Fachhochschule Südwestfalen
University of Applied Sciences

Advanced Control Technology

SoSe 2019

Bioreactor System (5)

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Objectives

- To build a model of the system in Matlab/Simulink.
- Linearize the system model in a suitable operation point.
- Analyze the system behavior, i.e. stability, controllability, observability.
- Design a state space controller by pole placement approach.
- Analyze obtained results.

Bioreactor

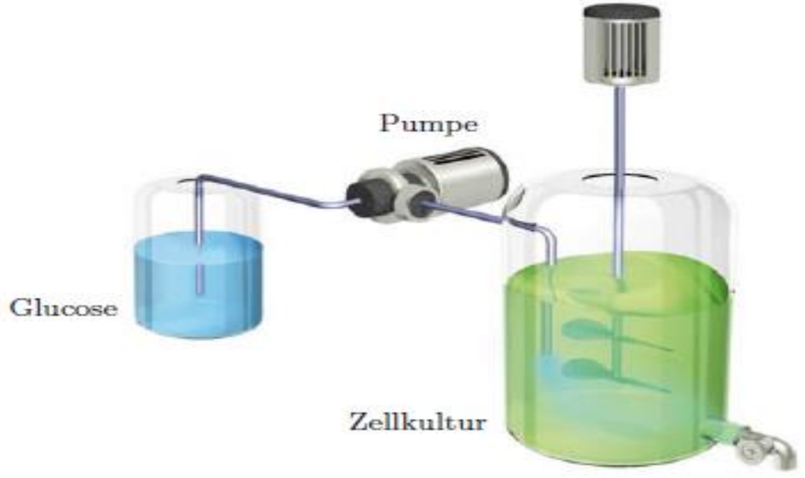

Fig. 1: Bioreactor system

The Bioreactor system can be modeled by the following state space equation:

$$\dot{\boldsymbol{x}} = \boldsymbol{a}(\boldsymbol{x}) + \boldsymbol{b}(\boldsymbol{x}) \cdot u = \begin{bmatrix} \mu(x_2) \cdot x_1 \\ -\frac{1}{\alpha}\mu(x_2) \cdot x_1 \end{bmatrix} + \begin{bmatrix} -x_1 \\ K - x_2 \end{bmatrix} u,$$

$$y = g(\boldsymbol{x}) = \begin{bmatrix} 1 & 0 \end{bmatrix} \boldsymbol{x}\,.$$

Where, $\mu(x_2) = \dfrac{\mu_0 x_2}{k_1 + x_2 + k_2 x_2^2}$

Where:
Maximal growth rate, $\mu_0 = 2$
Affinity constant, $k_1 = 0.06$
Affinity constant, $k_2 = 0.3$
Feed concentration of glucose, K = 2
Yield constant, $\alpha = 0.7$

Concentration of **biomass** =**x1**
Concentration of **substrate** = **x2**

Linearization of the System Model in a Suitable Operation Point

By finding suitable operating points, the system can be linearized

$$\dot{\boldsymbol{x}}\,(t) = \boldsymbol{A}x(t) + \boldsymbol{B}u(t)$$
$$\boldsymbol{y}(t) = \boldsymbol{C}x(t) + \boldsymbol{D}u(t)$$

Where,
A is the state matrix,
B is the input matrix,
C is the output matrix,
D is the direct transmission matrix

- By taking the system dynamics as zero, the operating points are calculated
- The input value of u is arbitrarily chosen equal to 1.3

```
syms x1 x2
u0 = 2;
u = 1.3;
k1 = 0.06;
k2 = 0.3;
k = 2;
alpha = 0.7;

fn1 = ((u0*x1*x2)/(k1+x2+k2*(x2^2))) - (x1*u);
fn2 = (-(u0*x1*x2)/(alpha*(k1+x2+k2*(x2^2)))) + ((k-x2)*u);
[solx1, solx2] = solve([fn1 == 0, fn2 == 0], [x1,x2], 'real', true);
x1_roots = zeros;
x1_roots = vpa(solx1);
x2_roots = zeros;
x2_roots = vpa(solx2);
x1_value = x1_roots(end);
x2_value = x2_roots(end);
fprintf('The operating points are x1 = %f, x2 = %f\n', x1_value, x2_value);
```

