

N-queens Problem using backtracking

①

Q₁ Q₂ Q₃ Q₄

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | | | | |
| 2 | | | | |
| 3 | | | | |
| 4 | | | | |

2

| 1 | 2 | 3 | 4 |
|---|---|---|---|
| | | | |

sol'n
vector

- 1) Horizontal
- 2) vertical
- 3) diagonally

Place all queens on the board so that no queen is under attack.

When the queen is under attack if they are in same row, same column or same diagonal

¹⁶
C₄ — Lot many possibilities

- ① The constraint to be added to reduce the possibilities
- 1st row — 1st queen
 - 2nd row — 2nd queen

Now we just need to decide the column for each queen so that they are not under attack.

Problem is reduced.

- ② Two queens should not be in the same column.

So attacks

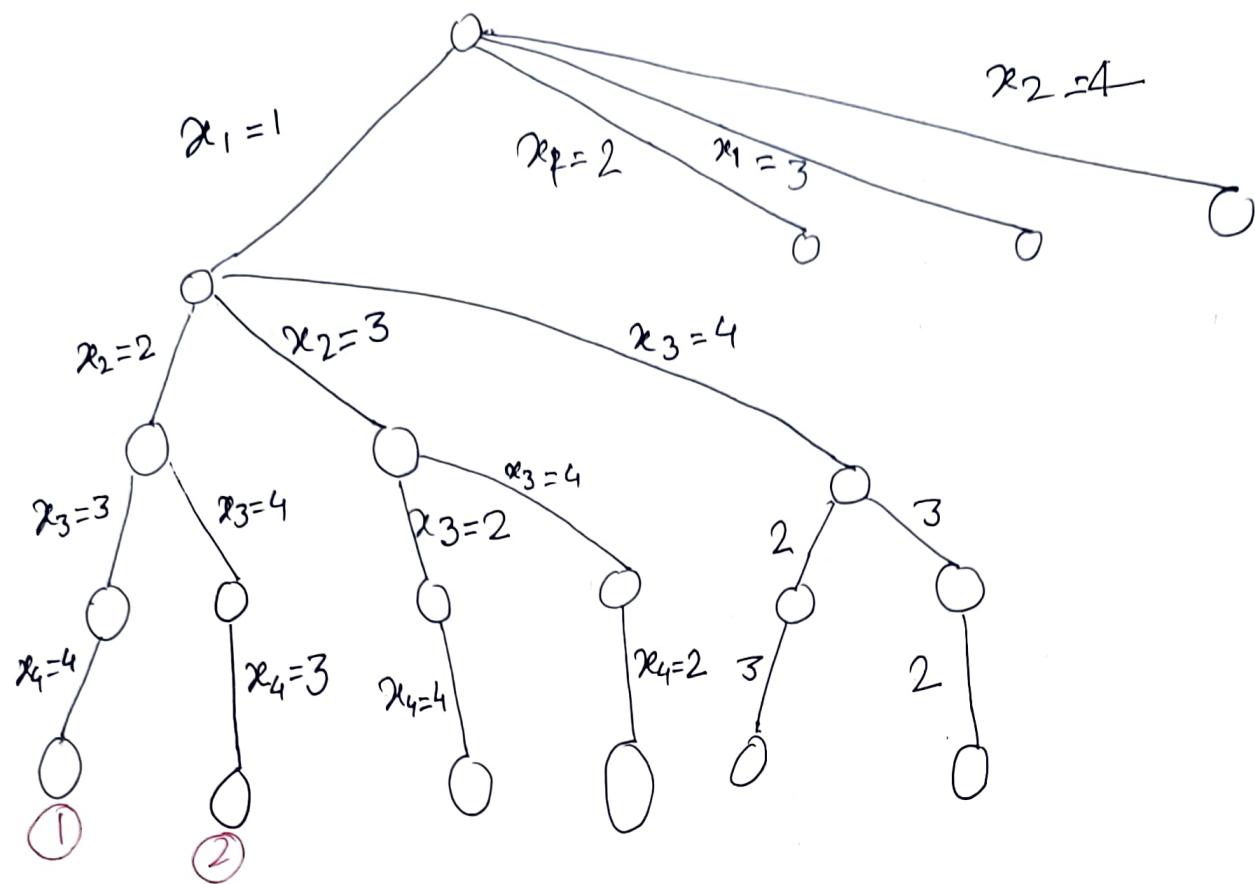
~~Same row~~

~~Same column~~

same diagonal

Now check if the queens are on the same diagonal or not.

