Introduction to Cost-Benefit Analysis

Lecture 4

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CBA: Lecture 4

Today's Lecture

- Dealing with Uncertainty.
- Risk preferences.
- Bayes' Theorem.
- Optimal testing strategy to prevent disease.
- Optimal testing strategy to reduce speeding.

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Uncertainty

- Suppose we do not know how much the bridge will cost exactly. Assume we believe it will cost $25,000\$$ with probability $1/4$, and $35,000\$$ with probability $3/4$.
- How should we adjust our CBA so as to decide whether the project is worthwhile?
- A natural solution consists in computing the expected cost by taking the weighted probability average:

$$\text{Expected Cost} = \frac{1}{4} \times 25,000\$ + \frac{3}{4} \times 35,000\$ = 32,500\$$$

- So we now pretend the cost to be a certain $32,500\$$ and conduct the same analysis as previously.

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Uncertainty

- What assumption are we implicitely making here?
- Remember that individuals will have to pay the cost of the bridge eventually, and that we are interested in the net impact of the project on their welfare.
- But most individuals are not indifferent between an uncertain outcome and a certain outcome of same expected value: some prefer the sure outcome (risk averse), others prefer the uncertain outcome (risk lovers).
- So what we should be really looking for is the change in expected ***utility*** for each individual.
- The expected net present value might be positive, yet the expected effect on social welfare might be negative. Let's try to understand why.

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Uncertainty

- To simplify, assume that each individual's utility writes $u_i(x_i, P)$, where x_i is i's consumption (income) and $P \in \{0,1\}$ indicates whether the project is implemented or not.
- If the project costs c (per person), then the individual's utility from implementing the project is equal to $u_i(x_i - c, 1)$.
- Then the project is worthwhile if:

$$\sum_{i \in N} u_i(x_i - c, 1) > \sum_{i \in N} u_i(x_i, 0) = \sum_{i \in N} u_i(x_i - \Delta_i, 1)$$

- Where Δ_i is individual i's willingness to pay for the bridge.

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Uncertainty

- Assume now there is uncertainty: the project costs $\underline{c}$ with probability p and $\overline{c}$ with probability $1-p$. From now on, we write $u(x_i, 1) = u(x_i)$.
- The individual's expected utility is then equal to:

$$pu_i(x_i - \underline{c}) + (1-p)u_i(x_i - \overline{c})$$

- Therefore, the expected social welfare from implementing the project is equal to:

$$\sum_{i \in N} pu_i(x_i - \underline{c}) + (1-p)u_i(x_i - \overline{c})$$

- The decision is to implement the project if:

$$\sum_{i \in N} pu_i(x_i - \underline{c}) + (1-p)u_i(x_i - \overline{c}) > \sum_{i \in N} u_i(x_i - \Delta_i)$$

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Uncertainty

- Instead, if we had taken the expected cost as a certain, the decision would be to implement the project if:

$$\sum_{i \in N} u_i(x_i - p\underline{c} - (1-p)\overline{c}) > \sum_{i \in N} u_i(x_i - \Delta_i)$$

- In general the two expressions will differ, and may lead to different recommendations.
- If $u(.)$ is concave then:

$$pu_i(x_i - \underline{c}) + (1-p)u_i(x_i - \overline{c}) < u_i(x_i - p\underline{c} - (1-p)\overline{c})$$

- In that case, we say that the individual is ***risk averse***: she prefers a sure outcome to an uncertain outcome of equal expected value.
- Using the expected cost as a certainty equivalent will lead to adopting the project too often.

