Identifying dominated strategies

In **non-cooperative games**, identifying **dominated strategies** is a crucial step for simplifying the game and predicting outcomes. A strategy is **dominated** if there is another strategy that always provides a better or at least an equal payoff, regardless of what the other players do. Players will never choose dominated strategies if they are rational.

- **Dominant Strategy**: A strategy that always results in a higher payoff, no matter what the other players choose.
- **Dominated Strategy**: A strategy that is always worse than another strategy, no matter what the other players choose.

There are two types of dominated strategies:

1. **Strictly Dominated Strategy**: A strategy is strictly dominated if another strategy always gives a higher payoff, no matter what the other player(s) choose.
2. **Weakly Dominated Strategy**: A strategy is weakly dominated if another strategy gives an equal or higher payoff in all cases and a strictly higher payoff in at least one case.

Identifying Dominated Strategies:

1. **Compare Strategies Across Payoffs**: For each player, compare the payoffs of their strategies across all possible choices of the other players. If one strategy is strictly worse than another across the board, it is dominated.
2. **Iterated Elimination of Dominated Strategies (IEDS)**: After identifying dominated strategies, you can iteratively remove them from the game to simplify it. Each player eliminates their dominated strategies, reducing the strategy set until no dominated strategies remain.

   Consider a **two-player game** with the following payoff matrix:

   Payoff Matrix (Player 1's payoffs, Player 2's payoffs):

	Player 2: X	Player 2: Y
Player 1: A	(2, 1)	(3, 0)
Player 1: B	(1, 2)	(0, 1)
Player 1: C	(4, 3)	(2, 1)

- **Player 1** has three strategies: A, B, C.
- **Player 2** has two strategies: X, Y.

Step 1: Compare Player 1's Strategies

- **Compare A and B**:
  - When Player 2 plays X, A gives a payoff of 2 and B gives a payoff of 1.
  - When Player 2 plays Y, A gives a payoff of 3 and B gives a payoff of 0.

In both cases, strategy A strictly dominates B, since A gives a higher payoff than B regardless of what Player 2 does.

- **Compare A and C**:
    - When Player 2 plays X, C gives a payoff of 4 and A gives a payoff of 2.
    - When Player 2 plays Y, C gives a payoff of 2 and AAA gives a payoff of 3.

Here, neither A nor C dominates the other. C is better when Player 2 plays X, but A is better when Player 2 plays Y.

So, **strategy B is strictly dominated by strategy A** for Player 1 and can be eliminated.

Step 2: Compare Player 2's Strategies

- **Compare X and Y** for Player 2:
    - When Player 1 plays A, X gives a payoff of 1 and Y gives a payoff of 0.
    - When Player 1 plays B, X gives a payoff of 2 and Y gives a payoff of 1.
    - When Player 1 plays C, X gives a payoff of 3 and Y gives a payoff of 1.

In all cases, strategy X gives a higher payoff than strategy Y, so **Y is strictly dominated by X** for Player 2.

Step 3: Iterated Elimination of Dominated Strategies

1. **Eliminate Player 1's Strategy B**.
    - After removing B, Player 1 is left with strategies A and C.
2. **Eliminate Player 2's Strategy Y**.
    - After removing Y, Player 2 is left with strategy X only.

Now the game is reduced, and both players are left with their dominant strategies:

- Player 1's choice is between A and C.
- Player 2 will always choose X, since Y has been eliminated.

In this simplified game, Player 1 will now select the best response to Player 2's strategy X. Between A and C, C gives a higher payoff (4 compared to 2), so Player 1 will choose C. The final outcome of the game is:

- Player 1 chooses C, and Player 2 chooses X.

Identifying and eliminating dominated strategies helps in:

- **Simplifying complex games**: Reduces the number of strategies and narrows down the choices for players.
- **Finding Nash equilibria**: Once dominated strategies are removed, the remaining strategies can often lead directly to Nash equilibria.