C & Data Structures

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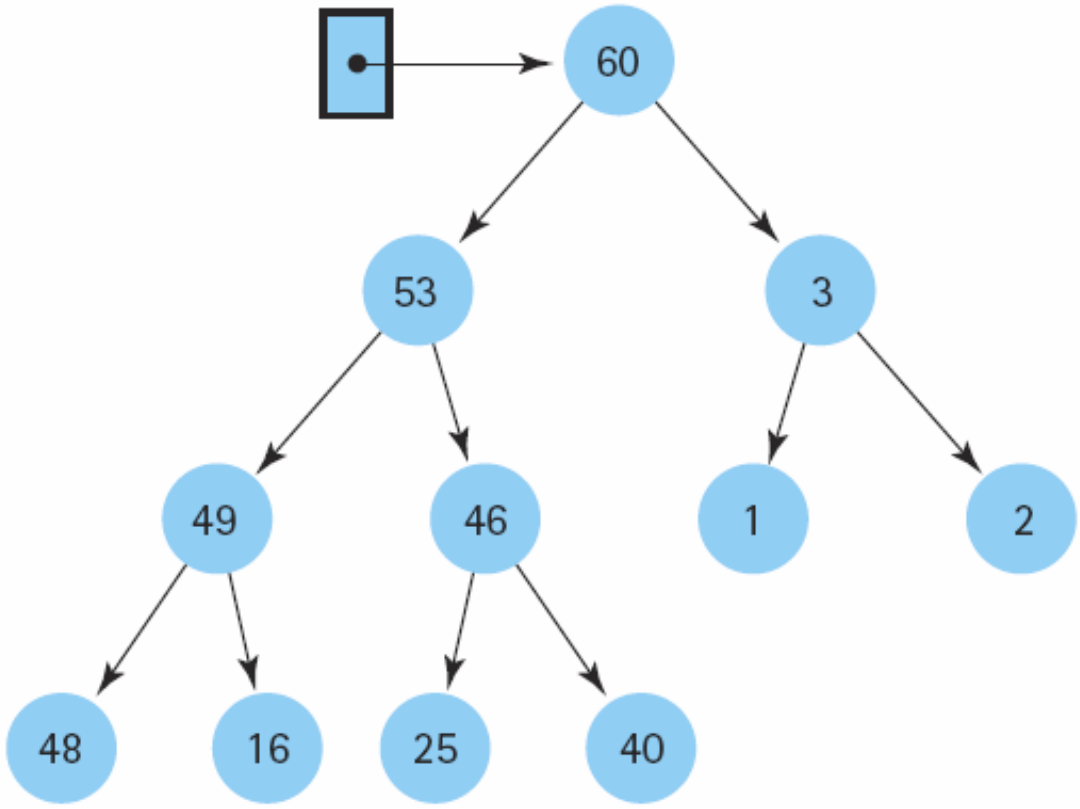

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CHAPTER 0: INTRODUTION

1. What This Book Is About

This book is about data structures and algorithms as used in computer programming. Data structures are ways in which data is arranged in your computer's memory (or stored on disk). Algorithms are the procedures a software program uses to manipulate the data in these structures.

Almost every computer program, even a simple one, uses data structures and algorithms. For example, consider a program that prints address labels. The program might use an array containing the addresses to be printed, and a simple for loop to step through the array, printing each address.

The array in this example is a data structure, and the for loop, used for sequential access to the array, executes a simple algorithm. For uncomplicated programs with small amounts of data, such a simple approach might be all you need. However, for programs that handle even moderately large amounts of data, or which solve problems that are slightly out of the ordinary, more sophisticated techniques are necessary. Simply knowing the syntax of a computer language such as C isn't enough.

This book is about what you need to know *after* you've learned a programming language. The material we cover here is typically taught in colleges and universities as a second-year course in computer science, after a student has mastered the fundamentals of programming.

2. What Do We Mean by Data?

When we talk about the function of a program, we use words such as "add," "read," "multiply," "write," "do," and so on. The function of a program describes what it does in terms of the verbs in the programming language.

The data are the nouns of the programming world: the objects that are manipulated, the information that is processed by a computer program. In a sense, this information is just a collection of bits that can be turned on or off. The computer itself needs to have data in this form. Humans, however, tend to think of information in terms of somewhat larger units such as numbers and lists, so we want at least the human-readable portions of our programs to refer to data in a way that makes sense to us. To separate the computer's view of data from our own view, we use data abstraction to create other views. Whether we use functional decomposition to produce a hierarchy of tasks or object-oriented design to produce a hierarchy of cooperating objects, data abstraction is essential.

Data abstraction The separation of a data type's logical properties from its implementation.

3. Data Abstraction

Many people feel more comfortable with things that they perceive as real than with things that they think of as abstract. As a consequence, "data abstraction" may seem more forbidding than a more concrete entity such as an "integer." But let's take a closer look at that very concrete-and very abstract-integer you've been using since you wrote your earliest programs.

Just what is an integer? Integers are physically represented in different ways on different computers. In the memory of one machine, an integer may be a binary-coded decimal. In a second machine, it may be a sign-and-magnitude binary. And in a third one, it may be represented in one's complement or two's complement notation. Although you may not know what any of these terms mean, that lack of knowledge hasn't stopped you from using integers. (You learn about these terms in an assembly

language course, so we do not explain them here.) Figure shows several representations of an integer number.

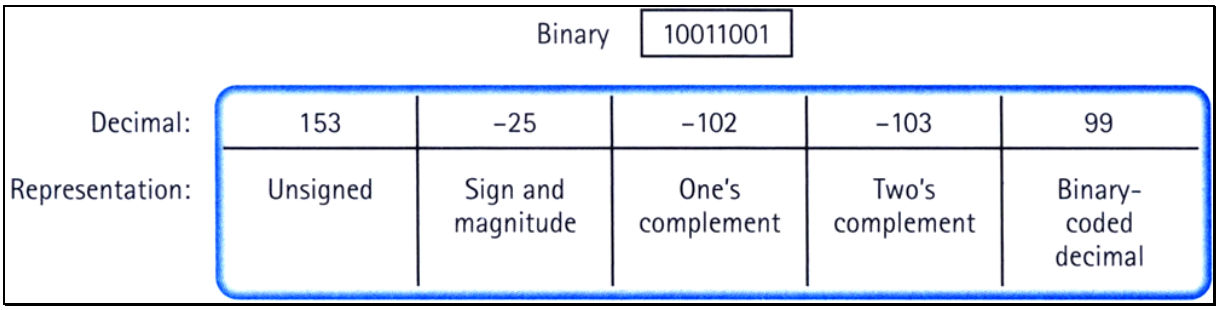


Figure: The decimal equivalents of an 8-bit binary number

The way that integers are physically represented determines how the computer manipulates them. As a C++ programmer, you rarely get involved at this level; instead, you simply use integers. All you need to know is how to declare an int type variable and what operations are allowed on integers: assignment, addition, subtraction, multiplication, division, and modulo arithmetic.

Consider the statement

distance = rate * time;

It's easy to understand the concept behind this statement. The concept of multiplication doesn't depend on whether the operands are, say, integers or real numbers, despite the fact that integer multiplication and floating-point multiplication may be implemented in very different ways on the same computer. Computers would not be so popular if every time we wanted to multiply two numbers we had to get down to the machine-representation level. But that isn't necessary: C++ has surrounded the int data type with a nice, neat package and has given you just the information you need to create and manipulate data of this type.

Another word for "surround" is "encapsulate." Think of the capsules surrounding the medicine you get from the pharmacist when you're sick. You don't have to know anything about the chemical composition of the medicine inside to recognize the big blue-and-white capsule as your antibiotic or the little yellow capsule as your decongestant. Data encapsulation means that the physical representation of a program's data is surrounded. The user of the data doesn't see the implementation, but deals with the data only in terms of its logical picture-its abstraction.

Data encapsulation The separation of the representation of data from the applications that use the data at a logical level; a programming language feature that enforces information hiding

If the data are encapsulated, how can the user get to them? Operations must be provided to allow the user to create, access, and change data. Let's look at the operations C provides for the encapsulated data type int. First, you can create ("construct") variables of type int using declarations in your program. Then you can assign values to these integer variables by using the assignment operator or by reading values into them and perform arithmetic operations using +, -, *, /, and %. Figure shows how C has encapsulated the type int in a tidy package.

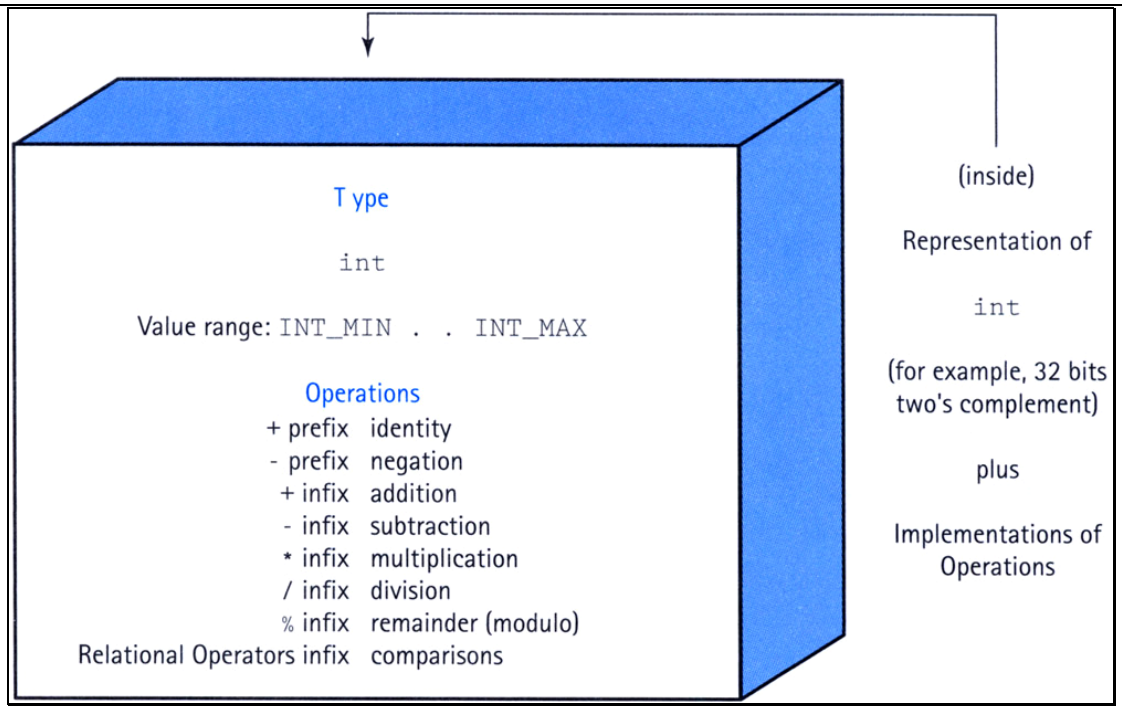


Figure: A black box representing an integer

The point of this discussion is that you have been dealing with a logical data abstraction of "integer" since the very beginning. The advantages of doing so are clear: You can think of the data and the operations in a logical sense and can consider their use without having to worry about implementation details. The lower levels are still there-they're just hidden from you.

Remember that the goal in design is to reduce complexity through abstraction. We can extend this goal further: to protect our data abstraction through encapsulation. We refer to the set of all possible values (the domain) of an encapsulated data "object," plus the specifications of the operations that are provided to create and manipulate the data, as an abstract data type (ADT for short).

Abstract data type (ADT) A data type whose properties (domain and operations) are specified independently of any particular implementation

4. Data Structures

A single integer can be very useful if we need a counter, a sum, or an index in a program, but generally we must also deal with data that have lots of parts, such as a list. We describe the logical properties of such a collection of data as an abstract data type; we call the concrete implementation of the data a data structure. When a program's information is made up of component parts, we must consider an appropriate data structure. Data structures have a few features worth noting. First, they can be "decomposed" into their component elements. Second, the arrangement of the elements is a feature of the structure that affects how each element is accessed. Third, both the arrangement of the elements and the way they are accessed can be encapsulated.

Data structure A collection of data elements whose organization is characterized by accessing operations that are used to store and retrieve the individual data elements; the implementation of the composite data members in an abstract data type

Let's look at a real-life example: a library. A library can be decomposed into its component elements-books. The collection of individual books can be arranged in a number of ways, as shown in Figure. Obviously, the way the books are physically arranged on the shelves determines how one would go about looking for a specific volume. The particular library with which we're concerned doesn't let its patrons get their own books, however; if you want a book, you must give your request to the librarian, who retrieves the book for you.

