

Subject: Applied Mathematics IV

SEM:IV

Relationship b/w Primal & Dual Optimal Solutions

$$Z_{\max} = W_{\min}$$

① Using duality solve the following LPP

$$\text{Maximise } Z = 5x_1 - 2x_2 + 3x_3$$

$$\text{subject to } 2x_1 + 2x_2 - x_3 \geq 2$$

$$3x_1 - 4x_2 \leq 3$$

$$x_1 + 3x_3 \leq 5$$

$$x_1, x_2, x_3 \geq 0$$

Soln:-

Since the objective function is of maximisation type, we first write the constraints in less than or equal to form.

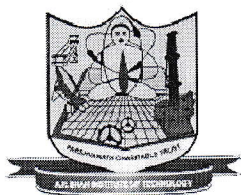
$$\text{Maximise } Z = 5x_1 - 2x_2 + 3x_3$$

$$\text{subject to } -2x_1 - 2x_2 + x_3 \leq -2$$

$$3x_1 - 4x_2 + 0x_3 \leq 3$$

$$0x_1 + x_2 + 3x_3 \leq 5$$

$$x_1, x_2, x_3 \geq 0$$



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∴ The dual is

$$\text{Minimise } w = -2y_1 + 3y_2 + 5y_3$$

$$\text{a) Maximise } w' = -w = 2y_1 - 3y_2 - 5y_3$$

$$\text{Subject to } -2y_1 + 3y_2 + 0y_3 \geq 5$$

$$-2y_1 - 4y_2 + y_3 \geq -2$$

$$\text{a) } 2y_1 + 4y_2 - y_3 \leq 2$$

$$y_1 + 0y_2 + 3y_3 \geq 3$$

$$y_1, y_2, y_3 \geq 0$$

By adding the slack variables and artificial variables,

$$\text{Maximise } w' = -w = 2y_1 - 3y_2 - 5y_3 - 0s_1 - 0s_2 - 0s_3 - MA_1$$

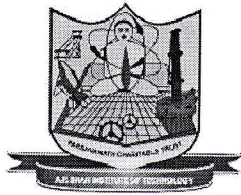
$\xrightarrow{\text{①}} -MA_3$

subject to

$$-2y_1 + 3y_2 + 0y_3 - s_1 + 0s_2 + 0s_3 + A_1 + 0A_3 = 5 \rightarrow \text{②}$$

$$2y_1 + 4y_2 - y_3 + 0s_1 + s_2 + 0s_3 + 0A_1 + 0A_3 = 2 \rightarrow \text{③}$$

$$y_1 + 0y_2 + 3y_3 + 0s_1 + 0s_2 - s_3 + 0A_1 + A_3 = 3 \rightarrow \text{④}$$



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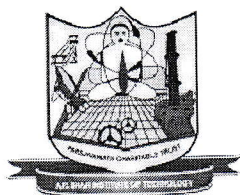
To eliminate MA_1 & MA_3 from the objective function Multiply (2) & (4) by M and add it to (1).

$$\therefore w' = 2y_1 - 3y_2 - 5y_3 - 0S_1 - 0S_2 - 0S_3 - MS_1 - MS_3 - My_1 + 3My_2 + 3My_3 - 0A_1 - 0A_3 - 8M.$$

$$w' = (2-M)y_1 + (-3+3M)y_2 + (-5+3M)y_3 - MS_1 - 0S_2 - MS_3 - 0S_3 - 0A_1 - 0A_3 - 8M.$$

$$\therefore w' + (-2+M)y_1 + (3-3M)y_2 + (5-3M)y_3 + MS_1 + 0S_2 + MS_3 + 0A_1 + 0A_3 = -8M.$$

Simplex table

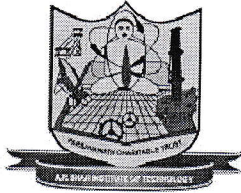


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Iteration	Basic Variables	y_1	y_2	y_3	Co-effs of s_1, s_2, s_3	A_1	A_2	RHS soln	Ratio
0	w'	$-24M$	$3-3M$	$5-3M$	M	0	0	$-8M$	
	A_1	-2	3	0	-1	0	1	5	$5/3$
	S_2	2	4	-1	0	1	0	2	$1/2$
	A_3	1	0	3	0	-1	0	3	$-$

Iteration	Basic Variables	y_1	y_2	y_3	Co-effs of s_1, s_2, s_3	A_1	A_2	RHS soln	Ratio
1	w'	$\frac{-7+5M}{2}$	0	$\frac{23-15M}{4}$	M	$\frac{-3+3M}{4}$	M	0	$\frac{-3-13M}{2}$
	A_1	$-7/2$	0	$3/4$	-1	$-3/4$	0	1	$7/2$
	y_2	$1/2$	1	$-1/4$	0	$1/4$	0	0	$1/2$
	A_3	1	0	3	0	0	-1	0	1



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2	w_1	$-65 + 45M$ 12	0	0	M	$-3 + 3M$ 4	$23 - 3M$ 12	0	$-29 - 11M$ 4
A ₁ leaves	A ₁	-15/4	0	0	-1	3/4	1/4	1	11/4
S ₃ enters	y ₂	7/12	1	0	0	1/4	-1/2	0	-3/4
y ₃	y ₃	1/3	0	1	0	0	-1/3	0	1

3	w'	-70/3	0	0	23/3	5	0	$-2 + 3M$ 3	$-85/3$
S ₃	S ₃	-15	0	0	-4	-3	1	4	11
y ₂	y ₂	-2/3	1	0	-1/3	0	0	1/3	-5/3
y ₃	y ₃	-14/3	0	1	-4/3	-1	0	4/3	-14/3

$$S_1 = \frac{23}{3}, \quad S_2 = 5, \quad S_3 = 0 \quad \therefore w_{\max} = -\frac{85}{3}$$

$$\therefore x_1 = \frac{23}{3}, \quad x_2 = 5, \quad x_3 = 0, \quad z_{\max} = \frac{85}{3}$$

$$\therefore w_{\min} = \frac{85}{3}$$