The values are obtained as follows:

x1= 0.22714
x2 =1.675505

Linearization of the System Model in a Suitable Operation Point

With the help of Taylor´s Series we can linearize the non-linear model around the operating points

$$A=\begin{bmatrix} \left.\frac{\partial f_1}{\partial x_1}\right|_{x_1,x_2,u} & \left.\frac{\partial f_1}{\partial x_2}\right|_{x_1,x_2,u} \\ \left.\frac{\partial f_2}{\partial x_1}\right|_{x_1,x_2,u} & \left.\frac{\partial f_2}{\partial x_2}\right|_{x_1,x_2,u} \end{bmatrix}$$

$$A=\begin{bmatrix} \frac{\mu_0\, x_2}{k_1+x_2+k_2 x^2{}_2}-u & \frac{(k_1+x_2+k_2 x^2{}_2)\mu_o\, x_1-\mu_o\, x_1\, x_2(1+2k_2 x_2)}{(k_1+x_2+k_2 x^2{}_2)^2} \\ \frac{-1}{\alpha}\,\frac{\mu_0\, x_2}{k_1+x_2+k_2 x^2{}_2} & \frac{-\alpha(k_1+x_2+k_2 x^2{}_2)\mu_o\, x_1-(-\mu_o\, x_1\, x_2)(\alpha+\alpha 2k_2 x_2)}{(k_1+x_2+k_2 x^2{}_2)^2\alpha^2}-u \end{bmatrix}$$

$$B=\begin{bmatrix} -x_1 \\ k-x_2 \end{bmatrix}$$

C= [1 0]

D=0

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix}=\begin{bmatrix} -5.1193 * 10^{-8} & -0.0535 \\ -1.8571 & -1.2236 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}+\begin{bmatrix} -0.227147 \\ x_2\, 0.324495 \end{bmatrix}u$$

