Indian Institute of Technology Bhubaneswar

School of Infrastructure

Mid Semester (Autumn) Examination – 2025

Subject Name : Solid Mechanics	Subject Code: CE2L001

Date: September 23, 2025

Duration: 2 Hours　　　　Full Marks : 50

Instructions:

(1) All questions are mandatory.

(2) Assume a reasonable value of any missing data.

(3) Provide neatly drawn figures whenever needed.

(4) Zeroth-order tensors or scalars are represented by small letters. For eg. a.

(5) First-order tensors or vectors are represented by bold small letters. For eg. $\boldsymbol{a}$.

(6) Second-order tensors are represented by bold capital letters. For eg. $\boldsymbol{A}$.

(7) Second-order identity tensor is represented by $\boldsymbol{I}$.

1. (a) Explain the following statement through theoretical derivations and examples with illustrative figures. "A tensor is independent of the coordinate system, but its components are not".

   (b) Consider a two-dimensional orthonormal basis $\{\boldsymbol{e}_1, \boldsymbol{e}_2\}$ in which a two-dimensional tensor $\boldsymbol{T}$ has the representation $\boldsymbol{T} = T_{ij}\,\boldsymbol{e}_i \otimes \boldsymbol{e}_j$, where $i, j = 1, 2$. The component matrix of $\boldsymbol{T}$ has values

$$[\boldsymbol{T}]_{\{\boldsymbol{e}_i\}} = \begin{bmatrix} 50 & 25 \\ 25 & 100 \end{bmatrix}.$$

   Consider a second basis $\{\boldsymbol{e}_1^*, \boldsymbol{e}_2^*\}$ which is related to $\{\boldsymbol{e}_1, \boldsymbol{e}_2\}$ by

$$\boldsymbol{e}_1^* = \frac{1}{2}\boldsymbol{e}_1 - \frac{\sqrt{3}}{2}\boldsymbol{e}_2, \qquad \boldsymbol{e}_2^* = \frac{\sqrt{3}}{2}\boldsymbol{e}_1 + \frac{1}{2}\boldsymbol{e}_2.$$

   Find the values of $[\boldsymbol{T}]_{\{\boldsymbol{e}_i^*\}}$ in the $\{\boldsymbol{e}_1^*, \boldsymbol{e}_2^*\}$ basis.

[$5 + 5 = 10$]

2. (a) Considering δ_{ij} is the Kronecker delta symbol and ϵ_{ijk} the Permutation symbol, simplify the following expressions:

(i) $\epsilon_{ijk}u_i u_j v_k$ and (ii) $\epsilon_{ijk}\delta_{jk}$.

(b) Suppose the vector field $\boxed{\boldsymbol{v}\,(x_1, x_2) = -x_1\, x_2 \boldsymbol{e}_1 + x_2\, \boldsymbol{e}_2}$ (where $x_1, x_2 \in \mathbb{R}$ and $x_2 > 0$) models the flow of a fluid. Is more fluid flowing into point (1,4) than flowing out? Plot the vector field and justify your answer.

(c) Vector field $\boxed{\boldsymbol{v}\,(x_1, x_2) = -\left(\dfrac{x_2}{x_1^2 + x_2^2}\right)\, \boldsymbol{e}_1 + \left(\dfrac{x_1}{x_1^2 + x_2^2}\right)\, \boldsymbol{e}_2}$ models the flow of a fluid. Show that if you drop a leaf into this fluid, as the leaf moves over time, the leaf does not rotate.

$[4 + 4 + 2 = 10]$

3. (a) Explain the physical meaning of the components of the stress tensor using figures and examples.

(b) The stress distribution in a body $\mathcal{B}$ is given as

$$\begin{aligned}\boldsymbol{\sigma} &= (3x_1^2 + Ax_1x_2 - 8x_2^2)\, \boldsymbol{e}_1 \otimes \boldsymbol{e}_1 + (-Bx_1^2 - 6x_1x_2 - 2x_2^2)\, (\boldsymbol{e}_1 \otimes \boldsymbol{e}_2 + \boldsymbol{e}_2 \otimes \boldsymbol{e}_1)\\ &\quad + (2x_1^2 + x_1x_2 + Cx_2^2)\, \boldsymbol{e}_2 \otimes \boldsymbol{e}_2,\end{aligned}$$

where all scalar multipliers and the constants A, B, and C have units of force per area-squared. For what values of A, B, and C does this stress distribution represent an equilibrium stress distribution? Assume zero body forces and no accelerations.

