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Article *in* Studies in Computational Intelligence · December 2015

DOI: 10.1007/978-3-319-11017-2_7

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Abstract Bio-inspired computation is a field of study that connects together numerous subfields of connectionism (neural network), social behavior, emergence field of artificial intelligence and machine learning algorithms for complex problem optimization. Bio-inspired computation is motivated by nature and over the last few years, it has encouraged numerous advance algorithms and set of computational tools for dealing with complex combinatorial optimization problems. Black Hole is a new bio-inspired metaheuristic approach based on observable fact of black hole phenomena. It is a population based algorithmic approach like genetic algorithm (GAs), ant colony optimization (ACO) algorithm, particle swarm optimization (PSO), firefly and other bio-inspired computation algorithms. The objective of this book chapter is to provide a comprehensive study of black hole approach and its applications in different research fields like data clustering problem, image processing, data mining, computer vision, science and engineering. This chapter provides with the stepping stone for future researches to unveil how metaheuristic and bio-inspired commutating algorithms can improve the solutions of hard or complex problem of optimization.

Keywords Metaheuristic · Black hole · Swarm intelligence · K-means · Clustering

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A.T. Azar and S. Vaidyanathan (eds.), *Computational Intelligence Applications in Modeling and Control*, Studies in Computational Intelligence 575,
DOI 10.1007/978-3-319-11017-2_7

1 Introduction

Bio-inspired computation and swarm intelligence based algorithms have attracted significant attention in recent years for solving the complex and combinatorial optimization problems of data clustering, feature selection and maximization of matching scores for authentication of human in biometrics [1] computer vision, data mining and machine learning based algorithms. Motivated from the natural and social behavioral phenomena, bio-inspired computation algorithms have significant research area during the recent years from both multidisciplinary research and the scientific research purpose. In the last 30 years, a great interest has been devoted to bio-inspired metaheuristics and it has encouraged and provides successful algorithms and computational simulated tools for dealing with complex and optimization problems (ISA Trans [2]). Several of these approaches are motivated from natural processes and generally start with an initial set of variables and then evolve to obtain the global minimum or maximum of the objective function and it has been an escalating interest in algorithms motivated by the behaviors of natural phenomena which are incorporated by many scientists and researchers to solve hard optimization problems. Hard problems cannot be solved to optimality, or to any guaranteed bound by any exact (deterministic) method within a '*reasonable*' time limit [3–10]. It is computational problems such as optimization of objective functions [11, 12] pattern recognition [1, 13] control objectives [2, 4, 14], image processing [15, 16] and filter modeling [17, 18] etc. There are different heuristic approaches that have been implemented by researches so far, for example Genetic algorithm [10] is the most well-known and mostly used evolutionary computation technique and it was developed in the early 1970s at the University of Michigan by John Holland and his students, whose re-search interests were devoted to the study of adaptive systems [19]. The basic genetic algorithm is very general and it can be implemented differently according to the problem: representation of solution (chromosomes), selection strategy, type of crossover (the recombination operator) and mutation operators. The fixed-length binary string is the most common representation of the chromosomes applied in GAs and a simple bit manipulation operation allow the implementation of crossover and mutation operations. Emphasis is mainly concentrated on crossover as the main variation operator that combines multiple (generally two) individuals that have been selected together by exchanging some of their parts. An exogenous parameter pc$\in$ [0.6, 1.0] (crossover rate) indicates the probability per individual to undergo crossover. After evaluating the fitness value of each individual in the selection pool, Individuals for producing offspring are selected using a selection strategy. A few of the popular selection schemes are mainly roulette-wheel selection, tournament selection and ranking selection, etc. After crossover operation, individuals are subjected to mutation process. Mutation initiates some randomness into the search space to prevent the optimization process from getting trapped into local optima. Naturally, the mutation rate is applied with less than 1 % probability but the appropriate value of the mutation rate for a given optimization problem is an open issue in research.

Simulated Annealing [9] is inspired by the annealing technique used by the different metallurgists to get a "well ordered" solid state of minimal energy (while avoiding the "meta stable" structures, characteristic of the local minima of energy), Ant Colony optimization (ACO) algorithm is a metaheuristic technique to solve problems that has been motivated by the ants' social behaviors in finding shortest paths. Real ants walk randomly until they find food and return to their nest while depositing pheromone on the ground in order to mark their preferred path to attract other ants to follow [6, 20, 21], Particle Swarm Optimization (PSO) was introduced by James Kennedy and Russell Eberhart as a global optimization technique in 1995. It uses the metaphor of the flocking behavior of birds to solve optimization problems [22], firefly algorithm is a population based metaheuristic algorithm. It has become an increasingly important popular tool of Swarm Intelligence that has been applied in almost all research area so of optimization, as well as science and engineering practice. Fireflies have their flashing light. There are two fundamental functions of flashing light of firefly: (1) to attract mating partners and (2) to warn potential predators. But, the flashing lights comply with more physical rules. On the one hand, the light intensity of source (I) decrease as the distance (r) increases according to the term $I \propto 1/r^2$. This phenomenon inspired [23] to develop the firefly algorithm [23–25], Bat-inspired algorithm is a metaheuristic optimization algorithm. It was invented by Yang et al. [26–28] and it is based on the echolocation behavior of microbats with varying pulse rates of emission and loudness. And honey bee algorithm [29] etc. Such algorithms are progressively analyzed, deployed and powered by different researchers in many different research fields [3, 5, 27, 28, 30–32]. These algorithms are used to solve different optimization problems. But, there is no specific algorithm or direct algorithms to achieve the best solution for all optimization problems. Numerous algorithms give a better solution for some particular problems than others. Hence, searching for new heuristic optimization algorithms is an open problem [29] and it requires a lot of exploration of new metaheuristic algorithms for solving of hard problems.

Recently, one of the metaheuristic approaches has been developed for solving the hard or complex optimization and data clustering problem which is NP-hard problem known as black hole heuristic approach. Black Hole heuristic algorithm is inspired by the black hole phenomenon and black hole algorithm (BH) starts with an initial population of candidate solutions to an optimization problem and an objective function that is calculated for them similar to other population-based algorithms.

2 Heuristics Algorithm and Metaheuristic Algorithm

2.1 Heuristics Algorithm

The "*heuristic*" is Greek word and means *"to know*", "*to find*", "*to discover*" or "to guide an investigation" by trial and error methodology [33]. Specifically, heuristics are techniques which search for near-optimal solutions of problem at a reasonable computational cost without being able to guarantee either feasibility or optimality, or even in many cases to state how close to optimality a particular feasible solution is? [34]. The main algorithmic characteristic of heuristic is based on mimic physical or biological processes which are motivated by nature phenomena. Quality solution to complex optimization problems can be founded in reasonable amount of time however, there is no guarantee that is optimal solutions are reached. A heuristic is a technique designed for solving a problem more quickly when classic methods are too slow, or for finding an approximate solution when classic methods or deterministic approaches fail to provides any exact solution of a hard or complex problem. This is achieved by trading optimality, completeness, accuracy or precision for speed. Heuristic search exploits additional knowledge about the problem that helps direct search to more promising paths [23].

