2.1 Text Clustering

2.1.1 Feature Selection and Transformation Methods for Text Clustering

Feature selection and transformation are essential preprocessing steps in text clustering to enhance clustering quality and efficiency by reducing noise and dimensionality. These methods focus on identifying relevant features or creating new representations that capture meaningful patterns in the data.

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1. Feature Selection Methods

Feature selection aims to identify and retain only the most informative features (words or terms) for clustering. It reduces irrelevant, redundant, or noisy features to improve computational efficiency and clustering quality.

1.1 Document Frequency-Based Selection

- **Concept:** Removes words that occur too frequently (stop words) or too infrequently in the corpus.
- **Methodology:**
  - Stop words like "the," "is," and "of" are removed as they add noise but no meaningful information.
  - Very rare words, which contribute little to similarity calculations, are also eliminated.
- **Applications:** Commonly uses predefined stop-word lists and frequency thresholds for pruning.

1.2 Term Strength

- **Concept:** Measures the ability of a term to indicate the similarity between two documents.
- **Formula:**

$$s(t) = \frac{\text{Number of pairs where } t \text{ occurs in both}}{\text{Number of pairs where } t \text{ occurs in the first document}}$$

- **Procedure:**
  - Compute term strength by identifying related document pairs using cosine similarity or manual annotations.

    - Prune terms with low strength (compared to randomly distributed terms).
- **Advantage:** Does not require supervision, making it suitable for unsupervised clustering.

1.3 Entropy-Based Ranking

- **Concept:** Ranks terms based on the entropy reduction when the term is removed.
- **Formula for Entropy:**

$$E(t) = -\sum_{i=1}^{n}\sum_{j=1}^{n}(S_{ij} \cdot \log S_{ij} + (1 - S_{ij}) \cdot \log(1 - S_{ij}))$$

Where $S_{ij} = 2^{-\mathrm{dist}(i,j)/\mathrm{dist}}$ represents document similarity after removing the term t.

- **Procedure:**
    - Calculate the entropy for each term.
    - Remove terms that contribute the least to entropy reduction.
- **Efficiency:** Sampling methods can be used for large corpora to reduce computational overhead.

1.4 Term Contribution

- **Concept:** Measures how much a term contributes to the similarity between document pairs.
- **Procedure:**
    - Compute the similarity between documents using dot products of term frequencies.
    - Sum the contributions of each term across all document pairs.
- **Criticism:** May favor frequent terms without considering discriminative power.

---

2. Feature Transformation Methods

Feature transformation creates new representations of data by combining original features to form a reduced set of informative features.

2.1 Latent Semantic Indexing (LSI)

- **Concept:** Uses Singular Value Decomposition (SVD) to reduce the dimensionality of the term-document matrix.
- **Procedure:**
    - Compute SVD of the term-document matrix AA:

$$A = U\Sigma V^T$$

Where U, Σ, V^T are matrices representing terms, singular values, and documents.

 - Retain the top kk singular values and their corresponding components to form a reduced representation.
- **Benefits:** Reduces noise from synonymy and polysemy, enhancing semantic understanding in clustering.

2.2 Probabilistic Latent Semantic Analysis (PLSA)

- **Concept:** Models documents as mixtures of topics, with topics being distributions over words.
- **Procedure:**
  - Estimate the probabilities of words given topics P(w∣z) and topics given documents P(z|d) using Expectation-Maximization (EM).
  - Soft-cluster documents by assigning them probabilistic memberships across topics.
- **Application:** Suitable for capturing semantic relationships and overlapping clusters.

2.3 Non-Negative Matrix Factorization (NMF)

- **Concept:** Factorizes the term-document matrix A into two non-negative matrices U (document-cluster membership) and V (term-cluster relationship).
- **Objective Function:**

$$J = \frac{1}{2}\|A - UV^T\|_F^2 \quad \text{subject to } U \geq 0, V \geq 0$$

- **Procedure:**
  - Iteratively update U and V using multiplicative update rules:

$$u_{ij} \leftarrow u_{ij}\frac{(AV)_{ij}}{(UV^TV)_{ij}}, \quad v_{ij} \leftarrow v_{ij}\frac{(A^TU)_{ij}}{(VU^TU)_{ij}}$$

- **Benefits:** Produces interpretable clusters as each document is expressed as an additive combination of topics.

