Module-1

1(a). Evaluate $\int_0^\infty e^{-t} \int_0^t \frac{e^{-2u}\sin u}{u}\,du\,dt$

Solution:

Let $\int_0^\infty e^{-t} \int_0^t \frac{e^{-2u}\sin u}{u}\,du\,dt = L\left[\int_0^t \frac{e^{-2u}\sin u}{u}\,du\right] put\ s = 1$

$\therefore \mathrm{L}[\sin \mathrm{u}] = \frac{1}{s^2+1}$

By effect of division by t

$\therefore \mathrm{L}\left[\frac{\sin u}{u}\right] = \int_s^\infty \frac{1}{s^2+1}\,ds$

$\because \int \frac{1}{s^2+a^2}\,ds = \frac{1}{a}\tan^{-1}\left(\frac{s}{a}\right)$

$\therefore \mathrm{L}\left[\frac{\sin u}{u}\right] = [\tan^{-1}(s)]_s^\infty$

Substituting the limits

$\therefore \mathrm{L}\left[\frac{\sin u}{u}\right] = \tan^{-1}(\infty) - \tan^{-1}(s)$

$= \frac{\pi}{2} - \tan^{-1}(s)$

$\therefore \mathrm{L}\left[\frac{\sin u}{u}\right] = \cot^{-1}(s)$

$\therefore \mathrm{L}\left[\frac{e^{-2u}\sin u}{u}\right] = \cot^{-1}(s+2)$

$\therefore \mathrm{L}\left[\int_0^t \frac{e^{-2u}\sin u}{u}\,du\right] = \frac{1}{s}\cot^{-1}(s+2)$

$\therefore \int_0^\infty e^{-t} \int_0^t \frac{e^{-2u}\sin u}{u}\,du\ \ dt = \left[\frac{1}{s}\cot^{-1}(s+2)\right] put\ s = 1$

$= \left[\frac{1}{1}\cot^{-1}(1+2)\right]$

$= \cot^{-1}(3)$

OR

1(a). Find $L\left[\frac{e^{-2t}\sin 2t\cosh t}{t}\right]$.

Solution:

$L\left[\frac{e^{-2t}\sin 2t\cosh t}{t}\right]$

$= L\left[\frac{e^{-2t}\sin 2t}{t}\ \frac{e^t+e^{-t}}{2}\right]$

$= \frac{1}{2}L\left[\frac{e^{-t}\sin 2t+e^{-3t}\sin 2t}{t}\right]$

$= \frac{1}{2}\left\{L\left[\frac{e^{-t}\sin 2t}{t}\right] + L\left[\frac{e^{-3t}\sin 2t}{t}\right]\right\}$

$\because \mathrm{L}[\sin at] = \frac{a}{s^2+a^2}$

$$\therefore L[\sin 2t] = \frac{2}{s^2+4}$$

By effect of division by t

$$\therefore L\left[\frac{\sin 2t}{t}\right] = \int_s^{\infty} \frac{2}{s^2+4} ds$$

$$\because \int \frac{1}{s^2+a^2} ds = \frac{1}{a}\tan^{-1}\left(\frac{s}{a}\right)$$

$$\therefore L\left[\frac{\sin 2t}{t}\right] = \frac{2}{2}\left[\tan^{-1}\left(\frac{s}{2}\right)\right]_s^{\infty}$$

Substituting the limits

$$\therefore L\left[\frac{\sin 2t}{t}\right] = \left\{\tan^{-1}(\infty) - \tan^{-1}\left(\frac{s}{2}\right)\right\}$$

$$= \frac{\pi}{2} - \tan^{-1}\left(\frac{s}{2}\right)$$

$$\therefore L\left[\frac{\sin 2t}{t}\right] = \cot^{-1}\left(\frac{s}{2}\right)$$

$$\therefore L\left[e^{-t}\frac{\sin 2t}{t}\right] = \cot^{-1}(s+1)$$

$$\therefore L\left[e^{-3t}\frac{\sin 2t}{t}\right] = \cot^{-1}(s+3)$$

$$L\left[\frac{e^{-2t}\sin 2t \cosh t}{t}\right] = \frac{1}{2}\{\cot^{-1}(s+1) + \cot^{-1}(s+3)\}$$