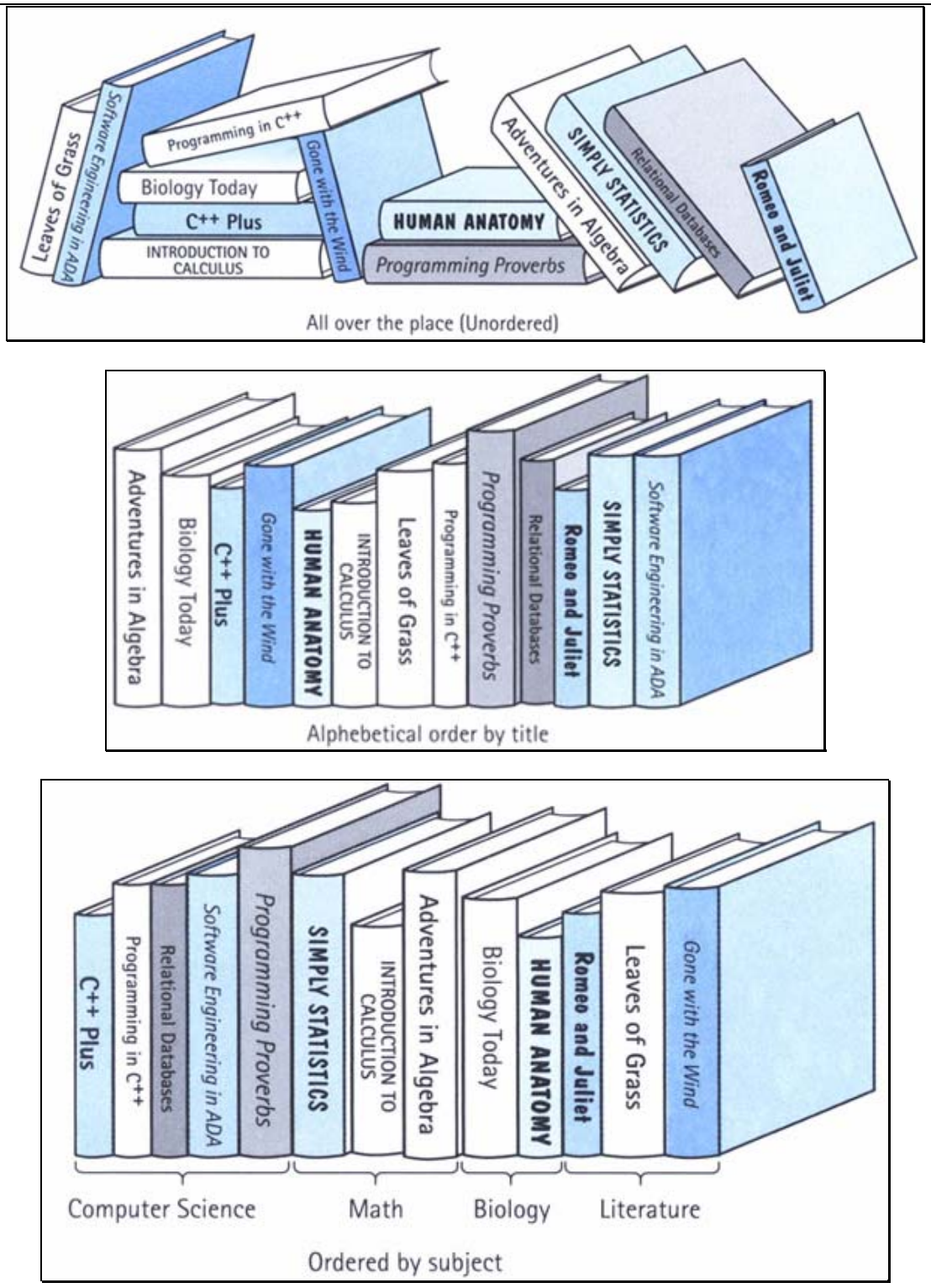


Figure: A collection of books ordered in different ways

The library "data structure" is composed of elements (books) in a particular physical arrangement; for instance, it might be ordered on the basis of the Dewey decimal system. Accessing a particular book requires knowledge of the arrangement of the books. The library user doesn't have to know about the structure, however, because it has been encapsulated: Users access books only through the librarian. The physical structure and the abstract picture of the books in the library are not the same. The card catalog provides logical views of the library-ordered by subject, author, or title-that differ from its physical arrangement.

We use the same approach to data structures in our programs. A data structure is defined by (1) the logical arrangement of data elements, combined with (2) the set of operations we need to access the elements.

Notice the difference between an abstract data type and a data structure. The former is a high-level description: the logical picture of the data and the operations that manipulate them. The latter is concrete: a collection of data elements and the operations that store and retrieve individual elements. An abstract data type is implementation independent, whereas a data structure is implementation dependent. A data structure is how we implement the data in an abstract data type whose values have component parts. The operations on an abstract data type are translated into algorithms on the data structure.

Another view of data focuses on how they are used in a program to solve a particular problem-that is, their application. If we were writing a program to keep track of student grades, we would need a list of students and a way to record the grades for each student. We might take a by-hand grade book and model it in our program. The operations on the grade book might include adding a name, adding a grade, averaging a student's grades, and so on. Once we have written a specification for our grade book data type, we must choose an appropriate data structure to implement it and design the algorithms to implement the operations on the structure.

In modeling data in a program, we wear many hats. That is, we must determine the logical picture of the data, choose the representation of the data, and develop the operations that encapsulate this arrangement. During this process, we consider data from three different perspectives, or levels:

1. *Application (or user) level:* A way of modeling real-life data in a specific context; also called the problem domain

2. *Logical (or abstract) level:* An abstract view of the data values (the domain) and the set of operations to manipulate them

3. *Implementation level:* A specific representation of the structure to hold the data items, and the coding of the operations in a programming language (if the operations are not already provided by the language)

In our discussion, we refer to the second perspective as the "abstract data type." Because an abstract data type can be a simple type such as an integer or character, as well as a structure that contains component elements, we also use the term "composite data type" to refer to abstract data types that may contain component elements. The third level describes how we actually represent and manipulate the data in memory: the data structure and the algorithms for the operations that manipulate the items on the structure.

Let's see what these different viewpoints mean in terms of our library analogy. At the application level, we focus on entities such as the Library of Congress, the Dimsdale Collection of Rare Books, and the Austin City Library.

At the logical level, we deal with the "what" questions. What is a library? What services (operations) can a library perform? The library may be seen abstractly as "a collection of books" for which the following operations are specified:

- Check out a book
- Check in a book
- Reserve a book that is currently checked out
- Pay a fine for an overdue book
- Pay for a lost book

How the books are organized on the shelves is not important at the logical level, because the patrons don't have direct access to the books. The abstract viewer of library services is not concerned with how the librarian actually organizes the books in the library. Instead, the library user needs to know only the correct way to invoke the desired operation. For instance, here is the user's view of the

operation to check in a book: Present the book at the check-in window of the library from which the book was checked out, and receive a fine slip if the book is overdue.

At the implementation level, we deal with the "how" questions. How are the books cataloged? How are they organized on the shelf? How does the librarian process a book when it is checked in? For instance, the implementation information includes the fact that the books are cataloged according to the Dewey decimal system and arranged in four levels of stacks, with 14 rows of shelves on each level. The librarian needs such knowledge to be able to locate a book. This information also includes the details of what happens when each operation takes place. For example, when a book is checked back in, the librarian may use the following algorithm to implement the check-in operation:

CheckInBook

```
Examine due date to see whether the book is late.
if book is late
    Calculate fine.
    Issue fine slip.
Update library records to show that the book has been returned.
Check reserve list to see if someone is waiting for the book.
if book is on reserve list
    Put book on the reserve shelf.
else
    Replace book on the proper shelf, according to the library's shelf arrangement scheme.
```

All of this activity, of course, is invisible to the library user. The goal of our design approach is to hide the implementation level from the user.

Picture a wall separating the application level from the implementation level, as shown in Figure 2.4. Imagine yourself on one side and another programmer on the other side. How do the two of you, with your separate views of the data, communicate across this wall? Similarly, how do the library user's view and the librarian's view of the library come together? The library user and the librarian communicate through the data abstraction. The abstract view provides the specification of the accessing operations without telling how the operations work. It tells *what* but not *how.* For instance, the abstract view of checking in a book can be summarized in the following specification:

CheckInBook (library, book, fineSlip)

Function:	Check in a book
Preconditions:	book was checked out of this library; book is presented at the check-in desk
Postconditions:	fineSlip is issued if book is overdue; contents of library is the original contents + book

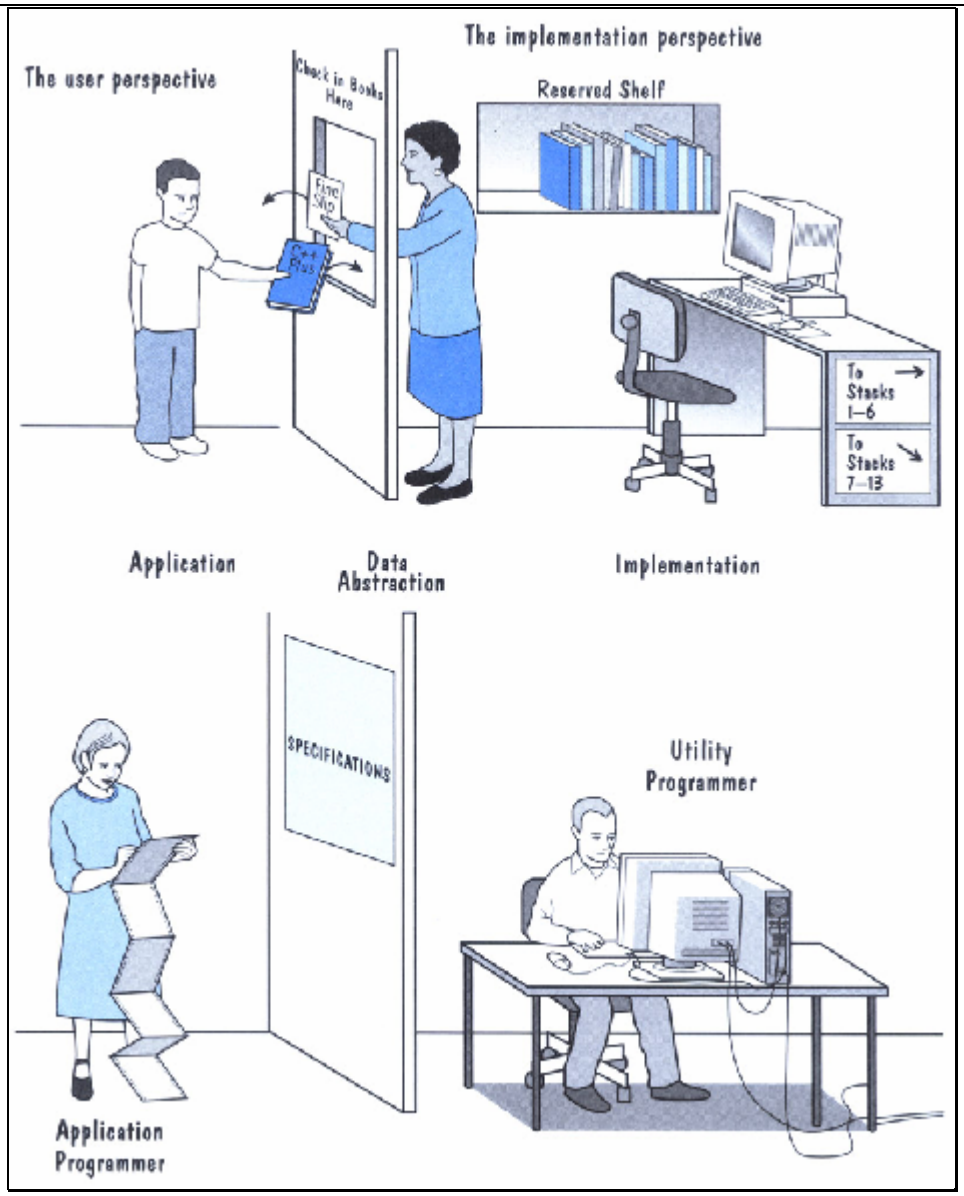


Figure: Communication between the application level and implementation level

The only communication from the user into the implementation level occurs in terms of input specifications and allowable assumptions-the preconditions of the accessing routines. The only output from the implementation level back to the user is the transformed data structure described by the output specifications, or postconditions, of the routines. The abstract view hides the data structure, but provides windows into it through the specified accessing operations.

When you write a program as a class assignment, you often deal with data at all three levels. In a job situation, however, you may not. Sometimes you may program an application that uses a data type that has been implemented by another programmer. Other times you may develop "utilities" that are called by other programs. In this book we ask you to move back and forth between these levels.

Abstract Data Type Operator Categories

In general, the basic operations that are performed on an abstract data type are classified into four categories: *constructors, transformers* (also called *mutators*), *observers,* and *iterators.*

A constructor is an operation that creates a new instance (object) of an abstract data type. It is almost always invoked at the language level by some sort of declaration. Transformers are operations that change the state of one or more of the data values, such as inserting an item into an object, deleting an item from an object, or making an object empty. An operation that takes two objects and merges them into a third object is a binary transformer.

An observer is an operation that allows us to observe the state of one or more of the data values without changing them. Observers come in several forms: *predicates* that ask if a certain property is true, *accessor* or *selector* functions that return a copy of an item in the object, and *summary* functions that return information about the object as a whole. A Boolean function that returns true if an object is empty and false if it contains any components is an example of a predicate. A function that returns a copy of the last item put into the structure is an example of an accessor function. A function that returns the number of items in the structure is a summary function.

An iterator is an operation that allows us to process all components in a data structure sequentially. Operations that print the items in a list or return successive list items are iterators. Iterators are only defined on structured data types.

In later, we use these ideas to define and implement some useful data types that may be new to you. First, however, let's explore the built-in composite data types C provides for us.

Definition of Data Structure: An organization of information, usually in memory, for better algorithm efficiency, such as queue, stack, linked list, heap, dictionary, and tree, or conceptual unity, such as the name and address of a person. It may include redundant information, such as length of the list or number of nodes in a subtree.

Definition of Algorithm: A computable set of steps to achieve a desired result.

Note: Most data structures have associated algorithms to perform operations, such as search, insert, or balance, that maintain the properties of the data structure.

The subjects of this book are data structures and algorithms. A *data structure* is an arrangement of data in a computer's memory (or sometimes on a disk). Data structures include linked lists, stacks, binary trees, and hash tables, among others. *Algorithms* manipulate the data in these structures in various ways, such as inserting a new data item, searching for a particular item, or sorting the items. You can think of an algorithm as a recipe: a list of detailed instructions for carrying out an activity.

5. Overview of Data Structures

Another way to look at data structures is to focus on their strengths and weaknesses. This section provides an overview, in the form of a table, of the major data storage structures discussed in this book. This is a bird's-eye view of a landscape that we'll be covering later at ground level, so don't be alarmed if it looks a bit mysterious. Table shows the advantages and disadvantages of the various data structures described in this book.

- A data structure is the organization of data in a computer's memory (or in a disk file).
- The correct choice of data structure allows major improvements in program efficiency.
- Examples of data structures are arrays, stacks, and linked lists.
- An algorithm is a procedure for carrying out a particular task.

TABLE CHARACTERISTICS OF DATA STRUCTURES

Data Structure	*Advantages*	*Disadvantages*
Array	Quick insertion, very fast access if index known.	Slow search, slow deletion, fixed size.
Ordered array	Quicker search than unsorted array.	Slow insertion and deletion, fixed size.
Stack	Provides last-in, first-out access.	Slow access to other items.
Queue	Provides first-in, first-out access.	Slow access to other items.
Linked list	Quick insertion, quick deletion.	Slow search.
Binary tree	Quick search, insertion, deletion (if tree remains balanced).	Deletion algorithm is complex.
Red-black tree	Quick search, insertion, deletion. Tree always balanced.	Complex.
2-3-4 tree	Quick search, insertion, deletion. Tree always balanced. Similar trees good for disk storage.	Complex.
Hash table	Very fast access if key known. Fast insertion.	Slow deletion, access slow if key not known, inefficient memory usage.
Heap	Fast insertion, deletion, access to largest item.	Slow access to other items.

