

Semester : 1Subject : Statistics for AIDS Academic Year: 2023-2024.KRUSKAL-WALLIS TEST (H-TEST).

A clinical study was conducted for 3 different COVID-19 vaccines and observed for complete immunization by a suitable method in days as given below:

X-Shield (X)	Yo Vaxin (Y).	Zo Vax (Z)
30	34	67
27	35	54
34	19	27
15	15	27
18	27	79
	78	85
	45	

Use the H-Test (Kruskal-Wallis Test) at the 5% level of significance whether the three COVID-19 vaccines are equally effective (with reference to immunization) or not? ( $\chi^2_{0.05,2} = 5.99$ ).

Solution: $H_0$ : X, Y, Z are equally effective. $H_a$ : X, Y, Z are not equally effective.

} Steps: Define Null and alternate hypothesis.

Step 2:- Assign rank for each value.

(P.T.O).



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X-shield (X)	R <sub>1</sub>	Y-Varin (Y)	R <sub>2</sub>	Z-Varx (Z)	R <sub>3</sub>
30	9	34	10.5	67	15
27	6.5	35	12	54	14
34	10.5	19	4	27	6.5
15	1.5	15	1.5	27	6.5
18	3	27	6.5	79	17
		78	16	85	18
		45	18		

$$\frac{1+2}{2} = 1.5$$

$$\frac{5+6+7+8}{4} = \frac{26}{4} = 6.5$$

$$\frac{10+11}{2} = 10.5$$

$$n_1 = 5 \quad \sum R_1 = 30.5 \quad n_2 = 7 \quad \sum R_2 = 68.5 \quad n_3 = 6 \quad \sum R_3 = 77$$

$$N = n_1 + n_2 + n_3 = 18 \quad \text{Step 3: Calculate H value.}$$

$$H = \frac{12}{N(N+1)} \times \left( \frac{\sum R_1^2}{n_1} + \frac{\sum R_2^2}{n_2} + \frac{\sum R_3^2}{n_3} + \dots + \frac{\sum R_k^2}{n_k} \right) - 3(N+1)$$

$$= \frac{12}{18 \times (18+1)} \left( \frac{(30.5)^2}{5} + \frac{(68.5)^2}{7} + \frac{(77)^2}{6} \right) - 3(18+1)$$

$$H = \frac{12}{342} \times \left( \frac{930.25}{5} + \frac{4692.25}{7} + \frac{5929}{6} \right) - 57$$

$$H = 0.035 \times (186.05 + 670.32 + 988.2) - 57$$

$$H = 0.035 \times 1750.29 - 57$$

$$= 61.25 - 57$$

$$H_{cal} = 4.41$$



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$$df = \text{no. of Groups} - 1 \\ = 3 - 1 = 2.$$

$$\chi_{0.05, 2} = 5.99$$

Step 4: Compare values and take decision.

$$H_{\text{cal}} = 4.25 < \chi_{0.05, 2}^2 = 5.99.$$

The null hypothesis is accepted.  
There is no difference in the effectiveness of vaccine.

Example 2:

Consider there are three groups and their reaction time is measured. Check whether there is difference between the groups using Kruskal-Wallis Test at 5% level of significance.

Group A	Group B	Group C
34	44	35
36	37	39
41	45	42
43	33	46

Solution:

$H_0$ : There is no difference between the groups. } Step 2.  
 $H_a$ : There is difference between the groups.

Step 3 ∴ Assign Ranks



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Group A	$R_1$	Group B	$R_2$	Group 3	$R_3$
34	2	44	10	35	3
36	4	37	5	39	6
41	7	45	11	42	8
43	9	33	4	46	12
$n_1 = 4$	$\sum R_1 = 22$	$n_2 = 4$	$\sum R_2 = 27$	$n_3 = 4$	$\sum R_3 = 29$

$$N = n_1 + n_2 + n_3 = 4 + 4 + 4 = \boxed{12}$$

Steps: Calculate H value.

$$H = \frac{12}{N(N+1)} \times \left( \frac{\sum R_1^2}{n_1} + \frac{\sum R_2^2}{n_2} + \frac{\sum R_3^2}{n_3} + \dots + \frac{\sum R_k^2}{n_k} \right) - 3(N+1)$$

$$= \frac{12}{12(12+1)} \times \left( \frac{(22)^2}{4} + \frac{(27)^2}{4} + \frac{(29)^2}{4} \right) - 3(12+1)$$

$$= 0.076 \times \left( \frac{484}{4} + \frac{729}{4} + \frac{841}{4} \right) - 3(13)$$

$$= 0.076 \times (121 + 182.25 + 210.25) - 39$$

$$= 0.076 (513.5) - 39$$

$$= 39.026 - 39$$

$$\boxed{H = 0.026}$$

$$0.026 < \chi_{0.05,2} = 5.991$$

Null hypothesis is accepted.

Degree of freedom =  $3-1=2$