Games in extensive form (dynamic or sequential games)

An extensive from game specifies:

1) The players.

2) The order of the game.

3) The choices available to each player at each turn of play (at each decision node).

4) The information held by each player at each turn of play (at each decision node).

5) The payoffs of each player as a function of the movements selected.

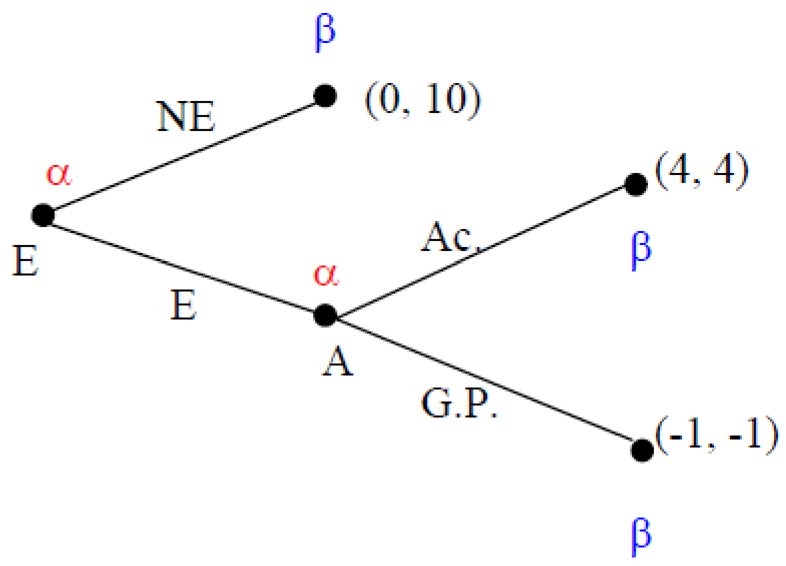
6) Probability distributions for movements made by nature.

An extensive form game is represented by a decision tree. A decision tree comprises nodes and branches. There are two types of node: decision nodes and terminal nodes. We have to assign each decision node to one player. When the decision node of a player is reached the player chooses a move. When a terminal node is reached the players obtain payoffs: an assignment of payoffs for each player.

EXAMPLE 1: Entry game

Consider a market where there are two firms: an incumbent firm, A, and a potential entrant, E.

At the first stage, the potential entrant decides whether or not to enter the market. If it decides "not to enter" the game concludes and the players obtain payoffs (firm A obtains the monopoly profits) and if it decides "to enter" then the incumbent firm, A, has to decide whether to accommodate entry (that is, to share the market with the entrant) or to start a mutually injurious war price. The extensive form game can be represented as follows:



Players: E and A.

Actions: *E* (to enter), *NE* (not to enter), *Ac.* (to accommodate), *G.P.* (price war).

Decision nodes: α .

Terminal nodes: β .

(x, y): vector of payoffs. x: payo of of player E; y: payoff of player A.

At each terminal node we have to specify the payoffs of each player (even though some of them have not actually managed to play).

Assumptions:

(i) All players have the same perception of how the game is.

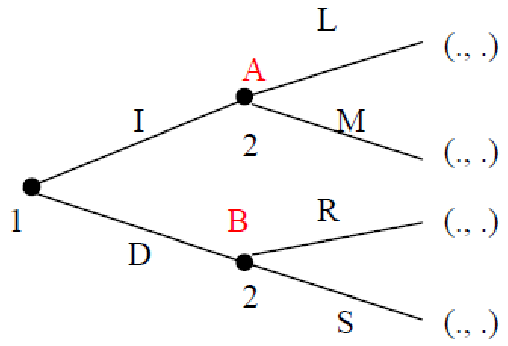
(ii) Complete information: each player knows the characteristics of the other players:

preferences and strategy spaces.

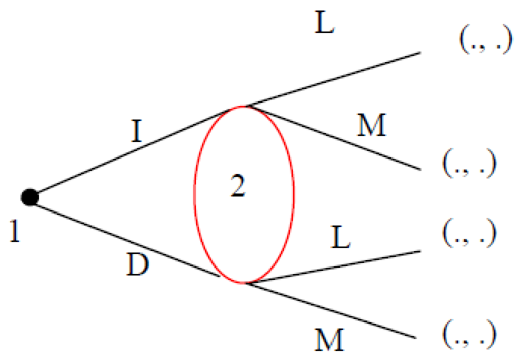
(iii) Perfect recall: each player remembers his/her previous behaviour in the game

Definition 1: **Information set**

"The information available to each player at each one of his/her decision nodes".



