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JEE Advanced 2019 - Paper 1 - Physics 10

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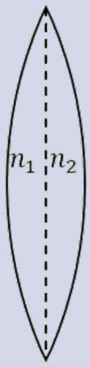
Problem *[Multiple Choice Multiple Correct]*

A thin convex lens is made of two materials with refractive indices n_1 and n_2, as shown in figure. The radius of curvature of the left and right spherical surfaces are equal. f is the focal length of the lens when $n_1 = n_2 = n$. The focal length is $f + \Delta f$ when $n_1 = n$ and $n_2 = n + \Delta n$. Assuming that $\Delta n \ll (n-1)$ and $1 < n < 2$, the correct statement(s) is/are:



(A) $\left|\frac{\Delta f}{f}\right| < \left|\frac{\Delta n}{n}\right|$

(B) For $n = 1.5$, $\Delta n = 10^{-3}$ and $f = 20\,\text{cm}$, the value of $|\Delta f|$ is 0.02 cm (round off to 2^{nd} decimal place).

(C) If $\frac{\Delta n}{n} < 0$ then $\frac{\Delta f}{f} > 0$

(D) The relation between $\frac{\Delta f}{f}$ and $\frac{\Delta n}{n}$ remains unchanged if both the convex surfaces are replaced by concave surfaces of the same radius of curvature.

What to Observe:

- A thin convex lens made of two materials with refractive indices n_1 and n_2.
- Radius of curvature of both spherical surfaces is equal.
- Focal length is f when $n_1 = n_2 = n$.
- Focal length becomes $f + \Delta f$ when $n_1 = n$, $n_2 = n + \Delta n$.
- $\Delta n \ll (n-1)$
- Refractive index range: $1 < n < 2$

My Approach:

Determining the relationship between Δn and Δf

Thought

The question may seem complicated at first, but it can be solved easily with logical reasoning.
All the options involve Δn and Δf, and their respective relationships with n and f. Therefore, if we determine the relationship between Δn and Δf, we can solve the question easily.

We can use the concept of the Lens Maker's formula to establish the connection between the refractive index n and the focal length f.
The Lens Maker's formula is given by:

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

where f is the focal length, n is the refractive index of the lens material, and R_1, R_2 are the radii of curvature of the two lens surfaces.

Case 1: $\Delta n = 0$ and $n = n_1 = n_2$

$$\begin{aligned}\frac{1}{f} &= (n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)\\ &= (n-1)\left(\frac{1}{R}-\frac{1}{-R}\right)\\ &= (n-1)\left(\frac{2}{R}\right)\\ &= \frac{2(n-1)}{R}\end{aligned}$$

Case 2: $\Delta n \neq 0$, with $n_1 = n$ and $n_2 = n + \Delta n$

Treat this situation as a combination of two lenses placed together, each with a different refractive index.

$$\frac{1}{f_{\text{eq}}} = \frac{1}{f_1} + \frac{1}{f_2}$$

Here, f_1 is the focal length of the lens with refractive index n, and f_2 is the focal length of the lens with refractive index $n + \Delta n$.

$$\frac{1}{f_{\text{eq}}} = \frac{1}{f + \Delta f}$$

where Δf represents the change in focal length due to the change in refractive index.

Now using the lens maker's formula for both lenses:

$$\begin{aligned}\frac{1}{f_1} &= (n-1)\left(\frac{1}{R}-\frac{1}{\infty}\right)\\ &= \frac{(n-1)}{R}\\ \frac{1}{f_2} &= (n+\Delta n-1)\left(\frac{1}{\infty}-\frac{1}{-R}\right)\\ &= \frac{(n+\Delta n-1)}{R}\end{aligned}$$

Now substituting these values into the equation for f_{eq}:

$$\begin{aligned}\frac{1}{f_{\text{eq}}} &= \frac{1}{f_1}+\frac{1}{f_2}\\ &= \frac{(n-1)}{R}+\frac{(n+\Delta n-1)}{R}\\ &= \frac{(2n-2+\Delta n)}{R}\\ &= \frac{2(n-1)}{R}+\frac{\Delta n}{R}\\ &= \frac{1}{f}+\frac{\Delta n}{R}\end{aligned}$$

