ANSWER SCHEME **[100 MARKS]**

This paper consists of 10 structured questions. Answer all questions in the space provided. The marks for each part of the question is as indicated.

Question 1 (10 marks)

a) Write a regular expression for the following languages. (6 marks)

ii) The language L = {$w \in \{a, b\}$: w contains the substring aa}. (a + b)*aa(a + b)*

ii) The language of all strings over {a, b} in which *b* is the second letter and *a* is the second last letter. (a + b)b(a + b)*a(a + b)

b) Give a description of the following languages in your own words. (4 marks)

i) $(ab)^*ba$ The language of all strings over {a, b} that start with zero or more substring *ab* and end with substring *ba*

ii) $(a + b)^*(b + aa)(a + b)^*$ The language of all strings over {a, b} that contain letter *b* or substring *aa*.

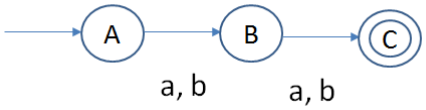
Question 2 (10 marks)

a) List down all the strings with length at most three and state whether the sets are finite or infinite.
{ε, a, b, aa, ab, ba, bb, aaa, aab, aba, abb, baa, bab, bba, bbb}, FINITE

b) Give a regular expression of language A* that start and end with different symbol
a(a + b)*b + b(a + b)*a

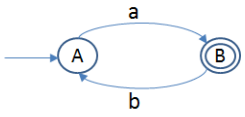
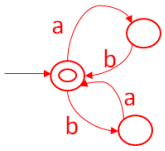
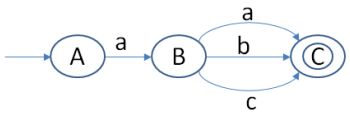
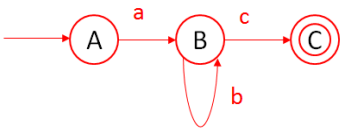
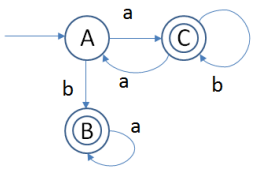
c) Given B = {a, b, c}. Write the language of AB^2
A = {a, b}
B = {a, b, c}
B^2= {a, b, c} {a, b, c} = {aa, ab, ac, ba, bb, bc, ca, cb, cc}
AB^2= {a, b} { aa, ab, ac, ba, bb, bc, ca, cb, cc}
AB^2 = {aaa, aab, aac, aba, abb, abc, aca, acb, acc, baa, bab, bac, bba, bbb, bbc, bca, bcb, bcc}

d) Draw a state diagram of DFA which accepts set of all string over A* of length two.
L = {aa, ab, ba, bb}



Question 3 (10 marks)

Complete the table below with the corresponding finite automata or regular expression.

Finite Automata	Regular Expression
	a(ba)*
	(ab + ba)*
	a(a + b + c)
	ab*c
	(ab*a)* + ba*

Question 4 (10 marks)

a) (4 marks)

Table 1.

IN the language	NOT IN the language
a, aa, ab, aab, abab, abaab	*ba, abb*

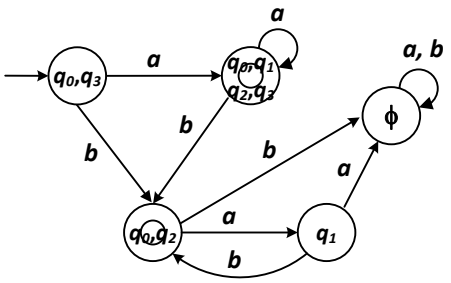
b) Convert the NFA to the equivalent DFA. (4 marks)

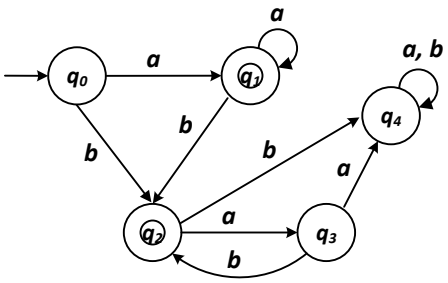
NFA

δ	*a*	*b*	λ*
$\{q_0\}$	$\{q_1\}$	ϕ	$\{q_0, q_3\}$
$\{q_1\}$	ϕ	$\{q_2\}$	$\{q_1\}$
$\{q_2\}$ - final	ϕ	ϕ	$\{q_0, q_2\}$
$\{q_3\}$	$\{q_0, q_2\}$	ϕ	$\{q_3\}$