(c) Prove the following:

(i) $(\boldsymbol{e}_i \otimes \boldsymbol{e}_i) = \boldsymbol{I}$ and (ii) $(\boldsymbol{SF})^T = \boldsymbol{F}^T \boldsymbol{S}^T$.

$[3 + 3 + 4 = 10]$

4. (a) To test a glue, two plates are glued together as shown in Fig. 1. The bar formed by the joined plates is then subjected to tensile axial loads of 200 N. Using stress states and related **Mohr's circle**, determine the normal and shear stresses act on the plane where the plates are glued together (In other words, what stresses must the glue support?).

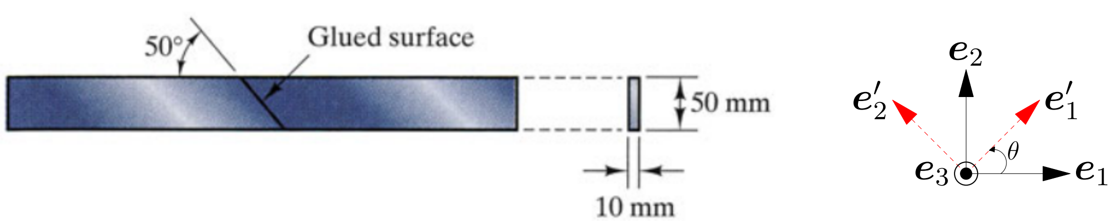

Figure 1

(b) Consider the unit ball $\mathcal{B} = \{\boldsymbol{x} | \; |\boldsymbol{x}| < 1\}$ with boundary $\partial\mathcal{B} = \{\boldsymbol{x} | \; |\boldsymbol{x}| = 1\}$. Using the divergence theorem, compute

$$\int_{\mathcal{B}} \operatorname{div}\left(\frac{\boldsymbol{x}}{|\boldsymbol{x}|}\right) dV.$$

(c) Express the following divergence theorem in terms of components using index notation.

$$\int_{\partial\mathcal{B}} \boldsymbol{\sigma}\, \boldsymbol{n}\, dS = \int_{\mathcal{B}} \operatorname{div}(\boldsymbol{\sigma})\; dV.$$

$[5 + 3 + 2 = 10]$

5. The components of stress at a critical location in a structural member are given by:

$$[\boldsymbol{\sigma}] = 150 \times \begin{bmatrix} 3 & 2 & 0 \\ 2 & 3 & 0 \\ 0 & 0 & -1 \end{bmatrix} \text{MPa}$$

with respect to a rectangular Cartesian coordinate system with base vectors $\{\boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3\}$.

(a) Evaluate the mean normal pressure, $p = -(1/3)\,\mathrm{tr}\,(\boldsymbol{\sigma})$.

(b) Evaluate the components of the stress deviator, $\boldsymbol{\sigma}^{\mathrm{dev}} = \boldsymbol{\sigma} + p\boldsymbol{I}$.

(c) Evaluate the traction vector $\boldsymbol{t}$ associated with the unit normal $\boxed{\boldsymbol{n} = (1/\sqrt{2})\,\boldsymbol{e}_1 + (1/\sqrt{2})\,\boldsymbol{e}_2}$. Show that $\boldsymbol{n}$ is a principal direction. What is the corresponding principal value of stress?

(d) Determine all the principal values and principal directions of stress.

(e) Evaluate the Von-Mises equivalent stress,

$$\sigma^{\mathrm{eq}} := \sqrt{\frac{(\sigma_I - \sigma_{II})^2 + (\sigma_{II} - \sigma_{III})^2 + (\sigma_{III} - \sigma_I)^2}{2}}$$

where σ_I, σ_{II} and σ_{III} are the principal stresses.

$[1 + 1 + 2 + 5 + 1 = 10]$