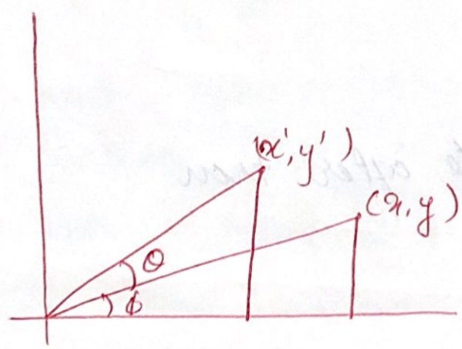
Rotation :-

- It is a transformation that used to reposition the object along the circular path in xy-plane.
- To generate a rotation we specify a rotation angle θ and the position of the rotation point (pivot point) (x_r, y_r) about which the object is to be rotated.
- Positive value of rotation angle defines counter clockwise rotation and negative value of rotation angle defines clockwise rotation.
- we first find the equation of rotation when pivot point is at co-ordinate origin $(0,0)$.



From figure we can write

$$x = r\cos\phi$$
$$y = r\sin\phi$$

and

$x' = r\cos(\theta+\phi)$

$= r\cos\theta\cos\phi - r\sin\theta\sin\phi$

$= x\cos\theta - y\sin\theta$

$y' = r\sin(\theta+\phi)$

$= \cancel{r\sin\theta\sin\phi + r\sin\theta\sin\phi}$

$= r\cos\phi\sin\theta + r\sin\phi\cos\theta$

$= x\sin\theta + y\cos\theta$

- we can write it in the form of column vector matrix equation as

$$P' = R \cdot P$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

- Rotation is also rigid body transformation so we need to rotate each point of object.

Example :-

Q. Locate the new position of the traingle A(5,4) B(8,3) C(8,8) after its rotation by 90° clockwise about the origin.

As rotation is clockwise we will take $\theta = -90°$

$$P' = R \cdot P$$

$$P' = \begin{bmatrix} \cos(-90) & -\sin(90) \\ \sin(-90) & \cos(90) \end{bmatrix} \begin{bmatrix} 5 & 8 & 8 \\ 4 & 3 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} \cos 90 & \sin 90 \\ -\sin 90 & \cos 90 \end{bmatrix} \begin{bmatrix} 5 & 8 & 8 \\ 4 & 3 & 8 \end{bmatrix}$$

$\because \cos(-\theta) = \cos\theta$
$\sin(-\theta) = -\sin\theta$

$$= \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 5 & 8 & 8 \\ 4 & 3 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 3 & 8 \\ -5 & -8 & -8 \end{bmatrix}$$

Final co-ordinates after rotation are

A' (4, -5) B' (3, -8) C' (8, -8)