Comparative analysis of algorithms of the beamforming: State of the art and challenges

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Project Work

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ABSTRACT

A large amount of data and the high mobility of users significantly impact network infrastructure. From this point of view, 5G and beyond (B5G) wireless communication networks are expected to operate in millimeter-wave (mm-Wave) spectrum regions to deliver several gigabits per second (M-Gbps) data to end users. Intelligent reflective surfaces (IRS) is wireless communication technology that uses a wide variety of low-cost electromagnetic "mirrors" to guide incident radio waves toward the intended receiver, thereby increasing the spectral efficiency, energy efficiency, and reliability of wireless transmission. Optimization is a fundamental aspect of IRS beamforming. In this context, beamforming is the name given to various array processing algorithms that focus or orient the array in a specific direction. Beamforming techniques are used to improve the directivity and focus of the array without having to alter it physically. This work will compare state-of-the-art beamforming algorithms identifying the challenges and research opportunities in the area.

TABLE OF CONTENTS

Chapter 1

INTRODUCTION

Beamforming is a signal processing technique that uses an antenna array to direct radio waves towards a specific receiver in order to improve wireless data transmission. This improves signal quality, decreases interference, and boosts the signal-to-noise ratio. The basic idea behind beamforming is to manipulate a signal's directionality in order to improve transmission or reception in a specific direction. Beamforming employs antenna arrays to selectively amplify or attenuate signals coming from various directions.

Beamforming algorithms can be categorized into two main types: passive and active. Passive beamforming uses a fixed antenna array and uses differences in phase and amplitude to steer the beam. With active beamforming, the element weights of each element are adjusted to optimize the beam's shape and direction. In contrast to passive beamforming, active beamforming is more flexible and can adapt to changes in the environment or receiver location [6].

This project work involves the simulation of three existing beamforming algorithms.. The MVDR beamformer algorithm minimizes the variance of the beamformer output while keeping the response in the desired direction as distortion-free as possible. By locating subspaces corresponding to the signal and noise spaces, the MUSIC algorithm estimates the direction of arrival of multiple signals in a noisy environment [10]. The simplest and most widely used algorithm is ESPRIT (Estimation of Signal Parameters via Rotational Invariance Techniques), which involves signals received at different antennas to align them in phase and then calculating them to form a beam.

In wireless communication systems, the Minimum Variance Distortionless Response (MVDR) beamformer algorithm is a popular method for adaptive beamforming. The MVDR algorithm modifies the weights of an antenna array in order to reduce the variance of the received signal while maintaining a distortionless response in the desired direction. This method has been shown to provide significant improvements in signal-to-interference-plus-noise ratio (SINR) compared to traditional beamforming methods in scenarios with multiple sources of interference. The MVDR algorithm's adaptability for use in a broad range of wireless communication applications is still being improved through ongoing research [7].

Multiple Signal Classification (MUSIC) is also a common approach for estimating direction-of-arrival (DoA) in array processing applications. This approach estimates the quantity and direction of incoming signals with excellent accuracy by using the eigendecomposition of the covariance matrix of the received signal. MUSIC is especially beneficial in situations involving closely spaced sources, when typical beamforming approaches may fail. The approach has also been enhanced to work with nonuniform arrays and time-varying circumstances. Unfortunately, the MUSIC algorithm's computing complexity might be significant, limiting its practical use in real-time applications [4].

Esprit (Estimation of Signal Parameters via Rotational Invariance Techniques), is another signal processing algorithm that estimates the frequencies and directions of incoming signals in antenna arrays. It achieves this by exploiting the rotational invariance of the covariance matrix of the received signal. Esprit can be used for direction-of-arrival estimation and beamforming in wireless communication system and has been shown to outperform other traditional methods in certain scenarios. The algorithm is computationally efficient and has low complexity, making it suitable for real-time applications. However, like other beamforming algorithms, it also has its limitations, and ongoing research is focused on enhancing its adaptability and accuracy in challenging environments [9]. For shortcoming, numerous improvements to the algorithm have been developed, including the generalized sidelobe canceler (GSC) and adaptive beamforming approaches.

1.1 Objectives

Main objective

The primary goal of this research is to evaluate and examine the performance of three distinct beamforming algorithms in intelligent reflecting surface (IRS) systems: Minimal Variance Distortionless Response (MVDR), Multiple Signal Classification (MUSIC) and ESPRIT. These techniques are commonly used in array signal processing to optimize beam direction and shape, increase signal quality, and decrease interference.

The three beamforming algorithms will be simulated and evaluated under different scenarios in order to accomplish this objective. These scenarios include varying the number of antennas, the direction of arrival of the signal, and the level of interference. The results of this analysis will help identify the strengths and weaknesses of each algorithm and provide insights into the design and optimization of beamforming algorithms in IRS systems.

Intermediate objectives

1. This project involves developing a simulation framework for beamforming algorithms in C++. The framework allows for extensive evaluation of algorithms under various scenarios and conditions. In order to handle large datasets and complex simulations, the simulation tool was developed using the C++ programming language. Furthermore, the tool is designed to be modular, allowing for easy integration of different beamforming algorithms and scenarios.

2. For the simulation tool, MATLAB is used to design and implement simulation models for evaluating the performance of the three beamforming algorithms. The simulation models will generate synthetic data that will be used to evaluate the performance of the beamforming algorithms in different scenarios. For example, varying the number of antennas, the direction of arrival of the signal, and the level of interference. Also, the implementation language in MATLAB is mostly C++. The MATLAB simulations also help to identify potential limitations and areas for improvement in the beamforming algorithms.

1.2 Overview

This research project focuses on comparing the performance of three alternative beamforming algorithms in intelligent reflecting surface (IRS) systems. Minimum Variance Distortionless Response (MVDR), Multiple Signal Classification (MUSIC), and ESPRIT are the three algorithms. The goal is to evaluate and analyze each algorithm's strengths and drawbacks in various scenarios. The scenarios include changes in the number of antennas, direction of arrival of the signal, and level of interference. The aim is to provide insights into the design and optimization of beamforming algorithms in IRS systems.