$$= \frac{1}{2}\left\{\tan^{-1}\left(\frac{1}{s+1}\right) + \tan^{-1}\left(\frac{1}{s+1}\right)\right\}$$

$$= \frac{1}{2}\tan^{-1}\left(\frac{\frac{1}{s+1}+\frac{1}{s+3}}{1-\frac{1}{s+1}\frac{1}{s+3}}\right)$$

$$= \frac{1}{2}\tan^{-1}\left(\frac{s+3+s+1}{(s+1)(s+3)-1}\right)$$

$$= \frac{1}{2}\tan^{-1}\left(\frac{2s+4}{s^2+4s+3-1}\right)$$

$$= \frac{1}{2}\tan^{-1}\left(\frac{2s+4}{s^2+4s+2}\right)$$

1(b). If $f(t) = \begin{cases} t+1 & , \quad 0 \le t \le 2 \\ 3 & , \quad t > 2 \end{cases}$ then find $L[f'(t)]$.

Solution:

$$\because L[f(t)] = \int_0^{\infty} e^{-st} f(t) dt$$

$$= \int_0^2 e^{-st}(t+1)dt + \int_2^{\infty} 3e^{-st}dt$$

$$\therefore L[f(t)] = \left[(t+1)\left(\frac{e^{-st}}{-s}\right) - (1)\left(\frac{e^{-st}}{(-s)^2}\right)\right]_0^2 + \left[(3)\left(\frac{e^{-st}}{-s}\right)\right]_2^{\infty}$$

$$= \left[(2+1)\left(\frac{e^{-2s}}{-s}\right) - (1)\left(\frac{e^{-2s}}{(-s)^2}\right)\right] - \left[(0+1)\left(\frac{e^{-0}}{-s}\right) - (1)\left(\frac{e^{-0}}{(-s)^2}\right)\right]$$

$$+ \left[(3)\left(\frac{e^{-\infty \times s}}{-s}\right) - (3)\left(\frac{e^{-2s}}{-s}\right)\right]$$

$$= -\frac{3e^{-2s}}{s} - \frac{e^{-2s}}{s^2} + \frac{1}{s} + \frac{1}{s^2} - \frac{3}{s} + \frac{3e^{-2s}}{s}$$

$$= -\frac{e^{-2s}}{s^2} + \frac{1}{s^2} - \frac{2}{s}$$

$$\therefore \mathrm{L}[\mathrm{f}(\mathrm{t})] = -\frac{\mathrm{e}^{-2s}}{s^2} + \frac{1}{s^2} - \frac{2}{s}$$

$$\because L[f'(t)] = s\,\phi(s) - f(0)$$

$$\therefore L[f'(t)] = s\left(-\frac{\mathrm{e}^{-2s}}{s^2} + \frac{1}{s^2} - \frac{2}{s}\right) - 1$$

$$= -\frac{\mathrm{e}^{-2s}}{s} + \frac{1}{s} - 2 - 1$$

$$= -\frac{\mathrm{e}^{-2s}}{s} + \frac{1}{s} - 3$$

OR

1(b). Find the Laplace Transform of $sin^5 t$

Solution:

Method:1 $\because \mathrm{L}[\sin^5 \mathrm{t}] = \mathrm{L}[\sin^3 \mathrm{t} \cdot \sin^2 \mathrm{t}]$

$$\because \sin^3 \mathrm{t} = \frac{3\mathrm{sint} - \sin 3\mathrm{t}}{4} \text{ and } \sin^2 \mathrm{t} = \frac{1 - \cos 2\mathrm{t}}{2}$$

$$\therefore \mathrm{L}[\sin^5 \mathrm{t}] = \mathrm{L}\left[\frac{3\mathrm{sint} - \sin 3\mathrm{t}}{4} \cdot \frac{1 - \cos 2\mathrm{t}}{2}\right]$$

$$\therefore \mathrm{L}[\sin^5 \mathrm{t}] = \frac{\mathrm{L}}{8}[3\mathrm{sint} - 3\mathrm{sintcos2t} - \sin 3\mathrm{t} + \sin 3\mathrm{tcos2t}]$$