Key Takeaways

1. **Feature Selection** reduces irrelevant or noisy terms to simplify clustering.
2. **Feature Transformation** creates new, lower-dimensional representations that better capture semantic or structural patterns.
3. Both methods are essential for handling high-dimensional, sparse text data effectively.

2.2.2 Distance-Based Clustering Algorithm

Distance-based clustering algorithms group documents based on how similar or distant they are from each other. These algorithms often measure the distance between data points (in this case, documents) to form clusters of related items. One of the most common approaches to distance-based clustering is **K-means**, but there are other methods too, such as **DBSCAN**, **Agglomerative Hierarchical Clustering**, and **K-medoids**.

Basic Concept of Distance-Based Clustering:

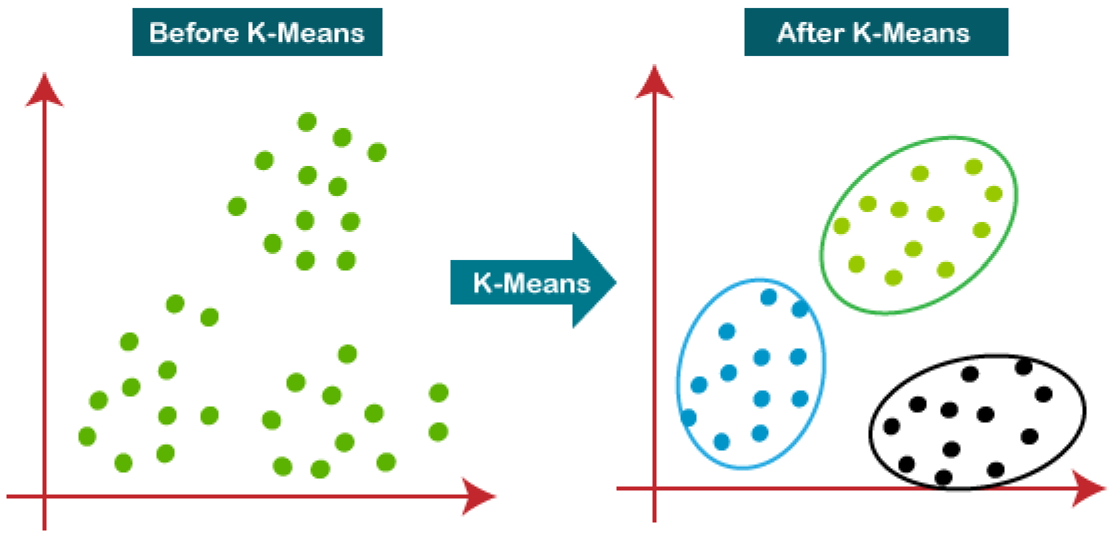
1. **Distance Measure**:
    - The central idea behind distance-based clustering is that documents that are closer to each other in some feature space (typically a vector space model) are likely to belong to the same cluster.
    - The most common distance measures used in text clustering are **Euclidean distance**, **Cosine similarity**, and **Manhattan distance**. Each of these measures calculates how far apart two documents are in the feature space.
2. **Document Representation**:
    - Each document is represented as a vector in a high-dimensional space. This space is created using term-frequency or more advanced models like TF-IDF or word embeddings.
    - For example, a document may be represented as a vector of term frequencies, where each dimension corresponds to a specific term in the vocabulary.

Common Distance-Based Clustering Algorithms:

1. K-means Clustering:

- **Initialization**: Choose k initial cluster centers (either randomly or using some heuristic).
- **Assignment Step**: Assign each document to the nearest cluster center using a distance measure (like Euclidean distance).
- **Update Step**: Calculate the new cluster centers by taking the average of all the documents in a cluster (for Euclidean space).
- **Repeat**: The assignment and update steps are repeated until convergence, meaning the cluster centers no longer change significantly.

Distance Metric: Typically, **Euclidean distance** is used in K-means. If cosine similarity is used, it's often referred to as **Cosine K-means**.



Let's cluster a small set of text documents into groups based on their similarity.

Dataset

Suppose we have the following documents:

1. "Cats are playful and friendly."
2. "Dogs are loyal and friendly."
3. "Birds can fly and are colorful."
4. "Airplanes can fly long distances."
5. "Cars are fast and used for transportation."

Steps

1. Preprocessing

- Convert text to lowercase.
- Remove stop words like "are", "and", "can", "for".
- Apply stemming or lemmatization.