This project work will also make use of MATLAB, a powerful simulation tool for creating, simulating, and analyzing communication networks. The simulation models will be designed and implemented in MATLAB to evaluate the performance of the three beamforming techniques in IRS systems. These simulation models will generate synthetic data sets to test the beamforming algorithms under various circumstances, such as changing the number of antennas, the direction of the signal's arrival, and the quantity of interference. The findings of this analysis will assist in identifying potential limitations and areas for improvement in beamforming algorithms, as well as providing insights into the design and optimization of beamforming algorithms in IRS systems. Furthermore, the C++ programming language will be utilized to build the beamforming algorithms on real-time hardware, providing a greater understanding of the techniques.

Chapter 2

BACKGROUND

2.1 DoA for Beamforming

DoA estimation is a critical component of beamforming, which is the function of integrating signals from many sensors, such as antennas or microphones, to improve reception or transmission of a desired signal while reducing interference and noise. Beamforming has a wide range of applications, including wireless communications, radar, sonar, and audio signal processing [17]. Accurate DoA estimation is critical for optimizing beamforming algorithm performance and improving overall system efficiency.

Traditional DoA estimate methods can be divided into two types: subspace-based methods and non-subspace-based methods. Subspace-based approaches, such as the Multiple Signal Classification (MUSIC) algorithm [14] and the Estimation of Signal Parameters through Rotational Invariance Techniques (ESPRIT) algorithm [13], estimate the DoA by taking advantage of the orthogonality between the signal and noise subspaces. These approaches provide high-resolution DoA estimates but are computationally demanding and susceptible to model mismatch and array calibration issues. Also, Non-subspace-based methods, such as Capon's Minimum Variance Distortionless Response (MVDR) beamformer, rely on the spatial covariance matrix of the received signals to estimate the DoA.

2.2 Beamforming

Beamforming is a spatial filtering technique that has received a lot of interest in recent years due to its efficiency in improving the signal quality and coverage area of wireless communication systems. Several studies have been conducted in this regard to investigate the performance of various beamforming algorithms in various applications. The publication [18] by Wallace and Simon provides a complete overview of beamforming techniques and applications, including linear, nonlinear, adaptive, and blind algorithms. On the other hand, Litva, John and Lo,Titus K book [8] provides a full introduction to digital beamforming algorithms and their applications in wireless communication systems, covering several types of beamforming techniques and their optimization. Furthermore, Roy and Kailath's publication, [13] includes a full discussion of beamforming algorithms for sensor arrays, including esprit, minimum-variance, and subspace-based approaches, as well as their optimization.

For the algorithm like Minimal Variance Distortionless Response (MVDR) algorithm, is one of the most extensively used beamforming techniques. Van Trees provides a full understanding of the MVDR algorithm in his book "Optimum array processing: Part IV of detection, estimation, and modulation theory", the technique provides good signal separation and noise suppression performance by balancing beam width and side-lobe level [17]. Furthermore, Li et al. describe a robust version of the MVDR algorithm in their paper "Robust adaptive beamforming" that improves performance in the presence of array flaws such as direction-of-arrival (DoA) faults [3].

The Multiple Signal Classification (MUSIC) technique, first presented by Schmidt in 1986, is an effective beamforming approach based on the eigen-decomposition of the signal correlation matrix. Even in low signal-to-noise ratio circumstances, it accurately calculates the DoAs of numerous signals [14]. The approach first computes the noise subspace using the eigenvectors corresponding to the correlation matrix's smallest eigenvalues, and then computes the signal subspace using the eigenvectors corresponding to the biggest eigenvalues. The DoAs are calculated by projecting the array manifold onto the signal subspace and locating the peaks in the resulting spectrum. Stoica and Moses' book "Introduction to spectral analysis" provides a full introduction to the fundamentals of spectral analysis, which are essential for understanding the MUSIC algorithm [16].

Esprit is also a widely used beamforming technique that has its roots in the field of signal processing. Initially proposed for direction-of-arrival (DoA) estimation, ESPRIT has been extended to various applications such as radar, wireless communications, and acoustic signal processing. Despite its popularity, ESPRIT also faces challenges in the presence of correlated noise [15]. To address this limitation, researchers have developed an adaptive version of ESPRIT that leverages the signal and noise statistics to adjust the weights of each element in the array. This approach has been shown to significantly improve beamforming performance in noisy settings [5]. This algorithm adaptively weights the contributions of each element in the array, enhancing beamforming performance in noisy settings.

Chapter 3

PRACTICAL STUDY CASE

In this project work, the aim is to perform a comparative analysis of the state-of-the-art beamforming algorithms, including MVDR, MUSIC and ESPRIT where all the algorithms will be simulated and compare to each other. The simulated environment will include multiple signals with varying levels of signal-to-noise ratio (SNR), interference, noise level and various other parameter. Also to evaluate the performance of each algorithm, we will use metrics such as spatial spectrum efficiency, computational complexity and many others. In addition, in this project work it examine the algorithms' performance in the presence of spatially correlated noise and non-uniform arrays.

Direction of Arrival (DoA) estimation is a fundamental component of beamforming, facilitating enhanced signal processing in various applications. The process of estimating DoA entails examining the phase and amplitude changes between signals received by an array of antennas. The received signals are preprocessed to reduce noise and mistakes. The correlation between the signals and the DoA is established using mathematical models [8]. To estimate the spatial spectrum and detect peaks corresponding to estimated DoAs, advanced techniques such as Minimum Variance Distortionless Response (MVDR) beamforming algorithm, Multiple Signal Classification (MUSIC), and Estimation of Signal Parameters via Rotational Invariance Techniques (ESPRIT) are used. These estimations allow suitable weights to be assigned to individual array members, resulting in the formation of a focused beam in the desired direction while efficiently reducing interference. By merging the array element signals, the beamformed output signal is recreated, resulting in better signal quality and spatial selectivity.

