Data Communications and Networking Solutions
Chapter 6 Practice Problems - Multiplexing

Reference: Data Communications and Networking by Behrouz A. Forouzan

1 Problem P6-1: FDM Bandwidth Calculation

Problem: Assume that a voice channel occupies a bandwidth of 4 kHz. We need to multiplex 10 voice channels with guard bands of 500 Hz using FDM. Calculate the required bandwidth.

Solution:

$$\begin{aligned} \text{Given:} \quad & \text{Voice channel bandwidth} = 4 \text{ kHz} && (1) \\ & \text{Number of channels} = 10 && (2) \\ & \text{Guard band} = 500 \text{ Hz} = 0.5 \text{ kHz} && (3) \end{aligned}$$

$$\begin{aligned} \text{Total bandwidth} &= (\text{Number of channels} \times \text{Channel BW}) + (\text{Number of guard bands} \times \text{Guard band BW}) && (4) \\ &= (10 \times 4) + (9 \times 0.5) && (5) \\ &= 40 + 4.5 && (6) \\ &= 44.5 \text{ kHz} && (7) \end{aligned}$$

Answer: $\boxed{44.5 \text{ kHz}}$

Explanation: In FDM, we need 9 guard bands between 10 channels (not 10 guard bands). Each guard band prevents interference between adjacent channels.

2 Problem P6-2: Digital Voice Transmission

Problem: We need to transmit 10 digitized voice channels using a passband channel of 20 KHz. What should be the ratio of bits/Hz if we use no guard band?

Solution:

$$\begin{aligned} \text{Given:} \quad & \text{Number of channels} = 10 && (8) \\ & \text{Available bandwidth} = 20 \text{ kHz} && (9) \\ & \text{Standard PCM parameters: 8000 samples/sec, 8 bits/sample} && (10) \end{aligned}$$

$$\begin{aligned} \text{Data rate per channel} &= 8000 \times 8 = 64 \text{ kbps} && (11) \\ \text{Total data rate} &= 10 \times 64 = 640 \text{ kbps} && (12) \\ \text{Bits/Hz ratio} &= \frac{\text{Total data rate}}{\text{Available bandwidth}} && (13) \\ &= \frac{640 \text{ kbps}}{20 \text{ kHz}} = 32 \text{ bits/Hz} && (14) \end{aligned}$$

Answer: $\boxed{32 \text{ bits/Hz}}$

3 Problem P6-4: Synchronous TDM Analysis

Problem: We need to use synchronous TDM and combine 20 digital sources, each of 100 Kbps. Each output slot carries 1 bit from each digital source, but one extra bit is added to each frame for synchronization. Answer the following questions:

Solution:

$$\text{Given:} \quad \text{Number of sources} = 20 \tag{15}$$
$$\text{Data rate per source} = 100 \text{ kbps} \tag{16}$$
$$\text{Bits per slot per source} = 1 \text{ bit} \tag{17}$$
$$\text{Synchronization bits per frame} = 1 \text{ bit} \tag{18}$$

a) Size of output frame in bits:

$$\text{Frame size} = (\text{Number of sources} \times \text{Bits per slot}) + \text{Sync bits} \tag{19}$$
$$= (20 \times 1) + 1 = 21 \text{ bits} \tag{20}$$

b) Output frame rate:

$$\text{Since each source contributes 1 bit per frame:} \tag{21}$$
$$\text{Frame rate} = \frac{\text{Input rate per source}}{\text{Bits per source per frame}} \tag{22}$$
$$= \frac{100,000 \text{ bps}}{1 \text{ bit}} = 100,000 \text{ frames/sec} \tag{23}$$

c) Duration of output frame:

$$\text{Frame duration} = \frac{1}{\text{Frame rate}} \tag{24}$$
$$= \frac{1}{100,000} = 10 \times 10^{-6} \text{ sec} = 10 \text{ s} \tag{25}$$

d) Output data rate:

$$\text{Output data rate} = \text{Frame size} \times \text{Frame rate} \tag{26}$$
$$= 21 \times 100,000 = 2,100,000 \text{ bps} = 2.1 \text{ Mbps} \tag{27}$$

e) Efficiency of the system:

$$\text{Total useful bits} = 20 \times 100,000 = 2,000,000 \text{ bps} \tag{28}$$
$$\text{Efficiency} = \frac{\text{Useful bits}}{\text{Total bits}} \tag{29}$$
$$= \frac{2,000,000}{2,100,000} = \frac{20}{21} = 0.952 = 95.2\% \tag{30}$$