- If $u(.)$ is convex then:

$$pu_i(x_i - \underline{c}) + (1-p)u_i(x_i - \overline{c}) > u_i(x_i - p\underline{c} - (1-p)\overline{c})$$

- In that case, we say that the individual is ***risk lover***: she prefers an uncertain outcome to a sure outcome equal to its expected value.
- Using the expected cost as a certainty equivalent will lead to adopting the project too little

- If $u(.,1)$ is linear then:

$$pu_i(x_i - \underline{c}) + (1-p)u_i(x_i - \overline{c}) = u_i(x_i - p\underline{c} - (1-p)\overline{c})$$

- In that case, we say that the individual is ***risk neutral***: she is indifferent between a sure outcome and an uncertain outcome of equal expected value.
- Replacing the cost by the expected cost is only appropriate when the individual is risk neutral!
- However, in practice, if the cost of the project is spread out over a very large number of individuals, the interval $[x_i - \underline{c}, x_i - \overline{c}]$ will be very small.
- It may then be acceptable to assume the utility function to be linear, and use the expected cost as a certainty equivalent.

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Uncertainty

- Let's go back to our original example. The bridge is expected to cost $25,000$ with probability $1/4$ and $35,000$ with probability $3/4$.
- Assume there are $1,000$ taxpayers and they each get to pay an equal share of the realized cost. Furthermore, assume they all have the same income (before paying for the bridge) of 100.
- Assume $u(z) = \sqrt{z}$. How much do individuals need to value the bridge for the project to be profitable?
- Assume $u(z) = z^2$. How much do individuals need to value the bridge for the project to be profitable?
- Assume $u(z) = z$. How much do individuals need to value the bridge for the project to be profitable?

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Uncertainty

- If $u(x) = \sqrt{x}$, then the project is profitable if:

$$\frac{1}{4}\sqrt{100 - \frac{25,000}{1,000}} + \frac{3}{4}\sqrt{100 - \frac{35,000}{1,000}} \geq \sqrt{100 - \Delta}$$

- which implies:

$$\Delta \geq 100 - \left(\frac{1}{4}\sqrt{75} + \frac{3}{4}\sqrt{65}\right)^2 \approx 32.57$$

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Uncertainty

- If $u(x) = x^2$, then the project is profitable if:

$$\frac{1}{4}(100-25)^2 + \frac{3}{4}(100-35)^2 \geq (100-\Delta)^2$$

- which implies:

$$\Delta \geq 100 - \sqrt{\frac{1}{4}75^2 + \frac{3}{4}65^2} \approx 32.36$$

- If $u(x) = x$, then the project is profitable if:

$$\frac{1}{4}(100 - 25) + \frac{3}{4}(100 - 35) \geq (100 - \Delta)$$

- which implies:

$$\Delta \geq 100 - \left(\frac{1}{4}75^2 + \frac{3}{4}65\right) = 32.5$$

- Here, because the project is risky, it takes a larger willingness to pay Δ for the project to be profitable if the individuals are risk averse than if they are risk neutral or risk lover.

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Uncertainty

- Assume that the agency can choose to insure against the risk of the project being more costly than expected.
- A (risk neutral) insurance company offers the following contract: pay a premium r, and if the cost of the project ends up being $35,000$, receive a payment of p.
- Assume the individuals are risk averse with utility function $u(x) = \sqrt{x}$.
- Under what conditions on r and p, is the insurance company indeed willing to offer the contract?
- Under what conditions on r and p, is the agency willing to accept the contract (and adopt the project)?

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Uncertainty

- The insurance company gets the following expected revenue from contract (r, p):

$$ER = r - \frac{3}{4}p$$

- The insurance company is risk neutral which means it is willing to offer the contract if it brings a positive expected revenue.

$$ER = r - \frac{3}{4}p \geq 0 \quad \Leftrightarrow \quad r \geq \frac{3}{4}p$$

- Under the insurance contract, each individual gets an expected utility of:

$$\frac{1}{4}\sqrt{75-\frac{r}{1000}}+\frac{3}{4}\sqrt{65+\frac{p-r}{1000}}$$

- Therefore, the agency chooses to accept the contract (and adopt the project) if:

$$\frac{1}{4}\sqrt{75-\frac{r}{1000}}+\frac{3}{4}\sqrt{65+\frac{p-r}{1000}}>\sqrt{100-\Delta}$$

