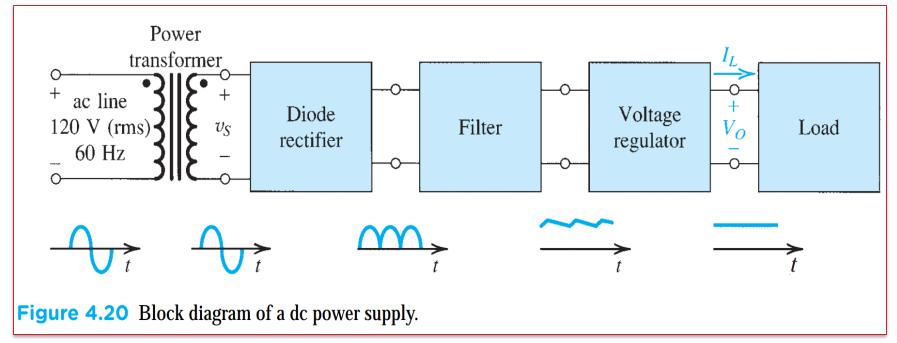
#### **Rectifier Circuits**

One of the most important applications of diodes is in the design of rectifier (AC to DC) circuits. A diode rectifier forms an essential building block of the dc power supplies required to power electronic equipment. A block diagram of such a power supply is shown in Fig. 4.20.



### The Half-Wave Rectifier

half-wave The rectifier utilizes alternate half-cycles of the input sinusoid. **Figure** 4.21(a) shows the circuit of a half-wave rectifier. **Figure** 4.21(b) shows the voltage output obtained when the input  $v_s$  is a sinusoid.

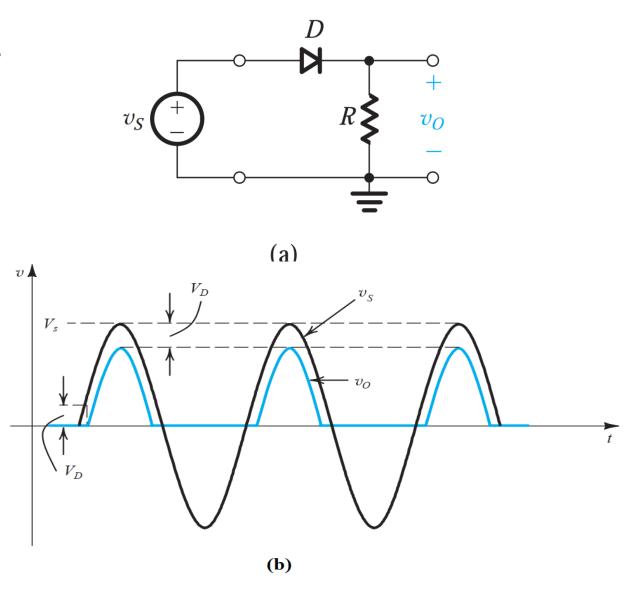
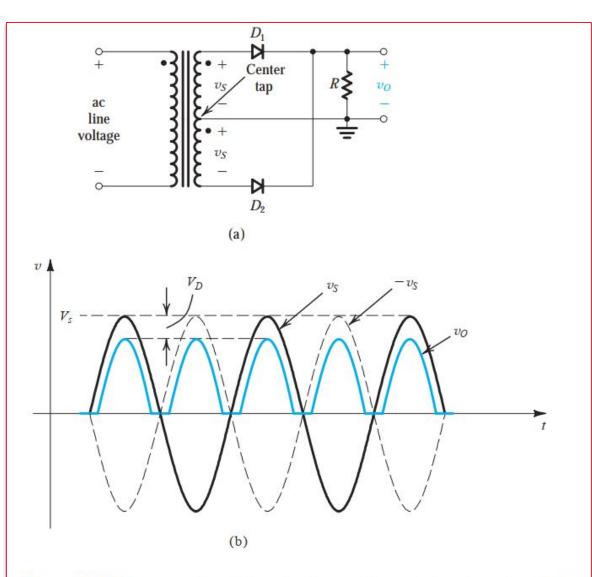


Figure 4.21 (a) Half-wave rectifier. (b) Input and output waveforms.

#### The Full-Wave Rectifier

The full-wave rectifier utilizes halves of the both input sinusoid. To provide a unipolar output, it inverts the negative halves of the sine wave. One implementation possible is shown in Fig. 4.22(a). Here the transformer secondary winding is **center-tapped** to provide two equal voltages  $v_s$  across the two halves of the secondary winding with the polarities indicated.



**Figure 4.22** Full-wave rectifier utilizing a transformer with a center-tapped secondary winding: **(a)** circuit; **(b)** input and output waveforms.

