Indian Institute of Technology Bhubaneswar

School of Infrastructure

Session: Autumn 2025

Solid Mechanics (CE2L001) — Assignment No. 2

Date: September 06, 2025 — Total Marks: 100

<u>Instructions:</u>

(1) If two or more answer scripts appear identical, each of them will be awarded ZERO.

(2) Solve the questions using indicial notations.

(3) Provide neatly drawn figures to explain the concepts behind the problems whenever possible.

(4) Provide practical examples corresponding to a problem whenever possible.

(5) For plotting purposes, you may use any programming language such as Julia, MATLAB, Python, etc.

(6) Submit your answer script by September 18, 2025 (drop it in my department mailbox).

<u>Notations :</u>

Zeroth-order tensors or scalars are represented by small letters. For eg. a

First-order tensors or vectors are represented by bold small letters. For eg. $\boldsymbol{a}$.

Second-order tensors are represented by bold capital letters. For eg. $\boldsymbol{A}$.

1. The three principal invariants of a second-order tensor $\boldsymbol{T} = T_{ij}\boldsymbol{e}_i \otimes \boldsymbol{e}_j$ are defined as $I_1 = \text{trace}\,(\boldsymbol{T}) = T_{ii}$, $I_2 = (1/2)\left((\text{trace}\,(\boldsymbol{T}))^2 - \text{trace}\left(\boldsymbol{T}^2\right)\right) = (1/2)\left(T_{ii}T_{jj} - T_{ij}T_{ij}\right)$ and $I_3 = \det(\boldsymbol{T}) = (1/6)\,\epsilon_{lmn}\,\epsilon_{ijk}\,T_{li}\,T_{mj}\,T_{nk}$.
   (a) Define the eigenvalue problem associated with the second-order tensor, $\boldsymbol{T}$.
   (b) Show that the correspondence between the principal invariants and the characteristic polynomial of the second-order tensor, $\boldsymbol{T}$, can be given by
   $$\lambda^3 - I_1\lambda^2 + I_2\lambda - I_3 = 0,$$
   where λ's are the eigenvalues of the second-order tensor, $\boldsymbol{T}$.
   (c) Explain the physical meaning of the three principal invariants of the second-order tensor, $\boldsymbol{T}$. [10]

2. (a) Derive the relationship between the invariants of the stress tensor and the invariants of the deviatoric stress tensor.
   (b) Show that the deviatoric stress tensor has zero first invariant.
   (c) A material is subjected to a stress state with principal stresses $\sigma_I = 100\,\text{MPa}$, $\sigma_{II} = 50\,\text{MPa}$, and $\sigma_{III} = -20\,\text{MPa}$. Calculate the invariants of the stress tensor. [10]

3. At a particular point in a wooden member, the state of stress is as shown in Fig. 1. The direction of the grain in the wood makes an angle of $+30°$ with the x-axis (i.e, horizontal axis). The allowable shear stress parallel to the grain is 150 psi for this wood. Is this state of stress permissible? Verify your answer by calculations. [10]

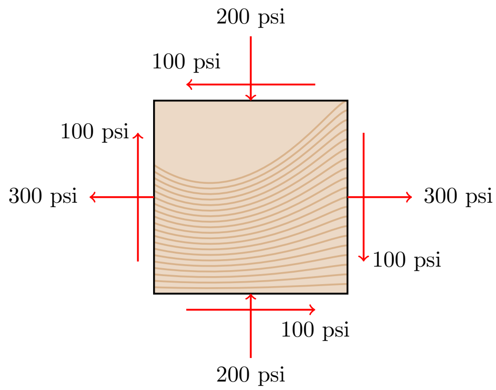


Figure 1

4. Consider a stress field whose matrix of scalar components in the vector basis $\{\boldsymbol{e}_i \mid i = 1, 2, 3\}$ is

$$[\boldsymbol{\sigma}] = \begin{bmatrix} 4x_1x_3 & 0 & -2x_3^2 \\ 0 & 1 & 2 \\ -2x_3^2 & 2 & 3x_1^2 \end{bmatrix} \text{ MPa},$$

where the constants are given with appropriate units so as to be compatible with Cartesian coordinates x_i in meters.

(i) For the static case (no inertial forces) plus assuming no body forces, is this stress field in equilibrium?

(ii) Determine the traction vector acting at a point $\boldsymbol{x} = 2\boldsymbol{e}_1 + \boldsymbol{e}_2 + \boldsymbol{e}_3$ on the plane $x_1 + x_2 - x_3 = 2$. Note the unit normal to a plane defined by $a_i x_i = b$ is,

$$\boldsymbol{n} = \pm \frac{a_i \boldsymbol{e}_i}{\sqrt{a_j a_j}}$$

(iii) Find the magnitude of the normal and shear traction on this plane at the given point.

(iv) Determine the principal stresses and directions at the given point. [10]

5. Provide qualitative stress state and the corresponding Mohr's circle considering an example for the following practical applications:

(a) Analysis of Structural Members.

(b) Design of Mechanical Components.

(c) Material Failure Analysis.

(d) Rock Mechanics.

