An Earthly Model of the Divine Coincidence

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January 2024

Recent evidence shows that the divine coincidence might prevail in the United States. That is, inflation is stable, on target whenever the labor market is efficient; inflation rises whenever the labor market is inefficiently tight; and inflation falls whenever the labor market is inefficiently tight. Since the US labor market has been inefficiently tight in the aftermath of the coronavirus pandemic, from the middle of 2021 to today, the flare-up in inflation in 2021–2023 might have been partly caused by excessive tightness. The divine coincidence is also important for policy. If it holds, the full-employment and price-stability mandates of the Federal Reserve would just coincide—greatly simplifying the Fed's task. This paper proposes a simple, earthly model in which the divine coincidence holds. The model proves that the phenomenon of divine coincidence might not be entirely surprising. The model also highlights features of the economy that might lead to the divine coincidence.

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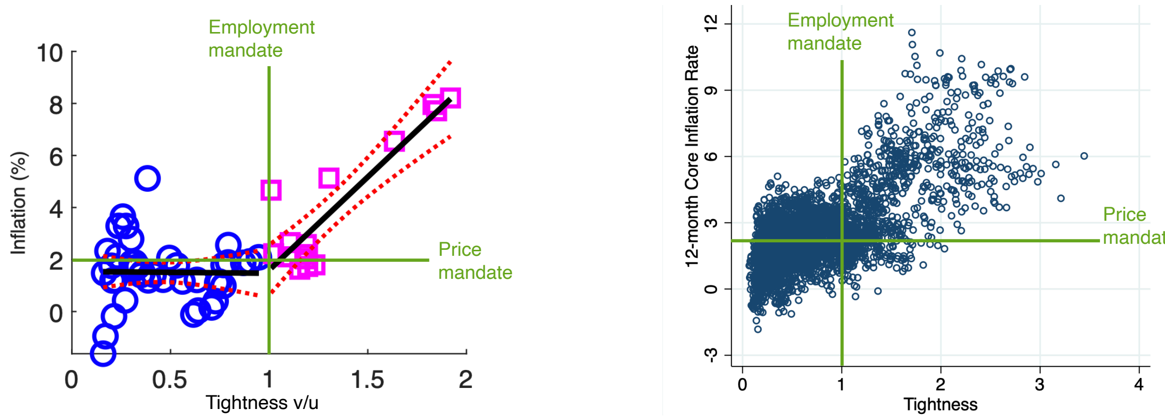
1. Introduction

Divine coincidence in the New Keynesian model. The textbook New Keynesian model features a divine coincidence. In the model, stabilization inflation is equivalent to stabilizing the gap between the actual and efficient levels of output (Blanchard and Gali 2007). That divine coincidence implies that stabilizing inflation is the optimal monetary policy, once appropriate taxes and subsidies have been put in place to ensure that the output is efficient on average. This property of the New Keynesian model is generally seen as unrealistic, so several modification have been introduced into the model to remove the coincidence (Blanchard and Gali 2007, 2010).

Stronger divine coincidence in US data. Recent evidence suggests, however, that a strong form of the divine coincidence might prevail in the United States: maintaining inflation on target is equivalent to maintaining unemployment at its efficient level (figure 1). That is, inflation is on target whenever the labor market is efficient; inflation is above target whenever the labor market is inefficiently tight; and inflation falls below target whenever the labor market is inefficiently tight. This form of divine coincidence is stronger than that in the New Keynesian model: here keeping inflation on target guarantees not only that the gap between actual and efficient unemployment is constant, but in fact that the gap is zero.

The evidence of divine coincidence. The evidence of divine coincidence in the United States comes from the work of Benigno and Eggertsson (2023) and Gitti (2023). Benigno and Eggertsson use aggregate data for inflation and labor market tightness (the number of vacant jobs per unemployed workers). The divine coincidence appears clearly in the 2008–2022 period (figure 1A). When the labor market is efficient, which corresponds to a tightness of 1 (Michaillat and Saez 2023), inflation is on target at 2%. Gitti use data at the metropolitan-area (MSA) level for 2001–2022. Using metropolitan data has several advantages. First, the Phillips curve has many more data points, since the data come from 21 MSAs instead of one country. Second, variations in labor-market tightness are much more pronounced at the metropolitan level, so that there are many more examples of inefficiently tight markets (tightness above 1). The divine coincidence appears again at the metropolitan level, although in a more noisy fashion: when the labor market tightness is 1, inflation is around 2% (figure 1B).



A. Aggregate data, 2008–2022

B. Metropolitan data, 2001–2022

FIGURE 1. Evidence of divine coincidence in the United States

Sources: Panel A was produced by Benigno and Eggertsson (2023, figure 4). Panel B was produced by Gitti (2023, figure 1). The lines marking the employment and price mandates were added.

Implications of divine coincidence. The divine coincidence has important implications. On the positive front, it might explain the flare-up in inflation observed after the pandemic in the United States. Indeed, the US labor market has been inefficiently tight in the aftermath of the coronavirus pandemic, from the middle of 2021 to today (Michaillat and Saez 2023). The 2021–2023 inflation flare-up might have been partly caused by excessive tightness. On the normative front, the divine coincidence implies that the full-employment and price-stability mandates of the Federal Reserve coincide—greatly simplifying the Fed's task.

Earthly model of divine coincidence. This paper proposes a simple, earthly model in which the divine coincidence holds. The model proves that the divine coincidence might arise in theory under fairly common assumptions. So it might not be as surprising as it first seemed that the divine coincidence appears in the data. More generally, the simple model developed here shows how the joint movements of inflation, unemployment, and tightness can be studied via the Euler, Phillips, and Beveridge curves.

Structure of the earthly model. The model uses the structure of the economical business-cycle model developed by Michaillat and Saez (2022). In that model, however, inflation is constant. To generate price dynamics, we introduce price competition through directed search (Moen 1997). Furthermore, to ensure that unemployment fluctuates, we introduce price rigidity through quadratic price-adjustment costs (Rotemberg 1982).

2. Model

This section develops a simple, earthly model of the divine coincidence.

2.1. People

The size of the population is normalized to 1. People are organized in large households. The households are all initially identical and indexed by $j \in [0,1]$. Household $j \in [0,1]$ has l_j workers. The aggregate labor force is $l = \int_0^1 l_j(t)\, dj$.

2.2. Matching between workers and customers

Of the l_k workers of household k, y_{jk} work for household j, and a total $y_k = \int_0^1 y_{jk}(t)\, dk$ are employed across all households. Not all workers are employed, however. $U_k = l_k - y_k$ workers remain unemployed. The unemployment rate in household k is

$$u_k = \frac{U_k}{l_k}.$$

Services are sold through long-term worker-household relationships. Once a worker has matched with a household, she becomes a full-time employee of the household. She remains so until they separate, which occurs at rate $s > 0$.

To recruit workers from household k, household j sends V_{jk} of their own employees to visit household k's shop. These V_{jk} employees advertise the vacancies open by their employer, read applications from household k's workers, and interview and select suitable candidates. A total $V_k = \int_0^1 V_{jk}(t)\, dj$ employed workers are at shop k to recruit unemployed workers from household k. The recruiting rate at shop k is

$$v_k = \frac{V_k}{l_k}.$$

A matching function determines the flow of new matches at shop k based on the number of unemployed workers and recruiters: $h_k = h(U_k, V_k)$ where

$$h(U_k, V_k) = \omega \cdot \sqrt{U_k \cdot V_k} - s \cdot U_k. \tag{1}$$

The matching function h satisfies standard assumptions (Petrongolo and Pissarides 2001): it is 0 when $U = 0$ and $V = 0$, it has constant returns to scale, is increasing in V, and it is increasing in U as long as the market is not too slack—a condition that will be satisfied as long the unemployment rate is below 50%.

The partial derivative of the matching function with respect to U_k is

$$\frac{\partial h}{\partial U_k} = \frac{\omega}{2} \cdot \sqrt{\frac{V_k}{U_k}} - s = \frac{\omega}{2} \cdot \sqrt{\theta_k} - s,$$

where

$$\theta_k = \frac{V_k}{U_k} = \frac{v_k}{u_k}$$

is the tightness of market k. The partial derivative is positive for any $\theta_k \geq \underline{\theta}$ where the lower bound $\underline{\theta}$ is given by

$$\underline{\theta} = 4\left(\frac{s}{\omega}\right)^2. \tag{2}$$

We impose

$$\omega > 2s$$

so $\underline{\theta} < 1$. In the model the condition $\theta_k \geq \underline{\theta}$ is verified for any $u_k \in [0, 1/2]$. So as long the local unemployment rate is below 50%, the matching function will be increasing in the unemployment rate. The matching function is also concave in U and concave in V.

Although it satisfies all standard properties, the matching function h takes an unusual form. We specify the function as such to obtain an hyperbolic Beveridge curve in the model, and therefore be consistent with the empirical Beveridge curve observed in US data (Michaillat and Saez 2023).

The tightness on market k is the ratio of the number of recruiters (buyers) and unemployed workers (sellers):

$$\theta_k = \frac{V_k}{U_k}.$$

Equivalently, tightness on market k is the ratio of the recruiting and unemployment rates: $\theta_k = v_k/u_k$. Tightness on market k is also the number of recruiters per unemployed workers. We impose $\theta_k \geq \underline{\theta}$ to ensure that the matching function is increasing in U, which also ensures that the number of matches is positive.

Tightness determines all trading rates. The customer-finding rate is

$$f(\theta_k) = \frac{h_k}{U_k} = \omega \cdot \sqrt{\theta_k} - s. \tag{3}$$

For $\theta_k \in [\underline{\theta}, \infty)$, the customer-finding rate f is positive since $\sqrt{\theta_k} \geq 2s/\omega$ so $\omega\sqrt{\theta_k} - s \geq s > 0$. The customer-finding rate is also increasing in θ_k. In fact, $f(\underline{\theta}) = s$ and $f(\infty) = \infty$. Hence, when tightness is higher, it is easier to find a job and sell services.

The worker-finding rate is

$$q(\theta_k) = \frac{h_k}{V_k} = \frac{\omega}{\sqrt{\theta_k}} - \frac{s}{\theta_k}. \tag{4}$$

For $\theta_k \in [\underline{\theta}, \infty)$, the worker-finding rate is positive since $\sqrt{\theta_k} \geq 2s/\omega$ so $s/\sqrt{\theta_k} \leq \omega/2$ and $\omega - s/\sqrt{\theta_k} \geq \omega/2 > 0$. The worker-finding rate is also decreasing in θ_k.

The derivative of the worker-finding rate with respect to θ_k is

$$\frac{dq}{d\theta_k} = -\frac{\omega}{2} \cdot \theta_k^{-3/2} + s\theta_k^{-2} = -\theta_k^{-3/2} \cdot \left[\frac{\omega}{2} - \frac{s}{\sqrt{\theta_k}}\right] < 0.$$

Indeed, we showed that for $\theta_k \in [\underline{\theta}, \infty)$,

$$\omega - \frac{s}{\sqrt{\theta_k}} \geq \frac{\omega}{2},$$

so that

$$\frac{\omega}{2} - \frac{s}{\sqrt{\theta_k}} \geq 0,$$

which implies that $dq/d\theta_k < 0$. In fact, $q(\underline{\theta}) = \omega^2/4s > s$ and $q(\infty) = 0$. This means that when tightness is higher, it is harder to find a worker and buy services.

Note also that as usual, $f(\theta_k) = \theta_k \cdot q(\theta_k)$.

2.3. Cost of unemployment and hiring

Unemployed workers wait in their shop to be hired. During that time, they do not receive any income and cannot engage in home production, which explain why unemployment is individually and socially costly.

There are costs not only on the selling side of the market, but also on the buying side. Hiring workers is indeed costly. Each recruiter looking to hire a worker on behalf of their employer cannot produce utility-providing services. Because the recruiters employed by household j to hire workers from household k do not provide direct utility but are used in the process of hiring other workers, consumption c_{jk} is less than output y_{jk}. Consumption c_{jk} is the number of workers from household k working for household

j, minus the number of workers employed by household j to recruit workers from household k:

$$c_{jk} = y_{jk} - V_{jk}.$$

2.4. Balanced flows and unemployment

We assume that flows on each individual market are balanced. This assumption is motivated by the fact that on the US labor market, flows are always approximately balanced (Michaillat and Saez 2021a, p. 7).

The number of employed workers in household k is given by a differential equation:

$$\dot{y}_k = f(\theta_k) \cdot U_k - s \cdot y_k = f(\theta_k) \cdot U_k - s \cdot [l_k - U_k] = l_k \cdot \left[f(\theta_k) \cdot u_k - s \cdot [1 - u_k]\right].$$

We assume that flows are balanced in all (j, k) cells. In particular flows are balanced in all household k: $\dot{y}_k = 0$. This assumption implies that the local unemployment rate is a function of local tightness: $u_k = u(\theta_k)$, where

$$u(\theta_k) = \frac{s}{s + f(\theta_k)} = \frac{s/\omega}{\sqrt{\theta_k}}. \tag{5}$$

From this expression we see that for any $\theta_k \geq \underline{\theta} = 4(s/\omega)^2$, $u(\theta_k) \leq 1/2$. The analysis focuses on this range of tightness—which is without real loss of generality since in practice unemployment rates are not above 50%.

The unemployment function (5) has the following properties when $\theta_k \in [\underline{\theta}, \infty)$: $u(\underline{\theta}) = 1/2$, $u(\infty) = 0$, and u is decreasing in θ_k. When the market is tighter, workers find jobs more rapidly, so the unemployment rate is lower.

Thanks to the shape of the matching function (1), the unemployment rate is an isoelastic function of tightness, and the Beveridge curve is isoelastic as well. In fact the Beveridge curve is a rectangular hyperbola, just like in US data (Michaillat and Saez 2023):

$$v(u_k) = \frac{(s/\omega)^2}{u_k}. \tag{6}$$

From the Beveridge curve we can also express the recruiting rate as a function of local tightness:

$$v(\theta_k) = \frac{s}{\omega} \cdot \sqrt{\theta_k}. \tag{7}$$

The recruiting function (7) has the following properties when $\theta_k \in [\underline{\theta}, \infty)$: $v(\underline{\theta}) = 2(s/\omega)^2$, $v(\infty) = \infty$, and v is increasing in θ_k. When the market is tighter, it takes longer to hire workers, so the recruiting rate is higher.

2.5. Balanced flows and recruiter-producer ratio

Next we compute the recruiter-producer ratio for household k. The number of employed workers from household j in household k follows a differential equation:

$$\dot{y}_{jk} = q(\theta_k) \cdot V_{jk} - s \cdot y_{jk} = q(\theta_k) \cdot [y_{jk} - c_{jk}] - s \cdot y_{jk}.$$

We assume that flows are balanced in all (j, k) cells, so $\dot{y}_{jk} = 0$. This means that tightness determines the gap between consumption and output:

$$y_{jk} = [1 + \tau(\theta_k)] \cdot c_{jk},$$

where the recruiter-producer ratio is a function of tightness:

$$\tau(\theta_k) = \frac{s}{q(\theta_k) - s}. \tag{8}$$

By definition, $\tau(\theta_k) = (y_{jk} - c_{jk})/c_{jk}$. The quantity $y_{jk} - c_{jk}$ is the number of workers hired by household j to recruit new workers from household k, while c_{jk} is the number of workers from household k hired by household j for producing services. Hence the function τ gives the recruiter-producer ratio required for any local tightness θ_k.

