Some examples

1. Eight unbiased coins are tossed 256 times and the number of heads observed in the throws is shown below:

No. of heads	0	1	2	3	4	5	6	7	8
Frequency	2	6	4	24	63	50	36	10	1

Fit a binomial distribution to the above data and find the expected frequencies.

Solution: Let the random variable X denote the number of heads when 8 unbiased coins are tossed 256 times. Suppose X follows a binomial distribution with parameters $n=8$ and $p=P(head\ appears\ in\ a\ single\ coin)=0.5$ (Since the coins are unbiased) i.e

$X \sim B(n=8, p=\frac{1}{2})$ We shall calculate the probabilities

$P(X=x)=8C_x p^x q^{8-x}$, $x=0,1,2,\ldots,8$ with $p=q=1/2$ Then we shall calculate the expected frequencies $f_x = N*P(X=x) = 256*P(X=x)$, $x=0,1,2,\ldots,8$

Table for expected frequencies

No.of Heads (x)	$P(X=x)=p_x=8C_x\left(\frac{1}{2}\right)^x\left(\frac{1}{2}\right)^{8-x}$	Expected frequency $=N*p_x=256*p_x$
0	$\frac{1}{256}$	$256(\frac{1}{256})=1$
1	$\frac{8}{256}$	$256(\frac{8}{256})=8$
2	$\frac{28}{256}$	$256(\frac{28}{256})=28$
3	$\frac{56}{256}$	$256(\frac{56}{256})=56$
4	$\frac{70}{256}$	$256(\frac{70}{256})=70$
5	$\frac{56}{256}$	$256(\frac{56}{256})=56$
6	$\frac{28}{256}$	$256(\frac{28}{256})=28$
7	$\frac{8}{256}$	$256(\frac{8}{256})=8$
8	$\frac{1}{256}$	$256(\frac{1}{256})=1$
	Total	256

2. Five dice are thrown together 96 times and the number of times 4, 5 Or 6 was observed is given below:

No. of times 4,5 or 6 was obtained	0	1	2	3	4	5
Frequency	1	10	24	35	18	8

Fit a binomial distribution to the above data if (i) dice are unbiased and (ii) the nature of the dice is not known.

Solution: Let the random variable X denote the number of times 4, 5 Or 6 was observed.

Case 1: Suppose dice are unbiased

Let X follow a binomial distribution with parameters $n=5$ (Since 5 coins are thrown) and

$p = P(a\,4\,or\ 5\ or\ 6\ is\ observed) = \frac{3}{6} = \frac{1}{2}$ (Since the dice are unbiased) i.e

$X \simeq B(n=5, p=\frac{1}{2})$

We shall calculate the probabilities $P(X=x) = 5C_x p^x q^{5-x}$, $x = 0,1,2,...,5$

with $p = q = \frac{1}{2}$

Then we shall calculate the expected frequencies

$f_x = N * P(X=x) = 96 * P(X=x),\ x = 0,1,2,...,5$

Table for expected frequencies

No. of times 4 or 5 Or 6 is observed *(x)*	$P(X=x) = p_x = 5C_x\left(\frac{1}{2}\right)^x\left(\frac{1}{2}\right)^{5-x}$	Expected frequency $= N * p_x = 96 * p_x$
0	$\frac{1}{32}$	$96(\frac{1}{32}) = 3$
1	$\frac{5}{32}$	$96(\frac{5}{32}) = 15$
2	$\frac{10}{32}$	$96(\frac{10}{32}) = 30$
3	$\frac{10}{32}$	$96(\frac{10}{32}) = 30$
4	$\frac{5}{32}$	$96(\frac{5}{32}) = 15$
5	$\frac{1}{32}$	$96(\frac{1}{32}) = 3$
	Total	96

Case 2: Suppose nature of dice is not known

First to find $p = P(a\,4\,or\ 5\ or\ 6\ is\ observed)$

We know that the mean of a binomial distribution is np. Now we can find the mean of the given data by the usual formula $mean = \frac{\sum fx}{\sum f}$ and then equate this to np. Since we have $n = 5$, we can find p.

Now, $\sum fx = [1(0)+10(1)+24(2)+35(3)+18(4)+8(5)] = 275$ And

$\sum f = 1+10+24+35+18+8 = 96$

$\therefore Mean = \frac{275}{96} = 2.8646$

$\Rightarrow np = 2.8646$

$\Rightarrow 5p = 2.8646$

$\Rightarrow p = \frac{2.8646}{5} = 0.5729$

So let X follow a binomial distribution with parameters $n = 5$ and

$p = P(a\,4\,or\ 5\ or\ 6\ is\ observed) = 0.5729$ i.e $X \sim B(n=5, p=\frac{1}{2})$

We shall calculate the probabilities $P(X=x) = 5C_x (0.5729)^x (0.4271)^{5-x}$, $x = 0,1,2,\ldots,5$

Then we shall calculate the expected frequencies

$f_x = N * P(X=x) = 96 * P(X=x)$, $x = 0,1,2,\ldots,5$

Table for expected frequencies

No. of times 4 or 5 Or 6 is observed (x)	$P(X=x)$ $= p_x$ $= 5C_x (0.5729)^x (0.4271)^{5-x}$	Expected frequency $= N * p_x = 96 * p_x$
0	0.0142	$1.3643 \approx 1$
1	0.0953	$9.1504 \approx 9$
2	0.2557	$24.5481 \approx 25$
3	0.3430	$32.9281 \approx 33$
4	0.2300	$22.0844 \approx 22$
5	0.0617	$5.9247 \approx 6$
	Total	96

Prof. Anushri Tambe

3. Fit a Poisson distribution to the following data:

X	0	1	2	3	4
Frequency	192	100	24	3	1

Solution: Let us assume that $X \sim P(\lambda)$ where λ is the mean (and parameter) of the distribution.
To find mean:

We have $mean = \frac{\sum fx}{\sum f} = \frac{[192(0)+100(1)+24(2)+3(3)+1(4)]}{192+100+24+3+1} = \frac{161}{320} = 0.5031$

$\Rightarrow \lambda = 0.5031$

We shall calculate the probabilities

$$P(X=x) = \frac{e^{-\lambda}\lambda^x}{x!} = \frac{e^{-0.5031}(0.5031)^x}{x!}, \; x = 0,1,2,...$$

Then we shall calculate the expected frequencies

$$f_x = N * P(X=x) = 320 * P(X=x), \; x = 0,1,2,...$$

Table for expected frequencies

x	$P(X=x) = p_x = \frac{e^{-\lambda}\lambda^x}{x!} = \frac{e^{-0.5031}(0.5031)^x}{x!}$	Expected frequency $= N * p_x = 320 * p_x$
0	0.6046	$193.472 \approx 193$
1	0.3042	$97.344 \approx 97$
2	0.0765	$24.48 \approx 25$
3	0.0128	$4.096 \approx 4$
4	0.0016	$0.512 \approx 1$
	Total	320

4. The following mistakes per page were observed in a book:

No. of mistakes	0	1	2	3	4
No. of pages	211	90	19	5	0

Fit a Poisson distribution

Ans: $\lambda = 0.44$;

No. of mistakes	0	1	2	3	4
Expected frequencies	209	92	20	3	1

Prof. Anushri Tambe