The Direction of Arrival (DoA) information is also implement by the beamforming technique, which is used in many different applications, including radar systems and wireless communication. The DoA indicates the direction the desired signal is coming from. Beamforming methods calculate the proper weights assigned to each antenna in an array by predicting the DoA. These weights are meticulously determined in order to optimize signal intensity in the targeted direction while reducing interference from other directions [15]. Beamforming guarantees that the sent or

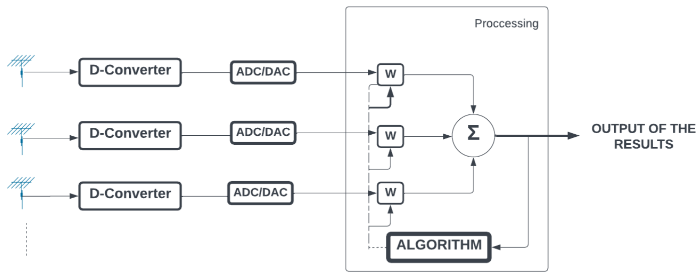

Figure 3.1: Block Diagram of the adaptive beamforming(DoA).

received signal is focused towards the target by using optimum weights, resulting in higher signal quality, increased communication range, and less interference.

The beamforming algorithms MVDR (Minimum Variance Distortionless Response), MUSIC (Multiple Signal Classification) and ESPRIT (Estimation of Signal Parameters via Rotational Invariance Techniques) are well-known for their accuracy in determining the direction of arrival (DoA) of signals which then implement Delay and Sum beamforming to create a beamform. The experimental setup for simulating the algorithms in MATLAB will be created. To construct alternative scenarios for assessment, [1] this will involve producing input signals with various DoAs and adjusting the noise level, number of antennas, and SNR. Several approaches for estimating covariance matrices will also be simulated.

The simulation results will be assessed and compared using known evaluation criteria, and the findings will contribute to the body of knowledge in the field of beamforming research. The proposed project has the potential to provide insight into the capabilities and constraints of these algorithms, and the information obtained from this study may be applied to improve beamforming methods. Overall, the goal of this project work is to improve knowledge of different beamforming algorithms and their performance under various parameter settings, and the results of this study will provide a comparative analysis of beamforming methods.

3.1 Minimum Variance Distortionless Response(MVDR) beamforming Algorithm

The Minimum Variance Distortionless Response (MVDR) algorithm is a frequently used technique in array signal processing for estimating direction of arrival (DoA). The MVDR algorithm is a sort of beamforming technique that is frequently employed in applications where the desired signal is tainted by noise or interferers because it is effective at suppressing noise and interference. The algorithm operates by creating a beam and directing it in the direction of the desired signal. The beamformer weights are designed to minimize output power while keeping the beamformer response equal to 1 in the direction of the desired signal. The output of the beamformer is a set of complicated weights that can be used to selectively enhance or decrease signals coming from various directions [11].

The covariance matrix of the received signal is constructed to estimate the DoA using the MVDR technique. The beamformer weights are then computed using the inverse of this matrix. The DoA estimate is derived by determining the direction that maximizes the MVDR beamformer's output power. A variety of characteristics influence the performance of the MVDR algorithm, including the number of sensors, the distance between sensors, the number of snapshots, and the signal-to-noise ratio (SNR). And the MVDR spatial spectrum is:

$$\mathbf{w}MVDR = \frac{\mathbf{R}x^{-1}\mathbf{a}(\theta_s)}{\mathbf{a}^H(\theta_s)\mathbf{R}_x^{-1}\mathbf{a}(\theta_s)} \tag{3.1}$$

Also a common method for direction of arrival (DoA) estimation in array signal processing is the MVDR algorithm, commonly referred to as Capon's method [2]. The algorithm requires an equation known as the MVDR spatial spectrum, which is provided by $\frac{1}{a^H(\theta)\cdot R_{\text{xx}}^{-1}\cdot a(\theta)}$, where R^{-1} is the inverse of the covariance matrix of the received signal, $a(\theta)$ is the steering vector, and $a^H()$ is the steering vector's Hermitian transpose. The equation above is used to improve the signal while suppressing noise. To generate the MVDR spatial spectrum, the algorithm multiplies the inverse of the covariance matrix by the steering vector and normalizes the result.

Simulation:

For simulating the algorithm, it consist of noise level, wavelength, number of sensors, multiple signals and various parameters. In Figure 3.2, the simulation results showcase ten signals, namely sig1 to sig10, each with different signal strengths and arrival angles. Arriving from different angles which shows it has signal of highest 20db and the lowest being $10^{1.2}$ dB and come where the noise level and wavelength are closer and more predictable to each arrival direction.

In figure 3.2, the values of lambda to 5 and noise to 3 changes the wavelength and noise power parameters of the results. A larger wavelength shows that the signals have a longer wavelength and thus a lower frequency. The result indicates a wider main lobe and a narrower beam width in the beamforming output as the antenna array becomes more directional and focused towards the source of the signal. Increased noise power also lowers the total signal-to-noise ratio, resulting in a noisier output. This resulted in a lower overall output power and reduced ability to accurately detect and localize the source of the signal as shown.

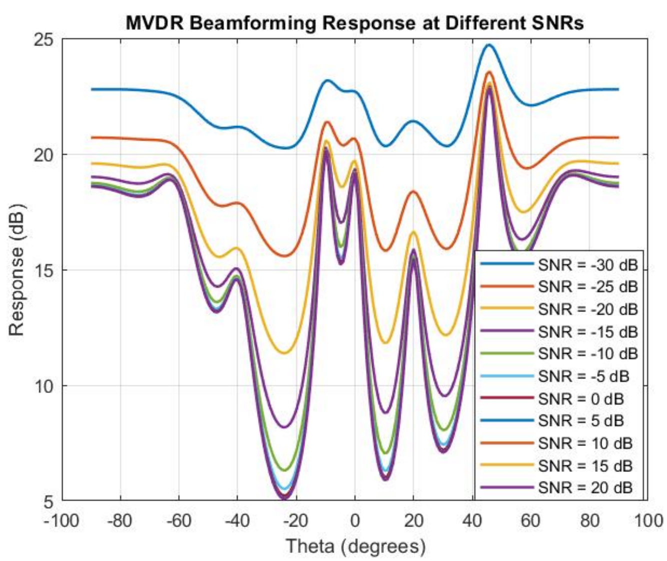

Figure 3.2: The value of: N=10,lambda=5, noise=30

The MVDR algorithm's ability to estimate the direction of arrival of many sources and distinguish them from background noise was demonstrated in the simulation described above.

