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JEE Advanced 2019 - Paper 1 - Physics 08

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Problem *[Multiple Choice Multiple Correct]*

A charged shell of radius R carries a total charge Q. Given Φ as the flux of electric field through a closed cylindrical surface of height h, radius r and with its center same as that of the shell. Here, center of the cylinder is a point on the axis of the cylinder which is equidistant from its top and bottom surfaces. Which of the following option(s) is/are correct?
[ϵ_0 is the permittivity of free space]

(A) If $h > 2R$ and $r > R$ then $\Phi = \frac{Q}{\epsilon_0}$
(B) If $h < \frac{8R}{5}$ and $r = \frac{3R}{5}$ then $\Phi = 0$
(C) If $h > 2R$ and $r = \frac{3R}{5}$ then $\Phi = \frac{Q}{5\epsilon_0}$
(D) If $h > 2R$ and $r = \frac{4R}{5}$ then $\Phi = \frac{Q}{5\epsilon_0}$

What to Observe:

- A spherical shell of radius R carries a total charge Q.
- A closed cylindrical surface is considered for calculating electric flux.
- Properties of the cylindrical surface:
  - Height of the cylinder is h.
  - Radius of the cylinder is r.
  - The center of the cylinder coincides with the center of the spherical shell.
  - The center refers to the midpoint along the axis, equidistant from the top and bottom surfaces.
- Electric flux Φ through the cylindrical surface is being evaluated.

My Approach:

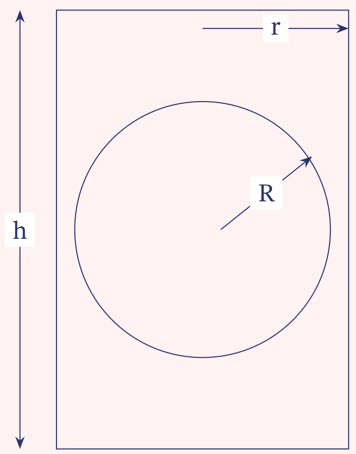
Dissecting Option A

Thought

When $h > 2R$ and $r > R$, this implies that the sphere is completely enclosed within the cylinder.
Since the charge is distributed on the surface of the sphere, the entire charge is contained inside the cylinder. Therefore, by Gauss's Law, the total electric flux through the cylinder is:

$$\Phi = \frac{Q}{\epsilon_0}$$

The following diagram illustrates the situation:



Therefore, **Option A is correct.**

Dissecting Option B

Thought

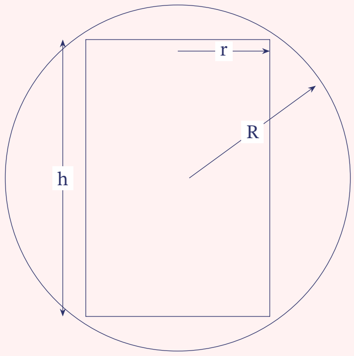
When $h < \frac{8R}{5}$ and $r = \frac{3R}{5}$, this implies that the cylinder is completely enclosed within the sphere.

Starting from the center of the sphere, the top of the cylinder lies at a distance less than $\frac{4R}{5}$. If it were exactly $\frac{4R}{5}$, the top (and bottom) edge of the cylinder would be just touching the inner surface of the sphere. Since the height is less, the cylinder is entirely contained within the sphere.

Since the charge resides only on the surface of the sphere, there is no enclosed charge within the cylinder. Therefore, by Gauss's Law, the electric field inside the cylinder is zero, and the total electric flux through it is:

$$\Phi = 0$$

The following diagram illustrates the situation:



Therefore, **Option B is correct.**

Dissecting Option C and D

Thought

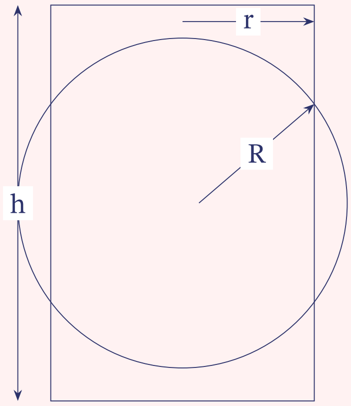
Before making any conclusions, observe that in both Option C and Option D, the condition $h > 2R$ is satisfied. However, the radius differs: $r = \frac{3R}{5}$ in Option C, and $r = \frac{4R}{5}$ in Option D.

