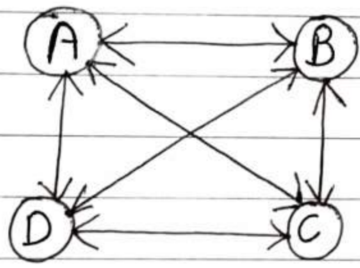
Travelling Salesman Problem

~~A salesman~~ wants ~~to travel a~~ set ~~of n cities starting from homecities. He wants to travel~~ all ~~the cities star~~

A salesman wants to travel n cities. We need to find the path for salesman to start his journey from his home city, Visiting all the cities only once & returning to his home city at the end of the journey.



$$\begin{array}{c|cccc|} & A & B & C & D \\ \hline A & 0 & 16 & 11 & 6 \\ B & 8 & 0 & 13 & 16 \\ C & 4 & 7 & 0 & 9 \\ D & 5 & 12 & 2 & 0 \end{array}$$

Adj. Matrix

Salesman can visit all the other cities other than his home city only once.

Let's A is the home city of salesman.

The problem statement ~~says~~ is expecting the find the route of journey with minimum cost. 2

First method is brute force method. Where we check for all possible ways and find the minimum cost route.

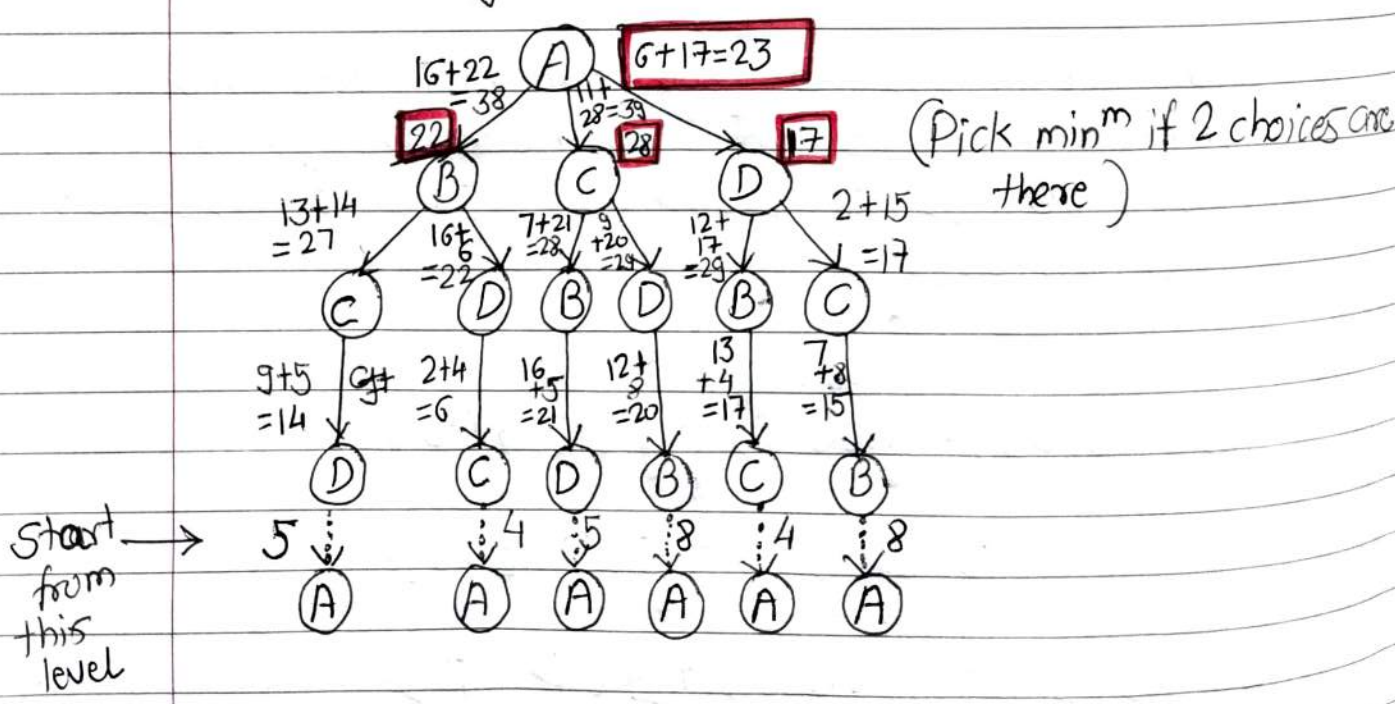
The second way to solve this problem is dynamic programming approach.

Considering A as source or home city, draw a tree for all possible vertices.

From A, salesman go to A B or C or D. ~~Fr~~ After reaching A to B further he can go to C or D.

After reaching A to C further he can ~~B~~ go to B or D. After reaching A to D further he can go to B or C.

Further after reaching from A to B to C he can go to D and from D he can go to A back. Like wise we need to draw tree for all possible ways.



Formula of Travelling Salesman problem

$$g(i, S) = \min_{j \in S} \left(w(i,j) + g(j, \{S - j\}) \right)$$

where,

i = starting vertexe
S = set of vertices salesman will visit only once.

$i = A$

$$g(A, \{B, C, D\}) = \min[W(A,B) + g(B, \{C, D\})]$$

$$\text{or} = \min[W(A,C) + g(C, \{B, D\})]$$

$$\text{or} \ \min[W(A,D) + g(D, \{B, C\})]$$

Out of these 3 we can not find the minimum value as we have recursive calls to $g(B, \{C, D\})$, $g(C, \{B, D\})$ & $g(D, \{B, C\})$

$$g(B, \{C, D\}) = W(B,C) + g(C, \{D\})$$

$$\text{or} \ W(B,C) + g(D, \{C\})$$

$$g(C, \{D\} = W(C,D) + g(D, \phi)$$

As $g(D, \phi)$ which means from D we are going no where. Which means we are returing to home city A.

So $g(D, \phi) = w(D, A)$ ...... $[w(S, i)]$
$= 5$

$g(C, \{D\}) = w(C, D) + g(D, \phi)$
$= w(C, D) + w(D, A)$
$= 9 + 5$
$= \boxed{14}$

There is no min^m as we have only one value 14.

$g(D, \{C\}) = w(D, C) + g(C, \phi)$
$= w(D, C) + w(C, A)$
$= 2 + 4$
$= \boxed{6}$

There is no min^m as we have only one value 6.

And accordingly recursive calls will work.