Algorithms + Data Structures = Programs

Algorithms ↔ Data Structures

6. *Exercises*

E1. Give an example of a relationship between Data Structure and Algorithm

E2. Write a program to finf maximum value of 4 numbers. Using 2 types of data structures: array of 4 numbers, 4 int numbers seperated.

E3. Imagine a group of data you would like to put in a computer so it could be accessed and manipulated. For example, if you collect old CDROMs, you might want to catalog them so you could search for a particular author or a specific date...

CHAPTER 1: C LANGUAGE

1. ADDRESS

Introduction

For every variable declared in a program there is some memory allocation. Memory is specified in arrays of bytes, the size of which depending on the type of variable. For the integer type, 2 bytes are allocated, for floats, 4 bytes are allocated, etc. For every variable there are two attributes: *address* and *value*, described as follows:

Program

```
#include <stdio.h>
main ()
{
    int i, j, k;      //A
    i = 10;           //B
    j = 20;           //C
    k = i + j; //D
    printf ("Value of k is %d\n", k);
}
```

Explanation

Memory allocations to the variables can be explained using the following variables:

```
1.    100,i      10
2.    200, j     20
3.    300,k      30
```

When you declare variables `i`, `j`, `k`, memory is allocated for storing the values of the variables. For example, 2 bytes are allocated for `i`, at location 100, 2 bytes are allocated for `j` at location 200, and 2 bytes allocated for `k` at location 300. Here 100 is called the address of `i`, 200 is called address of `j`, and 300 is called the address of `k`.

4. When you execute the statement `i = 10`, the value 10 is written at location 100, which is specified in the figure. Now, the address of `i` is 100 and the value is 10. During the lifetime of variables, the address will remain fixed and the value may be changed. Similarly, value 20 is written at address 200 for `j`.

During execution, addresses of the variables are taken according to the type of variable, that is, local or global. Local variables usually have allocation in stack while global variables are stored in runtime storage.

Points to Remember

- Each variable has two attributes: address and value.
- The address is the location in memory where the value of the variable is stored.
- During the lifetime of the variable, the address is not changed but the value may change.

2. POINTERS

Introduction

A *pointer* is a variable whose value is also an address. As described earlier, each variable has two attributes: address and value. A variable can take any value specified by its data type. For example, if the variable `i` is of the integer type, it can take any value permitted in the range specified by the integer data type. A pointer to an integer is a variable that can store the address of that integer.

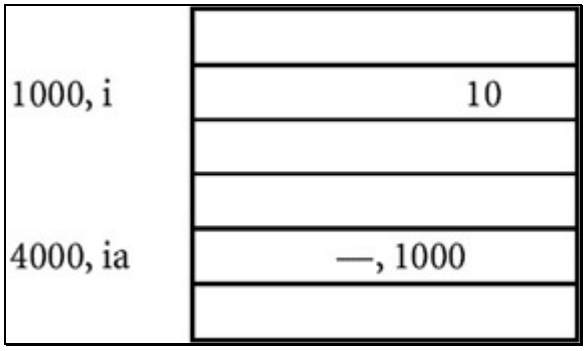
Program

```
#include <stdio.h>
main ()
{
   int i;          //A
   int * ia;       //B
   i = 10;         //C
   ia = &i;   //D

    printf (" The address of i is %8u \n", ia);          //E
    printf (" The value at that location is %d\n", i);   //F
    printf (" The value at that location is %d\n", *ia); //G
  *ia = 50;                                  //H
    printf ("The value of i is %d\n", i);                //I
}
```

Explanation

1. The program declares two variables, so memory is allocated for two variables. `i` is of the type of `int`, and `ia` can store the address of an integer, so it is a pointer to an integer.
2. The memory allocation is as follows:



3. `i` gets the address 1000, and `ia` gets address 4000.
4. When you execute `i = 10`, 10 is written at location 1000.
5. When you execute `ia = &i` then the address and value are assigned to `i`, thus `i` has the address of 4000 and value is 1000.
6. You can print the value of `i` by using the format `%au` because addresses are usually in the format unsigned long, as given in statement E.
7. Statement F prints the value of `i`, (at the location 1000).
8. Alternatively, you can print the value at location 1000 using statement G. `*ia` means you are printing the value at the location specified by `ia`. Since `i` has the value for 1000, it will print the value at location 1000.

9. When you execute `*ia = 50`, which is specified by statement H, the value 50 is written at the location by `ia`. Since `ia` specifies the location 1000, the value at the location 1000 is written as 50.
10. Since `i` also has the location 1000, the value of `i` gets changed automatically from 10 to 50, which is confirmed from the `printf` statement written at position `i`.

Points to Remember

1. Pointers give a facility to access the value of a variable indirectly.
2. You can define a pointer by including a* before the name of the variable.
3. You can get the address where a variable is stored by using &.

3. ARRAYS

Introduction

An *array* is a data structure used to process multiple elements with the same data type when a number of such elements are known. You would use an array when, for example, you want to find out the average grades of a class based on the grades of 50 students in the class. Here you cannot define 50 variables and add their grades. This is not practical. Using an array, you can store grades of 50 students in one entity, say grades, and you can access each entity by using subscript as grades[1], grades[2]. Thus you have to define the array of grades of the float data type and a size of 50. An array is a composite data structure; that means it had to be constructed from basic data types such as array integers.

Program

```
#include <stdio.h>
main()
{
    int a[5];  \\A
    for(int i = 0;i<5;i++)
    {
       a[i]=i;\\B
    }
    printarr(a);
}
void printarr(int a[])
{
    for(int i = 0;i<5;i++)
    {
        printf("value in array %d\n",a[i]);
    }
}
```

Explanation

1. Statement A defines an array of integers. The array is of the size 5—that means you can store 5 integers.
2. Array elements are referred to using subscript; the lowest subscript is always 0 and the highest subscript is (size –1). If you refer to an array element by using an out-of-range subscript, you will get an error. You can refer to any element as `a[0]`, `a[1]`, `a[2]`, etc.
3. Generally, you can use a `for` loop for processing an array. For the array, consecutive memory locations are allocated and the size of each element is same.

4. The array name, for example, a, is a pointer constant, and you can pass the array name to the function and manipulate array elements in the function. An array is always processed element by element.
5. When defining the array, the size should be known.

Note The array subscript has the highest precedence among all operators thus a[1] * a[2] gives the multiplication of array elements at position 1 and position 2.

Points to Remember

1. An array is a composite data structure in which you can store multiple values. Array elements are accessed using subscript.
2. The subscript operator has the highest precedence. Thus if you write `a[2]++`,it increments the value at location 2 in the array.
3. The valid range of subscript is 0 to size −1.

4. ADDRESS OF EACH ELEMENT IN AN ARRAY

Introduction

Each element of the array has a memory address. The following program prints an array limit value and an array element address.

Program

```
#include <stdio.h>
void printarr(int a[]);
main()
{
    int a[5];
    for(int i = 0;i<5;i++)
    {
        a[i]=i;
    }
    printarr(a);
}
void printarr(int a[])
{
    for(int i = 0;i<5;i++)
    {
        printf("value in array %d\n",a[i]);
    }
}
void printdetail(int a[])
{
    for(int i = 0;i<5;i++)
    {
        printf("value in array %d and address is %16lu\n",a[i],&a[i]);
\\ A
```

```
    }
}
```

Explanation

1. The function `printarr` prints the value of each element in `arr`.
2. The function `printdetail` prints the value and address of each element as given in statement A. Since each element is of the integer type, the difference between addresses is 2.
3. Each array element occupies consecutive memory locations.
4. You can print addresses using place holders `%16lu` or `%p`.

Point to Remember

For array elements, consecutive memory locations are allocated.

5. ACCESSING AN ARRAY USING POINTERS

Introduction

You can access an array element by using a pointer. For example, if an array stores integers, then you can use a pointer to integer to access array elements.

Program

```
#include <stdio.h>
void printarr(int a[]);
void printdetail(int a[]);
main()
{
    int a[5];
    for(int i = 0;i<5;i++)
    {
        a[i]=i;
    }
    printdetail(a);
}
void printarr(int a[])
{
    for(int i = 0;i<5;i++)
    {
        printf("value in array %d\n",a[i]);
    }
}
void printdetail(int a[])
{
    for(int i = 0;i<5;i++)
    {
        printf("value in array %d and address is %8u\n",a[i],&a[i]);
    }
```

```
}
void print_usingptr(int a[]) \\ A
{
    int *b;    \\ B
    b=a;               \\ C
    for(int i = 0;i<5;i++)
    {
        printf("value in array %d and address is %16lu\n",*b,b); \\ D
        b=b+2; \\E
    }
}
```

Explanation

1. The function `print_using` pointer given at statement A accesses elements of the array using pointers.
2. Statement B defines variable b as a pointer to an integer.
3. Statement C assigns the base address of the array to b, thus the array's first location (a[0]) is at 100; then b will get the value 100. Other elements of the array will add 102,104, etc.
4. Statement D prints two values: *b means the value at the location specified by b, that is, the value at the location 100. The second value is the address itself, that is, the value of b or the address of the first location.
5. For each iteration, b is incremented by 2 so it will point to the next array location. It is incremented by 2 because each integer occupies 2 bytes. If the array is long then you may increment it by 4.

Points to Remember

1. Array elements can be accessed using pointers.
2. The array name is the pointer constant which can be assigned to any pointer variable.

6. MANIPULATING ARRAYS USING POINTERS

Introduction

When the pointer is incremented by an increment operator, it is always right incremented. That is, if the pointer points to an integer, the pointer is incremented by 2, and, if it is long, it is incremented by 4.

Program

```
#include <stdio.h>
void printarr(int a[]);
void printdetail(int a[]);
void print_usingptr(int a[]);
main()
{
    int a[5];
    for(int i = 0;i<5;i++)
    {
        a[i]=i;
    }
```

```
    print_usingptr(a);
}
void printarr(int a[])
{
    for(int i = 0;i<5;i++)
    {
        printf("value in array %d\n",a[i]);
    }
}
void printdetail(int a[])
{
    for(int i = 0;i<5;i++)
    {
        printf("value in array %d and address is %8u\n",a[i],&a[i]);
    }
}
void print_usingptr(int a[])
{
    int *b;
    b=a;
    for(int i = 0;i<5;i++)
    {
        printf("value in array %d and address is %16lu\n",*b,b);
        b++;                    // A
    }
}
```

Explanation

1. This function is similar to the preceding function except for the difference at statement A. In the previous version, b = b+2 is used. Here b++ is used to increment the pointer.
2. Since the pointer is a pointer to an integer, it is always incremented by 2.

Point to Remember

The increment operator increments the pointer according to the size of the data type.

7. ANOTHER CASE OF MANIPULATING AN ARRAY USING POINTERS

Introduction

You can put values in the memory locations by using pointers, but you cannot assign the memory location to an array to access those values because an array is a pointer constant.

Program

```
#include <stdio.h>
void printarr(int a[]);
```

```
void printdetail(int a[]);
void print_usingptr_a(int a[]);
main()
{
    int a[5];
    int *b;
    int *c;
    for(int i = 0;i<5;i++)
    {
        a[i]=i;
    }
    printarr(a);
    *b=2;            \\ A
    b++;             \\ B
    *b=4;            \\ C
    b++;
    *b=6;            \\ D
    b++;
    *b=8;            \\ E
    b++;
    *b=10;
    b++;
    *b=12;
    b++;
    a=c; //error     \\F
    printarr(a);


}
void printarr(int a[])
{
    for(int i = 0;i<5;i++)
    {
        printf("value in array %d\n",a[i]);
    }
}
void printdetail(int a[])
{
    for(int i = 0;i<5;i++)
    {
        printf("value in array %d and address is %16lu\n",a[i],&a[i]);
    }
}

void print_usingptr_a(int a[])
{
```

```
    for(int i = 0;i<5;i++)
    {
        printf("value in array %d and address is %16lu\n",*a,a); \\ F
        a++; // increase by 2 bytes         \\ G
    }
}
```

Explanation

1. You can assign a value at the location specified by b using statement A.
2. Using statement B, you can point to the next location so that you can specify a value at that location using statement C. Using this procedure, you can initialize 5 locations.
3. You cannot assign the starting memory location as given by statement F to access those elements because a is a pointer constant and you cannot change its value.
4. The function `print_usingptr_a` works correctly even though you are writing `a++`. This is because when you pass a as a pointer in an actual parameter, only the value of a is passed and this value is copied to the local variable. So changing the value in the local variable will not have any effect on the outside function.

Point to Remember

The array limit is a pointer constant and you cannot change its value in the program.

8. TWO-DIMENSIONAL ARRAY

Introduction

You can define two- or multi-dimensional arrays. It is taken as an array of an array. Logically, the two-dimensional array 3 X 2 is taken as

```
 3     1
 5     2
 8     7
```

Here there are three arrays, i.e. one array in each row. The values are stored as

```
3   1  5   2    8  7
```

This style is called *row measure form*. Each row array is represented as `a[0]`, which consists of elements 3 and 1. `a[1]` consists of 5 2 and `a[2]` consists of 8 7. Each element of `a[0]` is accessed as a `[0] [0]` and `a[0] [1]`, thus the value of `a[0][0]` and `a[0][1]` is 1.

Program

```
#include <stdio.h>
void printarr(int a[][]);
void printdetail(int a[][]);
void print_usingptr(int a[][]);
main()
{
    int a[3][2];      \\ A
    for(int i = 0;i<3;i++)
```

```
        for(int j=0;j<2 ;j++)
        {
                {
                        a[i]=i;
                }
        }
    printdetail(a);
}
void printarr(int a[][])
{
    for(int i = 0;i<3;i++)
        for(int j=0;j<2;j++)
        {
                {
                        printf("value in array %d\n",a[i][j]);
                }
        }
}
void printdetail(int a[][])
{
    for(int i = 0;i<3;i++)
        for(int j=0;j<2;j++)
        {
                {
                        printf(
                        "value in array %d and address is %8u\n",
                        a[i][j],&a[i][j]);
                }
        }
}
void print_usingptr(int a[][])
{
    int *b;   \\ B
    b=a;                \\ C
    for(int i = 0;i<6;i++)   \\ D
    {
        printf("value in array %d and address is %16lu\n",*b,b);
        b++; // increase by 2 bytes \\ E
    }
}
```

Explanation

1. Statement A declares a two-dimensional array of the size 3 × 2.
2. The size of the array is 3 × 2, or 6.
3. Each array element is accessed using two subscripts.
4. You can use two `for` loops to access the array. Since `i` is used for accessing a row, the outer loop prints elements row-wise, that is, for each value of `i`, all the column values are printed.

