Module lanoisnemiD eerhT -5 ,snoitamrofsnarT cirtemoeG slatcarF dna sevruC

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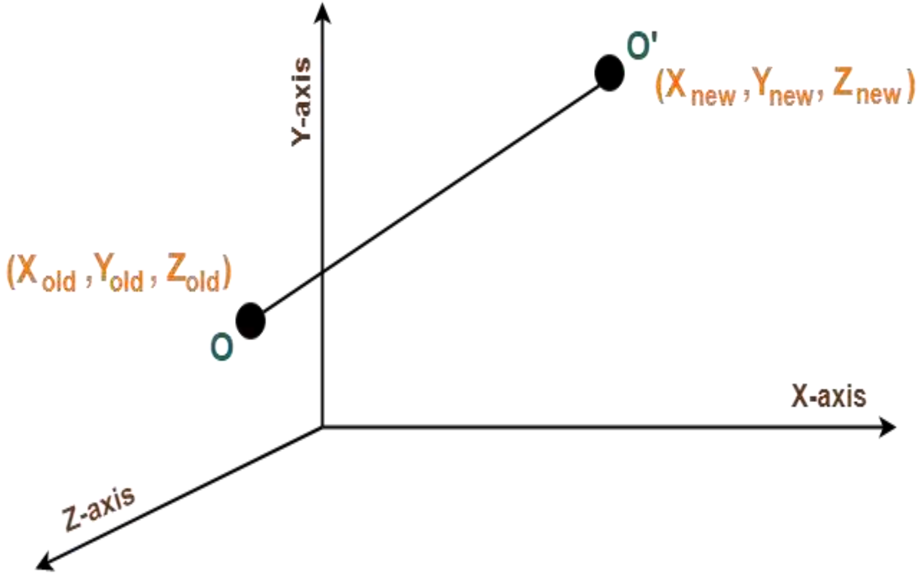
Translation

- Let us consider the original point P(x,y,z) which becomes P'(x',y',z') after translation where,
  - $x' = x + t_x$
  - $y' = y + t_y$
  - $z' = z + t_z$
- Equivalent homogeneous matrix representation

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & Tx \\ 0 & 1 & 0 & Ty \\ 0 & 0 & 1 & Tz \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

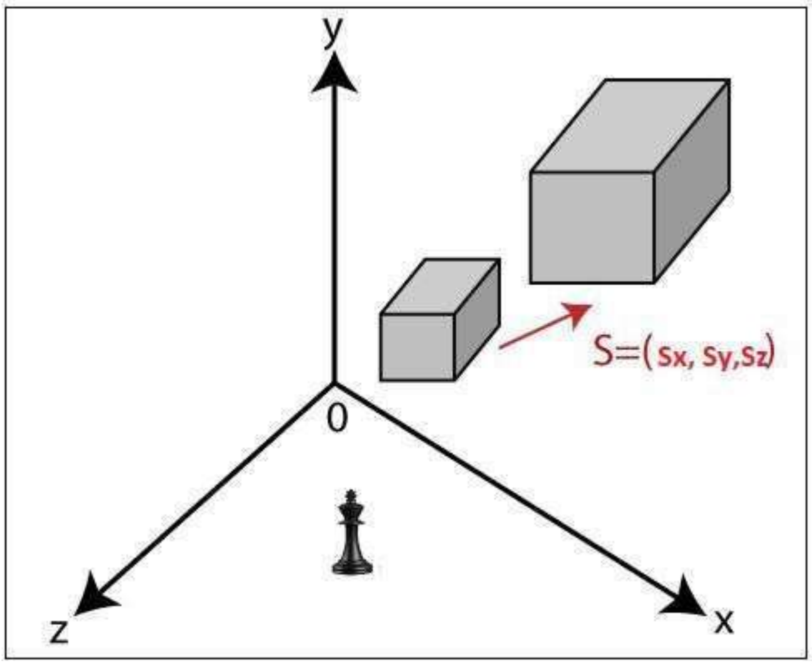


3D Translation in Computer Graphics

Scaling

- Let us consider the original point P(x,y,z) which becomes P'(x',y',z') after scaling where,
  - $x' = S_x \cdot x$
  - $y' = S_y \cdot y$
  - $z' = S_z \cdot z$
- Equivalent homogeneous matrix representation

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$



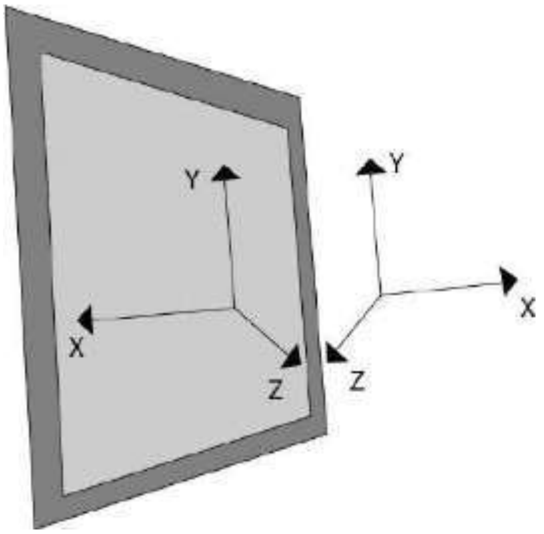
Reflection

- Let us consider the original point P(x,y,z) which becomes P'(x',y',z') after rotation by 180 about the X, Y, Z axes.

- $Ref_{(x=0)} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

- $Ref_{(y=0)} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

- $Ref_{(z=0)} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$



Rotation

- In 3D rotation, we have to specify along with the axis of rotation. We can perform 3D rotation about x,y,z axes.
- About X-axis:
  - x’ = x
  - y’ = y cos ϴ - z sin ϴ
  - z’ = y sin ϴ + z cos ϴ

Rotation about x-axis   Rotation about y-axis   Rotation about z-axis

$$R_x(\Theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\Theta & -sin\Theta & 0 \\ 0 & sin\Theta & cos\Theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rotation

- About Y-axis:
  - x’ = z sin ϴ + x cos ϴ
  - y’ = y
  - z’ = z cos ϴ - x sin ϴ

x y z 1

$$R_y(\Theta) = \begin{bmatrix} cos\Theta & 0 & sin\Theta & 0 \\ 0 & 1 & 0 & 0 \\ -sin\Theta & 0 & cos\Theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rotation

- About Z-axis:
  - x’ = x cos ϴ - y sin ϴ
  - y’ = x sin ϴ + y cos ϴ
  - z’ = z

$$R_z(\Theta) = \begin{bmatrix} cos\Theta & -sin\Theta & 0 & 0 \\ sin\Theta & \cos\Theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Numerical

A cube is defined by 8 vertices, A (0,0,0) B(2,0,0) C(2,2,0) D(0,2,0) E(0,0,2) F (0,2,2) G (2,0,2) H(2,2,2)

Perform the following transformations on this cube-

Translation Vector [5 3 4]

Scaling Factor [1 2 0.5]

Rotation about X-axis by 90 degree in clockwise direction

1	0	0	0
0	0	1	0
0	-1	0	0
0	0	0	1

3D Rotation around arbitrary axis

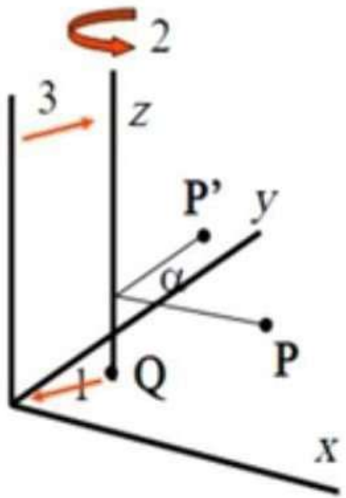
Rotation around axis, parallel to coordinate axis, through point **Q**.

