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Subject: Applied Mathematics III **SEM: III**

Topic: LAPLACE TRANSFORM

Laplace Transform is an integral transform method which is perticularly useful in solving linear differential equations. Laplace transform techniques are widely used in engineering fields. The Laplace Transforms can be interpreted as a transformation from the time domain where inputs and outputs are functions of time (t) to the frequency domain where inputs and outputs are functions of complex angular frequency (s).

- **Definition: Laplace Transform (L.T.)**

let, $f(t)$ be a given function defined for all $t \geq 0$. The Laplace Transform of $f(t)$ denoted by $L[f(t)]$ is defined as,

$$L[f(t)] = \int_0^\infty e^{-st} \cdot f(t) \cdot dt = \phi(s).$$

- **Problems using definition of Laplace Transform:**

1] Find Laplace Transform of

$$f(t) = \begin{cases} t^2, & 0 < t < 3 \\ 6, & t > 3. \end{cases}$$

Sol^n: W.k.t.

$$L[f(t)] = \int_0^\infty e^{-st} \cdot f(t)\, dt$$

$$= \int_0^3 e^{-st} \cdot f(t)\, dt + \int_3^\infty e^{-st} \cdot f(t) \cdot dt$$

$$= \int_0^3 e^{-st} \cdot t^2 \cdot dt + \int_3^\infty e^{-st} \cdot 6 \cdot dt$$

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$$= \int_0^3 t^2 \cdot e^{-st} \cdot dt + \int_3^\infty 6 \cdot e^{-st} \cdot dt$$

• Note: $\int u \cdot v = u \int v - \int [u' \cdot \int v]$

$$\therefore L[f(t)] = \int_0^3 t^2 \cdot e^{-st} dt + 6 \int_3^\infty e^{-st} \cdot dt$$

$$= \left[t^2 \frac{e^{-st}}{-s} - (2t)\left(\frac{e^{-st}}{s^2}\right) + 2\left(\frac{e^{-st}}{-s^3}\right)\right]_0^3 + 6 \cdot \left[\frac{e^{-st}}{-s}\right]_3^\infty$$

$$= 9\frac{e^{-3s}}{-s} - \frac{6e^{-3s}}{s^2} + \frac{2 \cdot e^{-3s}}{-s^3} - 0 + 0 - 2\left(\frac{1}{-s^3}\right) + 6\left[0 - \frac{e^{-3s}}{-s}\right] \ldots (\because e^{-\infty} = 0)$$

$$= -\frac{9e^{-3s}}{s} - \frac{6e^{-3s}}{s^2} - \frac{2e^{-3s}}{s^3} + \frac{2}{s^3} + \frac{6e^{-3s}}{s}$$

$$= -\frac{3e^{-3s}}{s} - \frac{6e^{-3s}}{s^2} - \frac{2e^{-3s}}{s^3} + \frac{2}{s^3}.$$

• Note: $\int e^{ax} \cos bx \, dx = \frac{e^{ax}}{a^2+b^2}[a \cos bx + b \sin bx]$

$\int e^{ax} \sin bx \, dx = \frac{e^{ax}}{a^2+b^2}[a \sin bx - b \cos bx]$

2] Find the Laplace Transform of

$$f(t) = \cos t, \quad 0 < t < \pi$$
$$= \sin t, \quad t > \pi.$$

Sol^n: $L[f(t)] = \int_0^\infty e^{-st} f(t) \, dt$

$$= \int_0^{\pi} e^{-st} \cos t \, dt + \int_{\pi}^{\infty} e^{-st} \sin t \, dt$$

