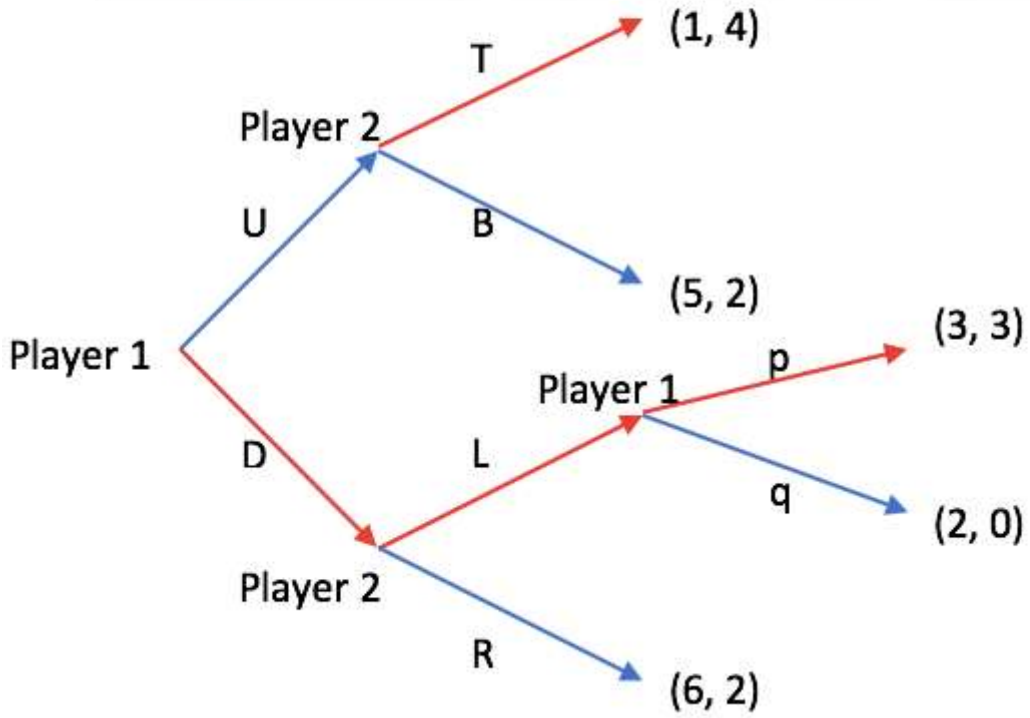
Subgame perfect equilibrium

In game theory, a subgame perfect equilibrium (or subgame perfect Nash equilibrium) is a refinement of a Nash equilibrium used in dynamic games. A strategy profile is a subgame perfect equilibrium if it represents a Nash equilibrium of every subgame of the original game. Informally, this means that at any point in the game, the players' behavior from that point onward should represent a Nash equilibrium of the continuation game (i.e. of the subgame), no matter what happened before. Every finite extensive game with perfect recall has a subgame perfect equilibrium ".

A common method for determining subgame perfect equilibria in the case of a finite game is backward induction. Here one first considers the last actions of the game and determines which actions the final mover should take in each possible circumstance to maximize his/her utility. One then supposes that the last actor will do these actions, and considers the second to last actions, again choosing those that maximize that actor's utility. This process continues until one reaches the first move of the game. The strategies which remain are the set of all subgame perfect equilibria for finite-horizon extensive games of perfect information.However, backward induction cannot be applied to games of imperfect or incomplete information because this entails cutting through non-singleton information sets.

Example

Determining the subgame perfect equilibrium by using backward induction is shown below in Figure 1. Strategies for Player 1 are given by {Up, Uq, Dp, Dq}, whereas Player 2 has the strategies among {TL, TR, BL, BR}. There are 4 subgames in this example, with 3 proper subgames.

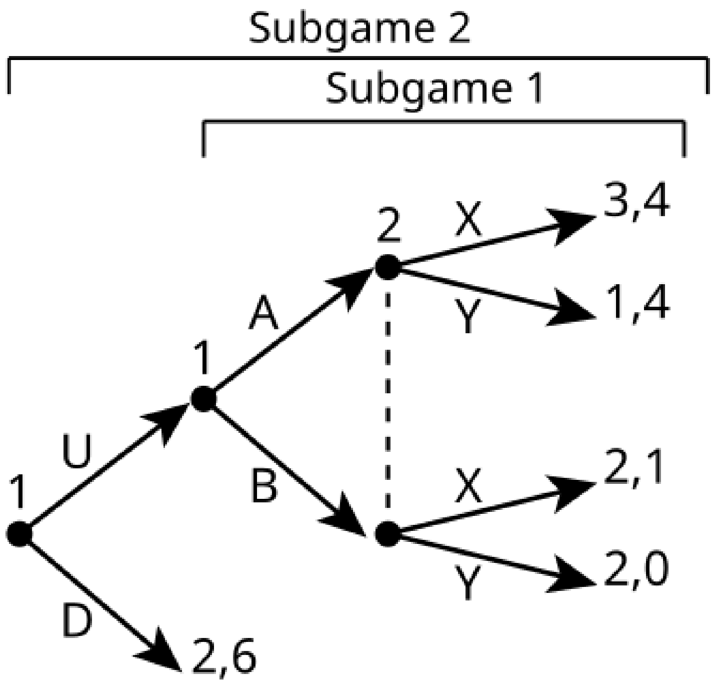


Using the backward induction, the players will take the following actions for each subgame:

- Subgame for actions p and q: Player 1 will take action p with payoff (3, 3) to maximize Player 1's payoff, so the payoff for action L becomes (3,3).
- Subgame for actions L and R: Player 2 will take action L for 3 > 2, so the payoff for action D becomes (3, 3).
- Subgame for actions T and B: Player 2 will take action T to maximize Player 2's payoff, so the payoff for action U becomes (1, 4).
- Subgame for actions U and D: Player 1 will take action D to maximize Player 1's payoff.