Given the properties of the worker-finding rate q, we infer the properties the properties of the recruiter-producer ratio τ. Since $q(\underline{\theta}) = \omega^2/4s$, $\tau(\underline{\theta}) = 1/[(\omega/2s)^2{-}1] = 1/(1/\underline{\theta}{-}1) > 0$. Since q is decreasing in θ for $\theta \geq \underline{\theta}$, τ is increasing in θ for $\theta \geq \underline{\theta}$. Furthermore, $\tau \to \infty$ when $\theta \to \bar{\theta}$ when $\bar{\theta} > \underline{\theta}$ is defined by $q(\bar{\theta}) = s$. The upper tightness bound is well defined because $q(\underline{\theta}) > s$, q is decreasing in θ, and $q(\infty) \to 0$. In fact it is possible to express $\bar{\theta}$ as a function of $\underline{\theta}$ alone:

$$\bar{\theta} = \frac{\underline{\theta}}{\left[1 - \sqrt{1 - \underline{\theta}}\right]^2} > \underline{\theta}.$$

Indeed, we can write $q(\bar{\theta}) = s$ as

$$\frac{\omega}{\sqrt{\theta}} - \frac{s}{\theta} - s = 0.$$

With a change of variable $x = 1/\sqrt{\theta}$, this is equivalent to solving the second-order polynomial equation

$$-sx^2 + \omega x - s = 0.$$

The determinant of the equation is $\Delta = \omega^2 - 4s^2 > 0$. The two solutions of the equation are

$$x' = \frac{\omega \pm \sqrt{\Delta}}{2s} = \frac{\omega}{2s} \cdot \left[1 \pm \sqrt{1 - \frac{4s^2}{\omega^2}}\right] = \frac{1 \pm \sqrt{1-\underline{\theta}}}{\sqrt{\underline{\theta}}}.$$

Accordingly, the tightnesses that solve $q(\theta) = s$ are given by $1/(x')^2$ or

$$\theta = \frac{\underline{\theta}}{\left[1 \pm \sqrt{1-\underline{\theta}}\right]^2}$$

The only solution that is larger than $\underline{\theta}$ is

$$\bar{\theta} = \frac{\underline{\theta}}{\left[1 - \sqrt{1-\underline{\theta}}\right]^2}.$$

It is also helpful to write the recruiter-producer ratio as a function of the unemployment rate. In a local market k, the recruiter-producer $\tau(\theta_k)$ ratio is the same in all households that hire workers there. So the recruiter-producer ratio is all the ratio between all the recruiters hired from household k and all the producers from household k:

$$\tau_k = \frac{y_k - c_k}{c_k}, \qquad y_k - c_k = v_k, \qquad c_k = l_k - u_k - v_k.$$

Therefore we can write the recruiter-producer as a function of the unemployment rate:

$$\tau(u_k) = \frac{v(u_k)}{1 - [u_k + v(u_k)]}. \tag{9}$$

2.6. Some elasticities

The elasticity of the customer-finding rate f given by (3) is

$$\frac{d\ln(f)}{d\ln(\theta)} = \frac{\omega\sqrt{\theta}}{\omega\sqrt{\theta} - s} \cdot \frac{1}{2} = \frac{1}{1 - (s/\omega)/\sqrt{\theta}} \cdot \frac{1}{2} = \frac{1/2}{1-u},$$

where the unemployment rate is a function of tightness given by (5). Since $1 - u \approx 1$, the elasticity is never far from 1/2, as it would be with a more common Cobb-Douglas matching function.

Since $q(\theta) = f(\theta)/\theta$, we infer the elasticity of the worker-finding rate q:

$$\frac{d\ln(q)}{d\ln(\theta)} = \frac{d\ln(f)}{d\ln(\theta)} - 1 = -\frac{1/2 - u}{1 - u},$$

where the unemployment rate is a function of tightness given by (5). Since $1 - u \approx 1$ and $1/2 - u \approx 1/2$, the elasticity is never far from $-1/2$, as it would be with a more common Cobb-Douglas matching function.

The elasticity of the unemployment rate (5) simply is

$$\frac{d\ln(u)}{d\ln(\theta_k)} = -(1-u) \cdot \frac{d\ln(f)}{d\ln(\theta)} = -\frac{1}{2}. \tag{10}$$

The elasticity of the recruiter-producer ratio (8) is given by

$$\frac{d\ln(\tau)}{d\ln(\theta_k)} = -(1+\tau) \cdot \frac{d\ln(q)}{d\ln(\theta)} = \frac{(1+\tau) \cdot (1/2 - u)}{1-u}. \tag{11}$$

From (9), we have

$$1 + \tau(\theta_k) = \frac{1 - u(\theta_k)}{1 - u(\theta_k) - v(\theta_k)},$$

so we can simplify the elasticity of the recruiter-producer ratio:

$$\frac{d\ln(\tau)}{d\ln(\theta_k)} = \frac{1}{2} \cdot \frac{1 - 2u}{1 - u - v}. \tag{12}$$

These elasticities will be important when we solve the model.

2.7. Productive efficiency at shop k

What is the efficient allocation of labor at shop k? We are interested in productive efficiency, that is the allocation of labor that maximizes the amount of services from household k that are consumed. The amount of services consumed is

$$c_k = y_k - V_k = l_k - U_k - V_k = l_k \cdot \left[1 - u_k - v_k\right].$$

Maximizing that amount is equivalent to minimizing the sum of the unemployment and recruiting rates, $u_k + v_k$, subject to the Beveridge curve (6). This is exactly the problem studied by Michaillat and Saez (2023). The solution is

$$u_k^* = \sqrt{u_k v_k} = s/\omega, \qquad v_k^* = u_k^*, \qquad \theta_k^* = 1. \tag{13}$$

Furthermore, the economy is inefficiently tight whenever there are more recruiters than jobseekers, $u_k < v_k$, inefficiently slack whenever there are more jobseekers than recruiters, $u_k > v_k$, and of course efficient whenever there are as many jobseekers as recruiters, $u_k = v_k$.

2.8. Directed search and price-tightness competition

All workers from household k charge a price p_k per unit time. The expenditure by household j on workers k therefore is

$$p_k \cdot y_{jk} = p_k \cdot [1 + \tau(\theta_k)] \cdot c_{jk}.$$

The relevant price of services is not just p_k but $p_k \cdot [1 + \tau(\theta_k)]$. The price involves the price per unit time as well as the time it takes to replace a worker.

All workers are perfectly substitutable, so households hires workers from the household that offers the cheapest consumption. All households are aware of this fact, so all households price their services to compete with other households:

$$p_k \cdot [1 + \tau(\theta_k)]$$

must be the same across all households k. Just as in Moen (1997), buyers direct their search toward the most attractive sellers, which induces competition across all sellers. Through competition, sellers set prices so buyers are indifferent across all sellers. If sellers set a higher price, then cheaper workers would be available, or workers could be hired with less wait, so they would not get any customers.

Accordingly, there is a price level p such that for all k,

$$p_k \cdot [1 + \tau(\theta_k)] = p \cdot [1 + \tau(\theta)], \tag{14}$$

where the aggregate market tightness is the ratio of the aggregate number of recruiters to the aggregate number of unemployed workers, given by

$$\theta = \frac{\sum_k V_k}{\sum_k U_k}.$$

2.9. Effect of local price on local tightness

The price chosen by household j determines the tightness θ_j it faces, and therefore the pace at which workers from the household find employment. From (14), we see that the local tightness is given by

$$\theta_j(p_j) = \tau^{-1}\left(\frac{p}{p_j}[1+\tau(\theta)] - 1\right).$$

The function τ^{-1} is increasing, so the local tightness $\theta_j(p_j)$ is decreasing in the local price p_j. A high price leads to low tightness and high unemployment. A low price leads to high tightness and low unemployment. In that way, households face downward-sloping demand curves in a price-tightness plane.