(from above Note, $a = -s$ & $b = 1$)

$$\therefore L[f(t)] = \left[\frac{e^{-st}}{s^2+1} (-s\cos t + \sin t) \right]_0^{\pi} + \left[\frac{e^{-st}}{s^2+1} (-s\sin t - \cos t) \right]_{\pi}^{\infty}$$

$$= \left[\frac{e^{-\pi s}}{s^2+1} (-s\cos\pi + \sin\pi) - \frac{1}{s^2+1} (-s(1) + 0) \right]$$

$$+ \left[0 - \frac{e^{-\pi s}}{s^2+1} (-s \cdot \sin\pi - \cos\pi \right]$$

$$= \left[\frac{e^{-\pi s}}{s^2+1} (-s(-1) + 0) - \frac{1}{s^2+1} \cdot (-s) - \frac{e^{-\pi s}}{s^2+1} (0 - (-1)) \right]$$

$$= \frac{e^{-\pi s}}{s^2+1} \cdot s + \frac{s}{s^2+1} - \frac{e^{-\pi s}}{s^2+1}$$

Ex: Find the Laplace Transform of

1) $f(t) = (t-1)^2$, $0 < t < 1$
$\quad\;\; = 3$, $t > 1$

2) $f(t) = \sin 2t$, $0 < t < \pi$
$\quad\;\; = 0$, $t > \pi$

- Linearity Property of Laplace Transform

$$L[\alpha \cdot f(t) + \beta \cdot g(t)] = \alpha \cdot L[f(t)] + \beta \cdot L[g(t)]$$

..... (Where, $\alpha, \beta \in \mathbb{R}$)

- Laplace Transform of standard functions.

1) $L[e^{at}] = \frac{1}{s-a}$

2) $L[e^{-at}] = \frac{1}{s+a}$.

3) $L[1] = \frac{1}{s}$

4) $L[\sin at] = \frac{a}{s^2+a^2}$.

5) $L[\cos at] = \frac{s}{s^2+a^2}$.

6) $L[\sinh at] = \frac{a}{s^2-a^2}$.

7) $L[\cosh at] = \frac{s}{s^2-a^2}$.

8) $L[t^n] = \frac{\Gamma n+1}{s^{n+1}}$

Note: $\Gamma n = (n-1)\,\Gamma n-1$.
$\Gamma n = (n-1)!$ if n is natural number.
$\Gamma \frac{1}{2} = \sqrt{\pi}$.

9) $L[\operatorname{erf}(\sqrt{t})] = \frac{1}{s\sqrt{s+1}}$ ..... (We will prove this later on)

(Note: The above formulae can be proved by using definition of Laplace Transform).

- Note:

1) $L[c^{at}] = L[e^{\log_e c^{at}}] = L[e^{at\log c}] = \frac{1}{s-a\log c}$.

2) $L[c^{-at}] = \frac{1}{s+a\log c}$ (using above method)

3) $L[K] = K \cdot L[1] = \frac{K}{s}$. ..... (Where, K is constant.

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SEM: III

Subject: Applied Mathematics III

Examples :

1) Find $L[\cos 2t + 3\cdot \sin t + 4e^{-2t} + 2]$

Sol^n: consider,

$$L[\cos 2t + 3\sin t + 4\cdot e^{-2t} + 2] =$$

$$= L[\cos 2t] + 3\cdot L[\sin t] + 4\cdot L[e^{-2t}] + 2\,L[1]$$

$$= \frac{s}{s^2+4} + \frac{3}{s^2+1} + \frac{4}{s+2} + \frac{2}{s}.$$

2) find $L[2\cosh 2t + 3e^{4t} + 4t^3 + 2^{3t}]$

Sol^n: consider, $L[2\cosh 2t + 3e^{4t} + 4t^3 + 2^{3t}]$

$$= 2\cdot L[\cosh 2t] + 3\cdot L[e^{4t}] + 4\cdot L[t^3] + L[2^{3t}]$$

$$= \frac{2\cdot s}{s^2-4} - \frac{3}{s-4} + \frac{4\cdot 3!}{s^4} + L[e^{\log_e 2^{3t}}]$$

$$= \frac{2s}{s^2-4} - \frac{3}{s-4} + \frac{24}{s^4} + L[e^{3t\log 2}]$$

$$= \frac{2s}{s^2-4} - \frac{3}{s-4} + \frac{24}{s^4} + L[e^{(3\log 2)t}]$$

$$= \frac{2s}{s^2-4} - \frac{3}{s-4} + \frac{24}{s^4} + \frac{1}{s-3\log 2}.$$

3) Find $L[t^{3/2}]$.

