- **Propositional Logic (PL): Syntax, Semantics, Formal logic-connectives, truth tables, tautology, validity, well-formed-formula**

Propositional logic (PL) is the simplest form of logic where all the statements are made by propositions. A proposition is a declarative statement which is either true or false. It is a technique of knowledge representation in logical and mathematical form.

Example:

a) It is Sunday.

b) The Sun rises from West (False proposition)

c) 3+3= 7(False proposition)

d) 5 is a prime number.

Following are some basic facts about propositional logic:

- Propositional logic is also called Boolean logic as it works on 0 and 1.
- In propositional logic, we use symbolic variables to represent the logic, and we can use any symbol to represent a proposition, such as A, B, C, P, Q, R, etc.
- Propositions can be either true or false, but it cannot be both.
- Propositional logic consists of an object, relations or function, and logical connectives.
- These connectives are also called logical operators.
- The propositions and connectives are the basic elements of the propositional logic.
- Connectives can be said as a logical operator which connects two sentences.
- A proposition formula which is always true is called tautology, and it is also called a valid sentence.
- A proposition formula which is always false is called Contradiction.

- A proposition formula which has both true and false values is called Statements which are questions, commands, or opinions that are not propositions such as "Where is Rohini", "How are you", "What is your name", are not propositions.

Syntax of propositional logic:

The syntax of propositional logic defines the allowable sentences for the knowledge representation. There are two types of Propositions:

- Atomic Propositions
- Compound propositions

Atomic Proposition: Atomic propositions are the simple propositions. It consists of a single proposition symbol. These are the sentences which must be either true or false.

Example:

a) 2+2 is 4, it is an atomic proposition as it is a true fact.

b) "The Sun is cold" is also a proposition as it is a false fact.

Compound proposition: Compound propositions are constructed by combining simpler or atomic propositions, using parenthesis and logical connectives.

Example:

a) "It is raining today, and street is wet."

b) "Ankit is a doctor, and his clinic is in Mumbai."

Logical Connectives:

Logical connectives are used to connect two simpler propositions or representing a sentence logically. We can create compound propositions with the help of logical connectives. There are mainly five connectives, which are given as follows:

Negation: A sentence such as $\neg$ P is called negation of P. A literal can be either Positive literal or negative literal.

Conjunction: A sentence which has $\wedge$ connective such as, P $\wedge$ Q is called a conjunction.

Example: Rohan is intelligent and hardworking. It can be written as, P= Rohan is intelligent,

Q= Rohan is hardworking. → P ∧ Q.

Disjunction: A sentence which has ∨ connective, such as P ∨ Q. is called disjunction, where P and Q are the propositions.

Example: "Ritika is a doctor or Engineer",

Here P= Ritika is Doctor. Q= Ritika is Doctor, so we can write it as P ∨ Q.

Implication: A sentence such as P → Q, is called an implication. Implications are also known as if-then rules. It can be represented as

If it is raining, then the street is wet.

Let P= It is raining, and Q= Street is wet, so it is represented as P → Q

Biconditional: A sentence such as P⇔ Q is a Biconditional sentence, example If I am breathing, then I am alive

P= I am breathing, Q= I am alive, it can be represented as P ⇔ Q.

Following is the summarized table for Propositional Logic Connectives:

Connective symbols	Word	Technical term	Example
∧	AND	Conjunction	A ∧ B
∨	OR	Disjunction	A ∨ B
→	Implies	Implication	A → B
⇔	If and only if	Biconditional	A⇔ B
¬ or ~	Not	Negation	¬ A or ¬ B

Properties of Operators:

<u>Commutativity:</u>

P ∧ Q= Q ∧ P, or

P ∨ Q = Q ∨ P.

<u>Associativity:</u>

$(P \land Q) \land R = P \land (Q \land R)$,

$(P \lor Q) \lor R = P \lor (Q \lor R)$

Identity element:

$P \land True = P$,

$P \lor True = True$.

