Indian Institute of Technology Bhubaneswar

School of Infrastructure

Subject Name : Solid Mechanics	Subject Code: CE2L001

Tutorial No. 1 Date: July 31, 2025

Instructions:
Provide neatly labelled diagrams whenever necessary.

1. The vector field is represented by

   a) $\boldsymbol{f} = -y\boldsymbol{e}_1 + x\boldsymbol{e}_2$

   b) $\boldsymbol{f} = (e^x y^2)\boldsymbol{e}_1 + (x + 2y)\boldsymbol{e}_2$

   Draw the vector field diagrammatically and determine the divergence and curl of the vector field.

2. A hill is described by the scalar function h(x, y) = 200 - x^2 - $2y^2$, where h is the height of the hill at any point (x, y) on the $x - y$ plane. If you are standing at the point $(1, 2)$, in which direction should you walk to climb the hill most rapidly, and what is the rate of increase of height in that direction?

3. a) Plot the following vector fields and determine which of them acts as source, sink, or solenoidal.

   1. $\boldsymbol{f} = x\boldsymbol{e}_1 + y\boldsymbol{e}_2$
   2. $\boldsymbol{f} = -x\boldsymbol{e}_1 - y\boldsymbol{e}_2$
   3. $\boldsymbol{f} = \boldsymbol{e}_1 + \boldsymbol{e}_2$

   Consider the following coordinates for x and y for plotting vector field

$[x, y]$	[0,1]	[1,0]	[1,1]	[0,-1]	[-1,0]	[-1,-1]	[1,-1]	[-1,1]	[0,0]

   b) Determine which of the following vector fields acts as source, sink, or solenoidal @ co-ordinate (1,2): (**No need to plot**)

   1. $\boldsymbol{f} = (2x + y^3)\boldsymbol{e}_1 + (x^3 + 2 * y)\boldsymbol{e}_2$
   2. $\boldsymbol{f} = (e^x)\boldsymbol{e}_1 + (e^y)\boldsymbol{e}_2$

   Hints:

   - Divergence of a vector is positive: Source (diverging).

- Divergence of a vector is zero: Solenoidal (divergence-free).
- Divergence of a vector is negative: Source (converging).

4. Consider that a paddle wheel is being placed on flowing water with the following velocity vector field of water particles on the pond's surface. Determine whether the paddle wheel will undergo any rotation or not. If rotation occurs, specify the direction of rotation. Consider a X-Y cartesian coordinate system.

   - a) $\boldsymbol{f} = (y+1)\boldsymbol{e}_1$
   - b) $\boldsymbol{f} = (x+1)\boldsymbol{e}_2$
   - c) $\boldsymbol{f} = \boldsymbol{e}_1 + \boldsymbol{e}_2$

   Consider the following coordinates for x and y for plotting vector field

$[x, y]$	[0,0]	[0,0.5]	[0.5,0]	[0.5,1.5]	[1,1.5]	[1.5,1.5]	[2,1.5]
$[x, y]$	[0.5,0.5]	[1,0.5]	[1.5,0.5]	[2,0.5]	[0,1]	[0.5,1]	[1,1]
$[x, y]$	[1.5,1]	[2,1]	[1,0]	[1,1]	[0,2]	[0.5,2]	[1,2]
$[x, y]$	[1.5,2]	[2,2]	[2,0]	[2,2]	[0,1.5]	[1.5,0]	[1.5,1.5]

5. Consider a cyclone in the northern hemisphere described by the velocity vector field of the wind:

$$\boldsymbol{v}(x, y) = -y\boldsymbol{e}_1 + (x^2 + y^2)\boldsymbol{e}_2$$

   where x and y are the coordinates in the horizontal plane, and $\boldsymbol{e}_1$ and $\boldsymbol{e}_2$ are unit vectors in the x- and y-directions, respectively.

   a) Calculate the divergence and curl of the vector field $\boldsymbol{v}(x, y)$.
   b) Explain the physical significance of the divergence and curl in the context of a cyclone.
   c) Based on the curl, determine the direction of rotation of the cyclone.

6. In Geo-technical engineering, understanding the flow of water in a dam's vicinity is crucial. The potential function $\phi(x, y)$ of a water flow around a dam is given by:

$$\phi(x, y) = x^2 - y^2$$

   a) Calculate the velocity vector field $\boldsymbol{v}(x, y)$ from the potential function $\phi(x, y)$.
   b) Determine the divergence and curl of the velocity vector field.
   c) Draw the vector field and discuss the behavior of water flow around the dam.

7. Consider the velocity vector field of water flow in an open channel given by:

$$\mathbf{v}(x, y) = (y, x)$$

   where x and y are the coordinates in the horizontal plane, and $\mathbf{i}$ and $\mathbf{j}$ are unit vectors in the x- and y-directions, respectively.

a) Calculate the divergence and curl of the vector field $\mathbf{v}(x, y)$.

b) Explain the physical significance of the divergence and curl in the context of open channel flow.

c) Plot the vector field and discuss the flow characteristics in the channel based on the vector field and curl.

8. Consider the airflow around an aircraft wing described by the velocity vector field:

$$\mathbf{v}(x, y) = \left(-ye^{-x^2-y^2}, (x-y)e^{-x^2-y^2}\right)$$

where x and y represent coordinates in the horizontal plane, and $\mathbf{i}$ and $\mathbf{j}$ are unit vectors in the x- and y-directions, respectively.

a) Calculate the divergence and curl of the vector field $\mathbf{v}(x, y)$.

b) Explain the physical significance of the divergence and curl in the context of aerodynamics around the aircraft wing.

c) Plot the vector field and discuss the flow characteristics around the wing based on the vector field and curl.