Indian Institute of Technology Bhubaneswar

School of Infrastructure

Session: Autumn 2025

Solid Mechanics (CE2L001) | Compensatory Class Test 1

Date: September 13, 2025 | Total Marks: 50

> Instructions:
> (1) Zeroth-order tensors or scalars are represented by small letters. For eg. a.
> (2) First-order tensors or vectors are represented by bold small letters. For eg. $\boldsymbol{a}$.
> (3) Second-order tensors are represented by bold capital letters. For eg. $\boldsymbol{A}$.

1. (a) Simplify the following expressions:
   (i) $\delta_{ij}\,\delta_{ik}\,\delta_{jk}$, (ii) $\epsilon_{ijk}\,\epsilon_{ijk}$ and (iii) $\epsilon_{ijk}u_iu_jv_k$.
   (b) Show that

$$\mathbf{a}\times(\mathbf{b}\times\mathbf{c}) = (\mathbf{a}\cdot\mathbf{c})\mathbf{b} - (\mathbf{a}\cdot\mathbf{b})\mathbf{c}$$

   . $[6+4=10]$

2. Which of the following vector fields acts as source, sink, or solenoidal @ co-ordinate (1, 2):
   (a) $2x\boldsymbol{e}_1 + 5y\boldsymbol{e}_2$,
   (b) $-6x\boldsymbol{e}_1 - 3y\boldsymbol{e}_2$,
   (c) $(4x+y^2)\,\boldsymbol{e}_1 + (x^4+2y)\,\boldsymbol{e}_2$.
   Justify your answer using plots of the vector fields. Consider the following Cartesian coordinates (x, y) for plotting the vector field.

$[x, y]$	[-1,0]	[-1,-1]	[-1,1]	[0,0]	[0,-1]	[0,1]	[1,-1]	[1,0]	[1,1]

   [10]

3. Let $\boldsymbol{r} = x\ \boldsymbol{e}_1 + y\ \boldsymbol{e}_2 + z\ \boldsymbol{e}_3$ be the position vector field in $\mathbb{R}^3$. Find
   (a) The gradient of $f(\boldsymbol{r}) = \boldsymbol{r}\cdot\boldsymbol{r}$,
   (b) The divergence of $\boldsymbol{r}$,
   (c) The curl of $\boldsymbol{r}$. [10]

4. (a) Consider the second-order tensor $\boldsymbol{A}$ given by

$$\boldsymbol{A} = 3\,(\boldsymbol{e}_1\otimes\boldsymbol{e}_1) - 4\,(\boldsymbol{e}_1\otimes\boldsymbol{e}_2) + 2\,(\boldsymbol{e}_2\otimes\boldsymbol{e}_1) + (\boldsymbol{e}_2\otimes\boldsymbol{e}_2) + (\boldsymbol{e}_3\otimes\boldsymbol{e}_3)\,.$$

   Determine the image of the vector $\boldsymbol{v} = 4\boldsymbol{e}_1 + 2\boldsymbol{e}_2 + 5\boldsymbol{e}_3$ when $\boldsymbol{A}$ operates on it.

(b) Consider a two-dimensional orthonormal basis $\{\boldsymbol{e}_1, \boldsymbol{e}_2\}$ in which a two-dimensional tensor $\boldsymbol{T}$ has the representation

$$\boldsymbol{T} = T_{ij}\, \boldsymbol{e}_i \otimes \boldsymbol{e}_j \qquad i, j = 1, 2,$$

and the component matrix of $\boldsymbol{T}$ has values

$$[\boldsymbol{T}] = \begin{bmatrix} 5 & 10 \\ 15 & 20 \end{bmatrix}.$$

Consider a second basis $\{\boldsymbol{e}_1^*, \boldsymbol{e}_2^*\}$ which is related to $\{\boldsymbol{e}_1, \boldsymbol{e}_2\}$ by

$$\boldsymbol{e}_1^* = \frac{1}{2}\boldsymbol{e}_1 + \frac{\sqrt{3}}{2}\boldsymbol{e}_2, \qquad \boldsymbol{e}_2^* = -\frac{\sqrt{3}}{2}\boldsymbol{e}_1 + \frac{1}{2}\boldsymbol{e}_2,$$

Find the value of T_{11}^* in the $\{\boldsymbol{e}_1^*, \boldsymbol{e}_2^*\}$ basis. [4 + 6 = 10]

5. (a) Prove that Mohr's circle of stress is a graphical representation corresponding to the coordinate transformation of stress components.

(b) The components of plane stress on an element of an industrial robot are shown in Fig. 0.1. Determine the stresses σ and τ by using Mohr's circle.

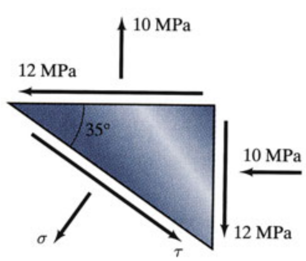

Figure 0.1

[5 + 5 = 10]