Fachhochschule Südwestfalen
University of Applied Sciences

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Assignment on Bioreactor (Group- 05)

Course Name: Advanced Control Technology

Assignment Part: 02

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1. Introduction

The model and parameters for the Bioreactor system remains the same as it was in Assignment part I. The mechanism and schematic of the bioreactor system is shown in Fig. 1.

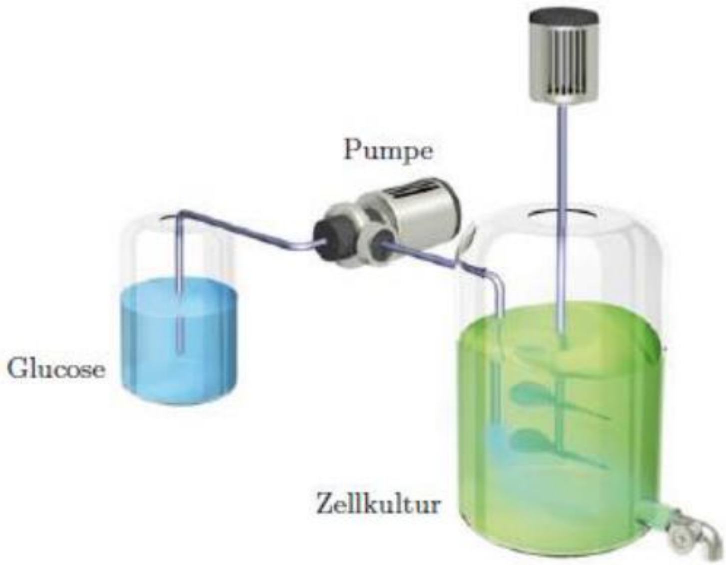

Fig. 01: Bioreactor System.

The given state space system is modeled as follows:

$$\dot{x} = a\,(x) + b\,(x).u = \begin{bmatrix} \mu\,(x_2).\ x_1 \\ -\frac{1}{\alpha}\,\mu(x_2).\ x_1 \end{bmatrix} + \begin{bmatrix} -x_1 \\ K - x_2 \end{bmatrix}.u$$

$$y = g(x) = \begin{bmatrix} 1 & 0 \end{bmatrix}.x$$

where,

$$\mu(x_2) = \frac{\mu_0 x_2}{k_1 + x_2 + k_2 x_2^2}$$

- Maximal growth rate, $\mu_o = 2$
- Affinity constant, $k_1 = 0.06$
- Affinity constant, $k_2 = 0.3$
- Feed concentration of glucose, $K = 2$
- Yield constant, $\alpha = 0.7$
- Concentration of biomass and substrate = x1 and x2 respectively.
- Feed Rate = u.

The tasks to be performed for this assignment are briefly summarized as follows:

1. Choose an appropriate control method for the nonlinear system derived in the first assignment.
2. Design the controller for the system.
3. Compare the results of the derived controller with the results obtained with the linear controller derived in assignment 1. Particularly, analyze the controller performance in the operation point for which the linear controller is designed as well as in regions of the state space not covered by the linear controller.

2. Choosing a Control method for Nonlinear System

I. Detecting Internal Dynamics using Lie Derivation:

In order to find the presence of internal dynamics in a system, Lie Derivation is performed. A system has internal dynamics if its relative degree is less than the dimension, Conversely, if its relative degree is equal to the system dimension, then the system has no internal dynamics [1]. The dimension of the provided bioreactor system is n=2.
The system can be represented in nonlinear form as follows:

$$\dot{x} = f(x) + g(x).u$$
$$y = c(x)$$

Where,
x = Dimensional vector of state variables
y = Dimensional vector of Controller output
u = Dimensional vector of controller input
We can write for the given state space model,

$$a(x) = \begin{bmatrix} \mu(x_2).x_1 \\ -\frac{1}{\alpha}\ \mu(x_2).x_1 \end{bmatrix}$$

$$b(x) = \begin{bmatrix} -x_1 \\ k - x_2 \end{bmatrix}$$

$$c(x) = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

We know for Lie Derivation [2],

$$\dot{y} = L_a c(x) + L_b c(x).u$$

And,

$$\dot{y} = \frac{\partial c(x)}{\partial x}.a(x) + \frac{\partial c(x)}{\partial x}.b(x).u$$

∴ For the 1st derivative, $\delta = 1$,

$$L_a{}^1 c(x) = \frac{\partial c(x)}{\partial x}.a(x)$$

$$= [1 \quad 0]\begin{bmatrix} \mu(x_2).x_1 \\ -\frac{1}{\alpha}\ \mu(x_2).x_1 \end{bmatrix}$$

$$\therefore L_a{}^1 c(x) = \ \mu(x_2).x_1$$

And, $L_b c(x) = \frac{\partial c(x)}{\partial x}.b(x)$

$$= [1 \quad 0]\begin{bmatrix} -x_1 \\ k - x_2 \end{bmatrix}$$

$$\therefore L_b c(x) = -x_1 \neq 0$$

Hence,

$\dot{y} = L_a c(x) + L_b c(x).u$

$= \mu(x_2).x_1 - x_1.u$; which is nonzero.

Therefore, it can be stated that the relative degree of the system ($\delta = 1$) is less than the system dimension ($n = 2$) i.e. $\delta < \mathrm{n}$. Thus, the system has internal dynamics and the system is not observable.
Hence, to design a controller for this nonlinear system, the Fuzzy Logic control approach is chosen because of its less-complicated mechanism and ability to manipulate the outcome.

II. Fuzzy Logic Controller (Mamdani)

Fuzzy logic is widely used in concepts where the outcome cannot be simply expressed by a "TRUE" or "FALSE". It is an experience-based control approach where the mechanism logic can be applied based on the experience of a human operator. The controller processes analog input with continuous values between 0 and 1 and provides outcome as per the conditions set by the operator [3]. The Fuzzy logic controller type to be used for this nonlinear system is the Mamdani Fuzzy controller scheme. The fuzzy controller with the input, process and output is illustrated in Fig. 02.

