Indian Institute of Technology Bhubaneswar

School of Infrastructure

Subject Name : Solid Mechanics	Subject Code: CE2L001

Solution of Tutorial No. 7

1. Consider a simply supported beam of length L subjected to a point load P applied at the center. Calculate the reaction forces, the shear force and the bending moment at any point along the beam.

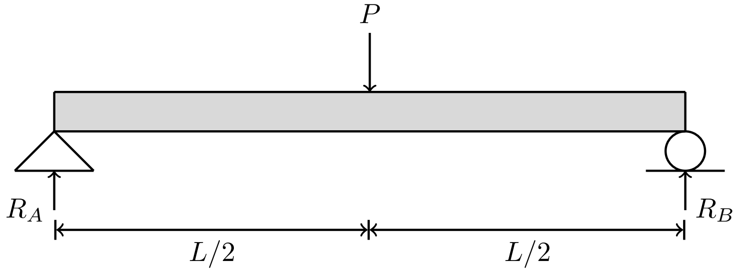


Figure 1: Simply Supported Beam with Center Load

Solution:

Step 1: Equilibrium and Reaction Forces

The beam is in static equilibrium, which means:

- Sum of vertical forces $\sum F_y = 0$
- Sum of moments about any point $\sum M = 0$

Vertical Force Equilibrium:

$$R_A + R_B - P = 0$$

Since the point load P is applied symmetrically at the center of the beam:

$$R_A = R_B = \frac{P}{2}$$

Moment Equilibrium (taking moments about point A):

$$M_A = 0$$

The moment equilibrium equation gives:

$$R_B \times L - P \times \frac{L}{2} = 0$$

Solving for R_B, we again get:

$$R_B = \frac{P}{2}$$

Step 2: Shear Force Calculation

The shear force at any point x along the beam can be calculated as:

For $0 \leq x < \frac{L}{2}$:

$$V(x) = R_A = \frac{P}{2}$$

For $\frac{L}{2} < x \leq L$:

$$V(x) = R_A - P = -\frac{P}{2}$$

The shear force diagram has a constant positive value of $\frac{P}{2}$ from the left support to the center, and a constant negative value of $-\frac{P}{2}$ from the center to the right support.

Step 3: Bending Moment Calculation

The bending moment at any point x along the beam can be calculated using:

$$M(x) = R_A \times x \quad \text{for} \quad 0 \leq x \leq \frac{L}{2}$$

Therefore, the bending moment is:

$$M(x) = \frac{P}{2} \times x$$

At $x = \frac{L}{2}$ (the center of the beam):

$$M\left(\frac{L}{2}\right) = \frac{P}{2} \times \frac{L}{2} = \frac{PL}{4}$$

For the second half of the beam $\left(\frac{L}{2} < x \leq L\right)$, the bending moment is:

$$M(x) = \frac{PL}{4} - \frac{P}{2} \times (x - \frac{L}{2})$$

Simplifying this expression:

$$M(x) = \frac{PL}{4} - \frac{P}{2} \times x$$

Step 4: Conclusion

The maximum bending moment occurs at the center of the beam and is given by:

$$M_{\max} = \frac{PL}{4}$$

The shear force has constant positive and negative values of $+\frac{P}{2}$ and $-\frac{P}{2}$ on the two halves of the beam.

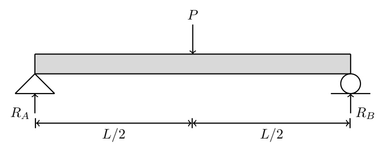
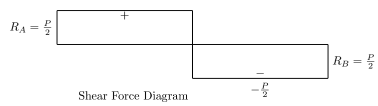
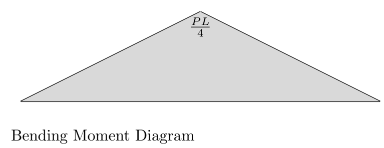


Figure 2: Simply Supported Beam with Center Load





2. Consider a simply supported beam of length L subjected to two equal point loads P applied at $L/4$ and $3L/4$. Calculate the reaction forces, shear force, and bending moment at any point along the beam.

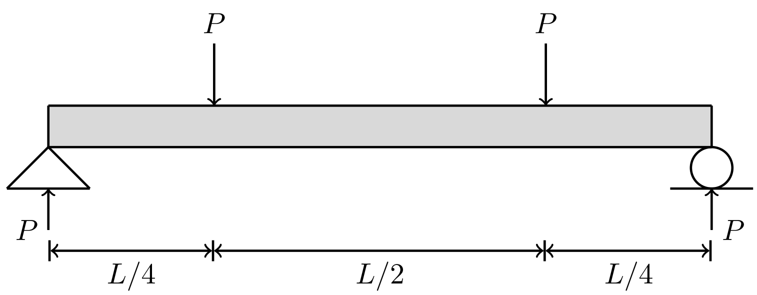


Figure 3: Simply Supported Beam with Four-Point Loads

Solution:

Step 1: Equilibrium and Reaction Forces

The beam is in static equilibrium, which means:

- Sum of vertical forces $\sum F_y = 0$
- Sum of moments about any point $\sum M = 0$

Vertical Force Equilibrium:

$$R_A + R_B - 2P = 0$$

Since the loads P are symmetrically placed:

$$R_A = R_B = P$$

Moment Equilibrium (taking moments about point A):

$$M_A = 0$$

The moment equilibrium equation gives:

$$R_B \times L - P \times \frac{L}{4} - P \times \frac{3L}{4} = 0$$

Solving for R_B, we again find:

$$R_B = P$$

Similarly, $R_A = P$ as derived earlier.

Step 2: Shear Force Calculation

The shear force at any point x along the beam can be calculated as:

For $0 \leq x < \frac{L}{4}$:

$$V(x) = R_A = P$$

For $\frac{L}{4} < x < \frac{3L}{4}$:

$$V(x) = R_A - P = 0$$

For $\frac{3L}{4} < x \leq L$:

$$V(x) = R_A - 2P = -P$$

The shear force diagram has a constant positive value of P from the left support to $L/4$, a value of 0 between $L/4$ and $3L/4$, and a constant negative value of $-P$ from $3L/4$ to the right support.

Step 3: Bending Moment Calculation

The bending moment at any point x along the beam can be calculated using:

For $0 \leq x < \frac{L}{4}$:

$$M(x) = R_A \times x = P \times x$$

For $\frac{L}{4} < x < \frac{3L}{4}$:

$$M(x) = P \times \frac{L}{4} \quad \text{(constant value)}$$

For $\frac{3L}{4} < x \leq L$:

$$M(x) = P \times \frac{L}{4} - P \times (x - \frac{3L}{4}) = P \times \left(\frac{L}{4} - (x - \frac{3L}{4})\right)$$

This simplifies to:

$$M(x) = P \times (L - x)$$

Step 4: Conclusion

The maximum bending moment occurs at the center of the load application and is given by:

$$M_{\text{max}} = P \times \frac{L}{4}$$

The shear force has a positive value of P from the left support to $L/4$, zero between $L/4$ and $3L/4$, and a negative value of $-P$ from $3L/4$ to the right support.