Game 1

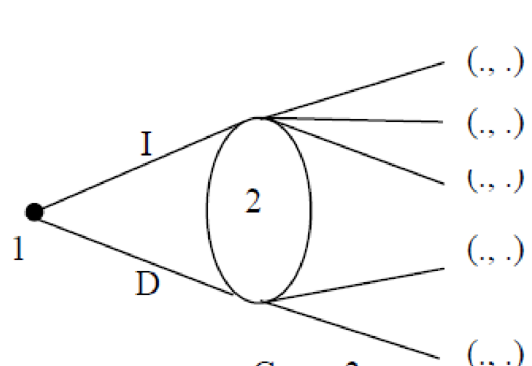


Game 2

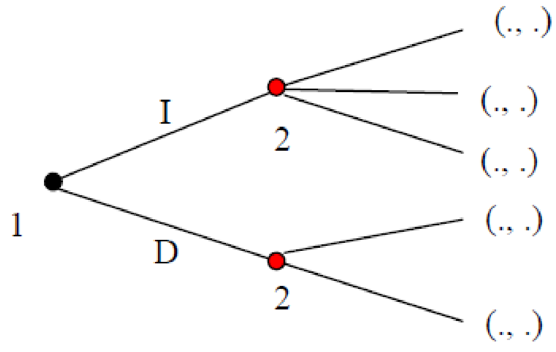
In game 1, player 2 has different information at each one of his/her decision nodes. At node *A*, if he/she is called upon to play he/she knows that the player 1 has played *I* and at *B* he/she knows that player 1 has played *D*. We say that these information sets are singleton sets consisting of only one decision node. *Perfect information game*: a game where all the information sets are singleton sets or, in other words, a game where all the players know everything that has happened previously in the game. In game 2, the player 2 has the same information at both his/her decision nodes. That is, the information set is composed of two decision nodes. Put differently, player 2 does not know which of those nodes he or she is at. A game in which there are information sets with two or more decision nodes is called an *imperfect information game*: atleast one player does not observe the behaviour of the other(s) at one or more of his/her decision nodes.

The fact that players know the game that they are playing and the perfect recall assumption

restrict the situations where we can find information sets with two or more nodes.

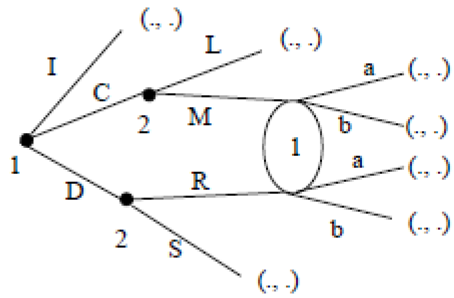


Game 3

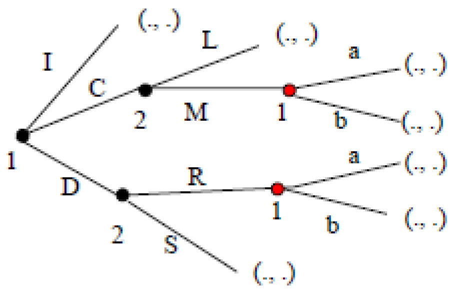


Game 4

Game 3 is poorly represented because it would not be an imperfect information game. Assuming that player 2 knows the game, if he/she is called on to move and faces three alternatives he/she would inmediatley deduce that the player 1 has played I. That is, the game should be represented like game 4. Therefore, *if an information set consists of two or more nodes the number of alternatives, actions or moves at each one should be the same.*



Game 5



Game 6

The assumption of perfect recall avoids situations like that in game 5. When player 1 is called on

to play at his/her second decision node perfectly recall his/her behaviour at his/her first decision

node. The extensive form should be like that of game 6.