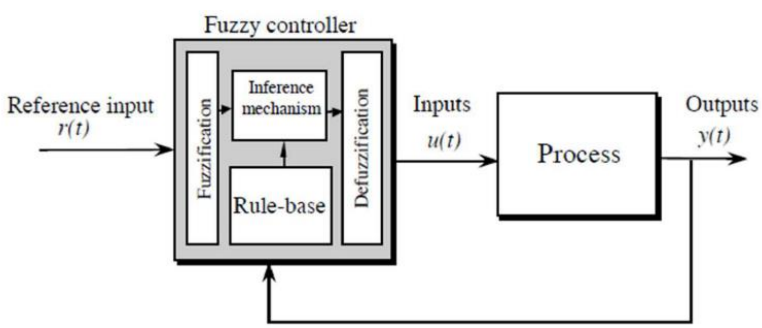

Fig. 02: Fuzzy Logic Controller scheme [4].

The steps for the Mamdani Fuzzy logic controller are stated as follows:

i. **Fuzzification**: Membership functions (MF) are created from crisp data and a reference input is processed through fuzzy controller based on the MF.

ii. **Rule Base**: A rule base is defined based on the MFs. These are the desired outputs set by the human operator for different combinations of input variables.

iii. **Defuzzification**: An output is obtained in real numbers after from the system with Fuzzy controller. Several Defuzzification exists and the one used for this task is Centroid Defuzzification.

3. Design of the Mamdani Fuzzy Controller using MATLAB

Step 1- Assigning Inputs and Output: The first step is to assign the input and output variables in the MATLAB Fuzzy Logic Designer App. The provided Bioreactor system consists of two inputs namely, *Biomass Concentration, x1* and *Substrate Concentration, x2*. The output for this step is the *Flow Rate, u*. The assigned inputs and outputs in the designer app are shown in Fig. 03.

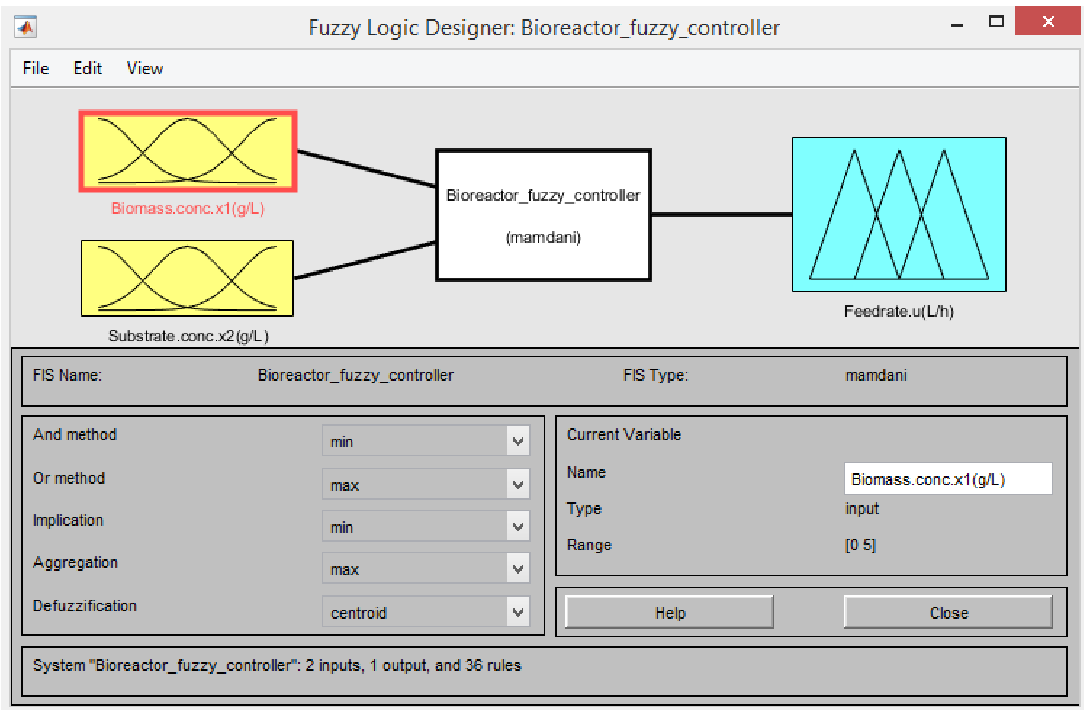

Fig. 03: Assigning Inputs and Output in MATLAB Fuzzy Logic Designer App.

Step 2- Membership Functions and Linguistic variables: The type and number of membership functions (MFs) and linguistic variables for the two inputs (x1 and x2) and output (u) is illustrated in Table I. Furthermore, the assignment of the membership functions for each input and output in MATLAB Fuzzy Logic Designer is shown in Fig. 04, 05 and 06 respectively.

Table I: *Assigning Membership Functions (MFs) and Linguistic Variables to Input and Output*

Parameter		Type of MFs	Linguistic Variables	
Input	Biomass Concentration (x1)	Triangular	Low	L
			More than low	MTL
			Medium	M
			High	H
			Very high	VH
			Very very high	VVH
	Substrate Concentration (x2)	Triangular	Very low	VL
			Low	L
			Medium	M
			High	H
			Very high	VH
			Very very high	VVH
Output	Feed Rate (u)	Triangular	Zero	Zero
			Very low	VL
			Low	L
			Medium	M
			High	H
			Very high	VH

