Semester : III   Subject : DSGT   Academic Year: 2022-2023

Module No. 2

* Relations and Functions *

* Sets - A set is any well defined collection of objects, called the "elements" or "members" of the set.

e.g. A set generally denoted by capital letters A, B, C, ..., X, Y, Z.
elements of a set are denoted by lower case letters a, b, c, ... x, y, z.

If x is an element of a set A.
$\therefore x \in A$.
x is not a member of a set A
$x \notin A$.

ex. $A = \{x, y, z\}$ or $A = \{1, 2, 3\}$

* Representation of a set -

i) Listing method. - ex. $A = \{a, b, c, d\}$

ii) Statement form - ex. The set of all prime no.

iii) Set-Builder notation - ex. $A = \{x \mid x > 10\}$
so A is a set of all x such that x is greater than 10.

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Finite set - $A = \{x \mid x < 10\}$
Infinite set - $A = \{x \mid x > 10\}$

* Set properties -

i) Every set A is a subset of the universal set 'U' since all the elements of A belongs to 'U', empty set ϕ is a subset of A

$\phi \subseteq A \subseteq U$

ii) Every set A is a subset of itself,

$\therefore A \subseteq A$.

iii) If every element of A belongs to set B, and every element of B belongs to set C, then every element of A belongs to C.

$\therefore$ if $A \subseteq B$ and $B \subseteq C$
then $A \subseteq C$

iv) If $A \subseteq B$ and $B \subseteq A$ then A and B have the same elements that is $A = B$.

if $A = B$ then $A \subseteq B$ and $B \subseteq A$.

* Partitions of set.

Let A be a set. A partition of A is any set of non-empty subset $A_1, A_2, \cdots$ of A such that

i) $A_1 \cup A_2 \cup \dots = A$ (union)

ii) The subset A_i are mutually disjoint, that is

$A_i \cap A_j = \phi$ for $i \neq j$ (intersection)

May 17 similar ex

examples :-

1. Let $A = \{a, b, c\}$

then $A_1 = \{a\}$ $A_2 = \{b, c\}$

i) $A = A_1 \cup A_2 = \{a\} \cup \{b, c\}$

$= \{a, b, c\}$

$A_1 \cup A_2 = A$

ii) $A_1 \cap A_2 = \{a\} \cap \{b, c\}$

$= \phi$

$A_1 \cap A_2 = \phi$

2. Let $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ Determine whether or not each of the following is a partition of S.

i) $\{\{1, 3, 5\} \{2, 6\} \{4, 8, 9\}\}$

ii) $\{\{1, 3, 5\} \{2, 4, 6, 8\} \{5, 7, 9\}\}$

iii) $\{\{1, 3, 5\} \{2, 4, 6, 8\} \{7, 9\}\}$

iv) $\{\{S\}\}$

i) $\{\{1, 3, 5\} \{2, 6\} \{4, 8, 9\}\}$

$S_1 = \{1, 3, 5\}$ $S_2 = \{2, 6\}$ $S_3 = \{4, 8, 9\}$

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a) $S_1 \cup S_2 \cup S_3 = \{1,3,5\} \cup \{2,6\} \cup \{4,8,9\}$
$= \{1,2,3,4,5,6,8,9\}$
$S_1 \cup S_2 \cup S_3 \neq S$

b) $S_1 \cap S_2 \cap S_3 = \{1,3,5\} \cap \{2,6\} \cap \{4,8,9\}$
$= \phi$

Hence S1, S2 and S3 is not a partition of S.

(ii) $\{\{1,3,5\} \{2,4,6,8\} \{5,7,9\}\}$
$S_1 = \{1,3,5\}$
$S_2 = \{2,4,6,8\}$
$S_3 = \{5,7,9\}$

a) $S_1 \cup S_2 \cup S_3 = \{1,3,5\} \{2,4,6,8\} \{5,7,9\}$
$= \{1,2,3,4,5,6,7,8,9\}$
$= S$

b) $S_1 \cap S_2 \cap S_3 = \{1,3,5\} \cap \{2,4,6,8\} \cap \{5,7,9\}$
$= \{5\}$

Hence S1, S2, S3 is not a partition of S.

