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A.P. SHAH INSTITUTE OF TECHNOLOGY

Department of Computer Science and Engineering

Data Science

University Exam Paper Solution

Academic Year: 2022 2023

Year: - SE

Semester: - III

Subject: - DSGT

Date of Exam: - 23/11/2022

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DSGT Nov 2022 Solution:

Q. 1 Solve any 4 of the following questions:

a) **prove that n^3+2n is divisible by 3 for all integers n**

ANSWER: Step 1: Show true for n=1

For n=1, $n^3+2n=(1)^3+2(1)$

$n^3+2n=3$

3 is definitely divisible by 3 so the statement is true for n=1.

Step 2: Assume true for n=k

We assume that for any integer k, n^3+2n is divisible by 3. We can write this mathematically as:

$n^3+2k=3m$, where m is an integer

Step 3: Show true for k+1

For n=k+1,

$n^3+2n=(k+1)^3+2(k+1)$

$=(k^3+3k^2+3k+1)+2k+2$

$=(k^3+2k)+3(k^2+k+1)$

Subbing in from part 2 for (k^3+2k), we get:

$n^3+2n=3m+3(k^2+k+1)$

$=3(m+k^2+k+1)$

which is divisible by 3.

This means that the statement being true for n=k implies the statement is true for n=k+1, and as we have shown it to be true for n=1 the proof of the statement follows by induction.

b) Explain the following terms with suitable example.

i) Partition set

Partition of a set, say *S*, is a collection of *n* disjoint subsets, say P_1, P_1, ... P_n that satisfies the following three conditions –

P_i does not contain the empty set.

$[\,P_i \neq \{\emptyset\}$ for all $0 < i \leq n\,]$

The union of the subsets must equal the entire original set.

$[\,P_1 \cup P_2 \cup ... \cup P_n = S\,]$

The intersection of any two distinct sets is empty.

$[\,P_a \cap P_b = \{\emptyset\}$, for $a \neq b$ where $n \geq a, b \geq 0\,]$

Example

Let S = { a, b, c, d, e, f, g, h }

One probable partitioning is { a }, { b, c, d }, { e, f, g, h }

Another probable partitioning is { a, b }, { c, d }, { e, f, g, h }

A collection of disjoint subsets of a given set. The union of the subsets must equal the entire original set.

For example, one possible partition of {1, 2, 3, 4, 5, 6} is {1, 3}, {2}, {4, 5, 6}.

ii) Power set

A power set includes all the subsets of a given set including the empty set. The power set is denoted by the notation P(S) and the number of elements of the power set is given by 2^n. A **power set** can be imagined as a place holder of all the subsets of a given set or, in other words, the subsets of a set are the members or elements of a power set.

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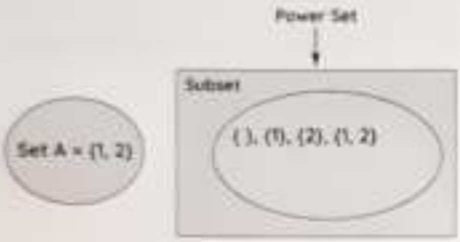
A power set is defined as the set or group of all subsets for any given set, including the empty set, which is denoted by {}, or, ϕ. A set that has 'n' elements has 2^n subsets in all. For example, let Set A = {1,2,3}, therefore, the total number of elements in the set is 3. Therefore, there are 2^3 elements in the power set. Let us find the power set of set A.

Set A = {1,2,3}

Subsets of set A = {}, {1}, {2}, {3}, {1,2}, {2,3}, {1,3}, {1,2,3}

Power set P(A) = { {}, {1}, {2}, {3}, {1,2}, {2,3}, {1,3}, {1,2,3} }



c) State the Pigeonhole Principle and show that if any five numbers from 1 to 8 are chosen, then two of them will add to 9.