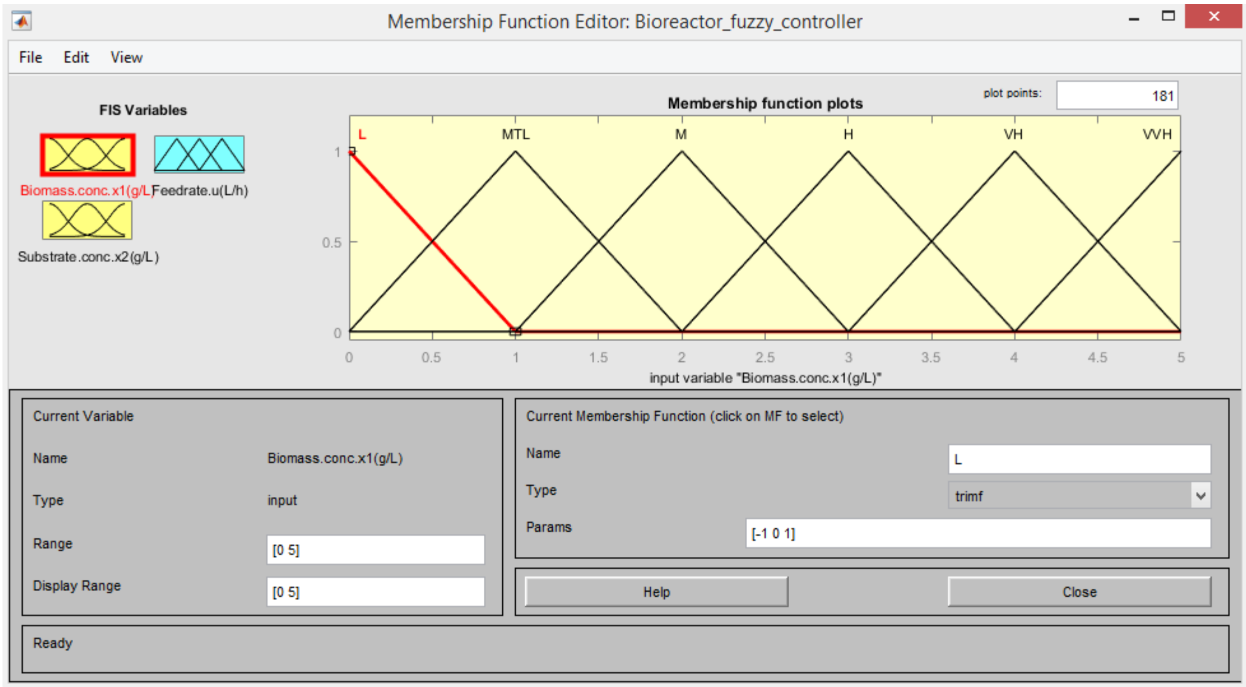

Fig. 04: Membership Functions for input Biomass Concentration (x1).

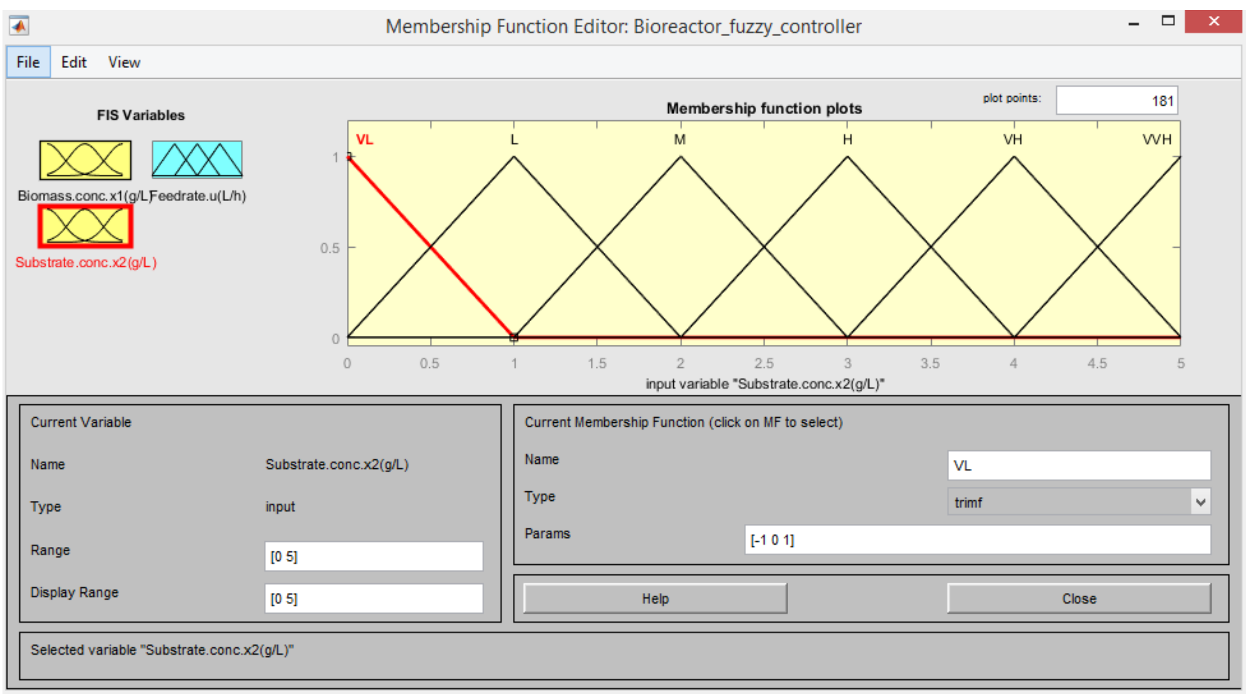

Fig. 05: Membership Functions for input Substrate Concentration (x2).

