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Subject: Applied Mathematics III **SEM: III**

- Properties of Laplace Transform:

We will see proofs of only first three properties then we will list all the properties together.

1] Change of scale property:

If $L[f(t)] = \phi(s)$ then $L[f(at)] = \frac{1}{a}\phi\left(\frac{s}{a}\right)$

Proof: Given $L[f(t)] = \phi(s)$

$\therefore$ By def^n, $\phi(s) = \int_0^\infty e^{-st} f(t)\, dt$ —— (1)

consider, $L[f(at)] = \int_0^\infty e^{-st} f(at)\, dt$

put, $at = u \Rightarrow t = \frac{u}{a} \Rightarrow dt = \frac{du}{a}$

as $t \to 0,\ u \to 0,$

$t \to \infty,\ u \to \infty.$

$$L[f(at)] = \int_0^\infty e^{-(s/a)u} f(u) \cdot \frac{du}{a}$$

$$= \frac{1}{a}\int_0^\infty e^{-(s/a)u} \cdot f(u) \cdot du$$

$$= \frac{1}{a} \cdot \phi\left(\frac{s}{a}\right). \quad \cdots \text{(from (1))}.$$

2] First shifting theorem:

If $L[f(t)] = \phi(s)$ then $L[e^{-at} f(t)] = \phi(s+a)$.

Proof: Given $L[f(t)] = \phi(s)$

By def^n, $\phi(s) = \int_0^\infty e^{-st} \cdot f(t)\, dt$ —— (1)

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consider, $L[e^{-at} f(t)] = \int_0^\infty e^{-st} e^{-at} f(t)\, dt$.

$$= \int_0^\infty e^{-(s+a)t} f(t)\, dt$$

$$= \phi(s+a) \quad \ldots\ldots \quad (\text{from } 1))$$

Similarly, one can prove,

$$L[e^{at} f(t)] = \phi(s-a)$$

3] Second shifting theorem :

IF $L[g(t)] = \phi(s)$ and $f(t) = g(t-a)$, $t>a$
$= 0$, $t<a$

then prove that $L[f(t)] = e^{-as}\phi(s)$.

Sol^n: W.k.t. $L[f(t)] = \int_0^\infty e^{-st} f(t)\cdot dt$

$$= \int_0^a e^{-st} f(t)\cdot dt + \int_a^\infty e^{-st} f(t)\, dt$$

$$= \int_0^a e^{-st} (0)\, dt + \int_a^\infty e^{-st} g(t-a)\, dt$$

put, $t-a=u \Rightarrow t=a+u \Rightarrow dt=du$.

As $t\to a$, $u\to 0$, as $t\to\infty$, $u\to\infty$

$$\therefore L[f(t)] = \int_0^\infty e^{-(sa+su)} \cdot g(u)\, du$$

$$= e^{-sa} \int_0^\infty e^{-su} g(u)\, du$$

$$L[f(t)] = e^{-as} L[g(u)] = e^{-as}\phi(s).$$

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List of all properties of Laplace Transform

If $L[f(t)] = \phi(s)$ then

1] Change of scale:

$$L[f(at)] = \frac{1}{a}\,\phi\left(\frac{s}{a}\right).$$

2] First shifting Theorem:

$$L[e^{at} f(t)] = \phi(s-a).$$

$$L[e^{-at} f(t)] = \phi(s+a).$$

3] Effect of multiplication by t:

$$L[t^n f(t)] = (-1)^n \frac{d^n}{ds^n}\cdot\phi(s).$$

4] Effect of Division by t:

$$L\left[\frac{f(t)}{t}\right] = \int_s^\infty \phi(s)\cdot ds.$$

5] Laplace Transform of Derivative:

$$L\left[\frac{d}{dt} f(t)\right] = L[f'(t)] = s\cdot L[f(t)] - f(0).$$

$$L\left[\frac{d^2}{dt^2}\cdot f(t)\right] = L[f''(t)] = s^2 L[f(t)] - s\cdot f(0) - f'(0).$$

$$L\left[\frac{d^n}{dt^n} f(t)\right] = L[f^{(n)}(t)] = s^n L[f(t)] - s^{n-1} f(0) \ldots, - f^{(n-1)}($$

