**Standard integral formula**

1. $\int \frac{dx}{x^2+a^2} = \frac{1}{a} tan^{-1} \frac{x}{a}$+c,

2. $\int \frac{dx}{x^2-a^2} = \frac{1}{2a} ln \frac{x-a}{x+a}$+c,

3. $\int \frac{dx}{a^2-x^2} = \frac{1}{2a} ln \frac{a+x}{a-x}$+c ,

**Type-4** $\int \frac{\boldsymbol{dx}}{\boldsymbol{ax^2+bx+c}}$

Type equation here.

**Procedure** $\int \frac{\boldsymbol{dx}}{\boldsymbol{ax^2+bx+c}} = \frac{1}{\mathrm{a}} \int \frac{dx}{(x+\frac{b}{2a})^2+\frac{4ac-b^2}{4a^2}}$

**Example:** Workout $\int \frac{dx}{4x^2+4x+5}$

Solution: $I = \int \frac{dx}{4x^2+4x+5} = \int \frac{dx}{4\left(x^2+x+\frac{5}{4}\right)}$

$$= \frac{1}{4} \int \frac{dx}{(x+\frac{1}{2})^2 + \frac{5}{4} - \frac{1}{4}} == \frac{1}{4} \int \frac{dx}{(x+\frac{1}{2})^2 + 1}$$

$$= \frac{1}{4} tan^{-1} \frac{x+\frac{1}{2}}{1} + c = \frac{1}{4} tan^{-1}(x+\frac{1}{2}) + c$$

**Example:** Workout $\int \frac{dx}{2x^2+3x+1}$

Solution: $I = \int \frac{dx}{2x^2+3x+1} = \int \frac{dx}{2\left(x^2+\frac{3}{2}x+\frac{1}{2}\right)}$

$$= \frac{1}{2}\int \frac{dx}{(x+\frac{3}{4})^2+\frac{1}{2}-\frac{9}{16}} = \frac{1}{2}\int \frac{dx}{(x+\frac{3}{4})^2-(\frac{1}{4})^2}$$

$$= \frac{1}{2.\frac{1}{2}} ln \frac{x+\frac{3}{4}-\frac{1}{4}}{x+\frac{3}{4}+\frac{1}{4}} + c = ln \frac{4x+2}{4x+4} + c$$

**Type-5** $\int \frac{px+q}{ax^2+bx+c} dx$

**Procedure** $\int \frac{px+q}{ax^2+bx+c} dx = \int \frac{\frac{p}{2a}\frac{d}{dx}(ax^2+bx+c)+(q-\frac{pb}{2a})}{ax^2+bx+c} \quad dx$

**Example:** Workout $\int \frac{3x+4}{2x^2+3x+1} dx$

Solution: $I = \int \frac{3x+4}{2x^2+3x+1} dx = \int \frac{\frac{3}{4}(4x+3)+4-\frac{9}{4}}{2x^2+3x+1} dx$

$= \frac{3}{4}\int \frac{4x+3}{2x^2+3x+1} dx + \frac{7}{4}\int \frac{dx}{2x^2+3x+1}$

$= \frac{3}{4}\int \frac{dz}{z} + \frac{7}{4}\int \frac{dx}{2x^2+3x+1} = \frac{3}{4}\ln z + \frac{7}{4}\int \frac{dx}{2x^2+3x+1} + C$

**FORMULA**

4. $\int tanx \;\; dx =$ ln(secx)+c

5. $\int cotx \;\; dx =$ ln(sinx)+c

6. $\int secx \;\; dx =$ ln (secx+tan x)+c

7. $\int cosecx \;\; dx =$ ln tan(x/2 )+c

8. $\int \frac{dx}{\sqrt{x^2+a^2}}$ =ln(x+$\sqrt{x^2+a^2}) + c = \sinh^{-1}\frac{x}{a}$ +c, x=atan$\theta$,

9. $\int \frac{dx}{\sqrt{x^2-a^2}}$ =ln(x+$\sqrt{x^2-a^2}$ )+c $= cosh^{-1}\frac{x}{a} + c$ , x=asec$\theta$,

10. $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a}$ +c x=asin$\theta$,

$\int \frac{dx}{\sqrt{x^2+a^2}}$ x=atan$\theta$ dx=$a\sec^2\theta$ d$\theta$

$\int \frac{a\sec^2\theta d\theta}{\sqrt{a\tan^2\theta+a^2}} = \int \sec\theta d\theta$=ln(sec$\theta$+tan$\theta$)+c= ln(x+$\sqrt{x^2+a^2}$ )+c

