Types of Algebraic structure

There are various types of algebraic structure, which is described as follows:

- Semigroup
- Monoid
- Group
- Abelian Group

All these algebraic structures have wide application in particular to binary coding and in many other disciplines.

Semi Group

Suppose there is an algebraic structure (G, *), which will be known as semigroup if it satisfies the following condition:

- **Closure:** The operation * is a closed operation on G that means (a*b) belongs to set G for all a, b ∈
- **Associative:** The operation * shows an association operation between a, b, and c that means a*(b*c) = (a*b)*c for all a, b, c in G.

Example 1:

The examples of semigroup are (Matrix, *) and (Set of integer, +).

Example 2:

The semigroup contains a set of positive integers with an additional or multiplication operation. The positive integers will not contain zero. **For example:** Suppose we have a set G, which contains some positive integers except zero such as 1, 2, 3, and so on like this:

G = {1, 2, 3, 4, 5, …..}

- This set contains the closure property because according to closure property (a * b) belongs to G for every element a, b. So in this set, (1*2) = 2 ∈
- This set also contains the associative property because according to associative property (a + b) + c = a + (b + c) belongs to G for every element a, b, c. So in this set, (1 + 2) + 3 = 1 + (2 + 3) = 6 ∈

Monoid:

A monoid is a semigroup, but it contains an extra **identity element** (E or e). An algebraic structure (G, *) will be known as a monoid if it satisfies the following condition:

- **Closure:** G is closed under operation * that means (a*b) belongs to set G for all a, b ∈
- **Associative:** Operation * shows an association operation between a, b, and c that means a*(b*c) = (a*b)*c for all a, b, c in G.
- **Identity Element:** There must be an identity in set G that means a * e = e * a = a for all x.

Note: An algebraic structure and a semigroup are always shown by a monoid.

Group:

A Group is a monoid, but it contains an extra **inverse element**, which is denoted by 1. An algebraic structure (G, *) will be known as a group if it satisfies the following condition:

- **Closure:** G is closed under operation * that means (a*b) belongs to set G for all a, b ∈
- **Associative:** * shows an association operation between a, b, and c that means a*(b*c) = (a*b)*c for all a, b, c in G.
- **Identity Element:** There must be an identity in set G that means a * e = e * a = a for all a.

- **Inverse Element:** It contains an inverse element that means $a * a^{-1} = a^{-1} * a = e$ for a ∈

Abelian Group

An abelian group is a group, but it contains **commutative law**. An algebraic structure (G, *) will be known as an abelian group if it satisfies the following condition:

- **Closure:** G is closed under operation * that means (a*b) belongs to set G for all a, b ∈
- **Associative:** * shows an association operation between a, b, and c that means a*(b*c) = (a*b)*c for all a, b, c in G.
- **Identity Element:** There must be an identity in set G that means a * e = e * a = a for all a.
- **Inverse Element:** It contains an inverse element that means $a * a^{-1} = a^{-1} * a = e$ for a ∈
- **Commutative Law:** There will be a commutative law such that a * b = b * a such that a, b belongs to G.

Note: (Z, +) is an Abelian group because it is commutative, but matrix multiplication is not commutative that's why it is not an abelian group.

Suppose we have a set G, which contains some positive integers except zero such as 1, 2, 3, and so on with additional operations like this:

G = {1, 2, 3, 4, 5, …..}

- This set contains the **closure property** because according to closure property (a + b) belongs to G for every element a, b. So in this set, (1 + 2) = 2 ∈ G and so on.
- This set also contains the **associative property** because according to associative property (a + b) + c = a + (b + c) belongs to G for every element a, b, c. So in this set, (1 + 2) + 3 = 1 + (2 + 3) = 6 ∈ G and so on.
- This set also contains the **identity property** because according to this property (a * e) = a, where a ∈ So in this set, (2 × 1) = 2, (3 × 1) = 3, and so on. In our case, 1 is the identity element.
- This set also contains the **commutative property** because according to this property (a * b) = (b * a), where a, b ∈ So in this set, (2 × 3) = (3 × 2) = 6 and so on.

Semigroup

An algebraic structure **(G, *)** is said to be a semigroup. If the binary operation * is associated in **G** i.e. **if (a*b) *c = a *(b*c) a,b,c e G**. For example, the set of **N** of all natural number is semigroup with respect to the operation of addition of natural number.

Obviously, addition is an associative operation on **N**. similarly, the algebraic structure **(N, .)(I, +)** and **(R, +)** are also semigroup.

Monoid

A group which shows property of an identity element with respect to the operation * is called a monoid. In other words, we can say that an algebraic system **(M,*)** is called a monoid if **x, y, z E M**.

(x *y) * z = x * (y * z)

And there exists an elements **e E M** such that for any **x E M**

e * x = x * e = x where **e** is called identity element.

- **Closure property**

The operation + is closed since the sum of two natural number is a natural number.

- **Associative property**

The operation + is an associative property since we have **(a+b) + c = a + (b+c) a, b, c E I**.

- **Identity**

There exist an identity element in a set **I** with respect to the operation +. The element 0 is an identity element with respect to the operation since the operation + is a closed, associative and there exists an identity. Since the operation + is a closed associative and there exists an identity. Hence the algebraic system **(I, +)** is a **monoid**.

Group

A system consisting of a non-empty set **G** of element **a, b, c** etc with the operation is said to be group provided the following postulates are satisfied:

1. Closure property

For all a, b E G => a, b E G
i.e G is closed under the operation '.'

2. Associativity

(a,b).c = a.(b.c) a, b, c E G.
i.e the binary operation '.' Over g is associative.

3. Existence of identity

There exits an unique element in G. Such that e.a = a = a.e
for every a E G. This element e is called the identity.

4. Existence of inverse

For each a E G , there exists an element a^-1 E G
such that a. a^-1 = e = a^-1.a
the element a^-1 is called the inverse of a .

Abelian group/Commutative Group

- A group **G** is said to be abelian or commutative if in addition to the above four postulates the following postulate is also satisfied.
  Commutativity

a.b = b.a for every a, b E G.

Cyclic Group

A group **G** is called cyclic. If for some **aEG**, every element **xEG** is of the form **a^n**. where **n** is some integer. Symbolically we write **G = {a^n : n E I}**. The single element a is called a generator of **G** and as the cyclic group is generated by a single element, so the cyclic group is also called **monogenic**.

Subgroup

A non- empty subset **H** of a set group **G** is said to be a subgroup of **G**, if **H** is stable for the composition * and **(H, *)** is a group. The additive group of even integer is a subgroup of the additive group of all integer.