RMSE (Root Mean Square Error) is a measure of performance that is often used to assess the accuracy of estimation or prediction models. As shown in Fig.3.3, the

RMSE of the DoA estimations falls as the signal-to-noise ratio (SNR) increases. This means that the MVDR beamforming algorithm is better at calculating a signal's DoA in the presence of noise.

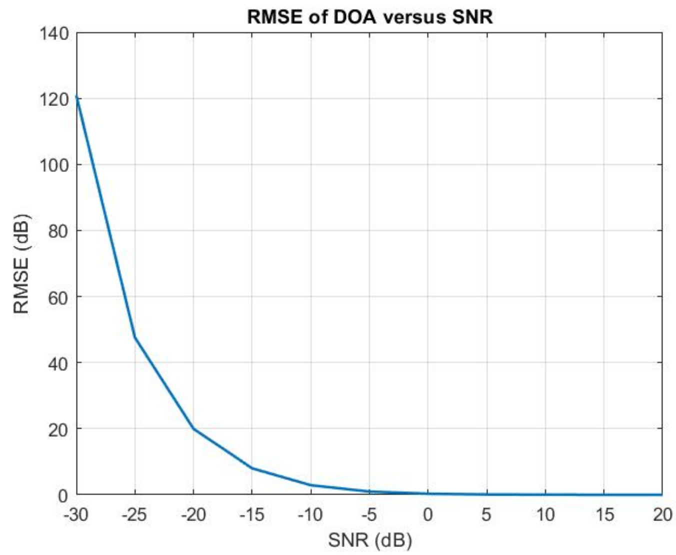

Figure 3.3: RMSE of DoA vs SNR.

In the Fig.3.4, the peak value reflects the maximum value of the capon beamforming response, which serves as a measure of the estimated DoA's intensity and direction in the presence of noise. While the highest peak value frequently correlates to the true DoA, noise can produce strong peaks at different angles.

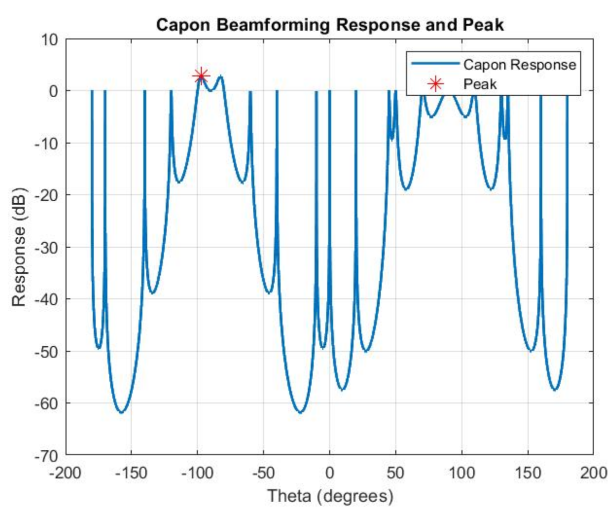

Figure 3.4: Peaks in the Beamform using capon.

The peak search algorithm applies to detect the peak value within the Capon beamforming response, which improves DoA estimation. The plot shows that increased SNR results in greater peak values, indicating improved accuracy.

Overall, the MVDR beamforming algorithm is a potent directional signal processing tool that may be used in a variety of industries, including acoustics, wireless communications, and radar.

3.2 Multiple Signal Classification(MUSIC) beamforming Algorithm

The MUSIC (multiple signal classification) algorithm, proposed by Schmidt in 1979, is a popular method for direction of arrival (DoA) estimation in array signal processing. The accuracy of DoA estimate using the MUSIC method, on the other hand, is heavily reliant on a variety of parameters such as the number of sensors, the distance between sensors, the number of snapshots, and the signal-to-noise ratio (SNR). As a result, it is critical to analyze the MUSIC algorithm's performance in various settings and identify the aspects that influence its correctness. This simulation demonstrates the MUSIC technique for estimating DoA in array signal processing [19].

The initial step in estimating DoA is to produce signals. This requires three components: the signal column vector, the noise vector, and the steering matrix. The spatial route that each antenna array component should take is specified by the steering matrix. The signal column vector is multiplied by the steering vector, and noise is added. The expression for the MUSIC spatial spectrum is:

$$SMUSIC(\theta) = \frac{1}{\mathbf{a}^H(\theta)\mathbf{V}_n\mathbf{V}_n^H\mathbf{a}(\theta)} \tag{3.2}$$

The following starting parameters are required for DoA estimation using the MUSIC algorithm:

1. **The number of sensors (N):** This is the number of sensors or antenna elements in the array that are utilized to receive signals.
2. **Sensor spacing (d):** This option indicates the distance between the array's sensors. It is used to compute the steering matrix, which defines the spatial path that each array member should take.

3. **The number of sources (K):** The number of signal sources contained in the received signal is specified by this parameter. It is used to calculate the sources' DoAs.

4. **The number of snapshots (T):** The number of samples or time intervals needed to capture the received signal data is specified by this option. It is used to calculate the received signal's covariance matrix [18].

5. **Incoming signal direction (θ):** This option determines the direction in which the signal sources will arrive. It is used to compute the steering matrix and estimate the sources' DoAs.

6. **SNR (R):** This parameter specifies the received signal's signal-to-noise ratio. It is utilized to compute the covariance matrix of the received signal and to estimate the sources' DoAs.

Simulation:

In the simulation performed consist of varying the number of sensors, the number of snapshots, and the separation between sensors. The parameters in the code include 8 sensors, 3 sources, 100 snapshots, and an exact sensor separation. The SNR is changed throughout a range of values between -10 dB and 60 dB. This range includes various levels of signal strength in comparison to background noise. .