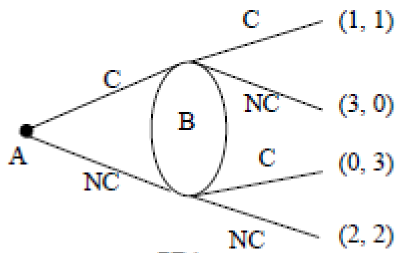
Definition 2: **Subgame**

"It is what remains to be played from a decision node with the condition that what remains to be played does not form part of an information set with two or more decision nodes. To build subgames we look at parts of the game tree that can be constructed without breaking any information sets. An information set starts at a singleton information set and all the decision nodes of the same information set must belong to the same subgame."

EXAMPLE 2: The Prisoner's Dilemma

Two prisoners, A and B, are being held by the police in separate cells. The police know that the two (together) committed a crime but lack sufficient evidence to convict them. So the police offer each of them separately the following deal: each is asked to implicate his partner. Each prisoner can "confess" (*C*) or "not confess" (*NC*). If neither confesses then each player goes to jail for one month. If both players confess each prisoner goes to jail for three months. If one prisoner confesses and the other does not confess, the first player goes free while the second goes to jail for six months.

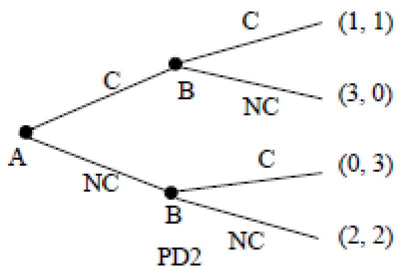
- **Simultaneous case**: each player takes his decision with no knowledge of the decision of the other.



PD1

There is an information set with two decision nodes. This is an imperfect information game. There is a subgame which coincides with the proper game.

Sequential game: the second player observes the choice made by the first.



PD2

Game PD2 is a perfect information game and there are three subgames. "In perfect information games there are as many subgames as there are decision nodes".

Definition 3: **Strategy**

"A player's strategy is a complete description of what he/she would do if he/she were called on to play at each one of his/her decision nodes. It needs to be specified even in those nodes not attainable by him/her given the current behavior of the other(s) player(s)".

It is a *behaviour plan* or *conduct plan*. (Examples: consumer demand, supply from a competitive firm.). It is a player's function which assigns an action to each of his/her decision nodes (or to each of his/her information sets). A player's strategy has as many components as the player has information sets.

Definition 4: **Action**

"A choice (decision or move) at a decision node".

Actions are physical while strategies are conjectural.

Definition 5: **Combination of strategies or strategy profile**

"A specification of one strategy for each player". The result (the payoff vector) must be unequivocally determined.

EXAMPLE 1: The entry game

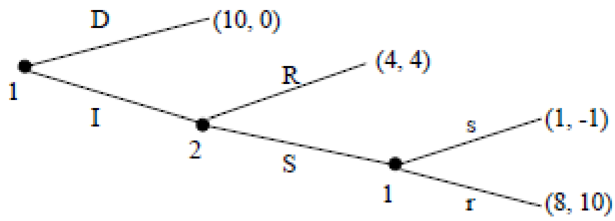
This is a perfect information game with two subgames. Each player has two strategies: $SE = \{NE,E\}$ and $SA = \{Ac.,G.P.\}$. Combinations of strategies: (*NE*, *Ac.*), (*NE*, *G.P.*), (*E*, *Ac.*) and (*E*, *G.P.*).

EXAMPLE 2: The Prisoner's Dilemma

PD1: This is an imperfect information game with one subgame. Each player has two strategies: $SA = \{C,NC\}$ and $SB = \{C,NC\}$. Combinations of strategies: (*C*, *C*), (*C*, *NC*), (*NC*, *C*) and (*NC*, *NC*).

PD2: This is a perfect information game with three subgames. Player A has two strategies $SA = \{C,NC\}$ but player B has four strategies $SB = \{CC,CNC,NCC, NCNC\}$. Combinations of strategies: (*C*, *CC*), (*C*, *CNC*), (*C*, *NCC*), (*C*, *NCNC*), (*NC*, *CC*), (*NC*,*CNC*), (*NC*, *NCC*) and (*NC*, *NCNC*).

EXAMPLE 3



Player 1 at his/her first node has two possible actions, *D* and *I*, and two actions also at his/her second: *s* and *r*. $S1 = \{Ds,Dr, Is, Ir\}$ and $S2 = \{R, S\}$.