For example, the z - as. Similar as 2D rotation :

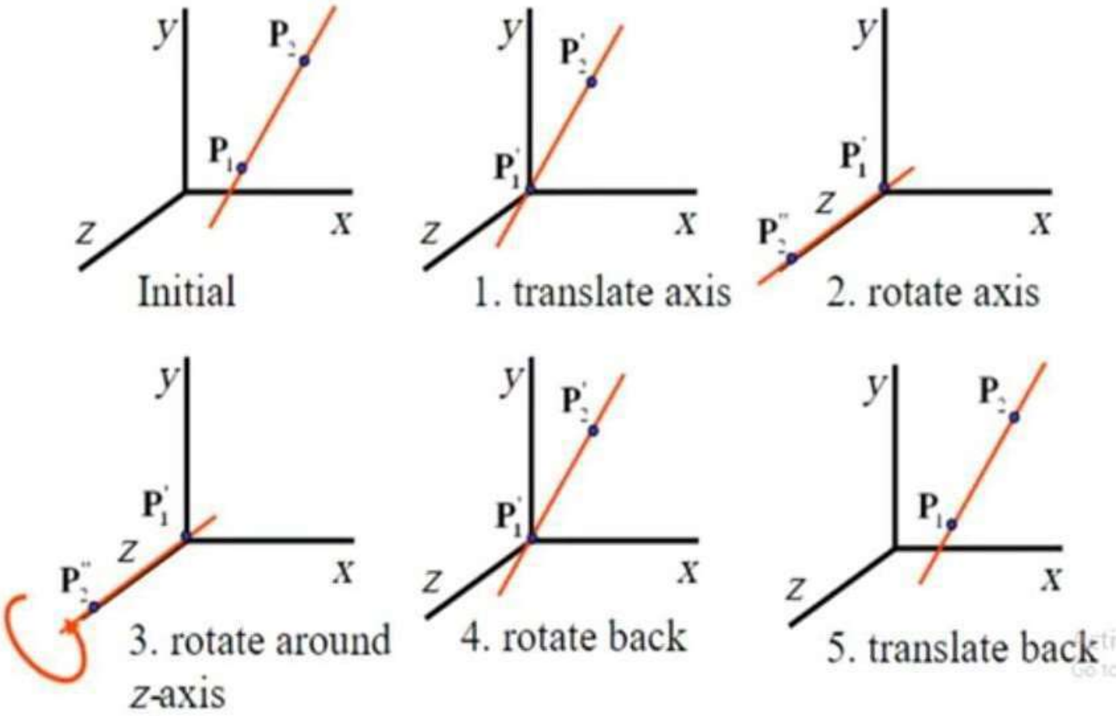
1. Translate over -**Q**;
2. Rotate around z - axis;
3. Translate back over **Q**.

Or :

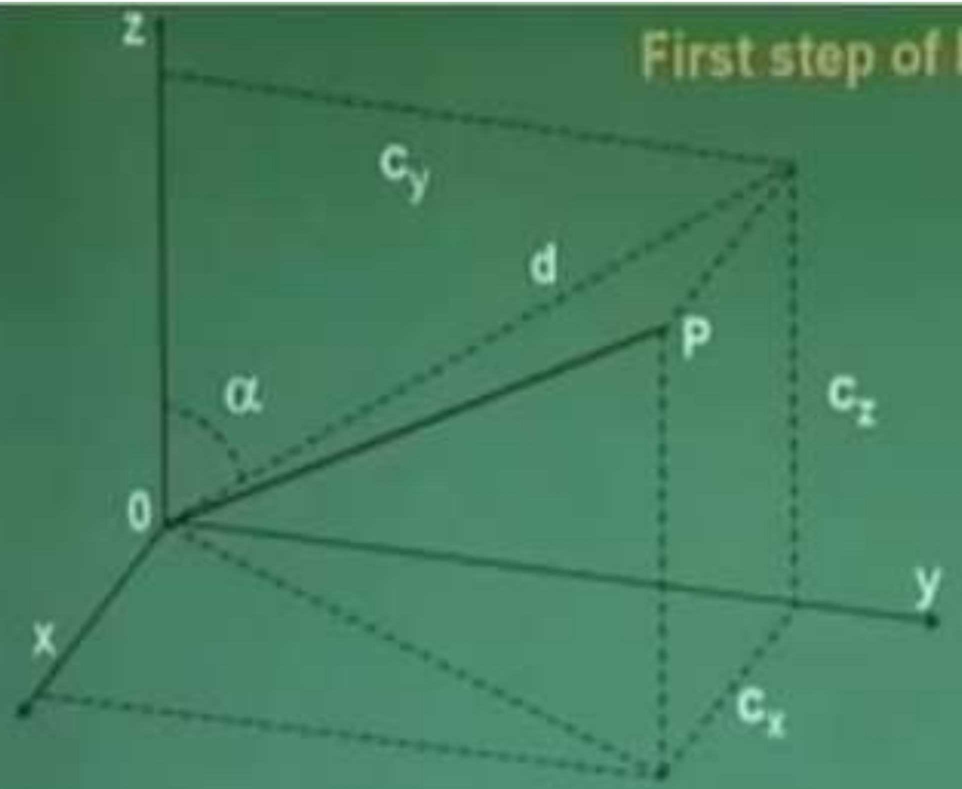
$$\mathbf{P}' = \mathbf{T}(\mathbf{Q})\mathbf{R}_z(\alpha)\mathbf{T}(-\mathbf{Q})\mathbf{P}$$



3D Rotation around arbitrary axis



First step of Rotation:



Rotation about x by α:

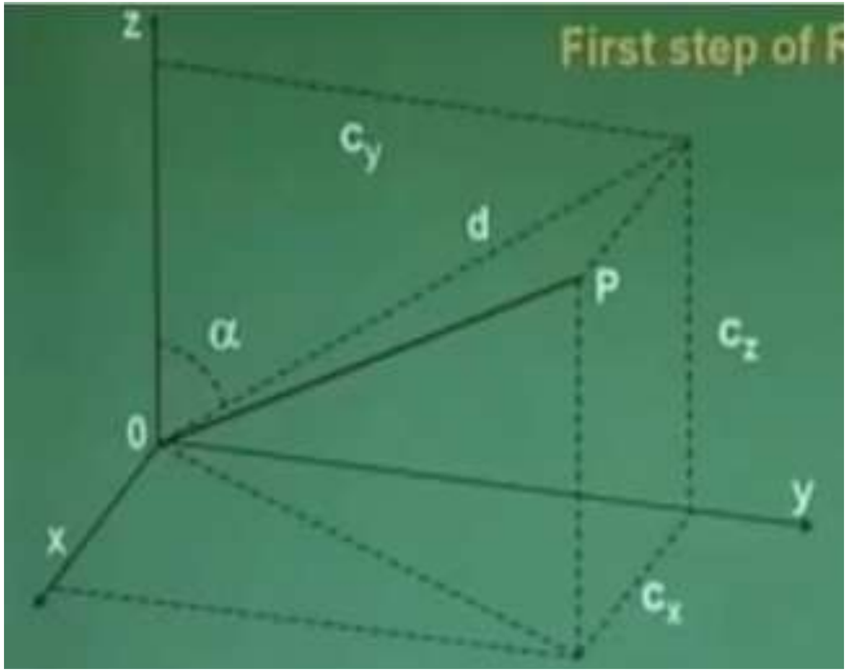
How do we determine α?

