Introduction to Cost-Benefit Analysis

Lecture 5

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Today's Lecture

- Charging a toll?
- Modelling Congestion.

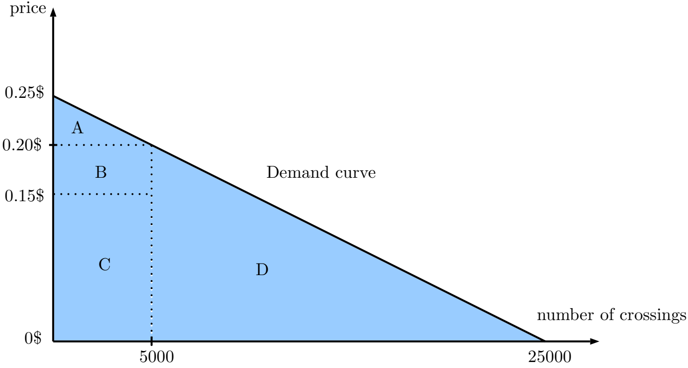
Charging a toll?

- Let's go back to our bridge example.
- Could the project be made more profitable by charging a toll on the bridge?
- Let's recalculate the CBA for the case where the government levies a toll of 0.05$ per crossing.
- What is changed?
- Now only individuals whose willingness to pay exceeds 0.05$ will be willing to use the bridge. The toll thus affects the number of bridge users, and therefore the consumer surplus.
- Compute the consurmer surplus and toll revenue associated to this policy.

A first go at CBA

Social Welfare

Without a toll:



$$\text{Social Welfare} = \text{Consumer Surplus} + \text{Profits}$$
$$= 3,125 + 0 = 3,125.$$

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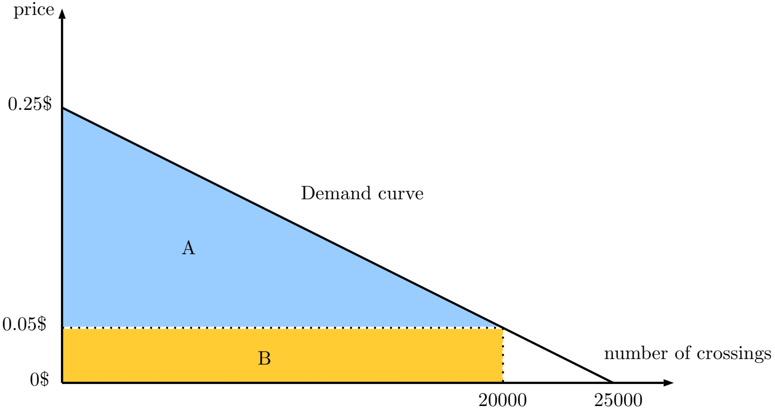
A first go at CBA:charging a toll?



$$\text{Consumer Surplus} = \text{Area A}$$
$$= \frac{1}{2} \times 20,000 \times 0.20 = 2000.$$

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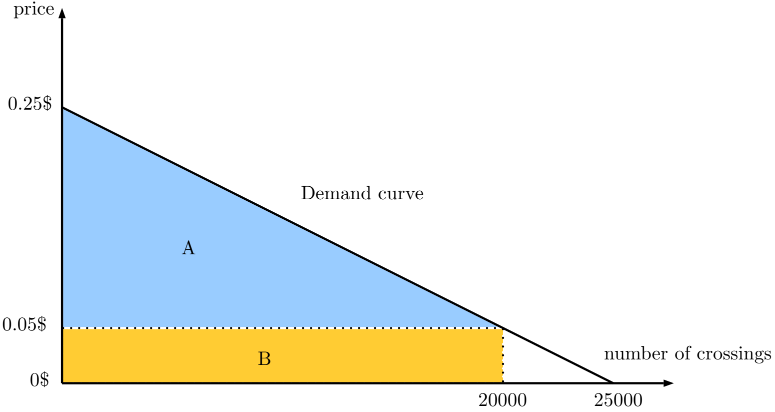
A first go at CBA:charging a toll?



$$\text{Toll Revenue (Profit)} = \text{Area B}$$
$$= 20,000 \times 0.05 = 1000.$$

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A first go at CBA:charging a toll?

- What's the new Social Welfare?
- Social Welfare = Consumer Surplus + Profit = 3000 < 3125.