**Type**6 $\int \frac{dx}{\sqrt{ax^2+bx+c}}$

**Procedure :** $\int \frac{dx}{\sqrt{ax^2+bx+c}} = \frac{1}{\sqrt{a}} \int \frac{dx}{\sqrt{\left(x+\frac{b}{2a}\right)^2+\frac{4ac-b^2}{4a^2}}}$

**Example:** Workout $\int \frac{dx}{\sqrt{2x^2+3x+1}}$

Solution: $I = \int \frac{dx}{\sqrt{2x^2+3x+1}} = \int \frac{dx}{\sqrt{2(x^2+\frac{3}{2}x+\frac{1}{2})}} = \frac{1}{\sqrt{2}} \int \frac{dx}{\sqrt{(x+\frac{3}{4})^2+\frac{1}{2}-\frac{9}{16}}}$

$$= \frac{1}{\sqrt{2}} \int \frac{dx}{\sqrt{(x+\frac{3}{4})^2-(\frac{1}{4})^2}} = \frac{1}{\sqrt{2}} \ln\left(x+\frac{3}{4}+\sqrt{(x+\frac{3}{4})^2-(\frac{1}{4})^2}\right)+c$$

**Example:** Workout $\int \frac{dx}{\sqrt{6+11x-10x^2}}$

Solution: $I = \int \frac{dx}{\sqrt{6+11x-10x^2}} = \frac{1}{\sqrt{10}} \int \frac{dx}{\sqrt{\frac{3}{5}+\frac{11}{10}x-x^2}}$

$$= \frac{1}{\sqrt{10}} \int \frac{dx}{\sqrt{\frac{3}{5}-(x-\frac{11}{20})^2+\frac{121}{400}}}$$

$$= \frac{1}{\sqrt{10}} \int \frac{dx}{\sqrt{\left(\sqrt{\frac{19}{20}}\right)^2-(x-\frac{11}{20})^2}} = \frac{1}{\sqrt{10}} \sin^{-1} \frac{x-\frac{11}{20}}{\sqrt{\frac{19}{20}}} + C$$

**Type-7** $\int \frac{dx}{\sqrt{(x-a)(x-b)}}$

**Procedure** substitute $\boldsymbol{x - b = z^2}$

**Example:** Workout $\int \frac{dx}{\sqrt{(x-3)(x-4)}}$

Solution: $I = \int \frac{dx}{\sqrt{(x-3)(x-4)}}$ put $x - 4 = z^2$ , dx=2zdz

$=\int \frac{2zdz}{\sqrt{(z^2+1^2)z^2}} = 2\int \frac{dz}{\sqrt{(z^2+1^2)}}$ =2ln(z+$\sqrt{z^2+1}$)+c

=2ln($\sqrt{x-4} + \sqrt{x-4+1}$)+c

=2ln($\sqrt{x-4} + \sqrt{x-3}$ )+c

**Type-8** $\int \frac{px+q}{\sqrt{ax^2+bx+c}}dx$

**Procedure** $\int \frac{px+q}{\sqrt{ax^2+bx+c}}dx = \int \frac{\frac{p}{2a}(2ax+b)+\frac{2aq-pb}{2a}}{\sqrt{ax^2+bx+c}}dx$

**Example:** Workout $\int \frac{3x+4}{\sqrt{2x^2+3x+1}}dx$

Solution: $\mathrm{I} = \int \frac{3x+4}{\sqrt{2x^2+3x+1}}dx = \int \frac{\frac{3}{4}(4x+3)+4-\frac{9}{4}}{\sqrt{2x^2+3x+1}}dx$

$= \frac{3}{4}\int \frac{4x+3}{\sqrt{2x^2+3x+1}}dx + \frac{7}{4}\int \frac{dx}{\sqrt{2x^2+3x+1}} = I_1 + I_2$

$I_1 = \frac{3}{4}\int \frac{4x+3}{\sqrt{2x^2+3x+1}}dx, \quad \frac{7}{4}\int \frac{dx}{\sqrt{2x^2+3x+1}} = I_2$

$I_1 = \frac{3}{4}\int \frac{4x+3}{\sqrt{2x^2+3x+1}}dx$ put z= $2x^2 + 3x + 1 => dz = (4x+3)dx$

$= \frac{3}{4}\int \frac{dz}{\sqrt{z}} = \frac{3}{2}\sqrt{z} = \frac{3}{2}\sqrt{2x^2+3x+1} + c_1$

$I_2 = \frac{7}{4}\int \frac{dx}{\sqrt{2x^2+3x+1}} = \int \frac{dx}{\sqrt{2(x^2+\frac{3}{2}x+\frac{1}{2})}} = \frac{1}{\sqrt{2}}\int \frac{dx}{\sqrt{(x+\frac{3}{4})^2+\frac{1}{2}-\frac{9}{16}}}$

