Indian Institute of Technology Bhubaneswar

School of Infrastructure

End Semester (Autumn) Examination – 2025

Subject Name : Solid Mechanics	Subject Code: CE2L001

Date: November 25, 2025

Duration: 3 Hours | Full Marks : 50

Instructions:

(1) Assume a reasonable value of any missing data.

(2) Provide neatly drawn figures whenever needed.

(3) Zeroth-order tensors or scalars are represented by small letters. For eg. a.

(4) First-order tensors or vectors are represented by bold small letters. For eg. $\boldsymbol{a}$.

(5) Second-order tensors are represented by bold capital letters. For eg. $\boldsymbol{A}$.

1. Explain the following concepts briefly.

   (a) Free index, dummy index, Kronecker delta symbol and the Permutation symbol.

   (b) Traction vector and stress tensor.

   (c) Displacement vector and strain tensor.

   (d) Eigenvalue problem corresponding to a second order tensor.

   (e) Invariants of a second order tensor. $[5 \times 2 = 10]$

2. (a) A square material element with side length 1 unit undergoes a volume-preserving deformation given by:

$$x_1' = \alpha x_1, \qquad x_2' = \frac{x_2}{\alpha}, \qquad \text{where } \alpha = 1.5.$$

   (i) Determine the deformation gradient tensor $\boldsymbol{F}$.

   (ii) Plot the reference and current configurations.

(iii) Calculate the stretch ratio in the x_1 direction and the contraction ratio in the x_2 direction.

(b) Consider a 60° strain gauge rosette to be mounted on the surface of a specimen as shown in Fig. 1 with $\theta_a = 0°$, $\theta_b = 60°$, and $\theta_c = 120°$. The strain gauge rosette measures strains of 500 μm/m, 700 μm/m, and 900 μm/m. Determine the principal strains, principal stresses, and maximum shear stress. [$4 + 6 = 10$]

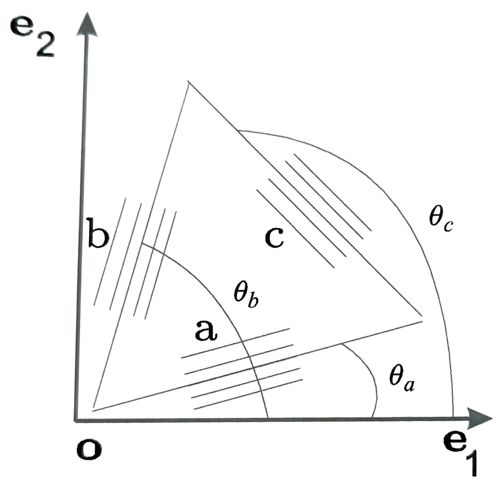


Figure 1: Schematic representation of a 60° strain-gauge rosette.

3. (a) Explain the following statement through theoretical derivations and illustrative figures.
"Vector and second-order tensor are independent of the coordinate system, but their components are not".

(b) The components of linearized strain tensor is given by

$$E_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right).$$

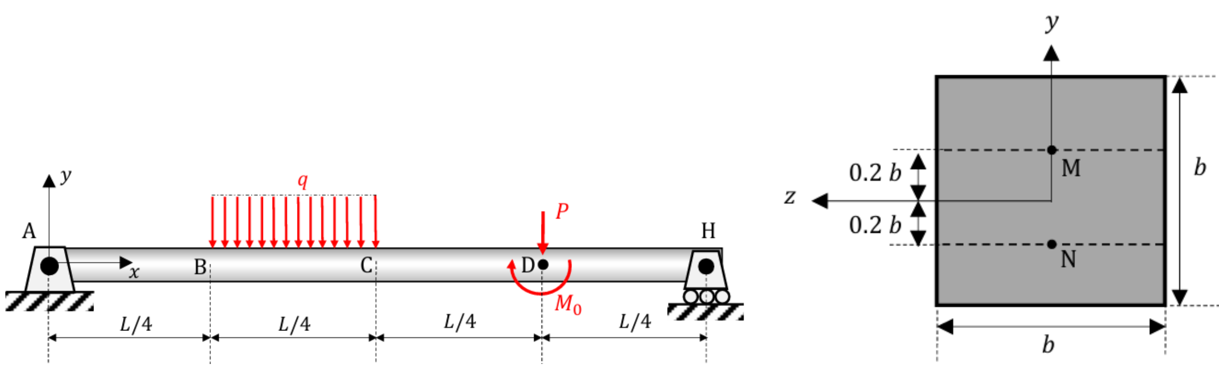
Consider E_{ij} as the components of a two-dimensional (2D) strain tensor in a basis $\{\boldsymbol{e_i}\}$.
(i) Derive the principal strains expression and show the principal direction of strain forms an angle θ_p with respect to $\boldsymbol{e_1}$ such that $\tan 2\theta_p = 2E_{12}/\left(E_{11} - E_{22}\right)$.
(ii) Derive the maximum shear strain expression and show that the normals of the planes of maximum shear form angles θ_s with respect to $\boldsymbol{e_1}$ such that $\tan 2\theta_s = -\left(E_{11} - E_{22}\right)/2E_{12}$.
(iii) Conclude that the direction of maximum shear is always oriented at an angle equal to 45° with respect to the principal directions of strain. [$4 + 6 = 10$]