5. You can access the element of the array by using a pointer.
6. Statement B assigns the base address of the array to the pointer.
7. The `for` loop at statement C increments the pointer and prints the value that is pointed to by the pointer. The number of iterations done by the `for` loop, 6, is equal to the array.
8. Using the output, you can verify that C is using row measure form for storing a two-dimensional array.

Points to Remember

1. You can define a multi-dimensional array in C.
2. You have to provide multiple subscripts for accessing array elements.
3. You can access array elements by using a pointer.

9. POINTER ARRAYS

Introduction

You can define a *pointer array* similarly to an array of integers. In the pointer array, the array elements store the pointer that points to integer values.

Program

```
#include <stdio.h>
void printarr(int *a[]);
void printarr_usingptr(int *a[]);
int *a[5];      \\ A
main()
{

    int i1=4,i2=3,i3=2,i4=1,i5=0;        \\ B
    a[0]=&i1;                          \\ C
    a[1]=&i2;
    a[2]=&i3;
    a[3]=&i4;
    a[4]=&i5;

    printarr(a);
    printarr_usingptr(a);
}
void printarr(int *a[])                  \\ D
{
    printf("Address        Address in array         Value\n");
    for(int  j=0;j<5;j++)
    {
        printf("%16u           %16u                %d\n",
        a[j],a[j],a[j]);           \\E
    }
}
```

```
void printarr_usingptr(int *a[])
{
    int j=0;
    printf("using pointer\n");
    for( j=0;j<5;j++)
    {
        printf("value of elements   %d %16lu %16lu\n",**a,*a,a); \\ F
        a++;
    }
}
```

Explanation

1. Statement A declares an array of pointers so each element stores the address.
2. Statement B declares integer variables and assigns values to these variables.
3. Statement C assigns the address of `i1` to element `a[0]` of the array. All the array elements are given values in a similar way.
4. The function `print_arr` prints the address of each array element and the value of each array element (the pointers and values that are pointed to by these pointers by using the notations `&a[i]`, `a[i]` and `*a[i]`).
5. You can use the function `printarr_usingptr` to access array elements by using an integer pointer, thus a is the address of the array element, `*a` is the value of the array element, and `**a` is the value pointed to by this array element.

Point to Remember

You can store pointers in arrays. You can access values specified by these values by using the * notations.

10. STRUCTURES

Introduction

Structures are used when you want to process data of multiple data types but you still want to refer to the data as a single entity. Structures are similar to records in Cobal or Pascal. For example, you might want to process information on students in the categories of name and marks (grade percentages). Here you can declare the structure 'student' with the fields 'name' and 'marks', and you can assign them appropriate data types. These fields are called members of the structure. A member of the structure is referred to in the form of `structurename.membername`.

Program

```
struct student      \\ A
{
    char name[30];   \\ B
    float marks;     \\ C
}   student1, student2;      \\ D

main ( )
{
    struct student student3; \\ E
```

```
    char s1[30];              \\ F
    float f;                  \\ G
    scanf ("%s", name);       \\ H
    scanf (" %f", & f);  \\ I
    student1.name = s1;       \\ J
    student2.marks = f;       \\ K
    printf (" Name is %s \n", student1.name);      \\ L
    printf (" Marks are %f \n", student2.marks);   \\ M
}
```

Explanation

1. Statement A defines the structure type `student`. It has two members: `name` and `marks`.
2. Statement B defines the structure member `name` of the type `character 30`.
3. Statement C defines the structure member `marks` of the type `float`.
4. Statement D defines two structure variables: `structure1` and `structure2`. In the program you have to use variables only. Thus `struct student` is the data type, just as `int` and `student1` is the variable.
5. You can define another variable, `student3`, by using the notations as specified in statement E.
6. You can define two local variables by using statements F and G.
7. Statement J assigns `s1` to a member of the structure. The structure member is referred to as structure `variablename.membername`. The member `student1.name` is just like an ordinary string, so all the operations on the string are allowed. Similarly, statement J assigns a value to `student1.marks`
8. Statement L prints the marks of `student1` just as an ordinary string.

Points to Remember

1. Structures are used when you want to process data that may be of multiple data types.
2. Structures have members in the form: structurename.membername.

11. STRUCTURE POINTERS

Introduction

You can process the structure using a *structure pointer*.

Program

```
struct student       \\ A
{
    char name[30];   \\ B
    float marks;     \\ C
};              \\ D

main ( )
{
    struct student *student1;      \\ E
    struct student student2; \\ F
    char s1[30];
    float f;
```

```
    student1 = &student2;    \\ G
    scanf ("%s", name);          \\ H
    scanf (" %f", & f);      \\ I
    *student1.name = s1;         \\ J student1-> name = f;
    *student2.marks = f;         \\ K student1-> marks = s1;

    printf (" Name is %s \n", *student1.name);    \\ L
    printf (" Marks are %f \n", *student2.marks); \\ M
}
```

Explanation

1. Statement E indicates that `student1` is the pointer to the structure.
2. Statement F defines the structure variable `student2` so that memory is allocated to the structure.
3. Statement G assigns the address of the structure `student2` to the pointer variable structure `student1`.
4. In the absence of statement G, you cannot refer to the structure using a pointer. This is because when you define the pointer to the structure, the memory allocation is done only for pointers; the memory is not allocated for structure. That is the reason you have to declare a variable of the structure type so that memory is allocated to the structure and the address of the variable is given to the point.
5. Statement J modifies a member of the structure using the * notation. The alternative notation is
6. `student1-> name = f;`
7. `student1-> marks = s1;`

Points to Remember

1. You can access members of the structure using a pointer.
2. To access members of the structure, you have to first create a structure so that the address of the structure is assigned to the pointer.

12. Exercises

E1. Write a function to check if an integer is negative; the declaration should look like

bool is_positive(int i);

E2. Write a function to calculate if a number is prime.

E3. Write a program to input an int array. Display the numbers of negative elements, prime elements in this array.

E4. a. Write a function that can find the largest element in the array that is passed to the function as a parameter. b. Write a program that inputs an array and invokes the above function to find the largest element in the array and print it out.

E4. Given the following function that can find the largest element in an integer array. Notice that the function uses pointer arithmetic:

```
int findMax(int * vals, int numEls)

{
```

```
int j, max = *vals;

for (j = 1; j < numEls; j++)

  if (max < *(vals + j))

    max = *(vals + j);

return max;

}
```

Write a C++ program that inputs an integer array and invokes the above function to find the largest element in that array and displays the result out.

E5. a) Define a structure with 2 fields: width and height for rectangle. Input an array of rectangle, then display each area and perimeter of each rectanle in array. b) Using pointer to loop around array.

E6. The hexadecimal digits are the ordinary, base-10 digits '0' through '9' plus the letters 'A' through 'F'. In the hexadecimal system, these digits represent the values 0 through 15, respectively. Write a function named hexValue that uses a switch statement to find the hexadecimal value of a given character. The character is a parameter to the function, and its hexadecimal value is the return value of the function. You should count lower case letters 'a' through 'f' as having the same value as the corresponding upper case letters. If the parameter is not one of the legal hexadecimal digits, return -1 as the value of the function.

E7. Write a program: Read in a series of numbers representing tst tubes and readings. The sentinel is a single minus one. Your program should keep the product of the readings for each test tube. At the end, it display the test tube numbers and product of each reading. Be sure not to display values for test tubes that have no value. After this is done, display the minimum of the products. If the input were:

```
3 100

5 6

2 50

5 12

4 4

4 7

3 500

2 75

4 3

4 2

5 3

5 5

-1
```

The output would be

```
Test Tube Number 2 Product 3750
```

```
Test Tube Number 3 Product 50000

Test Tube Number 4 Product 168

Test Tube Number 5 Product 1080

 Min is 168
```

CHAPTER 2: FUNCTION & RECURSION

1. FUNCTION

Introduction

Functions are used to provide modularity to the software. By using functions, you can divide complex tasks into small manageable tasks. The use of functions can also help avoid duplication of work. For example, if you have written the function for calculating the square root, you can use that function in multiple programs.

Program

```
#include <stdio.h>
int add (int x, int y)      //A
{
    int z;           //B
    z = x + y;
    return (z);      //C
}
main ()
{
    int i, j, k;
    i = 10;
    j = 20;
    k = add(i, j);        //D
    printf ("The value of k is%d\n", k);   //E
}
```

Explanation

1. This function adds two integers and returns their sum.
2. When defining the name of the function, its return data type and parameters must also be defined. For example, when you write
3. `int add (int x, int y)`

   `int` is the type of data to be returned, `add` is the name of the function, and `x` and `y` are the parameters of the type `int`. These are called *formal parameters.*
4. The body of a function is just like the body of main. That means you can have variable declarations and executable statements.
5. A function should contain statements that return values compatible with the function's return type.
6. The variables within the function are called *local variables*.
7. After executing the return statement, no further statements in the function body are executed.
8. The name of the function can come from the arguments that are compatible with the formal parameters, as indicated in statement D.
9. The arguments that are used in the call of the function are called *actual parameters.*
10. During the call, the value of the actual parameter is copied into the formal parameter and the function body is executed.
11. After the return statement, control returns to the next statement which is after the call of the function.

Points to Remember

1. A function provides modularity and readability to the software.
2. To define the function, you have to define the function name, the return data type and the formal parameters.
3. Functions do not require formal parameters.
4. If the function does not return any value, then you have to set the return data type as void.
5. A call to a function should be compatible with the function definition.

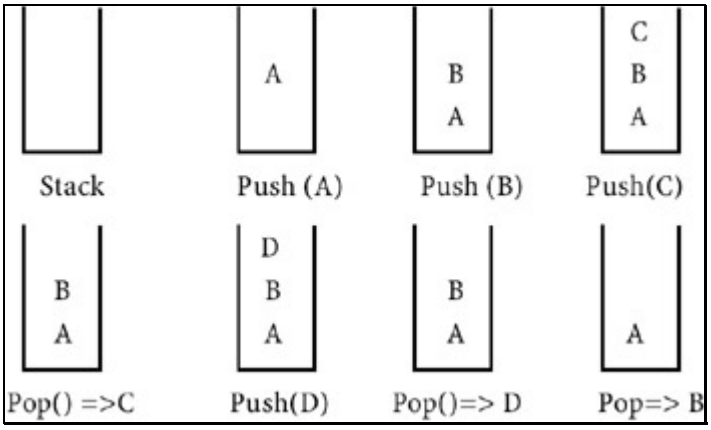
2. THE CONCEPT OF STACK

Introduction

A *stack* is memory in which values are stored and retrieved in "last in first out" manner by using operations called *push* and *pop*.

Program

Suppose you want to insert values in a stack and retrieve values from the stack. The operations would proceed in the following manner:



Explanation

1. Initially, the stack is empty. When you start `push A`, `A` is placed in the stack.
2. Similarly, `push B` and `push C` put these elements in the stack; the last element pushed is `C`.
3. The `pop` operation takes the topmost element from the stack. Thus the element `C`, which was put in last, is retrieved first. This method is called *last-in first-out (LIFO)*.
4. The `push D` operation puts element `D` in the stack above `B`.
5. Thus `push` puts the element on the top of the stack and `pop` takes the element from the top of the stack. The element `A` which is pushed is the last element taken from the stack.

Point to Remember

The last-in first-out retrieval from the stack is useful for controlling the flow of execution during the function call.

3. THE SEQUENCE OF EXECUTION DURING A FUNCTION CALL

Introduction

When the function is called, the current execution is temporarily stopped and the control goes to the called function. After the call, the execution resumes from the point at which the execution is stopped.

To get the exact point at which execution is resumed, the address of the next instruction is stored in the stack. When the function call completes, the address at the top of the stack is taken.

Program

```
main ( )
{
    printf ("1 \n"); // 1
    printf ("2 \n"); // 2
    printf ("3 \n"); // 3
    printf ("4 \n"); // 4
    printf ("5 \n"); // 5
    f1 ( );
    printf ("6 \n"); // 6
    printf ("7 \n"); // 7
    printf ("8 \n"); // 8
}
void f1 (void)
{
    printf ("f1-9 \n");     // 9
    printf ("f1-10 \n");    // 10
    f2 ( );
    printf ("f1-11 \n");    // 11
    printf ("f1-12 \n");    // 12
}
void f2 (void)
{
    printf ("f2-13 \n");    // 13
    printf ("f2-14 \n");    // 14
    printf ("f3-15 \n");    // 15
}
```

Explanation

1. Statements 1 to 5 are executed and function `f1( )` is called.
2. The address of the next instruction is pushed into the stack.
3. Control goes to function `f1( )`, which starts executing.
4. After the 10th statement, fuction `f2` is called and address of the next instruction, 11, is pushed into the stack.
5. Execution begins for function `f2` and statements 13, 14, and 15 are executed.
6. When `f2` is finished, the address is popped from the stack. So address 11 is popped.
7. Control resumes from statement 11.
8. Statements 11 and 12 are executed.
9. After finishing the `f1` address is popped from the stack, i.e. 6.

10. Statements 6, 7, and 8 are executed.
11. The execution sequence is `1 2 3 4 5 f1_9 f1_10 f2_13 f2_14 f2_15 f1_11 f1_12 6 7 8.`

Points to Remember

1. Functions or sub-programs are implemented using a stack.
2. When a function is called, the address of the next instruction is pushed into the stack.
3. When the function is finished, the address for execution is taken by using the pop operation.

4. PARAMETER PASSING

Introduction

Information can be passed from one function to another using parameters.