In fact, the demand curve $\theta_j(p_j)$ has the following properties for $p_j \in (0, p[1+\tau(\theta)])$: $\theta_j(0) = \bar{\theta}$, $\theta_j(p) = \theta$, $\theta_j(p[1+\tau(\theta)]) = 0$. The derivative and elasticity of the demand curve are:

$$\frac{d\theta_j}{dp_j} = -\frac{p}{p_j^2} \cdot [1+\tau(\theta)] \cdot \frac{1}{\tau'(\theta_j)} = -\frac{1+\tau(\theta_j)}{p_j \cdot \tau'(\theta_j)}$$

$$\frac{d\ln\left(\theta_j\right)}{d\ln\left(p_j\right)} = -\frac{1+\tau(\theta_j)}{\theta_j \cdot \tau'(\theta_j)} = \frac{-1}{d\ln(1+\tau(\theta_j))/d\ln(\theta_j)}.$$

From (11), we infer that

$$\frac{d\ln(1+\tau)}{d\ln(\theta)} = \frac{\tau}{1+\tau}\frac{d\ln(\tau)}{d\ln(\theta)} = \frac{\tau \cdot (1/2 - u)}{1-u}.$$

Hence the elasticity of the demand curve is

$$\frac{d\ln\left(\theta_j\right)}{d\ln\left(p_j\right)} = -\frac{1-u(\theta_j)}{\tau(\theta_j) \cdot [1/2 - u(\theta_j)]}. \tag{15}$$

2.10. Efficiency without price-adjustment costs

As a benchmark, we consider the case without any price-adjustment cost. In that case, seller k is free to set any price she wants to maximize labor income. That is, she choose p_k to maximize $p_k \cdot y_k$ subject to the demand constraint (14). Because of the demand constraint, labor income can be written

$$p_k \cdot y_k = p \cdot [1 + \tau(\theta)] \cdot \frac{y_k}{1 + \tau(\theta_k)} = p \cdot [1 + \tau(\theta)] \cdot \frac{1 - u(\theta_k)}{1 + \tau(\theta_k)} \cdot l_k.$$

The variables τ, u, v are linked by (9), so

$$\frac{1 - u(\theta_k)}{1 + \tau(\theta_k)} = 1 - u(\theta_k) - v(\theta_k).$$

Accordingly, seller k sets local tightness θ_k so as to minimize $u(\theta_k) + v(\theta_k)$. This is equivalent to choosing the unemployment rate u_k so as to minimize $u_k + v(u_k)$, where the unemployment and recruiting rates are related by the Beveridge curve (6). The local tightness θ_k and unemployment rate u_k are therefore chosen efficiently: (13) holds, so that in particular $\theta_k = 1$. Here we have just recovered the central efficiency result of Moen (1997).

2.11. Price rigidity

Generally, tightness and unemployment rate are not efficient because prices are somewhat rigid. The local inflation for household k is

$$\pi_k(t) = \frac{\dot{p}_k(t)}{p_k(t)}. \tag{16}$$

Changing prices is costly. As in Rotemberg (1982), households incur a quadratic price-adjustment cost when local inflation departs from normal inflation $\bar{\pi}$. This quadratic cost appears in their utility function, reflecting two phenomena.

When prices fall, or increase less than normal, workers in household k feel short-changed. Indeed, the price p_k is their hourly salary. And Bewley (1999, 2005) has shown that workers' morale dips when their wage does not grow as expected. So here we assume that workers incur a quadratic cost when wage growth $\pi_k(t)$ falls short of the normal growth $\bar{\pi}$.

When prices rise, or increase more than normal, it is the customers of household k that are unhappy. Shiller (1996) shows that higher-than-normal inflation upsets customers, who feel unfairly treated when they go to the store. In fact, such inflation makes customers angry at the sellers. So here we assume that sellers internalize the anger of customers that is directed at them, and incur a quadratic cost when wage growth $\pi_k(t)$

is above the normal growth $\bar{\pi}$.[1]

Formally, the flow disutility caused by prices deviating from the norm is

$$\rho(\pi_k) = \frac{\kappa^+}{2} \cdot [\pi_k - \bar{\pi}]^2$$

if $\pi_k > \bar{\pi}$, and

$$\rho(\pi_k) = \frac{\kappa^-}{2} \cdot [\pi_k - \bar{\pi}]^2$$

if $\pi_k < \bar{\pi}$. The parameters $\kappa^+, \kappa^- > 0$ govern the price-adjustment costs. Since the costs come from different sources when inflation is too high and too low, we allow the cost parameters κ^+ and κ^- to be different. Even with different parameters, the cost function $\rho(\pi)$ is continuous and differentiable at $\bar{\pi}$ since

$$\lim_{\pi \to \bar{\pi}^+} \rho(\pi) = \lim_{\pi \to \bar{\pi}^-} \rho(\pi) = 0$$
$$\lim_{\pi \to \bar{\pi}^+} \rho'(\pi) = \lim_{\pi \to \bar{\pi}^-} \rho'(\pi) = 0,$$

so that we can complete the definition of the cost function at $\bar{\pi}$ by $\rho(\bar{\pi}) = 0$ and $\rho'(\bar{\pi}) = 0$. Initially we consider the symmetric case $\kappa^+ = \kappa^- = \kappa$. We later look at the asymmetric case $\kappa^+ < \kappa^-$ to create a kink in the Phillips curve, as observed in figure 1A.

2.12. People's preferences

People care about two things: their consumption of services and their social status, measured by their relative wealth. In addition people incur a cost from price changes. Each household maximizes the discounted sum of flow utilities,

$$\int_0^\infty e^{-\delta t} \left\{ \ln\left(c_j(t)\right) + \sigma \cdot \left[\frac{b_j(t)}{p(t)} - \frac{b(t)}{p(t)}\right] - \frac{\kappa}{2} \cdot [\pi_j - \bar{\pi}]^2 \right\} dt,$$

where $\delta > 0$ is the time discount rate, $\sigma > 0$ indicates concerns for social status, $c_j(t) = \int_0^1 c_{jk}(t)\, dk$ is total consumption of services, $b_j(t)$ is saving in government bonds, and $b(t) = \int_0^1 b_j(t)\, dj$ is aggregate wealth in the economy.

[1]This is a reduced-form way to capture how sellers internalize customers' anger at price increases. For a complete model of why price increases anger customers, and how firms internalize such anger to maximize profits, see Eyster, Madarasz, and Michaillat (2021).

2.13. People's budget constraint

People are subject to a budget constraint. This constraint takes the form of a law of motion of government bond holdings. For household j, the law of motion is

$$\dot{b}_j = i \cdot b_j - \int_0^1 p_k y_{jk}\, dk + p_j y_j.$$

Because of the matching process and the equalization of prices achieved through directed search, the household's expenditure on services can be rewritten as follows:

$$\int_0^1 p_k y_{jk}\, dk = \int_0^1 p_k [1+\tau(\theta_k)] c_{jk}\, dk = p \cdot [1+\tau(\theta)] \cdot \int_0^1 c_{jk}\, dk = p \cdot [1+\tau(\theta)] \cdot c_j.$$

Then, because of the matching process, the household's income becomes

$$p_j \cdot y_j = p_j \cdot [1 - u(\theta_j(p_j))] \cdot l_j.$$

Accordingly, the the law of motion can be written as

$$\dot{b}_j = i \cdot b_j - p \cdot [1+\tau(\theta)] \cdot \int_0^1 c_{jk}\, dk + p_j \cdot [1 - u(\theta_j(p_j))] \cdot l_j. \tag{17}$$

3. Model solution

We now solve the model by Hamiltonian.

3.1. Construction of the Hamiltonian

The main step is to solve household j's maximization problem, which we do by Hamiltonian. The Hamiltonian of household j's problem is

$$\begin{aligned}\mathcal{H}_j = {} & \ln\left(c_j\right) + \sigma \cdot \left[\frac{b_j}{p} - \frac{b}{p}\right] - \frac{\kappa}{2} \cdot [\pi_j - \bar{\pi}]^2 \\ & + \mathcal{A}_j \cdot \left[i \cdot b_j - p \cdot [1+\tau] \cdot c_j + p_j \cdot [1 - u(\theta_j(p_j))] \cdot l_j\right] \\ & + \mathcal{B}_j \cdot \pi_j \cdot p_j.\end{aligned}$$

The control variables are consumption c_j and inflation π_j. The state variables are bond holdings b_j and price level p_j. The costate variables are $\mathcal{A}_j$, which applies to the law of motion of bond holdings (17), and $\mathcal{B}_j$, which applies to the law of motion of the price level (16).

