Heaven's Light is Our Guide

Department of Computer Science & Engineering
Rajshahi University of Engineering & Technology, Bangladesh

Course Code: CSE 2203
Course Title: Digital Techniques

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Common Number Systems

System	Base	Symbols	Used by humans?	Used in computers?
Decimal	10	0, 1, … 9	Yes	No
Binary	2	0, 1	No	Yes
Octal	8	0, 1, … 7	No	No
Hexa-decimal	16	0, 1, … 9, A, B, … F	No	No

Quantities/Counting (1 of 3)

Decimal	Binary	Octal	Hexa-decimal
0	0	0	0
1	1	1	1
2	10	2	2
3	11	3	3
4	100	4	4
5	101	5	5
6	110	6	6
7	111	7	7

Quantities/Counting (2 of 3)

Decimal	Binary	Octal	Hexa-decimal
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

Quantities/Counting (3 of 3)

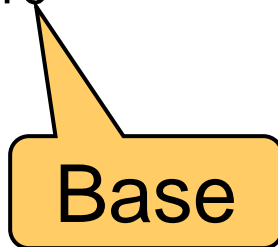| Decimal	Binary	Octal	Hexa-decimal
16	10000	20	10
17	10001	21	11
18	10010	22	12
19	10011	23	13
20	10100	24	14
21	10101	25	15
22	10110	26	16
23	10111	27	17

Conversion Among Bases

The possibilities:
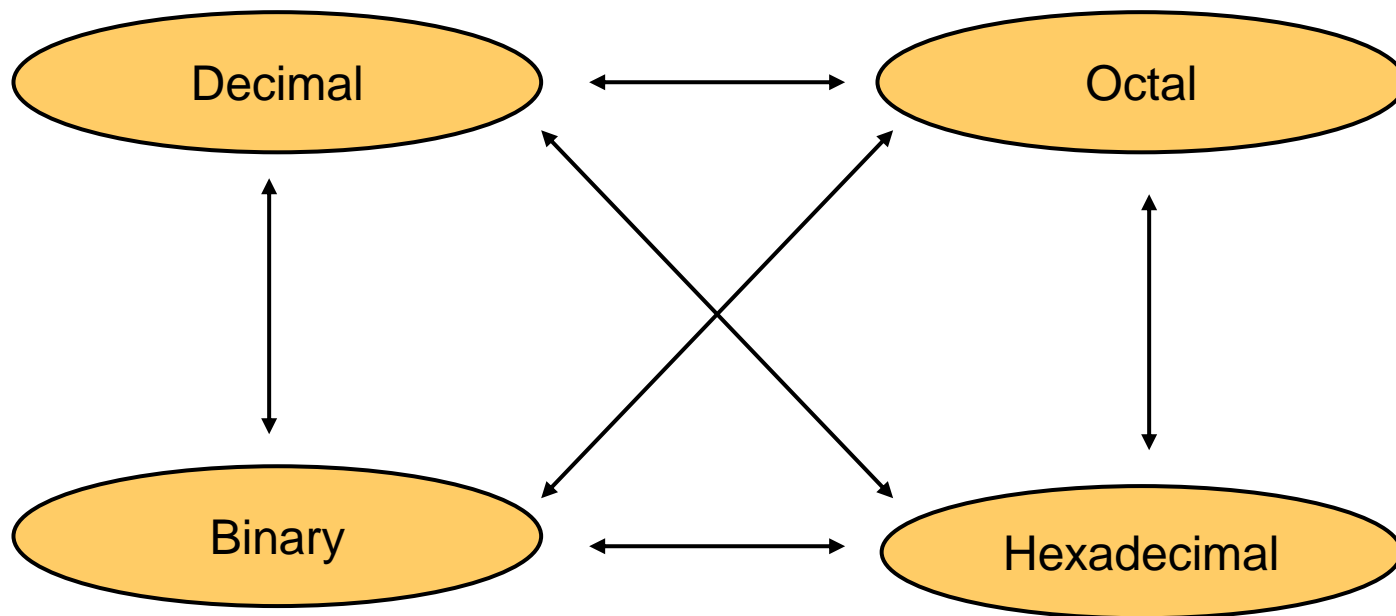
Quick Example

$$25_{10} = 11001_2 = 31_8 = 19_{16}$$
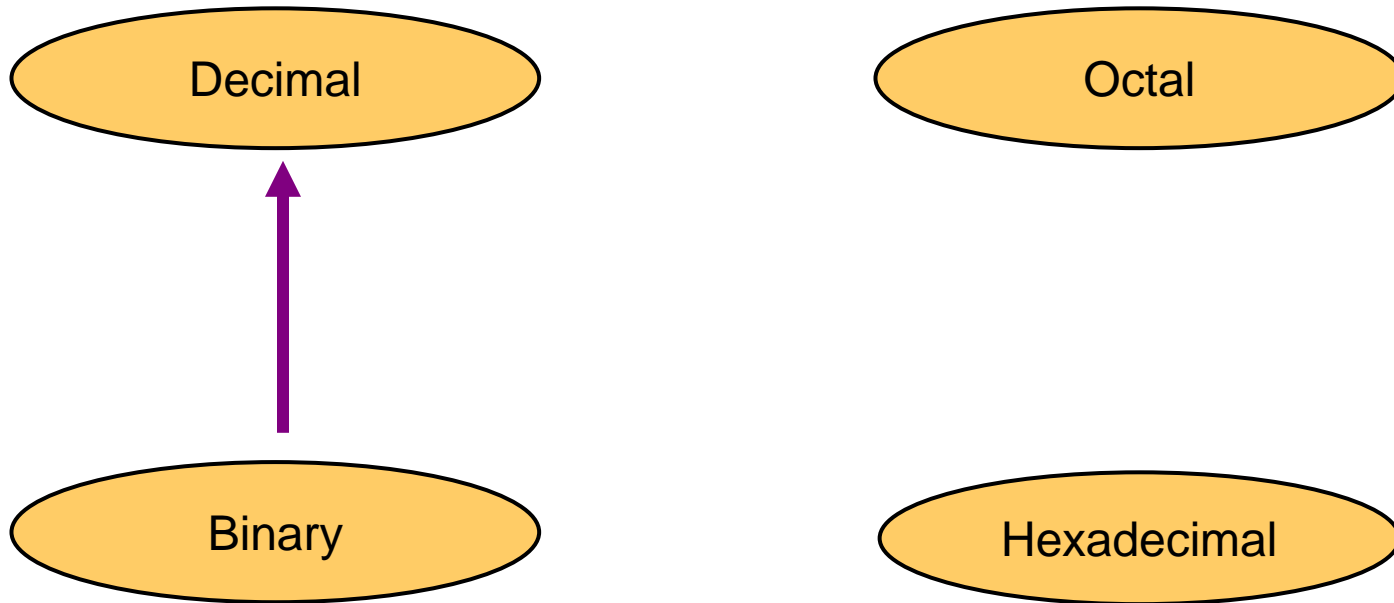
Base

Decimal to Decimal (just for fun)

Decimal

Octal

Binary

Hexadecimal
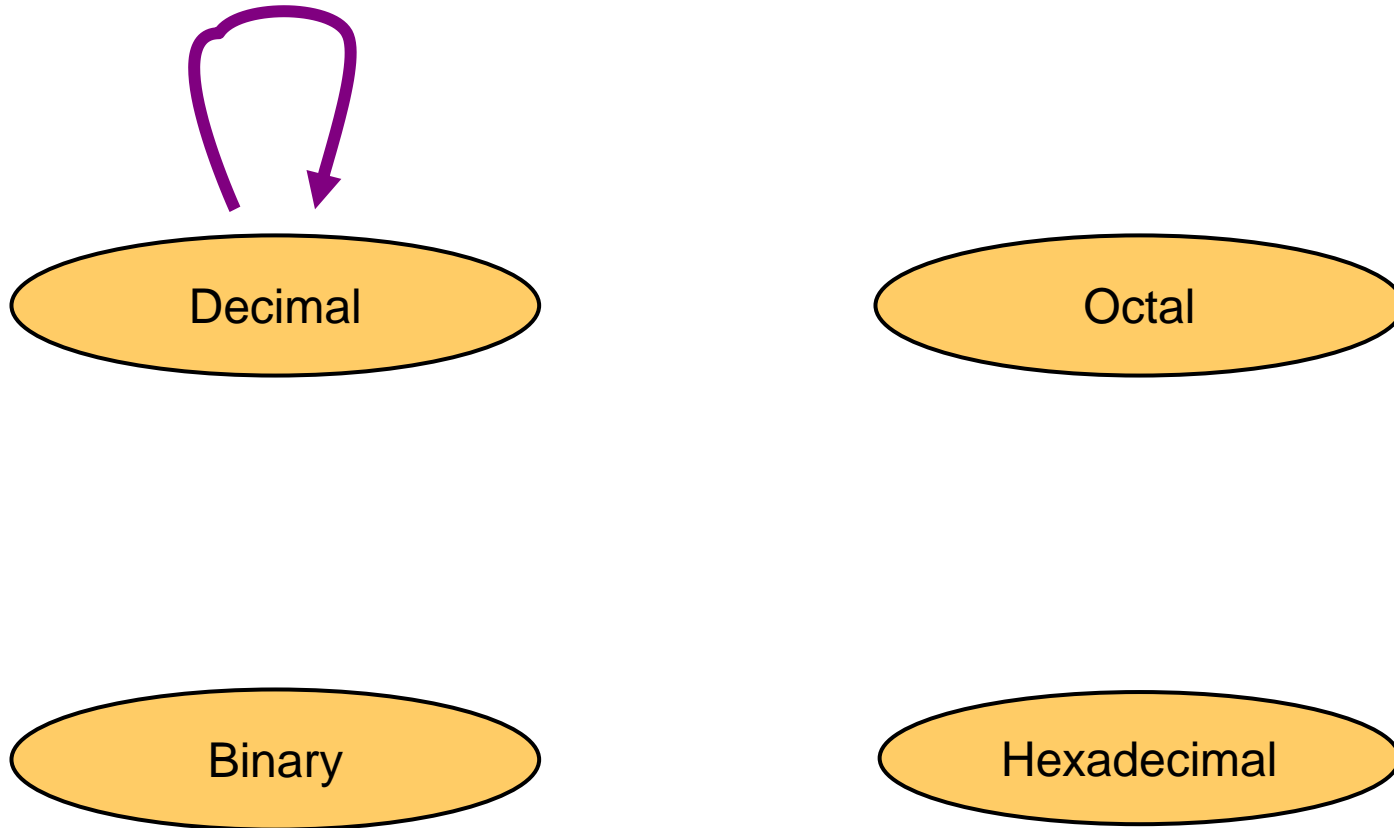
Next slide…

$$125_{10} \Rightarrow$$

$$5 \times 10^0 \quad = \quad 5$$
$$2 \times 10^1 \quad = \quad 20$$
$$1 \times 10^2 \quad = \quad \underline{100}$$
$$125$$

Weight

Base

Binary to Decimal

Binary to Decimal

Technique

- Multiply each bit by 2^n, where n is the "weight" of the bit.

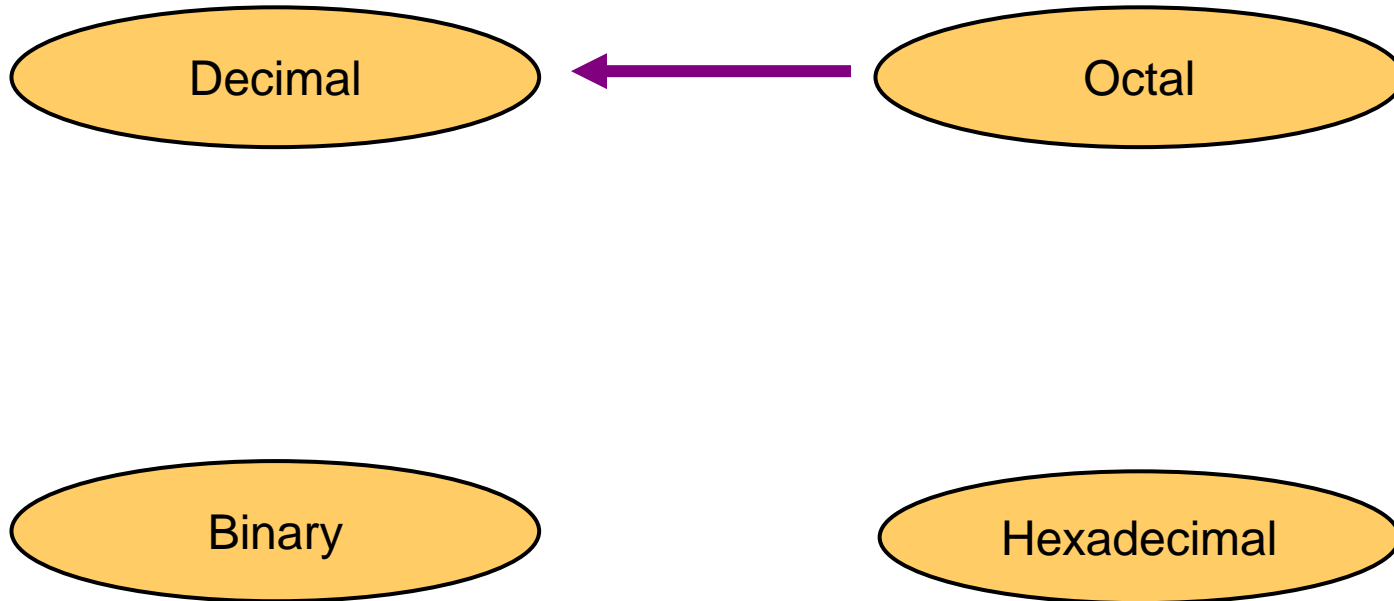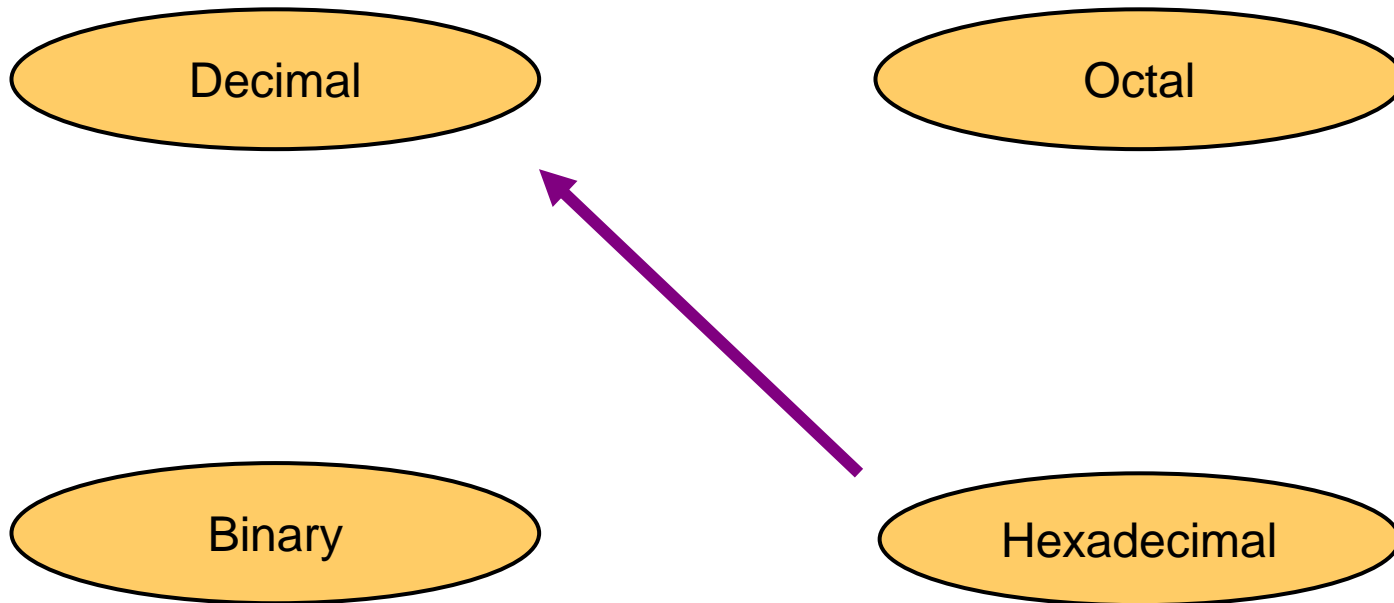- The weight is the position of the bit, starting from 0 on the right