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A first go at CBA: Present value

- The yearly increase in social welfare due to the construction of the bridge is now equal to:

$$\begin{aligned}\Delta SW &= \text{SW(after the bridge) - SW(before the bridge)}\\ &= 2,000 - 375 = 2,625\end{aligned}$$

- The *net present value* of the bridge is equal to:

$$\begin{aligned}NPV &= \Delta SW \left(\frac{1+r}{r}\right) - C\\ &= 2,625 \left(\frac{1+0.1}{0.1}\right) - 30,000 = -1,125 < 0\end{aligned}$$

- Introducing a toll of 0.05$ per crossing actually makes the NPV become negative! Therefore, the CBA recommendation would now be not to build the bridge.

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A first go at CBA: charging a toll?

- Is that result surprising?
- Actually no, a toll only reduces the number of journeys and hence reduces the gain in the real value of additional consumption, without any corresponding reduction in cost.
- The lesson of this is that the prices charged for the output of a project may profoundly affect its economic desirability.
- Under what additional assumption would a toll be profitable?

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Revisiting the toll idea

- In absence of congestion a toll would not increase the total welfare.
- However, a toll might still be desirable as a way to pay for the cost of the bridge. Why?
- That way, only individuals who actually use the bridge get to pay for it. It will decrease the consumer's surplus but might be more fair.

Questions:

- What is the maximum revenue that could be obtained by imposing a toll?
- What is the associated loss of welfare from funding the bridge by maximizing the total revenue?

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Revisiting the toll idea

- The demand function tells us how many people will cross the bridge for any level of toll.

$$D(p) = 25,000 - 100,000p$$

- We want to find the toll p that will maximize revenue:

$$\max_{p \geq 0} D(p) \times p = (25,000 - 100,000p)p$$

- Equating the derivative to 0 yields:

$$25,000 - 200,000p^* = 0 \quad \Rightarrow \quad p^* = \frac{25}{200} = 0.125$$

- We conclude that the maximum (yearly) revenue is equal to $D(0.125) \times 0.125 = 12,500 \times 0.125 = 1,562$.

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Revisiting the toll idea

- Compounded over 40 years with a discount rate of $r = 10\%$, this represents $17,187 < 30,000$. Therefore, we conclude that the bridge cannot be financed entirely by toll revenue.
- If the toll is 0.125, then the total number of crossings is equal to $12,500$.
- Social Welfare is now equal to $SW = 2343$.
- So the loss compounded over the 40 years of the project is equal to:

$$\text{Net Present Loss} = (3125 - 2343) \times \frac{1+0.1}{0.1} = 8593.75$$

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Today's Lecture

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Modelling Congestion.

- What if there is congestion on the bridge?
- Assume the more people use the bridge the smaller the utility it brings to each user.
- Individual i 's utility from using the bridge now writes:

$$u_i(x_i - t, m, 1)$$

  where m denotes the number of individuals who use the bridge, and $t \in [0, 1]$ denotes the toll charged to each user.
- Individual i's utility from not using the bridge is still equal to $u_i(x_i, 0)$.
- So for a given toll t, the number of individuals m using the bridge is such that:

$$m = \#\left\{ i \in N \mid u_i(x_i - t, m, 1) > u_i(x_i, 0) \right\}$$

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Modelling Congestion.

- Assume for example that the utility function takes the following form:

$$u_i(x_i, 0) = x_i \qquad \text{(without the bridge)}$$

$$u_i(x_i, m, 1) = x_i + v_i - \left(\frac{m}{n}\right)^2 \qquad \text{(with the bridge)}$$

  where v_i reflects how much individual i values crossing the river.

- Then individual i decides to use the bridge as long as:

$$x_i - t + v_i - \left(\frac{m}{n}\right)^2 > x_i \iff v_i > t + \left(\frac{m}{n}\right)^2$$

- Assume furthermore that individuals' valuations v_i are uniformly distributed over $[0, 1]$:

$$N(x) = \#\left\{i \in N \mid v_i > x\right\} \approx n \int_x^1 dv = (1 - x)n$$

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Modelling Congestion.

- If the number of individuals using the bridge is equal to m, then the number of individuals who get a larger utility from using the bridge than from not using the bridge is equal to:

$$\left(1 - t - \left(\frac{m}{n}\right)^2\right) n$$

- Therefore, at equilibrium, we must have:

$$\left(1 - t - \left(\frac{m}{n}\right)^2\right) n = m$$

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Modelling Congestion.