We focus on a symmetric solution of model, in which all households behave the same. In this symmetric situation, we can drop the index j.w

3.2. First-order condition with respect to consumption

We begin with the first-order condition $d\mathcal{H}_j/dc_j = 0$. It gives

$$
\begin{aligned}
1/c_j &= \mathcal{A}_j \cdot p \cdot [1+\tau] \\
1/\mathcal{A} &= p \cdot [1+\tau] \cdot c \\
1/\mathcal{A} &= p \cdot y.
\end{aligned}
\tag{18}
$$

Taking the log and then time derivative of this last equation yields

$$
\begin{aligned}
-\ln(\mathcal{A}) &= \ln(p) + \ln(y) \\
-\frac{\dot{\mathcal{A}}}{\mathcal{A}} &= \pi + \frac{\dot{y}}{y}.
\end{aligned}
\tag{19}
$$

3.3. First-order condition with respect to inflation

Next we turn to the first-order condition $d\mathcal{H}_j/d\pi_j = 0$. It yields

$$
\begin{aligned}
\mathcal{B}_j \cdot p_j &= \kappa \cdot (\pi_j - \bar{\pi}) \\
\mathcal{B} &= \frac{\kappa}{p} \cdot (\pi - \bar{\pi}).
\end{aligned}
\tag{20}
$$

Taking the log and then time derivative of the last equation yields

$$
\begin{aligned}
\ln(\mathcal{B}) &= \ln(\kappa) - \ln(p) + \ln(\pi - \bar{\pi}). \\
\frac{\dot{\mathcal{B}}}{\mathcal{B}} &= -\pi + \frac{\dot{\pi}}{\pi - \bar{\pi}}.
\end{aligned}
\tag{21}
$$

3.4. First-order condition with respect to saving

The next first-order condition is $d\mathcal{H}_j/db_j = \delta \cdot \mathcal{A}_j - \dot{\mathcal{A}}_j$. It gives

$$
\frac{\sigma}{p} + \mathcal{A}_j \cdot i = \delta \cdot \mathcal{A}_j - \dot{\mathcal{A}}_j.
$$

Reshuffling the terms yields

$$\frac{\dot{\mathcal{A}}}{\mathcal{A}} = \delta - i - \frac{\sigma}{p \cdot \mathcal{A}}$$

Using $1/(p \cdot \mathcal{A}) = y$, which comes from (18), we obtain

$$\frac{\dot{\mathcal{A}}}{\mathcal{A}} = \delta - (i + \sigma \cdot y) \tag{22}$$

3.5. First-order condition with respect to price

The final first-order condition is $d\mathcal{H}_j/dp_j = \delta \cdot \mathcal{B}_j - \dot{\mathcal{B}}_j$. This condition becomes

$$\mathcal{A}_j \cdot (1 - u_j) \cdot l_j - \mathcal{A}_j \cdot p_j \cdot l_j \cdot u'(\theta_j) \cdot \theta'(p_j) + \mathcal{B}_j \cdot \pi_j = \delta \cdot \mathcal{B}_j - \dot{\mathcal{B}}_j.$$

From the elasticity (10), we have the following derivative:

$$u'(\theta_j) = -\frac{u(\theta_j)}{2 \cdot \theta_j}.$$

And from the elasticity (15), we have

$$\theta'(p_j) = -\frac{\theta_j[1 - u(\theta_j)]}{\tau(\theta_j) \cdot p_j \cdot [1/2 - u(\theta_j)]}.$$

Hence,

$$p_j \cdot u'(\theta_j) \cdot \theta'(p_j) = \frac{1 - u(\theta_j)}{1 - 2u(\theta_j)} \cdot \frac{u(\theta_j)}{\tau(\theta_j)}.$$

Reshuffling terms gives:

$$(\delta - \pi_j) \cdot \mathcal{B}_j - \dot{\mathcal{B}}_j = \mathcal{A}_j \cdot y_j \cdot \left[1 - \frac{u(\theta_j)}{\tau(\theta_j)[1 - 2u(\theta_j)]}\right]$$

$$-\frac{\dot{\mathcal{B}}}{\mathcal{B}} = \pi - \delta + \frac{\mathcal{A} \cdot y}{\mathcal{B}} \cdot \left[1 - \frac{u(\theta)}{\tau[1 - 2u(\theta)]}\right].$$

Using $y \cdot \mathcal{A} = 1/p$, which comes from (18), and $\mathcal{B} = \kappa(\pi_j - \bar{\pi})/p$, which comes from (20), we link inflation to the unemployment rate:

$$-\frac{\dot{\mathcal{B}}}{\mathcal{B}} = \pi - \delta + \frac{1}{\kappa} \cdot \frac{1}{\pi - \bar{\pi}} \cdot \left[1 - \frac{u}{\tau[1 - 2u]}\right].$$

Then, using the expression for τ given by (9), we conclude that

$$-\frac{\dot{\mathcal{B}}}{\mathcal{B}} = \pi - \delta + \frac{1}{\kappa} \cdot \frac{1}{\pi - \bar{\pi}} \cdot \left[1 - \frac{u}{v(u)} \cdot \frac{1 - u - v(u)}{1 - 2u}\right], \tag{23}$$

where u is the unemployment rate and $v(u)$ is the recruiting rate, given by (6).

3.6. Aggregate demand: discounted Euler equation

We now derive the aggregate demand from optimal consumption and saving. Combining the first-order conditions (19) and (22), we obtain an Euler equation:

$$\frac{\dot{y}}{y} = (i - \pi + \sigma \cdot y) - \delta. \tag{24}$$

In the Euler equation, $i - \pi$ is the real interest rate, which gives the financial return on saving, while $\sigma \cdot y$ is the marginal rate of substitution between wealth and consumption, which gives the hedonic return on saving. Just as in the New Keynesian model developed by Michaillat and Saez (2021b), the presence of wealth in the utility function produces a discounted Euler equation (McKay, Nakamura, and Steinsson 2017).

In steady state ($\dot{y} = 0$), the Euler equation yields a nondegenerate aggregate-demand curve:

$$y = \frac{\delta - i + \pi}{\sigma} \tag{25}$$

The steady-state Euler equation gives the amount of output demanded by households when they optimally save over time. The preference over social status and wealth, σ, determines the slope of the curve.

When we solve the model we will focus on inflation π and unemployment u, so we rewrite the Euler equation in terms of the unemployment rate u instead of output y. Since $y = (1 - u)l$, we have $\dot{y} = -\dot{u} \cdot l$ and

$$\frac{\dot{y}}{y} = -\frac{\dot{u}}{1 - u}.$$

Accordingly, the Euler equation (24) becomes

$$\frac{\dot{u}}{1 - u} = \delta - \left[i - \pi + \sigma \cdot (1 - u) \cdot l\right] \tag{26}$$

The steady-state Euler equation (25) becomes

$$1-u=\frac{\delta-i+\pi}{\sigma\cdot l} \tag{27}$$

3.7. Aggregate supply: Phillips curve

Next we derive the aggregate supply from optimal pricing. Combining the first-order conditions (21) and (23), we obtain a Phillips curve linking inflation to unemployment:

$$\dot{\pi}=\delta\cdot(\pi-\bar{\pi})-\frac{1}{\kappa}\cdot\left[1-\frac{u}{v(u)}\cdot\frac{1-u-v(u)}{1-2u}\right]. \tag{28}$$

In the Phillips curve, the parameter κ is the price-adjustment cost. The term in square bracket measures the inefficiency of the labor market. When the economy is inefficiently tight, $v>u$ so the term is positive. When the economy is efficient, $v=u$ so the term is zero. When the economy is inefficiently slack, $v<u$ so the term is negative.

In steady state ($\dot{\pi}=0$), the Phillips curve yields an aggregate-supply curve:

$$\kappa\cdot\delta\cdot(\pi-\bar{\pi})=1-\frac{u}{v(u)}\cdot\frac{1-u-v(u)}{1-2u} \tag{29}$$

The steady-state Phillips curve gives the inflation chosen by households given the competition they face from other households, and the cost they face in changing prices. The price-adjustment cost, κ, determines the slope of curve.

If we assume that the price-adjustment cost is asymmetric, then the left-hand side would be $\kappa^{+}\cdot\delta\cdot(\pi-\bar{\pi})$ when $\pi>\bar{\pi}$ and $\kappa^{-}\cdot\delta\cdot(\pi-\bar{\pi})$ when $\pi<\bar{\pi}$. In that case, the curve would have a kink at $\pi=\bar{\pi}$.