Suppose that a flock of 20 pigeons flies into a set of 19 pigeonholes to roost. Because there are 20 pigeons but only 19 pigeonholes, a least one of these 19 pigeonholes must have at least two pigeons in it. To see why this is true, note that if each pigeonhole had at most one pigeon in it, at most 19 pigeons, one per hole, could be accommodated. This illustrates a general principle called the pigeonhole principle, which states that if there are more pigeons than pigeonholes, then there must be at least one pigeonhole with at least two pigeons in it.

At least one pigeon hole contains **ceil[A]** (smallest integer greater than or equal to A) pigeons

Remaining pigeon holes contains at most **floor[A]** (largest integer less than or equal to A) pigeons

Given: if any five numbers from 1 to 8 are chosen, then two of them will add to 9.

Let us consider 1-8 numbers given (1,2,3,4,5,6,7,8)

Now let's take any 5 numbers from 1 to 8 such as (1,3,4,7,8)

As it given that any two of the numbers out of the 5 numbers we have chosen should be equal to sum 9.

Let's add every two numbers so that we can get one such pair of numbers whose sum would be 9.

Case 1>. 1 + 3 = 4

Case 2>. 3 + 4 = 7

Case 3>. 4 + 7 = 11

Case 4>. 7 + 8 = 15

Case 5>. 8 + 1 = [9]
→Hence in Case 5 we get a pair of numbers
8 and 1 whose sum is equal to 9 , so we
present them together in a same set as {8,1}.
✓ So according to Pigeonhole Principle ,We can take any 5 numbers and there will always exist one pair whose sum is equal to 9.

d) Consider the function f(x)=2x-3. Find a composition functions

i) f^2 = fof
ii) f^3 = fofof
Given:
f(x)=2x-3
f^2 = fof
=f(f(x))
=f(2x-3)
=2(2x-3)-3
=4x-6-3
=4x-9
f^3 = fofof
=f(f(f(x)))
=f(f(2x-3))
=f(2(2x-3)-3)
=f(4x-9)
=4(2x-3)-9
=8x-12-9
=8x-21

e) Explain the bipartite graph with suitable example.

Bipartite Graph - If the vertex-set of a graph G can be split into two disjoint sets, V_1 and V_2 , in such a way that each edge in the graph joins a vertex in V_1 to a vertex in V_2 , and there are no edges in G that connect two vertices in V_1 or two vertices in V_2 , then the graph G is called a bipartite graph.
A bipartite graph is a special kind of graph with the following properties-
It consists of two sets of vertices X and Y.
The vertices of set X join only with the vertices of set Y.
The vertices within the same set do not join.
The following graph is an example of a bipartite graph-

Example of Bipartite Graph

Q. 2

a) What is a transitive closure? Find the transitive closure of R using Warshall's algorithm where A={1,2,3,4,5) and R={(x,y)|x-y=+-1}

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Transitive closure In mathematics, the transitive closure of a binary relation R on a set X is the smallest relation on X that contains R and is transitive. For finite sets, "smallest" can be taken in its usual sense, of having the fewest related pairs; for infinite sets it is the unique minimal transitive superset of R.

Let $A = \{1, 2, 3, 4, 5\}$

$R = \{(x, y) \mid x - y = \pm 1\}$

$R = \{(1,2)(2,1)(3,2)(2,3)(3,4)(4,3)(4,5)(5,4)\}$

$$M_R = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \left[\begin{array}{ccccc} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right] \end{array} = W_0 = M_R$$

C (2) R (2) $C \times R = \{(2,2)\}$

$$W_1 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \left[\begin{array}{ccccc} 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right] \end{array}$$

C (1, 2, 3) R (1, 2, 3)

$C \times R = \{(1,1)(1,2)(1,3)(2,1)(2,2)(2,3)(3,1)(3,2)($

Subject Incharge ________ Page No ______ Department of CSE-Data Science | APSIT

Semester ________ Subject ________ Academic Year 20 - 20

$$W_2 = \begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

C (1,2,3,4)   R (1,2,3,4)

