Parshvanath Charitable Trust's
A. P. SHAH INSTITUTE OF TECHNOLOGY
(Approved by AICTE New Delhi & Govt. of Maharashtra, Affiliated to University of Mumbai)
(Religious Jain Minority)

SUBJECT : AM IV

SEM : IV

χ^2-Test :

The statistic χ^2 is defined by karl pearson as,

$$\chi^2 = \Sigma\left[\frac{(O-E)^2}{E}\right],$$

E = Expected frequency.
O = observed frequency.

Yate's correction : $\chi^2 = \Sigma\left[\frac{(|O-E|-0.5)^2}{E}\right]$

• Applications :

1) To test independence of attributes.
2) To test the goodness of fit.
3) To test hypothesis about variance (σ^2).
4) To test equality of several proportions.

Type I] Independence of attributes :

Examples :

1) Investigate the association between the darkness of eyes colour in father and son from following data.

colour of father's eyes

Colour of son's eyes	Dark	Not Dark	Total
Dark	48	90	138
Not Dark	80	782	862
Total	128	872	1000

⇒ soln.

i) Null Hypothesis H_0 : There is no association between darkness of eye colour in father & son.

ii) Alternative Hypothesis H_a : There is an association.

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ii) Calculation of test statistic :

Based on above hypothesis, the expected frequency of dark eyed sons with dark eyed fathers.

$$= \frac{A \times B}{N},$$

A : Number of dark eyed father (Total of first coloumn)

B : No. of dark eyed sons (Total of first row).

N : Total No. of observations.

Expected Frequency $= \frac{128 \times 138}{1000} = 18.$

Now,

Colour of son's eye	Colour of fathers eyes		
	Dark	Not Dark	Total
Dark	18	120	138
NotDark	110	752	862
Total	120	872	1000

Calculation of $(O-E)^2/E$.

O	E	$(O-E)^2$	$(O-E)^2/E$
48	18	900	50
80	110	900	8.18
90	120	900	7.50
782	752	900	1.20
		Total	$\chi^2 = 66.88$

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iii) Level of significance : $\alpha = 0.05$.

Degrees of freedom : $(r-1)(c-1) = (2-1)(2-1) = 1$

(where; $r \to$ No. of rows, $c \to$ No. of coloumns).

iv) critical value : For 1 d.f. at 5% LOS the table value of χ^2_α is 3.84.

v) Decision : since, the calculated value $\chi^2 = 66.68$ is greater than $\chi^2_\alpha = 3.84$.

The Null Hypothesis rejected.

$\therefore$ There is an association between darkness of colour of fathers and sons.

2] A sample of 400 students of under-graduate and 400 students of post-graduate classes was taken to know their opinion about autonomous colleges. 290 of under graduate and 310 of post-graduate students favoured the autonomous status. Present these facts in the form of table and test at 5% level, that the opinion regarding autonomous status of colleges is independent of the level of classes of students.

→ soln :

opinion about autonomous colleges.

	Favoured	Not-Favoured	Total.
under-graduate	290	110	400
post graduate	310	90	400
Total	600	200	800

i) Null Hypothesis : There is no association between the classes and opinion.
Alternative Hypothesis : There is an association.

ii) calculation of test statistic :

Expected Frequency $= \frac{A \times B}{N} = \frac{400 \times 600}{800} = 300$, this is frequency in first cell and the remaining frequencies are given by, $400 - 300 = 100$
$600 - 300 = 300$
$400 - 300 = 100$

∴ New table :

	Favoured	Not-Favoured	Total.
under graduate	300	100	400
post-graduate	300	100	400
Total	600	200	800.

calculation of $(O-E)^2/E$.

O	E	$(O-E)^2$	$(O-E)^2/E$
290	300	100	0.33
310	300	100	0.33
110	100	100	1.00
90	100	100	1.00.
		Total	2.66

iii) Level of significance : $\alpha = 0.05$.

Degrees of freedom : $(r-1)(c-1) = (2-1)(2-1) = 1$.

iv) Critical value : $\chi^2_\alpha = 3.84$.

v) Decision : since, $\chi^2 < \chi^2_\alpha$.

Null Hypothesis accepted.

∴ There is no association between opinion and level of classes.

(3) Two batches of 12 animals each are given test of inoculation. one batch was inoculated and the other was not. The number of dead and surviving animals are given in the following table for both cases. can the inoculation be regarded as effective against the disease at 5% level of significance?