- Indeed, if $m > \left(1 - t - \left(\frac{m}{n}\right)^2\right) n$, then it means that some of the m individuals who are using the bridge are actually better off not using the bridge (given the level of congestion m).
- Conversely, if $m < \left(1 - t - \left(\frac{m}{n}\right)^2\right) n$, it means that some of the $n - m$ individuals who are not using the bridge are actually better off using the bridge (given the level of congestion m).
- So the equilibrium level of congestion m^* is a solution to the following second order equation:

$$m^2 + nm - n^2(1 - t) = 0$$

- Discriminant $\Delta = n^2 (5 - 4t)$.

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Modelling Congestion.

- Only positive solution:

$$m^* = \frac{n}{2}\left[\sqrt{5-4t}-1\right]$$

- Note that we have assumed $t \in [0,1]$, so that $m^* \geq 0$.
- As expected, the equilibrium level of congestion decreases as the toll t increases.
- When the toll is 0, $m^* = \frac{n}{2}(\sqrt{5}-1) \approx 0.61n$.
- When the toll is 1, $m^* = 0$.
- Let's look at the equilibrium level of congestion on a graph.

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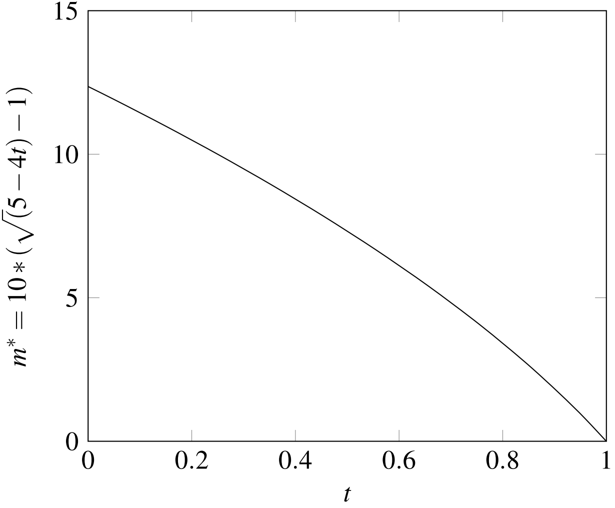
Modelling Congestion.

For $n = 20$ we get:



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Modelling Congestion.

- What level of toll t maximizes social welfare?
- Increasing t reduces congestion, leading to two opposite effects on social welfare.
- As congestion is reduced, the utility of bridge users is increased, thus increasing social welfare.
- But as congestion is reduced, the number of bridge users is also reduced, thus decreasing social welfare.
- The social welfare associated to toll t is equal to:

$$
\begin{aligned}
SW(t) &= \sum_{v_i > t + \left(\frac{m^*}{n}\right)^2} \left[x_i + v_i - \left(\frac{m^*}{n}\right)^2 \right] + \sum_{v_i \leq t + \left(\frac{m^*}{n}\right)^2} x_i \\
&= -\, m^* \left(\frac{m^*}{n}\right)^2 + \sum_{i \in N} x_i + \sum_{v_i > t + \left(\frac{m^*}{n}\right)^2} v_i
\end{aligned}
$$

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Modelling Congestion.

- Note that the revenue from the toll does not enter into the social welfare because it is a transfer.
- Since v_i is uniformly distributed over $[0, 1]$, we have:

$$\sum_{v_i > t + \left(\frac{m^*}{n}\right)^2} v_i \approx n \int_{t+\left(\frac{m^*}{n}\right)^2}^{1} v dv = \frac{n}{2}\left(1 - t - \left(\frac{m^*}{n}\right)^2\right) = \frac{m^*}{2}$$

- So that:

$$SW(t) \approx \sum_{i \in N} x_i - \left(\frac{m^{*3}}{n^2}\right) + \frac{m^*}{2}$$

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Modelling Congestion.

- Replacing m^* by the previous expression we get:

$$SW(t) \approx \sum_{i \in N} x_i - \frac{n}{8}\left(\sqrt{5-4t}-1\right)^3 + \frac{n}{4}\left(\sqrt{5-4t}-1\right)$$

- Let's plot this on a graph!

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Revisiting the toll idea

For $n = 20$ we get: