Semester: VIII Subject: Recommendation system Academic Year:2024-2025

Q] Given below is a set table that contains some items and the user who have rated those items. The rating is explicit and is on a scale of 1 to 5. Each entry in the table denotes the rating even by a 'i th' user to a 'j th' item. In most cases majority of cells are empty as a user rates only for few items. Here we have taken 4 users and 3 items. We need to find the missing ratings for the respective user.

User/Item	Item 1	Item 2	Item 3
User 1	2	–	3
User 2	5	2	–
User 3	3	3	1
User 4	–	2	2

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Solution:-

Step 1) Finding similarities of all the item pairs. In above example, there are 3 item-pairs, (Item 1, Item 2), (Item 1, Item 3) and (Item 2, Item 3). Select each item to pair one by one. After this, we find all the users who have rated for both the items in the item pair. Form a vector for each item and calculate the similarity between the two items using the cosine formula stated above.

1] Sim (Item 1, Item 2)

In the table, we can see only User 2 and User 3 have rated for both items 1 and 2.

Let I_1 be vector for Item 1 and I_2 be for Item 2. Then,

$$I_1 = 5U_2 + 3U_3$$

$$I_2 = 2U_2 + 3U_3$$

$$\text{Similarity}(I_1, I_2) = \frac{(5*2) + (3*3)}{\sqrt{5^2+3^2}\sqrt{2^2+3^2}}$$

$$= 0.90$$

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ii] Sim (Item 2, Item 3)

In the table, we can see only User 3 and User 4 have rated for both the item 2 and 3.

Let I_2 be vector for Item 2 and I_3 be for Item 3.

$I_2 = 3U_3 + 2U_4$

$I_3 = 1U_3 + 2U_4$

$$\text{Similarity}(I_2, I_3) = \frac{(3*1)+(2*2)}{\sqrt{3^2+2^2}\sqrt{1^2+2^2}}$$

$$= \frac{3+4}{\sqrt{9+4}\sqrt{1+4}}$$

$$= \frac{7}{\sqrt{13}\sqrt{5}}$$

$$= \frac{7}{\sqrt{65}}$$

$$= 0.869$$

iii] Sim (Item 1, Item 3)

In the table, we can see only User 1 and User 3 have rated for both the items 1 and 2.

Let I_1 be the vector for Item 1 and I_3 be for Item 3.

$I_1 = 2U_1 + 3U_3$

$I_3 = 3U_1 + 1U_3$

Step 2] Generating the missing ratings in the table
In this step, we calculate the ratings that are missing in the table.

[i] Rating of Item 2 for User 1

$$r(U_1, I_2) = \frac{r(U_1, I_1) * S_{I_1 I_2} + r(U_1, I_3) * S_{I_2 I_3}}{S_{I_1 I_2} + S_{I_2 I_3}}$$

$$= \frac{(2 * 0.9) + (3 * 0.869)}{(0.9 + 0.869)}$$

$$= 2.49$$

[ii] Rating of Item 3 for User 2

$$r(U_2, I_3) = \frac{r(U_2, I_1) * S_{I_1 I_3} + r(U_2, I_2) * S_{I_2 I_3}}{S_{I_1 I_3} + S_{I_2 I_3}}$$

$$= \frac{(5 * 0.789) + (2 * 0.869)}{(0.789 + 0.869)}$$

$$= 3.43$$

iii] Rating of Item 1 for User 4

$$r(U_4, I_1) = \frac{r(U_4, I_2) * S_{I_1 I_2} + r(U_4, I_3) * S_{I_1 I_3}}{S_{I_1 I_2} + S_{I_1 I_3}}$$

$$= \frac{(2 * 0.9) + (2 * 0.789)}{(0.9 + 0.789)}$$

$$= 2.0$$

Thus, we have understood the basic working of item-to-item collaborative filtering with the help of above example.