



**Indian Institute of Technology Bhubaneswar**  
School of Infrastructure

**Subject Name:** Solid Mechanics

**Subject Code:** CE2L001

**Tutorial No.:** 5

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**Instructions:**

Provide neatly labeled diagrams whenever necessary.

1. Derive the expressions for the strain compatibility conditions.
2. a) Given the following two-dimensional, infinitesimal strain field:

$$E_{11} = c_1 x_1 (x_1^2 + x_2^2),$$

$$E_{22} = \frac{1}{3} c_2 x_1^3,$$

$$E_{12} = E_{21} = c_3 x_1^2 x_2,$$

where  $c_1$ ,  $c_2$ , and  $c_3$  are constants, determine if the strain field is compatible.

- b) The strain field is given as:

$$E_{11} = 2 \alpha x_1 x_2,$$

$$E_{22} = 2 \beta x_1 x_2,$$

$$E_{12} = E_{21} = \frac{1}{2} (\alpha x_1^2 + \beta x_2^2),$$

Check whether the strain field is compatible or not, where  $\alpha$  and  $\beta$  are constants.

3. In two dimensions, let us consider two basis vectors  $\mathbf{e}_i$  and  $\mathbf{e}_j^*$  such that  $\mathbf{e}_1^*$  is oriented at an angle  $\theta$  with respect to  $\mathbf{e}_1$ .  $E_{ij}$  and  $E_{ij}^*$  are, respectively, the components of a strain tensor  $E$  expressed in the  $\mathbf{e}_i$  and  $\mathbf{e}_j^*$  bases (corresponding to the same state of deformation).

Using the following expression:

$$E_{ij}^* = E_{lk} (\mathbf{e}_l \cdot \mathbf{e}_i^*) (\mathbf{e}_j^* \cdot \mathbf{e}_k),$$

derive the following relations:

$$\begin{aligned} E_{11}^* &= E_{11} \cos^2 \theta + E_{22} \sin^2 \theta + E_{12} \sin 2\theta, \\ E_{22}^* &= E_{11} \sin^2 \theta + E_{22} \cos^2 \theta - E_{12} \sin 2\theta, \\ E_{12}^* &= -\frac{E_{11} - E_{22}}{2} \sin 2\theta + E_{12} \cos 2\theta. \end{aligned}$$

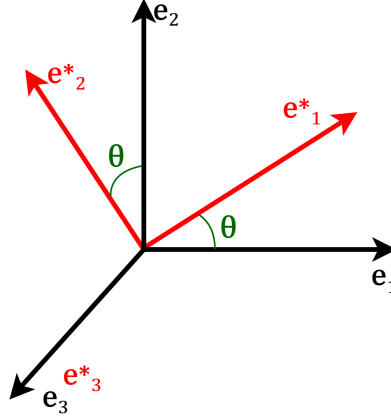


Figure 1: Representation of the two basis vectors

4. Using the relations  $E_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$ , show that, given the components  $E_{ij}$  of a 2D strain tensor in a basis  $\mathbf{e}_i$ :

a) Derive the principal strains expression:

$$E_{1,2} = \frac{E_{11} + E_{22}}{2} \pm \sqrt{\left( \frac{E_{11} - E_{22}}{2} \right)^2 + E_{12}^2},$$

and the principal direction of strain for the angle with respect to  $\mathbf{e}_1$  satisfies:

$$\tan 2\theta^p = \frac{2E_{12}}{E_{11} - E_{22}}.$$

b) Derive the maximum shear strain expression:

$$E_{12}^{\max} = \sqrt{\left( \frac{E_{11} - E_{22}}{2} \right)^2 + E_{12}^2},$$

and the normals of the planes of maximum shear form angles with respect to  $\mathbf{e}_1$ :

$$\tan 2\theta^s = -\frac{E_{11} - E_{22}}{2E_{12}}.$$

Conclude that the direction of maximum shear is always oriented at an angle equal to  $45^\circ$  with respect to the principal directions of strain.

