Indian Institute of Technology Bhubaneswar

School of Infrastructure

Subject Name: Solid Mechanics	**Subject Code:** CE2L001

Tutorial No.: 5 **Date:** October 23, 2025

Instructions:
Provide neatly labeled diagrams whenever necessary.

1. Derive the expressions for the strain compatibility conditions.

2. a) Given the following two-dimensional, infinitesimal strain field:

$$E_{11} = c_1\, x_1\, (x_1^2 + x_2^2),$$
$$E_{22} = \frac{1}{3} c_2\, x_1^3,$$
$$E_{12} = E_{21} = c_3\, x_1^2\, x_2,$$

where c_1, c_2, and c_3 are constants, determine if the strain field is compatible.

b) The strain field is given as:

$$E_{11} = 2\,\alpha x_1\, x_2,$$
$$E_{22} = 2\,\beta x_1\, x_2,$$
$$E_{12} = E_{21} = \frac{1}{2}(\alpha\, x_1^2 + \beta x_2^2),$$

Check whether the strain field is compatible or not, where α and β are constants.

3. In two dimensions, let us consider two basis vectors $\boldsymbol{e_i}$ and $\boldsymbol{e_j^*}$ such that $\boldsymbol{e_1^*}$ is oriented at an angle θ with respect to $\boldsymbol{e_1}$. E_{ij} and E_{ij}^* are, respectively, the components of a strain tensor E expressed in the $\boldsymbol{e_i}$ and $\boldsymbol{e_j^*}$ bases (corresponding to the same state of deformation).

Using the following expression:

$$E_{ij}^* = E_{lk}(\boldsymbol{e_l} \cdot \boldsymbol{e_i^*})(\boldsymbol{e_j^*} \cdot \boldsymbol{e_k}),$$

derive the following relations:

$$
\begin{aligned}
E^*_{11} &= E_{11}\cos^2\theta + E_{22}\sin^2\theta + E_{12}\sin 2\theta,\\
E^*_{22} &= E_{11}\sin^2\theta + E_{22}\cos^2\theta - E_{12}\sin 2\theta,\\
E^*_{12} &= -\frac{E_{11}-E_{22}}{2}\sin 2\theta + E_{12}\cos 2\theta.
\end{aligned}
$$

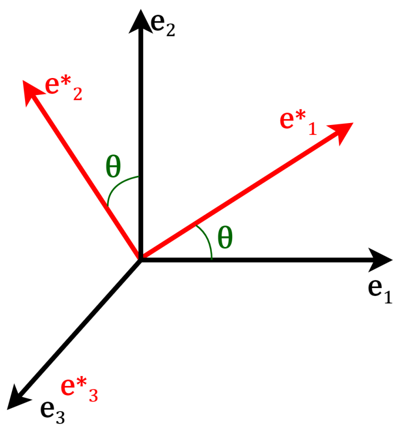

Figure 1: Representation of the two basis vectors

4. Using the relations $E_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$, show that, given the components E_{ij} of a 2D strain tensor in a basis $\boldsymbol{e_i}$:

   a) Derive the principal strains expression:

   $$
   E_{1,2} = \frac{E_{11}+E_{22}}{2} \pm \sqrt{\left(\frac{E_{11}-E_{22}}{2}\right)^2 + E_{12}^2},
   $$

   and the principal direction of strain for the angle with respect to $\boldsymbol{e_1}$ satisfies:

   $$
   \tan 2\theta^p = \frac{2E_{12}}{E_{11}-E_{22}}.
   $$

   b) Derive the maximum shear strain expression:

   $$
   E_{12}^{\max} = \sqrt{\left(\frac{E_{11}-E_{22}}{2}\right)^2 + E_{12}^2},
   $$

   and the normals of the planes of maximum shear form angles with respect to $\boldsymbol{e_1}$:

   $$
   \tan 2\theta^s = -\frac{E_{11}-E_{22}}{2E_{12}}.
   $$

   Conclude that the direction of maximum shear is always oriented at an angle equal to 45° with respect to the principal directions of strain.