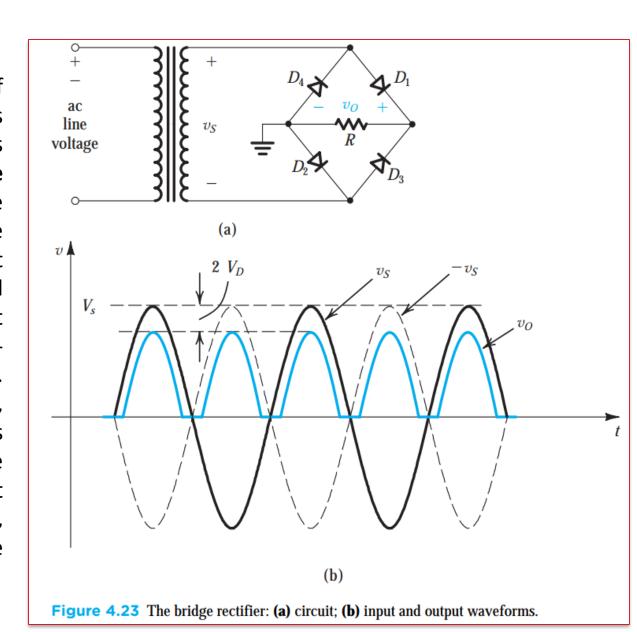
### The Center-Tapped rectifier circuit operates as follows:

Note that when the input line voltage (feeding the primary) is positive, both of the signals labeled  $v_S$  will be positive. In this case  $D_1$  will conduct and  $D_2$  will be reverse biased. The current through  $D_1$  will flow through R and back to the center tap of the secondary. The circuit then behaves like a half-wave rectifier.

Now, during the negative half-cycle of the ac line voltage, both of the voltages labeled  $v_S$  will be negative. Thus  $D_1$  will be cut off while  $D_2$  will conduct. The current conducted by  $D_2$  will flow through R and back to the center tap. It follows that during the negative half-cycles while  $D_2$  conducts, the circuit behaves again as a half-wave rectifier. The important point, however, is that the current through R always flows in the same direction, and thus  $v_O$  will be unipolar, as indicated in Fig. 4.22(b).

### The Bridge Rectifier

An alternative application of the full-wave rectifier shown in Fig. 4.23(a). This circuit, known as the bridge **rectifier** because of the similarity with the Wheatstone bridge, does not a center-tapped require transformer, a distinct advantage over the centertapped rectifier circuit of Fig. 4.22. The bridge rectifier, four diodes requires compared to two in the previous circuit. This is not much of a disadvantage, diodes because are inexpensive.



### The bridge rectifier circuit operates as follows:

During the positive half-cycles of the input voltage,  $v_s$  is positive, and thus current is conducted through diode  $D_1$ , resistor R, and diode  $D_2$ . Meanwhile, diodes  $D_3$  and  $D_4$  will be reverse biased. Observe that there are two diodes in series in the conduction path, and thus  $v_0$  will be lower than  $v_s$  by two diode drops (compared to center-tapped). This is somewhat of a disadvantage of the bridge rectifier.

Next, consider the situation during the negative half-cycles of the input voltage. The secondary voltage  $v_s$  will be negative, and thus  $-v_s$  will be positive, forcing current through  $D_3$ , R, and  $D_4$ . Meanwhile, diodes  $D_1$  and  $D_2$  will be reverse biased. The important point to note, though, is that during both half-cycles, current flows through R in the same direction (from right to left), and thus  $v_o$  will always be positive, as indicated in Fig. 4.23(b).

### The average value of Half wave rectifier circuit

$$V_{o} = \frac{1}{T} \int_{0}^{T} v(t)dt$$

$$= \frac{1}{2\pi} \int_{0}^{2\pi} V_{m} \sin\theta \ d\theta$$

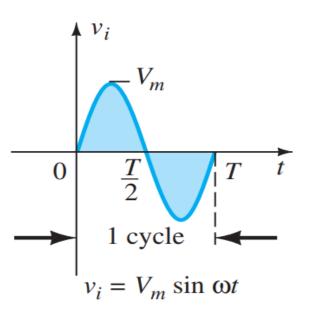
$$= \frac{V_{m}}{2\pi} \left[ \int_{0}^{\pi} \sin\theta \ d\theta + \int_{\pi}^{2\pi} \sin\theta \ d\theta \right]$$

