### Lecture – 10

### **Combinational Array Multiplier**

- Composed of arrays of simple combinational elements, each of which implements an add/sub and shift operation for small slices of the multiplicand operands.
- $X = x_{n-1}x_{n-2}.....x_1x_0$  and  $Y = y_{n-1}y_{n-2}.....y_1y_0$  where both X and Y are unsigned integers. Now  $P = X \times Y$  can be expressed as  $P = \sum_{i=0}^{n-1} 2^i x_i Y$  which can be rewritten as  $P = \sum_{i=0}^{n-1} 2^i \left(\sum_{i=0}^{n-1} x_i y_i 2^i\right)$
- It requires  $n \times n$  array of 2-input AND gate.
- The product terms are summed by an array of n(n-1) 1bit full adders.



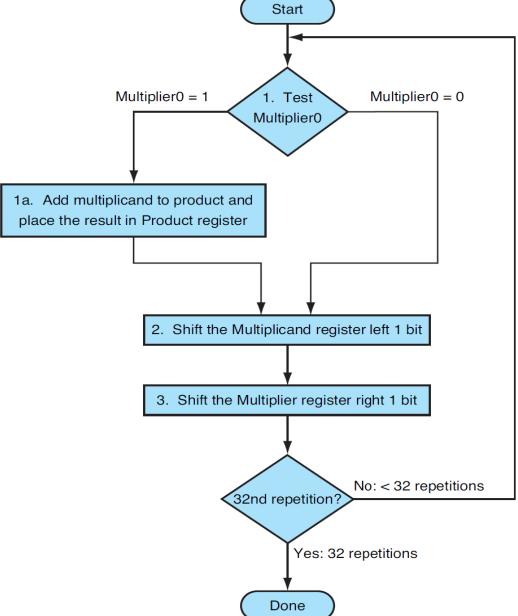
#### Multiplication

$$\begin{array}{c}
1000 \text{ Multiplicand Y} \\
1001 \overline{)} & \text{Multiplier } X = x_3 x_2 x_1 x_0 \\
\hline
1000 \\
0000 \\
1000 \\
\hline
1001000 & \text{Product } P = \sum_{j=0}^{3} x_j 2^j Y
\end{array}$$

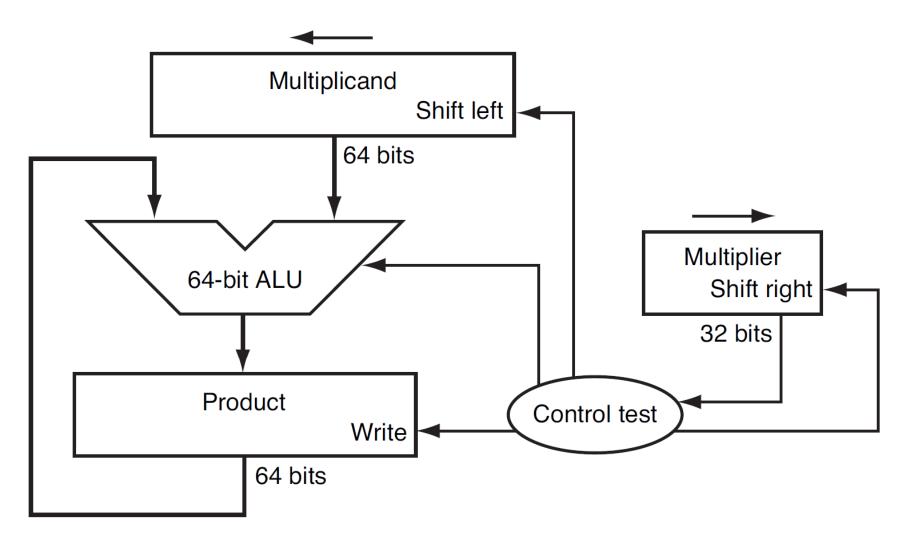
$$P_{i+1} = P_i + x_j 2^i Y$$

- If multiplicand = n bits and multiplier = m bits then product = n + m bits.
- Two rules:
- Place a copy of multiplicand in the proper place if multiplier bit=1.
- Place 0 in the proper place if multiplier bit = 0.

