Department of Mathematics and Natural Science

MAT110: Differential Calculus and Co–ordinate Geometry

Section ______, Quiz ______(Fall'22)

Name (PRINT): ______________________________ **ID:**____________________

Time: 25 minutes **Total Marks: 20**

Problem 1 A function f is said to be **continuous at** $x = c$ provided the following conditions are satisfied:

- $f(c)$ is defined.
- $\lim_{x\to c} f(x)$ exists.
- $\lim_{x\to c} f(x) = f(c)$

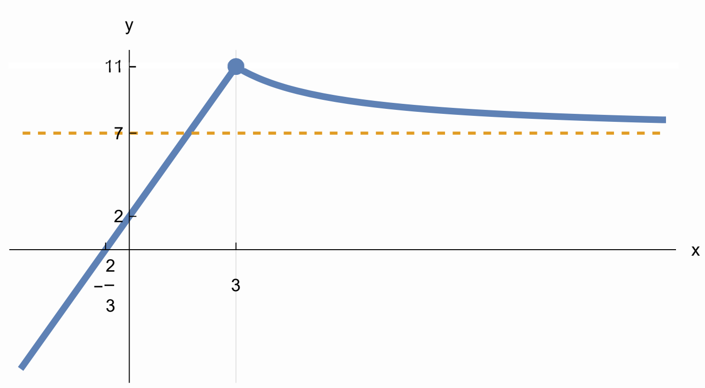
(a) Using the idea of above definition, **find values** of x, if any, at which function is **NOT continuous** and then **show the function and discontinuity**(if present) **in a graph**:

(i) $f(x) = \begin{cases} 3x+2, & x \le 3 \\ 7 + \dfrac{12}{x} & x > 3 \end{cases}$ (ii) $g(x) = \begin{cases} \dfrac{3}{x-1}, & x \ne 1 \\ 3 & x = 1 \end{cases}$

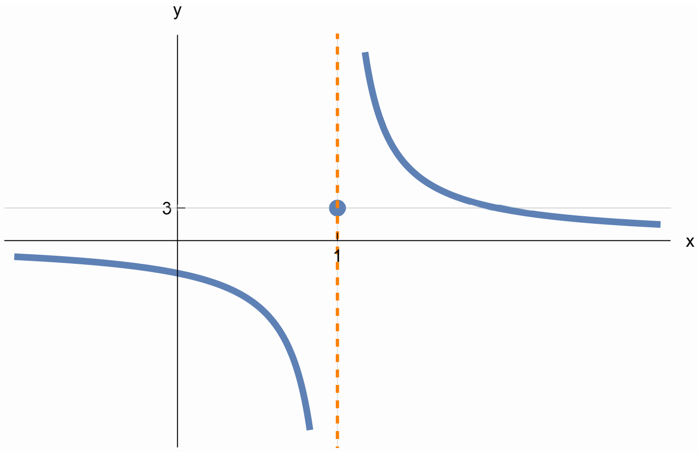
(b) Find a **non-zero value** for the constant k that makes $h(x) = \begin{cases} \dfrac{\sin kx}{x}, & x < 0 \\ 3x + 2k^2 & x \ge 0 \end{cases}$ continuous at $x = 0$

[(1 + 2) + (1 + 2) + 4 = 10]

Solution: (a) (i) Continuous for all x.



(ii) Discontinuous at $x = 1$



(b) For $h(x)$ to be continuous,

$$\lim_{x\to 0^-} h(x) = \lim_{x\to 0^+} h(x) = h(0) = 3(0) + 2k^2$$

$$\text{Or, } \lim_{x\to 0^-} \frac{\sin(kx)}{x} = 2k^2$$

$$\text{Or, } \lim_{x\to 0^-} \frac{k\sin(kx)}{kx} = 2k^2$$

$$\text{Or, } k = 2k^2 \qquad \left(\text{as } \lim_{x\to 0^-} \frac{\sin(kx)}{kx} = 1\right)$$

$$\text{Or, } 1 = 2k \qquad (\text{as } k \neq 0)$$

$$\implies k = 1/2$$

Problem 2 Complete the statement for **The Squeezing Theorem**

Solution: Let $f(x), g(x),$ and $h(x)$ be functions satisfying

$$\boxed{g(x) \leq f(x) \leq h(x)}$$

for **all** x in some **open interval containing the number** c, with the possible exception that the inequalities need not hold at c. If limit of $g(x)$ and $h(x)$ as x approaches c, is given by,

$$\boxed{\lim_{x\to c} g(x) = \lim_{x\to c} h(x) = L}$$

then $f(x)$ also has this limit as x approaches c, that is,

$$\boxed{\lim_{x\to c} f(x) = L}$$

[2+1+1 = 4]

Problem 3 Find the limits

$$\text{(a)}\ \lim_{x\to 0} \frac{\tan(3x^2) + \sin^2(5x)}{x^2} \qquad\qquad \text{(b)}\ \lim_{x\to 0} \frac{x^2 - 3\sin(x)}{x}$$

[3 + 3 = 6]

(a)

$$\begin{aligned}
\lim_{x\to 0} \frac{\tan(3x^2) + \sin^2(5x)}{x^2} &= \lim_{x\to 0} \frac{\tan(3x^2)}{x^2} + \lim_{x\to 0} \frac{\sin^2(5x)}{x^2} \\
&= 3 \lim_{x\to 0} \frac{\tan(3x^2)}{3x^2} + 5^2 \lim_{x\to 0} \left(\frac{\sin(5x)}{5x}\right)^2 \\
&= 3 + 25 = 28
\end{aligned}$$

(b)

$$\begin{aligned}
\lim_{x\to 0} \frac{x^2 - 3\sin(x)}{x} &= \lim_{x\to 0} \frac{x^2}{x} - \lim_{x\to 0} \frac{3\sin x}{x} \\
&= \lim_{x\to 0} x - 3 \lim_{x\to 0} \frac{\sin x}{x} \\
&= 0 - 3 = -3
\end{aligned}$$