



● Inference using Full Joint Distribution

Bayesian network is based on Joint probability distribution and conditional probability.

If we have variables $x_1, x_2, x_3, \dots, x_n$, then the probabilities of a different combination of $x_1, x_2, x_3, \dots, x_n$, are known as Joint probability distribution.

$P[x_1, x_2, x_3, \dots, x_n]$, it can be written as the following way in terms of the joint probability distribution.

$$= P[x_1 | x_2, x_3, \dots, x_n] P[x_2, x_3, \dots, x_n]$$

$$P(X_i | X_{i-1}, \dots, X_1) = P(X_i | \text{Parents}(X_i))$$

Example:

(b) Find the probabilistic inference by enumeration of entries in a full joint distribution table shown in figure 1. 10

(i) No cavity when toothache is there

(ii) $p(\text{Cavity} | \text{toothache or catch})$

	toothache		\neg toothache	
	catch	\neg catch	catch	\neg catch
cavity	.108	.012	.072	.008
\neg cavity	.016	.064	.144	.576

Solution

1. No cavity when toothache is there

$$\begin{aligned} P(\neg \text{cavity} | \text{toothache}) &= P(\neg \text{cavity} \wedge \text{toothache}) / P(\text{toothache}) \\ &= (0.016 + 0.064) / (0.108 + 0.012 + 0.016 + 0.064) \\ &= 0.4 \end{aligned}$$

2. $p(\text{Cavity} | \text{toothache or catch})$

$$\begin{aligned} &= P(\text{Cavity} | \text{toothache}) + p(\text{catch}) \\ &= [P(\text{cavity} \wedge \text{toothache}) / P(\text{toothache})] + P(\text{catch}) \\ &= [(0.108 + 0.012)] \\ &\quad / [(0.108 + 0.012 + 0.016 + 0.064) + (0.108 + 0.016 + 0.072 + 0.144)] \\ &= 0.96 \end{aligned}$$