2.2 Metaheuristic Algorithm

Metaheuristic algorithms are the master strategy key that modify and update the other heuristic produced solution that is normally generated in the quest of local optimal. These nature-inspired algorithms are becoming popular and powerful in solving optimization problems. The suffix "meta" Greek means upper level methodology, beyond or higher level and it generally perform better than simple heuristic approach. Metaheuristic algorithms are conceptual set of all heuristic approach which is used to find the optimal solution of a combinatorial optimization problem. The term "metaheuristic" was introduced by Sir F. Glover in research paper. In addition, metaheuristic algorithms use certain trade-off of randomization and local search for finding the optimal and near optimal solution. Local search is a general methodology for finding the high quality solutions to complex or hard combinatorial optimization problems in reasonable amount of time. It is basically an iterative based search approach to diversification of neighbor of solutions trying to enhance the current solution by local changes [23, 35].

2.2.1 Characteristics of Metaheuristics

Each meta-heuristic search process depends on balancing between two major components which is involved through-out search of optimal solution. Two main components are known as diversification (exploration) and intensification (exploitation) [35].

2.2.2 Diversification or Exploration

Diversification phase ensures that the algorithm explores the search space more efficiently and thoroughly and helps to generate diverse solutions. On other hand, when diversification is too much, it will increase the probability of finding the true optimality globally solutions. But, it will often to slow the exploration process with much low rate of convergence of problem.

2.2.3 Intensification or Exploitation

It uses the local information in search process to generate better solutions of problems. If there is too much intensification, it will may lead to converge rapidly often to a local optimum or a wrong solution with respect to problem and reduce the probability of finding the global optimum solutions of a complex problem. Therefore, there is requirement of fine tuning or a fine balance and trade-off between intensification and diversification characteristics of metaheuristic approach. These metaheuristic techniques are in combination with the solutions of the best solutions of the complex combinatorial optimization problems. The main objective behind the best solution of metaheuristic ensures that the solution will converge to the optimization, while the diversification via randomization avoids the solution beings trapped or struck at local minima at the same time and increase the diversity of the solutions of hard problems and solving optimization problems with multiple often conflicting objectives is generally a very difficult target. The good combinations of these two major components (diversification and intensification) will usually ensure that the global optimality of given hard or complex is achievable and it provides a way solve large size population based problem instances by delivering the efficient solution in reasonable amount of time.

In short, metaheuristics are high level strategies for diversifying search spaces by using different algorithmic approach. Of great importance hereby is that a dynamic balance is given between diversification and intensification. The term diversification generally refers to the exploration of the search space, whereas the term intensification refers to the exploitation of the accumulated search experience [36].

3 Classification of Bio-inspired Metaheuristic Algorithm

Bio-inspired metaheuristic algorithms can be classified as: population based algorithm, trajectory based, swarm intelligence based, artificial intelligence based and bio-insect behavior based approaches. Some of the most famous algorithms are Genetic Algorithm (GAs), Simulated Annealing (SA) [9], Artificial Immune System (AIS) [7], Ant Colony Optimization (ACO) [6], Particle Swarm Optimization (PSO) [22] and Bacterial Foraging Algorithm (BFA) [37]. Genetic Algorithm (GAs) are enthused from Darwinian evolutionary theory [10], simulated annealing method mimics the thermodynamics process of cooling of molten metal for getting the minimum free energy state. It works with a point and at each iteration builds a point according to the Boltzmann probability distribution [9, 38]. Artificial immune systems (AIS) simulate biological immune systems [7], ant colony optimization (ACO) field study a model derived from the observation of real ant's behavior and uses these models as source of inspiration for the design of innovative and novel approaches for solution of optimization and distributed control problems. The main objective of ant colony algorithm is that the self organizing principle which allows the highly coordinated behavior of ants can be exploited to coordinate transport, Bacterial foraging algorithm (BFA) comes from search and optimal foraging of bacteria and particle swarm optimization (PSO) simulates the behavior of flock of birds and fish schooling which search for best solution in both local and global search space [5, 22, 38]. Based on bio-inspired characteristics, various algorithms are illustrated as below (Table 1).

Unlike exact algorithm methodologies (it is guaranteed to find the optimal solution and to prove its optimality for every finite size instance of a combinatorial optimization problem within an instance dependent run time.). The metaheuristic algorithms ensure to find the optimal solution of a given hard problem and reasonable amount of time. The application of metaheuristic falls into a large number of area some of them are as follows:

- Engineering design, topological optimization, structural optimizations in electronics and VLSI design, aerodynamics based structural design.
- Fluid dynamics, telecommunication field, automotives and robotics design and robotic roadmap planning optimization.
- In data mining and machine learning: Data mining in bioinformatics, computational biology.
- System modeling simulations and identification in chemistry, physics and biology.
- Images processing and control signal processing: Feature extraction from data and selection of feature with help of metaheuristic approach.
- Planning in routing based problems, robotic planning, scheduling and production based problems, logistics and transportation, supply chain management and environmental.

4 Black Hole Phenomena

In the eighteens-century, Dr. John Michel and Pierre Pierre Simon de Laplace were established to blemish the idea of black holes. Based on Newton's law, they invented the concept of a star turning into invisible to the human eye but during that period it was not able to recognize as a black hole in 1967, John Wheeler the American physicist first named the phenomenon of mass collapsing as a black hole [42]. A black hole in space is a form when a star of massive size collapses named the development of mass collapsing as apart. The gravitational power of the black hole is too strong that even the any light cannot escape from it. The gravity of such body is so

Table 1 Description of bio-inspired algorithms

S.No.	Metaheuristic algorithms	Description of metaheuristic algorithms
1.	Genetic algorithms (GAs) [10]	Genetic algorithm is a search and optimization based techniques that evolve a population of candidate solutions to a given problem, using natural genetic variation and natural selection operators
2.	Simulated annealing(SA) algorithm [9]	Simulated Annealing is developed by modeling the steel annealing process and gradually decreases the temperature (T)
3.	Ant colony optimization (ACO) [6]	Ant Colony Optimization is inspired from the behavior of a real ant colony, which is able to find the shortest path between its nest and a food source (destination)
4.	Particle swarm optimization (PSO) algorithm [22]	Particle Swarm Optimization is developed based on the swarm behavior such as fish and bird schooling in nature
5.	The gravitational search algorithm (GSA) [39]	It is constructed based on the law of gravity and the notion of mass interactions. In the GSA algorithm, the searcher agents are a collection of masses that interact with each other based on the Newtonian gravity and the laws of motion
6.	Intelligent water drops algorithm [40]	It is inspired from observing natural water drops that flow in rivers and how natural rivers find almost optimal paths to their destination. In the IWD algorithm, several artificial water drops cooperate to change their environment in such a way that the optimal path is revealed as the one with the lowest soil on its links
7.	Firefly algorithm (FA) [23, 41]	The firefly algorithm (FA) was inspired by the flashing behavior of fireflies in nature. FA is nature inspired optimization algorithm that imitates or stimulates the flash pattern and characteristics of fireflies. It is used data analysis and to identify homogeneous groups of objects based on the values of their attributes
8.	Honey bee mating optimization (HBMO) algorithm [29]	It is inspired by the process of marriage in real honey bees
9.	Bat algorithm (BA)	It is inspired by the echolocation behavior of bats. The capability of the echolocation of bats is fascinating as they can find their prey and recognize different types of insects even in complete darkness
10.	Harmony search optimization algorithm	It is inspired by the improvising process of composing a piece of music. The action of finding the harmony in music is similar to finding the optimal solution in an optimization process
11.	Big Bang–Big Crunch (Bb–By) optimization	It is based on one of the theories of the evolution of the universe. It is composed of the big bang and big crunch phases. In the big bang phase the candidate solutions are spread at random in the search space and in the big crunch phase a contraction procedure calculates a center of mass for the population