Program

```
main ( )
{
    int i;
    i = 0;
    printf (" The value of i before call %d \n", i);
    f1 (i);
    printf (" The value of i after call %d \n", i);
}
void f1 (int k)
{
    k = k + 10;
}
```

Explanation

1. The parameter used for writing the function is called the formal parameter, `k` in this case.
2. The argument used for calling the function is called the actual parameter.
3. The actual and formal parameters may have the same name.
4. When the function is called, the value of the actual parameter is copied into the formal parameter. Thus `k` gets the value 0. This method is called *parameter passing by value*.
5. Since only the value of `i` is passed to the formal parameter `k`, and `k` is changed within the function, the changes are done in `k` and the value of `i` remains unaffected.
6. Thus `i` will equal 0 after the call; the value of `i` before and after the function call remains the same.

Points to Remember

1. C uses the method of parameter passing by value.
2. In parameter passing by value, the value before and after the call remains the same.

5. CALL BY REFERENCE

Introduction

Suppose you want to pass a parameter under the following conditions:

1. You need to change the value of the parameter inside the function.
2. You are interested in the changed value after the function completes.

In languages such as Pascal, you have the option of passing the parameter by reference. C, however, does not support this. As explained in the previous example, you cannot have a changed value after the function call because C uses the method of parameter passing by value. Instead, you'll have to implement the function indirectly. This is done by passing the address of the variable and changing the value of the variable through its address.

Program

```
main ( )
{
    int i;
    i = 0;
   printf (" The value of i before call %d \n", i);
    f1 (&i);      // A
   printf (" The value of i after call %d \n", i);
}
void (int *k)         // B
{
    *k = *k + 10;     // C
}
```

Explanation

1. This example is similar to the previous example, except that the function is written using a pointer to an integer as a parameter.
2. Statement C changes the value at the location specified by `*k`.
3. The function is called by passing the address of `i` using notation `&i`.
4. When the function is called, the address of `i` is copied to `k`, which holds the address of the integer.
5. Statement C increments the value at the address specified by `k`.
6. The value at the address of `i` is changed to 10. It means the value of `i` is changed.
7. The `printf` statements after the function call prints the value 10, that is, the changed value of `i`.

Points to Remember

1. Call by reference is implemented indirectly by passing the address of the variable.
2. In this example, the address of `i` is passed during the function call. It does not change; only the value of the address is changed by the function.

6. RESOLVING VARIABLE REFERENCES

Introduction

When the same variable is resolved using both local definition and global definition, the local definition is given preference. This is called the rule of inheritance. It says that when you can resolve a reference to the variable by using multiple definitions, the nearest definition is given preference. Since local definition is the nearest, it gets preference.

Program

```
int i =0;            //Global variable  /A
main()
{
    int i ;                 // local variable for main  / B
    void f1(void) ;         //C
    i =0;                   // D
    printf("value of i in main %d\n",i);   // E
    f1();                                  // F
    printf("value of i after call%d\n",i); // G
}
void f1(void)                       // H
{
    int i=0;          //local variable for f1      // I
    i = 50;                                 // J
}
```

Explanation

1. Here `i` is declared globally and locally in function main and in function `f1`, respectively, as given in statements A, B and I.
2. Statement D refers to `i`, which can be resolved by using both local definition and global definition. Local definition is given more preference. So statement D refers to the definition at statement B and all the statements in main refer to the definition at statement B, that is, the local definition.
3. When a function is called, statement `i = 50` refers to the local definition in that function (definition at statement I).
4. Using statement G, the value of `i` is 0 because both main and function `f1` refer to their local copies of `i`. So the changed value of `f1` is not reflected in main.
5. Even if you comment local definition of function `f1` at statement I the value printed remains the same. This is because main refers to its local copy while `f1` refers to the global variable `i` — the two are different.

Point to Remember

When a variable can be resolved by using multiple references, the local definition is given more preference.

7. RECURSION

Introduction

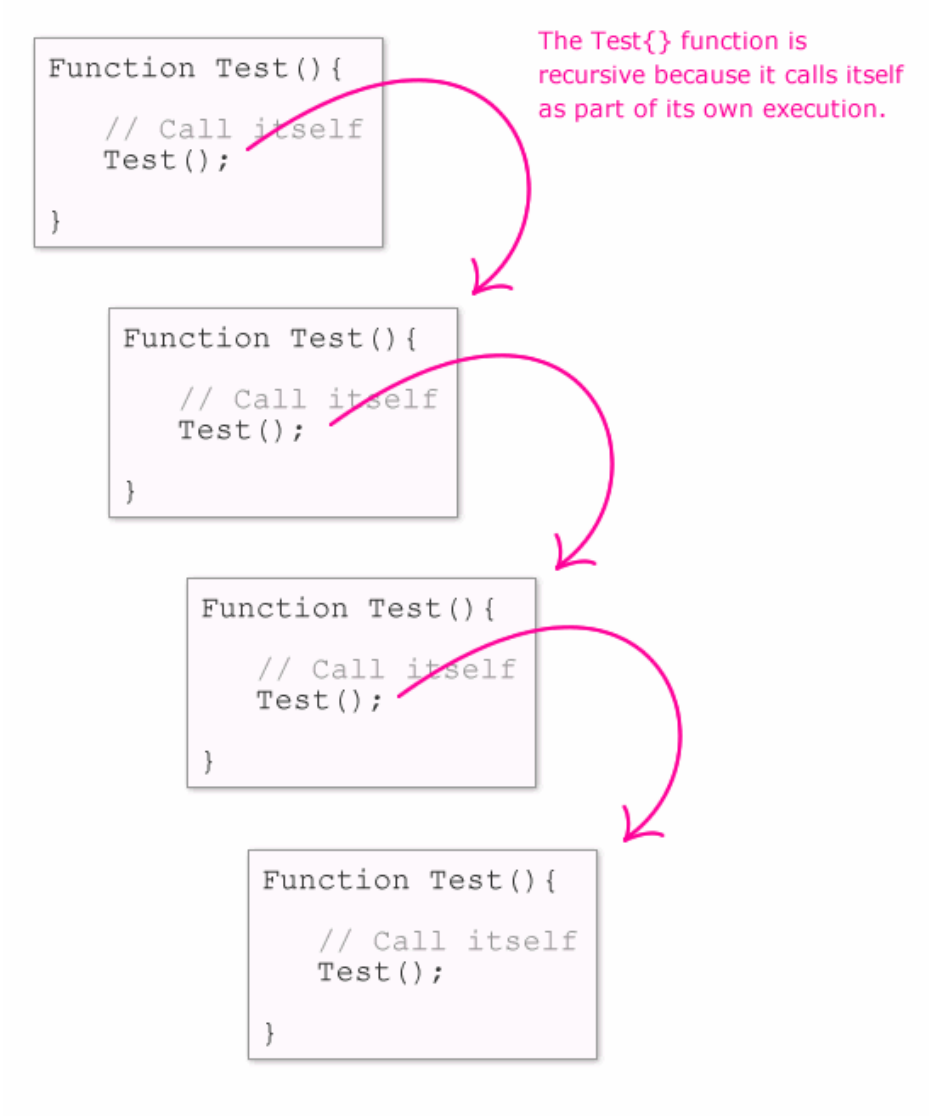
You can express most of the problems in the following program by using *recursion.* We represent the function add by using recursion. RECURSION - A method of programming whereby a function directly or indirectly calls itself. Recursion is often presented as an alternative to iteration.

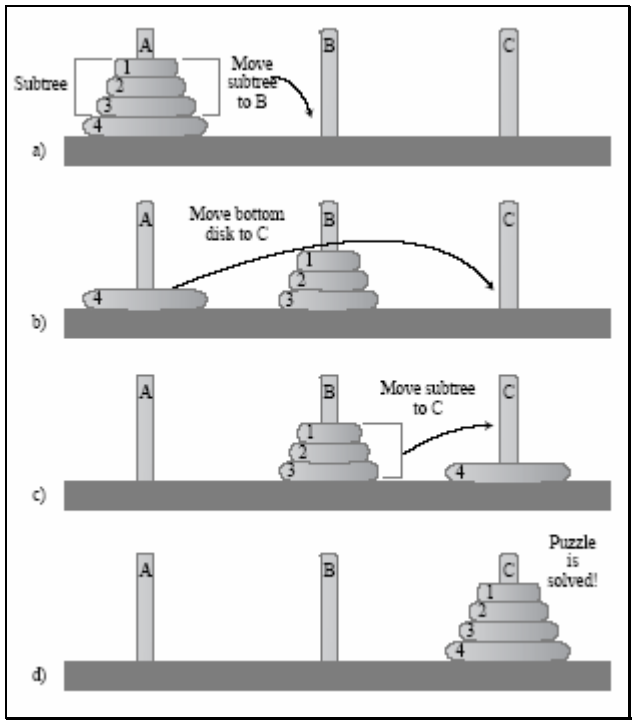
So now, let's step back and look at what recursion in. Recursion is simply the use of a function which, as part of it's own execution code, invokes itself (as seen in this diagram).

As you can see in the picture, the Test() method keeps calling itself. With every Test() method invocation, a new local scope is set up for that function body and a set of parameters (arguments) is passed to it (although, no arguments are used in our example). For each currently executing Test() function, the function cannot return (finish executing) until its nested Test() method call returns.

You might think that you can replace a recursive function with several nested FOR loops, but you would soon find out that this is not true. The beauty behind recursion is that you can search through a dynamic number of levels in your problem. FOR loops do not have this benefit. In order to mimic recursion, you'd have to have an infinite number of nested FOR loops.

Recursion can be used to do something repeatedly (similar to loops). For many problems, it is much easier to use recursion than loops to solve the problems. This is especially true for many problems which can be defined recursively.





Example: Hanoi tower

Example: Factorial

- The factorial is defined in this way:

$$n! = \begin{cases} 1 & n = 0 \\ 1 \times 2 \times \cdots \times n & n > 0 \end{cases}$$

This can be computed by a loop.

- We can also define the factorial as:

$$n! = \begin{cases} 1 & n = 0 \\ n \times (n-1)! & n > 0 \end{cases}$$

So, if we know how to compute the factorial of $n-1$, we know how to compute the factorial of n.

Example: Factorial

```
int factorial(int n)  // assumes n >= 0
{
  if (n == 0)
    return 1;
  else
    return n * factorial(n-1);
}
```

To see how the computation is done, trace `factorial(3)`:

```
factorial(3) = 3 * factorial(2)
             = 3 * (2 * factorial(1))
             = 3 * (2 * (1 * factorial(0)))
             = 3 * (2 * (1 * 1))
```

Program

```
#include <stdio.h>
int add(int pk,int pm);
main()
{
    int k ,i,m;
    m=2;
    k=3;
    i=add(k,m);.
    printf("The value of addition is %d\n",i);
}
int add(int pk,int pm)
{
    if(pm==0) return(pk);          \\ A
    else return(1+add(pk,pm-1));   \\ B
}
```

Explanation

1. The add function is recursive as follows:

```
2.    add (x, y) = 1 + add(x, y-1)      y > 0
3.          = x                  y = 0
4.    for example,
5.        add(3, 2) = 1 + add(3, 1)
6.        add(3, 1) = 1 + add(3, 0)
7.        add(3, 0) = 3
8.        add(3, 1) = 1+3 = 4
9.        add(3, 2) = 1+4 = 5
```

10. The recursive expression is `1+add(pk, pm-1)`. The terminating condition is `pm = 0` and the recursive condition is `pm > 0`.

8. STACK OVERHEADS IN RECURSION

Introduction

If you analyze the address of local variables of the recursive function, you will get two important results: the depth of recursion and the stack overheads in recursion. Since local variables of the function are pushed into the stack when the function calls another function, by knowing the address of the variable in repetitive recursive call, you will determine how much information is pushed into the stack. For example, the stack could grow from top to bottom, and the local variable `j` gets the address 100 in the stack in the first column. Suppose stack overheads are 16 bytes; in the next call `j` will have the address 84, in the call after that it will get the address 16. That is a difference of 16 bytes. The following program uses the same principle: the difference of the address in consecutive calls is the stack overhead.

Program

```
#include <stdio.h>
int fact(int n);
long old=0;   \\E
long current=0;      \\F
main()
{
    int k = 4,i;
    long diff;
    i =fact(k);
    printf("The value of i is %d\n",i);
    diff = old-current;
    printf("stack overheads are %16lu\n",diff);
}
int fact(int n)
}
    int j;
    static int m=0;
    if(m==0) old =(long) &j; \\A
    if(m==1) current =(long) &j;    \\B
    m++;              \\C
    printf("the address of j and m is %16lu %16lu\n",&j,&m);   \\D
    if(n<=0)
```

```
        return(1);
    else
        return(n*fact(n-1));
}
```

Explanation

1. The program calculates factorials just as the previous program.
2. The variable to be analyzed is the local variable `j`, which is the automatic variable. It gets its location in the stack.
3. The static variable `m` is used to track the number of recursive calls. Note that the static variables are stored in memory locations known as data segments, and are not stored in stack. Global variables such as `old` and `current` are also stored in data segments.
4. The program usually has a three-segment text: first, storing program instructions or program code, then the data segment for storing global and static variables, and then the stack segment for storing automatic variables.
5. During the first call, `m` is 0 and the value of `j` is assigned to the global varable `old`. The value of `m` is incremented.
6. In the next call, `m` is 1 and the value of `j` is stored in current.
7. Note that the addresses of `j` are stored in long variables of type castings.
8. `old` and `current` store the address of `j` in consecutive calls, and the difference between them gives the stack overheads.
9. You can also check the address of `j` and check how the allocation is done in the stack and how the stack grows.

   Note You can also check whether the address of m is constant.

Points to Remember

1. The recursive program has a stack overhead.
2. You can calculate stack overheads by analyzing the addresses of local variables.

9. WRITING A RECURSIVE FUNCTION

Introduction

A recursive function is a function that calls itself. Some problems can be easily solved by using recursion, such as when you are dividing a problem into sub- problems with similar natures. Note that recursion is a time-consuming solution that decreases the speed of execution because of stack overheads. In recursion, there is a function call and the number of such calls is large. In each call, data is pushed into the stack and when the call is over, the data is popped from the stack. These push and pop operations are time-consuming operations. If you have the choice of iteration or recursion, it is better to choose iteration because it does not involve stack overheads. You can use recursion only for programming convenience. A sample recursive program for calculating factorials follows.

