Indian Institute of Technology Bhubaneswar

School of Infrastructure

Session: Autumn 2025

Solid Mechanics (CE2L001) — Assignment No. 3

Date: October 15, 2025 — Total Marks: 100

<u>Instructions:</u>

(1) If two or more answer scripts appear identical, each of them will be awarded ZERO.

(2) Provide neatly drawn figures to explain the concepts behind the problems.

(3) Submit your answer script by October 28, 2025.

<u>Notations :</u>

Zeroth-order tensors or scalars are represented by small letters. For eg. a

First-order tensors or vectors are represented by bold small letters. For eg. $\boldsymbol{a}$.

Second-order tensors are represented by bold capital letters. For eg. $\boldsymbol{A}$

1. Consider the following two-dimensional transformation

$$x_1' = 4 - 2x_1 - x_2, \qquad x_2' = 2 + \frac{3x_1}{2} - \frac{x_2}{2}$$

   (a) Is the transformation linear?

   (b) Calculate the components of the deformation gradient $\boldsymbol{F}$. Compute $\det(\boldsymbol{F})$, and $\boldsymbol{F}^{-1}$, where $\det(\cdot)$ and $(\cdot)^{-1}$ denote the determinant and the inverse of a second order tensor, respectively.

   (c) Study the transformation over a unit square defined through the following corner points (0,0), (1,0), (1,1), (0,1).

[10]

2. Determine the linear strain tensor $\boldsymbol{E}_s = (1/2)(\nabla \boldsymbol{u} + \nabla \boldsymbol{u}^T)$ for the following displacement fields, given in a Cartesian basis:

   (a) $\boldsymbol{u} = a\,(x_2 - x_1)\,\boldsymbol{e_1} + a\,(x_1 - x_2)\,\boldsymbol{e_2} + a\,x_1\,x_3\,\boldsymbol{e_3}$

   (b) $\boldsymbol{u} = a\,x_2\,x_3\,\boldsymbol{e_1} + b\,x_3\,x_1\,\boldsymbol{e_2} + c\,x_1\,x_2\,\boldsymbol{e_3}$

   (c) $\boldsymbol{u} = -a\,x_1\,x_2\,\boldsymbol{e_1} + (bx_1^2 + cx_2^2 - cx_3^2)\,\boldsymbol{e_2} + c\,x_2\,x_3\,\boldsymbol{e_3}$

   (d) $\boldsymbol{u} = a\,(3x_1^2 + x_2)\,\boldsymbol{e_1} + a\,(2x_2^2 + x_3)\,\boldsymbol{e_2} + a\,(4x_3^2 + x_1)\,\boldsymbol{e_3}$

   where a, b and c are positive constants. Also for each of the above displacement fields, determine the normal strain at the point $(x_1, x_2, x_3) = (1, 1, 1)$ for the direction $\boldsymbol{n} = \frac{1}{\sqrt{3}}(\boldsymbol{e}_1 + \boldsymbol{e}_2 + \boldsymbol{e}_3)$. [10]

3. The displacement field in a turbine blade of a jet engine shown in Fig. 1 can be given in a Cartesian reference frame by $\boldsymbol{u} = u_1\boldsymbol{e_1} + u_2\boldsymbol{e_2} + u_3\boldsymbol{e_3}$, where

$$
\begin{aligned}
u_1 &= c\,(x_1^2 + 10)\\
u_2 &= 2\,c\,x_2\,x_3\\
u_3 &= c\,(-x_1x_2 + x_3^2),
\end{aligned}
$$

with $c = 10^{-4}$ mm. Determine the components of the strain tensor $\boldsymbol{E} = (1/2)(\boldsymbol{F}^T\boldsymbol{F} - \boldsymbol{I})$ and linear strain tensor $\boldsymbol{E}_s = (1/2)(\nabla\boldsymbol{u} + \nabla\boldsymbol{u}^T)$ at $(x_1,\, x_2,\, x_3) = (0,\, 2,\, 1)$ mm. Comment on the results.

