Semester : III Subject : DSGT Academic Year: 2022-2023

* Semi Group :-

An algebraic structure $(S, *)$ is called a semigroup if it follows associative property.

i.e. $(a*b)*c = a*(b*c) \quad \forall\ a, b, c \in S$.

ex. (1) $(N, +)$

$N = \{1, 2, 3, \ldots \infty\}$

i.e. $a = 5$, $b = 10$, $c = 15$

i) first check for closure property

$a+b = 5+10 = 15$

$15 \in N$

hence it is closed rel^n

ii) put values

$(a+b)+c = a+(b+c)$

$(5+10)+15 = 5+(10+15)$

$15+15 = 5+25$

$30 = 30$

hence this algebraic structure $(N, +)$ is a semigroup.

(2) $(Z, *)$

$Z = \{\ldots -3, -2, -1, 0, 1, 2, 3, \ldots\}$

$a = 10$

$b = 10$

$c = 1$

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First check for closure property.

$10 * 10 = 100$

$100 \in Z$ hence it is closed set

So $(Z, *)$ is algebraic structure.

Now check for associative property

$(a * b) * c = a * (b * c)$

$(10 * 10) * 1 = 10 * (10 * 1)$

$100 = 100$

hence it is a semigroup.

(3) $\{2^n \mid n$ is any integer is a semigroup with multiplication$\}$.

$2^5 * 2^{10} = 2^{(5+10)}$

$= 2^{15}$

$\therefore \forall\ 2^{15} \in Z$

it is algebraic structure

$a = 5 \quad b = 10 \quad c = 15$

$2^5 * (2^{10} * 2^{15}) = (2^5 * 2^{10}) * 2^{15}$

$2^5 * 2^{25} = 2^{15} * 2^{15}$

$2^{30} = 2^{30}$

it satisfies associative property.

Hence it is semigroup.