Prisoner's Dilemma

Question 1: Classic Prisoner's Dilemma

Problem:

Two suspects, Alice and Bob, are arrested for a crime. They are interrogated separately and cannot communicate with each other. Each has the option to betray the other or stay silent. The outcomes are:

- If both betray, each receives 5 years in prison.
- If one betrays and the other stays silent, the betrayer goes free, and the silent one receives 10 years in prison.
- If both stay silent, each receives 1 year in prison.

a) **Construct the payoff matrix.**

b) **Identify the Nash equilibrium.**

c) **Discuss why the dominant strategy leads to a suboptimal outcome for both players.**

Solution:

a) Payoff Matrix Construction:

	Bob: Silent	**Bob: Betray**
Alice: Silent	(-1, -1)	(-10, 0)
Alice: Betray	(0, -10)	(-5, -5)

b) Nash Equilibrium:

The Nash equilibrium occurs when both players betray each other, resulting in each receiving 5 years in prison. This is because betraying is the dominant strategy for both players.

c) Discussion:

Despite the fact that mutual cooperation (both staying silent) would lead to a better outcome for both players (1 year each), the fear of the other player betraying drives both players to betray. This results in a worse outcome for both, demonstrating the classic dilemma where rational self-interest leads to a suboptimal collective outcome.

Question 2: Iterated Prisoner's Dilemma

Problem:

Consider an iterated version of the Prisoner's Dilemma, where Alice and Bob play the game multiple times. In each round, they can see the results of the previous round before making a decision. Suppose they play 5 rounds.

a) **What strategy might each player adopt if they aim to maximize their long-term outcomes?**
b) **How does this differ from the single-shot Prisoner's Dilemma?**
c) **What are potential strategies like "Tit for Tat" and how do they influence the outcome?**

Solution:

a) **Strategy in Iterated Games:**

In an iterated game, players might adopt cooperative strategies like "Tit for Tat," where they start by cooperating and then replicate the other player's previous move. This encourages cooperation because betrayal in one round leads to retaliation in the next.

b) **Difference from Single-Shot Game:**

In a single-shot game, the dominant strategy is to betray. However, in an iterated game, the possibility of future rounds incentivizes cooperation, as long-term punishment for betrayal can outweigh the short-term gain.

c) **"Tit for Tat" Strategy:**

"Tit for Tat" is a simple and effective strategy where a player starts by cooperating and then mimics the opponent's last move. This strategy tends to promote cooperation and can lead to mutual cooperation over multiple rounds, reducing overall prison time for both players.

Question 3: Variation in Payoffs

Problem:

Consider a variation of the Prisoner's Dilemma where the payoffs are slightly different:

- If both betray, each receives 4 years in prison.
- If one betrays and the other stays silent, the betrayer receives 2 years, and the silent one receives 8 years.
- If both stay silent, each receives 3 years in prison.

a) Construct the payoff matrix.

b) **Identify the Nash equilibrium.**

c) Discuss how slight changes in payoffs might influence the players' strategies.

Solution:

a) Payoff Matrix Construction:

	Bob: Silent	**Bob: Betray**
Alice: Silent	(-3, -3)	(-8, -2)
Alice: Betray	(-2, -8)	(-4, -4)

b) Nash Equilibrium:

The Nash equilibrium remains at both players betraying each other, leading to 4 years in prison each. This is because betrayal remains the dominant strategy for both players, despite the change in payoffs.

c) **Impact of Payoff Changes:**

While the dominant strategy remains the same, slight changes in payoffs can influence players' perception of risk and reward. For example, reducing the punishment for mutual betrayal might make betrayal seem less risky, reinforcing the incentive to betray.

Mixed Strategy Nash Equilibrium

Problem 1: Matching Pennies

Two players play a game where each player chooses heads or tails. If both choices match, Player 1 wins $1, and Player 2 loses $1. If the choices don't match, Player 2 wins $1, and Player 1 loses $1. Identify the Nash equilibrium in mixed strategies and explain why there is no pure strategy Nash equilibrium.

Ans.

a) Nash Equilibrium in Mixed Strategies:

In this game, both players are trying to outguess each other. Each player is indifferent between choosing heads or tails because the payoff depends entirely on the choice of the other player.

To find the mixed strategy Nash equilibrium, let p be the probability that Player 1 chooses heads and q be the probability that Player 2 chooses heads. The Nash equilibrium occurs when each player is indifferent to the other player's choice, meaning they have no incentive to change their strategy.

Let's assign the following actions:

- Player 1 can choose Heads (H) or Tails (T).
- Player 2 can also choose Heads (H) or Tails (T).

The payoffs are given as:

- If both players choose the same (HH or TT), Player 1 wins $1, and Player 2 loses $1.
- If the players choose differently (HT or TH), Player 2 wins $1, and Player 1 loses $1.

The payoff matrix can be represented as:

	Player 2: H	Player 2: T
Player 1: H	(1, -1)	(-1, 1)
Player 1: T	(-1, 1)	(1, -1)

- Since there's no pure strategy equilibrium, let's consider a mixed strategy where each player randomizes their choices between Heads and Tails.
- Let p be the probability that Player 1 chooses Heads.
- Let q be the probability that Player 2 chooses Heads.

We calculated the expected payoffs for both strategies in the mixed strategy Nash equilibrium analysis. To make each player indifferent to choosing Heads or Tails, we found:

P=1/2

Q=1/2

Thus, the Nash equilibrium in mixed strategies is where both players choose Heads and Tails with a probability of 1/2

b) No Pure Strategy Nash Equilibrium:

In a pure strategy Nash equilibrium, each player would consistently choose either heads or tails. However, in this game:

- If Player 1 consistently chooses heads, Player 2 will always choose tails to win.
- If Player 1 consistently chooses tails, Player 2 will always choose heads.

The same logic applies in reverse for Player 2. This creates a situation where each player's best response changes based on the other's choice, leading to no stable pure strategy.

Therefore, the game has no pure strategy Nash equilibrium because the players keep trying to outguess each other, which leads to continuous changes in their choices. The only stable solution is a mixed strategy where both players randomize their choices, making them unpredictable.

Problem 2: Rock-Paper-Scissors Game

Problem: Two players engage in a game of Rock-Paper-Scissors. The rules are:

- Rock beats Scissors.
- Scissors beat Paper.
- Paper beats Rock.

