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JEE Advanced 2019 - Paper 1 - Physics 07

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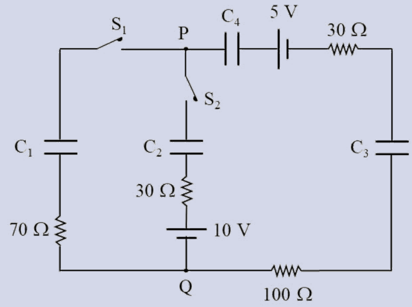
Problem *[Multiple Choice Multiple Correct]*

In the circuit shown, initially there is no charge on capacitors and keys S_1 and S_2 are open. The values of the capacitors are $C_1 = 10\,\mu\text{F}$, $C_2 = 30\,\mu\text{F}$ and $C_3 = C_4 = 80\,\mu\text{F}$. Which of the statement(s) is/are correct.



(A) At time $t = 0$, the key S_1 is closed, the instantaneous current in the closed circuit will be 25 mA.
(B) If key S_1 is kept closed for long time such that capacitors are fully charged, the voltage across the capacitor C_1 will be 4 V.
(C) The key S_1 is kept closed for long time such that capacitors are fully charged. Now key S_2 is closed, at this time, the instantaneous current across 30 Ω resistor (between points P and Q) will be 0.2 A (round off to 1^{st} decimal place).
(D) If key S_1 is kept closed for long time such that capacitors are fully charged, the voltage difference between point P and Q will be 10 V.

What to Observe:

- Initially, all capacitors are uncharged.
- Switches S_1 and S_2 are both open at the start.
- Capacitor values:
  - $C_1 = 10\,\mu\text{F}$
  - $C_2 = 30\,\mu\text{F}$
  - $C_3 = 80\,\mu\text{F}$
  - $C_4 = 80\,\mu\text{F}$

My Approach:

Dissecting Option A

Thought

Since the capacitors are initially uncharged, closing switch S_1 allows current to flow without hindrance. The current can be calculated using Ohm's Law.

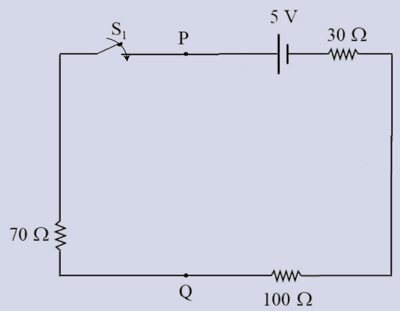


Figure 1. Circuit Diagram for Option A

Given:

- Voltage source: $V = 5\,\text{V}$

- Resistors in series: $R_{eq} = 30\,\Omega + 70\,\Omega + 100\,\Omega = 200\,\Omega$

Using Ohm's Law:

$$I = \frac{V}{R_{eq}} = \frac{5\,\mathrm{V}}{200\,\Omega} = 0.025\,\mathrm{A} = 25\,\mathrm{mA}$$

Therefore, **Option A is correct**.

Dissecting Option B and D

Thought

After closing switch S_1 and waiting a long time, the capacitors reach a steady state where no current flows through them. In this state, the capacitor acts as an open circuit.
Capacitors C_1, C_3, and C_4 are connected in series, so they will all hold the same charge. We can replace them with an equivalent series capacitor.
The equivalent capacitance for capacitors in series is given by:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_3} + \frac{1}{C_4}$$

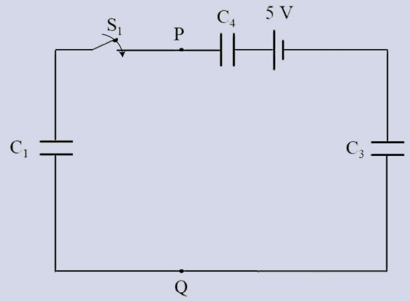


Figure 2. Circuit Diagram for Option B and D

Given:

- $C_1 = 10\,\mu\mathrm{F}$, $C_3 = 80\,\mu\mathrm{F}$, $C_4 = 80\,\mu\mathrm{F}$
- Voltage source: $V = 5\,\mathrm{V}$

Calculating equivalent capacitance:

$$\frac{1}{C_{eq}} = \frac{1}{10} + \frac{1}{80} + \frac{1}{80} = \frac{1}{10} + \frac{2}{80} = \frac{1}{10} + \frac{1}{40} = \frac{4+1}{40} = \frac{5}{40}$$

$$C_{eq} = \frac{40}{5} = 8\,\mu\mathrm{F}$$

Charge on the equivalent capacitor:

$$Q = C_{eq} \cdot V = 8\,\mu\mathrm{F} \cdot 5\,\mathrm{V} = 40\,\mu\mathrm{C}$$

Since capacitors in series share the same charge, $Q_{C_1} = 40\,\mu\mathrm{C}$.
Voltage across C_1:

$$V_{C_1} = \frac{Q}{C_1} = \frac{40\,\mu\mathrm{C}}{10\,\mu\mathrm{F}} = 4\,\mathrm{V}$$

Therefore, **Option B is correct**.

