Indian Institute of Technology Bhubaneswar

School of Infrastructure

Subject Name : Solid Mechanics	Subject Code: CE2L001
Tutorial No. 3	Date: Sep 04, 2025

1. The three principal invariants of a second-order tensor $\boldsymbol{T} = T_{ij}\boldsymbol{e}_i \otimes \boldsymbol{e}_j$ are defined as
   (i) $I_1 = \text{trace}\,(\boldsymbol{T}) = T_{ii}$,
   (ii) $I_2 = (1/2)\left((\text{trace}\,(\boldsymbol{T}))^2 - \text{trace}\,(\boldsymbol{T}^2)\right) = (1/2)\,(T_{ii}T_{jj} - T_{ij}T_{ij})$
   (iii) and $I_3 = \det(\boldsymbol{T}) = (1/6)\,\epsilon_{lmn}\,\epsilon_{ijk}\,T_{li}\,T_{mj}\,T_{nk}$.
   Prove that I_1, I_2 and I_3 are indeed invariants, i.e., they do not change with the change in the coordinate system.

2. In two dimensions, let us consider two orthogonal basis vectors $\boldsymbol{e_i}$ and $\boldsymbol{e_i^*}$ such that $\boldsymbol{e_1^*}$ is oriented at an angle θ with respect to $\boldsymbol{e_1}$. σ_{ij} and σ^*_{ij} are, respectively, the components of a stress tensor $\boldsymbol{\sigma} = \sigma_{ij}\boldsymbol{e_i}\otimes\boldsymbol{e_j} = \sigma^*_{ij}\boldsymbol{e_i^*} \otimes \boldsymbol{e_j^*}$ expressed in the $\boldsymbol{e_i}$ and $\boldsymbol{e_i^*}$ bases. Using the expression: $\sigma^*_{ij} = \sigma_{lk}\,(\boldsymbol{e_l} \cdot \boldsymbol{e_i^*})\left(\boldsymbol{e_j^*} \cdot \boldsymbol{e_k}\right)$,

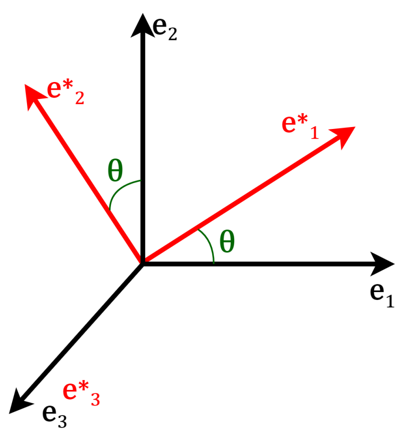

Figure 1: Transformation of coordinate system.

   (a) Derive the following relations:

$$\sigma^*_{11} = \sigma_{11}\cos^2\theta + \sigma_{22}\sin^2\theta + \sigma_{12}\sin 2\theta$$
$$\sigma^*_{22} = \sigma_{11}\sin^2\theta + \sigma_{22}\cos^2\theta - \sigma_{12}\sin 2\theta$$
$$\sigma^*_{12} = -\frac{\sigma_{11} - \sigma_{22}}{2}\sin 2\theta + \sigma_{12}\cos 2\theta$$

   (b) Show that these equations are related to Mohr's circle for plane stress problems.

3. Suppose we have a material under plane stress conditions with the following stress state: $\sigma_{11} = 80$ MPa, $\sigma_{22} = 20$ MPa, and $\sigma_{12} = 40$ MPa. Draw the Mohr's circle for this state of stress.

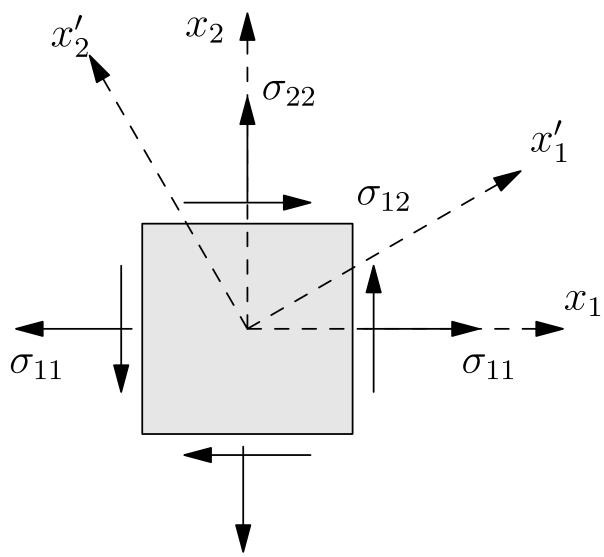

Figure 2: Two-dimensional (2D) stress element corresponding to the given stress state.

4. Give practical examples corresponding to each of the stress states shown in Fig. 3 and sketch the Mohr's circles. For each case, explain the usefulness of the Mohr's circle. Assume suitable numerical values for $\sigma_0 > 0$ and $\tau_o > 0$.

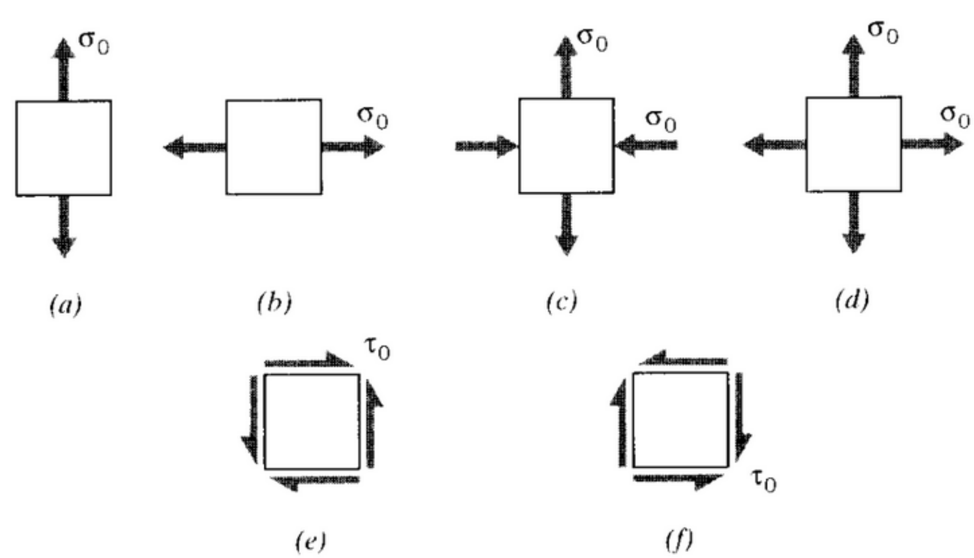

Figure 3: Different state of stress

5. To test a glue, two plates are glued together as shown in Fig. 4. The bar formed by the joined plates is then subjected to tensile axial loads of 200 N. Using stress states and related Mohr's circle, determine the normal and shear stresses act on the plane where the plates are glued together (In other words, what stresses must the glue support?).

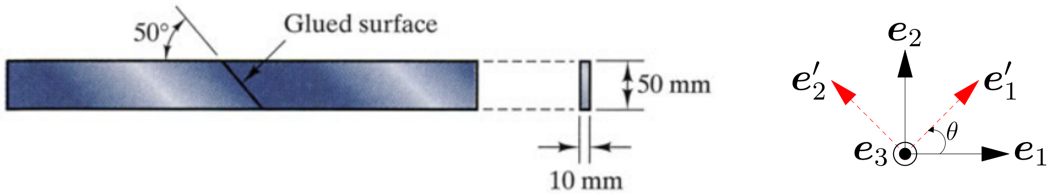

Figure 4

6. A point p of the car's frame is subjected to the components of plane stress $\sigma_{x'x'} = 32$ MPa, $\sigma_{y'y'} = -16$ MPa, and $\sigma_{x'y'} = -24$ MPa. If $\theta = 35°$, what are the stresses σ_{xx}, σ_{yy}, and σ_{xy} at p?

Figure 5: Stress transformation problem for the car frame at point p.

7. For the stress tensor given below, determine the principal stresses and their corresponding principal directions.

$$[\boldsymbol{\sigma}] = \begin{pmatrix} 10 & 2 & 0 \\ 2 & 20 & 0 \\ 0 & 0 & 30 \end{pmatrix} \text{ MPa.}$$

8. A point (say, P) of a solid body is subjected to the state of stress (in MPa):

$$[\boldsymbol{\sigma}] = \begin{bmatrix} 4 & 2 & 1 \\ 2 & 2 & 1 \\ 1 & 1 & 3 \end{bmatrix}.$$

Determine the principal stresses and the absolute maximum shear stress at point P.