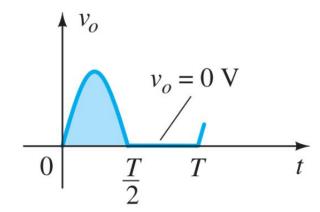
$$= \frac{V_{m}}{2\pi} \left[ -\cos\theta \right]_{0}^{\pi}$$

$$= -\frac{V_{m}}{2\pi} \left[ -1 - 1 \right] = \frac{V_{m}}{2\pi} [2]$$

$$\therefore V_0 = \frac{V_m}{\pi}$$

Similarly, 
$$I_0=rac{I_m}{\pi}$$





The R.M.S value of Half wave rectifier circuit,

$$\begin{split} V_{rms} &= \sqrt{\frac{1}{T}} \int_{0}^{T} \{v(t)\}^{2} dt &= \sqrt{\frac{1}{2\pi}} \int_{0}^{2\pi} V_{m}^{2} \sin^{2}\theta d\theta \\ &= \sqrt{\frac{V_{m}^{2}}{2\pi}} \left[ \int_{0}^{\pi} \sin^{2}\theta d\theta + \int_{\pi}^{2\pi} \sin^{2}\theta d\theta \right] \\ &= \sqrt{\frac{V_{m}^{2}}{2\pi}} \int_{0}^{\pi} \sin^{2}\theta d\theta &= \sqrt{\frac{V_{m}^{2}}{4\pi}} \int_{0}^{\pi} 2\sin^{2}\theta d\theta \\ &= \sqrt{\frac{V_{m}^{2}}{4\pi}} \int_{0}^{\pi} (1 - \cos 2\theta) d\theta &= \sqrt{\frac{V_{m}^{2}}{4\pi}} \{ \int_{0}^{\pi} (1) d\theta - \int_{0}^{\pi} (\cos 2\theta) d\theta \} \\ &= \sqrt{\left( \frac{V_{m}^{2}}{4\pi} \times [\theta]_{0}^{\pi} \right)} &= \sqrt{\left( \frac{V_{m}^{2}}{4\pi} \times \pi \right)} &= \sqrt{\left( \frac{V_{m}^{2}}{4} \right)} \end{split}$$

$$V_{rms} = \frac{V_m}{2}$$

Similarly, 
$$I_{rms} = \frac{I_m}{2}$$

### The average value of Full wave rectifier circuit

$$V_{o} = \frac{1}{2\pi} \int_{0}^{2\pi} v(t)dt$$

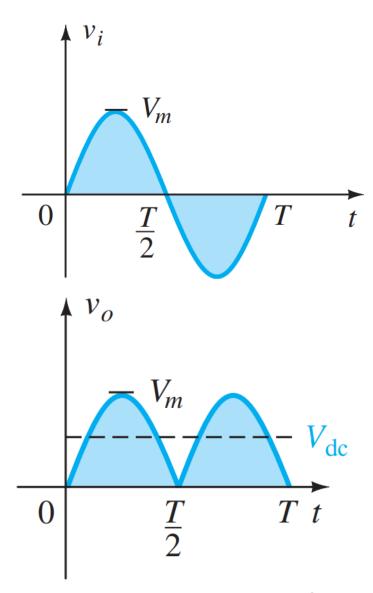
$$= \frac{1}{2\pi} \left[ \int_{0}^{\pi} V_{m} \sin\theta \ d\theta + \int_{\pi}^{2\pi} V_{m} \sin\theta \ d\theta \right]$$

$$= \frac{1}{\pi} \int_{0}^{\pi} V_{m} \sin\theta \ d\theta = \frac{V_{m}}{\pi} \left[ -\cos\theta \right]_{0}^{\pi}$$

$$= -\frac{V_{m}}{\pi} \left[ -1 - 1 \right]$$

$$\boldsymbol{V_o} = \frac{2V_m}{\pi}$$

Similarly, 
$$I_o = \frac{2I_m}{\pi}$$



The RMS value of full wave rectifier circuit,

$$\begin{split} V_{rms} &= \sqrt{\frac{1}{\pi}} \int_0^{\pi} \{v(t)\}^2 d\theta \\ &= \sqrt{\frac{1}{\pi}} \int_0^{\pi} V_m^2 \sin^2\theta d\theta \\ &= \sqrt{\frac{V_m^2}{2\pi}} \int_0^{\pi} 2 \sin^2\theta d\theta \\ &= \sqrt{\frac{V_m^2}{2\pi}} \int_0^{\pi} (1 - \cos 2\theta) d\theta \\ &= \sqrt{\frac{V_m^2}{2\pi}} \left[ \left(\theta - \frac{\sin 2\theta}{2}\right) \right]_0^{\pi} \quad = \sqrt{\frac{V_m^2}{2\pi}} \times \pi \end{split}$$