After preprocessing:

1. "cats playful friendly"
2. "dogs loyal friendly"
3. "birds fly colorful"
4. "airplanes fly long distances"
5. "cars fast transportation"

---

2. Feature Extraction

- Use **TF-IDF** to convert text into vectors.

For simplicity, assume the following TF-IDF matrix for the 5 documents:

Term	**Doc1**	**Doc2**	**Doc3**	**Doc4**	**Doc5**
cats	0.7	0.0	0.0	0.0	0.0
dogs	0.0	0.8	0.0	0.0	0.0
friendly	0.5	0.5	0.0	0.0	0.0
fly	0.0	0.0	0.6	0.7	0.0

colorful	0.0	0.0	0.7	0.0	0.0
airplanes	0.0	0.0	0.0	0.8	0.0
cars	0.0	0.0	0.0	0.0	0.9
fast	0.0	0.0	0.0	0.0	0.7
transportation	0.0	0.0	0.0	0.0	0.8

---

3. Choose a Distance Metric

- Use **Cosine Similarity** to measure the similarity between documents.

---

4. Apply Clustering Algorithm

- Use **K-Means** with k=3 (assume we want 3 clusters).

After running K-Means, we get the following clusters:

- **Cluster 1**: Doc1 ("Cats are playful and friendly"), Doc2 ("Dogs are loyal and friendly").
- **Cluster 2**: Doc3 ("Birds can fly and are colorful"), Doc4 ("Airplanes can fly long distances").
- **Cluster 3**: Doc5 ("Cars are fast and used for transportation").

---

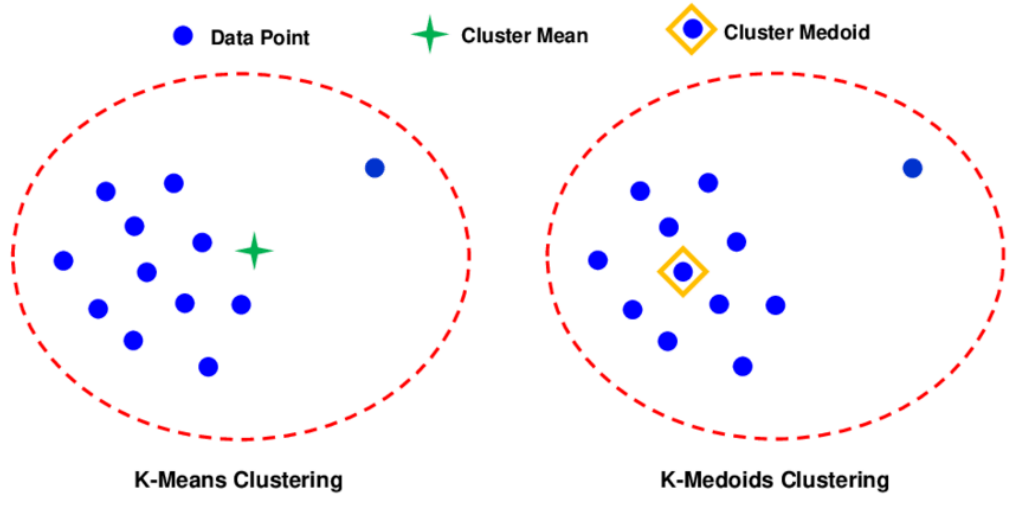
5. Analysis

- **Cluster 1**: Groups animals known for being "friendly."
- **Cluster 2**: Groups things that can "fly."
- **Cluster 3**: Groups vehicles focused on "transportation."

2. K-medoids (Partitioning Around Medoids):

- Similar to K-means but instead of using the centroid of the cluster, K-medoids chooses an actual data point (medoid) as the center of the cluster.
- The **medoid** is the point in the cluster that minimizes the sum of distances to all other points in the cluster.
- This method is more robust to outliers than K-means since the medoid is less affected by extreme values.

Distance Metric: Can use **any distance metric** (like Manhattan or Euclidean distance), making it more flexible.



Steps for K-Medoids

1. **Initialize Medoids**: Randomly select k data points as medoids.
2. **Assign Points to Clusters**: Assign each point to the nearest medoid using a distance metric (e.g., cosine similarity or Euclidean distance).
3. **Update Medoids**:
    - For each cluster, calculate the total distance between all points in the cluster and each point in the cluster.
    - Select the point that minimizes the total distance as the new medoid.
4. **Repeat**: Reassign points and update medoids until the medoids no longer change or a maximum number of iterations is reached.
5. **Output**: Final clusters and medoids.

