Department of Mathematics and Natural Science

MAT110: Differential Calculus and Co–ordinate Geometry

Section ______, Quiz ______(Fall'22)

Name (PRINT): ______________________________________ **ID:**______________________

Time: 25 minutes **Total Marks: 27**

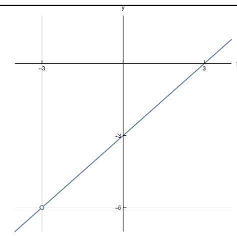
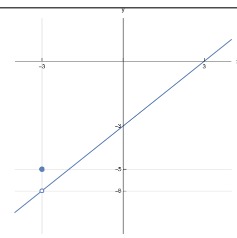
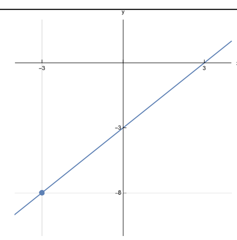
Problem 1 Complete the definition, then use it to answer the question.

a) A function f is said to be **continuous at** $\mathbf{x} = \mathbf{c}$ provided the following conditions are satisfied:

1. $\underline{f(x) \text{ is defined at } x = c}$
2. $\underline{\lim_{x\to c} f(x) \text{ exists.}}$
3. $\underline{\lim_{x\to c} f(x) = f(c)}$

Write in each case if the function is continuous or not. If not, which of the condition is hampered?

Draw the graph of the functions in the assigned space-

function	Continuous or not (Yes/No)	which condition was broken? (1/2/3/ No condition)	Graph
$f(x) = \dfrac{x^2-9}{x+3}$	No	1	
$g(x) = \begin{cases} \dfrac{x^2-9}{x+3}, & x \neq -3 \\ -5, & x = -3 \end{cases}$	No	3	
$h(x) = \begin{cases} \dfrac{x^2-9}{x+3}, & x \neq -3 \\ -6, & x = -3 \end{cases}$	Yes	no condition	

[3 + 3 + 3 + 3 = 12]

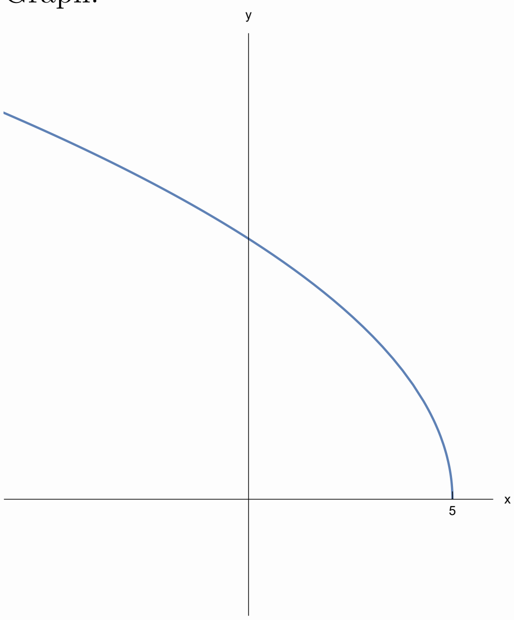
Problem 2 Answer the following:

a. Find the limit and plot the graph showing the end behaviour: for $\lim\limits_{x\to-\infty}\sqrt{5-x}$ [1+1 = 2]

b. Find the limit: $\lim\limits_{x\to+\infty}\left(\sqrt{x^2-3x}-x\right)$ [3]

Solution: (a) $\lim\limits_{x\to-\infty}\sqrt{5-x}=+\infty$

Graph:



(b)

$$\begin{aligned}
\lim_{x\to+\infty}\left(\sqrt{x^2-3x}-x\right) &= \lim_{x\to+\infty}\left(\sqrt{x^2-3x}-x\right)\times\frac{\left(\sqrt{x^2-3x}+x\right)}{\left(\sqrt{x^2-3x}+x\right)}\\
&= \lim_{x\to+\infty}\frac{x^2-3x-x^2}{\left(\sqrt{x^2-3x}+x\right)}\\
&= \lim_{x\to+\infty}\frac{-3x}{\left(\sqrt{x^2\left(1-\frac{3}{x}\right)}+x\right)}\\
&= \lim_{x\to+\infty}\frac{-3x}{\left(x\sqrt{\left(1-\frac{3}{x}\right)}+x\right)}\\
&= \lim_{x\to+\infty}\frac{-3x}{x\left(\sqrt{\left(1-\frac{3}{x}\right)}+1\right)}\\
&= \lim_{x\to+\infty}\frac{-3}{\left(\sqrt{\left(1-\overset{0}{\cancel{\frac{3}{x}}}\right)}+1\right)}=-\frac{3}{2}
\end{aligned}$$

Problem 3 Let

$$g(t) = \begin{cases} t-2, & t<0 \\ t^2, & 0 \le t \le 2 \\ 2t, & t>2 \end{cases}$$

Find (a) $\lim_{t\to 0} g(t)$ (b) $\lim_{t\to 1} g(t)$ (c) $\lim_{t\to 2} g(t)$

and **plot the graph**. [7+3 = 10]

Solution: (a) $\lim_{t\to 0^-} g(t) = \lim_{t\to 0^-}(t-2) = -2$ $\lim_{t\to 0^+} g(t) = \lim_{t\to 0^+} t^2 = 0$

$$\lim_{t\to 0^-} g(t) \neq \lim_{t\to 0^+} g(t)$$

$\therefore$ limit does not exist

(b) $\lim_{t\to 1} g(t) = \lim_{t\to 1} t^2 = 1^2 = 1$

(c) $\lim_{t\to 2^-} g(t) = \lim_{t\to 2^-} t^2 = 4$ $\lim_{t\to 2^+} g(t) = \lim_{t\to 2^+} 2t = 4$

$$\lim_{t\to 2^-} g(t) = \lim_{t\to 2^+} g(t) = \lim_{t\to 2} g(t) = 4$$

Grpah: