Semester : III   Subject : DSGT   Academic Year: 2022-2023

* Coding theory :-

Examples :-

(1) Given the parity check matrix

$$H = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Find the minimum distance of the code generated by H. How many errors it can detect and correct?

⇒ Given

$$H = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

d = hamming distance.

use $\oplus$ opⁿ.

In given parity check matrix all columns are distinct and non-zero, $d \geq 3$

we can use property that the minimum distance of a binary linear code is equal to the smallest number of columns of the parity check matrix H that sum upto 0.

sum of first 3 columns is zero.

so minimum distance. $d_{min} = 3$.

It can detect
$d_{min} - 1 = 3 - 1 = 2$ errors.

It can correct
$(d_{min} - 1)/2 = 1$ error

(2) Consider the encoding function $e: B^2 \to B^6$ defined as follows.
$e(00) = 001000$ $e(01) = 010100$
$e(10) = 100010$ $e(11) = 110001$
How many errors it can detect & correct.
→ i) compute distances betn pairs of code words.
$d(001000, 010100) = 3$ ∴ XOR
$$\begin{array}{r} 001000 \\ \oplus\ 010100 \\ \hline 011100 \end{array}$$
$d(001000, 100010) = 3$
$d(001000, 110001) = 4$
$d(010100, 100010) = 4$
$d(010100, 110001) = 3$
$d(100010, 110001) = 3$

This code will detect k errors iff its min distance is at least k+1.
min. distance = 3
it can detect $(3-1) = 2$ or less errors.
& correct $(3-1)/2 = 1$ error.

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ex. on Group codes.

(1) Show that the (2,5) encoding function $e: B^2 \to B^5$ defined by

$e(00) = 00000$ $e(01) = 01110$

$e(10) = 10101$ $e(11) = 11011$

is a group code. How many errors will it detect and correct?

$\Rightarrow$

Let $N = \{00000, 01110, 10101, 11011\}$ be the set of all code words.

$\oplus$	00000	01110	10101	11011
00000	00000	01110	10101	11011
01110	01110	00000	11011	10101
10101	10101	11011	00000	01110
11011	11011	10101	01110	00000

i) For any, $a, b \in N$, $a \oplus b \in N$

N is closed under $\oplus$ op^n.

ii) Identity element of B^5

$00000 \oplus 00000 = 00000 \oplus 00000$

$10101 \oplus 00000 = 00000 \oplus 10101$

$01110 \oplus 00000 = 00000 \oplus 01110$

$11011 \oplus 00000 = 00000 \oplus 11011$

iii) $\oplus$ is associative operation

iv) Each element of N is its own inverse

00000 $\oplus$ 00000 = 00000 $\oplus$ 00000 = 00000

01110 $\oplus$ 01110 = 01110 $\oplus$ 01110 = 00000

10101 $\oplus$ 10101 = 10101 $\oplus$ 10101 = 00000

11011 $\oplus$ 11011 = 11011 $\oplus$ 11011 = 00000

$\therefore$ N is subgroup of B^5 and the given encoding function is a group code.

d(00000, 01110) = 3
d(00000, 10101) = 3
d(00000, 11011) = 4
d(01110, 10101) = 4
d(01110, 11011) = 3
d(10101, 11011) = 3

$\therefore$ Minimum distance is 3.

The code can detect k errors if its min dist is atleast k+1.

min dist = 3

The code can detect 3 - 1 = 2 errors.

The code will correct

(3-1)/2 = 1 error.

$\therefore$ Note :- code can ~~dete~~ correct errors iff its min distance is atleast 2k+1.