Measurement Data Interpretation using Uncertain Models (MeDIUM)

for

Safe Predictions and Sustainable Asset Management

Developed by

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MeDIUM

MeDIUM supports evaluation of a population of models to provide predictions that are compatible with observations (measurements) in the presence of uncertainties. It is particularly useful when uncertainties have non-zero mean distributions.

The task of data interpretation using uncertain models is complex and requires significant expertise and technical knowledge to obtain accurate solutions. This software simplifies a few of the steps involved and provides tools for interpreting measurement data and validating interpretations in order to assess accuracy.

In Section 2, the layout of the software is explained along with of the data-interpretation steps that are supported by the software. Section 2 also discusses pre-processing steps that the user has to perform to generate input for the software.

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Terms and definitions

Definitions of a few commonly used terms are:

Measurement	Data recorded directly from a sensor (accelerations, strain, deflection etc.) or processed observations from sensor data (frequency, equivalent stress range etc.)
Sensor location	Spatial position of a sensor used to record data. Also called measurement location
Physics-based model	Approximate model of a structure developed using principles of structural mechanics
Model parameter	Parameters of the physics-based model
Model-parameter values	Values of model parameters provided as inputs to a physics-based model
Model instance	Set of parameters describing the state of the system
Model prediction	Simulated predictions (output) of a model instance at sensor locations
Residual	Difference between model predictions and measurement values
Prior	Probability distribution function (PDF) quantifying prior (before measurements) uncertainty in model parameters
Model-class selection	Selection of the parameterised physics-based model. Involves choices on parameterisation of model and modelling approximations. A model class includes the parameters of a model (as variables), which have to be updated using measurements
Structural identification	Task of interpreting measurements using a physics-based model to improve knowledge related to uncertainty distributions of model parameters
Posterior	Probability distribution function (PDF) quantifying the posterior (after measurements) uncertainty in model parameters
Error-domain	A multi-dimensional space of errors, where each dimension corresponds to the uncertainty in error between model predictions and measurements. Dimensionality of this space is equal to the number of measurements

Error-domain model falsification	Abbreviated as EDMF, it is a multi-model probabilistic data interpretation methodology, which rejects model instances that provide model predictions that are incompatible with measurements (observations)
Candidate model set	Set of models that are compatible with observations
Calibration	Inference of optimal model-parameter values using observed data. Also called model updating
Covariance	Measure of how changes in one random variable are associated with changes in a second variable
Correlation	Degree of linear relationship between either quantities or random variables
Error/residual	Difference between a prediction and a measurement
Extrapolation	Prediction of a quantity outside the domain of data
Falsification	Process of discarding hypotheses and models using empirical evidence
Inference	Process of reaching logical conclusions from evidence
Initial model set	Set of model instances generated prior to interpreting data
Interpolation	Prediction of a quantity inside the domain of data
Secondary parameter	Parameter of a model, which is not included in the model class for structural identification, and contributes to prediction uncertainty
Sensor placement	Selection of good combinations of sensor types and locations optimizing some criterion of data-interpretation performance.
Structural identification	Task of improving knowledge of structural behaviour using information provided by measurements
Systematic error	A non-zero mean error that either remains constant or that varies in a predictable manner. This error may arise due to simplifications and omissions in the modelling process and alter the degree of spatial correlation between measurement locations
Threshold bounds	Bounds on estimated uncertainties, used as criteria to falsify models

Uncertainty	Description of the possible values that a variable or an error can take.

1 Background

MeDIUM is a software (tool) to interpret monitoring data primarily from civil infrastructure systems such as bridges, tunnels, dams and excavations. Monitoring such infrastructure systems and interpreting this data using physics-based models helps improve understanding of system behaviour. Models that are updated with monitoring data may be used to simulate possible future system behaviour due to changes such as increased loading and retrofit scenarios to support management actions and related decisions.

Practical implementation of data-interpretation methodologies is feasible today due to availability of inexpensive sensing [1], [2] and computing tools [3]. The inverse task of interpreting monitoring data using physics-based models is ill-posed. This implies that the solutions to this task are sensitive to presence of noise (uncertainties) and there is no unique solution. Therefore, to obtain solutions for this inverse task, appropriate probabilistic methods of data interpretation are necessary. However, due to challenges in using probabilistic methodologies for practical applications, the use of monitoring to support infrastructure management has been limited.

MeDIUM provides a user-interface-based implementation of error-domain model falsification (EDMF), which is a probabilistic data-interpretation methodology. This methodology was developed by Goulet and Smith [4] and builds on more than a decade of research [5]–[7], with applications to over ten full-scale case studies [8]. A brief explanation of this methodology is provided in Section 4.

2 Interface Layout and Instructions

Welcome screen of the software, shown in Figure 1, presents information about developers and contributors to the software. Users may contact the developers for questions related to algorithms and implementations in the software.

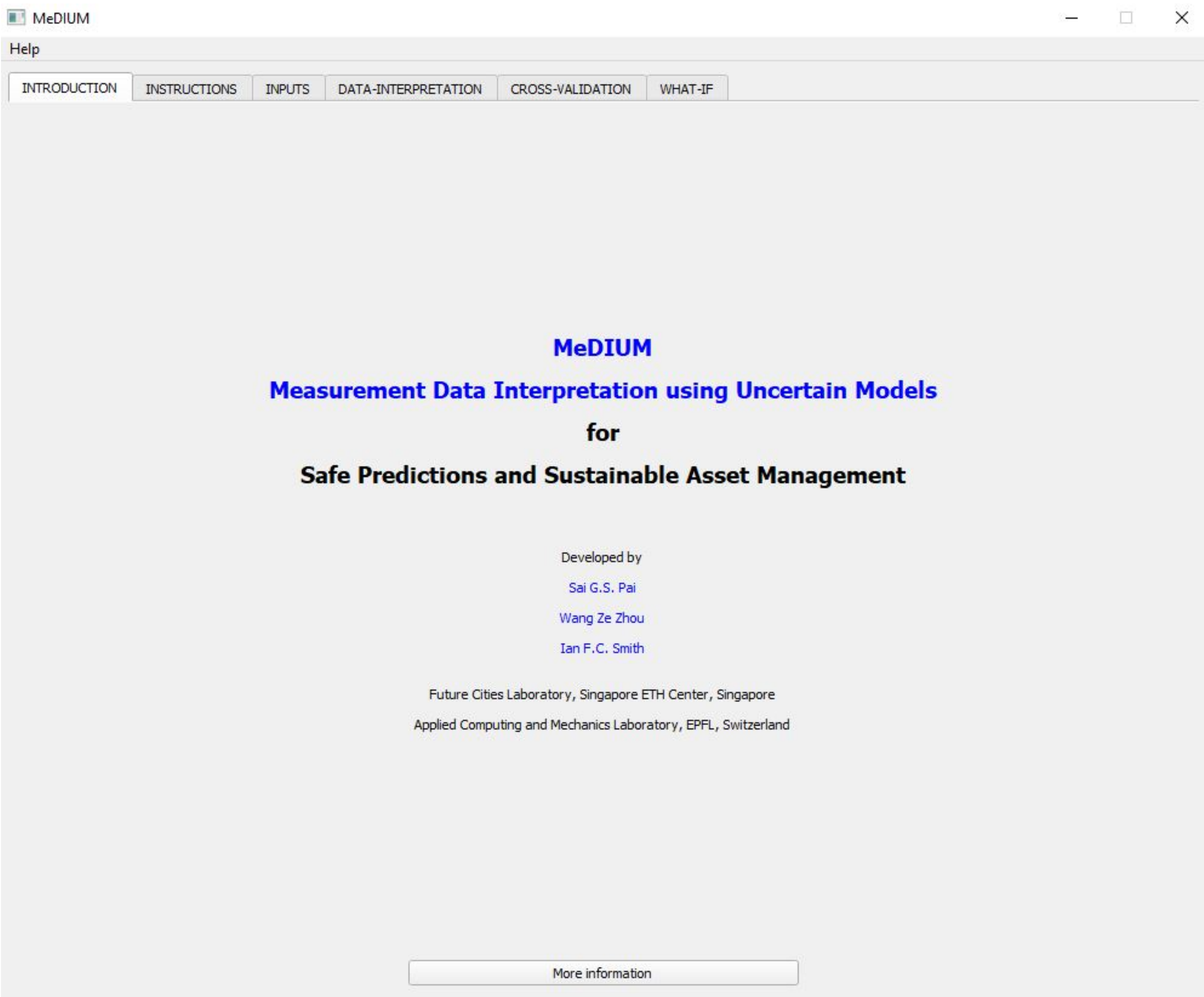


Figure 1 Welcome tab of MeDIUM software. Click on more information to learn more about contributors to this software.

The second tab in the software is the instructions tab, which presents the steps involved in using the software. These steps are shown in Figure 2. Step 1 (inputs) is mandatorily the first step, where the user provides information required to the software for interpretation. Steps 2 to 4 are tools for interpreting data and assessing validity of interpretations and assumptions. These steps do not have to be followed in order. However, to begin using the software, it may be useful to follow these steps in the order provided.

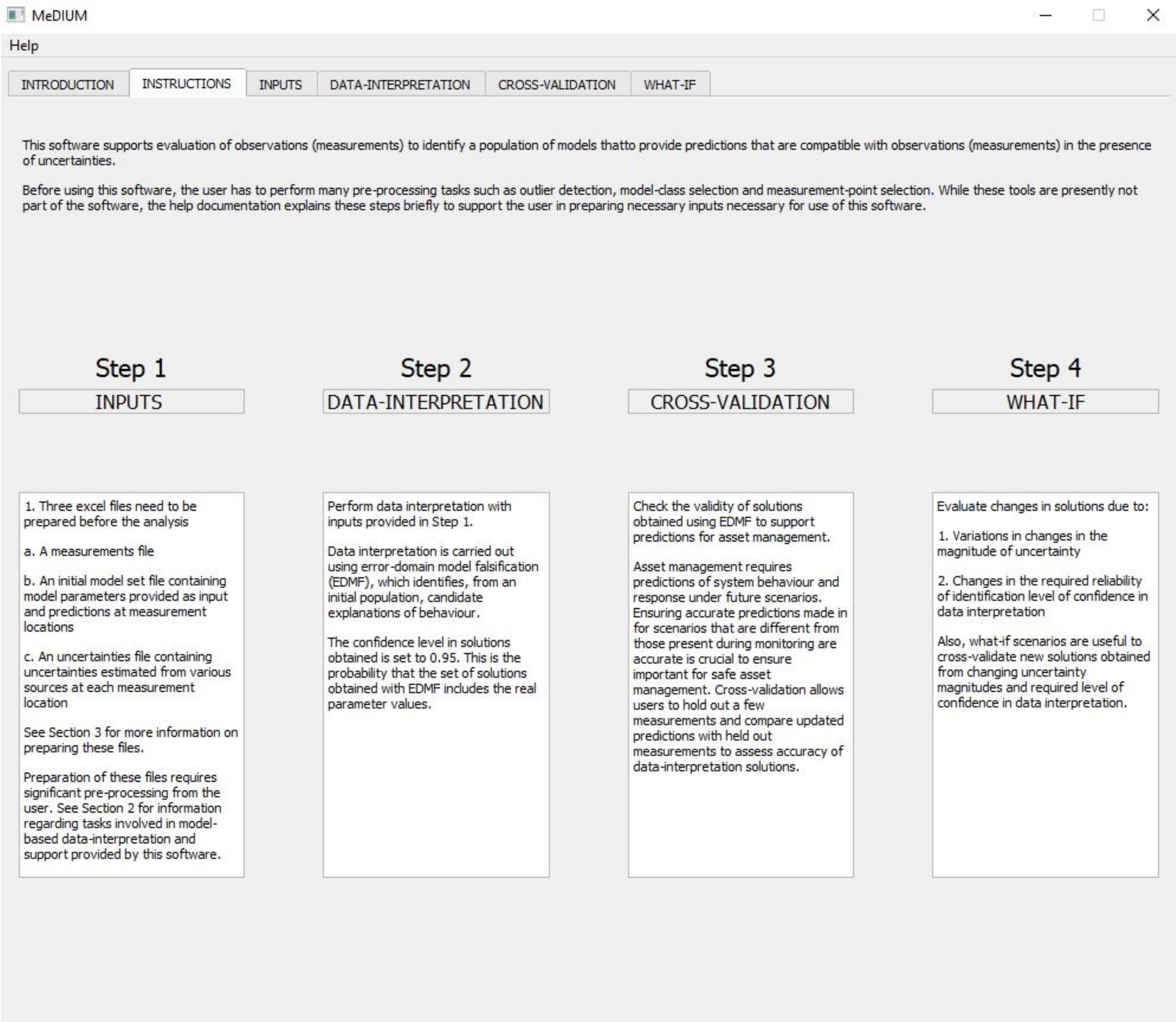


Figure 2 Instructions tab outlines the steps involved in using MeDIUM.

2.1 Step 1: Inputs

The user needs to provide three excel files as input to upload information such as measurements, model instances and predictions and estimations of uncertainties affecting the task of data interpretation. These excel files have to be prepared before the analysis by the user. These three files are:

a) A measurement file

b) An initial model set file containing model parameters provided as input and predictions at measurement location
c) An uncertainties file containing uncertainties estimated from various sources at each measurement location

Preparation of these files requires significant pre-processing from the user. See Section 3 for more information on tasks related to preparing these files including formatting instructions.

2.2 Step 2: Data interpretation

MeDIUM supports interpretation of data using error-domain model falsification (EDMF). EDMF is a multi-model probabilistic data interpretation methodology developed by Goulet and Smith (2013) and supported by more than ten years of research and development [8].

In EDMF, model instances that provide predictions that are incompatible with measurements are rejected. Model instances not rejected (falsified) are called candidate model instances and form the candidate model set (CMS).

See Section 4 for more information related to EDMF and use of MeDIUM to perform EDMF.

2.3 Step 3: Cross-validation

Check validity of solutions obtained using data interpretation to support predictions for asset management. Safe asset management requires accurate predictions under possible future scenarios. Accurate interpretations of data support accurate predictions.

In this step, the user may assess the accuracy of data interpretation using data-driven cross-validation methods. Updated candidate model instances are used to predict response at new measurement locations (data not used during updating). If candidate model predictions include measured values (within bounds), then accurate data interpretation is possible.

See Section 5 for more information on performing cross-validation and various cross-validation techniques available to the user.

2.4 Step 4: What-if scenarios

Steps 1, 2, and 3 involve few user-defined and default assumptions related to:

- Magnitude of uncertainty. It is defined by the user with the uncertainties file provided as input in Step 1 (Section 2.1).
- The level of confidence (probability that the candidate model set includes the correct solution). It is, by default, set equal to 0.95.

These two factors may be varied in this step with the help of sliders provided in the what-if tab. Changing these factors changes the solutions of data interpretation. The user may visualize and validate solutions (similar to Step 2 and Step 3). See Section 6 for more information related to evaluating what-if scenarios and significance of this step.

3 Inputs

This is Step 1 of using MeDIUM. The user provides inputs to the software in the form of excel files.

IMPORTANT:

Before uploading any data, please use the checkbox available at the top of the "Inputs" tab in MeDIUM to select/unselect the geotechnical excavation option. The difference between default and geotechnical excavation application arises due to variations in forms of uncertainties and measurements. Unchecking the box leads to MeDIUM carrying out the "default" analysis option. Checking the box will enable application to typical geotechnical excavation analysis.

Default analysis option includes applications such as load-test data interpretation [9], [10], continuous monitoring (acceleration) data interpretation [11], [12] and occupant localization [13] among others [8].

The geotechnical analysis option includes interpretation of data from excavation cases, which require different forms of uncertainty estimation [14], as explained in Section 3.3.2.

3.1 Measurement data

Monitoring a system using various types of sensors is an important task that has to be performed before using this software. Measurements may be data that are either recorded by sensors or retrieved by analysis of full-field techniques such as photogrammetry and interferometry.

For example, a bridge may be monitored using sensors such as strain gauges and inclinometers placed at various locations along its span. Typically, a load test is carried out on the bridge to obtain measurements. A load test involves placing a known load (such as a truck whose weight is known) on the bridge and recording strain generated in the bridge using the installed strain gauges. Other sensors such as accelerometers may also be used to continuously monitor a bridge over a period of time, which helps obtain dynamic characteristics such as natural frequencies that may also be used as measurements. In the context of an excavation, typical sensors include inclinometers and settlement markers. An inclinometer can be installed on the wall or in soil to measure lateral movement while a settlement marker is typically used to measure the vertical movement at ground level.

Figure 3 shows an example measurements file to be provided as input.

	A	B	C	D	E	F	G
1	22.00	-13.00	-8.00	-10.00	-4.00	-5.00	80.00

Figure 3 Excel file with measurements in one row. Each column is a new measurement value to be used to falsify model instances as described in Section 0.

As shown in Figure 3, the measurements file includes a list of measurements (one measurement per column) in a single row. Each measurement corresponds to structural response recorded at a sensor for a load test. These values are a structural response such as strain, deflection, inclination, acceleration, velocity and natural frequency.

Use the “Browse” button under section “Measurement Data” section in “Inputs” tab of MeDIUM software to load the excel file of measurements.

Significant effort is necessary to obtain good measurements. Prior activities such as design of measurement system [15], planning of load tests [16], assessment criteria [17] and outlier removal are important. A well designed measurement system helps obtain informative measurements that improve understanding of structural behaviour and support effective asset management [18].

3.2 Initial Model Instances and Predictions

A physics-based model is used to interpret monitoring data (measurements) uploaded using the measurements file. A physics-based model is a behaviour model, which takes instances of model-parameter values as input and provides predictions at measurement locations as output.

For example, if a bridge is monitored using seven sensors (see Figure 3 for an example of measurements from sensors), then using the physics-based model, predictions of response at these sensor locations are simulated. Multiple predictions are made by varying the input model parameters.

Selecting the right parameterisation of the model (what parameters are in the model) [19], the right parameters to vary (called as model class selection) [20] and a strategy for sampling (obtaining samples of input parameter values) [9], [21] are important to generate the model instances and predictions file.

Figure 4 shows the format of the instances of model-parameter values and predictions file. Instances of model-parameter values uploaded using this file are termed the initial model set (IMS), which implies that these are samples of prior parameter values (instances). They are prior because this is before information from measurements is used to update knowledge related to these parameter values.

The first row of the instances of model-parameter values and predictions file is a header indicating whether the column contains input parameter values or output prediction values. Let the number of parameters selected to be varied be p and let m be the number of measurement locations at which model response has to be simulated. Then, from the second row onwards, each row in the model instances and predictions file includes values of p parameters provided as input to the physics-based model and the subsequent model predictions at m measurement locations.

	A	B	C	D	E	F	G	H	I	J	K	L
1	Parameter	Parameter	Parameter	Parameter	Parameter	Prediction	Prediction	Prediction	Prediction	Prediction	Prediction	Prediction
2	4.00	3.00	4.00	2.00	4.00	37.24	-31.79	-29.03	-29.18	-23.97	-23.75	104.60
3	5.33	3.00	4.00	2.00	4.00	29.07	-29.63	-26.91	-27.01	-21.97	-21.64	87.56
4	6.67	3.00	4.00	2.00	4.00	28.23	-29.09	-26.42	-26.51	-21.66	-21.31	84.10
5	8.00	3.00	4.00	2.00	4.00	28.21	-29.05	-26.38	-26.47	-21.64	-21.29	83.89
6	4.00	4.33	4.00	2.00	4.00	38.96	-21.60	-18.84	-18.98	-13.78	-13.54	84.12
7	5.33	4.33	4.00	2.00	4.00	34.56	-18.20	-15.46	-15.60	-10.51	-10.24	62.01
8	6.67	4.33	4.00	2.00	4.00	34.02	-17.25	-14.55	-14.69	-9.76	-9.50	56.87
9	8.00	4.33	4.00	2.00	4.00	34.02	-17.18	-14.49	-14.62	-9.72	-9.46	56.57
10	4.00	5.67	4.00	2.00	4.00	39.20	-20.68	-17.92	-18.06	-12.88	-12.63	82.33
11	5.33	5.67	4.00	2.00	4.00	35.21	-17.01	-14.26	-14.41	-9.33	-9.06	59.37
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
1013	8.00	3.00	9.00	6.00	8.00	23.42	-9.38	-6.69	-6.91	-2.02	-2.05	59.70
1014	4.00	4.33	9.00	6.00	8.00	36.50	-11.45	-8.67	-8.91	-3.69	-3.74	74.02
1015	5.33	4.33	9.00	6.00	8.00	27.16	-9.36	-6.61	-6.84	-1.73	-1.78	61.72
1016	6.67	4.33	9.00	6.00	8.00	26.24	-8.88	-6.18	-6.40	-1.48	-1.52	58.99
1017	8.00	4.33	9.00	6.00	8.00	26.21	-8.84	-6.14	-6.37	-1.47	-1.51	58.82
1018	4.00	5.67	9.00	6.00	8.00	36.78	-11.24	-8.47	-8.71	-3.50	-3.54	74.00
1019	5.33	5.67	9.00	6.00	8.00	27.79	-9.23	-6.48	-6.72	-1.61	-1.66	61.60
1020	6.67	5.67	9.00	6.00	8.00	26.94	-8.74	-6.04	-6.27	-1.36	-1.40	58.80
1021	8.00	5.67	9.00	6.00	8.00	26.92	-8.70	-6.01	-6.24	-1.35	-1.39	58.63
1022	4.00	7.00	9.00	6.00	8.00	36.85	-11.22	-8.45	-8.69	-3.49	-3.53	73.97
1023	5.33	7.00	9.00	6.00	8.00	27.90	-9.21	-6.46	-6.70	-1.61	-1.65	61.57
1024	6.67	7.00	9.00	6.00	8.00	27.06	-8.72	-6.02	-6.25	-1.35	-1.39	58.77
1025	8.00	7.00	9.00	6.00	8.00	27.04	-8.68	-5.99	-6.22	-1.34	-1.38	58.59
1026												

Figure 4 Initial model instances and predictions file. Each row in the file includes parameter values provided as input to a physics-based model and corresponding predictions at the measurement locations. The header of the file indicates whether a column contains either input-parameter values or recorded predictions.

In the model instances and predictions file, it is important to report the predictions at measurement locations in the same order as order of measurements in the measurements file. This ensures that predictions are compared with the correct measurement value during the data interpretation step. Additionally, ensure that predictions have the same units (such as mm and Hz) as the input measurements.

To upload this model instances and predictions file, use the “Browse” button available in the “Inputs” tab under section ‘Initial Model Parameter Instances and Predictions”.

3.3 Uncertainty Estimations

Interpreting monitoring data with physics-based models supports use of data-informed models to predict future scenarios for decision-making. Physics-based models developed to simulate the behaviour of real-world systems such as bridges involve many assumptions, which lead to uncertainty in model behaviour [19]. Additionally, models to simulate behaviour of civil infrastructure involve safe assumptions (biased) due to the high perception of risk, which leads to biased uncertainties [4], [22], [23].

While modelling and model-class selection assumptions contribute the most to the uncertainties affecting the task of data interpretation, uncertainties may be present from other sources such as sensors noise and environmental effects. Appropriate estimation of uncertainty sources, magnitude and bias are important for accurate interpretation of monitoring data [4], [22], [24], [25].

In the next two sections, the format of uncertainties file to be uploaded is described. The format of uncertainties file for geotechnical excavation applications is different from the format of uncertainties file for default applications. This difference is indicative of the nature of uncertainties, which differ from one case to another and one application to another. Therefore, the task of uncertainty estimation is important and requires engineering expertise and heuristic knowledge.

3.3.1 <u>Uncertainty estimation for default applications</u>

The uncertainties file for default data-interpretation applications (data from bridges, buildings etc.) is an excel file. Each excel sheet in this file includes information regarding uncertainty from one source (for example model uncertainty, measurement uncertainty etc.). Number of excel sheets in the uncertainty file should be equal to the (expected) sources of uncertainty affecting the task of data interpretation.

In each excel sheet, the user should include information about the uncertainty distribution at each measurement location. The first row in each excel sheet is a header. The header for each column includes the following:

- Sensor ID: Index of sensor. List the sensors in the same order as they are reported in the measurements file.
- Type: Type of uncertainty distribution. Presently, the software supports uniform and normal uncertainty distributions, which are typically used to quantify most uncertainty sources affecting civil system identification.
- Dist_param_1: First parameter defining the uncertainty distribution.

  If type of uncertainty is “uniform”, then this column should include the minimum value of the uncertainty distribution.

  If type of uncertainty is “normal”, then this column should include the mean value of the uncertainty distribution.

- Dist_param_2: Second parameter defining the uncertainty distribution.

  If type of uncertainty is “uniform”, then this column should include the maximum value of the uncertainty distribution.

  If type of uncertainty is “normal”, then this column should include the standard deviation value of the uncertainty distribution.

Figure 5 shows an example of multiple excel sheets which include information about the uncertainty distribution from each source, for all measurement locations. It is important that the uncertainty distributions are estimated in same units as measurements.

1	Sensor ID	Type	dist_param_1	dist_param_2
2	1	Uniform	-2.229	1.1145
3	3	Uniform	1.35	-0.675
4	4	Uniform	0.81	-0.405
5	5	Uniform	0.97	-0.485
6	6	Uniform	0.403	-0.2015
7	7	Uniform	0.532	-0.266
8	8	Uniform	-8.107	4.0535
9				

(a) Model uncertainty

1	Sensor ID	Type	dist_param_1	dist_param_2
2	1	Uniform	-2.78625	0.80244
3	3	Uniform	1.242	0
4	4	Uniform	0.891	0
5	5	Uniform	1.0573	0
6	6	Uniform	0.16926	-0.54405
7	7	Uniform	0.17024	-0.79268
8	8	Uniform	-0.64856	2.26996
9				

(d) Secondary parameter uncertainty

1	Sensor ID	Type	dist_param_1	dist_param_2
2	1	Normal	0	1
3	3	Normal	0	1
4	4	Normal	0	1
5	5	Normal	0	1
6	6	Normal	0	1
7	7	Normal	0	1
8	8	Normal	0	1
9				

(b) Measurement uncertainty

1	Sensor ID	Type	dist_param_1	dist_param_2
2	1	Uniform	-0.02229	0.02229
3	3	Uniform	0.0135	-0.0135
4	4	Uniform	0.0081	-0.0081
5	5	Uniform	0.0097	-0.0097
6	6	Uniform	0.00403	-0.00403
7	7	Uniform	0.00532	-0.00532
8	8	Uniform	-0.08107	0.08107
9				

(e) Load magnitude uncertainty

1	Sensor ID	Type	dist_param_1	dist_param_2
2	1	Uniform	0	5.3496
3	3	Uniform	0.2835	0
4	4	Uniform	0.05103	0
5	5	Uniform	0.3686	0
6	6	Uniform	0.09672	0
7	7	Uniform	0.2926	0
8	8	Uniform	0	1.86461
9				

(c) Load position uncertainty

Figure 5 Sheets of excel file including estimations of uncertainties from a multitude of sources. Most uncertainties are biased and not normally distributed. All uncertainty values are in the same units as those used for measurement input.

3.3.2 Uncertainty estimation in geotechnical applications

The uncertainties file for geotechnical excavation analysis is an excel file. The excel contains two sheets. The first sheet is related to 3D effects. Due to the nature of EDMF, multiple simulations of the physics-based model are needed to generate the initial model-parameter-value set and their predictions. In this regard, a plane-strain model is typically adopted to model an excavation from a practicality point of view. However, the plane-strain model may yield biased predictions in the case where the excavation 3D effect is significant. Therefore, Wang et al. [11] proposes a technique to quantify the 3D effect as an uncertainty term that corrects the predictions obtained from the plane-strain model. In this arrangement, the bulk computations can still be performed using a plane-strain model and the accuracy of the predictions is attained by including the uncertain error term that represents the 3D effects.

Following the procedure described in Wang et al. [11], users are expected to perform two realizations of the full 3D models with the upper bound and lower bound of the initial parameter values. In the "3D Effects" sheet, users are then expected to input four sets of data, i.e. 3D upper bound predictions (mm), 3D lower bound prediction (mm), 2D upper bound prediction (mm), 2D lower bound prediction (mm). The number of columns corresponds to the number of measurement data shown in Figures 3 and 4.

Measurement ID	3D upper bound prediction(mm)	3D lower bound prediction(mm)	2D upper bound prediction(mm)	2D lower bound prediction(mm)
1	2.32	28.47	2.66	30.43
2	2.40	28.90	2.77	31.54
3	2.48	29.26	2.88	32.65
4	2.56	29.73	2.99	33.76
5	2.61	30.00	3.10	34.86
6	2.67	30.46	3.19	35.91
7	2.70	30.76	3.27	36.91
8	2.73	31.07	3.33	37.84
9	2.74	31.29	3.37	38.69
10	2.73	31.68	3.40	39.44
11	2.72	31.75	3.40	40.11
12	2.68	31.85	3.40	40.66
13	2.65	31.87	3.37	41.12
14	2.61	31.92	3.34	41.48
15	2.52	31.84	3.30	41.73
16	2.48	31.73	3.25	41.88
17	2.41	31.57	3.19	41.95
18	2.35	31.34	3.13	41.92
19	2.28	31.07	3.05	41.78
20	2.21	30.79	3.00	41.62
21	2.15	30.47	2.94	41.41
22	2.09	30.14	2.85	41.01

Figure 6 Examples of uncertainty input for 3D effects.

In the "Measurement Uncertainty" sheet, users are expected to input one set of data, for example, Vertical distance from toe of wall to the measurement point(m). This is for the purpose of deriving uncertainties pertaining to inclinometer errors. The errors associated with inclinometers are a function of the vertical distance from toe of the wall to the measurement point of interest.

Measurement ID	Vertical distance from toe of wall (m)
1	40
2	39
3	38
4	37
5	36
6	35
7	34
8	33
9	32
10	31
11	30
12	29
13	28
14	27

Figure 7 Examples of uncertainty input for measurement uncertainty.

After the uncertainty data is specified, the user is expected to use the "browser" button to load this excel file to the software. For geotechnical excavation analysis, an intermediate calculation, taking the information in this excel, is carried out to arrive at the final uncertainty values for the subsequent EDMF analysis. This step is performed automatically.

A popup window, shown in Figure 8, appears when all inputs have been provided in the correct format.

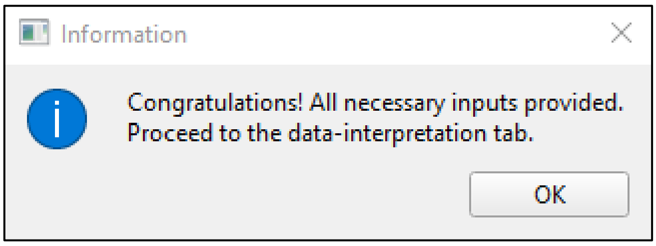


Figure 8 Popup window appears to inform the user that all necessary inputs have been uploaded successfully.

4 Data Interpretation

MeDIUM is a software implementation of a probabilistic data-interpretation methodology called EDMF. In this methodology, model instances (uploaded using the model instances and predictions file) that provide predictions incompatible with measurements (uploaded using the measurements) are falsified (rejected). Compatibility is assessed using thresholds (tolerance) on residuals between model predictions and measurements. These threshold values are computed based on uncertainty associated with the interpretation task at each measurement location (uploaded using the uncertainties file).

The process of performing EDMF is shown in Figure 9. Input provided using the model instances and predictions file (see Section 3.2) includes an initial population of parameter values ($\boldsymbol{\theta}$) and corresponding predictions ($g_i(\theta)$, $i \in [1, \ldots, m]$) obtained using a model at measurement locations (m). These model predictions are compared with measurements (y_i) uploaded using the measurements file (see Section 3.1). The comparisons are made based on the magnitude of uncertainty associated with model predictions and measurements. Estimations of these uncertainties are uploaded using the uncertainties file (see Section 3.3).

As shown in Figure 9, the output of EDMF is a set of candidate model instances, which form the candidate model set (CMS). All model instances in the CMS are assumed to be equally likely solutions, *i.e.*, no instance is more likely to be the correct solution than others.

CMS is a subset of the initial model set (IMS) of parameters. Model instances from the CMS provide predictions that are compatible with measurements, $\boldsymbol{y}$. This assessment of compatibility is carried out on the basis of Eq. (1).

$$T_{\mathrm{low},i} \leq g_i(\theta) - y_i \leq T_{\mathrm{high},i} \qquad i \in [1, \ldots, m] \tag{1}$$

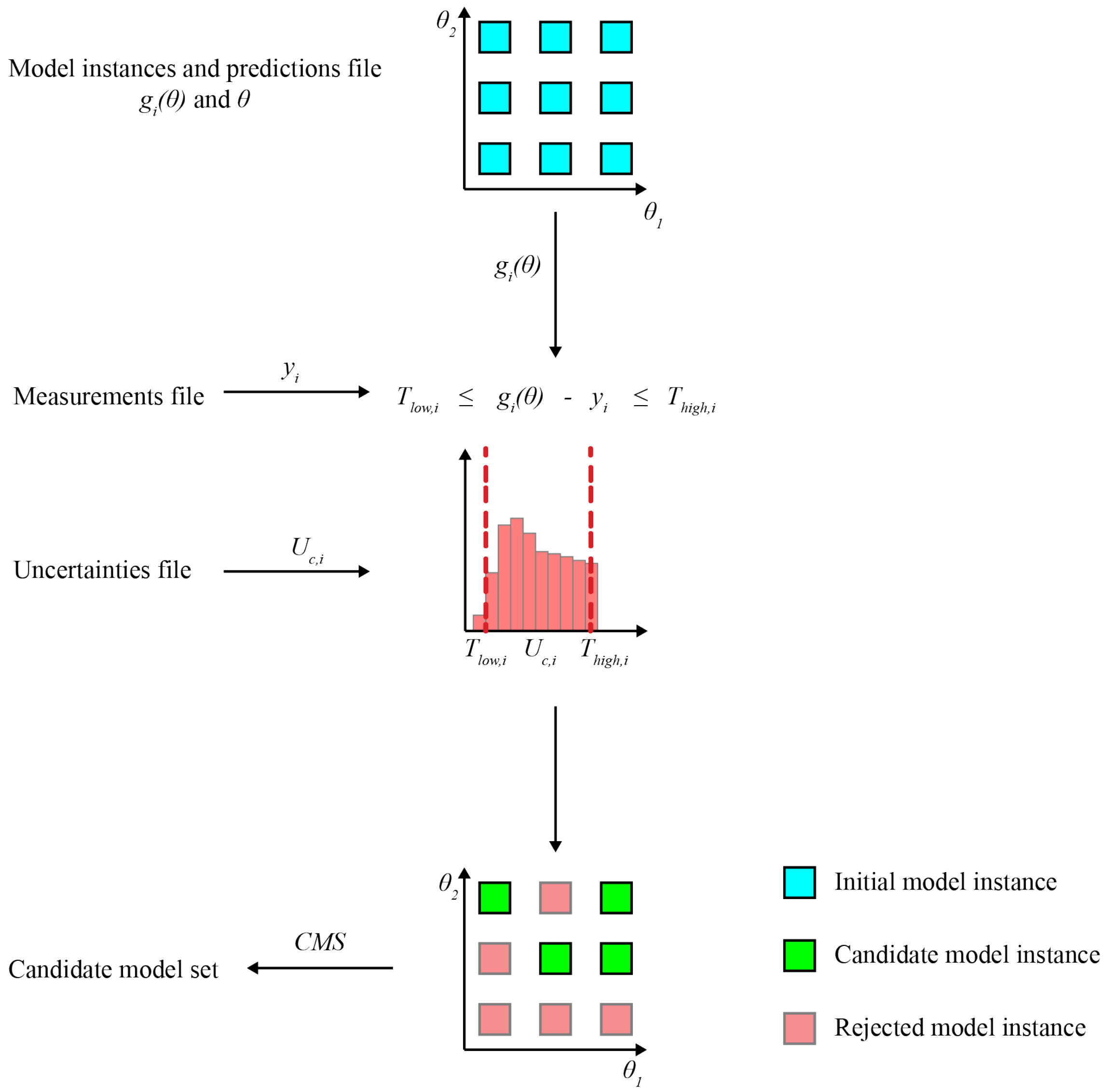


Figure 9 Steps involved in EDMF

In Eq. (1), $g_i(\theta)$-y_i, is the residual between measurement and model predictions at a measurement location, i. $T_{\text{low},i}$ and $T_{\text{high},i}$ are compatibility thresholds calculated for measurement location i. These compatibility thresholds are calculated based on the modelling and measurement uncertainties that are affecting the task of structural identification, which are uploaded by the user as described in Section 3.3.

Let ϵ_i be the combined uncertainty at a measurement location i. This combined uncertainty is calculated by combining uncertainty from various sources using Monte Carlo sampling. Using this combined uncertainty at a measurement location i, thresholds for falsification, $T_{low,i}$ and $T_{high,i}$, are calculated using Eq. (2).

$$\varphi^{1/m} = \int_{T_{\text{low},i}}^{T_{\text{high},i}} f(\varepsilon_i) \quad d\varepsilon_i \tag{2}$$

In Eq. (2), $f(\varepsilon_i)$ is the PDF of combined uncertainty at measurement location i and φ is the target reliability of identification. In Eq. (2), $\varphi \in [0,1]$ is the desired target reliability of identification [4]. While Eq.2 has an infinite number of solutions, the thresholds, $T_{high,i}$ and $T_{low,i}$, are computed as the ones that provide the shortest interval. In Eq. (2), the term $1/m$ is the Sidak correction [26], which accounts for m independent measurements used in identification of model parameters (uploaded using the measurements file in Step 1).

The target reliability of identification (φ) in Eq. (2) is a metric that determines the probability that the true parameter values are included in the CMS. A larger value φ indicates a higher probability that the data interpretation is accurate. However, to ensure this, the data interpretation methodology has to accept more instances from the IMS as possible explanation of observed behaviour. This leads to a larger CMS and less precise interpretation of data. In MeDIUM, the target reliability of identification (φ) is set to a default value of 0.95.

Falsification rate is the percentage of model instances rejected from the IMS. It is an indication of reduction in parametric uncertainty using information from measurements.

Falsification rate = (Number of initial model instances - Number of candidate models) * 100 / Number of initial model instances

A high falsification rate is indicative of:

- Informative measurements and good selection of parameters for data interpretation
- Outliers in measurement data
- Poor estimation of uncertainties and prior distribution of parameters
- Poor exploration of parameter space for solutions (insufficient samples of initial model instances)
- The wrong model parameterization

A low falsification rate is indicative of:

- Uninformative measurements with respect to parameters chosen for data interpretation
- Conservative estimation of uncertainties
- Inaccurate estimation of prior model-parameter distributions

If the falsification rate is either too high or too low, then the user is advised to re-evaluate the inputs provided.

EDMF, when compared with other data-interpretation methodologies, has been shown to provide more accurate identification [4] and prediction [22]. EDMF is more accurate due to its robustness to correlation assumptions and explicit estimation of model bias based on engineering heuristics [4], [22].

While using EDMF with MeDIUM, the resulting output is a parallel axis plot. A parallel axis plot is a tool to visualize a multi-dimensional plot in two dimensions. Figure 10 shows an example of a parallel axis plot.

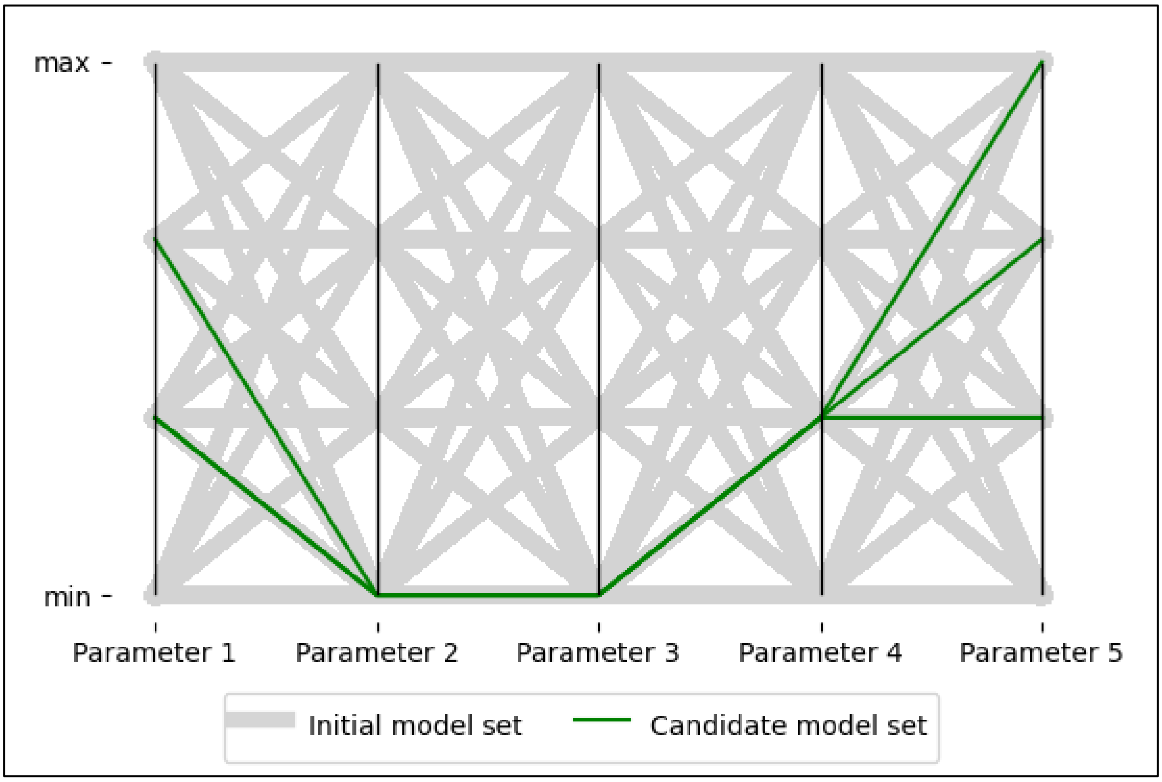


Figure 10 Example of a parallel axis plot that is generated as an output of data interpretation using MeDIUM.

In a parallel axis plot, along the X-axis, there are multiple Y-axis. Each Y-axis is parallel to the others and represents one parameter (dimension) among all parameters chosen for interpretation. Points along the Y-axis indicate values of a parameter. For example, along Y-axis = Parameter 5 in Figure 10, the lower most point on this axis (Y = 0) represents the smallest value attributed to Parameter 5.

Figure 10 shows the IMS and CMS. Each instance in either model set is a line connecting the different parallel Y-axis. The point where the line intersects a Y-axis is the parameter value provided as an input to the model. For example, Y = 0 for axis Parameter 1 to Parameter 5 is a line that represents a model instances with minimum values of Parameter 1 to Parameter 5. A parallel axis plot, as shown in Figure 10, helps in visualizing the CMS relative to the IMS.

5 Cross-validation

EDMF has been shown to provide accurate model updating for theoretical cases using simulated measurements [4], [22]. In theoretical comparisons, the ground truth values are known. For assessment of accuracy of full-scale structures, data-driven methods may be used for quantitative validation.

EDMF solutions have been validated for full-scale case studies using leave-one-out cross-validation [9] and hold-out cross-validation [24]. In these comparisons, one or more measurements (data points) are excluded during identification. Subsequent to identification, the updated parameter values are used to predict response at measurement locations that were excluded. If the predicted response is similar to the measurement value, then structural identification is assumed to be validated [27].

In MeDIUM a "Cross-validation" tab is provided to assess the accuracy of data interpretation solutions. Cross-validating solutions provides insights into potential inaccuracies in uncertainty estimations leading to inaccurate data-interpretation.

5.1 Hold-out cross-validation

In Step 1, the user has uploaded measurements from "m" locations using the "measurements file". In the cross-validation step, few of these measurements may be left out from being used while performing EDMF. In the 'Cross-validation" tab, there is a text box available, where the user can input the index of measurement locations to be left out.

For example, if 8 measurements were uploaded, the user may input: 1-3, 5. With this input in the text box, measurements 1, 2, 3 and 5 will be excluded and EDMF will be performed with the remaining measurements.

Let the number of measurements held out from a set of m measurements be h. EDMF is performed with m-h measurements. Predictions corresponding to these measurements from the “model instances and predictions file” and uncertainties from the “uncertainties” file are used by the software to perform EDMF.

Predictions at h measurement locations held out corresponding to the CMS instances are compared with the measurements at these h locations. If the bounds of these updated-prediction distributions include the measured values held out, then solutions obtained are accurate. If the updated prediction bounds do not include the measured values, then structural identification is inaccurate, as shown in Eq. (3).

$$\text{Accuracy}, \Psi_i = \begin{cases} 1 & \text{for } y_i \in \left[\min\left[g_i(\theta''), \max[g_i(\theta'')]\right]\right] \\ 0 & \text{for } y_i \notin \left[\min\left[g_i(\theta''), \max[g_i(\theta'')]\right]\right] \end{cases} \quad \textbf{(3)}$$

In Eq. (3), Ψ_i is a binary variable with value equal to 1 for accurate structural identification and 0 for inaccurate structural identification at a measurement data point i, which is held out for validation. In the equation, $\boldsymbol{\theta''}$, are instances from CMS obtained using EDMF.

Using cross validation, precision of the data interpretation can be quantified in addition to accuracy. Precision is a measure of variability either in updated model-parameter distributions or model predictions. Using cross-validation, precision, ϕ, is estimated as the relative (to measurement) average (over all measurements held out) reduction in prediction uncertainty using measurements [9].

A precision index, ψ, equal to one implies perfect data-interpretation where updated parameters and consequently the prediction distributions have zero variability. Conversely, ψ equal to zero implies that no information was acquired from measurements regarding the model-parameter distributions.

In MeDIUM, when the user performs cross-validation, the following results are provided:

- Number of initial model instances uploaded in Step 1.
- Number of candidate models identified using measurements included for data interpretation (*m-h*)
- Falsification rate = (Number of initial model instances - Number of candidate models) * 100 / Number of initial model instances
- Updated bounds (minimum and maximum) of model parameter distributions obtained with EDMF
- Precision (ψ) of data interpretation (Average reduction in prediction uncertainty)
- Accuracy of data interpretation (Number of cases where updated predictions include measurement value out of all measurements left out)

The falsification rate is directly related to the level of precision, which is a measure of reduction in uncertainty. Greater falsification of model instances reduces parametric uncertainty, which reduces prediction uncertainty that is used to calculate precision. Accuracy of data interpretation is the number of comparisons where predictions include measurement value out of the *h* measurements held out for validation.

Two figures are plotted when the user clicks on the “Validate” button:

- Parallel Axis Plot – This figure shows the candidate model set obtained without using measurements retained for validation.
- Cross-Validation Plot - This figure shows the results of leaving out measurements.

Validated accurate CMS may be used to predict structural response at unmeasured locations under new and future scenarios to support decision making.

5.2 Leave-one-out cross-validation

In hold-out cross-validation, h measurements were left out from m available (uploaded) measurements. In leave-one-out cross-validation, $h = 1$, *i.e.* only one sensor is left out. Comparisons involve repeating the process of cross-validation with one measurement left-out for all measurements. Therefore, in each iteration $h = 1$ and m-1 measurements are used to perform EDMF and m-1 cases of validation are performed.

In MeDIUM, there is no explicit option available to perform leave-one-out cross-validation. Instead, the user may repeatedly perform hold-out cross-validation as described in Section 5.1 with only one measurement held out.

6 What-if scenarios

Step 4 and Step 5 involve interpreting measurement data and validating solutions of data-interpretation. However, these interpretations involve assumptions. In this step, the user may assess the sensitivity of data-interpretation solutions to two important assumptions:

- Magnitude of uncertainty

  The magnitude of uncertainty affecting the data interpretation task is estimated by the user and uploaded using the uncertainties file in Step 1. However, the uncertainty value is not completely known and these estimations are based partly on the user's expert judgement.

  In this tab, the user is provided with a slider, as shown in Figure 11. Using this slider, the user may choose a multiplier value. The uncertainty estimations provided by the user are multiplied by this value to either decrease or increase the magnitude of uncertainty.

  Using the slider shown in Figure 11, the user may re-estimate uncertainty affecting the task of data-interpretation. Changing uncertainty affects the EDMF thresholds, which consequently leads to a new CMS.

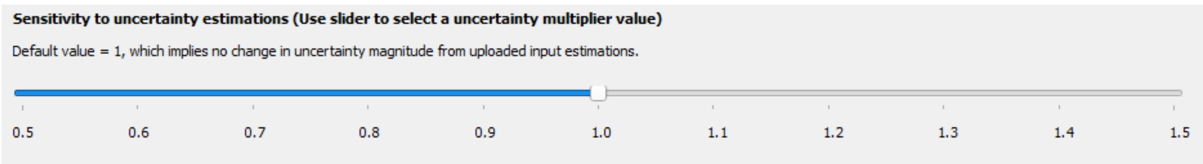


Figure 11 Slider to select value of uncertainty multiplier

- Target reliability of identification

  The target reliability of identification is a user-defined metric that is used to calculate the EDMF thresholds using Eq. (2). It represents the confidence (lower bound of probability) that the identified CMS includes parameter values representing true system behaviour.

In this what-if tab, the user is provided with a slider, as shown in Figure 11. Using this slider, the user may vary the target reliability of identification. As the value is varied, the threshold bounds calculated using Eq. (2) change. Correspondingly, a new CMS is obtained. Varying the target reliability of identification, the user may assess sensitivity of CMS to this input metric.

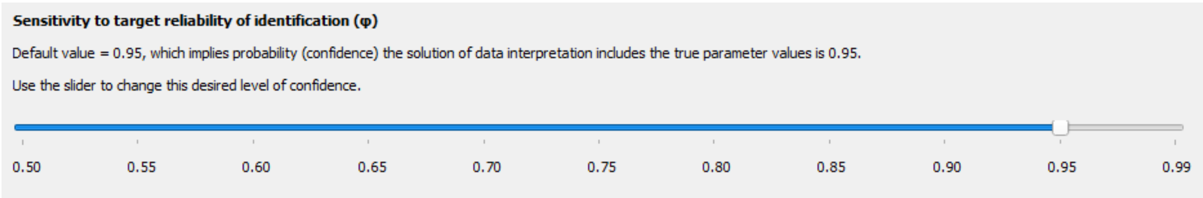


Figure 12 Slider to select value of target reliability of identification.

Varying these two metrics, the user obtains new CMSs. In this tab, functionality to perform cross-validation (as explained in Section 5) is also included.

Varying these metrics, the user may perform checks such as:

- Assess if uncertainty estimations are the cause for high falsification rate using EDMF
- Detect spurious measurements that lead to unusually high falsification rate, even complete falsification of model class in some cases [28]
- Verify uncertainty estimations based on validation results [9]

7 Examples

Two examples, a bridge case study and a geotechnical excavation case study are provided in this section to explain use of MeDIUM.

7.1 Bridge case study

In this example, data interpretation from the Crêt de l'Anneau Bridge, shown in Figure 13, is described. Deflection and strain measurements were recorded during a load-test on this bridge. The Crêt de l'Anneau Bridge was built in 1969 and serves an elevated section of Route de la Promenade close to Neuchatel in Switzerland.

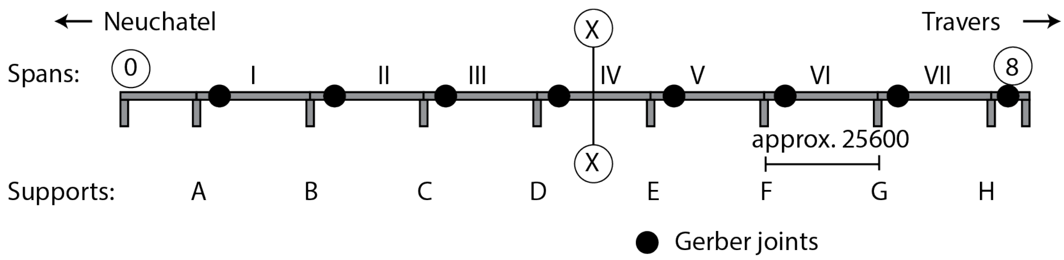
Figure 13. Crêt de l'Anneau viaduct, case study

Crêt de l'Anneau Bridge is a steel-concrete composite bridge with nine spans. The first and last spans connect the bridge to the roadway. The bridge is curved and the total length of the bridge along the inner arc is 195m. A Gerber joint (articulation) is placed in each span, 5m after support in direction Neuchatel-Travers, as shown in Figure 14 (a). Sensors for measuring response are placed in span IV at section X-X shown in Figure 14 (a). Placement of these sensors in the transverse section is shown in Figure 14 (b).

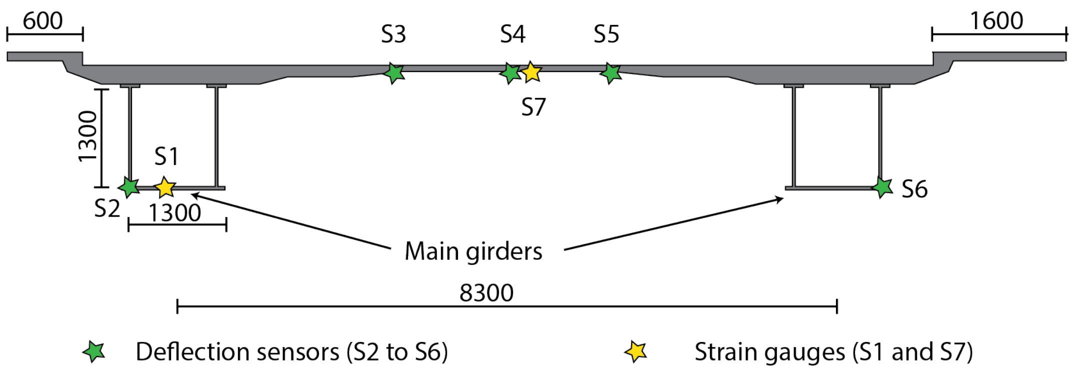
Strain and deflection sensors placed on the bridge to record the response as a 40-tonne truck moves across the bridge span. The truck moves along the centre of the bridge slowly and stops at middle of span IV for 20seconds. The response of the bridge recorded by the sensors when the truck stops at middle of span IV is used for structural identification of the Cret de l'Anneau bridge.

A finite element model of the bridge was developed using ANSYS R18 (external commercial application). Table 1 lists parameters included in the model and prior distribution ranges of these parameters. Refer to Bayane et al. [29] for more details on model development.

A total of 10 parameters were included in the FE model. The model class for structural identification is selected using forward-variable selection to search for a globally relevant model class. Five parameters are included for identification, which are listed in Table 2.



(a) Bridge elevation (Span 0 and 8 are special spans that connect the bridge to the road.)



(b) Cross-section of a typical span (X-X)

Figure 14 Schematic drawing of Cret de l'Anneau bridge and layout of sensors installed on the bridge

Table 1. Prior probability distributions of model parameters

Parameter	Description	Range
E_c	Young's modulus of concrete	30– 50
K_{sg_x}	Stiffness of the connection slab-girder - x direction	3 – 9
K_{sg_z}	Stiffness of the connection slab-girder - z direction	4 - 8
K_{sup_y}	Stiffness of the support D - y direction	3 - 7
K_{sup_z}	Stiffness of the support D - z direction	4 - 9
R_{sup_x}	Stiffness of the support D - x direction	7 -12
K_{art1_y}	Stiffness of the articulation (Gerber join in Span IV) - y direction	2 - 6
K_{art1_z}	Stiffness of the articulation Gerber join in Span IV) - z direction	4 - 8
K_{art2_y}	Stiffness of the articulation (Gerber join in Span V) - y direction	2 - 6
K_{art2_z}	Stiffness of the articulation (Gerber join in Span V) - z direction	4 - 8

Table 2. Parameters selected for structural identification

Parameter	Description
K_{sg_z}	Stiffness of the connection slab-girder - z direction
K_{sup_y}	Stiffness of the support D - y direction
K_{sup_z}	Stiffness of the support D - z direction
K_{art1_y}	Stiffness of the articulation (Gerber join in Span IV) - y direction
K_{art2_z}	Stiffness of the articulation (Gerber join in Span V) - z direction

In the next section, use of MeDIUM to update knowledge of these structural parameters is explained.

7.1.1 Interpretation using MeDIUM

Interpretation of data recorded using sensors during a load test can be performed using MeDIUM.

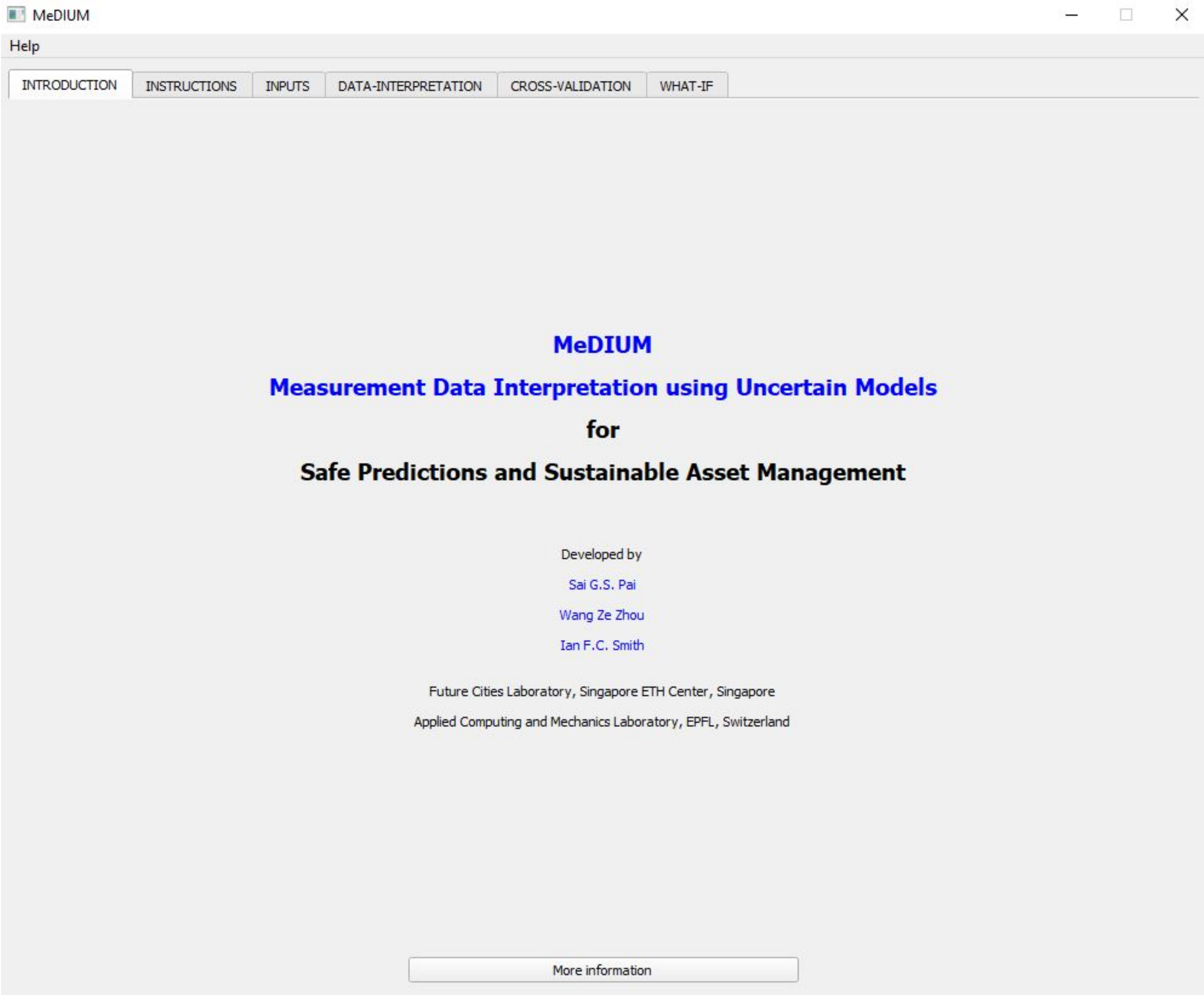


Figure 15 MeDIUM Home Page

The steps involved in using MeDIUM for data interpretation are shown in Figure 16. In the first step, user provides the software with inputs related to measurements, initial model instances (samples from prior parameter distributions) and uncertainties pertaining to the data interpretation task. See Section 3 for more details on input definitions.

In the second step, after providing all necessary inputs, the user may choose to perform EDMF using the data-interpretation tab. See Section 4 for more details on performing EDMF. In the third step, the user may cross-validate solutions of EDMF using leave-one-out and hold out cross-validation methods. See Section 5 for more details related to performing cross-validation using MeDIUM. In the fourth step, the user may repeat EDMF and cross-validation with new settings related to magnitude of uncertainty (see Section 3.1 and Section 6) and target reliability of identification (φ, see Section 4 and Section 6).

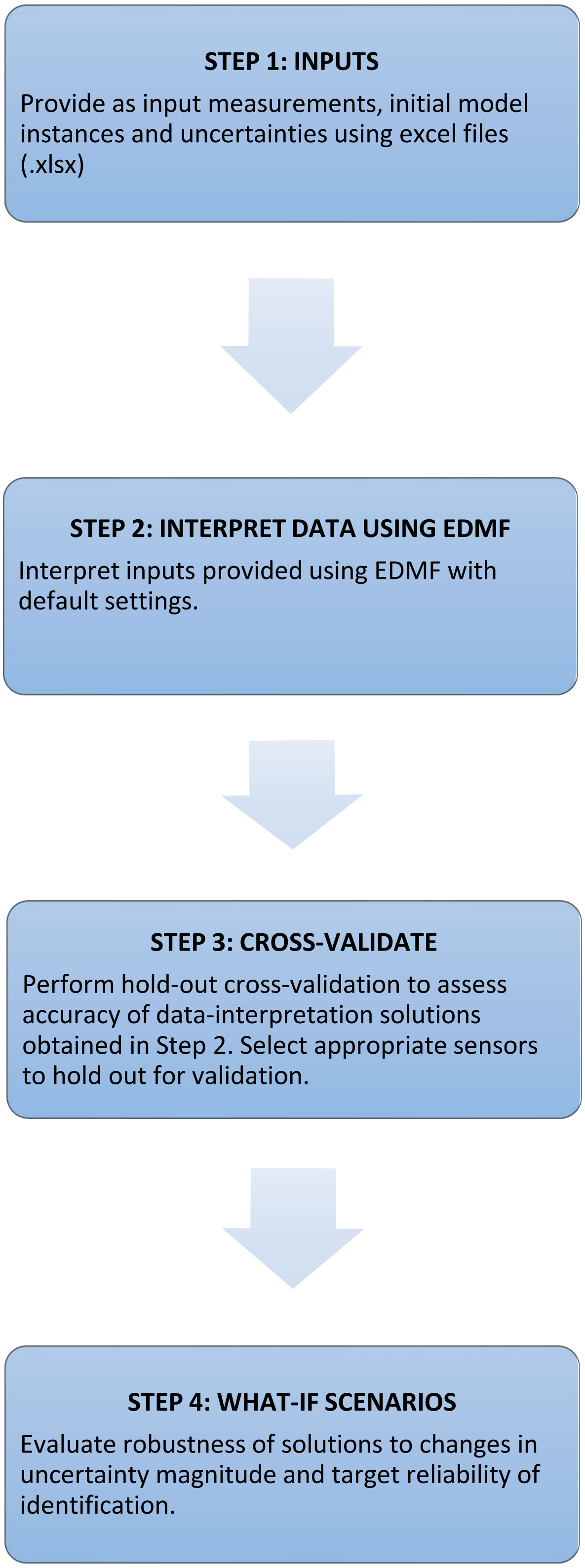


Figure 16 Steps involved in using MeDIUM to interpret measurement data using uncertain models.

7.1.1.1 Step 1: Input measurements file, initial model instances file and uncertainties file

In the inputs tab, use the browse buttons to search for appropriate excel files to provide as input.

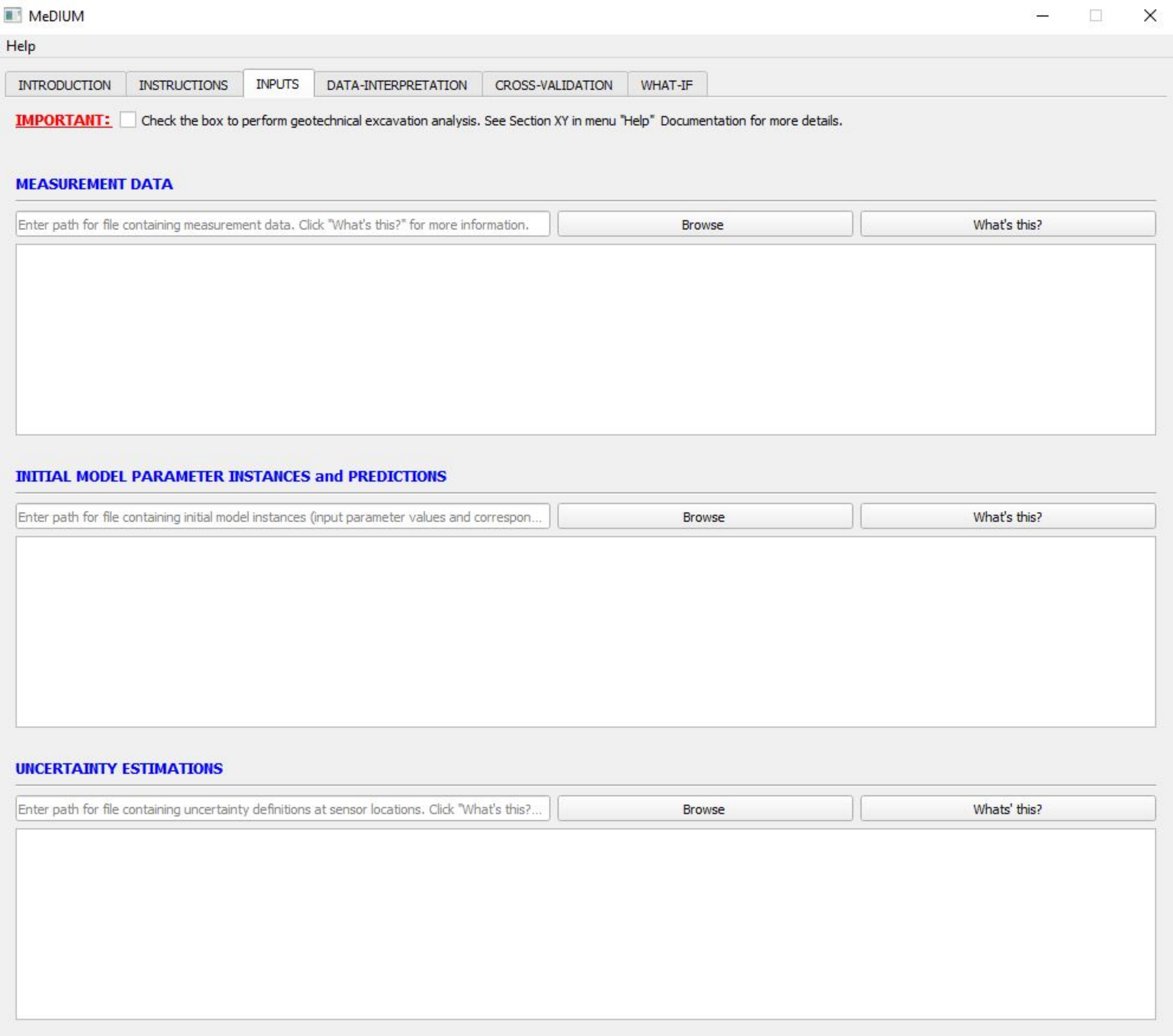


Figure 17 Browse and upload input files with information related to measurements, initial model set and uncertainty estimations.

If any of the excel files are not provided or the data in the excel files is not provided in the correct format, then the user will be directed to an error message while using MeDIUM.

The measurements file to be uploaded is an excel file with one row. In this row, provide measurements recorded using the monitoring system to be used for model-based data interpretation.

For the Cret de l'Anneau Bridge, measurements from seven sensors are available, which are recorded in the excel file. Each column in the row lists measurements from sensors S1 to S7, in that order, as shown in Figure 18. See Section 3.1 for more details on formatting the measurements file.

	A	B	C	D	E	F	G
1	22.00	-13.00	-8.00	-10.00	-4.00	-5.00	80.00

Figure 18 Excel file with measurements in one row. Each column is a new measurement value to be used to falsify model instances as described in Section 3.1.

In the initial model instances file, the first p (number of parameters) columns include instances of model parameters provided as input to a physics-based model, where p is the to be interpreted using data. In the next m (number of measurement points) columns, the predictions from the model at each of the measurement points is tabulated.

For the Cret de l'Anneau Bridge, five parameters have to be interpreted using data as listed in

Table 2 (See Table 1 for prior parameter distributions). Samples of these parameters are provided as input to the FE model of the bridge to predict responses at the seven sensor locations, S1 to S7. This information of inputs to the FE model and predictions are included in the excel file for initial model instances. The first row of this file lists whether a column was an input to the FE model (parameter) or an output (prediction), as shown in Figure 19. See Section 3.2 for more details on formatting the initial model instances and predictions excel file.

	A	B	C	D	E	F	G	H	I	J	K	L
1	Parameter	Parameter	Parameter	Parameter	Parameter	Prediction	Prediction	Prediction	Prediction	Prediction	Prediction	Prediction
2	4.00	3.00	4.00	2.00	4.00	37.24	-31.79	-29.03	-29.18	-23.97	-23.75	104.60
3	5.33	3.00	4.00	2.00	4.00	29.07	-29.63	-26.91	-27.01	-21.97	-21.64	87.56
4	6.67	3.00	4.00	2.00	4.00	28.23	-29.09	-26.42	-26.51	-21.66	-21.31	84.10
5	8.00	3.00	4.00	2.00	4.00	28.21	-29.05	-26.38	-26.47	-21.64	-21.29	83.89
6	4.00	4.33	4.00	2.00	4.00	38.96	-21.60	-18.84	-18.98	-13.78	-13.54	84.12
7	5.33	4.33	4.00	2.00	4.00	34.56	-18.20	-15.46	-15.60	-10.51	-10.24	62.01
8	6.67	4.33	4.00	2.00	4.00	34.02	-17.25	-14.55	-14.69	-9.76	-9.50	56.87
9	8.00	4.33	4.00	2.00	4.00	34.02	-17.18	-14.49	-14.62	-9.72	-9.46	56.57
10	4.00	5.67	4.00	2.00	4.00	39.20	-20.68	-17.92	-18.06	-12.88	-12.63	82.33
11	5.33	5.67	4.00	2.00	4.00	35.21	-17.01	-14.26	-14.41	-9.33	-9.06	59.37
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
1013	8.00	3.00	9.00	6.00	8.00	23.42	-9.38	-6.69	-6.91	-2.02	-2.05	59.70
1014	4.00	4.33	9.00	6.00	8.00	36.50	-11.45	-8.67	-8.91	-3.69	-3.74	74.02
1015	5.33	4.33	9.00	6.00	8.00	27.16	-9.36	-6.61	-6.84	-1.73	-1.78	61.72
1016	6.67	4.33	9.00	6.00	8.00	26.24	-8.88	-6.18	-6.40	-1.48	-1.52	58.99
1017	8.00	4.33	9.00	6.00	8.00	26.21	-8.84	-6.14	-6.37	-1.47	-1.51	58.82
1018	4.00	5.67	9.00	6.00	8.00	36.78	-11.24	-8.47	-8.71	-3.50	-3.54	74.00
1019	5.33	5.67	9.00	6.00	8.00	27.79	-9.23	-6.48	-6.72	-1.61	-1.66	61.60
1020	6.67	5.67	9.00	6.00	8.00	26.94	-8.74	-6.04	-6.27	-1.36	-1.40	58.80
1021	8.00	5.67	9.00	6.00	8.00	26.92	-8.70	-6.01	-6.24	-1.35	-1.39	58.63
1022	4.00	7.00	9.00	6.00	8.00	36.85	-11.22	-8.45	-8.69	-3.49	-3.53	73.97
1023	5.33	7.00	9.00	6.00	8.00	27.90	-9.21	-6.46	-6.70	-1.61	-1.65	61.57
1024	6.67	7.00	9.00	6.00	8.00	27.06	-8.72	-6.02	-6.25	-1.35	-1.39	58.77
1025	8.00	7.00	9.00	6.00	8.00	27.04	-8.68	-5.99	-6.22	-1.34	-1.38	58.59
1026												

Figure 19 Initial model instances and predictions file for the bridge case study. See Section 3.2 for more information on formatting requirements for this file.

The uncertainties file includes quantification of uncertainties related to the data interpretation task from multiple sources. Uncertainties affecting structural identification arise from modelling assumptions such as geometry, modelling of supports, connections and loading and measurement noise. Quantification of these uncertainties is important for accurate structural identification.

In the uncertainties file, each sheet corresponds to uncertainty from a particular source. In each sheet, note sensor number in the first column, type of uncertainty distribution in the second column (uniform, normal etc.) and features (for example mean and variance) of uncertainty distribution in the third and fourth columns. See Section 3.3.1 for more details on formatting the excel uncertainties file. Figure 20 shows multiple excel sheets quantifying uncertainty related to structural identification of Cret de l'Anneau bridge.

1	Sensor ID	Type	dist_param_1	dist_param_2
2	1	Uniform	-2.229	1.1145
3	3	Uniform	1.35	-0.675
4	4	Uniform	0.81	-0.405
5	5	Uniform	0.97	-0.485
6	6	Uniform	0.403	-0.2015
7	7	Uniform	0.532	-0.266
8	8	Uniform	-8.107	4.0535
9				

(a) Model uncertainty

1	Sensor ID	Type	dist_param_1	dist_param_2
2	1	Normal	0	1
3	3	Normal	0	1
4	4	Normal	0	1
5	5	Normal	0	1
6	6	Normal	0	1
7	7	Normal	0	1
8	8	Normal	0	1
9				

(b) Measurement uncertainty

1	Sensor ID	Type	dist_param_1	dist_param_2
2	1	Uniform	0	5.3496
3	3	Uniform	0.2835	0
4	4	Uniform	0.05103	0
5	5	Uniform	0.3686	0
6	6	Uniform	0.09672	0
7	7	Uniform	0.2926	0
8	8	Uniform	0	1.86461
9				

(c) Load position uncertainty

1	Sensor ID	Type	dist_param_1	dist_param_2
2	1	Uniform	-2.78625	0.80244
3	3	Uniform	1.242	0
4	4	Uniform	0.891	0
5	5	Uniform	1.0573	0
6	6	Uniform	0.16926	-0.54405
7	7	Uniform	0.17024	-0.79268
8	8	Uniform	-0.64856	2.26996
9				

(d) Secondary parameter uncertainty

1	Sensor ID	Type	dist_param_1	dist_param_2
2	1	Uniform	-0.02229	0.02229
3	3	Uniform	0.0135	-0.0135
4	4	Uniform	0.0081	-0.0081
5	5	Uniform	0.0097	-0.0097
6	6	Uniform	0.00403	-0.00403
7	7	Uniform	0.00532	-0.00532
8	8	Uniform	-0.08107	0.08107
9				

(e) Load magnitude uncertainty

Figure 20 Sheets of excel file including estimations of uncertainties from a multitude of sources. Most uncertainties are biased and not normally distributed. All uncertainty values are in the same units as those used for measurement input.

7.1.1.2 Step 2: Interpret data using EDMF

Once all necessary inputs are provided to MeDIUM, user may proceed with interpretation of measurements to update knowledge of model parameters. To perform EDMF, click the button "*Interpret data using EDMF*". See Section 4 for details on EDMF application.

The "Summary" text box provides results of interpretation using EDMF, such as number of model instances accepted as candidates (solutions) from the initial set of model instances.

Results of EDMF may be visualized using a parallel axis plot. Parallel axis plot to visualize results of EDMF for the Cret de l'Anneau Bridge is shown in Figure 21. From an initial population of 1024 model instances provided, MeDIUM performs EDMF to provide 5 candidate model instances that provide responses that are compatible with measurements.

See Section 4 for details on understanding a parallel axis plot.

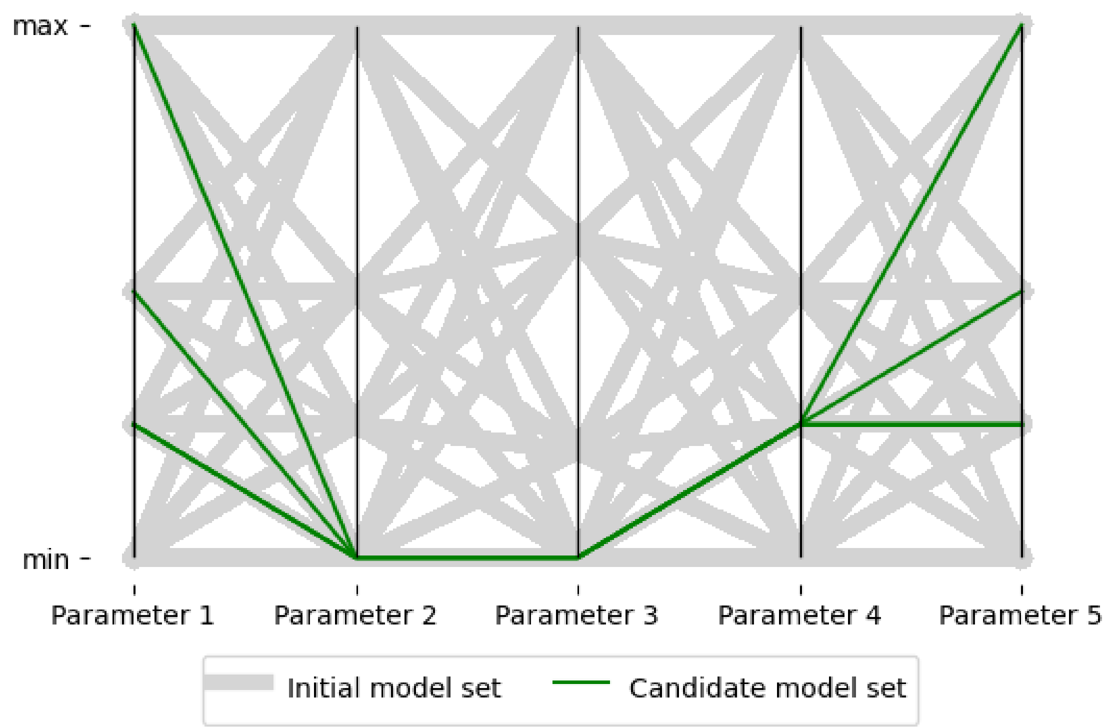


Figure 21 Parallel axis plot showing the initial model instances and candidate model instances. Section 4 provides a brief explanation on interpreting parallel axis plots.

7.1.1.3 Step 3: Cross-validation

In the cross-validation tab, input measurement indices to be held out for validation. These values are not included while performing EDMF again.

For the Cret de l'Anneau bridge, cross-validation is performed by interpreting data using deflection measurements to update knowledge of model parameters. The measurements held out are strains (S1 and S7). Using updated knowledge of model parameters, predictions at these measurement locations are compared with measured values.

Predictions for held-out measurement locations are provided in the initial model instances file uploaded by the user (see Figure 19). Updated predictions are a subset of these uploaded predictions corresponding to only the accepted (candidate) model-parameter instances.

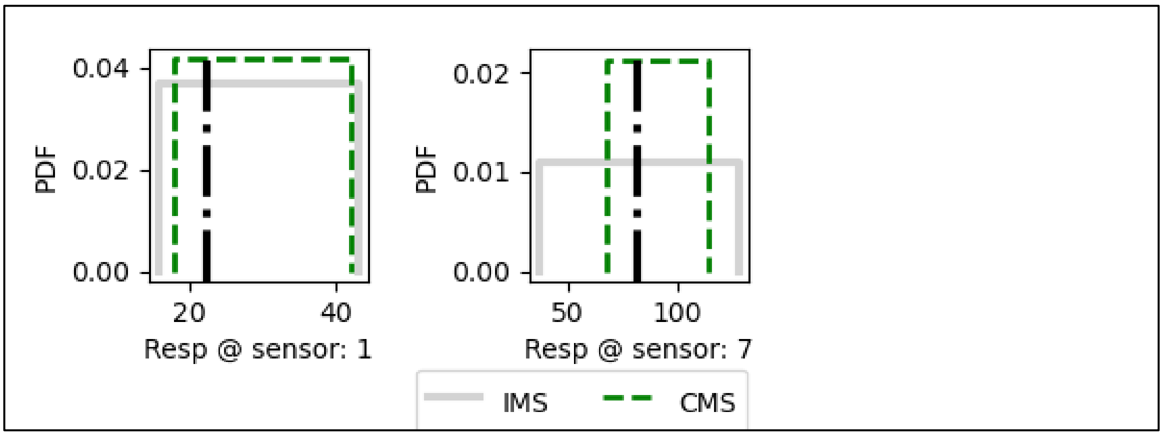


Figure 22 Measurements from S1 and S7 were held out and EDMF is repeated. Using the new CMS (with data from sensors S2 to S6) responses at locations of S1 and S7 are predicted. The updated predictions bounds include the measured value. Hence, identification is accurate for this validation scenario.

In Figure 22, updated predictions at sensors S1 and S7 include the measured value within its bounds. Therefore, data interpretation is accurate for predictions at these sensor locations.

See Section 5 for details on interpreting results of cross-validation. See Section 5.2 for details on performing leave-one-out cross-validation.

7.1.1.4 Step 4: What-if scenarios

In this tab, the user is able to assess other scenarios of data interpretation to assess sensitivity of results to few inputs provided.

A crucial input provided by the user is the uncertainty estimations file, where uncertainties from many sources are quantified. As uncertainty estimations are approximate, users may be interested to study how changes in uncertainty magnitudes affect results of data interpretation. To enable this study, a slider to change uncertainty magnitudes is provided, as shown in Figure 23. Use this slider to select a multiplication factor. For example, if a value of 0.5 is chosen, then uncertainties calculated based on inputs provided in Step 1 are multiplied by a factor of 0.5.

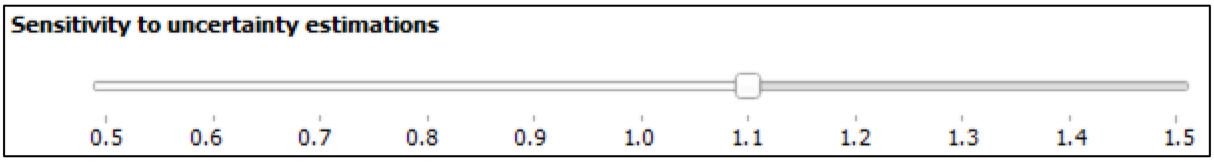


Figure 23 Increase or decrease uncertainty magnitudes provided as input in Step 1. Factor selected using the slider is multiplied with uncertainty provided as input in Step 1. Default value is 1.

Another crucial input is the target reliability of identification, φ. The default value of φ is 0.95. However, users may be interested to assess changes to EDMF results by changing the value of φ. To facilitate this, a slider, as shown in Figure 24, is provided to select requisite values of φ.

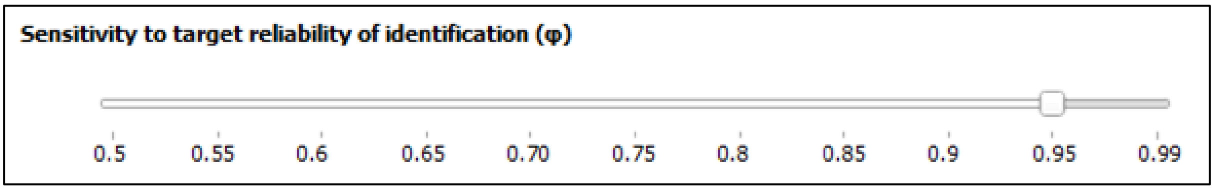


Figure 24 Select the target reliability of identification for performing EDMF. The default value is 0.95.

For the input values changed by the user using the sliders shown in Figure 23 and Figure 24, EDMF may be performed again using the "*Redo EDMF*" button. This creates a parallel axis plot as shown in Figure 25. This parallel axis plot illustrates the CMS obtained using EDMF when the uncertainty provided as input in Step 1 is multiplied by a factor of 1.5 and the φ value is set to 0.99. Larger uncertainties and greater requirement of reliability of identification leads to a significantly larger CMS compared to the one shown in Figure 21.

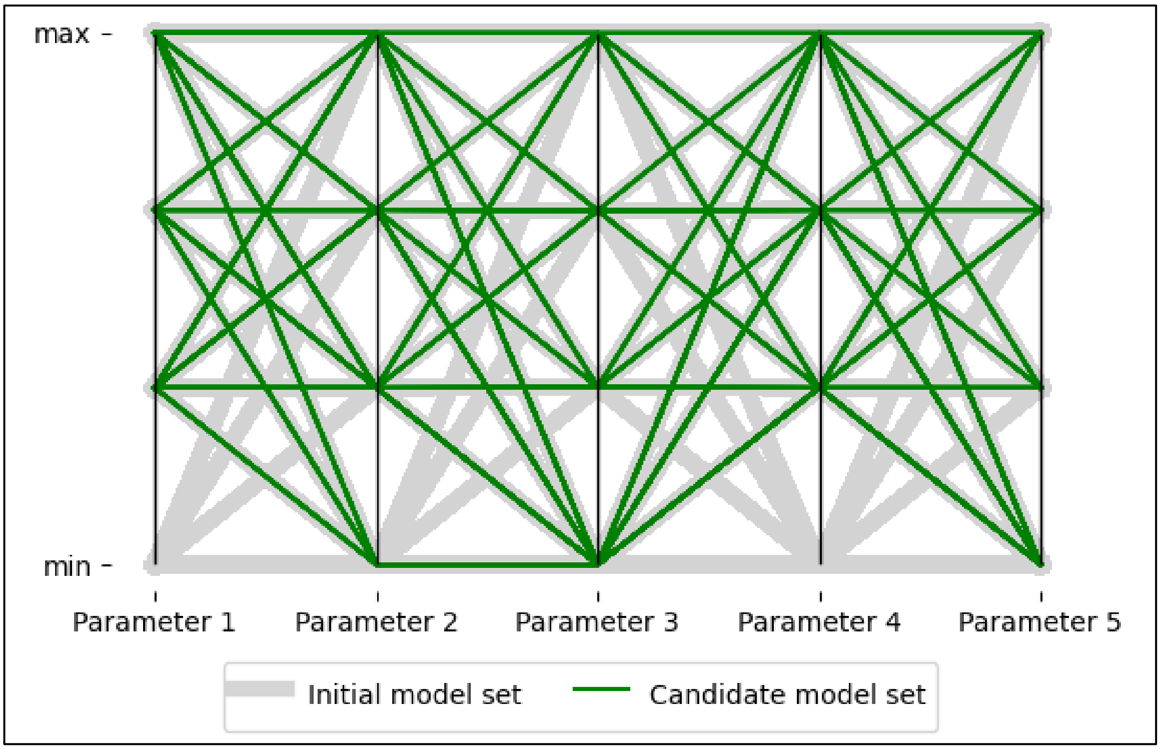


Figure 25 Parallel axis plot of CMS obtained with uncertainty multiplied by a factor of 1.5 and a target reliability of identification set to 0.99.

Results of data-interpretation after changes to uncertainty magnitude and target reliability of identification may be cross-validated. To perform cross-validation, click on "*Redo validation*" button. Clicking on this button will prompt a pop-up, where the user can input the indices of measurements to be held out for cross-validation.

Figure 26 shows a comparison of updated predictions and initial predictions with measurements at sensors S1 and S7. The updated prediction bounds in Figure 26 are wider than in Figure 22 as the CMS size has increased to input of a larger magnitude of uncertainty and a greater target reliability of identification. However, the updated prediction bounds include the measured value, indicating that structural identification is accurate for this scenario.

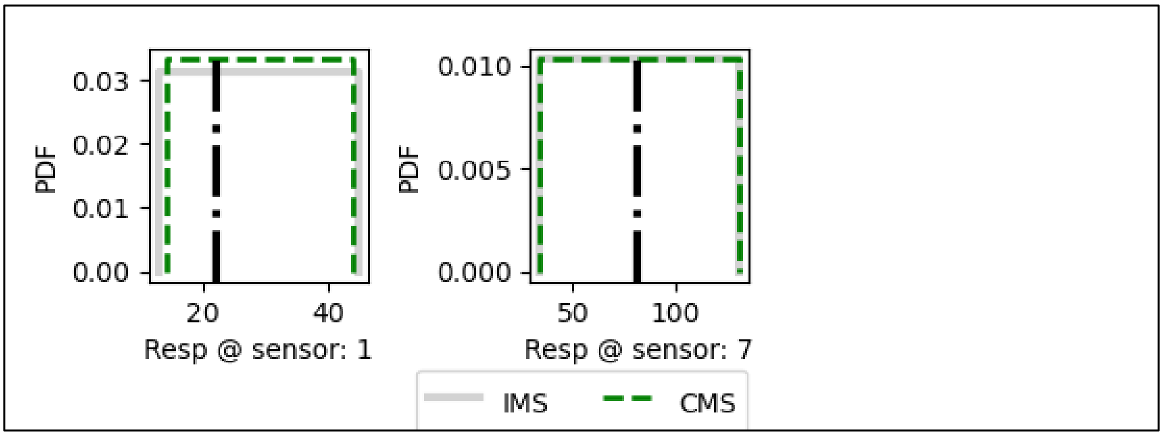


Figure 26 Holdout validation with sensors 1 and 7 with uncertainty multiplied by a factor of 1.5 and target reliability of identification set to 0.99. Increase in uncertainty magnitudes and reliability of identification leads to updated prediction bounds similar to initial values as more instances are accepted as possible solutions using EDMF.

MeDIUM supports interpretation of measurements data based on user preferences. While the user may utilise the default settings to interpret and validate data interpretation, the what-if scenarios tab provides the user with tools to assess robustness of data interpretation solutions to few important input quantities (uncertainties and ϕ).

7.2 Excavation Case Study

A simulated excavation example with excel input files is provided in this section.

As shown in Figures 26, this analysis considers a 25 m deep excavation whose plan is 50 m in length and 40 m wide. The excavation is supported by concrete diaphragm walls embedded to a depth of 40 m in a single layer of soil, together with four layers of centre and corner struts. The walls and struts are modelled using elastic plate elements and node-to-node anchors respectively, while the soil is modelled as an elastic-perfectly plastic Mohr Coulomb material whose stiffness varies linearly with depth. The structural and soil material parameters are summarised in Table 3. The soil response is modelled as undrained, using the "almost incompressible" effective stress approach, also known as Undrained Method A.

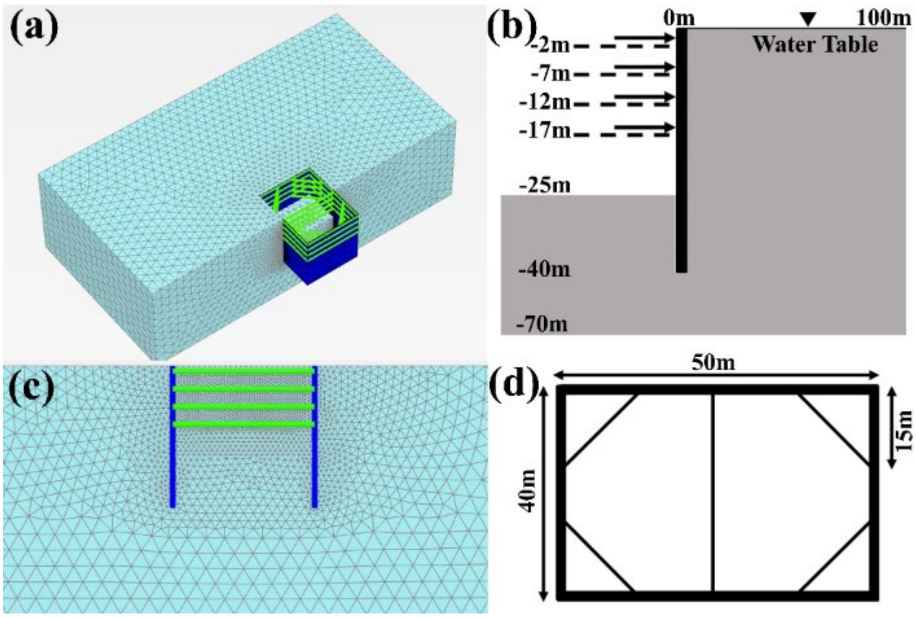


Figure 27 Detailed Configurations of the Synthetic Excavation Problem.

The excavation comprises 5 stages in total, with stage 1 corresponding to the initial cantilever phase and stage 5 corresponding to the excavation to formation level. This is illustrated in Figure 2. Each strutting phase and excavation phase are combined into a single phase. For example, stage 2 refers to the installation of the first layer of strut and the excavation to 7 m below ground level. This setting is reasonable because there is no pre-stress in the struts. The installation of the struts will not cause any significant stress change in the system.

