5 a) Minimize $f(x_1, x_2) = 4x_1 + 8x_2 - x_1^2 - x_2^2$

subject to $x_1 + x_2 = 4$, $x_1, x_2 \geq 0$

$\rightarrow$ $f(x_1, x_2) = 4x_1 + 8x_2 - x_1^2 - x_2^2$

$\phi = x_1 + x_2 - 4 = 0$

Let $L = f + \lambda \phi$

$L = 4x_1 + 8x_2 - x_1^2 - x_2^2 + \lambda(x_1 + x_2 - 4)$

$\frac{\partial L}{\partial x_1} = 4 - 2x_1 + \lambda = 0 \Rightarrow x_1 = \frac{\lambda + 4}{2}$ — (1)

$\frac{\partial L}{\partial x_2} = 8 - 2x_2 + \lambda = 0 \Rightarrow x_2 = \frac{8 + \lambda}{2}$ — (2)

$\frac{\partial L}{\partial \lambda} = x_1 + x_2 - 4 = 0 \Rightarrow x_1 + x_2 = 4$ — (3)

Substituting eqn (1) & (2) in eqn (3),

$\frac{\lambda + 4}{2} + \frac{8 + \lambda}{2} = 4$

$\frac{2\lambda + 12}{2} = 4$

$2\lambda = -4 \Rightarrow \boxed{\lambda = -2}$

From (1), $x_1 = \frac{4-2}{2} = 1$

From (2), $x_2 = \frac{8-2}{2} = 3$

Point of minima is $(1, 3)$.

5b) Find minimizer, $f(x) = x^2 + \frac{54}{x}$ using bisection method in $(2, 5)$ within a range of 0.3.

$f(x) = x^2 + \frac{54}{x}$, $f'(x) = 2x - \frac{54}{x^2}$

Iteration	a	b	$c = \frac{a+b}{2}$	$f'(c)$	Remark
1	2	5	$c = 3.5$	$2.5918 > 0$	Replace b by c
2	2	3.5	$c = 2.75$	$-1.6404 < 0$	Replace a by c
3	2.75	3.5	$c = 3.125$	$0.7204 > 0$	Replace b by c.
4	2.75	3.125	$\boxed{c = 2.9375}$	~~[illegible]~~	

Minimizer of $f(x)$ within range of 0.3 is 2.9375.