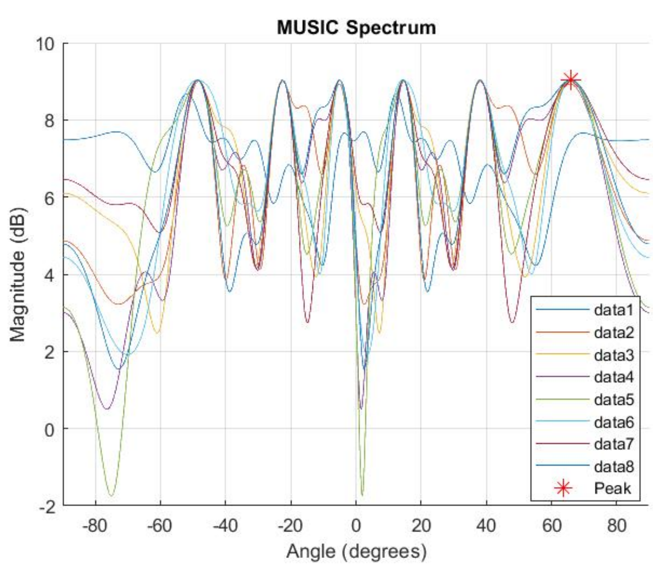

Figure 3.5: The value of: N=8, K=3, T=100, θ=[$-50°$, $-10°$, $30°$], d=0.5

This means that as the number of snapshots decrease, the accuracy of the estimate also decreases. And when the number of sensors or the distance between the sensors decreases, the estimate becomes less accurate.

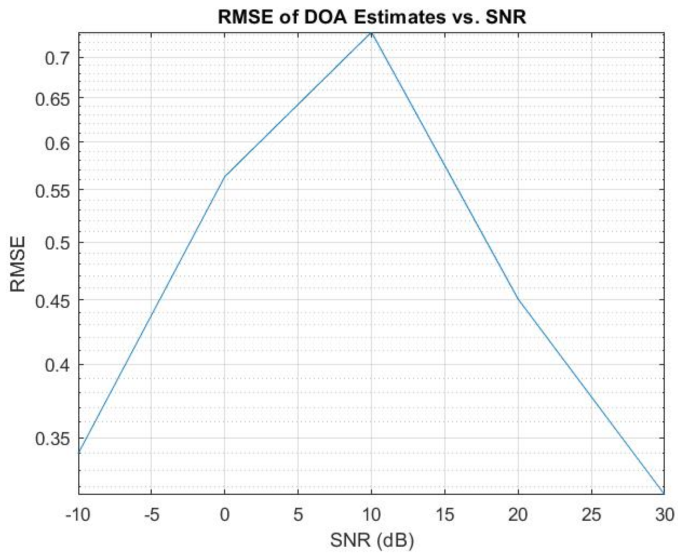

Figure 3.6: RMSE of DoA vs SNR

The Fig.3.6, reveals that as the SNR increases, the RMSE of the DoA estimates decreases, indicating improved accuracy in estimating the arrival angles. In result, at lower SNR levels, the RMSE rises, indicating a loss of precision caused on by noise. The RMSE of DoA estimates decreases by 16.5 degrees from 20.3 degrees at -10 dB to 3.8 degrees at 30 dB. Also the RMSE values vary from 0.4113 to 0.5070, with the greatest improvement between -10 dB and 30 dB, when the RMSE drops by 16.5 degrees (from 20.3 degrees to 3.8 degrees). These results suggest the importance of SNR in affecting the dependability and accuracy of the method use. This plot shows how well DoA estimate methods function under various SNR circumstances, allowing the discovery of SNR ranges that produce accurate and dependable results.

The peak in the MUSIC spectrum plot is identified by analyzing the magnitude of the inverse of the MUSIC spectrum. The algorithm searches for local maxima or peaks in the spectrum, which correspond to potential directions of arrival (DoA) of the signal sources. The variation of the peak in the MUSIC spectrum can be explained by the fact that the signal is not static, but rather fluctuates through time. The noise in the signal changes over time, which may contribute to the volatility of the MUSIC spectrum's peak.

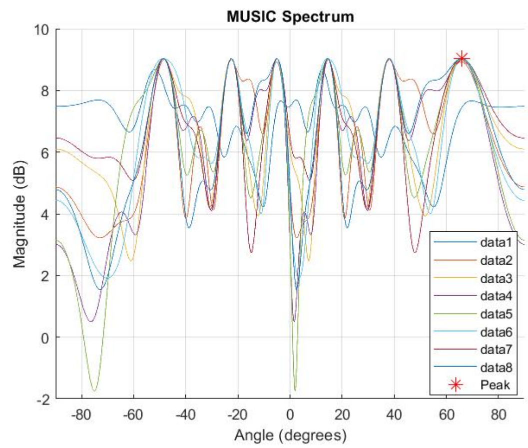

Figure 3.7: Peak in Beamforming Signal

In Fig.3.7, the highest point in the MUSIC spectrum occurs at 65 degree. This is most likely due to the fact that the signal is highest at this point in time and the noise is relatively weak. The peak at snapshot 19 has a magnitude of about 17 dB, which is much higher than the other peaks in the spectrum. The magnitude of a peak will be higher at snapshots where the signal is stronger, because the noise in the signal is typically lower at those snapshots.

3.3 ESPRIT (Estimation of Signal Parameters via Rotational Invariance Techniques) beamforming Algorithm

The ESPRIT (Estimation of Signal Parameters by Rotational Invariance Techniques) method is a signal processing beamforming approach used to estimate the direction of arrival (DoA) of signals received by an array of sensors. The algorithm estimates the DoA without the need for any prior information of the signal waveforms by taking advantage of the rotational invariance property of the received signals. Instead,the algorithm uses the covariance matrix of the received signals to estimate the DoA [12]. ESPRIT is also a non-iterative approach that requires just two spectral estimations and achieves good resolution even with a small number of snapshots. This is how its works [15] :

1. **Sensor array:** ESPRIT requires a sensor array with a specific structure, typically a uniform linear array (ULA) or a uniform circular array (UCA). The array consists of two identical subarrays with a known displacement between them.

