



Module 2: System of Linear equations

A **system of linear equations** (or **linear system**) is a collection of two or more linear equations involving the same set of variables.

Representation of linear equations in matrix and vector forms:

Lets take a system of three linear equations in three variables x, y, and z

$$a_1x + b_1y + c_1z = d_1$$

$$a_2x + b_2y + c_2z = d_2$$

$$a_3x + b_3y + c_3z = d_3$$

Matrix Representation

The equations can be written as matrix multiplication form -

$$\begin{matrix} A \\ \left[\begin{array}{ccc} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{array} \right] \end{matrix} \begin{matrix} X \\ \left[\begin{array}{c} x \\ y \\ z \end{array} \right] \end{matrix} = \begin{matrix} D \\ \left[\begin{array}{c} d_1 \\ d_2 \\ d_3 \end{array} \right] \end{matrix}$$

or,

$$AX = D$$

Matrix A is called the **coefficient matrix**. If we append the columns of matrix D with matrix A like below the resultant matrix is called **augmented matrix**, denoted by $[A|D]$

$$[A|D] = \left[\begin{array}{cccc} a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \end{array} \right]$$



Vector Representation

The above linear equations can also be represented in vector form as :

$$x \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} + y \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} + z \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

or,

$$ax + by + cz = D$$

where $a = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$, $b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$, $c = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}$, $D = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$ are the column vectors

Now look carefully, we can write from equation 4 and 5 -

$$AX = ax + by + cz = D$$

or,

$$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = x \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} + y \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} + z \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}$$

The above relation gives us a another way to think about matrix multiplication

Solution set of linear equations :

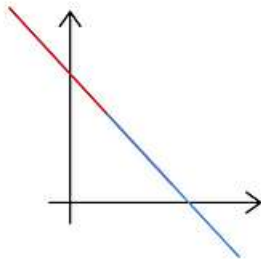
Every linear system may have only one of three possible number of solutions:



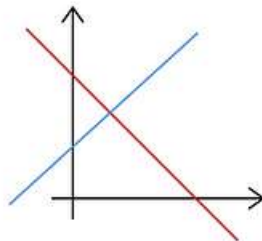
1. *The system has a single unique solution.*
2. *The system has infinitely many solutions.*
3. *The system has no solution.*

Geometrical Representation :

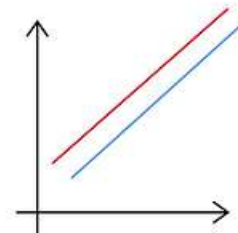
For a system of two variables (x and y), each linear equation determines a line on the xy -plane. The solution set is the intersection of these lines, and is hence either a line, a single point or don't have any common point.



Infinite no of Solution : Line



Unique Solution : point



No Solution