# An Earthly Model of the Divine Coincidence

Pascal Michaillat, Emmanuel Saez

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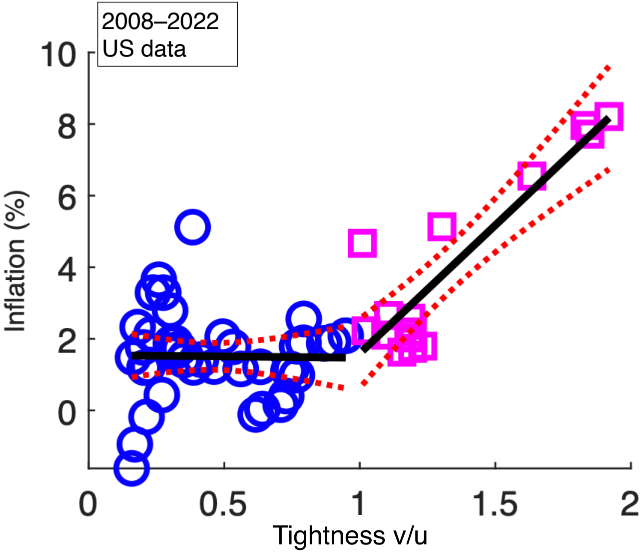
## BENIGNO, EGGERTSSON (2023): DIVINE COINCIDENCE?



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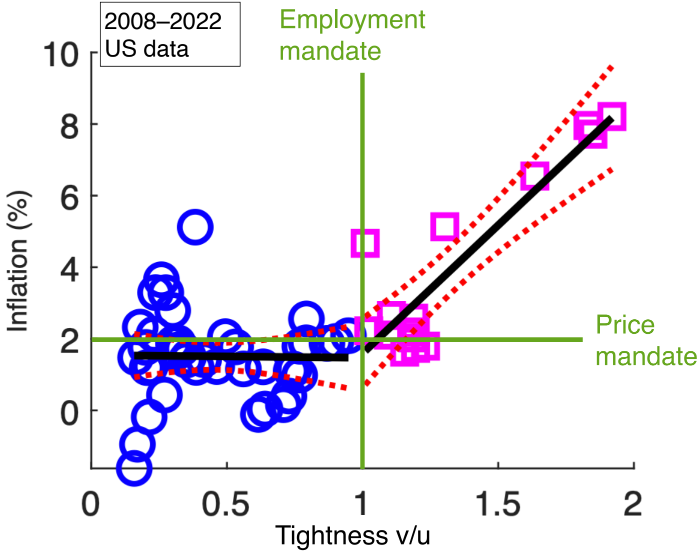

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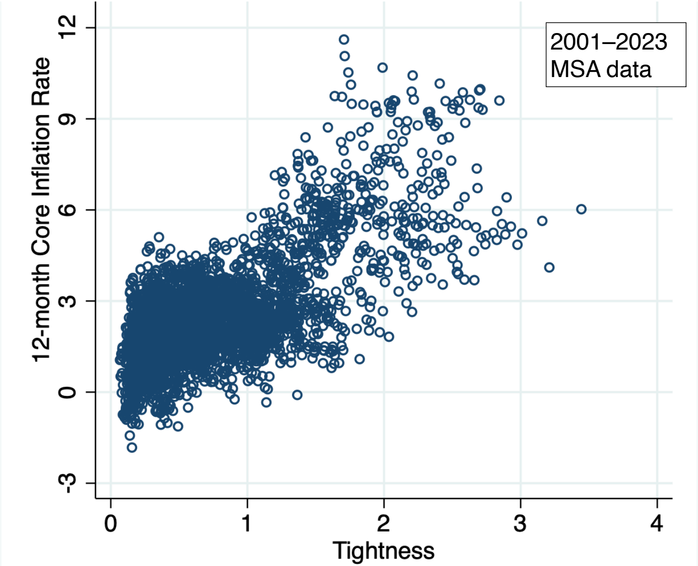
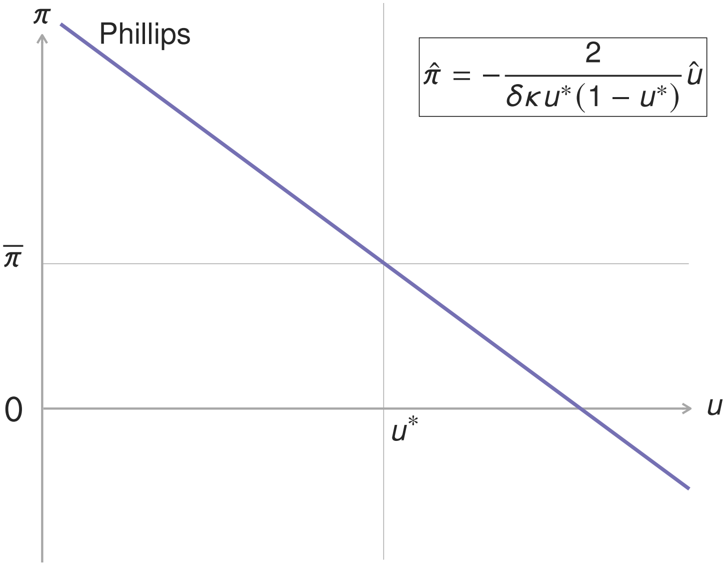


# GITTI (2023): DIVINE COINCIDENCE?

# AN EARTHLY MODEL OF DIVINE COINCIDENCE

- economical business-cycle model structure (Michaillat, Saez 2022)
  - identical households sell and buy chauffeur services
  - drivers find customers through matching $\Rightarrow$ unemployment
  - utility from being chauffeured and wealth $\Rightarrow$ AD curve
- price competition through directed search (Moen 1997)
  - chauffeurs with higher prices are hired more slowly
  - chauffeurs with lower prices are hired more quickly
- price rigidity from quadratic price-adjustment costs (Rotemberg 1982)
- divine coincidence appears: $\pi = \bar{\pi} \iff u = u^*$