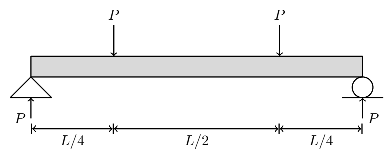


Figure 4: Simply Supported Beam with Four-Point Loads

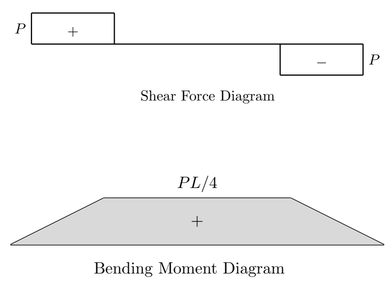


Figure 5: Bending Moment Diagram

3. Consider a simply supported beam of length L subjected to applied load as shown in Fig. 6. Calculate the reaction forces, shear force, and bending moment at any point along the beam.

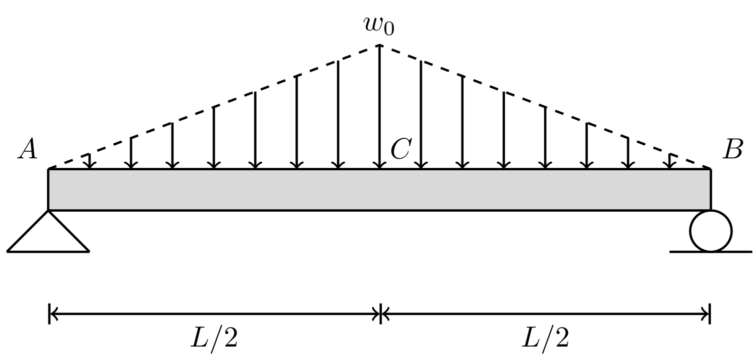


Figure 6

Consider a simply supported beam of length L subjected to a symmetric triangular load as shown in Fig. 6, where the load intensity is maximum (w_0) at the center and zero at both ends. Determine the reaction forces, shear force, and bending moment at any point along the beam.

Solution

Loading Description

Let the coordinate x be measured from the left support A ($x = 0$) toward the right support B ($x = L$).

The distributed load intensity varies linearly as:

$$w(x) = \begin{cases} \dfrac{2w_0}{L}\, x, & 0 \le x \le \dfrac{L}{2}, \\ \dfrac{2w_0}{L}\,(L - x), & \dfrac{L}{2} \le x \le L. \end{cases}$$

1. Total Load and Support Reactions

The total load on the beam is the area of the symmetric triangular load:

$$W = \int_0^L w(x)\, dx = \frac{1}{2}\, w_0\, L = \frac{w_0 L}{2}.$$

By symmetry, both supports carry equal reactions:

$$R_A = R_B = \frac{W}{2} = \frac{w_0 L}{4}.$$

2. Shear Force and Bending Moment

We analyze the left half ($0 \le x \le L/2$) and then use symmetry for the right half.

(a) Left Half ($0 \le x \le L/2$)

The resultant of the distributed load from 0 to x is:

$$W_x = \int_0^x \frac{2w_0}{L} s\, ds = \frac{w_0}{L} x^2.$$

The centroid of this triangular load from A is located at $x/3$.

Shear Force:

$$V(x) = R_A - W_x = \frac{w_0 L}{4} - \frac{w_0}{L} x^2.$$

Bending Moment:

$$M(x) = R_A x - W_x \left(\frac{x}{3}\right) = \frac{w_0 L}{4} x - \frac{w_0}{3L} x^3.$$

(b) Right Half ($L/2 \le x \le L$)

By symmetry about the midspan:

$$V(x) = -\left(\frac{w_0 L}{4} - \frac{w_0}{L}(L - x)^2\right) = -\frac{w_0 L}{4} + \frac{w_0}{L}(L - x)^2,$$

$$M(x) = \frac{w_0 L}{4}(L - x) - \frac{w_0}{3L}(L - x)^3.$$

3. Maximum Bending Moment

At midspan, $x = L/2$:

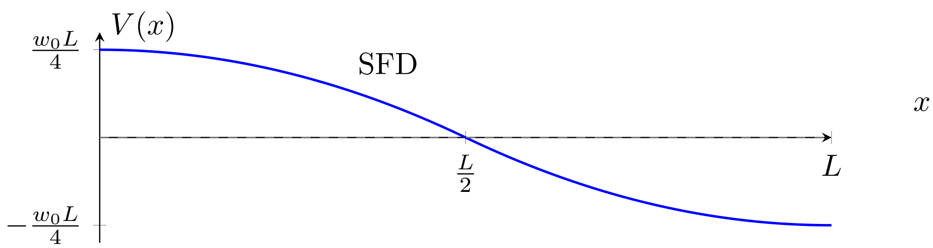
$$V\left(\frac{L}{2}\right) = 0, \qquad M\left(\frac{L}{2}\right) = \frac{w_0 L}{4}\left(\frac{L}{2}\right) - \frac{w_0}{3L}\left(\frac{L}{2}\right)^3 = \frac{w_0 L^2}{12}.$$

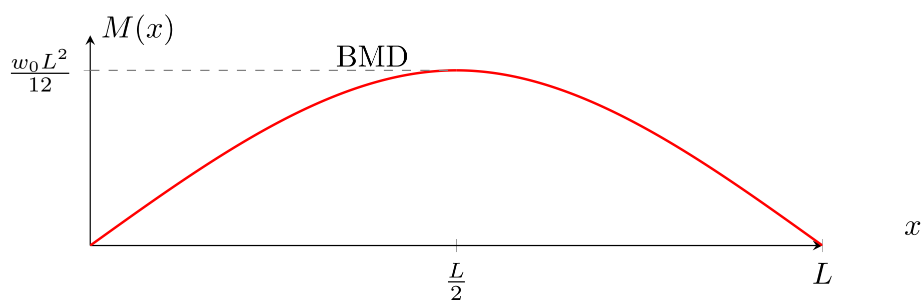
Hence, the maximum bending moment occurs at the midspan and is given by:

$$\boxed{M_{\max} = \frac{w_0 L^2}{12}}.$$

4. Summary of Results

$$\begin{aligned}
&\text{Total Load:} && W = \frac{w_0 L}{2}, \\
&\text{Support Reactions:} && R_A = R_B = \frac{w_0 L}{4}, \\
&\text{For } 0 \le x \le \tfrac{L}{2}: && \begin{cases} V(x) = \dfrac{w_0 L}{4} - \dfrac{w_0}{L}x^2, \\ M(x) = \dfrac{w_0 L}{4}x - \dfrac{w_0}{3L}x^3, \end{cases} \\
&\text{For } \tfrac{L}{2} \le x \le L: && \begin{cases} V(x) = -\dfrac{w_0 L}{4} + \dfrac{w_0}{L}(L-x)^2, \\ M(x) = \dfrac{w_0 L}{4}(L-x) - \dfrac{w_0}{3L}(L-x)^3, \end{cases} \\
&\text{Maximum Moment:} && M_{\max} = \frac{w_0 L^2}{12} \text{ at } x = \frac{L}{2}.
\end{aligned}$$





4. Consider a simply supported beam of length L subjected to applied load as shown in Fig. 7. Calculate the reaction forces, shear force, and bending moment at any point along the beam.

