# Department of Mathematics and Natural Science

## MAT110: Differential Calculus and Co–ordinate Geometry

## Section ______, Quiz ______(Fall'22)

**Name (PRINT):** ________________________________ **ID:**____________________

**Time: 25 minutes** **Total Marks: 20**

**Problem 1** A function $f$ is said to be **continuous everywhere** if the following conditions are satisfied for **every value** of $x = c \in (-\infty, \infty)$

- $f(c)$ is defined.
- $\lim\limits_{x \to c} f(x)$ exists.
- $\lim\limits_{x \to c} f(x) = f(c)$

(a) Using the idea of above definitions, **find a value** of the **constant** $k$, if possible, that will make the function **continuous everywhere** and then **show the function in a graph**:

(i) $f(x) = \begin{cases} 9 - x^2, & x \geq -3 \\ k/x^2, & x < -3 \end{cases}$ (ii) $g(x) = \begin{cases} 2x + 1, & x \leq 1 \\ kx^2 & x > 1 \end{cases}$

(b) Find a **non-zero value** for the constant $k$ that makes $h(x) = \begin{cases} \dfrac{\tan kx}{x}, & x < 0 \\ 3x + 2k^2 & x \geq 0 \end{cases}$ continuous at $x = 0$

[(2 + 2) + (2 + 2) + 4 = 12]

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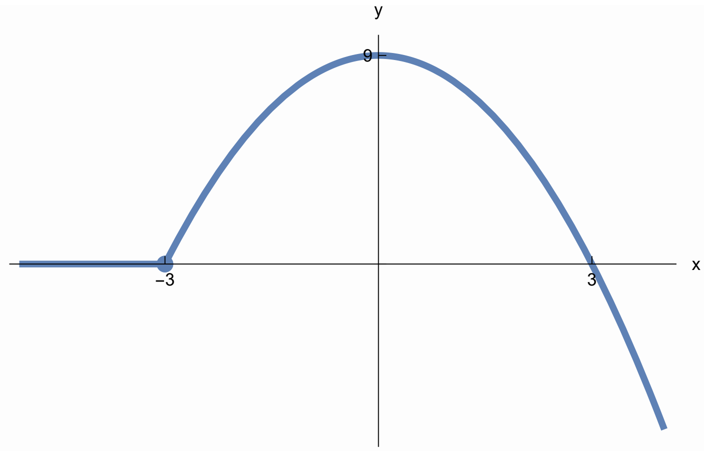
*Solution:* (a) (i) For continuity, we need

$$\lim_{x \to -3^-} f(x) = \lim_{x \to -3^+} f(x) = f(-3) = 9 - (-3)^2 = 0$$

$$\text{Or, } 0 = \frac{k}{(-3)^2}$$

$$\implies k = 0$$

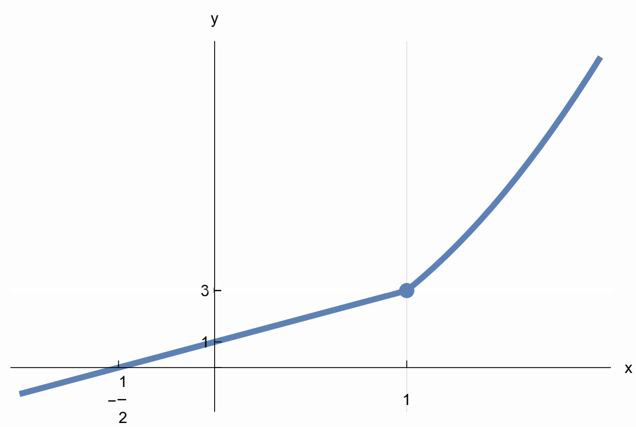
So, $f(x) = \begin{cases} 9 - x^2, & x \geq -3 \\ 0, & x < -3 \end{cases}$




(ii) For continuity, we need

$$\lim_{x\to 1^-} g(x) = \lim_{x\to 1^+} g(x) = g(1) = 2(1) + 1 = 3$$
$$\text{Or, } 3 = k(1)^2$$
$$\implies k = 3$$

So, $g(x) = \begin{cases} 2x+1, & x \le 1 \\ 3x^2, & x > 1 \end{cases}$




(b) For $h(x)$ to be continuous,

$$\lim_{x\to 0^-} h(x) = \lim_{x\to 0^+} h(x) = h(0) = 3(0) + 2k^2$$
$$\text{Or, } \lim_{x\to 0^-} \frac{\tan kx}{x} = 2k^2$$
$$\text{Or, } \lim_{x\to 0^-} \frac{k\tan kx}{kx} = 2k^2$$
$$\text{Or, } k = 2k^2 \qquad \left(\text{as } \lim_{x\to 0^-} \frac{\tan kx}{kx} = 1\right)$$
$$\text{Or, } 1 = 2k \qquad (\text{as } k \neq 0)$$
$$\implies k = 1/2$$

**Problem 2** Complete the statement for **The Squeezing Theorem**

*Solution:* Let $f(x), g(x),$ and $h(x)$ be functions satisfying

$$\boxed{g(x) \leq f(x) \leq h(x)}$$

for **all** $x$ in some **open interval containing the number** $c$, with the possible exception that the inequalities need not hold at $c$. If limit of $g(x)$ and $h(x)$ as $x$ approaches $c$, is given by,

$$\boxed{\lim_{x \to c} g(x) = \lim_{x \to c} h(x) = L}$$

then $f(x)$ also has this limit as $x$ approaches $c$, that is,

$$\boxed{\lim_{x \to c} f(x) = L}$$

[2+1+1 = 4]

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**Problem 3** Given that, $-1 \leq \sin\left(\frac{1}{x}\right) \leq 1$, prove **using the squeezing theorem** that

$$\lim_{x \to 0} x \sin\left(\frac{1}{x}\right) = 0$$

[4]

*Solution:* Given, $-1 \leq \sin\left(\frac{1}{x}\right) \leq 1$

for $x > 0$, $-x \leq \sin\left(\frac{1}{x}\right) \leq x$

for $x < 0$, $-x \geq \sin\left(\frac{1}{x}\right) \geq x$

combining both, $-|x| \leq \sin\left(\frac{1}{x}\right) \leq |x|$

Now, as $\lim_{x \to 0} |x| = \lim_{x \to 0} -|x| = 0$

So, by squeezing theorem, $\lim_{x \to 0} x \sin\left(\frac{1}{x}\right) = 0$