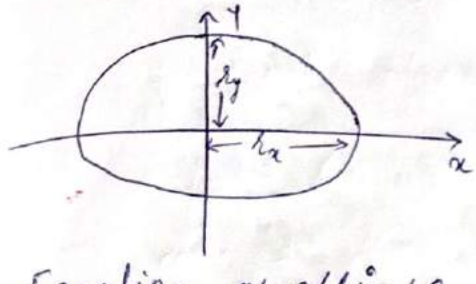
# MID POINT ELLIPSE DRAWING ALGO




Ellipse is elongated circle.

semi major axis(a) = $r_x$

semi minor axis (b) = $r_y$

Equation of ellipse centered at (0,0)

$$\frac{x^2}{r_x^2} + \frac{y^2}{r_y^2} = 1$$

$$\frac{x^2 r_y^2 + y^2 r_x^2}{r_x^2 r_y^2} = 1$$

$$r_y^2 x^2 + r_x^2 y^2 = r_x^2 r_y^2$$

$$r_y^2 x^2 + r_x^2 y^2 - r_x^2 r_y^2 = 0$$

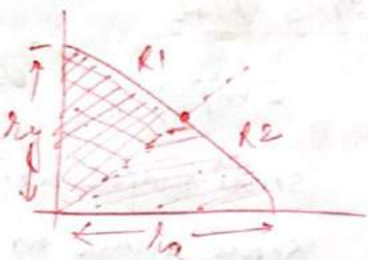
$$f_{ellipse} = r_y^2 x^2 + r_x^2 y^2 - r_x^2 r_y^2$$

If we put midpoint in this eq^n then we can get

$$f_{ellipse} = \begin{cases} <0 & \text{point lies inside ellipse boundary.} \\ =0 & \text{point lies on the ellipse boundary.} \\ >0 & \text{point lies outside ellipse boundary.} \end{cases}$$

## Difference between circle & ellipse.

- circle has 8-way symmetry
  ellipse has 4-way symmetry.
- In circle we need to plot only 1 octant of any quadrant, but in ellipse we need to plot 2 octants i.e. 1 complete quadrant to plot entire ellipse.




quadrant 1 → Region 1

* start point is $(0, r_y)$
* slop of curve $< -1$
* Take unit steps in positive $x$ direction till boundary b/w the 2 region is reached and calculate respective y values.

Quadrant 1 → Region 2.

* slop of curve $> -1$
* Take unit step in negative y direction i.e. $\Delta y = -1$ till the end of quadrant and calculate respective $x$ values.

# On the boundary b/w the 2 region, the slop of curve is -1.

Slop of curve.

If $f(x,y)$ represents an implicit function then $\frac{dy}{dx} = \frac{-\partial f / \partial x}{\partial f / \partial y}$

where $\partial f/\partial x$ is partial derivative of $f$ wrt $x$ treating $y$ as constants.

& $\partial f / \partial y$ is partial derivative of $f$ wrt to $y$ treating $x$ as constant.

Here, $f_{ellipse}(x,y) = r_y^2 x^2 + r_x^2 y^2 - r_x^2 r_y^2$

$$\therefore \frac{\partial f_{ellipse}}{\partial x} = 2x r_y^2$$

$$\& \ \frac{\partial f_{ellipse}}{\partial y} = 2y r_x^2$$

$$\therefore \frac{dy}{dx} = -\frac{\partial f_{ellipse}/\partial x}{\partial f_{ellipse}/\partial y}$$

$$= -\frac{2x r_y^2}{2y r_x^2}$$

Finding $P_{k+1}$ for region 1.

In region 1 we have two possiblities to choose next pixel position to plot.

Those are $(x_k+1, y_k)$ $(x_k+1, y_k-1) \Rightarrow \underbrace{(x_k+1, y_k-\frac{1}{2})}_{\text{midpoint}}$

eq$^n$ $\Rightarrow r_y^2 x^2 + r_x^2 y^2 - r_x^2 r_y^2$

putting values of $x$ & $y$

$$P1_k = r_y^2(x_k+1)^2 + r_x^2(y_k-\tfrac{1}{2})^2 - r_x^2 r_y^2 \quad \text{--- (1)}$$

$$P1_{k+1} = r_y^2(x_{k+1}+1)^2 + r_x^2(y_{k+1}-\tfrac{1}{2})^2 - r_x^2 r_y^2 \quad \text{--- (2)}$$