- Add the results

Example

Bit "0"

$$101011_2 \Rightarrow$$

$$1 \times 2^0 = 1$$
$$1 \times 2^1 = 2$$
$$0 \times 2^2 = 0$$
$$1 \times 2^3 = 8$$
$$0 \times 2^4 = 0$$
$$1 \times 2^5 = 32$$
$$\overline{}$$
$$43_{10}$$

Octal to Decimal

Octal to Decimal

Technique

- Multiply each bit by 8^n, where n is the "weight" of the bit
- The weight is the position of the bit, starting from o on the right
- Add the results

Example

$$724_8 \Rightarrow$$

$$4 \times 8^0 = 4$$
$$2 \times 8^1 = 16$$
$$7 \times 8^2 = \underline{448}$$
$$468_{10}$$

Hexadecimal to Decimal

Decimal

Octal

Binary

Hexadecimal

Hexadecimal to Decimal

Technique

- Multiply each bit by 16^n, where n is the "weight" of the bit
- The weight is the position of the bit, starting from o on the right
- Add the results

Example

ABC_{16} =>   C x 16^0 = 12 x   1 =   12
               B x 16^1 = 11 x   16 =   176
               A x 16^2 = 10 x 256 = 2560
                                        —————
                                      2748_{10}

Decimal to Binary

Decimal

Octal

Binary

Hexadecimal

Decimal to Binary

Technique

- Divide by two, keep track of the remainder
- First remainder is bit 0 (LSB, least-significant bit)
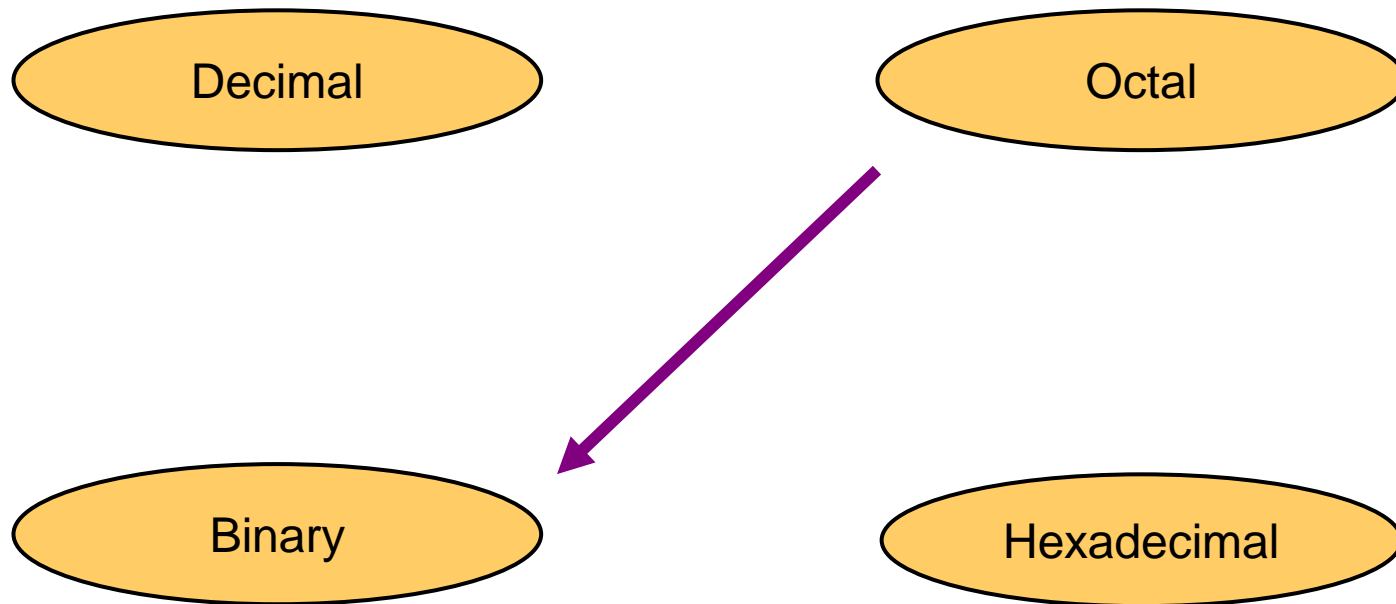- Second remainder is bit 1
- Etc.

Example

$125_{10} = ?_2$

```
2 | 125
2 |  62     1
2 |  31     0
2 |  15     1
2 |   7     1
2 |   3     1
2 |   1     1
        0     1
```
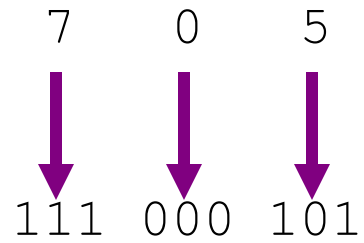
$125_{10} = 1111101_2$

