



SELF STUDY
BASED ON MY UNDERSTANDING OF THE SUBJECT

Functional Analysis

Compilation of my study materials

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Fundamentals

Hahn-Banach Theorems

The theorem is named for the mathematicians Hans Hahn and Stefan Banach, who proved it independently in the late 1920s. The special case of the theorem for the space $C[a, b]$ of continuous functions on an interval was proved earlier (in 1912) by Eduard Helly, and a more general extension theorem, the M. Riesz extension theorem, from which the Hahn–Banach theorem can be derived, was proved in 1923 by Marcel Riesz.

Definition 1: Sublinear Function

Let X be a vector space over a field \mathbb{K} , where \mathbb{K} is either the real numbers \mathbb{R} or complex numbers \mathbb{C} . A real-valued function $p : X \rightarrow \mathbb{R}$ on X is called a sublinear function (or a sublinear functional if $\mathbb{K} = \mathbb{R}$), and also sometimes called a quasi-seminorm or a Banach functional, if it has these two properties:

1. **Positive homogeneity/Nonnegative homogeneity:** $p(rx) = rp(x)$ for all real $r \geq 0$ and all $x \in X$. This condition holds if and only if $p(rx) \leq rp(x)$ for all positive real $r > 0$ and all $x \in X$.
2. **Subadditivity/Triangle inequality:** $p(x+y) \leq p(x) + p(y)$ for all $x, y \in X$. This subadditivity condition requires p to be real-valued.

A real-valued function $f : M \rightarrow \mathbb{R}$ defined on a subset $M \subseteq X$ is said to be dominated (above) by a function $p : X \rightarrow \mathbb{R}$ if $f(m) \leq p(m)$ for every $m \in M$. Hence the reason why the following version of the Hahn-Banach theorem is called the dominated extension theorem [1].

Theorem 2.0.1: Helly, Hahn-Banach Analytic Form

Bibliography

- [1] Wikipedia contributors. *Hahn–Banach theorem* — *Wikipedia, The Free Encyclopedia*. [Online; accessed 24-May-2022]. 2022. URL: https://en.wikipedia.org/w/index.php?title=Hahn%E2%80%93Banach_theorem&oldid=1089256573 (page - 2).