Boolean laws

1. $A + 0 = A$
2. $A + 1 = 1$
3. $A \cdot 0 = 0$
4. $A \cdot 1 = A$
5. $A + A = A$
6. $A + \overline{A} = 1$
7. $A \cdot A = A$
8. $A \cdot \overline{A} = 0$
9. $\overline{\overline{A}} = A$
10. $A + AB = A$

$$A + AB = A(1+B)$$
$$= A \cdot 1$$
$$= A$$

11. $A + \overline{A}B = A + B$

$$A + \overline{A}B = A + AB + \overline{A}B \quad \because A = A + AB$$
$$= AA + AB + \overline{A}B \quad \because A = AA$$
$$= AA + AB + A\overline{A} + \overline{A}B \quad \because A\overline{A} = 0$$
$$= (A + \overline{A})(A + B)$$
$$= 1 \cdot (A + B)$$
$$= A + B$$

Commutative Law:

$$A + B = B + A$$

The order in which variables are ORed makes no difference.

$$A \cdot B = B \cdot A$$

The order in which variables are ANDed makes no difference.

Associative law:

$$A + (B+C) = (A+B)+C$$

$$A(BC) = (AB) \cdot C$$

Distributive law

$$A(B+C) = AB + AC$$

De Morgan's Theorem

Ist theorem $\overline{AB} = \overline{A} + \overline{B}$

The compliment of two or more variables ANDed is equivalent to OR of the compliment of individual variable.

II theorem. $\overline{A+B} = \overline{A} \cdot \overline{B}$

The compliment of two variables ORed is equivalent to ANDing of compliment of individual variable.

Simplification of Boolean expression

(I) Simplify $AB + A(B+C) + B(B+C)$

$AB + AB + AC + BB + BC$; Apply distributive law

$= AB + AC + B + BC$; $AB+AB=AB$, $BB=B$

$= AB + AC + B(1+C)$;

$= AB + AC + B$; $(1+C)=1$

$= B(1+A) + AC$

$= B + AC$; $(1+A)=1$

(II) Simplify $[A\bar{B}(C+BD)+\bar{A}\bar{B}]C$

$= (A\bar{B}C + A\bar{B}BD + \bar{A}\bar{B})C$

$= (A\bar{B}C + 0 + \bar{A}\bar{B})C$; $B\bar{B}=0$

$= (A\bar{B}CC + \bar{A}\bar{B}C)$

$= (A\bar{B}C + \bar{A}\bar{B}C)$; $CC=C$

$= \bar{B}C(A+\bar{A})$

$= \bar{B}C$; $A+\bar{A}=1$

Simplify :

① $(A+C)(AD+A\bar{D})+AC+C$

$= (A+C)\,A(D+\bar{D})+AC+C$

$= (A+C)A+AC+C$

$= (A+C)A+(A+1)C$

$= (A+C)A+C$

$= A\cdot A+AC+C$

$= A+AC+C$

$= A(C+1)+C$

$= A+C$

② $\bar{A}(A+B)+(B+A)(A+\bar{B})$

$= \bar{A}A+\bar{A}B+AB+B\cdot\bar{B}+A\cdot A+A\bar{B}$

$= \bar{A}B+AB+A+A\bar{B}$

$= B(\bar{A}+A)+A(\bar{B}+1)$

$= B+A$

$= A+B.$

$Y = \overline{AB}(B+C) + AB(\overline{B+C})$

$= (\bar{A}+\bar{B})(B+C) + AB(\bar{B}\cdot\bar{C})$

$= \bar{A}B + \bar{A}C + \bar{B}\cdot B + \bar{B}C + AB\bar{B}\bar{C}$

$= \bar{A}B + \bar{A}C + \bar{B}C + 0$

$= \bar{A}B + \bar{A}C + \bar{B}C$

Simplify using boolean theorems and draw logic diagram for the following:

1. $\bar{A}BC + A\bar{B}C + AB\bar{C} + ABC$

$= \bar{A}BC + A\bar{B}C + AB(\bar{C}+C)$

$= \bar{A}BC + A\bar{B}C + AB$ $\quad \bar{C}+C=1$

$= \bar{A}BC + A(B+\bar{B}C)$ $\quad (A+\bar{A}B) = A+B$

$= \bar{A}BC + A(B+C)$

$= \bar{A}BC + AB + AC$

$= B(A+\bar{A}C) + AC$

$= B(A+C) + AC$

$= AB + BC + AC$