Octal to Binary

Octal to Binary

Technique

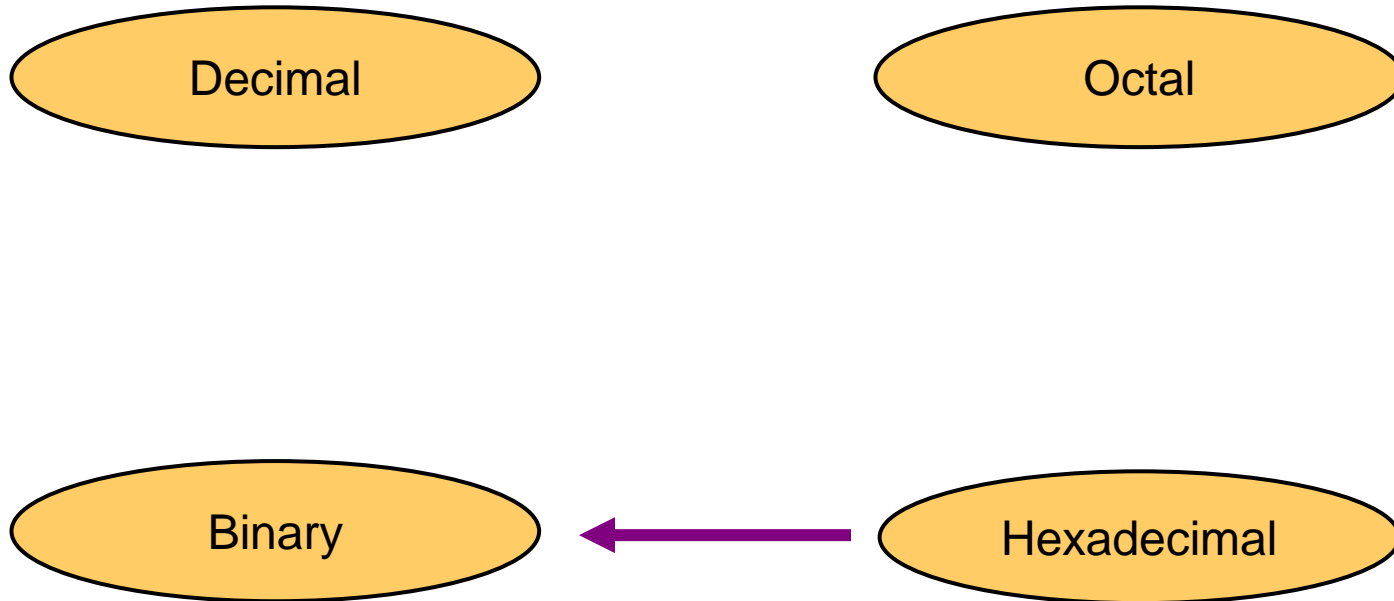Convert each octal digit to a 3-bit equivalent binary representation

Example

$705_8 = ?_2$

$$7 \quad 0 \quad 5$$

$$\downarrow \quad \downarrow \quad \downarrow$$

$$111 \quad 000 \quad 101$$

$705_8 = 111000101_2$

Hexadecimal to Binary

Decimal

Octal

Binary ← Hexadecimal

Hexadecimal to Binary

Technique

Convert each hexadecimal digit to a 4-bit equivalent binary representation

Example

$10AF_{16} = ?_2$
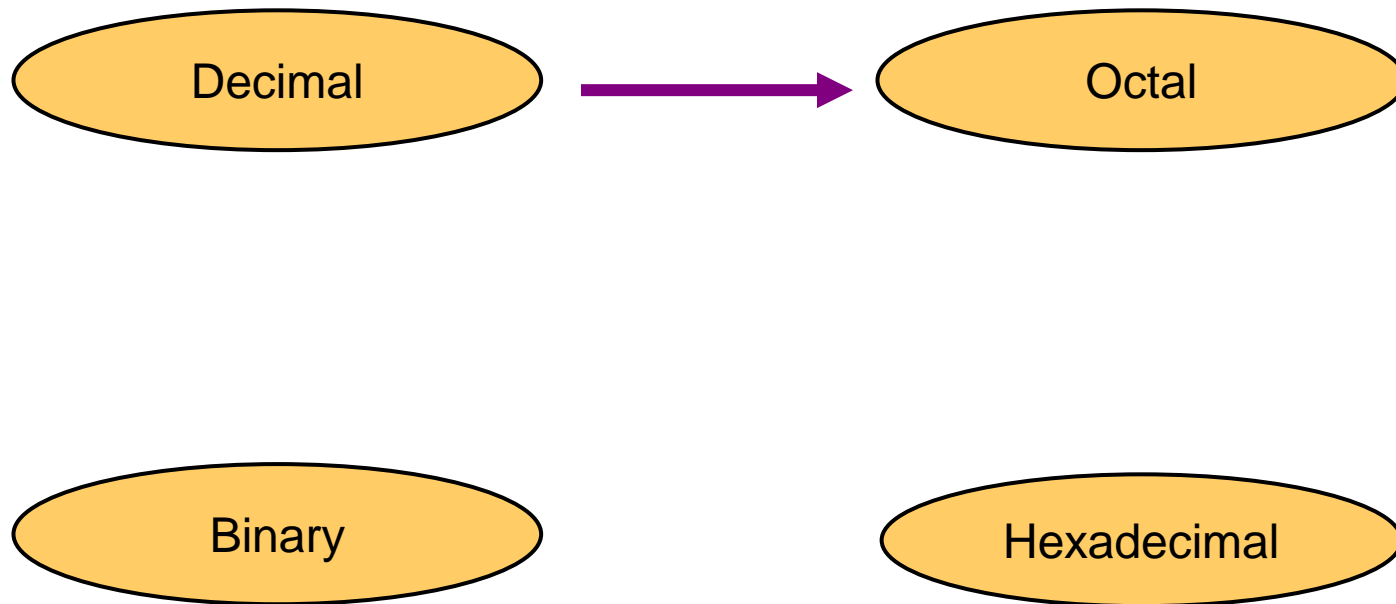
1	0	A	F
↓	↓	↓	↓
0001	0000	1010	1111

$10AF_{16} = 0001000010101111_2$

Decimal to Octal

Decimal → Octal

Binary

Hexadecimal

Decimal to Octal
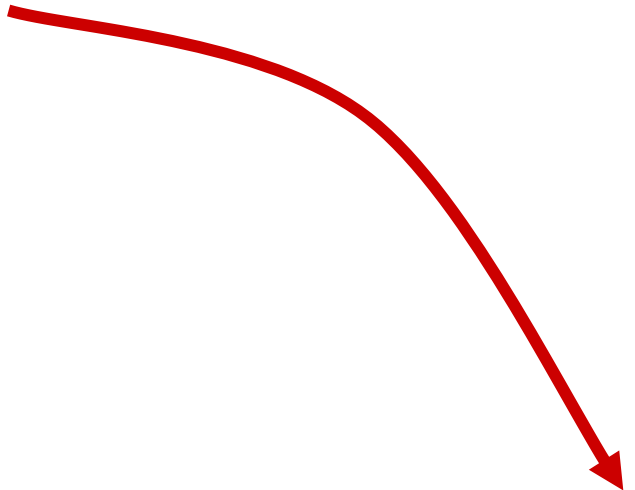
Technique

- Divide by 8
- Keep track of the remainder

Example

$1234_{10} = ?_8$

$$
\begin{array}{r|l|l}
8 & 1234 & \\
8 & 154 & 2 \\
8 & 19 & 2 \\
8 & 2 & 3 \\
& 0 & 2 \\
\end{array}
$$