# DIVINE COINCIDENCE IN THE EARTHLY MODEL



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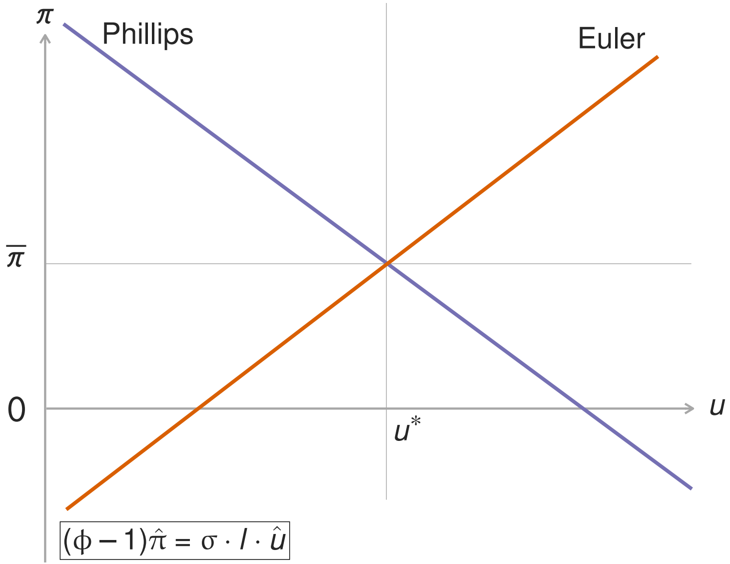

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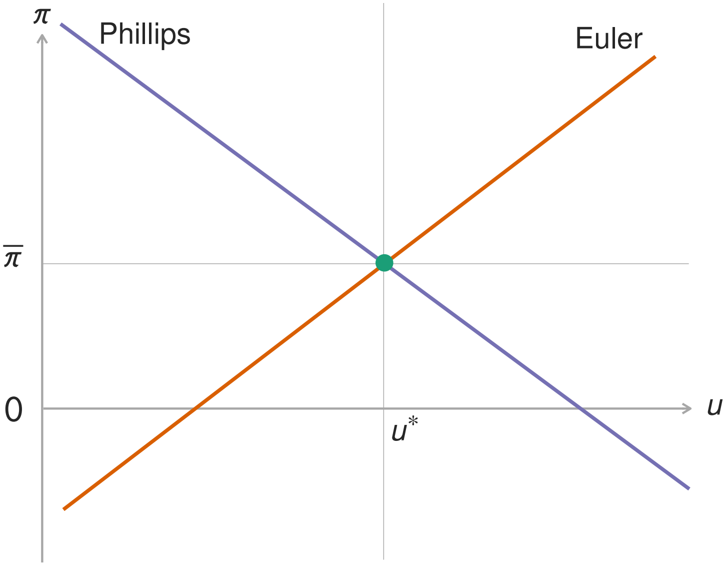
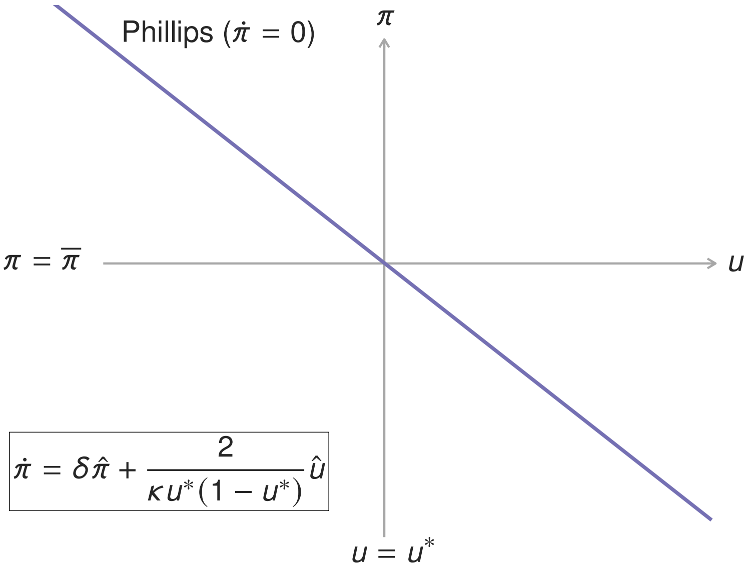


