Semester: III Subject: DSGT Academic Year: 2022-2023

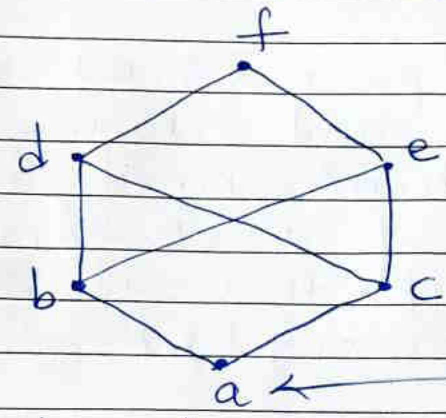
Minimal Elements and Maximal Elements -

Let say we have a poset (S, R) where S is some arbitrary set and relation R is a partial order relation defined on set S.

Minimal element -

An element x of set S is called a minimal element if there is no y ∈ S such that yRS (or $(y, x) \notin R$) and $y \neq x$.

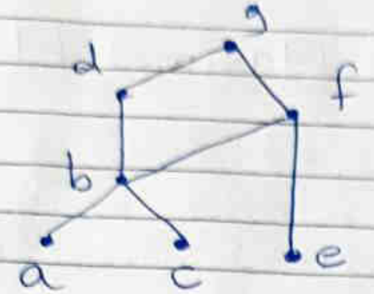
ex: Consider the following Hasse Diagram -



a is minimal element.

Here, b and c are not minimal because aRb and aRc.
similarly d and e are not minimal because, bRd and cRd.
also cRe and bRe
also f is not minimal element as dRf, eRf.
but a is a minimal element as no element is related a.

ex. 2



Here,
a, c and e are minimal elements.
no element related to a, c and e.

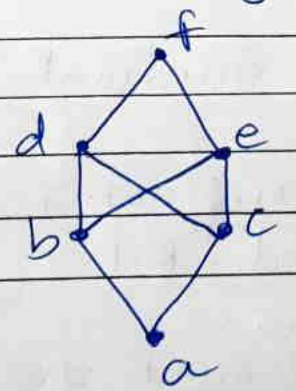
A poset can have more than 1 minimal element.

Maximal element -

Let say we have a poset (S, R) where S is some arbitrary set and relation R is a partial order relation defined on sets.

An element x of a set S is called a maximal element if there is no $y \in S$ such that xRy (or $(x, y) \notin R$) & $x \neq y$.

ex.

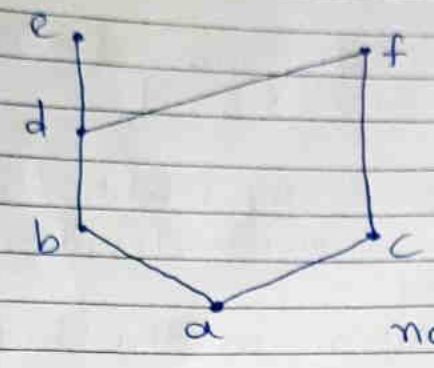


a is not maximal element as aRb, aRc
d, e, b, c also not maximal element as bRd, bRe, cRd, cRe, dRf, eRf.

but f is maximal ele. as f is not related to any element.

Semester : III Subject : DSGT Academic Year: 2022-2023

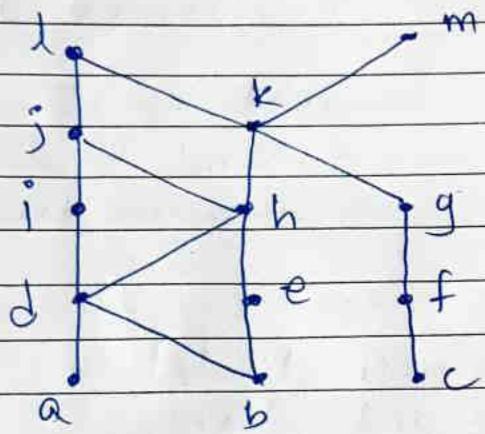
ex. 2



e and f are maximal elements because e is not related to any element & f is also not related to any element.

A poset may have more than one maximal elements.

ex. 3



Find minimal and maximal elements.

⇒ Minimal elements = a, b, c

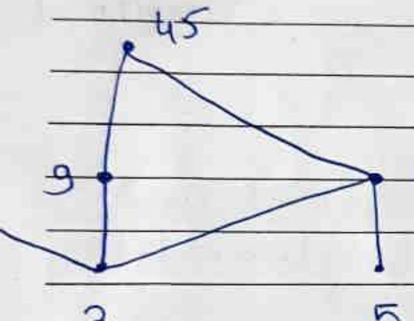
Maximal elements = l, m

ex. 4 What are the minimal and maximal elements for the poset
$(\{3, 5, 9, 15, 24, 45\}, |)$

Semester : III Subject : DSGT Academic Year: 2022-2023

⇒ lets draw Hasse diagram for the poset
({3, 5, 9, 15, 24, 45}, |) ← divide rel^n

3 and 5 are non comparable / incomparable element and no element divides 3 and 5 in the given set.
So lets put them in the bottom most place.



Next element is 9
3 divides 9

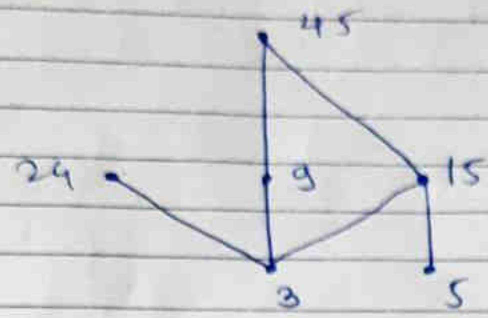
next is 15
3 and 5 divides 15
9 does not divide 15.

next is 24
9 does not divide 24
5, 15 does not divide 24
only 3 divide 24.

next element 45.
Top elements are 24, 9, 15
24 not divide 45 but 9 divides 45
also 15 divides 45.

Semester : III　　Subject : DSGT　　Academic Year: 2022-2023

Resultant Hasse diagram.



~~Minimal elements 24 & 45~~

Minimal elements = 3 and 5

Maximal elements = 24 & 45.

* Lower Bound and Upper Bound)

Let say we have a poset (S, R) such that S is an arbitrary set and R is a partial order defined on set S.
Also, let say $T \subseteq S$.

Lower Bound -
An element $x \in S$ is a lower bound of set T if $\forall y \in T \; (x, y) \in R$.

Upper Bound -
An element $x \in S$ is an upper bound of set T if $\forall y \in T \; (y, x) \in R$.

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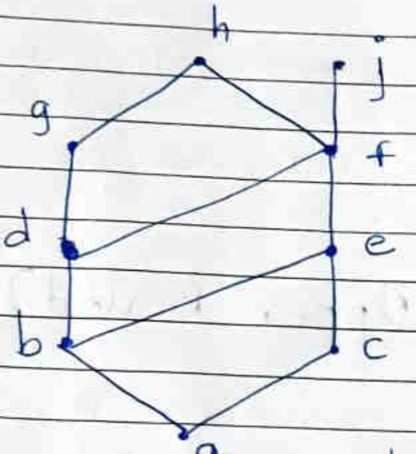
Semester: III Subject: DSGT Academic Year: 2022-2023

ex ① Find the lower and upper bounds of the subsets {a, b, c}, {j, h} and {a, c, d, f} in the poset with the Hasse diagram shown below.



go in upward direction

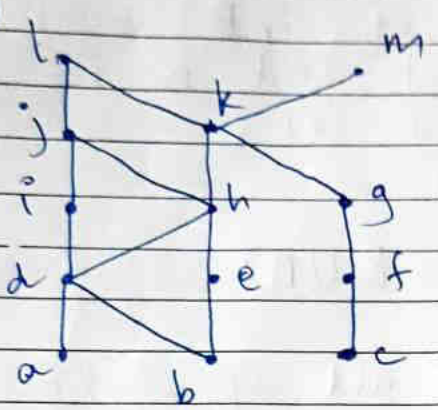
subsets = {a, b, c}
{j, h}
{a, c, d, f}

i) {a, b, c}
lower bound =
as a is lower bound
aRb, aRc
but b & c not lower bound
b does not relate to a & c
$b \not R a$, $b \not R c$.
upper bound -
h, j, f, e is upper bound.
as bRe & cRe.

ii) {j, h}
lower bound = f, d, e, b, c, a
upper bound = j & h are not related to any element. = ϕ.

iii) {a, c, d, f}
lower bound = a
upper bound = f, j, h

Ex. (2)

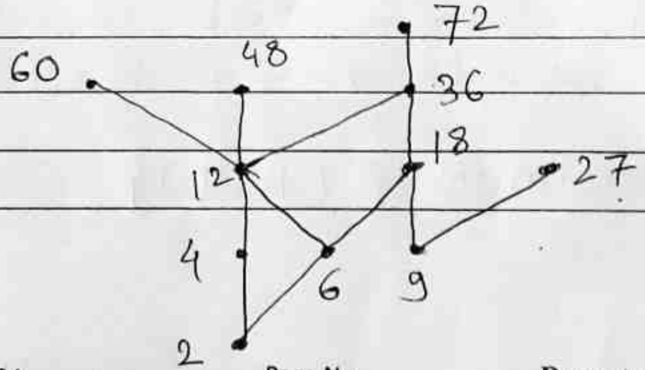


Find upper bounds of {a, b, c} also find all the lower bounds of {f, g, h}.

⇒ upper bounds of {a, b, c} is k, l, m

lower bounds of {f, g, h} is ϕ

(3) Find all upper bounds of {2, 9} and all lower bounds of {60, 72} for the poset ({2, 4, 6, 9, 12, 18, 27, 36, 48, 60, 72}, |)



upper bounds of {2, 9} are 18, 36, 72

lower bounds of {60, 72} = 12, 6, 4, 2