Rotate line OP

- Cx, Cy & Cz is the distance of point P from x-y-z axis

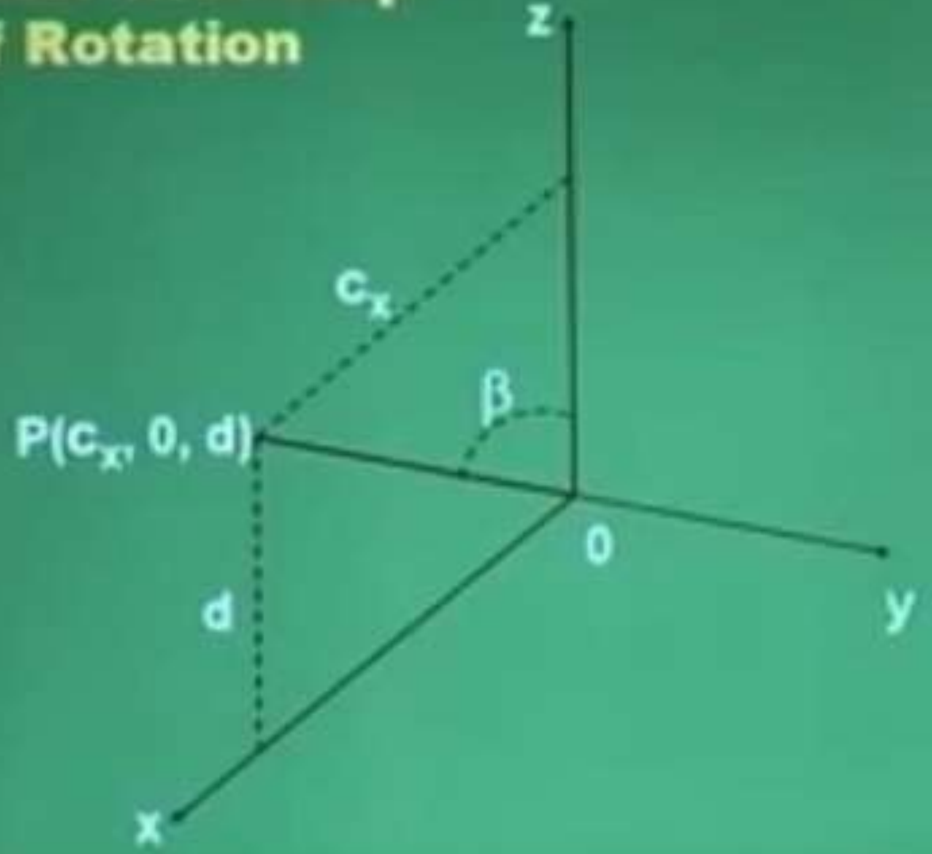
- Translate line in y-z axis named as d.

- By Pythagorean Theorem,
  - $d^2 = Cy^2 + Cz^2$
  - $d = \sqrt{(Cy^2 + Cz^2)}$



- $\cos \alpha = Cz/d$
- $\sin \alpha = Cy/d$
- $\alpha = \sin^{-1} (Cy/d)$

After first step of Rotation



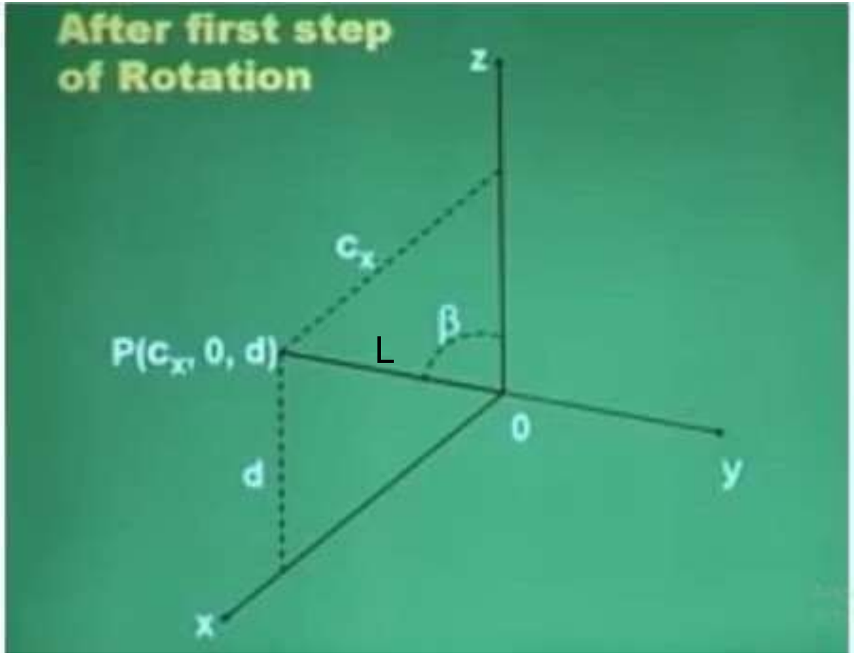
- The line is present in x-z plane .
- The line will form β angle with the z-axis.
- Its x-component is Cx & z-coordinate is d.
- ∴ cos β = d/l = d
- ∴ sin β = Cx/l = Cx

Final Transformation for 3D rotation

$M = |T|\ |Rx|\ |Ry|\ |Rz|\ |Ry|^{-1}\ |Rx|^{-1}\ |T|^{-1}$

where, T =

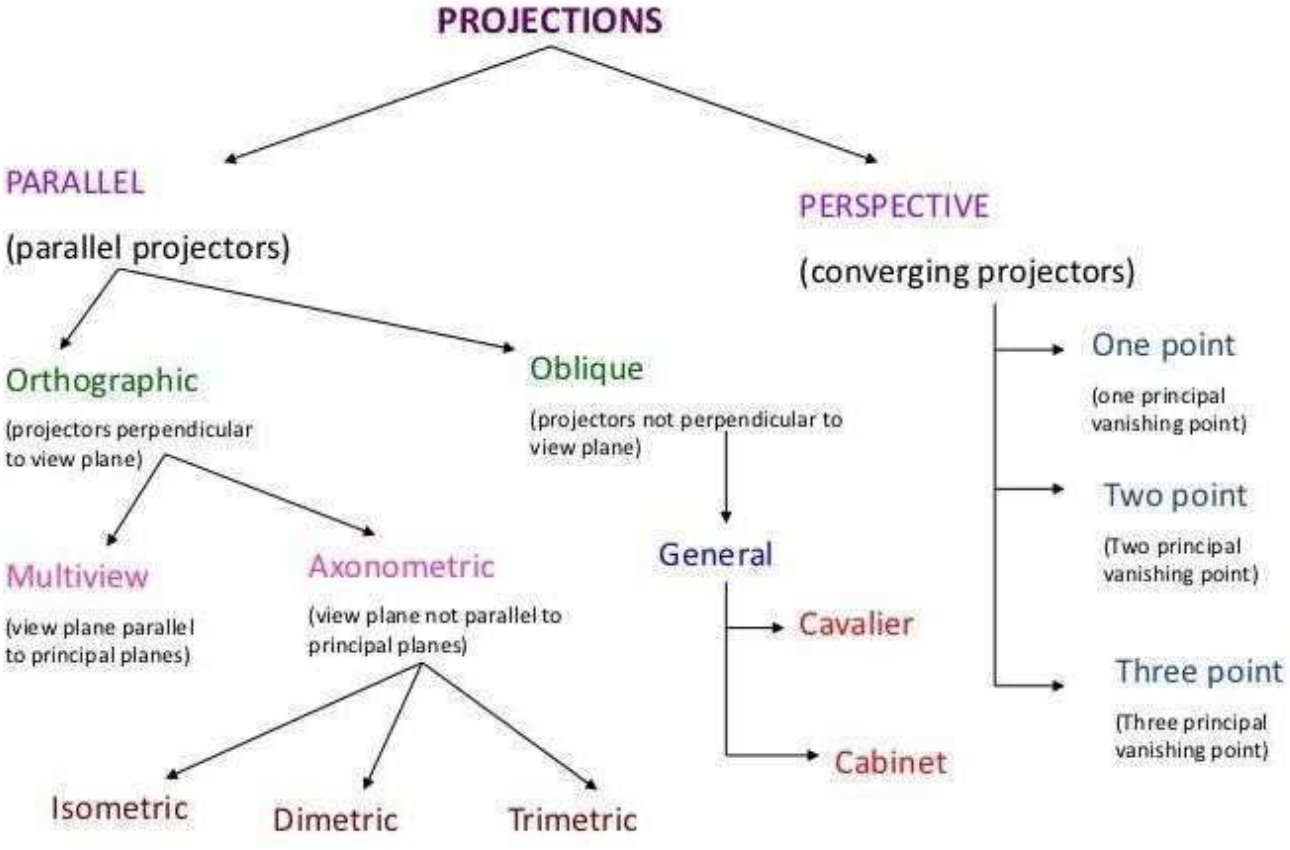
$$\begin{bmatrix} 1 & 0 & 0 & -x_0 \\ 0 & 1 & 0 & -y_0 \\ 0 & 0 & 1 & -z_0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



