

SE, sem. III, COMPUTER, 5H2018  
(choice base) (3 Hours)

04/11/18 [Total Marks: 80]

N.B (1) Question No. 1 is compulsory.

- (2) Solve any three questions out of remaining five questions.  
(3) Assumptions made should be clearly stated.  
(4) Figures to the right indicate full marks.

Q.1 (a) Two dice are rolled, find the probability that the sum is  
(i) Equal to 1 (ii) Equal to 4 (iii) Less than 13

[6M]

(b) Use the laws of logic to show that  
 $[(p \rightarrow q) \wedge \sim q] \rightarrow \sim p$  is a tautology

[6M]

(c) Determine the matrix of the partial order of divisibility on the set A. Draw the Hasse diagram of the Poset. Indicate those which are chains

[8M]

- (1)  $A = \{1, 2, 3, 5, 6, 10, 15, 30\}$   
(2)  $A = \{3, 6, 12, 36, 72\}$

Q.2 (a) Find the complement of each element in  $D_{42}$

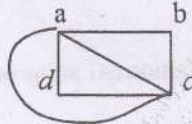
[6M]

(b) Let Q be the set of positive rational numbers which can be expressed in the form  $2^a 3^b$ , where a and b are integers. Prove that algebraic structure  $(Q, \cdot)$  is a group. Where  $\cdot$  is multiplication operation.

[6M]

(c) Define isomorphic graphs. Show whether the following graphs are isomorphic or not.

[8M]



G1  
Fig (a)

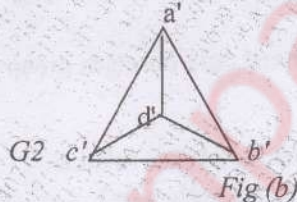
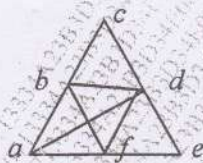


Fig (b)

Q.3 (a) Determine which of the following graph contains an Eulerian or Hamiltonian circuit.

[6M]



Fig(a)



Fig(b)

(b) For all sets A, X and Y show that  
 $A \times (X \cap Y) = (A \times X) \cap (A \times Y)$

[6M]

(c) Let  $f(x) = x+2$ ,  $g(x) = x-2$  and  $h(x) = 3x$  for  $x \in \mathbb{R}$ , Where  $\mathbb{R}$  = Set of real numbers. Find  
 $(g \circ f)$ ,  $(f \circ g)$ ,  $(f \circ f)$ ,  $(g \circ g)$ ,  $(f \circ h)$ ,  $(h \circ g)$ ,  $(h \circ f)$ ,  $(f \circ h \circ g)$

[8M]

Q.4 (a) Let R is a binary relation. Let  $S = \{(a, b) \mid (a, c) \in R \text{ and } (c, b) \in R \text{ for some } c\}$  Show that if R is an equivalence relation then S is also an equivalence relation.

[6M]

[TURN OVER]



- (b) Determine the generating function of the numeric function  $a_r$ , where

$$(i) a_r = 3^r + 4^{r+1}, r \geq 0$$

$$(ii) a_r = 5, r \geq 0$$

[6M]

- (c) Consider the (3, 6) encoding function  $e: B^3 \rightarrow B^6$  defined by

$$e(000) = 000000 \quad e(001) = 001100 \quad e(010) = 010011 \quad e(011) = 011111$$

$$e(100) = 100101 \quad e(101) = 101001 \quad e(110) = 110110 \quad e(111) = 111010$$

[8M]

Decode the following words relative to a maximum likelihood decoding function.

$$(i) 000101 \quad (ii) 010101$$

- Q.5 (a) Determine the number of positive integers  $n$  where  $1 \leq n \leq 100$  and  $n$  is not divisible by 2, 3 or 5.

[6M]

- (b) Use mathematical induction to show that  
 $1+5+9+\dots+(4n-3) = n(2n-1)$

[6M]

- (c) Find the greatest lower bound and least upper bound of the set  $\{3, 9, 12\}$  and  $\{1, 2, 4, 5, 10\}$  if they exist in the poset  $(Z^+, /)$ . Where  $/$  is the relation of divisibility.

[8M]

- Q.6 (a) Let  $A = \{1, 2, 3, 4\}$  and Let  $R = \{(1,1) (1,2) (1,4) (2,4) (3,1) (3,2) (4,2) (4,3) (4,4)\}$ . Find transitive closure by Warshall's algorithm.

[6M]

- (b) Let  $H = \{[0]_6, [3]_6\}$  find the left and right cosets in group  $Z_6$ . Is  $H$  a normal subgroup of group of  $Z_6$ .

[6M]

- (c) Find the complete solution of the recurrence relation

$$a_n + 2a_{n-1} = n+3 \text{ for } n \geq 1 \text{ and with } a_0 = 3$$

[8M]