Hidden Markov Model (HMM)

→ It is one of the most popular model for sequential and temporal data.

→ discrete time series data ≃ temporal data (similar to)

→ It is used in many AI, ML & NLP algorithm

→ It is used to compute the probability & distribution of event that can be observed.

→ Markov Process
chain
Property

★ Markov Process :-

→ It is a stochastic process in which the distribution of events is based on the present state / present event without knowing how the present state has arrived.

randomly changing system
future state depends on current state

x_1	x_2	x_3
past	current	future
$t-1$	t	$t+1$

x_3 depends on x_2
x_2 arrived from any other state / event which does not matter here.

$t+1$ depends on t event

$S = \{x_1, x_2, x_3\}$
State space / Set of states

N distinct state, $N = 3$

Event / System / state

Weather
- Sunny S
- Rainy R
- cloudy C

Today is sunny, tomorrow it can be rainy or cloudy.
If it is rainy, then next is cloudy then cloudy state depends only on rainy state, & not any other state.

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If one wants to predict that customer will buy which mobile product as next one, then

Customer product
- Nokia
- Samsung
- Iphone

So,
If currently the person is having Samsung model, which he will buy as next one either Iphone, Nokia or of Samsung only.
This is an example of Markov model.

Infant
- playing
- Eating
- Sleeping
- Crying.

With foll. events, if the baby is currently eating, what will be the probability that next he/she will be sleeping / crying / playing?

Tourist city visit
- A city
- B city
- C city

Probability of visit to cities A or B or C

Game result — Lose / Draw / Win

3 Possibilities, either lose / draw / win the game.

If I have following states / events

$$\begin{tikzcd}
x_{t-2} \arrow[r] & x_{t-1} \arrow[r] & x_t \arrow[r] \arrow[l, bend right, dashed] & x_{t+1} \arrow[r] \arrow[l, bend right, dashed] & x_{t+2} \\
t-2 & t-1 & t & t+1 & t+2
\end{tikzcd}$$

current

Sequence of events that are independent of time, not continuous, discrete.

$P(x_{t+1})$ depends on $P(x_t)$ — Conditional probability.

$P(x_t)$ depends on $P(x_{t-1})$

$P(x_t) =$

$$\therefore P_{ij} = P(x_t = j \mid x_{t-1} = i)$$

$P(x_{t+1}) =$

$P(x_{t+1} \mid x_t)$

This is called 'Markov Property'

Any chain of event following Markov property, that chain is called 'Markov Chain'.

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Let $\{x_0, x_1, x_2, \ldots\}$ be a sequence of discrete random variable, then $\{x_0, x_1, x_2, \ldots\}$ is a Markov chain if it satisfies the Markov property which is

$$P(X_{t+1} = S \mid x_t = S_t, \cancel{\ldots\ldots X_0 = S_0})$$

$$= P(X_{t+1} = S \mid X_t = S_t)$$

This is called First Order Markov Model
x_{t+1} depends on immediate X_t

Second order MM.

