Assignment 1 - Formal languages & computability

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September 2023

1.

$(1+2+\ldots+n)^2 > 1^2+2^2+\ldots+n^2$ for $n \geq 2$, we find the sum of both series.

Sum for n is:

$$1+2+\ldots+n = \frac{n(n+1)}{2}$$

The sum of n squared:

$$1^2+2^2+\ldots+n^2 = \frac{n(n+1)(2n+1)}{6}$$

We square the n to get the left half $1+2+\ldots+n$:

$$\left(\frac{n(n+1)}{2}\right)^2 = \frac{n^2(n+1)^2}{4}$$

That gives us:

$$\frac{n^2(n+1)^2}{4} > \frac{n(n+1)(2n+1)}{6}$$

$$6n^2(n+1)^2 > 4n(n+1)(2n+1)$$

$$6n^2(n^2+2n+1) > 8n^2+4n^2+4n$$

$$6n^3+12n^2+6n^2 > 12n^2+4n^2+4n$$

$$6n^3 > 4n$$

$$n^3 > \frac{2}{3}n$$

Since we're considering $n \geq 2$, both n^3 and n are positive,n^3 is greater than $\frac{2}{3}n$.

The inequality $(1+2+\ldots+n)^2 > 1^2+2^2+\ldots+n^2$ holds for $n \geq 2$.

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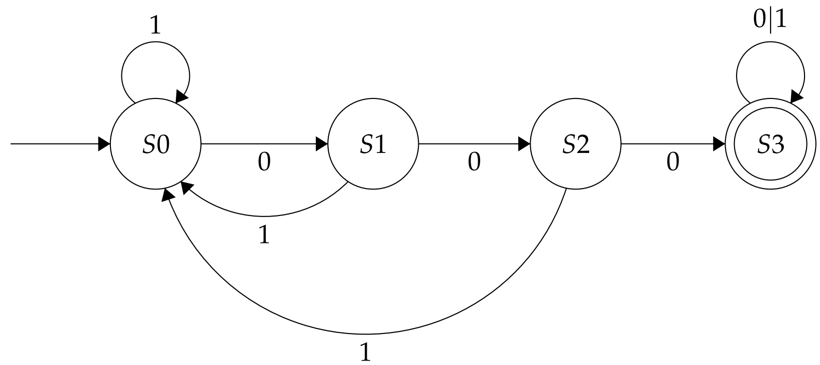

3.

If A is a regular language, then there exists a finite automaton M that recognizes A. Let's construct a finite automaton M' that recognizes B.

M' will have the same states, alphabet, and transition function as M. The only difference will be in the set of accept states.

Let's define the set of accept states of M' to be those states of M that are reached by an odd number of transitions from the start state. Formally, a state q is in the set of accept states of M' if and only if there exists a string w in Σ * such that δ (q0, w) = q and ∣w∣ is odd.

Now, let's prove that L(M') = B.

First, suppose w is in B. Then w is in A, and ∣w∣ is odd. Since M recognizes A, there is a sequence of transitions in M from the start state to some accept state that spells out w. Since ∣w∣ is odd, this accept state is in the set of accept states of M'. Therefore, M' accepts w, and w is in L(M').

Conversely, suppose w is in L(M'). Then there is a sequence of transitions in M' from the start state to some accept state that spells out w. Since this state is in the set of accept states of M', ∣w∣ must be odd. Since M and M' have the same transition function, this sequence of transitions is also valid in M, and so w is in A. Therefore, w is in B.

Therefore, L(M') = B, and so B is regular.

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a) Let's break down how the checksum is computed. The checksum c of a string $a_1, a_2 ... a_n$ is computed as the bitwise XOR of all the a_i. That is, $c = a_1 x \oplus a_2 x \oplus ...a_n$

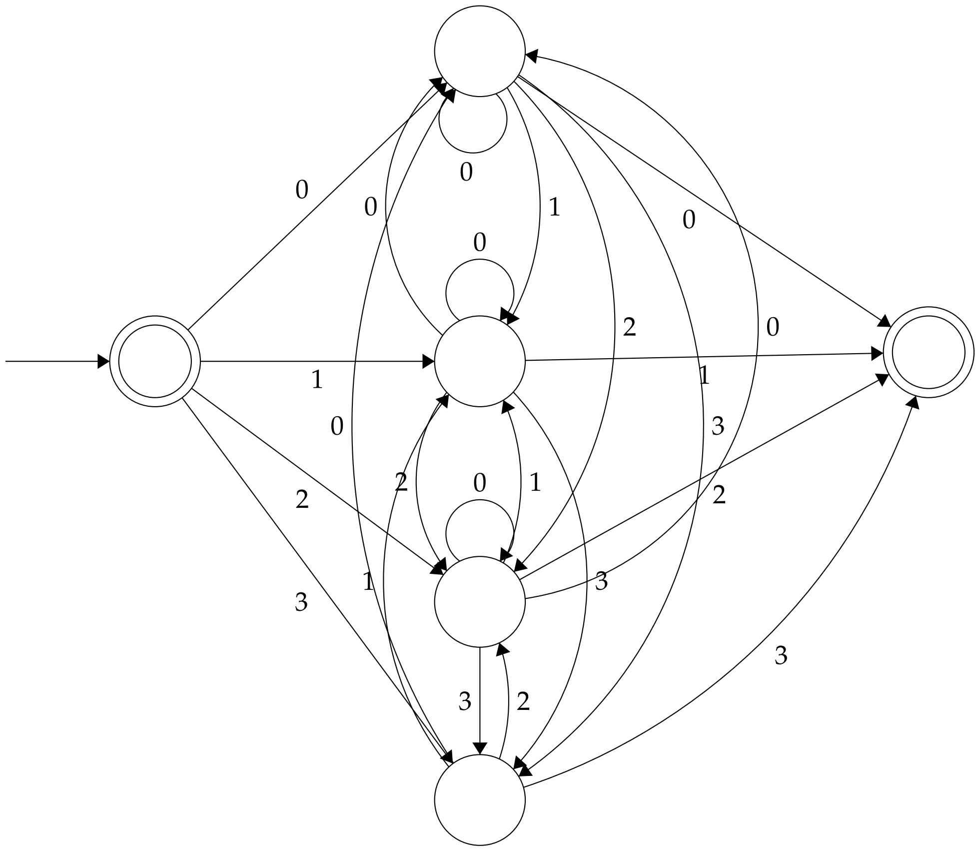
The XOR operation has two important properties:

- It is associative: $ax \oplus (bx \oplus c) = (ax \oplus b)x \oplus c$ - It is commutative: $ax \oplus b = bx \oplus a$

Using these two properties, we can compute the checksum of w^R by reversing the order of the XOR operations used to compute the checksum of w.

Because the $X \oplus R$ operation is commutative and associative, we have $c^R = a_n x \oplus a_{n1} x \oplus ...x \oplus a_1 = a_1 x \oplus a_2 x \oplus ...x \oplus a_n = c$

So the checksum of w^R is the same as the checksum of w, and therefore if w is in A, then w^R is also in A b)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