$1234_{10} = 2322_8$

Decimal to Hexadecimal
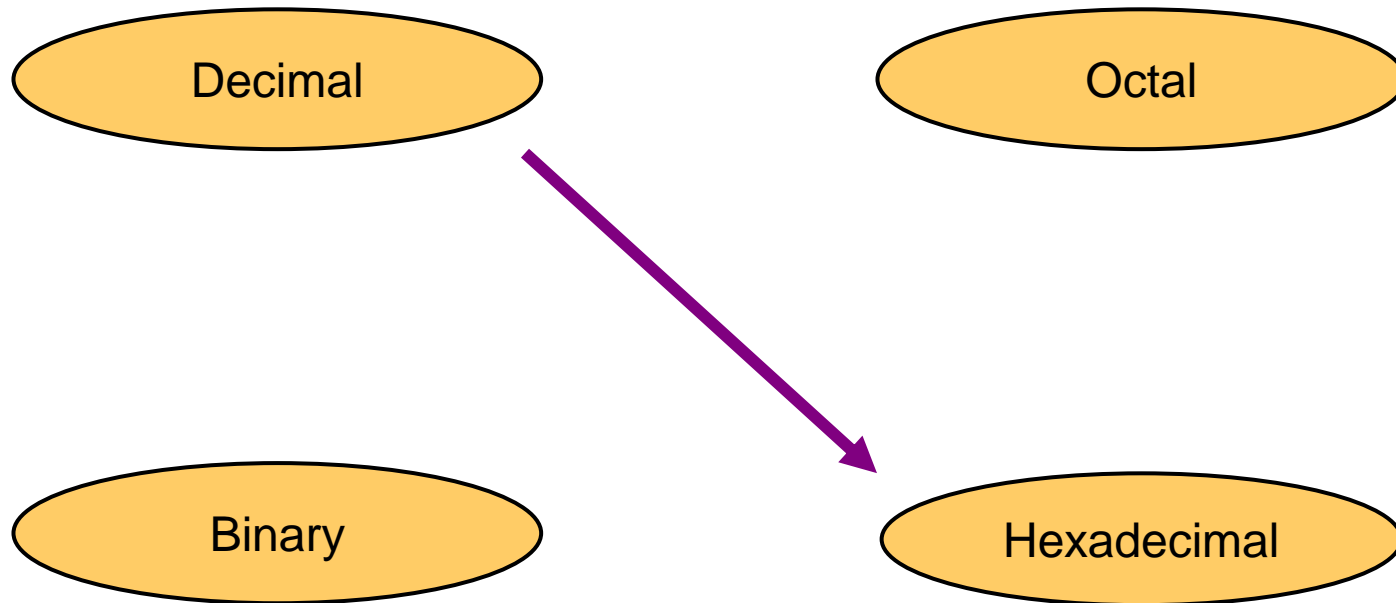
Decimal to Hexadecimal
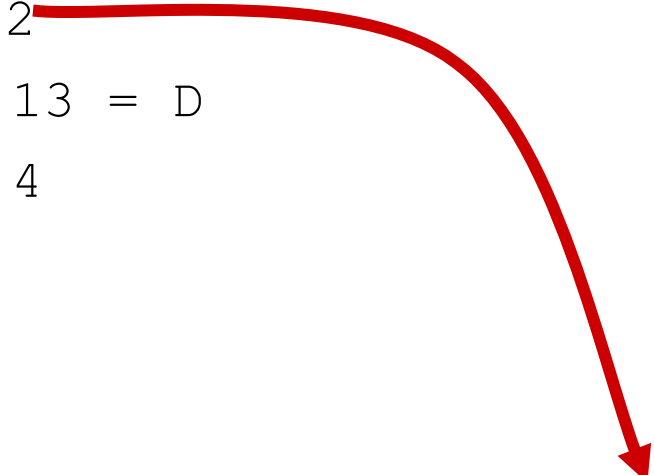
Technique

