

Subject: Applied Mathematics IV

SEM:IV

Duality:-

Definition:-

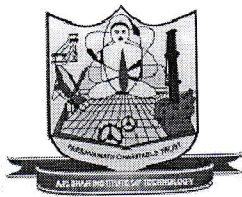
The phenomenon occurring in linear programming that given a problem there exists another closely related problem with the same set of data, and with the same solution is called the principle of duality.

Procedure:-

* Write the objective function in maximisation type, if not.

* Write all the constraints in \leq type. If any constraint is in \geq type multiply the inequality by -1 and change the inequality \leq .

* If the constraint is of equality type, change it into inequality type,



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For example $3x_1 + 6x_2 = 7$ is equivalent to

$$3x_1 + 6x_2 \leq 7 \text{ \& } 3x_1 + 6x_2 \geq 7$$

$$\text{Now, } 3x_1 + 6x_2 \geq 7 \Rightarrow -3x_1 - 6x_2 \leq -7.$$

$\therefore 3x_1 + 6x_2 = 7$ is now equivalent to

$$3x_1 + 6x_2 \leq 7 \text{ \& }$$

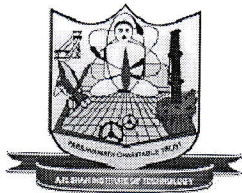
$$-3x_1 - 6x_2 \leq -7.$$

* All decision variables must be ' \geq ' type. If any variable is unrestricted say x_2 , then write x_2 as

$$x_2 = x_2' - x_2''$$

* In the primal in the standard form

- (i) the objective function is of maximisation type with the constraints in ' \leq ' type &
- (ii) the objective function is of minimisation type with the constraints in ' \geq ' type.



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Now write the dual by
* Write the objective function from
maximisation to minimisation

① Construct the dual of the following problem

$$\text{Minimise } Z = x_1 + 3x_2$$

$$\text{Subject to } 2x_1 + x_2 \leq 3$$

$$x_1 + 2x_2 + 6x_3 \geq 5$$

$$-x_1 + x_2 + 2x_3 = 2$$

$$x_1, x_2, x_3 \geq 0.$$

Soln:-

Since the objective function is of
minimisation type, the constraints should be ' \geq ' type

$$\therefore 2x_1 + x_2 \leq 3 \Rightarrow -2x_1 - x_2 \geq -3$$

$-x_1 + x_2 + 2x_3 = 2$ can be written as

$$\Rightarrow -x_1 + x_2 + 2x_3 \leq 2 \quad \& \quad -x_1 + x_2 + 2x_3 \geq 2$$

$$\Rightarrow x_1 - x_2 - 2x_3 \geq -2 \quad \& \quad -x_1 + x_2 + 2x_3 \geq 2.$$



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∴ The ^{Primal} dual is

Minimize $Z = 0x_1 + x_2 + 3x_3$

subject to $-2x_1 - x_2 + 0x_3 \geq -3$

$$x_1 + 2x_2 + 6x_3 \geq 5$$

$$-x_1 + x_2 + 2x_3 \geq 2$$

$$x_1 - x_2 - 2x_3 \geq -2$$

$$x_1, x_2, x_3 \geq 0.$$

Since the last constraint is now expressed as 2 constraints, we have $y_3 = y_3' - y_3''$.

∴ The dual is

Maximize $w = -3y_1 + 5y_2 + 2y_3' - 2y_3''$

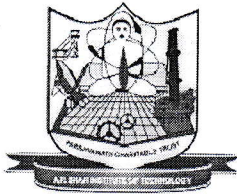
subject to $-2y_1 + y_2 - y_3' + y_3'' \leq 0$

$$-y_1 + 2y_2 + y_3' - y_3'' \leq 1$$

$$0y_1 + 6y_2 + 2y_3' - 2y_3'' \leq 3.$$

Since $y_3 = y_3' - y_3''$, y_3 is unrestricted,

∴ The dual is



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$$\text{Maximise } W = -3y_1 + 5y_2 + 0y_3$$

$$\text{subject to } -2y_1 + y_2 - y_3 \leq 0$$

$$-y_1 + 2y_2 + y_3 \leq 1$$

$$6y_2 + 2y_3 \leq 3$$

$$y_1, y_2 \geq 0, y_3 \text{ is unrestricted.}$$

② Find the dual of the follow: LPP.

$$\text{Maximise } Z = 2x_1 - x_2 + 3x_3$$

$$\text{subject to } x_1 - 2x_2 + x_3 \geq 4$$

$$2x_1 + x_3 \leq 10$$

$$x_1 + x_2 + 3x_3 = 20$$

$$x_1, x_3 \geq 0, x_2 \text{ unrestricted.}$$

Soln:-

$$x_2 \text{ is unrestricted, } \therefore x_2 = x_2' - x_2''$$

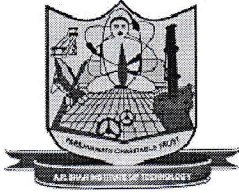
$$\text{Maximise } Z = 2x_1 - (x_2' - x_2'') + 3x_3$$

$$\text{subject to } -[x_1 - 2(x_2' - x_2'') + x_3] \leq -4$$

$$2x_1 + 0x_2 + x_3 \leq 10$$

$$x_1 + (x_2' - x_2'') + 3x_3 \leq 20$$

$$-[x_1 + (x_2' - x_2'') + 3x_3] \leq -20$$



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∴ The Primal is

$$\text{Maximise } Z = 2x_1 - x_2' + x_2'' + 3x_3$$

$$\text{subject to } -x_1 + 2x_2' - 2x_2'' - x_3 \leq -4$$

$$2x_1 + 0x_2' - 0x_2'' + x_3 \leq 10$$

$$x_1 + x_2' - x_2'' + 3x_3 \leq 20$$

$$-x_1 - x_2' + x_2'' - 3x_3 \leq -20$$

$$x_1, x_2', x_2'', x_3 \geq 0.$$

The dual is

$$\text{Minimise } w = -4y_1 + 10y_2 + 20y_3' - 20y_3''$$

$$\text{subject to } -y_1 + 2y_2 + y_3' - y_3'' \geq 2$$

$$2y_1 + 0y_2 + y_3' - y_3'' \geq -1$$

$$-2y_1 + 0y_2 - y_3' + y_3'' \geq 1$$

$$-y_1 + y_2 + 3y_3' - 3y_3'' \geq 3$$

$$\text{e) Minimise } w = -4y_1 + 10y_2 + 20y_3$$

$$\text{subject to } -y_1 + 2y_2 + y_3 \geq 2$$

$$2y_1 + 0y_2 + y_3 = -1$$

$$-y_1 + y_2 + 3y_3 \geq 3$$

$$y_1, y_2 \geq 0$$

y_3 unrestricted.