Refer python notebook for implementation

Assuming `k = 2`, the output could look like:

1. **Cluster Labels:** `[0, 0, 1, 1, 1, 0, 1]`
2. **Medoids:**
   - Cluster 0 Medoid: `"Love this blue and bright sky!"`
   - Cluster 1 Medoid: `"The quick brown fox jumps over the lazy dog"`
3. **Clustered Documents:**
   - **Cluster 0:**
     - "The sky is blue and beautiful"
     - "Love this blue and bright sky!"
     - "The fox is quick and the sky is blue"
   - **Cluster 1:**
     - "The quick brown fox jumps over the lazy dog"
     - "A king's breakfast has sausages, ham, and bacon"
     - "Breakfast is the most important meal of the day"
     - "The lazy dog is sleeping"

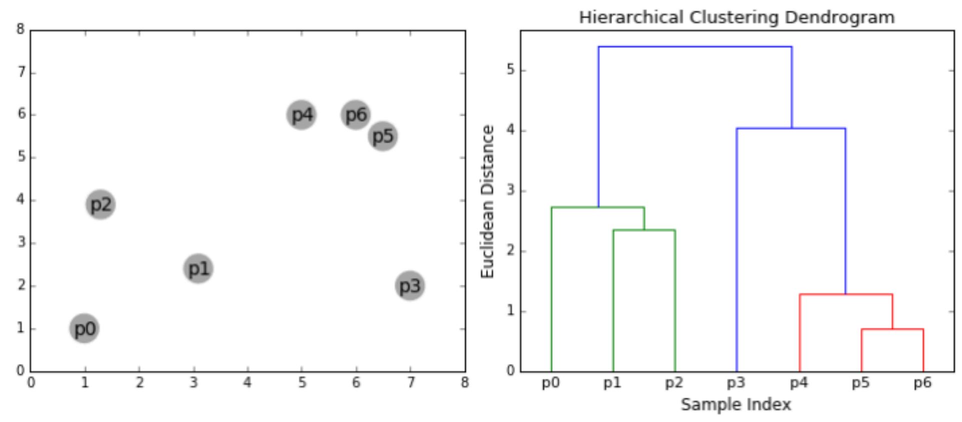
Why K-Medoids for Text Clustering?

- **Robust to Noise**: Unlike K-Means, medoids are less affected by outliers.
- **Flexible Metrics**: Allows custom distance metrics like cosine similarity, which is ideal for text data.
-

3. Agglomerative Hierarchical Clustering:

- **Agglomerative** means starting with each document as its own cluster and iteratively merging the closest clusters.
- The distance between clusters is calculated using methods such as:
  - **Single linkage**: Distance between the closest pair of points in two clusters.
  - **Complete linkage**: Distance between the farthest pair of points in two clusters.
  - **Average linkage**: Average distance between all pairs of points in two clusters.
- This process continues until all documents are merged into a single cluster, forming a **dendrogram** (a tree-like diagram) to represent the hierarchy.

Distance Metric: Typically uses **Euclidean distance** or **Manhattan distance** but can work with other metrics.



Agglomerative Hierarchical Clustering is a bottom-up clustering approach where each data point starts as its own cluster, and clusters are merged step by step based on a similarity or distance metric until all points belong to a single cluster or a specified number of clusters is reached.

Steps in Agglomerative Hierarchical Clustering

1. **Start with Individual Clusters**:
   - Treat each data point as its own cluster.
2. **Calculate Distance Between Clusters**:
   - Use a distance metric (e.g., Euclidean, Manhattan, or Cosine) to calculate the distance between clusters.
3. **Merge Clusters**:
   - Merge the two closest clusters based on a linkage criterion (e.g., single, complete, average).
4. **Repeat**:
   - Recalculate distances and merge clusters iteratively until only one cluster remains or a specific condition is met.
5. **Visualize**:
   - Create a **dendrogram** to visualize the hierarchical clustering process.

Linkage Criteria

1. **Single Linkage**: Distance between the closest points in two clusters.
2. **Complete Linkage**: Distance between the farthest points in two clusters.
3. **Average Linkage**: Average distance between all points in two clusters.
4. **Ward's Method**: Minimizes variance within clusters.

Expected Output

1. **Dendrogram**:
    - Shows how documents are merged step by step.
    - Documents with high similarity (low distance) are merged early.
2. **Cluster Assignment**:
    - Use a threshold or number of clusters to cut the dendrogram and assign points to clusters.

