Department of Mathematics and Natural Science

MAT110: Differential Calculus and Co–ordinate Geometry

Section ______, Quiz ______(Fall'22)

Name (PRINT): ______________________________	**ID:**____________________

Time: 25 minutes **Total Marks: 20**

Problem 1

Find limits: (a) $\lim_{x\to-\infty} \frac{\sqrt{5x^2-2}}{x+3}$ (b) $\lim_{y\to+\infty} \frac{2-y}{\sqrt{7+6y^2}}$ [3 + 3 = 6]

Solution:

(a)

$$
\begin{aligned}
&\lim_{x\to-\infty} \frac{\sqrt{5x^2-2}}{x+3} \\
&= \lim_{x\to-\infty} \frac{\sqrt{x^2(5-2/x^2)}}{x(1+3/x)} \\
&= \lim_{x\to-\infty} \frac{|x|\sqrt{(5-2/x^2)}}{x(1+3/x)} \\
&= \lim_{x\to-\infty} \frac{|x|}{x} \times \lim_{x\to-\infty} \frac{\sqrt{(5-2/x^2)}}{(1+3/x)} \\
&= -1 \times \frac{\sqrt{5}}{1} = -\sqrt{5}
\end{aligned}
$$

(b)

$$
\begin{aligned}
&\lim_{y\to+\infty} \frac{y(2/y-1)}{\sqrt{y^2(7/y^2+6)}} \\
&= \lim_{y\to+\infty} \frac{y(2/y-1)}{|y|\sqrt{(7/y^2+6)}} \\
&= \lim_{y\to+\infty} \frac{y}{|y|} \times \lim_{\to+\infty} \frac{(2/y-1)}{\sqrt{(7/y^2+6)}} \\
&= 1 \times \frac{-1}{\sqrt{6}} = -\frac{1}{\sqrt{6}}
\end{aligned}
$$

Problem 2

Let

$$f(x) = \begin{cases} \dfrac{2x^2+5}{x^2-1}; & x < 0 \\ \dfrac{3-5x^3}{1+4x+x^3}; & x \geq 0 \end{cases}$$

Find $(a) \lim_{x \to -\infty} f(x)$ $(b) \lim_{x \to +\infty} f(x)$ [2 + 2 = 4]

Solution:

(a) When $x < 0$, $f(x) = \dfrac{2x^2+5}{x^2-1}$

$$\lim_{x \to -\infty} \frac{x^2(2+5/x^2)}{x^2(1-1/x^2)}$$
$$= \lim_{x \to -\infty} \frac{(2+5/x^2)}{(1-1/x^2)} = 2$$

(b) When $x > 0$, $f(x) = \dfrac{3-5x^3}{1+4x+x^3}$

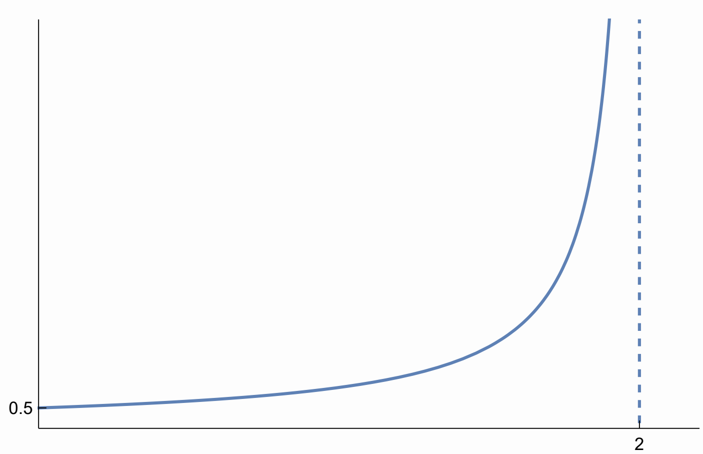
$$\lim_{x \to +\infty} \frac{x^3(3/x^3-5)}{x^3(1/x^3+4/x^2+1)}$$
$$\lim_{x \to +\infty} \frac{(3/x^3-5)}{(1/x^3+4/x^2+1)} = -5$$