Answers:

$$\text{a)} \ \boxed{21 \text{ bits}} \tag{31}$$
$$\text{b)} \ \boxed{100,000 \text{ frames/sec}} \tag{32}$$
$$\text{c)} \ \boxed{10 \text{ s}} \tag{33}$$
$$\text{d)} \ \boxed{2.1 \text{ Mbps}} \tag{34}$$
$$\text{e)} \ \boxed{95.2\%} \tag{35}$$

4 Problem P6-5: Modified Synchronous TDM

Problem: Repeat Problem 6-4 if each output slot carries 2 bits from each source.
Solution:

$$\begin{aligned}\text{Given:}\quad & \text{Number of sources} = 20 && (36)\\ & \text{Data rate per source} = 100 \text{ kbps} && (37)\\ & \text{Bits per slot per source} = 2 \text{ bits} && (38)\\ & \text{Synchronization bits per frame} = 1 \text{ bit} && (39)\end{aligned}$$

a) Size of output frame in bits:

$$\text{Frame size} = (20 \times 2) + 1 = 41 \text{ bits} \tag{40}$$

b) Output frame rate:

$$\text{Frame rate} = \frac{100,000 \text{ bps}}{2 \text{ bits}} = 50,000 \text{ frames/sec} \tag{41}$$

c) Duration of output frame:

$$\text{Frame duration} = \frac{1}{50,000} = 20 \times 10^{-6} \text{ sec} = 20 \text{ s} \tag{42}$$

d) Output data rate:

$$\text{Output data rate} = 41 \times 50,000 = 2,050,000 \text{ bps} = 2.05 \text{ Mbps} \tag{43}$$

e) Efficiency:

$$\text{Efficiency} = \frac{2,000,000}{2,050,000} = \frac{40}{41} = 0.976 = 97.6\% \tag{44}$$

Answers:

$$\text{a) } \boxed{41 \text{ bits}} \tag{45}$$

$$\text{b) } \boxed{50,000 \text{ frames/sec}} \tag{46}$$

$$\text{c) } \boxed{20 \text{ s}} \tag{47}$$

$$\text{d) } \boxed{2.05 \text{ Mbps}} \tag{48}$$

$$\text{e) } \boxed{97.6\%} \tag{49}$$

5 Problem P6-6: Statistical TDM

Problem: We have 14 sources, each creating 500 8-bit characters per second. Since only some of these sources are active at any moment, we use statistical TDM to combine these sources using character interleaving. Each frame carries 6 slots at a time, but we need to add 4-bit addresses to each slot.
Solution:

$$\begin{aligned}\text{Given:}\quad & \text{Number of sources} = 14 && (50)\\ & \text{Characters per source} = 500 \text{ char/sec} && (51)\\ & \text{Bits per character} = 8 && (52)\\ & \text{Slots per frame} = 6 && (53)\\ & \text{Address bits per slot} = 4 && (54)\end{aligned}$$

a) Size of output frame in bits:

$$\text{Data per slot} = 8 + 4 = 12 \text{ bits} \tag{55}$$
$$\text{Frame size} = 6 \times 12 = 72 \text{ bits} \tag{56}$$

b) Output frame rate:

$$\text{Total input rate} = 14 \times 500 = 7000 \text{ characters/sec} \tag{57}$$
$$\text{Frame rate} = \frac{7000}{6} = 1166.67 \text{ frames/sec} \tag{58}$$

c) Duration of output frame:

$$\text{Frame duration} = \frac{1}{1166.67} = 857.14 \times 10^{-6} \text{ sec} = 857.14 \text{ s} \tag{59}$$

d) Output data rate:

$$\text{Output data rate} = 72 \times 1166.67 = 84,000 \text{ bps} = 84 \text{ kbps} \tag{60}$$

Answers:

$$\text{a)}\ \boxed{72 \text{ bits}} \tag{61}$$
$$\text{b)}\ \boxed{1166.67 \text{ frames/sec}} \tag{62}$$
$$\text{c)}\ \boxed{857.14 \text{ s}} \tag{63}$$
$$\text{d)}\ \boxed{84 \text{ kbps}} \tag{64}$$

6 Problem P6-7: Multilevel TDM

Problem: Ten sources, six with a bit rate of 200 kbps and four with a bit rate of 400 kbps, are to be combined using multilevel TDM with no synchronizing bits.

