$v_2 = -\frac{1}{4}x$

$v_3 = \frac{1}{8}\ln|\cos 2x|$

$y = c_1 + c_2\cos 2x + c_3\sin 2x + \frac{1}{8}\ln|\sec 2x + \tan$
$-\frac{1}{4}x\cos 2x + \frac{1}{8}\sin 2x\ln|\cos 2x|$

CLO-05

Ex # 11.2

Q#7:-

$f(x) = x + \pi \qquad -\pi < x < \pi$

Sol:-

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n\cos\frac{n\pi}{p}x + b_n\sin\frac{n\pi}{p}x\right)$$

$$a_0 = \frac{1}{p}\int_{-p}^{p} f(x)\,dx$$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{0} f(x) + \int_{0}^{\pi} f(x)$$

$$a_0 = \frac{1}{\pi}\left[\int_{-\pi}^{0} x+\pi + \int_{0}^{\pi} x+\pi\right]$$

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$$= \frac{1}{\pi}\left[\left.\frac{x^2}{2}+\pi x\right|_{-\pi}^{0}\right] + \frac{1}{\pi}\left[\left|\frac{x^2}{2}+\pi x\right|_{0}^{\pi}\right]$$

$$= \frac{1}{\pi}\left[-\left(\frac{\pi^2}{2}-\pi^2\right)\right] + \frac{1}{\pi}\left[\frac{\pi^2}{2}+\pi^2\right]$$

$$= \frac{1}{\cancel{\pi}} \times \frac{\pi^{\cancel{2}}}{2} + \frac{1}{\cancel{\pi}} \times \frac{3\pi^{\cancel{2}}}{2}$$

$$= \frac{\pi}{2} + \frac{3\pi}{2}$$

$$= 2\pi$$

$$a_n = \frac{1}{p}\int_{-p}^{p} f(x)\cos\frac{n\pi}{p}x\,dx.$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} (x+\pi)\cos\frac{n\cancel{\pi}}{\cancel{\pi}}x\,dx.$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} (x+\pi)\cos nx\,dx$$

$$a_n = 0$$

$$b_n = \frac{1}{p}\int_{-p}^{p} f(x)\sin\frac{n\pi}{p}x\,dx.$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} (x+\pi)\sin\frac{n\cancel{\pi}}{\cancel{\pi}}x\,dx.$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} (x+\pi)\sin nx\,dx.$$

$$b_n = \frac{2}{n}(-1)^{n+1}.$$

$$f(x) = \pi + \sum_{n=1}^{\infty} \frac{2}{n}(-1)^{n+1} \sin nx.$$

So

$$a_0 = 2\pi$$

$$a_n = 0$$

$$b_n = \frac{2}{n}(-1)^{n+1}$$

Q# 9

$$f(x) = \begin{cases} 0 & , -\pi < x < 0 \\ \sin x & , 0 \le x < \pi \end{cases} \qquad [-\pi, \pi]. \quad p =$$

Sol:-

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi}{p} x + b_n \sin \frac{n\pi}{p} x \right)$$

$$a_0 = \frac{1}{p} \int_{-p}^{p} f(x)\, dx.$$

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)\, dx.$$

$$a_0 = \frac{1}{\pi} \left[\int_{-\pi}^{0} f(x) + \int_{0}^{\pi} f(x) \right]$$

$$a_0 = \frac{1}{\pi} \left[\int_{-\pi}^{0} 0 + \int_{0}^{\pi} \sin x \right]$$

$$a_0 = \frac{1}{\pi} \left| \left[-\cos x \right] \right|_0^{\pi}$$

$$a_0 = \frac{1}{\pi} - (-1 - 1)$$

$$a_0 = \frac{2}{\pi}$$

$$a_n = \frac{1}{P} \int_{-P}^{P} f(x) \cos \frac{n\pi}{P} x$$

$$a_n = \frac{1}{\pi} \left[\int_{-\pi}^{0} 0 \cdot \cos \frac{n\pi}{P} x + \int_0^{\pi} \sin x \cdot \cos \frac{n\pi}{P} x \cdot dx \right]$$

$$a_n = \frac{1}{\pi} \left[\int_0^{\pi} \sin x \cdot \cos \frac{n\cancel{\pi}}{\cancel{\pi}} x \, dx \right]$$