Applications

- **Topic Modeling**: Group documents into themes.
- **Document Similarity Analysis**: Identify closely related documents.
- **Customer Segmentation**: Analyze textual survey responses.

Advantages

- No need to predefine the number of clusters.
- Dendrogram provides a clear visualization of clustering hierarchy.

Drawbacks

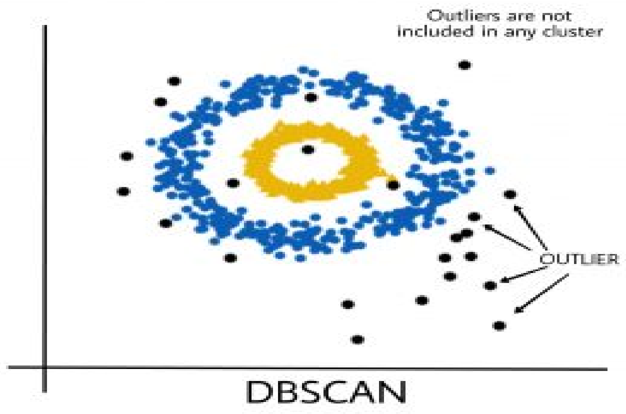
- Computationally expensive for large datasets.
- Sensitive to noise and outliers.

4. DBSCAN (Density-Based Spatial Clustering of Applications with Noise):

- DBSCAN focuses on the density of data points and can discover clusters of arbitrary shape.
- It works by grouping together points that are close to each other and have a minimum number of neighboring points (density).
- Points that don't have enough neighbors are classified as **noise**.

Distance Metric: **Euclidean distance** is commonly used, though other metrics can be used depending on the application.

Advantages: Unlike K-means, DBSCAN does not require specifying the number of clusters beforehand. It also can handle clusters of varying shapes and sizes and can identify outliers as noise.



Key Steps in a Distance-Based Clustering Algorithm:

1. **Choose a Distance Metric**: This is crucial, as the way you define "distance" influences how clusters are formed. Common choices for document clustering include:
    - **Cosine Similarity**: Measures the cosine of the angle between two vectors. It's often used when the magnitude of the vectors is less important than the direction (e.g., when dealing with text data).
    - **Euclidean Distance**: Measures the straight-line distance between two points in space. Suitable for numerical data.
    - **Manhattan Distance**: Measures the sum of the absolute differences of their coordinates.
2. **Initialize Clusters**: Depending on the algorithm, initial cluster centers (for K-means or K-medoids) are chosen either randomly or through other methods (e.g., using the **Buckshot** or **Fractionation** algorithms to select initial centroids in scatter-gather clustering).
3. **Assign Documents to Clusters**: For each document, calculate its distance from each cluster center and assign it to the closest cluster based on the chosen distance measure.
4. **Recompute Centers (if applicable)**: After assigning documents to clusters, update the cluster centers (mean of the points for K-means, medoid for K-medoids, etc.).
5. **Repeat Until Convergence**: Continue the process of assignment and updating cluster centers until the algorithm reaches a stable state where cluster centers no longer change significantly.

Distance-Based Clustering: Advantages and Challenges

- **Advantages**:
    - **Simple and efficient**: Especially for smaller datasets, algorithms like K-means can be very fast.

    - **Scalability**: Many distance-based clustering algorithms scale well with large datasets, especially if you use dimensionality reduction techniques like **PCA** or **Latent Semantic Indexing (LSI)**.
    - **Flexibility**: Can work with different types of distance metrics depending on the application (e.g., text, image, or numerical data).
- **Challenges**:
    - **Choice of K**: For algorithms like K-means or K-medoids, you must predefine the number of clusters kkk, which might not be easy in many real-world scenarios.
    - **Outliers**: Algorithms like K-means are sensitive to outliers, which can distort the cluster centers.
    - **Distance Metric Sensitivity**: The choice of distance metric can significantly affect the clustering result. For example, Cosine similarity may work well for text data but may not be suitable for numerical data.

Note : For all Distance based algorithm example refer python notebook shared on moodle it is just for reference to understand example, in exam there will be no coding required only logic is expected

2.1.3 Word and Phrase Based Clustering

Word-Based Clustering and Phrase-Based Clustering are two different approaches used for clustering textual data based on words or phrases. Both methods focus on grouping similar documents based on their content, but the way they handle textual features (words or phrases) differs. Let me explain each one in detail:

---

1. Word-Based Clustering:

In **word-based clustering**, the clustering algorithm groups documents based on individual words in the text. Each document is represented as a vector of words, and clustering algorithms such as **K-Means** or **DBSCAN** are applied to these word vectors to group similar documents together.