- Assume there is perfect competition on the insurance market, and the agency chooses to fully insure itself, How much do individuals need to value the bridge for the project to be profitable?
- Perfect competition means that the profit of the insurance company is equal to 0:

$$r = \frac{3}{4}p$$

- Furthermore assume the agency chooses to fully insure itself so that $p = 10,000$. As a result:

$$r = \frac{3}{4}10,000$$

- Therefore, the project is now profitable if:

$$\sqrt{75-\frac{30}{4}} > \sqrt{100-\Delta} \quad \leftrightarrow \quad \Delta \geq 100-\frac{270}{4} = 32.5$$

- We're back to the case where individuals are risk neutral!
- If there is a prefectly competitive insurance market, then the government can transfer the risk to the risk neutral insurance company at no cost.
- If the insurance market is not perfectly competitive, getting rid of the risk will come at some cost.

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Uncertainty

- The initial cost of constructing a permanent dam (i.e., a dam that is expected to last forever) is 425 million.
- The annual net benefits will depend on the amount of rainfall: 18 million in a "dry" year, 29 million in a "wet" year, and 52 million in a "flood" year.
- Meteorological records indicate that over the last 100 years there have been 86 "dry" years, 12 "wet" years, and 2 "flood" years.
- Assume the annual benefits begin to accrue at the end of the first year.
- The government is assumed to be risk neutral: appropriate to use expected benefits/costs in the NPV formula.

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Uncertainty

- Using the meteorological records as a basis for prediction, and assuming individuals are risk neutral, conduct CBA for a discount rate of 5%?
- Find the internal rate of return.

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Uncertainty: exercise.

- The expected yearly benefit is equal to:

$$\mathbb{E}(NB_t) = 0.86 \times 18.10^6 + 0.12 \times 29.10^6 + 0.02 \times 52.10^6 = 20.10^6$$

- For a discount rate of r, the NPV is equal to:

$$\begin{aligned} NPV &= \sum_{t=1}^{+\infty} \frac{20.10^6}{(1+r)^t} - 425.10^6 \\ &= 20.10^6 \left(\sum_{t=0}^{+\infty} \frac{1}{(1+r)^t} - 1 \right) - 425.10^6 \\ &= 20.10^6 \left(\frac{1+r}{r} - 1 \right) - 425.10^6 \\ &= \frac{20.10^6}{r} - 425.10^6 \end{aligned}$$

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Uncertainty: exercise.

- For $r = 5\%$, we get a NPV of $-25.10^6 < 0$. Therefore the project is not worthwhile.
- The internal rate of return is the minimum discount rate below which the project becomes profitable:

$$\frac{20.10^6}{r} - 425.10^6 > 0 \quad \Leftrightarrow \quad r < \frac{20}{425} = 4.7\%$$

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Today's Lecture

- Dealing with Uncertainty.
- Risk preferences.
- Bayes' Theorem
- Optimal testing strategy to prevent disease.
- Optimal testing strategy to reduce speeding.

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Bayes' Theorem: reminder on probability.

- If A and B are two events, and the probability of B is positive, $P(B) > 0$, then the probability of A conditional on B is given by:

$$\mathbb{P}(A \mid B) = \frac{\mathbb{P}(A \text{ and } B)}{\mathbb{P}(B)}$$

- Therefore,

$$\mathbb{P}(B \mid A) = \frac{\mathbb{P}(A \text{ and } B)}{\mathbb{P}(A)} = \frac{\mathbb{P}(A \mid B)\mathbb{P}(B)}{\mathbb{P}(A)}$$

- This is Bayes's Theorem.

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Bayes' Theorem: reminder on probability.

- Let $\overline{A}$ be the complement event of A: $\overline{A}$ happens if and only if A does not happen.
- We have: $\mathbb{P}(\overline{A}) = 1 - \mathbb{P}(A)$.
- Then:

$$\begin{aligned}
\mathbb{P}(B) &= \mathbb{P}(B \text{ and } A) + \mathbb{P}(B \text{ and } \overline{A}) \\
&= \mathbb{P}(B \mid A)\mathbb{P}(A) + \mathbb{P}(B \mid \overline{A})\mathbb{P}(\overline{A}) \\
&= \mathbb{P}(B \mid A)\mathbb{P}(A) + \mathbb{P}(B \mid \overline{A})\,(1 - \mathbb{P}(A))
\end{aligned}$$

- This is the Law of Total Probability.

