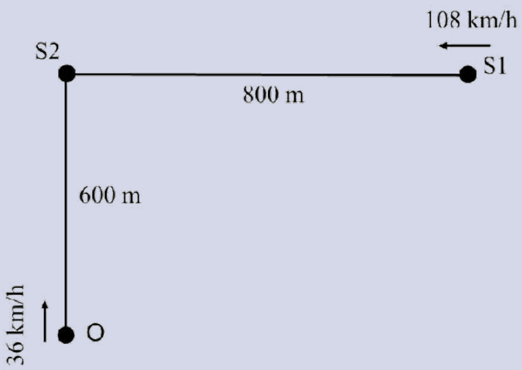
The Bored IITian

JEE Advanced 2019 - Paper 1 - Physics 15

Senan

Problem *[Numerical Value]*

A train S_1, moving with a uniform velocity of 108 km/h, approaches another train S_2 standing on a platform. An observer O moves with a uniform velocity of 36 km/h towards S_2, as shown in figure. Both the trains are blowing whistles of same frequency 120 Hz. When O is 600 m away from S_2 and distance between S_1 and S_2 800 m, the number of beats heard by O is _______.
[Speed of the sound = 330 m/s]



What to Observe:

- Train S_1 is moving with a uniform velocity of 108 km/h.
- Train S_2 is stationary on a platform.
- Observer O is moving with a uniform velocity of 36 km/h towards S_2.
- Both trains are blowing whistles of the same frequency 120 Hz.
- Distance between observer O and train S_2 is 600 m.
- Distance between trains S_1 and S_2 is 800 m.
- Speed of sound is given as 330 m/s.

My Approach:

What I Know

Thought

The Doppler effect formula is:

$$f' = \left(\frac{v \pm v_o}{v \mp v_s}\right) f$$

where:

- f' is the observed frequency,
- f is the source frequency,
- v is the speed of sound in the medium,
- v_o is the speed of the observer (positive if moving towards the source),
- v_s is the speed of the source (positive if moving away from the observer).

Given:

$$\text{Velocity of observer} = 36\,\text{km/h} = \frac{36 \times 1000}{60 \times 60} = 10\,\text{m/s} \quad (\text{towards source } S_2)$$
$$\text{Velocity of source } S_1 = 108\,\text{km/h} = \frac{108 \times 1000}{60 \times 60} = 30\,\text{m/s} \quad (\text{towards source } S_2)$$
$$\text{Speed of sound} = 330\,\text{m/s}$$

Observed Frequency at Point O due to Source S_2

The formula for observed frequency is:

$$f' = \left(\frac{v + v_o}{v - v_s}\right) f$$

Given:

$$\begin{aligned} v &= 330\,\text{m/s} \\ v_o &= 10\,\text{m/s} \quad \text{(observer moving towards the source)} \\ v_s &= 0 \quad \text{(source is stationary)} \\ f &= 120\,\text{Hz} \end{aligned}$$

Substituting into the formula:

$$f' = \left(\frac{330+10}{330-0}\right)\cdot 120 = \left(\frac{340}{330}\right)\cdot 120$$

Simplifying:

$$f' = \frac{34}{33}\cdot 120 = \frac{4080}{33} \approx 123.64\,\text{Hz}$$

Observed Frequency at Point O due to Source S_1

The Doppler effect formula is:

$$f' = \left(\frac{v+v_o^{\parallel}}{v-v_s^{\parallel}}\right) f$$

Given:

$$\begin{aligned} v &= 330\,\text{m/s} \\ f &= 120\,\text{Hz} \\ v_o &= 10\,\text{m/s}, \quad \theta_o = \tan^{-1}\left(\frac{800}{600}\right) \\ v_s &= 30\,\text{m/s}, \quad \theta_s = \tan^{-1}\left(\frac{600}{800}\right) \end{aligned}$$

Calculate the components along the line joining O and S_1:

$$v_o^{\parallel} = 10\cos\left(\tan^{-1}\left(\frac{800}{600}\right)\right) = 10\cdot\frac{600}{\sqrt{600^2+800^2}} = 10\cdot\frac{600}{1000} = 6\,\text{m/s}$$

$$v_s^{\parallel} = 30\cos\left(\tan^{-1}\left(\frac{600}{800}\right)\right) = 30\cdot\frac{800}{\sqrt{600^2+800^2}} = 30\cdot\frac{800}{1000} = 24\,\text{m/s}$$

Now plug into the Doppler formula:

$$f' = \left(\frac{330+6}{330-24}\right)\cdot 120 = \left(\frac{336}{306}\right)\cdot 120$$

Simplify:

$$f' = \frac{336\cdot 120}{306} = \frac{40320}{306} = \frac{4480}{34} \approx 131.76\,\text{Hz}$$

Beats Observed

Given:

$$\begin{aligned} f_1 &= 131.76\,\text{Hz} \\ f_2 &= 123.64\,\text{Hz} \end{aligned}$$

Therefore, the beat frequency is:

$$f_{\text{beat}} = |131.76 - 123.64| = 8.12\,\text{Hz}$$

Conclusion

Based on detailed analysis and the computed expression for beat frequency:

$$\boxed{f_{beat} = 8.12\,\text{Hz}}$$

Support the Channel:

If you found this explanation helpful and it helped you think like a JEE Advanced ranker, consider supporting the channel!

- **Subscribe** to *The Bored IITian* for more such high-quality JEE problem breakdowns.
- **Share it with 3 friends** who have **never heard of the channel**. Help them level up too!
- Drop a comment or suggestion - your feedback helps improve future content.

Note from Senan: The way I solve and explain problems in this PDF is simply how I personally approach and think through JEE questions. This is **not** the only way to solve them. What I'm sharing here is my mindset and thought process - use it as inspiration, not as a prescription. Develop your own flow, but always stay curious and logical!

YouTube Channel: youtube.com/@bored_iitian