Morgan Hodge , 10779528 New Due date(extenuating circumstances): 27/03/2023

Q1 Functions

Consider the function $f: R \to R$, $f(x) = (x-2)^2(x+2)$

A) Turn F into a Polynomial (2)

First
Outside
Inside
Last

first expand

$F(x) = (x-2)^2(x+2)$

$(x-2)(x-2)(x+2)$

$(x^2-4x+4)(x+2) =$

$x(x^2) + x(-4)x + x(4) + 2(x^2) + 2(-4x) + 2(4)$

$= x^3 - 4x^2 + 4x + 2x^2 - 8x + 8$

$= \underline{x^3 - 2x^2 - 4x + 8}$

B) Compute the derivative and second derivative of F (4)

$F(x) = x^3 - 2x^2 - 4x + 8$

$F'(x) = d/dx\,[x^3] - d/dx\,[2x^2] - d/dx\,(4x) + d/dx\,[8]$

First derivative $= \underline{F'(x) = 3x^2 - 4x - 4}$

Second derivative $= \underline{F''(x) = 6x - 4}$

( ) Compute the zero crossings, minima and maxima of f (if any)

$(x-2)^2(x+2)=0$

$x-2=0 \qquad x+2=0$

$x=2 \qquad x=-2$

Roots → crossing with x-axis

$x=2 \qquad x=-2$

$\underline{(2,0)} \qquad \underline{(-2,0)}$

Crossings with y-axies $(0,8)$

$3x^2-4x-4=0$

$(3x+2)(x-2)=0$

$3x+2=0 \qquad x=2$

$3x=-2$

$\underline{x=\frac{-2}{3}}$

$y-(-\frac{2}{3}-2)^2(\frac{-2}{3}+2)=(-\frac{8}{3})^2$

$(-\frac{8}{3})^2 \times \frac{4}{3}=\frac{64}{9} \times \frac{4}{3}=\frac{256}{27}=\underline{9\frac{13}{27}}$

$\underline{MAX=(\frac{-2}{3}, 9\frac{13}{27})} \quad \underline{MIN=(2,0)}$

D) Compute the indefinite integral of F.

$$\int (x^3 - 2x^2 - 4x + 8)\, dx =$$

$$\tfrac{1}{4}x^4 - \tfrac{2}{3}x^3 - 2x^2 + 8x + c$$

E) Compute the integral of F between -4 and 3

$$\int_{-4}^{3} (x^3 - 2x^2 - 4x + 8)\, dx =$$

$$\left[\tfrac{1}{4}x^4 - \tfrac{2}{3}x^3 - 2x^2 + 8x\right]_{-4}^{3} = \tfrac{1}{4} \times 81 - \tfrac{2}{3} \times 27 - 2 \times 9 + 8 \times 3 - \tfrac{1}{4} \times 256 + \tfrac{2}{3} \times 64 + 2 \times 16 - 8 \times 4 =$$

$$\tfrac{81}{4} - 18 - 18 + 24 - 64 + \tfrac{178}{3} + 32 - 32 =$$

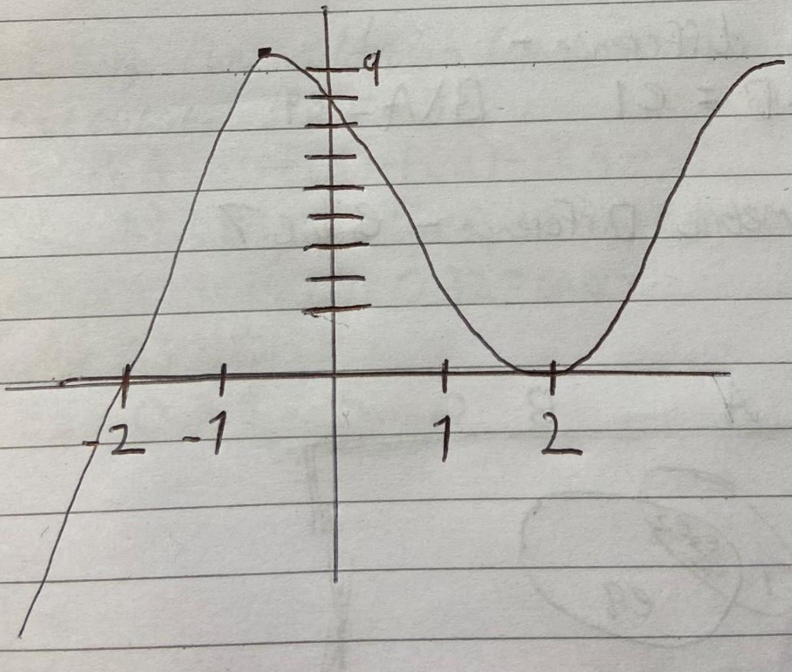
$$-76 + \tfrac{255}{12} = \tfrac{157}{12} = -13\tfrac{1}{12}$$