CXR = {(1,1)(1,2)(1,3)(1,4)(2,1)(2,2)(2,3)(2,4)(3,1)(3,2)(3,3)(3,4)(4,1)(4,2)(4,3)(4,4)}

$$W_3 = \begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

C (1,2,3,4,5)   R (1,2,3,4,5)

CXR = {(1,1)(1,2)(1,3)(1,4)(1,5)(2,1)(2,2)(2,3)(2,4)(2,5)(3,1)(3,2)(3,3)(3,4)(3,5)(4,1)(4,2)(4,3)(4,4)(4,5)(5,1)(5,2)(5,3)(5,4)(5,5)}

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$$W_4 = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 1 & 1 & 1 & 1 & 1 \\ 3 & 1 & 1 & 1 & 1 & 1 \\ 4 & 1 & 1 & 1 & 1 & 1 \\ 5 & 1 & 1 & 1 & 1 & 1 \end{array}$$

C (1,2,3,4,5) R (1,2,3,4,5)

$C \times R = \{(1,1)(1,2)(1,3)(1,4)(1,5)(2,1)(2,2)(2,3)(2,4)(2,5)(3,1)(3,2)(3,3)(3,4)(3,5)(4,1)(4,2)(4,3)(4,4)(4,5)(5,1)(5,2)(5,3)(5,4)(5,5)\}$

$$W_4 = W_5 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

b) What is a ring? Let A={0,1,2,3,4,5,6,7}. Determine whether a set A with addition module 8 and multiplication modulo 8 is a commutative ring? Justify your answer.

The ring is a type of algebraic structure (R, +, .) or (R, *, .) which is used to contain non-empty set R. Sometimes, we represent R as a ring. It usually contains two binary operations that are multiplication and addition.

An algebraic system is used to contain a non-empty set R, operation o, and operators (+ or *) on R such that:

- (R, 0) will be a semigroup, and (R, *) will be an algebraic group.
- The operation o will be said a ring if it is distributive over operator *.

Semester: ______ Subject: ______ Academic Year: 20 - 20

Q.2 b - Let $A = \{0,1,2,3,4,5,6,7\}$

composition table wrt addition modulo 8

$\oplus_8$	0	1	2	3	4	5	6	7
0	0	1	2	3	4	5	6	7
1	1	2	3	4	5	6	7	0
2	2	3	4	5	6	7	0	1
3	3	4	5	6	7	0	1	2
4	4	5	6	7	0	1	2	3
5	5	6	7	0	1	2	3	4
6	6	7	0	1	2	3	4	5
7	7	0	1	2	3	4	5	6

composition table wrt multi^n modulo 8

$\otimes_8$	0	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7
2	0	2	4	6	0	2	4	6
3	0	3	6	1	4	7	2	5
4	0	4	0	4	0	4	0	4
5	0	5	2	7	4	1	6	3
6	0	6	4	2	0	6	4	2
7	0	7	6	5	4	3	2	1

1. Closure Property

In the closure property, the set R will be called for composition '+' like this:

$x \in R, y \in R \Rightarrow x+y \in R$ for all $x, y \in R$

2. Association

In association law, the set R will be related to composition '+' like this:

$(x+y) + z = x + (y+z)$ for all $x, y, z \in R$.

3. Existence of identity

Here, R is used to contain an additive identity element. That element is known as zero elements, and it is denoted by 0. The syntax to represent this is described as follows:

$x + y = x = 0 + x, x \in R$

4. Existence of inverse

In existence of inverse, the elements $x \in R$ is exist for each $x \in R$ like this:

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$x + (-x) = 0 = (-x) + x$

5. Commutative of addition

In the commutative law, the set R will represent for composition + like this:

$x + y = y + x$ for all $x, y \in R$

Q. 3

a) A survey in 1986 asked household whether they had a VCR, a CD player or cable TV. 140 had a VCR. 60 had a CD player, and 50 had cable TV. 25 owned VCR and CD player. 30 owned a CD player and had cable TV. 35 owned a VCR and had cable TV. 10 households had all three. How many households had at least one of the three? How many of them had only CD player?

let V be the set of households with a VCR.