If both players choose the same option, it's a tie. The payoff matrix is as follows:

	Player 2: Rock	Player 2: Paper	Player 2: Scissors
Player 1: Rock	(0, 0)	(-1, 1)	(1, -1)
Player 1: Paper	(1, -1)	(0, 0)	(-1, 1)
Player 1: Scissors	(-1, 1)	(1, -1)	(0, 0)

a) Identify the mixed strategy Nash equilibrium.

b) Explain why no pure strategy Nash equilibrium exists.

Solution:

a) Mixed Strategy Nash Equilibrium:

Let:

p1.p2, and p3 be the probabilities that Player 1 chooses Rock, Paper, and Scissors, respectively.

q1, q2, and q3 be the probabilities that Player 2 chooses Rock, Paper, and Scissors, respectively.

For the players to be indifferent among their strategies, the expected payoffs from choosing Rock, Paper, and Scissors must be equal. Given the symmetry of the game:

Player 1 should mix between Rock, Paper, and Scissors with equal probabilities: p1=p2=p3=1/3

Player 2 should mix between Rock, Paper, and Scissors with equal probabilities: q1=q2=q3=1/3

So, the mixed strategy Nash equilibrium is (1/3,1/3,1/3) for both players.

b) No Pure Strategy Nash Equilibrium:

There is no pure strategy Nash equilibrium in Rock-Paper-Scissors because:

- If one player knows the other's strategy, they can always choose an option that beats it.
- This leads to continuous switching of strategies, meaning that no stable pure strategy exists.

Problem 3: Battle of the Sexes

Problem: A couple is deciding how to spend their evening. The husband prefers to go to a football game, while the wife prefers to go to a ballet. However, both prefer to be together rather than going to their preferred event alone. The payoffs are as follows:

- If both go to the football game, the husband gets 3, and the wife gets 2.
- If both go to the ballet, the wife gets 3, and the husband gets 2.
- If they go to different events, each gets 0.
- the payoff matrix is:

	Wife: Football	**Wife: Ballet**
Husband: Football	(3, 2)	(0, 0)

	Wife: Football	**Wife: Ballet**
Husband: Ballet	(0, 0)	(2, 3)

a) Determine the mixed strategy Nash equilibrium.

b) Discuss the existence of pure strategy Nash equilibria.

Solution:

a) Mixed Strategy Nash Equilibrium:

Let:

- p be the probability that the husband chooses football.
- q be the probability that the wife chooses football.

To find the mixed strategy Nash equilibrium, consider the indifference condition:

- For the husband:
  - Expected payoff from Football: $3q + 0(1-q) = 3q$.
  - Expected payoff from Ballet: $0q + 2(1-q) = 2 - 2q$.

Setting these equal:

$3q = 2 - 2q$

$5q = 2$

$q = \frac{2}{5}$

- For the wife:
  - Expected payoff from Football: $2p + 0(1-p) = 2p$.
  - Expected payoff from Ballet: $0p + 3(1-p) = 3 - 3p$.

Setting these equal:

$2p = 3 - 3p$

$5p = 3$

$p = \frac{3}{5}$

Thus, the mixed strategy Nash equilibrium is $p = \frac{3}{5}$ and $q = \frac{2}{5}$.

b) Pure Strategy Nash Equilibria: The game has two pure strategy Nash equilibria:

1. Both go to the football game (Husband, Wife) = (Football, Football).
2. Both go to the ballet (Husband, Wife) = (Ballet, Ballet).

These equilibria exist because each player prefers to be with the other, even if it's not their preferred event.

Problem 4: Coin Matching Game

Problem: Two players each flip a coin, secretly choosing between Heads (H) or Tails (T). If both coins match (HH or TT), Player 1 wins $2, and Player 2 loses $2. If the coins do not match (HT or TH), Player 2 wins $2, and Player 1 loses $2.

	Player 2: H	**Player 2: T**
Player 1: H	(2, -2)	(-2, 2)
Player 1: T	(-2, 2)	(2, -2)

a)Determine the mixed strategy Nash equilibrium.

b) Explain why no pure strategy Nash equilibrium exists.

Solution:

a) Mixed Strategy Nash Equilibrium:

Let p be the probability that Player 1 chooses Heads, and q be the probability that Player 2 chooses Heads.

For Player 1:

- Expected payoff for Heads: $2q + (-2)(1 - q) = 4q - 2$.
- Expected payoff for Tails: $-2q + 2(1 - q) = 2 - 4q$.

Setting them equal:

$4q - 2 = 2 - 4q$

$8q = 4$

$q = \frac{1}{2}$

For Player 2:

- Expected payoff for Heads: $-2p + 2(1 - p) = 2 - 4p$.
- Expected payoff for Tails: $2p + (-2)(1 - p) = 4p - 2$.

Setting them equal:

$2 - 4p = 4p - 2$

$8p = 4$

$p = \frac{1}{2}$

Thus, the mixed strategy Nash equilibrium is $p = q = \frac{1}{2}$.

b) No Pure Strategy Nash Equilibrium:

The game lacks a pure strategy Nash equilibrium because:

- If Player 1 chooses Heads, Player 2's best response is Tails.
- If Player 1 chooses Tails, Player 2's best response is Heads.
- This leads to continuous switching of strategies, making it impossible to have a stable pure strategy.

Strategic game

- Refer the below payoff matrix and identify the strategic game involved in it and describe it in brief.

Strategic games

Two competing companies, X and Y, are deciding whether to launch a new advertising campaign. Each company can either "Advertise" or "Not Advertise." If both companies advertise, they cancel each other out, and neither gains any significant advantage, resulting in moderate profits. If one company advertises while the other does not, the advertising company captures a larger market share and earns higher profits, while the non-advertising company loses market share and earns lower profits. If neither company advertises, they maintain their current market share and profits.

Payoffs:

- If both advertise: 2 for each.
- If one advertises and the other doesn't: 3 for the advertiser, 1 for the non-advertiser.
- If neither advertises: 3 for each.

a) Construct the payoff matrix for this scenario.

b) Identify the Nash equilibrium and explain the strategic reasoning behind it.

Solution:

a) Payoff Matrix:

	Company Y: Advertise	**Company Y: Not Advertise**
Company X: Advertise	(2, 2)	(3, 1)

	Company Y: Advertise	Company Y: Not Advertise
Company X: Not Advertise	(1, 3)	(3, 3)

b) Nash Equilibrium:

- **Analyze Company X's Best Responses:**
  - If Company Y advertises, Company X's best response is to advertise (2 vs. 1).
  - If Company Y does not advertise, Company X's best response is not to advertise (3 vs. 3).
- **Analyze Company Y's Best Responses:**
  - If Company X advertises, Company Y's best response is to advertise (2 vs. 1).
  - If Company X does not advertise, Company Y's best response is not to advertise (3 vs. 3).

Nash Equilibrium:

There are two Nash equilibria:

- **(Advertise, Advertise):** Both companies advertise, resulting in moderate profits (2, 2).
- **(Not Advertise, Not Advertise):** Neither company advertises, resulting in stable profits (3, 3).

Strategic Reasoning:

The second equilibrium (Not Advertise, Not Advertise) is Pareto superior because it results in higher profits for both companies. However, if both companies suspect the other might advertise, they may choose to advertise as well, leading to the first equilibrium (Advertise, Advertise). This reflects the tension between individual incentives and collective outcomes.

Dominant Strategy

Problem:

Consider a game between two firms, Firm A and Firm B, that are competing in the market by setting their prices. Each firm can choose one of three pricing strategies: Low, Medium, or High. The payoffs (in millions of dollars) for each combination of strategies are given in the payoff matrix below. The first number in each cell represents the payoff for Firm A, and the second number represents the payoff for Firm B.

	Firm B: Low Price	Firm B: Medium Price	Firm B: High Price
Firm A: Low Price	(4, 4)	(3, 5)	(2, 6)
Firm A: Medium Price	(5, 3)	(6, 6)	(3, 7)
Firm A: High Price	(6, 2)	(7, 3)	(8, 8)

a) Define the concept of a dominant strategy.
b) Identify if either Firm A or Firm B has a dominant strategy. Justify your answer.
c) Find the Nash equilibrium or equilibria of this game. Explain your reasoning.

Solution:

a) Dominant Strategy

A **dominant strategy** is a strategy that results in the highest payoff for a player, regardless of the strategies chosen by the other players. In other words, a strategy is dominant if, no matter what the other players do, the player choosing this strategy will always get a higher payoff compared to choosing any other strategy.

b) Identifying Dominant Strategies

To determine if either firm has a dominant strategy, we need to compare the payoffs for each firm when they choose different strategies.

For Firm A:

- **If Firm B chooses Low Price:**
  - If Firm A chooses Low Price, the payoff is 4.
  - If Firm A chooses Medium Price, the payoff is 5.
  - If Firm A chooses High Price, the payoff is 6.
  - **Best response**: High Price (6 > 5 > 4)
- **If Firm B chooses Medium Price:**
  - If Firm A chooses Low Price, the payoff is 3.
  - If Firm A chooses Medium Price, the payoff is 6.
  - If Firm A chooses High Price, the payoff is 7.
  - **Best response**: High Price (7 > 6 > 3)
- **If Firm B chooses High Price:**
  - If Firm A chooses Low Price, the payoff is 2.
  - If Firm A chooses Medium Price, the payoff is 3.
  - If Firm A chooses High Price, the payoff is 8.
  - **Best response**: High Price (8 > 3 > 2)

Conclusion for Firm A: High Price is always the best response, regardless of Firm B's choice. Therefore, High Price is a **dominant strategy** for Firm A.

For Firm B:

- **If Firm A chooses Low Price:**
  - If Firm B chooses Low Price, the payoff is 4.
  - If Firm B chooses Medium Price, the payoff is 5.
  - If Firm B chooses High Price, the payoff is 6.
  - **Best response**: High Price (6 > 5 > 4)
- **If Firm A chooses Medium Price:**
  - If Firm B chooses Low Price, the payoff is 3.
  - If Firm B chooses Medium Price, the payoff is 6.
  - If Firm B chooses High Price, the payoff is 7.
  - **Best response**: High Price (7 > 6 > 3)
- **If Firm A chooses High Price:**
  - If Firm B chooses Low Price, the payoff is 2.
  - If Firm B chooses Medium Price, the payoff is 3.
  - If Firm B chooses High Price, the payoff is 8.
  - **Best response**: High Price (8 > 3 > 2)

Conclusion for Firm B: High Price is always the best response, regardless of Firm A's choice. Therefore, High Price is a **dominant strategy** for Firm B.

c) Nash Equilibrium

A **Nash equilibrium** occurs when each player's strategy is the best response to the other player's strategy. In other words, no player can benefit by unilaterally changing their strategy given the strategy of the other player.

From the analysis above, we found that both Firm A and Firm B have High Price as their dominant strategy. Thus, when both firms choose High Price:

- Firm A's payoff is 8.
- Firm B's payoff is 8.

Since both firms choosing High Price is the best response for each given the strategy of the other, the pair (High Price, High Price) is a Nash equilibrium.

Problem 1: Production Levels

Two companies, A and B, are competing in a market where each must decide independently whether to adopt a high-price or low-price strategy. If both companies choose a high-price strategy, they maintain their current market shares and profits. However, if one company adopts a low-price strategy while the other sticks to a high-price strategy, the company with the low-price strategy captures a larger market share, leading to increased profits for them and losses for the other. If both companies adopt a low-price strategy, they engage in a price war, leading to lower profits for both.

a) Construct the payoff matrix for this scenario.

b) Identify the Nash equilibrium in this game and explain the strategic reasoning behind it.

a) Constructing the Payoff Matrix

To construct the payoff matrix, we need to define the payoffs for both companies, A and B, based on their pricing strategies. Let's assume:

- **High-price strategy** gives a payoff of 3 for each company if both choose it, representing a stable market and profit.

- **Low-price strategy** gives a payoff of 1 for the company that chooses it and a payoff of 0 for the other company if they stick to the high-price strategy, representing a market share shift.
- **Low-price strategy** for both companies results in a payoff of 2 for each, representing a price war with lower profits.

The payoff matrix for the two companies, A and B, can be constructed as follows:

	Company B: High Price	**Company B: Low Price**
Company A: High Price	(3, 3)	(0, 1)
Company A: Low Price	(1, 0)	(2, 2)

Explanation of Payoffs:

- **(3, 3):** Both companies choose the high-price strategy. They maintain their current market shares and profits.
- **(0, 1):** Company A chooses a high-price strategy, and Company B chooses a low-price strategy. Company B captures a larger market share, leading to increased profits for B and losses for A.
- **(1, 0):** Company A chooses a low-price strategy, and Company B chooses a high-price strategy. Company A captures a larger market share, leading to increased profits for A and losses for B.
- **(2, 2):** Both companies choose the low-price strategy. They engage in a price war, leading to lower profits for both.

b) Identifying the Nash Equilibrium and Strategic Reasoning

Nash Equilibrium:

- A Nash equilibrium occurs when no player has an incentive to deviate unilaterally from their chosen strategy.

- To find the Nash equilibrium, we analyze the best responses for each company given the strategy of the other.

Step 1: Analyze Company A's Best Responses

- If Company B chooses **High Price**, Company A's best response is **High Price** (payoff = 3 vs. 1).
- If Company B chooses **Low Price**, Company A's best response is **Low Price** (payoff = 2 vs. 0).

Step 2: Analyze Company B's Best Responses

- If Company A chooses **High Price**, Company B's best response is **High Price** (payoff = 3 vs. 0).
- If Company A chooses **Low Price**, Company B's best response is **Low Price** (payoff = 2 vs. 1).

Nash Equilibrium Identification:

- The only situation where both companies are playing their best responses is when **both companies choose the Low-Price strategy**.
- This gives the payoff (2, 2), meaning both companies are engaged in a price war but have no incentive to deviate unilaterally because the alternative would result in a lower payoff (1 or 0).

Strategic Reasoning:

- Even though adopting a low-price strategy leads to lower profits overall, each company knows that if it alone adopts a high-price strategy while the other adopts a low-price strategy, it will suffer a significant loss in market share and profits.
- Thus, both companies anticipate that the other will adopt a low-price strategy, leading them to also choose a low-price strategy to avoid being undercut, despite the fact that it leads to a price war and lower profits.

Problem 2: Advertising Campaign

Problem: Two companies, Company A and Company B, are deciding whether to launch an expensive advertising campaign or not. The payoffs are as follows:

- If both companies launch the campaign, each earns $4 million.
- If one company launches the campaign and the other does not, the company that advertises earns $7 million, and the other earns $2 million.
- If neither company advertises, each earns $5 million.

a) Construct the payoff matrix.
b) Determine if either company has a dominant strategy.
c) Identify the Nash equilibrium of this game.

Solution:

a) Payoff Matrix:

	Company B: Advertise	**Company B: Not Advertise**
Company A: Advertise	(4, 4)	(7, 2)
Company A: Not Advertise	(2, 7)	(5, 5)

b) Dominant Strategy:

- **Company A:**
  - If Company B advertises, Company A earns $4 million by advertising and $2 million by not advertising.
  - If Company B does not advertise, Company A earns $7 million by advertising and $5 million by not advertising.

   - **Conclusion:** Company A should always choose to advertise because it yields a higher payoff regardless of Company B's choice. Therefore, Company A has a **dominant strategy** to advertise.
- **Company B:**
   - If Company A advertises, Company B earns \$4 million by advertising and \$2 million by not advertising.
   - If Company A does not advertise, Company B earns \$7 million by advertising and \$5 million by not advertising.
   - **Conclusion:** Company B should also always choose to advertise because it yields a higher payoff regardless of Company A's choice. Therefore, Company B has a **dominant strategy** to advertise.

c) Nash Equilibrium:

- Since both companies have a dominant strategy to advertise, the Nash equilibrium of the game is **(Advertise, Advertise)** with payoffs **(4, 4)**.

Problem 3: Production Levels

Problem: Two factories, Factory 1 and Factory 2, are deciding on production levels. Each factory can choose to produce either "High Output" or "Low Output." The payoffs are as follows:

- If both choose high output, each earns \$6 million.
- If one chooses high output and the other chooses low output, the factory with high output earns \$8 million, and the one with low output earns \$3 million.
- If both choose low output, each earns \$7 million.

a) Construct the payoff matrix.

b)Determine if either factory has a dominant strategy.

c) Identify the Nash equilibrium of this game.

Solution:

a) Payoff Matrix:

	Factory 2: High Output	**Factory 2: Low Output**
Factory 1: High Output	(6, 6)	(8, 3)
Factory 1: Low Output	(3, 8)	(7, 7)

b) Dominant Strategy:

- **Factory 1:**
    - If Factory 2 chooses high output, Factory 1 earns \$6 million by choosing high output and \$3 million by choosing low output.
    - If Factory 2 chooses low output, Factory 1 earns \$8 million by choosing high output and \$7 million by choosing low output.
    - **Conclusion:** Factory 1 should always choose high output because it yields a higher payoff regardless of Factory 2's choice. Therefore, Factory 1 has a **dominant strategy** to choose high output.
- **Factory 2:**
    - If Factory 1 chooses high output, Factory 2 earns \$6 million by choosing high output and \$3 million by choosing low output.
    - If Factory 1 chooses low output, Factory 2 earns \$8 million by choosing high output and \$7 million by choosing low output.
    - **Conclusion:** Factory 2 should also always choose high output because it yields a higher payoff regardless of Factory 1's choice. Therefore, Factory 2 has a **dominant strategy** to choose high output.

c) Nash Equilibrium:

- Since both factories have a dominant strategy to choose high output, the Nash equilibrium of the game is **(High Output, High Output)** with payoffs **(6, 6)**.

Problem 4: Product Launch Timing

Problem: Two software companies, Software A and Software B, are deciding on the timing of their new product launch. Each company can either launch early or late. The payoffs are as follows:

- If both companies launch early, they both capture the market simultaneously, earning $6 million each.
- If one company launches early while the other launches late, the early launcher earns $9 million, and the late launcher earns $4 million.
- If both companies launch late, they maintain the market and earn $7 million each.

a) **Construct the payoff matrix.**

b)Determine if either company has a dominant strategy.

c) Identify the Nash equilibrium of this game.

Solution:

a) Payoff Matrix:

	Software B: Launch Early	**Software B: Launch Late**
Software A: Launch Early	(6, 6)	(9, 4)
Software A: Launch Late	(4, 9)	(7, 7)

b) Dominant Strategy:

- **Software A:**

  - If Software B launches early, Software A earns \$6 million by launching early and \$4 million by launching late.
  - If Software B launches late, Software A earns \$9 million by launching early and \$7 million by launching late.
  - **Conclusion:** Software A should always launch early because it yields a higher payoff regardless of Software B's choice. Therefore, Software A has a **dominant strategy** to launch early.
- **Software B:**
  - If Software A launches early, Software B earns \$6 million by launching early and \$4 million by launching late.
  - If Software A launches late, Software B earns \$9 million by launching early and \$7 million by launching late.
  - **Conclusion:** Software B should also always launch early because it yields a higher payoff regardless of Software A's choice. Therefore, Software B has a **dominant strategy** to launch early.

c) Nash Equilibrium:

- Since both companies have a dominant strategy to launch early, the Nash equilibrium of the game is **(Launch Early, Launch Early)** with payoffs **(6, 6)**.

non cooperative games

Q. Identify the game in which no form of negotiation and binding contracts and explain with one example.

Ans. The game in which no form of negotiation or binding contracts is allowed is called a **non-cooperative game**. In non-cooperative games, players make decisions independently, and there are no enforceable agreements or cooperation between players. Each player seeks to maximize their own payoff without the possibility of forming alliances or binding agreements with other players.

Examples

Problem 1: The Battle of the Sexes

Problem: A couple, Alex and Jordan, want to decide on an evening activity. Alex prefers going to a football game, while Jordan prefers going to a movie. However, they both prefer spending the evening together rather than apart. The payoffs are as follows:

- If they both choose the football game, Alex is very happy (3) and Jordan is somewhat happy (2).
- If they both choose the movie, Jordan is very happy (3) and Alex is somewhat happy (2).
- If they end up in different places, they both are unhappy and get 0.

a) **Construct the payoff matrix.**

b)Identify the Nash equilibria in this game.

c) Discuss why this is a non-cooperative game.

Solution:

a) Payoff Matrix:

	Jordan: Football	**Jordan: Movie**
Alex: Football	(3, 2)	(0, 0)
Alex: Movie	(0, 0)	(2, 3)

b) Nash Equilibria:

- **(Football, Football):** Alex and Jordan both choose football. Alex is happy with a payoff of 3, and Jordan is less happy with a payoff of 2. Neither has an incentive to deviate, as doing so would result in a payoff of 0.

- **(Movie, Movie):** Alex and Jordan both choose the movie. Jordan is happy with a payoff of 3, and Alex is less happy with a payoff of 2. Neither has an incentive to deviate, as doing so would result in a payoff of 0.

c) Why This is a Non-Cooperative Game:

- **No Binding Agreement:** Alex and Jordan must make their decisions independently without any binding contract or negotiation to enforce cooperation.
- **Individual Rationality:** Each player seeks to maximize their own payoff given the other player's decision. This self-interest may lead to outcomes where the players end up at different locations, yielding a payoff of 0 for both.

Problem 2: Cournot Competition

Problem:

Two firms, Firm A and Firm B, compete in a market by deciding how much quantity q_A and q_B to produce of a homogenous good. The market price P is determined by the total quantity produced, $P = 100 - (q_A + q_B)$. Both firms have no production costs. Each firm's profit is $\pi_A = q_A \times P$ for Firm A and $\pi_B = q_B \times P$ for Firm B.

a) Find the best response function for each firm.

b) Identify the Nash equilibrium quantities q_A and q_B.

Solution:

a) Best Response Functions:

- Firm A's profit:

$$\pi_A = q_A \times (100 - (q_A + q_B))$$

Maximizing π_A with respect to q_A gives:

$$\frac{d\pi_A}{dq_A} = 100 - 2q_A - q_B = 0 \implies q_A = \frac{100 - q_B}{2}$$

This is Firm A's best response function.

- Similarly, Firm B's best response function is:

$$q_B = \frac{100 - q_A}{2}$$

b) Nash Equilibrium:

- Solving the two best response functions simultaneously:

$$q_A = \frac{100 - q_B}{2} \quad \text{and} \quad q_B = \frac{100 - q_A}{2}$$

Substituting one into the other:

$$q_A = \frac{100 - \frac{100 - q_A}{2}}{2} \implies q_A = \frac{100}{3} \approx 33.33$$

Similarly:

$$q_B = \frac{100}{3} \approx 33.33$$

So, the Nash equilibrium quantities are $q_A = q_B \approx 33.33$.

Problem 3: Matching Pennies

Problem: Two players, Player 1 and Player 2, are playing a game called "Matching Pennies." Each player has a penny and must secretly turn it to heads or tails. Afterward, both players reveal their choices:

- If the pennies match (both heads or both tails), Player 1 wins and takes Player 2's penny.
- If the pennies do not match, Player 2 wins and takes Player 1's penny.

a) Construct the payoff matrix.

b)Determine if there is a pure strategy Nash equilibrium.

c) Find the mixed strategy Nash equilibrium for this game.

Solution:

a) Payoff Matrix:

	Player 2: Heads	**Player 2: Tails**
Player 1: Heads	(1, -1)	(-1, 1)
Player 1: Tails	(-1, 1)	(1, -1)

b) Pure Strategy Nash Equilibrium:

- There is no pure strategy Nash equilibrium in this game. If Player 1 knows Player 2's choice, they will choose the same (if the pennies are heads and Player 2 chooses heads, Player 1 wins) or the opposite (if the pennies are tails and Player 2 chooses tails, Player 1 wins), making the outcome unpredictable.

c) Mixed Strategy Nash Equilibrium:

- Let p be the probability that Player 1 chooses heads, and q the probability that Player 2 chooses heads.
- Player 1 is indifferent between choosing heads or tails if:

$$1 \times q + (-1) \times (1 - q) = -1 \times q + 1 \times (1 - q)$$

Solving gives $q = \frac{1}{2}$.

- Similarly, Player 2 is indifferent between choosing heads or tails if:

$$(-1) \times p + 1 \times (1 - p) = 1 \times p + (-1) \times (1 - p)$$

Solving gives $p = \frac{1}{2}$.

- The mixed strategy Nash equilibrium is therefore:

$$p = q = \frac{1}{2}$$

This means each player chooses heads or tails with equal probability (50%).

Problem 4: Voting Game

Problem: Three candidates (A, B, and C) are running for office, and three voters must choose which candidate to support. Each voter can vote for one candidate, and the candidate with the most votes wins. If a voter's preferred candidate wins, they get a payoff of 3. If their second-choice wins, they get a payoff of 2. If their least favorite wins, they get a payoff of 0. Each voter's preferences are as follows:

- Voter 1: A > B > C
- Voter 2: B > C > A
- Voter 3: C > A > B

  a) What are the possible outcomes of the election?

  b) Is there a pure strategy Nash equilibrium?

  c) Discuss why this is a non-cooperative game.

Solution:

a) Possible Outcomes:

- Candidate A wins if Voter 1 votes for A, and either Voter 2 or Voter 3 votes for A.
- Candidate B wins if Voter 2 votes for B, and either Voter 1 or Voter 3 votes for B.
- Candidate C wins if Voter 3 votes for C, and either Voter 1 or Voter 2 votes for C.

b) Pure Strategy Nash Equilibrium:

- One possible pure strategy Nash equilibrium is where each voter votes for their first choice (Voter 1 for A, Voter 2 for B, Voter 3 for C), resulting in a tie with each candidate receiving one vote. This outcome does not change if any single voter deviates, as the deviating voter would not improve their payoff.

c) Why This is a Non-Cooperative Game:

- **Independent Decision-Making:** Each voter independently chooses their vote without any enforceable agreement or coordination with other voters.
- **No Binding Agreement:** Voters cannot form a binding coalition to agree on a candidate.
- **Self-Interest:** Each voter seeks to maximize their own utility based on their preferences, leading to potential conflicts of interest.