Sol^n: consider, $L[t^{3/2}] = \dfrac{\Gamma\left(\frac{3}{2}+1\right)}{s^{3/2+1}}$

$$= \frac{\Gamma(5/2)}{s^{5/2}}$$

but, $\Gamma n = (n-1)\,\Gamma(n-1)$

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$$\therefore L[t^{3/2}] = \frac{(5/2 - 1)\,\Gamma(5/2 - 1)}{s^{3/2}}$$

$$= \frac{3/2\,\Gamma 3/2}{s^{3/2}}$$

$$= \frac{3/2 \cdot (3/2 - 1)\,\Gamma(3/2 - 1)}{s^{3/2}}$$

$$= \frac{3/2 \cdot 1/2 \cdot \Gamma 1/2}{s^{3/2}}$$

$$= \frac{3/4 \cdot \sqrt{\pi}}{s^{3/2}}$$

$$L[t^{3/2}] = \frac{3\sqrt{\pi}}{4 \cdot s^{3/2}}$$

4) $L\left[\frac{1}{\sqrt{\pi t}}\right]$

<u>Soln</u>: $L\left[\frac{1}{\sqrt{\pi t}}\right] = L\left[\frac{1}{\sqrt{\pi}\cdot\sqrt{t}}\right] = \frac{1}{\sqrt{\pi}} L\left[t^{-1/2}\right] = \frac{1}{\sqrt{\pi}} \frac{\Gamma\left(-\frac{1}{2}+1\right)}{s^{-\frac{1}{2}+1}}$

$$= \frac{1}{\sqrt{\pi}} \cdot \frac{\Gamma 1/2}{s^{1/2}} = \frac{1}{\cancel{\sqrt{\pi}}} \cdot \frac{\cancel{\sqrt{\pi}}}{s^{1/2}} = \frac{1}{\sqrt{s}}$$

$$\therefore L\left[\frac{1}{\sqrt{\pi t}}\right] = \frac{1}{\sqrt{s}}.$$

5) $L\left[\sqrt{1+\sin t}\right]$

<u>Soln</u>: $L\left[\sqrt{1+\sin t}\right]$

$$= L\left[\sqrt{\sin^2 t/2 + \cos^2 t/2 + 2\sin t/2 \cdot \cos t/2}\right]$$

$$= L\left[\sqrt{(\sin t/2 + \cos t/2)^2}\right]$$

$$= L\left[\sin t/2 + \cos t/2\right]$$

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Subject: Applied Mathematics III

SEM: III

$$= \frac{1/2}{s^2+(\frac{1}{2})^2} + \frac{s}{s^2+(\frac{1}{2})^2}$$

$$= \frac{1}{2(s^2+\frac{1}{4})} + \frac{s}{s^2+(\frac{1}{4})}$$

6) $L[\cos t \cdot \cos 2t \cdot \cos 3t]$

Sol^n: consider,

$$L[\cos t \cdot \cos 2t \cdot \cos 3t]$$

$$= L[\cos t \cdot \tfrac{1}{2}(\cos(2t+3t) + \cos(2t-3t))]$$

$$= L[\tfrac{1}{2} \cdot (\cos t \cdot \cos 5t + \cos t \cdot \cos t)]$$

$$= \frac{1}{2} L[\cos t \cdot \cos 5t + \cos t \cdot \cos t]$$

$$= \frac{1}{2} L\left[\frac{\cos 6t + \cos(-4t)}{2} + \frac{\cos 2t + 1}{2}\right]$$

$$= \frac{1}{4} L[\cos 6t + \cos 4t + \cos 2t + 1]$$

$$= \frac{1}{4} \cdot \left[\frac{s}{s^2+36} + \frac{s}{s^2+16} + \frac{s}{s^2+4} + \frac{1}{s}\right]$$