Replacing R with $2f(n-1)$ from Case 1:

$$\begin{aligned}\frac{1}{f_{\text{eq}}} &= \frac{1}{f}+\frac{\Delta n}{R}\\ &= \frac{1}{f}+\frac{\Delta n}{2f(n-1)} \quad (\text{since } R = 2f(n-1))\end{aligned}$$

Now, we can express Δf and Δn in terms of f and n:

$$\begin{aligned}\frac{1}{f+\Delta f} &= \frac{1}{f}+\frac{\Delta n}{2f(n-1)}\\ \frac{1}{f+\Delta f}-\frac{1}{f} &= \frac{\Delta n}{2f(n-1)}\\ \frac{f-(f+\Delta f)}{f(f+\Delta f)} &= \frac{\Delta n}{2f(n-1)}\\ \frac{-\Delta f}{f+\Delta f} &= \frac{\Delta n}{2(n-1)}\end{aligned}$$

As Δn is small, therefore Δf is also small. Hence, we can ignore Δf in the denominator:

$$\frac{-\Delta f}{f} = \frac{\Delta n}{2(n-1)}$$
$$\frac{-\Delta f}{f} = \frac{\Delta n}{n} \cdot \frac{1}{2\left(1-\frac{1}{n}\right)}$$

Analysis of Options

Thought

Based on the derived relationship observed, if $\frac{\Delta n}{n} < 0$, then $\frac{\Delta f}{f} > 0$.
This implies that when the refractive index decreases, the focal length increases — which aligns with the known behavior of lenses.

Therefore, **Option C is correct**.

Thought

Suppose we performed the same derivation as above but used $-R$ instead of R in the Lens Maker's formula.
Even in that case, we would arrive at the same expression relating Δf and Δn. Which is the exact condition for concave lenses with same radius of curvature.

Therefore, **Option D is correct**.

Thought

As given in the question, $\Delta n \ll n-1$ and $1 < n < 2$.
Therefore, we can proceed as follows:

$$\begin{aligned}
& 1 < n < 2 \\
\implies & \frac{1}{2} < \frac{1}{n} < 1 \\
\implies & -1 < -\frac{1}{n} < -\frac{1}{2} \\
\implies & 1-1 < 1-\frac{1}{n} < 1-\frac{1}{2} \\
\implies & 0 < 1-\frac{1}{n} < \frac{1}{2} \\
\implies & 0 < 2\left(1-\frac{1}{n}\right) < 1
\end{aligned}$$

The expression we got is:

$$\frac{-\Delta f}{f} = \frac{\Delta n}{n} \cdot \frac{1}{\text{number between 0 and 1}}$$

Therefore, **Option A is incorrect**.

For $n = 1.5$, $\Delta n = 10^{-3}$, and $f = 20\,\text{cm}$, we can calculate the value of $|\Delta f|$ as follows:

$$\begin{aligned}
\left|\frac{\Delta f}{f}\right| &= \left|\frac{\Delta n}{2(n-1)}\right| \\
&= \frac{10^{-3}}{2(1.5-1)} \\
&= \frac{10^{-3}}{2 \times 0.5} \\
&= 10^{-3}
\end{aligned}$$

$$|\Delta f| = f \cdot \left|\frac{\Delta f}{f}\right| = 20 \times 10^{-3} = 0.02\,\text{cm}$$

Therefore, **Option B is correct**.

Thought

As stated in the question, the value is to be approximated to the second decimal place.
However, this level of approximation was not required in our case, since we already made an approximation during the derivation — specifically by neglecting Δf in the denominator.

Conclusion

Based on detailed analysis and the computed expression:

$$\boxed{\text{Options B, C, and D are correct.}}$$

Option A is incorrect because the inequality in the option is opposite.

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