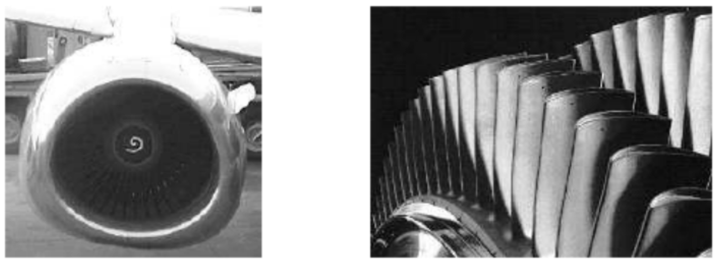

Figure 1: Turbine blades of a jet engine

[10]

4. Give an interpretation and determine the invariants of the strain tensors given in component form as

(a) $[\boldsymbol{E}] = \begin{bmatrix} \epsilon & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

(b) $[\boldsymbol{E}] = \begin{bmatrix} \epsilon & 0 & 0 \\ 0 & \epsilon & 0 \\ 0 & 0 & 0 \end{bmatrix}$

(c) $[\boldsymbol{E}] = \begin{bmatrix} -\epsilon & 0 & 0 \\ 0 & -\epsilon & 0 \\ 0 & 0 & 0 \end{bmatrix}$

(d) $[\boldsymbol{E}] = \begin{bmatrix} -\epsilon & 0 & 0 \\ 0 & -\epsilon & 0 \\ 0 & 0 & -\epsilon \end{bmatrix}$

(e) $[\boldsymbol{E}] = \begin{bmatrix} 0 & \frac{\gamma}{2} & 0 \\ \frac{\gamma}{2} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

where ϵ and γ are small positive numbers. [10]

5. In the direct extrusion process, a round billet is placed in a chamber and forced through a die opening by a hydraulic-driven ram in Fig. 2. The extrusion pressure is affected by the die angle, the reduction in cross-section, extrusion speed, billet temperature and lubrication. In the vicinity of the corner of the die a rectangular block of material is considered, with its axes oriented along an ortho-normal basis $\boldsymbol{e}_1$, $\boldsymbol{e}_2$ and $\boldsymbol{e}_3$. Its dimension along each of the three axes is l. The block is deformed as shown in Fig. 3 . The thickness remains unchanged. Determine the deformation gradient tensor $\boldsymbol{F}$. [10]

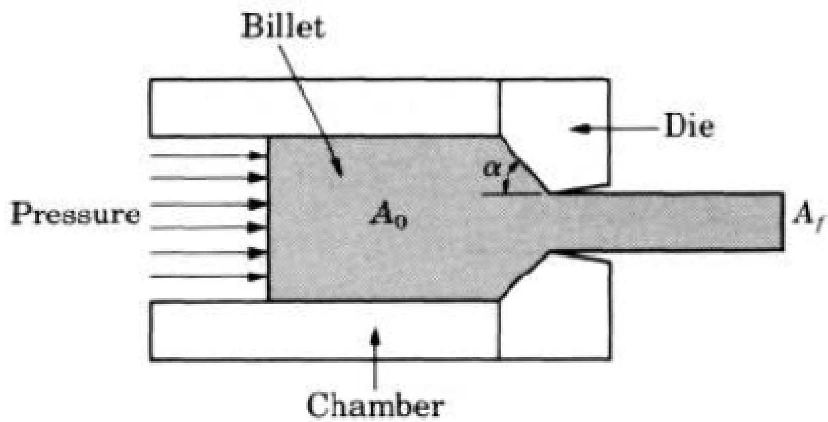

Figure 2: The direct extrusion process

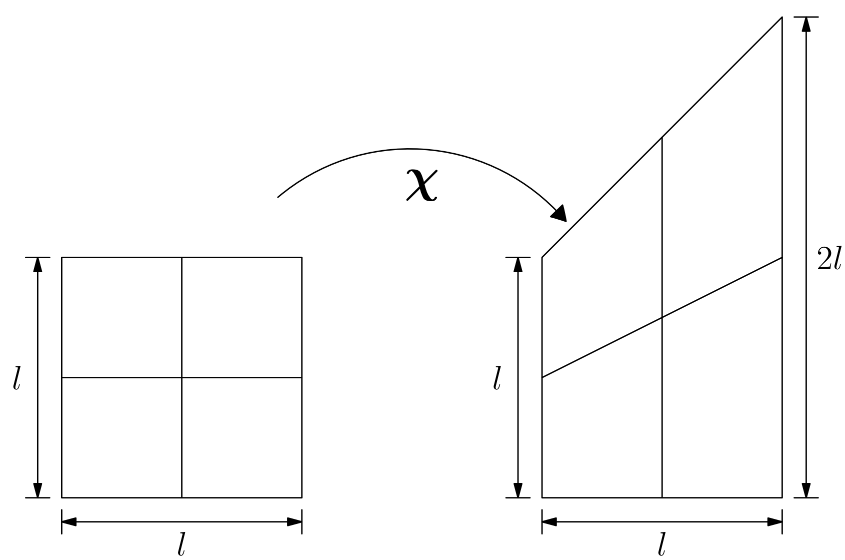

Figure 3: Deformation of body

6. Consider the following transformation

$$x_1' = x_1$$
$$x_2' = x_3,$$
$$x_3' = -x_2,$$

   (a) Is the transformation linear?

