University of Mumbai

Question Bank for May / June (summer) 2022 End Semester Theory Examination

(2^{nd} Half of A.Y. 2021-2022)

End Semester (Theory) Examinations Commencing from 17^{th} May 2022 to 31^{st} May 2022

Computer Engineering/Information Technology/Artificial Intelligence and Data Science/Artificial Intelligence and Machine Learning/Computer Science & Engineering (Data Science)

Curriculum Scheme: Rev 2019 'C' Scheme
Course Name: Engineering Mathematics IV
Semester: IV

	Multiple Choice Questions
1.	Eigen value of matrix A= $\begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}$ is,
Option A:	2 and 3
Option B:	2 and 1
Option C:	4 and 1
Option D:	4 and 2
2.	Two of the eigen values of 3×3 matrix are 3 and 4. IF the determinant of the matrix is 24, find its third eigen value.
Option A:	12
Option B:	2
Option C:	6
Option D:	3
3.	If the line integral $\int f(z)dz$ along the parabola $y = 2x^2$ then
Option A:	dz=(1-4xi)dx
Option B:	dz=(2+4xi)dx
Option C:	dz=(1+4xi)dx
Option D:	dz=(2-4xi)dx
4.	If $f(z) = \frac{3z+1}{z^2(z-2)}$ then the simple poles f(z) becomes
Option A:	0
Option B:	2
Option C:	3
Option D:	4
5.	In LPP if there are four variables and two constraints then the number of basic solutions becomes
Option A:	6
Option B:	3
Option C:	2
Option D:	4
6.	In NLPP which of the following is the Lagrange's function
Option A:	L=f/λh
Option B:	L=f+λh
Option C:	L=f*λh
Option D:	L=f-λh

7.	IF X follows Poisson distribution and P(x=2)=3P(x=1) then find the value of mean
Option A:	3
Option B:	4
Option C:	5
Option D:	6
8.	IF X is a random variable for the normal distribution with mean 10 and standard deviation 4 then find Z when X=16
Option A:	0.25
Option B:	1.5
Option C:	0.5
Option D:	0.8
9.	The z-transform of Discrete unit step function for k≥0 is
Option A:	$\frac{z}{(z-1)}$
Option B:	$\frac{z}{(z+2)}$
Option C:	$\frac{z}{(z-2)}$
Option D:	$\frac{z}{(z+1)}$
10.	Which of the following is Null hypothesis for two tail test
Option A:	$\mu \neq \mu 0$
Option B:	$\mu > \mu 0$
Option C:	$\mu = \mu 0$
Option D:	$\mu < \mu 0$
11.	Sum of eigen value of A $= \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ is
Option A:	11
Option B:	9
Option C:	6
Option D:	8
12.	If $A = \begin{bmatrix} 3 & 2 & 3 \\ 0 & -2 & 5 \\ 0 & 0 & -1 \end{bmatrix}$, find the eigen values of $A^3 + 5A + 8I$
Option A:	30, -12, 4
Option B:	25, 15, -10
Option C:	50, -10, 2
Option D:	40, -15, 12
13.	Find Z – Transform of $f(k) = 5^k, k < 0$
Option A:	$\frac{5}{z-5}$
Option B:	$\frac{z}{z-5}$

Option C:	$\frac{5}{5-z}$
Option D:	$\frac{z}{5-z}$
14.	If f(z)= $\frac{z+3}{(z-2)(z-4)}$ then $\int_C f(z)dz = 0$ if
Option A:	C is the circle $\lvert z\rvert = 1$
Option B:	C is the circle $\lvert z\rvert = 3$
Option C:	C is the circle $\lvert z-1\rvert = 2$
Option D:	C is the circle $\lvert z-4\rvert = 1$
15.	The singularities of f(z) $= \frac{z+3}{z^2+9}$ are
Option A:	3 and -3
Option B:	3 and 3i
Option C:	3 and -3i
Option D:	3i and -3i
16.	For a Poisson Distribution if P(X = 2) = P(X = 3) , then its variance is
Option A:	0
Option B:	3
Option C:	2
Option D:	1
17.	The probability density function f(x) of normal distribution is
Option A:	$\frac{1}{\sigma\sqrt{2\pi}} e^{-\left(\frac{X-\mu}{\sigma}\right)^2}$
Option B:	$\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}$
Option C:	$\frac{1}{\sigma\sqrt{\pi}} e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}$
Option D:	$\frac{1}{2\sigma\sqrt{\pi}} e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}$
18.	In the example of testing of hypothesis with 65 as mean of the population and n=10 , $\bar{x} = 67$, $s^2 = 8.8$ then the value of test statistics t is______
Option A:	2.65
Option B:	-1.75
Option C:	2.02
Option D:	-1.82
19.	Dual of Min. Z = $3x_1 + 7x_2$ subject to $x_1 + x_2 \geq 4$ $3x_1 + 2x_2 \geq 8$ where $x_1, x_2 \geq 0$ is
Option A:	Min. W = $4w_1 + 8w_2$ subject to $w_1 + 3w_2 \geq 3$ $w_1 + 2w_2 \geq 7$ where $w_1, w_2 \geq 0$
Option B:	Max. W = $4w_1 + 8w_2$ subject to

	$w_1 + 3w_2 \geq 3$ $w_1 + 2w_2 \geq 7$ where $w_1, w_2 \geq 0$
Option C:	Min. $W = 4w_1 + 8w_2$ subject to $w_1 + 3w_2 = 3$ $w_1 + 2w_2 = 7$ where $w_1, w_2 \geq 0$
Option D:	Max. $W = 4w_1 + 8w_2$ subject to $w_1 + 3w_2 \leq 3$ $w_1 + 2w_2 \leq 7$ where $w_1, w_2 \geq 0$
20.	If the problem is standard primal form of minimization then all the constraints involve the sign
[redacted]	[redacted]
Option B:	$\geq$
Option C:	$>$
Option D:	$<$

	Descriptive Questions
1	Find Eigen values of the matrix $A^3 - 4(adj.A) + 5I$ if $$A = \begin{bmatrix} 3 & 2 & 5 \\ 0 & -2 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$
2	Evaluate $\int_{1-i}^{2+i} (2x + iy + 1)dz$ along the curve $x=t+1$, $y=2t^2-1$
3	Find the z-transform of $\{\sin 5k\}$, $k \geq 0$
4	A random sample size of 16 from a normal population showed a mean of 103.75cm. and sum of squares of deviation from the mean 843.75cm2. Can we say that the population has a mean of 108.75cm?
5	Solve the following NLPP by Kuhn-Tucker conditions $Minimise \quad z = {x_1}^3 - 4x_1 - 2x_2$ subject to $x_1 + x_2 \leq 1, \quad x_1, x_2 \geq 0$
6	Write the dual of the following LPP $Maximise \quad z = 2x_1 - x_2 + 4x_3$ subject to $x_1 + x_2 + 2x_3 \leq 12,$ $2x_1 - x_3 \leq 4$ $2x_1 - x_2 - 3x_3 \leq 5, \quad , x_1, x_2, x_3 \geq 0$
7	Find Eigen values and Eigen vectors A if $$A = \begin{bmatrix} 2 & 3 & 4 \\ 0 & 2 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$
8	Evaluate $\oint_c \frac{(1-2z)}{z(z-2)(z-1)} dz$ where $C: \mid z \mid = 1.5$ using Cauchy Residue theorem.