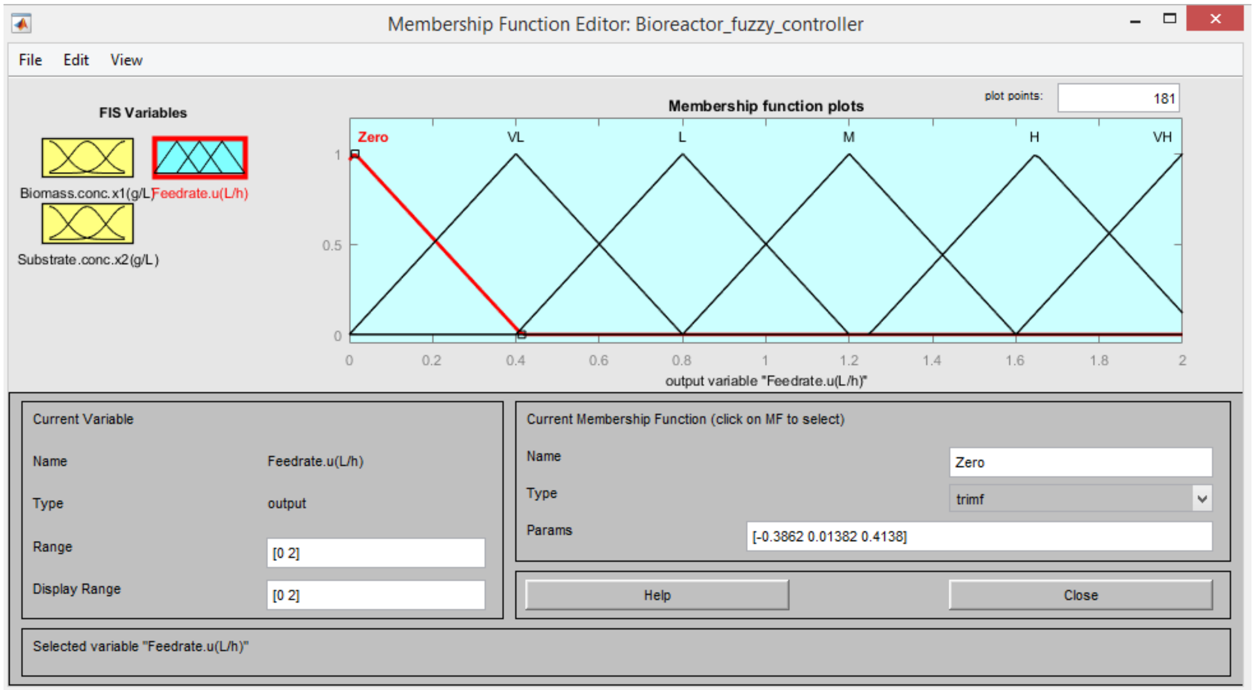

Fig. 06: Membership Functions for output Flow Rate (u).

Step 3- Creating Rule Base: The number of rules in a rule-based system is given by the equation [4],

$$R = \prod_{j=1}^{n} N_j$$

Where, N is the number of linguistic variables for j parameters.
In this Fuzzy model, each of the inputs x1 and x2 contain 6 linguistic variables respectively.
Hence, the number of rules in the system becomes, 6 x 6 = 36.
The rules are illustrated in Table II.

Table II: *Rule Base for Mamdani Fuzzy Logic Controller for Bioreactor System*

Rules
1. If (Biomass.conc.x1(g/L) is L) and (Substrate.conc.x2(g/L) is VL) then (Feedrate.u(L/h) is VH) (1)
2. If (Biomass.conc.x1(g/L) is L) and (Substrate.conc.x2(g/L) is L) then (Feedrate.u(L/h) is H) (1)
3. If (Biomass.conc.x1(g/L) is L) and (Substrate.conc.x2(g/L) is M) then (Feedrate.u(L/h) is M) (1)
4. If (Biomass.conc.x1(g/L) is L) and (Substrate.conc.x2(g/L) is H) then (Feedrate.u(L/h) is L) (1)
5. If (Biomass.conc.x1(g/L) is L) and (Substrate.conc.x2(g/L) is VH) then (Feedrate.u(L/h) is VL) (1)
6. If (Biomass.conc.x1(g/L) is L) and (Substrate.conc.x2(g/L) is VVH) then (Feedrate.u(L/h) is Zero) (1)
7. If (Biomass.conc.x1(g/L) is MTL) and (Substrate.conc.x2(g/L) is VL) then (Feedrate.u(L/h) is VH) (1)
8. If (Biomass.conc.x1(g/L) is MTL) and (Substrate.conc.x2(g/L) is L) then (Feedrate.u(L/h) is H) (1)
9. If (Biomass.conc.x1(g/L) is MTL) and (Substrate.conc.x2(g/L) is M) then (Feedrate.u(L/h) is M) (1)

10. If (Biomass.conc.x1(g/L) is MTL) and (Substrate.conc.x2(g/L) is H) then (Feedrate.u(L/h) is L) (1)
11. If (Biomass.conc.x1(g/L) is MTL) and (Substrate.conc.x2(g/L) is VH) then (Feedrate.u(L/h) is VL) (1)
12. If (Biomass.conc.x1(g/L) is MTL) and (Substrate.conc.x2(g/L) is VVH) then (Feedrate.u(L/h) is Zero) (1)
13. If (Biomass.conc.x1(g/L) is M) and (Substrate.conc.x2(g/L) is VL) then (Feedrate.u(L/h) is H) (1)
14. If (Biomass.conc.x1(g/L) is M) and (Substrate.conc.x2(g/L) is L) then (Feedrate.u(L/h) is H) (1)
15. If (Biomass.conc.x1(g/L) is M) and (Substrate.conc.x2(g/L) is M) then (Feedrate.u(L/h) is M) (1)
16. If (Biomass.conc.x1(g/L) is M) and (Substrate.conc.x2(g/L) is H) then (Feedrate.u(L/h) is M) (1)
17. If (Biomass.conc.x1(g/L) is M) and (Substrate.conc.x2(g/L) is VH) then (Feedrate.u(L/h) is VL) (1)
18. If (Biomass.conc.x1(g/L) is M) and (Substrate.conc.x2(g/L) is VVH) then (Feedrate.u(L/h) is Zero) (1)
19. If (Biomass.conc.x1(g/L) is H) and (Substrate.conc.x2(g/L) is VL) then (Feedrate.u(L/h) is H) (1)
20. If (Biomass.conc.x1(g/L) is H) and (Substrate.conc.x2(g/L) is L) then (Feedrate.u(L/h) is H) (1)
21. If (Biomass.conc.x1(g/L) is H) and (Substrate.conc.x2(g/L) is M) then (Feedrate.u(L/h) is M) (1)
22. If (Biomass.conc.x1(g/L) is H) and (Substrate.conc.x2(g/L) is H) then (Feedrate.u(L/h) is L) (1)
23. If (Biomass.conc.x1(g/L) is H) and (Substrate.conc.x2(g/L) is VH) then (Feedrate.u(L/h) is VL) (1)
24. If (Biomass.conc.x1(g/L) is H) and (Substrate.conc.x2(g/L) is VVH) then (Feedrate.u(L/h) is Zero) (1)
25. If (Biomass.conc.x1(g/L) is VH) and (Substrate.conc.x2(g/L) is VL) then (Feedrate.u(L/h) is M) (1)
26. If (Biomass.conc.x1(g/L) is VH) and (Substrate.conc.x2(g/L) is L) then (Feedrate.u(L/h) is M) (1)
27. If (Biomass.conc.x1(g/L) is VH) and (Substrate.conc.x2(g/L) is M) then (Feedrate.u(L/h) is L) (1)
28. If (Biomass.conc.x1(g/L) is VH) and (Substrate.conc.x2(g/L) is H) then (Feedrate.u(L/h) is L) (1)
29. If (Biomass.conc.x1(g/L) is VH) and (Substrate.conc.x2(g/L) is VH) then (Feedrate.u(L/h) is VL) (1)
30. If (Biomass.conc.x1(g/L) is VH) and (Substrate.conc.x2(g/L) is VVH) then (Feedrate.u(L/h) is Zero) (1)
31. If (Biomass.conc.x1(g/L) is VVH) and (Substrate.conc.x2(g/L) is VL) then (Feedrate.u(L/h) is Zero) (1)
32. If (Biomass.conc.x1(g/L) is VVH) and (Substrate.conc.x2(g/L) is L) then (Feedrate.u(L/h) is Zero) (1)
33. If (Biomass.conc.x1(g/L) is VVH) and (Substrate.conc.x2(g/L) is M) then (Feedrate.u(L/h) is Zero) (1)
34. If (Biomass.conc.x1(g/L) is VVH) and (Substrate.conc.x2(g/L) is H) then (Feedrate.u(L/h) is Zero) (1)
35. If (Biomass.conc.x1(g/L) is VVH) and (Substrate.conc.x2(g/L) is VH) then (Feedrate.u(L/h) is Zero) (1)
36. If (Biomass.conc.x1(g/L) is VVH) and (Substrate.conc.x2(g/L) is VVH) then (Feedrate.u(L/h) is Zero) (1)

