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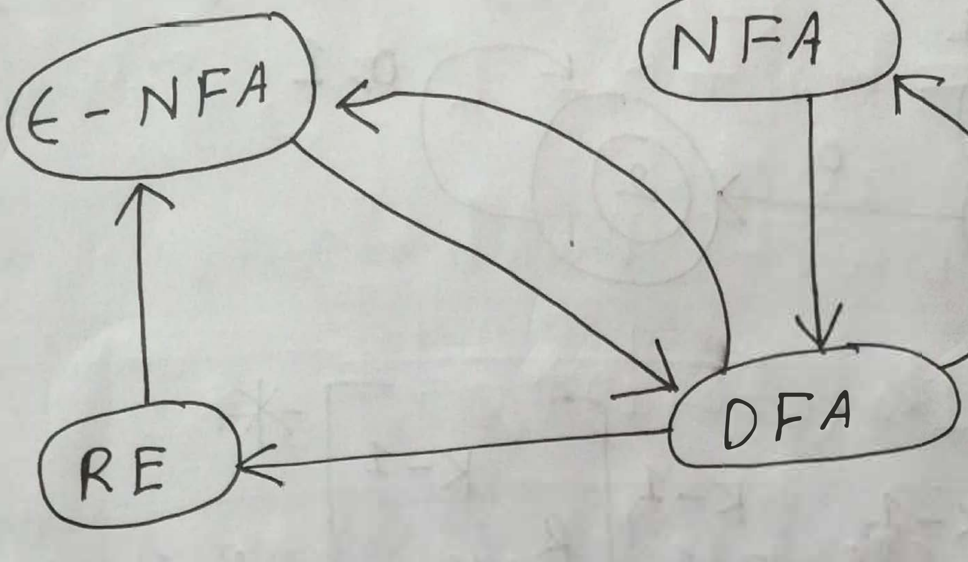
3.2 : Finite Automata & RE.



3.2.1 : DFA to RE

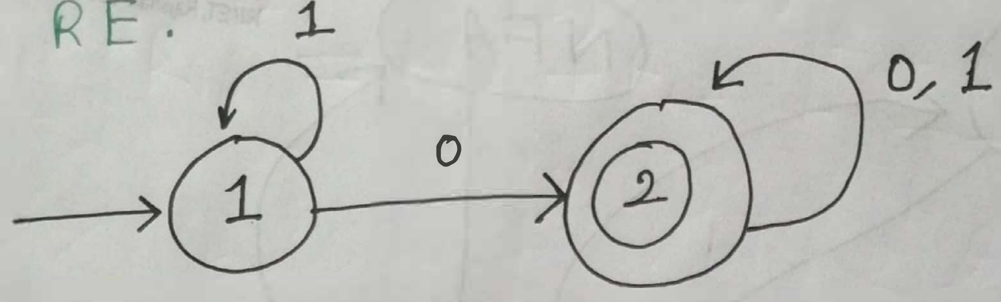
$$R_{ij}^{K} = R_{ij}^{K-1} + R_{iK}^{K-1}\left[R_{KK}^{(K-1)}\right]^{*} R_{Kj}$$

Let start at, $K=0$, and go through....

$K = n$.

n = total number of states.

Convert the following DFA to a RE.



Law :

$$R_{ij}^{K} = R_{ij}^{K-1} + R_{iK}^{K-1} \left[R_{KK}^{K-1} \right]^{*} R_{Kj}$$

Step-I : Calculate initial values.

	Expression
R_{11}^{0}	$\epsilon + 1$
R_{12}^{0}	0
R_{21}^{0}	$\emptyset$
R_{22}^{0}	$\epsilon + 0 + 1$

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Step-II : For $K = 1$

Law : $R_{ij}^{(1)} = R_{ij}^{(0)} + R_{i1}^{(0)} \left[R_{11}^{(0)} \right]^* R_{1j}^{(0)}$

	Direct substitution	simplified
$R_{11}^{(1)}$	$\epsilon + 1 + (\epsilon+1)(\epsilon+1)^*(\epsilon+1)$	1^*
$R_{12}^{(1)}$	$0 + (\epsilon+1)[\epsilon+1]^* 0$	$1^* 0$
$R_{21}^{(1)}$	$\phi + \phi(\epsilon+1)^*(\epsilon+1)$	ϕ
$R_{22}^{(1)}$	$\epsilon + 0 + 1 + \phi(\epsilon+1)^* 0$	$\epsilon + 0 + 1$

Some Hints:

(i) $(\epsilon + R)^* = R^*$

(ii) $(\epsilon + 1)^* = 1^*$

(iii) $(\epsilon + 1) 1^* = 1^*$

(iv) $\phi R = \phi$

(v) $\phi + R = R$

(vi) Example : $\epsilon + 1 + (\epsilon+1)(\epsilon+1)^*(\epsilon+1)$

$= \epsilon + 1 + \underbrace{(\epsilon+1) 1^*}(\epsilon+1) = \epsilon + 1 + 1^* + \epsilon + 1$