$$y=[1\ \ 0]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}+0$$

The linear state space representation

Simulink Model of Bioreactor System

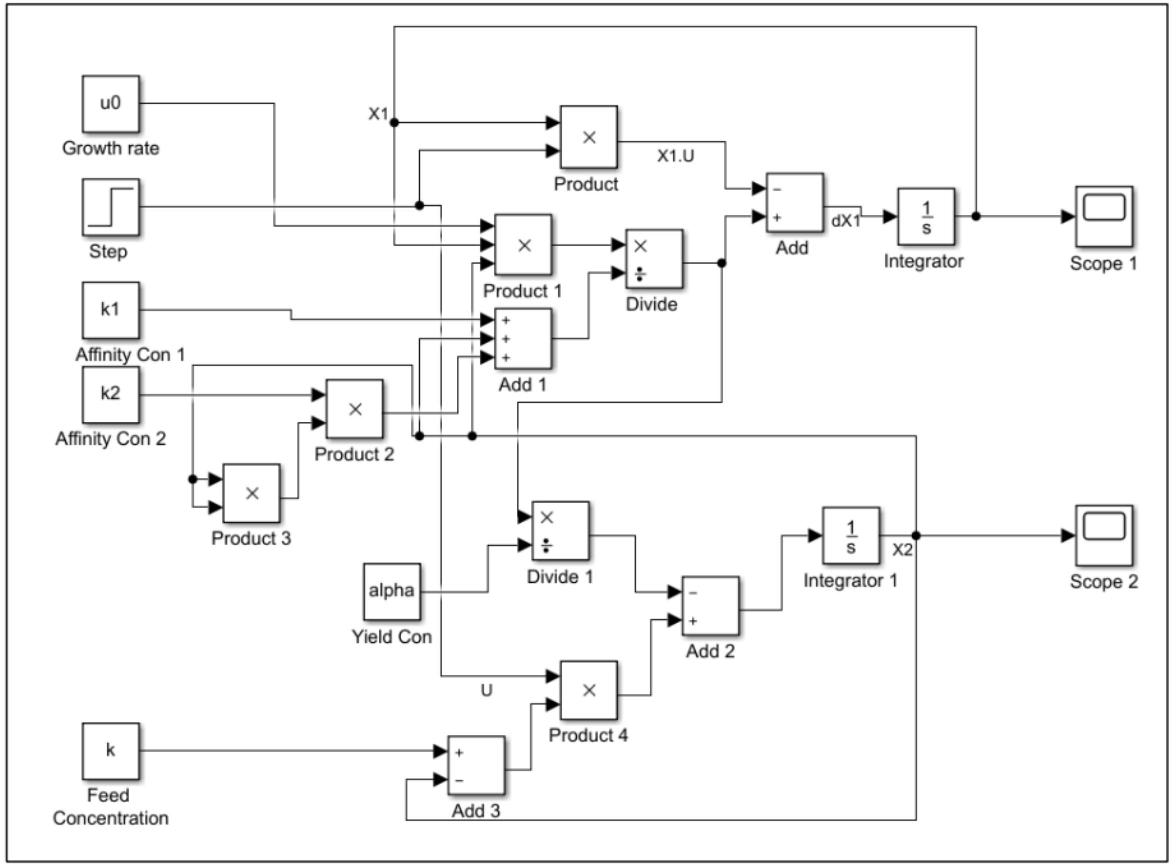

Fig. 2: Nonlinear model of the Bioreactor system in Simulink

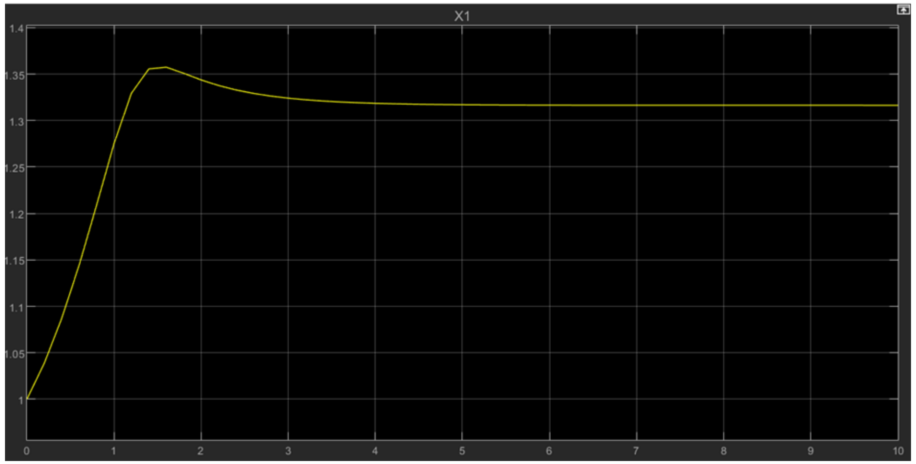

Fig. 3: Output graph of scope 1 (Value of x1)

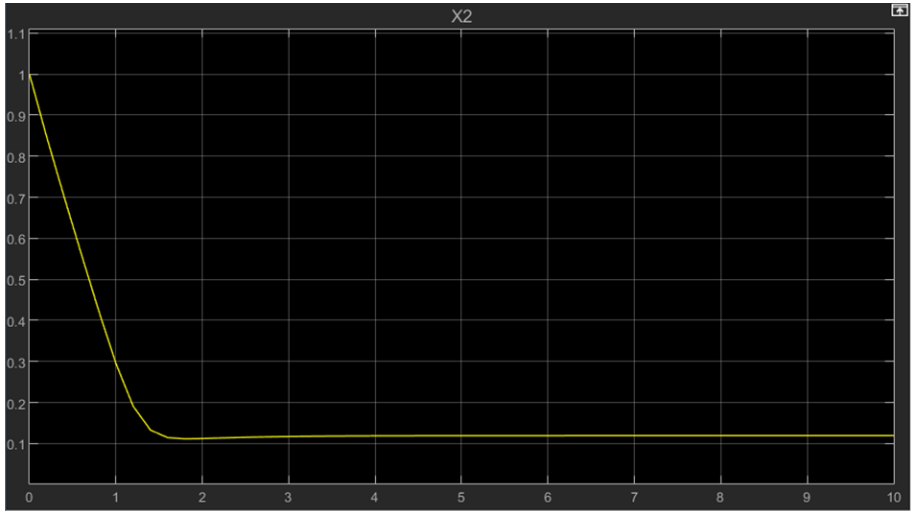

Fig. 4: Output graph of scope 2 (Value of x2)

- Stability

We compute the eigen values of A matrix

If : Eigen values are both negative → stable
Else : the system→ unstable

the eigenvalues of the matrix **A** are determined using the MATLAB script

```
A = [-5.1193e-08 -0.0535; -1.8571 -1.2236];
B = [-0.227147; 0.324495];
C = [1 0];
D = 0;

e=eig(A)                    %For Stability
```

The eigenvalues obtained are

```
e =

    0.0764
   -1.3000
```

This means the system is **unstable**

Analysis of the System Behavior (Contd.)