7) $L[\sin t \cdot \cos 4t \cdot \cos 3t]$

Sol^n: $L[\sin t \cdot \cos 4t \cdot \cos 3t]$

$$= L\left[\sin t \cdot \left(\frac{\cos 7t + \cos t}{2}\right)\right]$$

$$= \frac{1}{2} L[\sin t \cdot \cos 7t + \sin t \cdot \cos t]$$

$$= \frac{1}{2} \cdot L\left[\frac{\sin 8t + \sin(-6t)}{2} + \frac{2\sin t \cdot \cos t}{2}\right]$$

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$$= \frac{1}{4} L\left[\sin 8t - \sin 6t + \sin 2t\right]$$

$$= \frac{1}{4}\left[\frac{8}{s^2+64} - \frac{6}{s^2+36} + \frac{2}{s^2+4}\right]$$

- Note : 1) $\cos\theta = \frac{e^{i\theta}+e^{-i\theta}}{2}$; $\sin\theta = \frac{e^{i\theta}-e^{-i\theta}}{2i}$

2) $\cosh\theta = \frac{e^{\theta}+e^{-\theta}}{2}$; $\sinh\theta = \frac{e^{\theta}-e^{-\theta}}{2}$

3) We have, Binomial expansion as,

$$(a \pm b)^n = a^n \pm {}^nC_1 a^{n-1} b + {}^nC_2 a^{n-2} b^2 \pm \ldots \ldots b^n$$

One can also find ${}^nC_1, {}^nC_2, \ldots$ using the Pascal's triangle, given below,

1	$\rightarrow (a+b)^0 = 1$
1 1	$\rightarrow (a+b)$
1 2 1	$\rightarrow (a+b)^2 = a^2+2ab+b^2$
1 3 3 1	$\rightarrow (a+b)^3 = a^3+3a^2b+3ab^2+b^3$
1 4 6 4 1	$\rightarrow (a+b)^4$
1 5 10 10 5 1	$\rightarrow (a+b)^5$

- We'll use above formulae when we need to find the Laplace transform of powers of cosine, sine and also hyperbolic cosine & sine.

8] $L[\cosh^4 t]$

Soln: consider,

$$L[\cosh^4 t] = L\left[(\cosh t)^4\right]$$

$$= L\left[\left(\frac{e^t+e^{-t}}{2}\right)^4\right]$$

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$$= \frac{1}{2^4} L\left[(e^t + e^{-t})^4\right]$$

$$= \frac{1}{16} L\left[(e^t)^4 + 4(e^t)^3 e^{-t} + 6(e^t)^2(e^{-t})^2 + 4e^t(e^{-t})^3 + (e^{-t})^4\right]$$

$$= \frac{1}{16} L\left[e^{4t} + 4e^{2t} + 6 + 4e^{-2t} + e^{-4t}\right]$$

$$= \frac{1}{16}\left[\frac{1}{s-4} + \frac{4}{s-2} + \frac{6}{s} + \frac{4}{s+2} + \frac{1}{s+4}\right]$$

9) $L\left[\sinh^5 2t\right]$

Sol^n: consider,

$$L\left[\sinh^5 2t\right]$$

$$= L\left[(\sinh 2t)^5\right]$$

$$= L\left[\left(\frac{e^{2t} - e^{-2t}}{2}\right)^5\right]$$

$$= \frac{1}{32} L\left[(e^{2t} - e^{-2t})^5\right]$$

$$= \frac{1}{32} L\left[(e^{2t})^5 - 5\cdot(e^{2t})^4(e^{-2t}) + 10(e^{2t})^3\cdot(e^{-2t})^2 - 10(e^{2t})^2(e^{-2t})^3 + 5(e^{2t})(e^{-2t})^4 - (e^{-2t})^5\right]$$

$$= \frac{1}{32} L\left[e^{10t} - 5e^{6t} + 10e^{2t} - 10e^{-2t} + 5e^{-6t} - e^{-10t}\right]$$

$$= \frac{1}{32}\left[\frac{1}{s-10} - \frac{5}{s-6} + \frac{10}{s-2} - \frac{10}{s+2} + \frac{5}{s+6} - \frac{1}{s+10}\right]$$

