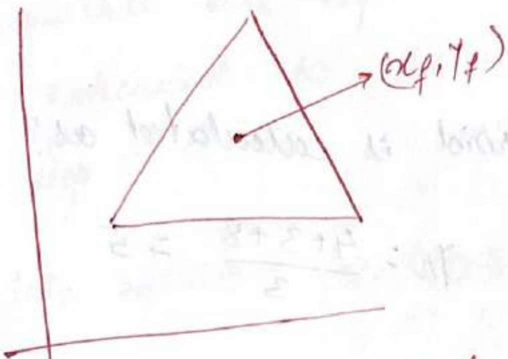
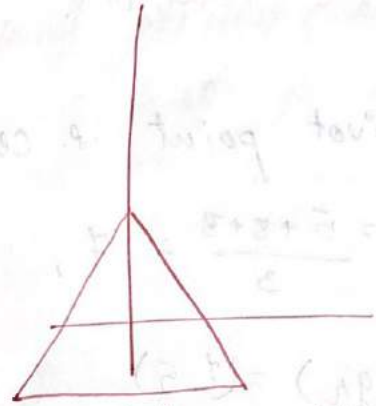
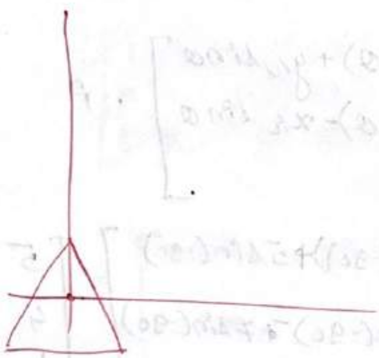
General fixed point scaling.



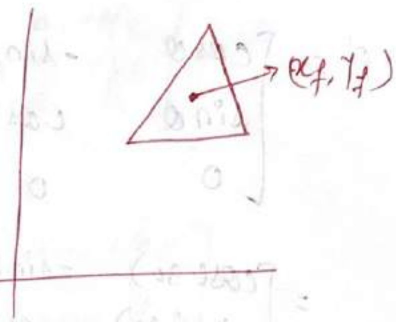
original position of object and fixed point.



Translate the object so that fixed point (x_f, y_f) is at origin



scale the object wrt origin



Translate the object so that fixed point is returned to position (x_f, y_f).

- For scaling object with position of one point called fixed point will remain same, we need to apply following sequence of transformations.

1. Translate the object so that the fixed point coincides with the co-ordinate origin.

2. Scale the object with respect to co-ordinate origin with specified scale factors

3. Translate the object so that fixed point is returned to its original position. (i.e. inverse of step 1)

. Matrix eqⁿ for this is:

$$P' = T(x_f, y_f) \cdot [S(s_x, s_y) \{T(-x_f, -y_f) \cdot P\}]$$

$$= \{T(x_f, y_f) \cdot S(s_x, s_y) \cdot T(-x_f, -y_f)\} \cdot P$$

$$= \begin{bmatrix} 1 & 0 & x_f \\ 0 & 1 & y_f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -x_f \\ 0 & 1 & -y_f \\ 0 & 0 & 1 \end{bmatrix} \cdot P$$

$$= \begin{bmatrix} s_x & 0 & x_f(1-s_x) \\ 0 & s_y & y_f(1-s_y) \\ 0 & 0 & 1 \end{bmatrix} \cdot P$$

$$P' = S(x_f, y_f, s_x, s_y) \cdot P$$

Here P' and P are column vector of final & initial point co-ordinates respectively and (x_f, y_f) are the co-ordinates of fixed point.

Example:

Q. Consider square with left-bottom corner at (2,2) and right-top corner at (6,6) apply the transformation which makes its size half such that its center remains same.

Sol.ⁿ center of square is:

$$x_f = 2 + \frac{6-2}{2} = 4, \quad y_f = 2 + \frac{6-2}{2} = 4$$

for the size to be half $s_x = 0.5$; $y_f = 0.5$

$$P' = \begin{bmatrix} s_x & 0 & x_f(1-s_x) \\ 0 & s_y & y_f(1-s_y) \\ 0 & 0 & 1 \end{bmatrix} \cdot P$$

$$= \begin{bmatrix} 0.5 & 0 & 4(1-0.5) \\ 0 & 0.5 & 4(1-0.5) \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 6 & 6 & 2 \\ 2 & 2 & 6 & 6 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.5 & 0 & 2 \\ 0 & 0.5 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 6 & 6 & 2 \\ 2 & 2 & 6 & 6 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 5 & 5 & 3 \\ 3 & 3 & 5 & 5 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

Final co-ordinates after scaling are A′(3,3)

B′(5,3)

C′(5,5)

D′(3,5)

Q. A traingle with vertex A(1,3) B(-2,4) C(3,-1) is scaled to its double size keeping point (-2,4) fixed. Find the transformation matrix to achive it.

Sol.ⁿ To double the size scaling factor will be

$S_x = S_y = 2$

$(x_f, y_f) = (-2, 4)$

$A(1,3) \quad B(-2,4) \quad C = (3,-1)$

$$P' = \begin{bmatrix} S_x & 0 & x_f(1-S_x) \\ 0 & S_y & y_f(1-S_y) \\ 0 & 0 & 1 \end{bmatrix} \cdot P$$

$$= \begin{bmatrix} 2 & 0 & -2(1-2) \\ 0 & 2 & 4(1-2) \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & -2 & 3 \\ 3 & 4 & -1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & 2 \\ 0 & 2 & -4 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -2 & 3 \\ 3 & 4 & -1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -2 & 8 \\ 2 & 4 & -6 \\ 1 & 1 & 1 \end{bmatrix}$$

Final co-ordinates after scaling are $A'(4,2)$

$B'(-2,4)$

$C'(8,6)$