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Bayes' Theorem: why it is useful.

- For example, consider the case of random drug testing in sports.
- Individuals can either be doped (A), or not doped ($\overline{A}$).
- If an athelete is tested for drugs, the test can be either positive (B) or negative ($\overline{B}$).
- We do not know whether a given athlete is doped or not, but we know whether his test is positive or negative.

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Bayes' Theorem: why it is useful.

- Here $\mathbb{P}(A)$ corresponds to the probability of any given athlete being doped.
- Here $\mathbb{P}(B)$ corresponds to the probability of any given athlete testing positive.
- Here $\mathbb{P}(A \mid B)$ corresponds to the probability of the athlete being doped conditional on testing positive for drugs.
- Here $\mathbb{P}(B \mid A)$ corresponds to the probability of testing positive conditional on being doped. It is the effectiveness of the drug testing.
- Using the Bayes' Theorem we can infer the probability that an athlete who tested positive is doped ($\mathbb{P}(A \mid B)$) from knowing $\mathbb{P}(B \mid A)$, $\mathbb{P}(A)$ and $\mathbb{P}(B)$!

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Bayes' Theorem: optimal testing strategy to prevent disease.

- The prevalence of a disease among a certain population is 40%. That is, there is a 40% chance that a person randomly selected from the population will have the disease.
- An imperfect test that costs 250$ is available to help identify those who have the disease before actual symptoms appear.
- Those who have the disease have a 90% of a positive test result; those who do not have the disease have a 5% percent chance of a positive test.
- Treatment of the disease before the appearance of symptoms costs $2,000$$ and inflicts additional costs of 200$ on those who do not actually have the disease. Treatment of the disease after symptoms have appeared costs $10,000$$.

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Bayes' Theorem: optimal testing strategy to prevent disease.

- The government is considering the following possible strategies with respect to the disease:
  - **Strategy 1**: Do not test and do not treat early.
  - **Strategy 2**: Do not test and treat early.
  - **Strategy 3**: Test and treat early if positive and do not treat early if negative.
- CBA: which treatment strategy has the lowest expected costs for a member of the population?

Bayes' Theorem: optimal testing strategy to prevent disease.

- From the description of the problem, we have:

$$\begin{aligned}
&\mathbb{P}(\text{sick}) = 0.4, \\
&\mathbb{P}(\text{test positive} \mid \text{sick}) = 0.9, \\
&\mathbb{P}(\text{test positive} \mid \text{healthy}) = 0.05
\end{aligned}$$

- What is the probability of a test being positive?
- The probability of a test being positive is equal to:

$$\begin{aligned}
\mathbb{P}(\text{test positive}) &= \mathbb{P}(\text{sick})\mathbb{P}(\text{test positive} \mid \text{sick}) \\
&\qquad + \mathbb{P}(\text{healthy})\mathbb{P}(\text{test positive} \mid \text{healthy}) \\
&= 0.4 \times 0.9 + 0.6 \times 0.05 \\
&= 0.39
\end{aligned}$$

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Bayes' Theorem: optimal testing strategy to prevent disease.

- In order to understand whether testing is worthwhile, we need to express the conditional probability of a patient actually being sick following a positive test.
- We use Bayes' Theorem:

$$\begin{aligned}\mathbb{P}(\text{sick} \mid \text{test positive}) &= \frac{\mathbb{P}(\text{test positive} \mid \text{sick})\mathbb{P}(\text{sick})}{\mathbb{P}(\text{test positive})} \\ &= \frac{0.9 \times 0.4}{0.39} = 0.92\end{aligned}$$

- Therefore we get:

$$\mathbb{P}(\text{healthy} \mid \text{test positive}) = 1 - \mathbb{P}(\text{sick} \mid \text{test positive}) = 0.08$$

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Bayes' Theorem: optimal testing strategy to prevent disease.