# PHASE DIAGRAM OF THE EARTHLY MODEL



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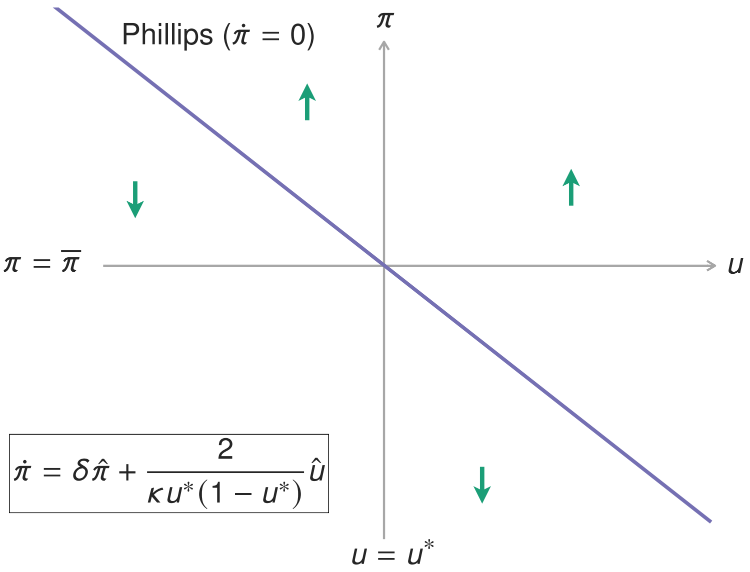
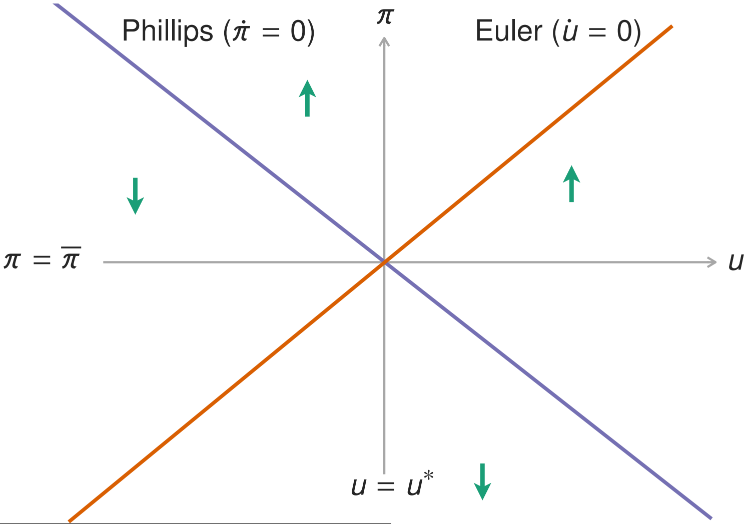


# PHASE DIAGRAM OF THE EARTHLY MODEL (TAYLOR)




$$\boxed{\dot{u} = (1 - u^*)\left[\sigma / \hat{u} + (1 - \phi)\hat{\pi}\right]}$$

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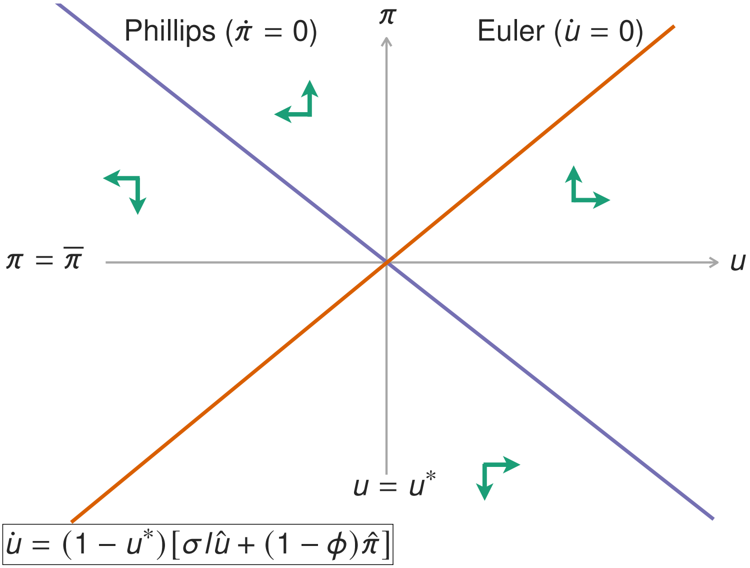
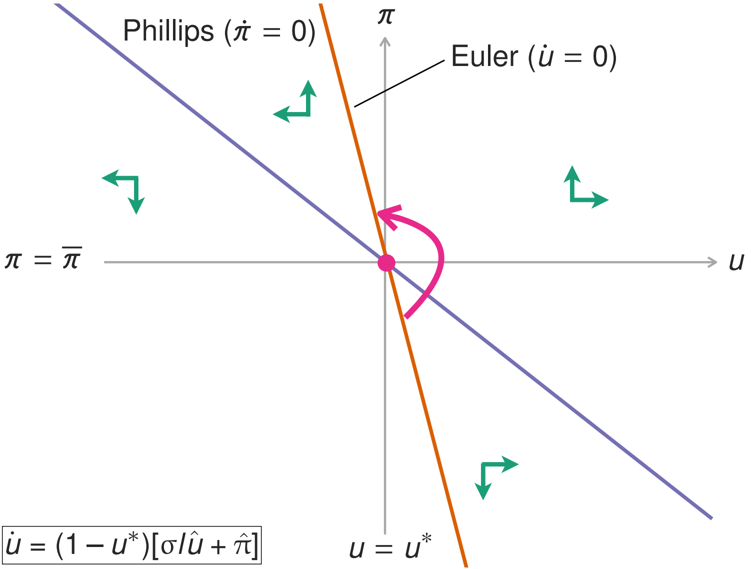
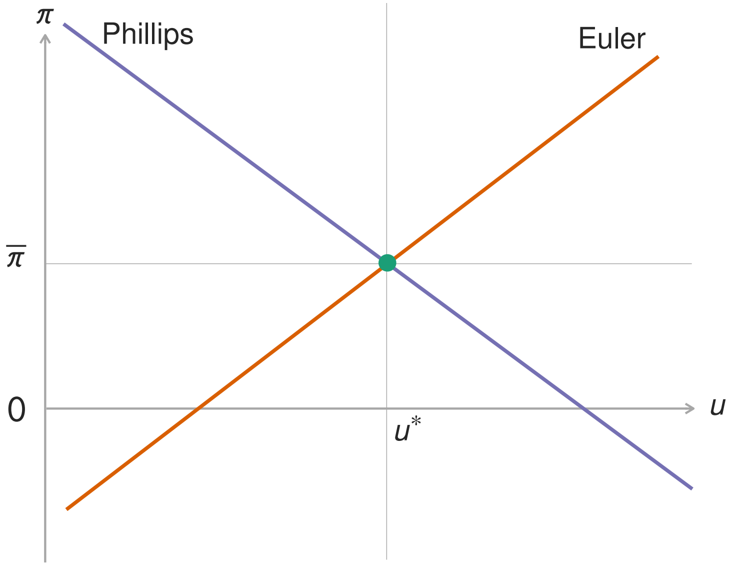