(continued)

Table 1 (continued)

S.No.	Metaheuristic algorithms	Description of metaheuristic algorithms
12.	Black hole (BH) algorithm	It is inspired by the black hole phenomenon. The basic idea of a black hole is simply a region of space that has so much mass concentrated in it that there is no way for a nearby object to escape its gravitational pull. Anything falling into a black hole, including light, is forever gone from our universe

strong because matter has been squeezed into a tiny space and anything that crosses the boundary of the black hole will be consumed or by it and vanishes and nothing can get away from its enormous power. The sphere-shaped boundary of a black hole in space is known as the event horizon. The radius of the event horizon is termed as the Schwarzschild radius. At this radius, the escape speed is equal to the speed of light, and once light passes through, even it cannot escape. Nothing can escape from within the event horizon because nothing can go faster than light. The Schwarzschild radius (R) is calculated by $R = \frac{2GM}{c^2}$, where G is the gravitational constant (6.67×10^{-11} N × (m/kg)^2), M is the mass of the black hole, and c is the speed of light. If star moves close to the event horizon or crosses the Schwarzschild radius it will be absorbed into the black hole and permanently disappear. The existence of black holes can be discerned by its effect over the objects surrounding it [43, 44].

4.1 Black Hole

A black hole is a region of space-time (x, y, t) whose gravitational field is so strong and powerful that nothing can escape from it. The theory and principle of general relativity predicts that a sufficiently compact mass will deform space-time to form a black hole. Around a black hole, there is a mathematically defined surface called an event horizon that marks the point of no return. If anything moves close to the event horizon or crosses the Schwarzschild radius, it will be absorbed into the black hole and permanently disappear. The existence of black holes can be discerned by its effect over the objects surrounding it [45]. The hole is called *black* because it absorbs all the light that hits the horizon, reflecting nothing, just like a perfect black body in thermodynamics [46, 47]. A black hole has only three independent physical properties: Black hole's mass (M), charge (Q) and angular momentum (J). A charged black hole repels other like charges just like any other charged object in given space. The simplest black holes have mass but neither electric charge nor angular momentum [48, 49].

4.2 Black Hole Algorithm

The basic idea of a black hole is simply a region of space that has so much mass concentrated in it that there is no way for a nearby object to escape its gravitational pull. Anything falling into a black hole, including light, is forever gone from our universe.

4.2.1 Terminology of Black Hole Algorithm

Black Hole: In black hole algorithm, the best candidate among all the candidates at each iteration is selected as a black hole.

Stars: All the other candidates form the normal stars. The creation of the black hole is not random and it is one of the real candidates of the population.

Movement: Then, all the candidates are moved towards the black hole based on their current location and a random number.

1. Black hole algorithm (black hole) starts with an initial population of candidate solutions to an optimization problem and an objective function that is calculated for them.
2. At each iteration of the Black Hole, the best candidate is selected to be the black hole and the rest form the normal stars. After the initialization process, the black hole starts pulling stars around it.
3. If a star gets too close to the black hole it will be swallowed by the black hole and is gone forever. In such a case, a new star (candidate solution) is randomly generated and placed in the search space and starts a new search.

4.3 Calculation of Fitness Value for Black Hole Algorithm

1. Initial Population: $P(x) = \{x_1^t, x_2^t, x_3^t, \ldots, x_n^t\}$ randomly generated population of candidate solutions (the stars) are placed in the search space of some problem or function.
2. Find the total Fitness of population:

$$f_i = \sum_{i=1}^{pop\_size} eval(p(t) \tag{1}$$

3.
$$f_{BH} = \sum_{i=1}^{pop\_size} eval(p(t)$$

where f_i and f_{BH} are the fitness values of black hole and i_{th} star in the initialized population. The population is estimated and the best candidate (from remaining stars) in the population, which has the best fitness value, f_i is selected to be the black hole and the remaining form the normal stars. The black hole has the capability to absorb the stars that surround it. After initializing the first black hole and stars, the black hole starts absorbing the stars around it and all the stars start moving towards the black hole.

4.3.1 Absorption Rate of Stars by Black Hole

The black hole starts absorbing the stars around it and all the stars start moving towards the black hole. The absorption of stars by the black hole is formulated as follows:

$$X_i(t) = X_i(t) + rand \times (X_{BH} - X_i(t)) \tag{3}$$

where i = 1, 2, 3, . . .n, X_i^t and X_i^{t+1} are the locations of the ith star at iterations t and (t + 1) respectively. X_{BH} is the location of the black hole in the search space and rand is a random number in the interval [0, 1]. N is the number of stars (candidate solutions). While moving towards the black hole, a star may reach a location with lower cost than the black hole. In such a case, the black hole moves to the location of that star and vice versa. Then the black hole algorithm will continue with the black hole in the new location and then stars start moving towards this new location.

4.3.2 Probability of Crossing the Event Horizon During Moving Stars

In block hole algorithm, the probability of crossing the event horizon of black hole during moving stars towards the black hole is used to gather the more optimal data point from search space of the problem. Every star (candidate solution) crosses the event horizon of the black hole will be sucked by the black hole and every time a candidate (star) dies it means it sucked in by the black hole, another candidate solution (star) is populated and distributed randomly over the search space of the defined problem and go for a new search in the search solution space. It is completed to remain the number of candidate solutions constant. The next iteration takes place after all the stars have been moved. The radius of the event horizon in the black hole algorithm is calculated using the following equation: The radius of horizon (R) of black hole is demonstrated as follow:

$$R = \frac{f_{BH}}{\sum_{i=1}^{N} f_i} \tag{4}$$

where f_i and f_{BH} are the fitness values of black hole and ith star. N is the number of stars (candidate solutions).When the distance between a candidate solution and the black hole (best candidate) is less than R, that candidate is collapsed and a new candidate is created and distributed randomly in the search space.

4.4 Pseudo Code for Black Hole Algorithm

1. Initialize a population of stars with random locations in the search space $P(t) = \{x_1^t, x_2^t, x_3^t \ldots x_n^t\}$. Randomly generated population of candidate solutions (the stars) are placed in the search space of some problem or function.

   Loop

2. For each ith star, evaluate the objective function

$$f_i = \sum_{i=1}^{pop\_size} eval(p(t)$$

$$f_{BH} = \sum_{i=1}^{pop\_size} eval(p(t)$$

3. Select the best star that has the best fitness value as the black hole.
4. Change the location of each star according to Eq. (3) as

$$X_i(t) = X_i(t) + rand \times (X_{BH} - X_i(t))$$

5. If a star reaches a location with lower cost than the black hole, exchange their locations.
6. If a star crosses the event horizon of the black hole
7. Calculate the event horizon radius (R)

$$R_{EventHorizon} = \frac{f_{BH}}{\sum_{i=1}^{N} f_i}$$

8. When the distance between a candidate solution and the black hole (best candidate) is less than R, that candidate is collapsed and a new candidate is created and distributed randomly in the search space.
9. Replace it with a new star in a random location in the search space
10. else
    break
11. If a termination criterion (a maximum number of iterations or a sufficiently good fitness) is met exit the loop.