- Controllability

we need to find the rank of the Controllability matrix Cm which is defined by

$$Cm = [B \mid AB]$$

If : rank of Cm = length of matrix **A** → controllable
Else : not controllable

```
Con_mat = ctrb(A,B);     %For Controllability
length(A);
rank(Con_mat);
fprintf('Length of Matrix A = %d\n', length(A));
fprintf('Rank of Controllability Matrix = %d\n', rank(Con_mat));
```

The result obtained are as follows:

```
Length of Matrix A = 2
Rank of Controllability Matrix = 2
```

Hence, the system is **controllable**.

Analysis of the System Behavior (Contd.)

- Observability

We need to find the rank of the Observability matrix Om,

$$Om = [C_* \mid A_* C_*]$$

If : rank of Om = rank of matrix **A** → observable
Else : not observable

```
Obs=obsv(A,C);              %For Observability
rank(A);
rank(Obs);
fprintf('Rank of Matrix A = %d\n', rank(A));
fprintf('Rank of Observability Matrix = %d\n', rank(Obs));
```

The result obtained are as follows:

```
Rank of Matrix A = 2
Rank of Observability Matrix = 2
```

Hence, the system is **observable**

To sum up, the bioreactor system is **unstable** but **controllable** and **observable**

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- Design of State Space Controller using Pole Placement Approach

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}x(t) + \boldsymbol{B}u(t)$$
$$\boldsymbol{y}(t) = \boldsymbol{C}x(t) + \boldsymbol{D}u(t)$$

control signal , u= -**K**x

K = State feedback gain matrix

The feedback gain matrix **K** is determined for various poles using MATLAB

*For Poles, **Pc*** **= -2+0.3j, -2-0.3j**

```
Pc = [-2+j*0.3 -2-j*0.3];     %POLES
K = place(A,B,Pc)            %Feedback Gain Matrix
```

The **K** matrix is, **K** = [-4.6024 -3.2208]

Similarly,

*for Poles, **Pc*** **= -2+5j, -2-5j**

The **K** matrix is, **K** = [-2.0223 -1.4156]

Design of State Space Controller (Contd.)

- Design of State Space Controller using Pole Placement Approach

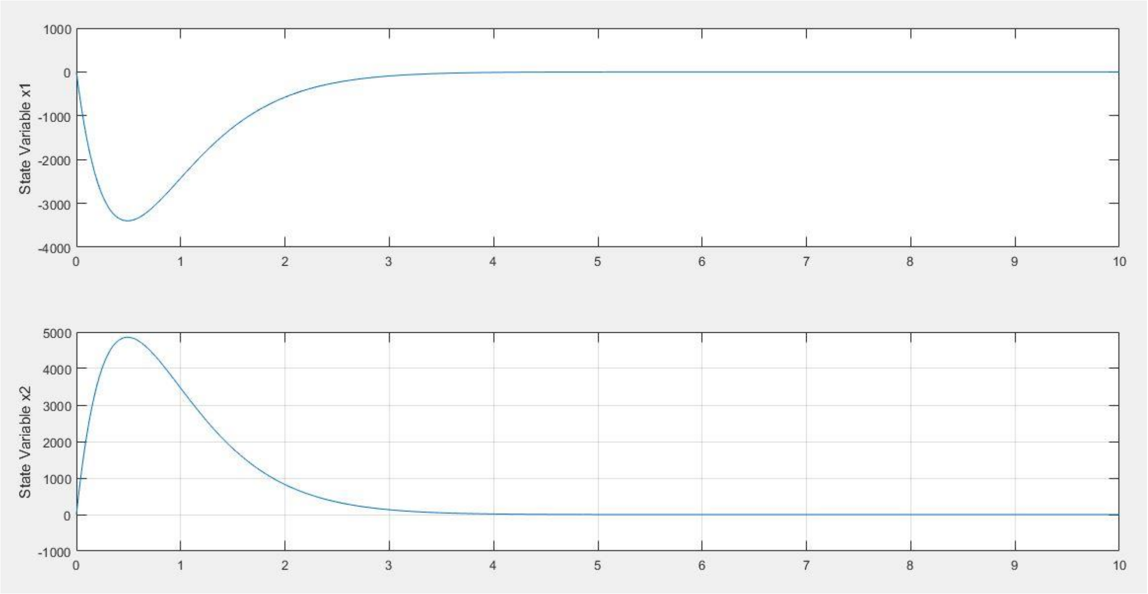

Fig. 5: Controller Response for Poles -2+0.3j, -2-0.3j

steady state at approx. 3.2 sec

Poles at 2+0.3j, -2-0.3j are selected for designing the state space controller.

