Indian Institute of Technology Bhubaneswar

School of Infrastructure

Session: Autumn 2025

Solid Mechanics (CE2L001) — Assignment No. 1

Date: August 09, 2025 — Total Marks: 100

Instructions:

(1) If two or more answer scripts appear identical, each of them will be awarded ZERO.

(2) Solve the questions using indicial notations.

(3) Provide neatly drawn figures to explain the concepts behind the problems whenever possible.

(4) Provide practical examples corresponding to a problem whenever possible.

(5) For plotting purposes, you may use any programming language such as Julia, MATLAB, Python, etc.

(6) Submit your answer script by August 18, 2025 (drop it in my department mailbox).

Notations :

Zeroth-order tensors or scalars are represented by small letters. For eg. a

First-order tensors or vectors are represented by bold small letters. For eg. $\boldsymbol{a}$.

Second-order tensors are represented by bold capital letters. For eg. $\boldsymbol{A}$.

1. Simplify the following expressions

   a) $\delta_{ij}\delta_{jk}\delta_{kl}\delta_{lm}\delta_{mn}$ (Use the contraction property of δ, $i \neq n$).

   b) $\epsilon_{jkq}\epsilon_{jkq}$ (Use the $\epsilon - \delta$ relation).

[10]

2. Consider two vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ whose matrix of components relative to an orthonormal basis $\{\boldsymbol{e_i}\}$ are

$$[\boldsymbol{a}] = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix} \quad \text{and} \quad [\boldsymbol{b}] = \begin{bmatrix} 1 \\ 4 \\ -2 \end{bmatrix}$$

. Compute

   a) the magnitude of the vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ denoted by $|\boldsymbol{a}|$ and $|\boldsymbol{b}|$, respectively.

   b) the angle between the vectors $\boldsymbol{a}$ and $\boldsymbol{b}$.

   c) the area of the parallelogram bounded by $\boldsymbol{a}$ and $\boldsymbol{b}$

   d) $\boldsymbol{b} \times \boldsymbol{a}$ and comment on the results. [10]

3. Rewrite the expression $\epsilon_{mni} a_i b_j c_m d_n \boldsymbol{e_j}$ in direct notation using the scalar and cross products of vectors. [5]

4. For a two-dimensional (2D) problem, let the components of a second-order tensor $\mathbf{A}$ be $A_{11} = 2$, $A_{12} = 4 = A_{21}$, $A_{22} = 5$. Let the components of a vector $\boldsymbol{v}$ be $v_1 = 3, v_2 = -1$, all in the same orthogonal basis. Compute the components of the vector, $\boldsymbol{w} = \boldsymbol{A}\,\boldsymbol{v}$ using the relation $w_i = A_{ij} v_j$. [5]

5. Show that

$$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c}$$

[5]

6. Show that $\mathbf{a} \times (\nabla \times \mathbf{a}) = \frac{1}{2} \nabla (\mathbf{a} \cdot \mathbf{a}) - (\mathbf{a} \cdot \nabla \mathbf{a})$ [5]

7. Prove the following identities

   a) $\nabla \cdot (\nabla \phi) = \nabla^2 \phi$, where ϕ is a scalar field .

   b) $\nabla \times (\nabla \times \mathbf{a}) = \nabla(\nabla \cdot \mathbf{a}) - \nabla^2 \mathbf{a}$, where $\mathbf{a}$ is a vector field.

[10]

8. Consider a cyclone in the northern hemisphere described by the velocity vector field of the wind:

$$\boldsymbol{v}(x, y) = x\,\boldsymbol{e}_1 - y^2\,\boldsymbol{e}_2$$

where x and y are the coordinates in the horizontal plane, and $\boldsymbol{e}_1$ and $\boldsymbol{e}_2$ are unit vectors in the x- and y-directions, respectively.

   a) Calculate the divergence and curl of the vector field $\boldsymbol{v}(x, y)$.

   b) Explain the physical significance of the divergence and curl in the context of a cyclone.

   c) Based on the curl, determine the direction of rotation of the cyclone.

[15]

9. In Geotechnical engineering, understanding the flow of water in a dam's vicinity is crucial. The potential function $\phi(x, y)$ of a water flow around a dam is given by:

$$\phi(x, y) = xy$$

   a) Calculate the velocity vector field $\boldsymbol{v}(x, y)$ from the potential function $\phi(x, y)$.

   b) Determine the divergence and curl of the velocity vector field.

   c) Draw the vector field and discuss the water flow behaviour around the dam.

[20]

10. Given a vector $\mathbf{a} = a_i \boldsymbol{e}_i = a_i^* \boldsymbol{e}_i^*$ defined with respect to the basis $\mathbf{e}_i$ by

$$\mathbf{a} = 3\mathbf{e}_1 - 2\mathbf{e}_2 + 4\mathbf{e}_3$$

Find the components a_i^* of $\mathbf{a}$ with respect to the basis $\mathbf{e}_i^*$ defined in Fig. 1.

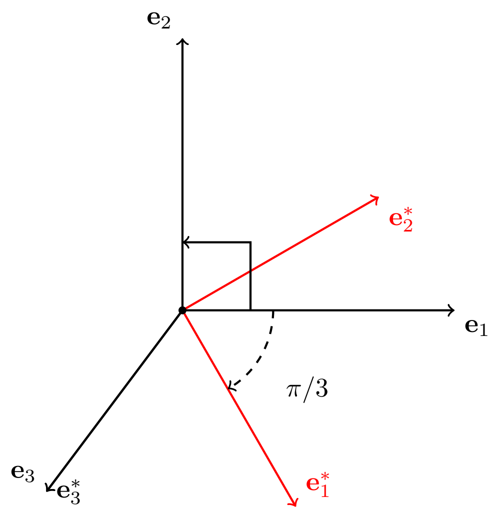

Figure 1: New ortho-normal basis $\mathbf{e}_i^*$ is obtained by a clockwise rotation of the ortho-normal basis $\mathbf{e}_i$ about the $\mathbf{e}_3$-axis.

[15]