Date: 21/10 $\frac{0.5}{0.5} = 1$ Day: M T W T F S

$$f(x) = 0$$

$x_0, x_1, x_2, \ldots, x_n$ iterations

Suppose 'c' is exact solution.

$$x_n - C = \varepsilon_n$$

approx soln. exact soln. Error at nth iteration.

$$|x_n - c| = |\varepsilon_n|$$

Rate of Convergence / Order of Convergence

"A method is said to be of order 'p' or it has rate of convergence p if

$$|\varepsilon_{n+1}| = k\,|\varepsilon_n|^p$$

$p=1$ Linear convergence

$p=2$ Quadratic

Rate of Convergence of Bisection Method:-

$f(x)=0$

$$x_{n+1} = \frac{x_n + x_{n-1}}{2} \quad \text{—(1)}$$

$x_n = \varepsilon_n + C$ —(2) $\quad x_{n-1} = \varepsilon_{n-1} + C$ —(4)

$x_{n+1} = \varepsilon_{n+1} + C$ —(3)

Put Eq (2,3,4) in Eq (1)

$$\varepsilon_{n+1} + C = \frac{\varepsilon_n + C + \varepsilon_{n-1} + C}{2}$$

$$\varepsilon_{n+1} + C = C + \frac{\varepsilon_n + \varepsilon_{n-1}}{2}$$

$$\varepsilon_{n+1} = \frac{\varepsilon_n + \varepsilon_{n-1}}{2}$$

$$\varepsilon_{n+1} = \frac{1\varepsilon_n}{2}\left[1 + \frac{\varepsilon_{n-1}}{\varepsilon_n}\right]$$

$$= \qquad \frac{\varepsilon_{n-1}}{\varepsilon_n} \approx 0$$

$$|\varepsilon_{n+1}| = \frac{1}{2}|\varepsilon_n|$$

$$k = \frac{1}{2}, \quad p = 1$$

So, bisection method has order of convergence 1.

Rate of Convergence of Newton Raphson method

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

$$x_k = \varepsilon_k + c$$

$$x_{k+1} = \varepsilon_{k+1} + c$$

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$\mathcal{E}_{k+1} + c = \mathcal{E}_k + c - \frac{f(\mathcal{E}_k + c)}{f'(\mathcal{E}_k + c)}$

$\mathcal{E}_{k+1} = \mathcal{E}_k - \frac{f(c+\mathcal{E}_k)}{f'(c+\mathcal{E}_k)}$ —— (1)

$\because f(x+h) = f(x) + hf'(x) + \frac{h^2}{2!} f''(x) + \cdots$

Tailor series

$f(c+\mathcal{E}_k) = f(c) + \mathcal{E}_k f'(c) + \frac{(\mathcal{E}_k)^2 f''(c)}{2}$

+ neglect higher terms

As $f(c)=0$ c is exact sol.

$f(c+\mathcal{E}_k) = \cancel{f(c)} + \mathcal{E}_k f'(c) + \frac{(\mathcal{E}_k)^2 f''(c)}{2}$ —— (2)

$f'(c+\mathcal{E}_k) = f'(c) + \mathcal{E}_k f''(c) + \underbrace{\frac{\mathcal{E}_k^2 f'''(c)}{2}}_{\text{neglect}}$

—— (3)

Put (2) & (3) in (1)

$\mathcal{E}_{k+1} = \mathcal{E}_k - \frac{\left[\mathcal{E}_k f'(c) + \frac{\mathcal{E}_k^2}{2} f''(c)\right]}{\left[f'(c) + \mathcal{E}_k f''(c)\right]}$

$\mathcal{E}_{k+1} = \mathcal{E}_k - \frac{\cancel{f'(c)}\left[\mathcal{E}_k + \frac{\mathcal{E}_k^2}{2}\frac{f''(c)}{f'(c)}\right]}{\cancel{f'(c)}\left[1 + \mathcal{E}_k \frac{f''(c)}{f'(c)}\right]}$

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$$\varepsilon_{k+1} = \varepsilon_k - \left[\varepsilon_k + \frac{\varepsilon_k^2}{2}\frac{f''(c)}{f'(c)}\right]\left[1+\varepsilon_k\frac{f''(c)}{f'(c)}\right]^{-1}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \cdots$$

