

Semester : IIISubject : DSGT

Academic Year: 2022-2023

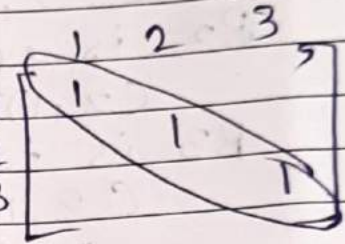
\* Reflexive closure and transitive closure -

\* Reflexive closure -

$$A = \{1, 2, 3\}$$

$$R = \{(1,1)(2,2)(3,3)\}$$

$$M_R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



diagonal value.

So for reflexive relation we must consider relation matrix  $M_R$

where for each  $a \in A$  we must have.

$\{ (a,a) \mid a \in A \}$  is a diagonal relation and this diagonal relation is  $\Delta$

e.g.

$$A = \{a, b\}$$

$$\Delta = \{ (a,a) (b,b) \}$$

$$R = \{ (a,b) (b,a) (a,a) \}$$

so reflexive closure of relation  $R$  on set  $A$  is

$$R \cup \Delta$$

Semester : IIISubject : DSGT

Academic Year: 2021-2023

## \* Closure of relation -

Let  $R$  be a relation on a set  $A$ .  $R$  may or may not have some property  $P$ , such as reflexivity, symmetry or transitivity.

If there is a relation  $S$  with property  $P$  containing  $R$  such that  $S$  is the subset of every relation with property  $P$  containing  $R$  then  $S$  is called the closure of  $R$  with respect to  $P$ .

### 1) Reflexive closure -

$\Delta = \{(a, a) \mid a \in A\}$  is the diagonal relation on set  $A$ . The reflexive closure of relation  $R$  on set  $A$  is  $R \cup \Delta$ .

### 2) Symmetric closure -

Let  $R$  be a relation on set  $A$ , and let  $R^{-1}$  be the inverse of  $R$ . The symmetric closure of relation  $R$  on set  $A$  is  $R \cup R^{-1}$ .

### 3) Transitive closure -

Let  $R$  be a relation on set  $A$ . The





Semester: \_\_\_\_\_

Subject: \_\_\_\_\_

Academic Year: 20 - 20

connectivity relation is defined as -

$$R^* = \bigcup_{n=1}^{\infty} R^n. \text{ The transitive closure of } R \text{ is } R^*.$$

Q.g.  $A = \{1, 2, 3, 4\}$

$R = \{(1,1), (1,4), (2,3), (3,1), (3,4)\}$

$$\Delta A = \{(1,1), (2,2), (3,3), (4,4)\}$$

$$R \cup \Delta A = \{(1,1), (2,2), (3,3), (4,4), (1,4), (2,3), (3,1), (3,4)\}$$

This is reflexive closure of  $R$ .

ii) Symmetric closure -

we need the inverse of  $R$ .

$$A = \{1, 2, 3, 4\}$$

$$R = \{(1,1), (1,4), (2,3), (3,1), (3,4)\}$$

$$R^{-1} = \{(1,1), (4,1), (3,2), (1,3), (4,3)\}$$

$$R \cup R^{-1} = \{(1,1), (1,4), (2,3), (3,1), (3,4), (4,1), (3,2), (1,3), (4,3)\}$$

This is symmetric closure of  $R$ .



Semester : III

Subject : DSGT

Academic Year: 2022-2023

iii) Transitive closure, we need to find  $R^*$

$\therefore$  we need to find  $R^1, R^2, R^3$  until  $R^n = R^{n-1}$  we stop when this condition is achieved since finding higher powers of  $R$  would be the same.

$$R^1 = \{(1,1)(1,4)(2,3)(3,1)(3,4)\}$$

$$R \circ R = R^2 = \{(1,1)(1,4)(2,1)(2,4)(3,1)(3,4)\}$$

$$R \circ R \circ R = R^3 = \{(1,1)(1,4)(2,1)(2,4)(3,1)(3,4)\}$$

we stop here

$$R^2 = R^3$$

Transitive closure

$$R^\infty = R^* = R^1 \cup R^2$$

$$= \{(1,1)(1,4)(2,1)(2,3)(2,4)(3,1)(3,4)\}$$

Note :-

$$\therefore R^2 = R \circ R = R = \{(1,1)(1,4)(2,3)(3,1)(3,4)\}$$

$$R = \{(1,1)(1,4)(2,3)(3,1)(3,4)\}$$

$$\checkmark R \circ R = \{(1,1)(1,4)(2,1)(2,4)(3,1)(3,4)\}$$