Department of Mathematics and Natural Science
MAT110: Differential Calculus and Co–ordinate Geometry
Section ______, Quiz ______(Fall'22)

Name (PRINT): ______________________________ **ID:**____________________

Time: 25 minutes **Total Marks: 20**

Problem 1

Find limit using factorization: $\lim_{t \to 1} \dfrac{t^3 + t^2 - 5t + 3}{t^3 - 3t + 2}$ [4]

Solution:

$$\begin{aligned}
&= \lim_{t \to 1} \frac{(t-1)(t^2+2t-3)}{(t-1)(t^2+t-2)} \\
&= \lim_{t \to 1} \frac{(t-1)(t-1)(t+3)}{(t-1)(t-1)(t+2)} \\
&= \lim_{t \to 1} \frac{(t+3)}{(t+2)} = \frac{1+3}{1+2} = \frac{4}{3}
\end{aligned}$$

Problem 2

$$f(x) = \frac{x^3 - 1}{x - 1}$$

[3 + 3 = 6]

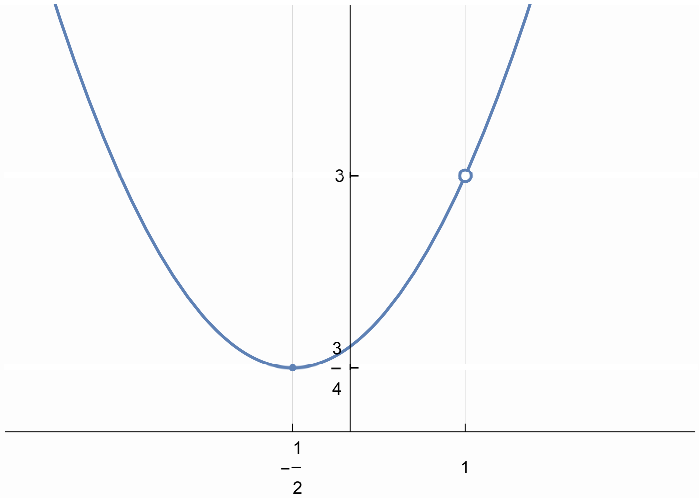
(a) Find $\lim_{x \to 1} f(x)$.
(b) Sketch the graph of $y = f(x)$

Solution:
(a)

$$\lim_{x \to 1} \frac{x^3 - 1}{x - 1}$$

$$\begin{aligned}
&= \lim_{x \to 1} \frac{(x-1)(x^2+x+1)}{x-1} \\
&= \lim_{x \to 1} (x^2+x+1) \\
&= 1 + 1 + 1 = 3
\end{aligned}$$

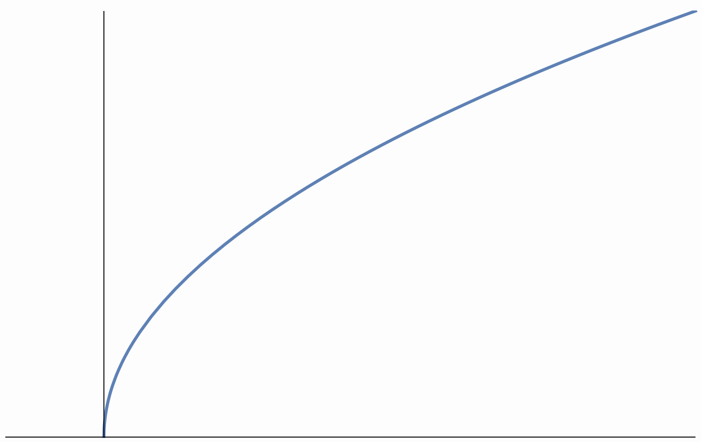
(b) $f(x)$ behaves like $(x^2 + x + 1)$ everywhere except at $x = 1$,
Now, $y = (x^2 + x + 1) = x^2 + 2\frac{1}{2}x + \left(\frac{1}{2}\right)^2 + \frac{3}{4} = \left(x + \frac{1}{2}\right)^2 + \frac{3}{4}$,
which is parabola $y = x^2$ but shifted $1/2$ unit left and $3/4$ unit up
or
which is upward concave parabola with axis parallel to y–axis, vertex at $(-1/2, 3/4)$



