Semester: VIII Subject: AIFB Academic Year: 2024-25

VARIANCE AND STANDARD DEVIATION:

VARIANCE:

Variance is a fundamental concept used to measure the risk or volatility of an asset or portfolio. The larger the variance the, the greater the risk. A smaller variance indicates that returns are more stable and predictable.

Formula for variance:

Variance is calculated by taking the average of the squared difference from the mean return. The formula for variance σ^2 for a sample is:

$$\boxed{\sigma^2 = \frac{1}{N-1} \sum_{i=1}^{N} (x_i - \bar{x})^2.}$$

where,

σ^2 = Variance

N = No. of data points (eg: montly returns).

X_i = Each individual data point (eg. individual return).

$\bar{X}$ = Mean (average) of the data points (eg. average return).

STANDARD DEVIATION:

In finance, the standard deviation is a statistical measure that represents the amount of variation or dispersion of a set of financial data, such as asset returns. or portfolio.

Formula for standard Deviation:

$$\sigma = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(X_i-\bar{X})^2}.$$

Variance in relation to Standard Deviation:

* Variance is the square of the standard deviation. Since variance is in squared units, it may be difficult to directly interpret in terms of the actual returns.
* To make variance more interpretable, we often take the square root of variance to get the standard deviation, which brings the measure back to the same units as the returns.

$$\sigma = \sqrt{\sigma^2}$$

Example:

Suppose we are analyzing the monthly returns of a stock over the past 5 months. The returns are as follows: 5%, -3%, 7%, 2%, -4%.

Calculate the standard deviation of these monthly returns to understand how volatile the stock is.

Solution:

Step 1: Calculate the mean (Average) Return.

$$\text{Mean return} = (\bar{X}) = \frac{5\% + (-3)\% + 7\% + 2\% + (-4\%)}{5}$$

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$$\bar{X} = \frac{5-3+7+2-4}{5} = \frac{7}{5} = \boxed{1.4\%}$$

Step 2: Calculate the squared Deviation from the Mean:

We calculate how far each return is from the mean (the deviation), and then square those deviations.

For Month 1: $(5\% - 1.4\%)^2 = (3.6\%)^2 = 0.1296\%$.

For Month 2: $(-3\% - 1.4\%)^2 = (-4.4\%)^2 = 0.1936\%$.

For Month 3: $(7\% - 1.4\%)^2 = (5.6)\%^2 = 0.3136\%$

For Month 4: $(2\% - 1.4\%)^2 = (0.6\%)^2 = 0.0036\%$

For Month 5: $(-4\% - 1.4\%)^2 = (-5.4\%)^2 = 0.2916\%$

Step 3: Calculate the Variance:

$$\text{Variance } (\sigma^2) = \frac{0.1296\% + 0.1936\% + 0.3136\% + 0.0036\% + 0.2916\%}{4}$$

$$\boxed{\sigma^2 = \frac{0.932\%}{4} = 0.2330\%}$$

Step 4: Calculate the standard Deviation:

$$\sigma = \sqrt{0.2330\%} \approx 0.483\%$$

Conclusion:

The standard deviation of the stocks' monthly return is approximately 0.483%. This means that, on average the stocks' monthly return deviates from the mean by about 0.483%. In this case, the stock has a relatively moderate volatality based on the 5-Month data.

Applications of Variance and standard Deviation:

(1) Volatility indicator:

The standard deviation helps in measuring the volatality of an asset's price or the returns of a portfolio. A higher standard deviation indicates higher volatility, meaning the asset or portfolio has a wider range of possible returns and is therefore riskier. A lower standard deviation indicates more stable returns.

(2) Risk Measurement:

In finance, risk is often associated with the uncertainity of returns, and standard deviation is a direct way to quantify this uncertainty. For instanse, if an asset's annual returns have a high standard deviation, there is a larger expected fluctuation in its returns from year to year.

(3) Comparisons:

Investors often compare the standard deviation of a stock or portfolio to its expected return to assess the risk-adjusted return.

(4) Normal Distribution Asumption:

In many cases, asset returns are assumed to follow a normal distribution, and the standard deviation serves to quantify how much actual returns are expected to deviate from the mean.