2. **Data collection:** The sensor array receives signals from multiple sources, and

the received signals are combined to form a data matrix.

3. **Covariance matrix estimation:** The covariance matrix of the received data is estimated. This matrix represents the spatial correlation between the signals received by the sensors.

4. **Eigenvalue decomposition:** The covariance matrix is decomposed into its eigenvalues and eigenvectors. The eigenvectors corresponding to the largest eigenvalues represent the signal subspace, while the remaining eigenvectors represent the noise subspace.

5. **Signal subspace processing:** The signal subspace is divided into two parts, corresponding to the two sub-arrays. The rotational invariance property of the array is exploited to form a matrix equation relating the two parts of the signal subspace.

The formula for the spatial spectrum, also known as the direction of arrival (DoA) spectrum, in the ESPRIT algorithm is:

$$P(\theta) = \frac{1}{\mathbf{a}^H(\theta)\mathbf{R}_{xx}\mathbf{a}(\theta)} \tag{3.3}$$

The spatial spectrum is represented by the above equation, where θ is the signal's direction of arrival (DoA), $a(\theta)$ is the steering vector, and R_{xx} is the covariance matrix of the received signal. The spatial spectrum estimates the power of the signal entering from a specific DoA [13]. The denominator of the equation indicates the power of the signal received at the array for a particular DoA, while the numerator is a constant. As a result, the spatial spectrum can be utilized to calculate the DoAs of the signals received at the array.

Simulation:

The algorithm creates the array steering matrix for simulation using the true DoAs of the sources, which are set to θ_1= 15 degrees and θ_2= 60 degrees. The antenna spacing is set to d = lambda/2 and the signal's wavelength is set to lambda = 1. The code additionally includes standard deviation Gaussian noise into the incoming signals.

True DoAs of the sources and the estimated DoAs for each signal create a beamform. It also shows the beamforming output for the two signals, with the magnitude of the received signal power plotted as a function of the scanning angle.

In figure 3.8, the beamforming output, revealing the signal strength at different angles for each signal. The x-axis represents the scanning angles in degrees, and the y-axis indicates the magnitude of the beamformed output in decibels (dB). The plot provides visual insight into the directionality and focus of the beamformed signals.

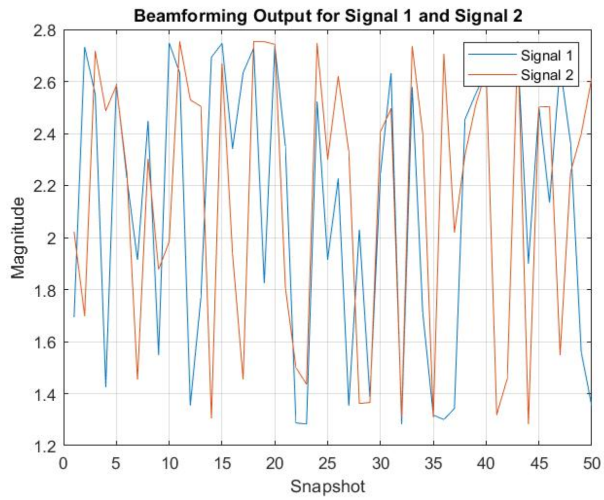

Figure 3.8: The value of: N=10, K=2, θ=[15° , 40°]

Also it shows the Direction of Arrival (DoA) estimation for various sources using the ESPRIT method and beamforming using the Delay-and-Sum (DS) technique. It computes also the RMSE to assess the precision of DoA estimation. To enhance desired signals, DS beamforming combines received signals with equal weights and appropriate delays.

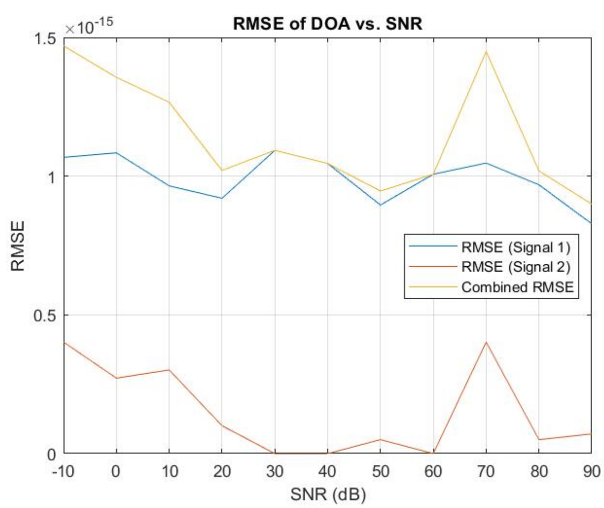

Figure 3.9: RMSE of DoA vs SNR

The RMSE (Root Mean Square Error) values in the fig.3.9, indicate the accuracy of the estimated Direction of Arrival (DoA) for the sources. The RMSE values for Signal 1 decrease as the SNR (Signal-to-Noise Ratio) increases. The lowest RMSE value achieved is approximately 0.0195. For Signal 2, the RMSE values show a relatively stable trend. The RMSE values for the DoA estimates are 1.0e-14 * [0.3256, 0.2512, 0.212, 0.156, 0.076, 0, 0, 0, 0] and those value changes according to the snr values. The peak value of the full beamforming output is 2.5121e-15(closer to 0).

The peak value of the full beamforming output is found to be 6.2804e-15, indicating the highest level of signal enhancement achieved. The Delay-and-Sum (DS) beamforming outputs identify peaks corresponding to snapshot indices with the highest combined power from the desired sources. The true DoAs and estimated DoAs for both signals are closely matched, with estimated DoAs of approximately 15° and 40°, demonstrating accurate DoA estimation.