5. Consider the problem of the isotropic cantilever beam bent by a load  $P$  at the free end, as shown in Fig.  
 2. From the elementary beam theory, we have the following strains:

$$\begin{aligned} S_{11} &= -\frac{Px_1x_2}{EI}, \\ S_{22} &= \nu\frac{Px_1x_2}{EI}, \\ S_{12} &= -\frac{(1+\nu)P}{2EI}(h^2 - x_2^2). \end{aligned} \quad (1)$$

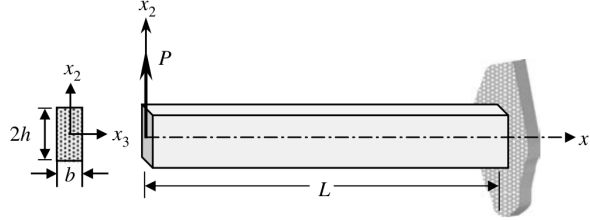


Figure 2: Cantilever beam bent by a point load,  $P$

where  $I$  is the second moment of area about the  $x_3$ -axis,  $\nu$  is the Poisson ratio,  $E$  is Young's modulus, and  $2h$  is the height of the beam.

- Determine whether the strains are compatible.
  - If compatible, find the displacement field using the linearized strain-displacement relations.
  - Determine the constants of integration using suitable boundary conditions.
6. A rectangular loaded plate is clamped along the  $x_1$  and  $x_2$  axes (see Fig. 3). On the basis of measurements, the strain components are determined as:

$$E_{11} = a(x_1^2x_2 + x_2^3), \quad E_{22} = b x_1 x_2^2.$$

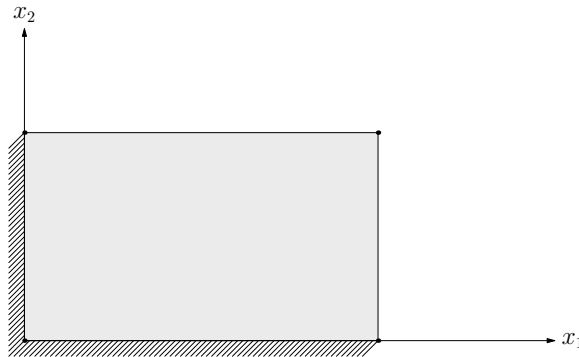


Figure 3: Rectangular plate

Determine the following:

- (a) Find the displacement field.
  - (b) Compute the shear strain component,  $E_{12}$ .
  - (c) Check for strain compatibility condition.
7. Find the linearized strain field associated with the following displacements:

$$u_1 = x_1^3 x_2 + 2c_1 c_2^3 x_1 + 3c_1 c_2^2 x_1 x_2 - c_1 x_1 x_2^3, \quad (2)$$

$$u_2 = -2c_2^3 x_2 - \frac{3}{2} c_2^2 x_2^2 + \frac{1}{4} x_2^4 - \frac{3}{2} c_1 x_1^2 x_2^2, \quad (3)$$

where  $c_1, c_2$ , and  $c_3$  are constants.

8. The displacement field in a material is given by:

$$u_1 = ax_2 x_3^3, \quad u_2 = bx_1 x_2^2,$$

where  $a$  is a small constant. Determine:

- (a) The components of the linearized strain tensor.
  - (b) The components of the linearized rotation tensor.
  - (c) Whether the compatibility condition is satisfied.
9. A continuous body undergoes a homogeneous plane deformation. A material point initially at position  $\mathbf{x} = (x_1, x_2)$  moves to a new position  $\mathbf{x}' = (x'_1, x'_2)$  according to the following mapping:

$$x'_1 = 3x_1 + x_2, \quad x'_2 = 2x_1 + 2x_2$$

Calculate the following:

- (a) The deformation gradient tensor,  $\mathbf{F}$ .
  - (b) The right Cauchy-Green deformation tensor,  $\mathbf{C}$ .
  - (c) The principal stretches,  $\lambda_1$  and  $\lambda_2$ .
10. A body undergoes simple shear deformation, where  $\gamma$  is a constant representing the shear. The deformation mapping is given by:

$$x'_1 = x_1 + \gamma x_2, \quad x'_2 = x_2$$

Determine the following:

- (a) The deformation gradient tensor,  $\mathbf{F}$ .
- (b) The right Cauchy-Green deformation tensor,  $\mathbf{C}$ .
- (c) The principal stretches,  $\lambda_1$  and  $\lambda_2$ , in terms of  $\gamma$ .