Semester : VIII Subject : AIFB Academic Year: 2024-25

SAMPLE SKEWNESS AND KURTOSIS :-

Skewness and kurtosis are statistical measures used to describe the distribution of asset returns or portfolio returns.

Skewness :

It refers to the asymmetry of the distribution of returns. It tells us whether the returns are skewed more to the left (negative skew) or to the right (positive skew) of the mean.

Positive skew :- If the skewness is greater than 0, the distribution is positively skewed, meaning there are more frequent smaller negative returns and fewer extreme positive returns.

Negative skew :- If the skewness value is less than 0 the distribution is negatively skewed, meaning there are more frequent smaller positive returns and fewer extreme negative returns.

No skew (Symmetrical): A skewness value of 0 suggests a symmetric distribution of returns around the mean.

Formula for ~~Kurtosis~~ skewness ν_2 is:

$$\boxed{\nu_2 = \frac{N}{(N-1)(N-2)} \cdot \sum_{i=1}^{N} \left(\frac{X_i - \bar{X}}{\sigma}\right)^3}$$

N = No. of datapoints
X_i = Each individual datapoint (return)
$\bar{X}$ = Mean return
σ = Standard Deviation of returns.

KURTOSIS:

Kurtosis measures the "tailedness" of the distribution of returns. It looks at the frequency of extreme return (outliers) compared to normal distribution.

Types of Kurtosis:

(1) Leptokurtic (High Kurtosis): When the kurtosis is greater than 3, the distribution has fatter tails and a higher peak than a normal distribution. This suggest more outliers indicating higher risk.

(2) Playkurtic (Low kurtosis): When the kurtosis is less than 3, the distribution ~~has~~ thinner tails and a flatter peak than a normal distribution. This suggest fewer extreme returns, indicating lower risk.

(3) Mesokurtic (Normal kurtosis): A kurtosis of exactly 3 suggests that the distribution is normal

Formula for kurtosis:-

$$\boxed{\gamma_2 = \frac{N(N+1)}{(N-1)(N-2)(N-3)} \cdot \sum_{i=1}^{N} \left(\frac{X_i - \bar{X}}{\sigma}\right)^4 - \frac{3(N-1)^2}{(N-2)(N-3)}}$$

where:

N = No. of data points

X_i = Each individual data point (return)

$\bar{X}$ = Mean return

σ = Standard Deviation of returns.

~~Example~~ **Application of skewness and kurtosis in Finance:**

(1) Risk Management:

Skewness and kurtosis are useful for identifying the tail risk of an asset. Investors can use these measures to understand the probability of extreme outcomes.

(2) Portfolio Construction:

Investors who are risk-averse might prefer assets with low kurtosis (fewer extreme outliers) and less negative skewness (fewer extreme losses).

Both skewness and kurtosis are important for understanding risk in financial markets, as they provide insights into the likelihood of extreme events and help investors manage tail risk.

Example:

Suppose we have the following monthly returns for a stock over 7 months:

Month 1: 5%, Month 2: 2%, Month 3: -1%, Month 4: 8%, Month 5: -4%, Month 6: 3%, Month 7: 10%

Calculate the skewness and kurtosis of the stock's returns to assess the shape of the return distribution.

Solution:

Step 1: Calculate the Mean (Average) Return:

$$\bar{X} = \frac{5+2+(-1)+8+(-4)+3+10}{7} = \frac{23}{7} = \boxed{3.29\%}$$

Step 2: Calculate the Skewness:

$$\gamma_1 = \frac{N}{(N-1)(N-2)} \cdot \sum_{i=1}^{N} \left(\frac{X_i - \bar{X}}{\sigma}\right)^3$$