Binomial Theorem

$$\left[1+\frac{\varepsilon_k f''(c)}{f'(c)}\right]^{-1} = 1 + (-1)\left[\frac{\varepsilon_k f''(c)}{f'(c)}\right]$$

$$= 1 - \frac{\varepsilon_k f''(c)}{f'(c)}$$

$$\varepsilon_{k+1} = \varepsilon_k - \left[\varepsilon_k + \frac{\varepsilon_k^2}{2}\frac{f''(c)}{f'(c)}\right]\left[1 - \frac{\varepsilon_k f''(c)}{f'(c)}\right]$$

$$\varepsilon_{k+1} = \varepsilon_k - \left[\varepsilon_k - \varepsilon_k^2\frac{f''(c)}{f'(c)} + \frac{\varepsilon_k^2}{2}\frac{f''(c)}{f'(c)} - \underbrace{\frac{\varepsilon_k^3}{2}\frac{[f''(c)]^2}{[f'(c)]^2}}_{\text{neglect}}\right]$$

$$\varepsilon_{k+1} = \varepsilon_k - \left[\varepsilon_k - \frac{1}{2}\varepsilon_k^2\frac{f''(c)}{f'(c)}\right]$$

$$\varepsilon_{k+1} = \cancel{\varepsilon_k} - \cancel{\varepsilon_k} + \frac{1}{2}\varepsilon_k^2\frac{f''(c)}{f'(c)}$$

$$\varepsilon_{k+1} = \frac{1}{2}\varepsilon_k^2\frac{f''(c)}{f'(c)}$$

$$\varepsilon_{k+1} = \frac{f''(c)}{2f'(c)} \varepsilon_k^2$$

$$k = \frac{f''(c)}{2f'(c)} \qquad p = 2$$

So, Newton Raphson is of order of converge of 2.

Order of Convergence

may or may not convergent	Newton Raphson = 2		More speadily converge to actual sol.
	Secant Method = 1.618		
Always converge	Bisection = 1		comparatively less speadily converge
	Regula false = 1		

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Romberg Integration:

By Trapezoidal Rule:

$$I = \int_a^b f(x)\,dx$$

$$I = I_1 + E_1$$

$$E_1 = \frac{-(b-a) h_1^2 f''(\xi)}{12}$$

for h_2 (diff. interval from h_1)

$$I = I_2 + E_2$$

$$E_2 = \frac{-(b-a) h_2^2 f''(\xi)}{12}$$

$$\frac{E_1}{E_2} = \frac{h_1^2 \cancel{(-(b-a) f''(\xi))}}{h_2^2 \cancel{(-(b-a) f''(\xi))}}$$

$$\frac{E_1}{E_2} = \frac{h_1^2}{h_2^2} \quad \text{--- (1)}$$

$$\frac{E_1}{E_2 - E_1} = \frac{h_1^2}{h_2^2 - h_1^2} \quad \text{--- (2)}$$

$$I_1 + E_1 = I_2 + E_2$$ (As I_1 and I_2 are approx. of same interval)

$$I_1 - I_2 = E_2 - E_1 \quad \text{--- (3)}$$

$$E_1 = (E_2 - E_1) \times \frac{h_1^2}{h_2^2 - h_1^2}$$

$$E_1 = (I_2 - I_1) \times \frac{h_1^2}{h_2^2 - h_1^2}$$

$$E_1 = \frac{I_1 h_1^2 - I_2 h_1^2}{h_2^2 - h_1^2}$$

$$I = I_1 + E_1$$

$$= I_1 + \frac{I_1 h_1^2 - I_2 h_1^2}{h_2^2 - h_1^2}$$

$$= \frac{I_1 h_2^2 - I_2 h_1^2}{h_2^2 - h_1^2} \quad \text{--- (4)}$$

Now take $h_1 = h$ and $h_2 = \frac{h}{2}$

$$\boxed{I = I_2 + \frac{(I_2 - I_1)}{3}}$$

$\int_0^1 \frac{1}{1+x}\,dx$ by Romberg Integration

Here $h_1 = 0.5$

x	0	0.5	1
$f(x)$	1	0.6669	0.5

$$I_1 = \frac{h_1}{2}\left[f(x_0) + f(x_n) + 2(f(x_1) + \dots f(x_{n-1}))\right]$$

$$= \frac{0.5}{2}\left[1 + 0.5 + 2(0.6669)\right]$$

$$= 0.7083$$

for $h_2 = \frac{h}{2} = 0.25$

x	0	0.25	0.5	0.75	1
$f(x)$	1	0.8	0.6669	0.38	0.5

$$I_2 = \frac{h_2}{2}\left[f(x_0) + f(x_1) + 2f(x_2) + \dots + f(x_{n-1})\right]$$

