Trigonometric Ratios

Cheat Sheet

Version 1.0

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Trigonometric Values

	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$\sin\theta$	0	$1/2$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\ \theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$1/2$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞
$\csc\theta$	∞	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
$\sec\theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞
$\cot\theta$	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

Domain

$$\sin\theta : \theta \in \mathbb{R}$$
$$\cos\theta : \theta \in \mathbb{R}$$
$$\tan\theta : \theta \in \mathbb{R} - (n + \frac{1}{2})\pi,\ where\ n \in \mathbb{Z}$$
$$\sec\theta : \theta \in \mathbb{R} - (n + \frac{1}{2})\pi,\ where\ n \in \mathbb{Z}$$
$$\csc\theta : \theta \in \mathbb{R} - n\pi,\ where\ n \in \mathbb{Z}$$
$$\cot\theta : \theta \in \mathbb{R} - n\pi,\ where\ n \in \mathbb{Z}$$

Conversions

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$$
$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$$
$$\csc\left(\frac{\pi}{2} - \theta\right) = \sec\theta$$
$$\sec\left(\frac{\pi}{2} - \theta\right) = \csc\theta$$
$$\tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta$$
$$\cot\left(\frac{\pi}{2} - \theta\right) = \tan\theta$$

Range

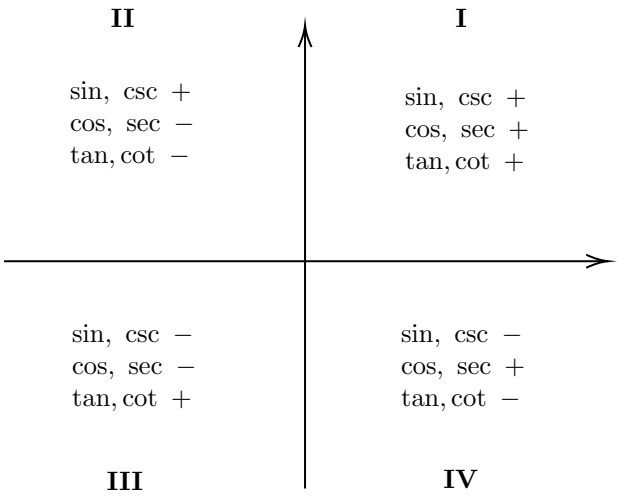
$$\sin\theta \in [-1, 1]$$
$$\cos\theta \in [-1, 1]$$
$$\tan\theta \in \mathbb{R}$$
$$\sec\theta \in (-\infty, -1] \cup [1, \infty)$$
$$\csc\theta \in (-\infty, -1] \cup [1, \infty)$$
$$\cot\theta \in \mathbb{R}$$

In any trigonometric function, if an odd multiple of $\frac{\pi}{2}$ is added to the angle. The following conversion takes place:

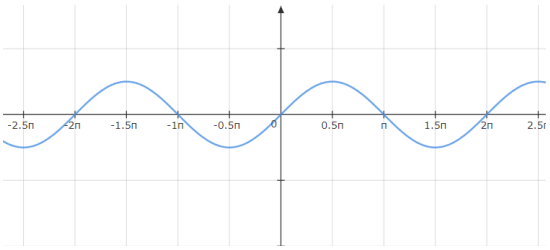
$$\sin \leftrightarrow \cos$$
$$\csc \leftrightarrow \sec$$
$$\tan \leftrightarrow \cot$$

The sign depends on the initial trigonometric function to which odd multiple of $\frac{\pi}{2}$ is added.

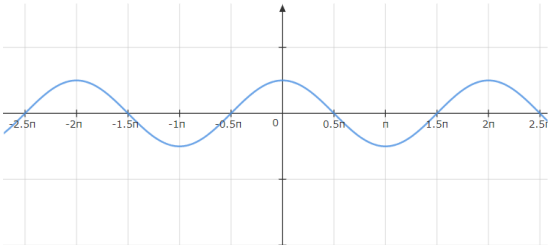
Sign of Trigonometric Ratios in Different Quadrants