4. (a) The stress components in a material body are given by:

$$\sigma_{11} = a_1\, x_1 x_2, \qquad \sigma_{22} = a_2\, x_2 x_3, \qquad \sigma_{12} = a_3\, x_1 x_3,$$

where a_1, a_2, and a_3 are small constants. Determine:
(i) The body force components.
(ii) The strain components using Hooke's law.
(iii) Whether the compatibility condition is satisfied.

(b) A simply supported beam AH is subjected to a constant distributed load q over the section BC, a moment M_0 and a concentrated force P at D. The cross section of the beam is shown below (See Fig. 2). The parameters are following: L=8 ft., q=10 lb/ft, M_0 = 40 lb-ft, P=10 lb, b= 2 in.
(i) Draw the shear force and bending moment diagrams. Mark the values at the cross sections A, B, C, D, and H, and the maximum and minimum values along the beam.
(ii) Determine the stress state at the points M and N which are located at the cross section C. Sketch their stress state on the given stress elements.
(iii) Provide Mohr's circle for the stress state at the points M and N. [$4 + 6 = 10$]



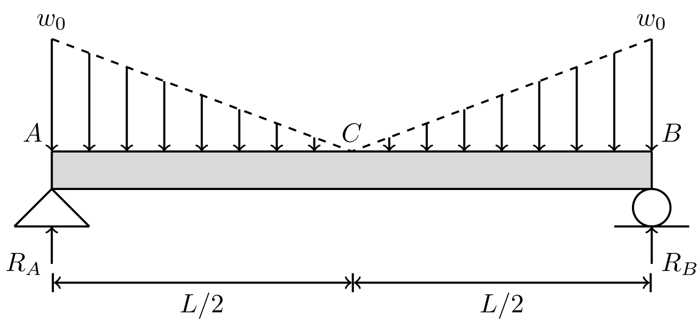
(a) (b) cross-section

Figure 2

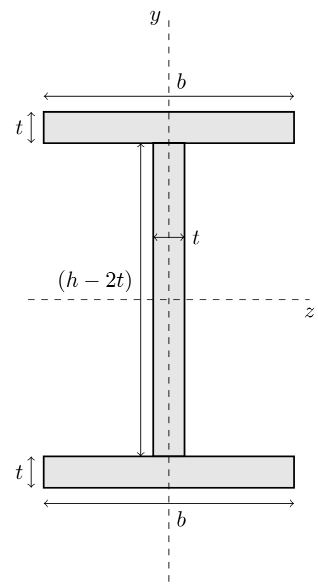
5. (a) (i) Explain the concept of shear center and torsion using figures and examples.

(ii) Comment on the symmetrical and unsymmetrical bending depending on the location of the shear center and centroid for different types of cross sections.

(b) Consider a simply supported I-beam of length L subjected to an applied load as shown in Fig. 3.



(a)



(b) cross-section

Figure 3

(i) Draw the bending moment and shear force diagram.

(ii) Find the slope at point A and deflection at point C of the I-beam shown in Fig. 3.

(iii) Determine the distribution of shear stress across the cross section of the I-beam.

$[4+6=10]$