The candidate solution to the clustering problem corresponds to one dimensional (1-D) array while applying black hole algorithm for data clustering. Every candidate solution is considered as k initial cluster centers and the individual unit in the array as the cluster center dimension. Figure 1 illustrates a candidate solution of a problem with three clusters and all the data objects have four features.

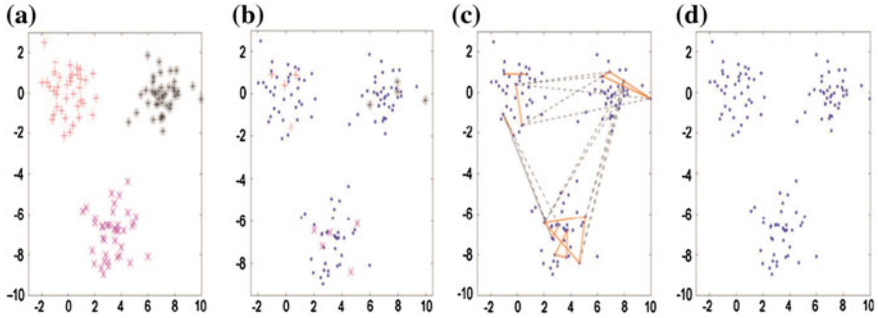

Fig. 1 Learning problems: dots correspond to points without any labels. Points with labels are denoted by *plus signs*, *asterisks*, and *crosses*. In (**c**), the must-link and cannot link constraints are denoted by *solid* and *dashed lines*, respectively [50]. **a** Supervised. **b** Partially labelled. **c** Partially constrained. d Unsupervised

4.5 Advantage of Black Hole Algorithm

- It has a simple structure and it is easy to implement.
- It is free from tuning parameter issues like genetic algorithm local search utilizes the schemata(S) theorem of higher order O(S) (compactness) and longer defining length δ(S). In Genetic Algorithm, to improve the fine tuning capabilities of a genetic algorithm, which is a must for high precision problem over the traditional representation of binary string of chromosomes? It was required a new mutation operator over the traditional mutation operator however, it only use only local knowledge i.e. it stuck into local minimum optimal value.

The Black Hole algorithm converges to global optimum in all the runs while the other heuristic algorithms may get trapped in local optimum solutions like genetic algorithm, Ant colony Optimization algorithm simulated Annealing algorithm.

5 Application of Black Hole Metaheuristic Algorithm

Nature-inspired metaheuristic algorithms have been used in many fields such as computer science [51–53] clustering analysis, industry [54] agriculture [55], computer vision [56–58] is about computing visual properties from the real world and automatic circle detection in digital still images has been considered an important and complex task for the computer vision community that has devoted a tremendous amount of research, seeking for an optimal circle detector. Electro-magnetism Optimization (EMO) bio-inspired algorithm based circle detector method which assumes the overall detection process as optimization problem, forecasting [59], medicine and biology [60], scheduling [61], data mining [62, 63], economy [64] and engineering [65]. There are following applications of black hole algorithm in data clustering and its performance analysis.

5.1 Cluster Analysis

Clustering is an important unsupervised classification approach, where a set of patterns are usually vectors (observations, data items, or feature vectors) into in multi-dimensional space are grouped into clusters or groups, based on some similarity metrics between data objects; the distance measurement is used to find out the similarity and dissimilarity of different object of our database [66]. The main idea is to classify a given data set through a certain number of clusters by minimizing distances between objects inside each cluster. Cluster analysis is the organization of a collection of patterns (usually represented as a vector of measurements, or a point in a multidimensional space) into clusters based on similarity [50, 67]. Cluster is often used for different type of application in image processing, data statistical analysis, medical imaging analysis and other field of science and engineering research field. In addition, it is a main task of exploratory data mining and a common technique for statistical data analysis used in many fields including machine learning, pattern recognition, image analysis, information retrieval and bioinformatics.

5.1.1 Data Analysis Problem Is Specified as Follows

Given N objects, assign each object to one of K clusters and minimize the sum of squared euclidean distances between each object and the center of the cluster that belongs to every allocated object:

$$F(O,Z) = \sum\nolimits_{i=1}^{N} \sum\nolimits_{j=1}^{K} Wij(O_i - Z_j)^2$$

where $(O_i - Z_j)^2 = \|\mathrm{O_i} - \mathrm{Z_j}\|$ is the Euclidean distance between a data object O_i and the cluster center Z_j. N and K are the number of data objects and the number of clusters, respectively. W_{ij} is the association weight of data object $O_i$with cluster j.

$$W_{ij} = \begin{cases} 1 & \text{if object i is assign to cluster j.} \\ 0 & \text{if object i is not assigned to cluster j.} \end{cases}$$

5.2 Data Clustering

The goal of data clustering also known as cluster analysis is to discover the natural grouping of a set of patterns, points or objects. An operational definition of clustering can be stated as follows: Given a representation of n objects, find K groups based on a measure of similarity such that the similarities between objects in the same group are high while the similarities between objects in different groups are

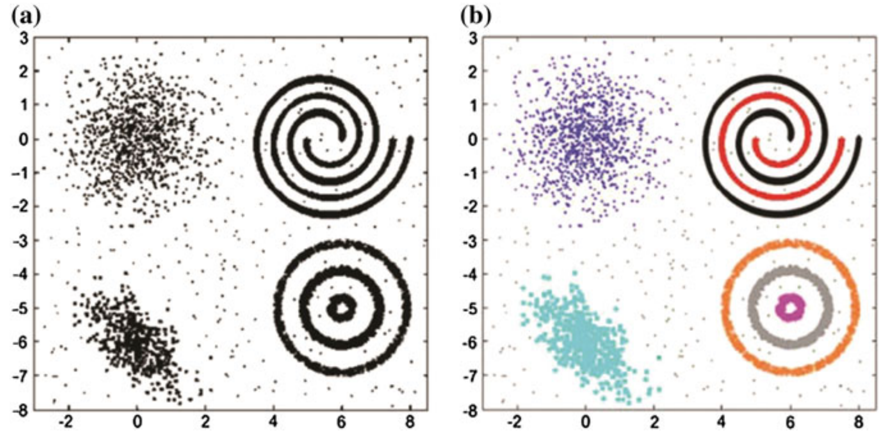

Fig. 2 Diversity of clusters. The seven clusters in (**a**) [denoted by seven different colors in 1(**b**)] differ in *shape*, *size* and *density*. Although these clusters are apparent to a data analyst, none of the available clustering algorithms can detect all these clusters. (*Source* [50]. **A** Input data. **b** Desired clustering

low. Figure 2 demonstrates that clusters may differ in terms of their shape, size, and density. The presence of noise in the data makes the detection of the clusters even more difficult and ideal cluster can be defined as a set of points that is compact and isolated. While humans are excellent cluster seekers in two and possibly three dimensions, we need automatic algorithms for high-dimensional data. It is this challenge along with the unknown number of clusters for the given data that has resulted in thousands of clustering algorithms that have been published and that continue to appear. An example of clustering is shown in Fig. 2. In pattern recognition, data analysis is concerned with predictive modeling: given some training data and to predict the behavior of the unseen test data. This task is also referred to as learning.

