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# JEE Advanced 2019 - Paper 1 - Physics 05

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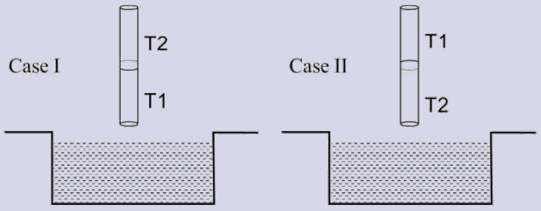
### Problem *[ Multiple Choice Multiple Correct ]*

A cylindrical capillary tube of 0.2 mm radius is made by joining two capillaries $T_1$ and $T_2$ of different materials having water contact angles of 0° and 60° respectively. The capillary tube is dipped vertically in water in two different configurations, case I and II as shown in figure. Which of the following option(s) is(are) correct?
[ Surface tension of water = 0.075 N/m, density of water = 1000 kg/m$^3$, take $g = 10\,\text{m/s}^2$ ]




(A) The correction in the height of water column raised in the tube, due to weight of water contained in the meniscus, will be different for both cases.
(B) For case II, the capillary joint is 5 cm above the water surface, the height of water column raised in the tube will be 3.75 cm. (Neglect the weight of the water in the meniscus)
(C) For case I, if the joint is kept at 8 cm above the water surface, the height of water column in the tube will be 7.5 *cm*. (Neglect the weight of the water in the meniscus)
(D) For case I, capillary joint is 5 cm above the water surface, the height of water column raised in the tube will be more than 8.75 cm. (Neglect the weight of the water in the meniscus)

## What to Observe:

- Capillary radius: $r = 0.2\,\text{mm} = 0.2 \times 10^{-3}\,\text{m}$
- Two contact angles: $\theta_1 = 0°, \theta_2 = 60°$
- Surface tension: $T = 0.075\,\text{N/m}$
- Density of water: $\rho = 1000\,\text{kg/m}^3$
- Acceleration due to gravity: $g = 10\,\text{m/s}^2$
- Configurations (I and II): *Orientation of tubes (which contact angle faces water) changes the rise*
- "Neglect weight of water in meniscus": simplifies calculations in options B, C, D
- Capillary rise formula: $h = \frac{2S\cos\theta}{r\rho g}$

## My Approach:

### Elephant In The Room

**Thought**

I know what you're thinking — there's an elephant in the room. The question feels a bit confusing, and you might already be guessing that scoring a full +4 could be tricky. Partial marks seem more likely.

So, what's the elephant? It's this: What exactly is meant by the "correction in height" in option A?

The contact angles are different for the two cases, and since the height of the liquid column is measured from the lower end of the meniscus, this difference becomes significant. Although the radius is the same for both tubes, the curvature of the meniscus varies with the contact angle. As a result, each meniscus holds a different volume of water above its base.

This additional volume means there is extra mass being supported, and to balance this added weight, the height of the water column must adjust slightly. Therefore, a **correction in height** is necessary, and it will differ between the two tubes due to the difference in meniscus curvature and the associated mass of water.

**Given below is a diagram:** Observe carefully — the mass of water above the meniscus is different in each case because the radius of curvature of the meniscus changes with the contact angle.

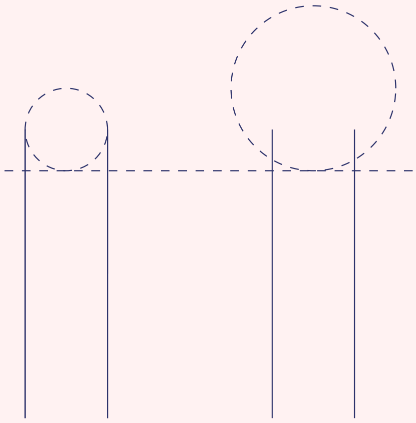


**Figure 1.** Logical diagram for observing height correction

Therefore, we can conclude that **Option A is correct** as the correction in the height due to the weight of water in the meniscus will indeed be different for the two cases.

### First Steps

Let us first consider capillaries $T_1$ and $T_2$ independently and calculate the capillary rise in each case.

The formula for capillary rise is given by:

$$h = \frac{2S\cos\theta}{r\rho g}$$

For $T_1$, with $\theta_1 = 0°$, we have:

$$h_1 = \frac{2 \times 0.075 \times \cos(0°)}{0.2 \times 10^{-3} \times 1000 \times 10} = 0.075\,\text{m} = 7.5\,\text{cm}$$

For $T_2$, with $\theta_2 = 60°$, the only change is in the cosine term:

$$\cos(60°) = \frac{1}{2} \Rightarrow h_2 = \frac{7.5}{2}\,\text{cm} = 3.75\,\text{cm}$$

From this, we can clearly conclude that **Options B and C are correct**, since in both cases, the height of water rise is less than the position of the capillary joint. This means the liquid never reaches the second tube, and thus the type of tube connected above becomes irrelevant.

### Trickiest Option

**Thought**

Option D is both the most logical and the most subtle. At first glance, it might seem that the water would rise through $T_1$ and then partially into $T_2$. However, that is not the case.

This is actually a classic case of a **capillary of insufficient length**. Once the water column reaches the capillary joint at the top of $T_1$, it does not continue to rise into $T_2$. Instead, the rise halts, and the system begins to adjust the contact angle at the interface to maintain equilibrium.

Think of it this way: if the water were to rise above $T_1$ and into $T_2$, then in theory, you could remove $T_2$ altogether and still observe the same height.

This would imply that water is rising higher than what the contact angle of $T_1$ allows on its own — essentially creating a system where water appears to gain energy without input, resembling a form of **perpetual motion**, which is physically impossible.

Therefore, this thought experiment confirms that the water column **cannot** exceed the capillary height dictated by $T_1$ alone, and hence, **Option D is incorrect**.

### Equilibrium Condition for Insufficient Length

When a capillary is not long enough for the liquid to rise to its full theoretical height, the system adjusts the curvature of the meniscus to maintain equilibrium. This is described by the relation:

$$hR = h'R'$$

Where:

- $h$ is the theoretical capillary rise for a tube of sufficient length
- $R$ is the original radius of curvature of the meniscus
- $h'$ is the actual (insufficient) rise of liquid in the shorter tube
- $R'$ is the new radius of curvature that adjusts to compensate

Since $h' < h$, it follows that $R' > R$. The meniscus becomes flatter (less curved) to support the reduced height while preserving the pressure balance. This explains why the water does **not** continue into $T_2$ in Option D, and the liquid instead stabilizes at the joint.

### Conclusion

Based on detailed analysis, physical reasoning, and capillary theory:

$$\boxed{\text{Options A, B, and C are correct.}}$$

Option D is incorrect due to the physical constraint imposed by the capillary rise limit of $T_1$ and the concept of insufficient capillary length.

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