F) Determine the limits of $f(x)$ when x approaches ~~[illegible]~~ infinity and -infinity

$$\lim_{x \to \infty} f(x) = \infty$$

$$\lim_{x \to -\infty} f(x)$$

G) Draw the graph of f indicating the zero crossings, minima, maxima and limits



Q2 Probability

P(e1) = P(e2) = 0.08, P(e3) = P(e4) = P(e5) = 0.1, P(e6) = P(e7) = 0.2, P(e8) = P(e9) = 0.07

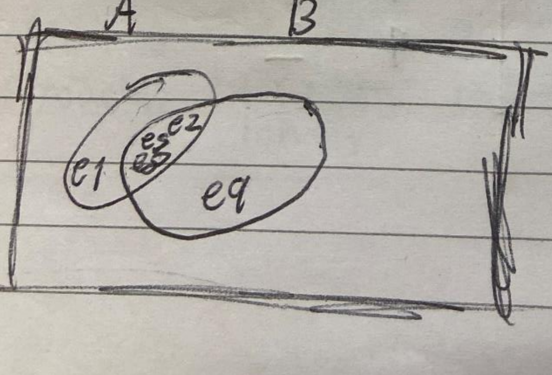
A = e1, e2, e5, e8 B = e2, e5, e8, e9

A) Union - e1, e2, e5, e8, e9 = $A \cup B$

Intersec - e2, e5, e8 = $A \cap B$

Set difference -
$A \setminus B$ = e1 $B \setminus A$ = e9

Symmetric Difference - e1, e7



B) Calculate $P(A)$, $P(B)$ and $P(A \cap B)$

$P(A) = e1, e2, e5, e8 =$
$+ 0.08, 0.08, 0.1, 0.07 = 0.33$

$P(B) = e2, e5, e8, e9 =$
$+ 0.08, 0.1, 0.07, 0.07 = 0.32$

$P(A \cap B) = e2, e5, e8 =$
$0.08, 0.1, 0.07 = 0.25$

C) Using the addition law of Probability,
Calculate $P(A \cup B)$
LAW: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$P(A \cup B) = e1, e2, e5, e8, e9 = 0.40$
$P(A) = 0.33 + P(B) = 0.32 = 0.65$
$P(A \cap B) = 0.25$
$0.65 - 0.25 = \underline{0.40}$

~~D) Calculate P(B') from P(B), also calculate P(B') directly from~~
elementary outcomes of B'

D) List the composition of the event $A \cup B$, and calculate $P(A \cup B)$ by adding the probabilities of the elementary outcomes.

Composition = what its made of

$A \cup B = e1, e2, e5, e8, e9$

$P(A \cup B) = 0.08, 0.08, 0.1, 0.07, 0.07 = \underline{0.4}$

E) Calculate P(B') from P(B), also calculate P(B') directly from the elementry outcomes of B'

- "B'/A'" is the complement of outcomes that are not in "B'/A'"
- To find value I must subtract P(B) by 1 since the sum of all elementry outcomes must be 1

$B' = e1, e3, e4, e6, e7$

Calculate $P(B')$ from $P(B)$

$P(B') = 1 - P(B)$
$P(B) = 0.32$
$1 - 0.32 = \underline{0.68}$

Calculate $P(B')$ directly from B'
$B = e2, e5, e8, e9$
$B' = e1, e3, e4, e6, e7$
$P(B') = 0.08, 0.1, 0.1, 0.2, 0.2 = \underline{0.68}$

F) Calculate the Probability of the Symmetric difference between A and B
- Formular $P(A \Delta B) = P(A \cup B) - P(A \cap B)$

$P(A \cup B) = 0.40$ -
$P(A \cap B) = 0.25$

$\underline{0.15}$