(iii) $\{\{1,3,5\} \{2,4,6,8\} \{7,9\}\}$
$S_1 = \{1,3,5\}$
$S_2 = \{2,4,6,8\}$
$S_3 = \{7,9\}$

a) $S_1 \cup S_2 \cup S_3 = \{1,3,5\} \cup \{2,4,6,8\} \cup \{7,9\}$
$= \{1,2,3,4,5,6,7,8,9\}$
$= S$

b) $S1 \cap S2 \cap S3 = \{1,3,5\} \cap \{2,4,6,8\} \cap \{7,9\}$
$= \emptyset$

Hence S1, S2, S3 partitions of set S.

iv) $\{\{S\}\}$, $S = \{1,2,3,4,5,6,7,8,9\}$
$\{\{S\}\} = \{\{1,2,3,4,5,6,7,8,9\}\}$
$\{\{S\}\}$ is a partition of S.

(2)

Determine whether or not each of the following is a partition of the set of positive integers.

i) $\{\{n : n > 5\}, \{n : n < 5\}\}$
ii) $\{\{n : n > 5\} \{0\} \{1,2,3,4,5\}\}$
iii) $\{\{n : n^2 > 11\}, \{n : n^2 < 11\}\}$

* Power Set :-

Let A be a set, then the set of all subsets of A is called the power set of A and is denoted by P(A). Power set is an example of set of sets. that is, a set whose elements are themselves one. Power set of A has 2^n elements where n is number of elements in A.

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ex. Let $A = \{1, 2, 3\}$. Determine the power set A.

$\Rightarrow$ P(A) consists of the following subsets of A
$\{\}$, $\{1\}$, $\{2\}$, $\{3\}$, $\{1, 2\}$, $\{2, 3\}$, $\{1, 3\}$ and $\{1, 2, 3\}$ (or A).

P(A) has $2^3 = 8$ elements

$P(A) = \{\{\}, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$

(2)

ex. Let set $A = \{a, b, c, d\}$ Determine the power set A.

$\Rightarrow P(A) = \{\{\}, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\},$
$\{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\},$
$\{a, b, d\}, \{a, c, d\}, \{b, c, d\}, \{a, b, c, d\}\}$

P(A) has $2^4 = 16$ elements.

(3)

ex. Determine the power set P(A) of A $\{a, b, c\}$

$P(A) = \{\{\}, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\},$
$\{a, b, c\}\}$

P(A) is a set of $2^3 = 8$ elements.

ex.

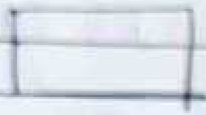
(4) If $A \{\phi, a\}$ then construct the sets $A \cup P(A)$, $A \cap P(A)$

$P(A) = \{\phi, \{a\}, \{\phi\}, \{\phi, a\}\}$

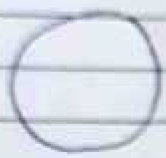
$A \cup P(A) = \{\phi, a, \{\phi\}, \{a\}, \{\phi, a\}\}$

$A \cap P(A) = \{\phi\}$

<u>Venn</u> <u>diagram</u> - it is used to represent set diagramatically

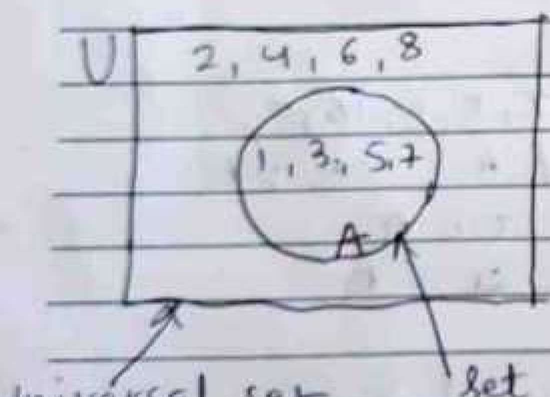


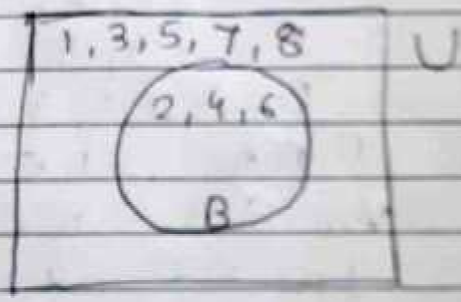
· Rectangle : used to represent Universal set.

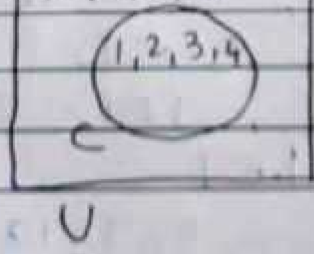


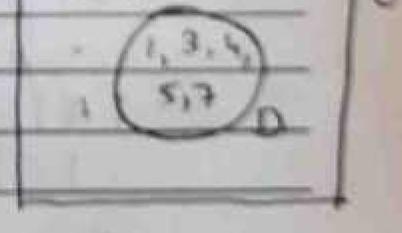
: Circle : To represent a set.

<u>ex</u> $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$
$A = \{1, 3, 5, 7\}$
$B = \{2, 4, 6\}$
$C = \{1, 2, 3, 4\}$
$D = \{1, 3, 4, 5, 7\}$









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Examples on set -

ex. (1) If $A = \{a, b, c\}$ then

$P(A) = \{\phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$

$\because |P(A)| = 2^{|A|}$

$|A| = 3$

then $|P(A)| = 2^3 = 8$

(2) consider the following sets

$A = \{1, 2, 3, 4, 5\}$

$B = \{2, 3, 4\}$

$C = \{4, 5, 6, 7\}$

$D = \{4, 2, 3\}$

$E = \{1, 2, 3, 4\}$

$F = \{4, 5\}$

$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$.

$\therefore 1 \in A, \; 1 \in E, \; 6 \notin A, \; 5 \notin B$

$\forall a \in B$ we have $a \in A$

$\exists a \in C$ we have $a \in B$

$B \subseteq A, \; B \subset A$.

$C \not\subseteq A, \; E \subseteq A, \; F \subseteq C$

$C \neq F, \; B = D, \; D \subseteq B,$

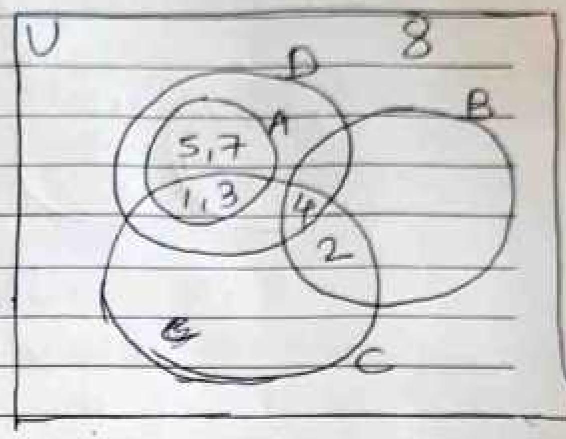
$|A| = 5, \; |F| = 2$

$|C| = |E|$

$\bar{A} = \{6, 7, 8, 9, 10\}$ $\bar{C} = \{1, 2, 3, 8, 9, 10\}$

To represent all the above sets in a single Venn diagram, we have

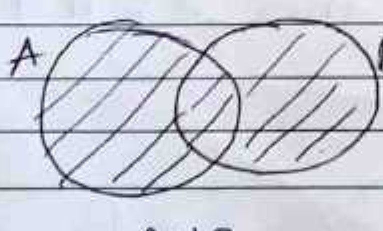
1 ∈ A, C, D
2 ∈ B, C
3 ∈ A, C, D
4 ∈ B, C, D
5 ∈ A, D
6 ∈ B
7 ∈ A, D
8 ∈ A, B, C, D



* Set Operations -

① Union of set - ∪

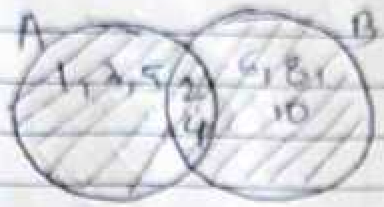
Union of two or more sets results into a set that includes all elements of those sets, i.e. if $a \in A$ or $a \in B$ then $a \in A \cup B$.



A∪B

If an element belongs to both sets, then it is present only once in the union set.

ex $A = \{1, 2, 3, 4, 5\}$
$B = \{2, 4, 6, 8, 10\}$
$A \cup B = \{1, 2, 3, 4, 5, 6, 8, 10\}$



consider

$$A \cup B = B \cup A$$

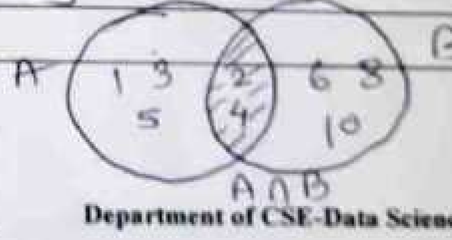
Consider n different sets $A_1, A_2, A_3, \ldots A_n$
The union of these sets is represented as

$$\bigcup_{i=1}^{n} A_i = A_1 \cup A_2 \cup A_3 \cup \ldots \cup A_n$$

(2) Intersection of sets - $\cap$

Intersection of two or more sets results into a set that includes all elements that are present in all the sets.
i.e. if $a \in A$ and $a \in B$ then $a \in A \cap B$

e.g. $A = \{1, 2, 3, 4, 5\}$
$B = \{2, 4, 6, 8, 10\}$
$A \cap B = \{2, 4\}$



$$\bigcap_{i=1}^{n} A_i = A_1 \cap A_2 \cap \ldots \cap A_n$$

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3) Complement of set –

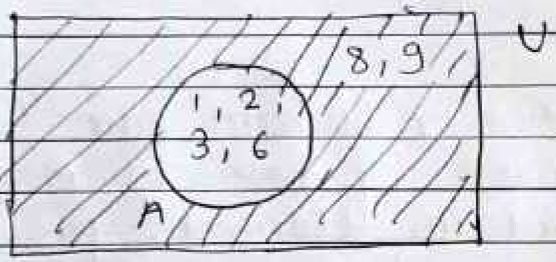
The complement of a set is a set that includes all elements that are not present in the given set.

i.e. if $a \notin A$ then $a \in \bar{A}$ or $a \in A'$

e.g. $A = \{1, 2, 3, 6\}$ $U = \{1, 2, 3, 6, 8, 9\}$

$B = \{3, 6, 8, 9\}$

$\therefore \bar{A} = \{8, 9\}$



4) Set difference

Consider two sets A and B. The diff. opⁿ A − B is a set that includes all elements in A but not in B.

$A - B = \{x \mid x \in A \text{ and } x \notin B\} = A \cap \bar{B}$

if $A \neq B$ then $A - B \neq B - A$

if $A = B$ then $A - B = B - A = \phi$

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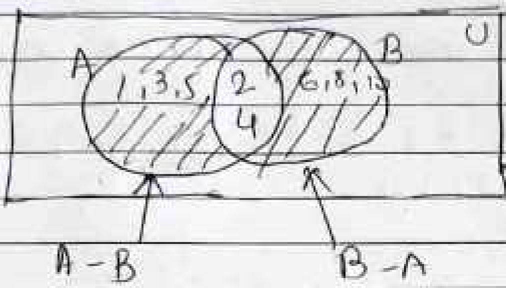
e.g. $A = \{1, 2, 3, 4, 5\}$
$B = \{2, 4, 6, 8, 10\}$
$A - B = \{1, 3, 5\}$
$B - A = \{6, 8, 10\}$



(5) Symmetric Difference or Boolean sum $\oplus$

Symmetric difference of two sets A and B is the set that contains elements present either in A or in B but not in both sets. This is called Boolean sum.

i.e. $A \oplus B = \{x \mid (x \in A \text{ or } x \in B)\}$ and $(x \notin A \cap B)$

$\therefore A \oplus B = (A - B) \cup (B - A)$
$= (A \cap \bar{B}) \cup (B \cap \bar{A})$

e.g. $A = \{1, 2, 3, 4, 5\}$ $B = \{2, 4, 6, 8, 10\}$
$A \oplus B = \{1, 3, 5, 6, 8, 10\}$