Thus, the subgame perfect equilibrium is {Dp, TL} with the payoff (3, 3).

An extensive-form game with incomplete information is presented below in Figure 2. Note that the node for Player 1 with actions A and B, and all succeeding actions is a subgame. Player 2's nodes are not a subgame as they are part of the same information set.



1 \ 2	X	Y
UA	3,4	1,4
UB	2,1	2,0
DA	2,6	2,6
DB	2,6	2,6

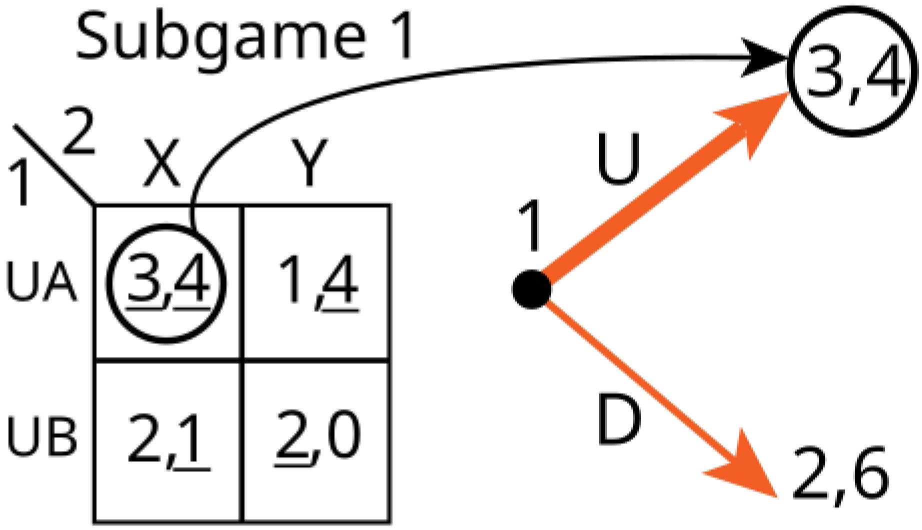
The first normal-form game is the normal form representation of the whole extensive-form game. Based on the provided information, (UA, X), (DA, Y), and (DB, Y) are all Nash equilibria for the entire game.

The second normal-form game is the normal form representation of the subgame starting from Player 1's second node with actions A and B. For the second normal-form game, the Nash equilibrium of the subgame is (A, X).

For the entire game Nash equilibria (DA, Y) and (DB, Y) are not subgame perfect equilibria because the move of Player 2 does not constitute a Nash equilibrium. The Nash equilibrium (UA, X) is subgame perfect because it incorporates the subgame Nash equilibrium (A, X) as part of its strategy.

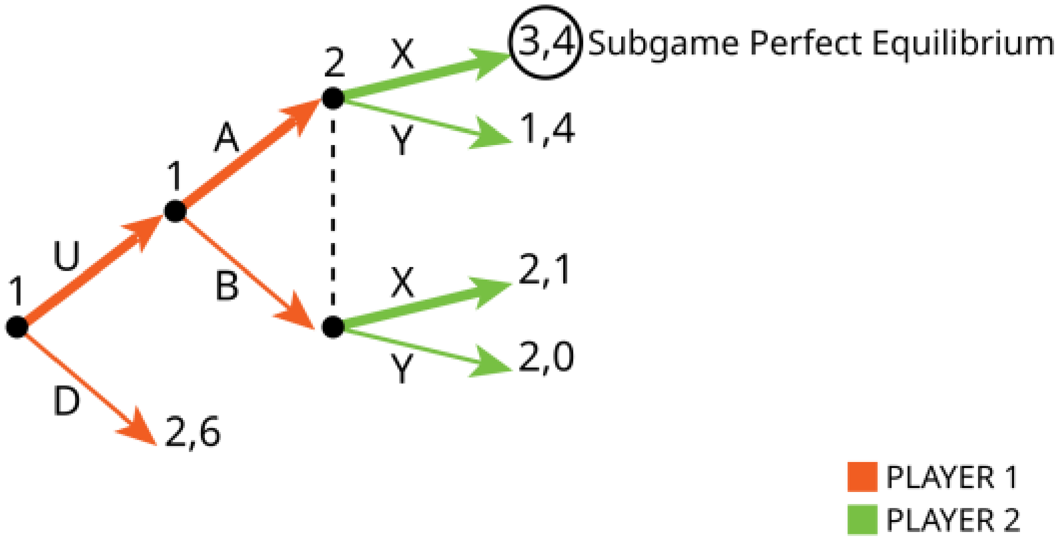
To solve this game, first find the Nash Equilibria by mutual best response of Subgame 1. Then use backwards induction and plug in (A,X) → (3,4) so that (3,4) become the payoffs for Subgame 2.

The dashed line indicates that player 2 does not know whether player 1 will play A or B in a simultaneous game.



Subgame 1 is solved and (3,4) replaces all of Subgame 1 and player one will choose U -> (3,4)Solution for Subgame 1

Player 1 chooses U rather than D because 3 > 2 for Player 1's payoff. The resulting equilibrium is (A, X) → (3,4).



Solution of Subgame Perfect Equilibrium

Thus, the subgame perfect equilibrium through backwards induction is (UA, X) with the payoff (3, 4).