Bayesian Games:

So far we have assumed that each player knows the exact payoff of other players. However this assumption might not hold true in several scenarios.

For Eg: In an auction, with the several bidders, each bidder might not know the valuation & therefore, the payoffs to the other players.

In several real life scenarios, there is UNCERTAINTY regarding the payoffs of other players, such games are known as Bayesian Games.

Simple Bayesian Game Example :-

Bayesian version of BoS (Battle of Sexes):

BoS is a game in which Player 1 (Boy) and Player 2 (Girl) can either choose B (Bach show) or S (Stravinsky show).

Now we are going to slightly modify the scenario where boy is uncertain about the mood of the girl!
(payoff)

for instance,
the girl can be either interested or not interested in going either to B or S.

So there are two types of girl player P_2, represented by I and U

Girl → Interested (I)
Girl → Not interested (U)
Types

So we characterize it by probability. Lets say it is given that

$$P(I) = 1/2 \quad \text{(Prob. of girl interested)}$$
$$P(U) = 1/2 \quad \text{(Prob. — uninterested)}$$

In this example, we are assuming that there is only one TYPE of player 1 (Boy)

Boy \ Girl	B	S
B	10,5	0,0
S	0,0	5,10

⇓

Each prefers watching B or S with the other.
This is game table for conventional BoS.

Game table for uninterested girl can be modelled as

girl want to avoid boy

Boy \ Girl	B	S
B	10, 0	0, 10
S	0, 5	5, 0

⇐ Game table correspoing to girl of type U.

* Girl prefers to watch B or S alone.
Boy prefers to ——— together.

(B, B) - Boy gets payoff of 10, girl get payoff of 0 because she prefers not to watch B with boy.

(B, S) - Because the girl is doing things differently from the boy, the girl gets a payoff of 10 & boy gets 0.

(S, B) - Boy's payoff is 0 because boy watching S but girl watching S, girl prefers to watch something different (but she is watching B which prefers less), so girl payoff is 5.

(S, S) - Because Boy watching S with girl, so his payoff is 5 (cause Boy prefers Bach). Girl prefers not to watch anything together with boy, so her payoff is zero.

30/ Bayesian BoS Game Table :-

Girl (type I):

Boy \ Girl	B	S
B	10,5	0,0
S	0,0	5,10

girl wish to watch together

$P(I) = \frac{1}{2}$

Girl (type U):

Boy \ Girl	B	S
B	10,0	0,10
S	0,5	5,0

girl wish to avoid (watch alone)

$P(U) = \frac{1}{2}$

* The most important thing to remember in bayesian game is to assign a strategy to each player or each type.

So, lets consider →

Boy choosing → B

Girl of type I choosing → B

——— U → B

This can be represented as.

(B, B)

Strategy or action of girl of type I

Strategy or action of girl of type U

Now we want to compute the payoff of Boy corresponding to (B,B) strategy of Girl.

$U_b\,(B,\,(B,B))$

Payoff — boy — Boy is choosing B — girl of Type I — girl of type U.

Hence, we have to compute average payoff of Boy, average w.r.t. probabilities of type of girl players

$$U_b(B,(B,B)) = P(I) * U_b(B,B) + P(U) * U_b(B,B)$$
$$= \tfrac{1}{2} * 10 + \tfrac{1}{2} * 10 = 10$$

$$U_b(S,(B,B)) = P(I) * U_b(S,B) + P(U) * U_b(S,B)$$
$$= \tfrac{1}{2} * 0 + \tfrac{1}{2} * 0 = 0$$

$$U_b(B,(B,S)) = P(I) * U_b(B,B) + P(U) * U_b(B,S)$$
$$= \tfrac{1}{2} * 10 + \tfrac{1}{2} * 0$$
$$= 5$$

$$U_b(S,(B,S)) = \tfrac{1}{2} * 0 + \tfrac{1}{2} * 5 = 2.5$$
$$U_b(B,(S,B)) = \tfrac{1}{2} * 0 + \tfrac{1}{2} * 10 = 5$$
$$U_b(S,(S,B)) = \tfrac{1}{2} * 5 + \tfrac{1}{2} * 0 = 2.5$$

$u_b(B,(S,S)) = \frac{1}{2} * 0 + \frac{1}{2} * 0 = 0$

$u_b(S,(S,S)) = \frac{1}{2} * 5 + \frac{1}{2} * 5 = 5$

So, Average payoff table for Boy :

Girl

Boy	(B,B)	(B,S)	(S,B)	(S,S)
B	10	5	5	0
S	0	2.5	2.5	5

To figure out the payoff of the girl of each type →

For type(I) → standard table

	B	S
B	10, $\boxed{5}$	0, 0
S	0, 0	5, $\boxed{10}$

For type(U) →

	B	S
B	10, 0	0, $\boxed{10}$
S	0, $\boxed{5}$	5, 0

Here, we have illustrated the best response of the girl of each type by putting □ on it.

* Find the Bayesian Nash Equilibirium (BNE) of Game →

	(B,B)	(B,S)	(S,B)	(S,S)
B	$\boxed{10}$	$\boxed{5}$	$\boxed{5}$	0
S	0	2.5	2.5	$\boxed{5}$

□ Best responses of Boy

* consider (B,B) in Boy table ; Best Response for Boy is B.

let's check B,(B,B) is BNE ?

If Boy choosing B, then Best Response (BR) of Girl (I) is B.
B, Girl (U) is S.

So, [B,(B,B)] Not BNE.

* Now consider (B,S) in Boy Table : ~~Best Response is B.~~

lets check if (B,(B,S)) is BNE ?

If Boy choosing B, BR of girl of type I is B
If B, BR of girl of type U is S.

so (C,H) is the BR of girl of each type for B of Boy.

Therefore Player2 (girl of each type) is playing her Best Response. So, (B,(B,S)) is BNE.

(B,(B,S))

B is BR of Boy to (B,S) of girl

BR of girl of type I to B of boy

BR of girl of type U to B of Boy.

* Now consider (S, B) in Boy table – Best Response is B.

Check (B, (S, B)) is BNE or not?

If Boy choosing B, BR of girl of Type I is – B
Type U is – S

girl of Type I is not playing to Best Response of Boy (B), same with girl of Type U.

So, (B(S, B)) is not BNE.

* Now consider (S, S) is boy table – BR is S.

check (S, (S, S)) is BNE?

If Boy choosing S, BR of girl of type I = S
type U = B

So Here girl of type U is not playing to Best response of S of Boy.

so (S, (S, S)) is not BNE.