	Dead	Surviving	Total
inoculated	2	10	12
Non-inoculated	8	4	12
Total	10	14	24.

⇒ Solution :

1) Null Hypothesis : There is no association

Alternative Hypo. : There is association.

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ii) calculation of test statistic :

Expected frequency = $\frac{A \times B}{N} = \frac{10 \times 12}{24} = 5$. This frequency will be in the first cell.

By using Yates correction,

O	E	$\lvert O - E \rvert - 0.5$	$\frac{\{\lvert O - E \rvert - 0.5\}^2}{E}$
2	5	2.5	1.25
10	7	2.5	0.89
8	5	2.5	1.25
4	7	2.5	0.89.
		Total	$\chi^2 = 4.29$.

iii) level of significance : $\alpha = 0.05$

D. of freedom : $(r-1)(c-1) = (2-1)(2-1) = 1$.

iv) critical value : d.o.f = for 5% LOS is 1.

$\therefore \chi^2_\alpha = 3.81$.

v) Decision : since $\chi^2 > \chi^2_\alpha$.

$\therefore$ Null Hypothesis rejected.

$\therefore$ There is an association between inoculation & death ie. inoculation is effective against disease.

Type II] Goodness of fit :

(1) A die was thrown 132 times and the following frequencies were observed.

No. obtained	1	2	3	4	5	6	Total
Frequency	15	20	25	15	29	28	132.

Test the hypothesis that the die is unbiased.

⇒ Soln :

i) Null Hypothesis H_0 : The die is unbiased.

Alternative Hypothesis H_a : The die is not unbiased.

ii) calculation of test statistic :

Here, the expected frequency is,

$$E = \frac{132}{6} = 22.$$

No.	O	E	$(O-E)^2$
1	15	22	49
2	20	22	4
3	25	22	9
4	15	22	49
5	29	22	49
6	28	22	36.
		Total	196.

$$\therefore \chi^2 = \Sigma \frac{(O-E)^2}{E} = \frac{196}{22} = 8.91.$$

iii) Level of significance : $\alpha = 0.05$

Degrees of freedom : $n-1 = 6-1 = 5$

iv) Critical value : $\chi^2_\alpha = 11.07$

v) Decision : since $\chi^2 < \chi^2_\alpha$,

Null Hypothesis accepted.

i.e. The die is unbiased.

(2) The number of car accidents in a metropolitan city was found to be 20, 17, 12, 6, 7, 15, 8, 5, 16, 14. per month resp. Use χ^2-test to check whether these frequencies are in agreement with the belief that occurence of accidents was the same during 10 months period. Test at 5% level of significance.

(value of χ^2 at 9 d.f. is 16.9).

⇒ Sol^n :

1) Null Hypo. H_0 : Accidents occur equally on all months

Alt Hypo. H_a : Accidents do not occur equally on all months.

ii) calculation of test statistic :

No. of accidents per month i.e. expected frequency,

$$E = \frac{\text{Total}}{10} = \frac{20+17+\cdots+16+14}{10} = \frac{120}{10} = 12$$

$$\text{Now, } \chi^2 = \Sigma \frac{(O-E)^2}{E} = \frac{(20-12)^2 + (17-12)^2 + \cdots + (14-12)^2}{12}$$

$$= \frac{244}{12} = 20.33.$$

iii) level of significance : $\alpha = 0.05$.

degrees of freedom : $n-1 = 9$.

iv) critical value : $\chi^2_\alpha = 16.92$.

v) Decision : since, $\chi^2 > \chi^2_\alpha$.

$\therefore$ Null Hypo. rejected.

$\therefore$ Accidents do not occur equally on all months.

(3) In an experiment on pea breeding the following frequencies were obtained.

Round & Yellow	Wrinkled & Yellow	Round & green	wrinkled & green	Total
315	101	108	32	556.

Theory predicts that the frequencies should be in proportion of 9:3:3:1. Examine the correspondence between theory and experiment using chi-square Test.

Soln : i) Null Hypothesis : H_0 : The proportion of the peas in the four groups say A, B, C, D is in the given proportion 9:3:3:1.

Alternative Hypothesis H_a : The proportion is not as given above.

ii) calculation of test statistic :

Sum of Ratios $= 9+3+3+1 = 16$.

$A = \frac{9}{16} \times 556 = 312.75 = 313.$

$B = \frac{3}{16} \times 556 = 104.25 = 104.$

$C = \frac{3}{16} \times 556 = 104.25 = 104.$

$D = \frac{1}{16} \times 556 = 34.75 = 35.$

$$\therefore \chi^2 = \frac{\Sigma(O-E)^2}{E} = \frac{(315-313)^2}{313} + \frac{(101-104)^2}{104} + \frac{(108-104)^2}{104} + \frac{(32-35)^2}{35}$$

$$= \underline{\underline{0.571}}$$

iii) LOS : $\alpha = 0.05$

D.O.F : $n-1 = 3.$

iv) critical value : $\chi^2_\alpha = 7.81$

v) Decision : since $\chi^2 < \chi^2_\alpha$.

$\therefore$ Null hypo. accepted.

$\therefore$ The proportion 9:3:3:1 is correct.

(4) The no. of defects in printed circuit board is hypothesised to follow poisson distribution. A random sample of 60 printed boards showed the foll. data.

No. of defects :	0	1	2	3
observed frequency :	32	15	9	4.

Does the hypothesis of poisson distribution seen appropriate

$\Rightarrow$ Soln :

i) Null Hypothesis H_0 : The defects follow poisson distribution.
Alternative Hypo. H_a : The defects do not follow poisson dist.

ii) calculation of test statistic :

Expected frequency for poisson distribution is given by,

$$E = Np = N \times \frac{e^{-m} m^x}{x!}$$

where, m = mean of distribution, x = random variable &
N = No. of observations.

Here, $m = \frac{\sum f x}{\sum f} = \frac{32(0) + 15(1) + 9(2) + (4)(3)}{35+15+9+4} = \underline{0.75}$

Expected frequency (En) $= \frac{60 \times e^{-0.75} (0.75)^x}{x!}$, $x = 0,1,2,3,\ldots$

Let, E_n = Expected frequency of n defects.

$E_0 = 28.32$, $E_1 = 21.25$, $E_2 = 7.97$,

$E_3 = 60 -$ (sum of above freq.) $= 60 - 57.54 = 2.46$.

No. of defects	O	E	$(O-E)^2/E$
0	32	28.32	0.4782
1	15	21.25	1.8382
2	9	7.97	0.6332
3	4	2.46	0.6332
			Total : 2.9494

$\therefore \chi^2 = \Sigma \frac{(O-E)^2}{E} = 2.95$

iii) level of significance : $\alpha = 0.05$

Degrees of freedom : $4-(1+2) = 1$.

($\because$ No. of d.o.f. for each class is one. There are originally 4 classes. Hence, the degrees of freedom originally is 4. But we reduce the classes by one, thus, reducing the degree by one. Further, while calculating the parameter m, we used two sums Σf_i & $\Sigma f_i x_i$ thus, reducing the d.o.f by 2).

iv) critical value : $\chi^2_\alpha = 3.84$

v) Decision : Since, $\chi^2 < \chi^2_\alpha$. Null Hypothesis accepted.

$\therefore$ The defects follow poisson Distribution.

5) Weights in kgs. of 10 students are given below.

38, 40, 45, 53, 47, 43, 55, 48, 52, 49.

Can we say that the variance of normal distribution from which the above sample is drawn is 20 kg ?

$\Rightarrow$ Sol^n :

X :	38	40	45	53	47	43	55	48	52	49
$(x_i-47)^2$:	81	49	4	36	0	16	64	1	25	04

$\bar{X} = \frac{\Sigma x_i}{n} = \frac{470}{10} = 47.$, $\Sigma (x_i-\bar{x})^2 = 280.$

i) Null Hypothesis : $H_0 : \sigma = \sqrt{20}$

Alternative Hypo. : $H_a : \sigma \neq \sqrt{20}$.

ii) calculation of test statistic : $\chi^2 = \frac{\Sigma (x_i-\bar{x})^2}{\sigma^2}$

$= \frac{280}{20} = 14.$

iii) LOS : $\alpha = 0.05$

iv) D.O.F. : $10-1 = 9$

v) critical value : $\chi^2_\alpha = 16.99$

vi) Decision : $\chi^2 = 14 < 16.99 = \chi^2_\alpha$.

Null Hypo. accepted.

$\therefore$ The sample was drawn from normal popn with variance 20.