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Subject: Applied Mathematics III SEM: III

10] $L[\sin^3 t]$

Sol^n: consider,

$L[\sin^3 t]$

$= L[(\sin t)^3]$

$= \frac{1}{(2i)^3} L\left[(e^{it} - e^{-it})^3\right]$

$= \frac{1}{8i^3} L\left[(e^{it})^3 - 3(e^{it})^2(e^{-it}) + 3(e^{it})(e^{-it})^2 - (e^{-it})^3\right]$

$= \frac{-1}{8i} L\left[e^{3it} - 3e^{it} + 3e^{-it} - e^{-3it}\right]$

$= \frac{-1}{8i}\left[\frac{1}{s-3i} - \frac{3}{s-i} + \frac{3}{s+i} - \frac{1}{s+3i}\right]$

#] Examples for practice

1) $L[\cos^4 2t]$ 2] $L[\cosh^3 3t]$ 3] $L[\sinh^3 5t]$

Note : 1] $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$

2] $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$

3] $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots$

11] Find $L[\sin(\sqrt{t})]$

Sol^n: As $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$

$\sin\sqrt{t} = \sqrt{t} - \frac{(\sqrt{t})^3}{3!} + \frac{(\sqrt{t})^5}{5!} - \cdots$

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Subject: Applied Mathematics III **SEM: III**

$$\therefore L[\sin\sqrt{t}] = L\left[t^{1/2} - \frac{t^{3/2}}{3!} + \frac{t^{5/2}}{5!} - \cdots\right]$$

$$= L[t^{1/2}] - \frac{1}{3!}L[t^{3/2}] + \frac{1}{5!}L[t^{5/2}] - \cdots$$

$$= \frac{\Gamma(\frac{1}{2}+1)}{s^{\frac{1}{2}+1}} - \frac{1}{3!}\frac{\Gamma(\frac{3}{2}+1)}{s^{\frac{3}{2}+1}} + \frac{1}{5!}\frac{\Gamma(\frac{5}{2}+1)}{s^{\frac{5}{2}+1}} - \cdots$$

$$= \frac{\Gamma(3/2)}{s^{3/2}} - \frac{\Gamma(5/2)}{3!\cdot s^{5/2}} + \frac{\Gamma(7/2)}{5!\, s^{7/2}} - \cdots$$

As $\Gamma(n) = (n-1)\,\Gamma(n-1)$

$$\therefore \Gamma(3/2) = \frac{1}{2}\Gamma(1/2) = \frac{1}{2}\sqrt{\pi} = \frac{\sqrt{\pi}}{2}.$$

$$\Gamma(5/2) = \frac{3}{2}\Gamma(3/2) = \frac{3}{2}\cdot\frac{1}{2}\Gamma(\tfrac{1}{2}) = \frac{3\sqrt{\pi}}{4}$$

$$\Gamma(7/2) = \frac{5}{2}\Gamma(\tfrac{5}{2}) = \frac{5}{2}\cdot\frac{3}{2}\Gamma(\tfrac{3}{2}) = \frac{5}{2}\cdot\frac{3}{2}\cdot\frac{1}{2}\cdot\Gamma(\tfrac{1}{2}) = \frac{15\sqrt{\pi}}{8}$$

& so on.