- Similarly, we can compute the conditional probability of being sick if the test comes back negative.

$$\begin{aligned}\mathbb{P}(\text{sick} \mid \text{test negative}) &= \frac{\mathbb{P}(\text{test negative} \mid \text{sick})\mathbb{P}(\text{sick})}{\mathbb{P}(\text{test negative})} \\ &= \frac{0.1 \times 0.4}{0.61} = 0.07\end{aligned}$$

- We can now compute the net expected benefit under each of the three strategies.

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Bayes' Theorem: optimal testing strategy to prevent disease.

- Under **Strategy 1**, only sick patients are treated, but after their symptoms appear. Therefore, the expected cost is equal to:

$$\mathbb{E}(c) = 0.4 \times 10,000 = 4,000\$$$

- Under **Strategy 2**, all patients are treated before any symptoms appear. This means that some of the patients are treated even though they are not sick. The expected cost is now equal to:

$$\mathbb{E}(c) = 0.4 \times 2000 + 0.6 \times 2,200 = 2,120\$$$

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Bayes' Theorem: optimal testing strategy to prevent disease.

- Under **Strategy 3**, only patients who took a positive tests are treated early.
- Because the test is not perfect, some of the sick patients get a negative test while some of the healthy patients get a positive test.
- Sick patients who took a negative test are only treated once their symptoms appear.
- The expected cost is now equal to:

$$\begin{aligned}
\mathbb{E}(c) &= \mathbb{P}(\text{test positive})\mathbb{P}(\text{sick} \mid \text{test positive}) \times 2,000 \\
&+ \mathbb{P}(\text{test positive})\mathbb{P}(\text{healthy} \mid \text{test positive}) \times 2,200 \\
&+ \mathbb{P}(\text{test negative})\mathbb{P}(\text{sick} \mid \text{test negative}) \times 10,000 + 250 \\
&= 0.39 \times 0.92 \times 2,000 + 0.39 \times 0.08 \times 2,200 \\
&+ 0.61 \times 0.07 \times 10,000 + 250 \\
&= 1463.24\$
\end{aligned}$$

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Bayes' Theorem: optimal testing strategy to prevent disease.

- Does testing remain optimal if the prevalence of the disease in the population is only .05?
- Find under which prevalence level of the disease in the population strategy $S3$ is indeed the most cost effective policy.

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Bayes' Theorem: optimal testing strategy to prevent disease.

- Let's denote by p the prevalence of the disease in the population. Then, the previous probabilities now write:

$$\mathbb{P}(\text{test positive}) = 0,9p + 0,05(1-p)$$

$$\mathbb{P}(\text{sick} \mid \text{test positive}) = \frac{0,9p}{0,9p + 0,05(1-p)}$$

$$\mathbb{P}(\text{healthy} \mid \text{test positive}) = \frac{0,05(1-p)}{0,9p + 0,05(1-p)}$$

$$\mathbb{P}(\text{sick} \mid \text{test negative}) = \frac{0.1p}{1 - 0,9p - 0,05(1-p)}$$

- We can now compute the expected cost of each of the three strategies.

- Under **Strategy 1**, only sick patients are treated, but after their symptoms appear. Therefore, the expected cost is equal to:

$$\mathbb{E}(c) = 10,000p$$

- Under **Strategy 2**, all patients are treated before any symptoms appear. This means that some of the patients are treated even though they are not sick. The expected cost is now equal to:

$$\mathbb{E}(c) = 2000p + 2,200(1-p) = 2,200 - 200p$$

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Bayes' Theorem: optimal testing strategy to prevent disease.

- Under **Strategy 3**, the expected cost is now equal to:

$$
\begin{aligned}
\mathbb{E}(c) &= (0,9p + 0,05(1-p))\frac{0.9p}{0,9p + 0,05(1-p)} \times 2,000 \\
&+ (0,9p + 0,05(1-p))\frac{0,05(1-p)}{0,9p + 0,05(1-p)} \times 2,200 \\
&+ (1 - 0,9p - 0,05(1-p))\frac{0.1p}{1 - 0,9p - 0,05(1-p)} \times 10,000 + 250 \\
&= 0.9p \times 2,000 + 0,05(1-p) \times 2,200 + 0.1p \times 10,000 + 250 \\
&= 360 + 2690p
\end{aligned}
$$

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Bayes' Theorem: optimal testing strategy to prevent disease.

