Poisson Distribution

1) Between the hours of 2 & 4 p.m, the average number of phone calls per minute coming into the switch board of a company is 2.5. Find the prob. that during a particular minute there will be

(i) no phone call at all

(ii) 4 or less calls.

→ Let x : number of phone calls per minute

Given $\lambda = 2.5$, $x \sim P(\lambda = 2.5)$

i) ∴ P (no phone calls) = $P(x=0) = \frac{e^{-\lambda}\lambda^{x}}{x!} = \frac{e^{-2.5}(2.5)^{0}}{0!}$

$\therefore \; \underline{P(x=0) = 0.0821}$

(ii) $P(x \le 4) = P(x=0) + P(x=1) + P(x=2) + P(x=3) + P(x=4)$

$= \frac{e^{-2.5}(2.5)^{0}}{0!} + \frac{e^{-2.5}(2.5)^{1}}{1!} + \frac{e^{-2.5}(2.5)^{2}}{2!} + \frac{e^{-2.5}(2.5)^{3}}{3!} + \frac{e^{-2.5}(2.5)^{4}}{4!}$

$= 0.0821 + 0.2052 + 0.2566 + 0.2138 + 0.1336$

$\underline{P(x \le 4) = 0.8913}$

2) Using Poisson Distribution, find the probability that the ace of spades will be drawn from a pack of well-shuffled cards atleast once in 104 consecutive trials.

x : no. of spades ace drawn in 104 trails

→ Let p : prob of the ace of spades $= \frac{1}{52}$

& $n = 104$

$\therefore$ mean $= \lambda = np = 104\left(\frac{1}{52}\right) = 2$

$X \sim P(\lambda = 2)$

$\therefore P(X=x) = \frac{e^{-\lambda}\lambda^{x}}{x!} = \frac{e^{-2}2^{x}}{x!}$

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$\therefore P(X \geq 1) = 1 - P(X < 1)$

$= 1 - P(X=0)$

$= 1 - \frac{e^{-2} \cdot 2^0}{0!} = 1 - \frac{1}{e^2} = 1 - 0.135$

$\therefore \boxed{P(X \geq 1) = 0.865}$

3) Suppose that a book of 600 pages contains 40 printing mistakes. Assume that these errors are randomly distributed throughout the book. what is the probability that 10 pages selected at random will be free of errors?

$\rightarrow$ Let X : number of errors per page.

$\therefore p$ = prob of errors per page $= \frac{40}{600} = \frac{1}{15}$

& $n = 10$

$\therefore$ mean $= \lambda = np = 10\left(\frac{1}{15}\right) = \frac{2}{3}$

Let $X \sim P(\lambda = \frac{2}{3})$

$\therefore$ Required prob. $= P(X=0)$ = P(10 pages free of errors

$= \frac{e^{-\lambda} \lambda^x}{x!} = \frac{e^{-2/3} (2/3)^0}{0!} = e^{-2/3}$

$\therefore \boxed{P(X=0) = 0.51}$

4) A manufacturer knowns that the condensers he
HW makes contain of an average 1% of defectives. He packs them in boxes of 100. what is the probability that a box picked at random will contain 4 or more faulty condensers?

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5) If the mean of the poisson distribution is 4. Find $P(\lambda - 2\sigma < X < \lambda + 2\sigma)$

→ Let $X \sim P(\lambda)$ & $P(X=x) = \dfrac{e^{-\lambda}\lambda^{x}}{x!}$, $x = 0, 1, 2, \ldots$

Here Mean = variance = 4

$\therefore \lambda = 4$ & $\sigma = \sqrt{4} = 2$

$\therefore P(\lambda - 2\sigma < X < \lambda + 2\sigma) = P(0 < X < 8)$

$= P(X=1) + P(X=2) + \cdots + P(X=7)$

$= e^{-4}\left[\dfrac{4}{1!} + \dfrac{4^2}{2!} + \cdots + \dfrac{4^7}{7!}\right]$

$= 0.93$

6) Using Poisson distribution find the approximate value of $^{300}C_2\,(0.02)^2\,(0.98)^{298} + {}^{300}C_3\,(0.02)^3\,(0.98)^{297}$

→ Compare it with probability of binomial distribution.

$P(X=x) = {}^{n}C_{x}\, p^{x}\, q^{n-x}$

$\therefore n = 300$, $p = 0.02$, $q = 0.98$, $x = 2$ & $x = 3$

binomial distn is related to Poisson distn $\therefore \lambda = np = 6$

$\therefore$ Poisson distribution is,

$P(X=2) + P(X=3) = \dfrac{e^{-6}\,6^2}{2!} + \dfrac{e^{-6}\,.\,6^3}{3!}$

$= 0.0446 + 0.0892 = 0.1338$

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