$= 1^* = \epsilon + 1 + 11 + \cdots$

Step- II : $K = 2$

Law: $R_{ij}^{(2)} = R_{ij}^{(1)} + R_{i2}^{(1)} \left[R_{22}^{(1)} \right]^* R_{2j}^{(1)}$

	By Direct Substitution	Simplified
$R_{11}^{(2)}$	$1^* + 1^* 0 (\epsilon + 0 + 1)^* \phi$	1^*
$R_{12}^{(2)}$	$1^* 0 + 1^* 0 (\epsilon + 0 + 1)^* (\epsilon + 0 + 1)$	$1^* 0 (0+1)^*$
$R_{21}^{(2)}$	$\phi + (\epsilon + 0 + 1)(\epsilon + 0 + 1)^* \phi$	ϕ
$R_{22}^{(2)}$	$\epsilon + 0 + 1 + (\epsilon + 0 + 1)(\epsilon + 0 + 1)^* (\epsilon + 0 + 1)$	$(0+1)^*$

So, start state $= 1$
Final state $= 2$

$\left\} \begin{aligned} K &= n \\ &= 2 \end{aligned} \right.$

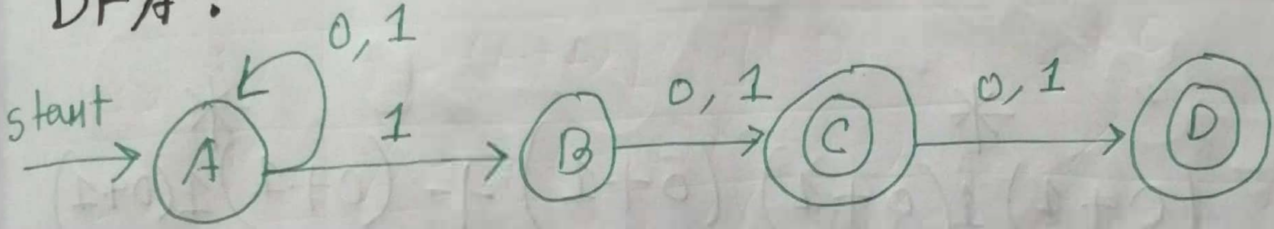
So, $RE = R_{12}^{(2)} = \underline{1^* 0 (0+1)^*}$

3.2.2 : DFA $\longrightarrow$ RE

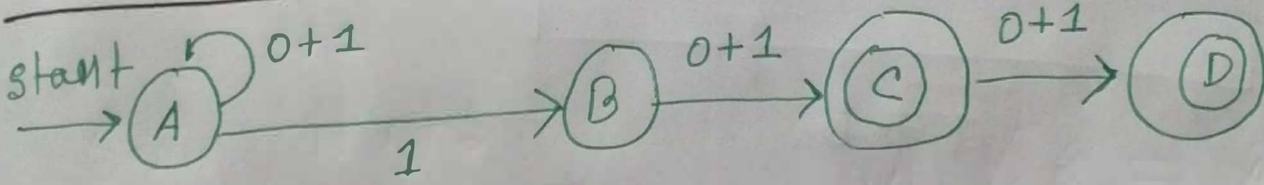
[State Elimination Method]

☐ Converting by ----

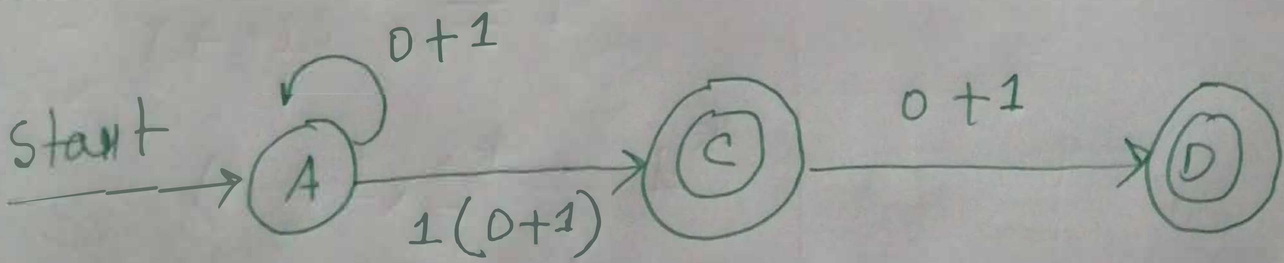
DFA :



