Senan

$$I_n = \int \sin^n(x)\,dx = \int \underbrace{\sin^{n-1}(x)}_{u} \cdot \underbrace{\sin(x)\,dx}_{dv}$$

Notes

$n \geq 2$

Integration by Parts

$$\int u\,dv = uv - \int v\,du$$

$$= \sin^{n-1}(x) \cdot (-\cos(x))$$
$$- \int (-\cos(x)) \cdot (n-1) \sin^{n-2}(x) \cdot \cos(x)$$

$$= -\sin^{n-1}(x)\cos(x) + (n-1)\int \sin^{n-2}(x) \cdot \cos^2(x)\,dx$$

$$\cos^2(x) = 1 - \sin^2 x$$

$$= -\sin^{n-1}(x)\cos(x) + (n-1)\int \sin^{n-2}(x)\left(1 - \sin^2(x)\right)dx$$

$$= -\sin^{n-1}(x)\cos(x) + (n-1)\int \sin^{n-2}(x)\,dx$$
$$- (n-1)\int \sin^n(x)\,dx$$

$$I_n = -\sin^{n-1}(x)\cos(x) + (n-1) I_{n-2}$$
$$- (n-1) I_n$$

$$I_n + (n-1) I_n = -\sin^{n-1}(x)\cos(x) + (n-1) I_{n-2}$$

$$n I_n = -\sin^{n-1}(x)\cos(x) + (n-1) I_{n-2}$$

$$\boxed{\begin{gathered} I_n = \frac{-1}{n}\sin^{n-1}(x)\cos(x) + \left(\frac{n-1}{n}\right) I_{n-2} \\ I_n = \int \sin^n(x)\,dx \end{gathered}}$$