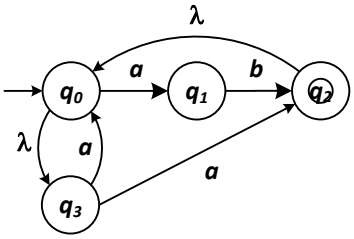
DFA

δ	***a*λ***	***b*λ***
$\{q_0, q_3\}$	$\{q_0, q_1, q_2, q_3\}$	$\{q_0, q_2\}$
$\{q_0, q_1, q_2, q_3\}$ - final	$\{q_0, q_1, q_2, q_3\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$ - final	$\{q_1\}$	ϕ
$\{q_1\}$	ϕ	$\{q_0, q_2\}$





OR



(ab*a)*(ab* + ba*) **(aa* + aa*b + b) (ab)***

c) Write the regular expression for the DFA. **(aa* + aa*b + b) (ab)*** (2 marks)

Question 5 (10 marks)

Given NFA state diagram below:

a) Which of the strings ababa, aababa, ababaabb, abaaba are accepted by the NFA? (4 marks)

ababa : $q_0 - q_2 - q_1 - q_2 - q_3 - q_1$: accepted

aababa: $q_0 - q_2$ - : rejected

ababaabb: $q_0 - q_2 - q_1 - q_2 - q_3 - q_1 - q_2 - q_3$ - : rejected

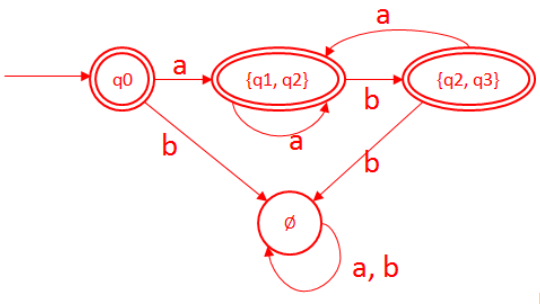
abaaba: $q_0 - q_2 - q_1 - q_1 - q_2 - q_3 - q_1$: accepted

b) Construct the transition table of above NFA. (2 marks)

δ	a	b
➢ q_0	$\{q_1, q_2\}$	Ø
q_1	$\{q_1, q_2\}$	Ø
q_2	Ø	$\{q_1, q_3\}$
q_3	$\{q_1, q_2\}$	Ø

c) Convert the NFA to equivalent DFA (4 marks)

δ	a	b
➢ q_0	$\{q_1, q_2\}$	Ø
$\{q_1, q_2\}$	$\{q_1, q_2\}$	$\{q_1, q_3\}$
$\{q_1, q_3\}$	$\{q_1, q_2\}$	Ø
Ø	Ø	Ø



Question 6 (10 marks)

a) Given a regular expression $a\Sigma^*b\Sigma^*c$ and set of alphabet $\Sigma = \{a, b, c\}$. List THREE strings that can be generated and TWO strings that cannot be generated by the regular expression. (5 marks)

Accepted strings { awc | w contains at least a b }	Rejected strings

abc	B
aabc	Abb
abac	

b) Write the regular expression over the alphabet Σ = {0, 1} for each of the following language:

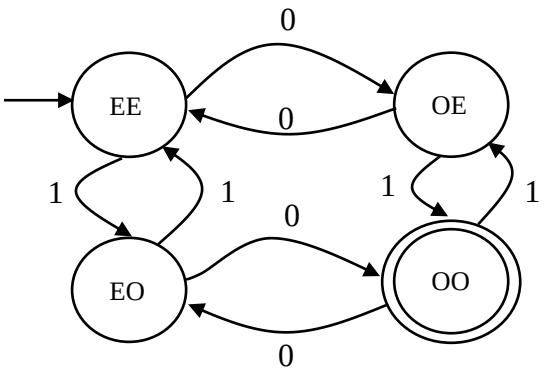
i) {w | w has *at most one 0*}. 1*(λ + 0)1* (1 mark)

ii) {w | w has *exactly two 0's*}. 1*01*01* (2 marks)

iii) {w | w has *no more than two 0's*}. (1*(λ + 0)1*) + (1*01*01*) (2 marks)

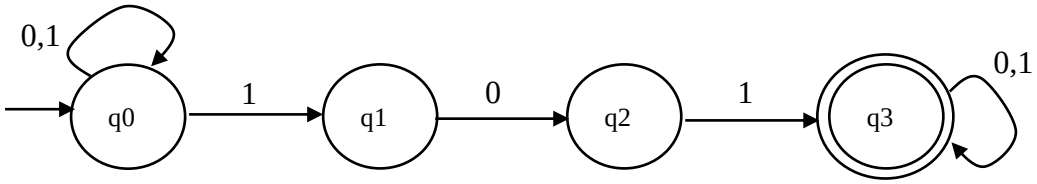
Questions 7 (10 marks)

a) Draw the DFA for the language L1 = {w | w has odd number of 0's and odd number of 1's}. Σ = {0, 1}. (5 marks)



