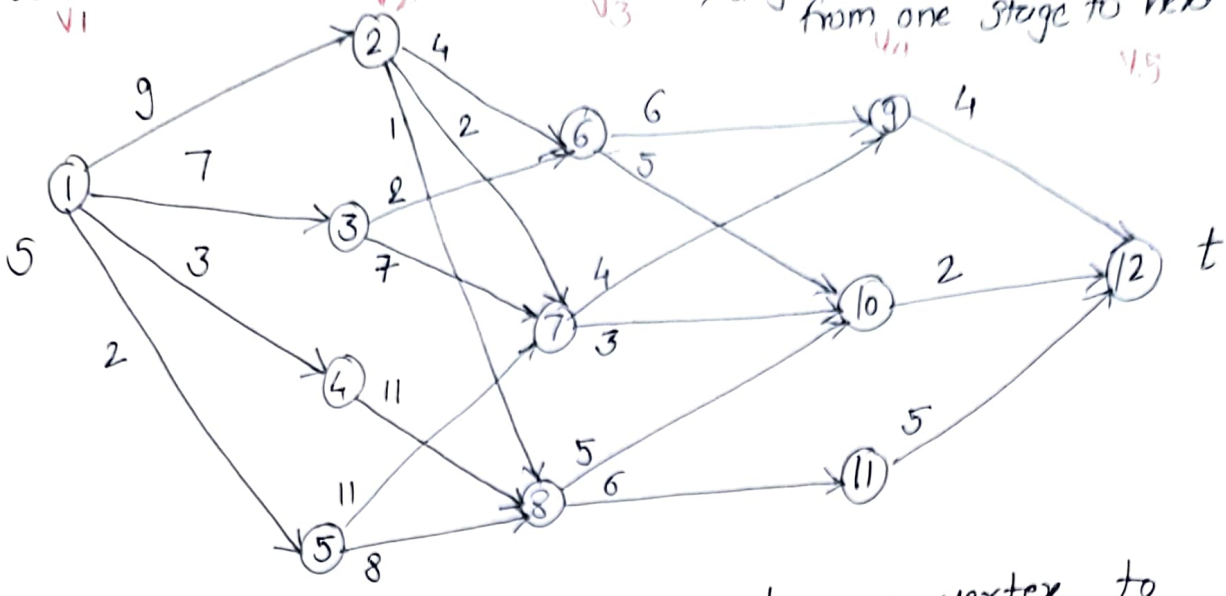
Multistage Graph → Directed, weighted Graph

vertices are divided into stages → adjacent vertices are connected from one stage to next stage



First & last stage will have only one vertex to represent start & end point. (Forward method)

Appl^{n} Resource allocation

Step 1 Find cost each vertex starting from t i.e. 12 to backwards.

v	1	2	3	4	5	6	7	8	9	10	11	12
cost	16	7	9	18	15	7	5	7	4	2	5	0
d	2/3	7	6	8	8	10	10	10	12	12	12	12

↑ Specify where are we reaching

stage ↓ vertex ↓ weight ↓

Step 1

Cost (5, 12) = 0

Step 2

Cost (4, 9) = 4

cost (4, 10) = 2

cost (4, 11) = 5

Step 3

$$cost(3,6) = \min \begin{cases} cost(6,9) + cost(4,9) \\ cost(6,10) + cost(4,10) \end{cases}$$

$$= \min \begin{cases} 6+4 = 10 \\ 5+2 = 07 \end{cases}$$

$$= \min \begin{cases} 10 \\ 07 \end{cases}$$

$$= 07 \quad \text{vertex } 10$$

$$cost(3,7) = \min \begin{cases} c(7,9) + cost(9,9) \\ c(7,10) + cost(4,10) \end{cases}$$

$$= \min \begin{cases} 4+4 = 08 \\ 3+2 = 05 \end{cases}$$

$$= \min \begin{cases} 8 \\ 5 \end{cases}$$

$$= 5 \quad \text{vertex } 10$$

$$cost(3,8) = \min \begin{cases} c(8,10) + cost(4,10) \\ c(8,11) + cost(4,11) \end{cases}$$

$$= \min \begin{cases} 5+2 = 7 \\ 6+5 = 11 \end{cases}$$

$$= 7 \quad \text{vertex } 10$$

$$cost(2,5)$$

$$= \min \begin{cases} c(5,7) + cost(3,7), \\ c(5,8) + cost(3,8), \end{cases}$$

$$= \min \begin{cases} 11+5 = 16 \\ 8+7 = (15) \end{cases}$$

$$= 15 \quad \text{vertex } 8$$

$$cost(2,3)$$

$$= \min \begin{cases} c(3,6) + cost(3,6) \\ c(3,7) + cost(3,7) \end{cases}$$

$$= \min \begin{cases} 2+7 = 9 \\ 7+5 = 12 \end{cases}$$

$$= 9 \quad \text{vertex } 6$$

Step 4

$$cost(2,2)$$

$$= \min \begin{cases} c(2,6) + cost(3,6) \\ c(2,9) + cost(3,7) \\ c(2,8) + cost(3,8) \end{cases}$$

$$= \min \begin{cases} 4+7 = 11 \\ 2+5 = 7 \\ 1+7 = 8 \end{cases}$$

$$= 7 \quad \text{vertex } 7$$

$$Cost(2,4)$$

$$= \min \{ c(4,8) + cost(3,8) \}$$

$$= \min \{ 11+7 \}$$

$$= 18 \quad \text{vertex } 8$$

Step 5

$$cost(1,1)$$

$$= \min \begin{cases} c(1,2) + cost(2,2) \\ c(1,3) + cost(2,3) \\ c(1,4) + cost(2,4) \\ c(1,5) + cost(2,5) \end{cases}$$

$$= \min \begin{cases} 9+7 = (16) \\ 7+9 = (16) \\ 3+18 = 21 \\ 2+15 = 17 \end{cases}$$

$$= 16 \quad \text{vertex } 2 \text{ or vertex } 3$$

Formula Multistage Graph

$$cost(i, j) = \min_{\langle j, l \rangle \in E,\ l \in V_{i+1}} \{ c(j, l) + cost(i+1) + l \}$$

(stage, vertex no; some vertex l)

Take decision based on table to reach from S to t

(stage, vertex)

$d(1,1) = 2$

$d(2,2) = 7$

$d(3,7) = 10$

$d(4,10) = 12$

Path $1 \rightarrow 2 \rightarrow 7 \rightarrow 10 \rightarrow 12$

$d(1,1) = 3$

$d(2,3) = 6$

$d(3,6) = 10$

$d(4,10) = 12$

$1 \rightarrow 3 \rightarrow 6 \rightarrow 10 \rightarrow 12$