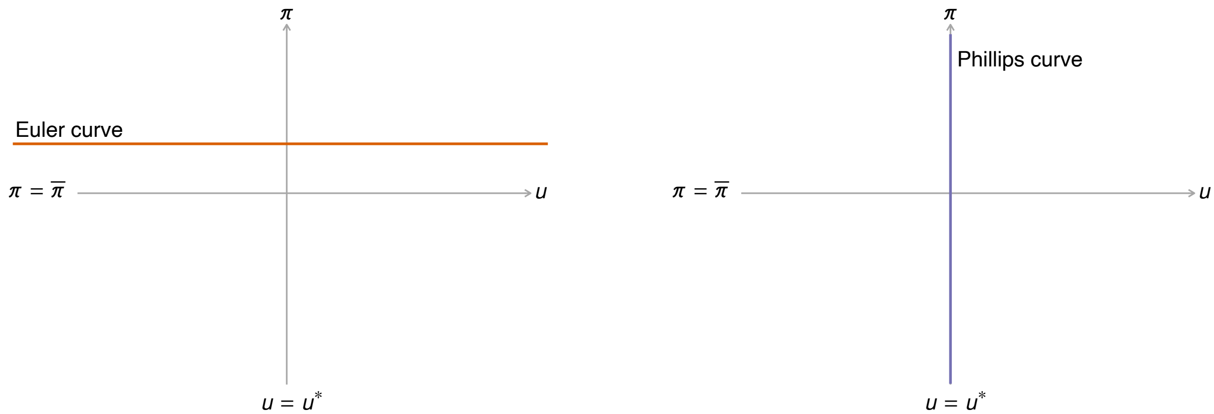
3.8. Special cases

Before moving forward, let's pause to examine a few special cases. Consider the steady-state Euler and Phillips curves in a standard unemployment-inflation (u,π) plane.

Without wealth in the utility ($\sigma=0$), the steady-state Euler curve (25) would be horizontal:

$$\pi=i-\delta. \tag{30}$$

This curve just imposes that the real interest rate equals the discount rate. Then inflation



A. No wealth in the utility

B. No price rigidity

FIGURE 2. Steady-state Euler and Phillips curves in special cases

π is determined one-for-one by the nominal interest rate i. This is the Fisher effect: a higher interest rate leads to higher inflation. The curve is degenerate because it does not involve unemployment u. The curve would also be horizontal in an output-inflation plane, since it does not involve output.

Without price rigidity ($\kappa = 0$), the steady-state Phillips curve (29) would be vertical:

$$u = u^*. \tag{31}$$

Indeed, without price rigidity, unemployment and recruiting rates must be equal, so the unemployment rate is efficient ($u = u^*$). The curve would also be vertical in an output-inflation plane, since the unemployment rate pins down the level of output irrespective of inflation: $y = (1 - u^*)l$.

3.9. Divine coincidence

The divine coincidence directly appears in the Phillips curve (29). The equation shows that inflation is on target ($\pi = \bar{\pi}$) if and only if the right-hand side is zero, which happens if and only if unemployment are efficient ($u = v$ so $u = u^*$). Therefore, if the government is able to bring unemployment to its efficient level, it will also automatically ensure that inflation is on target. In other words, when the government achieves its employment mandate, it also automatically achieves its price mandate.

Both mandates are achieved by moving the Euler curve along the Phillips curve to arrive at the point where $u = u^*$ and $\pi = \bar{\pi}$. This can be done for instance through monetary policy, which affects the nominal interest rate i and therefore the location of the Euler curve (25). The efficient nominal interest rate i^* ensures that inflation is on target ($\pi = \bar{\pi}$) and unemployment is efficient ($u = u^*$). From the Euler curve (25), we obtain an expression for the efficient nominal interest rate i^*:

$$1 - u^* = \frac{\delta - i^* + \bar{\pi}}{\sigma \cdot l}$$

so that the efficient nominal interest rate is

$$i^* = \bar{\pi} + \delta - \sigma \cdot (1 - u^*) \cdot l. \tag{32}$$

When the nominal interest rate is set to i^*, the model admits a steady-state solution in which the divine coincidence prevails: $(\pi, u) = (\bar{\pi}, u^*)$ satisfies both (29) and (25) when i is given by (32).

Such divine steady state exists only when $i^* \geq 0$. If $i^* < 0$, then the divine steady state is not a solution of the model since it would violate the zero lower bound constraint that $i \geq 0$. In that case the central bank would resort to setting $i = 0$.

4. Model dynamics

The model admits a steady-state solution in which the divine coincidence prevails. At that point, inflation is on target and the unemployment rate is efficient. To investigate the dynamics around that steady state, we now linearize the differential equations (26) and (28) around the divine steady state $(\bar{\pi}, u^*)$.

4.1. Linearized model around the divine steady state

We begin by introducing the deviations from efficient steady state: $\hat{u} = u - u^*$ and $\hat{\pi} = \pi - \bar{\pi}$. We also allow the nominal interest rate to follow a Taylor rule:

$$i = i^* + \phi(\pi - \bar{\pi}), \tag{33}$$

where $i^* \geq 0$ is the efficient nominal interest rate and $\phi \geq 0$ is the automatic response of the nominal interest rate to inflation. With the Taylor rule, the Euler equation (26) becomes

$$\frac{\dot{u}}{1 - u} = \delta + \phi\bar{\pi} - \left[i^* + (\phi - 1)\pi + \sigma \cdot (1 - u) \cdot l\right]$$

Using the value of i^* from (32), we can simplify the Euler equation:

$$\frac{\dot{u}}{1-u} = \sigma \cdot (u - u^*) \cdot l - (\phi - 1)(\pi - \bar{\pi}). \tag{34}$$

We start by linearizing differential equation (34) around $(u^*, \bar{\pi})$. The linearized version is easy to derive since the differential equation is almost linear:

$$\dot{u} = (1 - u^*) \cdot \left[\sigma \cdot l \cdot \hat{u} - (\phi - 1)\hat{\pi}\right].$$

The linearized version of differential equation (28) around $(u^*, \bar{\pi})$ is a little bit more complicated to derive. The key is to find the partial derivative of

$$\mathcal{P}(u) = -\frac{1}{\kappa} \cdot \left[1 - \frac{u}{v(u)} \cdot \frac{1 - u - v(u)}{1 - 2u}\right]$$

with respect to u at u^*. This will be the coefficient in front of $\hat{u}$ in the linearized equation. To do that, we need the derivative of

$$\mathcal{Q}(u) = \frac{u}{v(u)} \cdot \frac{1 - u - v(u)}{1 - 2u}$$

with respect to u at u^*. From (6), we know that the elasticity of $v(u)$ with respect to u is $d\ln(v)/d\ln(u) = -1$, so we have

$$\frac{d\ln(\mathcal{Q})}{d\ln(u)} = 1 + 1 + \frac{-u + v}{1 - u - v} + \frac{2u}{1 - 2u}.$$

When $u = u^*$, we also have $u = v$, so the elasticity simplifies to

$$\frac{d\ln(\mathcal{Q})}{d\ln(u)} = 2\left[1 + \frac{u^*}{1 - 2u^*}\right].$$

Moreover, $Q(u^*) = 1$ so we have the following derivative:

$$\frac{d\mathcal{Q}}{du} = \frac{2}{u^*}\left[1 + \frac{u^*}{1 - 2u^*}\right].$$

Since $\mathcal{P}'(u) = \mathcal{Q}'(u)/\kappa$, we finally get

$$\frac{d\mathcal{Q}}{du} = \frac{2}{\kappa u^*} \cdot \frac{1 - u^*}{1 - 2u^*}.$$

Accordingly, the linearized Phillips curve is

$$\dot{\pi} = \delta\hat{\pi} + \frac{2}{\kappa u^*} \cdot \frac{1 - u^*}{1 - 2u^*} \cdot \hat{u} \tag{35}$$

4.2. Classification of the linearized model

The Euler-Phillips system (26)-(28) is nonlinear, but we can determine its properties around the divine steady state from its linearized form. Combining (4.1) and (35), we find that around the divine steady state $[u, \pi] = [u^*, \bar{\pi}]$, the linearized Euler-Phillips system is

$$\begin{bmatrix} \dot{u}(t) \\ \dot{\pi}(t) \end{bmatrix} = \begin{bmatrix} \sigma y^* & -(\phi - 1)(1 - u^*) \\ 2(1 - u^*)/[\kappa u^*(1 - 2u^*)] & \delta \end{bmatrix} \begin{bmatrix} \hat{u}(t) \\ \hat{\pi}(t) \end{bmatrix}. \tag{36}$$

We denote by $\boldsymbol{M}$ the matrix in (36), and by $\mu_1 \in \mathbb{C}$ and $\mu_2 \in \mathbb{C}$ the two eigenvalues of $\boldsymbol{M}$, assumed to be distinct.