Maximin Principle

Problem 1:

Solve the following games by using Maximin (Minimax) principle whose payoff matrix are given below: (Include in your answer the strategy selection for each player and the value of game to each player)

Player A	**Player B**			
	B1	B2	B3	B4

A1	3	-5	0	6
A2	-4	-2	1	2
A3	5	4	2	3

Solution:

Maximin Strategy for Player A

1. **Calculate the minimum payoff for each strategy of Player A:**
   - For **A1**:
     - Minimum value = min(3, -5, 0, 6) = -5
   - For **A2**:
     - Minimum value = min(-4, -2, 1, 2) = -4
   - For **A3**:
     - Minimum value = min(5, 4, 2, 3) = 2
2. **Determine the maximum of these minimum values:**
   - Maximin value = max(-5, -4, 2) = 2

   So, Player A should choose A3 to maximize their minimum payoff.

Minimax Strategy for Player B

1. **Calculate the maximum payoff for each strategy of Player B:**

- For **B1**:
  - Maximum value = max(3, -4, 5) = 5
- For **B2**:
  - Maximum value = max(-5, -2, 4) = 4
- For **B3**:
  - Maximum value = max(0, 1, 2) = 2
- For **B4**:
  - Maximum value = max(6, 2, 3) = 6

2. **Determine the minimum of these maximum values:**
   - Minimax value = min(5, 4, 2, 6) = 2

   So, Player B should choose B3 to minimize their maximum payoff.

- **Strategy for Player A (row player):** A3
- **Strategy for Player B (column player):** B3
- **Value of the game:** 2

- Player A should choose A3 because it maximizes the minimum payoff they can secure.
- Player B should choose B3 because it minimizes the maximum payoff Player A can achieve.

Thus, the game value is 2, which is the outcome when both players use their optimal strategies: Player A chooses A3 and Player B chooses B3.

Problem 2: Two-Player Game

Payoff Matrix:

Player X / Player Y	**Y1**	**Y2**	**Y3**
X1	4	-1	2

Player X / Player Y	Y1	Y2	Y3
X2	3	2	-3
X3	5	-2	1

Solution:

1. **Determine the minimum payoff for each strategy of Player X:**
   - For **X1**:
     - Minimum value = min(4, -1, 2) = -1
   - For **X2**:
     - Minimum value = min(3, 2, -3) = -3
   - For **X3**:
     - Minimum value = min(5, -2, 1) = -2
2. **Determine the maximum of these minimum values:**
   - Maximin value = max(-1, -3, -2) = -1

Optimal strategy for Player X: X1

3. **Value of the game**: -1

4.

Problem 3: Zero-Sum Game

Payoff Matrix for Player A:

Player A / Player B	B1	B2	B3
A1	8	4	1
A2	2	5	7
A3	6	3	4

Solution:

1. **Determine the minimum payoff for each strategy of Player A:**
   - For **A1**:
     - Minimum value = min(8, 4, 1) = 1
   - For **A2**:
     - Minimum value = min(2, 5, 7) = 2
   - For **A3**:
     - Minimum value = min(6, 3, 4) = 3
2. **Determine the maximum of these minimum values:**
   - Maximin value = max(1, 2, 3) = 3

Optimal strategy for Player A: A3

3. **Value of the game**: 3

Problem 4: Mixed Strategy Game

Payoff Matrix for Player A:

Player A / Player B	**B1**	**B2**	**B3**	**B4**
A1	7	-1	6	4
A2	2	5	3	1
A3	8	4	-2	3

Solution:

1. **Determine the minimum payoff for each strategy of Player A:**
   - For **A1**:
     - Minimum value = min(7, -1, 6, 4) = -1
   - For **A2**:

- Minimum value = min(2, 5, 3, 1) = 1
- For **A3**:
  - Minimum value = min(8, 4, -2, 3) = -2

2. **Determine the maximum of these minimum values:**
   - Maximin value = max(-1, 1, -2) = 1

Optimal strategy for Player A: A2

3. **Value of the game**: 1
4.

Problem 5: Mixed Strategy Game with Different Matrix

Payoff Matrix for Player A:

Player A / Player B	**B1**	**B2**	**B3**
A1	10	2	7
A2	6	4	3
A3	8	5	9

Solution:

1. **Determine the minimum payoff for each strategy of Player A:**
   - For **A1**:
     - Minimum value = min(10, 2, 7) = 2
   - For **A2**:
     - Minimum value = min(6, 4, 3) = 3
   - For **A3**:
     - Minimum value = min(8, 5, 9) = 5
2. **Determine the maximum of these minimum values:**
   - Maximin value = max(2, 3, 5) = 5

Optimal strategy for Player A: A3

3. **Value of the game**: 5

Practice Problem:

Player A \ Player B	1	2	3	4	5
1	4	6	5	10	6
2	7	8	5	9	10
3	8	9	11	10	9
4	6	4	10	6	4

Find the solution of game by Pure strategy.

Bayesian game

Problem 1: Two firms are deciding whether to enter or stay out of a market. Each firm believes the market is favorable with probability 0.5 and unfavorable with probability 0.5. The payoffs for the firms are:

- Both Enter: Each earns 0.
- One Enters, One Stays Out: The entering firm earns 3 if the market is favorable, and 1 if the market is unfavorable. The staying-out firm earns 0.
- Both Stay Out: Each earns 1.

Assuming each firm believes the market is favorable with probability 0.5, construct a Bayesian game model and identify the Bayesian Nash Equilibrium.

1. Constructing the Bayesian Game Model

Players:

- Two firms: Firm A and Firm B.

Types:

- Each firm can believe that the market is either favorable (F) or unfavorable (U). Each firm has a 0.5 probability of believing the market is favorable or unfavorable.

Strategies:

- Each firm has two strategies: Enter (E) or Stay Out (S).

Payoffs: Given the market conditions, the payoffs are as follows:

- **Both Enter (E, E):**
    - Each firm earns 0 regardless of the market condition.
- **One Enters, One Stays Out (E, S) or (S, E):**
    - If the market is favorable (F):
        - The entering firm earns 3, and the staying-out firm earns 0.
    - If the market is unfavorable (U):
        - The entering firm earns 1, and the staying-out firm earns 0.
- **Both Stay Out (S, S):**
    - Each firm earns 1 regardless of the market condition.

