Radial Basis functions: (RBF)

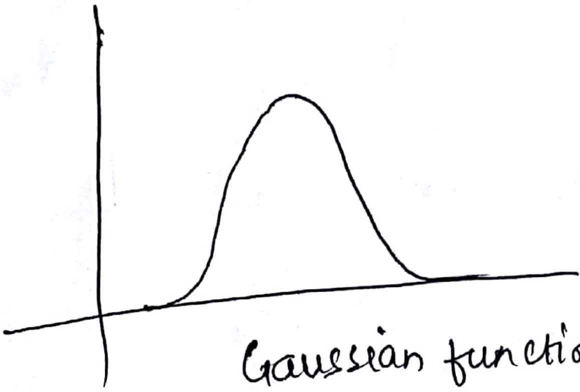
RBF are a special class of functions for nonlinear models. The basis of RBF is based on cover's theorem. As per cover's theorem, a nonlinear problem can be linearly seperable when the problem is elevated to higher dimensional space. This feature monotonically increase or decrease with distance from a central point.

- The center, the distance, the shape of the radial function are the parameters of the model.
- A typical radial function which is invariably used is Gaussian function which in the case of a scalar input is:

$$h(x) = \exp\left(-\frac{(x-c)^2}{r^2}\right)$$

where c = center, r = radius

- A gaussian RBF monotonically decreases with the distance from the center or in the other words the function has a single peak at the center and the edges tapering monotonically on both ends.



Gaussian function.

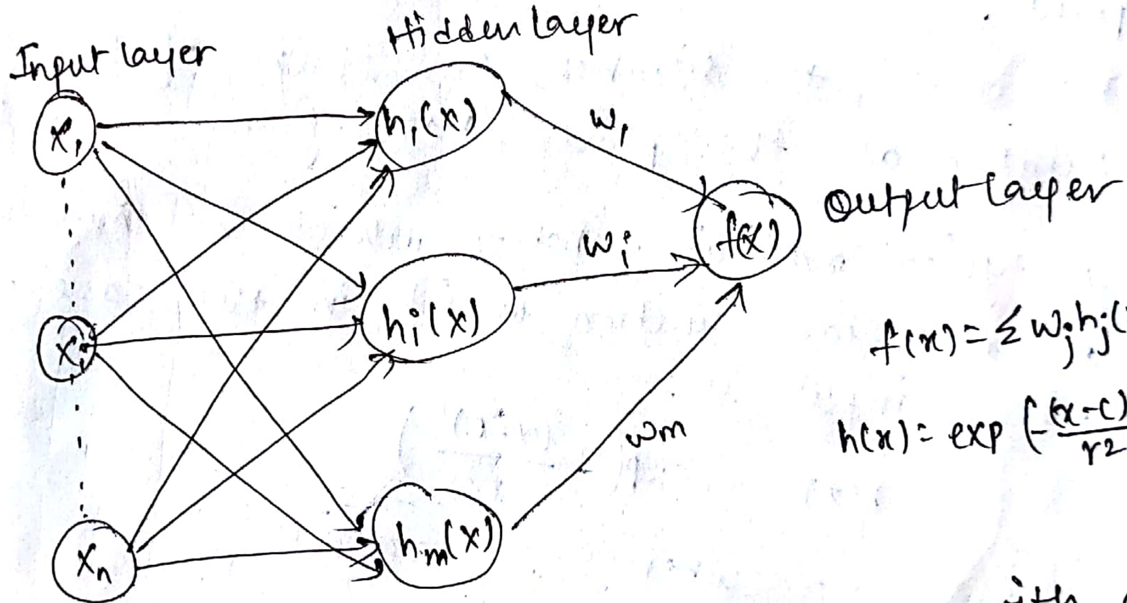
Radial Basis Function Networks :

→ This sort of functions can be employed in any sort of model, which can be either linear or nonlinear.

→ It can also be employed with a single layer or multilayered network.

→ The architecture of RBF has three layers – an Input layer, a hidden layer, and a output layer.

→ Basic Architecture of RBF is shown as-



$$f(x) = \sum w_j h_j(x)$$

$$h(x) = \exp\left(-\frac{(x-c)^2}{r^2}\right)$$

The focus here is on a single layer network with a single hidden layer where nonlinear mappings of higher dimensionality takes place.

