Indian Institute of Technology Bhubaneswar

School of Infrastructure

Subject Name : Solid Mechanics	Subject Code: CE2L001
Tutorial No. 4	Date: October 16, 2025

1. Different simple cases of transformations are illustrated in Fig. 1, where α and β are arbitrary scalar positive values. Consider the two-dimensional context and determine the deformation map, $\boldsymbol{\chi}(\boldsymbol{x})$, deformation gradient, $\boldsymbol{F}$, the strain tensor, $\boldsymbol{E} = (1/2)(\boldsymbol{F}^T\boldsymbol{F} - \boldsymbol{I})$, displacement vector, $\boldsymbol{u}$ and the linearized strain tensor, $\boldsymbol{E}_s$, for each case.

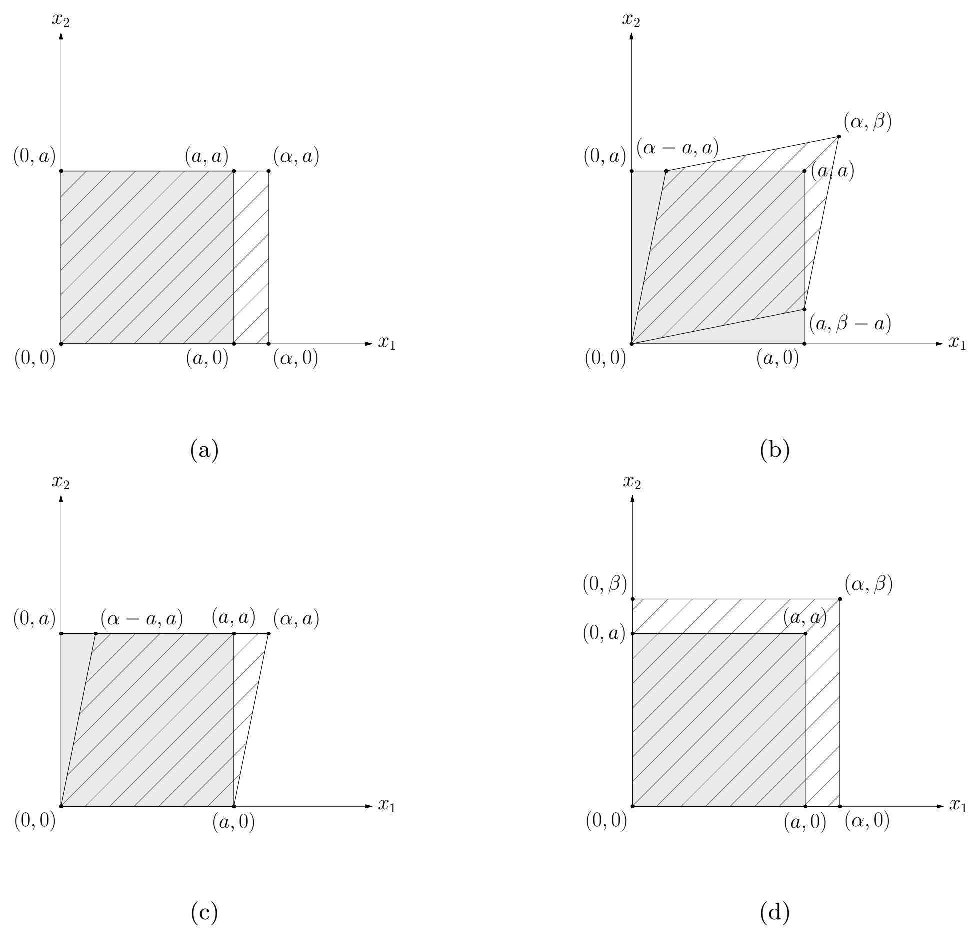

Figure 1: The undeformed and deformed configurations of a body under different cases of transformations with a, α, β denoting arbitrary scalar positive constants. The hatched area illustrates the deformed configurations.

2. Consider a homogeneous deformation corresponds to a strain field where the strain is the same at all points in a material body. Consider a prismatic, uniform thickness bar of initial length l_o undergoing a homogeneous deformation as shown in Fig. 2. Determine (a) deformation gradient, $\boldsymbol{F}$, (b) finite strain, $\boldsymbol{E}$ and (c) linearized strain, $\boldsymbol{E}_s$ (d) Demonstrate how the finite strain component reduces to linearized strain component through a numerical example.