Sequential Multiplication Algorithm



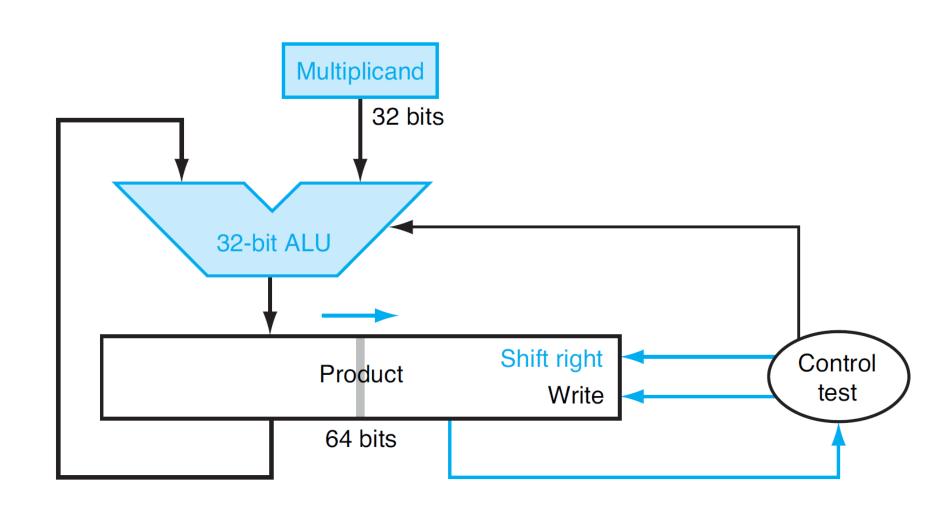
### Sequential Multiplication Hardware



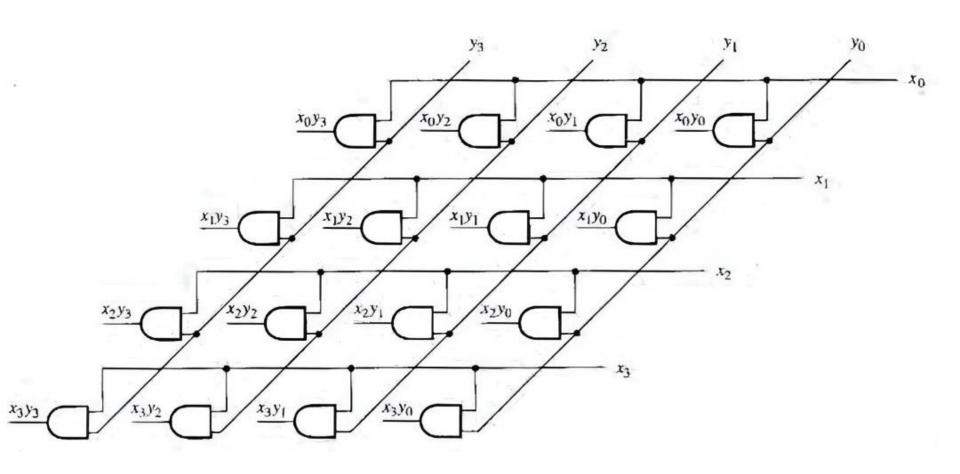
### **Sequential Multiplication Example**

Iteration	Step	Multiplier	Multiplicand	Product
0	Initial values	0011	0000 0010	0000 0000
1	1a: $1 \Rightarrow \text{Prod} = \text{Prod} + \text{Mcand}$	0011	0000 0010	0000 0010
	2: Shift left Multiplicand	0011	0000 0100	0000 0010
	3: Shift right Multiplier	0001	0000 0100	0000 0010
2	1a: $1 \Rightarrow \text{Prod} = \text{Prod} + \text{Mcand}$	0001	0000 0100	0000 0110
	2: Shift left Multiplicand	0001	0000 1000	0000 0110
	3: Shift right Multiplier	0000	0000 1000	0000 0110
3	1: $0 \Rightarrow \text{No operation}$	0000	0000 1000	0000 0110
	2: Shift left Multiplicand	0000	0001 0000	0000 0110
	3: Shift right Multiplier	0000	0001 0000	0000 0110
4	1: $0 \Rightarrow \text{No operation}$	0000	0001 0000	0000 0110
	2: Shift left Multiplicand	0000	0010 0000	0000 0110
	3: Shift right Multiplier	0000	0010 0000	0000 0110

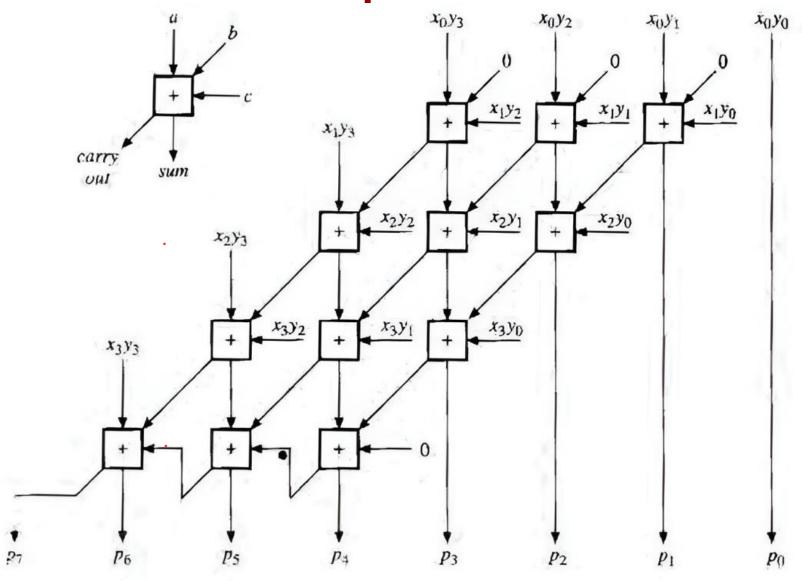
# Refined Version of Multiplication Hardware