Using $\mathrm{sinAcosB} = \frac{1}{2}[\sin(\mathrm{A}+\mathrm{B}) + \sin(\mathrm{A}-\mathrm{B})]$ and $\sin(-\mathrm{t}) = -\mathrm{sint}$

$$\therefore \mathrm{L}[\sin^5 \mathrm{t}] = \frac{\mathrm{L}}{8}\left[3\mathrm{sint} - \frac{3}{2}[\sin 3\mathrm{t} - \mathrm{sint}] - \sin 3\mathrm{t} + \frac{1}{2}(\sin 5\mathrm{t} + \mathrm{sint})\right]$$

$$= \frac{\mathrm{L}}{8}\left[5\mathrm{sint} - \frac{5}{2}\sin 3\mathrm{t} + \frac{1}{2}\sin 5\mathrm{t}\right]$$

$$= \frac{1}{8}\left[5\mathrm{L}[\mathrm{sint}] - \frac{5}{2}\mathrm{L}[\sin 3\mathrm{t}] + \frac{1}{2}\mathrm{L}[\sin 5\mathrm{t}]\right]$$

$$= \frac{1}{16}[10\mathrm{L}[\mathrm{sint}] - 5\mathrm{L}[\sin 3\mathrm{t}] + \mathrm{L}[\sin 5\mathrm{t}]]$$

Using $\mathrm{L}[\mathrm{sinat}] = \frac{\mathrm{a}}{\mathrm{s}^2 + \mathrm{a}^2}$

$$\therefore \mathrm{L}[\sin^5 \mathrm{t}] = \frac{1}{16}\left[10\left(\frac{1}{\mathrm{s}^2+1^2}\right) - 5\left(\frac{3}{\mathrm{s}^2+3^2}\right) + \left(\frac{5}{\mathrm{s}^2+5^2}\right)\right]$$

$$= \frac{1}{16}\left[\frac{10(\mathrm{s}^2+9)(\mathrm{s}^2+25) - 15(\mathrm{s}^2+1)(\mathrm{s}^2+25) + 5(\mathrm{s}^2+1)(\mathrm{s}^2+9)}{(\mathrm{s}^2+1)(\mathrm{s}^2+9)(\mathrm{s}^2+25)}\right]$$

$$= \frac{1}{16}\left[\frac{10(\mathrm{s}^4+34\mathrm{s}^2+225) - 15(\mathrm{s}^4+26\mathrm{s}^2+25) + 5(\mathrm{s}^4+10\mathrm{s}^2+9)}{(\mathrm{s}^2+1)(\mathrm{s}^2+9)(\mathrm{s}^2+25)}\right]$$

$$= \frac{1}{16}\left[\frac{10(\mathrm{s}^4+34\mathrm{s}^2+225) - 15(\mathrm{s}^4+26\mathrm{s}^2+25) + 5(\mathrm{s}^4+10\mathrm{s}^2+9)}{(\mathrm{s}^2+1)(\mathrm{s}^2+9)(\mathrm{s}^2+25)}\right]$$

$$= \frac{1}{16}\left[\frac{(10-15+5)\mathrm{s}^4 + (340-390+50)\mathrm{s}^2 + (2250-375+45)}{(\mathrm{s}^2+1)(\mathrm{s}^2+9)(\mathrm{s}^2+25)}\right]$$

$$= \frac{1}{16}\left[\frac{0+0+1920}{(\mathrm{s}^2+1)(\mathrm{s}^2+9)(\mathrm{s}^2+25)}\right]$$

$$= \frac{120}{(\mathrm{s}^2+1)(\mathrm{s}^2+9)(\mathrm{s}^2+25)}$$

$$\therefore \mathrm{L}[\sin^5 \mathrm{t}] = \frac{5!}{(\mathrm{s}^2+1)(\mathrm{s}^2+9)(\mathrm{s}^2+25)}$$

Method:2 $\therefore \mathrm{L}[\sin^5 \mathrm{t}] = \left(\frac{1}{2i}\right)^5 [(2i\sin 5t) - 5(2i\sin 3t) + 10(2i\sin t)]$

$$= \left(\frac{1}{2i}\right)^4 [(\sin 5t) - 5(\sin 3t) + 10(\sin t)]$$

$$= \frac{1}{16}[(\sin 5t) - 5(\sin 3t) + 10(\sin t)]$$

$$= \frac{1}{16}[10(\sin t) - 5(\sin 3t) + (\sin 5t)]$$

Using $\mathrm{L[sinat]} = \frac{\mathrm{a}}{\mathrm{s^2+a^2}}$

$$\therefore \mathrm{L}[\sin^5 \mathrm{t}] = \frac{1}{16}\left[10\left(\frac{1}{s^2+1^2}\right) - 5\left(\frac{3}{s^2+3^2}\right) + \left(\frac{5}{s^2+5^2}\right)\right]$$

$$= \frac{1}{16}\left[\frac{10(s^2+9)(s^2+25)-15(s^2+1)(s^2+25)+5(s^2+1)(s^2+9)}{(s^2+1)(s^2+9)(s^2+25)}\right]$$

$$= \frac{1}{16}\left[\frac{10(s^4+34s^2+225)-15(s^4+26s^2+25)+5(s^4+10s^2+9)}{(s^2+1)(s^2+9)(s^2+25)}\right]$$

$$= \frac{1}{16}\left[\frac{10(s^4+34s^2+225)-15(s^4+26s^2+25)+5(s^4+10s^2+9)}{(s^2+1)(s^2+9)(s^2+25)}\right]$$

$$= \frac{1}{16}\left[\frac{(10-15+5)s^4+(340-390+50)s^2+(2250-375+45)}{(s^2+1)(s^2+9)(s^2+25)}\right]$$

$$= \frac{1}{16}\left[\frac{0+0+1920}{(s^2+1)(s^2+9)(s^2+25)}\right]$$

$$= \frac{120}{(s^2+1)(s^2+9)(s^2+25)}$$

$$\therefore \mathrm{L}[\sin^5 \mathrm{t}] = \frac{5!}{(s^2+1)(s^2+9)(s^2+25)}$$

Module-2

2(a). Find $L^{-1}\left\{\frac{1}{(s^2+9)(s^2+1)}\right\}$ using convolution theorem

Solution:

$$\mathrm{L}^{-1}\left[\frac{1}{(s^2+9)(s^2+1)}\right]$$

$$= \mathrm{L}^{-1}\left[\frac{1}{(s^2+9)}\frac{1}{(s^2+1)}\right]$$

$$f_1(t) = \mathrm{L}^{-1}\left[\frac{1}{(s^2+9)}\right] = \frac{\sin 3t}{3}$$

$$f_2(t) = \mathrm{L}^{-1}\left[\frac{1}{(s^2+1)}\right] = \sin t$$

$$f_1(u) = \frac{\sin 3u}{3}$$

$$f_2(t-u) = \sin(t-u)$$

By using convolution theorem

$$\mathrm{L}^{-1}[\phi_1(s).\phi_2(s)] = \int_0^t f_1(u) f_2(t-u)du$$

$$\mathrm{L}^{-1}\left[\frac{1}{(s^2+9)(s^2+1)}\right]$$

$$= \frac{1}{3}\int_0^t \sin 3u \sin(t-u)\, du$$

$$= -\frac{1}{6}\int_0^t \{\cos(3u+t-u) + \cos(3u-t+u)\}du$$

$$= -\frac{1}{6}\int_0^t \{\cos(2u+t) + \cos(4u-t)\}du$$

$$= -\frac{1}{6}\left\{\int_0^t \{\cos(2u+t)\}du + \int_0^t \{\cos(4u-t)\}du\right\}$$

$$= -\frac{1}{6}\left\{\left[\frac{\sin(2u+t)}{2}\right]_0^t + \left[\frac{\sin(4u-t)}{4}\right]_0^t\right\}$$

$$= -\frac{1}{6}\left\{\left[\frac{\sin(2t+t)}{2} - \frac{\sin(t)}{2}\right] + \left[\frac{\sin(4t-t)}{4} - \frac{\sin(-t)}{4}\right]\right\}$$
$$= -\frac{1}{6}\left\{\left[\frac{\sin(3t)}{2} - \frac{\sin(t)}{2}\right] + \left[\frac{\sin(3t)}{4} + \frac{\sin(t)}{4}\right]\right\}$$
$$= -\frac{1}{6}\left\{\frac{\sin(3t)}{2} - \frac{\sin(t)}{2} + \frac{\sin(3t)}{4} + \frac{\sin(t)}{4}\right\}$$
$$= -\frac{1}{24}\{2\sin 3t - 2\sin t + \sin 3t + \sin t\}$$
$$= -\frac{1}{24}\{3\sin 3t - \sin t\}$$

OR

2(a). Find $L^{-1}\left\{\frac{5s^2+8s-1}{(s+3)(s^2+1)}\right\}$ using method of partial fraction

Solution:

$$\mathrm{L}^{-1}\left[\frac{5s^2+8s-1}{(s+3)(s^2+1)}\right]$$
$$= \mathrm{L}^{-1}\left[\frac{A}{(s+3)} + \frac{Bs+C}{(s^2+1)}\right]$$
$$= \mathrm{L}^{-1}\left[\frac{A}{(s+3)} + \frac{Bs}{(s^2+1)} + \frac{C}{(s^2+1)}\right]$$
$$= Ae^{-3t} + B\cos t + C\sin t$$
$$A = \frac{5s^2+8s-1}{(s^2+1)}|s = -3$$
$$A = \frac{45-24-1}{9+1} = 2$$

Now find B and C

$$A(s^2+1) + (Bs+C)(s+3) = 5s^2 + 8s - 1$$
$$As^2 + A + Bs^2 + Cs + 3Bs + 3C = 5s^2 + 8s - 1$$
$$S^2(A+B) + s(3B+C) + (A+3C) = 5s^2 + 8s - 1$$

Compare coefficients

$$A + B = 5$$
$$3B + C = 8$$
$$A + 3C = -1$$
$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 3 & 1 \\ 1 & 0 & 3 \end{pmatrix}\begin{pmatrix} A \\ B \\ C \end{pmatrix} = \begin{pmatrix} 5 \\ 8 \\ -1 \end{pmatrix}$$

$A = 2 \qquad B = 3 \qquad C = -1$

$$\mathrm{L}^{-1}\left[\frac{5s^2+8s-1}{(s+3)(s^2+1)}\right] = 2e^{-3t} + 3\cos t - \sin t$$

2(b). Find $L^{-1}\left\{\frac{6s-4}{s^2-4s+20}\right\}$ using Shifting

Solution:

$$\mathrm{L}^{-1}\left[\frac{6s-4}{s^2-4s+20}\right]$$
$$= \mathrm{L}^{-1}\left[\frac{6(s-2+2)-4}{(s-2)^2-2^2+20}\right]$$

$$= e^{2t}\mathrm{L}^{-1}\left[\frac{6(s+2)-4}{(s)^2-2^2+20}\right]$$

$$= e^{2t}\mathrm{L}^{-1}\left[\frac{6(s+2)-4}{(s)^2-2^2+20}\right]$$

$$= e^{2t}\mathrm{L}^{-1}\left[\frac{6(s)+12-4}{(s)^2+16}\right]$$

$$= e^{2t}\mathrm{L}^{-1}\left[\frac{6(s)}{(s)^2+16}+\frac{8}{(s)^2+16}\right]$$

$$= e^{2t}\left\{6\cos 4t + 8\times\frac{\sin 4t}{4}\right\}$$

$$= e^{2t}\{6\cos 4t + 2\sin 4t\}$$

OR

2(b). Find $L^{-1}\left\{\frac{3s-7}{s^2-6s+8}\right\}$ using method of partial fraction.

Solution:

$$L^{-1}\left\{\frac{3s-7}{s^2-6s+8}\right\}$$

$$= \mathrm{L}^{-1}\left[\frac{3s-7}{(s-2)(s-4)}\right]$$

$$= \mathrm{L}^{-1}\left[\frac{A}{s-2}+\frac{B}{(s-4)}\right]$$

$$= Ae^{2t} + Be^{4t}$$

$$A = \frac{3s-7}{(s-4)}|s = 2$$

$$A = \frac{1}{2}$$

$$B = \frac{3s-7}{(s-2)}|s = 4$$

$$B = \frac{5}{2}$$

$$L^{-1}\left\{\frac{3s-7}{s^2-6s+8}\right\} = Ae^{2t} + Be^{4t} = \frac{1}{2}e^{2t} + \frac{5}{2}e^{4t} = \frac{1}{2}\{e^{2t} + 5e^{4t}\}$$

Module-05

3(a). Calculate Spearman's coefficient of rank correlation from the following data.

