Rotations : -

- Two successive Rotations are performed as:

$$P' = R(\theta_2) \cdot \{R(\theta_1) \cdot P\}$$

$$= \{R(\theta_2) \cdot R(\theta_1)\} \cdot P$$

$$= \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 \\ \sin\theta_2 & \cos\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot P$$

$$= \begin{bmatrix} \cos\theta_2\cos\theta_1 - \sin\theta_2\sin\theta_1 & -\sin\theta_1\cos\theta_2 - \sin\theta_2\cos\theta_1 & 0 \\ \sin\theta_1\cos\theta_2 + \sin\theta_2\cos\theta_1 & \cos\theta_2\cos\theta_1 - \sin\theta_2\sin\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos(\theta_1+\theta_2) & -\sin(\theta_1+\theta_2) & 0 \\ \sin(\theta_1+\theta_2) & \cos(\theta_1+\theta_2) & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot P$$

$$P' = R(\theta_1+\theta_2) \cdot P$$

- Here P' and P are column vector of final and initial point co-ordinate respectively.
- This concept can be extended for any number of successive rotations.

Example:

Q. Obtain the final co-ordinates after two rotations on point $p(6,9)$ with rotation angles $30°$ & $60°$ resp.

$$p' = R(\theta_1+\theta_2) \cdot p$$

$$= \begin{bmatrix} \cos(\theta_1+\theta_2) & -\sin(\theta_1+\theta_2) & 0 \\ \sin(\theta_1+\theta_2) & \cos(\theta_1+\theta_2) & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot p$$

$$= \begin{bmatrix} \cos(30+60) & -\sin(30+60) & 0 \\ \sin(30+60) & \cos(30+60) & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 6 \\ 9 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos 90 & -\sin 90 & 0 \\ \sin 90 & \cos 90 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 6 \\ 9 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 6 \\ 9 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} -9 \\ 6 \\ 1 \end{bmatrix}$$

Final co-ordinates after rotations are $p'(-9,6)$.