We classify the Euler-Phillips system from the trace and determinant of $\boldsymbol{M}$ (Hirsch, Smale, and Devaney 2013, pp. 61–64). The classification relies on the property that $\mathrm{tr}(\boldsymbol{M}) = \mu_1 + \mu_2$ and $\det(\boldsymbol{M}) = \mu_1\mu_2$. Using (36), we compute the trace and determinant of $\boldsymbol{M}$:

$$\begin{aligned} \mathrm{tr}(\boldsymbol{M}) &= \delta + \sigma y^* \\ \det(\boldsymbol{M}) &= \delta\sigma y^* + \frac{2(\phi - 1)}{\kappa} \cdot \frac{(1 - u^*)^2}{u^*(1 - 2u^*)}. \end{aligned}$$

Clearly, $\mathrm{tr}(\boldsymbol{M}) > \delta > 0$. Further, since $\phi \geq 0$, we have $\phi - 1 \geq -1$, so that

$$\det(\boldsymbol{M}) \geq \delta\sigma y^* - \frac{2}{\kappa} \frac{(1 - u^*)^2}{u^*(1 - 2u^*)}.$$

Just as in Michaillat and Saez (2021b), we assume that the marginal utility of wealth is large enough to ensure that the determinant is positive:

$$\sigma \geq \frac{2}{\kappa\delta l} \cdot \frac{1 - u^*}{u^*(1 - 2u^*)}. \tag{37}$$

Under this assumption, $\mathrm{tr}(\boldsymbol{M}) > 0$ and $\det(\boldsymbol{M}) > 0$, so the Euler-Phillips system is a source for any $\phi \geq 0$. When prices are more flexible (lower κ), the marginal utility of wealth need to be larger to ensure that the determinant is positive and the system is a source.

Indeed $\det(\boldsymbol{M}) > 0$ indicates that μ_1 and μ_2 are either real, nonzero, and of the same sign; or complex conjugates. Since in addition $\mathrm{tr}(\boldsymbol{M}) > 0$, μ_1 and μ_2 must be either real and positive, or complex with a positive real part. Indeed, if μ_1 and μ_2 were real and negative, $\mathrm{tr}(\boldsymbol{M}) = \mu_1 + \mu_2 < 0$. If they were complex with a negative real part, $\mathrm{tr}(\boldsymbol{M}) = \mu_1 + \bar{\mu_1} = 2\,\mathrm{Re}(\mu_1) < 0$.

When μ_1 and μ_2 are real and positive, the solution of the linearized system is

$$[\hat{u}(t), \hat{\pi}(t)] = x_1 e^{\mu_1 t}\boldsymbol{v}_1 + x_2 e^{\mu_2 t}\boldsymbol{v}_2, \tag{38}$$

where $\boldsymbol{v}_1 \in \mathbb{R}^2$ and $\boldsymbol{v}_2 \in \mathbb{R}^2$ are the linearly independent eigenvectors respectively associated with the eigenvalues μ_1 and μ_2, and $x_1 \in \mathbb{R}$ and $x_2 \in \mathbb{R}$ are constants determined by the terminal condition (Hirsch, Smale, and Devaney 2013, p. 35). From (38), we see that the Euler-Phillips system is a source when $\mu_1 > 0$ and $\mu_2 > 0$. The solutions start at 0 when $t \to -\infty$ and go to infinity parallel to $\boldsymbol{v}_2$ when $t \to +\infty$.

When μ_1 and μ_2 are complex conjugates with a positive real part, we write the eigenvalues as $\mu_1 = \mu + i\beta$ and $\mu_2 = \mu - i\beta$ with $\mu > 0$. We also write the eigenvector associated with μ_1 as $\boldsymbol{v}_1 + i\boldsymbol{v}_2$, where the vectors $\boldsymbol{v}_1 \in \mathbb{R}^2$ and $\boldsymbol{v}_2 \in \mathbb{R}^2$ are linearly independent. Then the solution takes a more complicated form:

$$\begin{bmatrix} \hat{u}(t) \\ \hat{\pi}(t) \end{bmatrix} = e^{\mu t}\,[\boldsymbol{v}_1, \boldsymbol{v}_2] \begin{bmatrix} \cos(\beta t) & \sin(\beta t) \\ -\sin(\beta t) & \cos(\beta t) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix},$$

where $[\boldsymbol{v}_1, \boldsymbol{v}_2] \in \mathbb{R}^{2\times 2}$ is a 2×2 matrix, and $x_1 \in \mathbb{R}$ and $x_2 \in \mathbb{R}$ are constants determined by the terminal condition (Hirsch, Smale, and Devaney 2013, pp. 44–55). These solutions wind periodically around the steady state, moving away from it as $t \to +\infty$. Hence, the Euler-Phillips system is a spiral source.

Overall, when the marginal utility of wealth is large enough (equation (37)), the linearized model is source whether monetary policy is active ($\phi > 1$) or passive ($0 \geq \phi \geq 1$). This is just as in the New Keynesian model. That model is a source irrespective of monetary policy when the marginal utility of wealth is large enough (Michaillat and Saez 2021b, proposition 1).

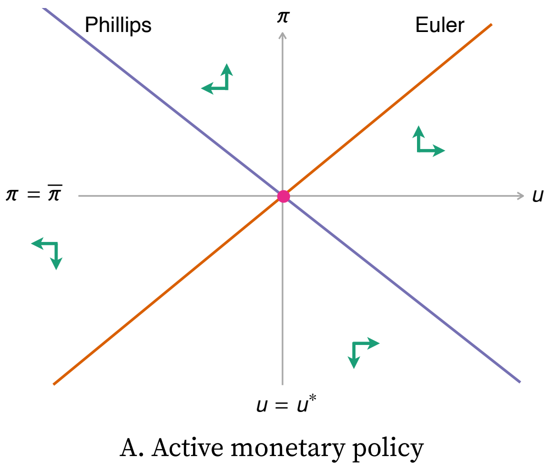


FIGURE 3. Phase Diagrams of the Linearized Model

The figure displays phase diagrams for the dynamical system generated by the linearized Euler equation (4.1) and Phillips equation (35): u is output; π is inflation; u^* is the efficient rate of unemployment; the Euler line is the locus $\dot{u} = 0$; and the Phillips line is the locus $\dot{\pi} = 0$. The monetary-policy rate is given by $i = i^* + \phi(\pi - \overline{\pi})$: when monetary policy is active, $\phi > 1$; when monetary policy is passive, $0 < \phi < 1$. The figure shows that the linearized model is always a source.

4.3. Local uniqueness of the model solution

We assume that the marginal utility of wealth is large enough: equation (37) holds. Therefore, the Euler-Phillips system is a source, which implies that the solution of the model is always locally unique—even when monetary policy is passive. The only solution in the vicinity of the divine steady state is to jump to the steady state and stay there. If the economy jumped somewhere else, unemployment or inflation would diverge away from the steady state.

Unlike in the New Keynesian model, indeterminacy is never a risk, so the central bank does not need to worry about how strongly its policy rate responds to inflation. The central bank can even follow an interest-rate peg without creating indeterminacy.

4.4. Phase diagram

We now construct the phase diagrams of the linearized model to understand its dynamics better.[2] The diagrams are displayed in figure 3.

