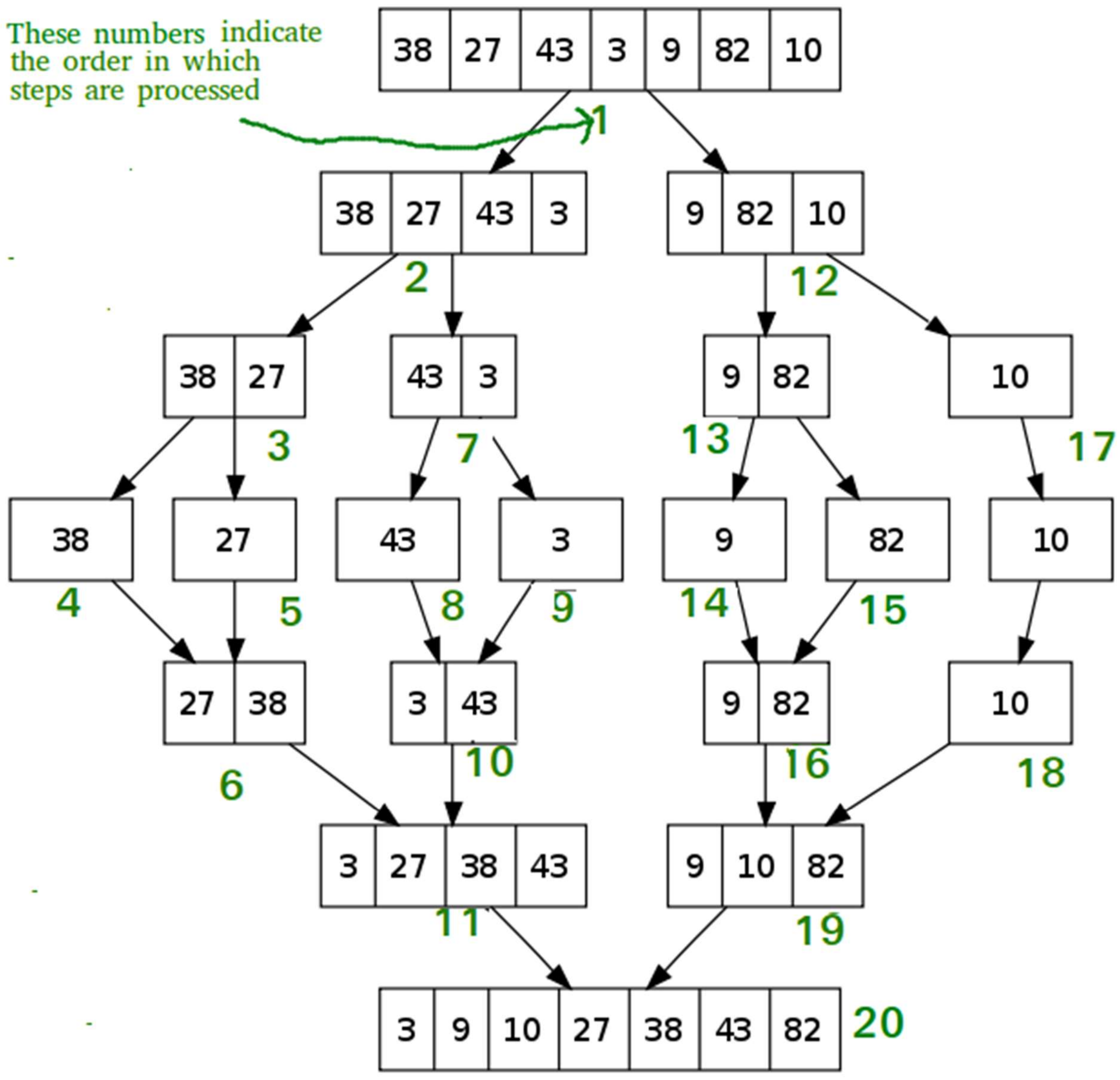
Subject :- ADSAA **Semester :- V**

Merge Sort

Merge Sort is a Divide and Conquer algorithm. It divides the input array into two halves, calls itself for the two halves, and then merges the two sorted halves. **The merge() function** is used for merging two halves. The merge(arr, l, m, r) is a key process that assumes that arr[l..m] and arr[m+1..r] are sorted and merges the two sorted sub-arrays into one. See the following C implementation for details.

```
MergeSort(arr[], l,  r)
If r > l
     1. Find the middle point to divide the array into two halves:
             middle m = (l+r)/2
     2. Call mergeSort for first half:
             Call mergeSort(arr, l, m)
     3. Call mergeSort for second half:
             Call mergeSort(arr, m+1, r)
     4. Merge the two halves sorted in step 2 and 3:
             Call merge(arr, l, m, r)
```

The following diagram from wikipedia shows the complete merge sort process for an example array {38, 27, 43, 3, 9, 82, 10}. If we take a closer look at the diagram, we can see that the array is recursively divided in two halves till the size becomes 1. Once the size becomes 1, the merge processes come into action and start merging arrays back till the complete array is merged.


These numbers indicate the order in which steps are processed
38 27 43 3 9 82 10
1
38 27 43 3
9 82 10
2
12
38 27
43 3
9 82
10
3
7
13
17
38
27
43
3
9
82
10
4
5
8
9
14
15
27 38
3 43
9 82
10
6
10
16
18
3 27 38 43
9 10 82
11
19
3 9 10 27 38 43 82
20

```
/* C program for Merge Sort */
#include <stdio.h>
#include <stdlib.h>

// Merges two subarrays of arr[].
// First subarray is arr[l..m]
// Second subarray is arr[m+1..r]
void merge(int arr[], int l, int m, int r)
{
        int i, j, k;
        int n1 = m - l + 1;
        int n2 = r - m;

        /* create temp arrays */
        int L[n1], R[n2];

        /* Copy data to temp arrays L[] and R[] */
        for (i = 0; i < n1; i++)
                L[i] = arr[l + i];
        for (j = 0; j < n2; j++)
                R[j] = arr[m + 1 + j];

        /* Merge the temp arrays back into arr[l..r]*/
```

```
i = 0; // Initial index of first subarray
j = 0; // Initial index of second subarray
k = l; // Initial index of merged subarray
while (i < n1 && j < n2) {
        if (L[i] <= R[j]) {
                arr[k] = L[i];
                i++;
        }
        else {
                arr[k] = R[j];
                j++;
        }
        k++;
}

/* Copy the remaining elements of L[], if there
are any */
while (i < n1) {
        arr[k] = L[i];
        i++;
        k++;
}

/* Copy the remaining elements of R[], if there
```

```
    are any */
    while (j < n2) {
        arr[k] = R[j];
        j++;
        k++;
    }
}

/* l is for left index and r is right index of the
sub-array of arr to be sorted */
void mergeSort(int arr[], int l, int r)
{
    if (l < r) {
        // Same as (l+r)/2, but avoids overflow for
        // large l and h
        int m = l + (r - l) / 2;

        // Sort first and second halves
        mergeSort(arr, l, m);
        mergeSort(arr, m + 1, r);

        merge(arr, l, m, r);
    }
}
```

```
/* UTILITY FUNCTIONS */
/* Function to print an array */
void printArray(int A[], int size)
{
        int i;
        for (i = 0; i < size; i++)
                printf("%d ", A[i]);
        printf("\n");
}

/* Driver code */
int main()
{
        int arr[] = { 12, 11, 13, 5, 6, 7 };
        int arr_size = sizeof(arr) / sizeof(arr[0]);

        printf("Given array is \n");
        printArray(arr, arr_size);

        mergeSort(arr, 0, arr_size - 1);

        printf("\nSorted array is \n");
        printArray(arr, arr_size);
```

```
    return 0;
}
```

```
Given array is
12 11 13 5 6 7
Sorted array is
5 6 7 11 12 13
```

Time Complexity: Sorting arrays on different machines. Merge Sort is a recursive algorithm and time complexity can be expressed as following recurrence relation.
$T(n) = 2T(n/2) + \theta(n)$
The above recurrence can be solved either using the Recurrence Tree method or the Master method. It falls in case II of Master Method and the solution of the recurrence is $\theta(n\text{Log}n)$. Time complexity of Merge Sort is $\theta(n\text{Log}n)$ in all 3 cases (worst, average and best) as merge sort always divides the array into two halves and takes linear time to merge two halves.

Auxiliary Space: O(n)
Algorithmic Paradigm: Divide and Conquer
Sorting In Place: No in a typical implementation
Stable: Yes

Applications of Merge Sort

1. Merge Sort is useful for sorting linked lists in O(nLogn) time.In the case of linked lists, the case is different mainly due to the difference in memory allocation of arrays and linked lists. Unlike arrays, linked list nodes may not be adjacent in memory. Unlike an array, in the linked list, we can insert items in the middle in O(1) extra space and O(1) time. Therefore, the merge operation of merge sort can be

implemented without extra space for linked lists.
In arrays, we can do random access as elements are contiguous in memory. Let us say we have an integer (4-byte) array A and let the address of A[0] be x then to access A[i], we can directly access the memory at (x + i*4). Unlike arrays, we can not do random access in the linked list. Quick Sort requires a lot of this kind of access. In a linked list to access i'th index, we have to travel each and every node from the head to i'th node as we don't have a continuous block of memory. Therefore, the overhead increases for quicksort. Merge sort accesses data sequentially and the need of random access is low.

2. Inversion Count Problem
3. Used in External Sorting