X	10	12	18	18	15	40
Y	12	18	25	25	50	25

Solution:

X	Y	Rx	Ry	d=Rx-Ry	d^2
10	12	6	6	0	0
12	18	5	5	0	0
18	25	3 2.5	2 3	-0.5	0.25
18	25	2 2.5	3 3	-0.5	0.25
15	50	4	1	3	9
40	25	1	4 3	-2	4

					$\sum d^2 = 13.5$

To find Correction Factor:

$c.f. = \frac{m(m^2-1)}{12}$

$c.f.(x = 18) = \frac{2(2^2-1)}{12} = 0.5$

$c.f.(y = 25) = \frac{3(3^2-1)}{12} = 2$

$\sum c.f. = 0.5 + 2 = 2.5$

$R = 1 - \left\{\frac{6[\sum d^2 + \sum c.f.]}{n(n^2-1)}\right\}$

$R = 1 - \left\{\frac{6[13.5+2.5]}{6(6^2-1)}\right\}$

$R = 1 - \frac{16}{35} = \frac{19}{35}$

OR

3(a). The regression lines of a sample are $3x + 2y = 26$ and $6x + y = 31$. Find the sample means and correlation coefficient between x and y. If the variance of y is 4, find the standard deviation of x.

Solution: **Let** $3x + 2y = 26 \ldots (1)$

$6x + y = 31 \ldots (2)$

On solving eq (1) and (2)

$X = 4, \quad Y = 7$

(i) **Case:1** Let $3x + 2y = 26$ represent line of regression of x on y

$x = -\frac{2}{3}y - \frac{26}{3}$

$\therefore b_{xy} = -\frac{2}{3}$

Let $6x + y = 31$ represent line of regression of Y on X

$y = -6x + 31$

$\therefore b_{yx} = -6$

$r = \pm\sqrt{b_{xy}\, b_{yx}} = -\sqrt{-\frac{2}{3} \times -6}$

$\therefore r = 2$

$\therefore$ r does not lie between -1 and 1

$\therefore$The assumption is wrong.

(ii) **Case:2** Let$3x + 2y = 26$ represent line of regression of y on x

$Y = -\frac{3}{2}X + \frac{26}{2}$

$\therefore b_{yx} = -\frac{3}{2}$

Let $6x + y = 31$ represent line of regression of X on Y

$X = -\frac{1}{6}Y + \frac{31}{6}$

$\therefore b_{xy} = -\frac{1}{6}$

$\therefore r = \pm\sqrt{b_{xy} b_{yx}}$

$= -\sqrt{-\frac{3}{2} \times -\frac{1}{6}}$

$\therefore \mathrm{r} = -0.5$

$\therefore$ r lies between-1 and 1

$\therefore$The assumption is right.

$\therefore \mathrm{r} = -0.5$

(iii) $\mathrm{b_{yx}} = \mathrm{r}\frac{\sigma_y}{\sigma_x}$

$\therefore -\frac{3}{2} = -\frac{1}{2}\frac{2}{\sigma_x}$

$\therefore \sigma_x = \frac{2}{3}$

OR

$\mathrm{b_{xy}} = \mathrm{r}\frac{\sigma_x}{\sigma_y}$

$\therefore -\frac{1}{6} = -\frac{1}{2}\frac{\sigma_x}{2}$

$\therefore \sigma_x = \frac{4}{6} = \frac{2}{3}$

3(b). Fit a second-degree curve to the following data.

X	1	2	3	4	5	6	7	8	9
Y	2	6	7	8	10	11	11	10	9

Solution:

Step-1: Write equation of parabolic curve

$y = a + bx + cx^2 \ldots\ldots (*)$

$Taking \sum\ on\ both\ sides$

$\sum y = \sum a + \sum bx + \sum cx^2$

$\sum y = a\sum 1 + b\sum x + c\sum x^2$

$\sum y = na + b\sum x + c\sum x^2 \ldots\ldots (i)$

Multiplying by x on both side in equation $(*)$

$xy = ax + bx^2 + cx^3$

$Taking \sum\ on\ both\ sides$

$\sum xy = \sum ax + \sum bx^2 + \sum cx^3$

$\sum xy = a\sum x + b\sum x^2 + c\sum x^3 \ldots\ldots (ii)$

multiplying by x^2 on both sides in euation $(*)$

$x^2y = ax^2 + bx^3 + cx^4$

$Taking \sum\ on\ both\ sides$

$\sum x^2y = \sum ax^2 + \sum bx^3 + \sum cx^4$

$\sum x^2y = a\sum x^2 + b\sum x^3 + c\sum x^4 \ldots\ldots (iii)$

Solve Equations $(i), (ii) and\ (iii)$

$$\begin{bmatrix} n & \sum x & \sum x^2 \\ \sum x & \sum x^2 & \sum x^3 \\ \sum x^2 & \sum x^3 & \sum x^4 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} \sum y \\ \sum xy \\ \sum x^2y \end{bmatrix}$$

