Simplex Method

Linear Programming Problem

Any L.P problem has three aspects (i) objective function (ii) constraints (iii) non-negative restrictions.

Consider the LPP in standard form

Maximise $Z = c_1x_1 + c_2x_2 + \dots + c_nx_n + 0s_1 + 0s_2 + \dots + 0s_m$

subject to
$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n + s_1 + 0s_2 + \dots + 0s_m = b_1$$
$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n + 0s_1 + s_2 + \dots + 0s_m = b_2$$
$$\vdots$$
$$a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n + 0s_1 + \dots + s_m = b_m$$

& $x_1, x_2, \dots x_n, s_1, s_2, \dots s_m \geq 0$.

Solution:- Any set of values $x_1, x_2 \dots x_n$ which satisfies the constraints is called a solution of the LPP.

Feasible Soln:-

Any solution which satisfies non-negativity restrictions is called feasible solution of the LPP.

Optimal Solution:-

Any feasible solution which maximises (minimises) the objective function is called optimal solution.

Basic Variables:-

When there are m constraints and $m+n$ variables, we start with setting any n variables equal to zero and solve the remaining m equations. The n variables which are equated to zero are called non-Basic variables. The remaining m variables are called basic variables.

Basic Solution:-

The solution obtained by putting any n variables equal to zero and solving the m

equations is called a basic solution.

Basic feasible Solution:-

If a basic solution satisfies the non-negativity restriction it is called basic feasible solution. If a basic solution contains negative values it is called a basic infeasible solution.

Degenerate and Non-degenerate Solutions:-

If all the m-values obtained in a basic feasible solution are non-zero, the solution is called the non-degenerate basic feasible solution. If some of the values obtained in a basic feasible solution are zero the solution is called degenerate basic feasible solution.

Optimal Solution:- The basic feasible solution which optimises the objective function and also satisfying the constraints and non-negativity restrictions is called the optimal soln.

1) Solve the following LPP by simplex method

Maximise $Z = x_1 + 4x_2$

subject to $2x_1 + x_2 \le 3$

$3x_1 + 5x_2 \le 9$

$x_1 + 3x_2 \le 5$

$x_1, x_2 \ge 0$.

Solution:-

Write the LPP in standard form

Maximise $Z = x_1 + 4x_2 + 0S_1 + 0S_2 + 0S_3$

$Z - x_1 - 4x_2 - 0S_1 - 0S_2 - 0S_3$

sub. to $2x_1 + x_2 + S_1 + 0S_2 + 0S_3 = 3$

$3x_1 + 5x_2 + 0S_1 + S_2 + 0S_3 = 9$

$x_1 + 3x_2 + 0S_1 + 0S_2 + S_3 = 5$

Simplex table

Iteration no.	Basic variables	co-effts of x_1	x_2	S_1	S_2	S_3	RHS Soln	Ratio
0	Z	-1	-4	0	0	0	0	
S_3 leaves	S_1	2	1	1	0	0	3	3/1 = 3
x_2 enters	S_2	3	5	0	1	0	9	9/5 = 1.8
	S_3	1	3^*	0	0	1	5	5/3 = 1.67 ←
1	Z	1/3	0	0	0	4/3	20/3	
	S_1	5/3	0	1	0	-1/3	4/3	
$Z+4x_2$	S_2	4/3	0	0	1	-5/3	2/3	
S_1-x_2, S_2-5x_2	x_2	1/3	1	0	0	1/3	5/3	

Since the co-effts of the objective function (Z) are positive, the optimal solution is obtained

$x_1 = 0$

$x_2 = 5/3$

$Z_{max} = 20/3$.

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2) Solve the following LPP by Simplex method

Maximise $Z = 3x_1 + 2x_2 + 5x_3$

subject to $x_1 + 2x_2 + x_3 \le 430$

$3x_1 + 2x_3 \le 460$

$x_1 + 4x_2 \le 420$

$x_1, x_2, x_3 \ge 0$.

Solution:- Write the LPP in standard form

$Z - 3x_1 - 2x_2 - 5x_3 + 0S_1 + 0S_2 + 0S_3$

sub. to $x_1 + 2x_2 + x_3 + S_1 + 0S_2 + 0S_3 = 430$

$3x_1 + 0x_2 + 2x_3 + 0S_1 + S_2 + 0S_3 = 460$

$x_1 + 4x_2 + 0x_3 + 0S_1 + 0S_2 + S_3 = 420$

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Simplex table

Iteration no.	Basic Variables	co-effts of x_1	x_2	x_3	s_1	s_2	s_3	RHS soln	Ratio
0	z	-3	-2	-5	0	0	0	0	
s_2 leaves	s_1	1	2	1	1	0	0	430	$430/1=430$
x_3 enters	s_2	3	0	2^*	0	1	0	460	$460/2=230 \leftarrow$
	s_3	1	4	0	0	0	1	420	—
1	z	$9/2$	-2	0	0	$5/2$	0	1150	
s_1 leaves	s_1	$-1/2$	2^*	0	1	$-1/2$	0	200	$200/2=100 \leftarrow$
x_2 enters	x_3	$3/2$	0	1	0	$1/2$	0	230	—
	s_3	1	4	0	0	0	0	420	$420/4=105$
2	z	4	0	0	1	0	0	1350	
	x_2	$-1/4$	1	0	$1/2$	$-1/4$	0	100	
	x_3	$3/2$	0	1	0	$1/2$	0	230	
	s_3	2	0	0	-2	1	0	20	

$\therefore x_1 = 0$

$x_2 = 2100$

$x_3 = 230$

$Z_{max} = 1350$.

3) Solve the following LPP by simplex method

Maximise $Z = 107x_1 + x_2 + 2x_3$

subject to $14x_1 + x_2 - 6x_3 + 3x_4 = 7$

$16x_1 + (1/2)x_2 - 6x_3 \le 5$.

$3x_1 - x_2 - x_3 \le 0$

$x_1, x_2, x_3, x_4 \le 0$.

Soln:- Write the LPP in standard form

Maximise $Z - 107x_1 - x_2 - 2x_3 + 0x_4 + 0S_1 + 0S_2$

subject to $\frac{14}{3}x_1 + \frac{1}{3}x_2 - 2x_3 + x_4 + 0S_1 + 0S_2 = 7/3$

$16x_1 + \frac{1}{2}x_2 - 6x_3 + 0x_4 + S_1 + 0S_2 = 5$

$3x_1 - x_2 - x_3 + 0x_4 + 0S_1 + S_2 = 0$

Simplex table

Iteration no	Basic Variables	co-effts of x_1	x_2	x_3	x_4	s_1	s_2	RHS soln	Ratio
0	z	-107	-1	-2	0	0	0	0	
	x_4	$14/3$	$1/3$	-2	1	0	0	$7/3$	$1/2$
	s_1	16	$1/2$	-6	0	1	0	5	$5/16$
	s_2	3	-1	-1	0	0	1	0	0 ←
		↑							
1	z	0	$-110/3$	$-113/3$	0	0	$107/3$	0	
	x_4	0	$17/9$	$-4/9$	1	0	$-14/9$	$7/3$	$-ve$
	s_1	0	$35/6$	$-2/3$	0	1	$-16/3$	5	$-ve$
	x_1	1	$-1/3$	$-1/3$	0	0	$1/3$	0	∞

Here x_3 is the incoming variables. But all the ratios in the ratio column are negative or infinite, no variable can leave and x_3 cannot enter $\therefore$ The solution is unbounded.