5.2.1 Classification of Machine Learning

Classification of data based on machine algorithms is follow:

Supervised Learning and Unsupervised Learning

Supervised learning is the machine learning approach of inferring a function from labeled training data. The training data consist of a set of training examples. Let a set of labeled training data $X = [(x_n, y_n)] \in X \times Y \leq n \leq N$ where x is input space and y a finite label set. It is assumed that each $[(x_n, y_n)]$ is drawn independently from a fixed, but unknown probability distribution p, where $[(x_n, y_n)] \in p(x, y)$. Unfortunately, supervised learning method is very expensive and time consuming to collect a huge amount of labeled data $[(x_n, y_n)]$. One of learning approach to deal such issues is to exploit unsupervised learning. The main aim object is to learn a classification model from both labeled $X = [(x_n, y_n)] \in X \times Y \leq n \leq N$ and $[x_j]_{j=N+1}^{N+M}$ unlabelled data where $N \leq M$. Clustering is a more difficult and challenging problem than classification.

Semi-supervised Learning

In semi-supervised classification, the labels of only a small portion of the training data set are available. The unlabeled data, instead of being discarded, are also used in the learning process. In semi-supervised clustering, instead of specifying the class labels, pair-wise constraints are specified, which is a weaker way of encoding the prior knowledge. A pair-wise must-link constraint corresponds to the requirement that two objects should be assigned the same cluster label, whereas the cluster labels of two objects participating in a cannot-link constraint should be different [50, 68]. Constraints can be particularly beneficial in data clustering, where precise definitions of underlying clusters are absent. Figure 1 illustrates this spectrum of different types of learning problems of interest in pattern recognition and machine learning.

5.3 K-means Clustering

Cluster analysis is prevalent in any discipline that involves analysis of multivariate data. Clustering algorithm K-means was first published in 1955. It is difficult to exhaustively list the numerous scientific fields and applications that have utilized clustering techniques as well as the thousands of published algorithms. Image segmentation an important problem in computer vision, can be formulated as a clustering problem. Documents can be clustered to generate topical hierarchies for efficient information access. Clustering is also used to group customers into different types for efficient marketing to group services delivery engagements for workforce management and planning as well as to study genome data in biology [50]. Data clustering has been used for the following three main purposes.

- Underlying structure to gain insight into data generates hypotheses, detect anomalies, and identify salient features.
- Natural classification to identify the degree of similarity among forms or organisms (phylogenetic relationship).
- Compression: as a method for organizing the data and summarizing it through cluster prototypes.

Among the classical clustering algorithms, K-means is the most well known algorithm due to its simplicity and efficiency.

5.3.1 Classification of Clustering Algorithms

Clustering algorithms are classified into can be broadly divided into two categories: (1): Hierarchical clustering and Partitional clustering.

Hierarchical Algorithm

Hierarchical clustering constructs a hierarchy of groups by splitting a large cluster into small ones and merging smaller cluster into a large cluster centroid [69]. In this, there are two main approaches:(1) *the divisive approach*, which splits a large cluster into two or more smaller clusters; (2) *the agglomerative approach,* which builds a larger cluster by merging two or more smaller clusters by recursively find nested clusters either in agglomerative mode (starting with each data point in its own cluster and merging the most similar pair of clusters successively to form a cluster hierarchy. Input to a hierarchical algorithm is an n × n similarity matrix, where n is the number of objects to be clustered [50].

Partitioned Algorithm

Partitioned clustering algorithms find all the clusters simultaneously as a partition of the data without hierarchical structure. The most widely used partitional clustering approaches are prototype-based clustering algorithm where each cluster is demonstrated by its centre. The objective function or square error function is sum of distance from the pattern to the centre [70]. Partitioned algorithm can use either an n × d pattern matrix, where n objects are embedded in a d-dimensional feature space, or an n × n similarity matrix. Note that a similarity matrix can be easily derived from a pattern matrix but ordination methods such as multi-dimensional scaling (MDS) are needed to derive a pattern matrix from a similarity matrix. The most well-known hierarchical algorithms are single-link and complete-link. The most popular and the simplest partitioned algorithm is K-means. Since partitioned algorithms are preferred in pattern recognition due to the nature of available data, our coverage here is focused on these algorithms [50].

5.4 K-means Algorithm

K-means has a rich and diverse history as it was independently discovered in different scientific fields by [71], Lloyd. It is a popular partitional clustering algorithm and essentially a function minimization technique, where the main objective function is the square error.

Let $X = \{x_i\}, i = 1, 2, 3, 4 \ldots, n$ be the set of n d-dimensional points to be clustered into a set of K clusters, $C = \{c_k\}$ where k = 1, 2, 3, 4….K. K-means algorithm finds a partition such that the squared error between the empirical mean of a cluster and the points in the cluster is minimized. Let μ_k be the mean of cluster C_k. The squared error between μ_k and the points in cluster C_k is defined as $J(c_k) = \sum_{x_i \in c_k} (\mathrm{X}_i - \mu_k)^2$. The goal of K-means is to minimize the sum of the squared error over all K clusters, $J(C) = \sum_{k=1}^{K} \sum_{xi \in c_k}^{N} (\mathrm{X}_i - \mu_k)^2$. Minimizing this objective function is known to be an NP-hard problem (even for K = 2). Thus K-means which is a greedy algorithm and can only converge to a local minimum, even though recent study has shown with a large probability, K-means could converge to the global optimum when clusters are well separated. K-means starts with an initial partition with K clusters and assign patterns to clusters so as to reduce the squared error. Since the squared error always decreases with an increase in the number of clusters K (with J(C) = 0 when K = n), it can be minimized only for a fixed number of clusters [64]. Recently efficient hybrid evolutionary and bio-inspired metaheuristic methods and K-means to overcome local problems in clustering are used [72, 73] (Niknam and Amiri 2010).

5.4.1 Steps of K-Means Algorithm

1. Select an initial partition with K clusters.
2. Repeat steps 2 and 3 until cluster membership stabilizes.
3. Generate a new partition by assigning each pattern to its closest cluster center.
4. Compute new cluster centers (Figs. 3, 4).

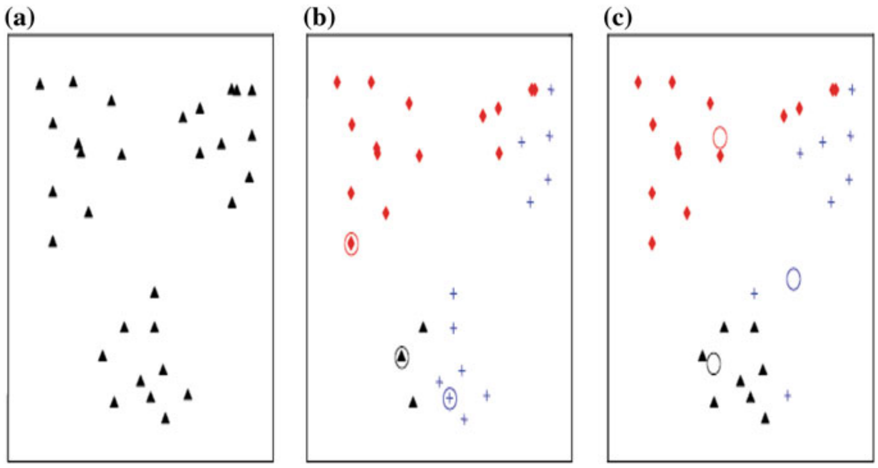

Fig. 3 Illustration of K-means algorithm **a** Two-dimensional input data with three clusters; **b** three seed points selected as cluster centers and initial assignment of the data points to clusters; and **c** updates intermediate cluster labels