- Divide by 16
- Keep track of the remainder

Example

$1234_{10} = ?_{16}$

```
16 |1234
16 |  77    2
16 |   4    13 = D
        0    4
```

$1234_{10} = 4D2_{16}$

Binary to Octal

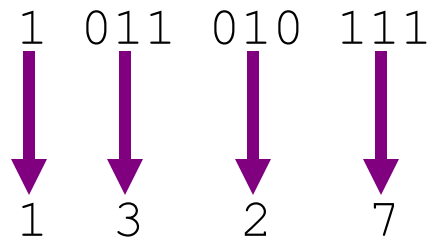Decimal

Octal

Binary
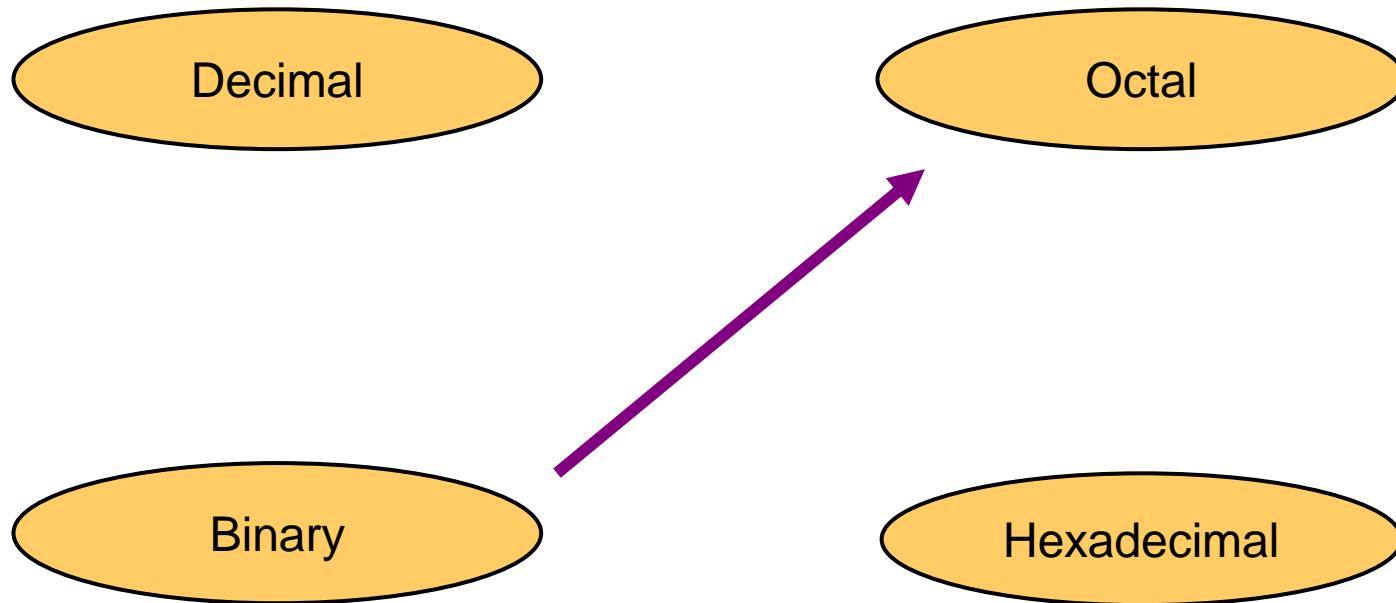
Hexadecimal

Binary to Octal

Technique

- Group bits in threes, starting on right
- Convert to octal digits

Example

$1011010111_2 = ?_8$

1 011 010 111

↓ ↓ ↓ ↓

1 3 2 7

$1011010111_2 = 1327_8$

Binary to Hexadecimal

Decimal

Octal

Binary → Hexadecimal
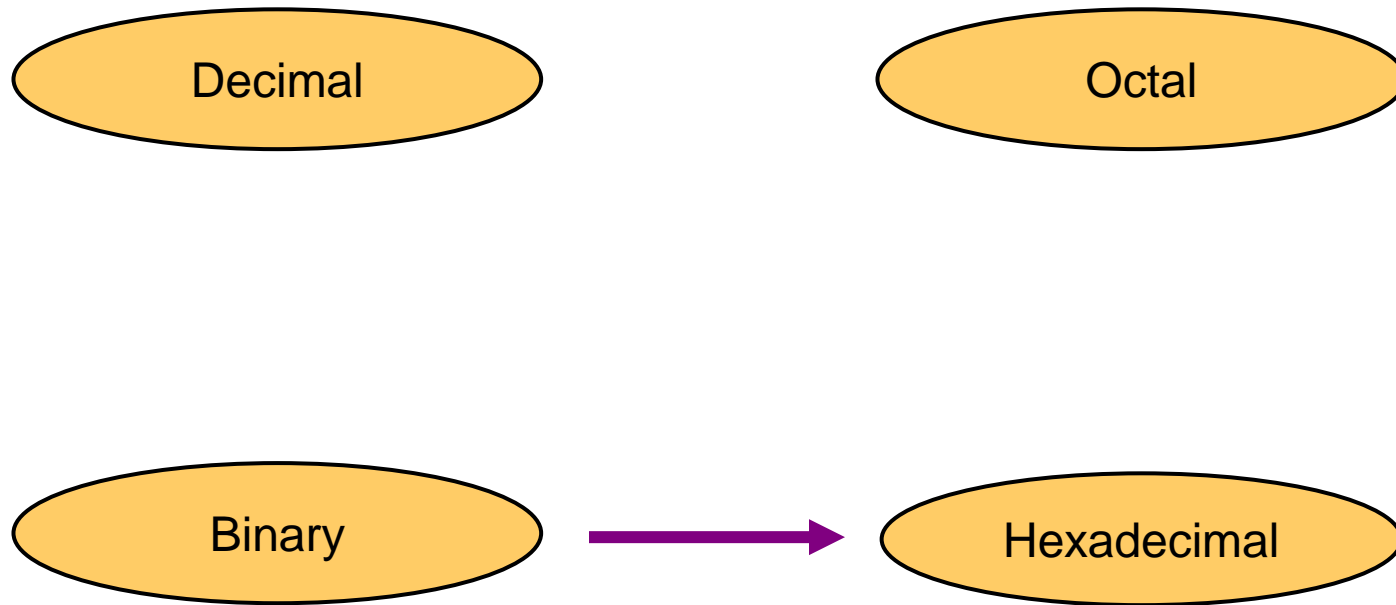
Binary to Hexadecimal