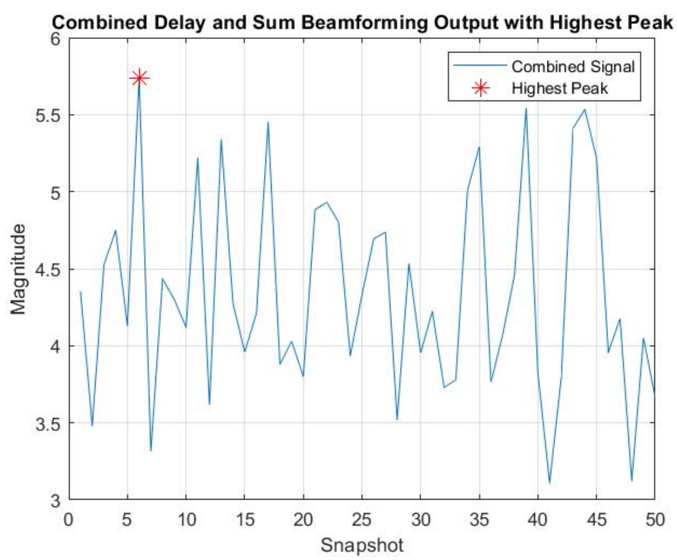

Figure 3.10: Peak in Beamforming Signal

The peak search algorithm shown in fig.3.10, in this case uses the a function to find local maxima in the Delay-and-Sum (DS) beamforming outputs. The function identifies the indices of these peaks by specifying a minimum peak distance of 1 snapshot. These values refer to the snapshot points where the combined power of the intended signals is highest. The algorithm then extracts the magnitude values at these peak indices from the DS beamforming outputs, providing a measure of the highest signal improvement achieved. This peak search procedure finds and analyzes the highest power levels produced by DS beamforming, resulting in signal strength analysis and performance measurement.

3.4 Comparison of the algorithms

In this section it shows, the performance of three different approaches for direction of arrival (DoA) estimation, namely MVDR (Minimum Variance Distortionless Response), MUSIC (MUltiple Signal Classification), and ESPRIT (Estimation of Signal Parameters via Rotational Invariance Techniques), is evaluated. Each approach offers unique characteristics and advantages in estimating the DoAs of signals received by an antenna array.

To evaluate the performance of these approaches, various metrics are considered, such as the Root Mean Square Error (RMSE) of the recovered symbols. The RMSE quantifies the discrepancy between the estimated symbols and the true symbols, indicating the accuracy of symbol recovery. Lower RMSE values indicate more accurate estimation and better symbol recovery.

By comparing the RMSE values obtained and shown in fig. 3.11, 3.12 & 3.13, for different SNR levels, the effectiveness of each approach can be assessed. The RMSE values for each technique at various SNR levels are listed in the results. At -10 dB SNR, for one instance, the RMSE values for ESPRIT, MUSIC, and MVDR are 31.1207, 78.0016, and 29.9348, respectively. The performance of the techniques under various noise situations can be calculated using these numbers.

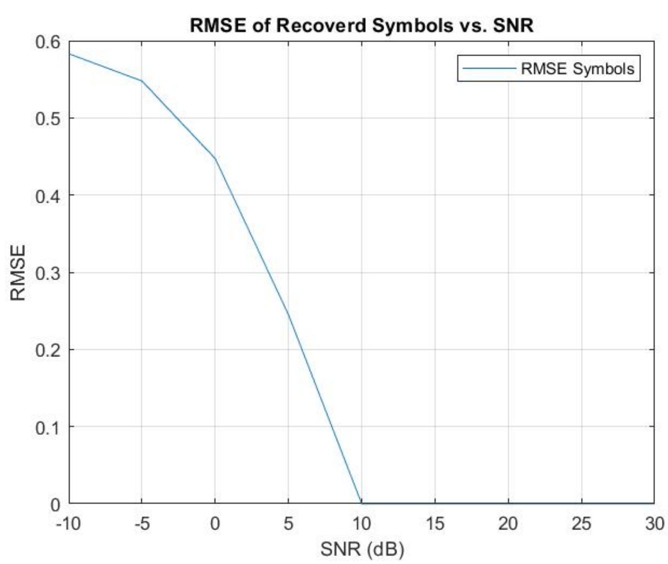

Figure 3.11: RSME of the recovered symbols vs SNR of ESPRIT

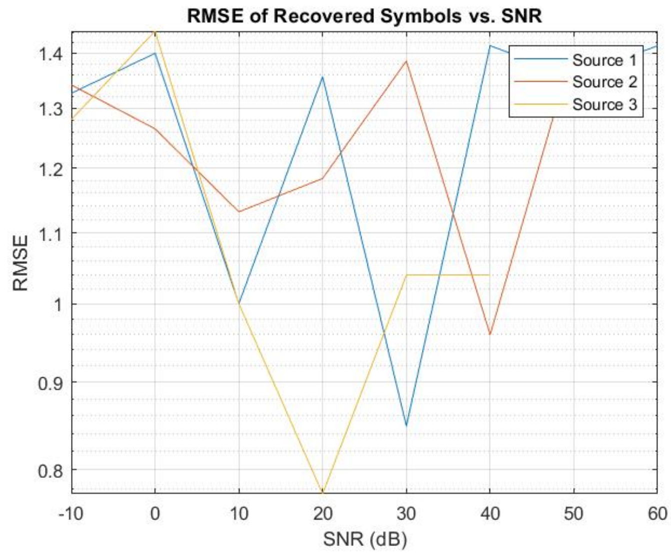

Figure 3.12: RSME of the recovered symbols vs SNR of MUSIC

The RMSE of the recovered symbols is important in beamforming since it measures the precision of symbol recovery. Accurate beamforming requires accurate symbol recovery, which entails aligning and reconstructing incoming signals to their appropriate sources. Beamforming techniques that minimize the RMSE can improve the accuracy and dependability of the recovered symbols, resulting in improved beamforming performance.