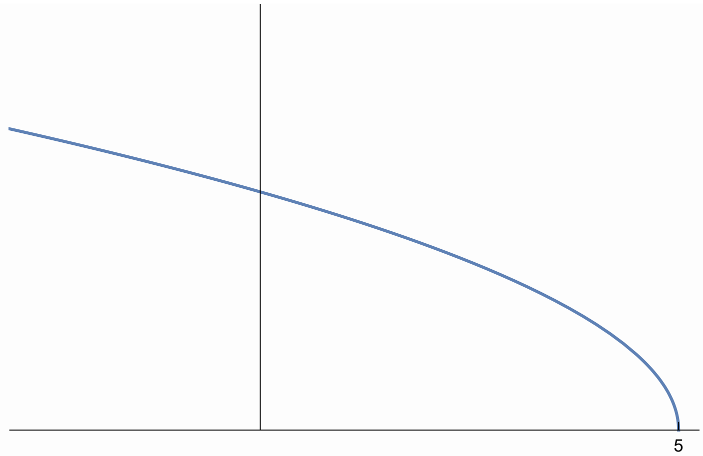
Problem 3
Mention the limit and **show the limit in a graph**:

(a) $\lim_{x\to\infty} \sqrt{x}$ (b) $\lim_{x\to-\infty} \sqrt{5-x}$ [2+2 = 4]

Solution:
(a) $\lim_{x\to\infty} \sqrt{x} = +\infty$



(b) $\lim_{x\to-\infty} \sqrt{5-x} = +\infty$



Problem 4

Let

$$f(x) = \begin{cases} \dfrac{x^2-4}{x+2}; & x \neq -2 \\ k; & x = -2 \end{cases}$$

(a) Find k so that $f(-2) = \lim_{x \to -2} f(x)$

(b) Sketch the function showing all significant points. Is the function continuous at $x = -2$?

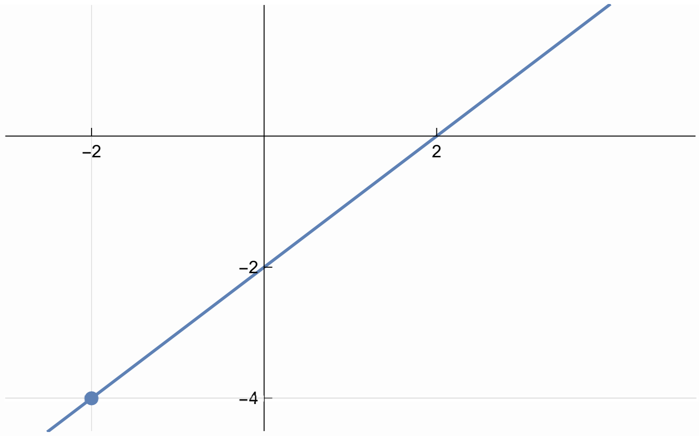
[3+3 = 6]

Solution:

(a) We need,

$$\begin{aligned} f(-2) &= \lim_{x \to -2} f(x) \\ \implies k &= \lim_{x \to -2} \frac{x^2-4}{x+2} \\ \text{or, } &= \lim_{x \to -2} \frac{(x-2)(x+2)}{x+2} \\ \text{or, } &= \lim_{x \to -2} (x-2) \\ \implies k &= -2-2 = -4 \end{aligned}$$

(b)



$f(x)$ is defined at $x = -2$, $\lim_{x \to -2} f(x)$ exists and $f(-2) = \lim_{x \to -2} f(x) = k = -4$. So the function is continuous at $x = -2$.