Semester : III Subject : DSGT Academic Year: 2022-2023

Module No - 4

Counting

* Basic Counting Principles -

* Sum Rule Principle -

We say a finite set S is partitioned into parts $S_1, \dots, S_k$ if the parts are disjoint and their union is S.

$S_i \cap S_j = \phi$ for $i \neq j$ & $S_1 \cup S_2 \cup \cdots S_k = S$

$|S| = |S_1| + |S_2| + \cdots + |S_k|$.

ex. Let S be the set of students attending the combination circuits lecture. It can be partitioned into parts S_1 & S_2 where,

S_1 = set of students those like easy examples

S_2 = set of students those don't like easy examples

if $|S_1| = 22$ and $|S_2| = 8$ then we can conclude $|S| = 30$.

Semester: III Subject: DSGT Academic Year: 2022-2023

* Product Rule Principle -

If S is a finite set S that is the product of S1, ..., Sk i.e. $S = S_1 \times S_2 \times \dots \times S_k$ then $|S| = |S_1| \times |S_2| \times \dots \times |S_k|$.

* Subtraction principle -

Let S be a subset of a finite set T. We define $\bar{S} = T \setminus S$

The complement of S in T.

Then $|\bar{S}| = |T| - |S|$.

e.g. If T is the set of students studying at MU and S the set of students studying neither math nor Computer science. If we know $|T| = 23905$ and $|S| = 20178$, then we can compute the number $|\bar{S}|$ of students studying either math or CS.

$|\bar{S}| = |T| - |S|$

$= 23905 - 20178$

$= 3727$

* Bijection Principle -

If S and T are finite sets, then $|S| = |T|$ there exist a bijection between S and T.

e.g. Let S be the set of students attending the

Semester: III Subject: DSGT Academic Year: 2022-2023

lecture and T the set of homework submissions for the first problem. If the no. of students and the number of submissions coincide, then there is a bijection between students & submissions.

* Principle of Inclusion-Exclusion -

The inclusion-exclusion principle is concerned with the number of elements in the set operations since in a set an element is to be counted once the principle states that while counting the elements in a situation if some elements are not counted already they are to be included and if some elements are already counted they are to be excluded.

e.g. If we want the no. of elements in A∪B denoted by n(A∪B).
we can add n(A) and n(B) & subtract n(A∩B) i.e in counting we 'include' n(A) and n(B) and 'exclude' n(A∩B) to avoid double counting because they are counted in n(A) as well as in n(B). This is inclusion-exclusion principle.

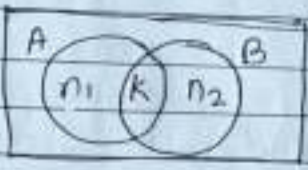
Semester: III Subject: DSGT Academic Year: 2022-2023

Inclusion-Exclusion principle -

Theorem 1 -

If A and B are two finite sets and $n(A)$, $n(B)$ denote the no. of elements in A and B then,

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$



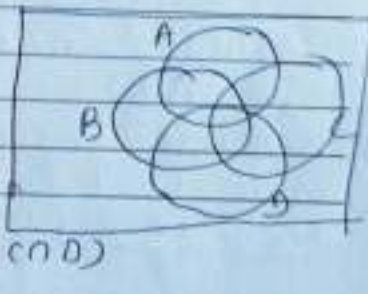
suppose, ~~A~~ A & B are 2 sets $n(A)$, $n(B)$ are the no. of elements in set A & B then.

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

If we have 3 sets then,

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$

If we have 4 sets - then,



$n(A \cup B \cup C \cup D) = n(A) + n(B) + n(C) + n(D) - n(A \cap B) - n(A \cap C) - n(A \cap D) - n(B \cap C) - n(B \cap D) - n(C \cap D) + n(A \cap B \cap C) + n(A \cap B \cap D) + n(A \cap C \cap D) + n(B \cap C \cap D) - n(A \cap B \cap C \cap D)$