The Rule Viewer generated in the designer app based on the Rule Base is shown in Fig. 07.

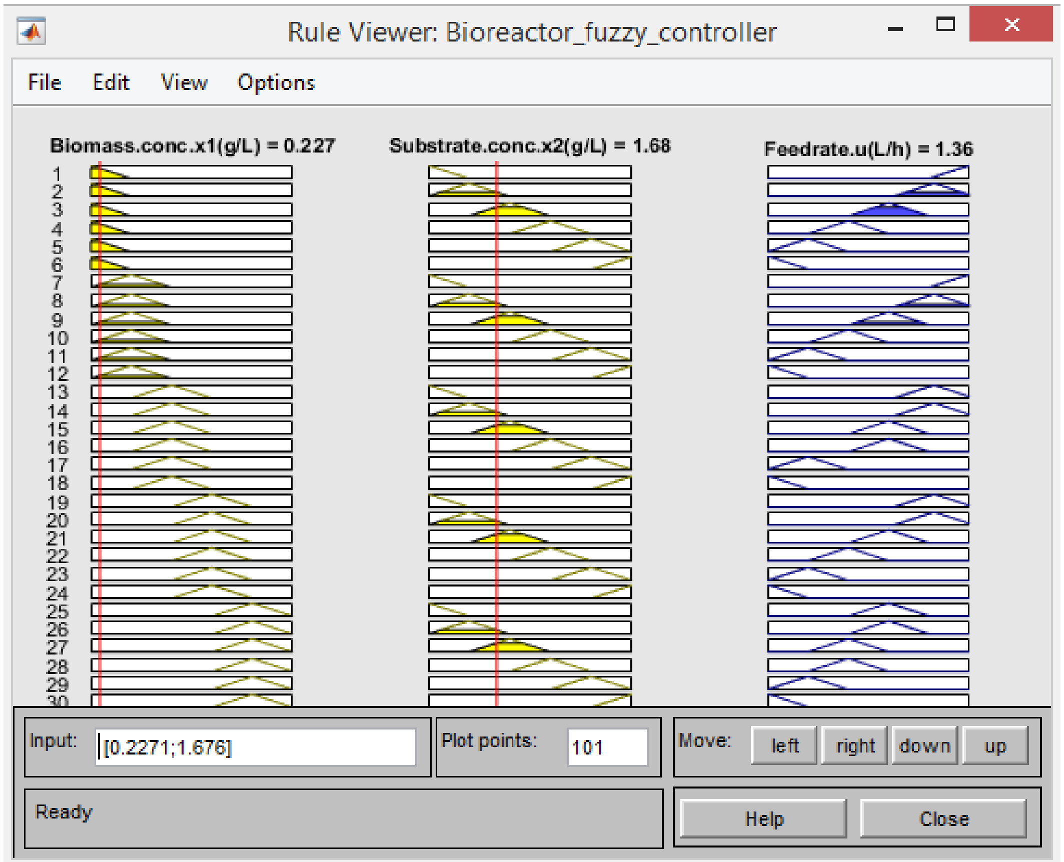

Fig. 07: Rule Viewer for Mamdani Fuzzy controller Rule base.

It is seen that, the assumed value of u (=1.3) during Assignment 1 for the operating points of linearization (x1= 0.227147, x2= 1.675505), is approximately equal to the value of u (=1.36) obtained from the Rule base of the Mamdani Fuzzy Controller.

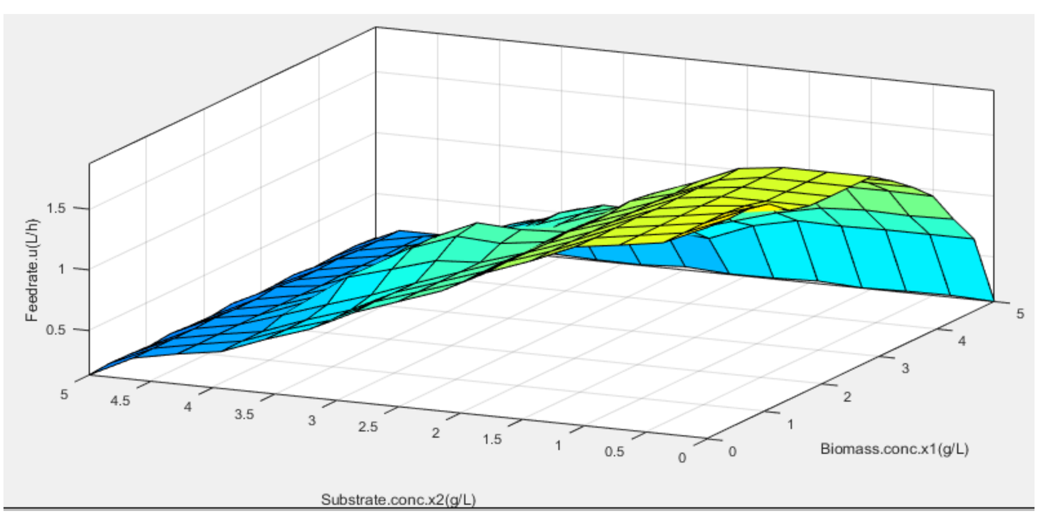

Fig. 08: Mamdani Fuzzy Controller Surface plot for Bioreactor system.