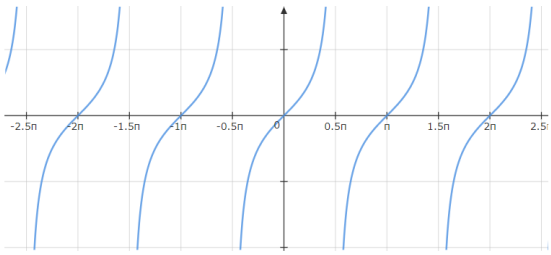
Graphs



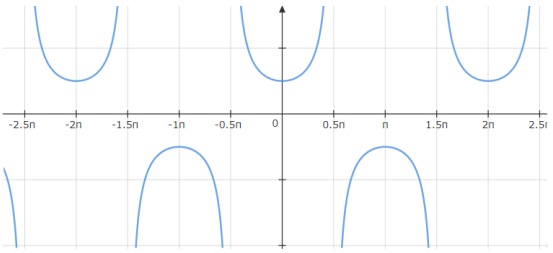
$\sin\theta$



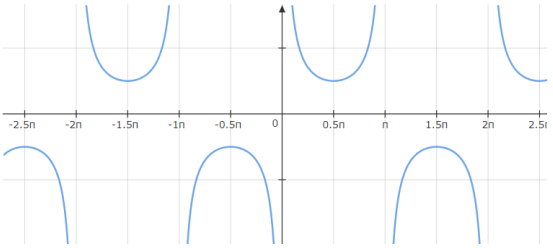
$\cos\theta$



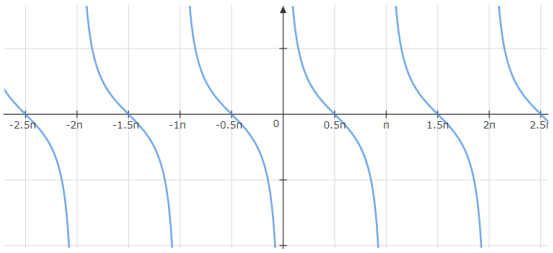
$\tan\theta$



$\sec\theta$



$\csc\theta$



$\cot\theta$

Standard Identities

$$\sin^2\theta + \cos^2\theta = 1$$
$$1 + \tan^2\theta = \sec^2\theta$$
$$1 + \cot^2\theta = \csc^2\theta$$

Tangent And Cotangent

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$
$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

Reciprocals

$$\sin\theta = \frac{1}{\csc\theta} \qquad \csc\theta = \frac{1}{\sin\theta}$$
$$\cos\theta = \frac{1}{\sec\theta} \qquad \sec\theta = \frac{1}{\cos\theta}$$
$$\tan\theta = \frac{1}{\cot\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

Odd Functions

$$\sin(-\theta) = -\sin(\theta)$$
$$\csc(-\theta) = -\csc(\theta)$$
$$\tan(-\theta) = -\tan(\theta)$$
$$\cot(-\theta) = -\cot(\theta)$$

Even Functions

$$\cos(-\theta) = \cos(\theta)$$
$$\sec(-\theta) = \sec(\theta)$$

Periodic Functions

For all $n \in \mathbb{Z}$:

$$\sin(\theta + 2\pi n) = \sin(\theta)$$
$$\csc(\theta + 2\pi n) = \csc(\theta)$$
$$\cos(\theta + 2\pi n) = \cos(\theta)$$
$$\sec(\theta + 2\pi n) = \sec(\theta)$$
$$\tan(\theta + \pi n) = \tan(\theta)$$
$$\cot(\theta + \pi n) = \cot(\theta)$$

Half Angle Formulas

$$\sin^2\theta = \frac{1 - \cos(2\theta)}{2}$$
$$\cos^2\theta = \frac{1 + \cos(2\theta)}{2}$$
$$\tan^2\theta = \frac{1 - \cos(2\theta)}{1 + \cos(2\theta)}$$

Double Angle Formulas

$$\begin{aligned}
\sin(2\theta) &= 2\sin\theta\cos\theta \\
&= \frac{2\tan\theta}{1 + \tan^2\theta} \\
&= \frac{2\cot\theta}{1 + \cot^2\theta} \\
&= (\sin\theta + \cos\theta - 1)(\sin\theta + \cos\theta + 1) \\
\cos(2\theta) &= \cos^2\theta - sin^2\theta \\
&= 2\cos^2\theta - 1 \\
&= 1 - 2\sin^2\theta \\
&= \frac{1 - \tan^2\theta}{1 + \tan^2\theta} \\
\tan(2\theta) &= \frac{2\tan\theta}{1 - \tan^2\theta} \\
\sec(2\theta) &= \frac{\sec^2\theta}{2 - \sec^2\theta} \\
\csc(2\theta) &= \frac{\csc\theta\sec\theta}{2} \\
\cot(2\theta) &= \frac{\cot^2\theta - 1}{2\cot\theta}
\end{aligned}$$

