max $z$ | 0 | 0 | 0 | 2 | 2 | 2 | 30 |
| $x_1$ | 1 | 0 | 0 | 3/10 | 1/5 | 0 | 5/2 |
| $x_2$ | 0 | 1 | 0 | 1/20 | 1/5 | 0 | 5/4 |
| $x_3$ | 0 | 0 | 1 | 1/20 | 0 | 1 | 25/4 |

This is optimal tableau.

From the $x_1$ - row

we make first cut.

$$x_1 + \frac{3}{10}x_4 + \frac{1}{5}x_5 = 5/2$$

min fraction from

$$x_1 + \left(\frac{3}{10}\right)x_4 + \left(\frac{1}{5}\right)x_5 = 2 + \frac{1}{2}.$$

$\frac{3}{10} = 0\frac{3}{10}$

$2\frac{1}{2} = 2 + \frac{1}{2}$

And cut is. $\left(0 + \frac{3}{10}\right)$

$$-\frac{3}{10}x_4 - \frac{1}{5}x_5 \le -\frac{1}{2}.$$

OR

$$-\frac{3}{10}x_4 - \frac{1}{5}x_5 + S_1 = -\frac{1}{2}.$$

so

Add this constrain into optimal tableau

| | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | $S_1$ | R.H.V |
|---|---|---|---|---|---|---|---|---|
| max $z$ | 0 | 0 | 0 | 2 | 2 | 2 | 0 | 30 |
| $x_1$ | 1 | 0 | 0 | 3/10 | 1/5 | 0 | 0 | 5/2 |
| $x_2$ | 0 | 1 | 0 | 1/20 | 1/5 | 0 | 0 | 5/4 |
| $x_3$ | 0 | 0 | 1 | 1/20 | 0 | 1 | 0 | 25/4 |
| $S_1$ | 0 | 0 | 0 | -3/10 | -1/5 | 0 | 1 | -1/2 |

20/3   10

$\frac{-10}{3}R_5$ , $R_1-2\left(\frac{-10}{3}R_5\right)$, $R_2+R_5$, $R_3-\frac{1}{20}\left(\frac{-70}{3}R_5\right)$, $R_4-\frac{1}{4}\left(\frac{-10}{3}R_5\right)$

$$\begin{array}{c} \\ \text{en. } z \\ x_1 \\ x_2 \\ x_3 \\ x_4 \end{array}\begin{array}{c} \begin{array}{cccccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & S_1 & R.H.V \end{array} \\ \left[\begin{array}{cccccccc} 0 & 0 & 0 & 0 & 2/3 & 2 & 20/3 & 80/3 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 2 \\ 0 & 1 & 0 & 0 & 1/6 & 0 & 1/6 & 7/6 \\ 0 & 0 & 1 & 0 & -1/6 & 1 & 5/6 & 35/6 \\ 0 & 0 & 0 & 1 & 2/3 & 0 & -10/3 & 5/2 \end{array}\right] \end{array}$$

Making cut at $x_2$

$$x_2+\frac{1}{6}x_5+\frac{1}{6}S_1 = 7/6.$$

$$x_2+\left(0+\frac{1}{6}\right)x_5+\left(0+\frac{1}{6}\right)S_1 = 1+\frac{1}{6}$$

The cut is

$$-\frac{1}{6}x_5-\frac{1}{6}S_1 \le -\frac{1}{6}$$

And

$$-\frac{1}{6}x_5 - \frac{1}{6}S_1+S_2 = -\frac{1}{6}$$

Add constrain into optimal tableau.

$$\begin{array}{c} \\ z \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ S_2 \end{array}\begin{array}{c} \begin{array}{ccccccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & S_1 & S_2 & R.H.V \end{array} \\ \left[\begin{array}{ccccccccc} 0 & 0 & 0 & 0 & 2/3 & 2 & 20/3 & 0 & 80/3 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 2 \\ 0 & 1 & 0 & 0 & 1/6 & 0 & 1/6 & 0 & 7/6 \\ 0 & 0 & 1 & 0 & -1/6 & 1 & 5/6 & 0 & 35/6 \\ 0 & 0 & 0 & 1 & 2/3 & 0 & -10/3 & 0 & 5/2 \\ 0 & 0 & 0 & 0 & -1/6 & 0 & -1/6 & 1 & -1/6 \end{array}\right] \\ \begin{array}{ccccccccc} & & & & |-4| & & |-40| & & \end{array} \end{array}$$

$R_1-\frac{2}{3}(-6R_6)$, $R_3+R_6$, $R_4-R_6$, $R_5-\frac{2}{3}(-6R_6)$

1

$$
\begin{array}{c|ccccccccc}
 & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & S_1 & S_2 & R.H.V \\
\hline
\text{Max } Z & 0 & 0 & 0 & 0 & 0 & 2 & 6 & 4 & 26 \\
x_1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 2 \\
x_2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\
x_3 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & -1 & 6 \\
x_4 & 0 & 0 & 0 & 1 & 0 & 0 & -4 & -4 & 1 \\
x_5 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & -6 & 1
\end{array}
$$

so

The solution is optimal and all are integers.