

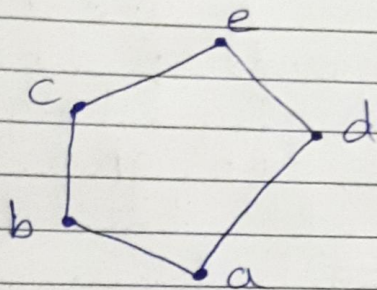
Semester : IIISubject : DSGT

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**\* Chain and Anti-chain -**

Let  $(A, \leq)$  be a partially ordered set. A subset of  $A$  is called a chain if every two elements in the subset are related. The number of elements in a chain is called the length of the chain.

A subset of  $A$  is called an 'anti-chain' if no two distinct elements in the subset are related.

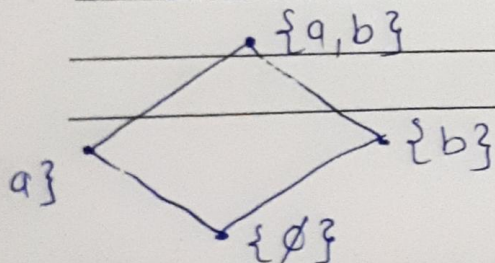
e.g. ①  $A = \{a, b, c, d, e\}$ 

chain =  $\{a, b, c, e\}$ ,  
 $\{a, b, c\}$ ,  $\{a, b\}$   
 $\{a, d, e\}$ ,  $\{a, d\}$ ,  $\{a\}$

anti-chain =  $\{b, d\}$ ,  $\{c, d\}$ .

② Let  $A = \{a, b\}$  and consider its poset  $(P(A), \subseteq)$  find chains & anti-chains

$\Rightarrow P(A) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$



chains =  $\{\emptyset, \{a\}, \{a, b\}\}$ ,  
 $\{\emptyset, \{b\}, \{a, b\}\}$ ,  $\{\emptyset, \{a\}\}$ ,  
 $\{\emptyset, \{b\}\}$ ,  $\{\emptyset, \{a, b\}\}$ ,  $\{\{a\}, \{a, b\}\}$ ,  $\{\{b\}, \{a, b\}\}$

anti-chain =  $\{\{a\}, \{b\}\}$