- Therefore, **Strategy 3** is preferable as long as:

$$360 + 2690p \leq 10,000p \quad \Leftrightarrow \quad p \geq \frac{360}{10000 - 2690} = 4.9\%$$

and

$$360 + 2690p \leq 2,200 - 200p \quad \Leftrightarrow \quad p \leq \frac{2,200 - 360}{2690 + 200} = 64\%$$

- The prevalence level needs to be in the interval $[4.9\%, 64\%]$ for **Strategy 3** to be effective.
- If it is below 4.9%, then **Strategy 1** is the most cost effective.
- If it is above 64%, then **Strategy 2** is the most cost effective.

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Bayes' Theorem: optimal testing strategy to prevent disease.

- Assume now that the government can invest so as to improve the testing technology.
- More specifically, assume the government can both increase the probability of a sick person getting a positive test by Δ and reduce the probability of a healthy person getting a positive test by Δ at a cost of $50,000 \times \Delta^2$.
- How much should the government invest in the testing technology if the prevalence of the disease is 40%?

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Bayes' Theorem: optimal testing strategy to prevent disease.

- If the government invests $50,000 \times \Delta^2$, the previous probabilities become equal to:

$$\mathbb{P}(\text{test positive}) = (0,9+\Delta) \times 0.4 + (0.05-\Delta) \times 0.6$$

$$\mathbb{P}(\text{sick} \mid \text{test positive}) = \frac{(0.9+\Delta) \times 0.4}{(0,9+\Delta) \times 0.4 + (0,05-\Delta) \times 0.6}$$

$$\mathbb{P}(\text{healthy} \mid \text{test positive}) = \frac{(0.05-\Delta) \times 0.6}{(0,9+\Delta) \times 0.4 + (0,05-\Delta) \times 0.6}$$

$$\mathbb{P}(\text{sick} \mid \text{test negative}) = \frac{(0.1-\Delta) \times 0.4}{1-(0,9+\Delta) \times 0.4 - (0,05-\Delta) \times 0.6}$$

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Bayes' Theorem: optimal testing strategy to prevent disease.

- Under **Strategy 3**, the expected cost (excluding the cost of investment Δ) is now equal to:

$$\begin{aligned}\mathbb{E}(c) &= (0.9+\Delta)\times 0.4\times 2,000 \\ &\quad + (0.05-\Delta)\times 0.6\times 2,200 \\ &\quad + (0.1-\Delta)\times 0.4\times 10,000 + 250 \\ &= 1436 - 4520\Delta\end{aligned}$$

- Therefore, the government must solve the following maximization program:

$$\min_{\Delta\geq 0}\ 1436 - 4520\Delta + 50,000\times\Delta^2$$

Bayes' Theorem: optimal testing strategy to prevent disease.

- We write the first order conditions:

$$-4520 + 50,000 \times 2\Delta = 0 \quad \Leftrightarrow \quad \Delta = 0.0452$$

- So the best strategy for the government is to improve the testing technology by an amount $\Delta = 0.0452$.

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Bayes' Theorem: optimal testing strategy to reduce speeding.

- The ministry of roads must choose whether to install radars to control drivers' speed.
- Radars would help cutting drivers' speed but they are costly to install. Is it worth the investment?
- The main issue is that radars are not fully reliable, especially at night.
- To simplify assume drivers can either choose to drive slow (below the speed limit), or fast (above the speed limit). There are a total of $100,000$ drivers.
- The probability of getting caught for a car going fast is 95%.
- The probability of getting caught for a car going slow is 8%.

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Bayes' Theorem: optimal testing strategy to reduce speeding.

- If a car is caught overspeeding (rightly or wrongly), then it gets a fine $F = 1000\$$.
- Individuals who get wrongly convicted of driving fast get an additional disutility equivalent to 500\$.
- The death rate on the road depends on how many drivers choose to go fast. If the proportion of cars driving fast is $q \in [0, 1]$, then the (expected) number of death every year is $100 \times q$.
- Individuals' utility from going fast v is uniformly distributed over the interval $[-2000, 2000]$. Individuals only care about their utility and disregard the risk of death.

