Semester : III Subject : DSGT Academic Year: 2022-2023

* Reflexive closure and transitive closure -

* Reflexive closure -

$A = \{1, 2, 3\}$

$R = \{(1,1)(2,2)(3,3)\}$

$$M_R = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 1 & & \\ 2 & & 1 & \\ 3 & & & 1 \end{array}$$

diagonal value.

So for reflexive relation we must consider relation matrix M_R where for each $a \in A$ we must have. $\{(a, a) \mid a \in A\}$ is a diagonal relation and this diagonal relation is Δ

e.g.

$A = \{a, b\}$

$\Delta = \{(a, a)(b, b)\}$

$R = \{(a,b)(b,a)(a,a)\}$

so reflexive closure of relation R on set A is

$$\boxed{R \cup \Delta A}$$

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* Closure of relation –

Let R be a relation on a set A. R may or may not have some property P, such as reflexivity, symmetry or transitivity.

If there is a relation S with property P containing R such that S is the subset of every relation with property P containing R then S is called the closure of R with respect to P.

1) Reflexive closure –

$\Delta = \{(a, a) \mid a \in A\}$ is the diagonal relation on set A. The reflexive closure of relation R on set A is $R \cup \Delta$.

2) Symmetric closure –

Let R be a relation on set A, and let R^{-1} be the inverse of R. The symmetric closure of relation R on set A is $R \cup R^{-1}$.

3) Transitive closure –

Let R be a relation on set A. The

connectivity relation is defined as -

$R^* = \bigcup_{n=1}^{\infty} R_n$. The transitive closure of R is R^*.

e.g. $A = \{1, 2, 3, 4\}$

i) $R = \{(1,1) (1,4) (2,3) (3,1) \cancel{(3,3)} (3,4)\}$

$\Delta A = \{(1,1) (2,2) (3,3) (4,4)\}$

$R \cup \Delta_A = \{(1,1) (2,2), (3,3), (4,4), (1,4) (2,3) (3,1) (3,4)\}$

This is reflexive closure of R.

ii) Symmetric closure -

we need the inverse of R.

$A = \{1, 2, 3, 4\}$

$R = \{(1,1) (1,4) (2,3) (3,1) (3,4)\}$

$R^{-1} = \{(1,1) (4,1) (3,2) (1,3) (4,3)\}$

$R \cup R^{-1} = \{(1,1) (1,4) (2,3) (3,1) (3,4) (4,1) (3,2) (1,3) (4,3)\}$

This is symmetric closure of R

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iii) Transitive closure, we need to find R^*

$\therefore$ we need to find R^1, R^2, R^3 until $R^n = R^{n-1}$ we stop when this condition is achieved since finding higher powers of R would be the same.

$R^1 = \{(1,1)(1,4)(2,3)(3,1)(3,4)\}$

$RoR = R^2 = \{(1,1)(1,4)(2,1)(2,4)(3,1)(3,4)\}$

$RoRoR = R^3 = \{(1,1)(1,4)(2,1)(2,4)(3,1)(3,4)\}$

we stop here

$R^2 = R^3$

Transitive closure

$R^\infty = R^* = R^1 \cup R^2$

$= \{(1,1)(1,4)(2,1)(2,3)(2,4)(3,1)(3,4)\}$

Note :-

$\therefore R^2 = RoR = \quad R = \{(1,1)(1,4)(2,3)(3,1)(3,4)\}$

$R = \{(1,1)(1,4)(2,3)(3,1)(3,4)\}$

$RoR = \{(1,1)(1,4)(2,1)(2,4)(3,1)(3,4)\}$