$$\therefore V_{rms} = \frac{V_m}{\sqrt{2}}$$

Similarly, 
$$I_{rms}=rac{I_m}{\sqrt{2}}$$

Efficiency of Half wave rectifier

$$\eta = \frac{P_{out}}{P_{in}} = \frac{P_{dc}}{P_{ac}} = \frac{P_{avg}}{P_{rms}}$$

$$P_{dc} = I_o^2 \times R = \left(\frac{I_m}{\pi}\right)^2 \times R_L$$

$$P_{ac} = I_{rms}^2 \times R = \left(\frac{I_m}{2}\right)^2 \times \left(R_L + r_f\right) \cong \left(\frac{I_m}{2}\right)^2 \times R_L$$

Since 
$$I_0 = \frac{I_m}{\pi}$$

Since 
$$I_{rms} = \frac{I_m}{2}$$

Efficiency of Full wave rectifier

$$\eta = \frac{P_{out}}{P_{in}} = \frac{P_{dc}}{P_{ac}} = \frac{P_{avg}}{P_{rms}}$$

$$P_{dc} = I_o^2 \times R = \left(\frac{2I_m}{\pi}\right)^2 \times R_L$$

$$P_{ac} = I_{rms}^2 \times R = \left(\frac{I_m}{\sqrt{2}}\right)^2 \times \left(R_L + 2r_f\right) \cong \left(\frac{I_m}{2}\right)^2 \times R_L$$

Since 
$$I_0 = \frac{2I_m}{\pi}$$

Since 
$$I_{rms} = \frac{I_m}{\sqrt{2}}$$

### **Rectifier with a Filter Capacitor**

At the end of the discharge interval, which lasts for almost the entire period T,

$$V_O = V_p - V_r$$
,

where  $V_r$  is the peak-to-peak ripple voltage.

The output dc voltage  $(V_0)$  can be obtained by taking the average values of  $v_0$ ,

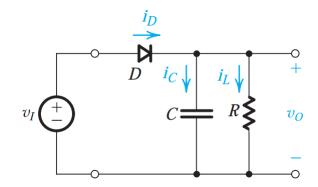
$$V_o = V_p - \frac{1}{2}V_r$$

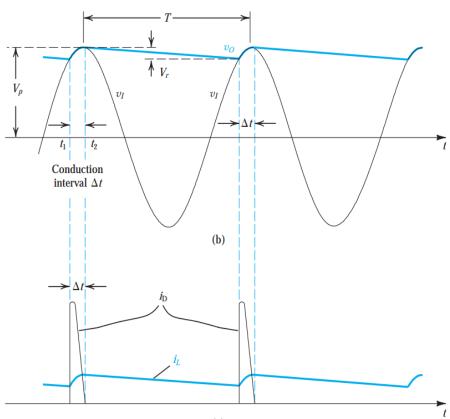
During the diode-off interval,  $v_O$  can be expressed as

$$v_o = V_p e^{-t/CR}$$

At the end of the discharge interval we have

$$V_p - V_r \cong V_p e^{-T/CR}$$





Now, since CR >>T, we can use the approximation  $e^{-T/CR}\cong 1-\frac{T}{CR}$  to obtain

$$V_p - V_r \cong V_p (1 - \frac{T}{CR})$$

$$V_p - V_r \cong V_p - V_p \frac{T}{CR}$$

$$V_r \cong V_p \frac{T}{CR}$$

$$\therefore V_r \cong \frac{V_p}{fCR}$$

Assuming that diode conduction ceases almost at the peak of  $v_p$ , we can determine the conduction interval  $\Delta t$  from

$$V_p \cos(\omega \Delta t) = V_p - V_r$$

Since  $(\omega \Delta t)$  is a small angle, we can employ the approximation

$$\cos(\omega \Delta t) = 1 - \frac{1}{2}(\omega \Delta t)^{2}$$

$$\therefore V_{p} \left(1 - \frac{1}{2}(\omega \Delta t)^{2}\right) = V_{p} - V_{r}$$

$$\Rightarrow V_{p} - \frac{1}{2}V_{p}(\omega \Delta t)^{2} = V_{p} - V_{r}$$

$$\Rightarrow \frac{1}{2}V_{p}(\omega \Delta t)^{2} = V_{r}$$

$$\Rightarrow V_{p}(\omega \Delta t)^{2} = 2V_{r}$$

$$\Rightarrow (\omega \Delta t) = \sqrt{\frac{2V_{r}}{V_{p}}}$$

$$\Rightarrow \Delta t = \frac{1}{\omega}\sqrt{\frac{2V_{r}}{V_{p}}}$$