State space tree (without considering attack)

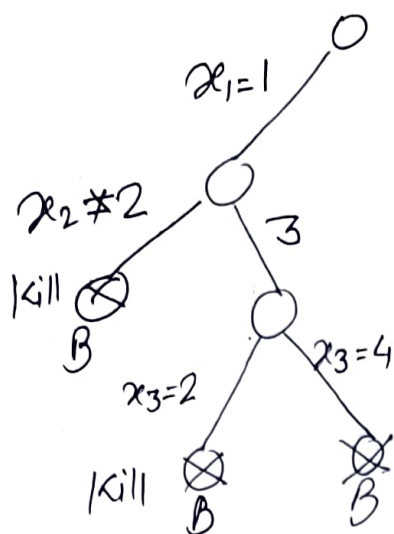


$$1 + 4 + 4 \times 3 + 4 \times 3 \times 2 + 4 \times 3 \times 2 \times 1$$

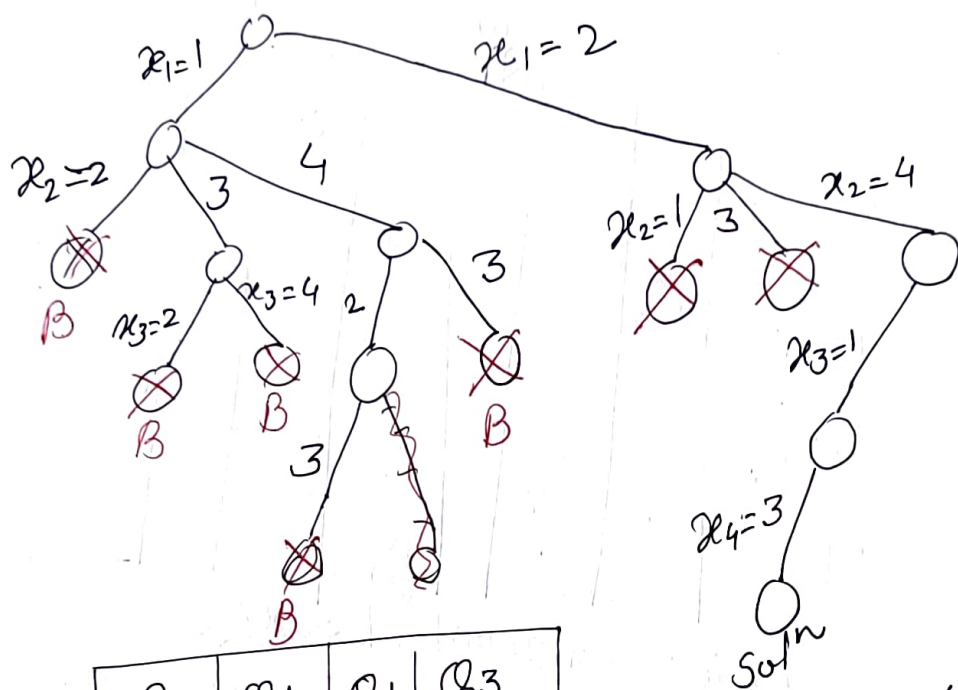
$$= 1 + \sum_{i=0}^3 4 + 12 + 24 + 24 = 65 \text{ (max}^m \text{ no of nodes)}$$

Bounding funⁿ = queens should not be in same row, same col^m & same diagonal.

Bounding funⁿ = row, col^m, diagonal



No place for queen 3 so move second queen



Solⁿ

| Q2 | Q4 | Q1 | Q3 |
|-------|-------|-------|-------|
| 2 | 4 | 1 | 3 |
| col 1 | col 2 | col 3 | col 4 |

| | 1 | 2 | 3 | 4 |
|----|----|----|----|----|
| Q1 | | | Q1 | |
| Q2 | Q2 | | | |
| Q3 | | | | Q3 |
| Q4 | | Q4 | | |

Solⁿ 1

| | 1 | 2 | 3 | 4 |
|---|----------------|----------------|----------------|----------------|
| 1 | | Q ₁ | | |
| 2 | | | | Q ₂ |
| 3 | Q ₃ | | | |
| 4 | | | Q ₄ | |

| 1 | 2 | 3 | 4 |
|---|---|---|---|
| 2 | 4 | 1 | 3 |

Solⁿ 2 mirror image of Solⁿ 1

| | 1 | 2 | 3 | 4 |
|---|----------------|----------------|----------------|----------------|
| 1 | | | Q ₁ | |
| 2 | Q ₂ | | | |
| 3 | | | | Q ₃ |
| 4 | | Q ₄ | | |

| 1 | 2 | 3 | 4 |
|---|---|---|---|
| 3 | 1 | 4 | 2 |

8 queen problem

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|------------------|----------------|------------------|------------------|------------------|---|------------------|------------------|
| 1 | | | | Q ₁ ① | . | . | Q ₂ ③ | . |
| 2 | | | | | | | | Q ₃ ④ |
| 3 | | | Q ₄ ⑤ | | | | | . |
| 4 | . | Q ₄ | | | | | Q ₅ ⑥ | . |
| 5 | . | | | | | | | |
| 6 | Q ₆ ⑦ | | Q ₇ ② | | | | | |
| 7 | | | | | Q ₈ ⑧ | | | |
| 8 | | | | | Q ₈ ② | | | |

explicit constraint $1 \leq x_i \leq 4$

implicit constraint

No 2 x_i can be same
or
all queens must be placed
on different columns
No 2 queens can be on
the same diagonal

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|
| 4 | 6 | 8 | 2 | 7 | 1 | 3 | 5 |