Normal Distribution – More Examples and M.G.F

Examples: (Continued)

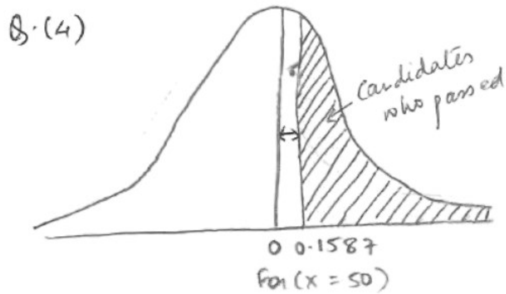
4. In an exam taken by 800 candidates the average and s.d of marks obtained (normally distributed) are 40% and 10%. (i) Find approximately the number of candidates who will pass if 50% is kept as minimum (ii) what should be the minimum score if 350 candidates are to be declared as passed? (iii) How many candidates have scored marks above 60%?

Solution: Let the r.v X denote the marks obtained by the candidates
Given: $X \sim N(\mu = 40, \sigma = 10)$ (*we can take* $X \sim N(\mu = 0.4, \sigma = 0.1)$ *also*)
(Check Figures 3.14.4)

(i) The number of candidates who will pass if 50% is kept as minimum is the number of candidates who will get marks more than 50. Now

$$\begin{aligned} P(X \geq 50) &= P\left(\frac{X-\mu}{\sigma} \geq \frac{50-\mu}{\sigma}\right) \\ &= P\left(Z \geq \frac{50-40}{10}\right) \\ &= P(Z \geq 1) \\ &= 0.5 - 0.3413 \ (\textit{from the tables}) \\ &= 0.1587 \end{aligned}$$

Hence no. of students who will pass $= 800 * 0.1587$
$= 126.96 \approx 127$



(ii) Let $X_{\min}$ be the minimum score such that 350 candidates are to be declared as passed

Then $P(a\,candidate\,passes) = \frac{350}{800} = 0.4375$

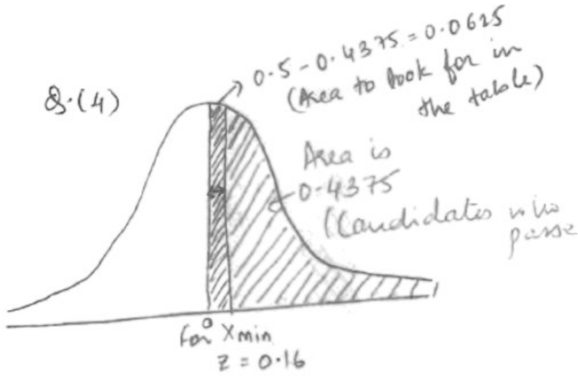
$$\Rightarrow P(X \geq X_{\min}) = 0.4375 \Rightarrow P\left(Z \geq \frac{X_{\min}-\mu}{\sigma}\right) = 0.4375$$

$$\Rightarrow P\left(0 \leq Z \leq \frac{X_{\min}-\mu}{\sigma}\right) = 0.5 - 0.4375$$

Let $\frac{X_{\min}-\mu}{\sigma} = z_m$ Then we have

$$P(0 \leq Z \leq z_m) = 0.5 - 0.4375 = 0.0625$$

From the tables we get $z_m = 0.16$



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$$\Rightarrow \frac{X_{\min} - \mu}{\sigma} = 0.16 \Rightarrow X_{\min} = 41.6$$

(iii) To find the number of students who have scored marks above 60%, we first find the probability that a student's score lies above 60%

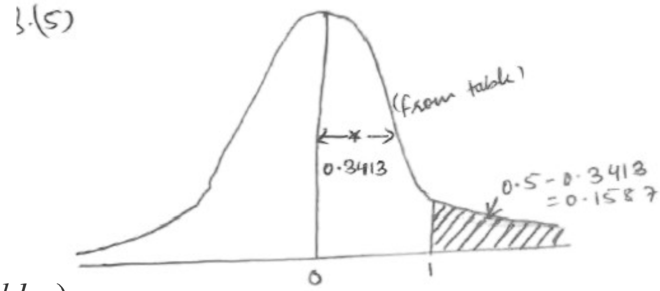
$$\therefore P(X \geq 60) = P(\frac{X-\mu}{\sigma} \geq \frac{60-\mu}{\sigma}) = P(Z \geq \frac{60-40}{10})$$
$$= P(Z \geq 2)$$
$$= 0.5 - 0.4772 \ (\textit{from the tables})$$
$$= 0.0228$$

$$\text{Hence no. of students whose score is above } 60\% = 800 * 0.0228$$
$$= 18.24 \approx 18$$

5. The marks obtained by a number of students in Physics are normally distributed with mean 65 and s.d 5. If 3 students are taken at random from this set of students, what is the probability that exactly 2 of them will have marks over 70?

Solution: We will do Binomial approximation to the normal distribution.
Let the r.v X denote the marks obtained by the students
Given: $X \sim N(\mu = 65, \sigma = 5)$

(Check Figures 3.14.5)



$$\therefore P(X > 70) = P(\frac{X-\mu}{\sigma} > \frac{70-\mu}{\sigma})$$
$$= P(Z > \frac{70-65}{5})$$
$$= P(Z \geq 1)$$
$$= 0.5 - 0.3413 \ (\textit{from the tables})$$
$$= 0.1587$$

We will do Binomial approximation to the normal distribution.

Then $X \sim B(n = 3, p = 0.1587)$

$$\therefore P(X = 2) = 3C_2 (0.1587)^2 (1 - 0.1587) = 0.06357$$

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