LOGARITHMS

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EXPONENT

DEFINITION

If a is any real number and n is a positive integer, then the product of n numbers is defined as

$$\underbrace{a \cdot a \cdot a \cdots a}_{n\,factors} = a^n$$

EXAMPLE:

Base
(real number)

Exponent
(integer)

$$a^n$$

Where n is the index or exponent or power and a is the base.

$$2^5, 3^{-2}, x^6, p^{-7} etc.$$

LOGARITHMS

Definition:

Logarithm is the inverse function to exponentiation

$$\boxed{b^x = N} \quad \cdots\cdots (1)$$

$$where\ N > 0, b > 0\ \&\ b \neq 1$$

$$\boxed{x = log_b(N)} \quad \cdots\cdots (2)$$

$$where\ N > 0, b > 0\ \&\ b \neq 1$$

(1) & (2) are equivalent

Example:

If $2^3 = 8$,

$\Rightarrow log_2(8) = 3.$

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LAWS OF LOGARITHMS

Formula	Example
$log_b(MN) = log_b(M) + log_b(N)$	$log_2(3 \cdot 5) = log_2 3 + log_2 5$
$log_b\left(\frac{M}{N}\right) = log_b(M) - log_b(N)$	$log_3\left(\frac{17}{24}\right) = log_3 17 - log_3 24$
$log_b(M^P) = P\, log_b(M)$	$log_3(5^7) = 7\, log_3 5$
$log_b b = 1$	$log_2 2 = 1$

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TYPES OF LOGARITHM

Common Logarithms

- The system of logarithms whose base is 10 is called the common logarithm system. When the base is omitted, it is understood that base 10 is to be used
- Thus, $log\,25 = log_{10}\,25$

Natural Logarithms:

- The system of logarithms whose base is the Eulerian constant e is called the natural logarithm system. When we want to indicate the base of a logarithm is e we write ln.
- Thus, $ln\,25 = log_e\,25$

NOTE: Since $10^{1.5377} = 34.49$ so $log\,34.49 = 1.5377$. Here the digit 1 before decimal point is called the characteristic and the digits .5377 after decimal point is called the mantissa of the log.

PROBLEM & SOLUTION

Express each of the following exponential form in logarithmic form:

1	2	3
$4^2 = 16$	$3^{-2} = \frac{1}{9}$	$8^{-\frac{2}{3}} = \frac{1}{4}$
Using log of base 4 we get	Using log of base 3 we get	Using log of base 8 we get
$log_4 4^2 = log_4 16$	$log_3 3^{-2} = log_3 \left(\frac{1}{9}\right)$	$log_8 8^{-\frac{2}{3}} = log_8 \left(\frac{1}{4}\right)$
$or, 2\, log_4 4 = log_4 16$	$or, -2\, log_3 3 = log_3 \left(\frac{1}{9}\right)$	$or, -\frac{2}{3} log_8 8 = log_8 \left(\frac{1}{4}\right)$
$or, 2 = log_4 16$	$or, -2 = log_3 \left(\frac{1}{9}\right)$	$or, -\frac{2}{3} = log_8 \left(\frac{1}{4}\right)$

PROBLEM & SOLUTION CONT..

Express each of the following logarithmic form in exponential form:

4	5	6
$\log_5 25 = 2$	$\log_2 64 = 6$	$\log_{\frac{1}{4}} \frac{1}{16} = 2$
By the definition of log, we get	By the definition of log, we get	By the definition of log, we get
$25 = 5^2$	$64 = 2^6$	$\frac{1}{16} = \left(\frac{1}{4}\right)^2$

PROBLEM & SOLUTION CONT..

7. Find the logarithms of 1728 to the base $2\sqrt{3}$
We have 1728
After factorization by prime number, we get
$1728 = 2^6 . 3^3$
$or, 2^6 . \left(\sqrt{3}\right)^6 = 1728$
$or, \left(2\sqrt{3}\right)^6 = 1728$
According to definition of log we get
$6 = \log_{2\sqrt{3}} 1728$
$\therefore \log_{2\sqrt{3}} 1728 = 6$

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PROBLEM & SOLUTION CONT..

8. Find x if $\frac{1}{2}\log_{10}\left(11+4\sqrt{7}\right) = \log_{10}(2+x)$

Given that,

$$\frac{1}{2}\log_{10}\left(11+4\sqrt{7}\right)=\log_{10}(2+x)$$

$$or,\ \log_{10}\sqrt{11+4\sqrt{7}}=\log_{10}(2+x)$$

$$or,\ \sqrt{11+4\sqrt{7}}=2+x$$

$$or,\ \left(\sqrt{11+4\sqrt{7}}\right)^2=(2+x)^2$$

$$or,\ 11+4\sqrt{7}=x^2+4x+4$$

$$or,\ x^2+4x-7=4\sqrt{7}$$

$$or,\ x^2+4x-\left(7+4\sqrt{7}\right)=0$$

$$\therefore\ x=\frac{-4\pm\sqrt{4^2-4\cdot1\cdot-\left(7+4\sqrt{7}\right)}}{2\cdot1}=\frac{-4\pm\sqrt{16+4\left(7+4\sqrt{7}\right)}}{2}$$

$$=\frac{-4\pm\sqrt{16+28+16\sqrt{7}}}{2}=\frac{-4\pm\sqrt{44+16\sqrt{7}}}{2}$$

$$=\frac{-4\pm2\sqrt{11+4\sqrt{7}}}{2}$$

$$\therefore x=-2\pm\sqrt{11+4\sqrt{7}}$$

$$x=-2+\sqrt{11+4\sqrt{7}},\ -2-\sqrt{11+4\sqrt{7}}$$

As $-2-\sqrt{11+4\sqrt{7}}<-2$ so $x\neq-2-\sqrt{11+4\sqrt{7}}$

$$\therefore x=-2+\sqrt{11+4\sqrt{7}}$$

PROBLEM & SOLUTION CONT..

9. Prove that $2\log x + 2\log x^2 + 2\log x^3 + \cdots + \cdots + 2\log x^n = n(n+1)\log x$

L.H.S. $= 2\log x + 2\log x^2 + 2\log x^3 + \cdots + 2\log x^n$

$= 2\log x + 4\log x + 6\log x + \cdots + 2n\log x$

$= (1 + 2 + 3 + \cdots + n)2\log x$

$= \dfrac{n(n+1)}{2}.2\log x$

$= n(n+1)\log x$

$= \text{R.H.S.}$

$$\therefore 2\log x + 2\log x^2 + 2\log x^3 + \cdots + 2\log x^n = n(n+1)\log x$$

(**Proved**)

EXERCISE

10. Express the logarithm of $\frac{\sqrt{a^3}}{c^5 b^2}$ $\boldsymbol{in\ terms\ of}$ $\log \boldsymbol{a} \log \boldsymbol{b}$ $\boldsymbol{and}$ $\log \boldsymbol{c}$

11. Find $\boldsymbol{x}$ from of equation $\boldsymbol{a^x . c^{-2x} = b^{3x+1}}$

12. Solve $\log_{10}(3x + 2) + \log_{10}(x - 1) = 1$

13. Solve the equation $\frac{e^x - 1}{e^{-x} - 1} = -3$

14. Calculate the value of P from $\log_{10} 4 + 2\log_{10} P = 2$

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