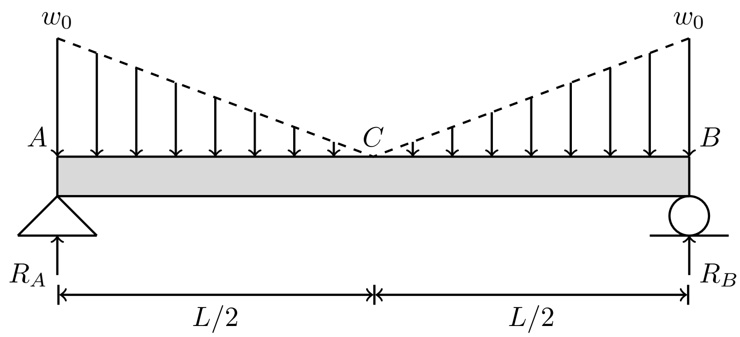


Figure 7

Solution

Load description

The distributed intensity (symmetric about $x = L/2$) can be written compactly as

$$w(x) = \frac{2w_0}{L}\,\left|\,x - \tfrac{L}{2}\,\right|,$$

or piecewise

$$w(x) = \begin{cases} w_0 - \dfrac{2w_0}{L}\,x, & 0 \le x \le \dfrac{L}{2}, \\ w_0 - \dfrac{2w_0}{L}\,(L - x) = -w_0 + \dfrac{2w_0}{L}x, & \dfrac{L}{2} \le x \le L. \end{cases}$$

1. Total load and support reactions

Total load (area of two identical end-triangles):

$$W = \int_0^L w(x)\,\mathrm{d}x = 2 \cdot \frac{1}{2}\left(\frac{L}{2}\right) w_0 = \frac{w_0 L}{2}.$$

By symmetry the support reactions are equal:

$$\boxed{R_A = R_B = \frac{W}{2} = \frac{w_0 L}{4}.}$$

2. Shear force and bending moment (piecewise)

We present expressions for the left half $0 \le x \le L/2$; by symmetry the right half follows by replacing x with $L - x$ and applying the sign rule for shear.

Left half: $0 \le x \le \dfrac{L}{2}$

For $0 \le x \le L/2$ the intensity is

$$w(s) = w_0 - \frac{2w_0}{L}s \quad (0 \le s \le x).$$

Resultant of the load on $[0, x]$:

$$W_x = \int_0^x \left(w_0 - \frac{2w_0}{L}s\right) \mathrm{d}s = w_0 x - \frac{w_0}{L}x^2 = \frac{w_0 x(L-x)}{L}.$$

Shear (equilibrium of left portion):

$$\boxed{V(x) = R_A - W_x = \frac{w_0 L}{4} - w_0 x + \frac{w_0}{L}x^2, \qquad 0 \le x \le \frac{L}{2}.}$$

Bending moment at the section (using $M(x) = R_A x - \displaystyle\int_0^x w(s)(x-s)\,ds$):

$$\begin{aligned} M(x) &= \frac{w_0 L}{4}x - \int_0^x \left(w_0 - \frac{2w_0}{L}s\right)(x-s)\,\mathrm{d}s \\ &= \frac{w_0 x(3L^2 - 6Lx + 4x^2)}{12L}. \end{aligned}$$

Thus

$$\boxed{M(x) = \frac{w_0\, x\,(3L^2 - 6Lx + 4x^2)}{12\,L}, \qquad 0 \le x \le \frac{L}{2}.}$$

One easily verifies the differential relations on this interval:

$$M'(x) = V(x), \qquad V'(x) = -w(x).$$

Right half: $\dfrac{L}{2} \le x \le L$

By symmetry,

$$V(x) = -V(L-x), \qquad M(x) = M(L-x).$$

Hence an explicit form for the right side is

$$\boxed{V(x) = -\frac{w_0 L}{4} + w_0(L-x) - \frac{w_0}{L}(L-x)^2, \qquad \frac{L}{2} \le x \le L,}$$

$$\boxed{M(x) = \frac{w_0(L-x)\big(3L^2 - 6L(L-x) + 4(L-x)^2\big)}{12L}, \qquad \frac{L}{2} \le x \le L.}$$

(Equivalently replace x by $L - x$ in the left-half expressions.)

3. Special values and checks

- End conditions: $M(0) = 0,\ M(L) = 0$ (simply supported).
- Shear at midspan: $V(L/2) = 0$ (by symmetry).
- Bending moment at midspan (maximum):

$$M\left(\frac{L}{2}\right) = \frac{w_0 L^2}{24}.$$

  So the maximum bending moment occurs at $x = L/2$ with

$$\boxed{M_{\max} = \frac{w_0 L^2}{24}}.$$

- Total load and reactions re-check:

$$\int_0^L w(x)\,dx = \frac{w_0 L}{2}, \qquad R_A + R_B = \frac{w_0 L}{2}.$$

- Differential checks (piecewise) hold:

$$M'(x) = V(x), \qquad V'(x) = -w(x).$$

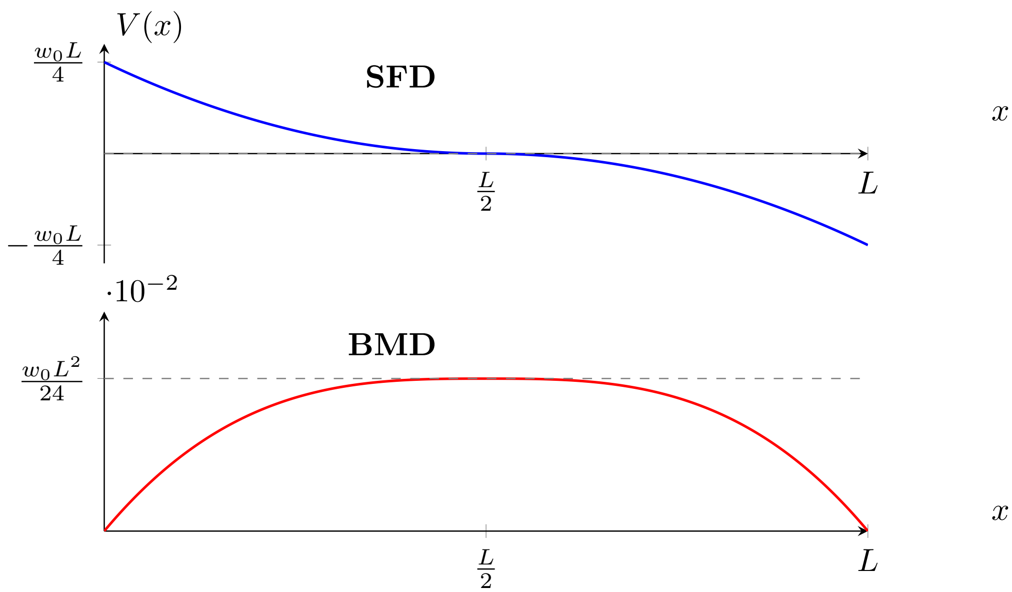
4. Compact summary

Total load: $W = \dfrac{w_0 L}{2}, \qquad R_A = R_B = \dfrac{w_0 L}{4},$

For $0 \le x \le \frac{L}{2}$: $\quad V(x) = \dfrac{w_0 L}{4} - w_0 x + \dfrac{w_0}{L} x^2, \quad M(x) = \dfrac{w_0 x(3L^2 - 6Lx + 4x^2)}{12L},$

For $\frac{L}{2} \le x \le L$: $\quad V(x) = -V(L - x), \quad M(x) = M(L - x),$

Maximum moment: $M_{\max} = \dfrac{w_0 L^2}{24}$ at $x = \frac{L}{2}$.



5. Consider a simply supported beam of length L subjected to applied load as shown in Fig. 8. Calculate the reaction forces, shear force, and bending moment at any point along the beam.

