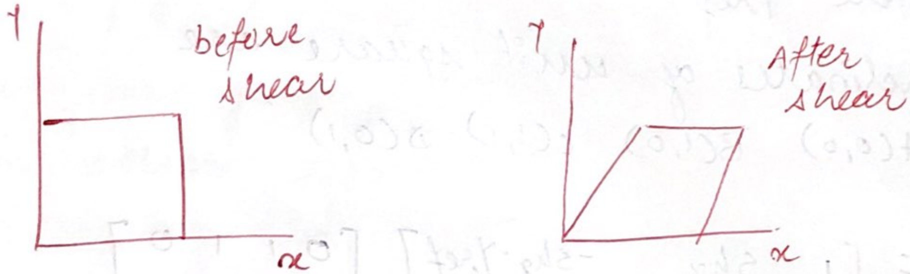
Shear :-

- A transformation that distorts the shape of an object such that the transformed shape appears as if the object were composed of internal layers that had been caused to slide over each other is called shear.

- Two common shearing transformations are those that shift co-ordinate x values and those that shift y values.

Shear in x-direction



- shear relative to x-axis that is y=0 can be produced by following equations.

$x' = x + sh_x \cdot y \quad , \quad y' = y$

Transformation matrix is given as

$$\begin{bmatrix} 1 & sh_x & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Here sh_x is shear parameter.

- we can generate x-direction shear relative to other refrence line $y = y_{ref}$ with following eq?

$$x' = x + sh_x(y - y_{ref}) \quad , \quad y' = y$$

Transformation matrix is given as:

$$\begin{bmatrix} 1 & sh_x & -sh_x \cdot y_{ref} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Example:

Q. shear the unit square parameter 1/2 relative to line $y = -1$.

Sol? Here $y_{ref} = -1$ & $sh_x = 0.5$

. co-ordinates of unit square are

A(0,0) B(1,0) C(1,1) D(0,1)

$$P' = \begin{bmatrix} 1 & sh_x & -sh_x \cdot y_{ref} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

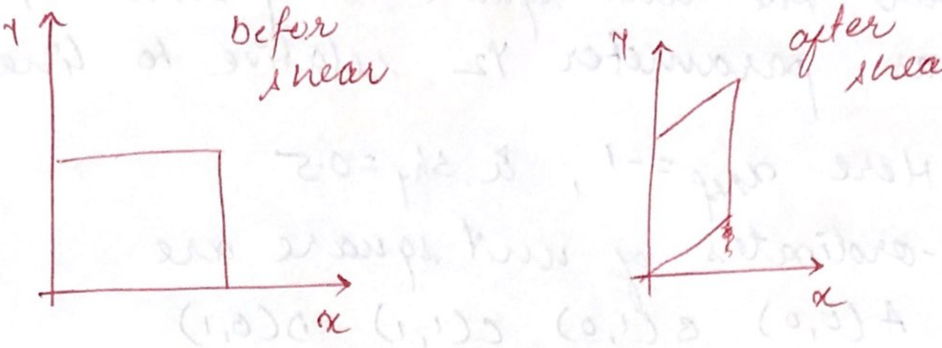
$$= \begin{bmatrix} 1 & 0.5 & -0.5(-1) \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0.5 & 0.5 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0.5 & 1.5 & 2 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

Final co-ordinate after shear are

A'(0.5, 0) B'(1.5, 0) C'(2, 1) D'(1, 1)//

shear in y-direction.



- shear relative to y-axis that is $x=0$ line can be produced by following equations:

$$x' = x, \quad y' = y + sh_y \cdot x.$$

- Transformation matrix is given as follows:

$$\begin{bmatrix} 1 & 0 & 0 \\ sh_y & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- we can generate y-direction shear relative to other reference line $x = x_{ref}$ with following eq.ⁿ

$$x' = x, \quad y' = y + sh_y(x - x_{ref})$$

- Transformation matrix for that is given as

$$\begin{bmatrix} 1 & 0 & 0 \\ sh_y & 1 & -sh_y \cdot ref \\ 0 & 0 & 1 \end{bmatrix}$$

Example:

Q. shear the unit square in y direction with shear parameter 1/2 relative to line $x = -1$

Sol.: Here $x_{ref} = -1$, & $sh_y = 0.5$

co-ordinates of unit square are

A(0,0) B(1,0) C(1,1) D(0,1)

$$P' = \begin{bmatrix} 1 & 0 & 0 \\ sh_y & 1 & -sh_y \cdot ref \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0.5 & 1 & -0.5(-1) \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0.5 & 1 & 0.5 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0.5 & 1 & 2 & 1.5 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

Final co-ordinates after shear are

A'(0, 0.5) B'(1, 1) C'(1, 2) D'(0, 1.5)