6(i) $\frac{d}{d(\tan^{-1}x)}(\ln\tan^{-1}x)$

$= \frac{1}{\tan^{-1}x}$

6(ii) $\frac{d}{d(2x)}(\sin 4x)$

$= \frac{d}{d(2x)}(\sin 2\cdot 2x)$

$= \cos 2x \cdot 2$

$= 2\cos 2x$

7(i) $y = 3x^3 + x^2 - 5x + 6$

$\therefore \frac{dy}{dx} = 9x^2 + 2x - 5$

(ii) Given that,

$x^3 + y^2 + 3y - 4x = 2$

$\therefore 3x^2 + 2y \cdot \frac{dy}{dx} + 3\frac{dy}{dx} - 4 = 0$

$\Rightarrow \frac{dy}{dx}(2y+3) = 4 - 3x^2$

$\therefore \frac{dy}{dx} = \frac{4-3x^2}{2y+3}$

(iii) $xy + 2y + 3x^2 = 0$

$\therefore x\frac{dy}{dx} + y\cdot 1 + 2\frac{dy}{dx} + 6x = 0$

$\Rightarrow \frac{dy}{dx}(x+2) = -6x - y$

$\therefore \frac{dy}{dx} = \frac{-(6x+y)}{(x+2)}$

(iv) $x^n + y^n = a^n$

derivating w.r.t x in both sides, we get

$nx^{n-1} + ny^{n-1}\cdot\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = \frac{-nx^{n-1}}{ny^{n-1}}$

$\therefore \frac{dy}{dx} = -\frac{x^{n-1}}{y^{n-1}}$

20. Given that,

$y = 1 - x + 2x^2 - 3x^3$

$\therefore \frac{dy}{dx} = -1 + 4x - 9x^2$

So, the slope at $(1, -1)$ is, $\left.\frac{dy}{dx}\right|_{1} = -1 + 4.1 - 9.1^2$

$\Rightarrow \frac{dy}{dx} = -6$

Therefore the tangent line becomes,

$y + 1 = \frac{dy}{dx}(x - 1)$

$\Rightarrow y + 1 = -6(x - 1)$

$\therefore 6x + y = 0$

Here,
Formula:
$y - y_1 = m(x - x_1)$

(11) Given that,

$x^2 + y^2 + 2x - 5y + 1 = 0$

$\therefore 2x + 2y \cdot \frac{dy}{dx} + 2 - 5\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx}(2y - 5) = -2x - 2$

$\therefore \frac{dy}{dx} = -\frac{2x + 2}{2y - 5}$

So, the slope at $(1, 1)$ is, $\frac{dy}{dx} = -\frac{2.1 + 2}{2.1 - 5}$

$\therefore \frac{dy}{dx} = \frac{4}{3}$

Now, the tangent line becomes,

$y - y_1 = \frac{dy}{dx}(x - x_1)$

$\Rightarrow y - 1 = \frac{4}{3}(x - 1)$

$\Rightarrow 3y - 3 = 4x - 4$

$\therefore 4x - 3y - 1 = 0$

(iii) Given that,

$$s = \frac{1}{2}t^3 - t^2 + 2t + 5$$

We know that,

the velocity is, $v = \frac{ds}{dt}$

and accelaration is $\frac{dv}{dt} = a$

$$\therefore \quad v = \frac{ds}{dt} = \frac{d}{dt}\left(\frac{1}{2}t^3 - t^2 + 2t + 5\right)$$

$$= \frac{3}{2}t^2 - 2t + 2$$

At the end of 5 sec,

$$v = \frac{3}{2} \cdot 5^2 - 2 \cdot 5 + 2.$$

$$\Rightarrow \quad v = \frac{45}{2} - 8$$

$$\therefore \quad v = \frac{29}{2} \ ms^{-1}$$

Again, $a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{3}{2}t^2 - 2t + 2\right)$

$$\Rightarrow \quad a = 3t - 2$$

At the end of 5 sec,

$$a = 3 \cdot 5 - 2$$

$$\therefore \quad a = 13 \ ms^{-2}.$$

Q① Given that,

$$f(x) = x^3 + 15x^2 + 48x + 6$$

$$\therefore \quad f'(x) = 3x^2 - 30x + 48$$

and, $f''(x) = 6x - 30$

For maximum or minimum $f'(x)$ must be equal to zero.

$$\therefore \quad f'(x) = 0$$

$$\Rightarrow \quad 3x^2 - 30x + 48 = 0$$

$$\Rightarrow \quad x^2 - 10x + 16 = 0$$

$\Rightarrow x^2 - 10x + 16 = 0$

$\Rightarrow x^2 - 8x - 2x + 16 = 0$

$\Rightarrow (x-8)(x-2) = 0$

$\therefore x = 2, 8$

Now, $f''(2) = 6.2 - 30 = -18 < 0$

So, at $x = 2$ the function $f(x)$ has maximum value.

$\therefore$ The maximum value is,

$$f(2) = 2^3 - 15.2^2 + 48.2 + 6$$

$$= 50$$

Again, $f''(8) = 6.8 - 30 = 18 > 0$

So, at $x = 8$ the function $f(x)$ has minimum value.

$$\therefore f(8) = 8^3 - 15.8^2 + 48.8 + 6$$

$$= -58$$

$\therefore$ The maximum value is 50 and the minimum value is -58.

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