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Matrices [10 - 12 Marks]

1 Q

A) Eigen values & Eigen vectors → 3 Examples

B) Cayley-Hamilton theorem → 1 Example

C) Diagonal Matrix → 2 Examples

History 100%

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(A) Eigen values & Eigen vectors

~~(1) Calculator~~
(2) Crammer's Rule

Ex. 1. Find Eigen values & Eigen vectors of

$$A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & \boxed{1} \end{bmatrix}_{3\times 3}$$

Diagonal

Sol^y:

a) Eigen values

i) $(A - \lambda I)X = 0$

$$\begin{bmatrix} 8-\lambda & -8 & -2 \\ 4 & -3-\lambda & -2 \\ 3 & -4 & 1-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

ii) $|A - \lambda I| = 0$

$$\begin{vmatrix} 8-\lambda & -8 & -2 \\ 4 & -3-\lambda & -2 \\ 3 & -4 & 1-\lambda \end{vmatrix} = 0$$

$\lambda^3 - (\text{sum of Diagonal elements})\lambda^2 + (\text{sum of minors of D.E})\lambda - |A| = 0$

$\lambda^3 - (6)\lambda^2 + (11)\lambda - 6 = 0$

$\lambda = 1, 3, 2 \leftarrow$ Eigen values

b) Eigen vector (X)

put $\lambda = 1$

$$\begin{bmatrix} 7 & -8 & -2 \\ 4 & -4 & -2 \\ 3 & -4 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$7x_1 - 8x_2 - 2x_3 = 0$

$4x_1 - 4x_2 - 2x_3 = 0$

$$\frac{+x_1}{\begin{vmatrix} -8 & -2 \\ -4 & -2 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 7 & -2 \\ 4 & -2 \end{vmatrix}} = \frac{+x_3}{\begin{vmatrix} 7 & -8 \\ 4 & -4 \end{vmatrix}}$$

$$\frac{x_1}{16-8} = \frac{-x_2}{-14-(-8)} = \frac{x_3}{-28-(-32)}$$

$$\frac{x_1}{8} = \frac{-x_2}{-6} = \frac{x_3}{4}$$

$$\frac{x_1}{4} = \frac{x_2}{3} = \frac{x_3}{2}$$

$$X_1 = \begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix}$$

put $\lambda = 3$

$$\begin{bmatrix} 5 & -8 & -2 \\ 4 & -6 & -2 \\ 3 & -4 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$5x_1 - 8x_2 - 2x_3 = 0$

$4x_1 - 6x_2 - 2x_3 = 0$

$$\frac{+x_1}{\begin{vmatrix} -8 & -2 \\ -6 & -2 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 5 & -2 \\ 4 & -2 \end{vmatrix}} = \frac{+x_3}{\begin{vmatrix} 5 & -8 \\ 4 & -6 \end{vmatrix}}$$

$$\frac{x_1}{16-12} = \frac{-x_2}{-10-(-8)} = \frac{x_3}{-30-(-32)}$$

$$\frac{x_1}{4} = \frac{-x_2}{-2} = \frac{x_3}{2}$$

$$\frac{x_1}{2} = \frac{x_2}{1} = \frac{x_3}{1}$$

$$X_2 = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$$

put $\lambda = 2$

Homework

$$X_3 = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

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(A) Eigen values & Eigen vectors ~~(1) Calculator~~ (2) Crammer's Rule

Find Eigen values and Eigen vectors of

$$A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}_{3\times 3}$$

Sol: (a) Eigen value

i) $(A-\lambda I)X=0$

$$\begin{bmatrix} 2-\lambda & 2 & 1 \\ 1 & 3-\lambda & 1 \\ 1 & 2 & 2-\lambda \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

ii) $|A-\lambda I| = 0$

$$\begin{vmatrix} 2-\lambda & 2 & 1 \\ 1 & 3-\lambda & 1 \\ 1 & 2 & 2-\lambda \end{vmatrix} = 0$$

$\lambda^3 - (\text{sum of DE})\lambda^2 + (\text{sum of minors of DE})\lambda - |A| = 0$

$\lambda^3 - (7)\lambda^2 + (11)\lambda - 5 = 0$

$\lambda = 5, 1, 1$

(b) Eigen vector

$\lambda = 5$

$$\begin{bmatrix} -3 & 2 & 1 \\ 1 & -2 & 1 \\ 1 & 2 & -3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-3x_1 + 2x_2 + 1x_3 = 0$

$1x_1 + 2x_2 - 3x_3 = 0$

$$\frac{+x_1}{\begin{vmatrix} 2 & 1 \\ 2 & -3 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} -3 & 1 \\ 1 & -3 \end{vmatrix}} = \frac{+x_3}{\begin{vmatrix} -3 & 2 \\ 1 & 2 \end{vmatrix}}$$

$$\frac{x_1}{-6-2} = \frac{-x_2}{9-1} = \frac{x_3}{-6-2}$$

$$\frac{x_1}{-8} = \frac{-x_2}{8} = \frac{x_3}{-8}$$

Divide by -8; $\dfrac{x_1}{1} = \dfrac{x_2}{1} = \dfrac{x_3}{1}$

$$X_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

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(A) Eigen values & Eigen vectors

~~(1) Calculator~~
~~(2) Crammer's Rule~~
(3) Assume

find Eigen values and Eigen vectors of

$$A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & [illegible] & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

Sol.

कितना values Assume करेंगे का ?

$= n - r$

$= 3 - 1$

$= 2$

(B) Eigen vector

$\lambda = 1$

$$\begin{array}{c} \\ R_2 \\ R_3 \end{array} \begin{bmatrix} 1 & 2 & 1 \\ 1 & 2 & 1 \\ 1 & 2 & 1 \end{bmatrix}_{3\times 3} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_2 - R_1, \; R_3 - R_1$

$$\begin{array}{c} NZ \\ Z \\ Z \end{array} \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

- r = rank: Number of Non-Zero rows = 1
- n = Number of unknown = 3

$x_1 + 2x_2 + x_3 = 0$

Assume $x_1 = s$, $x_2 = t$

$\therefore s + 2t + x_3 = 0$

$\therefore x_3 = -s - 2t$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} s \\ t \\ -s-2t \end{bmatrix} = \begin{bmatrix} 1s + 0t \\ 0s + t \\ -s - 2t \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} s + \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix} t$$

$$X_2 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \; X_3 = \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}$$

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(A) Eigen values & Eigen vectors

~~(1) Calculator~~
~~(2) Crammer's Rule~~
(3) Assume

Ex.3 The matrix A is given by

$$A = \begin{bmatrix} \boxed{1} & 2 & -3 \\ 0 & 3 & 2 \\ 0 & 0 & -2 \end{bmatrix}$$

Triangular Matrix
Eigen values = D.E.

Convert into λ

$\lambda^2 + 2\lambda + 1 - \frac{6}{\lambda}$

find Eigen values of $B = A^2 + 2A + I - 6A^{-1}$

Sol:

i) $(A - \lambda I)X = 0$

$$\begin{bmatrix} 1-\lambda & 2 & -3 \\ 0 & 3-\lambda & 2 \\ 0 & 0 & -2-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

ii) $|A - \lambda I| = 0$

$$\begin{vmatrix} 1-\lambda & 2 & -3 \\ 0 & 3-\lambda & 2 \\ 0 & 0 & -2-\lambda \end{vmatrix} = 0$$

$\lambda^3 - (\text{sum of D.E})\lambda^2 + (\text{sum of minors of D.E})\lambda - |A| = 0$

$\lambda^3 - (2)\lambda^2 + (-5)\lambda + 6 = 0$

$\lambda = 1, 3, -2$

Eigen values of $B = A^2 + 2A + I - 6A^{-1}$

$= -2, 14, 4$

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(A) Eigen values & Eigen vectors

Ex. 4 Find Eigen values of adj A.

Where $A = \begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix}$

Sol:

i) $(A - \lambda I)X = 0$

$$\begin{bmatrix} 2-\lambda & 3 \\ 0 & 1-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

ii) $|A - \lambda I| = 0$

$$\begin{vmatrix} 2-\lambda & 3 \\ 0 & 1-\lambda \end{vmatrix}$$

$\lambda^2 - (\text{sum of D.E})\lambda + |A| = 0$

$\lambda^2 - 3\lambda + 2 = 0$

$\lambda = 1, 2$

E.V. $[(\text{adj } A)] = \frac{|A|}{\lambda} = \frac{2}{\lambda}$

Eigen of adj A $= \frac{2}{1}, \frac{2}{2}$

$= 2, 1$

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Diagonal Matrix

100%

$$D = M^{-1} A M$$

M ↓ Modal matrix / Transforming M / Diagonalising M

$$M = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}$$

$$M^{-1} = \begin{bmatrix} \text{Calculator} \end{bmatrix}$$

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Diagonal Matrix

100%

*** Important ***

1. Eigen values are different : Matrix is diagonalisable
2. Eigen values are Repeated :

AM = GM → Matrix is diagonalisable

AM ≠ GM → Matrix is non-diagonalisable

A.M = Algebraic multiplicity
= λ कितनी बार आया है

G.M : Geometric multiplicity
= $n - r$

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Ex 1. Show that matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ is diagonalisable. Also find M and D.