Problem 3

Find the limit and **show the limit in a graph**: $\lim_{x \to 2^-} \dfrac{1}{|x-2|}$ [1+1 = 2]

Solution:

When $x \to 2^-$, $|x-2| \to 0^+$, so $\lim_{x \to 2^-} \dfrac{1}{|x-2|} = +\infty$



Problem 4 Let

$$g(x) = \begin{cases} \dfrac{1}{x+2}; & x \leq -2 \\ x^2 - 5; & -2 < x \leq 3 \\ x+1; & x > 3 \end{cases}$$

Find (a) $\lim\limits_{x\to -2} g(x)$ $\quad (b) \lim\limits_{x\to 0} g(x)$ $\quad (c) \lim\limits_{x\to 3} g(x)$

Plot the function $g(x)$ **in a graph** and **show the limits** at $x = -2, 0,$ and 3 [5+3 = 8]

Solution:
(a) As function changes on both sides of $x = -2$

$$\begin{aligned} LHL &= \lim_{x\to -2^-} \frac{1}{x+2} \\ &= -\infty \\ RHL &= \lim_{x\to -2^+} (x^2 - 5) \\ &= (-2)^2 - 5 = -1 \end{aligned}$$

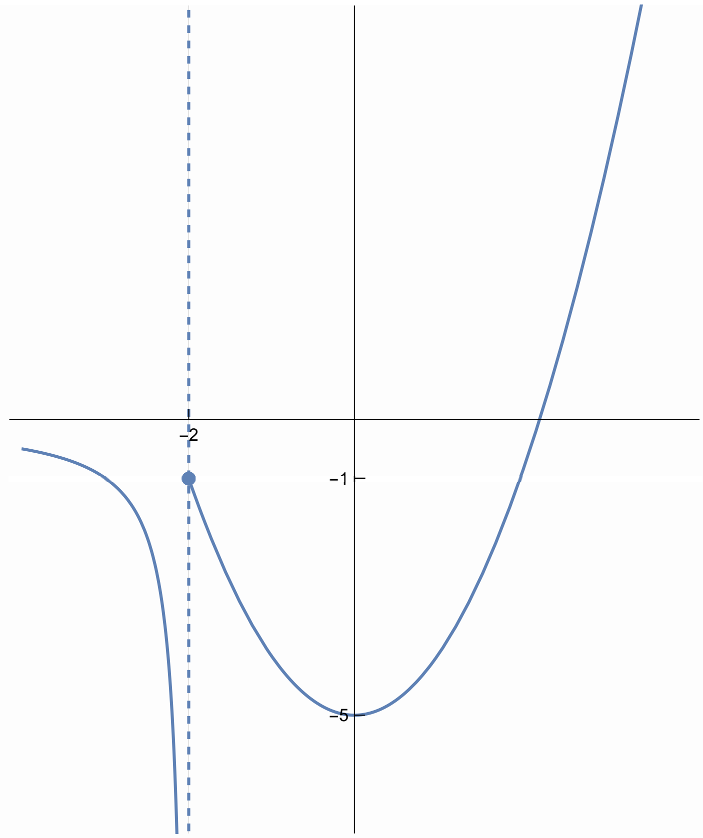
$LHL \neq RHL$, so limit does not exist, when $x \to -2$
(b) function remains same on both sides of $x = 0$

$$\begin{aligned} \lim_{x\to 0}(x^2 - 5) &= (0)^2 - 5 \\ &= -5 \end{aligned}$$

(c) As function changes on both sides of $x = 3$

$$\begin{aligned} LHL &= \lim_{x\to 3^-} (x^2 - 5) \\ &= (3)^2 - 5 = 4 \\ RHL &= \lim_{x\to 3^+} (x+1) = 3 + 1 = 4 \end{aligned}$$

$LHL = RHL$
so $\lim\limits_{x\to 3} g(x) = 4$



−2
−1
−5