Department of Mathematics and Natural Science

MAT110: Differential Calculus and Co–ordinate Geometry

Section _____, Quiz _____(Fall'22)

Name (PRINT): _____ **ID:**_____

Time: 25 minutes **Total Marks: 27**

Problem 1

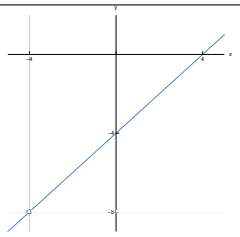
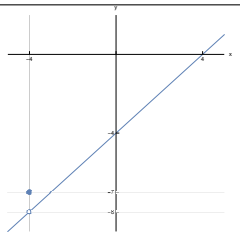
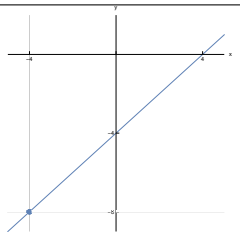
Complete the definition, then use it to answer the question.

a) A function f is said to be **continuous at x = c** provided the following conditions are satisfied:

1. $f(x)$ is defined at $x = c$
2. $\lim_{x \to c} f(x)$ exists.
3. $\lim_{x \to c} f(x) = f(c)$

Write in each case if the function is continuous or not. If not, which of the condition is hampered?

Draw the graph of the functions in the assigned space-

function	Continuous or not (Yes/No)	which condition was broken? (1/2/3/ No condition)	Graph
$f(x) = \dfrac{x^2-16}{x+4}$	No	1	
$g(x) = \begin{cases} \dfrac{x^2-16}{x+4}, & x \neq -4 \\ -7, & x = -4 \end{cases}$	No	3	
$h(x) = \begin{cases} \dfrac{x^2-16}{x+4}, & x \neq -4 \\ -8, & x = -4 \end{cases}$	Yes	no condition	

$[3 + 3 + 3 + 3 = 12]$

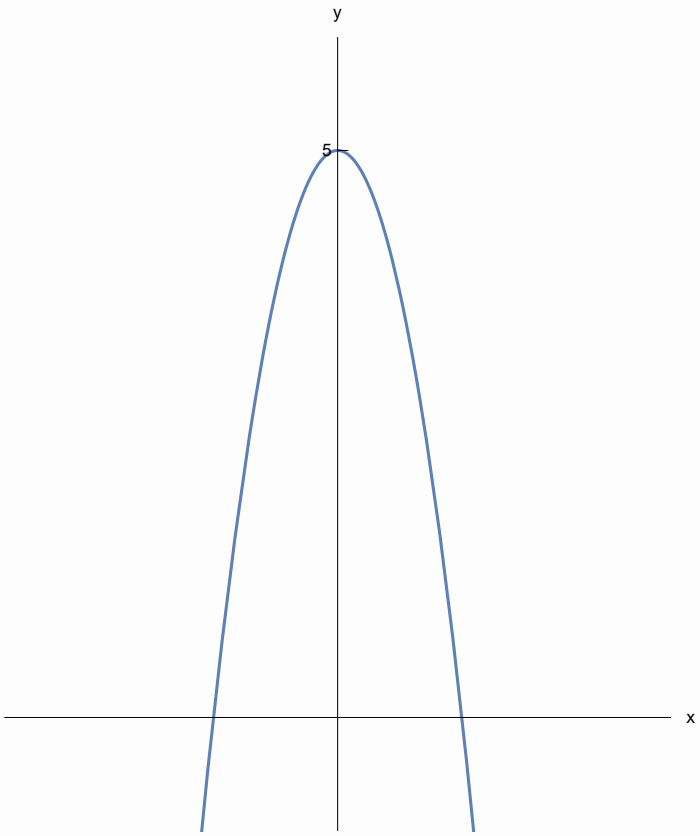
Problem 2
Answer the following:

a. Find the limit and plot the graph showing the end behaviour: for $\lim_{x\to-\infty} 5 - x^2$ [1+1 = 2]

b. Find the limit: $\lim_{x\to-\infty} \dfrac{e^x + e^{-x}}{e^x - e^{-x}}$ [3]

Solution: (a) $\lim_{x\to-\infty} 5 - x^2 = -\infty$

Graph:



(b)

$$\lim_{x\to-\infty} \frac{e^x + e^{-x}}{e^x - e^{-x}} = \lim_{x\to-\infty} \frac{e^{-x}\left(e^{2x} + 1\right)}{e^{-x}\left(e^{2x} - 1\right)}$$

$$= \lim_{x\to-\infty} \frac{\left(\cancel{e^{2x}}^{0} + 1\right)}{\left(\cancel{e^{2x}}^{0} - 1\right)}$$

$$= \frac{1}{-1} = -1$$

Problem 3

Let

$$g(x) = \begin{cases} x-3, & x < 0 \\ x^2 - 1, & 0 \le x \le 2 \\ 2x+1, & x > 2 \end{cases}$$

Find (a) $\lim_{x\to 0} g(x)$ (b) $\lim_{x\to 1} g(x)$ (c) $\lim_{x\to 2} g(x)$

and **plot the graph**. [7+3 = 10]

Solution: (a) $\lim_{x\to 0^-} g(x) = \lim_{x\to 0^-} (x-3) = -3$ $\lim_{x\to 0^+} g(x) = \lim_{x\to 0^+} (x^2-1) = -1$

$$\lim_{x\to 0^-} g(x) \neq \lim_{x\to 0^+} g(x)$$

$\therefore$ limit does not exist

(b) $\lim_{x\to 1} g(x) = \lim_{x\to 1}(x^2-1) = 1^2 - 1 = 0$

(c) $\lim_{x\to 2^-} g(x) = \lim_{x\to 1}(x^2-1) = 3$ $\lim_{x\to 2^+} g(x) = \lim_{x\to 2^+} (2x+1) = 5$

$$\lim_{x\to 2^-} g(x) \neq \lim_{x\to 2^+} g(x)$$

$\therefore$ limit does not exist

Grpah: