Relationship b/w Primal & Dual optimal solutions

$$\boxed{Z_{max} = W_{min}}$$

① Using duality solve the following LPP

Maximise $Z = 5x_1 - 2x_2 + 3x_3$

subject to $2x_1 + 2x_2 - x_3 \geq 2$

$3x_1 - 4x_2 \leq 3$

$x_1 + 3x_3 \leq 5$

$x_1, x_2, x_3 \geq 0$.

Soln:-

Since the objective function is of maximisation type, we first write the constraints in less than or equal to form.

Maximise $Z = 5x_1 - 2x_2 + 3x_3$

subject to $-2x_1 - 2x_2 + x_3 \leq -2$

$3x_1 - 4x_2 + 0x_3 \leq 3$

$0x_1 + x_2 + 3x_3 \leq 5$

$x_1, x_2, x_3 \geq 0$.

∴ The dual is

Minimise $w = -2y_1 + 3y_2 + 5y_3$

ⓐ) Maximise $w' = -w = 2y_1 - 3y_2 - 5y_3$

Subject to $-2y_1 + 3y_2 + 0y_3 \geq 5$

$-2y_1 - 4y_2 + y_3 \geq -2$

ⓘ) $2y_1 + 4y_2 - y_3 \leq 2$

$y_1 + 0y_2 + 3y_3 \geq 3$

$y_1, y_2, y_3 \geq 0$.

By adding the slack variables and artificial variables,

Maximise $w' = -w = 2y_1 - 3y_2 - 5y_3 - 0s_1 - 0s_2 - 0s_3 - MA_1 - MA_3$ → ①

subject to

$-2y_1 + 3y_2 + 0y_3 - s_1 + 0s_2 + 0s_3 + A_1 + 0A_3 = 5$ → ②

$2y_1 + 4y_2 - y_3 + 0s_1 + s_2 + 0s_3 + 0A_1 + 0A_3 = 2$ → ③

$y_1 + 0y_2 + 3y_3 + 0s_1 + 0s_2 - s_3 + 0A_1 + A_3 = 3$ → ④.

To eliminate MA_1 & MA_3 from the objective function Multiply (2) & (4) by M and add it to (1).

$$\therefore w' = 2y_1 - 3y_2 - 5y_3 - 0S_1 - 0S_2 - 0S_3 - MS_1 - MS_3 - My_1 + 3My_2 + 3My_3 - 0A_1 - 0A_3 - 8M.$$

$$w' = (2-M)y_1 + (-3+3M)y_2 + (-5+3M)y_3 - MS_1 - 0S_2 - MS_3 - 0S_3 - 0A_1 - 0A_3 - 8M.$$

$$\therefore w' + (-2+M)y_1 + (3-3M)y_2 + (5-3M)y_3 + MS_1 + 0S_2 + MS_3 + 0A_1 + 0A_3 = -8M.$$

Simplex table

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Iteration no	Basic Variables	y_1	y_2	y_3	s_1	s_2	s_3	A_1	A_3	RHS soln	Ratio
		-2M	3-3M	5-3M	M	0	M	0	0	-8M	
0	w'	$\frac{-7+5M}{2}$	0	$\frac{23-15M}{4}$	M	$\frac{-3+3M}{4}$	M	0	0	$\frac{-3-13M}{2}$	
	A_1	-2	3	0	-1	0	0	1	0	5	5/3
	s_2	2	4	-1	0	1	0	0	0	2	1/2
	A_3	1	0	3	0	0	-1	0	1	3	1
1	w'	-7/2	0	3/4	-1	-3/4	0	1	0	7/2	14/3
	A_1	-7/2	0	3/4	-1	-3/4	0	1	0	7/2	-2
	y_2	1/2	1	-1/4	0	1/4	0	0	0	1/2	
	A_3	1	0	3*	0	0	-1	0	1	3	1

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2	w'	$\frac{-65+45M}{12}$	0	0	M	$\frac{-3+3M}{4}$	$\frac{23-3M}{12}$	0	-	$\frac{-29-11M}{4}$	
A_1 leaves	A_1	$-15/4$	0	0	-1	$3/4$	$1/4$	1	-	$11/4$	11
S_3 enters	y_2	$7/12$	1	0	0	$1/4$	$-1/12$	0	-	$3/4$	-
	y_3	$1/3$	0	1	0	0	$-1/3$	0	-	1	-
3	w'	$70/3$	0	0	$23/3$	5	0	$\frac{-3+3M}{3}$	-	$-85/3$	
	S_3	-15	0	0	-4	-3	1	4	-	11	
	y_2	$-2/3$	1	0	$-1/3$	0	0	$1/3$	-	$5/3$	
	y_3	$-14/3$	0	1	$-4/3$	-1	0	$4/3$	-	$14/3$.	

$S_1 = \frac{23}{3}$, $S_2 = 5$, $S_3 = 0$ $w'_{max} = \frac{-85}{3}$ $\therefore w_{min} = \frac{85}{3}$

$\therefore x_1 = \frac{23}{3}$, $x_2 = 5$, $x_3 = 0$, $Z_{max} = \frac{85}{3}$.