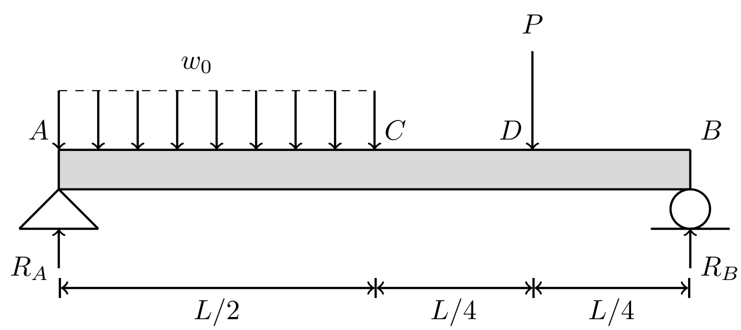


Figure 8

Solution:

Given: A simply-supported beam of length L with

- a uniform distributed load w_0 applied only on $0 \leq x \leq \dfrac{L}{2}$ (left half), and
- a concentrated load P at $x = \dfrac{3L}{4}$.

1. Resultant of the distributed load

Total resultant of the UDL (left half):

$$W_{udl} = w_0 \left(\frac{L}{2}\right) = \frac{w_0 L}{2},$$

acting at the centroid of the loaded patch, i.e. at $x = \dfrac{L}{4}$ from the left support.

2. Reactions R_A and R_B

Equilibrium of vertical forces:

$$R_A + R_B = W_{udl} + P = \frac{w_0 L}{2} + P.$$

Moment about A:

$$\sum M_A = 0 : \qquad R_B L - W_{udl}\left(\frac{L}{4}\right) - P\left(\frac{3L}{4}\right) = 0.$$

Thus

$$R_B = \frac{W_{udl}\,\frac{L}{4} + P\,\frac{3L}{4}}{L} = \frac{w_0 L}{8} + \frac{3P}{4}.$$

And

$$R_A = \left(\frac{w_0 L}{2} + P\right) - R_B = \frac{3w_0 L}{8} + \frac{P}{4}.$$

$$\boxed{R_A = \frac{3w_0 L}{8} + \frac{P}{4}, \qquad R_B = \frac{w_0 L}{8} + \frac{3P}{4}}$$

3. Shear force $V(x)$ — piecewise

There are three regions to consider:

$$\begin{aligned}
&\text{Region I: } 0 \le x \le \tfrac{L}{2} \quad \text{(inside the UDL)},\\
&\text{Region II: } \tfrac{L}{2} \le x < \tfrac{3L}{4} \quad \text{(to the right of the UDL, left of } P\text{)},\\
&\text{Region III: } \tfrac{3L}{4} < x \le L \quad \text{(to the right of } P\text{)}.
\end{aligned}$$

Region I: $(0 \le x \le L/2)$

$$V(x) = R_A - w_0 x = \left(\frac{3w_0 L}{8} + \frac{P}{4}\right) - w_0 x.$$

Region II: $(L/2 \le x < 3L/4)$ — UDL no longer contributes beyond $x = L/2$, so the total distributed load to the left is $W_{udl} = w_0 L/2$:

$$V(x) = R_A - W_{udl} = \left(\frac{3w_0 L}{8} + \frac{P}{4}\right) - \frac{w_0 L}{2} = \frac{w_0 L}{8} + \frac{P}{4}.$$

(Thus $V(x)$ is constant in Region II.)

Region III: $(3L/4 < x \le L)$ — include the concentrated load P:

$$V(x) = R_A - W_{udl} - P = \left(\frac{3w_0 L}{8} + \frac{P}{4}\right) - \frac{w_0 L}{2} - P = \frac{w_0 L}{8} - \frac{3P}{4}.$$

At $x = \frac{3L}{4}$ there is a downward jump in shear of magnitude P:

$$V\left(\tfrac{3L}{4}^{+}\right) = V\left(\tfrac{3L}{4}^{-}\right) - P.$$

4. Bending moment $M(x)$ — piecewise

Region I: $(0 \le x \le L/2)$

$$\begin{aligned} M(x) &= R_A x - \frac{w_0 x^2}{2} \\ &= \left(\frac{3w_0 L}{8} + \frac{P}{4}\right) x - \frac{w_0 x^2}{2}. \end{aligned}$$

Region II: $(L/2 \le x < 3L/4)$ – treat the UDL as a single resultant $W_{udl} = w_0 L/2$ acting at $x = L/4$:

$$M(x) = R_A x - W_{udl}\,(x - \tfrac{L}{4}) = \left(\frac{3w_0 L}{8} + \frac{P}{4}\right) x - \frac{w_0 L}{2}\Big(x - \frac{L}{4}\Big).$$

Region III: $(3L/4 \le x \le L)$ — include the concentrated moment from P:

$$M(x) = R_A x - W_{udl}\,(x - \tfrac{L}{4}) - P\Big(x - \frac{3L}{4}\Big).$$

All expressions satisfy $M(0) = 0$ and $M(L) = 0$ (can be verified by substitution).

5. Location of maximum bending moment

Different possibilities exist depending on the relative size of P and $w_0 L$.

- In Region I the derivative $M'(x) = V(x) = R_A - w_0 x$. Setting $M'(x) = 0$ gives the stationary point

  $$x^* = \frac{R_A}{w_0} = \frac{3L}{8} + \frac{P}{4w_0}.$$

  This is a candidate for $M_{\max}$ if $0 \le x^* \le \frac{L}{2}$, i.e. if

  $$\frac{3L}{8} + \frac{P}{4w_0} \le \frac{L}{2} \quad \Longleftrightarrow \quad P \le \frac{w_0 L}{2}.$$

- If $P > \dfrac{w_0 L}{2}$ then $x^* > \dfrac{L}{2}$ and the maximum will occur at an endpoint of Region I/II (usually at $x = \frac{L}{2}$) or in Region II/III depending on signs of shear there.

Therefore the maximum bending moment is

$$M_{\max} = \begin{cases} M(x^*) & \text{if } 0 \le x^* \le \dfrac{L}{2}, \\ \max\{M(L/2),\, M(3L/4)\} & \text{otherwise.} \end{cases}$$

(Explicit substitution gives $M(x^*)$ if required.)

6. Compact summary

$$\boxed{R_A = \frac{3w_0L}{8} + \frac{P}{4}, \qquad R_B = \frac{w_0L}{8} + \frac{3P}{4}}$$

$$V(x) = \begin{cases} \dfrac{3w_0L}{8} + \dfrac{P}{4} - w_0x, & 0 \le x \le \frac{L}{2}, \\ \dfrac{w_0L}{8} + \dfrac{P}{4}, & \frac{L}{2} \le x < \frac{3L}{4}, \\ \dfrac{w_0L}{8} - \dfrac{3P}{4}, & \frac{3L}{4} < x \le L, \end{cases}$$

$$M(x) = \begin{cases} \left(\dfrac{3w_0L}{8} + \dfrac{P}{4}\right)x - \dfrac{w_0x^2}{2}, & 0 \le x \le \frac{L}{2}, \\ \left(\dfrac{3w_0L}{8} + \dfrac{P}{4}\right)x - \dfrac{w_0L}{2}\left(x - \dfrac{L}{4}\right), & \frac{L}{2} \le x < \frac{3L}{4}, \\ \left(\dfrac{3w_0L}{8} + \dfrac{P}{4}\right)x - \dfrac{w_0L}{2}\left(x - \dfrac{L}{4}\right) - P\left(x - \dfrac{3L}{4}\right), & \frac{3L}{4} \le x \le L. \end{cases}$$

