saturday.

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② $\int_C \frac{z^2+4}{(z-2)(z+3i)} dz$  i> ~~dz~~ $|z+1|=2$

ii> $|z-2|=2$.

→

$f(z) = \frac{z^2+4}{(z-2)(z+3i)}$

$z = 2$ & $-3i$ are singularities.

$z = 2$ pole of order 1

$z = -3i$ pole of order 1.

i> - given C is $|z+1|=2$

$z = 2$   $|2+1| = 3 > 2$ → outside C

$z = -3i$   $|-3i+1| = \sqrt{10} > 2$ → outside C

By ~~z~~ $z=2$ & $z=-3i$ both are outside C

By Cauchy's Theorem,

$$\oint_C \frac{z^2+4}{(z-2)(z+3i)} dz = 0$$

ii> give C is $|z-2|=2$

$z = 2$   $|2-2| = 0 < 2$ → inside C

$z = -3i$   $|-3i-2| = \sqrt{13} > 2$ → outside. C

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Residue at $z = 2$.

$$= \lim_{z \to 2} (z-2) \frac{(z^2+4)}{(z-2)(z+3i)}$$

$$= \lim_{z \to 2} \frac{(z^2+4)}{(z+3i)}$$

$$= \frac{4+4}{2+3i}$$

$$= \frac{8}{2+3i}$$

By Cauchy's Residue Theorem,

$$\int_C \frac{z^2+4}{(z-2)(z+3i)} dz = 2\pi i \left( \frac{8}{2+3i} \right)$$

$$= 2\pi i \left( \frac{8}{2+3i} \right)$$

$$\int_C \frac{z^2+4}{(z-2)(z+3i)} dz = \frac{16\pi i}{2+3i}$$ //

(3) $\int_C \tan z \, dz$ $\quad |z| = 2$ f
$|z| = 1$

→

$$\int_C \frac{\sin z}{\cos z} dz$$

$$f(z) = \frac{\sin z}{\cos z}$$

~~$z = \frac{\pi}{2}$ is singular point~~
we'll get singularities if.
$\cos z = 0$
i.e., $z = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \ldots$

i) $|z| = 2$

$z = \pm\frac{\pi}{2}$ $\quad \left|\pm\frac{\pi}{2}\right| = \frac{\pi}{2} < 2 \rightarrow$ inside c

$z = \pm\frac{3\pi}{2}$ $\quad \left|\pm\frac{3\pi}{2}\right| = \frac{3\pi}{2} > 2 \rightarrow$ outside c

$z = \frac{\pi}{2}$

Residue at $z = \frac{\pi}{2}$..

$$= \lim_{z \to \frac{\pi}{2}} \left(z - \frac{\pi}{2}\right) \cdot \frac{\sin z}{\cos z}$$

$$= \lim_{z \to \frac{\pi}{2}} \left(z - \frac{\pi}{2}\right) \cdot \frac{\sin\left(\frac{\pi}{2}\right)}{\cos z.}$$

$$= \lim_{z \to \frac{\pi}{2}} \frac{\left(z - \frac{\pi}{2}\right)}{\cos z}$$

By L'Hospital's Rule

$$= \lim_{z \to \frac{\pi}{2}} \frac{1}{(-\sin z)}$$

$$= \frac{1}{-\sin\frac{\pi}{2}}$$

$$= -1$$

Residue at $z = -\frac{\pi}{2}$.

$$= \lim_{z \to -\frac{\pi}{2}} \left(z - \frac{\pi}{2}\right) \frac{\sin z}{\cos z}$$

$$= \lim_{z \to \frac{\pi}{2}} \frac{\left(z - \frac{\pi}{2}\right)\left(- \sin\frac{\pi}{2}\right)}{\cos z}$$

$$= (-1) \lim_{z \to \frac{\pi}{2}} \frac{\left(z - \frac{\pi}{2}\right)}{\cos z}$$

$$= (-1) \lim_{z \to \frac{\pi}{-2}} \frac{(1)}{(-\sin z)}$$

$$= (-1) \frac{1}{-\sin\left(-\frac{\pi}{2}\right)} = \frac{-1}{+\sin\left(\frac{\pi}{2}\right)} = -1$$

$$\cancel{\int_C \frac{\sin z}{\cos z}\, dz = 2\pi i \text{ sum of residue}}$$
$$\cancel{= 2\pi i}$$

~~given C: |z|=1~~ By Cauchy's Residue thm,

$$\int_C \frac{\sin z}{\cos z}\, dz = 2\pi i \text{ sum of residues}$$
$$= 2\pi i\, [-1-1]$$
$$= -4\pi i$$

ii) given C is $|z| = 1$

$\cancel{z = \pm\frac{\pi}{2}}$ $\quad \cancel{\left|\frac{\pi}{2}\right| =}$

$z = \pm\frac{\pi}{2}$ $\quad \left|\pm\frac{\pi}{2}\right| = \frac{\pi}{2} > 1$

$z = \pm\frac{3\pi}{2}$ $\quad \left|\pm\frac{3\pi}{2}\right| = \frac{3\pi}{2} > 1$

$z = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \ldots$ are all outside curve C.

$$\therefore \int_C \frac{\sin z}{\cos z}\, dz = 0,$$

④ $\int_C \frac{(\sin\pi z^2 + \cos\pi z^2)}{z^2+3z+2}\,dz$ i) $|z| = \frac{1}{2}$

ii) $|z| < 2$ $(|z| = 1.5)$

→ $f(z) = \frac{\sin\pi z^2 + \cos\pi z^2}{z^2+3z+2}$

$f(z) = \frac{\sin\pi z^2 + \cos\pi z^2}{(z+2)(z+1)}$

$z = -1, -2$ are singular points.

