The Bored IITian

JEE Advanced 2019 - Paper 1 - Physics 06

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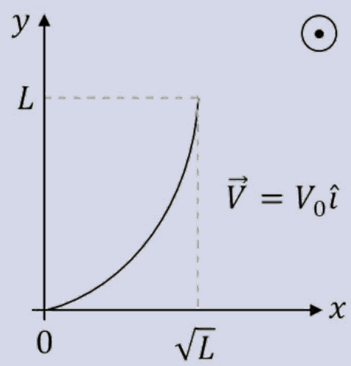
Problem *[Multiple Choice Multiple Correct]*

A conducting wire of parabolic shape, initially $y = x^2$, is moving with velocity $\vec{V} = V_0\hat{\imath}$ in a non-uniform magnetic field $\vec{B} = B_0\left(1 + \left(\frac{y}{L}\right)^\beta\right)\hat{k}$ as shown in figure. If V_0, B_0, L and β are positive constants and $\Delta\phi$ is the potential difference developed between the ends of the wire, then the correct statement(s) is/are:



(A) $|\Delta\phi| = \frac{1}{2}B_0V_0L$ for $\beta = 0$
(B) $|\Delta\phi| = \frac{4}{3}B_0V_0L$ for $\beta = 2$
(C) $|\Delta\phi|$ remains the same if the parabolic wire is replaced by a straight wire, $y = x$ initially, of length $\sqrt{2}L$.
(D) $|\Delta\phi|$ is proportional to the length of the wire projected on the y-axis.

What to Observe:

- Shape of the wire: $y = x^2$
- Velocity of the wire: $\vec{V} = V_0\hat{\imath}$
- Magnetic field: $\vec{B} = B_0\left(1 + \left(\frac{y}{L}\right)^\beta\right)\hat{k}$
- Constants:
  - $V_0 > 0$ (constant velocity magnitude)
  - $B_0 > 0$ (base magnetic field strength)
  - $L > 0$ (characteristic length)
  - $\beta > 0$ (exponent controlling variation of B with y)
- Potential difference between ends of the wire: $\Delta\phi$

My Approach:

What I Know

Thought

The question looks straightforward, but there's some vector physics making this complicated. Let's recall what we know about motional EMF. When a conductor moves in a magnetic field, the charges inside experience a Lorentz force given by:

$$\vec{F} = q(\vec{v} \times \vec{B})$$

This force sets up an electric field $\vec{E}_{\text{ind}} = \vec{v} \times \vec{B}$, which leads to a potential difference across the ends of the conductor. The general expression for motional EMF is:

$$\Delta\phi = \int (\vec{v} \times \vec{B}) \cdot d\vec{l}$$

So to solve this, we must carefully evaluate this line integral along the shape of the wire, keeping in mind the direction of motion and how $\vec{B}$ varies with position.

Let's break this down using the vector form of motional EMF:

$$\Delta\phi = \int (\vec{v} \times \vec{B}) \cdot d\vec{l}$$

Given:

$$\vec{v} = V_0\hat{i}, \quad \vec{B} = B(y)\hat{k}, \quad d\vec{l} = dx\,\hat{i} + dy\,\hat{j}$$

First, compute the cross product:

$$\vec{v} \times \vec{B} = V_0\hat{i} \times B(y)\hat{k} = -V_0 B(y)\hat{j}$$

Now take the dot product with $d\vec{l}$:

$$(\vec{v} \times \vec{B}) \cdot d\vec{l} = \left(-V_0 B(y)\hat{j}\right) \cdot \left(dx\,\hat{i} + dy\,\hat{j}\right) = -V_0 B(y)\,dy$$

Observe that the dx term vanishes because $\hat{j} \cdot \hat{i} = 0$. Therefore, the only contribution to the motional EMF comes from the dy component, and the expression simplifies to:

$$\Delta\phi = -\int V_0 B(y)\,dy$$

This shows clearly that the induced potential difference depends solely on the component of the wire along the y-axis, not its total length or shape in the xy-plane.
Therefore, **Option D is correct.**

Now, using the same logic, we consider a wire of length $\sqrt{2}L$ placed along the line $y = x$, which is at a 45° angle to the axes. The vertical (y-axis) component of this wire's length is:

$$L_y = \sqrt{2}L \cdot \sin(45°) = \sqrt{2}L \cdot \frac{1}{\sqrt{2}} = L$$

This means the effective contribution to the motional EMF, which depends only on the y-component of the wire, remains the same as in the original parabolic case.
Thus, **Option C is also correct.**

Final Calculations

Thought

Now, as the question only requests the **magnitude** of the EMF, we can drop the negative sign and evaluate the integral as follows:

$$|\Delta\phi| = \int_0^L V_0 B(y)\,dy = B_0 V_0 \int_0^L \left(1 + \left(\frac{y}{L}\right)^\beta\right) dy$$

Solving the integral,

$$\begin{aligned}
|\Delta\phi| &= B_0 V_0 \int_0^L \left(1 + \left(\frac{y}{L}\right)^\beta\right) dy \\
&= B_0 V_0 \left[\int_0^L 1\,dy + \int_0^L \left(\frac{y}{L}\right)^\beta dy\right] \\
&= B_0 V_0 \left[L + \int_0^L \left(\frac{y}{L}\right)^\beta dy\right] \\
&= B_0 V_0 \left[L + L\int_0^1 u^\beta du\right] \quad \left(\text{Let } u = \frac{y}{L},\ dy = L\,du\right) \\
&= B_0 V_0 \left[L + L \cdot \frac{1}{\beta + 1}\right] \\
&= B_0 V_0 L \left(1 + \frac{1}{\beta + 1}\right) \\
&= B_0 V_0 L \left(\frac{\beta + 2}{\beta + 1}\right)
\end{aligned}$$

Therefore,

$$|\Delta\phi| = \left(\frac{\beta + 2}{\beta + 1}\right) B_0 V_0 L$$

For specific values of β:

- When $\beta = 0$:

$$|\Delta\phi| = \left(\frac{0 + 2}{0 + 1}\right) B_0 V_0 L = 2B_0 V_0 L$$

- When $\beta = 2$:

$$|\Delta\phi| = \left(\frac{2+2}{2+1}\right)B_0V_0L = \frac{4}{3}B_0V_0L$$

Hence, **Option A is incorrect**, while **Option B is correct**.

Conclusion

Based on detailed analysis and the computed expression for $|\Delta\phi|$:

$$\boxed{\text{Options B, C, and D are correct.}}$$

Option A is incorrect because the calculated value of $|\Delta\phi|$ does not match the value stated in the option.

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