Semester : III   Subject : DSGT   Academic Year: 2022-2023

* Monoid :- A group which shows property of an identity element with respect to opⁿ *.
Let (A, *) be an algebraic system, where * is a binary operation on A. (A, *) is called a monoid if the following conditions are satisfied. $(a*e) = (e*a) = a \quad \forall a \in S.$
= * is a closed operation
= * is an associative operation
= There is an identity.

Examples -

(1) (Z, *) is monoid or not?
$\Rightarrow Z = \{ \ldots -3, -2, -1, 0, 1, 2, 3, \ldots \}$
$(a*e) = (e*a) = a$
consider $a = 2$
$2 * e = 2$ $e*a = e*2 = 2$
$e = 2/2$ $e = 2/2$
$\boxed{e = 1}$ $\boxed{e = 1}$

so it is monoid
we have to check for algebraic structure and semigroup.

Semester : III   Subject : DSGT   Academic Year: 2022-2023

(2) Given semigroup is monoid or not? $(N, \cdot)$ or $(N, *)$

⇒ check for closure property.

$N = \{1, 2, 3, \ldots \infty\}$

$a * b \in N$

$2 * 3 = 6 \qquad 6 \in N, \ 2 \& 3 \in N$

so it is an algebraic structure.

Now check for semigroup

associativity property.

$(a*b)*c = a*(b*c)$

$a = 2$

$b = 3$

$c = 1$

$(2*3)*1 = 2*(3*1)$

$6*1 = 2*3$

$6 = 6$

hence it is semigroup as it satisfies associativity property.

Now check for identity

$(a*e) = (e*a) = a$

$5*e = 5 \qquad e*5 = 5$

$e = 5/5 \qquad e = 5/5$

$\boxed{e = 1} \qquad \boxed{e = 1}$

identity property satisfied hence it is monoid also

Semester : ________ Subject : ________ Academic Year: 20 - 20

3) $(R, \div)$ is monoid or not ?

$R = \{$ set of real no $\}$
$= \{ \dots -3, -\sqrt{2}, -1/2, 0, 1, 4/5, 16, \dots \}$

First check for closure property.

$a * b \in R \quad a, b \in R$

$16 \div 1 = 16 \qquad 1 \div 16 = 0.06$

$0 \div 16 = 0 \qquad 16 \div 0 = \infty$

it is not a member of set of real numbers.

so it is not algebraic structure
so it is not semigroup
also not a monoid.

4) $(R^*, \div)$

R^* = set of all non-zero real numbers
$R^* = \{ \dots -3, -\sqrt{2}, -1/2, 1, 4/5, 16, \dots \}$

check for closure property
$a * b \in R$
$16 \div 1 = 16$

so it is algebraic structure.

Semester : ______ Subject : ______ Academic Year: 20 - 20

check for associativity property

$a = 2, \; b = 3, \; c = 5$

$(a * b) * c = a * (b * c)$

$a = 16 \quad (a*b)*c = a*(b*c)$

$b = 1$

$c = 1$

$(2*3)*5 = 2*(3*5)$

$30 = 30$

so it is semigroup

check for monoid, identity property

$(a*e) = (e*a) = e$

$a = 2$

$2*e = 2 \qquad e*2 = 2$

$e = 2/2 \qquad e = 2/2$

$\boxed{e = 1} \qquad \boxed{e = 1}$

so it satisfies identity property

so $(R^*, \div)$ is monoid.