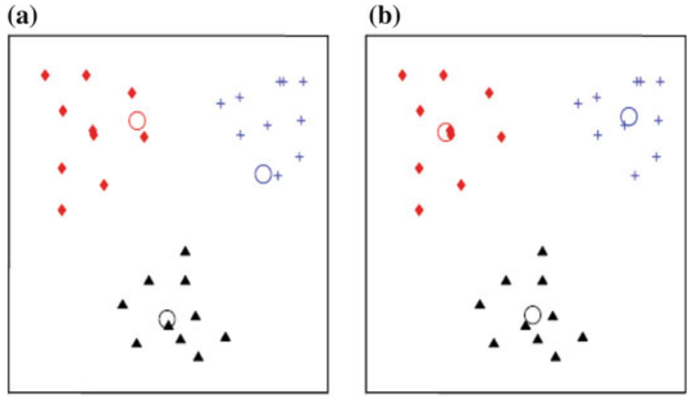

Fig. 4 **a** and **b** intermediate iterations updating cluster labels and their centers an final clustering obtained by K-means

The K-means algorithm requires three user-specified parameters: number of clusters K, cluster initialization, and distance metric. The most critical choice is K. While no perfect mathematical criterion exists, a number of heuristics are available for choosing K. Typically, K-means is run independently for different values of K and the partition that appears the most meaningful to the domain expert is selected. Different initializations can lead to different final clustering because K-means. One way to overcome the local minima is to run the K-means algorithm, for a given K, with multiple different initial partitions and choose the partition with the smallest squared error. K-means is typically used with the Euclidean metric for computing the distance between points and cluster centers. As a result, K-means finds spherical or ball-shaped clusters in data. K-means with mahalanobis distance metric has been used to detect hyper ellipsoidal clusters [74].

5.5 Advantages and Disadvantages of the K-Means Clustering

Among the all classical clustering algorithms, K-means clustering algorithm is the well known algorithm due to their simplicity and efficiency. It suffers from two problems: It needs number of cluster before starting i.e. the number of cluster must be known a priori. In addition, its performance strongly depends on the initial centroids and may get stuck in local optima solutions and its convergence rate are affected [56]. In order to overcome the shortcomings of K-means many heuristic approaches have been applied in the last two decades.

5.6 Evolutionary Computation Algorithms for Cryptography and Cryptanalysis

Cryptography is a methodology and study of techniques for secure communication in the presence of third parties and cryptanalysis is the study of analyzing information systems in order to study the hidden aspects of the systems. The cryptanalysis of different cipher problems can be formulated as NP-hard combinatorial problem. Metaheuristic algorithm provides a very powerful tool for the cryptanalysis of simple substitution ciphers using a cipher text only attack and automated cryptanalysis of classical simple substitution ciphers [75–77].

Recently, efficient hybrid evolutionary optimization algorithms based on combining evolutionary methods and swarm intelligence has significant role in the field of cryptography and demonstrates a dynamical system which is sensitive to initial condition and generates apparently random behavior but at the same time the system is completely deterministic. Hussein et al. presents a new encryption scheme based on a new chaotic map derived from a simplified model of Swarm Intelligence (SI) [78] and overcome the problem of cryptograph [79–81].

5.7 Short-Term Scheduling Based System Optimization by Black Hole Algorithm

Recently, a major challengeable subject that are facing the electric power system operator and how to manage optimally the power generating units over a scheduling horizon of one day considering all of the practical equality inequality and dynamic constraints. These constraints of system are comprised of load plus transmission losses balance, valve-point effects, prohibited operating zones, multi-fuel options, line flow constraints, operating reserve and minimum on/off time. There is not available any optimization for the short-term thermal generation scheduling (STGS). It has high-dimensional, high-constraints, non-convex, non-smooth and non-linear nature and needs an efficient algorithm to be solved. Then, a new optimization approach, known as gradient-based modified teaching–learning-based optimization combined with black hole (MTLBO–BH) algorithm has been planned to seek the optimum operational cost [82–84].

6 Discussion

The field of metaheuristic approaches for the application to combinatorial optimization problems is a rapidly growing field of research. This is due to a great importance of combinatorial optimization problems for the scientific as well as the industrial world. Since the last decade metaheuristics approaches have a significant role for solving the complex problem of different applications in science, computer vision, computer science, data analysis, data clustering, and mining, clustering analysis, industrial forecasting of weather, medical and biological research, economy and different multi-disciplinary engineering research field. In addition, meta-heuristics are useful in computer vision, image processing, machine learning and pattern recognition of any subject which can be deployed for finding the optimal set of discriminant values in form of Eigen vector (face recognition, fingerprint and other biometric characteristics) and incorporate these values for identification purpose. In biometrics and computer vision, face recognition has always been a major challenge for machine learning researchers and pattern recognition. Introducing the intelligence in machines to identifying humans from their face images (which is stored in template data base) deals with handling variations due to illumination condition, pose, facial expression, scale and disguise etc., and hence becomes a complex task in computer vision. Face recognition demonstrates a classification problem for human recognition. Face recognition classification problems can be solved by a technique for the design of the *Radial Basis Functions* neural network with metaheuristic approaches (like firefly, particle swarm intelligence and black hole algorithm). These algorithms can be used at match score level in biometrics and select most discriminant set of optimal features for identification of face and their classification.

Recently black hole methodology plays a major role of modeling and simulating natural phenomena for solving complex problems. The motivation for new heuristic optimization algorithm is based on the black hole phenomenon. Further, it has a simple structure and it is easy to implement and it is free from parameter tuning issues like genetic algorithm. The black hole algorithm can be applied to solve the clustering problem and can run on different benchmark datasets. In future research, the proposed algorithm can also be utilized for many different areas of applications. In addition, the application of BH in combination with other algorithms may be effective. Meta-heuristics support managers in decision-making with robust tools that provide high-quality solutions to important applications in business, engineering, economics and science in reasonable time horizons.

7 Conclusion and Future Direction

We conclude that new black hole algorithm approach is population based same as particle swarm optimization, firefly, genetic algorithm, BAT algorithm and other evolutionary methods. It is free from parameter tuning issues like genetic algorithm and other. It does not suffer from premature convergence problem. This implies that black hole is potentially more powerful in solving NP-hard (e.g. data clustering problem) problems which is to be investigated further in future studies. The further improvement on the convergence of the algorithm is to vary the randomization parameter so that it decreases gradually as the optima are approaching. In wireless sensor network, density of deployment, scale, and constraints in battery, storage device, bandwidth and computational resources create serious challenges to the developers of WSNs. The main issues of the node deployment, coverage and mobility are often formulated as optimization problems and moth optimization techniques suffer from slow or weak convergence to the optimal solutions for high performance optimization methods that produce high quality solutions by using minimum resources. Bio-inspired black hole algorithm can give a model to solve optimization problems in WSNs due to its simplicity, best solution, fast convergence and minimum computational complexity. These can be form important topics for further research in computer network. Furthermore, as a relatively straight forward extension, the black hole algorithm can be modified to solve multi objective optimization problems. In addition, the application of black hole in combination with other algorithms may form an exciting area for further research.