5. Consider the problem of the isotropic cantilever beam bent by a load P at the free end, as shown in Fig. 2. From the elementary beam theory, we have the following strains:

$$
\begin{aligned}
S_{11} &= -\frac{P x_1 x_2}{EI}, \\
S_{22} &= \nu \frac{P x_1 x_2}{EI}, \\
S_{12} &= -\frac{(1+\nu)P}{2EI}(h^2 - x_2^2).
\end{aligned}
\tag{1}
$$

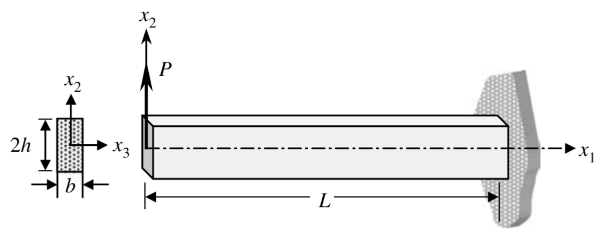

Figure 2: Cantilever beam bent by a point load, P

where I is the second moment of area about the x_3-axis, ν is the Poisson ratio, E is Young's modulus, and $2h$ is the height of the beam.

(a) Determine whether the strains are compatible.

(b) If compatible, find the displacement field using the linearized strain-displacement relations.

(c) Determine the constants of integration using suitable boundary conditions.

6. A rectangular loaded plate is clamped along the x_1 and x_2 axes (see Fig. 3). On the basis of measurements, the strain components are determined as:

$$E_{11} = a\,(x_1^2\,x_2 + x_2^3), \qquad E_{22} = b\,x_1\,x_2^2.$$

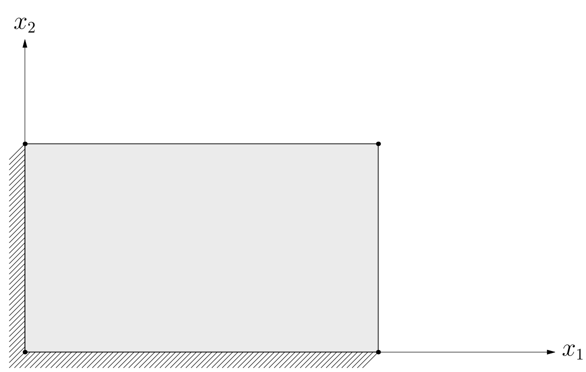

Figure 3: Rectangular plate

Determine the following:

(a) Find the displacement field.

(b) Compute the shear strain component, E_{12}.

(c) Check for strain compatibility condition.

7. Find the linearized strain field associated with the following displacements:

$$u_1 = x_1^3 x_2 + 2c_1 c_2^3 x_1 + 3c_1 c_2^2 x_1 x_2 - c_1 x_1 x_2^3, \tag{2}$$
$$u_2 = -2c_2^3 x_2 - \frac{3}{2} c_2^2 x_2^2 + \frac{1}{4} x_2^4 - \frac{3}{2} c_1 x_1^2 x_2^2, \tag{3}$$

where c_1, c_2, and c_3 are constants.

8. The displacement field in a material is given by:

$$u_1 = a x_2 x_1^3, \qquad u_2 = b x_1 x_2^2,$$

where a is a small constant. Determine:

(a) The components of the linearized strain tensor.

(b) The components of the linearized rotation tensor.

(c) Whether the compatibility condition is satisfied.

9. A continuous body undergoes a homogeneous plane deformation. A material point initially at position $\boldsymbol{x} = (x_1, x_2)$ moves to a new position $\boldsymbol{x}' = (x_1', x_2')$ according to the following mapping:

$$x_1' = 3\,x_1 + x_2, \qquad x_2' = 2\,x_1 + 2\,x_2$$

Calculate the following:

(a) The deformation gradient tensor, $\mathbf{F}$.

(b) The right Cauchy-Green deformation tensor, $\mathbf{C}$.

(c) The principal stretches, λ_1 and λ_2.

10. A body undergoes simple shear deformation, where γ is a constant representing the shear. The deformation mapping is given by:

$$x_1' = x_1 + \gamma\, x_2, \qquad x_2' = x_2$$

Determine the following:

(a) The deformation gradient tensor, $\mathbf{F}$.

(b) The right Cauchy-Green deformation tensor, $\mathbf{C}$.

(c) The principal stretches, λ_1 and λ_2, in terms of γ.