Subject: Applied Mathematics IV **SEM:IV**

Penalty (Big-M or charne's) Method :-

If any one or some of the constraints are of greater than or equal type to convert the inequality to equality we have to subtract the surplus variables i.e) $-S_1, -S_2, \ldots$. But we would not get a unit matrix. To overcome this difficulty in addition to surplus variables we introduce artificial variables $A_1, A_2, \ldots$ with +ve sign to the constraints. In the objective function we assign big penalty by subtracting $MA_1, MA_2, \ldots$ if the objective function is of maximisation type.

Remark :-

* The artificial variable leave the process and the optimality condition is satisfied by the basic variables.

* Atleast one of the artificial variables remains in the basis with zero value and the optimality condition is satisfied. This the optimal basic feasible solution.

* Atleast one of the artificial variables remains in the basis with non-zero value and the optimality condition is satisfied. This is not an optimal solution since it contains large penalty M. This is not a solution but a pseudo-solution.

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① Using Penalty (Big-M) method solve the following LPP.

Maximise $Z = 3x_1 - x_2$

subject to $2x_1 + x_2 \leq 2$

$x_1 + 3x_2 \geq 3$

$x_2 \leq 4$

$x_1, x_2 \geq 0$.

Soln:- Since the second constraint is '$\geq$' type we introduce Artificial variable A_2 and big penalty in the objective function.

Maximize $z = 3x_1 - x_2 - 0S_1 - 0S_2 - 0S_3 - MA_2 \rightarrow (1)$

Subject to $2x_1 + x_2 + S_1 + 0S_2 + 0S_3 + 0A_2 = 2 \rightarrow (2)$

$x_1 + 3x_2 + 0S_1 - S_2 + 0S_3 + A_2 = 3 \rightarrow (3)$

$0x_1 + x_2 + 0S_1 + 0S_2 + S_3 + 0A_2 = 4 \rightarrow (4)$.

$x_1, x_2, S_1, S_2, A_2 \geq 0$

We eliminate MA_2 from the objective function by adding $M \times 2$ constraint to it.

$(1) + (2)M \Rightarrow z = (3+M)x_1 - (1-3M)x_2 - 0S_1 - MS_2 - 0S_3 - 0A_2 - 3M$.

$z - (3+M)x_1 + (1-3M)x_2 + 0S_1 + MS_2 + 0S_3 + 0A_2 = -3M$.

Simplex table

Iteration no	Basic Var.	x_1	x_2	S_1	S_2	S_3	A_2	RHS soln	Ratio
0	z	$-3-M$	$1-3M$	0	M	0	0	$-3M$	
A_2 leaves	S_1	2	1	1	0	0	0	2	$2/1=2$
x_2 enters	A_2	1	3^*	0	-1	0	1	3	$3/3=1$ ⇐
	S_3	0	1	0	0	1	0	4	$4/1=4$

⇑

	1	Z	$-10/3$	0	0	$1/3$	0	–	-1	
s_1 leaves		s_1	$5/3^*$	0	1	$1/3$	0	–	1	$3/5 \Leftarrow$
x_1 enters		x_2	$1/3$	1	0	$-1/3$	0	–	1	3
		s_3	$-1/3$	0	0	$1/3$	1	–	3	-9
			$\Uparrow$							
	2	Z	0	0	2	1	0	–	1	
		x_1	1	0	$3/5$	$1/5$	0	–	$3/5$	
		x_2	0	1	$-1/5$	$-2/5$	0	–	$4/5$	
		s_3	0	0	$1/5$	$2/5$	1	–	$16/5$	

$\therefore x_1 = 3/5 \quad x_2 = 4/5 \quad Z_{max} = 1.$

2) Using the penalty (Big M) method solve the following LPP Maximise $Z = 3x_1 + 2x_2$

subject to $2x_1 + x_2 \le 2$

$3x_1 + 4x_2 \ge 12$

$x_1, x_2 \ge 0.$

Soln:-

Maximise $z = 3x_1 + 2x_2 - 0S_1 - 0S_2 - MA_2 \rightarrow (1)$

subject to $2x_1 + x_2 + S_1 + 0S_2 + 0A_2 = 2 \rightarrow (2)$

$3x_1 + 4x_2 + 0S_1 - S_2 + A_2 = 12 \rightarrow (3)$.

Eliminate M in (1) by (3) x M + (1).

$\therefore z = (3+3M)x_1 + (2+4M)x_2 - 0S_1 - MS_2 - 0A_2 = 12M$

$\therefore z - (3+3M)x_1 - (2+4M)x_2 + 0S_1 + MS_2 + 0A_2 = -12M$

Simplex table

Iteration no	Basic Variables	x_1	x_2	S_1	S_2	A_2	RHS Soln	Ratio
0	z	$-3-3M$	$-2-4M$	0	M	0	$-12M$	
	S_1	2	1^*	1	0	0	2	2 ←
	A_2	3	4	0	-1	1	12	3
1	z	$1+5M$	0	$2+4M$	M	0	$4-4M$	
	x_2	2	1	1	0	0	2	
	A_2	-5	0	-4	-1	1	4.	

Since the artificial variable A_2 appears not at zero level and all entries in the row of z have M with positive co-efficient, feasible solution does not exist. The solution is called pseudo-optimum basic feasible solution.