$= \frac{1}{\sqrt{2}}\int \frac{dx}{\sqrt{(x+\frac{3}{4})^2-(\frac{1}{4})^2}} = \frac{1}{\sqrt{2}}\ln\left(x+\frac{3}{4}+\sqrt{(x+\frac{3}{4})^2-(\frac{1}{4})^2}\right) = c_2$

**Type**-9 $\int \sqrt{\frac{(ax+b)}{cx+d}}dx$

Process $\int \sqrt{\frac{(ax+b)(ax+b)}{(cx+d)(ax+b)}}dx$

**Example:** Workout $\int \sqrt{\frac{2x+4}{3x+3}}dx$

Solution: $I = \int \sqrt{\frac{2x+4}{3x+3}}dx = \int \frac{2x+4}{\sqrt{(3x+3)(2x+4)}}dx = \int \frac{2x+4}{\sqrt{6x^2+18x+12}}dx$

**Type- 10** $\int \frac{dx}{(ax+b)\sqrt{cx+d}}$

**Procedure** Substitute $cx + d = z^2$

**Example:** Workout $\int \frac{dx}{(1-x)\sqrt{x}}$

Solution $I = \int \frac{dx}{(1-x)\sqrt{x}}$ put $x = z^2$ dx=2zdz

$$=\int \frac{2zdz}{(1-z^2)z}=\int \frac{2dz}{1-z^2}=\frac{1}{2}\, ln\, \frac{1+z}{1-z}+c=\frac{1}{2}\, ln\, \frac{1+x^2}{1-x^2}+c$$

**Type11** $\int \frac{dx}{(ax+b)\sqrt{px^2+qx+r}}$

**Procedure** Substitute $ax+b = \frac{1}{z}$

**Example:** Workout $\int \frac{dx}{(1+x)\sqrt{1+x-x^2}}$

Solution: $I = \int \frac{dx}{(1+x)\sqrt{1+x-x^2}}$

put $1+x=\frac{1}{z}$, $x = \frac{1}{z} - 1$ , $dx = -\frac{1}{z^2}dz$

$$1 + x - x^2=\frac{1}{z} - \left(\frac{1}{z} - 1\right)^2=\frac{1}{z} - \frac{1}{z^2} + \frac{2}{z} - 1=\frac{3}{z} - \frac{1}{z^2} - 1=\frac{3z-z^2-1}{z^2}$$

$$I = \int \frac{-\frac{1}{z^2}dz}{\frac{1}{z}\sqrt{\frac{3z-z^2-1}{z^2}}} = -\int \frac{dz}{\sqrt{3z-z^2-1}}$$

$$= -\int \frac{dz}{\sqrt{-1+3z-z^2}} = -\int \frac{dz}{\sqrt{-1-(z-\frac{3}{2})^2+\frac{9}{4}}}$$

$$= -\int \frac{dz}{\sqrt{\left(\frac{\sqrt{5}}{2}\right)^2-(z-\frac{3}{2})^2}} = \sin^{-1}\left(\frac{z-\frac{3}{2}}{\frac{\sqrt{5}}{2}}\right)+c$$

$$= \sin^{-1}\left(\frac{\frac{1}{x+1}-\frac{3}{2}}{\frac{\sqrt{5}}{2}}\right)+c$$

**Type12** $\int \frac{dx}{x(a+x^n)}$ , $\int \frac{dx}{x\sqrt{a+bx^n}}$

**Procedure** Substitute $x^n = \frac{1}{z^2}$, $lnx^n = ln\frac{1}{z^2}$

$$n\frac{dx}{x} = -\frac{2}{z}dz \Rightarrow \frac{dx}{x} = -\frac{2}{nz}dz$$

**Example:** Workout $\int \frac{dx}{x\sqrt{1+x^5}}$

Solution: $I = \int \frac{dx}{x\sqrt{1+x^5}}$

put $x^5 = \frac{1}{z^2}$ , $lnx^5 = lnz^{-2} \Rightarrow 5lnx = -2lnz$

$$\frac{dx}{x} = -\frac{2}{5z}dz$$

$$I = -\frac{2}{5}\int \frac{1}{\frac{1}{z}\sqrt{z^2+1}} \frac{1}{z}dz = -\frac{2}{5}\int \frac{1}{\sqrt{z^2+1}} dz$$

$$= -\frac{2}{5}\ln\left(z + \sqrt{z^2 + 1}\right) + c$$

$$= -\frac{2}{5}\ln\left(\frac{1}{\sqrt{x^5}} + \sqrt{\frac{1+x^5}{x^5}}\right) + c$$

$$= -\frac{2}{5}\ln\left(\frac{1+\sqrt{1+x^5}}{\sqrt{x^5}}\right) + c$$