Explain parallel and perspective projections and derive the matrix for perspective projection.

1. Projection operations convert the viewing-coordinate description (3D) to coordinate positions on the projection plane (2D).
2. There are 2 basic projection methods:

Parallel Projection:

i. In parallel projection, Z coordinate is discarded and parallel lines from each vertex on the object are extended until they intersect the view plane.

ii. The point of intersection is the projection of the vertex.

iii. We connect the projected vertices by line segments which correspond to connections on the original object.

iv. A parallel projection preserves relative proportions of objects.

v. Accurate views of the various sides of an object are obtained with a parallel projection. But not a realistic representation.

vi. Parallel projection is shown below in figure 30.

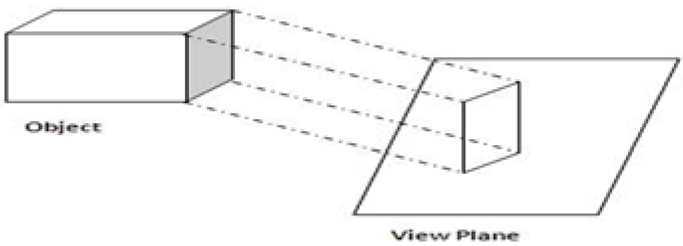

Figure 30

Perspective Projection:

i. In perspective projection, the lines of projection are not parallel.

ii. Perspective Projection transforms object positions to the view plane while converging to a center point of projection.

iii. In this all the projections are converge at a single point called the “center of projection” or “projection reference point”.

iv. Perspective projection produces realistic views but does not preserve relative proportions.

v. Projections of distant objects are smaller than the projections of objects of the same size that are closer to the projection plane.

vi. Perspective projection is shown below in figure 31

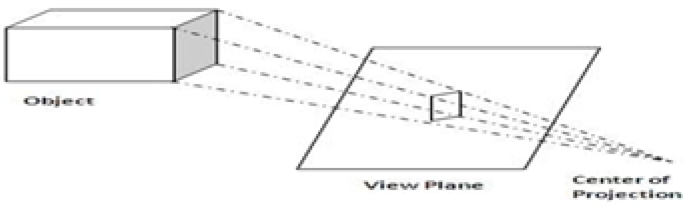

Figure 31

Matrix for perspective projection:

Matrix for perspective projection:

Let us consider the center of projection is at $P_c(X_c, Y_c, Z_c)$ and the point on object is $P_1(X_1, Y_1, Z_1)$, then the parametric equation for the line containing these points can be given as

$$X_2 = X_c + (X_1 - X_c)U$$

$$Y_2 = Y_c + (Y_1 - Y_c)U$$

$$Z_2 = Z_c + (Z_1 - Z_c)U$$

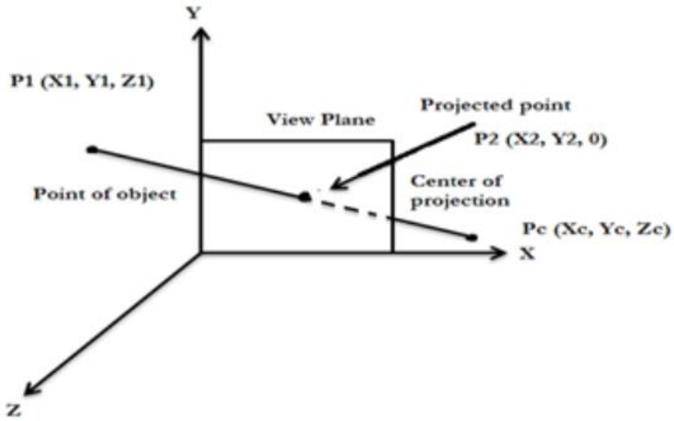

Figure 32

For projected point Z2 is 0, therefore the third equation can be written as

$$0 = Z_c + (Z_1 - Z_c)U$$

$$U = -Z_c/Z_1 - Z_c$$

Substituting the value of U in first two equations we get,

$$X2 = (X_c - Z_c) * (X_1 - X_c)/(Z_1 - Z_c)$$

$$= X_C Z_1 - X_C Z_C - X_1 Z_C + X_C Z_C / Z_1 - Z_C$$

$$= X_C Z_1 - X_1 Z_C / Z_1 - Z_C$$

$$Y2 = (Y_c - Z_c) * (Y_1 - Y_c)/(Z_1 - Z_c)$$

$$= Y_C Z_1 - Y_C Z_C - Y_1 Z_C + Y_C Z_C / Z_1 - Z_C$$

$$= Y_C Z_1 - Y_1 Z_C / Z_1 - Z_C$$

The above equations can be represented in the homogeneous matrix form as given below:

$$[X_2\ Y_2\ Z_2\ 1] = [X_1\ Y_1\ Z_1\ 1]\begin{bmatrix} -Zc & 0 & 0 & 0 \\ 0 & -Zc & 0 & 0 \\ Xc & Yc & 0 & 1 \\ 0 & 0 & 0 & -Zc \end{bmatrix}\begin{bmatrix} -Zc & 0 & 0 & 0 \\ 0 & -Zc & 0 & 0 \\ Xc & Yc & 0 & 1 \\ 0 & 0 & 0 & -Zc \end{bmatrix}$$

Here, we have taken the center of projection as $P_c(X_c,Y_c,Z_c)$Pc(Xc,Yc,Zc). If we take the center of projection on the negative Z – axis such that

X = 0

Y = 0

$Z = -Z_c$