- $R_x = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & Cz/d & -Cy/d & 0 \\ 0 & Cy/d & Cz/d & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
- $R_y = \begin{bmatrix} d & 0 & -Cx & 0 \\ 0 & 1 & 0 & 0 \\ Cx & 0 & d & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
- $R_z = \begin{bmatrix} cos\Theta & -sin\Theta & 0 & 0 \\ sin\Theta & \cos\Theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

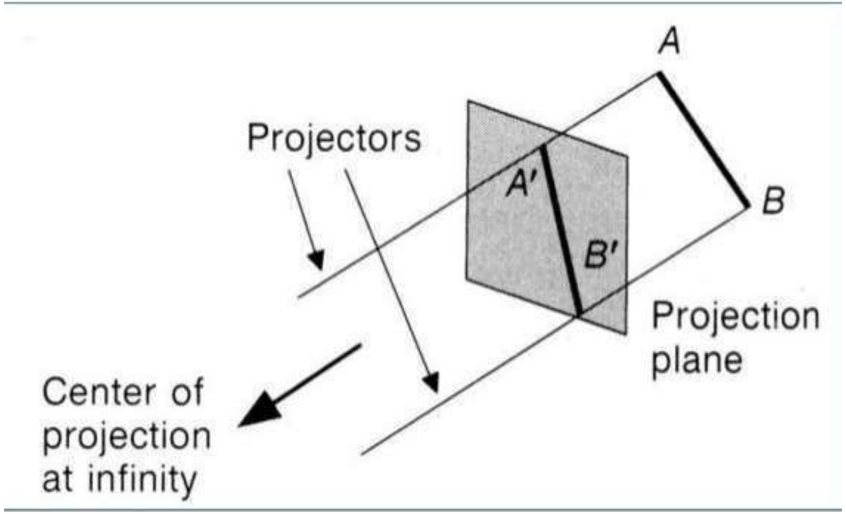
Projections

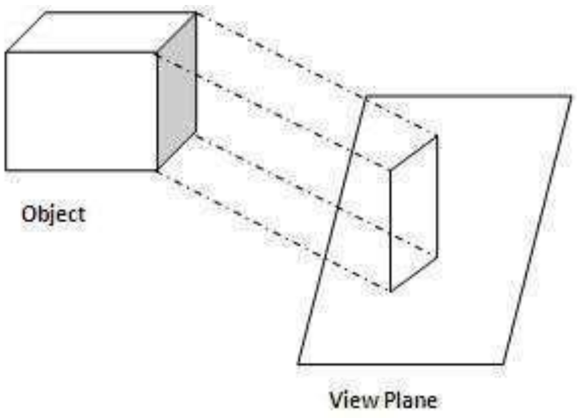
- It is a process of transforming an object representation from n-dimensional space to less than n-dimensional space.
- It converts a 3D object into a 2D object. It is also defined as mapping or transformation of the object in **projection plane or view plane**.
- View plane is the plane on which the object is projected.
- It is broadly classified into 2 categories:
  - Parallel Projection
  - Perspective Projection


PROJECTIONS
PARALLEL
(parallel projectors)
PERSPECTIVE
(converging projectors)
Orthographic
(projectors perpendicular to view plane)
Oblique
(projectors not perpendicular to view plane)
One point
(one principal vanishing point)
Two point
(Two principal vanishing point)
Three point
(Three principal vanishing point)
General
Cavalier
Cabinet
Multiview
(view plane parallel to principal planes)
Axonometric
(view plane not parallel to principal planes)
Isometric
Dimetric
Trimetric

Parallel Projection

- Parallel Projection use to display picture in its true shape and size. When projectors are perpendicular to view plane then is called orthographic projection.
- The parallel projection is formed by extending parallel lines from each vertex on the object until they intersect the plane of the screen. The point of intersection is the projection of vertex.
- Parallel projections are used by architects and engineers for creating working drawing of the object, for complete representations require two or more views of an object using different planes.

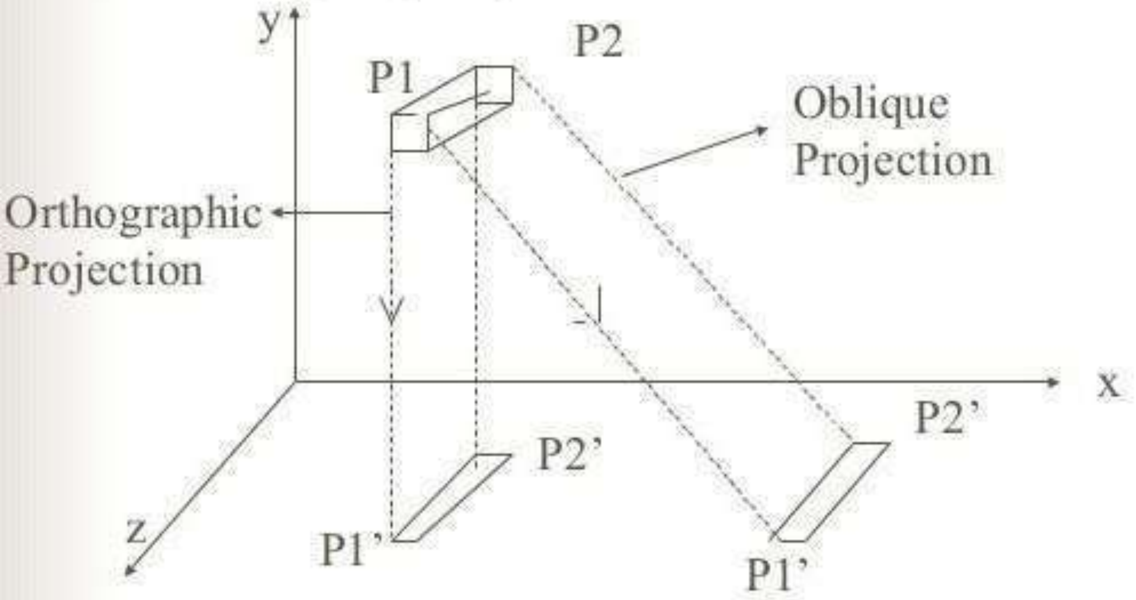

A
Projectors
A′
B′
B
Projection plane
Center of projection at infinity


Object
View Plane

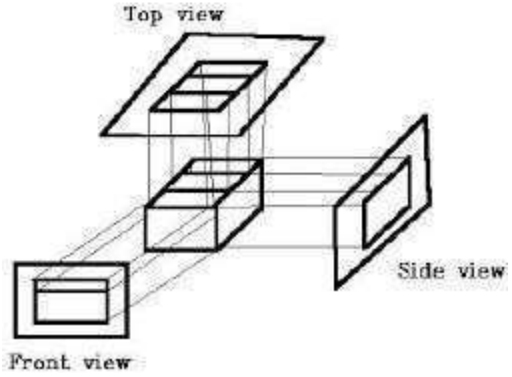
Types of Parallel Projections:

(i) **Orthographic Projection**

(ii) **Oblique projection**



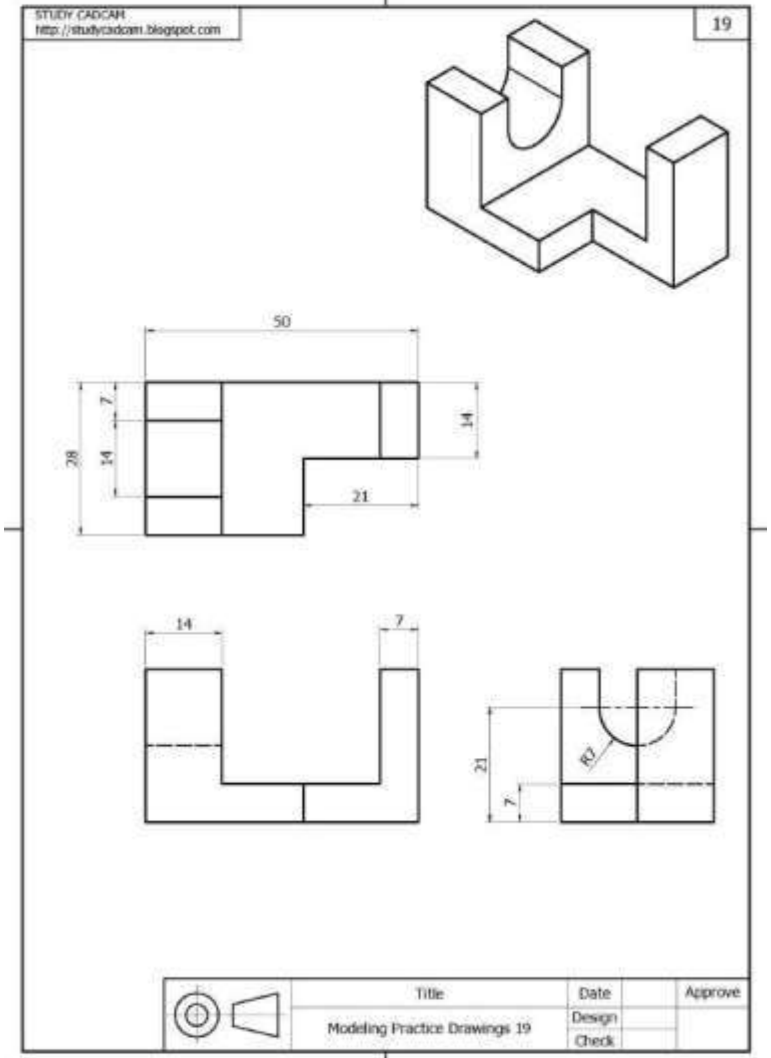
Orthographic Projection



- Orthographic Projection means of representing three-dimensional objects in two dimensions.
- It is a form of parallel projection, in which all the projection lines are orthogonal to the projection plane, resulting in every plane of the scene appearing in affine transformation on the viewing surface.
- The term orthographic is sometimes reserved specifically for depictions of objects where the principal axes or planes of the object are also parallel with the projection plane.

Multiview Orthographic Projection

- We can produce up to six pictures of an object, with each projection plane parallel to one of the coordinate axes of the object.
- The views are positioned relative to each other according to either of two schemes: *first-angle* or *third-angle projection*.
- The appearances of views may be thought of as being projected onto planes that form a six-sided box around the object.
- These views are known as front view, top view and end view. Other names for these views include plan, elevation and section.


STUDY CADCAM
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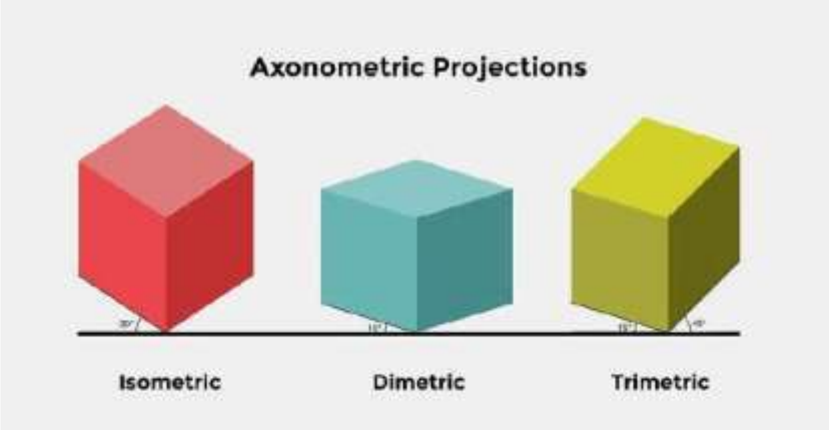
Title
Date
Approve
Modeling Practice Drawings 19
Design
Check

Axonometric Orthographic Projection

- Axonometric projection is used to describe the type of orthographic projection where the plane or axis of the object depicted is not parallel to the projection plane, and where multiple sides of an object are visible in the same image.
- It is further subdivided into three groups: **isometric, dimetric and trimetric** projection, depending on the exact angle at which the view deviates from the orthogonal.
- A typical characteristic of axonometric projection is that one axis of space is usually displayed as vertical.

1. **Isometric** - three axes of space appear equally foreshortened
2. **Dimetric** - two of the three axes of space appear equally foreshortened
3. **Trimetric** - all of the three axes of space appear unequally foreshortened.

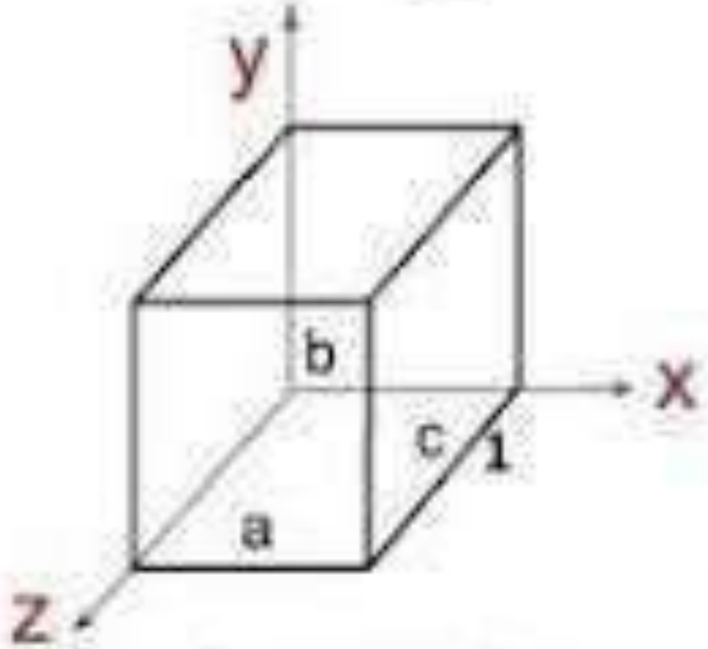


Oblique Projection

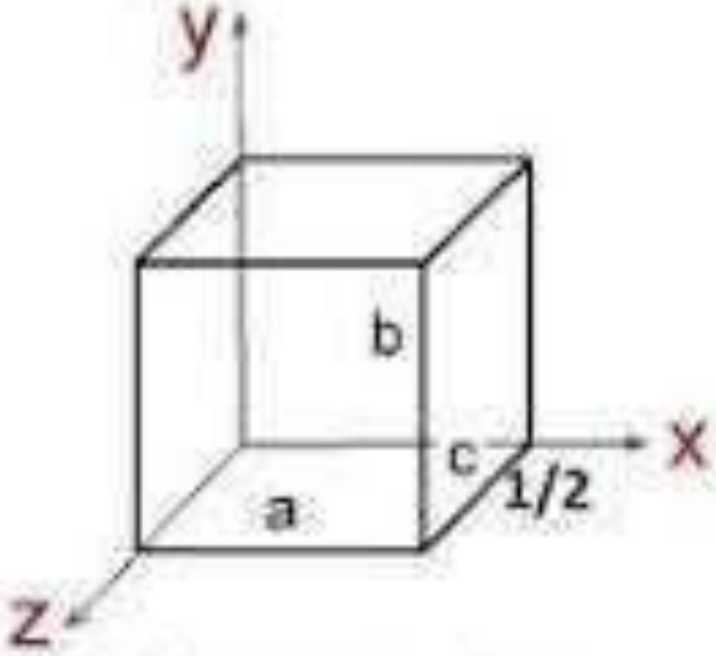
- In oblique projection, the direction of projection is not normal to the projection of plane. In oblique projection, we can view the object better than orthographic projection.
- The projectors in oblique projection intersect the projection plane at an oblique angle to produce the projected image, as opposed to the perpendicular angle used in orthographic projection.

1. **Cavalier** - A point of the object is represented by three coordinates, x, y and z. On the drawing, it is represented by only two coordinates, x″ and y″. On the flat drawing, two axes, x and z on the figure, are perpendicular and the length on these axes are drawn with a 1:1 scale; the third axis y, is drawn in diagonal, making an arbitrary angle with the x″ axis, usually 30 or 45°. The length of the third axis is not scaled.
2. **Cabinet** - Like cavalier perspective, one face of the projected object is parallel to the viewing plane, and the third axis is projected as going off at an angle. Unlike cavalier projection, where the third axis keeps its length, with cabinet projection the length of the receding lines is cut in half.