$$a_n = \frac{1}{\pi} \left[\int_0^{\pi} \sin x \cos nx \, dx \right]$$

$$a_n = \frac{1}{2\pi} \left[\int_0^{\pi} \left[\sin(1+n)x + \sin(1-n)x \right] \right]$$

$$a_n = \frac{1 + (-1)^n}{\pi(1-n^2)} \cdot \text{ for } n = 2, 3, 4.$$

$\therefore \sin 2A = 2 \sin A \cos A$

$\sin 2A = \sin(A+B) + \sin(A-B)$

$$a_1 = \frac{1}{2\pi} \int_0^{\pi} \sin 2x \, dx = 0.$$

$$b_n = \frac{1}{P} \int_{-P}^{P} f(x) \sin \frac{n\pi}{P} x.$$

$b_n = \frac{1}{\pi} \int$

$\Rightarrow b_n$

$$b_n = \frac{1}{\pi}\int_{0}^{\pi} \sin x \sin nx$$

$\therefore 2\sin A\sin B \Rightarrow \sin A \sin B = \cos(A-B) - \cos(A+B)$

$$\Rightarrow b_n = \frac{1}{2\pi}\int_0^{\pi} \cos(1-n)x - \cos(1+n)x \, dx$$

$b_n = 0$ for $n = 2, 3, 4 \ldots$ $\therefore (1-\cos 2x)$

$$b_1 = \frac{1}{2\pi}\int_0^{\pi} (1-\cos 2x)$$

$$b_1 = \frac{1}{2}.$$

$$f(x) = \frac{1}{\pi} + \frac{1}{2}\sin x + \sum_{n=2}^{\infty} \frac{1+(-1)^n}{\pi(1-n^2)} \cos nx.$$

Q#11:-

$$f(x) = \begin{cases} 0 & -2 < x < -1 \\ -2 & -1 \le x < 0 \\ 1 & 0 \le x < 1 \\ 0 & 1 \le x < 2. \end{cases}$$

$[-2, 2]$ $p = 2$

Sol:-

$$a_0 = \frac{1}{p}\int_{-p}^{p} f(x)\, dx.$$

$$a_0 = \frac{1}{2}\int_{-2}^{2} f(x).$$

$$= \frac{1}{2}\left[\int_{-2}^{-1} 0 + \int_{-1}^{0} -2 + \int_{0}^{1} 1 + \int_{1}^{2} 0\right]$$

$$\frac{1}{2}\left[\left[-2x\right]_{-1}^{0} + \left|x\right|_{0}^{1}\right]$$

$$a_0 = -\frac{1}{2}.$$

$$a_n = \frac{1}{P}\int_{-P}^{P} f(x)\cos\frac{n\pi}{2}x\,dx.$$

$$a_n = \frac{1}{2}\left[\int_{-1}^{0} -2\cos\frac{n\pi}{2}x + \int_{0}^{1}\cos\frac{n\pi}{2}x\,dx.\right]$$

$$a_n = \frac{1}{2}\left[\frac{-\cancel{2}\times\cancel{2}}{\cancel{2}n\pi}\left|\sin\frac{n\pi}{2}.x\right|_{-1}^{0} + \left|\frac{\cancel{2}}{\cancel{2}n\pi}\sin\frac{n\pi}{2}x\right|_{0}^{1}\right]$$

$$a_n = \frac{1}{2}\left[\frac{-4}{n\pi}\left(\sin\frac{n\pi}{2}\right)+2\left(\sin\frac{n\pi}{2}\right)\right]$$

$$= \frac{1}{\cancel{2}}\cdot\frac{-\cancel{2}}{n\pi}\left(\sin\frac{n\pi}{2}\right) = -\frac{1}{n\pi}\sin n\pi/2$$

$$b_n = \frac{1}{2}\left[\int_{-1}^{0} -2\sin n\pi/2\, x\,dx + \int_{0}^{1}\left(\sin n\pi/2\, x\,dx\right)\right]$$

$$b_n = \frac{3}{n\pi}\left(1-\cos\frac{n\pi}{2}\right)$$

$$f(x) = -\frac{1}{4} + \sum_{n=1}^{\infty}\left[-\frac{1}{n\pi}\sin\frac{n\pi}{2}\cos\frac{n\pi}{2}x + \frac{3}{n\pi}\left(1-\cos n\pi/2\right)\sin\frac{n\pi}{2}x\right]$$

Converges to -1 at $x=-1$, $-\frac{1}{2}$ at $x=0$ and $\frac{1}{2}$ at $x=1$.