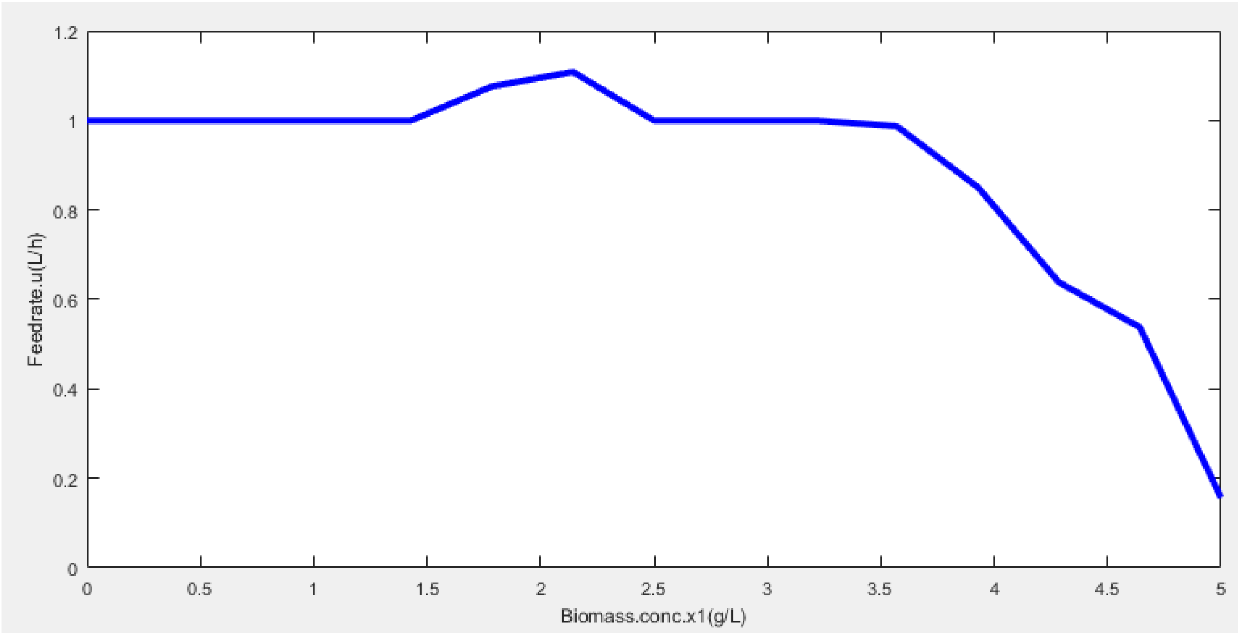

Fig. 09: Biomass concentration (x1) vs. Flow rate (u) plot.

It can be seen from Fig. 09 that, for a stable increase of x1, the level of u remains constant. After a slight increase, the flow rate (u) again becomes stable. For further increase of biomass concentration, the flow rate starts decreasing rapidly until it becomes approximately equal to zero.

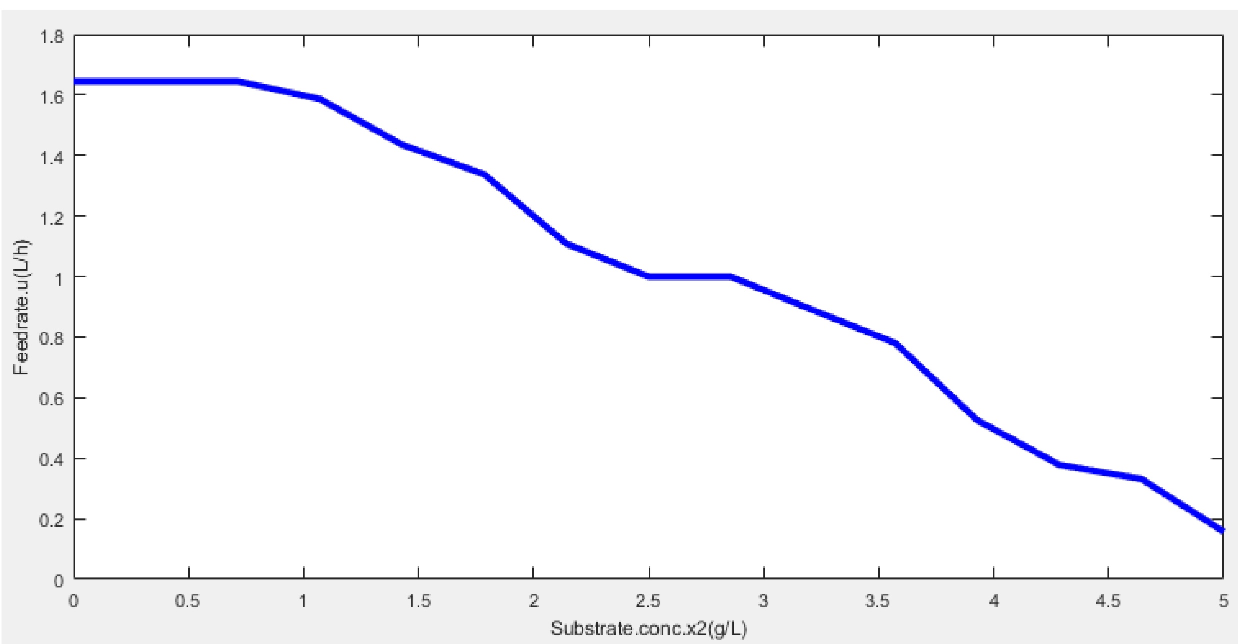

Fig. 10: Substrate concentration (x2) vs. Flow rate (u) plot.