OBLIQUE PROJECTION

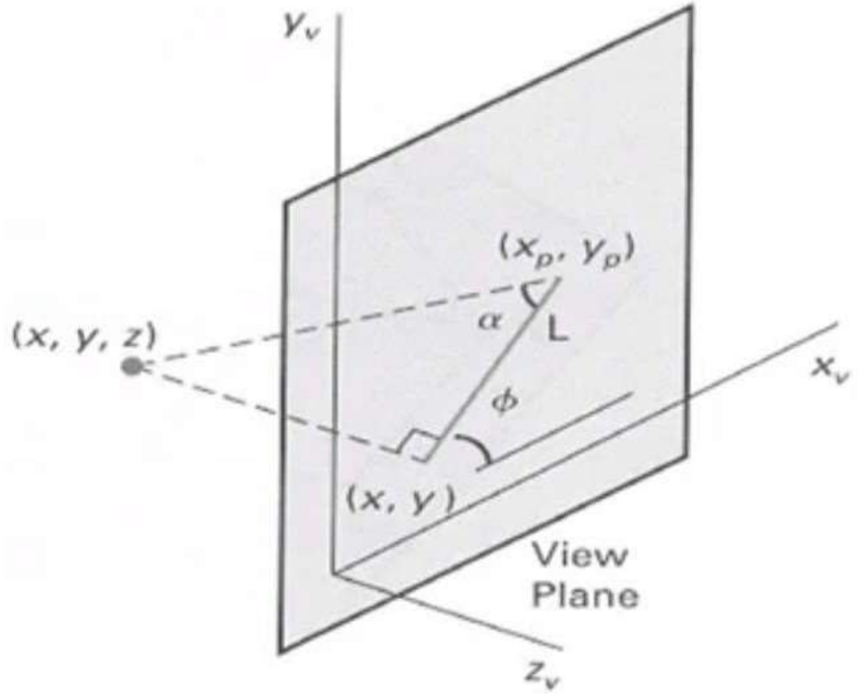


Cavalier Projection



Cabinet Projection

Oblique projections

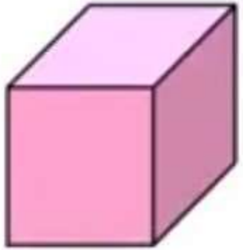


From the diagram

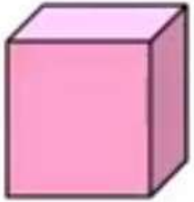
- $x_p = x + L \cos \Phi$
- $y_p = y + L \sin \Phi$
- $\tan \alpha = z/L$
- $L = z/\tan \alpha$
- $L = zL_1$
- $\therefore x_p = x + zL_1 \cos \Phi \quad \because L_1 = 1/\tan \alpha$
- $\therefore y_p = y + zL_1 \sin \Phi$

$$\begin{bmatrix} x_p \\ y_p \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & L_1 \cos \Phi & 0 \\ 0 & 1 & L_1 \sin \Phi & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Matrix Representation

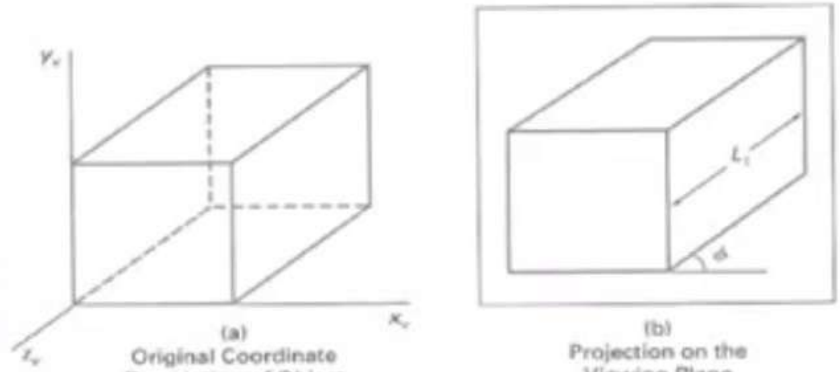


Cavalier
(DOP at 45°)



Cabinet
(DOP at 63.4°)

Parallel Projection Matrix



(a) Original Coordinate Description of Object

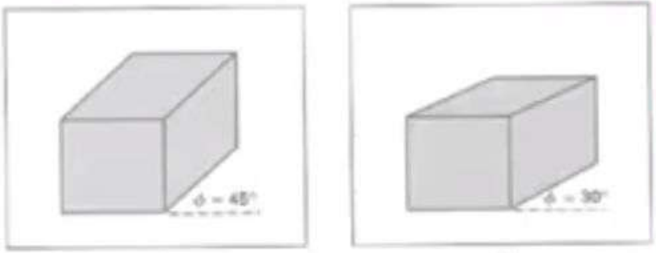
(b) Projection on the Viewing Plane

$$\mathbf{M}_{parallel} = \begin{bmatrix} 1 & 0 & L_1\cos\phi & 0 \\ 0 & 1 & L_1\sin\phi & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Oblique Projections

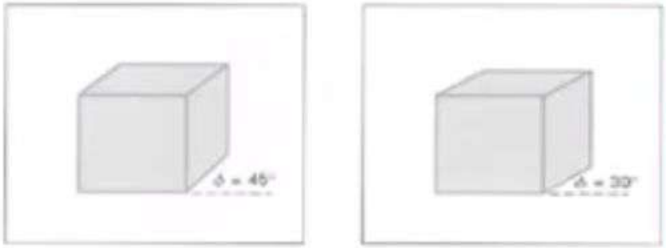
- DOP not perpendicular to view plane
  - **Cavalier projection**

$$\tan\alpha = 1, \quad \alpha = 45^\circ$$



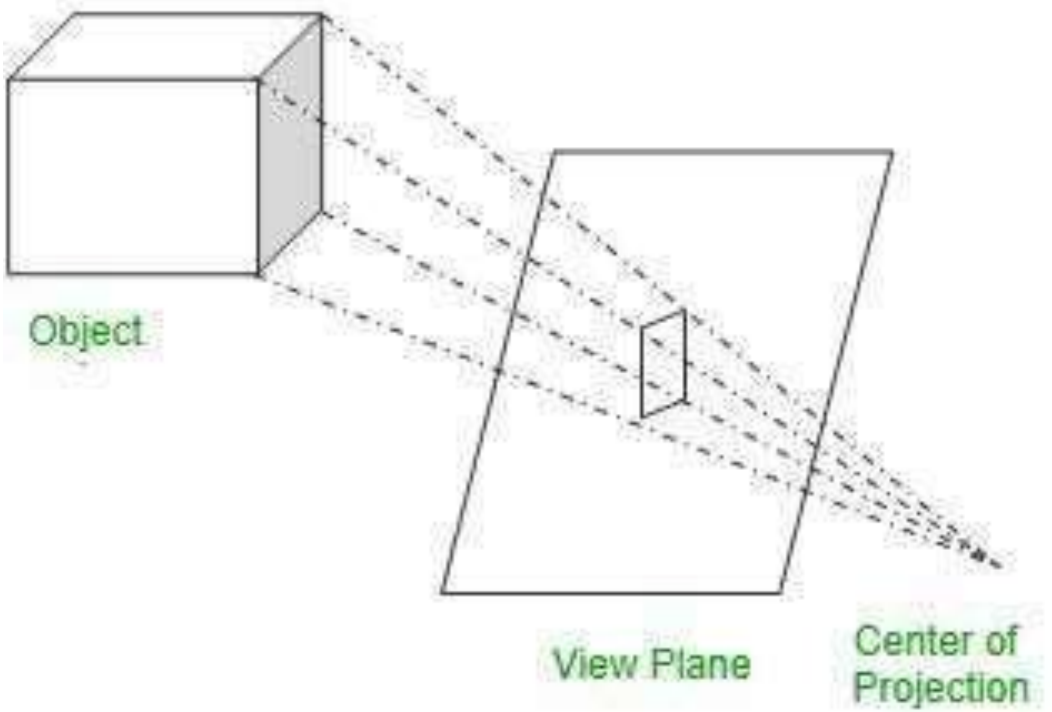
  - **Cabinet projection**

$$\tan\alpha = 2, \quad \alpha = 63.4^\circ$$



Perspective Projection

- In Perspective Projection the center of projection is at finite distance from projection plane. This projection produces realistic views but does not preserve relative proportions of an object dimensions.
- Projections of distant object are smaller than projections of objects of same size that are closer to projection plane. The perspective projection can be easily described by following figure.


