



Non-Cooperative Game theory

A game is a multi-person decision situation, in which a person's outcome is influenced by other people's decisions as well as his own.

Thus almost all economic interactions are games.

There are two leading frameworks for analyzing games: *cooperative* and *noncooperative*.

- **Cooperative game theory** assumes rational strategic behavior, unlimited communication, and unlimited ability to make agreements.
- It sidesteps the details of the structure by assuming that players reach a Pareto-efficient agreement, which is sometimes further restricted, e.g. by requiring symmetry of utility outcomes for symmetrically situated players.
- Its goal is to characterize the limits of the set of possible agreements that might emerge from rational (possibly implicit) bargaining.
- **Noncooperative game theory** also assumes strategic rationality.

But by contrast:

- Noncooperative game theory replaces cooperative game theory's assumptions of unlimited communication and ability to make agreements with a detailed model of the situation and of how rational players will behave in it.
- Its goal is to use rationality, augmented by the "rational expectations" notion of Nash equilibrium, to predict or explain outcomes in a situation.

But cooperative game theory is better suited to some applications, e.g. where the structure of the game is unclear or unobservable, and it is desired to make predictions that are robust to it.

Like the term "game" itself, the term "noncooperative" is a misnomer:

- Noncooperative game theory spans the entire range of *multi-person* or *interactive decision situations*.
- Although *zero-sum games*, whose players have perfectly conflicting goals, played a leading role in the development of the theory, most applications combine elements of conflict with elements of coordination, and some involve predicting which settings are better for fostering cooperation.
- This is done by making behavioral assumptions at the individual level

("methodological individualism"), thereby requiring cooperation to emerge (if it does) as the outcome of explicitly modeled, independent decisions by individuals in response to explicitly modeled institutions.

- By contrast, cooperative game theory makes the group-level assumption that the outcome will be Pareto-efficient, and (with some important exceptions) downplays the incentive and coordination issues that are the focus of noncooperative analyses of cooperation.

From the noncooperative point of view, a *game* is a multi-person decision situation defined by its *structure*, which includes:



- The *players*, independent decision makers (e.g. bridge has first four players, then three, not two, even though partners have the same goals)
- The *rules*, which specify the order of players' decisions, their feasible decisions at each decision point, and their information at each decision point.
- How players' decisions jointly determine the physical *outcome*
- Players' preferences over outcomes (or probability distributions of outcomes)

Players' preferences over outcomes are modeled just as in decision theory.

Preferences can be extended to handle shared uncertainty about how decisions determine the outcome just as in decision theory: by assigning von Neumann-Morgenstern utilities, or *payoffs*, to outcomes and assuming that players maximize their expected payoffs.

Something is *mutual knowledge* if all players know it.

Something is *common knowledge* if all players know it, all know that all know it, and so on.

Assume here, for simplicity, that the structure of the game is *common knowledge*, except possibly for shared uncertainty about how decisions determine the outcome, with the probability distributions common knowledge.

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Noncooperative game theory's methods for analyzing rational decisions in games in which players make simultaneous decisions can (and will, later) be extended to games in which some decisions are sequential, and reactions are possible.

Generalize the notion of a decision to a *decision rule* or *strategy*, a complete contingent plan for playing the game that allows a player's decisions to respond to others' decisions when he can observe them before making his own decisions.

Players must be thought of as choosing their strategies strategically simultaneously (without observing others' strategies) at the start of play.



Rational, perfect foresight (strong assumption!) implies that simultaneous choice of such strategies yields the same outcome as decision-making in real time.

Complete contingent plans are needed (even for decision points ruled out by prior decisions) to evaluate consequences of alternative strategies and formalize the idea that the predicted strategy choice is optimal. (Zero-probability events are endogenously determined by players' decisions, so cannot be ignored in games.)

Return to games in which players make single, simultaneous decisions.

- In these and other games, a player's decisions must be feasible *independent* of others' decisions; e.g. "wrestle with player 2" is not a well-defined decision, although "try to wrestle with player 2" could be, if what happens if 2 doesn't also try is clearly specified.
- Specifying all of each player's decisions must completely determine an outcome (or at least a probability distribution over outcomes).

If a game model does not pass these tests, it must be modified until it does.

E.g. If you object to my game analysis on the grounds that players don't really have to play "my" game, my (only!) remedy is to add to my game's rules a player's decision whether to participate, and then to insist that that decision be explained by the same behavioral assumptions as players' other decisions.