Calculate standard deviation:

$$SD = \sqrt{\frac{\sum (X_i - \bar{X})^2}{N-1}}$$

$$= \sqrt{\frac{(5-3.29)^2+(2-3.29)^2+(-1-3.29)^2+(8-3.29)^2+(-4-3.29)^2+(3-3.29)^2+(10-3.29)^2}{6}}$$

$$= \sqrt{\frac{143.5047}{6}} = \boxed{4.89\%}$$

Calculate skewness:

$$\left(\frac{5-3.29}{4.89}\right)^3 = (0.349)^3 = 0.042$$

$$\left(\frac{2-3.29}{4.89}\right)^3 = (-0.264)^3 = -0.018$$

$$\left(\frac{-1-3.29}{4.89}\right)^3 = (-0.870)^3 = -0.658$$

$$\left(\frac{8-3.29}{4.89}\right)^3 = (0.968)^3 = 0.912$$

$$\left(\frac{-4-3.29}{4.89}\right)^3 = (-1.469)^3 = -3.158$$

$$\left(\frac{3-3.29}{4.89}\right)^3 = (-0.059)^3 = -0.0002$$

$$\left(\frac{10-3.29}{4.89}\right)^3 = (1.373)^3 = 2.599$$

Sum of cubes $= 0.042 - 0.018 - 0.658 + 0.912 - 3.158 - 0.0002 + 2.599 = \boxed{-0.2802}$

Skewness Formula:

$$\gamma_1 = \frac{7}{(7-1)(7-2)} \cdot (-0.2802) = \frac{7}{30} \cdot (-0.2802) = \boxed{-0.0652}$$

The skewness of -0.0652 is negative, suggesting that the distribution of returns has a slightly longer tail on the left side, indicating a small tendency towards larger negative returns compared to positive ones.

Step3: Calculate the Kurtosis

$$\gamma_2 = \frac{N(N+1)}{(N-1)(N-2)(N-3)} \cdot \sum_{i=1}^{N} \left(\frac{x_i - \bar{x}}{\sigma}\right)^4 - \frac{3(N-1)^2}{(N-2)(N-3)}$$

Calculate the fourth powers of the Deviation:

$$\left(\frac{5 - 3.29}{4.89}\right)^4 = (0.349)^4 = 0.016$$

$$\left(\frac{2 - 3.29}{4.89}\right)^4 = (-0.264)^4 = 0.005$$

$$\left(\frac{-1 - 3.29}{4.89}\right)^4 = (-0.870)^4 = 0.592$$

$$\left(\frac{8 - 3.29}{4.89}\right)^4 = (0.968)^4 = 0.877$$

$$\left(\frac{-4 - 3.29}{4.89}\right)^4 = (-1.469)^4 = 5.128$$

$$\left(\frac{3 - 3.29}{4.89}\right)^4 = (-0.059)^4 = 0.0001$$

$$\left(\frac{10 - 3.29}{4.89}\right)^4 = (1.373)^4 = 3.574$$

Sum of fourth powers $= 0.016 + 0.005 + 0.592 + 0.877 + 5.128 + 0.0001 + 3.574 = 10.1921$

$$\gamma_2 = \frac{7(7+1)}{(7-1)(7-2)(7-3)} \times 10.1921 - \frac{3(7-1)^2}{(7-2)(7-3)}$$

$$\gamma_2 = \frac{56}{120} \times 10.1921 - \frac{3(6)^2}{20} = 4.7556 - 5.4 = \boxed{-0.6444}$$

The kurtosis value of -0.6444 indicates a playkurtic distribution (kurtosis less than 3), meaning the distribution has lighter tails and fewer ~~k~~ extreme outliers compared to a normal distribution.

COVARIANCE AND CORRELATION:

Covariance: Covariance measures the degree to which two variables (eg, asset returns) change together. It indicates whether an assets returns tends to move in the same direction as another assets return tends to move in the same direction as another assets return (positive covariance) or opposite directions (negative covariance).

Formula:

$$\boxed{cov(X,Y) = \frac{1}{N-1}\sum_{i=1}^{N}(X_i-\bar{X})(Y_i-\bar{Y})}$$

where,

* X_i and Y_i are the returns of the two assets for the i^{th} period.
* $\bar{X}$ and $\bar{Y}$ are the means (averages) of the returns of the two assets.
* N is the number of data points (periods).

Positive Covariance: The assets tends to move in the same direction. If one assets goes up, the another tends to go up as well.