# PHASE DIAGRAM OF THE EARTHLY MODEL (PEG)



NEGATIVE AD OR MONETARY-POLICY SHOCK



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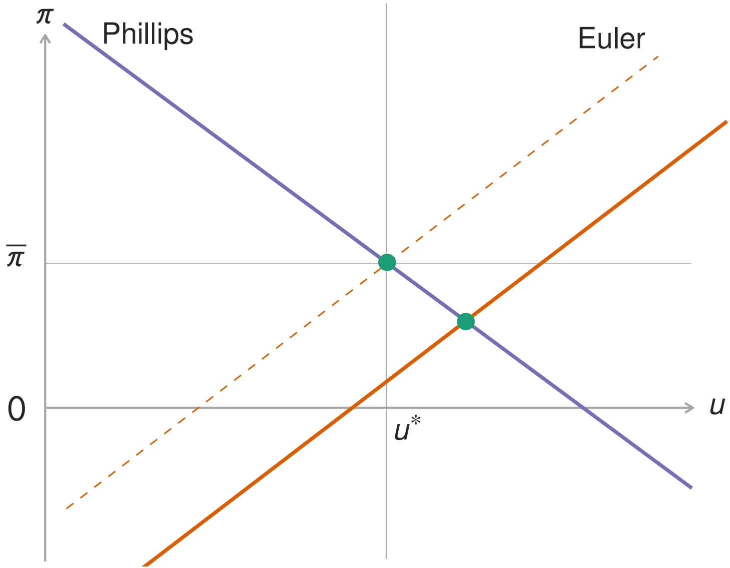
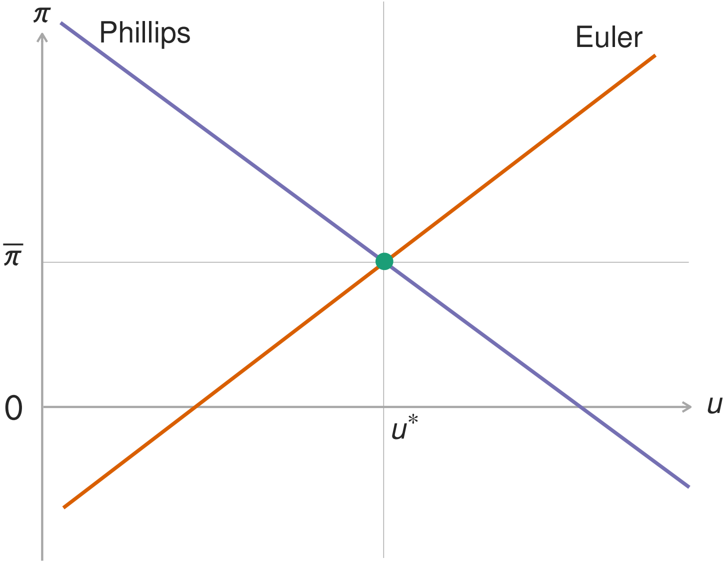


## OUTWARD SHIFT OF THE BEVERIDGE CURVE (PANDEMIC)



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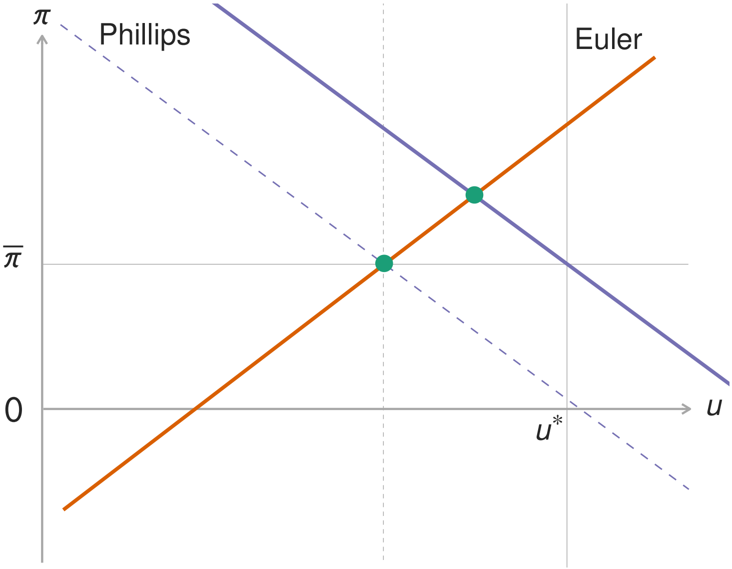
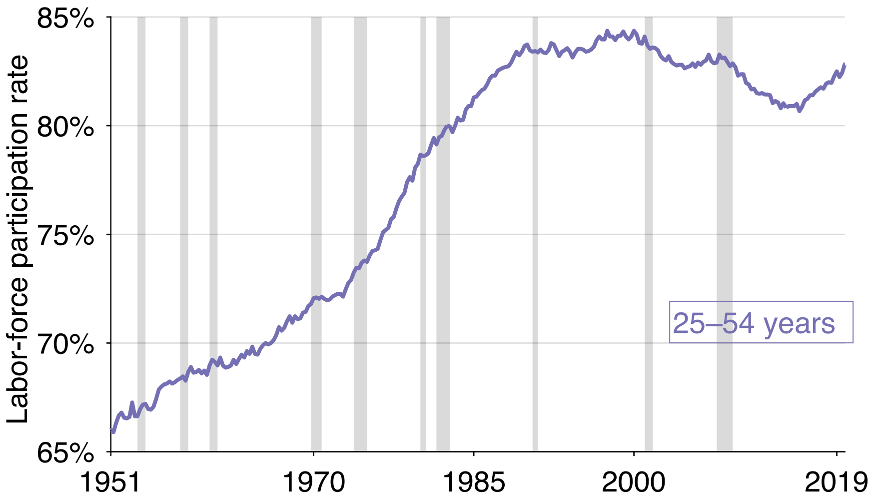