Distributive:

$P \land (Q \lor R) = (P \land Q) \lor (P \land R)$.

$P \lor (Q \land R) = (P \lor Q) \land (P \lor R)$.

DE Morgan's Law:

$\neg (P \land Q) = (\neg P) \lor (\neg Q)$

$\neg (P \lor Q) = (\neg P) \land (\neg Q)$.

Double-negation elimination:

$\neg (\neg P) = P$.

Limitations of Propositional logic:

- We cannot represent relations like ALL, some, or none with propositional logic. Example:
  - All the girls are intelligent.
  - Some apples are sweet.
- Propositional logic has limited expressive power.
- In propositional logic, we cannot describe statements in terms of their properties or logical relationships.

Rules of Inference in Artificial intelligence

Inference:

In artificial intelligence, we need intelligent computers which can create new logic from old logic or by evidence, so generating the conclusions from evidence and facts is termed as Inference.

Inference rules:

Inference rules are the templates for generating valid arguments. Inference rules are applied to derive proofs in artificial intelligence, and the proof is a sequence of the conclusion that leads to the desired goal.

In inference rules, the implication among all the connectives plays an important role. Following are some terminologies related to inference rules:

Implication: It is one of the logical connectives which can be represented as $P \rightarrow Q$. It is a Boolean expression.

Converse: The converse of implication, which means the right-hand side proposition goes to the left-hand side and vice-versa. It can be written as $Q \rightarrow P$.

Contrapositive: The negation of converse is termed as contrapositive, and it can be represented as $\neg Q \rightarrow \neg P$.

Inverse: The negation of implication is called inverse. It can be represented as $\neg P \rightarrow \neg Q$.

From the above term some of the compound statements are equivalent to each other, which we can prove using truth table:

P	Q	$P \rightarrow Q$	$Q \rightarrow P$	$\neg Q \rightarrow \neg P$	$\neg P \rightarrow \neg Q$.
T	T	T	T	T	T
T	F	F	T	F	T
F	T	T	F	T	F
F	F	T	T	T	T

Hence from the above truth table, we can prove that $P \rightarrow Q$ is equivalent to $\neg Q \rightarrow \neg P$, and $Q \rightarrow P$ is equivalent to $\neg P \rightarrow \neg Q$.

Types of Inference rules:

1. Modus Ponens:

The Modus Ponens rule is one of the most important rules of inference, and it states that if P and P → Q is true, then we can infer that Q will be true. It can be represented as:

Notation for Modus ponens: $\dfrac{P \rightarrow Q, \quad P}{\therefore Q}$

Example:

Statement-1: "If I am sleepy then I go to bed" ==> P→ Q

Statement-2: "I am sleepy" ==> P

Conclusion: "I go to bed." ==> Q.

Hence, we can say that, if P→ Q is true and P is true then Q will be true.

Proof by Truth table:

P	Q	$P \rightarrow Q$
0	0	0
0	1	1
1	0	0
1	1	1

2. Modus Tollens:

The Modus Tollens rule states that if P→ Q is true and ¬ Q is true, then ¬ P will also be true. It can be represented as:

Notation for Modus Tollens: $\dfrac{P \rightarrow Q, \quad \sim Q}{\sim P}$

Statement-1: "If I am sleepy then I go to bed" ==> P→ Q

Statement-2: "I do not go to the bed."==> ~Q

Statement-3: Which infers that "I am not sleepy" => ~P

Proof by Truth table:

P	Q	$\sim P$	$\sim Q$	$P \rightarrow Q$
0	0	1	1	1
0	1	1	0	1
1	0	0	1	0
1	1	0	0	1

3. Hypothetical Syllogism:

The Hypothetical Syllogism rule states that if P→R is true whenever P→Q is true, and Q→R is true.