(e) Biomechanical Applications. [10]

6. Find the traction-free planes (i.e., planes whose unit normal vectors make the traction vector vanish) passing through a point in a body subjected to the following stress state, expressed in the standard Cartesian basis:

$$[\boldsymbol{\sigma}] = \begin{bmatrix} 1 & 2 & 1 \\ 2 & \sigma_0 & 0 \\ 1 & 0 & -3 \end{bmatrix} \text{ MPa.}$$

Also, determine the value of σ_0. [10]

7. Suppose that at a point on the surface of a body the unit outward normal is

$$\boldsymbol{n} = \frac{\boldsymbol{e}_1 + \boldsymbol{e}_2 - \boldsymbol{e}_3}{\sqrt{3}}$$

and the traction vector is

$$\boldsymbol{t} = P(\boldsymbol{e}_1 + 2\boldsymbol{e}_2),$$

where P is a constant.

(a) Determine the normal traction vector $\boldsymbol{t}_{nn}$ and the shear traction vector $\boldsymbol{t}_{ns}$ at this point on the surface of the body.

(b) Determine the conditions between the stress tensor components and the traction vector components. [10]

8. At a certain point in a material under stress the intensity of the resultant stress on a vertical plane is 1000 N/cm^2, inclined at 30° to the normal to that plane, and the stress on a horizontal plane has a normal tensile component of intensity 600 N/cm^2 as shown in Fig. 2. Find the magnitude and direction of the resultant stress on the horizontal plane and the principal stresses. [10]

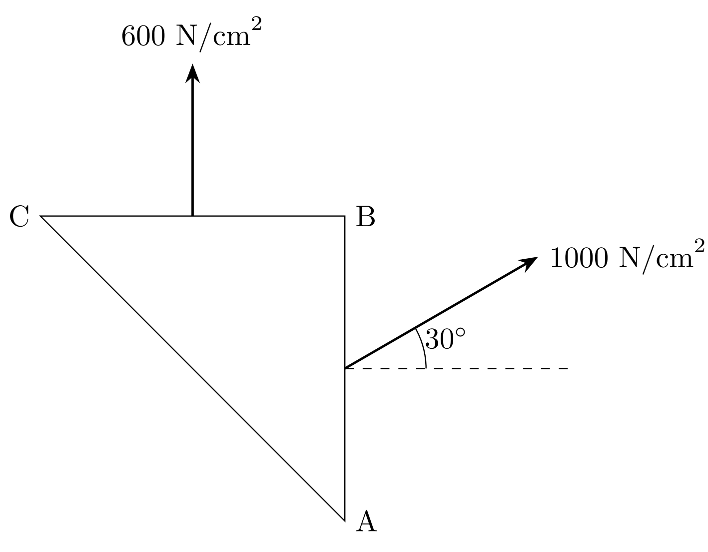


Figure 2

9. A thin walled pressure vessel shown in Fig. 3 (a) is subjected to an internal pressure which results in an axial normal stress $\sigma_a = 70$ MPa and a hoop stress $\sigma_h = 140$ MPa on its outer surface. An accidental event causes a torque to be applied on the entire structure causing a shear stress of magnitude equal to τ_{xy} as shown in Fig. 3 (b).

   (a) Use the stress element in Fig. 3 (b) to draw the Mohr's circle on the graph paper.

   (b) Use the Mohr's circle to calculate:

      i. The principal stresses at point A.

      ii. The maximum in-plane shear stress.

      iii. The absolute maximum shear stress.

      iv. The angle of rotation from the x-axis to the direction of the in-plane principal stress σ_{p1}.

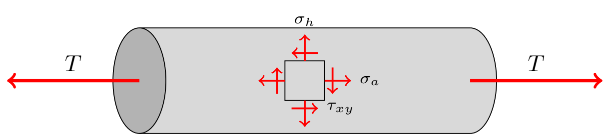
      v. Draw a stress element to show the in-plane principal stresses correctly oriented with respect to the x-axis.

      vi. Draw a stress element to show the in-plane maximum shear stress correctly oriented with respect to the x-axis.

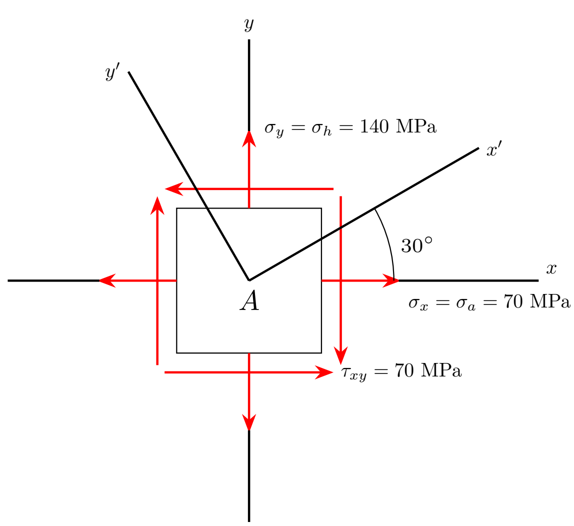
   (c) Use the Mohr's circle to calculate the normal and shear stresses in the x'–y' directions. Draw a stress element to show the calculated stresses, and mark the state of stress in the x'–y' directions on the Mohr's circle.
   Note: The x'–axis is oriented at 30° from the x–axis as shown in Fig. 3 (b).

   (d) What would the value of the principal stresses and maximum in-plane shear stresses acting at point A be if the accidental torque could be avoided? Explain your answer.

[20]



(a)



(b)

Figure 3