## SIMILARITIES WITH NEW KEYNESIAN DYNAMICS

- dynamical properties of the models are similar
  - dynamical system is a source with sufficient concerns for wealth
  - even with interest-rate peg and at the ZLB
  - required concerns for wealth are lower with higher price rigidity
- when dynamical system is a source:
  - economy jumps to steady state and remains there
  - no anomalies at the ZLB (Michaillat, Saez 2021)
- fluctuations in output caused by fluctuations in unemployment
  - instead of fluctuations in markups
- fluctuations in inflation caused by fluctuations in customer queues
  - instead of fluctuations in marginal costs

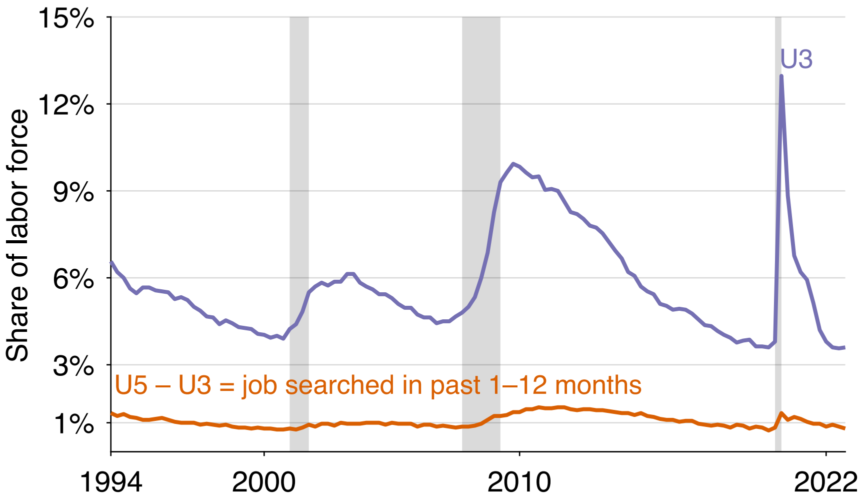
# Assumptions and derivation of the model

# US LABOR-FORCE PARTICIPATION ≈ ACYCLICAL



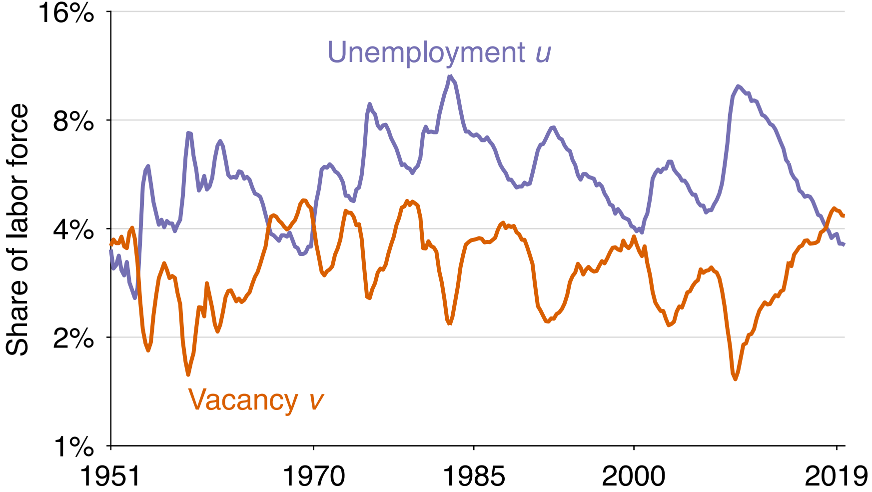

‣ Return to theory

# US MARGINAL ATTACHMENT RATE ≈ 1% LABOR FORCE




‣ Return to theory

## LOG UNEMPLOYMENT AND VACANCY RATES




▸ Return to Beveridge curve

# HOUSEHOLD UTILITY

- household $j \in [0,1]$ maximizes utility

$$\int_0^\infty e^{-\delta t} \left\{ \ln\left(c_j(t)\right) + \sigma \cdot \left[\frac{b_j(t)}{p(t)} - \frac{b(t)}{p(t)}\right] \right\} dt$$

- $\delta > 0$: time discount rate
- $\sigma > 0$: status concerns
- $c_j(t) = \int_0^1 c_{jk}(t)\, dk$: consumption of chauffeur services
- $b_j(t)$: saving in government bonds
- $b(t) = \int_0^1 b_j(t)\, dj$: aggregate wealth

## MATCHING BETWEEN CHAUFFEURS AND CUSTOMERS

- household $k \in [0,1]$ has $d_k$ chauffeurs
  - $y_{jk}$ chauffeurs work for household $j$
  - $y_k = \int_0^1 y_{jk}(t)\, dk$ chauffeurs are employed
  - $u_k = d_k - y_k$ chauffeurs are unemployed
- household $j$ sends $v_{jk}$ customers to parking lot $k$ to hire chauffeurs
  - $v_k = \int_0^1 v_{jk}(t)\, dj$ customers are hiring chauffeurs
- matching function determines flow of new matches on parking lot $k$:

$$h_k = \omega \cdot \sqrt{u_k \cdot v_k}$$

- market tightness $\theta_k = v_k/u_k$ determines trading rates
  - customer-finding rate: $f(\theta_k) = h_k/u_k = \omega \cdot \sqrt{\theta_k}$
  - chauffer-finding rate: $q(\theta_k) = h_k/v_k = \omega/\sqrt{\theta_k}$