Steps for Word-Based Clustering:

1. **Preprocessing**:
    - Remove stop words, punctuation, and perform stemming/lemmatization.
    - Tokenize the document into individual words.
2. **Vectorization**:
    - Represent each document as a vector of individual words. You can use techniques like:
        - **TF-IDF** (Term Frequency-Inverse Document Frequency)
        - **Bag of Words** (BoW)
        - **Word2Vec** (Word embeddings)
3. **Clustering**:
    - Apply a clustering algorithm such as **K-Means**, **DBSCAN**, or **Hierarchical clustering** on the word-based vectors.
    - These algorithms group documents based on the similarity between the word vectors of each document.
4. **Interpretation**:
    - The result will show clusters of documents where each cluster is defined by a collection of common words.

Example of Word-Based Clustering:

Documents:

1. "The car is fast and new."
2. "I love fast cars."
3. "The sun is bright and shining."
4. "I love the warm weather."

5. "Cars are great for fast transportation."

In **word-based clustering**, the clustering algorithm might group:

- **Cluster 1**: "The car is fast and new.", "I love fast cars.", "Cars are great for fast transportation."
- **Cluster 2**: "The sun is bright and shining.", "I love the warm weather."

Challenges:

- **Ambiguity**: Words like "bank" (financial institution vs. riverside) could create ambiguity.
- **Ignoring Context**: Word-based clustering doesn't capture relationships between words or the full context of the document.

---

2. Phrase-Based Clustering:

In **phrase-based clustering**, instead of focusing on individual words, the clustering algorithm uses **phrases** (combinations of words that appear together frequently) as features. This method can capture more context and semantic meaning than word-based clustering.

Steps for Phrase-Based Clustering:

1. **Preprocessing**:
   - Like word-based clustering, remove stop words and punctuation, perform stemming/lemmatization, and tokenize.
   - Extract meaningful phrases. This can be done by:
     - Using **n-grams** (e.g., bi-grams for pairs of words, tri-grams for triples of words).
     - Using **collocations** (frequent co-occurring words).
2. **Phrase Extraction**:
   - You can use **TF-IDF** to extract important phrases that appear frequently in the documents.
   - **Topic Modeling** techniques (e.g., **Latent Dirichlet Allocation (LDA)**) can also identify common phrases in the corpus.
3. **Vectorization**:
   - Represent each document as a vector of phrases (instead of words).
   - Each vector will have the frequency of occurrence for the important phrases extracted from the document.
4. **Clustering**:
   - Apply clustering algorithms such as **K-Means** or **DBSCAN** on the phrase-based vectors to identify groups of documents with similar phrase content.
5. **Interpretation**:

    - Clusters will be defined by common phrases (which may represent more complex ideas or topics).

Example of Phrase-Based Clustering:

Documents:

1. "The car is fast and new."
2. "I love fast cars."
3. "The sun is bright and shining."
4. "I love the warm weather."
5. "Cars are great for fast transportation."

In **phrase-based clustering**, the algorithm might detect key phrases like:

- **"fast cars"**
- **"warm weather"**
- **"bright sun"**

The clustering algorithm could group documents based on the occurrence of these key phrases:

- **Cluster 1**: "The car is fast and new.", "I love fast cars.", "Cars are great for fast transportation." (Common phrase: "fast cars")
- **Cluster 2**: "The sun is bright and shining.", "I love the warm weather." (Common phrase: "warm weather", "bright sun")

Advantages of Phrase-Based Clustering:

- **Contextual Understanding**: By considering phrases, the algorithm captures more context and meaning.
- **Semantic Relationships**: It can better understand the relationships between words and their occurrences in similar contexts.
- **Improved Accuracy**: Phrases that carry more meaning together can better differentiate clusters, as opposed to focusing on individual words.

Challenges:

- **Complexity**: Extracting and analyzing phrases increases the complexity of the model.
- **Phrase Identification**: Identifying relevant and meaningful phrases can be challenging, especially in large datasets.

Choosing Between Word and Phrase-Based Clustering:

- **Use Word-Based Clustering** if you want to cluster documents based on simple word occurrence or if your documents are short and relatively simple.
- **Use Phrase-Based Clustering** if you need to capture the meaning or topics of documents, especially when they contain more complex ideas or longer texts.