Let C be the set of households with a CD player.

Let T be the set of households with cable TV.

We have to find $|V \cup C \cup T|$

By inclusion-exclusion,

$|V \cup C \cup T| = |V| + |C| + |T| - |V \cap C| - |V \cap T| - |C \cap T| + |V \cap C \cap T|$

Therefore, $|V \cup C \cup T| = 140 + 60 + 50 - 25 - 30 - 35 + 10$

$|V \cup C \cup T| = 170$

b) Find the complete solution of a recurrence relation.

$a_n + 2a_{n-1} = n + 3$ for $n >= 1$ and with $a_0 = 3$

equation $= a_n + 2a_{n-1} = n + 3$

$n >= 1$

$a_0 = 3$

$\therefore n = 1$

general solution -

$a_{n_0} = C_1 1^n = C_1$

particular solution -

$a_n = (a_n + B)n = an^2 + bn$

$= a(n-1)^2 + b(n-1)$

Then

$an^2 + bn = a(n-1)^2 + b(n-1) + 2n + 3$

when

$a_n + 2a_{n-1} - n - 3 = 0$

$\therefore an^2 + bn = an^2 - 2an + a + bn - b + 2n + 3$

$2an - a + b = 2n + 3$

when $\boxed{a_0 = 3}$

$\therefore a_n = n^2 + 3n$

$a_n = (a_n)_0 + n + 3n$

$= c_1 + n + 3n$

$a_0 = 3$

$c_1 + 0 + 3 \cdot 0 = 3$

$c_1 + 0 + 0 = 3$

$c_1 = 3$

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c) Obtain CNF and DNF for the following expression:

p<->(~p V ~q)

$p \leftrightarrow (\sim p \vee \sim q)$

$= (\sim p \vee (\sim p \vee \sim q)) \wedge ((\sim p \vee \sim q) \vee p)$

$= (\sim p \vee \sim p \vee \sim q) \wedge ((\sim\sim p \wedge \sim\sim q) \vee p)$

$= (\sim p \wedge \sim q) \wedge (p \vee p) \wedge (q \vee p)$

$= (\sim p \wedge \sim q) \wedge p \wedge (q \vee p)$ ← CNF

$= (\sim p \wedge p) \vee (\sim q \wedge p)) \wedge (q \vee p)$

$= (F \vee (\sim q \wedge p)) \wedge (q \vee p)$

$= (\sim q \wedge p) \wedge (q \vee p)$

$= (\sim q \wedge p \wedge q) \vee (\sim q \wedge p \wedge p)$

$= F \vee (\sim q \wedge p)$

$= (\sim q \wedge p)$ ← DNF (single conjunct)

Q.4

a) What is a group? Let A={3,6,9,12}

i) Prepare the composition table w.r.t the operation of multiplication modulo 15.

ii) Whether it is an abelian group? Justify your answer.

iii) Find the inverses of all the elements.

iv) Whether it is a cyclic group?

Group:

A system consisting of a non-empty set G of element a, b, c etc with the operation is said to be group provided the following postulates are satisfied:

1. Closure property
   For all a, b E G => a, b E G
   i.e G is closed under the operation '.'
2. Associativity
   (a,b).c = a.(b.c) a, b, c E G.
   i.e the binary operation '.' Over g is associative.
3. Existence of identity
   There exits an unique element in G. Such that e.a = a = a.e
   for every a E G. This element e is called the identity.
4. Existence of inverse
   For each a E G , there exists an element a^-1 E G
   such that a. a^-1 = e = a^-1.a
   the element a^-1 is called the inverse of a .

Subject: ______ Academic Year 20 - 20

Q.4 a) Let $A = \{3, 6, 9, 12\}$
composition table w.r.t. the opⁿ of multiplication modulo 15.

$\otimes$	3	6	9	12
3	9	3	12	6
6	3	6	9	12
9	12	9	6	3
12	6	12	3	9

Inverses of all of the elements.
3 & 12 are inverses
(because $3 \times 12 = 6$)
6 is its own inverse
($6 \times 6 = 6$)
9 is its own inverse
($9 \times 9 = 9$)

multiplication modulo is an associative.
6 is the identity element

all elements having inverses
This composition is commutative because the elements equidistant from principal diagonal are equal each to each.
Hence this group is abelian group.

A group G is called cyclic. If for some aEG, every element xEG is of the form a^n. where n is some integer. Symbolically we write G = {a^n : n E I}. The single element a is called a generator of G and as the cyclic group is generated by a single element, so the cyclic group is also called monogenic.

b) What are the isomorphic graphs? Determine whether following graphs are isomorphic.

The isomorphism graph can be described as a graph in which a single graph can have more than one form. That means two different graphs can have the same number of edges, vertices, and same edges connectivity. These types of graphs are known as isomorphism graphs.

Q. 4

b) Isomorphic graph -

no. of edges of graph 1 = 10
no. of vertices of graph 1 : 8

no. of edges of graph 2 = 10
no. of vertices of graph 2 = 8

degree seq of graph 1 = {2, 3, 2, 3, 2, 3, 2, 3}
" — graph 2 = {3, 2, 2, 3, 3, 2, 2

mapping

a — t
b — s
c — u
d — v
e — x
f — w
g — y
h — z

hence we can say that both graphs are isomorphic.

Q.5

a) Let X={1,2,3,6,24,36} and R={(x,y) R|x divides y}

i) Write the pairs in a relation set R.

ii) Construct Hasse diagram.

iii) What are the Maximal and Minimal elements.

iv) Mention Chains and Anti chains from above set.

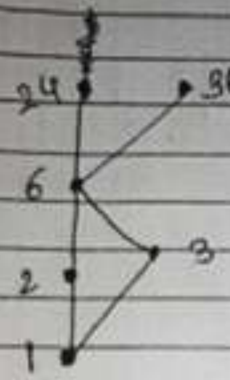
v) Is this poset a lattice?

Q.5 (a)

X = {1, 2, 3, 6, 24, 36}

R = {(x, y) | x divides y}

R = {(1,2) (1,1) (1,3) (1,6) (1,24) (1,36)
(2,6) (2,24) (2,36) (3,6) (3,24)
(3,36) (6,24) (6,36) (2,2) (3,3) (6,6)
(24,24) (36,36)}



Maximal elements: 24, 36

Minimal elements: 1

chain = {1, 2, 6, 24}

antichain = {2, 3}

Poset is a lattice.

GLB= 24 and 26

LUB=1

Hence it is lattice

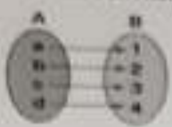
b) Define the term bijective function.

Let f:R->R-(7/5)->R-(2/5) be defined by f(x)= (2x-3)/(5x-7).

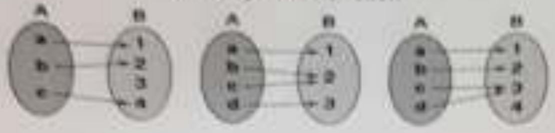
Whether a function is bijective? Justify your answer.

Bijective function connects elements of two sets such that, it is both one-one and onto function. The elements of the two sets are mapped in such a manner that every element of the range is in co-domain, and is related to a distinct domain element. In simple words, we can say that a function f: A→B is said to be a bijective function or bijection if f is both one-one (injective) and onto (surjective).

