Indian Institute of Technology Bhubaneswar

School of Infrastructure

Session: Spring 2026

Finite Element Method (CE6L314) Assignment No. 1

Date: February 06, 2026 Total Marks: 100

<u>Instructions:</u>

(1) If two or more answer scripts appear identical, each of them will be awarded ZERO.
(2) Provide neatly drawn figures to explain the concepts behind the problems whenever possible.
(3) Assume reasonable numerical values of the parameters for the comparison between two methods.
(4) For plotting purposes, you may use any programming language such as Julia, MATLAB, Python, etc.
(5) Provide the codes used for generating the plots.
(6) Submit your answer script by February 16, 2026.

1. (a) Explain the principle of minimum potential energy.
   (b) Show how the Rayleigh–Ritz method follows from this principle. [10]

2. (a) Define an admissible function in the Rayleigh–Ritz method.
   (b) Which boundary conditions must be satisfied exactly? Why?
   (c) Give two examples of admissible and two non-admissible trial functions for a fixed-fixed bar. [10]

3. (a) Explain the similarities between the Rayleigh-Ritz and Galerkin methods.
   (b) Why is the Rayleigh-Ritz method restricted to self-adjoint problems?
   (c) Explain why non-conservative forces cannot be handled directly using Rayleigh-Ritz method. [10]

4. Consider a uniform axial bar of length L, Young's modulus E, and cross-sectional area A. The bar is fixed at $x = 0$ and subjected to an axial force P at $x = L$.
   (a) Write the total potential energy functional of the system.
   (b) Assume a single-term admissible trial function and compute the Rayleigh-Ritz solution.
   (c) Repeat the analysis using two trial functions.
   (d) Compare the approximate solutions with the exact solution. Provide the plots. [10]

5. A bar of length L is fixed at both ends and subjected to a distributed load

$$f(x) = f_0 \sin\left(\frac{\pi x}{L}\right)$$

   (a) Formulate the Rayleigh–Ritz approximation using a one-term polynomial trial function.
   (b) Extend the solution using a two-term polynomial approximation.
   (c) Comment on the suitability of polynomial trial functions for this loading. Provide the plots. [10]

6. Solve the Poisson problem

$$-u''(x) = 1 \quad \text{in} \quad (0,1), \qquad u(0) = u(1) = 0$$

   (a) Apply the Rayleigh-Ritz method using polynomial trial functions.
   (b) Show explicitly what happens if the essential boundary conditions are violated.
   (c) Explain how the weak form and finite element method overcome this limitation. [10]

7. A simply supported beam of length L is subjected to a uniformly distributed load q.
   (a) Write the total potential energy functional for the beam.
   (b) Choose admissible trial functions based on:
   (i) polynomial functions, (ii) trigonometric functions.
   (c) Compare the convergence behavior of the two choices. Provide the plots. [10]

8. For a cantilever beam under tip load,
   (a) Derive the Rayleigh-Ritz equations using cubic polynomial trial functions.
   (b) Compute the tip deflection and strain energy.
   (c) Compare the results with the exact Euler-Bernoulli beam solution. provide the plots.
   (d) Comment on the improvement obtained with higher-order trial functions. [10]

9. (a) Show how the finite element method (FEM) can be interpreted as a piecewise Rayleigh-Ritz method.
   (b) Explain why FEM naturally supports h-refinement and complex geometries.
   (c) Give one example problem where the Rayleigh-Ritz method is more efficient than FEM. [10]

10. Consider

$$-u''(x) = 1 \quad \text{in} \quad (0,1), \qquad u(0) = 0, \qquad u'(1) = 0$$

   (a) Derive the weak form.
   (b) Identify essential and natural boundary conditions.
   (c) Solve the problem using the Galerkin method with two trial functions.
   (d) Compare the solution with the exact solution and comment on the results. Provide the plots. [10]