References

1. Tan, X., Bhanu, B.: Fingerprint matching by genetic algorithms. Pattern Recogn. **39**, 465–477 (2006)
2. Karakuzu, C.: Fuzzy controller training using particle swarm optimization for nonlinear system control. ISA Trans. **47**(2), 229–239 (2008)
3. Rajabioun, R.: Cuckoo optimization algorithm. Elsevier Appl. Soft Comput. **11**, 5508–5518 (2011)
4. Tsai Hsing, C., Lin, Yong-H: Modification of the fish swarm algorithm with particle swarm optimization formulation and communication behavior. Appl. Soft Comput. Elsevier **1**, 5367–5374 (2011)
5. Baojiang, Z., Shiyong, L.: Ant colony optimization algorithm and its application to neu ro-fuzzy controller design. J. Syst. Eng. Electron. **18**, 603–610 (2007)
6. Dorigo, M., Maniezzo, V., Colorni, A.: The ant system: optimization by a colony of cooperating agents. IEEE Trans. Syst. Man Cybern. Part B **26**(1), 29–41 (1996)
7. Farmer, J.D., et al.: The immune system, adaptation and machine learning. Phys. D Nonlinear Phenom. Elsevier **22**(1–3), 187–204 (1986)
8. Kim, D.H., Abraham, A., Cho, J.H.: A hybrid genetic algorithm and bacterial foraging approach for global optimization. Inf. Sci. **177**, 3918–3937 (2007)
9. Kirkpatrick, S., Gelatto, C.D., Vecchi, M.P.: Optimization by simulated annealing. Science **220**, 671–680 (1983)
10. Tang, K.S., Man, K.F., Kwong, S., He, Q.: Genetic algorithms and their applications. IEEE Sig. Process. Mag. **3**(6), 22–37 (1996)
11. Du, Weilin, Li, B.: Multi-strategy ensemble particle swarm optimization for dynamic optimization. Inf. Sci. **178**, 3096–3109 (2008)
12. Yao, X., Liu, Y., Lin, G.: Evolutionary programming made faster. IEEE Trans. Evol. Comput. **3**, 82–102 (1999)
13. Liu, Y., Yi, Z., Wu, H., Ye, M., Chen, K.: A tabu search approach for the minimum sum-of-squares clustering problem. Inf. Sci. **178**(12), 2680–2704 (2008)
14. Kim, T.H., Maruta, I., Sugie, T.: Robust PID controller tuning based on the constrained particle swarm optimization. J. Autom. Sciencedirect **44**(4), 1104–1110 (2008)
15. Cordon, O., Santamarı, S., Damas, J.: A fast and accurate approach for 3D image registration using the scatter search evolutionary algorithm. Pattern Recogn. Lett. **27**, 1191–1200 (2006)
16. Yang, X.S.: Firefly algorithms for multimodal optimization, In: Proceeding of Stochastic Algorithms: Foundations and Applications (SAGA), 2009 **(**2009**)**

17. Kalinlia, A., Karabogab, N.: Artificial immune algorithm for IIR filter design. Eng. Appl. Artif. Intell. **18**, 919–929 (2005)
18. Lin, Y.L., Chang, W.D., Hsieh, J.G.: A particle swarm optimization approach to nonlinear rational filter modeling. Expert Syst. Appl. **34**, 1194–1199 (2008)
19. Holland, J.H.: Adaptation in Natural and Artificial Systems. University of Michigan Press, Ann Arbor (1975)
20. Jackson, D.E., Ratnieks, F.L.W.: Communication in ants. Curr. Biol. **16**, R570–R574 (2006)
21. Goss, S., Aron, S., Deneubourg, J.L., Pasteels, J.M.: Self-organized shortcuts in the Argentine ant. Naturwissenschaften **76**, 579–581 (1989)
22. Kennedy, J., Eberhart, R.C.: Particle swarm optimization. Proc. IEEE Int. Conf. Neural Networks **4**, 1942–1948 (1995)
23. Yang, X. S.: 2010, 'Nature-inspired metaheuristic algorithms', Luniver Press
24. Tarasewich, p, McMullen, P.R.: Swarm intelligence: power in numbers. Commun. ACM **45**, 62–67 (2002)
25. Senthilnath, J., Omkar, S.N., Mani, V.: Clustering using firefly algorithm: performance study. Swarm Evol. Comput. **1**(3), 164–171 (2011)
26. Yang, X.S.: Firefly algorithm. Engineering Optimization, pp. 221–230 (2010)
27. Yang, X.S.: Bat algorithm for multi-objective optimization. Int. J. Bio-inspired Comput. **3**(5), 267–274 (2011)
28. Tripathi, P.K., Bandyopadhyay, S., Pal, S.K.: Multi-objective particle swarm optimization with time variant inertia and acceleration coefficients. Inf. Sci. **177**, 5033–5049 (2007)
29. Karaboga, D.: An idea based on honey bee swarm for numerical optimization. Technical Report TR06,Erciyes University (2005)
30. Ellabib, I., Calamari, P., Basir, O.: Exchange strategies for multiple ant colony system. Inf. Sci. **177**, 1248–1264 (2007)
31. Hamzaçebi, C.: Improving genetic algorithms performance by local search for continuous function optimization. Appl. Math. Comput. **96**(1), 309–317 (2008)
32. Lozano, M., Herrera, F., Cano, J.R.: Replacement strategies to preserve useful diversity in steady-state genetic algorithms. Inf. Sci. **178**, 4421–4433 (2008)
33. Lazar, A.: Heuristic knowledge discovery for archaeological data using genetic algorithms and rough sets, *Heuristic and Optimization for Knowledge Discovery*, IGI Global, pp. 263–278 (2014)
34. Russell, S.J., Norvig, P.: Artificial Intelligence a Modern Approach. Prentice Hall, Upper Saddle River (2010). 1132
35. Fred, W.: Glover, Manuel Laguna, Tabu Search, 1997, ISBN: 079239965X
36. Christian, B., Roli, A.: Metaheuristics in combinatorial optimization: Overview and conceptual comparison. ACM Comput. Surveys (CSUR) **35**(3), 268–308 (2003)
37. Gazi, V., Passino, K.M.: Stability analysis of social foraging swarms. IEEE Trans. Syst. Man Cybern. Part B **34**(1), 539–557 (2008)
38. Deb, K.: Optimization for Engineering Design: Algorithms and Examples, Computer-Aided Design. PHI Learning Pvt. Ltd., New Delhi (2009)
39. Rashedi, E.: Gravitational Search Algorithm. M.Sc. Thesis, Shahid Bahonar University of Kerman, Kerman (2007)
40. Shah-Hosseini, H.: The intelligent water drops algorithm: a nature-inspired swarm-based optimization algorithm. Int. J. Bio-inspired Comput. **1**(1), 71–79 (2009)
41. Dos Santos, C.L., et al.: A multiobjective firefly approach using beta probability. IEE Trans. Magn. **49**(5), 2085–2088 (2013)
42. Talbi, E.G.: Metaheuristics: from design to implementation, vol. 74, p. 500. Wiley, London (2009)
43. Giacconi, R., Kaper, L., Heuvel, E., Woudt, P.: Black hole research past and future. In: Black Holes in Binaries and Galactic Nuclei: Diagnostics. Demography and Formation, pp. 3–15. Springer, Berlin, Heidelberg (2001)
44. Pickover, C.: Black Holes: A Traveler's Guide. Wiley, London (1998)
45. Frolov, V.P., Novikov, I.D.: Phys. Rev. D. **42**, 1057 (1990)

