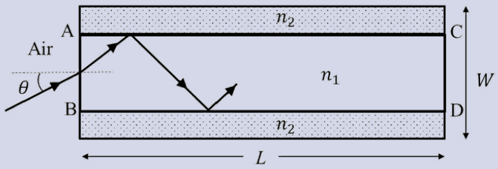
JEE Advanced 2019 - Paper 1 - Physics 18

Senan

Problem *[Numerical Value]*

A planar structure of length L and width W is made of two different optical media of refractive indices $n_1 = 1.5$ and $n_2 = 1.44$ as shown in figure. If $L \gg W$, a ray entering from end AB will emerge from end CD only if the total internal reflection condition is met inside the structure. For $L = 9.6$m, if the incident angle θ is varied, the maximum time taken by a ray to exit the plane CD is $t \times 10^{-9}$s, where t is ______.
[Speed of light $c = 3 \times 10^8$m/s]



What to Observe:

- A planar structure of length L and width W
- Structure is composed of two optical media with refractive indices $n_1 = 1.5$ and $n_2 = 1.44$
- $L \gg W$
- Ray enters from end AB and exits from end CD only if total internal reflection condition is satisfied
- Given length $L = 9.6$ m
- Incident angle θ is variable
- Speed of light $c = 3 \times 10^8$ m/s

My Approach:

Logical Steps

Thought

Note that in the two diagrams given below, the speed of light remains constant within a medium. Therefore, to maximize the time taken, the light must follow a longer path. Convince yourself that a greater number of reflections will result in a longer time taken.

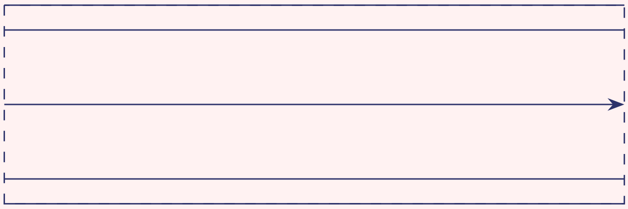


Figure 1. Light travels in a straight path

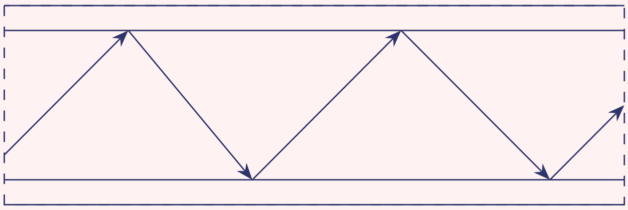


Figure 2. Light undergoes total internal reflection, resulting in a longer path

Thought

Now, inside the medium, the minimum angle at which total internal reflection occurs is the critical angle, θ_c. From Snell's Law:

$$\sin\theta_c = \frac{n_2}{n_1}$$

Given $n_1 = 1.5$ and $n_2 = 1.44$ as per the question.

Consider a right-angled triangle constructed as follows:

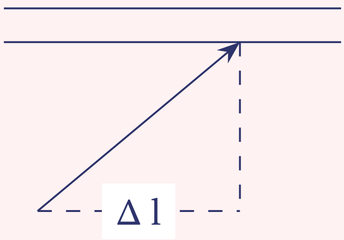


Figure 3. Elemental Triangle

Now, the distance travelled by light within one triangular segment is given by (θ_i is the angle of incidence):

$$\Delta l' = \frac{\Delta l}{\sin(\theta_i)}$$

Similarly, dividing the entire optical path into such triangular segments, and noting that:

$$\sum \Delta l = L$$

The total distance travelled by light becomes:

$$\sum_n \frac{\Delta l_n}{\sin(\theta_i)} = \frac{L}{\sin(\theta_i)}$$

The speed of light in a medium with refractive index n_1 is:

$$v = \frac{c}{n_1}$$

Thus, the total time taken by light is:

$$t = \frac{\text{distance}}{\text{speed}} = \frac{L}{\sin(\theta_i)} \cdot \frac{n_1}{c}$$

To maximize the time taken, we need to maximize this expression, which means minimizing $\sin(\theta_i)$, hence minimizing θ_i. However, θ_i cannot be smaller than the critical angle θ_c, otherwise total internal reflection will not occur.

$$\Rightarrow \text{Maximum time is taken when } \theta_i = \theta_c$$

Calculation

We use the formula:

$$t = \frac{L}{\sin(\theta_c)} \cdot \frac{n_1}{c}$$

Given:

$$c = 3 \times 10^8 \text{ m/s}, \quad n_1 = 1.5, \quad n_2 = 1.44, \quad L = 9.6 \text{ m}$$

From Snell's law at the critical angle:

$$\sin(\theta_c) = \frac{n_2}{n_1} = \frac{1.44}{1.5} = 0.96$$

Substituting into the time expression:

$$\begin{aligned} t &= \frac{9.6}{0.96} \cdot \frac{1.5}{3 \times 10^8} \\ &= 10 \cdot \frac{1.5}{3 \times 10^8} \\ &= \frac{15}{3 \times 10^8} \\ &= \frac{5}{10^8} \\ &= 50 \times 10^{-9} \end{aligned}$$

Conclusion

Based on detailed analysis and the computed expression for maximum time:

$$\boxed{\text{The value of } t \text{ is } 50.}$$

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