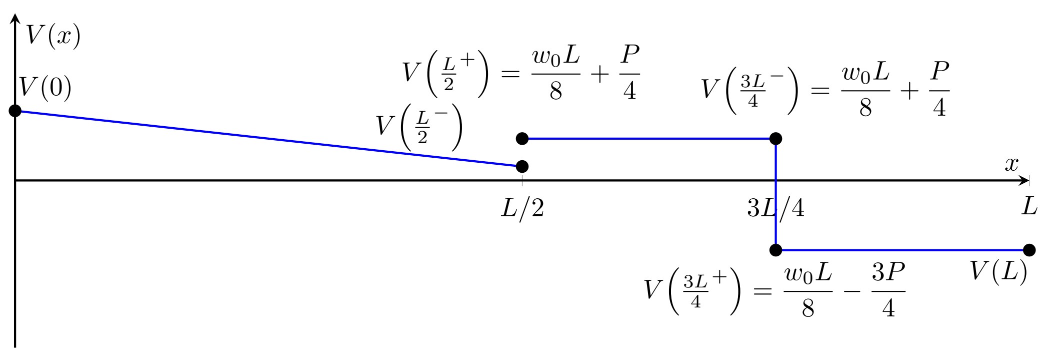


Figure 9: Shear Force Diagram with Critical Values

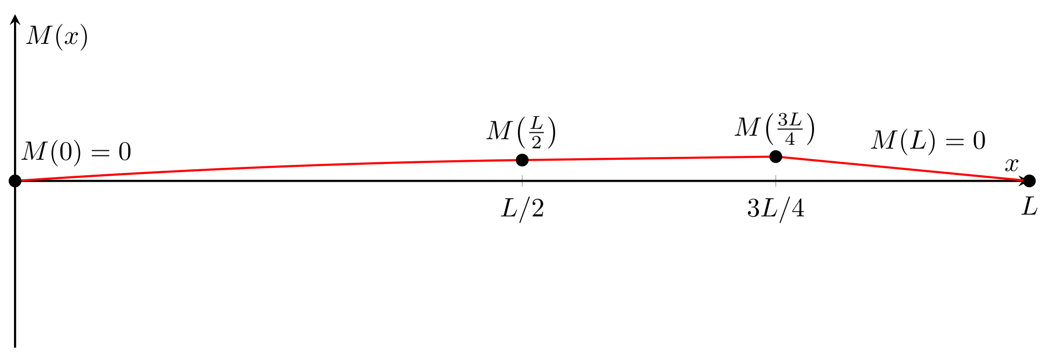


Figure 10: Bending Moment Diagram with Critical Values

6. Consider a simply supported beam of length L subjected to applied load as shown in Fig. 11. Calculate the reaction forces, shear force, and bending moment at any point along the beam.

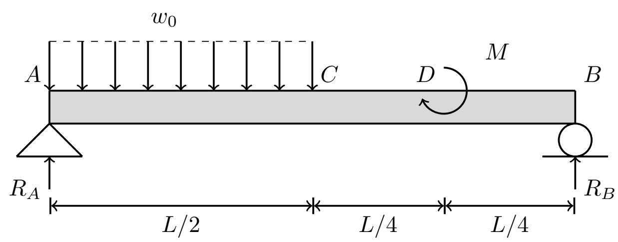


Figure 11

<u>**Solution**</u>:

Step 1: Calculate Reaction Forces

$$R_B = \frac{w_0 L}{8} + \frac{M}{L}$$
$$R_A = \frac{3w_0 L}{8} - \frac{M}{L}$$

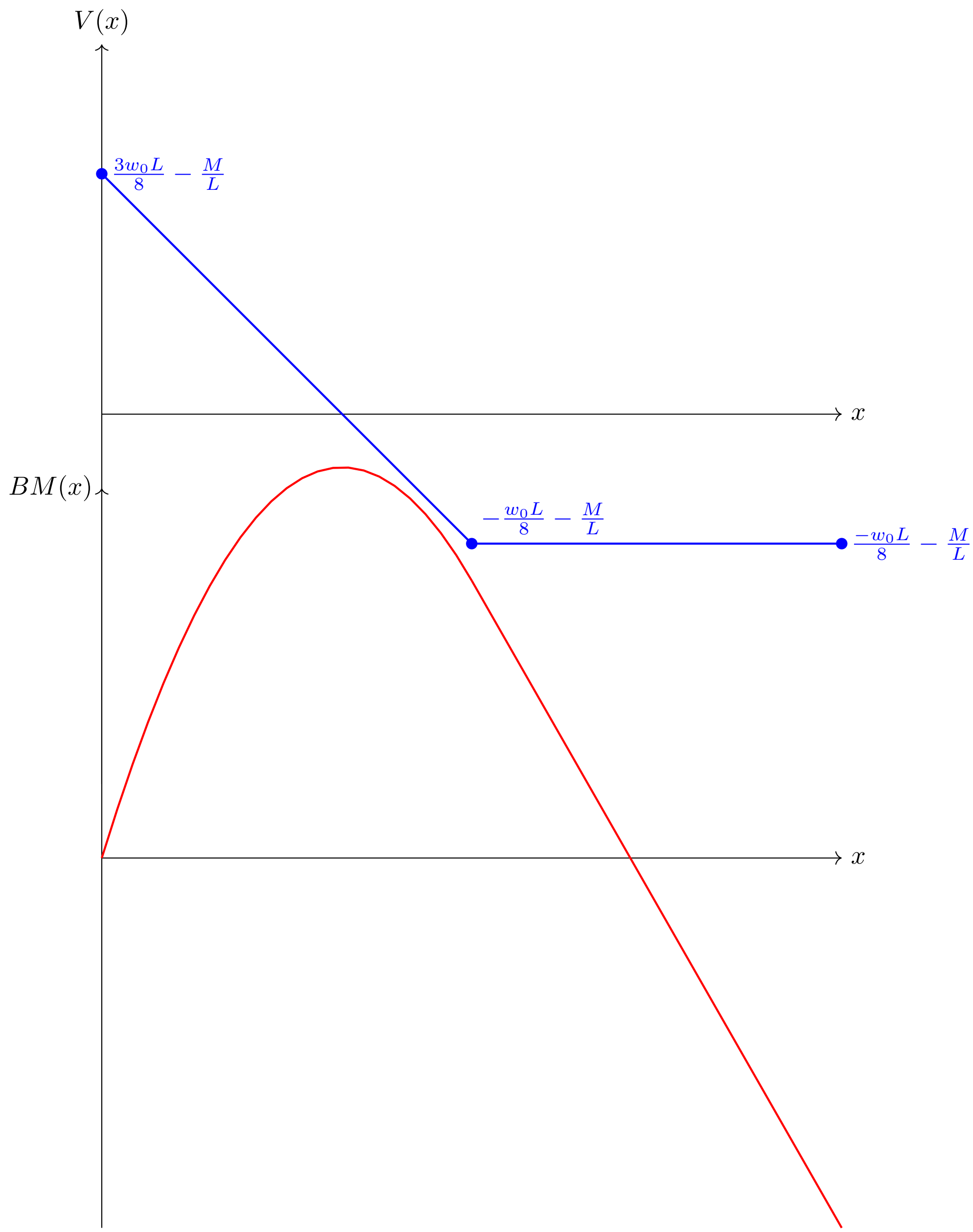
Step 2: SF and BM at any point

For $0 \leq x \leq \frac{L}{2}$:

$$V(x) = \left(\frac{3w_0 L}{8} - \frac{M}{L}\right) - w_0 x$$
$$BM(x) = \left(\frac{3w_0 L}{8} - \frac{M}{L}\right) x - \frac{w_0 x^2}{2}$$

