Gravity compensation of wrist tools

: Linear Regression Based Weight Compensation

[Physics Estimation Model]

$\vec{F}_s$: Summation force from the sensor base

$\vec{\tau}_s$: Summation moment from the sensor base

In the case of gravity force only (w/o any external forces)

$$\begin{cases} \vec{F}_s = m_t \cdot \vec{g}_b \\ \vec{\tau}_s = \vec{r}_t \times (m_t \cdot \vec{g}_b) \end{cases}$$

m_t: Tool weight

$\vec{r}_t$: Mass center vector of tool relative to sensor

$\vec{g}_b$: Gravity vector relative to sensor

[Regression Model]

(1) : Force → Tool Mass Estimation

* Force estimation at the pose "i"

$$\vec{F}_s(i) = m_t \cdot \vec{g}_b(i) \rightarrow \vec{g}_b(i) = \frac{1}{m_t} \cdot \vec{F}_s(i)$$

* Accumulated pose

$$\underbrace{\begin{bmatrix} \vec{F}_s(1) \\ \vec{F}_s(2) \\ \vdots \\ \vec{F}_s(N) \end{bmatrix}}_{\vec{F}_{stack} \in \mathbb{R}^{3N \times 1}} = m_t \cdot \underbrace{\begin{bmatrix} \vec{g}_b(1) \\ \vec{g}_b(2) \\ \vdots \\ \vec{g}_b(N) \end{bmatrix}}_{\vec{G}_{stack} \in \mathbb{R}^{3N \times 1}}$$

[Regression Model]

(1) : Force → Tool Mass Estimation

* Solution of least square

→ Scala linear regression problem estimating the single scala m_t

$$m_t = \underset{m}{\operatorname{argmin}} \left\| \vec{F}_{stack} - m \cdot \vec{G}_{stack} \right\|^2 \rightarrow m_t = \frac{\vec{G}_{stack}^T \vec{F}_{stack}}{\vec{G}_{stack}^T \vec{G}_{stack}}$$

[Regression Model]

(2) : moment → Mass Center Estimation

$$\vec{\tau}_s(i) = \vec{r}_t \times \left(m_t \cdot \vec{g}_b(i)\right) = [\vec{g}_b(i)]_\times \cdot (m_t \vec{r}_t)$$

Skew-Symmetric Matrix: $[\vec{g}_b(i)]_\times = \begin{bmatrix} 0 & -g_z & g_y \\ g_z & 0 & -g_x \\ -g_y & g_x & 0 \end{bmatrix}$

$\vec{\tau}_s(i) = A(i) \cdot \vec{x}$,with $A(i) = [\vec{g}_b(i)]_\times$ & $\vec{x} = m_t \vec{r}_t$

* Total Linear System

$$\underbrace{\begin{bmatrix} A(1) \\ A(2) \\ \vdots \\ A(N) \end{bmatrix}}_{A_{stack} \in \mathbb{R}^{3N\times 3}} \cdot \vec{x} = \underbrace{\begin{bmatrix} \vec{\tau}_s(1) \\ \vec{\tau}_s(2) \\ \vdots \\ \vec{\tau}_s(N) \end{bmatrix}}_{\vec{\tau}_{stack} \in \mathbb{R}^{3N\times 1}}$$

[Regression Model]

(2) : moment → Mass Center Estimation

* Solution of least square

$$\vec{x} = \left(A_{Stack}^{T} A_{Stack}\right)^{-1} A^{T} \vec{\tau}_{Stack} \rightarrow \vec{r}_t = \frac{\vec{x}}{m_t}$$