6] Laplace Transform of Integration:

$$L\left[\int_0^t f(u)\,du\right] = \frac{1}{s}\cdot\phi(s).$$

$$L\left[\int_0^t\int_0^t \cdots \int_0^t f(u)\,(du)^n\right] = \frac{1}{s^n}\cdot\phi(s).$$

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- Problems :

1] If $L[f(t)] = \frac{2}{s^3} \cdot e^{-s}$ find $L[f(2t)]$.

Sol^n: Given, $\phi(s) = L[f(t)] = \frac{2}{s^3} \cdot e^{-s}$.

By change of scale property,

$$L[f(at)] = \frac{1}{a} \phi\left(\frac{s}{a}\right).$$

$$\therefore L[f(2t)] = \frac{1}{2} \cdot \phi\left(\frac{s}{2}\right)$$

$$= \frac{1}{2} \cdot \frac{2}{(s/2)^3} \cdot e^{-s/2}$$

$$= \frac{8}{s^3} \cdot e^{-s/2}$$

2] If $L[f(t)] = \frac{20-4s}{s^2-4s+20}$ find $L[f(3t)]$

Sol^n: By change of scale property, $L[f(at)] = \frac{1}{a}\phi\left(\frac{s}{a}\right)$.

$$\therefore L[f(3t)] = \frac{1}{3}\phi\left(\frac{s}{3}\right).$$

$$= \frac{1}{3} \cdot \frac{20 - \frac{4s}{3}}{\left(\frac{s}{3}\right)^2 - 4\left(\frac{s}{3}\right) + 20}$$

$$= \frac{1}{3} \cdot \frac{60-4s}{3\left(\frac{s^2}{9} - \frac{4s}{3} + 20\right)}$$

$$= \frac{1}{9} \cdot \frac{(60-4s)\,9}{s^2 - 12s + 180}$$

$$= \frac{60-4s}{s^2 - 12s + 180}.$$

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3] If $L[f(t)] = \frac{s}{s^2+2s+3}$ then find $L[e^{-2t}\cdot f(t)]$

Soln: $L[e^{-2t}\cdot f(t)] = \frac{s+2}{(s+2)^2+2(s+2)+3}$

$$= \frac{s+2}{s^2+4s+4+2s+4+3}$$

$$= \frac{s+2}{s^2+6s+11}$$

4] $L[f(t)] = \frac{1}{s^2+4s+3}$ then find $L[e^{3t}\cdot f(2t)]$.

Soln: $L[f(2t)] = \frac{1}{2}\left[\frac{1}{(\frac{s}{2})^2+4(\frac{s}{2})+3}\right]$

$$= \frac{1}{2}\left[\frac{1}{s^2/4+2s+3}\right]$$

$$= \frac{1}{2}\left[\frac{4}{s^2+8s+12}\right]$$

$$= \frac{2}{s^2+8s+12}.$$

$$L[e^{3t}\cdot f(2t)] = \frac{2}{(s-3)^2+8(s-3)+12} = \frac{2}{s^2+2s-3}.$$

5] Find Laplace of sinhat·sinat.

Soln: consider,

$$\sinh at\cdot\sin at = \left(\frac{e^{at}-e^{-at}}{2}\right)\sin at.$$

$$= \frac{1}{2}\left[e^{at}\cdot\sin at - e^{-at}\cdot\sin at\right].$$

$$L[\sinh at\cdot\sin at] = \frac{1}{2}\left[L[e^{at}\cdot\sin at] - L[e^{-at}\cdot\sin at]\right]$$

$$= \frac{1}{2}\left[\frac{a}{(s-a)^2+a^2} - \frac{a}{(s+a)^2+a^2}\right]$$

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$$L[\sinh at \cdot \sin at] = \frac{1}{2}\left[\frac{a(s^2+2as+a^2+a^2)-a(s^2-2as+a^2+a^2)}{(s^2-2as+2a^2)(s^2+2as+2a^2)}\right]$$