Object
View Plane
Center of
Projection

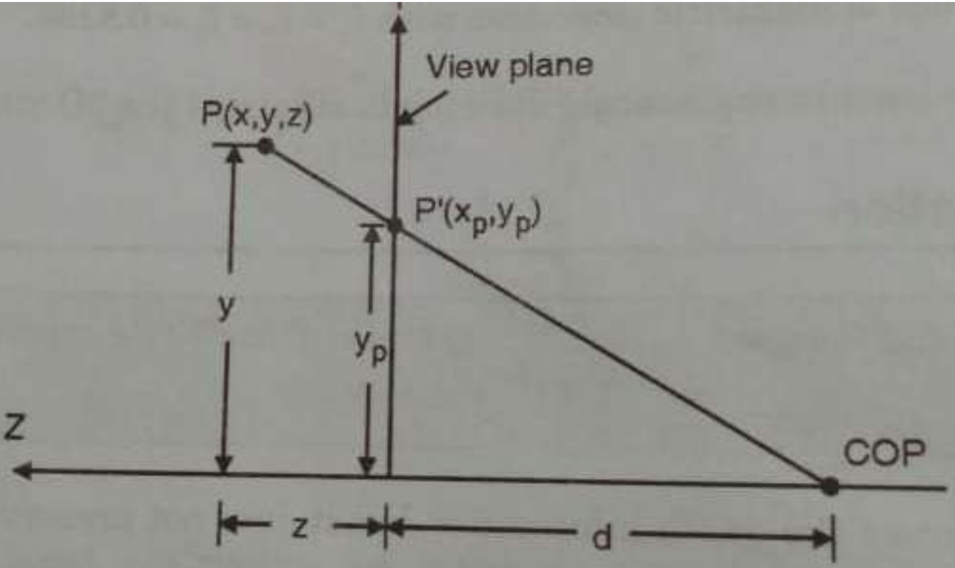


Fig. 7.3.3 : View from X-axis

From the triangle equality rule,

$$\frac{y_p}{d} = \frac{y}{z+d}$$

$$y_p = \frac{y \cdot d}{z+d}$$

If we look from the Y-axis, Fig. 7.3.2 looks as Fig. 7.3.3.

From the triangle equality rule,

$$\frac{x_p}{d} = \frac{x}{z+d}$$

$$\therefore x_p = \frac{x \cdot d}{z+d}$$

As view plane is positioned at z = 0, projected z coordinate would be 0

$$z_p = 0$$

From this, we can define a transformation matrix as,

$$M = \begin{bmatrix} d & 0 & 0 & 0 \\ 0 & d & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & d \end{bmatrix}$$

Projected Coordinates, $P' = M \cdot P$

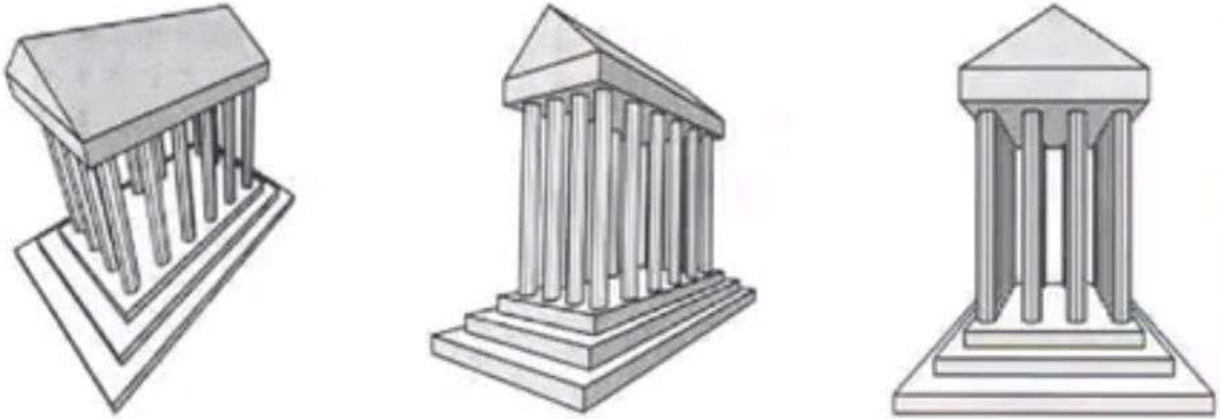
$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} x_p \\ y_p \\ z_p \\ 1 \end{bmatrix} = \begin{bmatrix} d & 0 & 0 & 0 \\ 0 & d & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & d \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

If view plane is not XY plane or it is not parallel to XY plane in that case, we need to perform composite transformation such that the normal of the plane get aligned with Z-axis.

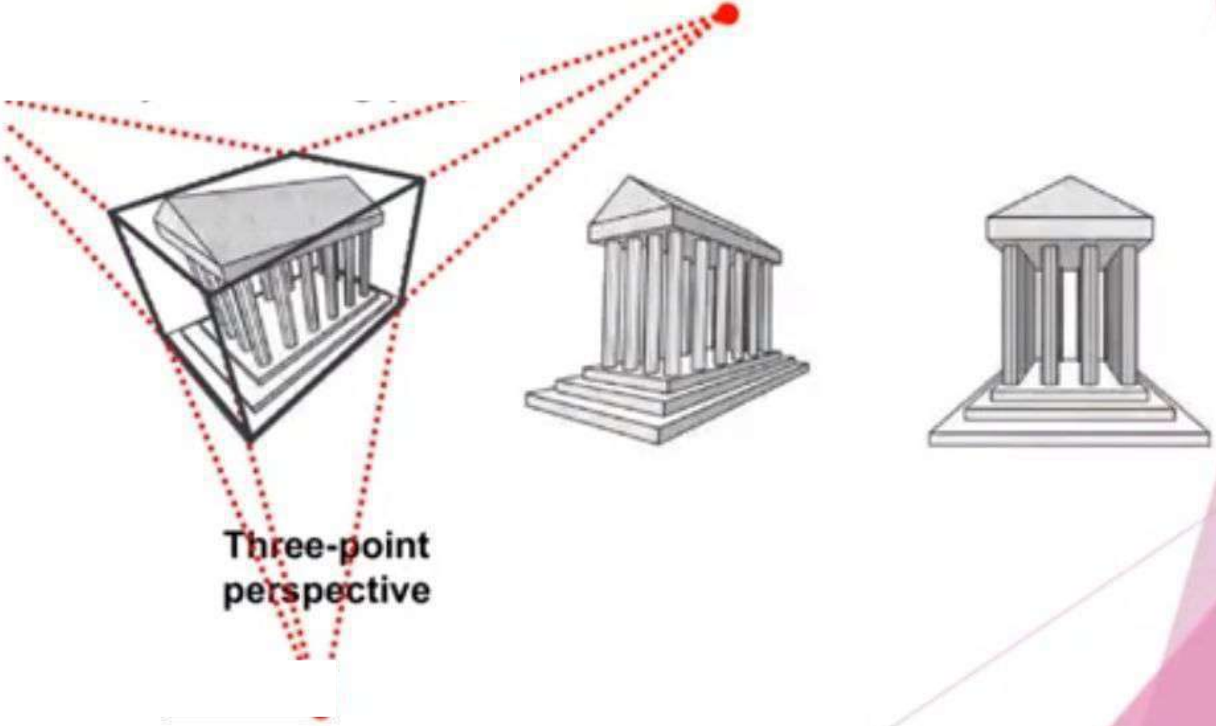
Vanishing point

- When a 3D object is projected onto a view plane using perspective transformation equations, any set of parallel lines in the object that are not parallel to the plane are projected into converging lines.
- Parallel line that are parallel to the view plane will be projected as parallel lines.
- The point at which a set of projected parallel lines appears to converge is called a vanishing point.