# AND Array for 4 X 4 bit Unsigned Multiplication

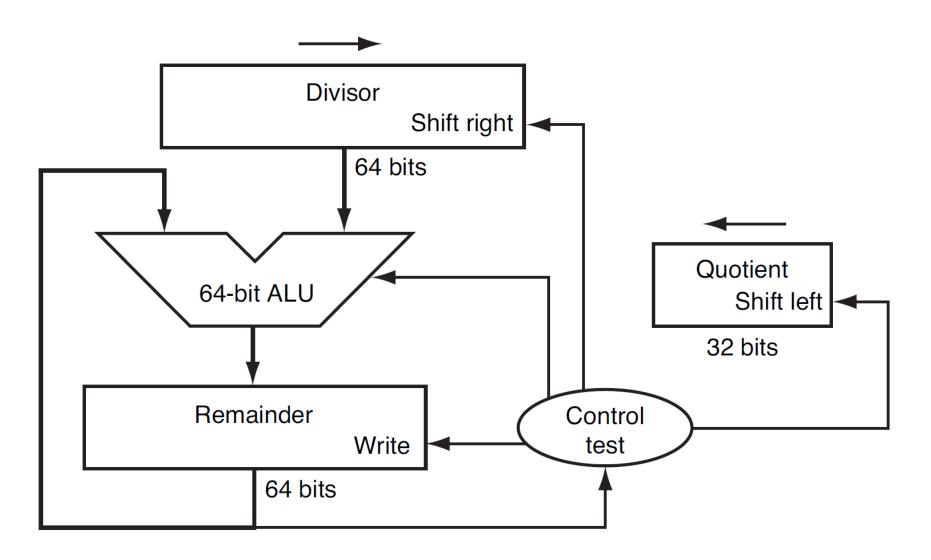


## Full Adder Array for 4 X 4 bit Unsigned Multiplication

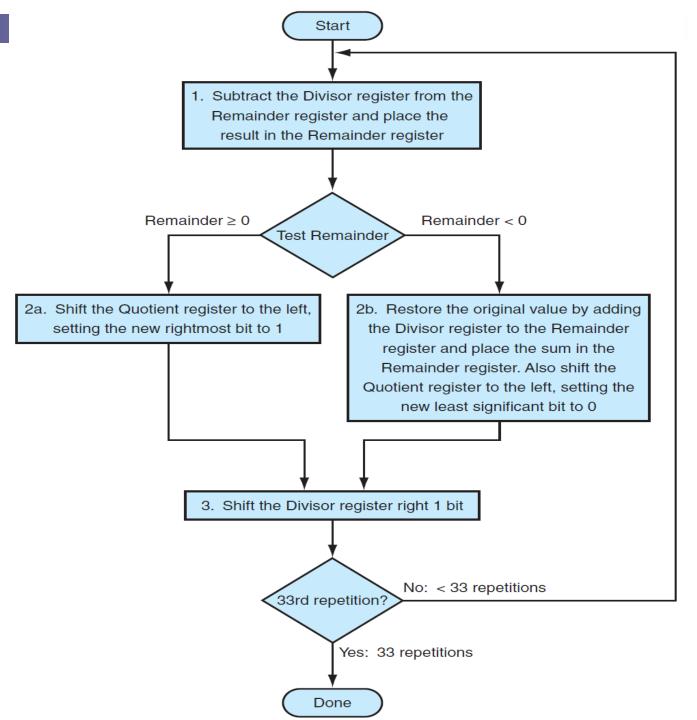


#### **Division**

#### **Division Hardware**



### Division Algorithm



### **Division Example**

Iteration	Step	Quotient	Divisor	Remainder
0	Initial values	0000	0010 0000	0000 0111
1	1: Rem = Rem - Div	0000	0010 0000	①110 0111
	2b: Rem $< 0 \implies$ +Div, sll Q, Q0 = 0	0000	0010 0000	0000 0111
	3: Shift Div right	0000	0001 0000	0000 0111
2	1: Rem = Rem - Div	0000	0001 0000	1111 0111
	2b: Rem $< 0 \implies$ +Div, sII Q, Q0 = 0	0000	0001 0000	0000 0111
	3: Shift Div right	0000	0000 1000	0000 0111
3	1: Rem = Rem - Div	0000	0000 1000	①111 1111
	2b: Rem $< 0 \implies$ +Div, sII Q, Q0 = 0	0000	0000 1000	0000 0111
	3: Shift Div right	0000	0000 0100	0000 0111
4	1: Rem = Rem - Div	0000	0000 0100	0000 0011
	2a: Rem $\geq 0 \implies$ sll Q, Q0 = 1	0001	0000 0100	0000 0011
	3: Shift Div right	0001	0000 0010	0000 0011
5	1: Rem = Rem - Div	0001	0000 0010	0000 0001
	2a: Rem $\geq 0 \implies$ sll Q, Q0 = 1	0011	0000 0010	0000 0001
	3: Shift Div right	0011	0000 0001	0000 0001