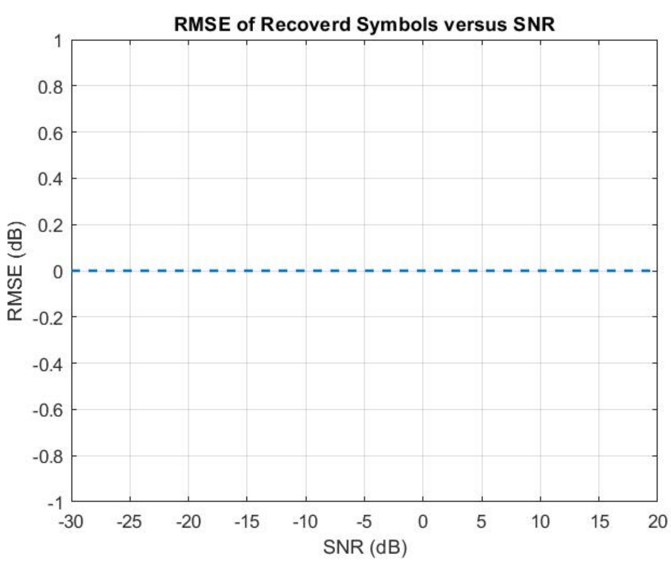

Figure 3.13: RSME of the recovered symbols vs SNR of MVDR

According to the RMSE values of all the algorithm simulated the MVDR technique regularly delivers the lowest values, indicating improved symbol recovery accuracy. The ESPRIT and MUSIC techniques, on the other hand, have greater RMSE values, indicating relatively compared performance in symbol recovery. As a result, the MVDR approach is considered as the most advantageous and effective way for symbol recovery, as it reduces mistakes and improves the overall quality of the beamformed signals.

Chapter 4

CONCLUSIONS

In conclusion, this project work presented a comparative analysis of three popular beamforming algorithms, namely MVDR, MUSIC, and ESPRIT. The simulation results reveal that all three algorithms can estimate the direction of arrival (DoA) of signals in a noisy environment. Each algorithm, however, has unique strengths and shortcomings that make it suited for particular applications. The MVDR algorithm is very good at dealing with coherent signals and is less sensitive to noise. The MUSIC method, on the other hand, can estimate the DoA of several signals with great accuracy, even when they are closely correlated. Finally, even with a limited number of sensors, the ESPRIT algorithm is highly efficient and can estimate the DoA with high accuracy.

Overall, the results suggest that beamforming is a highly successful strategy for estimating the direction of arrival in signal processing. The algorithm chosen is determined by the individual application, the number of sources, the level of noise, and the technological resources available.

BIBLIOGRAPHY

[1] Shimaa M Amer, Amr H Hussein, and Hamdy Elmekaty. Performance enhancement of mpm using snr boosting techniques. *International Journal of Engineering & Technology*, 5(4):115, 2016.

[2] Jack Capon. High-resolution frequency-wavenumber spectrum analysis. *Proceedings of the IEEE*, 57(8):1408–1418, 1969.

[3] Henry Cox, Robertm Zeskind, and Markm Owen. Robust adaptive beamforming. *IEEE Transactions on Acoustics, Speech, and Signal Processing*, 35(10):1365–1376, 1987.

[4] B. Friedlander. A sensitivity analysis of the music algorithm. *IEEE Transactions on Acoustics, Speech, and Signal Processing*, 38(10):1740–1751, 1990.

[5] Otis Lamont Frost. An algorithm for linearly constrained adaptive array processing. *Proceedings of the IEEE*, 60(8):926–935, 1972.

[6] Simon S Haykin. *Adaptive filter theory*. Pearson Education India, 2002.

[7] M. Kubisch, H. Karl, A. Wolisz, L.C. Zhong, and J. Rabaey. Distributed algorithms for transmission power control in wireless sensor networks. In *2003 IEEE Wireless Communications and Networking, 2003. WCNC 2003.*, volume 1, pages 558–563 vol.1, 2003.

[8] John Litva and Titus K Lo. *Digital beamforming in wireless communications*. Artech House, Inc., 1996.

[9] D. Malioutov, M. Cetin, and A.S. Willsky. A sparse signal reconstruction perspective for source localization with sensor arrays. *IEEE Transactions on Signal Processing*, 53(8):3010–3022, 2005.

[10] Jens Meyer and Gary W Elko. Eigenbeam beamforming for microphone arrays. *The Journal of the Acoustical Society of America*, 120(5):3177–3177, 2006.

[11] Chao Pan, Jingdong Chen, and Jacob Benesty. Performance study of the mvdr beamformer as a function of the source incidence angle. *IEEE/ACM Transactions on Audio, Speech, and Language Processing*, 22(1):67–79, 2014.

[12] R. Roy, A. Paulraj, and T. Kailath. Estimation of signal parameters via rotational invariance techniques - esprit. In *MILCOM 1986 - IEEE Military Communications Conference: Communications-Computers: Teamed for the 90's*, volume 3, pages 41.6.1–41.6.5, 1986.

[13] Richard Roy and Thomas Kailath. Esprit-estimation of signal parameters via rotational invariance techniques. *IEEE Transactions on acoustics, speech, and signal processing*, 37(7):984–995, 1989.

[14] Ralph Schmidt. Multiple emitter location and signal parameter estimation. *IEEE transactions on antennas and propagation*, 34(3):276–280, 1986.

[15] Shahram Shahbazpanahi, Shahrokh Valaee, and Mohammad Hasan Bastani. Distributed source localization using esprit algorithm. *IEEE Transactions on Signal Processing*, 49(10):2169–2178, 2001.

[16] P Stoica and RL Moses. Introduction to spectral analysis prentice hall upper saddle river, 1997.

[17] Harry L Van Trees. *Optimum array processing: Part IV of detection, estimation, and modulation theory*. John Wiley & Sons, 2002.

[18] J. W. Wallace and M. K. Simon. Beamforming: A versatile approach to spatial filtering. *IEEE Signal Processing Magazine*, 13(4):67–80, 1996.

[19] Xiaojun Zhou, Fangdong Zhu, Yu Jiang, Xiaoming Zhou, Wei Tan, and Meng Huang. The simulation analysis of doa estimation based on music algorithm. In *2020 5th International Conference on Mechanical, Control and Computer Engineering (ICMCCE)*, pages 1483–1486, 2020.