Semester : III Subject : DSGT Academic Year: 2022-2023

(3) Complemented lattice -

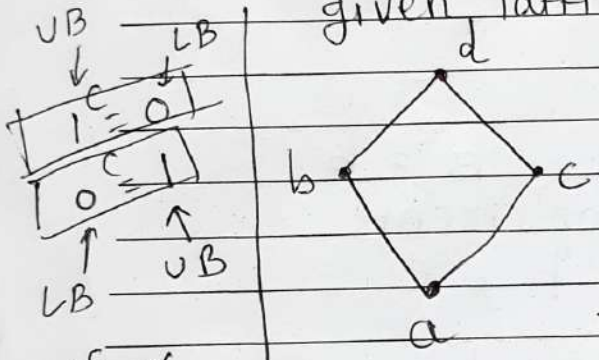
A bounded lattice 'P' is said to be complemented if every element in 'P' has atleast one complement.

⇒ Complement of an element -

An element 'x' in a bounded lattice 'P' is said to be complement of an element $a \in P$ if $a \vee x = 1$ and $a \wedge x = 0$

(∨ : join, 1 : upper bound, ∧ : meet, 0 : lower bound)

e.g. Find complement of an element in given lattice.



$\Rightarrow U^c = \phi$

$\phi^c = U$

For lattice :

UB = d

GLB = a

∴ for a, $a^c = d$

d, $d^c = a$

where $a \vee d = d$

$a \wedge d = a$

hence. a & d are complements

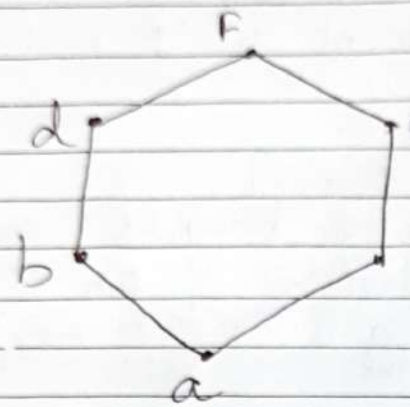
for b & c,

$b \vee c = d$ (i.e. our UB)

$b \wedge c = a$ (i.e. our LB)

hence. b and c are complements $b^c = c$, $c^c = b$

ex. (2) Find complement of elements in lattice.



i) for a and f

$a \vee f = f$

$a \wedge f = a$

$\therefore \boxed{a^c = f}$

$\boxed{f^c = a}$

for b and c

$b \vee c = f$

$b \wedge c = a$

$\boxed{b^c = c}, \boxed{c^c = b}$

ii) for b and e

$b \vee e = b$

$b \wedge e = a$

$\therefore b^c = e \quad e^c = b$

b has 2 complements

$b = c, \quad b^c = e$

iii) for d and c

$d \vee c = f$

$d \wedge c = a$

$\therefore d^c = c$

$c^c = d$

iv) for d and e

$d \vee e = f$

$d \wedge e = a$

$d^c = e$

$e^c = d$

v) for b and d

$b \vee d = d$ ← This is not UB hence.

$b \wedge d = b$ ← This is not LB d and b are not complements.

So elements on same edge, does not complement of each other.

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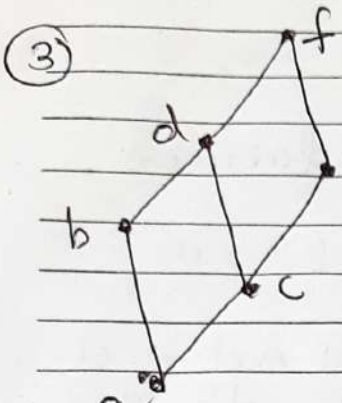
(3)



L3

UB = f
LB = a

for b and c, $b \vee c = d$
$b \wedge c = a$

for b and e, $b \vee e = f$
$b \wedge e = a$

$\therefore b^c = e$, $e^c = b$

for d and c, d and c are on same edge so they don't have complement.

d and e, $d \vee e = f$
$d \wedge e = c$

So d don't have complement.

Hence this lattice L3 is not complemented.

(4)