here x_t depends on x_{t-1} & x_{t-2}

$$P(X_t \mid \underbrace{x_{t-1}, X_{t-2}}_{2nd\ order})$$

$$P(X_t \mid \underbrace{\_, \_, \_}_{3rd\ order})$$

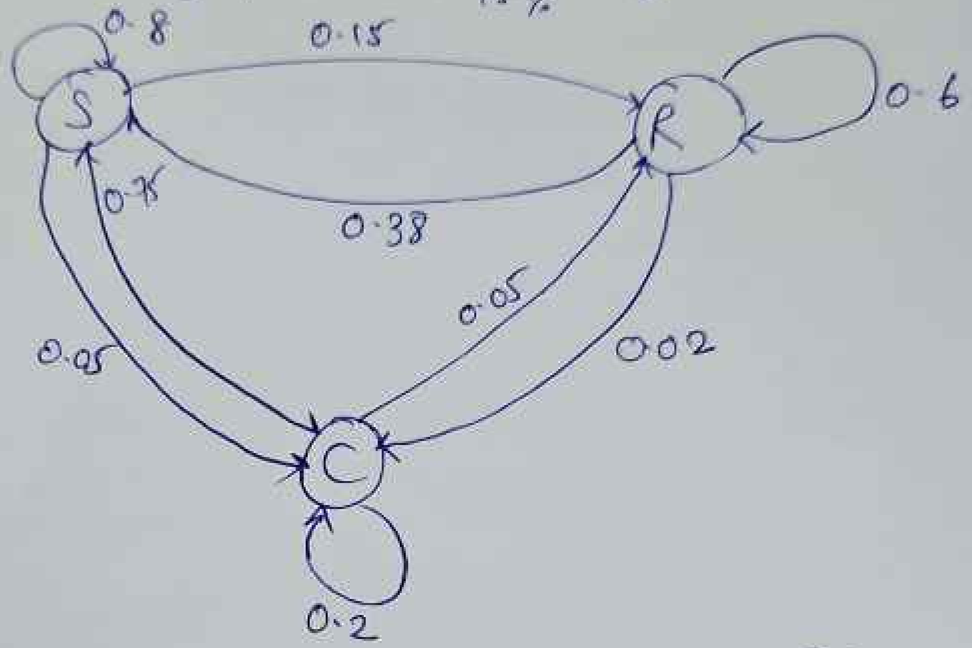
Example

Weather

3 event :- Sunny, Rainy, Cloudy.

15%

8%



$0.8 + 0.15 + —$
$= 1$

① This is called "State Transition Diagram"

Here,

② $S = \{S, R, C\}$

③ Initial State Distribution (/ Probability)

$$\pi = \{0.7, 0.25, 0.05\}$$
$\downarrow$ S, $\downarrow$ R, $\downarrow$ C

$$P(t_1 \dots t_n) = \prod_{i=1}^{n} P(t_i \mid t_{i-1})$$

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→ Future State →

$$
\begin{array}{c|c}
 & \begin{array}{ccc} S & R & C \end{array} \\
\begin{array}{c} S \\ R \\ C \end{array} & \begin{bmatrix} 0.8 & 0.15 & 0.05 \\ 0.38 & 0.6 & 0.02 \\ 0.75 & 0.05 & 0.2 \end{bmatrix}
\end{array}
$$

↓
current state

"Transition Matrix" of Markov chain

Each value ⇒ is called transition probability.

Eg. $P_{ij} = P(x_{t+1} = j \mid x_t = i)$

Each row = 1

N state ⇒ N×N Matrix here 3×3 matrix for 3 state.

Given that today the weather is sunny (S), what is the probability that tomorrow is Sunny (S) & day after is Rainy (R).

Soln.- $t_1 = S$ (Today is Sunny), $t_2 = S$, $t_3 = R$

hint
(Side Left - past
Side Right - future)

$$P(t_3 = R, t_2 = S \mid t_1 = S)$$

$$= P(t_3 = R \mid t_2 = S) \times P(t_2 = S \mid t_1 = S)$$

$$= 0.15 \times 0.8$$

$$= 0.120$$

= 12% probability to fulfil the question.

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Q. Given that today the weather is cloudy (C) & yesterday was Rainy (R). What is the probability that tomorrow would be Sunny (S) ?

Soln :-

$t_1 = R \qquad t_2 = C \qquad t_3 = S$

$P(t_3 = S \mid t_2 = C, t_1 = R)$

$= P(t_3 = S \mid t_2 = C) \times P(t_2 = C \mid t_1 = R)$

$= 0.75 \times 0.02$

$= 0.0150$

$= 1.5\%$

iii] What is probability of given series

$$\boxed{S}\ \boxed{R}\ \boxed{R}\ \boxed{R}\ \boxed{C}\ \boxed{C}$$

(Probability that this sequence would follow)

$$= P(S) \times P(R|S) \times P(R|R) \times P(R|R) \times P(C|R) \times P(C|C)$$

$$= 0.7 \times 0.15 \times 0.6 \times 0.6 \times 0.02 \times 0.2$$

$$= 0.0001512$$