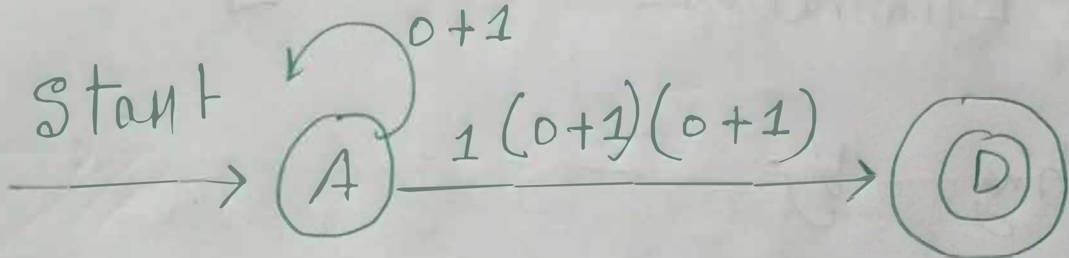
Step - I :



Step - II : Eliminate state - B



Step – III : Eliminate State - C



RE : $(0+1)^{*}1(0+1)(0+1) + (0+1)^{*}1(0+1)$

Convert RE ↘ ∈-NFA

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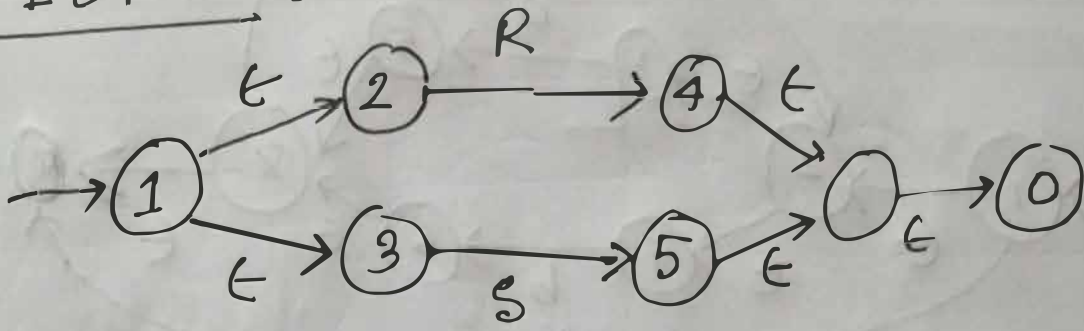
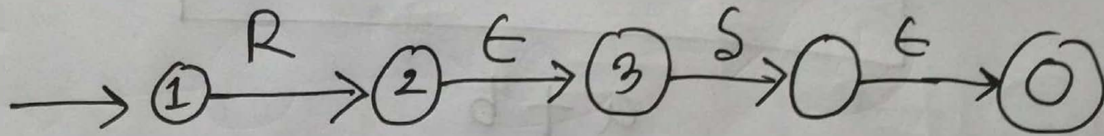
RE → ∈-NFA

INDUCTION :

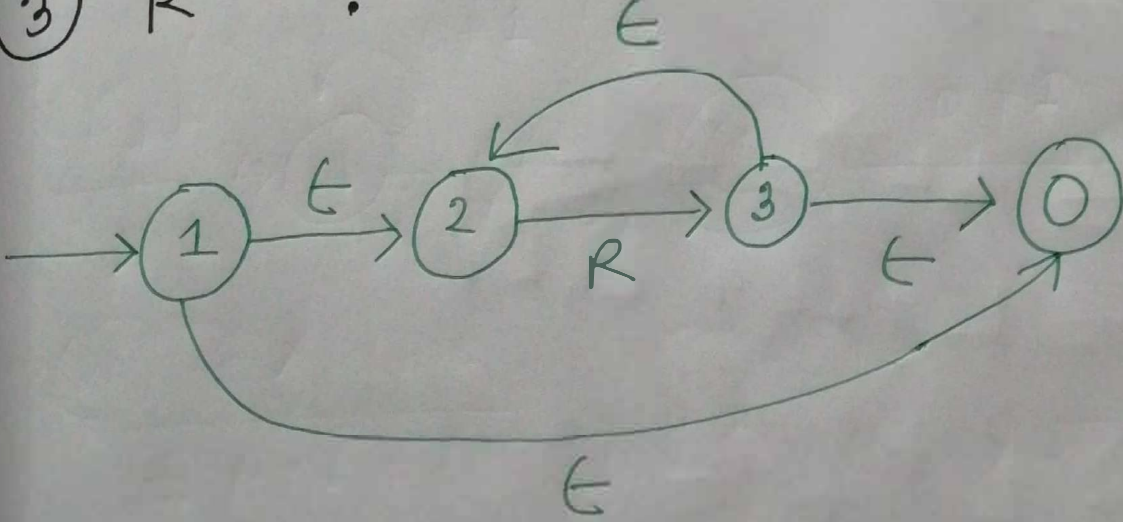
① R+S :



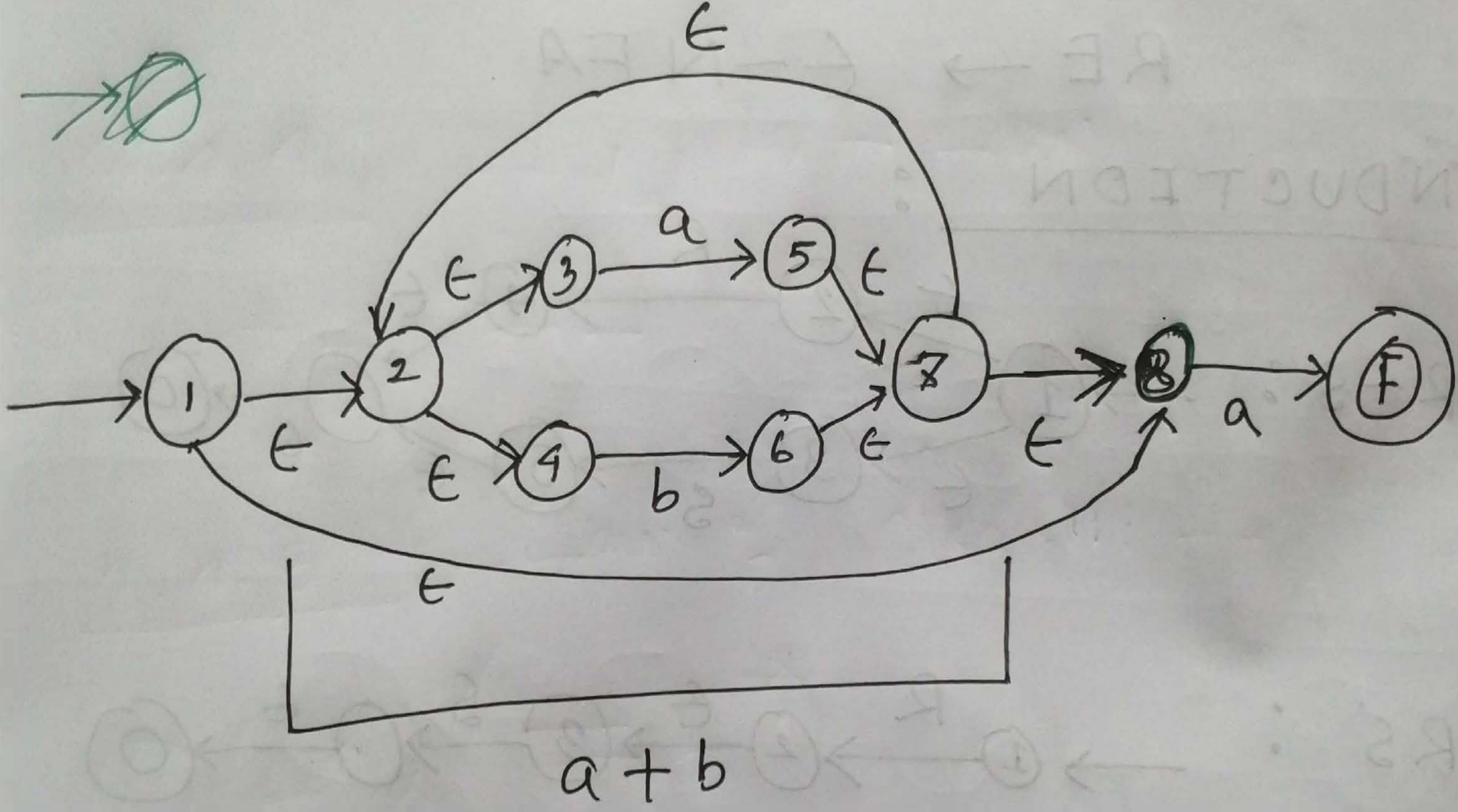
② RS :



③ R* :



Example: $(a+b)^* a$ [RE → ∈-NFA]



3.4.1 : Associativity & Commutativity

(i) Commutative law : $L+M + M+L$

(ii) Associative Law : $(L+M)+N$
$= L + (M+N)$

3.4.2 : Identities & Annihilators —

(i) $\phi + L = L + \phi = L$
↳ identity for union

(ii) $\epsilon R = R\epsilon = R$; identity for concatenation

(iii) $\phi L = L\phi = \phi$; ϕ is called annihilator for concatenation

3.4.3 Distribution laws

(i) $(M+N)L = ML + NL$

3.4.4 Idempotent Law

$L + L = L$

3.4.5 :- Law of closure —

(i) $(L^*)^* = L^*$

(ii) $\phi^* = \epsilon$

(iii) $\epsilon^* = \epsilon$

(iv) $L^+ = LL^* = L^*L$
$= L + LL + LLL + \text{----}$

(v) $L^* = L^+ + \epsilon$

(vi) $L? = \epsilon + L$