Program

```
#include <stdio.h>
int fact(int n);
main()
{
    int k = 4,i;
    i =fact(k);         \\ A
    printf("The value of i is %d\n",i);
}
```

```
int fact(int n)
{
    if(n<=0)          \\ B
    return(1);           \\ C
else
    return(n*fact(n-1)); \\ D
}
```

Explanation

1. You can express factorials by using recursion as shown:
2. `fact (5) = 5 * fact (4)`
3. `In general,`
4. `fact (N) = N * fact (N-1)`
5. `fact 5 is calculated as follows:`
6. `fact (5) = 5 * fact (4) i.e. there is call to fact (4) \\ A`
7. `fact (4) = 4 * fact (3)`
8. `fact (3) = 3 * fact (2)`
9. `fact (2) = 2 * fact (1)`
10. `fact (1) = 1 * fact (0)`
11. `fact (0) = 1 \\ B`
12. `fact (1) = 1 * 1, that is the value of the fact(0) is substituted in 1.`
13. `fact (2) = 2 * 1 = 2`
14. `fact (3) = 3 * 2 = 6`
15. `fact (4) = 4 * 6 = 24`
16. `fact (5) = 5 * 24 = 120 \\ C`
17. The operations from statements B to A are collectivelly called the *winding phase*, while the operations from B to C are called the unwinding phase. The winding phase should be the terminating point at some time because there is no call to function that is given by statement B; the value of the argument that equals 0 is the terminating condition. After the winding phase is over, the unwinding phase starts and finally the unwinding phase ends at statement C. In recursion, three entities are important: recursive expressions, recursive condition, and terminating condition. For example,
18. `fact ( N) = N * fact (N-1) N > 0`
19. `= 1 N = 0`
    - `N * fact (N-1)` indicates a recursive expression.
    - `N > 0` indicates a recursive condition.
    - `N = 0` indicates a terminating condition.
20. You should note that the recursive expression is such that you will get a terminating condition after some time. Otherwise, the program enters into an infinite recursion and you will get a stack overflow error. Statement B indicates the terminating condition, that is, `N = 0`.
21. The condition `N > 0` indicates a recursive condition that is specified by the `else` statement. The recursive expression is `n * fact(n-1)`, as given by statement D.

Points to Remember

1. Recursion enables us to write a program in a natural way. The speed of a recursive program is slower because of stack overheads.
2. In a recursive program you have to specify recursive conditions, terminating conditions, and recursive expressions.

10. TYPES OF RECURSION

LINEAR RECURSION

Recursion where only one call is made to the function from within the function (thus if we were to draw out the recursive calls, we would see a straight, or linear, path).

A linear recursive function is a function that only makes a single call to itself each time the function runs (as opposed to one that would call itself multiple times during its execution). The factorial function is a good example of linear recursion.

```
;Scheme
(define (factorial n)
  (if (= n 0)
      1
      (* n (factorial (- n 1)))
  )
)
//C++
int factorial (int n)
{
 if ( n == 0 )
   return 1;
 return n * factorial(n-1); // or factorial(n-1) * n
}
```

TAIL RECURSION

A recursive procedure where the recursive call is the last action to be taken by the function. Tail recursive functions are generally easy to transform into iterative functions.

Tail recursion is a form of linear recursion. In tail recursion, the recursive call is the last thing the function does. Often, the value of the recursive call is returned. As such, tail recursive functions can often be easily implemented in an iterative manner; by taking out the recursive call and replacing it with a loop, the same effect can generally be achieved. In fact, a good compiler can recognize tail recursion and convert it to iteration in order to optimize the performance of the code.

A good example of a tail recursive function is a function to compute the GCD, or Greatest Common Denominator, of two numbers:

```
int gcd(int m, int n)
{
        int r;

        if (m < n) return gcd(n,m);

        r = m%n;
        if (r == 0) return(n);
        else return(gcd(n,r));
}
```

BINARY RECURSION

A recursive function which calls itself twice during the course of its execution.

Some recursive functions don't just have one call to themself, they have two (or more). Functions with two recursive calls are referred to as binary recursive functions.

The mathematical combinations operation is a good example of a function that can quickly be implemented as a binary recursive function. The number of combinations, often represented as nCk where we are choosing n elements out of a set of k elements, can be implemented as follows:

```
int choose(int n, int k)
{
        if (k == 0 || n == k) return(1);
        else return(choose(n-1,k) + choose(n-1,k-1));
}
```

EXPONENTIAL RECURSION

Recursion where more than one call is made to the function from within itself. This leads to exponential growth in the number of recursive calls.

An exponential recursive function is one that, if you were to draw out a representation of all the function calls, would have an exponential number of calls in relation to the size of the data set (exponential meaning if there were *n* elements, there would be $O(a^n)$ function calls where a is a positive number).

A good example an exponentially recursive function is a function to compute all the permutations of a data set. Let's write a function to take an array of *n* integers and print out every permutation of it.

```
void print_array(int arr[], int n)
{
        int i;
        for(i=0; i<n; i++) printf("%d ", arr[i]);
        printf("\n");
}

void print_permutations(int arr[], int n, int i)
{
        int j, swap;
        print_array(arr, n);
        for(j=i+1; j<n; j++) {
                swap = arr[i]; arr[i] = arr[j]; arr[j] = swap;
                print_permutations(arr, n, i+1);
                swap = arr[i]; arr[i] = arr[j]; arr[j] = swap;
        }
}
```

NESTED RECURSION

In nested recursion, one of the arguments to the recursive function is the recursive function itself! These functions tend to grow extremely fast. A good example is the classic mathematical function, Ackermann's function. It grows very quickly (even for small values of x and y, Ackermann(x,y) is extremely large) and it cannot be computed with only definite iteration (a completely defined *for()* loop for example); it requires indefinite iteration (recursion, for example).

Try computing ackerman(4,2) by hand... have fun!

```
int ackerman(int m, int n)
{
        if (m == 0) return(n+1);
        else if (n == 0) return(ackerman(m-1,1));
        else return(ackerman(m-1,ackerman(m,n-1)));
}
```

MUTUAL RECURSION

A recursive function doesn't necessarily need to call itself. Some recursive functions work in pairs or even larger groups. For example, function A calls function B which calls function C which in turn calls function A.

A simple example of mutual recursion is a set of function to determine whether an integer is even or odd. How do we know if a number is even? Well, we know 0 is even. And we also know that if a number *n* is even, then *n* - 1 must be odd. How do we know if a number is odd? It's not even!

```
//C++
int is_even(unsigned int n)
{
        if (n==0) return 1;
        else return(is_odd(n-1));
}

int is_odd(unsigned int n)
{
        return (!is_even(n));
}
```

Recursion is powerful! Of course, this is just an illustration. The above situation isn't the best example of when we'd want to use recursion instead of iteration or a closed form solution. A more efficient set of function to determine whether an integer is even or odd would be the following:

```
int is_even(unsigned int n)
{
        if (n % 2 == 0) return 1;
        else return 0;
}

int is_odd(unsigned int n)
{
        if (n % 2 != 0) return 1;
        else return 0;
}
```

11. Exercises

E1. Write a program to compute: S = 1 + 2 + 3 + …n using recurson.

E2. Write a program to compute: n! = 1*2*3*…*n using recurson.

E3. Write a program to print a number as a character string using recursion.

Example: input n=12345. Print out: 12345.

E4. Write a recursion function to find the sum of every number in a int number. Example: n=1980 => Sum=1+9+8+0=18.

E5. Find the n(th) number of fibonacci numbers: fib(n) = fib(n-1) + fib(n-2). Fib(1)=1; Fib(2)=1;

E6. We can recursively define the number of combinations of m things out of n, denote C(n, m), for n >= 1 and 0 <= m <= n, by

C(n,m) = 1 if m = 0 or m=n

C(n, m) = C(n-1, m) + C(n-1, m-1) if 0 < m < n

+Write a recursive function to compute C(n, m).

+Write a complete program that reads two integers N and M and invokes the function to compute C(N, M) and prints the result out

E7. Write a program to display the moving of Towers of Hanoi problem.

Three pegs, n disks of different sizes. Originally all n disks are on peg 1, with the disks sorted by size (largest at the bottom). Rule: move one disk at a time, never putting a larger disk on top of a smaller disk. Goal: move all the disks to peg 3.

Recursive Solution

- If we want to move 1 disk from peg a to peg b, it is easy (just do it).
- If we want to move n disks from peg a to peg b, what we need to do is:
  - Move the top $n-1$ disks to a temporary peg $c \neq a, b$.
  - Move the largest disk from a to b.
  - Move the top $n-1$ disks from c to b.
- "Moving the top $n-1$ disks" is a smaller problem of the same type.

Tower of Hanoi

```
void hanoi(int n, int from, int to, int temp)
{
  if (n == 1)
    cout << from << " -> " << to << endl;
  else {
    hanoi(n-1, from, temp, to);
    cout << from << " -> " << to << endl;
    hanoi(n-1, temp, to, from);
  }
}
```

Start with

```
hanoi(n, 1, 3, 2);
```

E8. Define a recursion function named Ackermann :

Acker (m,n)= n+1 if m=0

Acker(m-1,1) if n=0

Acker(m-1, Acker(m,n-1)) (other cases)

Find Acker(1,2) by hand

Write function to find Acker(4,5)

E9. Write function (using recursion) to find Greatest common divisor of two numbers.

Example: GCD(9,12)=3. GCD(25,50)=25.

GCD(p,q)= r=p/q Q id r=0

GCD(p,q)= GCD(q,r)

CHAPTER 3: SEARCHING TECHNIQUES

1. SEARCHING TECHNIQUES: LINEAR OR SEQUENTIAL SEARCH

Introduction

There are many applications requiring a search for a particular element. Searching refers to finding out whether a particular element is present in the list. The method that we use for this depends on how the elements of the list are organized. If the list is an unordered list, then we use linear or sequential search, whereas if the list is an ordered list, then we use binary search.

The search proceeds by sequentially comparing the key with elements in the list, and continues until either we find a match or the end of the list is encountered. If we find a match, the search terminates successfully by returning the index of the element in the list which has matched. If the end of the list is encountered without a match, the search terminates unsuccessfully.

Program

```
#include <stdio.h>
#define MAX 10
void lsearch(int list[],int n,int element)
{
   int i, flag = 0;
   for(i=0;i<n;i++)
   if( list[i] == element)
   {
      printf(" The element whose value is %d is present at position %d
in list\n",
            element,i);
      flag =1;
      break;
   }
   if( flag == 0)
       printf("The element whose value is %d is not present in the
list\n",
             element);
}

void readlist(int list[],int n)
{
   int i;
   printf("Enter the elements\n");
   for(i=0;i<n;i++)
       scanf("%d",&list[i]);
```

```
}

void printlist(int list[],int n)
{
   int i;
   printf("The elements of the list are: \n");
   for(i=0;i<n;i++)
       printf("%d\t",list[i]);
}
void main()
{
    int list[MAX], n, element;
   printf("Enter the number of elements in the list max = 10\n");
   scanf("%d",&n);
   readlist(list,n);
   printf("\nThe list before sorting is:\n");
   printlist(list,n);
   printf("\nEnter the element to be searched\n");
   scanf("%d",&element);
   lsearch(list,n,element);
}
```

Example

Input

Enter the number of elements in the list, max = 10

```
10
```

Enter the elements

```
23
1
45
67
90
100
432
15
77
55
```

Output

The list before sorting is:

The elements of the list are:

```
23 1 45 67 90 100 432 15 77 55
```

Enter the element to be searched

```
100
```

The element whose value is 100 is present at position 5 in list

Input

Enter the number of elements in the list max = 10

```
10
```

Enter the elements

```
23
1
45
67
90
101
23
56
44
22
```

Output

The list before sorting is:

The elements of the list are:

```
23 1 45 67 90 101 23 56 44 22
```

Enter the element to be searched

```
100
```

The element whose value is 100 is not present in the list

Explanation

1. In the best case, the search procedure terminates after one comparison only, whereas in the worst case, it will do *n* comparisons.
2. On average, it will do approximately *n*/2 comparisons, since the search time is proportional to the number of comparisons required to be performed.
3. The linear search requires an average time proportional to O(*n*) to search one element. Therefore to search n elements, it requires a time proportional to O(n^2).
4. We conclude that this searching technique is preferable when the value of n is small. The reason for this is the difference between *n* and n^2 is small for smaller values of *n*.

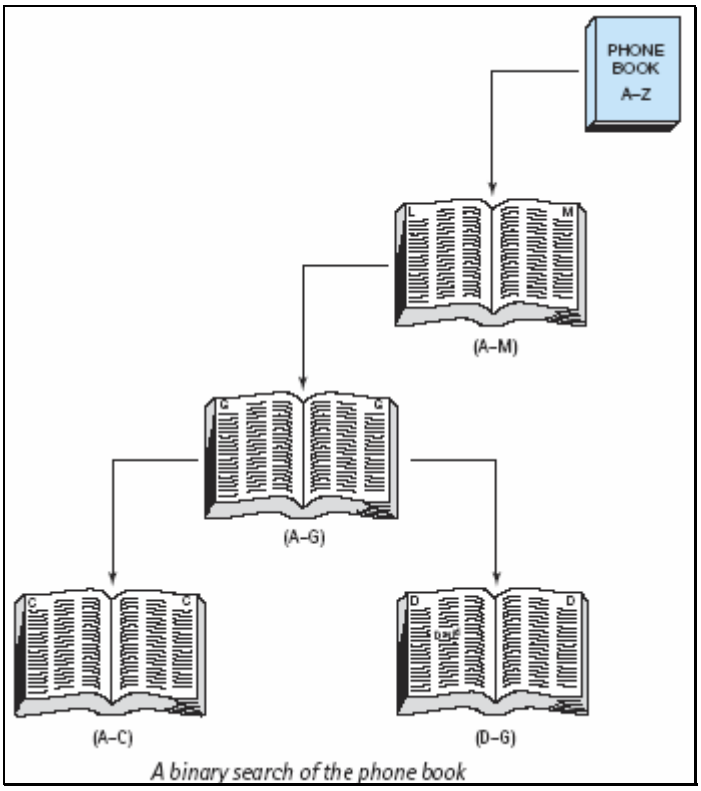
2. BINARY SEARCH

Introduction

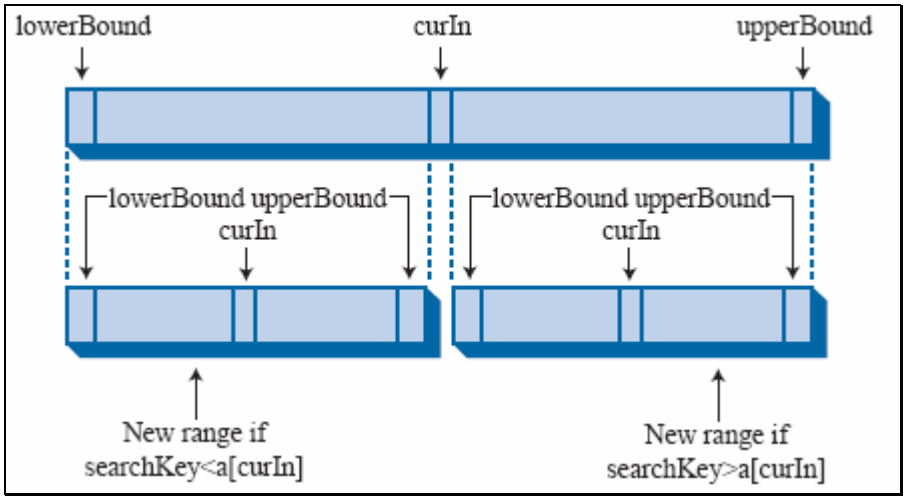
The prerequisite for using binary search is that the list must be a sorted one. We compare the element to be searched with the element placed approximately in the middle of the list.