Table 3. Parameter values used to generate synthetic excavation-induced wall deflection data.

Member	Property	Value
Plate	E_1 (MPa)	25E3
	E_2 (MPa)	25E3
	v_{12}	0.2
	Thickness (m)	1.0
Centre Strut	EA (MN)	20E3
Corner Strut	EA (MN)	10E3
Soil	E' (MPa)	35
	E'_{inc} (MPa/m)	0.5
	v_1	0.2
	c' (MPa)	0
	ϕ' (°)	30
	Ψ (°)	0
	R_{inter}	0.9

7.2.1 Interpretation using MeDIUM

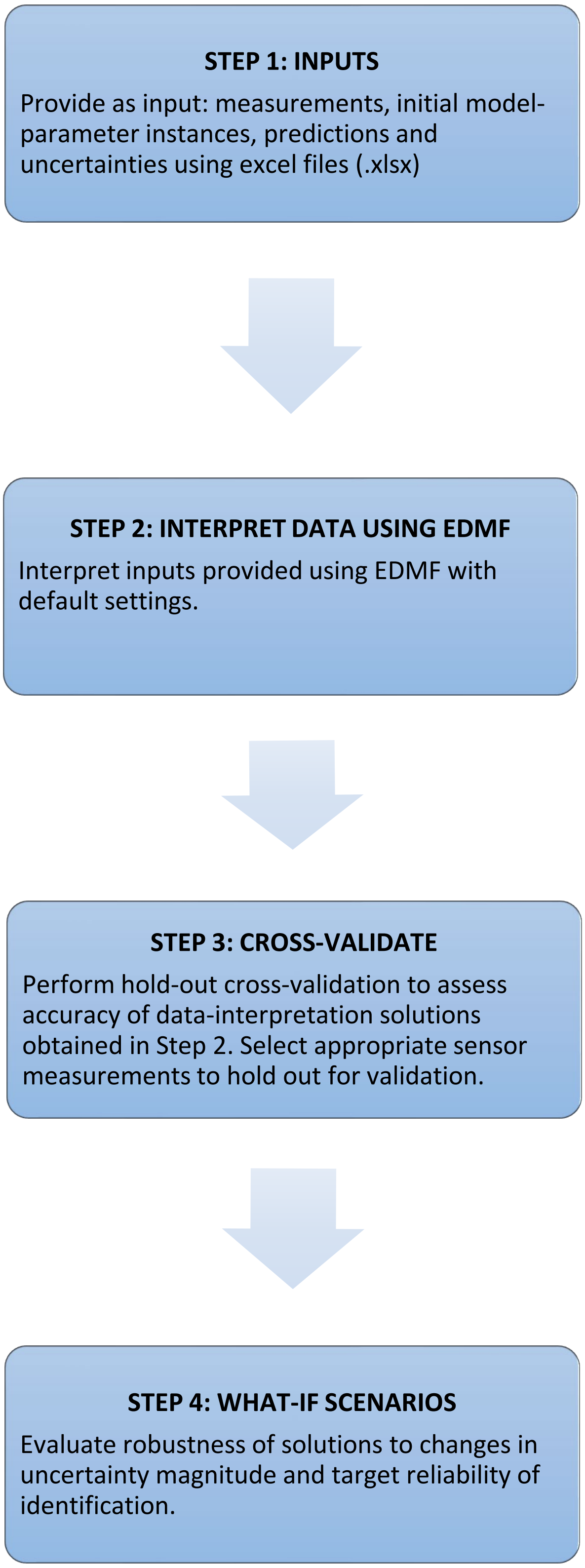


Figure 28 Steps involved in using MeDIUM to interpret measurement data using uncertain models.

Step 1: Inputs

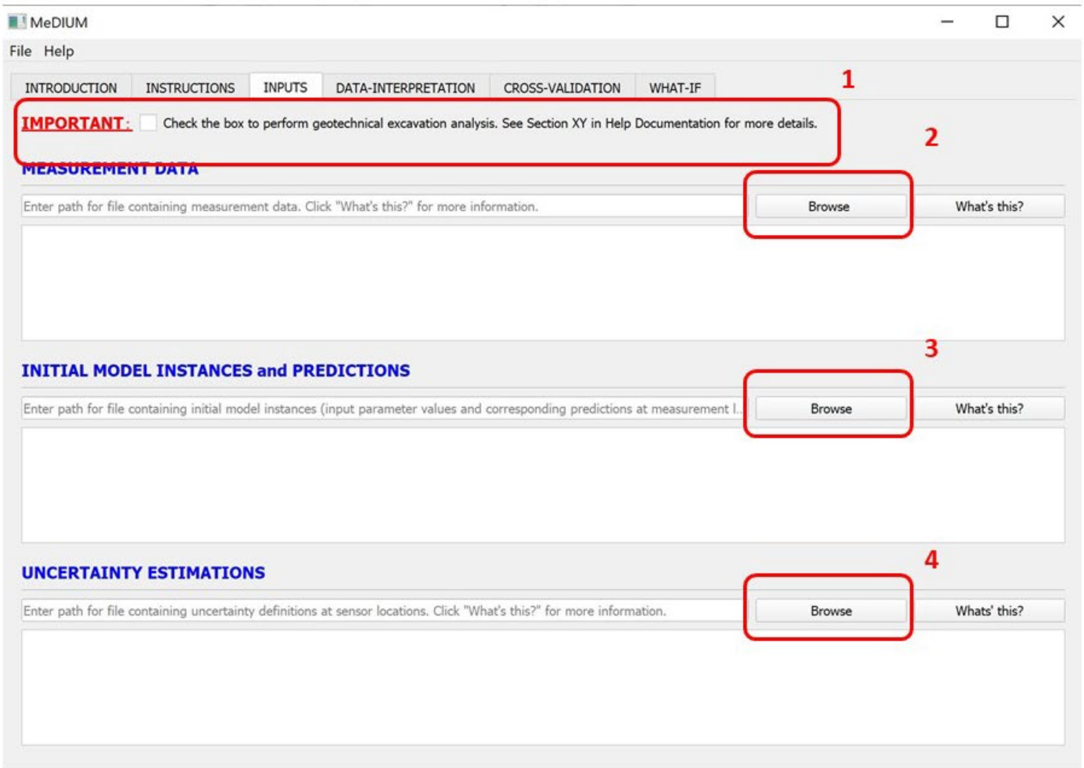


Figure 29 Input tab of the software.

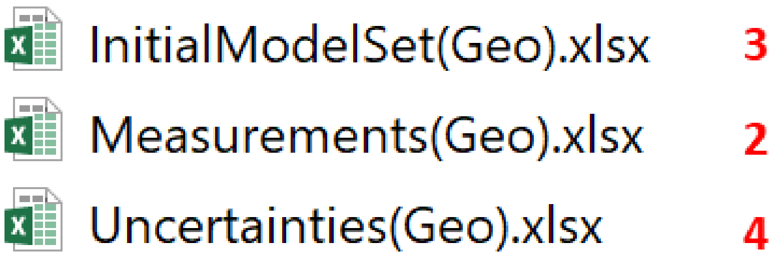


Figure 30 Input files required for the analysis.

To carry out this analysis, three excel files are required. Before uploading these files, the user has to check the box labelled by "1" shown in Figure 28. The user is then expected to click the "Browse" buttons labelled by "2" to "4" to upload the excel files shown in Figure 29.

“Measurements(Geo).xlsx”

This file contains the measurement data. Measurement data is expected to be placed in the first row of the excel file. The number of columns then equals the number of measurements involved in the interpretation. This tutorial example contains 200 measurements. They consist of 40 measurements per stage, and there are 5 excavation stages. The user is expected to arrange the 200 measurements in accordance to the sequence of excavation stages. For example, measurements 1 to 40 correspond to the 40 measurements of excavation stage 1 while measurements 41 to 80 correspond to the 40 measurements of excavation stage 2.

	A	B	C	D	E	F	G	H	I	J	K
1	9.50	7.12	9.36	8.85	8.40	6.26	4.10	5.28	6.58	6.38	6.58
2											

Figure 31 Examples of measurements.

“Prediction(Geo).xlsx”

This excel document contains information pertaining to initial model-parameter instances and the associated predictions. The first few columns contains parameter values. In this example, there are two parameters to be identified. The subsequent columns contain the predictions at all measurement locations. In this example, there are 200 measurements and therefore, there are 200 predictions. The 200 predictions are then expected to be arranged in a similar manner as in the measurement excel file. The number of rows equals the number of initial model instances. In the example, there are 1426 initial model instances

	A	B	C	D	E	F	G
1	Parameter	Parameter	Prediction	Prediction	Prediction	Prediction	Prediction
2	5000.00	0.00	30.43	31.54	32.65	33.76	34.86
3	6000.00	0.00	25.84	26.84	27.85	28.85	29.84
4	7000.00	0.00	21.38	22.29	23.19	24.10	24.99
5	8000.00	0.00	17.92	18.77	19.62	20.47	21.31
6	9000.00	0.00	15.84	16.62	17.41	18.19	18.96
7	10000.00	0.00	14.18	14.91	15.64	16.37	17.08
8	11000.00	0.00	12.71	13.40	14.09	14.77	15.45
9	12000.00	0.00	11.64	12.28	12.93	13.57	14.20
10	13000.00	0.00	10.62	11.23	11.84	12.45	13.05
11	14000.00	0.00	9.82	10.40	10.98	11.56	12.12
12	15000.00	0.00	9.12	9.66	10.21	10.77	11.31
13	16000.00	0.00	8.48	9.01	9.53	10.06	10.58
14	17000.00	0.00	7.96	8.46	8.96	9.47	9.96
15	18000.00	0.00	7.46	7.95	8.43	8.91	9.39
16	19000.00	0.00	7.05	7.51	7.98	8.44	8.89
17	20000.00	0.00	6.67	7.11	7.56	8.01	8.44
18	21000.00	0.00	6.32	6.75	7.18	7.61	8.03
19	22000.00	0.00	6.02	6.43	6.85	7.26	7.67
20	23000.00	0.00	5.73	6.13	6.53	6.93	7.33
21	24000.00	0.00	5.48	5.87	6.25	6.64	7.02

Figure 32 Example of initial model instances and predictions.

"Uncertainties(Geo).xlsx"

This file contains information pertaining to uncertainties. The excel contains two sheets. The first sheet is related to 3D effects. The second sheet is related to measurement uncertainty. In the "3D Effects" sheet, adopting uniform distributions, users are expected to input four sets of data, i.e. 3D upper bound predictions (mm), 3D lower bound predictions (mm), 2D upper bound predictions (mm), 2D lower bound predictions (mm). The number of columns corresponds to the number of measurements. In this example, there are 200 measurements. Please refer to [14] for more details pertaining to the "3D Effects".

Measurement ID	3D upper bound prediction(mm)	3D lower bound prediction(mm)	2D upper bound prediction(mm)	2D lower bound prediction(mm)
1	2.32	28.47	2.66	30.43
2	2.40	28.90	2.77	31.54
3	2.48	29.26	2.88	32.65
4	2.56	29.73	2.99	33.76
5	2.61	30.00	3.10	34.86
6	2.67	30.46	3.19	35.91
7	2.70	30.76	3.27	36.91
8	2.73	31.07	3.33	37.84
9	2.74	31.29	3.37	38.69
10	2.73	31.68	3.40	39.44
11	2.72	31.75	3.40	40.11
12	2.68	31.85	3.40	40.66
13	2.65	31.87	3.37	41.12
14	2.61	31.92	3.34	41.48
15	2.52	31.84	3.30	41.73
16	2.48	31.73	3.25	41.88
17	2.41	31.57	3.19	41.95
18	2.35	31.34	3.13	41.92
19	2.28	31.07	3.05	41.78
20	2.21	30.79	3.00	41.62
21	2.15	30.47	2.94	41.41
22	2.09	30.14	2.85	41.01

Figure 33 Examples of uncertainty input for 3D effects.

Measurement ID	Vertical distance from toe of wall (m)
1	40
2	39
3	38
4	37
5	36
6	35
7	34
8	33
9	32
10	31
11	30
12	29
13	28
14	27

Figure 34 Examples of uncertainty input for measurement uncertainty.

In the "Measurement Uncertainty" sheet, users are expected to input one set of data, i.e Vertical distance from toe of wall to the measurement point(m). This is for the purpose of deriving uncertainties pertaining to inclinometer errors. The errors associated with inclinometer errors are a function of the vertical distance from toe of the wall to the measurement point of interest. In this example, there are 200 measurements and therefore, users are expected to provide a vector with 200 rows.

After the three required excel files are prepared. Users are expected to load the files to the software. Component 1 is used to load “Measurements(Geo).xlsx”. Component 2 is used to load “Prediction(Geo).xlsx”. Component 3 is used to load “Input_for uncertainty(Geo).xlsx”. Users have to toggle the “Geotechnical Application” radio button before loading the documents.

Step 2: Interpret data with EDMF

Click on button “Interpret data using EDMF”. Click “What’s this?” button for more information.

The “Summary” text box provides results of interpretation using EDMF.

The following parallel axis plot is generated to visualize the candidate model set obtained.

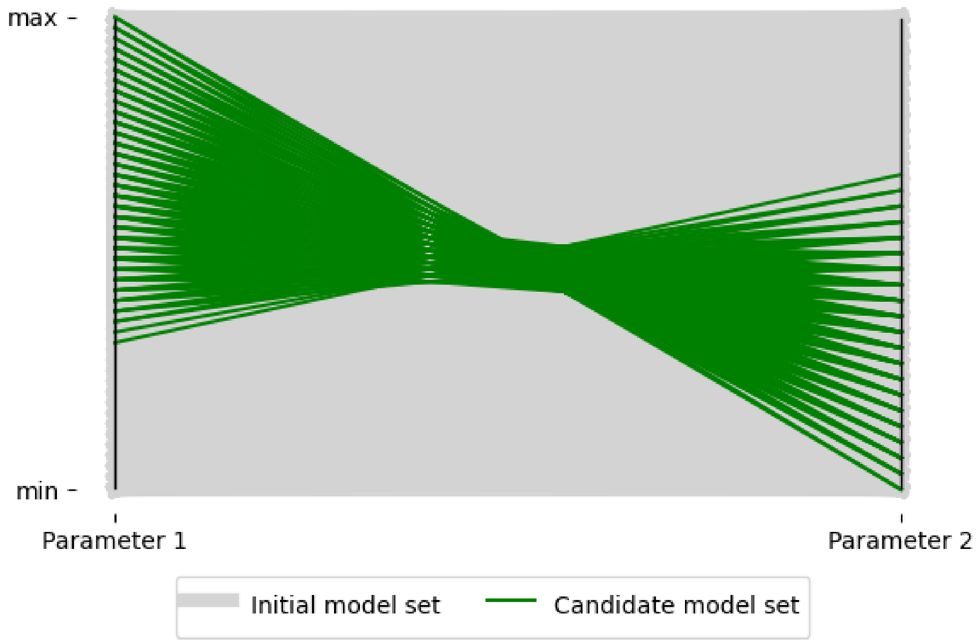


Figure 35 Parallel axis plot showing the initial model instances and candidate model instances. Section XX provides a brief explanation on interpreting parallel axis plots.

Results of EDMF may be visualized using a parallel axis plot. Parallel axis plot to visualize results of EDMF. From an initial population of 1426 model instances provided, MeDIUM performs EDMF to provide 125 candidate model instances that provide responses that are compatible with measurements.

Predictions can then be made with the candidate models. Figure 35 shows the comparisons of measurements and mean predicted wall deflections for all five excavation stages. The good agreement between predictions and measurements indicates that EDMF yields reasonable candidate models and the associated predictions. Figure 36 shows the mean and 3-standard deviation of wall deflections at excavation stage 5. EDMF also provides information about the variations of the predictions.