For $\frac{L}{2} \leq x \leq L$:

$$V(x) = \left(\frac{3w_0 L}{8} - \frac{M}{L}\right) - \frac{w_0 L}{2} = -\frac{w_0 L}{8} - \frac{M}{L}$$
$$BM(x) = \left(\frac{3w_0 L}{8} - \frac{M}{L}\right) x - \frac{w_0 L}{2}\left(x - \frac{L}{4}\right) - M$$



7. Members ABC and BD of the counter chair are rigidly connected at B and the smooth collar at D is allowed to move freely along the vertical slot. Draw the shear and moment diagrams for member ABC.

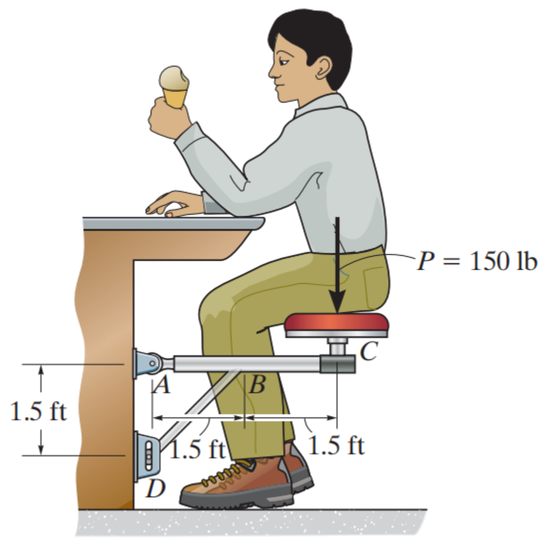


Figure 12

<u>**Solution**</u>:

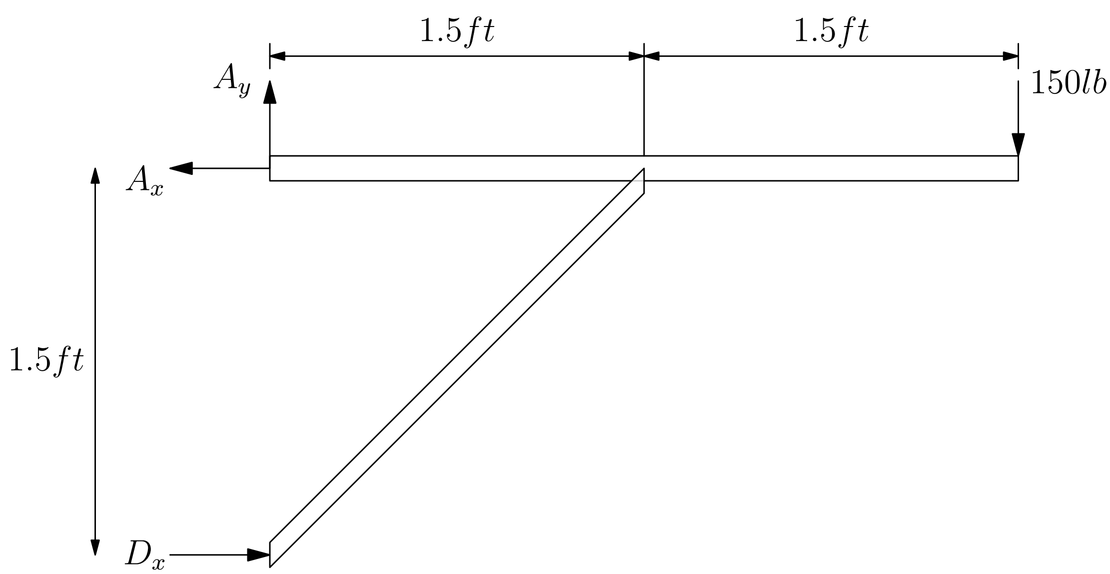


Figure 13

Equations of Equilibrium: Referring to the free-body diagram of the frame shown in Fig. 13,

$$+\uparrow \Sigma F_x = 0;$$
$$A_x = D_x$$

$$+\uparrow \Sigma F_y = 0;$$
$$A_y = \mathbf{150\,lb}$$

$$\Sigma M_A = 0;$$
$$D_x \times 1.5 = 150 \times 3$$
$$D_x = \mathbf{300\,lb}$$

Shear and Moment Diagram: The couple moment acting on B due to D_x is $M_B = 300 \times 1.5 =$ **450 lb-ft**. The loading acting on member ABC is shown in Fig. 14 and the shear and moment diagrams are shown in Figs. 15 and 17.

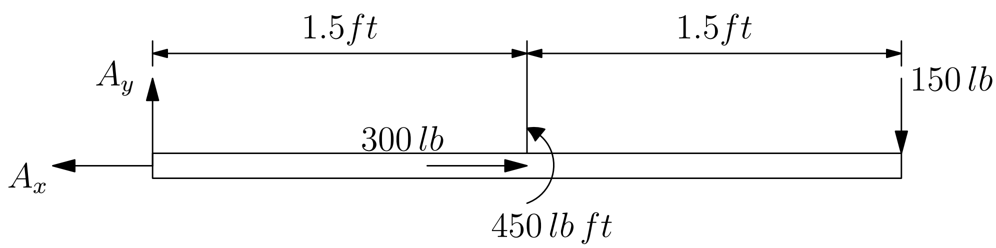


Figure 14

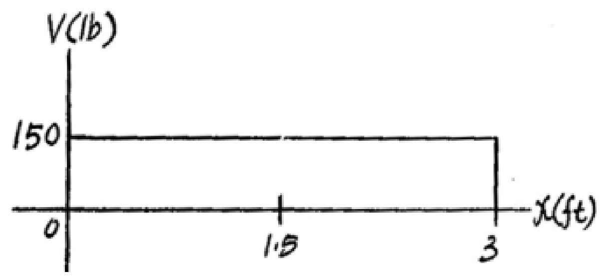


Figure 15

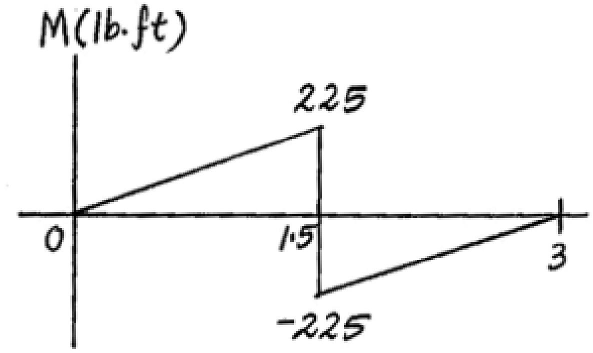


Figure 16

8. The beam is subjected to the uniform distributed load shown. Draw the shear and moment diagrams for the beam.

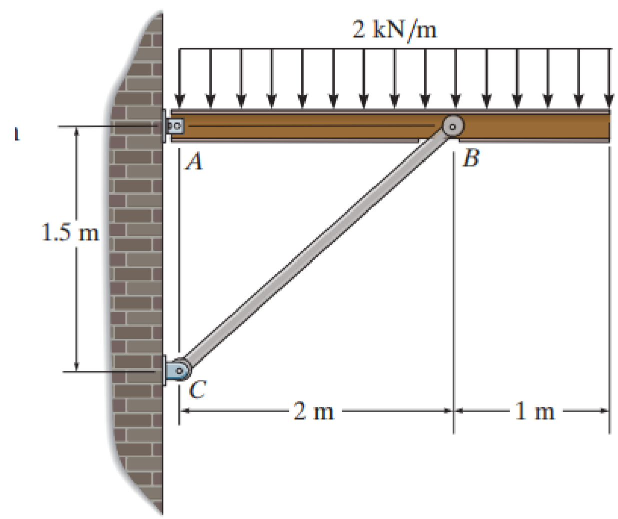


Figure 17

Solution:

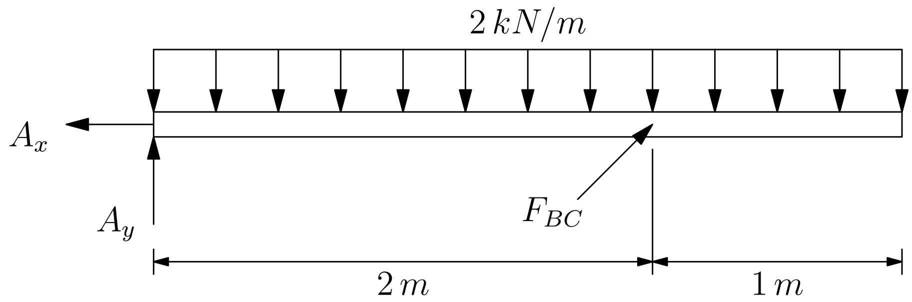


Figure 18

Equations of Equilibrium: Referring to the free-body diagram of the beam shown in Fig. 18,

$$+\Sigma M_A = 0;$$
$$F_{BC}(\frac{3}{5})(2) - 2(3)(1.5) = 0$$
$$F_{BC} = \mathbf{7.5\,kN}$$

$$+\uparrow \Sigma F_y = 0;$$
$$A_y + 7.5(\frac{3}{5}) - 2(3) = 0$$
$$A_y = 1.5\,kN$$

Shear and Moment Diagram: The vertical component of FBC is $(\mathrm{F}_{BC})_y = 7.5(\frac{3}{5}) = 4.5$ kN. The shear and moment diagrams are shown in Figs. 20 and 21.

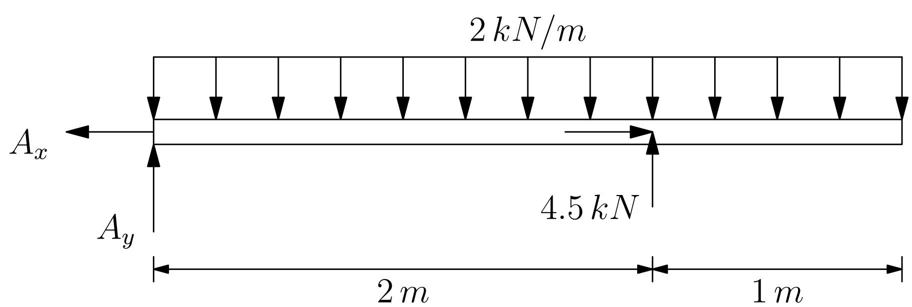


Figure 19

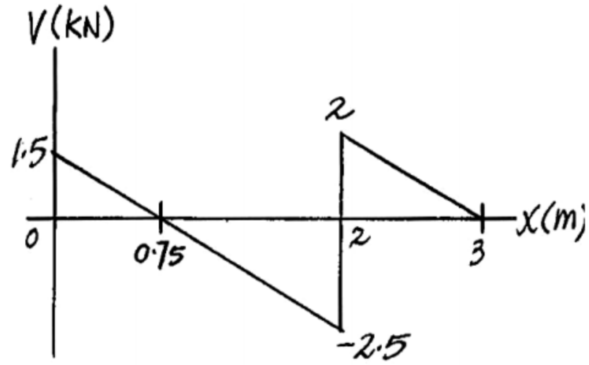


Figure 20

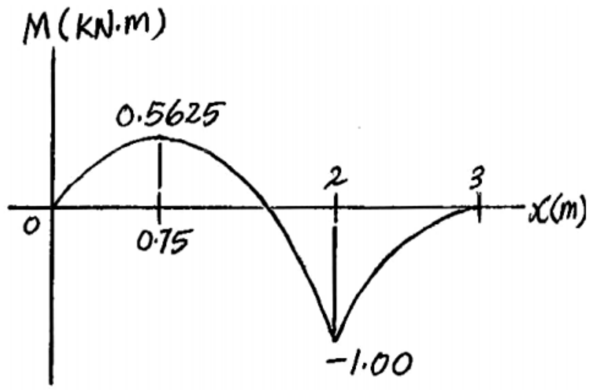


Figure 21

9. Draw the shear and moment diagrams for the overhang beam.

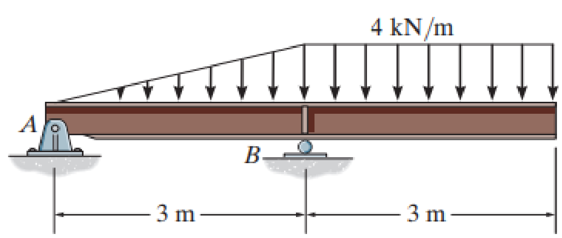


Figure 22

<u>**Solution**</u>:

Since the loading is discontinuous at support B, the shear and moment equations must be written for regions $0 \leqslant x < 3m$ and $3 < x \leqslant 6m$ of the beam. The free-body diagram of the beam's segment sectioned through an arbitrary point within these two regions is shown in Fig. 23 and Fig. 24.
Region $0 \leqslant x < 3m$, Fig. 23

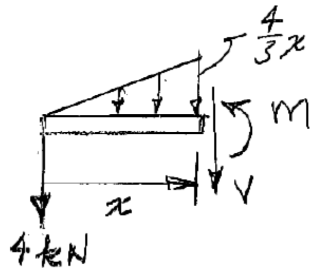


Figure 23

$$+\uparrow \Sigma F_x = 0;$$
$$-4 - \frac{1}{2} \times \frac{4}{3} \times x \times x - V = 0$$
$$V = (-\frac{2}{3} \times x^2 - 4)KN$$

$$+\uparrow \Sigma M = 0;$$
$$M + \frac{1}{2} \times \frac{4x}{3} \times x \times \frac{x}{3} + 4x = 0$$
$$M = (-\frac{2}{9} \times x^3 - 4x)KN.m$$

Region $3 < x \leqslant 6m$, Fig.24

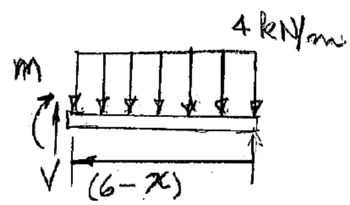


Figure 24

$$+\uparrow \Sigma F_y = 0;$$
$$V - 4(6 - x) = 0$$
$$V = (24 - 4x)KN$$

$$+\uparrow \Sigma M = 0;$$

$$M + -4(6-x)[\frac{1}{2}(6-x)] = 0$$

$$M = [-2(6-x)^2] KN.m$$

The shear diagram shown in Fig. 25 d is plotted using Eqs. (1) and (3). The value of shear just to the left and just to the right of the support is evaluated using Eqs. (1) and (3), respectively.

