Repeated Games

- **Definition:** Repeated games are a type of game in which the same game is played multiple times by the same players. Each repetition of the game is called a stage game.
- **Importance:** In the context of imperfect information, repeated games allow players to observe past actions and adjust their strategies accordingly, which can lead to different outcomes compared to a one-shot game.

Types of Repeated Games

1. **Finite Repeated Games:**
    - The game is played a specific number of times.
    - Players know when the game will end.
    - Often analysed using **backward induction**.
2. **Infinitely Repeated Games:**
    - The game has no predetermined endpoint.
    - Players don't know when the game will end, if at all.
    - Long-term strategies become more relevant, and cooperation can emerge as a stable outcome.

Imperfect Information in Repeated Games

- **Imperfect Information:** In the context of repeated games, this means that players do not have complete information about the entire history of the game or the payoffs, strategies, or actions of other players in every stage.
- **Role of Beliefs:** Players form beliefs about the actions of other players based on observed history and update these beliefs as the game progresses.

Strategies in Repeated Games

- **History-Dependent Strategies:** Strategies that depend on the entire history of the game, not just the current stage. These strategies allow players to punish or reward others based on past behavior.
- **Grim Trigger Strategy:** A common strategy where a player cooperates until the other player defects, after which they defect in all future stages.
- **Tit-for-Tat:** A strategy where a player starts by cooperating and then mimics the other player's previous action in subsequent stages.

Equilibrium Concepts

- **Subgame Perfect Nash Equilibrium (SPNE):** An equilibrium where the players' strategies form a Nash equilibrium in every subgame. In repeated games, SPNE takes into account the future consequences of current actions.
- **Folk Theorem:** A fundamental result in repeated games stating that a wide range of outcomes can be sustained as equilibrium outcomes, depending on how patient the players are (i.e., how much they value future payoffs).

Illustration: The Repeated Prisoner's Dilemma

- **One-Shot Game:** In a one-shot version of the Prisoner's Dilemma, both players have a dominant strategy to defect, leading to a suboptimal outcome for both.
- **Repeated Game:** When the Prisoner's Dilemma is repeated, cooperation can emerge as a rational strategy due to the threat of future punishment (e.g., using Grim Trigger or Tit-for-Tat strategies).
- **Imperfect Information:** If players have imperfect information about the history of the game, they may have to rely on beliefs and expectations, which can lead to different outcomes compared to perfect information scenarios.

Applications of Repeated Games

- **Collusion in Oligopolies:** Firms may repeatedly interact in a market and sustain collusion (e.g., maintaining high prices) by threatening to revert to competitive behavior if any firm undercuts the others.
- **Bargaining and Negotiation:** Repeated interactions in bargaining situations can lead to more cooperative outcomes, as parties seek to build trust over time.

Case Study: Repeated Games in Real-World Scenarios

- **Example 1: Price Wars in Markets** - Firms engage in price competition repeatedly. The threat of future price wars can deter firms from cutting prices today, leading to higher prices overall.
- **Example 2: International Trade Negotiations** - Countries engage in repeated negotiations, where the threat of future tariffs or trade barriers can enforce cooperation in current negotiations.

Example 1: Repeated Prisoner's Dilemma

Scenario: Two individuals, Alice and Bob, are arrested for a crime. They are interrogated separately and have two choices:

- **Cooperate (C):** Remain silent.
- **Defect (D):** Confess and testify against the other.

One-Shot Game Payoff Matrix:

	Bob Cooperates (C)	Bob Defects (D)
Alice Cooperates (C)	Alice: -1 year Bob: -1 year	Alice: -5 years Bob: 0 years
Alice Defects (D)	Alice: 0 years Bob: -5 years	Alice: -3 years Bob: -3 years

- **Dominant Strategy:** In a one-shot game, both Alice and Bob will choose to defect, leading to both serving 3 years.

Infinitely Repeated Game: Now, suppose this game is repeated infinitely, and both players care about future payoffs. They discount future payoffs using a **discount factor** δ\deltaδ where 0<δ<10 < \delta < 10<δ<1.

Strategies:

1. **Grim Trigger Strategy:**
    - **Definition:** Players cooperate until one defects; thereafter, they defect forever.
    - **Outcome Analysis:**
        - If both players follow Grim Trigger, they will continue to cooperate indefinitely, each receiving -1 year per round.
        - If a player defects once, they get an immediate gain but suffer worse payoffs in all future rounds.
    - **Condition for Cooperation to be Sustained:**
        - Cooperation is preferable if the present value of cooperating exceeds the temptation to defect.
        - **Inequality:** −1/(1−δ)≥0−1+(−3)δ/(1−δ)-1 / (1 - \delta) \geq 0 -1 + (-3) \delta / (1 - \delta)−1/(1−δ)≥0−1+(−3)δ/(1−δ)
            - Solve for δ\deltaδ to find the minimum discount factor needed to sustain cooperation.
2. **Tit-for-Tat Strategy:**
    - **Definition:** Start by cooperating; then, in each subsequent round, do what the other player did in the previous round.
    - **Outcome Analysis:**
        - Encourages cooperation while forgiving occasional defections.
        - Effective against various strategies and promotes mutual cooperation.
    - **Robustness:** Tit-for-Tat is known for its simplicity and effectiveness, especially when occasional mistakes (noise) occur.

Impact of Imperfect Information:

- **Observability Issues:** If actions are not perfectly observed, misunderstandings can occur.
    - **Example:** Alice defects accidentally due to miscommunication; Bob observes this as intentional and defects in the next round, leading to a breakdown of cooperation.
- **Strategies to Mitigate Imperfections:**
    - **Generous Tit-for-Tat:** Forgives occasional defections to account for possible errors in observation.
    - **Win-Stay, Lose-Shift:** Cooperate if mutual cooperation occurred; switch strategy if not.

