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JEE Advanced 2019 - Paper 1 - Physics 13

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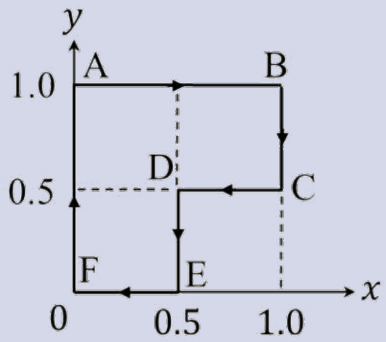
Problem ***[Numerical Value]***

A particle is moved along a path AB − BC − CD − DE − EF − FA, as shown in figure, in presence of a force $\vec{F} = (\alpha y\hat{i} + 2\alpha x\hat{j})$N, where x and y are in meter and $\alpha = -1\text{Nm}^{-1}$. The work done on the particle by this force $\vec{F}$ will be _____ Joule.



What to Observe:

- The particle moves along the path AB − BC − CD − DE − EF − FA.
- The force acting on the particle is $\vec{F} = (\alpha y\hat{i} + 2\alpha x\hat{j})$ N.
- The coordinates x and y are in meters.
- The constant α is given as -1 Nm^{-1}.

My Approach:

Definition of Work Done

Thought

We are given that the work done is defined as:

$$W = \int \vec{F} \cdot d\vec{l}$$

The force vector is:

$$\vec{F} = (\alpha y\hat{i} + 2\alpha x\hat{j})\,\text{N}$$

The displacement vector can be expressed differentially as:

$$d\vec{l} = dx\,\hat{i} + dy\,\hat{j}$$

Substituting into the work formula:

$$W = \int (\alpha y\,dx + 2\alpha x\,dy)$$

The motion occurs along a piecewise linear path: AB → BC → CD → DE → EF → FA. Since each segment of the path involves motion along either the x-axis or the y-axis (but not both simultaneously), we can evaluate the line integral over each segment individually.
On each segment, either $dx = 0$ or $dy = 0$, which allows us to treat x or y as constants within each integral. Therefore, for each segment:

$$W_{\text{segment}} = \int (\alpha y\,dx) + \int (2\alpha x\,dy)$$

$$W_{\text{segment}} = \alpha y\,\Delta x + 2\alpha x\,\Delta y$$

Where Δx and Δy are the changes in x and y coordinates along the segment.
We compute this for each segment and sum the contributions:

$$W = W_{AB} + W_{BC} + W_{CD} + W_{DE} + W_{EF} + W_{FA}$$

Calculating Work Done on Each Segment

We evaluate the work done along each segment using:

$$W_{\text{segment}} = \alpha y\,\Delta x + 2\alpha x\,\Delta y$$

Segment AB:

$$\Delta x = 1, \quad \Delta y = 0, \quad y = 1$$
$$W_{AB} = \alpha \cdot 1 \cdot 1 + 0 = \alpha$$

Segment BC:

$$\Delta x = 0, \quad \Delta y = -0.5, \quad x = 1$$
$$W_{BC} = 0 + 2\alpha \cdot 1 \cdot (-0.5) = -\alpha$$

Segment CD:

$$\Delta x = -0.5, \quad \Delta y = 0, \quad y = 0.5$$
$$W_{CD} = \alpha \cdot 0.5 \cdot (-0.5) + 0 = -0.25\alpha$$

Segment DE:

$$\Delta x = 0, \quad \Delta y = -0.5, \quad x = 0.5$$
$$W_{DE} = 0 + 2\alpha \cdot 0.5 \cdot (-0.5) = -0.5\alpha$$

Segment EF:

$$\Delta x = -0.5, \quad \Delta y = 0, \quad y = 0$$
$$W_{EF} = \alpha \cdot 0 \cdot (-0.5) + 0 = 0$$

Segment FA:

$$\Delta x = 0, \quad \Delta y = 1, \quad x = 0$$
$$W_{FA} = \alpha \cdot y \cdot 0 + 0 = 0$$

Total Work Done:

$$\begin{aligned} W_{\text{total}} &= W_{AB} + W_{BC} + W_{CD} + W_{DE} + W_{EF} + W_{FA} \\ &= \alpha - \alpha - 0.25\alpha - 0.5\alpha + 0 + 0 \\ &= -0.75\alpha \\ &= -0.75 \cdot (-1) \qquad (\alpha = -1) \\ &= 0.75 \end{aligned}$$

Conclusion

Based on detailed analysis and the computed expression for W:

Work done on the particle by this force is 0.75 Joule.

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