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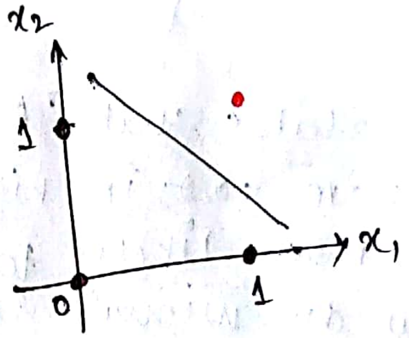
Single layer RBF Network :-

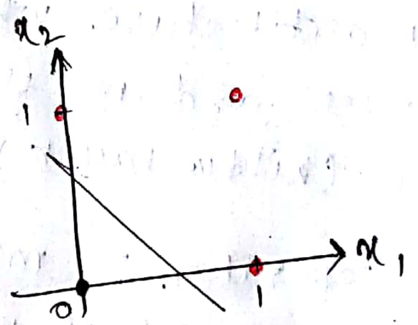
In a single perceptron, we can establish linear seperability, as in the case of AND and OR functions. for AND and OR functions, the outputs are either 1 or 0 and outputs can be linearly seperable. But this is not the case with EXOR function.

→ This can be understood dramatically -

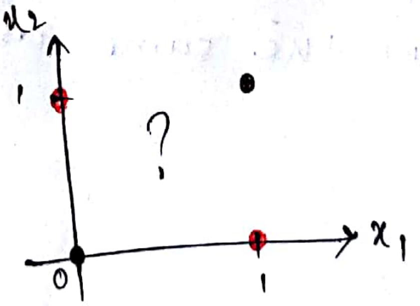
x_1	x_2	x_1 AND x_2
0	0	0 (Black dot)
0	1	0
1	0	0
1	1	1 (Red)



x_1	x_2	x_1 OR x_2
0	0	0 (Black)
0	1	1 Red
1	0	1
1	1	1



x_1	x_2	x_1 EXOR x_2
0	0	0
0	1	1
1	0	1
1	1	0



EXOR gate is not linearly seperable in contrast to AND gate and OR gate.

→ for seperating the input patterns, we need atleast one hidden layer.

→ The RBF network (RBFN) transforms the input single into another form and this is fed into the network to get linear seperability. RBFN is structurally same as perceptron.

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→ RBFN is composed of input, hidden and output layers.

→ If RBFN is having exactly one hidden layer, then this hidden layer is known as feature vector.

→ we apply nonlinear transfer function to the feature vector before we solve the classification problem.

→ when we increase the dimension of the feature vector, the linear seperability of feature vector increases.

Cover's Theorem →

cover's theorem states that "A complex pattern-classification problem cast in high-dimensional space non-linearly is more likely to be linearly seperable than in a low dimensional space."

- consider an RBFN architecture where n is the no. of input features/values and m is the no. of transformed vector dimensions (hidden layer width)

- for nonlinearity seperation: m≥n. Each node in the hidden layer performs a set of nonlinear an RBF.

- The output remains the same as for the classification problems.

Eg: Construct an RBF pattern classifier such that

1) (0,0) and (1,1) are mapped to class 0, class C_1

2) (1,0) and (0,1) ———————— 1, class C_2

Sol^n

① centers are selected at random. $\mu_1 = (0,0)$, $\mu_2 = (1,1)$

② Activation function of hidden neuron is computed as follows:

$$\phi_i(\|x\|) = \exp\left[-\frac{m_1}{d_{max}^2}\|x-\mu_i\|^2\right]$$

where m_1 = no of centers

d_{max} = max. dist between two centers.

$m_1 = 2$

$d_{max} = \sqrt{(0-1)^2 + (0-1)^2} = \sqrt{2} = 1.414$

$\phi_1(x) = \exp(-\|x-\mu_1\|^2)$

$\phi_2(x) = \exp(-\|x-\mu_2\|^2)$

x_1	x_2	$\phi_1(x)$	$\phi_2(x)$
0	0	1.000	0.1353
0	1	0.3679	0.3679
1	0	0.3679	0.3679
1	1	0.1353	1.000

Plotting them,