If a match is found, the search terminates successfully. Otherwise, we continue the search for the key in a similar manner either in the upper half or the lower half. If the elements of the list are arranged in ascending order, and the key is less than the element in the middle of the list, the search is continued in the lower half. If the elements of the list are arranged in descending order, and the key is greater

than the element in the middle of the list, the search is continued in the upper half of the list. The procedure for the binary search is given in the following program.



A binary search of the phone book



Program

```
#include <stdio.h>
#define MAX 10

void bsearch(int list[],int n,int element)
{
   int l,u,m, flag = 0;
   l = 0;
   u = n-1;
   while(l <= u)
   {
```

```
      m = (l+u)/2;
      if( list[m] == element)
      {
                printf(" The element whose value is %d is present at
position %d in list\n",
                      element,m);
                flag =1;
                break;
      }
      else
            if(list[m] < element)
                  l = m+1;
            else
                  u = m-1;
   }
   if( flag == 0)
   printf("The element whose value is %d is not present in the list\n",
       element);
}

void readlist(int list[],int n)
{
   int i;
   printf("Enter the elements\n");
   for(i=0;i<n;i++)
       scanf("%d",&list[i]);
}

void printlist(int list[],int n)
{
    int i;
   printf("The elements of the list are: \n");
   for(i=0;i<n;i++)
       printf("%d\t",list[i]);
}

void main()
{
   int list[MAX], n, element;
   printf("Enter the number of elements in the list max = 10\n");
   scanf("%d",&n);
   readlist(list,n);
   printf("\nThe list before sorting is:\n");
   printlist(list,n);
   printf("\nEnter the element to be searched\n");
   scanf("%d",&element);
```

```
   bsearch(list,n,element);
}
```

Example

Input

Enter the number of elements in the list, max = 10

```
10
```

Enter the elements

```
34
2
1
789
99
45
66
33
22
11
```

Output

The elements of the list before sorting are:

```
34   2   1   789   99   45   66   33   22   11
1    2   3    4     5    6    7    8    9   10
```

Enter the element to be searched

```
99
```

The element whose value is 99 is present at position 5 in the list

Input

Enter the number of elements in the list max = 10

```
10
```

Enter the elements

```
54
89
09
43
66
88
77
11
22
33
```

Output

The elements of the list before sorting are:

```
54 89 9 43 66 88 77 11 22 33
```

Enter the element to be searched

```
100
```

The element whose value is 100 is not present in the list.

In the binary search, the number of comparisons required to search one element in the list is no more than log2*n*, where n is the size of the list. Therefore, the binary search algorithm has a time complexity of O(n *(log2*n*.).)

Uses a recursive method to implement binary search

```
 public int Search (int[] data, int key, int left, int right) {
    if (left <= right) {
      int middle = (left + right)/2;
      if (key == data[middle])
        return middle;
      else if (key < data[middle])
        return Search(data,key,left,middle-1);
      else
        return Search(data,key,middle+1,right);
    }
    return -1;
  }


public static void Main(String[] args) {
    int key;       // the search key
    int index;     // the index returned
    int[10] data;

    for(int i = 0; i < 10; i++)
      data[i] = i;

    key = 9;
    index = Search(data, key, 0, 9);
    if (index == -1)   //"Key not found"
    else               //"Key found at index”
  }
```

3. COMPLEXITY OF ALGORITHMS

An algorithm is a procedure (a finite set of well-defined instructions) for accomplishing some task which, given an initial state, will terminate in a defined end-state. The computational complexity and efficient implementation of the algorithm are important in computing, and this depends on suitable data structures.

In Computer Science, it is important to measure the quality of algorithms, especially the specific amount of a certain resource an algorithm needs. Examples of such resources would be time or memory storage. Nowadays, memory storage is almost a non-essential factor when designing algorithms but be aware that several systems still have memory constraints, such as Digital Signal Processors in embedded systems.
Different algorithms may complete the same task with a different set of instructions in less or more time, space or effort than other. The analysis and study of algorithms is a discipline in Computer Science which has a strong mathematical background. It often relies on theoretical analysis of pseudo-code.

To compare the efficiency of algorithms, we don't rely on abstract measures such as the time difference in running speed, since it too heavily relies on the processor power and other tasks running in parallel. The most common way of qualifying an algorithm is the Asymptotic Notation, also called Big O.

Informal definition

The complexity of a given algorithm is expressed by the worst-case execution time, using the big O notation. It is generally written as $O(f(x_1, x_2, \ldots, x_n))$, where f is a function of one or more parameters xi of the system. The "O" is read "order" as in "order of magnitude" or "on the order of". It roughly states that in the asymptotic case (i.e. as one or more of the system parameters extend towards their extremes), the actual execution time of the algorithm is bounded by $K \times f(x_1, x_2, \ldots, x_n)$ for some positive constant K.

Sometimes one is interested in expected or best case execution time as well, or even the algorithms requirements of memory or other resources.

The big O notation expresses the complexity of an algorithm without restricting the statement to a particular computer architecture or time period; even if computers get faster, or if the algorithm is ported to another software platform, the worst case execution time of an algorithm does not change. Additionally, when comparing two algorithms, it is readily apparent which will execute faster in the extreme case.

Classes of complexity

Most commonly, one will see

- Polynomial time algorithms,
  - $O(1)$ --- Constant time --- the time necessary to perform the algorithm does not change in response to the size of the problem.
  - $O(n)$ --- Linear time --- the time grows linearly with the size (n) of the problem.
  - $O(n^2)$ --- Quadratic time --- the time grows quadratically with the size (n) of the problem. In big O notation, all polynomials with the same degree are equivalent, so $O(3n^2 + 3n + 7) = O(n^2)$
- Sub-linear time algorithms (grow slower than linear time algorithms).
  - $O(\log n)$ -- Logarithmic time
- Super-polynomial time algorithms (grow faster than polynomial time algorithms).
  - $O(n!)$
  - $O(2^n)$ --- Exponential time --- the time required grows exponentially with the size of the problem.

Despite the examples, with this notation one can express execution time as a function of multiple variables.

Expected case, best case

Similar analysis can be performed to find the expected and best case execution times of algorithms. The best case is generally written as $\Omega(f(x_1, x_2, \ldots, x_n))$.

Example: complexity of an algorithm

Let's start with a simple example:

```
void f ( int a[], int n )
{
    int i;
    printf ( "N = %dn", n );
    for ( i = 0; i < n; i++ )
```

```
        printf ( "%d ", a[i] );
    printf ( "n" );
}
```

In this function, the only part that takes longer as the size of the array grows is the loop. Therefore, the two printf calls outside of the loop are said to have a constant time complexity, or O(1), as they don't rely on N. The loop itself has a number of steps equal to the size of the array, so we can say that the loop has a linear time complexity, or O(N). The entire function f has a time complexity of 2 * O(1) + O(N), and because constants are removed, it's simplified to O(1) + O(N).

Now, asymptotic notation also typically ignores the measures that grow more slowly because eventually the measure that grows more quickly will dominate the time complexity as N moves toward infinity. So by ignoring the constant time complexity because it grows more slowly than the linear time complexity, we can simplify the asymptotic bound of the function to O(N), so the conclusion is that f has linear time complexity. If we say that a function is O(N) then if N doubles, the function's time complexity at most will double. It may be less, but never more. That's the upper bound of an algorithm, and it's the most common notation.

Linear Search: complexity of an algorithm

As before, linear search is a search algorithm that tries to find a certain value in a set of data. It operates by checking every element of a list (or array) one at a time in sequence until a match is found. The complexity of this algorithm is O(n).

Binary Search

Binary Search only works on sorted lists (or arrays). It finds the median, makes a comparison to determine whether the desired value comes before or after it and then searches the remaining half in the same manner.

We can call this an O(N) algorithm and not be wrong because the time complexity will never exceed O(N). But because the array is split in half each time, the number of steps is always going to be equal to the base-2 logarithm of N, which is considerably less than O(N). So an even better choice would be to set the upper bound to log N, which is the upper limit that we know we're guaranteed never to cross. Therefore, a more accurate claim is that binary search is a logarithmic, or O($\log_2$ N), algorithm.

Compare: Binary Search Linear Search

Which algorithm is more complex? Linear Search or Binary Search?

How do these functions compare?

n	$\log_2 n$
10	3
100	6
1,000	9
10,000	13
100,000	16

So for 100,00 items, binary search saves 999,984 comparisons compared to linear search. This is an amazing improvement!!

4. Exercises

E1. Write a program that performs the following steps:

* Input the integer array of N elements. (N is also an input data).

* Input a value you want to search on the array.

* Invoke the function linearSearch to find the element in the array which is equal to the specified value(assume that position is k).

E2. Write a program that performs the following steps:

* Input the integer array of N elements sorted. (N is also an input data).

* Input a value you want to search on the array.

* Invoke the function BinarySearch to find the element in the array which is equal to the specified value(assume that position is k).

E3. Write a program that performs the following steps:

* Input the array of N elements replace for N students. One student have 3 fields information: ID, MathGrade, ChemicalGrade.

* Input a ID value you want to search student have this ID. Display MathGrade, ChemicalGrade if finded student.

* Input a grade value. Find the first student who had average grage is bigger than input grade. Average = (MathGrade+ChemicalGrade)/2

CHAPTER 4: SORTING TECHNIQUES

We need to do sorting for the following reasons :

a) By keeping a data file sorted, we can do binary search on it.

b) Doing certain operations, like matching data in two different files, become much faster.

There are various methods for sorting: Bubble sort, Insertion sort, Selection sort, Quick sort, Heap sort, Merge sort…. They having different average and worst case behaviours:

	Average	Worst
Bubble sort	O(n2)	O(n2)
Insertion sort	O(n2)	O(n2)
Selection sort	O(n2)	O(n2)
Quick sort	O(nlogn)	O(n2)

The average and worst case behaviours are given for a file having n elements (records).

1. BUBBLE SORT

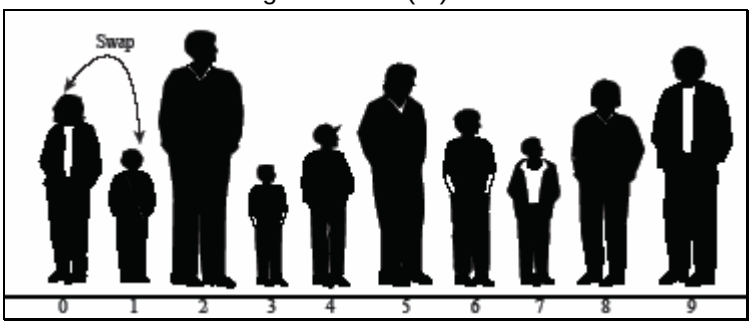
Introduction

Bubble sorting is a simple sorting technique in which we arrange the elements of the list by forming pairs of adjacent elements. That means we form the pair of the i^{th} and $(i+1)^{th}$ element. If the order is ascending, we interchange the elements of the pair if the first element of the pair is greater than the second element. That means for every pair (list[i],list[i+1]) for i :=1 to (n−1) if list[i] > list[i+1], we need to interchange list[i] and list[i+1]. Carrying this out once will move the element with the highest value to the last or n^{th} position. Therefore, we repeat this process the next time with the elements from the first to (*n* −*1*)*th* positions. This will bring the highest value from among the remaining (*n* −*1*) values to the $(n-1)^{th}$ position. We repeat the process with the remaining (*n*−2) values and so on. Finally, we arrange the elements in ascending order. This requires to perform (*n*−1) passes. In the first pass we

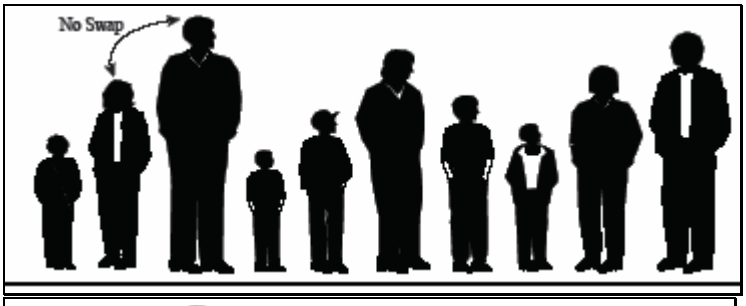
have $(n-1)$ pairs, in the second pass we have $(n-2)$ pairs, and in the last (or $(n-1)^{th}$) pass, we have only one pair. Therefore, the number of probes or comparisons that are required to be carried out is

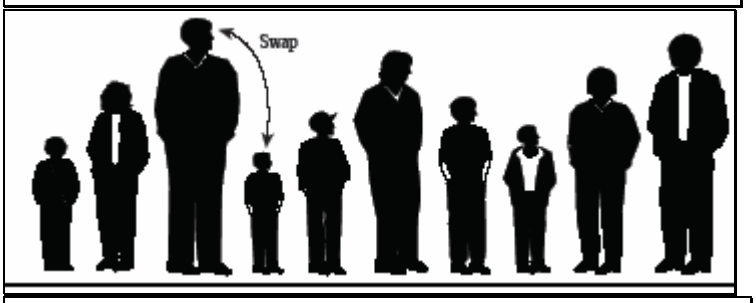
$$(n-1) + (n-2) + (n-3) + \ldots + 1$$
$$= n(n-1)/2,$$

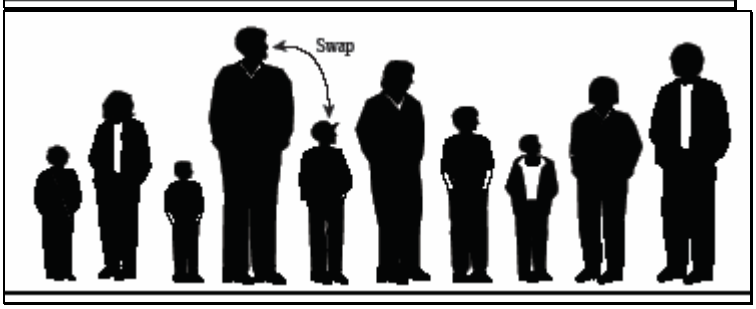
and the order of the algorithm is $O(n^2)$.



