Dual Simplex Method

Working Procedure :-

* Convert the problem into minimisation type.
* Convert all constraints in less than or equal to type. If any constraint is of greater than or equal to type multiply by '-1' & change the inequality sign
* form the simplex table
* In the RHS soln column select the row which contains the smallest (negative) number. This is the Key row & the corresponding variable is the outgoing variable
* Now find the ratios by dividing the Z row by the key row. & write these Ratios in another row below the table.

* Now the smallest ratio and the corresponding column is key column & the corresponding variable is the incoming variable.

* If all the co-efficients in the row of z are negative and all the right hand side constants ie) b's are positive then basic feasible solution is obtained.

If all the co-efficients in the row of z are negative and atleast one of the right hand side constants is negative, then continue the process.

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① Use the dual simplex method to solve the following LPP.

Minimise $Z = 2x_1 + 2x_2 + 4x_3$

Subject to $2x_1 + 3x_2 + 5x_3 \geq 2$

$3x_1 + x_2 + 7x_3 \leq 3$

$x_1 + 4x_2 + 6x_3 \leq 5$

$x_1, x_2, x_3 \geq 0.$

Soln:-

Minimise $Z = 2x_1 + 2x_2 + 4x_3 + 0S_1 + 0S_2 + 0S_3$

$Z - 2x_1 - 2x_2 - 4x_3 - 0S_1 - 0S_2 - 0S_3$

subject to $-(2x_1 + 3x_2 + 5x_3) + S_1 + 0S_2 + 0S_3 = -2$

(i) $-2x_1 - 3x_2 - 5x_3 + S_1 + 0S_2 + 0S_3 = -2$

$3x_1 + x_2 + 7x_3 + 0S_1 + S_2 + 0S_3 = 3$

$x_1 + 4x_2 + 6x_3 + 0S_1 + 0S_2 + S_3 = 5$

Simplex table

Iteration number	Basic Variables	Co-effts of x_1	x_2	x_3	S_1	S_2	S_3	RHS Solution
0	Z	-2	-2	-4	0	0	0	0
	S_1	-2	-3*	-5	1	0	0	-2 ⟸
S_1 leaves	S_2	3	1	7	0	1	0	3
x_2 enters	S_3	1	4	6	0	0	1	5
Ratio		-2/-2 = 1	-2/-3 = 2/3 ↑	-4/-5 = 4/5	-	-	-	-

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Z	$-2/3$	0	$-2/3$	$-2/3$	0	0	$4/3$
x_2	$2/3$	1	$5/3$	$-1/3$	0	0	$2/3$
S_2	$7/3$	0	$16/3$	$1/3$	1	0	$7/3$
S_3	$-5/3$	0	$-2/3$	$4/3$	0	1	$7/3$.

$\therefore x_1 = 0$

$x_2 = 2/3$

$x_3 = 0$

$Z_{min} = 4/3$.

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(2) Use the dual simplex method to solve the following LPP

Minimise $Z = x_1 + x_2$

subject to $2x_1 + x_2 \geq 2$

$-x_1 - x_2 \geq 1$

$x_1, x_2 \geq 0$.

Soln:-

Minimise $Z = 2x_1 + 2x_2 + 4x_3 + 0S_1 + 0S_2 + 0S_3$

$Z - 2x_1 - 2x_2 - 4x_3 - 0S_1 - 0S_2 - 0S_3$

subject to $-(2x_1 + 3x_2 + 5x_3) + S_1 + 0S_2 + 0S_3 = -2$

(i) $-2x_1 - 3x_2 - 5x_3 + S_1 + 0S_2 + 0S_3 = -2$

$3x_1 + x_2 + 7x_3 + 0S_1 + S_2 + 0S_3 = 3$

$x_1 + 4x_2 + 6x_3 + 0S_1 + 0S_2 + S_3 = 5$

Simplex table

Iteration number	Basic Variables	Co-effts of x_1	x_2	x_3	S_1	S_2	S_3	RHS Solution
0	Z	-2	-2	-4	0	0	0	0
	S_1	-2	-3*	-5	1	0	0	-2 ⟸
S_1 leaves	S_2	3	1	7	0	1	0	3
x_2 enters	S_3	1	4	6	0	0	1	5
	Ratio	-2/-2 = 1	-2/-3 = 2/3 ↑	-4/-5 = 4/5	-	-	-	

Soln:-

Minimise

$$Z = x_1 + x_2 + 0S_1 + 0S_2$$

$$Z - x_1 - x_2 - 0S_1 - 0S_2 = 0.$$

subject to $-2x_1 - x_2 + S_1 + 0S_2 = -2$

$$x_1 + x_2 + 0S_1 + S_2 = -1$$

Simplex table

Iteration no	Basic Variables	x_1	x_2	S_1	S_2	RHS Solution	
0	Z	-1	-1	0	0	0	
	S_1	-2^*	-1	1	0	-2	←
	S_2	1	1	0	1	-1	
	Ratio	1/2	1				
		↑					
1	Z	0	-1/2	-1/2	0	1	
	x_1	1	1/2	-1/2	0	1	
	S_2	0	1/2	1/2	1	-2	←
	Ratio	—	-1	-1	—		

Since all the ratios are negative, the given LPP has no feasible solution.

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