$$= \frac{1}{2}\left[\frac{as^2+2a^2s-as^2+2a^2s}{s^4+4a^4}\right]$$

$$\therefore = \frac{1}{2}\left[\frac{4a^2s}{s^4+4a^4}\right]$$

$$= \frac{2a^2s}{s^4+4a^4}.$$

6] Find $L[e^{-4t} \cdot \cosh t \cdot \sin t]$

Sol^n: consider,

$$L[e^{-4t} \cdot \cosh t \cdot \sin t] =$$

$$= L\left[e^{-4t} \cdot \left(\frac{e^t+e^{-t}}{2}\right)\sin t\right]$$

$$= L\left[\left(\frac{e^{-3t}+e^{-5t}}{2}\right)\sin t\right]$$

$$= \frac{1}{2}L\left[e^{-3t} \cdot \sin t + e^{-5t} \cdot \sin t\right].$$

$$= \frac{1}{2}\left[\frac{1}{(s+3)^2+1} + \frac{1}{(s+5)^2+1}\right]$$

$$= \frac{1}{2}\left[\frac{s^2+10s+26+s^2+6s+10}{(s^2+10s+26)(s^2+6s+10)}\right]$$

$$= \frac{1}{2}\left[\frac{2s^2+16s+36}{(s^2+10s+26)(s^2+6s+10)}\right]$$

- Examples for practice :

1) $e^{-3t} \cdot \cosh 4t \cdot \sin 3t$
2) $e^{2t} \cdot \cos 2t \cdot \cos t$
3) $e^{2t} \cdot (1+t)^2$.
4) $e^{-t} \cdot \sin^2 t$
5) $e^{2t} \cdot \sin^4 t$.

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7) Find $L\left[t\cdot e^{3t}\cdot \sin 2t\right]$

Sol^n: To find Laplace transform of $t\cdot e^{3t}\cdot \sin 2t$ one can proceed as follows,

$$L[\sin 2t] = \frac{2}{s^2+4}$$

$$L[t\cdot \sin 2t] = -\frac{d}{ds}\left[\frac{2}{s^2+4}\right] \quad \cdots\cdots \text{(Using effect of multiplication by } t\text{)}$$

$$= -2\cdot\frac{d}{ds}\left(\frac{1}{s^2+4}\right)$$

$$= -2\left[\frac{-1}{(s^2+4)^2}\cdot 2s\right]$$

$$L[t\cdot \sin 2t] = \frac{4s}{(s^2+4)^2}$$

$$L\left[e^{3t}\cdot t\cdot \sin 2t\right] = \frac{4(s-3)}{\left[(s-3)^2+4\right]^2} \quad \cdots\cdots \text{(Using first shifting thm)}$$

$$= \frac{4s-12}{(s^2-6s+12)^2}$$

8) Find $L\left[t\cdot\sqrt{1-\sin t}\right]$

Sol^n:

$$L\left[\sqrt{1-\sin t}\right] = L\left[\sqrt{(\cos t/2 - \sin t/2)^2}\right]$$

$$= L\left[\cos t/2 - \sin t/2\right]$$

$$= \frac{s}{s^2+\frac{1}{4}} - \frac{1/2}{s^2+1/4}.$$

$$= \frac{s-1/2}{s^2+1/4}.$$

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$$L\left[\sqrt{1-\sin t}\right] = \frac{2(2s-1)}{s^2+1}$$

$$L\left[t\cdot\sqrt{1-\sin t}\right] = -1\cdot\frac{d}{ds}\left[\frac{2(2s-1)}{s^2+1}\right] \quad \text{... (using effect of multiplication by t)}$$

$$= -2\left[\frac{(s^2+1)(2)-(2s-1)(2s)}{(s^2+1)^2}\right]$$

$$= -2\left[\frac{-2s^2+2s+2}{(s^2+1)^2}\right]$$

$$= 4\left[\frac{s^2-s-1}{(s^2+1)^2}\right].$$

- Examples for practice :

Find Laplace transforms of ,

1) $t\cdot e^{3t}\cdot\sin 3t$ 2) $t\cdot e^{t}\cdot\sin 2t\cdot\cos t$ 3) $t\cdot e^{-2t}\cdot\sinh 4t$

4) $t\cdot e^{3t}\cdot\sinh 2t$ 5) $e^{-2t}\cdot t\cdot\sin^2 t$ 6) $e^{-2t}\cdot t^3\cdot\sin t$

9] Find Laplace transform $\dfrac{\sin^2 t}{t}$.