Technique

- Group bits in fours, starting on right
- Convert to hexadecimal digits

Example

$1010111011_2 = ?_{16}$

10   1011   1011

↓   ↓   ↓

2   B   B
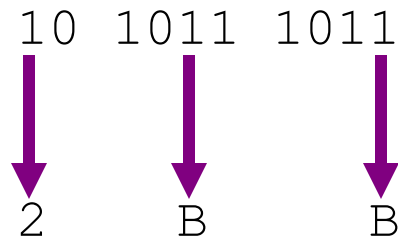
$1010111011_2 = 2BB_{16}$

Octal to Hexadecimal

Decimal

Octal

Binary
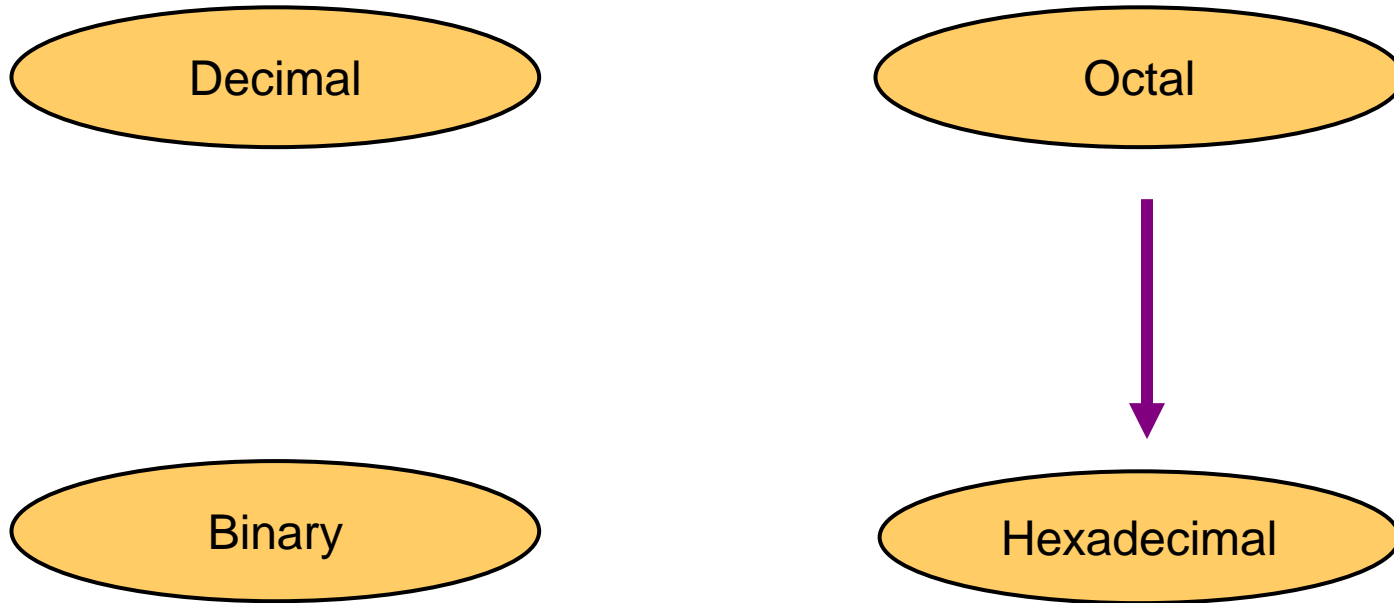
Hexadecimal

Octal to Hexadecimal

Technique

- Use binary as an intermediary

Example

$1076_8 = ?_{16}$

1	0	7	6
↓	↓	↓	↓
001	000	111	110

2 | 3 | E
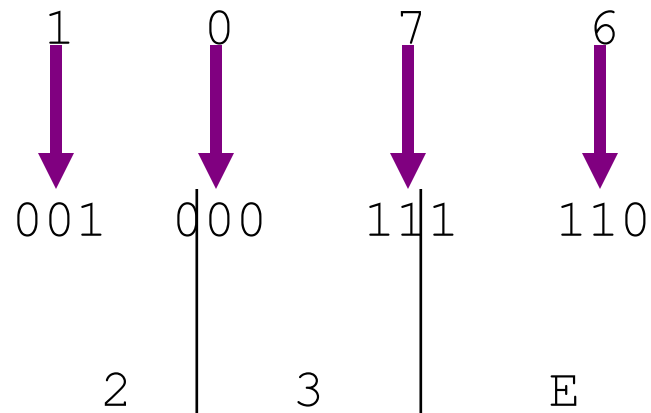
$1076_8 = 23E_{16}$

Hexadecimal to Octal
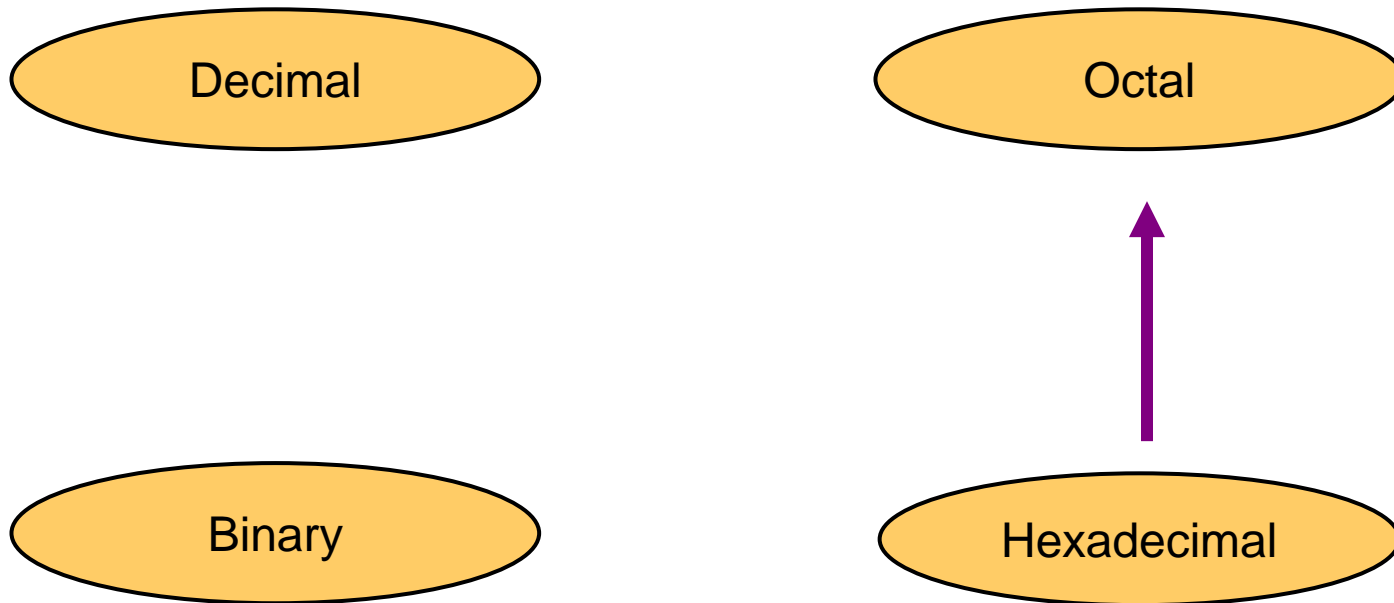
Hexadecimal to Octal

Technique
- Use binary as an intermediary

Example

$1F0C_{16} = ?_8$

1	F	0	C

$$0001 \quad 1111 \quad 0000 \quad 1100$$

$$1 \quad 7 \quad 4 \quad 1 \quad 4$$
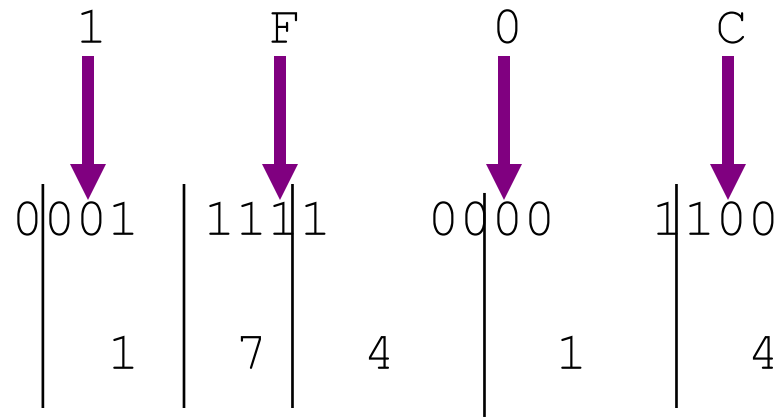
$1F0C_{16} = 17414_8$

Exercise – Convert …

Decimal	Binary	Octal	Hexa-decimal
33			
	1110101		
		703	
			1AF

Don't use a calculator!

Skip answer   Answer

Exercise – Convert …

Decimal	Binary	Octal	Hexa-decimal
33	100001	41	21
117	1110101	165	75
451	111000011	703	1C3
431	110101111	657	1AF

Common Powers (1 of 2)

Base 10

Power	Preface	Symbol	Value
10^{-12}	pico	p	.000000000001
10^{-9}	nano	n	.000000001
10^{-6}	micro	μ	.000001
10^{-3}	milli	m	.001
10^{3}	kilo	k	1000
10^{6}	mega	M	1000000
10^{9}	giga	G	1000000000
10^{12}	tera	T	1000000000000

Common Powers (2 of 2)

Base 2

Power	Preface	Symbol	Value
2^{10}	kilo	k	1024
2^{20}	mega	M	1048576
2^{30}	Giga	G	1073741824

- What is the value of "k", "M", and "G"?
- In computing, particularly w.r.t. <u>memory</u>, the base-2 interpretation generally applies

Example

(C:) Properties

General | Tools

Label: [ ]

Type: Local Disk

File system: FAT32

■ Used space: 1,977,475,072 bytes 1.84GB
■ Free space: 8,068,784,128 bytes 7.51GB

Drive C   Disk Cleanup...

OK   Cancel   Apply

Used space: 1,977,475,072 bytes   1.84GB
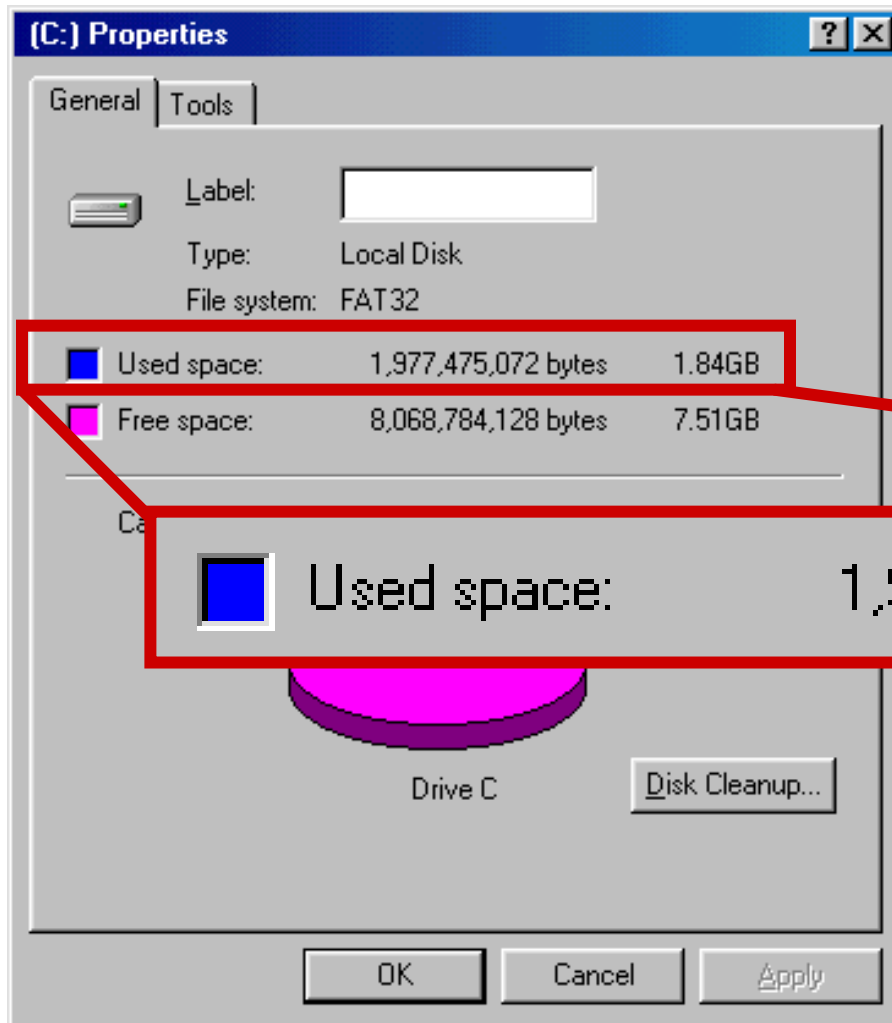
In the lab…
1. Double click on My Computer
2. Right click on C:
3. Click on Properties