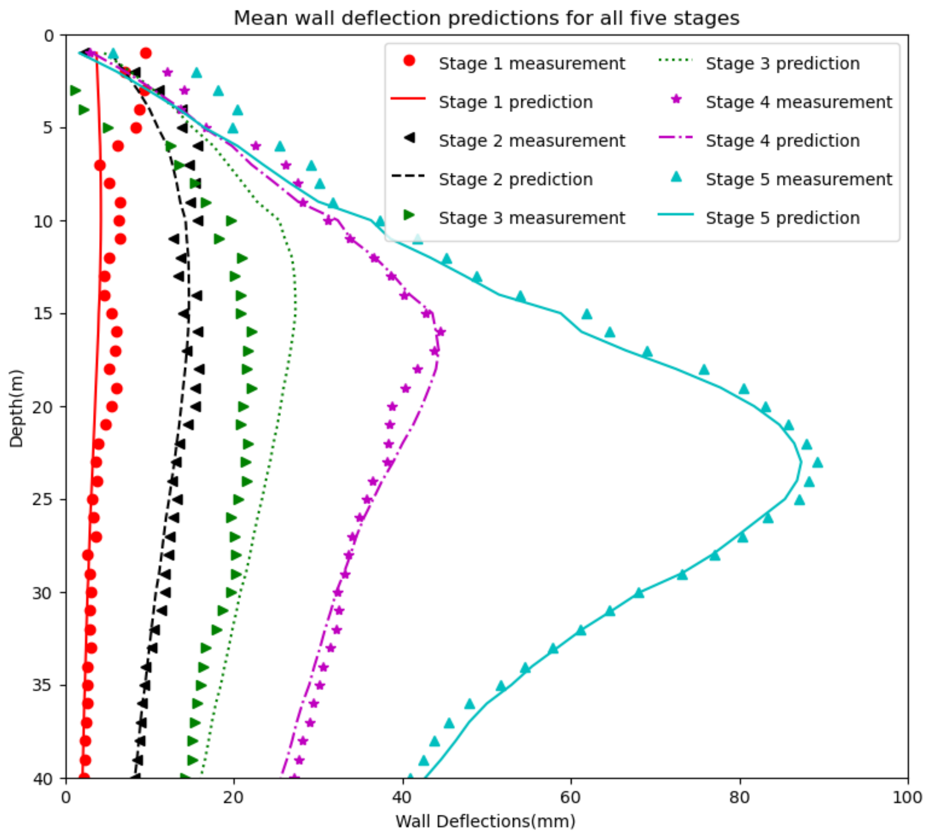


Figure 36 Mean predictions of wall deflections.

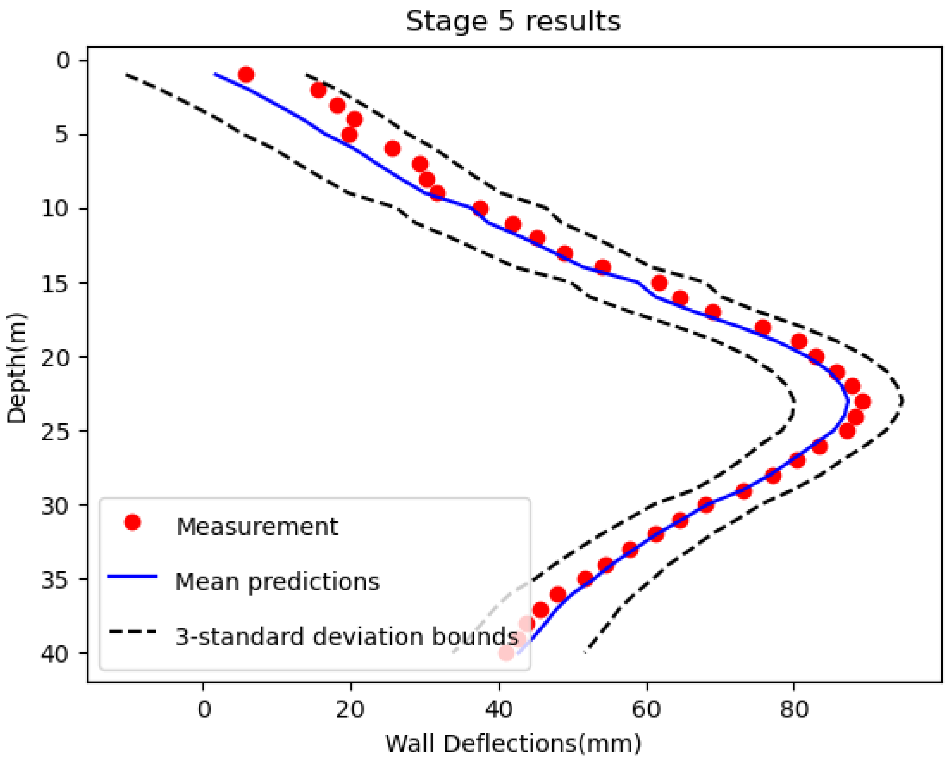


Figure 37 Mean and 3-standard deviation of wall deflections of stage 5.

Step 3: Cross-validation

Input validation indices to be held out for validation. These measurement values are not included for performing EDMF. Please refer to "What's this" for the format of measurement indices.

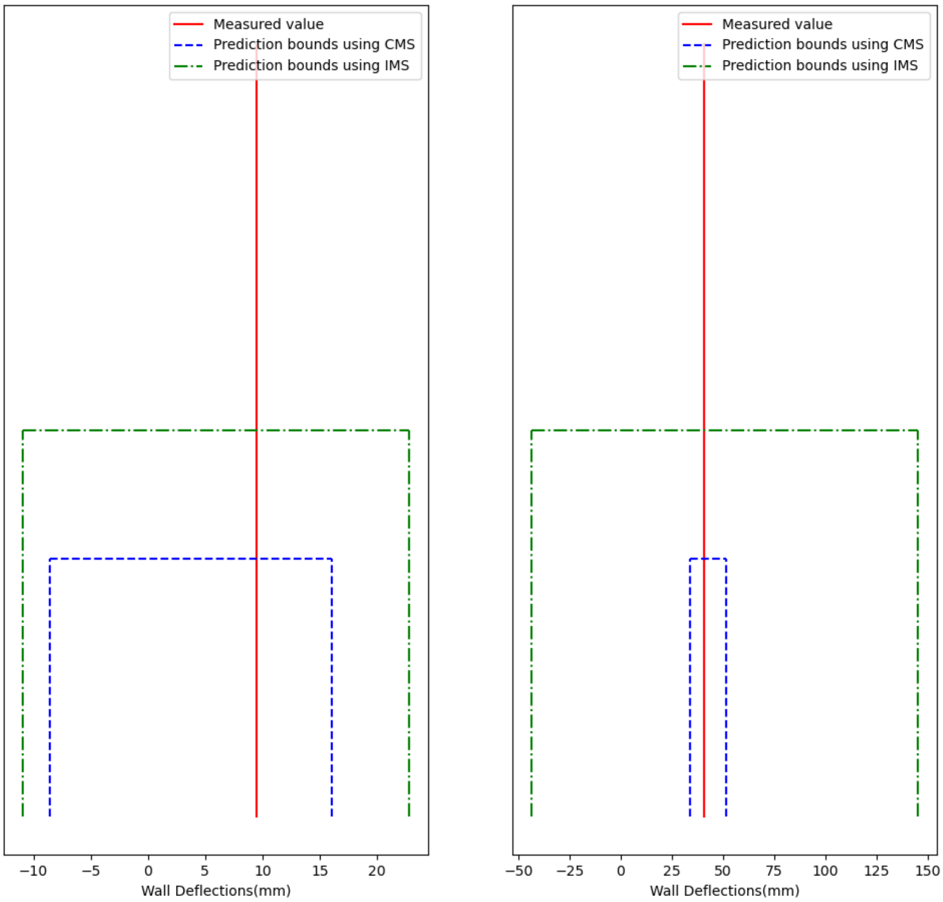


Figure 38 Sensors 1 and 200 were held out and EDMF is repeated. Using this new CMS obtained using the remaining sensors at locations of sensors 1 and 200 are predicted. The updated prediction bounds include the measured values. Hence identification is accurate using the remaining sensors for sensor locations 1 and 200.

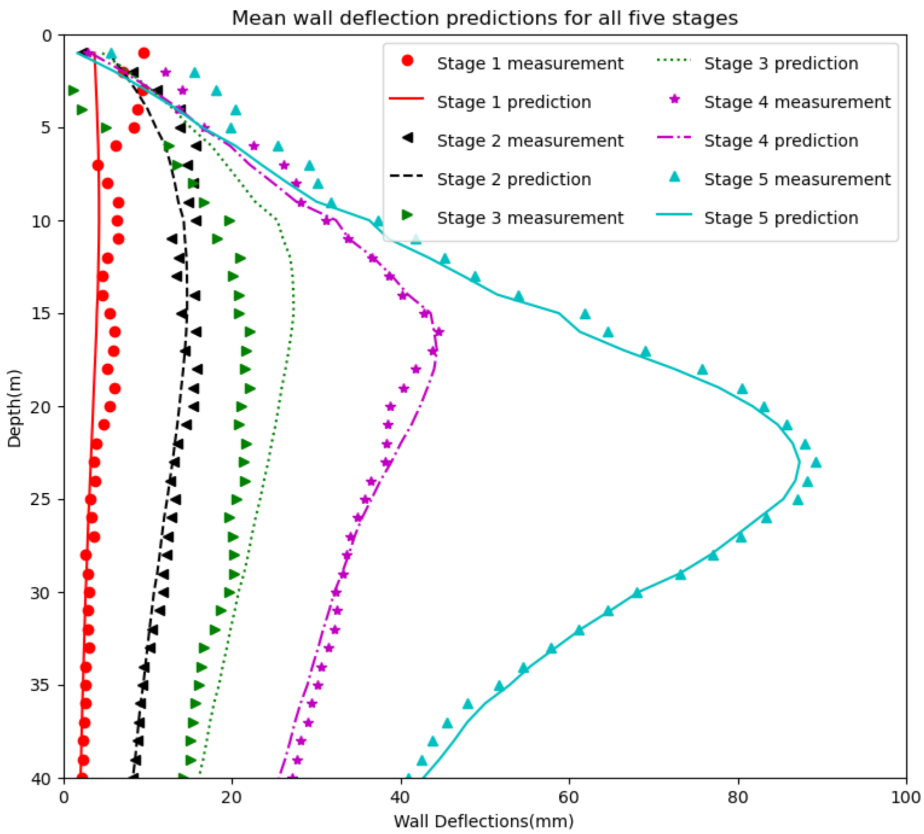


Figure 39 Mean predictions of wall deflections.

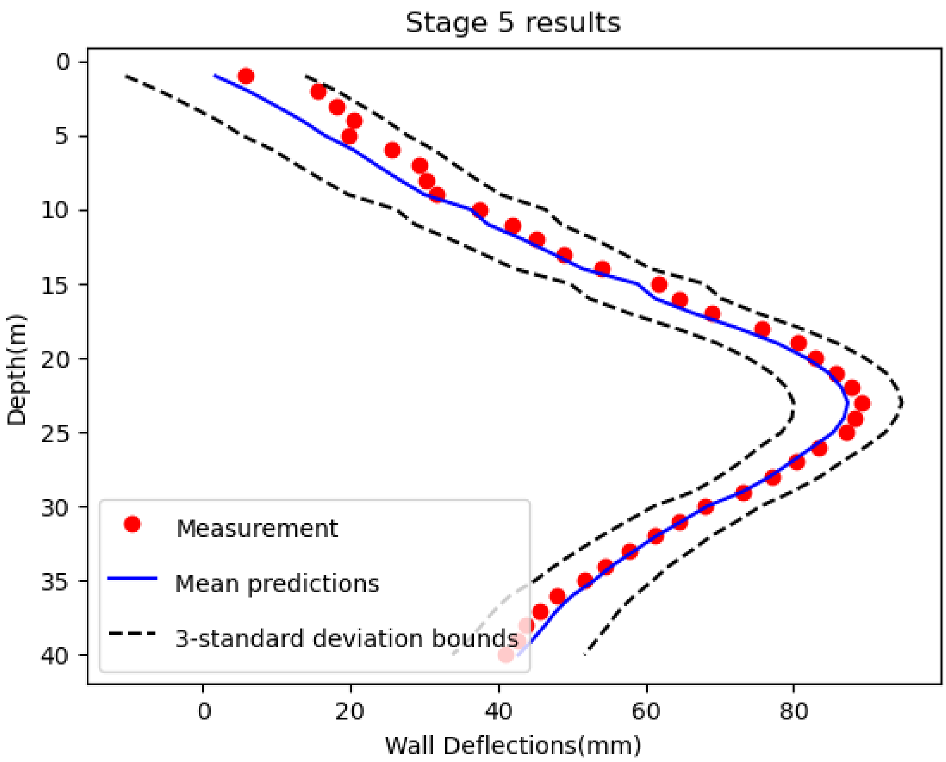


Figure 40 Mean and 3-standard deviation of wall deflections of stage 5.

Similarly, predictions can also be obtained with the remaining sensors. Figure 28 shows the mean wall deflection predictions for all excavation stages. Figure 29 shows the mean and 2-standard deviation of wall deflections at last excavation stages.

Step 4: What-if scenarios

Use the sliders to change the magnitude of uncertainty and target reliability of identification. Redo EDMF and validation as required. Results will then be plotted in the same manner as shown in Figures 34 to 39.

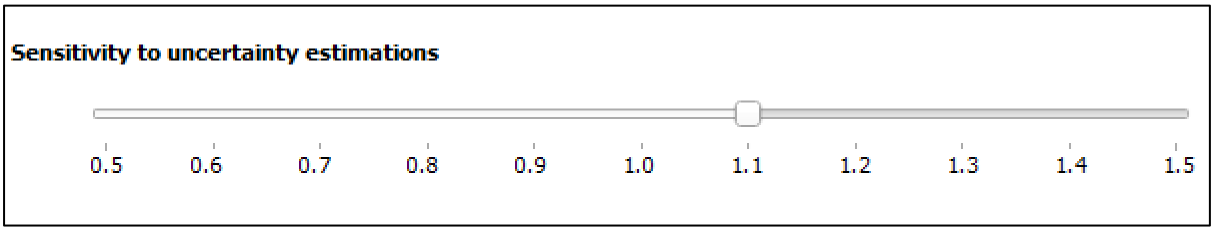


Figure 41 Increase or decrease uncertainty magnitudes provided as input in Step 1. Factor selected using the slider is multiplied with uncertainty provided as input in Step 1. Default value is 1.

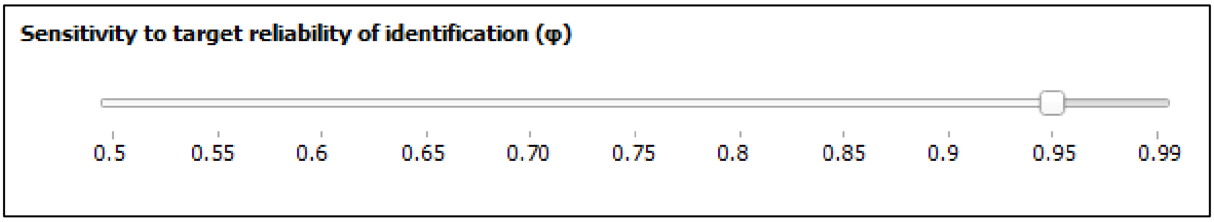


Figure 42 Select the target reliability of identification for performing EDMF. The default value is 0.95.

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