● Reasoning in Belief Networks

Harry installed a new burglar alarm at his home to detect burglary. The alarm reliably responds at detecting a burglary but also responds for minor earthquakes. Harry has two neighbors David and Sophia, who have taken a responsibility to inform Harry at work when they hear the alarm. David always calls Harry when he hears the alarm, but sometimes he got confused with the phone ringing and calls at that time too. On the other hand, Sophia likes to listen to high music, so sometimes she misses hearing the alarm. Here we would like to compute the probability of Burglary Alarm.

Problem:

Calculate the probability that the alarm has sounded, but there is neither a burglary, nor an earthquake occurred, and David and Sophia both called Harry.

Solution:

The Bayesian network for the above problem is given below. The network structure is showing that burglary and earthquake is the parent node of the alarm and directly affecting the probability of alarm's going off, but David and Sophia's calls depend on alarm probability.

The network is representing that our assumptions do not directly perceive the burglary and also do not notice the minor earthquake, and they also not confer before calling.

The conditional distributions for each node are given as conditional probabilities table or CPT.

Each row in the CPT must be sum to 1 because all the entries in the table represent an exhaustive set of cases for the variable.

In CPT, a boolean variable with k boolean parents contains 2K probabilities. Hence, if there are two parents, then CPT will contain 4 probability values

List of all events occurring in this network:

- Burglary (B)
- Earthquake(E)
- Alarm(A)
- David Calls(D)
- Sophia calls(S)

We can write the events of problem statement in the form of probability: P[D, S, A, B, E], can rewrite the above probability statement using joint probability distribution:

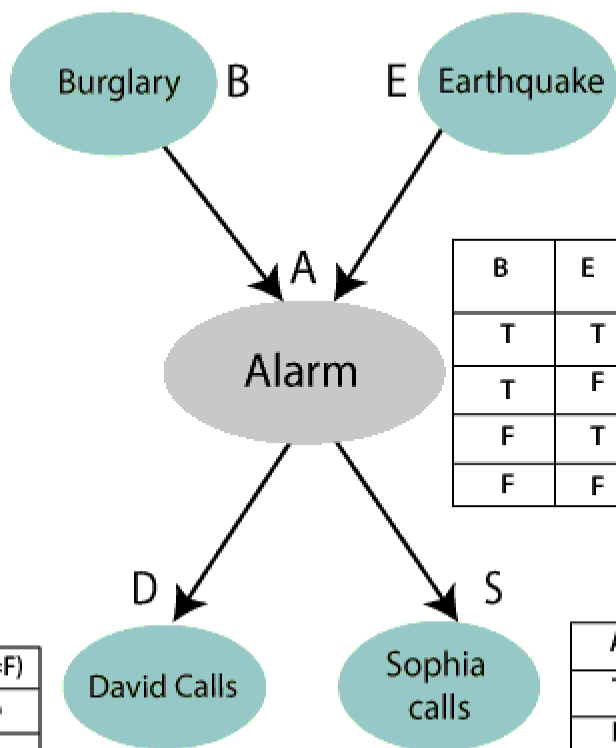
P[D, S, A, B, E]= P[D | S, A, B, E]. P[S, A, B, E]

=P[D | S, A, B, E]. P[S | A, B, E]. P[A, B, E]

= P [D| A]. P [S| A, B, E]. P[A, B, E]

= P[D | A]. P[S | A]. P[A| B, E]. P[B, E]

= P[D | A]. P[S | A]. P[A| B, E]. P[B |E]. P[E]



T	0.002
F	0.998

T	0.001
F	0.999

B	E	P(A=T)	P(A=F)
T	T	0.94	0.06
T	F	0.95	0.04
F	T	0.69	0.69
F	F	0.999	0.999

A	P (D=T)	P (D=F)
T	0.91	0.09
F	0.05	0.95

A	P (S=T)	P (S=F)
T	0.75	0.25
F	0.02	0.98

Let's take the observed probability for the Burglary and earthquake component:

P(B= True) = 0.002, which is the probability of burglary.

P(B= False)= 0.998, which is the probability of no burglary.

P(E= True)= 0.001, which is the probability of a minor earthquake

P(E= False)= 0.999, Which is the probability that an earthquake did not occur.

From the formula of joint distribution, we can write the problem statement in the form of probability distribution:

$P(S, D, A, \neg B, \neg E) = P(S|A) * P(D|A) * P(A|\neg B \wedge \neg E) * P(\neg B) * P(\neg E)$.

$= 0.75 * 0.91 * 0.001 * 0.998 * 0.999$

$= 0.00068045$.

Hence, a Bayesian network can answer any query about the domain by using Joint distribution.