Q#19:-

Sol:-

The function in problem 5 is discontinuous at $x = \pi$, so corresponding Fourier series converges to $\pi^2/2$ at $x = \pi$. That is:

$$\frac{\pi^2}{2} = \frac{\pi^2}{6} + \sum_{n=1}^{\infty}\left[\frac{2(-1)^n}{n^2}\cos n\pi + \left(\frac{\pi}{n}(-1)^{n+1} + \frac{2[(-1)^n - 1]}{n^3\pi}\right)\sin n\pi\right]$$

$$= \frac{\pi^2}{6} + \sum_{n=1}^{\infty}\frac{2(-1)^n(-1)^n}{n^2} = \frac{\pi^2}{6} + \sum_{n=1}^{\infty}\frac{2}{n^2}$$

$$= \frac{\pi^2}{6} + 2\left(1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots\right)$$

and

$$\frac{\pi^2}{6} = \frac{1}{2}\left(\frac{\pi^2}{2} - \frac{\pi^2}{6}\right) = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots$$

At $x = 0$ the series converges to 0.

$$0 = \frac{\pi^2}{6} + \sum_{n=1}^{\infty}\left[\frac{2(-1)^n}{n^2}\right] = \frac{\pi^2}{6} + 2\left(-1 + \frac{1}{2^2} - \frac{1}{3^2} + \frac{1}{4^2}\right)$$

So

$$\frac{\pi^2}{12} = 1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots$$

Q#20

Sol:-

$$\frac{\pi^2}{8} = \frac{1}{2}\left(\frac{\pi^2}{6} + \frac{\pi^2}{12}\right) = \frac{1}{2}\left(2 + \frac{2}{3^2} + \frac{2}{5^2} + \cdots\right)$$

$$= 1 + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$$

Ex # 11.3

Q#1:-

$f(x) = \sin 3x$

Sol:-

$f(-x) = \sin 3(-x)$

$f(-x) = -\sin 3x$.

$f(-x) = -f(x)$

This is an odd function.

Q#13

$f(x) = |x|, \quad -\pi < x < \pi$

Sol:-

$f(x)$ is an even function. we expand in cosine series.

$a_0 = \frac{2}{\pi} \int_0^{\pi} x \, dx = \pi$

$\frac{2}{\pi}(\pi - 0)$ $\quad a_n = \frac{2}{\pi} \int_0^{\pi} x \cos \frac{n\pi}{\pi} x \, dx$.

$a_n = \frac{2}{n^2 \pi} [(-1)^n - 1]$

$f(x) = \pi/2 + \sum_{n=1}^{\infty} \frac{2}{n^2 \pi} [(-1)^n - 1] \cos nx$.

Q#25:—

Sol:—

$$a_0 = 2\int_0^{1/2} 1\,dx = 1$$

$$a_n = 2\int_0^{1/2} 1\cdot\cos n\pi x\,dx = \frac{2}{n\pi}\sin n\pi/2$$

$$b_n = 2\int_0^{1/2} 1\cdot\sin n\pi x\,dx = \frac{2}{n\pi}(1-\cos n\pi/2)$$

$$f(x) = 1/2 + \sum_{n=1}^{\infty} \frac{2}{n\pi}\sin n\pi/2\,\cos n\pi x.$$

$$f(x) = \sum_{n=1}^{\infty} \frac{2}{n\pi}\left(1-\cos n\pi/2\right)\sin n\pi x.$$

Q#38:—

$$f(x) = 2-x.$$

$$a_0 = \frac{2}{2}\int_0^2 (2-x)\,dx = 2$$

$$a_n = \frac{2}{2}\int_0^2 (2-x)\cos n\pi x\,dx = 0.$$

$$b_n = \frac{2}{2}\int_0^2 (2-x)\sin n\pi x = \frac{2}{n\pi}$$

$$f(x) = 1 + \sum_{n=1}^{\infty}\frac{2}{n\pi}\sin n\pi x.$$