Module 2: System of Linear equations

*A **system of linear equations** (or **linear system**) is a collection of two or more linear equations involving the same set of variables.*

Representation of linear equations in matrix and vector forms:

Lets take a system of three linear equations in three variables x, y, and z

$$
\begin{aligned}
a_1x + b_1y + c_1z &= d_1 \\
a_2x + b_2y + c_2z &= d_2 \\
a_3x + b_3y + c_3z &= d_3
\end{aligned}
$$

Matrix Representation

The equations can be written as matrix multiplication form -

$$
\overbrace{\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}}^{A}
\overbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}^{X}
=
\overbrace{\begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}}^{D}
$$

or,

$$AX = D$$

Matrix A is called the **coefficient matrix.** If we append the columns of matrix D with matrix A like below the resultant matrix is called **augmented matrix**, denoted by [A|D]

$$
[A|D] = \begin{bmatrix} a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \end{bmatrix}
$$

Vector Representation

The above linear equations can also be represented in vector form as :

$$x\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} + y\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} + z\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

or,

$$ax + by + cz = D$$

where $a = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$, $b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$, $c = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}$, $D = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$ are the colum vectors

Now look carefully, we can write from equation 4 and 5 -

$$AX = ax + by + cz = D$$

or,

$$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = x\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} + y\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} + z\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}$$

The above relation gives us a another way to think about matrix multiplication

Solution set of linear equations :

Every linear system may have only one of three possible number of solutions:

1. ***The system has a single unique solution.***
2. ***The system has infinitely many solutions.***
3. ***The system has no solution.***

Geometrical Representation :

For a system of two variables (x and y), each linear equation determines a line on the xy-plane. The solution set is the intersection of these lines, and is hence either a line, a single point or don't have any common point.