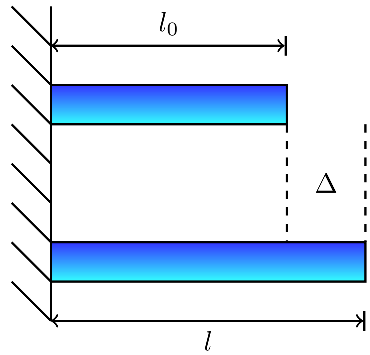

Figure 2: Undeformed and deformed element in the homogenous strain field in the bar.

3. Consider a two-dimensional (2D) square infinitesimal element in the $x_1 - x_2$ plane as shown in Fig. 3. The displacement field within the element is defined as,

$$[\boldsymbol{u}] = \begin{bmatrix} 0.1x_1 + 0.2x_2 \\ 0.2x_2 \end{bmatrix}.$$

(a) Plot the displacement field. (b) Compute the divergence of the displacement field $\boldsymbol{u}$. (c) Determine the strain $\boldsymbol{E}$ and linearized strain $\boldsymbol{E}_s$.

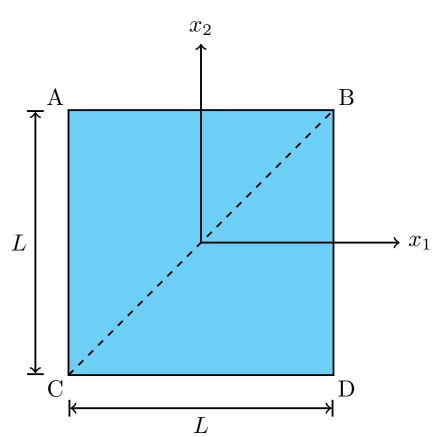

Figure 3: Inhomogeneous strain field

4. Consider the motion given by $[\boldsymbol{F}] = \begin{bmatrix} \lambda(t) & 0 & 0 \\ 0 & \lambda(t) & 0 \\ 0 & 0 & \lambda(t) \end{bmatrix}$, where $\lambda(t)$ is a time-dependent function. Determine the values of $\lambda(t)$ for which the motion is a rigid body motion.

5. Consider the motion given by $\boldsymbol{x}' = \boldsymbol{x} + t^2\, \boldsymbol{e}_1 + \sin(t)\, \boldsymbol{e}_2$. Determine if the motion is a rigid body motion by computing the strain tensor.

6. Consider the motion given by $\boldsymbol{x}' = \boldsymbol{Q}(t)\, \boldsymbol{x} + \boldsymbol{c}(t)$, where $\boldsymbol{Q}(t)$ is a time-dependent rotation matrix and $\boldsymbol{c}(t)$ is a time-dependent translation vector. Determine if the motion is a rigid body motion.

7. Consider the motion given by $\boldsymbol{u} = a\, x_1\, \boldsymbol{e}_1 + b\, x_2\, \boldsymbol{e}_2 + c\, x_3\, \boldsymbol{e}_3$. Determine the values of a, b, and c for which the motion is a rigid body motion by computing the strain tensor.

8. Consider the motion given by $\boldsymbol{x}' = \boldsymbol{x} + a\, \sin(\omega t)\, \boldsymbol{e}_1 + b\, \cos(\omega t)\, \boldsymbol{e}_2$. Determine if the motion is a rigid body motion by computing the strain tensor.

9. Consider the motion given by $\boldsymbol{F} = \boldsymbol{I} + \gamma(t)\, \boldsymbol{e}_1 \otimes \boldsymbol{e}_2$. Determine the values of $\gamma(t)$ for which the motion is a rigid body motion by computing the strain tensor.

10. Consider the motion given by $\boldsymbol{u} = \beta(t)\,(x_2\, \boldsymbol{e}_1 - x_1\, \boldsymbol{e}_2)$, where $\beta(t)$ is a time-dependent function. Determine the values of $\beta(t)$ for which the motion is a rigid body motion.