(2) - (1) : $P1_{k+1} - P1_k$

$$= r_y^2\{(x_k+1)+1\}^2 - r_y^2(x_k+1)^2 + r_x^2(y_{k+1}-\tfrac{1}{2})^2 - r_x^2(y_k-\tfrac{1}{2})^2 \cancel{- r_x^2 r_y^2} \cancel{+ r_x^2 r_y^2}$$

$$= r_y^2\{(x_k+1)^2 + 1 + 2(x_k+1)\} - r_y^2(x_k+1)^2 + r_x^2(y_{k+1}^2 + \tfrac{1}{4} - y_{k+1}) - r_x^2(y_k^2 + \tfrac{1}{4} - y_k)$$

$$= r_y^2 \{ \cancel{(x_k+1)^2} + 1 + 2(x_k+1) - \cancel{(x_k+1)^2} \} + r_x^2 \{ y_{k+1}^2 + \cancel{\tfrac{1}{4}} - y_{k+1} - y_k^2 - \cancel{\tfrac{1}{4}} + y_k \}$$

$$= r_y^2 \{ 2(x_k+1) + 1 \} + r_x^2 \{ y_{k+1}^2 - y_k^2 - y_{k+1} + y_k \}$$

$y_{k+1}$ can either be $y_k$ or $y_k - 1$

$\downarrow$ when $P1_k < 0$ $\qquad$ $\searrow$ when $P1_k \geq 0$

* when $P1_k < 0$ then $y_{k+1} = y_k$

$$\cancel{P1_{k+1} = P1_k + r_y^2 (2x_{k+1} + 1}$$

$$P1_{k+1} = P1_k + r_y^2 \{ 2\underbrace{(x_k+1)}_{x_{k+1}} + 1 \} + r_x^2 \{ \cancel{y_k^2} - \cancel{y_k^2} - \cancel{y_k} + \cancel{y_k} \}$$

$$\boxed{P1_{k+1} = P1_k + r_y^2 (2x_{k+1} + 1)}$$

* when $P1_k \geq 0$ then $y_{k+1} = y_k - 1$

$$P1_{k+1} = P1_k + r_y^2 \{ 2(x_k+1) + 1 \} + r_x^2 \{ (y_k-1)^2 - y_k^2 - (y_k-1) + y_k \}$$

$$= P1_k + r_y^2 \{ 2x_{k+1} + 1 \} + r_x^2 \{ \cancel{y_k^2} + 1 - 2y_k - \cancel{y_k^2} - \cancel{y_k} + 1 + \cancel{y_k} \}$$

$$= P1_k + 2r_y^2 x_{k+1} + r_y^2 + r_x^2 (-2y_k + 2)$$

$$= P1_k + 2r_y^2 x_{k+1} + r_y^2 - 2r_x^2 (y_k - 1)$$

$$\boxed{P1_{k+1} = P1_k + 2r_y^2 x_{k+1} + r_y^2 - 2r_x^2 y_{k+1}}$$

Initial decision parameter

put $(0, r_y)$ in

$$p1_k = r_y^2 (x_k+1)^2 + r_x^2 (y_k - \tfrac{1}{2})^2 - r_x^2 r_y^2$$

$$p1_0 = r_y^2 (0+1)^2 + r_x^2 (r_y - \tfrac{1}{2})^2 - r_x^2 r_y^2$$

$$p1_0 = r_y^2 + r_x^2 (r_y^2 + \tfrac{1}{4} - r_y) - r_x^2 r_y^2$$

$$= r_y^2 + \cancel{r_x^2 r_y^2} + \tfrac{1}{4} r_x^2 - r_x^2 r_y - \cancel{r_x^2 r_y^2}$$

$$\boxed{p1_0 = r_y^2 + r_x^2/4 - r_x^2 r_y}$$

Finding $p2_{k+1}$ for Region 2

In region 2 we have two possibilities to choose next pixel position to plot.

Those are $(x_k, y_k - 1)$ & $(x_k+1, y_k-1) \to \underbrace{(x_k + \tfrac{1}{2}, y_k - 1)}_{\text{midpoint}}$

eq.$^n$ $\Rightarrow r_y^2 x^2 + r_x^2 y^2 - r_x^2 r_y^2$

putting values of $x$ & $y$ in this eq.$^n$

$$p2_k = r_y^2 (x_k + \tfrac{1}{2})^2 + r_x^2 (y_k - 1)^2 - r_x^2 r_y^2 \quad \text{--- (1)}$$

$$p2_{k+1} = r_y^2 (x_{k+1} + \tfrac{1}{2})^2 + r_x^2 (y_{k+1} - 1)^2 - r_x^2 r_y^2$$

$$= r_y^2 (x_{k+1} + \tfrac{1}{2})^2 + r_x^2 ((y_k - 1) - 1)^2 - r_x^2 r_y^2 \quad \text{--- (2)}$$