Summary:

- **Word-Based Clustering** groups documents based on individual words and their occurrence in the text.
- **Phrase-Based Clustering** groups documents based on combinations of words or phrases, which can capture more meaningful relationships and topics in the text.

2.1.4 Probabilistic Document Clustering and Topic Models

Probabilistic document clustering and topic modeling are techniques that aim to group documents based on the topics they discuss. Topic modeling creates a probabilistic generative model for text documents, helping to uncover hidden thematic structures within a document corpus. The key idea is that each document can be modeled as a mixture of multiple topics, each represented as a distribution over terms in the vocabulary.

1. **Documents and Topics**:
   - A set of `n` documents in the corpus is assumed to belong to `k` topics.
   - A document can belong to multiple topics with varying probabilities, reflecting the reality that documents often cover multiple themes.
   - For a given document `Di`, the probability that it belongs to a topic `Tj` is expressed as P(Tj∣Di) , which indicates how much of `Di` is related to topic `Tj`.
2. **Topic Clusters**:
   - Topics act as soft clusters. Unlike deterministic clustering methods, where each document is assigned to a single cluster, topic modeling allows documents to belong to multiple topics.
   - This soft clustering is valuable when there are overlaps in the subject matter of documents.
3. **Topic-Document and Term-Topic Distributions**:
   - Each topic `Tj` has an associated probability distribution over words in the vocabulary, denoted P(tl∣Tj), where `tl` is a term in the vocabulary.
   - For a document `Di`, the probability of observing a term `tl` in it is a weighted combination of topic-word probabilities across all topics.

Probabilistic Model Equation:

- The probability of a term `tl` appearing in document `Di` can be computed as:

$$P(tl|Di) = \sum_{j=1}^{k} P(tl|Tj) \cdot P(Tj|Di)$$

Here, P(Tj∣Di)is the probability of document `Di` belonging to topic `Tj`, and P(tl∣Tj) is the probability of the term `tl` being associated with topic `Tj`.

Learning the Parameters:

- **Topic Modeling Methods**:

  - **Probabilistic Latent Semantic Indexing (PLSI)** and **Latent Dirichlet Allocation (LDA)** are two widely used topic modeling methods.
  - Both methods aim to estimate the topic distributions for each document and the term distributions for each topic.
- **Maximum Likelihood Estimation**:
  - The parameters of the model (e.g., P(Tj∣Di) and P(tl∣Tj) are estimated by maximizing the likelihood of the observed term-document occurrences.
  - The process involves constructing a likelihood function and optimizing it using methods like Expectation-Maximization (EM).

The Optimization Problem:

- The topic modeling algorithm optimizes the log-likelihood of observing the terms in each document using the formula:

$$\sum_{i,l} X(i,l) \cdot \log(P(tl|Di))$$

where `X(i, l)` is the frequency of term `tl` in document `Di`.

  - The solution involves updating two matrices: one for topic-document probabilities and another for term-topic probabilities.

Iterative Update Algorithm:

- The algorithm iteratively updates the matrices `P1` (topic-document matrix) and `P2` (term-topic matrix), using an EM approach.
  - **Update for P1**: P1(j,i)P1(j, i)P1(j,i) is updated based on the product of term-topic probabilities and the document-term occurrence matrix.
  - **Update for P2**: P2(l,j)P2(l, j)P2(l,j) is updated based on the document-topic matrix and the document-term occurrence matrix.

Dirichlet Distribution and LDA:

- In LDA (Latent Dirichlet Allocation), the topic and term distributions are modeled using Dirichlet priors, which are a type of probability distribution.
  - LDA is considered a **Bayesian version** of PLSI, which provides a more robust approach by using prior distributions to prevent overfitting.
  - This Bayesian framework allows LDA to better generalize and be more effective in modeling new, unseen documents.

PLSI vs LDA:

- **PLSI** has the disadvantage of being prone to overfitting and having a growing number of parameters as the dataset increases. This can make it less effective, particularly for small datasets.
- **LDA**, by contrast, uses Dirichlet priors to control overfitting and handles new documents more effectively, even if they weren't in the original dataset. This makes LDA more stable and scalable.

Applications:

- Topic modeling is widely used for text clustering, classification, and retrieval. By understanding the latent topics within a collection of documents, it aids in organizing and making sense of large amounts of unstructured text data.