Triple Angle Formulas

$$\sin(3\theta) = 3\sin\theta - 4\sin^3\theta$$
$$\cos(3\theta) = 4\cos^3\theta - 3\cos\theta$$
$$\tan(3\theta) = \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$$

Sum/Difference of Two Angles

$$\sin(a \pm b) = \sin a\cos b \pm \cos a\sin b$$
$$\cos(a \pm b) = \cos a\cos b \mp \sin a\sin b$$
$$\tan(a \pm b) = \frac{\tan a \pm \tan b}{1 \mp \tan a\tan b}$$
$$\cot(a \pm b) = \frac{\cot a\cot b \mp 1}{\cot b \pm \cot a}$$

Sum of Three Angles

$$\begin{aligned}
\sin(a+b+c) = {} & \sin a\cos b\cos c + \cos a\sin b\cos c + \\
& \cos a\cos b\sin c - \sin a\sin b\sin c \\
\cos(a+b+c) = {} & \cos a\cos b\cos c - \cos a\sin b\sin c - \\
& \sin a\cos b\sin c - \sin a\sin b\cos c \\
\tan(a+b+c) = {} & (\tan a + \tan b + \tan c - \tan a\tan b \\
& - \tan b\tan c - \tan a\tan c)/(1 - \tan a\tan b\tan c)
\end{aligned}$$

Sum and Difference of Trigonometric Functions

$$\cos a + \cos b = 2\cos\frac{a+b}{2}\cos\frac{a-b}{2}$$
$$\cos a - \cos b = 2\sin\frac{a+b}{2}\sin\frac{b-a}{2}$$
$$\sin a + \sin b = 2\sin\frac{a+b}{2}\cos\frac{a-b}{2}$$
$$\sin a - \sin b = 2\sin\frac{a-b}{2}\cos\frac{a+b}{2}$$

Some Derived Results

$$\begin{aligned}\sin(a+b)\sin(a-b) &= \sin^2 a - \sin^2 b \\ &= \cos^2 b - \cos^2 a \\ \cos(a+b)\cos(a-b) &= \cos^2 a - \sin^2 b \\ &= \cos^2 b - \sin^2 a\end{aligned}$$

Some Manipulations

$$\begin{aligned}a\sin x + b\cos x &= \sqrt{a^2+b^2}\ \sin(x+\theta) \\ &= \sqrt{a^2+b^2}\ \cos(x-\omega)\end{aligned}$$

where $\theta = \arctan\left(\frac{b}{a}\right)$, $\omega = \arctan\left(\frac{a}{b}\right)$

$$\cos(x)\cos(2x)\ldots\cos(2^{n-1}x) = \frac{\sin(2^n x)}{2^n\sin(x)}$$

Conditional Identities

Given $a+b+c=\pi$,

$$\sin 2a + \sin 2b + \sin 2c = 4\sin a\sin b\sin c$$
$$\cos 2a + \cos 2b + \cos 2c = -1 - 4\cos a\cos b\cos c$$
$$\sin a + \sin b + \sin c = 4\cos\frac{a}{2}\cos\frac{b}{2}\cos\frac{c}{2}$$
$$\cos a + \cos b + \cos c = 1 + 4\sin\frac{a}{2}\sin\frac{b}{2}\sin\frac{c}{2}$$
$$\tan a + \tan b + \tan c = \tan a\tan b\tan c$$

Angles in AP

$$\sin(a) + \sin(a+b) + \ldots + \sin(a+(n-1)b) = \frac{\sin\frac{nb}{2}}{\sin\frac{b}{2}}\sin\left(a+\frac{(n-1)b}{2}\right)$$

$$\cos(a) + \cos(a+b) + \ldots + \cos(a+(n-1)b) = \frac{\sin\frac{nb}{2}}{\sin\frac{b}{2}}\cos\left(a+\frac{(n-1)b}{2}\right)$$

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