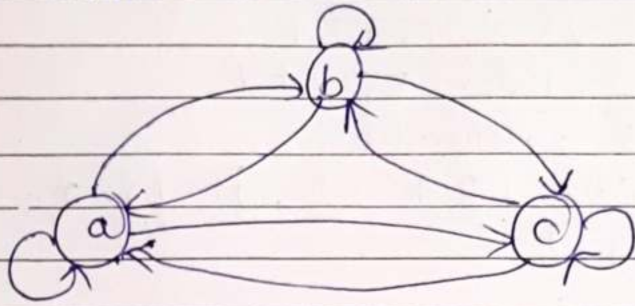
Semester : III Subject : DSGT Academic Year: 2022 - 2023

* Equivalence Relations -

A relation R on a set A is called an equivalence relation if it is reflexive, symmetric and transitive.



The digraph of an equivalence relation will have the following characteristics.

a) Every vertex will have a loop.

b) If there is an arc from a to b, there should be an arc from b to a

c) If there is an arc from a to b and arc from b to c, there should be an arc from a to c.

ex. (1) Let $A = \{a, b, c\}$ and let,

$$M_R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$ Determine whether R is an equivalence relation.

$\Rightarrow$ $R = \{(a,a)(b,b)(b,c)(c,b)(c,c)\}$

R is reflexive as

$(a,a)(b,b)(c,c) \in R$.

R is symmetric as

$(b,c) \in R \rightarrow (c,b) \in R$

R is transitive as

(b,b) & $(b,c) \in R$ implies $(b,c) \in R$

(b,c) & $(c,b) \in R$ implies $(b,b) \in R$

(c,c) & $(c,b) \in R$ implies $(c,b) \in R$

(c,b) & $(b,b) \in R$ implies $(c,b) \in R$

(c,b) & $(b,c) \in R$ implies $(c,c) \in R$

(b,c) & $(c,c) \in R$ implies $(b,c) \in R$

Hence R is an equivalence relation.

2) Let $A = \{a,b,c,d\}$

$R = \{(a,a)(b,a)(b,b)(c,c)(d,d)(d,c)\}$

Determine whether R is an equivalence relation

$\Rightarrow$ R is reflexive as $(a,a)(b,b)(c,c)(d,d) \in R$

R is not symmetric as

$(b,a) \in R$ but $(a,b) \notin R$

R is transitive since,

$(b,a),(a,a) \in R \Rightarrow (b,a) \in R$

$(b,b),(b,a) \in R \Rightarrow (b,a) \in R$

$(d,c),(c,c) \in R \Rightarrow (d,c) \in R$

$(d,d)(d,c) \in R \Rightarrow (d,c) \in R$

$\therefore$ R is not an equivalence relation.

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ex.
(3) $A = \{1, 2, 3\}$
$R_1 = \{(1,1)(2,2)(3,3)\}$
$R_2 = \{(1,1)(2,2)(3,3)(2,1)(1,2)\}$
$R_3 = \{(1,1)(2,2)(3,3)(3,2)(1,3)\}$
$R_4 = \{\ \}$
Find whether the given relations are equivalence relation or not.

⇒

Given $A = \{1, 2, 3\}$
i) $R_1 = \{(1,1)(2,2)(3,3)\}$
R_1 is reflexive as
$(1,1) \in R$
$(2,2) \in R$
$(3,3) \in R$
R_1 is symmetric as
~~(1,1)~~
$(x,y)\ (y,x) \in R$
we have
$(1,1)\ (1,1) \in R$
$(2,2)\ (2,2) \in R$
$(3,3)\ (3,3) \in R$.
R_1 is symmetric
R_1 is not transitive as

so R_1 is not equivalence relation.

ii) $R2 = \{(1,1)(2,2)(3,3)(2,1)(1,2)\}$
R2 is reflexive.
$(1,1), (2,2), (3,3) \in R$.
R2 is symmetric
$(2,1) \in R$ and $(1,2) \in R$
R3 is transitive
$(2,1) \in R$
$(1,2) \in R$
$(2,2) \in R$.
So Relation R2 is equivalence relation.

iii) $R3 = \{(1,1)(2,2)(3,3)(3,2)(1,3)\}$
R3 is reflexive.
$(1,1) \in R$
$(2,2) \in R$
$(3,3) \in R$
R3 is not symmetric
$(3,2) \in R$ but
$(2,3) \notin R$
hence not symmetric
Hence R3 is not equivalence relation.

iv) $R4 = \{\}$
R4 is not reflexive.
hence R4 is not equivalence relation.