Composite Transformation

- We can setup a matrix for any sequence of transformations as a composite transformation matrix by calculating the matrix by product of individual transformation.
- For column matrix representation of co-ordinate positions, we form composite transformations by multiplying matrix in order from right to left.

Translations

- Two successive translations are performed as:

$$P' = T(tx2, ty2) \cdot \{T(tx1, ty1)\} \cdot P$$

$$= \{T(tx2, ty2) \cdot T(tx1, ty1)\} \cdot P$$

$$= \begin{bmatrix} 1 & 0 & tx2 \\ 0 & 1 & ty2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & tx1 \\ 0 & 1 & ty1 \\ 0 & 0 & 1 \end{bmatrix} \cdot P$$

$$= \begin{bmatrix} 1 & 0 & tx1 + tx2 \\ 0 & 1 & ty1 + ty2 \\ 0 & 0 & 1 \end{bmatrix} \cdot P$$

$$P' = T(tx1 + tx2, ty1 + ty2) \cdot P$$

- Here P' and P are column vector of final and initial point co-ordinate respectively.
- This concept can be extended for any no of successive translations.

Example:

Q. Obtain the final co-ordinates after two translations on point P (2,3) with translation vector (4, 3) and (-1, 2) respectively.

$$P' = T(tx_1 + tx_2,\ ty_1 + ty_2) \cdot P$$

$$= \begin{bmatrix} 1 & 0 & tx_1 + tx_2 \\ 0 & 1 & ty_1 + ty_2 \\ 0 & 0 & 1 \end{bmatrix} \cdot P$$

$$= \begin{bmatrix} 1 & 0 & 4 + (-1) \\ 0 & 1 & 3 + 2 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \\ 1 \end{bmatrix}$$

Final co-ordinates after translations are $P'(5, 8)$