# COST OF UNEMPLOYMENT AND HIRING

- unemployed chauffeurs wait in their parking lot
  - no home production
  - no income
- each $v_{jk}$ customer looking for a chauffeur is driven to parking lot $k$ by one of the $y_{jk}$ chauffeurs from household $k$ working for household $j$
  - consumption $<$ output: $c_{jk} = y_{jk} - v_{jk}$

# BALANCED FLOWS AND UNEMPLOYMENT

- chauffeur-customer relationships separate at rate $s > 0$
- number of employed chauffeurs in household $k$:

$$\dot{y}_k = f(\theta_k) \cdot u_k - s \cdot y_k = f(\theta_k) \cdot u_k - s \cdot [d_k - u_k]$$

- US flows are always approximately balanced (Michaillat, Saez 2021)
  - assume that flows are balanced in all $(j, k)$ cells
  - in particular flows are balanced in all household $k$: $\dot{y}_k = 0$
- tightness determines unemployment:

$$u_k = \frac{s}{s + f(\theta_k)} \cdot d_k = u(\theta_k) \cdot d_k$$

# BALANCED FLOWS AND MATCHING WEDGE

- number of employed chauffeurs from household $j$ in household $k$:

$$\dot{y}_{jk} = q(\theta_k) \cdot v_{jk} - s \cdot y_{jk} = q(\theta_k) \cdot [y_{jk} - c_{jk}] - s \cdot y_{jk}$$

- flows are balanced in all $(j, k)$ cells: $\dot{y}_{jk} = 0$
- tightness determines the wedge between consumption and output:

$$y_{jk} = \frac{q(\theta_k)}{q(\theta_k) - s} \cdot c_{jk} = [1 + \tau(\theta_k)] \cdot c_{jk}$$

# PRODUCTIVE EFFICIENCY ON PARKING LOT $k$

- efficient allocation maximizes chauffeur services consumed

$$c_k = \frac{y_k}{1+\tau(\theta_k)} = \frac{1-u(\theta_k)}{1+\tau(\theta_k)} \cdot d_k$$

- efficient tightness $\theta_k^*$ maximizes $[1-u(\theta_k)]/[1+\tau(\theta_k)]$
- efficiency condition: share of unemployed workers $u(\theta_k^*)$ = share of consumption devoted to matching $\tau(\theta_k^*)$
- up to a first-order approximation: vacancy ≈ unemployment
  - just like in sufficient-statistic analysis

# DIRECTED SEARCH AND PRICE/TIGHTNESS COMPETITION

- all chauffeurs from household $k$ charge price $p_k$ per unit time
- expenditure by household $j$ on chauffeurs $k$ is

$$p_k \cdot y_{jk} = p_k \cdot [1 + \tau(\theta_k)] \cdot c_{jk}$$

- all chauffeurs are perfectly substitutable
- $p_k \cdot [1 + \tau(\theta_k)]$ must be the same across sellers (Moen 1997)
  - if not, there are cheaper chauffeurs available
  - or chauffeurs that can be hired with less wait

$\rightsquigarrow$ for all $k$, $p_k \cdot [1 + \tau(\theta_k)] = p \cdot [1 + \tau(\theta)]$

# EFFECT OF SELLING PRICE ON LOCAL TIGHTNESS

- price chosen by household $j$ determines the tightness $\theta_j$ it faces:

$$p_j[1+\tau(\theta_j)] = p[1+\tau(\theta)] \;\Rightarrow\; \theta_j = \tau^{-1}\left(\frac{p}{p_j}[1+\tau(\theta)] - 1\right)$$

- the function $\tau^{-1}$ is increasing, so $\theta_j$ is decreasing in $p_j$
  - a high price leads to low tightness, high unemployment
  - a low price leads to high tightness, low unemployment

# PRICE RIGIDITY

- inflation for household $k$: $\pi_k(t) = \dot{p}_k(t)/p_k(t)$
- changing prices is costly (Rotemberg 1982)
  - $\rho(\pi_k) = \frac{\kappa}{2} \cdot [\pi_k - \bar{\pi}]^2$: share of workers devoted to pricing
  - $\kappa > 0$: price-adjustment cost
  - unexpected price changes require communication with customers (Zbaracki et al 2004)
- $l_k$: labor-force participants from household $k$
- because of price-adjustment costs:

$$d_k = [1 - \rho(\pi_k)] \cdot l_k$$

## HOUSEHOLD BUDGET CONSTRAINT

- law of motion of government bond holdings for household $j$:

$$\dot{b}_j = i \cdot b_j - \int_0^1 p_k y_{jk} \, dk + p_j y_j$$

- because of matching and directed search, expenditure becomes:

$$\int_0^1 p_k y_{jk} \, dk = \int_0^1 p_k [1 + \tau(\theta_k)] c_{jk} \, dk$$
$$= p \cdot [1 + \tau(\theta)] \cdot \int_0^1 c_{jk} \, dk$$

- because of matching and price rigidity, income becomes:

$$p_j \cdot y_j = p_j \cdot [1 - u(\theta_j(p_j))] \cdot d_j = p_j \cdot [1 - u(\theta_j(p_j))] \cdot [1 - \rho(\pi_j)] \cdot l_j$$

# SOLUTION TO HOUSEHOLD MAXIMIZATION BY HAMILTONIAN

- Hamiltonian of household $j$'s maximization is

$$\mathcal{H}_j = \ln\left(c_j\right) + \sigma \cdot \left[\frac{b_j}{p} - \frac{b}{p}\right]$$
$$+ \mathcal{A}_j \cdot \left[i \cdot b_j - p \cdot [1 + \tau(\theta)] \cdot c_j + p_j \cdot [1 - u(\theta_j(p_j))] \cdot [1 - \rho(\pi_j)] \cdot l_j\right]$$
$$+ \mathcal{B}_j \cdot \pi_j \cdot p_j$$