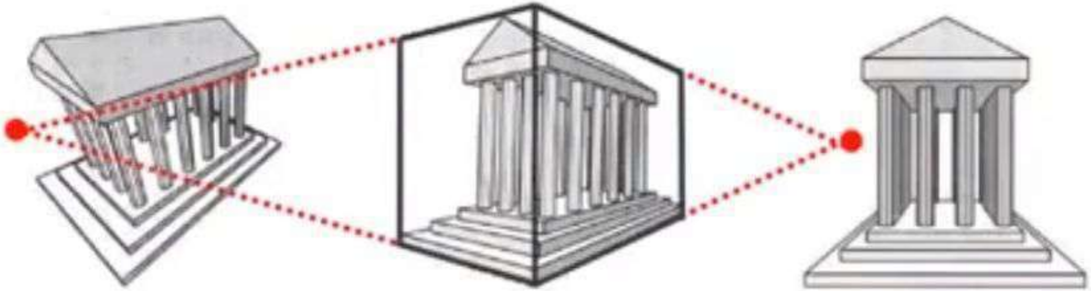
Perspective Projection



Perspective Projection



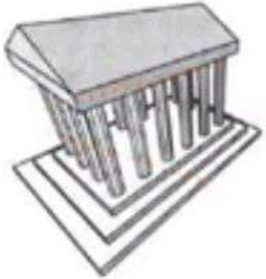
Perspective Projection



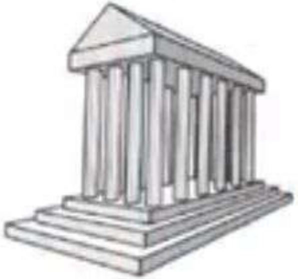
Three-point perspective

Two-point perspective

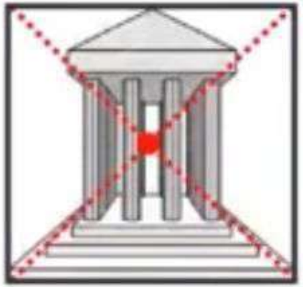
Perspective Projection



Three-point perspective

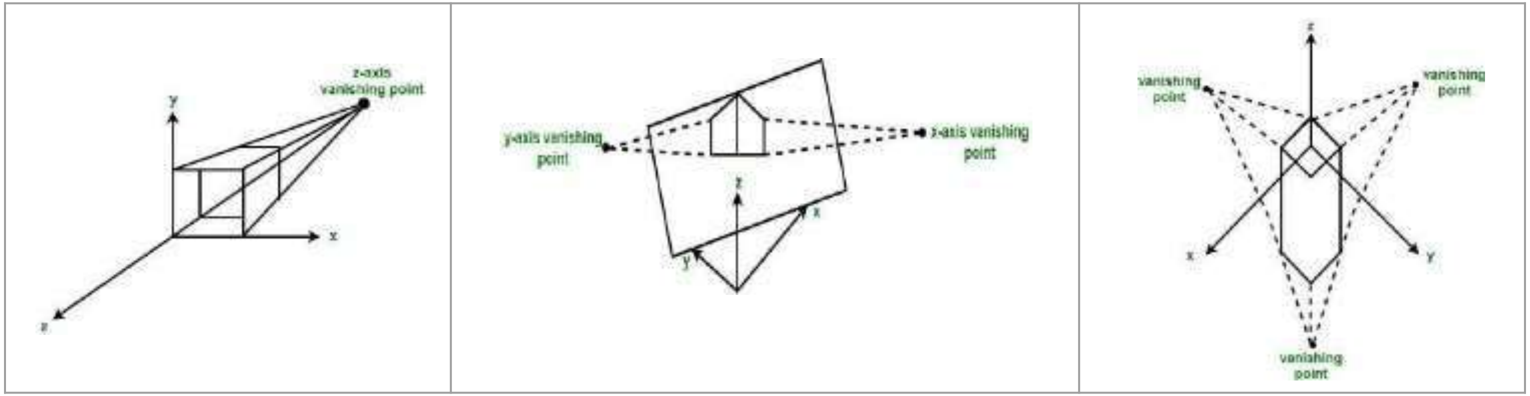


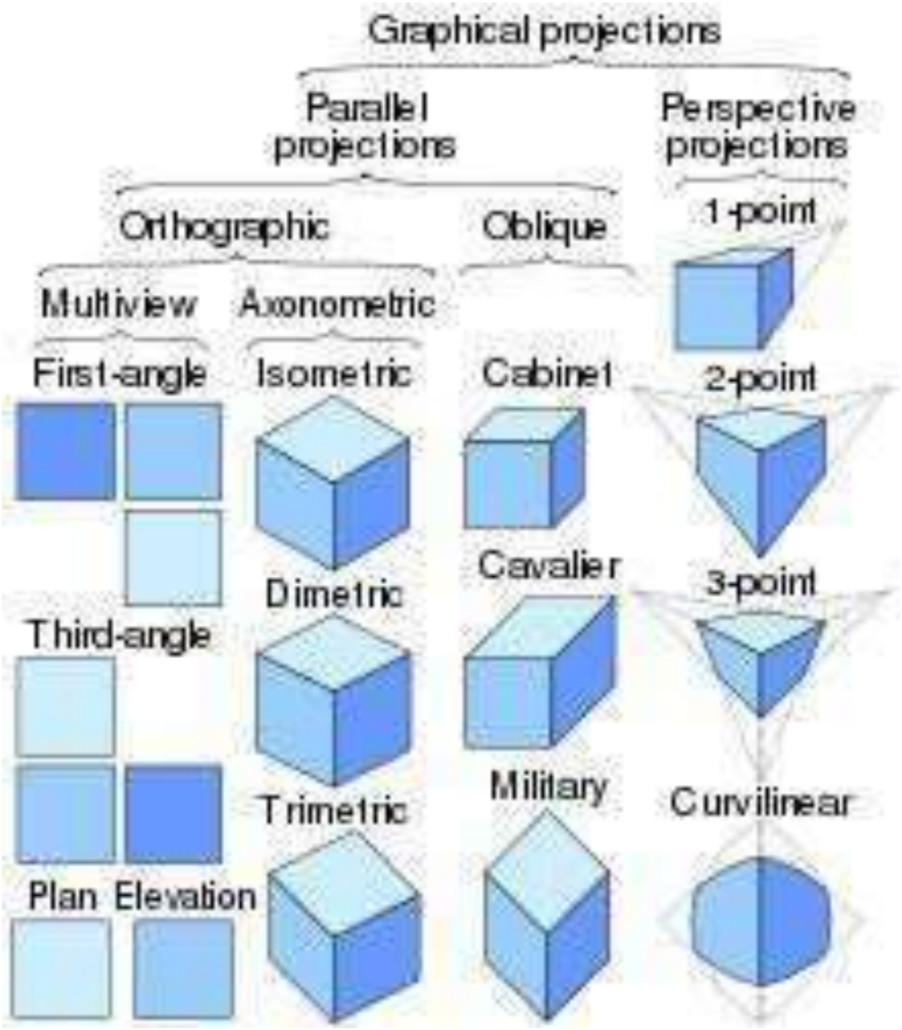
Two-point perspective



One-point perspective

1. **One point -** One point perspective projection occurs when any of principal axes intersects with projection plane or we can say when projection plane is perpendicular to principal axis.
2. **Two point -** Two point perspective projection occurs when projection plane intersects two of principal axis.
3. **Three point -** Three point perspective projection occurs when all three axis intersects with projection plane. There is no any principle axis which is parallel to projection plane.




Graphical projections
Parallel projections
Perspective projections
Orthographic
Oblique
1-point
Multiview
Axonometric
First-angle
Isometric
Cabinet
2-point
Dimetric
Cavalier
3-point
Third-angle
Military
Curvilinear
Trimetric
Plan
Elevation

References

- Hearn & Baker, “Computer Graphics C version”, 2nd Edition, Pearson Publication