It is evident that the potential difference across points P and Q is equal to the voltage across capacitor C_1 in Option D.
Therefore, **Option D is incorrect**.

Dissecting Option C

Thought

Since switch S_1 has been closed for a long time, the capacitors connected to it are fully charged. In this steady state, the capacitors behave like voltage sources, similar to the analysis in Option B.
Additionally, capacitor C_2 is initially uncharged, and immediately after closing switch S_2, it does not affect the circuit. Therefore, it can be ignored in the initial analysis.

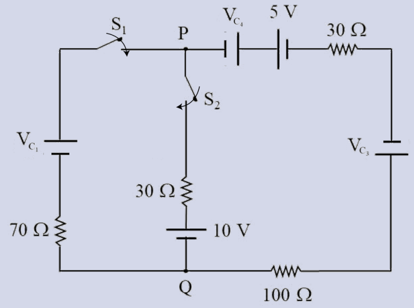


Figure 3. Circuit Diagram for Option C

We are given:

- $C_1 = 10\,\mu\text{F}, C_3 = 80\,\mu\text{F}, C_4 = 80\,\mu\text{F}$
- Charge on each capacitor: $Q = 40\,\mu\text{C}$ (as calculated previously)

Using the formula $V = \frac{Q}{C}$, we compute:

$$V_{C_1} = \frac{40\,\mu\text{C}}{10\,\mu\text{F}} = 4\,\text{V}$$
$$V_{C_3} = \frac{40\,\mu\text{C}}{80\,\mu\text{F}} = 0.5\,\text{V}$$
$$V_{C_4} = \frac{40\,\mu\text{C}}{80\,\mu\text{F}} = 0.5\,\text{V}$$

Proceed by applying Kirchhoff's Voltage Law (KVL) to analyze the left and right loops:

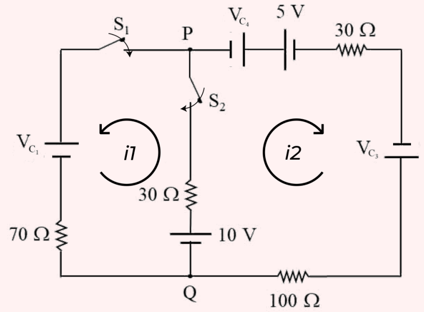


Figure 4. Circuit Diagram for KVL

Let the total current be (passing through 30 Ω resistor):
$$i = i_1 + i_2$$

Applying KVL in the left loop (starting and ending at point Q):

$$\Rightarrow 10 - 30(i_1 + i_2) - V_{C_1} - 70i_1 = 0$$
$$\Rightarrow 10 - 30(i_1 + i_2) - 4 - 70i_1 = 0$$
$$\Rightarrow 6 - 30(i_1 + i_2) - 70i_1 = 0$$
$$\Rightarrow 6 - 30i_1 - 30i_2 - 70i_1 = 0$$
$$\Rightarrow 6 - 100i_1 - 30i_2 = 0 \tag{1}$$

Applying KVL in the right loop (starting and ending at point Q):

$$\Rightarrow 10 - 30(i_1 + i_2) + V_{C_4} - 5 - 30i_2 + V_{C_3} - 100i_2 = 0$$
$$\Rightarrow 10 - 30(i_1 + i_2) + 0.5 - 5 - 30i_2 + 0.5 - 100i_2 = 0$$
$$\Rightarrow 6 - 30(i_1 + i_2) - 130i_2 = 0$$
$$\Rightarrow 6 - 30i_1 - 30i_2 - 130i_2 = 0$$
$$\Rightarrow 6 - 30i_1 - 160i_2 = 0 \tag{2}$$

Now solve equations (1) and (2) simultaneously:

$$(1): \quad 100i_1 + 30i_2 = 6$$
$$(2): \quad 30i_1 + 160i_2 = 6$$

We get:

$$i_1 = 0.05166\,\text{A}, \quad i_2 = 0.02781\,\text{A}$$

Therefore, the total current is:

$$i = i_1 + i_2 = 0.05166 + 0.02781 = 0.07947\,\text{A}$$

Therefore, **Option C is incorrect**.

Conclusion

Based on the detailed circuit analysis and calculated values:

$$\boxed{\text{Options A and B are correct.}}$$

Options C and D are incorrect, as their circuit behavior and resulting current or voltage values do not match the expected outcomes based on the given configurations.

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