Solution:

$$\text{Given:}\quad \text{6 sources at 200 kbps each} \tag{65}$$
$$\text{4 sources at 400 kbps each} \tag{66}$$
$$\text{No synchronizing bits} \tag{67}$$

a) Size of a frame in bits:

$$\text{For equal slot allocation, use LCM approach:} \tag{68}$$
$$\text{LCM}(200, 400) = 400 \tag{69}$$
$$\text{Time for one frame} = \frac{1}{400,000} \text{ sec} \tag{70}$$
$$\text{Bits from 200 kbps sources} = 6 \times 1 = 6 \text{ bits} \tag{71}$$
$$\text{Bits from 400 kbps sources} = 4 \times 2 = 8 \text{ bits} \tag{72}$$
$$\text{Frame size} = 6 + 8 = 14 \text{ bits} \tag{73}$$

b) Frame rate:

$$\text{Frame rate} = 400,000 \text{ frames/sec} \tag{74}$$

c) Duration of a frame:

$$\text{Frame duration} = \frac{1}{400,000} = 2.5 \times 10^{-6} \text{ sec} = 2.5 \text{ s} \tag{75}$$

d) Data rate:

$$\text{Total input rate} = (6 \times 200) + (4 \times 400) = 1200 + 1600 = 2800 \text{ kbps} \tag{76}$$
$$\text{Output data rate} = 2800 \text{ kbps} = 2.8 \text{ Mbps} \tag{77}$$

Answers:

$$\text{a) } \boxed{14 \text{ bits}} \tag{78}$$
$$\text{b) } \boxed{400,000 \text{ frames/sec}} \tag{79}$$
$$\text{c) } \boxed{2.5 \text{ s}} \tag{80}$$
$$\text{d) } \boxed{2.8 \text{ Mbps}} \tag{81}$$

7 Problem P6-8: Multiple-slot TDM

Problem: Four channels, two with a bit rate of 200 kbps and two with a bit rate of 150 kbps, are to be multiplexed using multiple-slot TDM with no synchronization bits.

Solution:

$$\text{Given: } \text{2 channels at 200 kbps each} \tag{82}$$
$$\text{2 channels at 150 kbps each} \tag{83}$$
$$\text{No synchronization bits} \tag{84}$$

a) Size of a frame in bits:

$$\text{LCM}(200, 150) = 600 \tag{85}$$
$$\text{Frame duration} = \frac{\text{LCM}}{200 \times 150 \times \text{gcd}(200, 150)} = \frac{1}{200,000} \text{ sec} \tag{86}$$
$$\text{Ratio } 200 : 150 = 4 : 3 \tag{87}$$
$$\text{Total ratio parts} = 4 + 4 + 3 + 3 = 14 \tag{88}$$
$$\text{Frame size} = 14 \text{ bits} \tag{89}$$

b) Frame rate:

$$\text{Frame rate} = \frac{200,000}{4} = 50,000 \text{ frames/sec} \tag{90}$$

c) Duration of a frame:

$$\text{Frame duration} = \frac{1}{50,000} = 20 \times 10^{-6} \text{ sec} = 20 \text{ s} \tag{91}$$

d) Data rate:

$$\text{Total input rate} = (2 \times 200) + (2 \times 150) = 400 + 300 = 700 \text{ kbps} \tag{92}$$
$$\text{Output data rate} = 700 \text{ kbps} \tag{93}$$

Answers:

$$\text{a) } \boxed{14 \text{ bits}} \tag{94}$$
$$\text{b) } \boxed{50,000 \text{ frames/sec}} \tag{95}$$
$$\text{c) } \boxed{20 \text{ s}} \tag{96}$$
$$\text{d) } \boxed{700 \text{ kbps}} \tag{97}$$

8 Problem P6-9: Pulse-stuffing TDM

Problem: Two channels, one with a bit rate of 190 kbps and another with a bit rate of 180 kbps, are to be multiplexed using pulse-stuffing TDM with no synchronization bits.

Solution:

$$\text{Given:} \quad \text{Channel 1: 190 kbps} \tag{98}$$
$$\text{Channel 2: 180 kbps} \tag{99}$$
$$\text{No synchronization bits} \tag{100}$$

For pulse-stuffing TDM, we use a common higher rate:

$$\text{Common rate} = \max(190, 180) = 190 \text{ kbps} \tag{101}$$
$$\text{Stuffing rate for channel 2} = 190 - 180 = 10 \text{ kbps} \tag{102}$$

a) Size of a frame in bits:

$$\text{Frame time} = \frac{1}{190,000} \text{ sec} \tag{103}$$
$$\text{Bits per channel per frame} = 1 \text{ bit} \tag{104}$$
$$\text{Frame size} = 2 \text{ bits} \tag{105}$$

b) Frame rate:

$$\text{Frame rate} = 190,000 \text{ frames/sec} \tag{106}$$

c Duration of a frame:

$$\text{Frame duration} = \frac{1}{190,000} = 5.26 \times 10^{-6} \text{ sec} = 5.26 \text{ s} \tag{107}$$

d) Data rate:

$$\text{Output data rate} = 2 \times 190 = 380 \text{ kbps} \tag{108}$$

Answers:

$$\text{a)} \ \boxed{2 \text{ bits}} \tag{109}$$
$$\text{b)} \ \boxed{190,000 \text{ frames/sec}} \tag{110}$$
$$\text{c)} \ \boxed{5.26 \text{ s}} \tag{111}$$
$$\text{d)} \ \boxed{380 \text{ kbps}} \tag{112}$$