$2 - (1)$ : $p2_{k+1} - p2_k$

$$\Rightarrow r_y^2 (x_{k+1} + \tfrac{1}{2})^2 - r_y^2 (x_k + \tfrac{1}{2})^2 + r_x^2 ((y_k - 1) - 1)^2 - r_x^2 (y_k - 1)^2$$
$$\cancel{- r_x^2 r_y^2} \cancel{+ r_x^2 r_y^2}$$

$$\Rightarrow r_y^2\left[x_{k+1}^2+\frac{1}{4}+x_{k+1}\right]-r_y^2\left[x_k^2+\frac{1}{4}+x_k\right]+r_x^2\left[(y_k-1)^2+1-2(y_k-1)\right]-r_x^2(y_k-1)^2$$

$$\Rightarrow r_y^2\left[x_{k+1}^2+\cancel{\frac{1}{4}}+x_{k+1}-x_k^2-\cancel{\frac{1}{4}}-x_k\right]+r_x^2\left[\cancel{(y_k-1)^2}+1-2(y_k-1)-\cancel{(y_k-1)^2}\right]$$

$$P2_{k+1}-P2_k=r_y^2\left[x_{k+1}^2+x_{k+1}-x_k^2-x_k\right]+r_x^2\left[1-2\underbrace{(y_k-1)}_{y_{k+1}}\right]$$

$$\Rightarrow P2_{k+1}-P2_k=r_y^2\left[x_{k+1}^2+x_{k+1}-x_k^2-x_k\right]+r_x^2\left[1-2y_{k+1}\right]$$

$$\Rightarrow P2_{k+1}-P2_k=r_y^2\left[x_{k+1}^2+x_{k+1}-x_k^2-x_k\right]-2y_{k+1}r_x^2+r_x^2$$

If $P2_k>0$ then $x_{k+1}=x_k$

$$\boxed{P2_{k+1}=P2_k-2y_{k+1}r_x^2+r_x^2}$$

If $P2_k\le 0$ then $x_{k+1}=x_k+1$

$$P2_{k+1}=P2_k+r_y^2\left[\cancel{x_k^2}+1+2x_k-\cancel{x_k^2}+\cancel{x_k}+1-\cancel{x_k}\right]-2y_{k+1}r_x^2+r_x^2$$

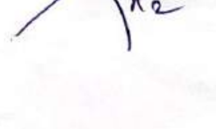
$$=P2_k+r_y^2(2x_k+2)-2y_{k+1}r_x^2+r_x^2$$

$$=P2_k+r_y^2\left[2(x_k+1)\right]-2y_{k+1}r_x^2+r_x^2$$

$$\boxed{P2_{k+1}=P2_k+r_y^2\left[2x_{k+1}\right]-2y_{k+1}r_x^2+r_x^2}$$

Initial decision Parameter

obtained by putting the last point of Region1 in the eq.$^n$ $P2_k=r_y^2\left(x_k+\frac{1}{2}\right)^2+r_x^2(y_k-1)^2-r_x^2r_y^2$




$$\boxed{P2_k=r_y^2\left(x+\frac{1}{2}\right)^2+r_x^2(y-1)^2-r_x^2r_y^2}$$

# Midpoint Ellipse Algorithm.

1. I/P $r_x, r_y$, and ellipse center $(x_c, y_c)$ and obtain the first point on an ellipse centered on origin as

$$(x_0, y_0) = (0, r_y)$$

2. Calculate the initial value of decision parameter in region 1 as

$$P1_0 = r_y^2 - r_x^2 r_y + \frac{1}{4} r_x^2$$

3. At each $x_k$ position in region 1, starting at $k=0$, do the following:

If $P1_k < 0$ then plot next point as $(x_{k+1}, y_k)$ & find

$$P1_{k+1} = P1_k + 2r_y^2 x_{k+1} + r_y^2$$

otherwise, the next point along the circle is $(x_k+1, y_k-1)$ and

$$P1_{k+1} = P1_k + 2r_y^2 x_{k+1} - 2r_x^2 y_{k+1} + r_y^2$$

4. Calculate the initial value of the decision parameter in region 2 using the last point $(x_0, y_0)$ calculated in region 1 as

$$P2_0 = r_y^2\left(x_0 + \frac{1}{2}\right)^2 + r_x^2(y_0 - 1)^2 - r_x^2 r_y^2$$

5. At each $y_k$ position in region 2, starting at $k=0$, do the following

If $P2_k > 0$, then plot next point as $(x_k, y_k - 1)$ & find

$$P2_{k+1} = P2_k - 2r_x^2 y_{k+1} + r_x^2$$

otherwise, plot next point as $(x_k+1, y_k-1)$ & find

$$P2_{k+1} = P2_k + 2r_y^2 x_{k+1} - 2r_x^2 y_{k+1} + r_x^2$$

6. Determine symmetry points in the other three quadrants

7. Move each calculated pixel position $(x,y)$ onto the ellipse path centered on $(x_c, y_c)$ and plot the coordinate values:

$$x = x + x_c ; \quad y = y + y_c$$

8. Repeat the steps of region 1 until $2r_y^2 x \geq 2r_x^2 y$.