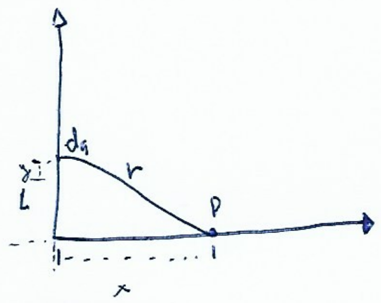
1º



$$dV = \frac{1}{4\pi\varepsilon_0} \cdot dq \qquad dV = \frac{1}{4\pi\varepsilon_0} \cdot \frac{1}{\sqrt{x^2+y^2}} \cdot \lambda\, dy$$

$$r = \sqrt{x^2+y^2}$$

$$dq = \lambda\, dy$$

$$V = \int_L^{2L} \frac{1}{4\pi\varepsilon_0} \cdot \frac{1}{\sqrt{x^2+y^2}} \cdot \lambda\, dy \rightarrow V = \frac{\lambda}{4\pi\varepsilon_0} \cdot \int_L^{2L} \frac{dy}{\sqrt{x^2+y^2}} \rightarrow \frac{\lambda}{4\pi\varepsilon_0} \cdot \ln\left(\sqrt{x^2+y^2}+y\right)\Big|_L^{2L}$$

$$V = \frac{\lambda}{4\pi\varepsilon_0}\left(\ln\left(\frac{\sqrt{x^2+y^2}+2L}{x}\right)\right) - \ln\left(\frac{\sqrt{x^2+L^2}+L}{x}\right) \rightarrow \frac{\lambda}{4\pi\varepsilon_0} \cdot \ln\left(\frac{\frac{\sqrt{x^2+4L^2}+2L}{x}}{\frac{\sqrt{x^2+L^2}+L}{x}}\right)$$

$$V = \frac{\lambda}{4\pi\varepsilon_0} \cdot \ln\left(\frac{\sqrt{x^2+4L^2}+2L}{\sqrt{x^2+L^2}+L}\right)$$

b)

$$V = \frac{Q}{4\pi\varepsilon_0}\left[\ln\left(\frac{2L+\sqrt{x^2+4L^2}}{L+\sqrt{x^2+L^2}}\right)\right] \qquad E_x = -\frac{dV}{dx}$$

$$E_x = \frac{-Q}{4\pi\varepsilon_0}\frac{d}{dx}\left[\ln\left(\frac{2L+\sqrt{x^2+4L^2}}{L+\sqrt{x^2+L^2}}\right)\right] \rightarrow \frac{-Q}{4\pi\varepsilon_0}\frac{d}{dx}\left[\ln\left(2L+\sqrt{x^2+4L^2}\right)-\ln\left(L+\sqrt{x^2+L^2}\right)\right]$$

$$E_x = \frac{-Q}{4\pi\varepsilon_0} \cdot \left[\frac{x}{\sqrt{x^2+4L^2}\left(2L+\sqrt{x^2+4L^2}\right)} - \frac{x}{\sqrt{x^2+L^2}\cdot\left(L+\sqrt{x^2+L^2}\right)}\right]$$

$$E_x = \frac{-Qx}{4\pi\varepsilon_0}\left[\frac{1}{\sqrt{x^2+4L^2}\cdot\left(2L+\sqrt{x^2+4L^2}\right)} - \frac{1}{\sqrt{x^2+L^2}\cdot\left(L+\sqrt{x^2+L^2}\right)}\right]$$

2|

$$\rho = \begin{cases} \rho_0 \left(\frac{R}{r} - \frac{r}{R}\right) \text{sen}\,\theta, & \text{para } r < R \\ 0, & \text{para } r > R \end{cases}$$

$dv = r^2 \cdot \text{sen}\,\theta \, d\theta \, d\phi \, dr$

$0 < \theta < \pi$

$0 < \phi < 2\pi$

$0 < r < R$

a) $\rho = \frac{dq}{dv} \rightarrow \int dq = \int \rho \, dv$

$$Q = \int_0^R \int_0^{2\pi} \int_0^{\pi} \rho_0 \left(\frac{R}{r} - \frac{r}{R}\right) \text{sen}\,\theta^2 \, r^2 \, d\theta \, d\phi \, dr \rightarrow \rho_0 \cdot \int_0^R \left(Rr - \frac{r^3}{R}\right) dr \cdot \int_0^{\pi} \text{sen}^2\theta \, d\theta \cdot \int_0^{2\pi} d\phi$$

