Equation of the Hyperplane :-

Eq^n of line is $y = ax + b$. However although hyperplane is line, its eq^n is

$$w^T x = 0$$

where w and x are the vectors and $w^T x$ is the computation of dot product of two vectors.

Given two vectors

$$w = \begin{pmatrix} -b \\ -a \\ 1 \end{pmatrix} \quad \text{and } x = \begin{pmatrix} 1 \\ x \\ y \end{pmatrix}$$

$$w^T \cdot x = y - ax - b$$

→ The Hyperplane eq^n $w^T x$ is used in place of $y = ax + b$ because it is easier to work ~~with~~ in more dimensions with this notation.

→ and vector w will always be normal to the
*** hyperplane.

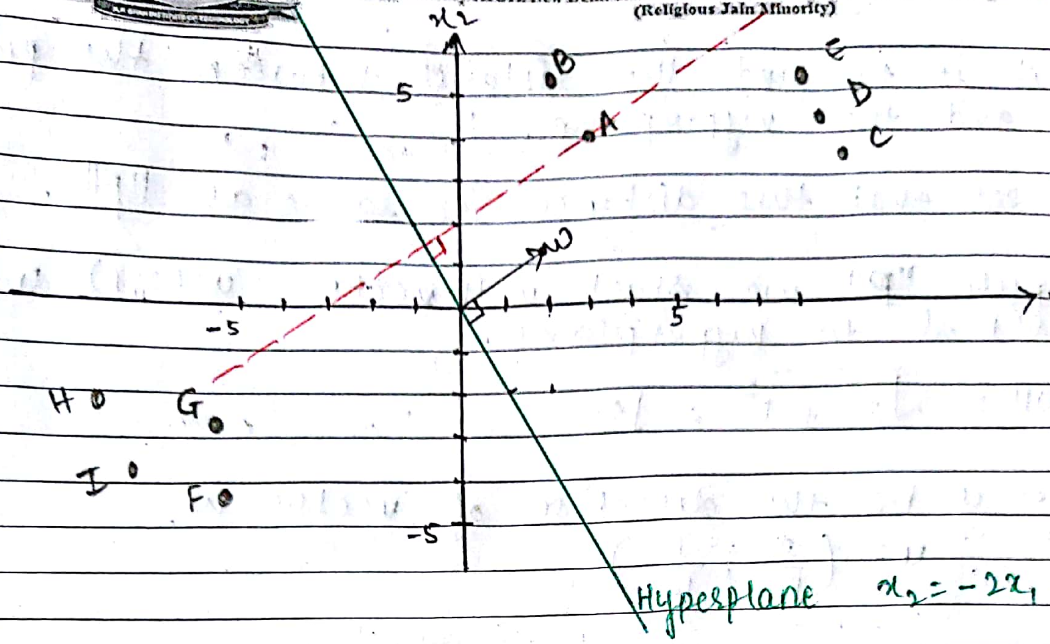
Computation of distance from a point to the Hyperplane :-

Consider Fig :-
To simplify the example, we have set $w_0 = 0$. The eq^n of hyperplane is given by