9 Problem P6-10: T-1 Line Analysis

Problem: Answer the following questions about a T-1 line:

Solution:

$$\text{T-1 specifications:} \quad \text{24 voice channels} \tag{113}$$
$$\text{8 bits per sample per channel} \tag{114}$$
$$\text{8000 samples per second per channel} \tag{115}$$
$$\text{1 framing bit per frame} \tag{116}$$

a) Duration of a frame:

$$\text{Frame rate} = 8000 \text{ frames/sec} \tag{117}$$

$$\text{Frame duration} = \frac{1}{8000} = 125 \times 10^{-6} \text{ sec} = 125 \text{ s} \tag{118}$$

b) Overhead (number of extra bits per second):

$$\text{Framing bits per second} = 8000 \times 1 = 8000 \text{ bits/sec} = 8 \text{ kbps} \tag{119}$$

Answers:

$$\text{a) } \boxed{125 \text{ s}} \tag{120}$$

$$\text{b) } \boxed{8 \text{ kbps}} \tag{121}$$

10 Problem P6-11: Synchronous TDM Character Interleaving

Problem: Show the contents of the five output frames for a synchronous TDM multiplexer that combines four sources sending the following characters. Note that the characters are sent in the same order that they are typed. The third source is silent.

Given:

Source 1 message: HELLO (122)

Source 2 message: HI (123)

Source 3 message: (silent) (124)

Source 4 message: BYE (125)

Solution:

Frame 1: H (S1), H (S2), _ (S3), B (S4) (126)

Frame 2: E (S1), I (S2), _ (S3), Y (S4) (127)

Frame 3: L (S1), _ (S2), _ (S3), E (S4) (128)

Frame 4: L (S1), _ (S2), _ (S3), _ (S4) (129)

Frame 5: O (S1), _ (S2), _ (S3), _ (S4) (130)

where _ represents empty slots due to sources being silent or having no more data.

11 Problem P6-12: TDM Multiplexer Output Stream

Problem: Figure 6.34 shows a multiplexer in a synchronous TDM system. Each output slot is only 10 bits long (3 bits taken from each input plus 1 framing bit). What is the output stream?

Given inputs:

Input 1: 101110111101 (131)

Input 2: 111111100000 (132)

Input 3: 101000001111 (133)

Solution:

$$\text{Frame 1: } 1(\text{framing}) + 101(\text{I1}) + 111(\text{I2}) + 101(\text{I3}) = 1101111101 \tag{134}$$

$$\text{Frame 2: } 1(\text{framing}) + 110(\text{I1}) + 111(\text{I2}) + 000(\text{I3}) = 1110111000 \tag{135}$$

$$\text{Frame 3: } 1(\text{framing}) + 111(\text{I1}) + 100(\text{I2}) + 001(\text{I3}) = 1111100001 \tag{136}$$

$$\text{Frame 4: } 1(\text{framing}) + 101(\text{I1}) + 000(\text{I2}) + 111(\text{I3}) = 1101000111 \tag{137}$$

Output stream: $\boxed{1101111101\ 1110111000\ 1111100001\ 1101000111}$

12 Problem P6-13: TDM Demultiplexer

Problem: Figure 6.35 shows a demultiplexer in a synchronous TDM. If the input slot is 16 bits long (no framing bits), what is the bit stream in each output?