$$Q = \rho_0 \cdot \left[R \cdot \frac{r^2}{2}\Big|_0^R - \frac{r^4}{4R}\Big|_0^R \right] \cdot \left[\frac{1}{2}(\theta - \text{sen}\,\theta \cos\theta)\right]\Big|_0^{\pi} \cdot 2\pi$$

$$Q = \rho_0 \cdot \left[\frac{R^3}{2} - \frac{R^3}{4}\right] \cdot \frac{1}{\cancel{2}} \cdot \pi \cdot \cancel{2}\pi \rightarrow Q = \rho_0 \cdot \frac{2R^3 - R^3}{4} \cdot \pi^2 \rightarrow \frac{\rho_0 \cdot R^3 \cdot \pi^2}{4}$$

$$\boxed{Q = \frac{\pi^2 \cdot \rho_0 \cdot R^3}{4}}$$

b) $\oint\!\!\oint \vec{E} \cdot \hat{n} \, dA = \frac{q_{int}}{\varepsilon_0} \rightarrow E \cdot A = \frac{Q}{\varepsilon_0} \rightarrow E = \frac{Q}{4\pi r^2 \varepsilon_0}$

Q para $r < R$:

$$Q = \int_0^r \int_0^{2\pi} \int_0^{\pi} \rho_0 \cdot \left(\frac{R}{r} - \frac{r}{R}\right) \text{sen}^2\theta \, r^2 \, d\theta \, d\phi \, dr \rightarrow \rho_0 \cdot \int_0^r \left(Rr - \frac{r^3}{R}\right) dr \cdot \int_0^{\pi} \text{sen}^2\theta \, d\theta \cdot \int_0^{2\pi} d\phi$$

$$Q = \rho_0 \cdot \left[R \cdot \frac{r^2}{2}\Big|_0^r - \frac{r^4}{4R}\Big|_0^r \right] \cdot \frac{1}{\cancel{2}} \pi \cdot \cancel{2}\pi$$

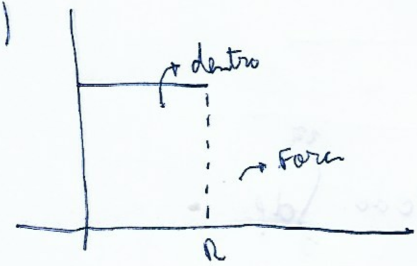
$$Q = \rho_0 \cdot \left(\frac{R \cdot r^2}{2} - \frac{r^4}{4R}\right) \cdot \pi^2 = \rho_0 \cdot r^2 \pi^2 \left(\frac{R}{2} - \frac{r^2}{4R}\right)$$

Substituindo Q e E:

$$E = \frac{\rho_0 \cdot \cancel{r^2} \cdot \pi^{\cancel{2}}}{4\cancel{\pi} \cancel{r^2} \varepsilon_0} \left(\frac{R}{2} - \frac{r^2}{4R}\right) \rightarrow \boxed{E = \frac{\pi \rho_0}{4\varepsilon_0} \left(\frac{R}{2} - \frac{r^2}{4R}\right)}$$

c) $E \cdot (4\pi r^2) = \frac{\rho_0 \pi^2 R^3}{4\varepsilon_0} \rightarrow E = \frac{\rho_0 \pi^{\cancel{2}} R^3}{16 \cancel{\pi} r^2} \rightarrow E = \frac{\rho_0 \pi R^3}{16 \cdot \varepsilon_0 \cdot r^2}$

d)



e)

$V = -\int_{ref}^{r} \vec{E} \cdot \vec{dl}$ × direção radial $\qquad \vec{dl} = dr\hat{r} + r\,d\theta\,\hat{\theta} + r \operatorname{sen} d\phi\,\hat{\phi}$

$\rightarrow V(r) - \int_{\infty}^{r} \frac{r \cdot \rho_0 R^3}{16\varepsilon_0 r^2}\,dr = \frac{r \rho_0 R^3}{16\varepsilon_0} \cdot \left(-\frac{1}{r}\right)\Big|_{\infty}^{r} \rightarrow \frac{r \rho_0 R^3}{16\varepsilon_0}\left(\frac{1}{r} - \frac{1}{\cancel{\infty}}\right)^0$

$$\boxed{V = \frac{r \rho_0 R^3}{16\varepsilon_0 r}}$$