steady state at approx. 2.5 sec

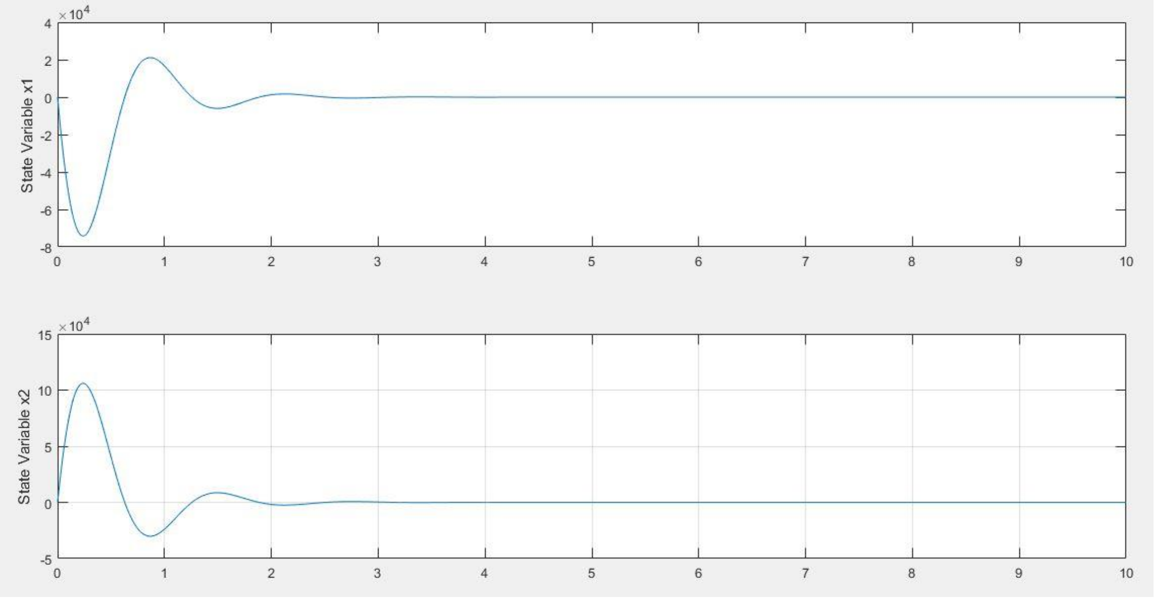

Fig. 6: Controller Response for Poles -2+5j, -2-5j.

Design of State Space Observer

- Design of State Space Observer using Pole Placement Approach

K=state feedback gain matrix
Ko= observer gain matrix

At first, the observer feedback gain matrix is determined for two arbitrarily chosen set of poles using the script

For Poles, P_o = -10, -11

```
Po = [-10 -11];
Ko = place(A',C',Po)'
```

$$\mathbf{K_o} = \begin{bmatrix} 0.0198 \\ -1.6056 \end{bmatrix}$$

Similarly for *Poles, P_o* = -5, -6

$$\mathbf{K_o} = \begin{bmatrix} 9.7764 \\ -339.0084 \end{bmatrix}$$

Design of State Space Observer (Contd.)