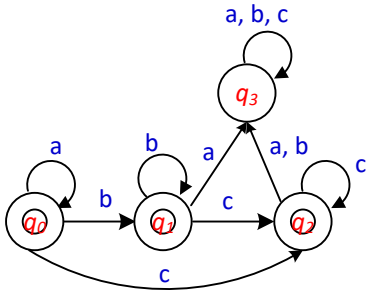
O = Odd
E = Even

b) Draw the NFA for the language L2 = {w | w contains the substring 101}. Σ = {0, 1}.(5 marks)



Questions 8 (10 marks)

a) Draw a state diagram for M. Is it a DFA or NFA? _____ **DFA** (3 mark)



b) Give a regular expression for the language accepted by M. **a*b*c*** (3 marks)

c) Give 2 possible strings accepted by M. **a, b, c, ab, ac, bc, abc, aa, bb, cc** (2 marks)

d) Trace the computations that process the strings *abbbbc*. (2 marks)

[q_0 , abbbbc] ⊢ **[q_0 , bbbbc]**

⊢ **[q_1 , bbbc]**

⊢ **[q_1 , bbc]**

⊢ **[q_1 , bc]**

⊢ **[q_1 , c]**

⊢ **[q_2 , λ]** **String Accepted**

Question 9 (10 marks)

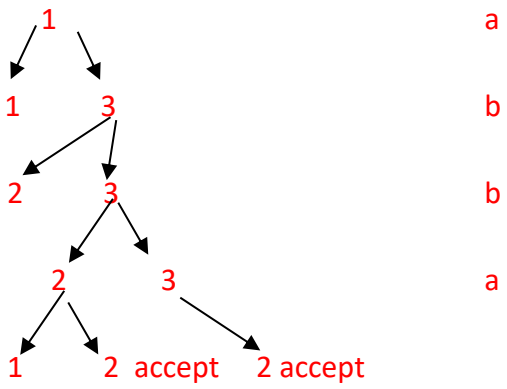
Based on the diagram below, answer the questions.

a) Write the transition table of the NFA above. (3 marks)

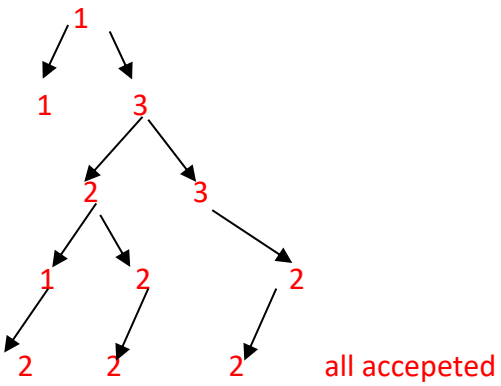
δ	*a*	*b*	*ε*
1	{3}	φ	{2}
2	{1}	φ	φ
3	{2}	{2,3}	φ

b) Draw the possibility tree for the computation of the following strings. (4 marks)

i. *abba*



ii. *abaa*

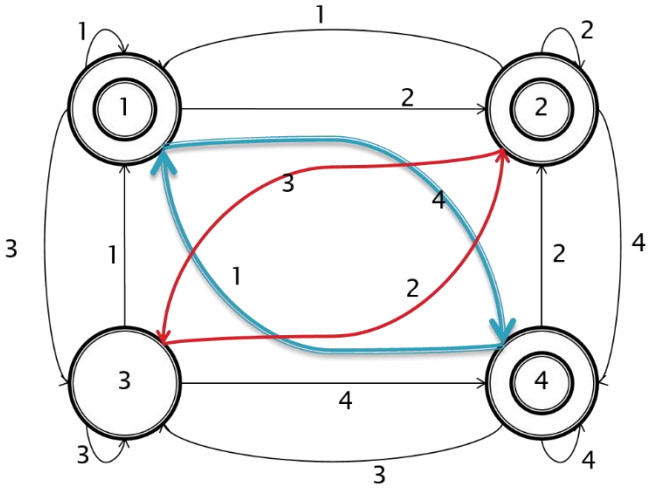


c) What is the regular expression of the FA above? **(a*ab*(a+b))*** (3 marks)

Question 10 (10 marks)

Given an FA description for Elevator/Lift.

a) Draw the state diagram for the Elevator. (3 marks)



b) Inputs : 1, 2, 3, 4 (5 marks)
States : 1, 2, 3, 4
Final States : 1, 2, 4
Starting States : 1, 2, 3, 4
Rules : 1 to 1st floor, 2 to 2nd floor, 3 to 3rd floor, 4 to 4th floor

c) 4 (2 marks)