Indian Institute of Technology Bhubaneswar

School of Infrastructure

Session: Autumn 2025

Solid Mechanics (CE2L001) | Class Test 1

Date: August 30, 2025 | Total Marks: 50

Instructions:

(1) Zeroth-order tensors or scalars are represented by small letters. For eg. a.

(2) First-order tensors or vectors are represented by bold small letters. For eg. $\boldsymbol{a}$.

(3) Second-order tensors are represented by bold capital letters. For eg. $\boldsymbol{A}$.

(4) Second-order identity tensor is represented by $\boldsymbol{I}$.

1. (a) Explain the following statement through theoretical derivations and examples with illustrative figures. "Vector is independent of the coordinate system, but its components are not".

   (b) Consider two vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ whose matrix of components relative to an orthonormal basis $\{\boldsymbol{e_i}\}$ are

$$[\boldsymbol{a}] = \begin{bmatrix} 1 \\ 4 \\ 6 \end{bmatrix} \quad \text{and} \quad [\boldsymbol{b}] = \begin{bmatrix} 2 \\ 0 \\ 4 \end{bmatrix}.$$

   Compute (i) the magnitude of the vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ denoted by $|\boldsymbol{a}|$ and $|\boldsymbol{b}|$, respectively.

   (ii) the angle between the vectors $\boldsymbol{a}$ and $\boldsymbol{b}$.

   (iii) the area of the parallelogram bounded by $\boldsymbol{a}$ and $\boldsymbol{b}$.

   (iv) $\boldsymbol{b} \times \boldsymbol{a}$ and comment on the results. [$5 + 5 = 10$]

2. (a) Define the Kronecker delta symbol (δ_{ij}) and the Permutation symbol (ϵ_{ijk}) that are frequently used in tensor calculus.

   (b) Simplify the following expressions:

   (i) $\delta_{ij}\, \delta_{jk}\, \delta_{kl}\, \delta_{lm}\, \delta_{mn}\, \delta_{nq}\, \delta_{qi}$.

   (ii) $\epsilon_{1jk}\delta_{3j}v_k$. [$4 + 6 = 10$]

3. (a) Explain the concept of dyadic product using theoretical derivations and illustrative figures.

   (b) Prove the following tensor identities

   (i) $\boldsymbol{T}(\boldsymbol{a} \otimes \boldsymbol{b}) = (\boldsymbol{T}\boldsymbol{a}) \otimes \boldsymbol{b}$.

   (ii) $(\boldsymbol{u} \otimes \boldsymbol{v})\, \boldsymbol{A} = \big(\boldsymbol{u} \otimes \boldsymbol{A}^T \boldsymbol{v}\big)$. [$4 + 6 = 10$]

4. (a) In two dimensions, any orthogonal tensor can be expressed as

$$\boldsymbol{R} = \cos\theta\, \boldsymbol{e}_1 \otimes \boldsymbol{e}_1 - \sin\theta\, \boldsymbol{e}_1 \otimes \boldsymbol{e}_2 + \sin\theta\, \boldsymbol{e}_2 \otimes \boldsymbol{e}_1 + \cos\theta\, \boldsymbol{e}_2 \otimes \boldsymbol{e}_2$$

(i) Show that $\boldsymbol{R}^{\mathrm{T}}\boldsymbol{R} = \boldsymbol{I}$ (where $\boldsymbol{I}$ is the second order identity tensor), i.e. prove that the inverse of an orthogonal tensor is its transpose.

(ii) Show that $|\boldsymbol{v}| = |\boldsymbol{R}\boldsymbol{v}|$ for all $\boldsymbol{v}$; i.e. an orthogonal tensor does not change the length of a vector.

(b) Consider a two-dimensional orthonormal basis $\{\boldsymbol{e}_1, \boldsymbol{e}_2\}$ in which a two-dimensional tensor $\boldsymbol{T}$ has the representation

$$\boldsymbol{T} = T_{ij}\, \boldsymbol{e}_i \otimes \boldsymbol{e}_j \qquad \text{where } \ i, j = 1, 2.$$

The component matrix of $\boldsymbol{T}$ has values

$$[\boldsymbol{T}]_{\{\boldsymbol{e}_i\}} = \begin{bmatrix} 20 & 10 \\ 10 & 10 \end{bmatrix}.$$

Consider a second basis $\{\boldsymbol{e}_1^*, \boldsymbol{e}_2^*\}$ which is related to $\{\boldsymbol{e}_1, \boldsymbol{e}_2\}$ by

$$\boldsymbol{e}_1^* = \frac{1}{2}\boldsymbol{e}_1 + \frac{\sqrt{3}}{2}\boldsymbol{e}_2, \qquad \boldsymbol{e}_2^* = -\frac{\sqrt{3}}{2}\boldsymbol{e}_1 + \frac{1}{2}\boldsymbol{e}_2.$$

Find the values of $[\boldsymbol{T}]_{\{\boldsymbol{e}_i^*\}}$ in the $\{\boldsymbol{e}_1^*, \boldsymbol{e}_2^*\}$ basis. $[5 + 5 = 10]$

5. (a) Explain the concept of traction vector. Explain the relation between the traction vector and the stress tensor for an ideal solid and an ideal fluid using theoretical derivations and illustrative figures.

(b) Explain the following questions using illustrative figures and practical examples.

To represent the stress state at a point of a solid body, why a second-order tensor called "stress tensor" is introduced? What was the difficulty to represent the stress state at a point of a solid body using a vector? $[4 + 6 = 10]$