We begin with the linearized Phillips equation (35), which gives $\dot{\pi}$. First, we plot the

[2]Michaillat and Saez (2021b) show for instance how to use the phases diagrams to study ZLB episodes of finite duration and forward guidance. They also show how to construct sample solutions to the Euler-Phillips system using the phase diagrams.

locus $\dot{\pi} = 0$, which we label "Phillips." The locus is given by

$$\hat{\pi} = -\frac{2}{\delta\kappa u^*} \cdot \frac{1-u^*}{1-2u^*} \cdot \hat{u}. \tag{39}$$

The line is downward sloping, and goes through the point $[u = u^*, \pi = \bar{\pi}]$. Second, we plot the arrows giving the directions of the trajectories solving the Euler-Phillips system. The sign of $\dot{\pi}$ is given by (35): any point above the Phillips line (where $\dot{\pi} = 0$) has $\dot{\pi} > 0$, and any point below the line has $\dot{\pi} < 0$. So inflation is rising above the Phillips line and falling below it.

We next turn to the linearized Euler equation (4.1), which gives $\dot{u}$. We plot the locus $\dot{u} = 0$, which we label "Euler." The locus is given by

$$\hat{u} = \frac{\phi - 1}{\sigma \cdot l} \cdot \hat{\pi}. \tag{40}$$

The line goes through the point $[u = u^*, \pi = \bar{\pi}]$. It is downward sloping if $\phi < 1$, vertical if $\phi = 1$, and upward sloping if $\phi > 1$.

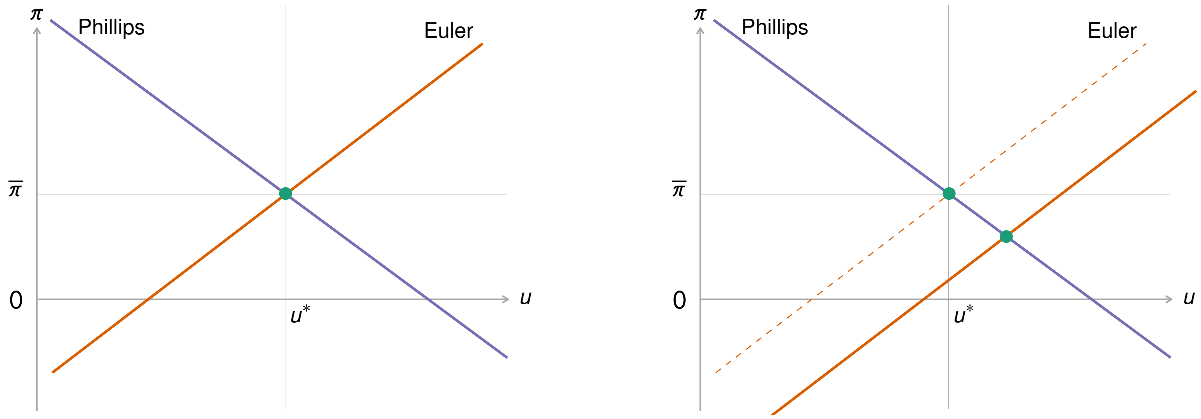
Next we use the Euler equation (4.1) to determine the sign of $\dot{u}$. We first consider an active monetary policy ($\phi > 1$), as showed in figure 3A. Any point above the Euler line has $\dot{u} < 0$, and any point below it has $\dot{u} > 0$. Hence, in all four quadrants of the phase diagram, the trajectories move away from the steady state. We conclude that the Euler-Phillips system is a source when monetary policy is active.

Second, we consider a passive monetary policy ($\phi \in [0, 1)$), as showed in figure 3B. Now any point above the Euler line has $\dot{u} > 0$, and any point below it has $\dot{u} < 0$. Nevertheless, in all four quadrants of the phase diagram, the trajectories move away from the steady state. We conclude that the Euler-Phillips system remains a source when monetary policy is passive.

The phase diagrams also illustrate the origin of the condition (37) on the marginal utility of wealth. The Euler-Phillips system remains a source with passive monetary policy as long as the Euler line is steeper than the Phillips line in figure 3B. The Euler line is the most flat with an interest-rate peg ($\phi = 0$), and then its slope is just the marginal utility of wealth. Thus, the marginal utility is required to be above a certain level—which is given by (37).

5. Response to shocks and application to the pandemic

Finally, we use the linearized model to study business-cycle shocks.



A. Initial position B. After negative demand shock

FIGURE 4. Response of the the linearized model to a negative demand shock

5.1. Typical recession: negative demand shock

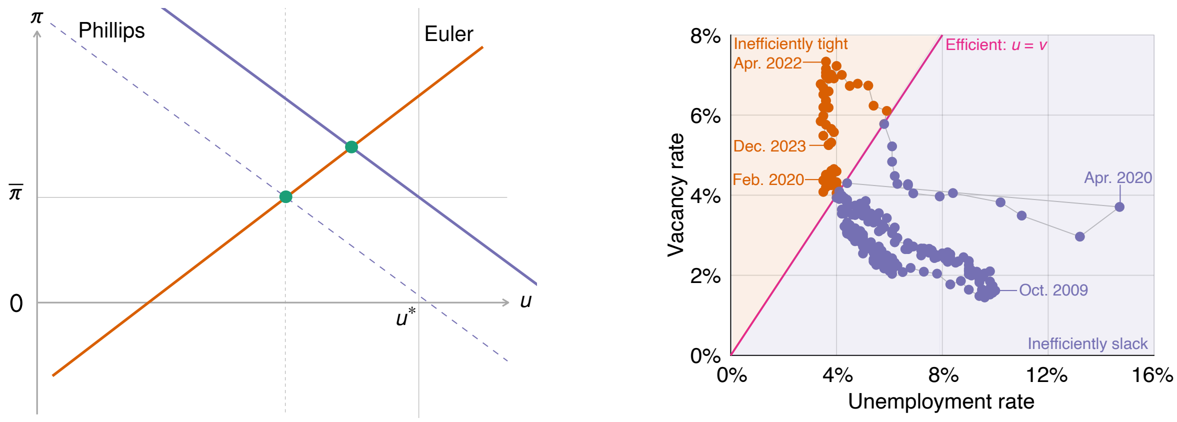
We consider first a traditional business-cycle shock: an aggregate-demand shock, which shifts the Euler curve. Such shock could be caused by a change in sentiment, reflected in a different marginal utility of wealth σ. A high σ indicates a low desire for consumption and therefore a low aggregate demand. The shock could also be a change in monetary policy, affecting the nominal interest rate i. A high i makes it more appealing to save and therefore leads to a low aggregate demand.

In the linearized model, a negative demand shock leads to an outward shift of the Euler line (figure 4). In response to the negative shock, unemployment is higher, the unemployment gap is also higher, and inflation is lower. The economy is moving along the Phillips curve so there is a negative correlation between unemployment and inflation.

5.2. Pandemic: negative supply shock

We consider next an unusual business-cycle shock: an aggregate-supply shock, which shifts the Phillips curve. Such shock is caused by a shift in the Beveridge curve (6), so either a change in the job-separation rate s or the matching efficacy ω. Both an increase in separation or a decrease in efficacy shift the Beveridge curve outward, which leads to an increase in the efficient unemployment rate u^*.

In the linearized model, a negative supply shock leads to an outward shift of the Phillips curve (figure 5). In response to the negative shock, unemployment is higher, and inflation is higher. But the key is that the unemployment gap is lower (it has become



A. After negative supply shock

B. Pandemic shift of the US Beveridge curve

FIGURE 5. Response of the the linearized model to a negative supply shock

negative) and inflation is higher. Indeed the efficient unemployment rate has increased more than actual unemployment, so the unemployment rate is now inefficiently low. Such excessive tightness leads to higher inflation.

If the Euler line remains the same—for instance because the central bank is not aware of the shift of the Beveridge curve—then the outward shift of the Phillips curve leads to burst of inflation (figure 5A). Since the US Beveridge curve has shifted dramatically outward in the aftermath of the coronavirus pandemic (figure 5B), the flare-up in inflation in 2021–2023 might have been partly caused by excessive tightness.

6. Conclusion

Recent evidence indicates that the divine coincidence might prevail in the United States (Benigno and Eggertsson 2023; Gitti 2023). That is, inflation is stable, on target whenever the labor market is efficient. The divine coincidence is critically important for policy. If it holds, the full-employment and price-stability mandates of the Federal Reserve overlap, which greatly simplifies the Fed's task. This paper proposes a simple model in which the divine coincidence holds. The model establishes that the phenomenon of divine coincidence might not be entirely surprising. The model also highlights features of the economy that produce divine coincidence.

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