Conclusion:

- **Sustained Cooperation:** Repeated interaction allows for the possibility of sustained cooperation, which is unattainable in a one-shot game.
- **Role of Discount Factor:** Players must value future payoffs sufficiently (high δ\deltaδ) for cooperation to be the optimal strategy.
- **Real-World Application:** This models situations like business partnerships or international treaties where ongoing relationships encourage cooperative behavior.

Example 2: Oligopoly Pricing – Repeated Bertrand Competition

Scenario: Two competing firms, Firm A and Firm B, sell identical products. Each chooses a price each period. Consumers buy from the firm with the lower price; if prices are equal, sales are split.

One-Shot Game Outcome:

- Both firms have an incentive to undercut the other's price to capture the entire market.
- Resulting in prices driven down to marginal cost, and zero economic profit.

Infinitely Repeated Game: Firms interact over infinite periods and can establish implicit collusion to keep prices high.

Strategies:

1. **Collusive Strategy:**
    - Both firms agree (implicitly) to set a high price, above marginal cost, to earn higher profits.
    - **Enforcement via Punishment:** If one firm undercuts, the other responds by setting low prices in future periods (price war), reducing profits.
2. **Trigger Strategies:**

- **Simple Trigger:** Maintain high prices until undercut; then revert to marginal cost pricing forever.
- **Tit-for-Tat Pricing:** Match the competitor's previous price.

Condition for Sustaining High Prices:

- **Profit Comparison:**
  - **Immediate Gain from Undercutting:** Short-term profit increase by capturing the entire market.
  - **Long-Term Loss:** Future profits decrease due to retaliatory pricing.
- **Discount Factor (δ\deltaδ) Requirement:**
  - Firms must value future profits enough to deter undercutting.
  - **Inequality:**
    High Price Profit/(1−δ)≥Undercut Profit+(Low Price Profit×δ/(1−δ))\text{High Price Profit} / (1 - \delta) \geq \text{Undercut Profit} + (\text{Low Price Profit} \times \delta / (1 - \delta))High Price Profit/(1−δ)≥Undercut Profit+(Low Price Profit×δ/(1−δ))
    - Solving for δ\deltaδ provides the minimum patience required for collusion.

Imperfect Information Considerations:

- **Market Fluctuations:** Demand shocks may be misinterpreted as price cuts.
- **Detection Lags:** Firms may not immediately observe competitor's prices.
- **Strategies to Handle Imperfections:**
  - **Price Matching Guarantees:** Commit to matching lower prices, reducing incentive to undercut.
  - **Monitoring Mechanisms:** Invest in better market intelligence to accurately observe competitor actions.

Real-World Observations:

- **Airline Pricing:** Airlines often maintain high prices on certain routes, sustained by repeated interaction and the threat of price wars.
- **Gas Stations:** Local gas stations may keep prices stable, knowing that aggressive price cuts can lead to mutual losses.

Example 3: International Trade Agreements

Scenario: Two countries, Country X and Country Y, trade goods and can choose to impose tariffs (T) or keep trade free (F).

One-Shot Game Outcome:

- Both imposing tariffs leads to reduced trade and welfare.
- Each has an incentive to impose tariffs while the other keeps trade free to protect domestic industries.

Repeated Interaction: Through repeated interactions, countries can sustain free trade agreements.

Strategies:

1. **Reciprocal Free Trade:**
    - Both countries keep trade free as long as the other does.
    - If one imposes tariffs, the other retaliates in future periods.
2. **Punishment Phases:**
    - After detecting a tariff, enter a punishment phase where both impose tariffs for a set number of periods before returning to free trade.

Ensuring Compliance:

- **Reputation:** Countries value their reputation for adhering to agreements, influencing future diplomatic and economic relations.
- **International Bodies:** WTO and other organizations facilitate monitoring and enforcement.

Dealing with Imperfect Information:

- **Detection Errors:** Sometimes, economic downturns may be mistaken for the effects of tariffs.
- **Communication Protocols:** Regular dialogues and transparent reporting help clarify intentions and reduce misunderstandings.

Outcomes:

- **Sustained Free Trade:** With appropriate strategies and sufficient valuation of future gains, countries maintain mutually beneficial free trade.
- **Real-World Example:** European Union trade agreements where member countries sustain low tariffs through repeated interactions and institutional frameworks.

Example 4: Employer-Employee Relationships

Scenario: An employer (Principal) and an employee (Agent) interact over multiple periods. The employee can choose to work hard (H) or shirk (S). The employer can choose to pay a high wage (W_H) or low wage (W_L).

One-Shot Game Outcome:

- Employee may shirk if effort isn't immediately rewarded.

- Employer may pay low wages if effort isn't observable.

Repeated Interaction: Ongoing employment allows for reputation and trust-building.

Strategies:

1. **Efficiency Wage Strategy:**
    - Employer pays a high wage to incentivize hard work.
    - Employee works hard to maintain employment and high wage.
2. **Performance-based Raises:**
    - Employer adjusts wages based on observed performance over time.
    - Employee puts in effort anticipating future wage increases.

Managing Imperfect Information:

- **Performance Measurement:** Imperfect monitoring can lead to uncertainty about effort levels.
- **Trust-based Relationships:** Building trust over time can compensate for monitoring imperfections.

Outcomes:

- **Mutual Benefit:** Employer gains from increased productivity; employee enjoys higher wages and job security.
- **Real-World Application:** Professional settings where long-term employment relationships foster high performance and loyalty.