Example 2, Substitution Method

Given recurrence relation is as follows,

$$T(n) = \begin{cases} 1 & \text{if } n=1 \\ n * T(n-1) & \text{if } n>1 \end{cases}$$

If $n=1$ $T(n)=1$ This is our base cond^n or termination ~~correction~~ condition where we have stop.

Otherwise if $n>1$ the recurrence relation is

$$T(n) = n * T(n-1) \quad \text{—— (1)}$$

The function here is decreasing by substraction.

Now we put $n-1$ as n

$$T(n-1) = (n-1) * T((n-1)-1)$$
$$= (n-1) * T(n-2) \quad \text{—— (2)}$$

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$$T(n-2) = (n-2) * T((n-2)-1)$$
$$= (n-2) * T(n-3) \quad \text{---} \quad (3)$$

Now we will use the substitution method

Substitute eq^n (2) in eq^n (1)

$$T(n) = n * (n-1) * T(n-2)$$

we have already calculated $T(n-2)$
So let's put eq^n (2) here

$$T(n) = n * (n-1) * (n-2) * T(n-3)$$

What we will get in next iteration,

$$T(n) = n * (n-1) * (n-2) * (n-3) * T(n-4)$$

So we have observed that our function
$T(n-4)$
⋮

It goes on decrementing
Our aim is to go on decrementing till the function terminates.

For this we will ~~you~~ use our base condition.

For this we have to go on ~~terminating~~ decrementing till n-1 steps

So after n-1 steps our equation becomes

$$= n * (n-1) * (n-2) * (n-3) \ldots T(n-(n-1))$$

$$= n * (n-1) * (n-2) * (n-3) \ldots * T(n-(n-1))$$

$$= n * (n-1) * (n-2) * (n-3) \ldots * T(n-n+1)$$

$$= n * (n-1) * (n-2) * (n-3) \ldots * T(\cancel{n}-\cancel{n}+1)$$

$$= n * (n-1) * (n-2) * (n-3) \ldots * T(1)$$

we know $T(1) = 1$ is our base condition

$$= n * (n-1) * (n-2) * (n-3) \ldots * 1$$

The series we have is as follows,

$$= n * (n-1) * (n-2) * (n-3) \ldots * 3 * 2 * 1$$

Take n out from each eq^n,

$$= n * n\left(1-\frac{1}{n}\right) * n\left(1-\frac{2}{n}\right) * \ldots n\left(\frac{3}{n}\right) * n\left(\frac{2}{n}\right) * n\left(\frac{1}{n}\right)$$

If we have n·n·n it is represented as n^3

So our eq^n can be written as

n^n we can write this as $O(n^n)$ which is our factorial time complexity