Semester : IV Subject : Statistics for AIDS Academic Year: 2023-2024

F-Distribution :-

The F-Distribution is used to compare two variance.
It comes under ANOVA (Analysis of Variance).
Consider two populations, population 1 and population 2, we have to calculate the IQ of these population.

Population 1

The population variance is denoted by σ_1^2
But we will be selecting a sample and calculate sample mean n_1.
* Calculate sample variance s_1^2

Population 2

* The population variance is denoted by σ_2^2
* Calculate sample mean n_2.
* Calculate sample variance s_2^2.

Hypothesis Test will be comparing two variances.

F Statistic :-

To calculate F statistics,

$$F = \frac{s_1^2}{s_2^2}, \quad s_1 > s_2$$

s_1 is derived from the sample n_1,
s_2 is derived from the sample n_2.

Degree of freedom :-

$$df = (n-1).$$

$$df_1 = (n_1 - 1), \quad df_2 = (n_2 - 1)$$

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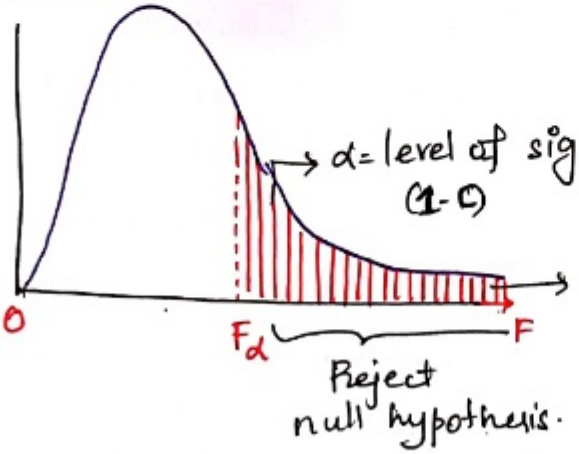
F-Distribution:



* F-Distribution is skewed to right.
* The values of $F > 0$.
* Shape of F distribution is determined by DOF_1 and dof_2.
  * DOF_1 → Degree of freedom of numerator
  * DOF_2 → Degree of freedom of denominator.

F-Test Problem:-

In a test given to two groups of students drawn from two normal populations, the marks obtained where as follows.

Group A	18	20	36	50	49	36	34	49	41
Group B	29	28	26	35	30	44	46		

Examine at 5% level of significance, whether two populations have the same variance.

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Solution :

Null Hypothesis, $H_0 : \sigma_1^2 = \sigma_2^2$. $[F = F_\alpha]$.

Alternate Hypothesis, $H_1 : \sigma_1^2 \neq \sigma_2^2$ (or) $[F \neq F_\alpha]$.
$\sigma_1^2 > \sigma_2^2$ (or) $[F > F_\alpha]$.

Calculate test statistics :

Group A X_1	$(X_1 - \bar{X}_1)$	$(X_1 - \bar{X}_1)^2$	Group B X_2	$(X_2 - \bar{X}_2)$	$(X_2 - \bar{X}_2)^2$
18	-19	361	29	-5	25
20	-17	289	28	-6	36
36	-1	1	26	-8	64
50	13	169	35	1	1
49	12	144	30	-4	16
36	-1	1	44	10	100
34	-3	9	46	12	144
49	12	144			
41	4	16			
$\Sigma X_1 = 333$		$\Sigma (X_1 - \bar{X}_1)^2 = 1134$	$\Sigma X_2 = 238$		$\Sigma (X_2 - \bar{X}_2)^2 = 386$

$\bar{X}_1 = \frac{\Sigma X_1}{n} = \frac{333}{9} = 37$, Variance, $s_1^2 = \frac{\Sigma (X_1 - \bar{X}_1)^2}{n-1} = \frac{1134}{9-1}$

$s_1^2 = 141.75$

$\bar{X}_2 = \frac{\Sigma X_2}{n} = \frac{238}{9} = 34$; Variance, $s_2^2 = \frac{\Sigma (X_2 - \bar{X}_1)^2}{n_2 - 1} = \frac{386}{7-1}$

$= \frac{386}{6} = 64.33$.

$s_2^2 = 64.33$

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F-Test :-

$$F = \frac{S_1^2}{S_2^2} = \frac{141.75}{64.33} = 2.203$$

Level of significance 0.05.

$dof_1 = 9-1 = 8$, $dof_2 = 7-1 = 6$.

$F_{0.05(8,7)} = 2.203$.

In this case $F = F_\alpha$.

$2.203 = 2.203$.

We accept the null hypothesis. The two population has the same variance.

Example 2:

In a sample of 9 observations, the sum of squared deviation of items from the mean was 64. In another sample of 11 observations, the value was found to be 88. Test whether the difference is significance at 5% level.

Solution:-

$n_1 = 9$, $\sum (x_1 - \bar{x_1})^2 = 64$.

$n_2 = 11$, $\sum (x_2 - \bar{x_2})^2 = 88$.

$H_0 : \sigma_1^2 = \sigma_2$

$H_1 : \sigma_1^2 \neq \sigma_2$.

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$$F = \frac{S_1^2}{S_2}, \quad S_1^2 > S_2^2.$$

$$S_1^2 = \frac{\Sigma (x_1 - \bar{x}_1)^2}{n-1} = \frac{64}{8} = 8.$$

$$S_2^2 = \frac{\Sigma (x_2 - \bar{x}_2)^2}{n-1} = \frac{88}{10} = 8.8.$$

Here $8.8 > 8$.

$$F = \frac{S_2^2}{S_1^2} = \frac{8.8}{8} = 1.1.$$

$$\boxed{F = 1.1}.$$

$dof_1 = 11 - 1 = 10$, $dof_2 = 10 - 1 = 9$.

$F_\alpha = 3.35$

$F < F_\alpha = 1.1 < 3.35$.

The null hypothesis is accepted.

Hence, the difference is not significant at 5% level.

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F-test (problem 3):

A research was conducted to understand whether women have a greater variation in attitude on political issue than men. Two independent samples of 31 men and 61 women were used for the study. The sample variance calculated were 130 and for women and 70 for men.
Test whether the difference in attitude towards political issues is difficult at 5% level of significance.

Solution:

$n_1 = 31$, $s_1^2 = 70$

$n_2 = 61$, $s_2^2 = 130$.

There is no significant diff. b/w the variance of women and variance of men.

$H_0 : \sigma_1^2 = \sigma_2^2$

$H_1 : \sigma_1^2 \neq \sigma_2^2$.

$F = \frac{s_1^2}{s_2^2}$, $s_1 > s_2$. in this case $\frac{s_2^2}{s_1^2}$

$F = \frac{130}{70} = 1.857$.

$dof_1 = 61 - 1 = 60$, $dof_2 = 31 - 1 = 30$.

$F_{0.05} = 1.74$.

$\Rightarrow F > F_{0.05}$ (i.e.) $1.857 > 1.74$. We reject null hypothesis. Hence, we understand that women has a greater variation in attitude on political issue than men.