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Big Oh (O) (upper bound)

→ max^m time the notations are represented in terms of O.

→ n is input values
t is time

→ we have fun^n f(n)
any problem which is solved using fun^n f(n)

→ I want to write f(n) in terms of Order of g(n)
Now
$f(n) = O \cdot g(n)$
↑ given

$f(n) \leq c \cdot g(n)$
↑ constant

$c > 0$
$n \geq k$
$k \geq 0$

Let's say we have
$f(n) = 16n^3 + 45n^2 + 12n$
Now we have to find
$f(n) = O(n)$

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$16 \cdot n^3 + 45n^2 + 12n \leq c \cdot g(n)$

this fun^n — This fun^n should be greater than

Big O represents upper bound that is the atmost value.

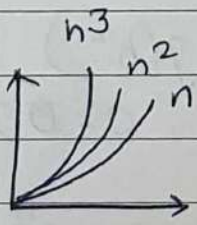
We need to find the largest term out of $16n^3 + 45n^2 + 12n$

$n^3, n^2 \rightarrow$ quadratic equation
$n \rightarrow$ linear equation

n^3 is largest term

If $n = 2$
value of $n^2 = 4$
& $n^3 = 8$

If $n = 100$
value of $n^2 = 10,000$
& $n^3 = 1,000,000$



not much change for smaller values.

But when we talk about upper bound we need least upper bound or closed upper bound

So, the closed upper bound is n^3. i.e.

$16n^3 + 45n^2 + 12n \leq c \cdot g(n^3)$

If I take value of $c = 2$

$16n^3 + 45n^2 + 12n \not\leq 2 \cdot g(n^3)$

So we should take $c = 42$

$16n^3 + 45n^2 + 12n \leq 3 \cdot g(n^3)$

so $n \geq 1$

& $c = 42$

From this we can say that

$16n^3 + 45n^2 + 12n \leq c \cdot g(n^3)$

$f(n) = O(n^3)$

Big Omega (Ω) (lower Bound)

$f(n) = \Omega g(n)$

$f(n) \geq c \cdot g(n)$

$f(n) = 2n^2 + n$

$2n^2 + n \geq c \cdot g(n)$

we can take n^2 or even n as lower bound but while selecting the lower bound always keep in mind select the greatest lower bound.

For example no.5 which are smaller than 5 are 4, 3, 2, 1, 0 but we should select the greatest smaller no. i.e. 4.

So we can write

$2n^2 + n$

$f(n) \geq c \cdot g(n)$

$2n^2 + n \geq c \cdot n^2$

Now we should select value for c, if $c = 2$

$2n^2 + n \geq 2 \cdot n^2$

$n \geq 2n^2 - 2n^2$

So, $n \geq 0$ & $c = 2$

③ Theta (Θ)

average case time complexity.

$C_1 \cdot g(n) \leq f(n) \leq C_2 \cdot g(n)$

$f(n) = 2n^2 + n$

So, $g(n) = n^2$

Now $C_1 = 2$ & $C_2 = 3$

So

$2 \cdot n^2 \leq 2n^2 + n \leq 3n^2$

Let's take an example if I have a notebook & I want to find a particular topic & there is no indexing or hashing in that case the sea situation is

→ If I found the topic on 1st page i.e. Best Case.

→ If I found the topic on last page i.e. Worst case.

→ If I found it in the middle or somewhere nearby i.e. average case.

④ Little Oh o

$f(n) < C \cdot g(n)$

⑤ Little Omega

$f(n) > C \cdot g(n)$