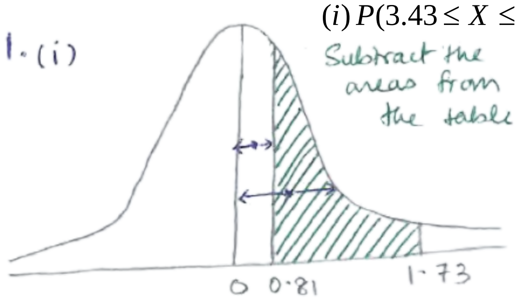
Problems on Normal Distribution

1. For a normally distributed variate X with mean 1 and s.d 3, find
$(i) P(3.43 \le X \le 6.19)$ (ii) $P(-1.43 \le X \le 2.3)$

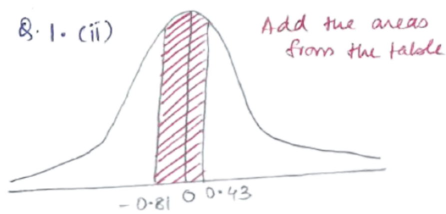
Solution: Given: $X \sim N(\mu = 1, \sigma = 3)$

(Check Figures 3.13.1)



$$(i)\, P(3.43 \le X \le 6.19) = P\left(\frac{3.43-\mu}{\sigma} \le \frac{X-\mu}{\sigma} \le \frac{6.19-\mu}{\sigma}\right)$$
$$= P\left(\frac{3.43-1}{3} \le Z \le \frac{6.19-1}{3}\right)$$
$$= P(0.81 \le Z \le 1.73)$$
$$= 0.4582 - 0.2910 \quad (\textit{from the tables})$$
$$= 0.1672$$

$$(ii)\ P(-1.43 \le X \le 2.3) = P\left(\frac{-1.43-\mu}{\sigma} \le \frac{X-\mu}{\sigma} \le \frac{2.3-\mu}{\sigma}\right)$$
$$= P\left(\frac{-1.43-1}{3} \le Z \le \frac{2.3-1}{3}\right)$$
$$= P(-0.81 \le Z \le 0.43)$$
$$= 0.2910 + 0.1664 \quad (\textit{from the tables})$$
$$= 0.4574$$



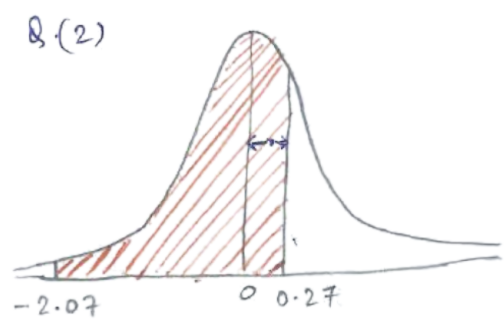
2. The mean height of 500 students is 151 cm and the s.d is 15 cm. Assuming that the heights are normally distributed, find the number of students whose heights lie between 120 and 155 cm.

Solution: To find the number of students whose heights lie between 120 and 155 cm, we first find the probability that a student's height lies between 120 and 155 cm.

Now, let X denote the heights of students

Given: $X \sim N(\mu = 151, \sigma = 15)$

(Check Figures 3.13.2)



$$\therefore P(120 \le X \le 155) = P\left(\frac{120-\mu}{\sigma} \le \frac{X-\mu}{\sigma} \le \frac{155-\mu}{\sigma}\right)$$
$$= P\left(\frac{120-151}{15} \le Z \le \frac{155-151}{15}\right)$$
$$= P(-2.07 \le Z \le 0.266)$$
$$= 0.4808 + 0.1064 \quad (\textit{from the tables})$$
$$= 0.5872$$

Hence no. of students whose heights lie between 120 and 155 cm $= 500 * 0.5872$
$$= 293.6 \approx 294$$

Prof. Anushri Tambe

3. In a normal distribution, 31% of the items are under 45 and 8% of items are over 64. Find the mean and s.d of the distribution.

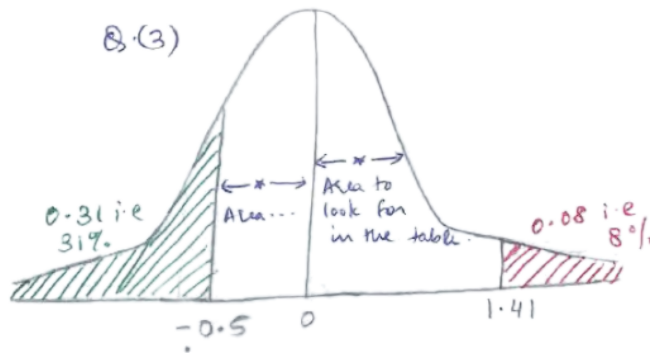
Solution: Let X denote the r.v. representing the distribution of the items. Suppose $X \sim N(Mean = \mu, s.d = \sigma)$. To find μ *and* σ

Now, given:

(Check Figures 3.13.3)

(i) 31% *of items are under* 45

$\Rightarrow P(X < 45) = 0.31 \quad .....(A)$



(ii) 8% *of items are over* 64

$\Rightarrow P(X > 64) = 0.08 \quad ....(B)$

$$(A) \Rightarrow P\left(\frac{X-\mu}{\sigma} < \frac{45-\mu}{\sigma}\right) = 0.31$$

$$\Rightarrow P\left(Z < \frac{45-\mu}{\sigma}\right) = 0.31$$

Let $z_1 = \dfrac{45-\mu}{\sigma} \quad ....(i)$

Then, $P(Z_1 < Z < 0) = 0.5 - .31 = 0.19$

From the tables, we get $z_1 = -0.5 \quad ......(1)$

$$(B) \Rightarrow P\left(\frac{X-\mu}{\sigma} > \frac{64-\mu}{\sigma}\right) = 0.08$$

$$\Rightarrow P\left(Z > \frac{64-\mu}{\sigma}\right) = 0.08$$

Let $z_2 = \dfrac{64-\mu}{\sigma} \quad .....(ii)$

Then, $P(0 < Z < Z_2) = 0.5 - .08 = 0.42$

From the tables, we get $z_2 = 1.41 \quad ......(2)$

Using (i),(1),(ii) and (2) we get,

$\dfrac{45-\mu}{\sigma} = -0.5 \quad ....(I)$ and $\dfrac{65-\mu}{\sigma} = 1.41 \quad ....(II)$

Solving (I) and (II) we get $\mu = 50$ *and* $\sigma = 10$

Prof. Anushri Tambe