46. Schutz, B. F.: Gravity from the Ground Up. Cambridge University Press, Cambridge. ISBN 0-521-45506-5 (2003)
47. Davies, P.C.W.: Thermodynamics of Black Holes. Reports on Progress in Physics, Rep. Prog. Phys. vol. 41 Printed in Great Britain (1978)
48. Heusler, M.: Stationary black holes: uniqueness and beyond. Living Rev. Relativity **1**(1998), 6 (1998)
49. Nemati, M., Momeni, H., Bazrkar, N.: Binary black holes algorithm. Int. J. Comput. Appl. **79** (6), 36–42 (2013)
50. Jain, A.K.: Data clustering: 50 years beyond K-means. Pattern Recogn. Lett. **31**(8), 651–666 (2010)
51. Akay, B., Karaboga, D.: A modified artificial bee colony algorithm for real-parameter optimization. Inf. Sci. **192**, 120–142 (2012)
52. El-Abd, M.: Performance assessment of foraging algorithms vs. evolutionary algorithms. Inf. Sci. **182**, 243–263 (2012)
53. Ghosh, S., Das, S., Roy, S., Islam, M.S.K., Suganthan, P.N.: A differential covariance matrix adaptation evolutionary algorithm for real parameter optimization. Inf. Sci. **182**, 199–219 (2012)
54. Fox, B., Xiang, W., Lee, H.: Industrial applications of the ant colony optimization algorithm. Int. J. Adv. Manuf. Technol. **31**, 805–814 (2007)
55. Geem, Z., Cisty, M.: Application of the harmony search optimization in irrigation. Recent Advances in Harmony Search Algorithm', pp. 123–134. Springer, Berlin (2010)
56. Selim, S.Z., Ismail, M.A.: K-means-type algorithms: a generalized convergence theorem and characterization of local optimality pattern analysis and machine intelligence. IEEE Trans. PAMI **6**, 81–87 (1984)
57. Wang, J., Peng, H., Shi, P.: An optimal image watermarking approach based on a multi-objective genetic algorithm. Inf. Sci. **181**, 5501–5514 (2011)
58. Picard, D., Revel, A., Cord, M.: An application of swarm intelligence to distributed image retrieval. Inf. Sci. **192**, 71–81 (2012)
59. Chaturvedi, D.: Applications of genetic algorithms to load forecasting problem. Springer, Berlin, pp. 383–402 (2008) (Journal of Soft Computing)
60. Christmas, J., Keedwell, E., Frayling, T.M., Perry, J.R.B.: Ant colony optimization to identify genetic variant association with type 2 diabetes. Inf. Sci. **181**, 1609–1622 (2011)
61. Guo, Y.W., Li, W.D., Mileham, A.R., Owen, G.W.: Applications of particle swarm optimization in integrated process planning and scheduling. Robot. Comput.-Integr. Manuf. Elsevier **25**(2), 280–288 (2009)
62. Rana, S., Jasola, S., Kumar, R.: A review on particle swarm optimization algorithms and their applications to data clustering. Artif. Intell. Rev. **35**, 211–222 (2011)
63. Yeh, W.C.: Novel swarm optimization for mining classification rules on thyroid gland data. Inf. Sci. **197**, 65–76 (2012)
64. Zhang, Y., Gong, D.W., Ding, Z.: A bare-bones multi-objective particle swarm optimization algorithm for environmental/economic dispatch. Inf. Sci. **192**, 213–227 (2012)
65. Marinakis, Y., Marinaki, M., Dounias, G.: Honey bees mating optimization algorithm for the Euclidean traveling salesman problem. Inf. Sci. **181**, 4684–4698 (2011)
66. Anderberg, M.R.: Cluster analysis for application. Academic Press, New York (1973)
67. Hartigan, J.A.: Clustering Algorithms. Wiley, New York (1975)
68. Valizadegan, H., Jin, R., Jain, A.K.: Semi-supervised boosting for multi-class classification. 19th European Conference on Machine Learning (ECM), pp. 15–19 (2008)
69. Chris, D., Xiaofeng, He: Cluster merging and splitting in hierarchical clustering algorithms. Proc. IEEE ICDM **2002**, 1–8 (2002)
70. Leung, Y., Zhang, J., Xu, Z.: Clustering by scale-space filtering. IEEE Trans. Pattern Anal. Mach. Intell. **22**, 1396–1410 (2000)
71. Révész, P.: On a problem of Steinhaus. Acta Math. Acad. Scientiarum Hung. **16**(3–4), 311–331(1965)

72. Niknam, T., et al.: An efficient hybrid evolutionary optimization algorithm based on PSO and SA for clustering. J. Zhejiang Univ. Sci. A **10**(4), 512–519 (2009)
73. Niknam, T., Amiri, B.: An efficient hybrid approach based on PSO, ACO and k-means for cluster analysis. Appl. Soft Comput. **10**(1), 183–197 (2011)
74. Ding, C., He, X.: K-means clustering via principal component analysis. Proceedings of the 21th international conference on Machine learning, pp. 29 (2004)
75. Uddin, M.F., Youssef, A.M.: Cryptanalysis of simple substitution ciphers using particle swarm optimization. IEEE Congress on Evolutionary Computation, pp. 677–680 (2006)
76. Clerc, M., Kennedy, J.: The particle swarm-explosion, stability, and convergence in a multidimensional complex space. IEEE Trans. Evol. Comput. **6**, 58–73 (2002)
77. Danziger, M., Amaral Henriques, M.A.: Computational intelligence applied on cryptology: a brief review. Latin America Transactions IEEE (Revista IEEE America Latina) **10**(3), 1798–1810 (2012)
78. Chee, Y., Xu, D.: Chaotic encryption using discrete-time synchronous chaos. Phys. Lett. A **348**(3–6), 284–292 (2006)
79. Hussein, R.M., Ahmed, H.S., El-Wahed, W.: New encryption schema based on swarm intelligence chaotic map. Proceedings of 7th International Conference on Informatics and Systems (INFOS), pp. 1–7 (2010)
80. Chen, G., Mao, Y.: A symmetric image encryption scheme based on 3D chaotic cat maps. Chaos Solutions Fractals **21**, 749–761 (2004)
81. Hongbo, Liu: Chaotic dynamic characteristics in swarm intelligence. Appl. Soft Comput. **7**, 1019–1026 (2007)
82. Azizipanah-Abarghooeea, R., et al.: Short-term scheduling of thermal power systems using hybrid gradient based modified teaching–learning optimizer with black hole algorithm. Electric Power Syst. Res. Elsevier **108**, 16–34 (2014)
83. Bard, J.F.: Short-term scheduling of thermal-electric generators using Lagrangian relaxation. Oper. Res. **36**(5), 756–766 (1988)
84. Yu, I.K., Song, Y.H.: A novel short-term generation scheduling technique of thermal units using ant colony search algorithms. Int. J. Electr. Power Energy Syst. **23**, 471–479 (2001)