Beliefs:

- Each firm believes with probability 0.5 that the market is favorable or unfavorable.

2. Construct the Payoff Matrix

To simplify, we will construct a matrix for each possible market condition and then average the payoffs considering each firm's belief.

Payoff Matrix for the Market Being Favorable (F):

Firm B / Firm A	Enter (E)	Stay Out (S)
Enter (E)	(0, 0)	(3, 0)
Stay Out (S)	(0, 3)	(1, 1)

Payoff Matrix for the Market Being Unfavorable (U):

Firm B / Firm A	Enter (E)	Stay Out (S)
Enter (E)	(0, 0)	(1, 0)
Stay Out (S)	(0, 1)	(1, 1)

3. Compute Expected Payoffs

Since each firm believes the market is favorable with probability 0.5 and unfavorable with probability 0.5, the expected payoffs for each strategy profile need to be calculated.

Expected Payoffs for Firm A:

1. **If Firm A chooses Enter (E):**
   - **If Firm B chooses Enter (E):**
     - Payoff = 0 (regardless of market condition)
   - **If Firm B chooses Stay Out (S):**
     - Payoff = 0.5 * 3 + 0.5 * 1 = 2
2. **If Firm A chooses Stay Out (S):**
   - **If Firm B chooses Enter (E):**
     - Payoff = 0.5 * 0 + 0.5 * 1 = 0.5
   - **If Firm B chooses Stay Out (S):**

- Payoff = 1 (regardless of market condition)

Expected Payoffs for Firm B:

1. **If Firm B chooses Enter (E):**
   - **If Firm A chooses Enter (E):**
     - Payoff = 0 (regardless of market condition)
   - **If Firm A chooses Stay Out (S):**
     - Payoff = 0.5 * 0 + 0.5 * 1 = 0.5
2. **If Firm B chooses Stay Out (S):**
   - **If Firm A chooses Enter (E):**
     - Payoff = 0.5 * 0 + 0.5 * 1 = 0.5
   - **If Firm A chooses Stay Out (S):**
     - Payoff = 1 (regardless of market condition)

4. Construct the Expected Payoff Matrix

For each firm, the expected payoffs can be summarized as follows:

Firm B / Firm A	**Enter (E)**	**Stay Out (S)**
Enter (E)	(0, 0)	(2, 0.5)
Stay Out (S)	(0.5, 2)	(1, 1)

5. Identify Bayesian Nash Equilibrium

We need to find the strategies where neither firm can improve their expected payoff given the strategy of the other firm.

1. **If Firm A chooses Enter (E):**

   - Firm B's expected payoff is 0 against Enter (E) or 0.5 against Stay Out (S). Firm B prefers Stay Out (S) with a payoff of 0.5.
2. **If Firm A chooses Stay Out (S):**
   - Firm B's expected payoff is 0.5 against Enter (E) or 1 against Stay Out (S). Firm B prefers Stay Out (S) with a payoff of 1.

From Firm A's perspective:

- If Firm B chooses Enter (E), Firm A prefers Stay Out (S) because 0.5 > 0.
- If Firm B chooses Stay Out (S), Firm A prefers Stay Out (S) because 1 > 2.

From Firm B's perspective:

- If Firm A chooses Enter (E), Firm B prefers Stay Out (S) because 0.5 > 0.
- If Firm A chooses Stay Out (S), Firm B prefers Stay Out (S) because 1 > 0.5.

Bayesian Nash Equilibrium

The Bayesian Nash Equilibrium for this game is (Stay Out, Stay Out).

- In this equilibrium, both firms stay out of the market, resulting in a payoff of 1 for each firm. Neither firm has an incentive to deviate given the other firm's strategy.

Problem 1: Investment Decision

Scenario: Two firms are deciding whether to invest in a new technology. Each firm believes the market conditions are either good (G) or bad (B). The probability that the market is good is 0.6. If both firms invest, each earns a payoff of 1 if the market is good and -1 if the market is bad. If one firm invests and the other does not, the investing firm earns 3 if the market is good and 0 if the market is bad. The non-investing firm earns 0 regardless of the market condition. If neither firm invests, each earns 2.

Solution:

1. **Construct Payoff Matrix:**

   For each firm, the payoffs can be summarized based on the market conditions:

   - **If Market is Good (G):**
     - Both Invest (I, I): Payoff = (1, 1)
     - One Invests, One Does Not (I, N) or (N, I): Payoff = (3, 0) or (0, 3)
     - Neither Invest (N, N): Payoff = (2, 2)
   - **If Market is Bad (B):**
     - Both Invest (I, I): Payoff = (-1, -1)
     - One Invests, One Does Not (I, N) or (N, I): Payoff = (0, 0) or (0, 0)
     - Neither Invest (N, N): Payoff = (2, 2)

2. **Expected Payoffs:**

   Each firm believes the market is good with probability 0.6 and bad with probability 0.4.

   - **Expected Payoff for Firm A:**
     - **If Firm A chooses Invest (I):**
       - **If Firm B chooses Invest (I):**
         - Payoff = 0.6 * 1 + 0.4 * (-1) = 0.6 - 0.4 = 0.2
       - **If Firm B chooses Not Invest (N):**
         - Payoff = 0.6 * 3 + 0.4 * 0 = 1.8
     - **If Firm A chooses Not Invest (N):**
       - **If Firm B chooses Invest (I):**
         - Payoff = 0.6 * 0 + 0.4 * 0 = 0
       - **If Firm B chooses Not Invest (N):**
         - Payoff = 0.6 * 2 + 0.4 * 2 = 2

- **Expected Payoff for Firm B:**
  - **If Firm B chooses Invest (I):**
    - **If Firm A chooses Invest (I):**
      - Payoff = 0.6 * 1 + 0.4 * (-1) = 0.2
    - **If Firm A chooses Not Invest (N):**
      - Payoff = 0.6 * 3 + 0.4 * 0 = 1.8
  - **If Firm B chooses Not Invest (N):**
    - **If Firm A chooses Invest (I):**
      - Payoff = 0.6 * 0 + 0.4 * 0 = 0
    - **If Firm A chooses Not Invest (N):**
      - Payoff = 0.6 * 2 + 0.4 * 2 = 2

3. **Bayesian Nash Equilibrium:**
   - If both firms choose Invest (I), the expected payoff for both is 0.2.
   - If one firm invests and the other does not, the investing firm earns 1.8.
   - If neither invests, each earns 2.

   Both firms prefer not investing when the other does not invest, given their expected payoffs are higher.