$/ \ 2^{30} =$

Exercise – Free Space

Determine the "free space" on all drives on a machine in the lab

Drive	Free space	
	Bytes	GB
A:		
C:		
D:		
E:		
etc.		

Review – multiplying powers

For common bases, add powers

$$a^b \times a^c = a^{b+c}$$

$$2^6 \times 2^{10} = 2^{16} = 65,536$$

or…

$$2^6 \times 2^{10} = 64 \times 2^{10} = 64k$$

Two n-bit values

    Add individual bits

    Propagate carries

    E.g.,

```
  1       1
   10101        21
+  11001     +  25
 101110         46
```

Decimal (just for fun)

```
        35
    x  105
    ------
       175
       000
        35
    ------
      3675
```

Multiplication (2 of 3)

Binary, two 1-bit values

A	B	A × B
0	0	0
0	1	0
1	0	0
1	1	1

Multiplication (3 of 3)

Binary, two *n*-bit values

    As with decimal values

    E.g.,

```
        1110
   x    1011
        1110
       1110
      0000
     1110
   10011010
```

Decimal to decimal (just for fun)

$$3.14 =>$$

$$4 \times 10^{-2} = 0.04$$
$$1 \times 10^{-1} = 0.1$$
$$3 \times 10^{0} = 3$$
$$\overline{3.14}$$

Fractions

Binary to decimal

$$10.1011 \Rightarrow$$

$$1 \times 2^{-4} = 0.0625$$
$$1 \times 2^{-3} = 0.125$$
$$0 \times 2^{-2} = 0.0$$
$$1 \times 2^{-1} = 0.5$$
$$0 \times 2^{0} = 0.0$$
$$1 \times 2^{1} = 2.0$$
$$2.6875$$

Fractions

Decimal to binary

```
      .14579
   x       2
   0 .29158
   x       2
   0 .58316
   x       2
   1 .16632
   x       2
   0 .33264
   x       2
   0 .66528
   x       2
   1 .33056

   etc.
```

3.14579
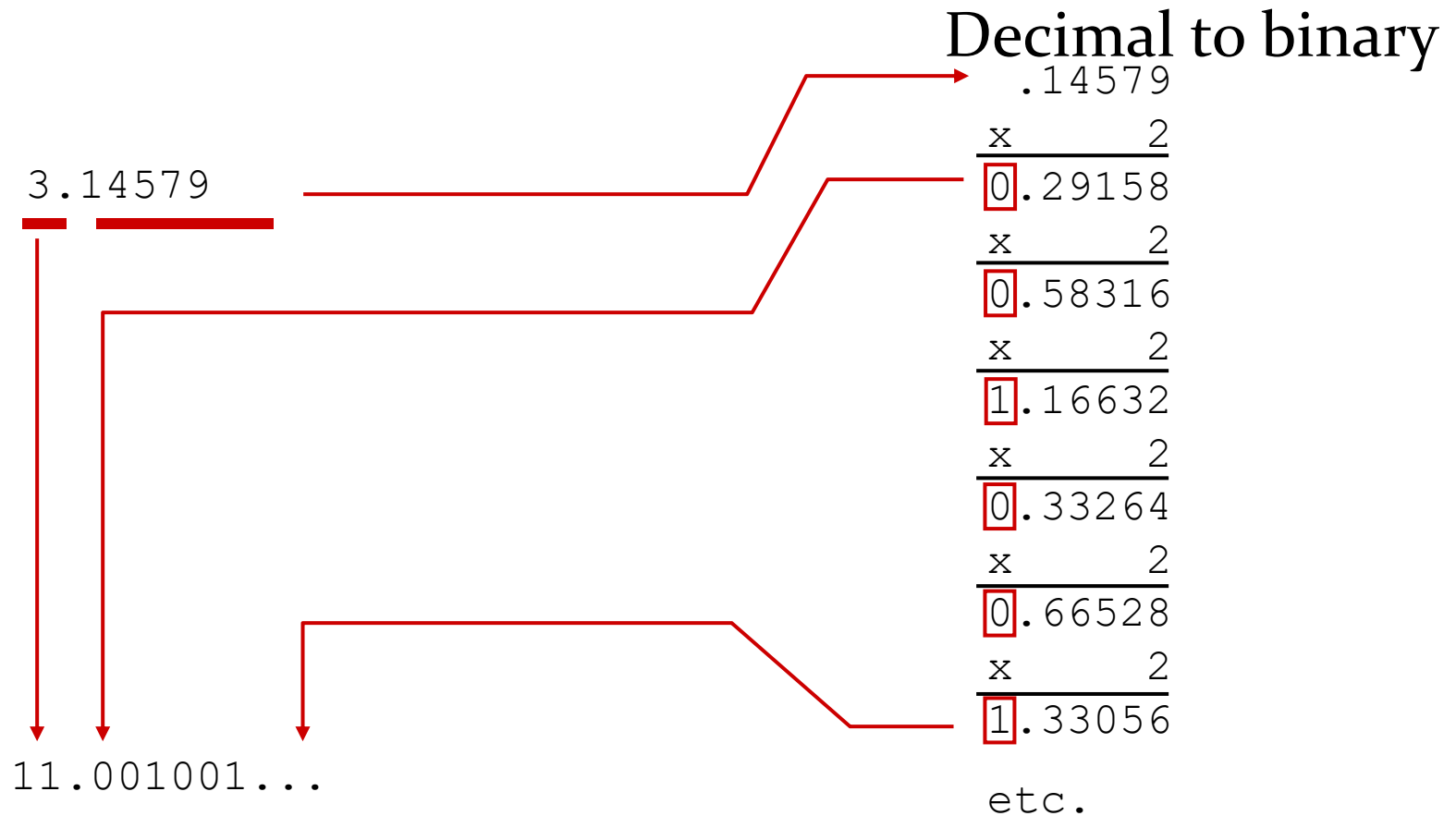
11.001001...

Exercise – Convert ...

Decimal	Binary	Octal	Hexa-decimal
29.8			
	101.1101		
		3.07	
			C.82

Don't use a calculator!

Answer

Exercise – Convert …

Answer

Decimal	Binary	Octal	Hexa-decimal
29.8	11101.110011…	35.63…	1D.CC…
5.8125	101.1101	5.64	5.D
3.109375	11.000111	3.07	3.1C
12.5078125	1100.10000010	14.404	C.82

☺

Thank You

Next topic