$$V(x = -3m) = -\frac{2}{3}(3^2) - 4 = -10KN$$

$$V(x = +3m) = 24 - 4(3) = 12KN$$

The moment diagram shown in Fig. 25 e is plotted using Eqs. (2) and (4). The value of the moment at support B is evaluated using either Eq. (2) or Eq. (4).

$$M(x = 3m) = -\frac{2}{3}(3^3) - 4(3) = -18KN.m$$

or

$$M(x = 3m) = -2(6-3)^2 = -18KN.m$$

Shear and Moment Diagram:

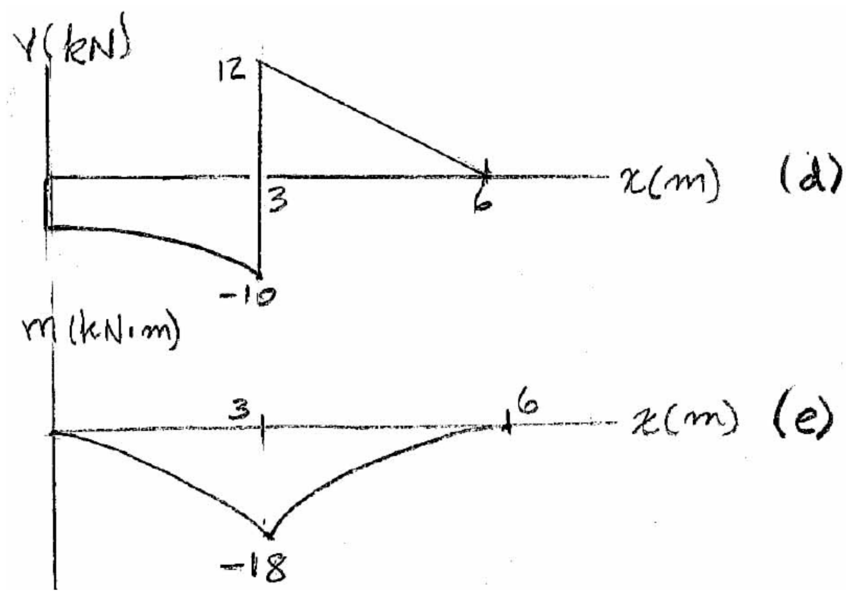


Figure 25

10. The dead-weight loading along the centerline of the airplane wing is shown. If the wing is fixed to the fuselage at A, determine the reactions at A, and then draw the shear and moment diagram for the wing.

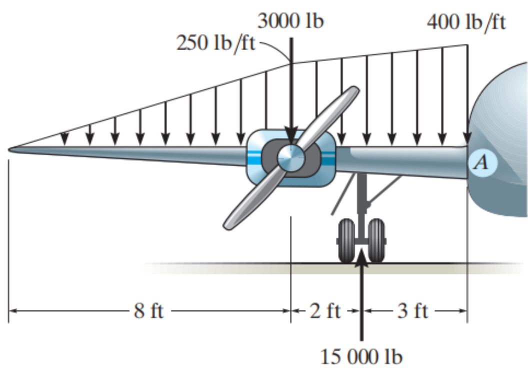


Figure 26

<u>**Solution**</u>:

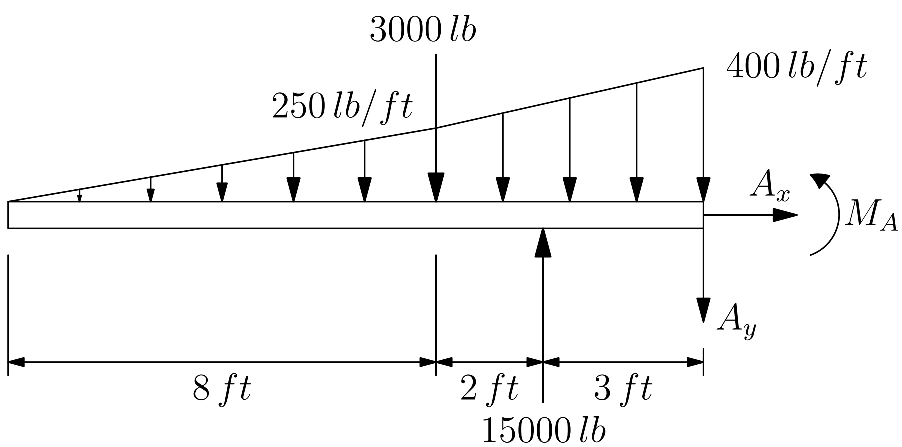


Figure 27

Support Reactions:

$$+\uparrow \Sigma F_y = 0;$$

$$A_y = -\frac{1}{2} \times 250 \times 8 - 3000 - 250 \times 5 - \frac{1}{2} \times 150 \times 5 + 15000$$

$$A_y = \mathbf{9375\,lb}$$

$\uparrow M_A = 0;$

$$M_A = -\frac{1}{2} \times 250 \times 8 \times (\frac{8}{3} + 5) - 3000 \times 5 - 250 \times 5 \times \frac{5}{2} - \frac{1}{2} \times 150 \times 5 \times \frac{5}{3} + 15000 \times 3$$

$$M_A = \mathbf{18583\,lb\text{-}ft}$$

Shear and Moment Diagram:

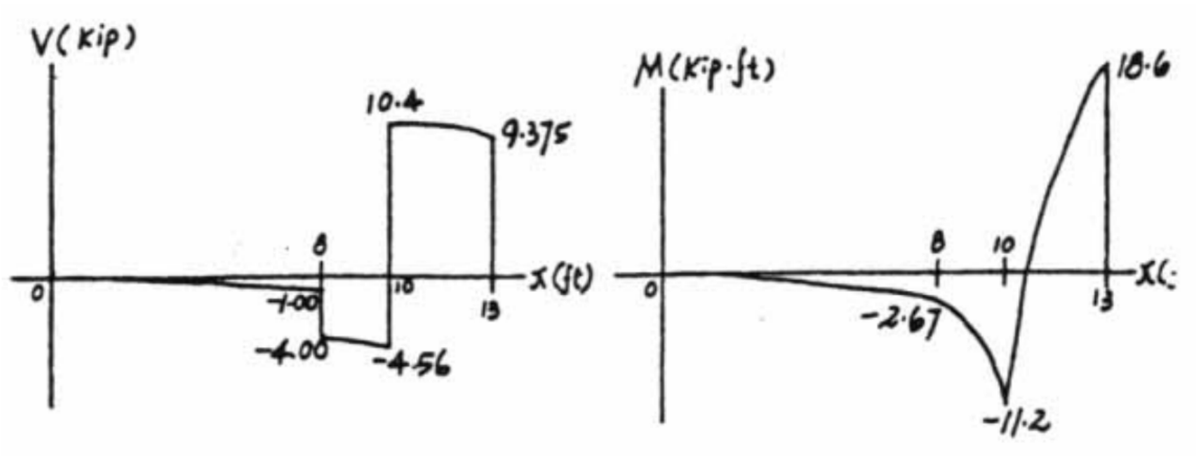


Figure 28