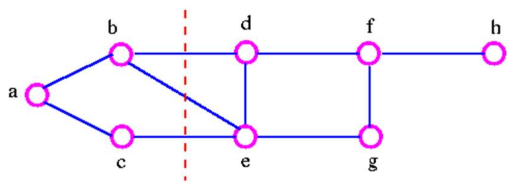
Cut Set

A cut set of a connected graph *G* is a set S of edges with the following properties

- The removal of all edges in S disconnects *G*.
- The removal of some (but not all) of edges in S does not disconnects *G*.

As an example consider the following graph



We can disconnect *G* by removing the three edges bd, bc, and ce, but we cannot disconnect it by removing just two of these edges. Note that a cut set is a set of edges in which no edge is redundant.

Cut-Vertex

A cut-vertex is a single vertex whose removal disconnects a graph.

It is important to note that the above definition breaks down if G is a complete graph, since we cannot then disconnects G by removing vertices. Therefore, we make the following definition.

Connectivity of Complete Graph

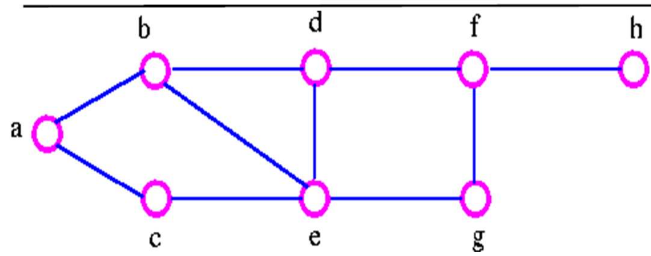
The connectivity $k(k_n)$ of the complete graph k_n is n-1. When $n-1 \geq k$, the graph k_n is said to be k-connected.

Vertex-Cut set

A vertex-cut set of a connected graph G is a set S of vertices with the following properties.

a. the removal of all the vertices in S disconnects G.
b. the removal of some (but not all) of vertices in S does not disconnects G.

Consider the following graph



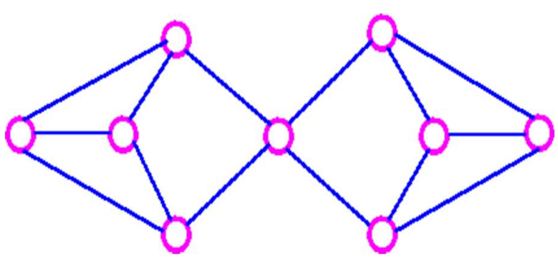
We can disconnects the graph by removing the two vertices b and e, but we cannot disconnect it by removing just one of these vertices. the vertex-cutset of G is {b, e}.

Note that the connectivity k(G) does not exceed the edge-connectivity $\lambda(G)$. This inequality holds for all connected graph.

Formally, for any connected graph G we have

$K(G) \leq \lambda(G) \leq \delta(G)$

where $\delta(G)$ is the smallest vertex-degree in G. But it is certainly possible for both inequality in above theorem to be strict inequalities (that is, $k(G) < \lambda(G) < \delta(G)$) For example, in the following graph,



$K(G)=1$, $\lambda(G) = 2$, and $\delta(G) = 3$.