Both are poles of order 1.

Given C is $|z| < 2$

$z = -1$ $|-1| = 1 > \frac{1}{2}$ → outside C

$z = -2$ $|-2| = 2 > \frac{1}{2}$ → outside C

∴ $z = -1$ & $z = -2$ both are outside

∴ By cauchy's residue theorem,

$\int_C \frac{(\sin\pi z^2 + \cos\pi z^2)}{(z+2)(z+1)}\,dz = 2\pi i$ (sum of residue)

$= 2\pi i(0)$

$= 0$

ii) Given C is $(|z| < 2)$ $(|z| = 1.5)$

$z = -1$ $|-1| = 1 < 2$ → inside

$z = -2$ $|-2| = 2 < 2$

$z = -2$ is on the curve.

∴ $z = -2$ outside curve.

Residue at $z = -1$

$$= \lim_{z \to -1} (z+1) \cdot \frac{\sin \pi z^2 + \cos \pi z^2}{(z+2)(z+1)}$$

$$= \lim_{z \to -1} \frac{\sin \pi z^2 + \cos \pi z^2}{(z+2)}$$

$$= \frac{\sin \pi (-1)^2 + \cos \pi (-1)^2}{(-1+2)}$$

$$= \frac{\sin \pi + \cos \pi}{(1)}$$

$$= 0 - 1$$

$$= -1$$

$$\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z+2)(z+1)} dz = 2\pi i \text{ (sum of residue)}$$

$$= 2\pi i \, (-1)$$

$$= -2\pi i$$

~~|z| = 1.5~~

~~z = -1~~ ~~|-1+.5|~~ ~~|-1| = 1 < 1.5 → inside~~

~~z =~~

⑥ $\int_C \frac{z-1}{(z+1)^2(z-2)} dz \quad |z-i|=2$

⑤ $\int_C \frac{15z+9}{(z^3-9z)} dz \quad |z-1|=3$

→

$f(z) = \frac{15z+9}{z(z^2-9)} = \frac{15z+9}{z(z+3)(z-3)}$

$z(z^2-9) = 0$

$z = 0 \quad , \quad z^2 = 9$

$z = \pm 3$

$z = 0, \pm 3$ are singularities
are all poles of order 1.
given C is $|z-1| = 3$

$z = 0 \quad |0-1| = 1 < 3 \rightarrow$ inside

$z = +3 \quad |3-1| = 2 < 3 \rightarrow$ inside

$z = -3 \quad |-3-1| = |-4| = 4 > 3 \rightarrow$ outside

Residue at $z = 0$

$= \lim_{z \to 0} (\cancel{z}) \frac{15z+9}{\cancel{z}(z+3)(z-3)}$

$= \lim_{z \to 0} \frac{15z+9}{(z+3)(z-3)}$

$= \frac{0+9}{(3)(-3)} = \frac{9}{-9} = -1$

Residue at $z = 3$

$$= \lim_{z \to 3} (z-3) \frac{15z+9}{z(z+3)(z-3)}$$

$$= \lim_{z \to 3} \frac{15z+9}{z(z+3)}$$

$$= \frac{15(3)+9}{3(3+3)}$$

$$= \frac{45+9}{18}$$

$$= \cancel{\frac{58}{18}} \quad \frac{54}{18}$$

$$= \cancel{3} \quad 3$$

$$\int_C \frac{15z+9}{z(z+3)(z-3)} dz = 2\pi i \text{ (sum of residues)}$$
$$= 2\pi i(-1+3)$$
$$= 2\pi i(2)$$
$$= 4\pi i$$

6 $\displaystyle\int_C \frac{z-1}{(z+1)^2(z-2)} dz \qquad |z-i| = 2$

→ $f(z) = \dfrac{z-1}{(z+1)^2(z-2)}$

$z = -1$ & $z = 2$ are singularities

~~(z+1)~~ $z = -1$ is pole of order 2.

$z = 2$ is pole of order 1.

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~~Residue at z = 1~~

$$= \lim_{z\to 1} \cancel{1/z} \; \frac{z-1}{}$$

$$= \lim_{\cancel{z\to} 1} \cancel{(z+1)^2}$$

$$= \lim_{z\to -1} \cancel{\frac{1}{2!}} \; d$$

$|z - i| = 2$

$z = -1 \qquad |-1 - i| = \sqrt{2} = 1.41 < 2 \to$ inside

$z = 2 \qquad |2 - i| = \sqrt{5} > 2 \to$ outside

$\sqrt{2^2+1^2}$
$\sqrt{4+1}$
$\sqrt{5}$

$\sqrt{1^2+1^2}$
$\sqrt{1+1}$
$\sqrt{2}$

$\sqrt{2^2+1^2}$
$\sqrt{4+1}$
$\sqrt{5}$

Residue at $z = -1$

$$= \lim_{z\to -1} \frac{1}{1!} \frac{d}{dz}\left[ (z+1)^2 \frac{(z-1)}{(z+1)^2(z-2)} \right]$$

$$= \lim_{z\to -1} \frac{d}{dz}\left[ \frac{(z-1)}{(z-2)} \right]$$

$$= \lim_{z\to -1} \frac{(z-2)(1) - (z-1)(1)}{(z-2)^2}$$

$$= \frac{(-1-2)(1) - (-1-1)}{(-1-2)^2}$$

$$= \frac{(-3) - (-2)}{(-3)^2} = \frac{-3+2}{9} = \frac{-1}{9}$$

$$\int_C \frac{z-1}{(z+1)^2(z-2)}\,dz = 2\pi i\left(-\frac{1}{9}\right)$$

$$= -\frac{2}{9}\pi i$$