Soln: First we find $L\left[\sin^2 t\right] = L\left[\frac{1-\cos 2t}{2}\right]$

$$= \frac{1}{2}L\left[1-\cos 2t\right]$$

$$= \frac{1}{2}\left[\frac{1}{s}-\frac{s}{s^2+4}\right] = \phi(s).$$

Now, $L\left[\frac{\sin^2 t}{t}\right] = \int_s^\infty \phi(s)\,ds$ ---- (using effect of division by t)

$$= \frac{1}{2}\int_s^\infty\left(\frac{1}{s}-\frac{s}{s^2+4}\right)ds.$$

$$L\left[\frac{\sin^2 t}{t}\right] = \frac{1}{2}\left[\log s - \frac{\log(s^2+4)}{2}\right]_s^\infty$$

$$= \frac{1}{2}\left[(\log\infty - \log\infty) - \left(\log s - \frac{\log(s^2+4)}{2}\right)\right]$$

$$= \frac{1}{2}\left[-\left(\frac{2\log s - \log(s^2+4)}{2}\right)\right]$$

$$= \frac{1}{4}\left[-\log s^2 + \log(s^2+4)\right]$$

$$= \frac{1}{4}\log\left(\frac{s^2+4}{s^2}\right).$$

10] $L\left[\frac{\cosh 2t \cdot \sin 2t}{t}\right]$

Sol^n: consider,

$$L\left[\frac{\cosh 2t \cdot \sin 2t}{t}\right] = L\left[\left(\frac{e^{2t}+e^{-2t}}{2}\right)\cdot\frac{\sin 2t}{t}\right]$$

$$= \frac{1}{2} L\left[e^{2t}\cdot\frac{\sin 2t}{t} + e^{-2t}\cdot\frac{\sin 2t}{t}\right]$$

$$= \frac{1}{2}\left[L\left[e^{2t}\cdot\frac{\sin 2t}{t}\right] + L\left[e^{-2t}\cdot\frac{\sin 2t}{t}\right]\right] \quad \text{--- (1)}$$

first calculating $L[\sin 2t] = \frac{2}{s^2+4}$

$$\therefore L\left[\frac{\sin 2t}{t}\right] = \int_s^\infty \frac{2}{s^2+4}\,ds$$

$$= \frac{2}{2}\cdot\left(\tan^{-1}(s/2)\right)_s^\infty$$

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$$\therefore L\left[\frac{\sin 2t}{t}\right] = \tan^{-1}\infty - \tan^{-1}(s/2)$$
$$= \frac{\pi}{2} - \tan^{-1}(s/2)$$
$$= \cot^{-1}(s/2).$$

$$\left.\begin{aligned} \therefore L\left[e^{2t}\cdot\frac{\sin 2t}{t}\right] &= \cot^{-1}\left(\frac{s-2}{2}\right) \\ \& \ L\left[e^{-2t}\cdot\frac{\sin 2t}{t}\right] &= \cot^{-1}\left(\frac{s+2}{2}\right)\end{aligned}\right\} - (2)$$

Substituting values from eq^n (2) in eq^n (1)

$$\therefore L\left[\frac{\cosh 2t\cdot\sin 2t}{t}\right] = \frac{1}{2}\left[\cot^{-1}\left(\frac{s-2}{2}\right) + \cot^{-1}\left(\frac{s+2}{2}\right)\right]$$

11] Find the laplace of $\frac{\cos at - \cos bt}{t}$.