$$= \frac{0.25}{2}\left[1 + 0.5 + 2[0.8 + 0.667 + 0.58]\right]$$

$$I_2 = 0.69702$$

$$I = 0.69702 + \frac{(0.69702 - 0.7083)}{3}$$

$$I = 0.69326$$

Romberg Integration by Simpson Rule

$$I = I_2 + \frac{I_2 - I_1}{15}$$

$$\int_a^b f(x)dx = \frac{h}{3}\Big[f(x_0) + f(x_n) +$$

$$4(f(x_1) + f(x_3) + \dots + f(x_{n-1})$$

$$+ 2(f(x_2) + f(x_4) + \dots + f(x$$

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$$I_1 = \frac{0.5}{3}\left[1 + 0.5 + 4(0.6667)\right]$$

$$I_1 = 0.6946$$

$$I_2 = \frac{0.25}{3}\left[1+0.5+4(\cancel{0.6669}\ 0.8 + 0.38) + 2(0.6667)\right]$$

$$I_2 = \cancel{0.62943}\ 0.66945$$

By Romberg Rule (Simpson Rule)

$$I = \cancel{0.62943}\ 0.66945 + \frac{\cancel{0.62943}\ 0.66945 - 0.6946}{15}$$

$$= \cancel{0.62513}\ 0.67112$$

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Solution of Equ. in 1 variable.

$x^{2/3} + x^{1/2} + 5 = 0$

$x \sin x = 0$

$\cos x e^{x} = 0$

→ Bisection

→ Regula false

→ ~~Section~~ Secant Method

→ Newton Raphson

→ fixed Point

: Root ~~equatio~~ (r) number r where

$f(r) = 0$

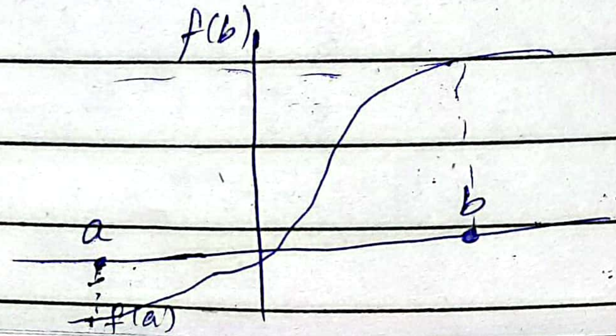
$\boxed{\text{Intermediate value Theorem}}$

Let f be continuous of $[a,b] \to [f(a), f(b)]$

$a < c < b$

$f(a) < k < f(b)$

$\Rightarrow f(c) = k$



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As $f(r) = 0$ (root)

$f(a) < 0 < f(b)$

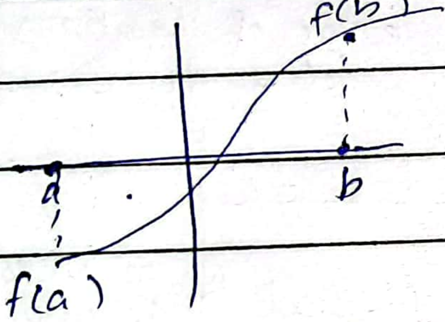
$a < r < b$

$\Rightarrow f(r) = 0$ Here,

For this $f(a) < 0$ $f(b) > 0$

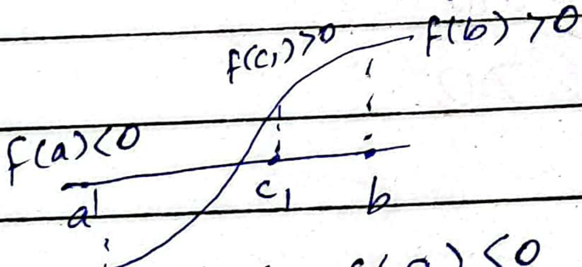
on is +ve and other is -ve

Bisection Method:



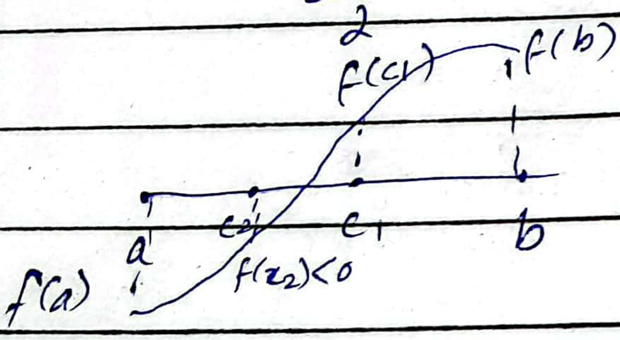
$f(a), f(b)$ opposite sign

$$\frac{a+b}{2} = c_1$$



Now, $f(a) < 0$ $f(c_1) > 0$ take this interval

$$\frac{a+c_1}{2} = c_2$$



Now take c_2, c_1 as $f(c_2) < 0$ $f(c_1) > 0$

$$c_3 = \frac{c_2 + c_1}{2}$$

Continue iterations as.

Criteria to stop iterations:

- $|c_1 - c_2| < \varepsilon$ — decimal values upto req. places of two iterations become equal
- $|f(c_1)| < \varepsilon$

$\therefore \varepsilon = 0.001$

- $|x - c_1| < \varepsilon$

Absolute error of iterations become so close to 0.

Solve by Bisection Method (Take intervals of opp. signs)

$f(x) = x^3 - 6x^2 + 11x - 6$

$f(2.5) = -0.375 < 0$

$f(4) = 6 > 0$

-ve	+ve	+ve
2.5	3.25	4

$$c_1 = \frac{a+b}{2} = \frac{4+2.5}{2} = 3.25$$

$f(3.25) = 0.703$

-ve	+ve	+ve
2.5	3.25	4

~~Now take 3.25 and 4~~

$$c_2 = \frac{c_1 + b}{2} = \frac{3.25 + \cancel{b}\, 2.5}{2} = 2.875$$

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$f(2.875) = -0.2051$

-ve -ve +ve

2.5 2.875 3.25

$$c_3 = \frac{c_1 + c_2}{2} = \frac{2.875 + 3.25}{2} = 3.0625$$

$f(3.0625) = 0.13$

-ve +ve +ve

2.875 3.0625 3.25

$$c_4 = \frac{2.875 + 3.25}{2} = 2.968$$

$f(2.968) = -0.06$

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Find minimum no. of iterations req. by bisection method to approximate root of $x^3 - 6x^2 + 11x - 6 = 0$ with error tolerance 10^{-3}. Interval is $[2.5, 4]$

$$N \geq \frac{\log(b-a) - \log \varepsilon}{\log 2}$$

$$N \geq \frac{\log(4-2.5) - \log 10^{-3}}{\log 2}$$

$$N \geq \cancel{\log}\ 10.56$$

$$N \geq 11$$

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Fixed Point Iteration Method:

Point 'p' of f where $f(p)=p$

$f(x)=\sin x$

~~$f(0)=\sin 0$~~

$=0$

Input is equal to output

$f(x)=x^3-6x^2+11x-6=0$

$11x=6+6x^2-x^3$

$x=\frac{6+6x^2-x^3}{11}$

$x=g(x)$

Fixed point of $g(x)$ will be root of $f(x)$. $g(x)$ is fixed point representation

There may be multiple fixed point representations of $f(x)$. But which $g(x)$ should be chosed:-

if $f(x)=0$ and $f(x)=x-g(x)$

s.t $x=g(x)$ and $g(x)$ must satisfy following conditions:

1. $\forall x\in[a,b]$

$\Rightarrow a<g(x)<b$

2. $g'(x)$ exists on (a,b)

$|g'(x)|\le r<1 \quad 0<r<1$

Solve $x - \cos x = 0$ by fixed point method on $[0, \pi/2]$

$$x - \cos x = 0$$

$$x = \cos x$$

$$g(x) = \cos x$$

$\rightarrow$ $x \in [0, \pi/2] = [0, 1.57]$ 1st cond.

$\cos x$ will be in interval $[0, 1.57]$

$\rightarrow$ $g'(x) = -\sin x$

$|g'(x)| = \sin x \leq 1$ 2nd cond.