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Bayes' Theorem: optimal testing strategy to reduce speeding.

- **Question 1**: What proportion of individuals choose to go fast with and without a radar?
- **Question 2**: What is the expected decrease in deaths from installing the radars?
- **Question 3**: What is the expected number of individuals wrongly convicted of driving fast?
- **Question 4**: What is the expected social welfare from installing the radar?

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Bayes' Theorem: optimal testing strategy to reduce speeding.

Question 1: What proportion of individuals choose to go fast with and without a radar?

- If there are no radars, individuals who get a positive utility from going fast choose to go fast, while individuals who get a negative utility choose to go slow. Therefore $q = 50\%$.
- If there are radars, individuals only choose to go fast if their utility from going fast exceeds the expected cost from possibly getting caught.

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Bayes' Theorem: optimal testing strategy to reduce speeding.

- The expected utility of going fast is now equal to:

$$v - \mathbb{P}(\text{getting caught} \mid \text{fast}) \times F = v - 0.95 \times 1000 = v - 950$$

- The expected utility of going slow is now equal to:

$$-\mathbb{P}(\text{getting caught} \mid \text{slow}) \times F = -0.08 \times 1500 = -120$$

- Therefore only individuals whose utility v exceeds $950 - 120 = 830$ choose to go fast. Because individuals' utilities are distributed uniformly over $[-2000, 2000]$,

$$q = \frac{2000 - 830}{4000} \approx 29.25\%$$

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Bayes' Theorem: optimal testing strategy to reduce speeding.

Question 2: What is the expected decrease in deaths from installing the radars?

- The expected decrease in road related deaths is equal to:

$$\begin{aligned}\Delta &= \mathbb{E}(\#\text{deaths} \mid \text{no radar}) - \mathbb{E}(\#\text{deaths} \mid \text{radar}) \\ &= 100 \times 50\% - 100 \times 29.25\% = 20.75\end{aligned}$$

- Assume we assign a value of $1,000,000$\$ to each life saved (the so-called value of a statistical life). Then the benefit in terms of saved lives is equal to:

$$20.75 \times 1,000,000 = 20,750,000\$$$

Bayes' Theorem: optimal testing strategy to reduce speeding.

Question 3: What is the expected cost of wrongly convicting slow drivers?

- We need to compute the probability of a driver going slow and getting caught:

$$\begin{aligned}\mathbb{P}(\text{slow \& caught}) &= \mathbb{P}(\text{caught} \mid \text{slow})\mathbb{P}(\text{slow}) \\ &= 8\%(1-0.2925) = 5.66\%\end{aligned}$$

- Since there are $100,000$ total drivers, the expected number of wrongly convicted drivers is equal to $5.66\% \times 100,000 = 5660$.
- Therefore the expected cost of wrongly convicting drivers is $5660 \times 500 = 2,830,000\$$.

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Bayes' Theorem: optimal testing strategy to reduce speeding.

Question 4: What is the expected social welfare from installing the radar?

- What are the benefits, what are the costs?
- **Benefit**: reducing mortality rate.
- **Cost**: wrongly convicting slow driver.
- **Cost**: lost utility from driving slow.
- Individuals whose utility from going fast is between 0 and 830$ now choose to drive slow, thus losing the enjoyment from fast driving. The loss of utility is equal to:

$$\int_0^{830} v \frac{100,000}{4000} dv = 25 \times \frac{830^2}{2} = 8,611,250$$

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Bayes' Theorem: optimal testing strategy to reduce speeding.

- What about the revenue from fines?
- Should not count because it is a transfer!
- Social welfare thus equal to:

$$\Delta SW = 20,750,000 - 2,830,000 - 8,611,250 = 9,308,750$$

- The project is only worthwhile if the cost of installing the radar is smaller than the expected change in social welfare $\Delta SW = 9,308,750\$$.