It can be seen from Fig. 10 that, the flow rate (u) initially remains stable for slight increase of substrate concentration (x2). Afterwards, the flow rate starts rapidly decreasing with increase of substrate concentration until it finally becomes approximately equal to zero.

Step 4- Fuzzy Logic Controller with Bioreactor System: The Mamdani Fuzzy logic controller connected with the Bioreactor system in MATLAB Simulink is shown in Fig. 11.

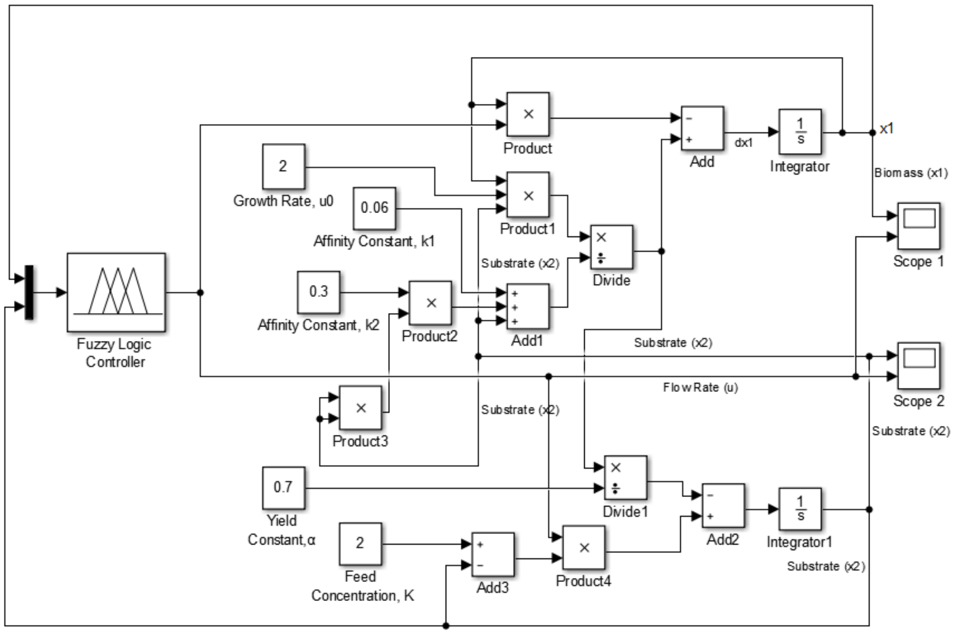

Fig. 11: MATLAB Simulink block of Bioreactor system with Mamdani Fuzzy controller.

4. Results and Analysis

i. Results

The graphs obtained from the Simulink block are shown as follows. Fig. 12 shows the response of Flow rate (u) and Biomass concentration (x1) of Bioreactor system with Mamdani controller. Fig. 13 shows the response of Flow rate and Substrate concentration (x2).

Fig. 12: Response of u and x1 of Bioreactor system with Mamdani Fuzzy controller.

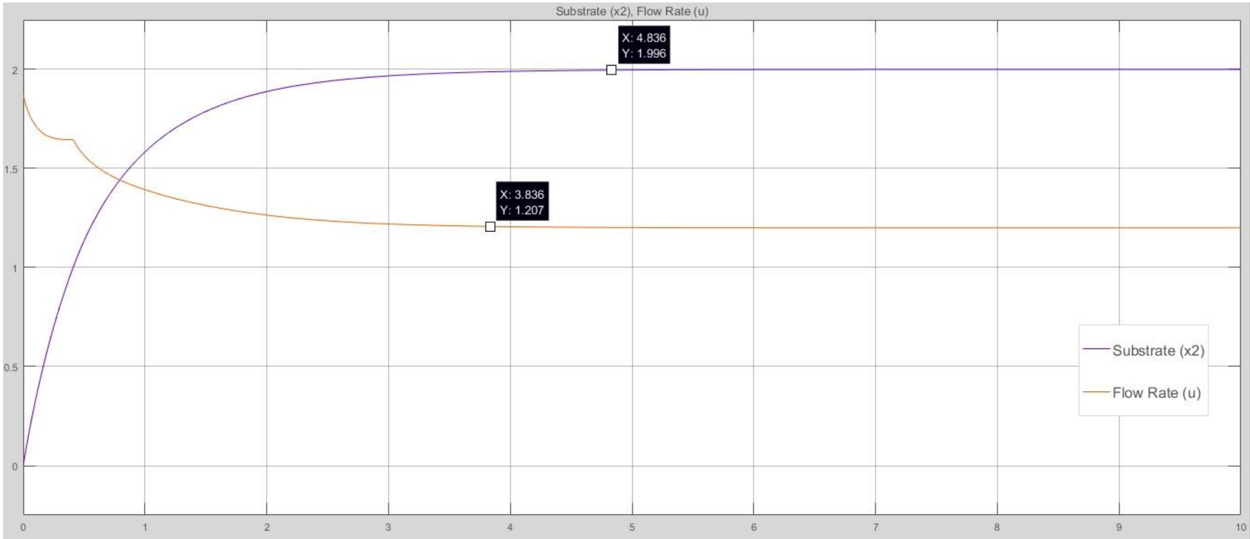

Fig. 13: Response of u and x2 of Bioreactor system with Mamdani Fuzzy controller.

ii. Analysis

It can be said that, the flow rate (u) stabilizes around 3.9 seconds at the value of 1.207. The value of u chosen in Assignment 1 is 1.3 and in the rule base, the value of u obtained is 1.36. Hence, the values of u in all cases are approximately similar.