Example:

Statement-1: If you have my home key then you can unlock my home. P→Q

Statement-2: If you can unlock my home then you can take my money. Q→R

Conclusion: If you have my home key then you can take my money. P→R

Proof by truth table:

P	Q	R	$P \rightarrow Q$	$Q \rightarrow R$	$P \rightarrow R$
0	0	0	1	1	1
0	0	1	1	1	1
0	1	0	1	0	1
0	1	1	1	1	1
1	0	0	0	1	1
1	0	1	0	1	1
1	1	0	1	0	0
1	1	1	1	1	1

4. Disjunctive Syllogism:

The Disjunctive syllogism rule state that if P∨Q is true, and ¬P is true, then Q will be true. It can be represented as:

Notation of Disjunctive syllogism: $\frac{P \vee Q, \quad \neg P}{Q}$

Example:

Statement-1: Today is Sunday or Monday. ==>$P \vee Q$

Statement-2: Today is not Sunday. ==> $\neg P$

Conclusion: Today is Monday. ==> Q

Proof by truth-table:

P	Q	$\neg P$	$P \vee Q$
0	0	1	0
0	1	1	1
1	0	0	1
1	1	0	1

5. Addition:

The Addition rule is one the common inference rule, and it states that If P is true, then $P \vee Q$ will be true.

Notation of Addition: $\frac{P}{P \vee Q}$

Example:

Statement: I have a vanilla ice-cream. ==> P

Statement-2: I have Chocolate ice-cream.

Conclusion: I have vanilla or chocolate ice-cream. ==> $(P \vee Q)$

Proof by Truth-Table:

P	Q	$P \vee Q$
0	0	0
1	0	1
0	1	1
1	1	1

6. Simplification:

The simplification rule state that if P$\wedge$ Q is true, then Q or P will also be true. It can be represented as:

Notation of Simplification rule: $\frac{P \wedge Q}{Q}$ Or $\frac{P \wedge Q}{P}$

Proof by Truth-Table:

P	Q	$P \wedge Q$
0	0	0
1	0	0
0	1	0
1	1	1

7. Resolution:

The Resolution rule state that if P$\vee$Q and ¬ P$\wedge$R is true, then Q$\vee$R will also be true. It can be represented as

Notation of Resolution $\frac{P \vee Q, \ \neg P \wedge R}{Q \vee R}$

Proof by Truth-Table:

P	¬ P	Q	R	$P \vee Q$	¬ P∧R	$Q \vee R$
0	1	0	0	0	0	0
0	1	0	1	0	0	1
0	1	1	0	1	1	1
0	1	1	1	1	1	1
1	0	0	0	1	0	0
1	0	0	1	1	0	1
1	0	1	0	1	0	1
1	0	1	1	1	0	1

Nature of Preposition

1. Tautology: A compound proposition is called tautology if and only if it is true for all possible truth values of its propositional variables. It contains only T (Truth) in last column of its truth table.

2. Contradiction: A compound proposition is called contradiction if and only if it is false for all possible truth values of its propositional variables. It contains only F (False) in last column of its truth table.

3. Contingency: A compound proposition is called contingency if and only if it is neither a tautology nor a contradiction. It contains both T (True) and F (False) in last column of its truth table.

4. Valid: A compound proposition is called valid if and only if it is a tautology. It contains only T (Truth) in the last column of its truth table.

5. Invalid: A compound proposition is called invalid if and only if it is not a tautology. It contains either only F (False) or both T (Truth) and F (False) in last column of its truth table.

6. Falsifiable: A compound proposition is called falsifiable if and only if it can be made false for some value of its propositional variables. It contains either only F (False) or both T (Truth) and F (False) in last column of its truth table.

7. Unfalsifiable: A compound proposition is called unfalsifiable if and only if it can never be made false for any value of its propositional variables. It contains only T (Truth) in last column of its truth table.

8. Satisfiable:A compound proposition is called satisfiable if and only if it can be made true for some value of its propositional variables. It contains either only T (Truth) or both T (True) and F (False) in last column of its truth table.

9. Unsatisfiable: A compound proposition is called unsatisfiable if and only if it can not be made true for any value of its propositional variables. It contains only F (False) in last column of its truth table.