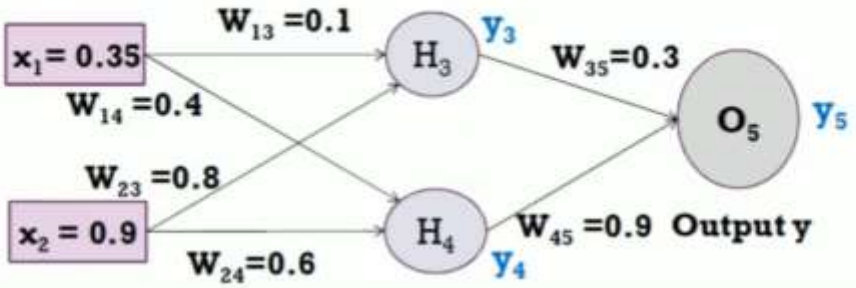
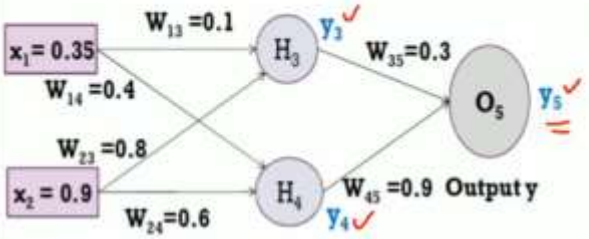
Back Propagation Solved Example - 1

- Assume that the neurons have a sigmoid activation function, perform a forward pass and a backward pass on the network. Assume that the actual output of y is 0.5 and learning rate is 1. Perform another forward pass.



Back Propagation Solved Example - 1



- Forward Pass: Compute output for y3, y4 and y5.

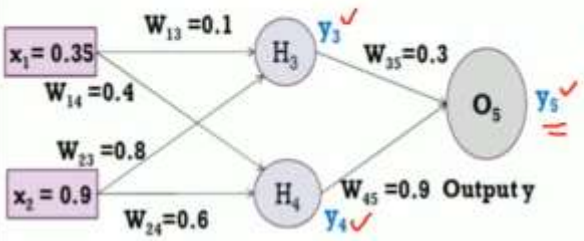
$$a_j = \sum_i (w_{i,j} * x_i) \qquad yj = F(aj) = \frac{1}{1 + e^{-a_j}}$$

$a_1 = (w_{13} * x_1) + (w_{23} * x_2)$
$= (0.1 * 0.35) + (0.8 * 0.9) = 0.755$
$y_3 = f(a_1) = 1/ (1 + e^{-0.755}) = 0.68$

$a_2 = (w_{14} * x_1) + (w_{24} * x_2)$
$= (0.4 * 0.35) + (0.6 * 0.9) = 0.68$
$y_4 = f(a_2) = 1/ (1 + e^{-0.68}) = 0.6637$

$a_3 = (w_{35} * y_3) + (w_{45} * y_4)$
$= (0.3 * 0.68) + (0.9 * 0.6637) = 0.801$
$y_5 = f(a_3) = 1/ (1 + e^{-0.801}) = \mathbf{0.69}$ (Network Output)

Back Propagation Solved Example - 1



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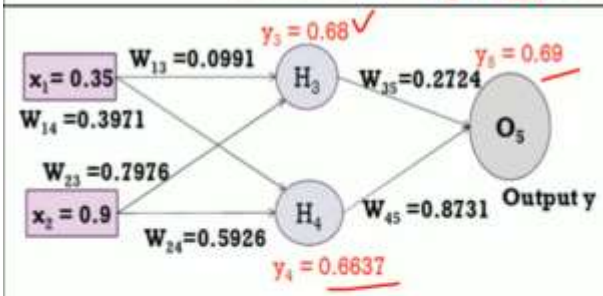
$y_5 = f(a_3) = 1/ (1 + e^{-0.801}) = \mathbf{0.69}$ (Network Output)

$\text{Error} = y_{target} - y_5 = -0.19$

0.5 - 0.69

Back Propagation Solved Example - 1



- Forward Pass: Compute output for y3, y4 and y5.

$$a_j = \sum_j (w_{i,j} * x_i) \qquad yj = F(aj) = \frac{1}{1 + e^{-a_j}}$$

$a_1 = (w_{13} * x_1) + (w_{23} * x_2)$
$= (0.0991 * 0.35) + (0.7976 * 0.9) = 0.7525$
$y_3 = f(a_1) = 1/ (1 + e^{-0.7525}) = 0.6797$

$a_2 = (w_{14} * x_1) + (w_{24} * x_2)$
$= (0.3971 * 0.35) + (0.5926 * 0.9) = 0.6723$
$y_4 = f(a_2) = 1/ (1 + e^{-0.6723}) = 0.6620$

$a_3 = (w_{35} * y_3) + (w_{45} * y_4)$
$= (0.2724 * 0.6797) + (0.8731 * 0.6620) = 0.7631$
$y_5 = f(a_3) = 1/ (1 + e^{-0.7631}) = \mathbf{0.6820}$ (Network Output)

$\text{Error} = y_{target} - y_5 = -0.182$

Back Propagation Solved Example - 1

- Each weight changed by:

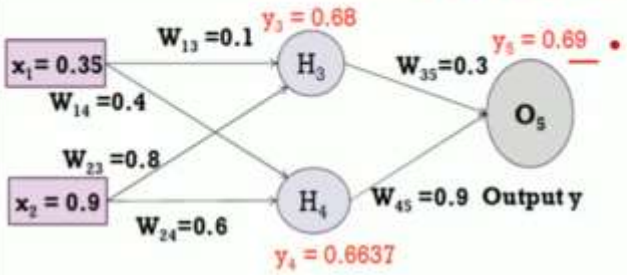
$$\Delta w_{ji} = \eta \delta_j o_i$$

$$\delta_j = o_j(1-o_j)(t_j - o_j) \qquad \text{if } j \text{ is an output unit}$$

$$\delta_j = o_j(1-o_j)\sum_k \delta_k w_{kj} \qquad \text{if } j \text{ is a hidden unit}$$

- where η is a constant called the learning rate
- tj is the correct teacher output for unit j
- δj is the error measure for unit j

Back Propagation Solved Example - 1



- Backward Pass: Compute δ3, δ4 and δ5.

For output unit:

$\delta_5 = y(1-y)\,(y_{target} - y)$

$= 0.69*(1-0.69)*(0.5-0.69) = -0.0406$

For hidden unit:

$\delta_3 = y_3(1-y_3)\, w_{35} * \delta_5$

$= 0.68*(1 - 0.68)*(0.3 * -0.0406) = -0.00265$

$\Delta w_{ji} = \eta \delta_j o_i$

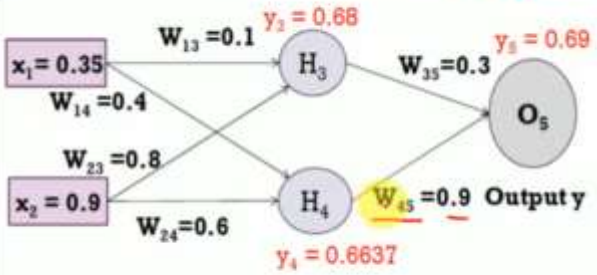
$\delta_j = o_j(1-o_j)(t_j - o_j)$ if j is an output unit

$\delta_j = o_j(1-o_j)\sum_k \delta_k w_{kj}$ if j is a hidden unit

$\delta_4 = y_4(1-y_4) w_{45} * \delta_5$

$= 0.6637*(1 - 0.6637)* (0.9 * -0.0406) = -0.0082$

Back Propagation Solved Example - 1



- Backward Pass: Compute δ3, δ4 and δ5.

For output unit:

$\delta_5 = y(1-y)\,(y_{target} - y)$

$= 0.69*(1-0.69)*(0.5-0.69) = -0.0406$

For hidden unit:

$\delta_3 = y_3(1-y_3)\, w_{35} * \delta_5$

$= 0.68*(1 - 0.68)*(0.3 * -0.0406) = -0.00265$

$\delta_4 = y_4(1-y_4) w_{45} * \delta_5$

$= 0.6637*(1 - 0.6637)* (0.9 * -0.0406) = -0.0082$

Compute new weights

$$\Delta w_{ji} = \eta \delta_j o_i$$

$\Delta w_{45} = \eta\, \delta_5\, y_4 = 1 * -0.0406 * 0.6637 = -0.0269$

w_{45} (new) = Δw_{45} + w_{45}(old) = -0.0269 + (0.9) = 0.8731

$\Delta w_{14} = \eta\, \delta_4\, x_1 = 1 * -0.0082 * 0.35 = -0.00287$

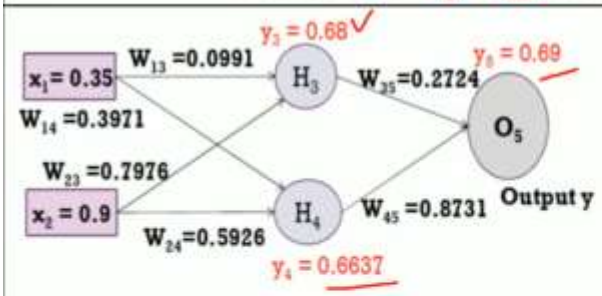
w_{14} (new) = Δw_{14} + w_{14}(old) = -0.00287+ 0.4 = 0.3971

Back Propagation Solved Example - 1

- Similarly, update all other weights

i	j	w_{ij}	δ_j	x_i	η	Updated w_{ij}
1	3	0.1	-0.00265	0.35	1	0.0991
2	3	0.8	-0.00265	0.9	1	0.7976
1	4	0.4	-0.0082	0.35	1	0.3971
2	4	0.6	-0.0082	0.9	1	0.5926
3	5	0.3	-0.0406	0.68	1	0.2724
4	5	0.9	-0.0406	0.6637	1	0.8731

Back Propagation Solved Example - 1



Error $= y_{target} - y_5 = -0.182$

- Forward Pass: Compute output for y3, y4 and y5.

$$a_j = \sum_i (w_{i,j} * x_i) \qquad yj = F(aj) = \frac{1}{1 + e^{-a_j}}$$

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$y_5 = f(a_3) = 1/ (1 + e^{-0.7631}) = \mathbf{0.6820}$ (Network Output)