Indian Institute of Technology Bhubaneswar

School of Infrastructure

Subject Name : Solid Mechanics	Subject Code: CE2L001
Tutorial No. 2	Date: Aug 14, 2025

1. Prove that:

$$\boldsymbol{u} \times \boldsymbol{v} = -\boldsymbol{v} \times \boldsymbol{u}$$

   and illustrate this property schematically.

2. Prove that:

$$\boldsymbol{u} \cdot (\boldsymbol{u} \times \boldsymbol{v}) = 0$$

   and illustrate it schematically.

3. Prove that $\boldsymbol{u} \cdot \boldsymbol{A}\boldsymbol{v} = \boldsymbol{A}^T\boldsymbol{u} \cdot \boldsymbol{v}$, where $\boldsymbol{u}$, $\boldsymbol{v}$ are vectors and $\boldsymbol{A}$ is a second order tensor with $(\cdot)^T$ denoting the transpose of a tensor. Draw the figure corresponding to the proof and illustrate it schematically.

4. Prove the following and illustrate it schematically:

$$\boldsymbol{e}_i = \frac{1}{2}\epsilon_{ijk}\boldsymbol{e}_j \times \boldsymbol{e}_k.$$

5. Consider two scalar functions f and g. Establish the following vector identities and explain them through illustrative figures:

   (a) $\nabla \times (\nabla f) = \mathbf{0}$,

   (b) $\nabla \cdot (\nabla f \times \nabla g) = 0$.

6. Prove that $\boldsymbol{Q}^T\boldsymbol{Q} = \boldsymbol{Q}\boldsymbol{Q}^T = \boldsymbol{I}$ where $\boldsymbol{Q}$ is orthogonal tensor and $\boldsymbol{I}$ is the second-order identity tensor. Draw the figure corresponding to the proof and explain it schematically.

7. For the vector field:

$$\mathbf{u} = 2x_1\,\mathbf{e}_1 + x_1x_2\,\mathbf{e}_2,$$

   compute the quantities $\nabla \cdot \mathbf{u}$, $\nabla \times \mathbf{u}$, $\nabla^2\mathbf{u}$, $\nabla\mathbf{u}$, and $\mathrm{tr}(\nabla\mathbf{u})$. Explain the physical meaning of the computed quantities.

8. Show that for any two differentiable vector fields $\mathbf{a}$ and $\mathbf{b}$, the following vector identity holds:

$$\nabla \cdot (\mathbf{a} \times \mathbf{b}) = \mathbf{b} \cdot (\nabla \times \mathbf{a}) - \mathbf{a} \cdot (\nabla \times \mathbf{b}).$$

   Explain the identity with illustrative figure.