   (b) Calculate the components of the deformation gradient $\boldsymbol{F}$. Compute $\det(\boldsymbol{F})$, and $\boldsymbol{F}^{-1}$, where $\det(\cdot)$ and $(\cdot)^{-1}$ denote the determinant and the inverse of a second order tensor, respectively.

   (c) Study the transformation over a unit cube defined by the coordinates of the corner points (0,0,0) , (1,0,0) , (1,1,0) , (0,1,0), (0,0,1) , (1,0,1) , (1,1,1) , (0,1,1).

[10]

7. The following transformation is assigned

$$x_1' = x_1 + \alpha x_2$$
$$x_2' = x_2,$$
$$x_3' = x_3,$$

   where α is a generic constant.

   (a) Study the deformation of a unit cube defined by the coordinates of the corner points (0,0,0) , (1,0,0) , (1,1,0) , (0,1,0), (0,0,1) , (1,0,1) , (1,1,1) , (0,1,1).

   (b) Calculate the components of tensor $\boldsymbol{C} = \boldsymbol{F}^T\boldsymbol{F}$.

[10]

8. The motion of the body is described as

$$x_1' = \frac{1}{\sqrt{2}}(x_1 - x_2 + 5)$$
$$x_2' = \frac{1}{\sqrt{2}}(x_1 + x_2 + 3),$$
$$x_3' = x_3 + 6,$$

   (a) Find the deformation gradient, $\boldsymbol{F}$, for the motion.

   (b) Calculate
   (i) $\boldsymbol{B} = \boldsymbol{F}\,\boldsymbol{F}^T$,
   (ii) $\boldsymbol{C} = \boldsymbol{F}^T\boldsymbol{F}$,
   (iii) $\boldsymbol{E} = (1/2)(\boldsymbol{F}^T\boldsymbol{F} - \boldsymbol{I})$ and
   (iv) $\boldsymbol{E}_s = (1/2)(\nabla\boldsymbol{u} + \nabla\boldsymbol{u}^T)$.
   Comment on the results.

[10]

9. Consider a linearized strain field $\boldsymbol{E}_s(\boldsymbol{x})$ whose components are given by

$$[\boldsymbol{E}_s(\boldsymbol{x})] = \begin{bmatrix} 3x_1 & 5x_2 + 6x_3 & (x_3)^3 \\ 5x_2 + 6x_3 & 0 & (x_1)^2 + (x_2)^2 \\ (x_3)^3 & (x_1)^2 + (x_2)^2 & \exp(x_1) \end{bmatrix} \times 10^{-6}$$

(a) Find the principal strains and directions at $x_i = (1, 2, 3)$.
(b) What is the normal strain in the direction $n_i = (1, 1, 1)$ at the point $x_i = (2, 2, 0)$?
(c) What is the change in angle between $v_i^{(1)} = (1, 1, 1)$ and $v_i^{(2)} = (2, 1, 3)$ at the point $x_i = (1, 1, 1)$?
(d) What is the volumetric strain at $x_i = (0, 0, 0)$? [10]

10. The linearized strain at a particular point in a body is given by

$$[\boldsymbol{E}_s] = \begin{bmatrix} 7 & 8 & 0 \\ 8 & 9 & 3 \\ 0 & 3 & 55 \end{bmatrix} \times 10^{-5}$$

in the $\{\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}\}$ basis where $\boldsymbol{a} = \frac{1}{\sqrt{3}}(\boldsymbol{e}_1 + \boldsymbol{e}_2 + \boldsymbol{e}_3)$, $\boldsymbol{b} = \frac{1}{\sqrt{2}}(\boldsymbol{e}_1 - \boldsymbol{e}_2)$, and $\boldsymbol{c} = \frac{1}{\sqrt{6}}(\boldsymbol{e}_1 + \boldsymbol{e}_2 - 2\boldsymbol{e}_3)$.
(a) Find the max normal and shear strains at this point.
(b) Find the normal strain in the $\boldsymbol{e}_1$ direction at this point.
(c) Find the angle change between the $\boldsymbol{e}_1$ and $\boldsymbol{e}_2$ directions at this point. [10]