Semester : VI **Subject :Machine Learning** **Academic Year: 2023 - 2024**

Multiple Linear Regresion

Example:-

- In linear regresion model we have one dependent and one independent variable
- Multiple regresion model involves multiple predictors or independent variables and one dependent variable.
- This is an extension of the linear regresion problem.

→ The multiple regresion of two variables x1 and x2 is given as follows:

$$y = f(x_1, x_2)$$
$$y = a_0 + a_1 x_1 + a_2 x_2$$

→ In general, this is given for 'n' independent variable as:-

$$y = f(x_1, x_2, \dots x_n)$$
$$y = a_0 + a_1 x_1 + a_2 x_2 + \dots + a_n x_n + \varepsilon$$

- Here $x_1, x_2, \dots x_n$ are predictor variables, y is the dependent variable $(a_0, a_1, a_2 \dots a_n)$ are the coefficients of the regresion equation and ε is error term

Example:- Here the matrices for Y an X are given as follows

$$X = \begin{bmatrix} 1 & 1 & 4 \\ 1 & 2 & 5 \\ 1 & 3 & 8 \\ 1 & 4 & 2 \end{bmatrix} \text{ and } Y = \begin{bmatrix} 1 \\ 6 \\ 8 \\ 12 \end{bmatrix}$$

X1 Project 1	X2 produ 2	Y weekly
1	4	1
2	5	6
3	8	8
4	2	12

→ The coefficient of multiple regression equation is given as.

$$a = \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix}$$

→ The regression coefficient for multiple regression is calculated the same way as linear regression

$$\hat{a} = ((X^T X)^{-1} X^T) Y$$

$$X^T X = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 4 & 5 & 8 & 2 \end{bmatrix} \begin{bmatrix} 1 & 1 & 4 \\ 1 & 2 & 5 \\ 1 & 3 & 8 \\ 1 & 4 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 10 & 19 \\ 10 & 30 & 46 \\ 19 & 46 & 109 \end{bmatrix}$$

– The regression coefficient for multiple regression is calculated the same way as linear regression.

$$\hat{a} = ((X^T X)^{-1} X^T) Y$$

$$(X^T X)^{-1} = \begin{bmatrix} 4 & 10 & 19 \\ 10 & 30 & 46 \\ 19 & 46 & 109 \end{bmatrix} = \begin{bmatrix} 3.15 & -0.59 & -0.30 \\ -0.59 & 0.20 & 0.016 \\ -0.30 & 0.016 & 0.054 \end{bmatrix}$$

- The regression coefficient for multiple regression is calculated the same way as linear regression:

$$\hat{a} = ((X^T X)^{-1} X^T) Y$$

$$(X^T X)^{-1} X^T = \begin{bmatrix} 3.15 & -0.59 & -0.30 \\ -0.59 & 0.20 & 0.016 \\ -0.30 & 0.016 & 0.054 \end{bmatrix} \times \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 4 & 5 & 8 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 0.05 & 0.47 & -1.02 & 0.19 \\ -0.32 & -0.098 & 0.155 & 0.26 \\ -0.065 & 0.005 & 0.185 & -0.125 \end{bmatrix}$$

→

$$\hat{a} = ((X^T X)^{-1} X^T) Y$$

$$= \begin{bmatrix} 0.05 & 0.47 & -1.02 & 0.19 \\ -0.32 & -0.098 & 0.155 & 0.26 \\ -0.065 & 0.005 & 0.185 & -0.125 \end{bmatrix} \times \begin{bmatrix} 1 \\ 6 \\ 8 \\ 12 \end{bmatrix} = \begin{bmatrix} -1.69 \\ 3.48 \\ -0.05 \end{bmatrix}$$

$$a_0 = -1.69$$

$$a_1 = 3.48$$

$$a_2 = -0.05$$

- $y = a_0 + a_1 x_1 + a_2 x_2$
- Hence, the constructed model is:
- $\boxed{y = -1.69 + 3.48x_1 - 0.05x_2}$