Sol^n: $L[\cos at - \cos bt] = L[\cos at] - L[\cos bt]$

$$= \frac{s}{s^2+a^2} - \frac{s}{s^2+b^2}.$$

$$L\left[\frac{\cos at - \cos bt}{t}\right] = \int_s^\infty\left(\frac{s}{s^2+a^2} - \frac{s}{s^2+b^2}\right)ds$$
$$= \frac{1}{2}\left[\log(s^2+a^2) - \log(s^2+b^2)\right]_s^\infty$$
$$= \frac{1}{2}\left[-\left(\log(s^2+a^2) - \log(s^2+b^2)\right)\right]$$
$$= \frac{1}{2}\log\left(\frac{s^2+b^2}{s^2+a^2}\right)$$

Examples for practice:

1) $\frac{e^{2t}\cdot\sin t}{t}$ 2) $\frac{2\sin t\cdot\sin 2t}{t}$ 3) $\frac{\sinh at}{t}$

4) $\frac{e^{2t}\cdot\sin^3 t}{t}$ [for easy solution in these type of examples one can apply first shifting theorem at last]

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- Examples Based on laplace transforms of Derivatives and Integration.

12] Find $L[f(t)]$ and $L[f'(t)]$

$$f(t) = t \quad , \quad 0 \le t < 3$$
$$= 6 \quad , \quad t > 3.$$

Sol^n:

As, $L[f(t)] = \int_0^\infty e^{-st} f(t)\, dt$

$$= \int_0^3 e^{-st} f(t)\, dt + \int_3^\infty e^{-st} f(t)\, dt$$

$$= \int_0^3 e^{-st} t \cdot dt + \int_3^\infty e^{-st} 6\, dt$$

$$= \left[t \cdot \frac{e^{-st}}{-s} - \int 1 \cdot \left[\frac{e^{-st}}{-s}\right] dt \right]_0^3 + 6 \cdot \left[\frac{e^{-st}}{-s}\right]_3^\infty$$

$$= \left[\frac{-t \cdot e^{-st}}{s} - \frac{e^{-st}}{s^2} \right]_0^3 + 6 \cdot \frac{e^{-3s}}{+s}$$

$$= \frac{-3e^{-3s}}{s} - \frac{e^{-3s}}{s^2} + \frac{e^0}{s^2} + \frac{6e^{-3s}}{s}$$

$$L[f(t)] = \frac{3e^{-3s}}{s} + \frac{1-e^{-3s}}{s^2}$$

Now, $L[f'(t)] = s \cdot L[f(t)] - f(0)$ ....(Using property laplace of Derivative)

$\therefore f(0) = 0.$

$$\therefore L[f'(t)] = s \cdot \left[\frac{3e^{-3s}}{s} + \frac{1-e^{-3s}}{s^2} \right] - 0$$

$$L[f'(t)] = 3e^{-3s} + \frac{1-e^{-3s}}{s}$$

13] Find $L\left[\frac{d}{dt}\left(\frac{1-\cos 2t}{t}\right)\right]$

Sol^n:

$$L[1-\cos 2t] = \frac{1}{s} - \frac{s}{s^2+4}$$

$$\therefore L\left[\frac{1-\cos 2t}{t}\right] = \int_s^\infty \left(\frac{1}{s} - \frac{s}{s^2+4}\right) ds$$

$$= \left[\log s - \frac{1}{2}\left(\log(s^2+4)\right)\right]_s^\infty$$

$$= 0 - \left(\log s - \frac{1}{2}\log(s^2+4)\right)$$

$$= \frac{-2\log s + \log(s^2+4)}{2}$$

$$= \frac{1}{2}\log\left(\frac{s^2+4}{s^2}\right).$$

Now, $L\left[\frac{d}{dt} f(t)\right] = s \cdot L[f(t)] - f(0).$

To find $f(0)$,

$$f(0) = \lim_{t\to 0}\left(\frac{1-\cos 2t}{t}\right)$$

$$= \lim_{t\to 0} \frac{2\sin^2 t}{t}$$

$$= \lim_{t\to 0} 2 \cdot \sin t \cdot \frac{\sin t}{t}$$

$$= 2 \cdot \lim_{t\to 0} \sin t \cdot \lim_{t\to 0} \frac{\sin t}{t}$$

$$= (0)(1)$$

$$= 0.$$

$$\therefore L\left[\frac{d}{dt}\left(\frac{1-\cos 2t}{t}\right)\right] = \frac{s}{2} \cdot \log\left(\frac{s^2+4}{s^2}\right) - 0$$