   Bayesian Nash Equilibrium: (N, N) – Both firms choose not to invest.

Problem 2: Public Goods Contribution

Scenario: Two individuals, Alice and Bob, decide whether to contribute to a public good. Each has private information about their valuation of the public good. Alice values the public good at $10 with probability 0.4 and $5 with probability 0.6. Bob has the same probabilities. If both contribute, each pays a cost of $3. If only one contributes, that person pays $3 and the other pays nothing. If neither contributes, the cost is $0. The public good is valued at $15 if at least one person contributes.

Solution:

1. **Construct Payoff Matrix:**
   - **If Both Contribute (C, C):**
     - Cost: -$3 (Alice) + -$3 (Bob) = -$6
     - Benefit: $15, shared equally: $7.50 each
     - Net Payoff: $7.50 - $3 = $4.50 each
   - **If One Contributes and One Does Not (C, N) or (N, C):**
     - Contributor:
       - Cost: -$3
       - Benefit: $15
       - Net Payoff: $15 - $3 = $12
     - Non-Contributor:
       - Benefit: $15
       - Net Payoff: $15
   - **If Neither Contributes (N, N):**
     - Benefit: $0
     - Net Payoff: $0
2. **Expected Payoffs:**

   Alice's Expected Payoff:

   - **If Alice contributes:**
     - **If Bob contributes:** $4.50
     - **If Bob does not contribute:** $12

     Expected payoff for Alice = 0.4 * 4.50 + 0.6 * 12 = 1.8 + 7.2 = 9

   - **If Alice does not contribute:**

- **If Bob contributes:** $15
- **If Bob does not contribute:** $0

Expected payoff for Alice = 0.4 * 15 + 0.6 * 0 = 6

Bob's Expected Payoff:

- **If Bob contributes:**
  - **If Alice contributes:** $4.50
  - **If Alice does not contribute:** $12

  Expected payoff for Bob = 0.4 * 4.50 + 0.6 * 12 = 1.8 + 7.2 = 9

- **If Bob does not contribute:**
  - **If Alice contributes:** $15
  - **If Alice does not contribute:** $0

  Expected payoff for Bob = 0.4 * 15 + 0.6 * 0 = 6

3. **Bayesian Nash Equilibrium:**

Each individual prefers to contribute if the other is not contributing (since 12 > 6), and both prefer not to contribute if the other is also not contributing.

Bayesian Nash Equilibrium: (C, C) or (N, N) – Both contribute or both do not contribute.

Problem 3: Job Offer Decision

Scenario: Two job applicants, John and Mary, are applying for a job where they can either accept an offer (A) or reject it (R). Each believes there's a 70% chance they will receive an offer from another company and a 30% chance they will not. The payoff for each depends on whether they receive an offer:

- **If both accept (A, A):**
  - Both get \$4 each.
- **If one accepts and the other rejects (A, R) or (R, A):**
  - The acceptor gets \$6 and the rejector gets \$2.
- **If both reject (R, R):**
  - Both get \$3.

Solution:

1. **Construct Payoff Matrix:**
   - **If Both Accept (A, A):**
     - Payoff: (\$4, \$4)
   - **If One Accepts and One Rejects (A, R) or (R, A):**
     - Payoff: (\$6, \$2) or (\$2, \$6)
   - **If Both Reject (R, R):**
     - Payoff: (\$3, \$3)
2. **Expected Payoffs:**

   John's Expected Payoff:

   - **If John accepts:**
     - **If Mary accepts:** \$4
     - **If Mary rejects:** \$6

Expected payoff for John = 0.7 * 4 + 0.3 * 6 = 2.8 + 1.8 = 4.6

- **If John rejects:**
  - **If Mary accepts:** $2
  - **If Mary rejects:** $3

  Expected payoff for John = 0.7 * 2 + 0.3 * 3 = 1.4 + 0.9 = 2.3

Mary's Expected Payoff:

- **If Mary accepts:**
  - **If John accepts:** $4
  - **If John rejects:** $6

  Expected payoff for Mary = 0.7 * 4 + 0.3 * 6 = 2.8 + 1.8 = 4.6

- **If Mary rejects:**
  - **If John accepts:** $2
  - **If John rejects:** $3

  Expected payoff for Mary = 0.7 * 2 + 0.3 * 3 = 1.4 + 0.9 = 2.3

3. **Bayesian Nash Equilibrium:**

   Each person prefers to accept if they believe the other will accept due to higher payoffs.

   Bayesian Nash Equilibrium: (A, A) – Both accept.

Game of Incomplete Information

Q. Two parties are involved in bargaining over a resource. Party 1 is uncertain about Party 2's valuation, which could be either high or low. Party 1 makes an offer, and Party 2 can either accept or reject it. If Party 2 accepts, the resource is split according to the offer. If Party 2 rejects, both parties receive nothing.

a) Model this situation as a game of incomplete information..

b) Identify the role of beliefs in this bargaining scenario and how they influence the outcome.

Ans,

a) Modeling the Situation as a Game of Incomplete Information

Players:

- Party 1 (P1)
- Party 2 (P2)

Types of Party 2:

- Party 2 can have two types:
  - **High Valuation (H):** Values the resource highly.
  - **Low Valuation (L):** Values the resource less.

  Assume Party 2's valuation is high with probability ppp and low with probability 1−p1-p1−p.

Strategies:

- **Party 1's Strategy:** Make an offer OOO for how to split the resource.
- **Party 2's Strategy:** Accept or reject the offer.

Payoffs:

- If Party 2 accepts an offer OOO from Party 1, the resource is split according to OOO. Suppose OOO is a fraction α\alphaα of the resource going to Party 1, and 1−α1 - \alpha1−α going to Party 2.
- If Party 2 rejects the offer, both parties get nothing.

Types and Beliefs:

- Party 2's valuation (high or low) is private information.
- Party 1 knows the probability distribution of Party 2's valuation but not the exact type.

c) Role of Beliefs in the Bargaining Scenario

Beliefs:

- Party 1 has beliefs about Party 2's type based on the probability distribution.
- Party 1's beliefs about Party 2's valuation influence the offer they make.
- The beliefs affect the decision of Party 2 whether to accept or reject the offer.

Influence on the Outcome:

- Party 1's offer is based on the expectation that Party 2 will accept if the offer meets or exceeds their reservation value.
- The more Party 1 believes Party 2 is likely to have a high valuation, the more Party 1 can afford to offer less and still expect acceptance.
- Party 2's acceptance depends on their private valuation and whether the offer meets their reservation value given their belief about the probability distribution.