Despite this difference, in both cases the value of the electric flux Φ is the same and equal to:

$$\Phi = \frac{Q}{5\epsilon_0}$$

This indicates that either one of the options is correct, or potentially neither, depending on constraints.

The following diagram illustrates the situation:



Thought

Based on the diagram, we can conclude that the electric flux through the cylinder depends on the intersection of the cylinder with the sphere, which is located inside the cylinder.

To solve this, we can use the concept of solid angles to determine the portion of the total charge enclosed by the cylinder. This allows us to calculate the charge present in the intersection region between the cylinder and the sphere.

If you have a cone with an apex angle θ (in radians), the solid angle Ω it subtends is given by:

$$\Omega = 2\pi\left(1 - \cos\left(\frac{\theta}{2}\right)\right)$$

where θ is the full apex angle of the cone, and Ω is the solid angle in steradians.

Given the diagram, the full apex angle is:

$$\theta = 2\sin^{-1}\left(\frac{r}{R}\right)$$

Therefore, the solid angle subtended by the cone is:

$$\Omega = 2\pi\left(1 - \cos\left(\sin^{-1}\left(\frac{r}{R}\right)\right)\right)$$

The total charge is Q (assuming uniform distribution over the surface of the sphere).

The total solid angle around a point is 4π steradians.

Therefore, the charge enclosed by the cylinder is proportional to the fraction of the solid angle it subtends. Hence, the enclosed charge is:

$$Q_{\text{cone}} = Q \cdot \frac{\Omega}{4\pi}$$

Substituting the expression for Ω:

$$Q_{\text{cone}} = Q \cdot \frac{2\pi\left(1 - \cos\left(\sin^{-1}\left(\frac{r}{R}\right)\right)\right)}{4\pi}$$

Simplifying:

$$Q_{\text{cone}} = \frac{Q}{2}\left(1 - \cos\left(\sin^{-1}\left(\frac{r}{R}\right)\right)\right)$$

Since there are 2 cones, the total charge enclosed by the cylinder is:

$$Q_{\text{enclosed}} = 2 \cdot Q_{\text{cone}} = Q\left(1 - \cos\left(\sin^{-1}\left(\frac{r}{R}\right)\right)\right)$$

For Option C, where $r = \frac{3R}{5}$:

$$\sin^{-1}\left(\frac{3R}{5R}\right) = \sin^{-1}\left(\frac{3}{5}\right) = 37^\circ$$

$$Q_{\text{enclosed}} = Q\left(1 - \cos(37^\circ)\right) = Q\left(1 - \frac{4}{5}\right)$$

$$Q_{\text{enclosed}} = \frac{Q}{5}$$

$$\implies \Phi = \frac{Q}{5\epsilon_0}$$

For Option D, where $r = \frac{4R}{5}$:

$$\sin^{-1}\left(\frac{4R}{5R}\right) = \sin^{-1}\left(\frac{4}{5}\right) = 53^\circ$$

$$Q_{\text{enclosed}} = Q\left(1 - \cos(53^\circ)\right) = Q\left(1 - \frac{3}{5}\right)$$

$$Q_{\text{enclosed}} = \frac{2Q}{5}$$

$$\implies \Phi = \frac{2Q}{5\epsilon_0}$$

Therefore, **Option C is correct** whereas **Option D is incorrect.**

Conclusion

Based on the detailed analysis and the computed expression for Q_{enclosed}:

$$\boxed{\text{Options A, B, and C are correct.}}$$

Option D is incorrect because, although the cylinder extends beyond the sphere vertically, the radius $r = \frac{4R}{5}$ results in a larger enclosed solid angle. Substituting this into the expression for Q_{enclosed} yields a value greater than what is stated in the option, which makes it inconsistent.

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