$$= \frac{s}{2} \cdot \log\left(\frac{s^2+4}{s^2}\right).$$

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Examples for practice

1) $f(t) = t+1, \quad 0 \le t \le 2$; find $L[f(t)]$ and $L[f'(t)]$
$\quad\;\; = 3, \quad t > 2$

2) Find $L\left[\frac{d^2}{dt^2}\left(\frac{\sin t}{t}\right)\right]$.

14] Find Laplace transform $\int_0^t u \cdot \cosh au \cdot du$.

Sol^n:

consider, $L\left[\int_0^t u \cdot \cosh au \cdot du\right] = L\left[\int_0^t u\left(\frac{e^{au}+e^{-au}}{2}\right)du\right]$

$= \frac{1}{2} L\left[\int_0^t e^{au} \cdot u\,du + \int_0^t e^{-au} \cdot u \cdot du\right]$ —— (1)

let, $f(u) = e^{au} \cdot u$ & $g(u) = e^{-au} \cdot u$.

$L[u] = \frac{1}{s^2}$

$L[e^{au} \cdot u] = \frac{1}{(s-a)^2}$ & $L[e^{-au} \cdot u] = \frac{1}{(s+a)^2}$.

$L\left[\int_0^t e^{au} \cdot u\,du\right] = \frac{1}{s(s-a)^2}$ & $L\left[\int_0^t e^{-au} \cdot u \cdot du\right] = \frac{1}{s(s+a)^2}$

substitute these values in eq^n (1)

$\therefore L\left[\int_0^t u \cdot \cosh au\,du\right] = \frac{1}{2} \cdot \left[\frac{1}{s(s-a)^2} + \frac{1}{s(s+a)^2}\right]$

$= \frac{1}{2s}\left[\frac{1}{(s-a)^2} + \frac{1}{(s+a)^2}\right]$

15] Find $L\left[e^{-3t} \int_0^t u \cdot \sin 3u \cdot du\right]$.

Sol^n: Let $f(u) = \sin 3u$.

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$L[\sin 3u] = \frac{3}{s^2+9}$

$L[u \cdot \sin 3u] = -\frac{6s}{(s^2+9)^2}$

$L\left[\int_0^t u \cdot \sin 3u \cdot du\right] = \frac{-6s}{s(s^2+9)^2} = \frac{-6}{(s^2+9)^2}$

$L\left[e^{-3t}\int_0^t u \cdot \sin 3u\, du\right] = \frac{-6}{((s+3)^2+9)^2}$.

16] $L\left[\int_0^t \frac{e^u \sin u}{u}\, du\right]$

Sol^n: Let $f(u) = \sin u$.

$L[\sin u] = \frac{1}{s^2+1}$

$L\left[\frac{\sin u}{u}\right] = \int_s^\infty \frac{1}{s^2+1}\, ds = \left[\tan^{-1}(s)\right]_s^\infty$

$= \cot^{-1}(s)$.

$L\left[e^u \cdot \frac{\sin u}{u}\right] = \cot^{-1}(s-1)$.

$L\left[\int_0^t e^u \cdot \frac{\sin u}{u}\, du\right] = \frac{1}{s} \cdot \cot^{-1}(s-1)$.

Examples for practice:

1) $L\left[\int_0^t \frac{1-e^{-au}}{u}\, du\right]$

2) $L\left[\int_0^t u \cdot e^{-3u} \sin^2 u \cdot du\right]$

3) $L\left[\int_0^t e^u \cdot \frac{\sin 4u}{u}\, du\right]$

4) $L\left[\cosh t \int_0^t e^u \cos hu \cdot du\right]$