Given input stream:

$$\text{Input: } 10100000\ 10101010\ 10100000\ 01110000\ 11110000 \tag{138}$$

Solution: Each 16-bit group is divided equally among outputs (assuming 4 outputs, 4 bits each):

$$\text{Group 1: } 10100000\ 10101010 \tag{139}$$
$$\text{Output 1: } 1010 \tag{140}$$
$$\text{Output 2: } 0000 \tag{141}$$
$$\text{Output 3: } 1010 \tag{142}$$
$$\text{Output 4: } 1010 \tag{143}$$

$$\text{Group 2: } 10100000\ 01110000 \tag{144}$$
$$\text{Output 1: } 1010 + 1010 = 10101010 \tag{145}$$
$$\text{Output 2: } 0000 + 0000 = 00000000 \tag{146}$$
$$\text{Output 3: } 1010 + 0111 = 10100111 \tag{147}$$
$$\text{Output 4: } 1010 + 0000 = 10100000 \tag{148}$$

13 Problem P6-15: FHSS Analysis

Problem: What is the minimum number of bits in a PN sequence if we use FHSS with a channel bandwidth of B = 4 KHz and B_{ss} = 100 KHz?

Solution:

$$\text{Given:} \quad B = 4 \text{ kHz (channel bandwidth)} \tag{149}$$
$$B_{ss} = 100 \text{ kHz (spread spectrum bandwidth)} \tag{150}$$

$$\text{Number of channels} = \frac{B_{ss}}{B} = \frac{100}{4} = 25 \text{ channels} \tag{151}$$
$$\text{Minimum PN bits} = \lceil \log_2(25) \rceil = \lceil 4.64 \rceil = 5 \text{ bits} \tag{152}$$

Answer: $\boxed{\text{5 bits}}$

14 Problem P6-16: FHSS System Analysis

Problem: An FHSS system uses a 4-bit PN sequence. If the bit rate of the PN is 64 bits per second, answer the following questions:

Solution:

$$\text{Given:} \quad \text{PN sequence length} = 4 \text{ bits} \tag{153}$$
$$\text{PN bit rate} = 64 \text{ bps} \tag{154}$$

a) Total number of possible channels:

$$\text{Number of channels} = 2^4 = 16 \text{ channels} \tag{155}$$

b) Time needed to finish a complete cycle of PN:

$$\text{Cycle time} = \frac{\text{PN sequence length}}{\text{PN bit rate}} \tag{156}$$
$$= \frac{4}{64} = 0.0625 \text{ sec} = 62.5 \text{ ms} \tag{157}$$

Answers:

$$\text{a) } \boxed{\text{16 channels}} \tag{158}$$
$$\text{b) } \boxed{\text{62.5 ms}} \tag{159}$$

15 Problem P6-17: Pseudorandom Number Generator

Problem: A pseudorandom number generator uses the following formula to create a random series: $N_{i+1} = (S + 7N_i) \bmod 17 - 1$

In which N_i defines the current random number and N_{i+1} defines the next random number. The term (mod 17) means the value of the remainder when dividing (S + $7N_i$) by 17. Show the sequence created by this generator to be used for spread spectrum.

Solution: Starting with $N_0 = S$ and using the formula $N_{i+1} = (S + 7N_i) \bmod 17 - 1$:

Let's assume $S = 1$ (seed value):

$$N_0 = 1 \tag{160}$$
$$N_1 = (1 + 7 \times 1) \bmod 17 - 1 = 8 - 1 = 7 \tag{161}$$
$$N_2 = (1 + 7 \times 7) \bmod 17 - 1 = 50 \bmod 17 - 1 = 16 - 1 = 15 \tag{162}$$
$$N_3 = (1 + 7 \times 15) \bmod 17 - 1 = 106 \bmod 17 - 1 = 4 - 1 = 3 \tag{163}$$
$$N_4 = (1 + 7 \times 3) \bmod 17 - 1 = 22 \bmod 17 - 1 = 5 - 1 = 4 \tag{164}$$
$$N_5 = (1 + 7 \times 4) \bmod 17 - 1 = 29 \bmod 17 - 1 = 12 - 1 = 11 \tag{165}$$

Sequence: $\boxed{1, 7, 15, 3, 4, 11, \ldots}$

16 Problem P6-18: DSSS Voice Channels

Problem: We have a digital medium with a data rate of 10 Mbps. How many 64-kbps voice channels can be carried by this medium if we use DSSS with the Barker sequence?

Solution:

$$\text{Given: Medium data rate} = 10 \text{ Mbps} \tag{166}$$
$$\text{Voice channel rate} = 64 \text{ kbps} \tag{167}$$
$$\text{Barker sequence length} = 11 \text{ chips} \tag{168}$$

$$\text{Spreading factor} = 11 \tag{169}$$
$$\text{Effective data rate per voice channel} = 64 \times 11 = 704 \text{ kbps} \tag{170}$$
$$\text{Number of voice channels} = \frac{10,000}{704} = 14.2 \tag{171}$$
$$\text{Maximum channels} = \lfloor 14.2 \rfloor = 14 \tag{172}$$

Answer: $\boxed{\text{14 voice channels}}$