Step-2: Prepare the table

x	Y	x^2	x^3	x^4	xy	x^2y
1	2	1	1	1	2	2
2	6	4	8	16	12	24
3	7	9	27	81	21	63

4	8	16	64	256	32	128
5	10	25	125	625	50	250
6	11	36	216	1296	66	396
7	11	49	343	2401	77	539
8	10	64	512	4096	80	640
9	9	81	729	6561	81	729
$\sum x = 280$	$\sum y = 74$	$\sum x^2 = 285$	$\sum x^3 = 2025$	$\sum x^4 = 15333$	$\sum xy = 421$	$\sum x^2 y = 2771$

Step-3: Put Values of

$\sum x = 45$

$\sum x^2 = 285$

$\sum x^3 = 2025$

$\sum x^4 = 153333$

$\sum y = 74$

$\sum xy = 421$

$\sum x^2 y = 2771$

in equation (i), (ii)&(iii)and find values of a, b & c

$$\begin{bmatrix} 9 & 45 & 285 \\ 45 & 285 & 2025 \\ 285 & 2045 & 15333 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 74 \\ 421 \\ 2771 \end{bmatrix}$$

$a = -0.9285, b = 3.5231 \;\&\; c = -0.2673$

Step-4: Put Values $a = -28.5, b = 5.7 \;\&\; c = -0.07$ in Equation $(*)$

$y = a + bx + cx^2 \;\ldots\ldots (*)$

$y = -0.9285 + 3.5231x - 0.2673x^2$

OR

3(b). Find equation of line of regressions of Y on X for the following data.

X	5	6	7	8	9	10	11
Y	11	14	14	15	12	17	16

Solution:

Prepare the table

X	Y	x^2	y^2	xy
5	11	25	121	55
6	14	36	196	84
7	14	49	196	98
8	15	64	225	120
9	12	81	144	108
10	17	100	289	170
11	16	121	256	176
$\sum x = 56$	$\sum y = 99$	$\sum x^2 = 476$	$\sum y^2 = 1427$	$\sum xy = 811$

Method:1

$n = 7$

$\bar{x} = 8$

$\bar{y} = 14.14$

$\sigma_x = 2$

$\sigma_y = 1.95$

$r = \frac{n\sum xy - \sum x \sum y}{\sqrt{n\sum x^2 - (\sum x)^2}\sqrt{n\sum y^2 - (\sum y)^2}} = 0.6928$

$b_{xy} = r\left(\frac{\sigma_x}{\sigma_y}\right) = 0.7105$

$b_{yx} = r\left(\frac{\sigma_y}{\sigma_x}\right) = 0.6754$

(i) Line of Regression of x on y is given as;

$(x - \bar{x}) = b_{xy}(y - \bar{y})$

$(x - 8) = 0.7105(y - 14.14)$

(ii) Line of Regression of y on x is given as;

$(y - \bar{y}) = b_{yx}(x - \bar{x})$

$(y - 14.14) = 0.6754(x - 8)$

Method:2

$b_{xy} = \frac{n\sum xy - \sum x \sum y}{(n\sum y^2 - (\sum y)^2)} = \frac{7\times 811 - 56\times 99}{7\times 1427 - 99^2} = 0.7105$

$b_{yx} = \frac{n\sum xy - \sum x \sum y}{(n\sum x^2 - (\sum x)^2)} = \frac{7\times 811 - 56\times 99}{7\times 476 - 56^2} = 0.6754$

(i) Line of Regression of x on y is given as;

$(x - \bar{x}) = b_{xy}(y - \bar{y})$

$(x - 8) = 0.7105(y - 14.14)$

(ii) Line of Regression of y on x is given as;

$(y - \bar{y}) = b_{yx}(x - \bar{x})$

$(y - 14.14) = 0.6754(x - 8)$