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* Group :-

A system consisting of a non-empty set G of element a, b, c etc with the operation is said to be group provided the following properties are satisfied.

- Closure
- Associativity
- Identity
- Inverse.

Let G be a non-void set with a binary operation * that assigns to each ordered pair (a, b) of elements of G an element of G denoted by a * b. G is a group under the binary operation * if the following properties are satisfied.

i) Closure: A binary operation* is closure i.e.

if $a \in X$, $b \in X$ then

$a * b \in X$.

ii) associativity:

A binary operation * is associative.

i.e. $a*(b*c) = (a*b)*c$

$\forall a, b, c \in X$.

iii) Identity :

There is an element e, called the identity such that $a * e = e * a = a \quad , \forall a \in X$.

iv) Inverse :

For each element a in X, there is an element b in X, called an inverse of a such that $a * b = b * a = e \quad , \forall a, b \in X$.

A monoid $(S, *)$ with identity element 'e' is called a group if to each element $a \in S$, there exist an $b \in S$ such that

$$(a * b) = (b * a) = e$$

here b is an inverse of a and denoted as a^{-1}. and $b^{-1} = a$.

ex.

1) $(Z, +)$ is group or not ?

$Z = \{-\infty, \ldots, -2, -1, 0, 1, 2, \ldots \infty\}$

$*$ is a binary operation $+$.

$$\left.\begin{aligned} a * b &\in Z \\ a + b &\in Z \\ 2 + 3 = 5 &\in Z. \end{aligned}\right\} \text{closure property}$$

hence it is algebraic structure.

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$$\left.\begin{array}{l} a * (b * c) = (a * b) * c \\ a + (b + c) = (a + b) + c \\ 2 + (3 + 5) = (2 + 3) + 5 \\ 10 = 10 \end{array}\right\} \text{associativity property}$$

hence it is semigroup

$$\left.\begin{array}{l} a * e = e * a = a \\ 0 \text{ is an identity element for } + \\ 3 + 0 = 0 + 3 = 3 \end{array}\right\} \text{identity}$$

hence it is monoid.

$$\left.\begin{array}{l} a * b = b * a = e \\ a + b = b + a = e \quad , e = 0 \\ 2 + (-2) = -2 + 2 = 0 \end{array}\right\} \text{inverse.}$$

so here $a = 2$, $b = a^{-1} = -2$
$e = 0$ inverse element is present.
it is a group.

2) $(\underline{R}^*, \cdot)$ R is set of real no.
$R = \{-2, -1, \frac{1}{2}, 6.2, 3.5, 1, 3, 6, 2\}$
i) $a * b$
$a \cdot b$
$2 \cdot 3 = 6$
$2, 3 \in R$ and $6 \in R$
It is an algebraic structure

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ii) $a \cdot (b \cdot c) = (a \cdot b) \cdot c$

$2 \cdot (3 \cdot 1) = (2 \cdot 3) \cdot 1$

$6 = 6$

it is a semigroup

iii)

$a * e = e * a = a$

for · multiplication, identity element = 1

$a = 5$

$5 * 1 = 1 * 5 = 5$

$a \in R$ and $e \in R$

hence it is monoid

iv)

$a * b = b * a = e$

$b = a^{-1}$

$a = 5$

$5 \cdot \frac{1}{5} = \frac{1}{5} \cdot 5 = 1$

1 is identity element

$a = 5$

$b = \frac{1}{5} = a^{-1}$

hence it is group.

Note:-

$R^* =$ all real no. except 0

ex. $0 \cdot \frac{1}{0} = \infty$

so not group

∞ is not real number.

if $(R, \cdot)$ then not group.