$$\therefore L[\sin\sqrt{t}] = \frac{\sqrt{\pi}/2}{s^{3/2}} - \frac{1}{3!}\cdot\frac{3\sqrt{\pi}/4}{s^{5/2}} + \frac{1}{5!}\cdot\frac{15\sqrt{\pi}/8}{s^{7/2}} - \cdots$$

$$= \frac{\sqrt{\pi}}{2}\left[\frac{1}{s^{3/2}} - \frac{1}{3!}\cdot\frac{3}{(2)\,s^{5/2}} + \frac{1}{5!}\cdot\frac{15}{(4)\,s^{7/2}} - \cdots\right]$$

$$= \frac{\sqrt{\pi}}{2\cdot s^{3/2}}\left[1 - \frac{\cancel{3}}{\cancel{3}\times 2\times 2\,s} + \frac{\cancel{15}}{\cancel{5}\times 4\times\cancel{3}\times 2\times 4\,s^2} - \cdots\right]$$

$$= \frac{\sqrt{\pi}}{2\cdot s^{3/2}}\left[1 - \frac{1}{4s} + \frac{1}{2!}\cdot\left(\frac{1}{4s}\right)^2 - \cdots\right]$$

$$= \frac{\sqrt{\pi}}{2s^{3/2}}\cdot e^{-1/4s} \quad \ldots\ldots \left(\text{from } 1\text{] (here } x = \frac{-1}{4s}\right)$$

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Subject: Applied Mathematics III

SEM: III

12] Prove that $L\left[\frac{\cos\sqrt{t}}{\sqrt{t}}\right] = \frac{\sqrt{\pi}}{s^{1/2}} \cdot e^{-1/4s}$

Sol^n: As $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots$

$$\cos\sqrt{t} = 1 - \frac{(\sqrt{t})^2}{2!} + \frac{(\sqrt{t})^4}{4!} + \dots$$

$$\cos\sqrt{t} = 1 - \frac{t}{2!} + \frac{t^2}{4!} - \dots$$

$$\frac{\cos\sqrt{t}}{\sqrt{t}} = \frac{1}{\sqrt{t}} - \frac{t}{2!\cdot\sqrt{t}} + \frac{t^2}{4!\cdot\sqrt{t}} - \dots$$

$$= t^{-1/2} - \frac{t^{1/2}}{2!} + \frac{t^{3/2}}{4!} - \dots$$

$$L\left[\frac{\cos\sqrt{t}}{\sqrt{t}}\right] = L\left[t^{-1/2} - \frac{t^{1/2}}{2!} + \frac{t^{3/2}}{4!} - \dots\right]$$

$$= \frac{\Gamma 1/2}{s^{1/2}} - \frac{1}{2!} L\left(t^{1/2}\right) + \frac{1}{4!} L\left(t^{3/2}\right)$$

$$= \frac{\Gamma 1/2}{s^{1/2}} - \frac{1}{2!} \cdot \frac{\Gamma 3/2}{s^{3/2}} + \frac{1}{4!} \cdot \frac{\Gamma 5/2}{s^{5/2}} - \dots$$

$$= \frac{\sqrt{\pi}}{s^{1/2}} - \frac{1}{2!} \frac{\sqrt{\pi}/2}{s^{3/2}} + \frac{1}{4!} \cdot \frac{3\sqrt{\pi}/4}{s^{5/2}}$$

.... (as we found in earlier problem)

$$= \frac{\sqrt{\pi}}{s^{1/2}} - \frac{\sqrt{\pi}}{2!} \cdot \frac{1}{(2)s^{3/2}} + \frac{3\cdot\sqrt{\pi}}{4!\cdot(4)\,s^{5/2}}$$

$$= \frac{\sqrt{\pi}}{s^{1/2}} \cdot \left[1 - \frac{1}{4s} + \frac{1}{2!} \cdot \frac{1}{(4s)^2}\right]$$

$$= \sqrt{\frac{\pi}{s}} \cdot e^{-1/4s} \quad \dots \text{(from 1), Here, } x = \frac{-1}{4s}$$

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