Sol

a) Eigen values

$\lambda = 0, 3, 15$

Since, E.V. are different, matrix is Diagonalisable.

b) Eigen vector

$\lambda = 0$, $\lambda = 3$, $\lambda = 15$

$X_1 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$, $X_2 = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$, $X_3 = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$

c) Diagonal Matrix

$D = M^{-1} A M$

$M = [X_1 \ X_2 \ X_3]$

$M = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$

$M^{-1} = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$

$D = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 15 \end{bmatrix}$

Ex 2. Show that matrix $A = \begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$ is diagonalisable. Also find M and D.

IMP

Sol: a) Eigen value.

i) $(A - \lambda I)X = 0$

$$\begin{bmatrix} -9-\lambda & 4 & 4 \\ -8 & 3-\lambda & 4 \\ -16 & 8 & 7-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

ii) $|A - \lambda I| = 0$

$$\begin{vmatrix} -9-\lambda & 4 & 4 \\ -8 & 3-\lambda & 4 \\ -16 & 8 & 7-\lambda \end{vmatrix} = 0$$

$\lambda^3 - (\text{sum of DE})\lambda^2 + (\text{sum of minors of DE})\lambda - |A| = 0$

$\lambda^3 - (1)\lambda^2 + (-5)\lambda - 3 = 0$

$\lambda = 3, -1, -1$

b) Eigen vector

$\lambda = 3$

$$\begin{bmatrix} -12 & 4 & 4 \\ -8 & 0 & 4 \\ -16 & 8 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\left.\frac{R_1}{-4}, \frac{R_2}{-4}, \frac{R_3}{-4}\right\}$ *

$$\begin{bmatrix} 3 & -1 & -1 \\ 2 & 0 & -1 \\ 4 & -2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$3x_1 - x_2 - x_3 = 0$

$2x_1 + 0x_2 - x_3 = 0$

$$\frac{+x_1}{\begin{vmatrix} -1 & -1 \\ 0 & -1 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 3 & -1 \\ 2 & -1 \end{vmatrix}} = \frac{+x_3}{\begin{vmatrix} 3 & -1 \\ 2 & 0 \end{vmatrix}}$$

$$\frac{x_1}{1-0} = \frac{-x_2}{-3-(-2)} = \frac{x_3}{0-(-2)}$$

$$\frac{x_1}{1} = \frac{x_2}{1} = \frac{x_3}{2}$$

$$X_1 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

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Ex2. Show that matrix $A=\begin{bmatrix}-9 & 4 & 4\\ -8 & 3 & 4\\ -16 & 8 & 7\end{bmatrix}$ is diagonalisable. Also find M and D.

IMP

Sol^n: (b) Eigen vector

$\boxed{\lambda=3,-1,-1}$

$\lambda=-1$ [AM & GM]

$$\begin{bmatrix}-8 & 4 & 4\\ -8 & 4 & 4\\ -16 & 8 & 8\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$\frac{R_1}{-4}, \frac{R_2}{-4}, \frac{R_3}{-8}$

$$\begin{array}{c} R_1\\ R_2\\ R_3\end{array}\begin{bmatrix}2 & -1 & -1\\ 2 & -1 & -1\\ 2 & -1 & -1\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

R_2-R_1, R_3-R_1

$$\begin{bmatrix}2 & -1 & -1\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$r=1, n=3$

AM = 2

GM = n − r = 3 − 1 = 2

AM = GM

∴ Matrix is diagonalisable.

$2x_1-x_2-x_3=0$

Assume, $x_1=s$, $x_2=t$

$\therefore 2s-t-x_3=0$

$\boxed{\therefore 2s-t=x_3}$

$$\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}s\\t\\2s-t\end{bmatrix}$$

$$=\begin{bmatrix}s+0t\\0s+t\\2s-t\end{bmatrix}$$

$$=\begin{bmatrix}1\\0\\2\end{bmatrix}s+\begin{bmatrix}0\\1\\-1\end{bmatrix}t$$

$X_2=\begin{bmatrix}1\\0\\2\end{bmatrix}, X_3=\begin{bmatrix}0\\1\\-1\end{bmatrix}$

Ex2. Show that matrix $A=\begin{bmatrix}-9 & 4 & 4\\ -8 & 3 & 4\\ -16 & 8 & 7\end{bmatrix}$ is diagonalisable. Also find M and D.

