Differentiation

$\sin 2x = 2\sin x \cos x$

$1 - \cos 2x = 2\sin^2 x$

$1 + \cos 2x = 2\cos^2 x$

Find the derivative of the followings with respect to x.

1. Basic type:

i) $2 + x + 2x + ax + x^3$

ii) $\frac{1}{x} + \frac{2}{x} + \frac{1}{3x^2} + \frac{a}{2b} + x^{-3}$

iii) $2\sin 2x + a\cos bx + a\tan x$

iv) $e^{2x} + 2\sin 2x + 2a^{2x} + 2^{2x}$

v) $\log_a x + 2\log_2 3x + 2\ln x + \log_{10} x$

2. Complex type:

i) $e^{2lnx} + a^{x^2} + a^{sinx}$

ii) $\sin\ln x + \cos\ln x + \ln\cos x$

iii) $\sqrt{\sin x^3} + a^{\ln\cos x}$

iv) $\sin^2\ln(x^2) + \cos^2\ln(\sec x)$

vi) $\ln\tan x + 10^{\ln\sin x}$

vii) $\sin^{-1}\tan^{-1} x + \tan^{-1} 3x$

viii) $\tan^{-1}\sqrt{\frac{1-\cos 2x}{1+\cos 2x}}$

3. By applying uv formula: $\frac{d}{dx}(uv) = u\frac{d}{dx}v + v\frac{d}{dx}u$

i) $x\sin x$

ii) $\sin x\cos x$

iii) $ax^3\sin x + bx^2\cos x$

iv) $ax^2\ln x + 6e^{2x}\ln a^x$

v) $xe^x\ln x + \sin 3x\cos 3x\tan 3x$

vi) $xe^{\cos x}\ln\sin x + xa^{\sin x}\ln a^x$

4. By applying $\frac{u}{v}$ formula: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{d}{dx}u - u\frac{d}{dx}v}{v^2}$

i) $\frac{1-\sin x}{1-\cos x}$

ii) $\frac{2x-\sin x}{x^2}$

ii) $\frac{\sin x+\cos x}{\sqrt{1+\sin 2x}}$

iv) $\frac{\sin x}{1-\cos x}$

v) $\frac{x\ln x}{\sin x\cos x}$

5. By applying u^v formula: $\frac{d}{dx}(u^v) = u^v[lnu\frac{d}{dx}v + v\frac{d}{dx}lnu]$

i) x^x

ii) x^{x^x}

iii) $\sin x^{\cos x}$

iv) $x^{\frac{1}{x}}$

v) x^{e^x}

vi) $\cot x^{\tan x}$

6. Miscellneous

i) Find the derivative of $\ln \tan^{-1} x$ with respect to $\tan^{-1} x$.

ii) Find the derivative of $\sin 4x$ with respect to $2x$.

7. Implicit Function:

i) $y = 3x^3 + x^2 - 5x + 6$

ii) $x^3 + y^2 + 3y - 4x = 2$

iii) $xy + 2y + 3x^2 = 0$

iv) $x^n + y^n = a^n$

8. Application of derivative:

i) Find the equation of the tangent line of the curve $y = 1 - x + 2x^2 - 3x^3$ at the point (1,-1).

ii) Find the equation of the tangent line of the circle $x^2 + y^2 + 2x - 5y + 1 = 0$ at the point (1, 1).

iii) A motors passes $s = \frac{1}{2}t^3 - t^2 + 2t + 5$ m at t sec. Find the velocity and acceleration at the end of 5 sec.

9. Find the maximum and minimum value.

i) $f(x) = x^3 - 15x^2 + 16x + 6$

ii) $f(x) = x^2 - 4x + 5$

iii) $f(x) = 4e^x + 9e^{-x}$