<table>
<tr><td>9</td><td>The number of accident on a particular highway in a month is a Poisson variate with parameter 5. Find the probability that more than 2 accidents have occurred on the road in a given month.</td></tr>
<tr><td>10</td><td>Find all basic solutions of the following system of equations. Also find basic feasible solutions 2x+y+4z=11, 3x+y+5z=14</td></tr>
<tr><td>11</td><td>Using the method of Lagrange's Multiplier's solve the following NLPP
$Optimise \quad z = 4x_1 + 8x_2 - {x_1}^2 - {x_2}^2$
subject to $x_1 + x_2 = 4, \quad x_1, x_2 \geq 0$</td></tr>
<tr><td>12</td><td>Find the z-transform of $\{ 2^{|k|} \}$</td></tr>
<tr><td>13</td><td>Verify Cayley Hamilton theorem and find inverse of A
If $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$</td></tr>
<tr><td>14</td><td>Obtain Laurent's series expansion $of\ f(z) = \frac{z-1}{z^2-2z-3}\ for\ 1 <\mid z \mid< 3.$</td></tr>
<tr><td>15</td><td>$Find\ inverse\ Z\text{-}transform\ of\ f(z) = \frac{z}{(z-1)(z-2)},\ \mid z \mid> 2$</td></tr>
<tr><td>16</td><td>The weight of 4000 students are found to be Normally distributed with mean 50kgs. And Standard deviation 5kgs. Find the probability that a student selected at random will have weight between 45 and 60kgs.</td></tr>
<tr><td>17</td><td>Solve the following LPP by using simplex method
$Maximise \quad z = 3x_1 + 2x_2,$
subject to $x_1 + x_2 \leq 4,$
$x_1 - x_2 \leq 2\ ,\ x_1, x_2 \geq 0$</td></tr>
<tr><td>18</td><td>Solve the following NLPP
$Miximise \quad z = 2x_1 + 3x_2 - {x_1}^2 - {x_2}^2$
subject to $x_1 + x_2 \leq 1,\ 2x_1 + 3x_2 \leq 6, \quad x_1, x_2 \geq 0$</td></tr>
<tr><td>19</td><td>Find eigen values and eigen vectors for the matrix $A = \begin{bmatrix} 3 & 1 & -1 \\ 2 & 2 & -1 \\ 2 & 2 & 0 \end{bmatrix}$</td></tr>
<tr><td>20</td><td>Evaluate $\int_0^{1+i} (x^2 - iy)\, dz$ along the path $y = x$</td></tr>
<tr><td>21</td><td>Find Z-Transform of $f(k) = \begin{cases} 4^k & ; \ k < 0 \\ 5^k & ; \ k \geq 0 \end{cases}$</td></tr>
<tr><td>22</td><td>A die was thrown 132 times and the following frequencies were observed
<table>
<tr><td>No. Obtained</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>Total</td></tr>
<tr><td>Frequency</td><td>15</td><td>20</td><td>25</td><td>15</td><td>29</td><td>28</td><td>132</td></tr>
</table>
Test the hypothesis that die is unbiased.</td></tr>
<tr><td>23</td><td>Use dual simplex method to solve the following L.P.P.
$\text{Manimize}\ \ z = x_1 + x_2$
$\text{subject to}\ \ 2x_1 + x_2 \geq 2$
$-x_1 - x_2 \geq 1$
$x_1,\ x_2 \geq 0$</td></tr>
<tr><td>24</td><td>Using Lagrange's Multipliers method, solve the following N.L.P.P.
Optimize $z = 6\,{x_1}^2 + 5{x_2}^2$</td></tr>
</table>

	Subject to $x_1 + 5x_2 = 7$ $x_1, x_2 \geq 0$
25	Find the matrix represented by the $A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$, where $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$
26	Find Laurent's series expansion of $f(z) = \frac{1}{(z-1)(z-2)}$ in the regions $\|z\| < 1$ & $\|z\| > 2$
27	Find $Z\{f(k)\}$, where $f(k) = \frac{1}{2^k} * \frac{1}{3^k}$; $k \geq 0$
28	An insurance company found that only 0.01% of the population is involved in a certain type of accident each year. If its 1000 policy holders were randomly selected from the population, what is the probability that no more than two of its clients are involved in such accident next year?
29	Determine all basic solutions to the following problem. Maximise $z = x_1 + 3x_2 + 3x_3$ subject to $x_1 + 2x_2 + 3x_3 = 4$ $2x_1 + 3x_2 + 5x_3 = 7$ Which of them are basic feasible, non – degenerate, infeasible basic and optimal basic feasible solutions?
30	Using Lagrange's Multipliers method, solve the following N.L.P.P. Optimize $z = x_1^2 + x_2^2 + x_3^2 - 10x_1 - 6x_2 - 4x_3$ Subject to $x_1 + x_2 + x_3 = 7$ $x_1, x_2, x_3 \geq 0$
31	Show that the matrix $A = \begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$ is diagonalizable. Find the diagonal form D and the diagonalizing matrix M.
32	Using Cauchy's Residue Theorem evaluate $\int_C \frac{z^2+3}{z^2-1}\, dz$, where C is the circle $\|z - 1\| = 1$
33	Find the Inverse Z – Transform of $\frac{1}{(z-2)(z-3)}$ in the region $2 < \|z\| < 3$
34	In a normal distribution 7% of items are under 35 and 89% items are under 63. What are the mean and standard deviation?
35	Solve the following linear programming problem by simplex method $\text{Max. } z = 3x_1 + 2x_2$ subject to $x_1 + x_2 \leq 4$ $x_1 - x_2 \leq 2$ $x_1, x_2 \geq 0$
36	Use Kuhn – Tucker conditions to solve the following N.L.P.P. Maximize $z = 2x_1^2 - 7x_2^2 + 12x_1x_2$ Subject to $2x_1 + 5x_2 \leq 98$ $x_1, x_2 \geq 0$