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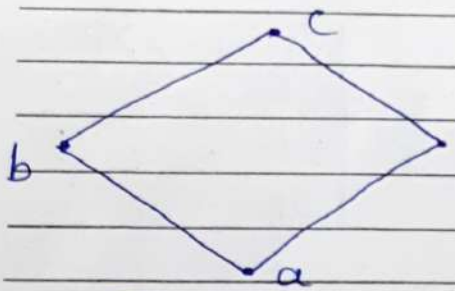
* Lattices :-

A lattice is a poset $(L, \leq)$ in which every subset $\{a, b\}$ consisting of two elements has a least upper bound and a greatest lower bound. We denote LUB $(\{a, b\})$ by $a \vee b$ and call it the join of a and b. Similarly we denote GLB $(\{a, b\})$ by $a \wedge b$ and call it the meet of a and b.

ex. ① Determine whether the following Hasse diagram represent lattice or Not.



Join

LUB: $\vee$	a	b	c	d
a	a	b	c	d
b	b	b	c	c
c	c	c	c	c
d	d	c	c	d

Meet

GLB: $\wedge$	a	b	c	d
a	a	a	a	a
b	a	b	b	a
c	a	b	c	d
d	a	a	d	d

every subset has a least upper bound and greatest lower bound hence it is a lattice.

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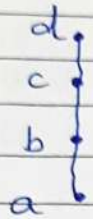
ex. (2)



Determine whether the following Hasse diagram represent lattice or not.

LUB : $\vee$

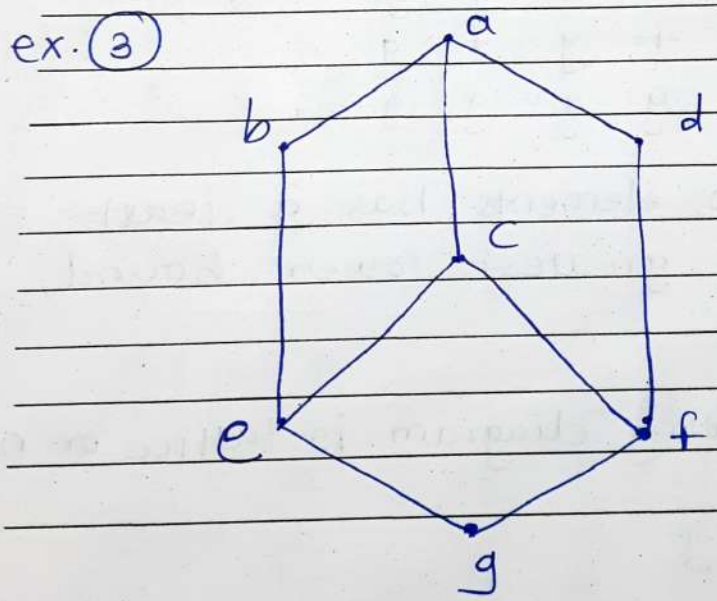
$\vee$	a	b	c	d
a	a	b	c	d
b	b	b	c	d
c	c	c	c	d
d	d	d	d	d

GLB : $\wedge$

$\wedge$	a	b	c	d
a	a	a	a	a
b	a	b	b	b
c	a	b	c	c
d	a	b	c	d

This is a lattice because every pair of elements has a least upper bound and a greatest lower bound.

ex. (3)



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⇒ LUB : $\vee$

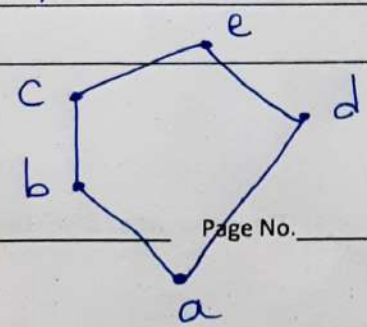
$\vee$	a	b	c	d	e	f	g
a	a	a	a	a	a	a	a
b	a	b	a	a	b	a	b
c	a	a	c	a	c	c	c
d	a	a	a	d	a	d	d
e	a	b	c	a	e	c	e
f	a	a	c	d	c	f	f
g	a	b	c	d	e	f	g

GLB : $\wedge$

$\wedge$	a	b	c	d	e	f	g
a	a	b	c	d	e	f	g
b	b	b	e	g	e	g	g
c	c	e	c	f	e	f	g
d	d	g	f	d	g	f	g
e	e	e	e	g	e	g	g
f	f	g	f	f	g	f	g
g	g	g	g	g	g	g	g

Each subset of two elements has a least upper bound and a greatest lower bound, so it is a lattice.

ex. (4) check if the following diagram is lattice or not.



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LUB:

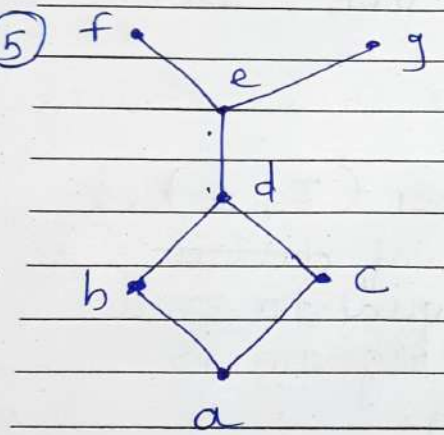
V	a	b	c	d	e
a	a	b	c	d	e
b	b	b	c	e	e
c	c	c	c	e	e
d	d	e	e	d	e
e	e	e	e	e	e

GLB:

∧	a	b	c	d	e
a	a	a	a	a	a
b	a	b	b	a	b
c	a	b	c	a	c
d	a	a	a	d	d
e	a	b	c	d	e

This is lattice because every pair of elements has a least upper bound and a greatest lower bound.

(5)



LUB: V

V	a	b	c	d	e	f	g
a	a	b	c	d	e	f	g
b	b	b	d	d	e	f	g
c	c	d	c	d	e	f	g
d	d	d	d	d	e	f	g
e	e	e	e	e	e	f	g
f	f	f	f	f	f	f	-
g	g	g	g	g	g	-	g

GLB: ∧

∧	a	b	c	d	e	f	g
a	a	a	a	a	a	a	a
b	a	b	a	b	b	b	b
c	a	a	c	c	c	c	c
d	a	b	c	d	d	d	d
e	a	b	c	d	e	e	e
f	a	b	c	d	e	f	e
g	a	b	c	d	e	e	g

This is not a lattice because f ∨ g is not exist.