$$x_2 = -2x_1$$

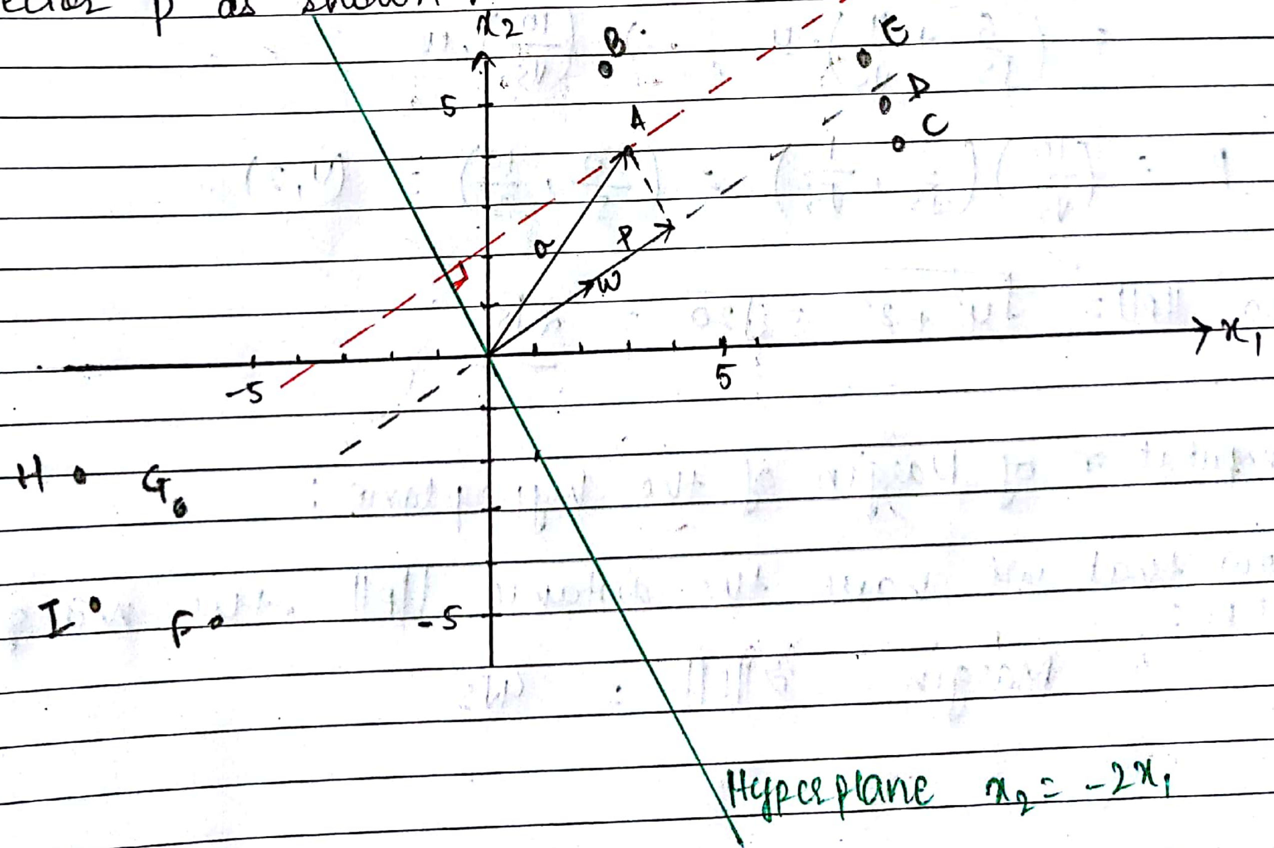
which is equivalent to $w^T x = 0$ with $w\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $x\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$.

→ Compute distance between $A(3,4)$ and hyperplane.



This is the distance between A and its projection onto the hyperplane (Red).

If we project A onto normal vector w, we get the vector p as shown:



our goal is to find the distance between the point A(3,4) and the hyperplane.

We can see that this distance is same as $\|p\|$.

To compute $\|p\|$, we start with vector $w(2,1)$ which is normal to hyperplane.

$$\|w\| = \sqrt{2^2 + 1^2} = \sqrt{5}$$

Let vector u be the direction of vector w.

$$u = \left(\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right)$$

→ p is orthogonal projection of A onto w, so

$$p = (u \cdot a) \cdot u$$

$$= \left[\left(\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right) \cdot (3,4)\right] \cdot u$$

$$= \left(\frac{6}{\sqrt{5}} + \frac{4}{\sqrt{5}}\right) \cdot u \Rightarrow \left(\frac{10}{\sqrt{5}}\right) \cdot u$$

$$p = \left(\frac{10}{\sqrt{5}}\right)\left(\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right) = \left(\frac{20}{5}, \frac{10}{5}\right) = (4,2)$$

so $\|p\| = \sqrt{4^2 + 2^2} = \sqrt{20} = 2\sqrt{5}$

Computation of Margin of the hyperplane:

Now that we have the distance $\|p\|$, the margin will be:

$$\text{Margin} = 2\|p\| = 4\sqrt{5}$$

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