Bijective Function



Not a Bijective Function



Q 5 b) Let $f: R \to R - (7/5)$ $R - (2/5)$
be defined by $f(x) = (2x-3)/(5x-7)$

$\frac{2x-3}{5x-7}$ for $R - \left(\frac{7}{5}\right)$

$y = \frac{(2x-3)}{(5x-7)}$

$x = \frac{(2y-3)}{(5y-7)}$

$y(5x-7) = (2x-3)$

$5xy - 7y = 2x - 3$

$x(5y-7) = (2y-3)$

$5xy - 7x = 2y - 3$

$y(5x-7) = 2x - 3$

$y = \frac{2x-3}{5x-7}$ $\frac{2}{5}$ $\frac{7}{5}$

$= \frac{2x-3}{5x-7}$ $\frac{7}{5}$

$= \frac{10x-15}{35x-49}$

c) Define minimum hamming distance. Consider $e: B^3 \to B^6$. Find the code words generated by the parity check matrix H given below.

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$$H = \begin{matrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}$$

Hamming distance is a metric for comparing two binary data strings. While comparing two binary strings of equal length, Hamming distance is the number of bit positions in which the two bits are different.

The Hamming distance between two strings, a and b is denoted as d(a,b).

It is used for error detection or error correction when data is transmitted over computer networks. It is also using in coding theory for comparing equal length data words.

Q.5 c) minimum hamming distance.

$e : B^3 \rightarrow B^6$

$$H = \begin{matrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}$$

→ we have,

$B^3 = \{000, 001, 010, 011, 100, 101, 110, 111\}$

Then

$e(000) = 000\,x_1 x_2 x_3$

$x_1 = 0 \cdot 1 + 0 \cdot 1 + 0 \cdot 0 = 0$

$x_2 = 0 \cdot 1 + 0 \cdot 1 + 0 \cdot 1 = 0$

$x_3 = 0 \cdot 1 + 0 \cdot 0 + 0 \cdot 1 = 0$

$e(000) = 000000$

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$e(001) = 001\,x_1x_2x_3$

$x_1 = 0\cdot1 + 0\cdot1 + 1\cdot0 = 0$

$x_2 = 0\cdot1 + 0\cdot1 + 1\cdot1 = 1$

$x_3 = 0\cdot1 + 0\cdot0 + 1\cdot1 = 1$

$e(001) = 001011$

$e(010) = 010\,x_1x_2x_3$

$x_1 = 0\cdot1 + 1\cdot1 + 0\cdot0 = 1$

$x_2 = 0\cdot1 + 1\cdot1 + 0\cdot1 = 1$

$x_3 = 0\cdot1 + 1\cdot0 + 0\cdot1 = 0$

$e(010) = 010110$

$e(011) = 011\,x_1x_2x_3$

$x_1 = 0\cdot1 + 1\cdot1 + 1\cdot0 = 1$

$x_2 = 0\cdot1 + 1\cdot1 + 1\cdot1 = 0$

$x_3 = 0\cdot1 + 1\cdot0 + 1\cdot1 = 1$

$e(011) = 011101$

$e(100) = 100\,x_1x_2x_3$

$x_1 = 1\cdot1 + 0\cdot1 + 0\cdot0 = 1$

$x_2 = 1\cdot1 + 0\cdot1 + 0\cdot1 = 1$

$x_3 = 1\cdot1 + 0\cdot0 + 0\cdot1 = 1$

$e(100) = 100111$

$e(101) = 101\,x_1x_2x_3$

$x_1 = 1\cdot1 + 0\cdot1 + 1\cdot0 = 1$

$x_2 = 1\cdot1 + 0\cdot1 + 1\cdot1 = 0$

$x_3 = 1\cdot1 + 0\cdot0 + 1\cdot1 = 0$

$e(101) = 101100$

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Semester ______ Subject ______ Academic Year 20 20