IMP

Sol: C P

$\lambda = 3, -1, -1$

$$D = M^{-1} A M$$

$$M = [x_1\ x_2\ x_3] = \begin{bmatrix}1 & 1 & 0\\ 1 & 0 & 1\\ 2 & 2 & -1\end{bmatrix}$$

$$M^{-1} = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$$

$$D = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}\begin{bmatrix}-9 & 4 & 4\\ -8 & 3 & 4\\ -16 & 8 & 7\end{bmatrix}\begin{bmatrix}1 & 1 & 0\\ 1 & 0 & 1\\ 2 & 2 & -1\end{bmatrix}$$

$$= \begin{bmatrix}3 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & -1\end{bmatrix}$$

Type-III Cayley-Hamilton theorem [E.V. & E.Vectors are not Required]

Verify C-H-T for $A=\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$. Also find A^{-1}, A^4

and matrix $A^8-5A^7+7A^6-3A^5+A^4-5A^3+8A^2-2A+I$

Matrix A Constant ke sath I Join करना

Sol

(I) $(A-\lambda I)X=0$

(II) $|A-\lambda I|=0$

$$\begin{vmatrix} 2-\lambda & 1 & 1 \\ 0 & 1-\lambda & 0 \\ 1 & 1 & 2-\lambda \end{vmatrix}=0$$

$\lambda^3-(\text{sum of D.E})\lambda^2+(\text{sum of minors of DE})\lambda-|A|=0$

$\lambda^3-5\lambda^2+7\lambda-3=0$

Characteristic Eqⁿ

a) C-H-T

Consider,

$A^3-5A^2+7A-3I$ | $14-25+14-3=0$

$$=\begin{bmatrix} 14 & 13 & 13 \\ 0 & 1 & 0 \\ 13 & 13 & 14 \end{bmatrix}-5\begin{bmatrix} 5 & 4 & 4 \\ 0 & 4 & 0 \\ 4 & 4 & 5 \end{bmatrix}+7\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}-3\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$=\begin{bmatrix} 14 & 13 & 13 \\ 0 & 1 & 0 \\ 13 & 13 & 14 \end{bmatrix}-\begin{bmatrix} 25 & 20 & 20 \\ 0 & 20 & 0 \\ 20 & 20 & 25 \end{bmatrix}+\begin{bmatrix} 14 & 7 & 7 \\ 0 & 7 & 0 \\ 7 & 7 & 14 \end{bmatrix}-\begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$=\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$A^3-5A^2+7A-3I=0$

C-H-T

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Type-III Cayley-Hamilton theorem. [E.V. & E.Vectors are not Required]

Verify C-H-T for $A=\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$. Also find A^{-1}, A^4

$A \times I = A$

$A A^{-1} = I$

and matrix $A^8-5A^7+\ A^6-3A^5+A^4-5A^3+8A^2-2A+I$

Sol^n ⓑ A^{-1}

By C-H-T

$A^3-5A^2+7A-3I=0$

Multiply by A^{-1}

$A^3A^{-1}-5A^2A^{-1}+7\underline{AA^{-1}}-3\underline{IA^{-1}}=0$

$A^2\cdot\underline{AA^{-1}}-5A\cdot\underline{AA^{-1}}+7I-3A^{-1}=0$

$A^2I-5AI+7I=3A^{-1}$

$A^2-5A+7I=3A^{-1}$

$A^{-1}=\frac{1}{3}\left[A^2-5A+7I\right]=\frac{1}{3}\left\{\begin{bmatrix} \ \end{bmatrix}-5\begin{bmatrix} \ \end{bmatrix}+7\begin{bmatrix} \ \end{bmatrix}\right\}$

Acting

$=\begin{bmatrix} \ \end{bmatrix}$

ⓒ A^4

By C-H-T

$A^3-5A^2+7A-3I=0$

Multiply by A

$A^4-5A^3+7A^2-3A=0$

$A^4=5A^3-7A^2+3A$

$=5\begin{bmatrix} \ \end{bmatrix}-7\begin{bmatrix} \ \end{bmatrix}+3\begin{bmatrix} \ \end{bmatrix}$

$=\begin{bmatrix} \ \end{bmatrix}-\begin{bmatrix} \ \end{bmatrix}+\begin{bmatrix} \ \end{bmatrix}$

$A^4=\begin{bmatrix} 41 & 40 & 40 \\ 0 & 1 & 0 \\ 40 & 40 & 41 \end{bmatrix}$

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Type-III **Cayley-Hamilton theorem.** [E.V. & E.Vectors are not Required]

Verify C-H-T for $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$. Also find A^{-1}, A^4

$A \times I = A$
$A \cdot A^{-1} = I$

and matrix $A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$

$A^3 - 5A^2 + 7A - 3I = 0$

Sol (D) $A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$

$= A^5[A^3 - 5A^2 + 7A - 3I] + A[A^3 - 5A^2 + 8A - 2I] + I$

$= A^5(0) + A[A^3 - 5A^2 + 7A + A - 3I + I] + I$

$= 0 + A[(A^3 - 5A^2 + 7A - 3I) + A + I] + I$

$= A[0 + A + I] + I$

$= A^2 + A + I = \begin{bmatrix} 5 & 4 & 4 \\ 0 & 1 & 0 \\ 4 & 4 & 5 \end{bmatrix} + \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 8 & 5 & 5 \\ 0 & 3 & 0 \\ 5 & 5 & 8 \end{bmatrix}$