<u>Integration by parts</u>

FORMULA: $\int \boldsymbol{uvdx} = \boldsymbol{u} \int \boldsymbol{vdx} - \int \left(\frac{\boldsymbol{du}}{\boldsymbol{dx}} \int \boldsymbol{vdx}\right) \boldsymbol{dx}$

Ex. $\int xe^x dx = x \int e^x dx - \int \left(\frac{dx}{dx} \int e^x dx\right) dx$

$=xe^x - \int e^x dx = xe^x - e^x + c$

Ex. $\int x^2 e^x dx = x^2 \int e^x dx - \int (2x \int e^x dx)\, dx$

$=x^2 e^x - 2[xe^x - \int e^x dx]$ $=x^2 e^x - 2xe^x + 2$+c

Exam Workout $\int \cos^{-1} \frac{1-x^2}{1+x^2} dx$

Solu $I = \int \cos^{-1} \frac{1-x^2}{1+x^2} dx,$ put $x = tan\theta, dx = \sec^2 \theta d\theta$

$$I = \int \cos^{-1} \frac{1 - tan^2\theta}{1 + tan^2\theta} \sec^2 \theta \, d\theta = \int \cos^{-1}(cos2\theta)\sec^2 \theta \, d\theta$$

$$= 2 \int \theta \sec^2 \theta \, d\theta = 2(\theta tan\theta - \int tan\theta \, d\theta)$$

$$= 2(\theta tan\theta + lncos\theta) + c = 2xtan^{-1}x + 2lncos\theta + c$$

$$= 2xtan^{-1}x - lnsec^2\theta + c = 2xtan^{-1}x - \ln(1 + tan^2\theta) + c$$

$$= 2xtan^{-1}x - \ln(1 + x^2) + c$$

Formula

1. $\int e^x(f(x)+f'(x))dx = e^x f(x) + c$

2. $\int e^{ax}\, cos\, b\, xdx = \frac{e^{ax}(a\, cos\, bx + b\, sin\, bx)}{a^2+b^2}$+c

3. $\int e^{ax} \sin b\, xdx = \frac{e^{ax}(a\, sin\, bx - b\, cos\, bx)}{a^2+b^2} + c$

Example: Workout $\int e^x \frac{x^2+1}{(x+1)^2} dx$

Sol. $I = \int e^x \frac{x^2+1}{(x+1)^2} dx = \int e^x \frac{x^2-1+2}{(x+1)^2} dx = \int e^x \frac{(x+1)(x-1)+2}{(x+1)^2} dx$

$= \int e^x \left[\frac{x-1}{x+1} + \frac{2}{(x+1)^2}\right] dx$

Let $f(x) = \frac{x-1}{x+1} => f'(x) = \frac{x+1-x+1}{(x+1)^2} = \frac{2}{(x+1)^2}$

$I = \int e^x [f(x) + f'(x)] d = e^x f(x) + c = e^x \frac{x-1}{x+1} + c$ Ans

Example: Workout $\int e^x \frac{1-sinx}{1-cosx} dx$

Sol. $I = \int e^x \frac{1-sinx}{1-cosx} dx = \int e^x \left[\frac{1}{1-cosx} - \frac{sinx}{1-cosx}\right] dx$

$= \int e^x \left[\frac{1}{2\sin^2(\frac{x}{2})} - \frac{2\sin\frac{x}{2}\, cos\frac{x}{2}}{2\sin^2(\frac{x}{2})}\right] dx = \int e^x \left[\frac{1}{2}\mathrm{cosec}^2(\frac{x}{2}) - cot\, \frac{x}{2}\right] dx$

$= -\int e^x \left[cot\, \frac{x}{2} - \frac{1}{2}\mathrm{cosec}^2(\frac{x}{2})\right] dx$

Let $f(x) = cot\, \frac{x}{2} => f'(x) = -\frac{1}{2}\mathrm{cosec}^2(\frac{x}{2})$

$I = \int e^x [f(x) + f'(x)] d = -e^x f(x) + c = -\, e^x cot\, \frac{x}{2} + c$ Ans

Example: Workout $\int xe^{2x} cosxdx$

Sol. $I = \int xe^{2x} cosx\, dx = x \int e^{2x} cosx\, dx - \int [\int e^{2x} cosx \quad dx\,] dx$

$$= \frac{xe^{2x}(2cosx + sinx)}{5} - \int \frac{e^{2x}(2cosx + sinx)}{5} dx$$

$$= \frac{xe^{2x}(2cosx + sinx)}{5} - \frac{2}{5} \int e^{2x} cosx\, dx - \frac{1}{5} \int e^{2x} sinx\, dx$$

$$= \frac{xe^{2x}(2cosx + sinx)}{5} - \frac{2}{5} \frac{e^{2x}(2cosx + sinx)}{5} - \frac{1}{5} \frac{e^{2x}(2sinx - cosx)}{5} + c$$

$$= \frac{xe^{2x}(2cosx+sinx)}{5} - \frac{e^{2x}(3cosx+4sinx)}{25} + c \text{ Ans}$$

Exam Workout $\int \frac{e^{3\tan^{-1}x}}{(1+x^2)^2} dx$

Solu $I = \int \frac{e^{3\tan^{-1}x}}{(1+x^2)^2} dx$, put $x = tan\theta, dx = \sec^2\theta d\theta$

$$I = \int \frac{e^{3\theta}}{(1 + tan^2 x)^2} \sec^2\theta\, d\theta = \int e^{3\theta} \cos^2\theta\, d\theta$$

$$= \frac{1}{2} \int e^{3\theta}(1 + cos2\theta) d\theta = \frac{1}{2} \int e^{3\theta} \quad d\theta + \frac{1}{2} \int e^{3\theta} cos2\theta\, d\theta$$

$$= \frac{1}{6} e^{3\theta} + \frac{e^{3\theta}(3cos2\theta + 2sin2\theta)}{26} + c$$

$$= \frac{1}{6} e^{3\tan^{-1}x} + \frac{e^{3\theta}}{26} \left\{ \frac{3(1-\tan^2\theta)}{1+\tan^2\theta} + \frac{4tan\theta}{1+\tan^2\theta} \right\} + c$$

$$= \frac{1}{6} e^{3\tan^{-1}x} + \frac{e^{3\tan^{-1}x}}{26} \left\{ \frac{3(1-x^2)}{1+x^2} + \frac{4x}{1+x^2} \right\} + c$$