$e(110) = 110\,x_1 x_2 x_3$

$x_1 = 1\cdot1 + 1\cdot1 + 0\cdot0 = 0$

$x_2 = 1\cdot1 + 1\cdot1 + 0\cdot1 = 0$

$x_3 = 1\cdot1 + 1\cdot0 + 0\cdot1 = 1$

$e(110) = 110001$

$e(111) = 111\,x_1 x_2 x_3$

$x_1 = 1\cdot1 + 1\cdot1 + 1\cdot0 = 0$

$x_2 = 1\cdot1 + 1\cdot1 + 1\cdot1 = 1$

$x_3 = 1\cdot1 + 1\cdot0 + 1\cdot1 = 0$

$e(111) = 111010$

$e_H : B^3 \to B^6$ is

$e(000) = 000000$ $\quad$ $e(100) = 100111$

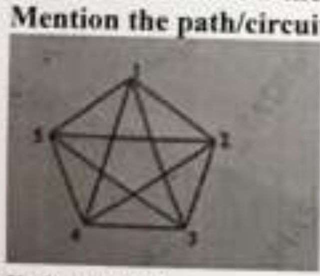
$e(001) = 001011$ $\quad$ $e(101) = 101100$

$e(010) = 010110$ $\quad$ $e(110) = 110001$

$e(011) = 011101$ $\quad$ $e(111) = 111010$

Q.6

a) Define with example Euler path, Euler circuit, Hamiltonian path, and Hamiltonian circuit. Determine if the following diagram has Euler circuit and Hamiltonian circuit. Mention the path/circuit.



Euler's Path

An Euler's path contains each edge of 'G' exactly once and each vertex of 'G' at least once. A connected graph G is said to be traversable if it contains an Euler's path.

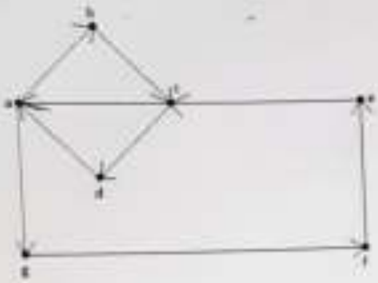
Example

Euler's Path = d-c-a-b-d-e.

Euler's Circuit

In an Euler's path, if the starting vertex is same as its ending vertex, then it is called an Euler's circuit.

Example



Euler's Path = a-b-c-d-a-g-f-e-c-a.

Hamiltonian Path

A connected graph is said to be Hamiltonian if it contains each vertex of G exactly once. Such a path is called a **Hamiltonian path**.

Example

Hamiltonian Path – e-d-b-a-c.

Prepared by: Rajashri C.

The diagram is having Euler circuit
= 1, 2, 3, 4, 5, 2, 4, 1, 3, 5, 1

Euler path := 1, 2, 3, 4, 5, 2, 4, 1, 3, 5

Hamiltonian circuit 1, 2, 3, 4, 5, 1
Hamiltonian path. 1, 2, 3, 4, 5

b) Let p denote the statement "The food is good", q denote the statement "The service is good" and r denote the statement "The rating is 3 star".
Write the following statements in a symbolic form.

i) Either food is good or service is good or both.
ii) The food is good but service is not good.
iii) If both food and service are good then the rating is 3 star.
iv) It is not true that a 3 star rating always means good food and good service.

Q. 6 (b)
p: The food is good
q: The service is good
r: The rating is 3 star.

i) Either food is good or service is good or both.
$\Rightarrow$ $p \vee q$

ii) The food is good but service is not good
$p \wedge \sim q$

iii) If ~~thoth~~ food and service are good then the rating is 3 star.
$(p \wedge q) \rightarrow r$

iv) It is not true that 3 star rating always means good food and good service.
$\sim (r \rightarrow (p \wedge q))$

c) Find out the incidence matrix of following graphs.

Incidence matrix :-

(1)

$$\begin{array}{c|cccccccc} & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 & e_8 \\ \hline a & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ b & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ c & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ d & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ e & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ f & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}$$

Semester: ______ Subject: ______ Academic Year: 20 - 20

(2)

$$\begin{array}{c|cccccc} & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \\ \hline a & 0 & 1 & 0 & 0 & 1 & 1 \\ b & 0 & 1 & 1 & 0 & 1 & 0 \\ c & 0 & 0 & 0 & 1 & 0 & 1 \\ d & 1 & 0 & 0 & 0 & 0 & 0 \\ e & 1 & 0 & 0 & 1 & 0 & 0 \end{array}$$

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