- Initial response of the observer for different pole position

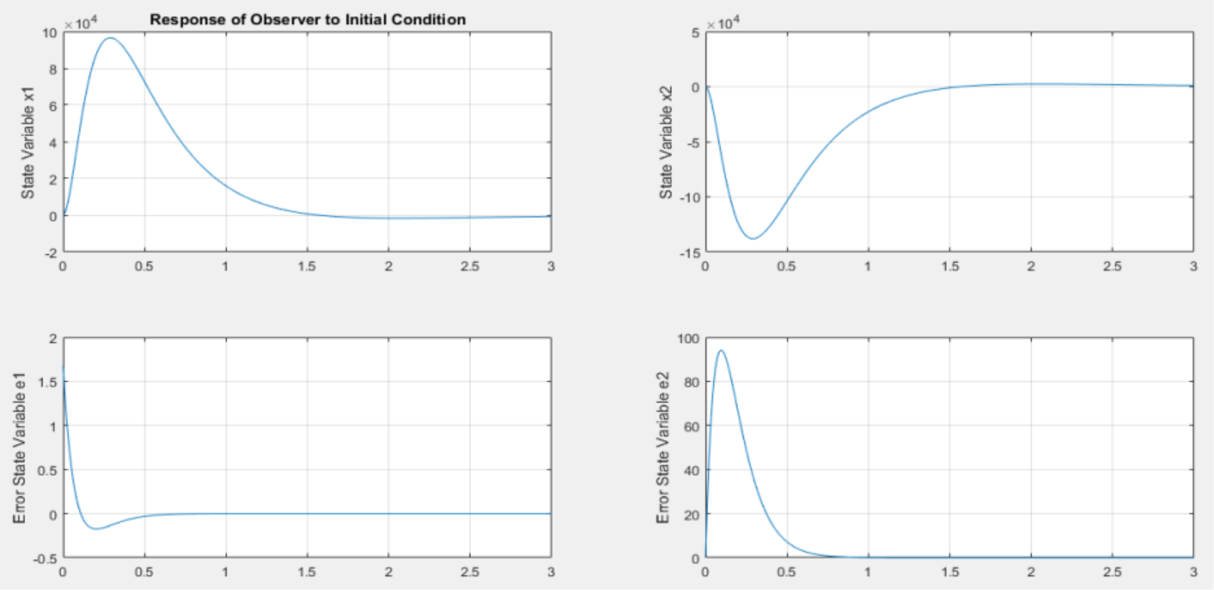

Fig. 7: Observer Response to Poles at -10, -11.

stabilize at approx. 1.5 sec

stabilize at approx. 2 sec

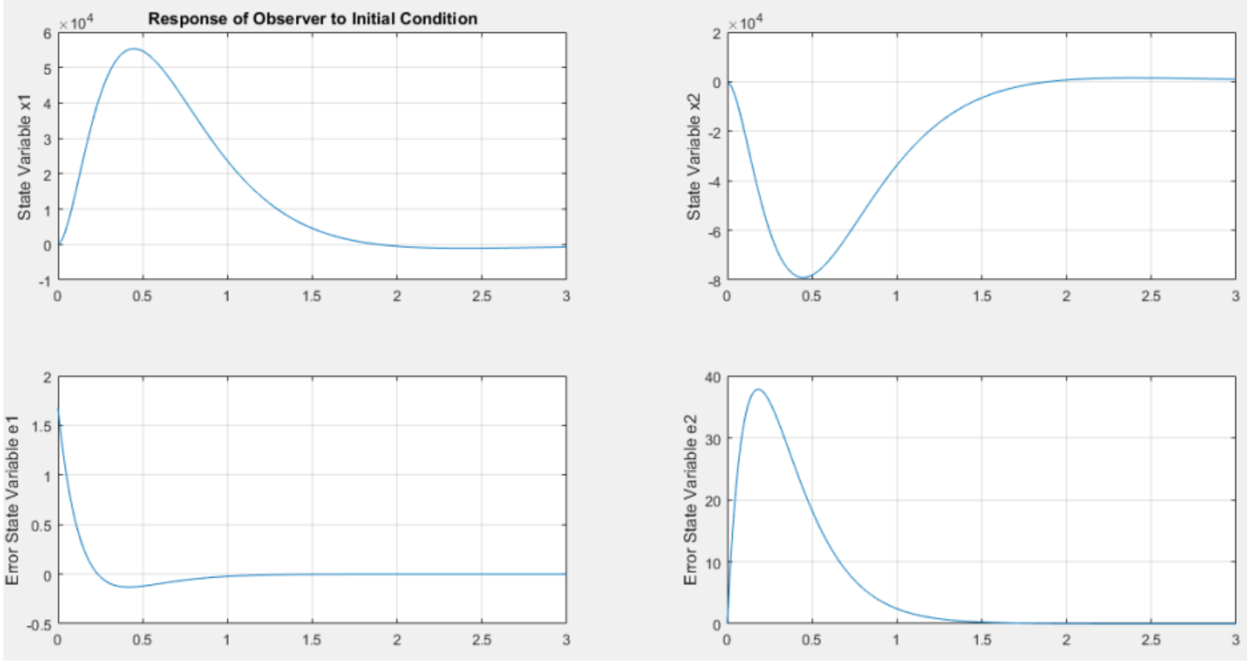

Fig. 8: Observer Response to Poles at -5, -6.

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THANK YOU!