- control variables: $c_j, \pi_j$
- state variables: $b_j, p_j$
- costate variables: $\mathcal{A}_j, \mathcal{B}_j$
- we focus on symmetric solution of model
  - all households behave the same, can drop $j$

# FIRST-ORDER CONDITION WITH RESPECT TO CONSUMPTION

- $d\mathcal{H}_j/dc_j = 0$

$\Leftrightarrow$ $1/c_j = \mathcal{A}_j \cdot p \cdot \left[1 + \tau(\theta)\right]$

$\Leftrightarrow$ $1/\mathcal{A} = p \cdot \left[1 + \tau(\theta)\right] \cdot c$

$\Leftrightarrow$ $1/\mathcal{A} = p \cdot y$

$\Leftrightarrow$ $-\ln(\mathcal{A}) = \ln(p) + \ln(y)$

- taking time derivative yields:

$$-\frac{\dot{\mathcal{A}}}{\mathcal{A}} = \pi + \frac{\dot{y}}{y}$$

# FIRST-ORDER CONDITION WITH RESPECT TO INFLATION

- $d\mathcal{H}_j/d\pi_j = 0$

$\Leftrightarrow$ $\mathcal{B}_j \cdot p_j = \mathcal{A}_j \cdot p_j \cdot [1 - u(\theta_j(p_j))] \cdot \rho'(\pi_j) \cdot l_j$

$\Leftrightarrow$ $\mathcal{B} = \mathcal{A} \cdot [1 - u(\theta)] \cdot \rho'(\pi) \cdot l$

$\Leftrightarrow$ $\mathcal{B} = \mathcal{A} \cdot [1 - u(\theta)] \cdot \kappa \cdot (\pi - \overline{\pi}) \cdot l$

$\Leftrightarrow$ $\mathcal{B} = \mathcal{A} \cdot \frac{\kappa[\pi - \overline{\pi}]}{1 - \rho(\pi)} \cdot y$

$\Leftrightarrow$ $\ln(\mathcal{B}) = \ln(\mathcal{A}) + \ln(y) + \ln(\kappa) + \ln(\pi - \overline{\pi}) - \ln(1 - \rho(\pi))$

- taking time derivative yields:

$$\frac{\dot{\mathcal{B}}}{\mathcal{B}} = \frac{\dot{\mathcal{A}}}{\mathcal{A}} + \frac{\dot{y}}{y} + \frac{\dot{\pi}}{\pi - \overline{\pi}} \cdot \frac{1 + \rho(\pi)}{1 - \rho(\pi)}$$

# FIRST-ORDER CONDITION WITH RESPECT TO SAVING

- $d\mathcal{H}_j/db_j = \delta \cdot \mathcal{A}_j - \dot{\mathcal{A}}_j$

$\Leftrightarrow$ $\frac{\sigma}{p} + \mathcal{A}_j \cdot i = \delta \cdot \mathcal{A}_j - \dot{\mathcal{A}}_j$

- reshuffling terms yields:

$$\frac{\dot{\mathcal{A}}}{\mathcal{A}} = \delta - i - \frac{\sigma}{p \cdot \mathcal{A}}$$

- using $1/\mathcal{A} = p \cdot y$ finally gives:

$$\frac{\dot{\mathcal{A}}}{\mathcal{A}} = \delta - (i + \sigma \cdot y)$$

# FIRST-ORDER CONDITION WITH RESPECT TO PRICE [1]

- $d\mathcal{H}_j/dp_j = \delta \cdot \mathcal{B}_j - \dot{\mathcal{B}}_j$

$\Leftrightarrow$ $\mathcal{A}_j \cdot (1-u_j) \cdot (1-\rho(\pi_j)) \cdot l_j - \mathcal{A}_j \cdot p_j \cdot (1-\rho(\pi_j)) \cdot l_j \cdot u'(\theta_j) \cdot \theta'(p_j) + \mathcal{B}_j \cdot \pi_j = \delta \cdot \mathcal{B}_j - \dot{\mathcal{B}}_j$

- we have the following derivatives:

$$u'(\theta_j) = -\frac{[1 - u(\theta_j)] \cdot u(\theta_j)}{2 \cdot \theta_j}, \qquad \theta'(p_j) = -\frac{2 \cdot \theta_j}{\tau(\theta_j) \cdot p_j}$$

- hence, $p_j \cdot u'(\theta_j) \cdot \theta'(p_j) = (1 - u(\theta_j)) \cdot$

# FIRST-ORDER CONDITION WITH RESPECT TO PRICE [2]

- reshuffling terms gives:

$$(\delta - \pi_j) \cdot \mathcal{B}_j - \dot{\mathcal{B}}_j = \mathcal{A}_j \cdot y_j \cdot \left[1 - \frac{u(\theta_j)}{\tau(\theta_j)}\right]$$

$$-\frac{\dot{\mathcal{B}}_j}{\mathcal{B}_j} = \pi_j - \delta + \frac{\mathcal{A}_j \cdot y_j}{\mathcal{B}_j} \cdot \left[1 - \frac{u(\theta_j)}{\tau(\theta_j)}\right]$$

- using $[1 - \rho(\pi_j)]/[\kappa(\pi_j - \overline{\pi})] = \mathcal{A}_j \cdot y_j/\mathcal{B}_j$, we finally get:

$$-\frac{\dot{\mathcal{B}}}{\mathcal{B}} = \pi - \delta + \frac{1}{\kappa} \cdot \frac{1 - \rho(\pi)}{\pi - \overline{\pi}} \cdot \left[1 - \frac{u(\theta)}{\tau(\theta)}\right]$$

