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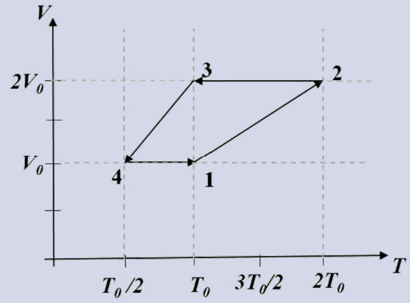
JEE Advanced 2019 - Paper 1 - Physics 09

Senan

Problem *[Multiple Choice Multiple Correct]*

One mole of a monatomic ideal gas goes through a thermodynamic cycle, as shown in the volume versus temperature (V – T) diagram. The correct statement(s) is/are:
[R is the gas constant]



(A) Work done in this thermodynamic cycle ($1 \to 2 \to 3 \to 4 \to 1$) is $|W| = \frac{1}{2}RT_0$

(B) The above thermodynamic cycle exhibits only isochoric and adiabatic processes.

(C) The ratio of heat transfer during processes $1 \to 2$ and $2 \to 3$ is $\left|\frac{Q_{1\to2}}{Q_{2\to3}}\right| = \frac{5}{3}$.

(D) The ratio of heat transfer during processes $1 \to 2$ and $3 \to 4$ is $\left|\frac{Q_{1\to2}}{Q_{3\to4}}\right| = \frac{1}{2}$.

What to Observe:

- One mole of a monatomic ideal gas undergoes a thermodynamic cycle shown in the V-T diagram.
- The cycle consists of four processes: $1 \to 2 \to 3 \to 4 \to 1$
- Process Types:
  - Analyze whether processes are isochoric, isothermal, adiabatic, etc.
- Ideal gas law: $PV = RT$
- Constants:
  - R: Universal gas constant
  - Monatomic ideal gas: $C_V = \frac{3}{2}R, \quad C_P = \frac{5}{2}R$

My Approach:

Understanding the Graph

Thought

Observing that the processes $4 \to 1$ and $2 \to 3$ occur at constant volume, we can conclude that these are **isochoric processes**.

Isochoric Process: A process in which the volume remains constant. In an isochoric process, the system does not perform any work on its surroundings. Hence, all the heat added to or removed from the system results in a change in its internal energy.

Furthermore, observe that in the processes $1 \to 2$ and $3 \to 4$, the ratio of volume to temperature remains constant. According to the ideal gas law,

$$\frac{V}{T} = \frac{nR}{P} = \text{constant},$$

which implies that these processes are **isobaric**, meaning the pressure remains constant.

Clearly, the processes are **isochoric** and **isobaric** in nature.
Therefore, **Option B is incorrect**.

Again, the processes $4 \to 1$ and $2 \to 3$ are **isochoric** processes. Since no work is done during an isochoric process, the work done in these two processes is zero.

Let the work done in the entire cycle be W. The work done in the cycle can be expressed as the sum of the work done in each process:

$$W = W_{1\to2} + W_{2\to3} + W_{3\to4} + W_{4\to1}$$
$$W_{2\to3} = 0 \quad , \quad W_{4\to1} = 0$$
$$\Rightarrow W = W_{1\to2} + W_{3\to4}$$

Now, we need to find $W_{1\to2}$ and $W_{3\to4}$. Both processes are **isobaric**, so we use the ideal gas law:

$$pV = RT \quad \text{with } n = 1 \quad \Rightarrow \quad p\Delta V = R\Delta T$$

Therefore, the work done is:

$$W = \int p\,dV = \int R\,dT = R\Delta T = R(T_{\text{final}} - T_{\text{initial}})$$

$$W_{1\to2} = R(T_2 - T_1) = R(2T_0 - T_0) = RT_0$$
$$W_{3\to4} = R(T_4 - T_3) = R\left(\frac{T_0}{2} - T_0\right) = -\frac{RT_0}{2}$$

$$W = W_{1\to2} + W_{3\to4} = RT_0 - \frac{RT_0}{2} = \frac{RT_0}{2}$$

$$\Rightarrow |W| = \frac{RT_0}{2}$$

Therefore, **Option A is correct**.

Finding the Heat Transfer

Thought

The heat transfer in an isochoric process is given by the equation:

$$Q = nC_V\Delta T$$

where C_V is the molar heat capacity at constant volume.

For a monatomic ideal gas, $C_V = \frac{3}{2}R$. Therefore, we can express the heat transfer as:

$$Q = nC_V\Delta T = n\left(\frac{3}{2}R\right)\Delta T$$

The heat transfer in an isobaric process is given by the equation:

$$Q = nC_P\Delta T$$

where C_P is the molar heat capacity at constant pressure.

For a monatomic ideal gas, $C_P = \frac{5}{2}R$. Therefore, we can express the heat transfer as:

$$Q = nC_P\Delta T = n\left(\frac{5}{2}R\right)\Delta T$$

Let's calculate the heat transfer in each process:

- For the process $1 \to 2$ (**isobaric**):

$$Q_{1\to2} = C_P\Delta T = C_P(T_2 - T_1) = C_P(2T_0 - T_0) = C_PT_0$$

- For the process $2 \to 3$ (**isochoric**):

$$Q_{2\to3} = C_V\Delta T = C_V(T_3 - T_2) = C_V\,(T_0 - 2T_0) = -C_VT_0$$

- For the process $3 \to 4$ (**isobaric**):

$$Q_{3\to4} = C_P\Delta T = C_P(T_4 - T_3) = C_P\left(\frac{T_0}{2} - T_0\right) = -C_P\left(\frac{T_0}{2}\right)$$

- For the process $4 \to 1$ (**isochoric**) *[not required]*:

$$Q_{4\to1} = C_V\Delta T = C_V(T_1 - T_4) = C_V(T_0 - \frac{T_0}{2}) = C_V\left(\frac{T_0}{2}\right)$$

Now for Option C:

$$\left|\frac{Q_{1\to 2}}{Q_{2\to 3}}\right| = \left|\frac{C_P T_0}{-C_V T_0}\right| = \frac{C_P}{C_V} = \gamma = \frac{5}{3}$$

Now for Option D:

$$\left|\frac{Q_{1\to 2}}{Q_{3\to 4}}\right| = \left|\frac{C_P T_0}{-C_P \frac{T_0}{2}}\right| = \left|\frac{T_0}{-\frac{T_0}{2}}\right| = 2$$

Therefore, **Option C is correct** but **Option D is incorrect**.

Conclusion

Based on the analysis of the processes and the calculations performed, we can conclude that:

$$\boxed{\text{Options A and C are correct.}}$$

Option B is incorrect because the cycle includes isobaric process. Option D is incorrect because the ratio of heat transfer during processes $1 \to 2$ and $3 \to 4$ does not equal $\frac{1}{2}$.

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