EM-Approach in Supervised Clustering:

- In cases where some labeled data is available, **supervised clustering** can be performed by combining topic modeling with classification techniques. The Expectation-Maximization (EM) approach helps optimize both labeled and unlabeled data in the clustering process.

In conclusion, probabilistic document clustering via topic modeling (PLSI, LDA) is a powerful tool for discovering hidden patterns within a large corpus of text. The approach allows for flexible and soft clustering of documents, capturing the complexity of real-world data where documents often contain multiple topics.

Latent Dirichlet Allocation (LDA), a popular probabilistic topic modeling method, using a small corpus of documents to illustrate how it works.

Example Corpus:

Consider the following three documents in a small corpus:

1. **Document 1 (D1)**: "Cats are great pets. They love to chase mice."
2. **Document 2 (D2)**: "Dogs are loyal companions. They like to play fetch."
3. **Document 3 (D3)**: "Cats and dogs are common pets. Many people love them."

Step 1: Preprocessing the Text

First, we preprocess the text by removing common stop words (e.g., "are", "to", "and"), stemming the words (e.g., "chase" becomes "chase"), and extracting the vocabulary.

Vocabulary (terms):

- `cats`, `great`, `pets`, `love`, `chase`, `mice`, `dogs`, `loyal`, `companions`, `play`, `fetch`, `common`, `many`, `people`

Step 2: Defining the Topics

Let's assume we want to find `k = 2` topics in the corpus (e.g., one for pets and another for dogs).

1. **Topic 1** might be about **cats** and **pets**.
2. **Topic 2** might be about **dogs** and **companions**.

Step 3: Initializing Topic Distributions

LDA assigns each document a distribution over topics and each term a distribution over topics. We start by randomly initializing these distributions.

For example, the topic distributions for documents might look something like this initially:

- D1 (Cats are great pets): 70% Topic 1, 30% Topic 2
- D2 (Dogs are loyal companions): 20% Topic 1, 80% Topic 2
- D3 (Cats and dogs are common pets): 50% Topic 1, 50% Topic 2

Similarly, each term gets an initial distribution over topics. For example:

- `cats` might have 80% probability for Topic 1, 20% for Topic 2.
- `dogs` might have 30% for Topic 1, 70% for Topic 2.

Step 4: Expectation-Maximization (EM) Algorithm

The Expectation-Maximization (EM) process alternates between two steps:

1. **E-Step (Expectation)**:
    - For each document, estimate the topic distribution (i.e., the probability that each topic generated each term in the document).
    - For example, if we look at Document 1, we update the probability that terms like "cats", "great", and "pets" come from Topic 1 (about pets and cats) vs. Topic 2 (about dogs and companions).
2. **M-Step (Maximization)**:
    - Based on the current topic assignments, update the term distributions for each topic. For example, words like "chase" and "mice" will have a higher probability of coming from Topic 1 (about pets) after several iterations because they are more likely associated with cats.

This process is repeated until convergence, meaning the topic distributions and term distributions stabilize.

Step 5: Final Output

After running the algorithm, LDA produces:

1. **Topic distributions for each document**: Each document has a probability distribution over topics. For example:
    - D1 (Cats are great pets): 90% Topic 1 (about cats), 10% Topic 2 (about dogs).
    - D2 (Dogs are loyal companions): 10% Topic 1 (about cats), 90% Topic 2 (about dogs).
    - D3 (Cats and dogs are common pets): 60% Topic 1, 40% Topic 2.
2. **Term distributions for each topic**: Each topic has a probability distribution over words. For example:
    - **Topic 1** (Pets and Cats): Higher probabilities for terms like `cats`, `pets`, `mice`, `chase`.
    - **Topic 2** (Dogs and Companions): Higher probabilities for terms like `dogs`, `loyal`, `companions`, `play`, `fetch`.

Interpretation of Results:

- **Topic 1** is about **cats** and **pets**, with words like `cats`, `pets`, and `mice` being more likely.
- **Topic 2** is about **dogs** and **companions**, with words like `dogs`, `loyal`, `companions`, and `fetch` being more likely.

In this way, LDA has successfully found two topics in the corpus: one about **cats and pets** and the other about **dogs and companions**, and has provided the topic distribution for each document and the term distribution for each topic.

Summary:

This process shows how **probabilistic topic modeling** works, where documents are represented as mixtures of topics, and the terms in each document are probabilistically linked to those topics. The result is a more nuanced, probabilistic clustering of the documents compared to traditional hard clustering methods.