# LINEARIZED SYSTEM

- set $i = i^* + \phi(\pi - \overline{\pi})$ and linearize differential equations around $(u^*, \overline{\pi})$
- deviations from efficient steady state: $\hat{u} = u - u^*$, $\hat{\pi} = \pi - \overline{\pi}$
- linearized AD curve: $\dot{u} = (1 - u^*) \cdot [\sigma \cdot l \cdot \hat{u} + (1 - \phi)\hat{\pi}]$
- steady-state linearized AD curve: $\hat{\pi} = -[\sigma/(1 - \phi)] \cdot l \cdot \hat{u}$
  - slope depends on policy rule
  - under Taylor principle ($\phi > 1$), standard increasing AD curve
- linearized AS curve: $\dot{\pi} = \delta\hat{\pi} + 2/[\kappa u^*(1 - u^*)]\hat{u}$
- steady state linearized AS curve: $\hat{\pi} = -2/[\delta\kappa u^*(1 - u^*)]\hat{u}$
  - recover downward-sloping Phillips curve in (unemployment, inflation) plane

# SPECIAL CASES

- consider AD and AS curves in $(y, \pi)$ plane
- AD curve without wealth in the utility ($\sigma = 0$)
  - horizontal: $\pi = i - \delta$
  - real rate = discount rate
  - inflation is determined one-for-one by policy rate (Fisher effect)
  - degenerate: does not involve output
- AS curve without price rigidity ($\kappa = 0$)
  - vertical: $y = y^* = (1 - u^*) \cdot l$
  - unemployment is always efficient, irrespective of inflation

# AGGREGATE DEMAND: DISCOUNTED EULER EQUATION

- from optimal consumption and saving:

$$\frac{\dot{y}}{y} = (i - \pi + \sigma \cdot y) - \delta$$

- $i - \pi$: real interest rate, financial return on saving
- $\sigma \cdot y$: MRS between wealth & consumption, hedonic return on saving
  - discounted Euler equation (McKay, Nakamura, Steinsson 2017)
  - from wealth in the utility function (Michaillat, Saez 2021)
- in steady state ($\dot{y} = 0$), nondegenerate AD curve:

$$y = \frac{\delta - i + \pi}{\sigma}$$

# AGGREGATE SUPPLY: PHILLIPS CURVE

- from optimal pricing:

$$\dot{\pi} \cdot \frac{1 + \rho(\pi)}{1 - \rho(\pi)} = \delta \cdot (\pi - \overline{\pi}) - \frac{1}{\kappa} \cdot [1 - \rho(\pi)] \cdot \left[1 - \frac{u(\theta)}{\tau(\theta)}\right]$$

- $\kappa$: price-adjustment cost
- $1 - \frac{u(\theta)}{\tau(\theta)}$: tightness gap
  - $u(\theta)$: share of idle drivers waiting for a job
  - $\tau(\theta)$: share of idle drivers waiting for a match
  - zero iff $\theta = \theta^*$
  - positive iff $\theta > \theta^*$
- in steady state ($\dot{\pi} = 0$), nonlinear AS curve:

$$\delta \cdot (\pi - \overline{\pi}) = \frac{1}{\kappa} \cdot [1 - \rho(\pi)] \cdot \left[1 - \frac{u(\theta)}{\tau(\theta)}\right]$$

# DIVINE COINCIDENCE APPEARS IN PHILLIPS CURVE

- recall steady-state Phillips curve:

$$\kappa \cdot \delta \cdot (\pi - \overline{\pi}) = [1 - \rho(\pi)] \cdot \left[1 - \frac{u(\theta)}{\tau(\theta)}\right]$$

- inflation is on target ($\pi = \overline{\pi}$) iff
  - $1 - \frac{u(\theta)}{\tau(\theta)} = 0$

$\Leftrightarrow$ $u(\theta) = \tau(\theta)$

$\Leftrightarrow$ tightness and unemployment are efficient ($\theta = \theta^*$, $u = u^*$)

# OPTIMAL MONETARY POLICY

- optimal nominal interest rate $i^*$ ensures:
  - inflation is on target: $\pi = \overline{\pi}$
  - unemployment is efficient: $u = u^*$
- optimal policy can take different forms:
  - interest-rate peg: $i = i^*$
  - Taylor rule with $\phi > 0$: $i = i^* + \phi \cdot (\pi - \overline{\pi})$
- from AD curve:
  - $\delta - i^* + \overline{\pi} = \sigma \cdot y^* = \sigma \cdot (1 - u^*) \cdot l$

  $\Leftrightarrow$ $i^* = \overline{\pi} + \delta - \sigma \cdot (1 - u^*) \cdot l$
- by divine coincidence, just need to target efficient unemployment
  - no need to compute $i^*$

# DYNAMICAL PROPERTIES OF THE LINEARIZED MODEL

$$\begin{bmatrix} \dot{u}(t) \\ \dot{\pi}(t) \end{bmatrix} = \begin{bmatrix} \sigma y^* & (1-\phi)(1-u^*) \\ \frac{2}{\kappa u^*(1-u^*)} & \delta \end{bmatrix} \begin{bmatrix} \hat{u}(t) \\ \hat{\pi}(t) \end{bmatrix}$$

- solution is unique iff dynamical system is a source

  $\Leftrightarrow$ trace > 0 and determinant > 0
- trace: $\delta + \sigma y^* > 0$
  - trace > 0 for any $\sigma \geq 0$
- determinant: $\delta\sigma y^* - 2(1-\phi)/(\kappa u^*)$
  - no wealth in utility ($\sigma = 0$): determinant > 0 iff $\phi > 1$ (Taylor)
  - interest-rate peg ($\phi = 0$): determinant > 0 iff $\sigma > 2/(\kappa\delta u^* y^*)$
  - fixed inflation ($\kappa \to \infty$): determinant > 0 iff $\sigma > 0$