Heimadæmi 4 - Formleg mál og reiknanleiki

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1.

$$A = \{0^n 1^n \mid n \leq 1\}$$
$$B = \{1^n 0^n \mid n \leq 1\}$$
$$A \cap B = \emptyset$$

2.

To show that DIR is nonregular, we can use the pumping lemma for regular languages. The pumping lemma states that for every regular language L, there exists a pumping length p such that for all strings s in L that are longer than p, we can divide s into three parts, u, v, and w, such that:

u and w are nonempty strings

v can be pumped any number of times (including zero times)

the string $uv^n w$ is still in L for all nonnegative integers n

We will show that no such pumping length exists for DIR.

Suppose that p is a pumping length for DIR. Let s be the string (usr() bin() home (alice () bob ())), which is in DIR and has length greater than p. We can divide s into three parts, u, v, and w, such that:

u = (

v = usr() bin() home

w = (alice () bob ()))

Now, let's consider the string $uv^0 w$. This is the empty string, which is not in DIR. Therefore, DIR is not regular.

Another way to show that DIR is nonregular is to use the following argument:

DIR contains the string (), which is the empty directory. If DIR were regular, then by the closure property of regular languages, DIR would also contain the strings ()()(), ()()()(), and so on. However, these strings do not represent valid directory structures. Therefore, DIR cannot be regular.

In conclusion, DIR is nonregular.

3.

a

The empty string has no symbols, so all three digits are equally frequent. By the tie-breaking rule, the checksum for the empty string is 00.

b

Suppose there exists an NFA N that recognizes the language A. Let $w1_* = 0110202$ be the string from the example in the problem. We construct a new string w by concatenating $w1_*$ with itself n times, for some large integer n.

The checksum of w is the same as the checksum of $w1_*$, which is 02. Therefore, $w \in A$.

However, w is not recognized by N. This is because N can only read a finite number of symbols at a time. For any finite value of n, N will eventually reach a state where it does not know whether the next symbol is a 0 or a 1. Therefore, N cannot recognize w.

This contradiction proves that no NFA exists that can recognize the language A.