Semester : III Subject : DSGT Academic Year: 2022-2023

* Cyclic Group-

A group $(G, *)$ is said to be a cyclic group if there exists an element $a \in G$ such that every element of G can be written as some power of a i.e. a^k for some integer k where by a^k, $a * a * a \ldots a$ (k times).

Then G is said to be generated by a or a generates G.

A cyclic group is always Abelian because commutativity is observed.

$\therefore$ if $a^r, a^s \in G$ then

$$a^r \times a^s = a^s \times a^r$$

ex. (1) Prove that $(G, *)$ is a cyclic group, where $G = \{1, \omega, \omega^2\}$.

$\Rightarrow$

composition table -

$*$	1	ω	ω^2
1	1	ω	ω^2
ω	ω	ω^2	1
ω^2	ω^2	1	ω

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Now we have $G = \{1, \omega, \omega^2\}$.
we have only 3 elements
now check for any generator element ?

$1^1 = 1$	$\omega^1 = \omega$
$1^2 = 1 * 1 = 1$	$\omega^2 = \omega * \omega = \omega^2$
$1^3 = 1 * 1 * 1 = 1$	$\omega^3 = \omega * \omega * \omega = \omega^2 * \omega = 1$
$1^4 = 1 * 1 * 1 * 1 = 1$	search in table.
	$\omega^4 = \omega^3 * \omega = 1 * \omega = \omega$
element 1 generates 1 only.	element ω generating all elements in a group hence ω is a generator element.
so 1 is not generating all elements hence 1 is not generator element	

$(\omega^2)^1 = \omega^2$
$(\omega^2)^2 = \omega^2 * \omega^2 = \omega^4 = \omega$ [see previous composition of ω]
$(\omega^2)^3 = \omega^6 = \omega^3 * \omega^3 = 1$
$(\omega^2)^4 = \omega^8 = \omega^3 * \omega^3 * \omega^2 =$
$= 1 . \omega^2$
$= \omega^2$

ω^2 also generates all elements in a group
so ~~it~~ ω^2 is also a generater.
& hence it is cyclic group.