$$p_n = g(p_{n-1})$$

$$p_0 = 0$$

$$p_1 = g(p_0)$$

$$p = \cos 0 = 1$$

$$p_2 = g(p_1) = \cos 1 = 0.5403$$

$$p_3 = g(p_2) = \cos(0.5403) = 0.8575$$

$$p_4 = g(p_3) = \cos(0.8575) = 0.6542$$

$$p_5 = g(p_4) = \cos(0.6542) = 0.7934$$

$$p_6 = g(p_5) = \cos(0.7934) = 0.7013$$

$$p_7 = g(p_6) = \cos(0.7013) = 0.7639$$

$$p_8 = g(p_7) = \cos(0.7639) = 0.7221$$

$$p_9 = 0.7504$$

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$P_{10} = 0.7314$

$P_{12} = 0.7356$

$P_{13} = 0.7414$

$P_{16} = 0.7383$

$P_{11} = 0.7442$

$P_{14} = 0.7375$

$P_{15} = 0.7401$

$P_{16} =$

Soln of system of linear Equ:

$a_1x_1 + b_1x_2 + c_1x_3 = d_1$

$a_2x_1 + b_2x_2 + c_2x_3 = d_2$

$a_3x_1 + b_3x_2 + c_3x_3 = d_3$

Direct Method

Give Exact Soln.

No error

Iterative Method

Approx. soln

Error of small value.

Gauss Elimination Method:

$$3x_1 + 5x_2 + 2x_3 = 8$$

$$8x_2 + 2x_3 = -7$$

$$6x_1 + 2x_2 + 8x_3 = 26$$

$$\underset{\sim}{R}: \begin{bmatrix} 3 & 5 & 2 & 8 \\ 0 & 8 & 2 & -7 \\ 6 & 2 & 8 & 26 \end{bmatrix} \quad R_3 - 2R_1$$

$$\underset{\sim}{R}: \begin{bmatrix} 3 & 5 & 2 & 8 \\ 0 & 8 & 2 & -7 \\ 0 & -8 & 4 & 10 \end{bmatrix} \quad R_1/3$$

$$\underset{\sim}{R}: \begin{bmatrix} 1 & 5/3 & 2/3 & 8/3 \\ 0 & 8 & 2 & -7 \\ 0 & -8 & 4 & 10 \end{bmatrix} \quad R_3 + R_2$$

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$$\underset{\sim}{R}: \begin{bmatrix} 1 & 5/3 & 2/3 & 8/3 \\ 0 & 8 & 2 & -7 \\ 0 & 0 & 6 & 3 \end{bmatrix}$$

$6x_3 = 3$, $x_3 = 1/2$

$8x_2 + 2x_3 = -7$

$8x_2 + 2 \cdot \frac{1}{2} = -7$

$x_2 = -1$

$x_1 + 5/3 x_2 + 2/3 x_3 = 8/3$

$x_1 =$

Partial Pivoting

Mag. in coln 1 should at top

$3x_1 - 4x_2 + 5x_3 = -1$

$-3x_1 + 2x_2 + x_3 = 1$

$6x_1 + 8x_2 - x_3 = 35$

$6x_1 + 8x_2 - x_3 = 35$

$3x_1 - 4x_2 + 5x_3 = -1$

$-3x_1 + 2x_2 + x_3 = 1$

$$\underset{\sim}{R}: \begin{bmatrix} 6 & 8 & -1 & 35 \\ 3 & -4 & 5 & -1 \\ -3 & 2 & 1 & 1 \end{bmatrix}$$

$\cancel{R_2 - R_1}$ $+ R_1 - 2R_2$

$R_1 + 2R_3$

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$$R: \begin{bmatrix} 6 & 8 & -1 & 35 \\ 0 & 16 & -11 & 37 \\ 0 & 12 & 1 & 37 \end{bmatrix} \quad \frac{R_1}{6}$$

$$R: \begin{bmatrix} 1 & 4/3 & -1/6 & 35/6 \\ 0 & 16 & -11 & 37 \\ 0 & 12 & 1 & 37 \end{bmatrix} \quad R_2/16$$

$$R: \begin{bmatrix} 1 & 4/3 & -1/6 & 35/6 \\ 0 & 1 & -11/16 & 37/16 \\ 0 & 12 & 1 & 37 \end{bmatrix} \quad -12R_2+R_3$$

$$\begin{bmatrix} 1 & 4/3 & -1/6 & 35/6 \\ 0 & 1 & -11/16 & 37/16 \\ 0 & 0 & 37/4 & 37/4 \end{bmatrix}$$

$\frac{37}{4} x_3 = 111/4$

$x_3 = 111/33$

$\frac{37}{4} x_3 = 37/4$

$\boxed{x_3 = 1}$

$x_1 + 4/3 x_2 - 1/6 x_3 = 35/6$

$x_1 + 4 - 1/6 = 35/6$

$x_1 = 35/6 + 1/6 - 4$

$\boxed{x_1 = 2}$

$x_2 - 11/16 x_3 = 37/16$

$x_2 = 37/16 + 11/16$

$\boxed{x_2 = \frac{48}{16} = 3}$

LU Factorization:

$a_1x_1 + b_1x_2 + c_1x_3 = d_1$

$a_2x_1 + b_2x_2 + c_2x_3 = d_2$

$a_3x_1 + b_3x_2 + c_3x_3 = d_3$

$$A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}, \quad X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \quad B = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

$AX = B$, $A = LU$ → U: upper Triangular, L: lower Triangular

$LUX = B$

let $UX = Z$

$LZ = B$

---

2 Methods:

Crouts Method (Diagonal entries of upper is 1)

① $$A = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

② $$A = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

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$$x_1 + x_2 + x_3 = 1$$
$$4x_1 + 3x_2 - x_3 = 6$$
$$3x_1 + 5x_2 + 3x_3 = 4$$

$$A = LU$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{bmatrix} = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{bmatrix} = \begin{bmatrix} l_{11} & l_{11}u_{12} & l_{11}u_{13} \\ l_{21} & l_{21}u_{12}+l_{22} & l_{21}u_{13}+l_{22}u_{23} \\ l_{31} & l_{31}u_{12}+l_{32} & l_{21}u_{13}+l_{32}u_{23}+l_{33} \end{bmatrix}$$

$l_{11} = 1$ $\quad l_{11}u_{12} = 1$ $\quad l_{11}u_{13} = 1$

$l_{21} = 4$ $\quad u_{12} = 1/1$ $\quad u_{13} = 1/l_{11} = 1$

$l_{31} = 3$ $\quad u_{12} = 1$

$l_{21}u_{12} + l_{22} = 3$

$4(1) + l_{22} = 3$

$l_{22} = -1$

$l_{21}u_{13} + l_{22}u_{23} = -1$

$4(1) + (-1)u_{23} = -1$

$-u_{23} = -5$

$u_{23} = 5$

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$l_{31}u_{12} + l_{32} = 5$

$3 \times 1 + l_{32} = 5$

$l_{32} = 2$

$l_{31}u_{13} + l_{32}u_{23} + l_{33} = 3$

$(3)(1) + 2 \times 5 + l_{33} = 3$

$l_{33} = 3 - 13 = -10$

~~$AX = B \Rightarrow LUX = B$~~

~~$UX = Z$~~

$\Rightarrow$ ~~$LZ = B$~~

$$\begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 4 & -1 & 0 \\ 3 & 2 & -10 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{bmatrix}$$

$AX = B \Rightarrow LUX = B$

$UX = Z \Rightarrow LZ = B$

$$\begin{bmatrix} 1 & 0 & 0 \\ 4 & -1 & 0 \\ 3 & 2 & -10 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 6 \\ 4 \end{bmatrix}$$

$z_1 = 1$

$4z_1 - z_2 = 6$

$4(1) - z_2 = 6$

$z_2 = -2$

$3z_1 + 2z_2 - 10z_3 = 4$

$3(1) + 2(-2) - 10z_3 = 4$

$3 - 4 - 10z_3 = 4$

$-10z = 5$

$z = -1/2$

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$UX = Z$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ -1/2 \end{bmatrix}$$

$x_3 = -1/2$

$x_2 + 5x_3 = -2$

$x_2 + 5(-1/2) = -2 \Rightarrow x_2 = -2 + 5/2 = 1/2$

$x_1 + x_2 + x_3 = 1$

$x_1 + \cancel{1/2} - \cancel{1/2} = 1$

$x_1 = 1$

$A = LU$

$$\begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

Date: Day: M T W T F S

Secant Method: Regula falsi

$f(x)=0$ $[a,b]$ $\boxed{f(x_1)f(x_2)<0}$

x_{k-1}, x_k given

$$x_{k+1} = x_k - \frac{f(x_k)(x_k-(x_{k-1}))}{\boxed{f(x_k) - f(x_{k-1})}} \quad f(x_k)$$

Newton Raphson Method:

$$x_{n+1} = \boxed{x_n - \frac{f(x_n)}{f'(x_n)}} \quad x_0 \text{ given}$$

$\boxed{f'(x_n) \neq 0}$

$f'(x_n) \neq 0$

a b

r

+ve -ve

$f'(x_n) = -3x^2+\sin x$ ✓ -ve +ve

$\boxed{-x^3-\cos x}$

✓

$f'(x)$