It can be seen from Fig. 12 that, the biomass concentration (x1) shows zero response with Mamdani Fuzzy controller. In Fig. 13, the substrate concentration (x2) stabilizes at around 4.9 seconds. The responses of x1 and x2 for linear feedback controller obtained during Assignment 1 are shown as follows:

Linear Feedback Controller responses for Bioreactor system:

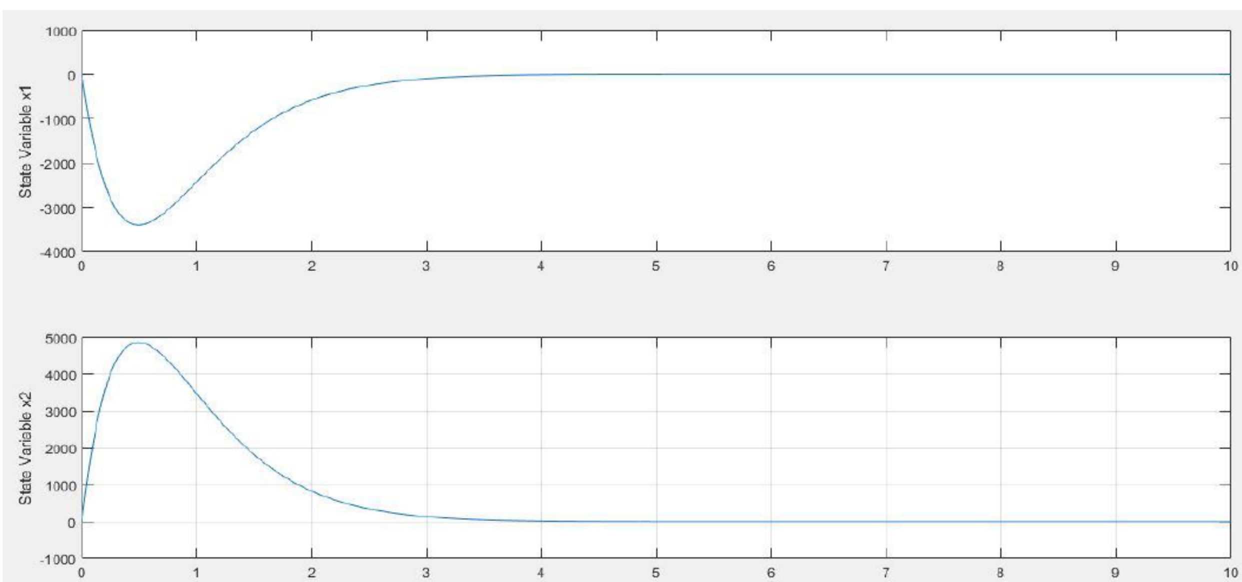

Fig. 14: Controller responses for poles at ***-2 ± 0.3j****.*

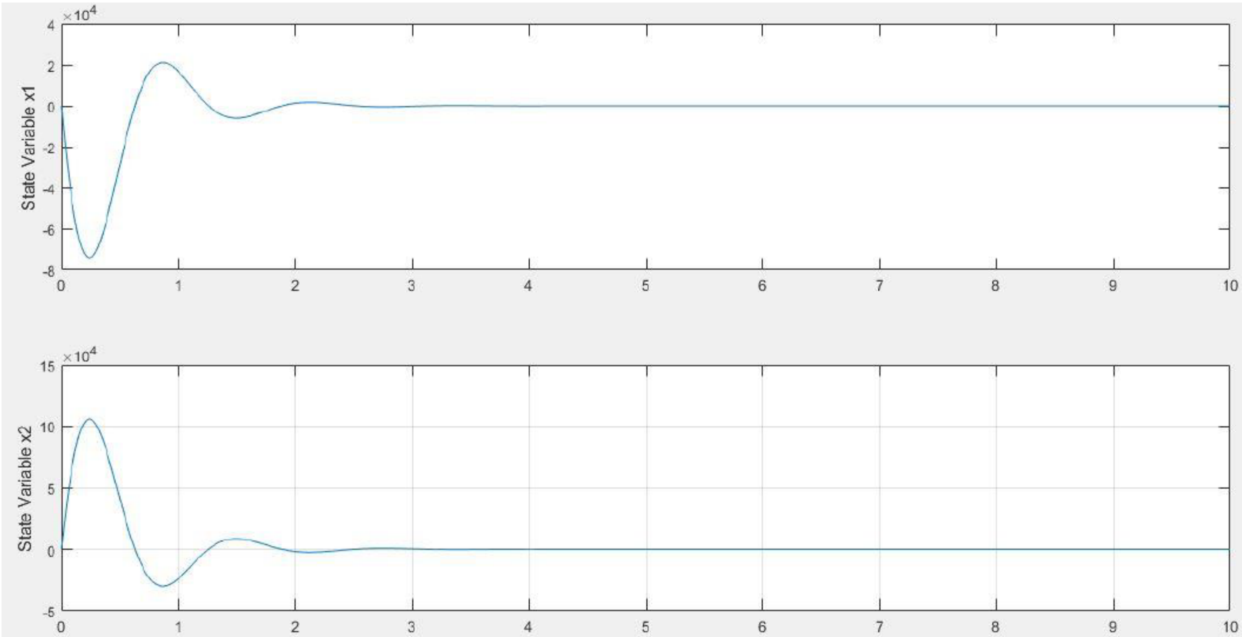

Fig. 15: Controller responses for poles at ***-2 ± 5j****.*

The complete analysis in terms of response and stabilization time of x1 and x2 can be summarized in Table III.

Table III: *Analysis of Biomass (x1) and Substrate (x2) concentration responses from Linear Feedback controller and Mamdani Fuzzy controller for Bioreactor system*

Stabilization time for x1 & x2			
	Linear Feedback Controller		Mamdani Fuzzy Controller
	Poles at -2 ± 0.3j	**Poles at -2 ± 5j**	
X1 (Biomass Concentration)	3.2 s	2.5 s	No response
X2 (Substrate Concentration)	3.2 s	2.5 s	4.9 s

5. Conclusion

In this assignment, an appropriate nonlinear controller was chosen for the Bioreactor system with the help of Lie Derivation. Fuzzy logic controller of Mamdani type was designed for the system using MATLAB Fuzzy Controller Designer app. A Rule Base was created based on user experience approach. The Fuzzy controller combined with the Bioreactor system provided response graphs for the two inputs- Biomass concentration (x1) and Substrate concentration (x2). A comparison of the responses was carried out against the responses obtained during assignment 1 for Linear Feedback controller with poles at -2 ± 0.3j and -2 ± 5j. The Bioreactor system stabilizes at 2.5 s with a Linear Feedback controller with poles at -2 ±

5j, and at 4.9 s with a Mamdani Fuzzy controller. Moreover, the Fuzzy logic controller provides no response for the Biomass concentration (x1).
Based on this analysis, it can be concluded that the Linear Feedback controller provides better performance for the Bioreactor system than the Mamdani Fuzzy controller.

6. References

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