Bubble sort: beginning of first pass.







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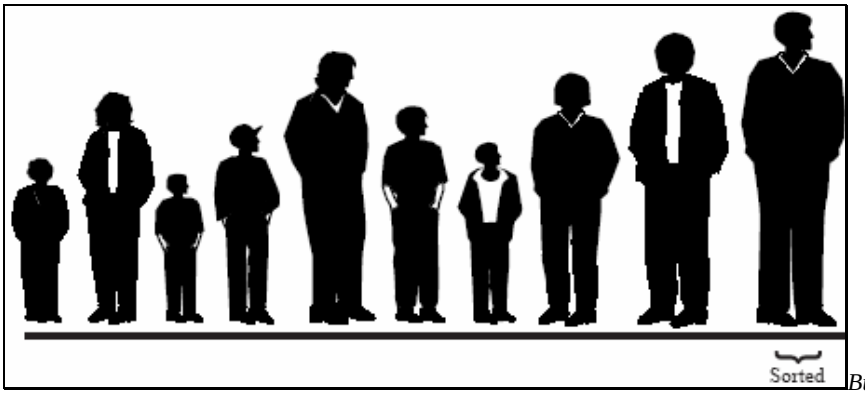


Bubble sort: end of First pass

Program

```
#include <stdio.h>
#define MAX 10
void swap(int *x,int *y)
{
   int temp;
   temp = *x;
   *x = *y;
   *y = temp;
}
void bsort(int list[], int n)
{
   int i,j;
   for(i=0;i<(n-1);i++)
      for(j=0;j<(n-(i+1));j++)
             if(list[j] > list[j+1])
                    swap(&list[j],&list[j+1]);
}
void readlist(int list[],int n)
{
   int i;
   printf("Enter the elements\n");
   for(i=0;i<n;i++)
       scanf("%d",&list[i]);
}

void printlist(int list[],int n)
{
   int i;
   printf("The elements of the list are: \n");
   for(i=0;i<n;i++)
      printf("%d\t",list[i]);
}

void main()
{
   int list[MAX], n;
   printf("Enter the number of elements in the list max = 10\n");
   scanf("%d",&n);
   readlist(list,n);
   printf("The list before sorting is:\n");
   printlist(list,n);
   bsort(list,n);
   printf("The list after sorting is:\n");
   printlist(list,n);
```

```
}
```

Example

Input

Enter the number of elements in the list, max = 10

5

Enter the elements

23

5

4

9

1

Output

The list before sorting is:

The elements of the list are:

```
23 5 4 9 1
```

The list after sorting is:

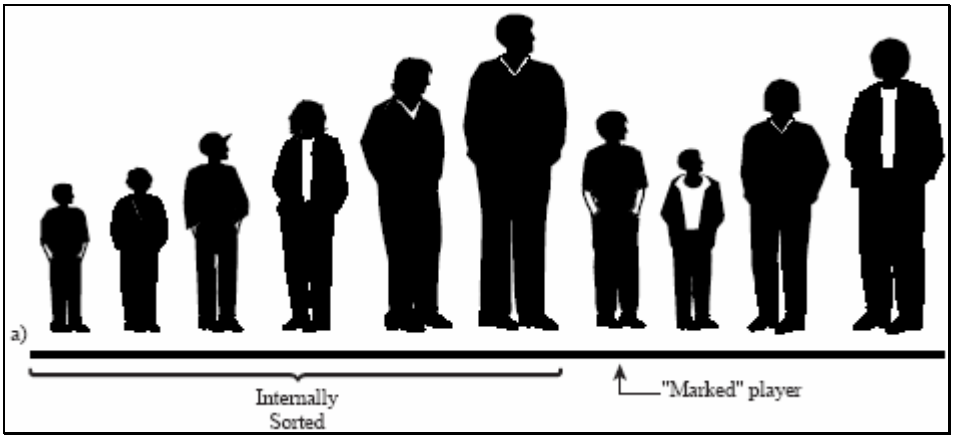
The elements of the list are:

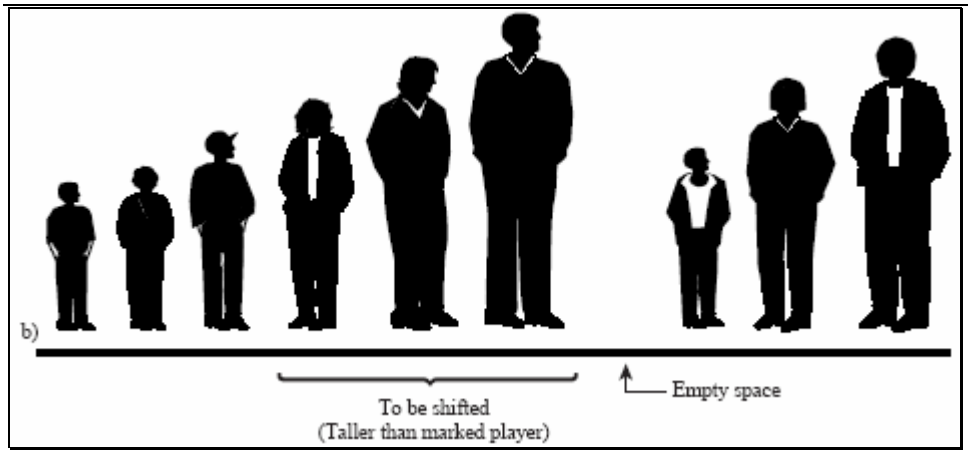
```
1 4 5 9 23
```

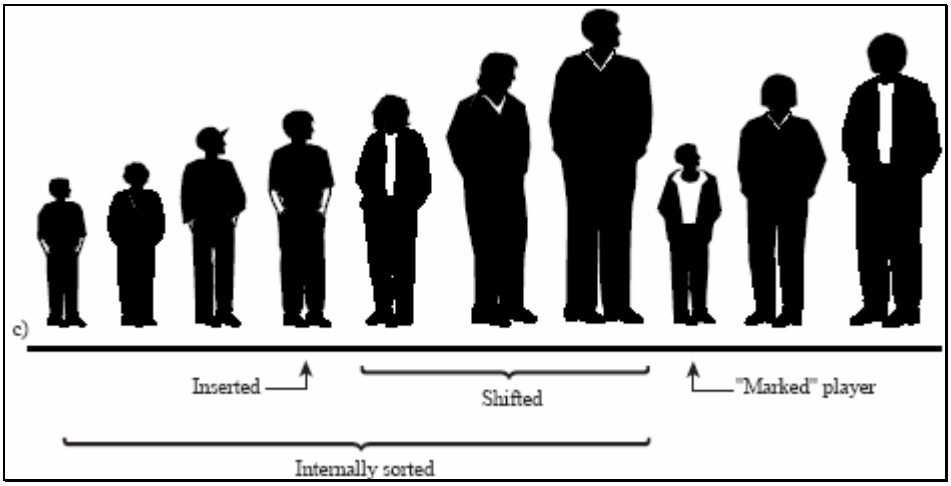
2. INSERTION SORT

Introduction

Basic Idea: Insert a record R into a sequence of ordered records: R1,R2,.., Ri with keys K1 <= K2 <= ... <= Ki , such that, the resulting sequence of size i+1 is also ordered with respect to key values.







The bubble sort, described in the previpus," is the easiest sort to understand, so it's a good starting place in our discussion of sorting. However, it's also the least sophisticated. The insertion sort is substantially better than the bubble sort (and various

other elementary sorts we don't describe here, such as the selection sort). It still executes in $O(N_2)$ time, but it's about twice as fast as the bubble sort. It's also not too complex, although it's slightly more involved than the bubble sort. It's often used as the final stage of more sophisticated sorts, such as quicksort.

Program

```
Algorithm Insertion_Sort; (* Assume Ro has Ko = -maxint *)

void InsertionSort( Item &list[])

{ // Insertion_Sort

Item r;

int i,j;

list[0].key = -maxint;

for (j=2; j<=n; j++)

{r=list[j];

i=j-1;
```

```
while ( r.key < list[i].key )

{// move greater entries to the right

list[i+1]:=list[i];

i:=i-1;

};

list[i+1] = r // insert into it's place

}
```

We start with R0,R1 sequence, here R0 is artificial. Then we insert records R2,R3,..Rn into the sequence. Thus, the file with n records will be ordered making n-1 insertions.

Example

Let m be maxint

a)

```
j          R0  R1  R2   R3   R4  R5
---------  --  --  --   --   --  ---
           -m   5   4    3    2   1
                 V
2          -m   4   5    3    2   1
                     v
3          -m   3   4    5    2   1
                         v
4          -m   2   3    4    5   1
                             v
5          -m   1   2    3    4   5
```

b)

```
j          R0  R1  R2   R3   R4  R5
---------  --  --  --   --   --  ---
           -m  12   7    5   10   2
                 v
2          -m   7  12    5   10   2
                     v
3          -m   5   7   12   10   2
                         v
4          -m   5   7   10   12   2
                             v
5          -m   2   5    7   10  12
```

3. SELECTION SORT

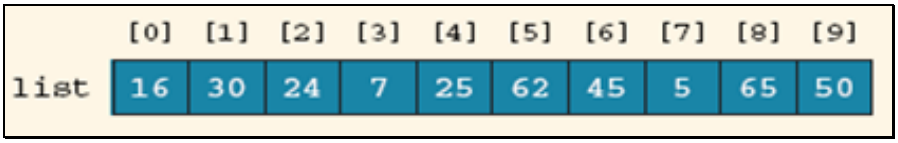
Introduction

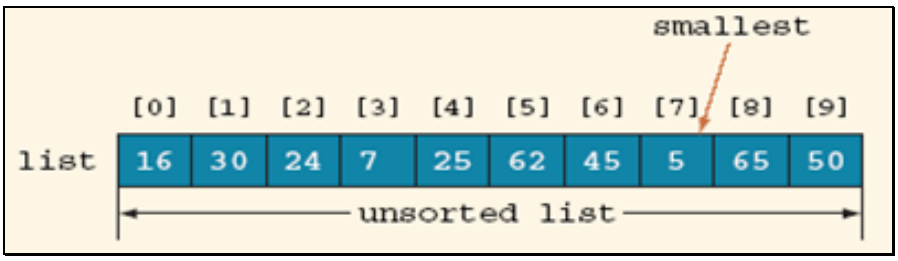
Selection sort is a simplicity sorting algorithm. It works as its name as it is. Here are basic steps of selection sort algorithm:

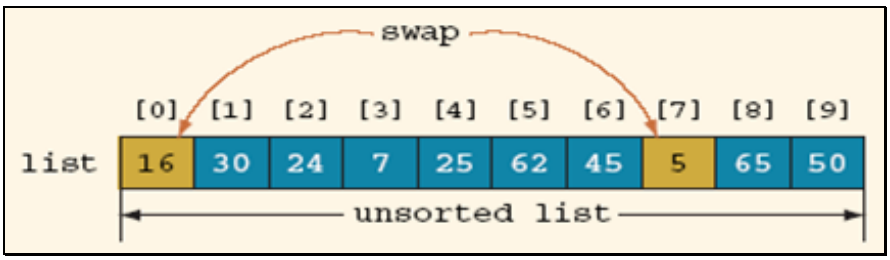
1. Find the minimum element in the list

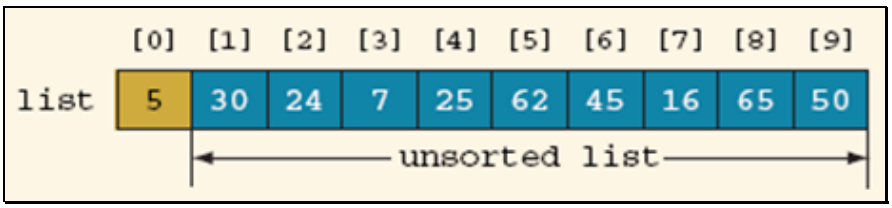
2. Swap it with the element in the first position of the list

3. Repeat the steps above for all remainder elements of the list starting at the second position.









Here is the selection sort algorithm implemented in C programming language:

Program

```
#include <stdio.h>
#include <stdlib.h>

void swap(int *x,int *y)
{
   int temp;
   temp = *x;
   *x = *y;
   *y = temp;
}
```

```
void selection_sort(int list[], int n)
{
  int i, j, min;

  for (i = 0; i < n - 1; i++)
  {
    min = i;
    for (j = i+1; j < n; j++)
    {
       if (list[j] < list[min])
       {
          min = j;
       }
    }
    swap(&list[i], &list[min]);
  }
}

void printlist(int list[],int n)
{
   int i;
   for(i=0;i<n;i++)
      printf("%d\t",list[i]);
}

void main()
{
   const int MAX_ELEMENTS = 10;
   int list[MAX_ELEMENTS];

   int i = 0;

   // generate random numbers and fill them to the list
   for(i = 0; i < MAX_ELEMENTS; i++ ){
          list[i] = rand();
   }
   printf("The list before sorting is:\n");
   printlist(list,MAX_ELEMENTS);

   // sort the list
   selection_sort(list,MAX_ELEMENTS);

   // print the result
   printf("The list after sorting:\n");
   printlist(list,MAX_ELEMENTS);
```

```
}
```

4. QUICK SORT

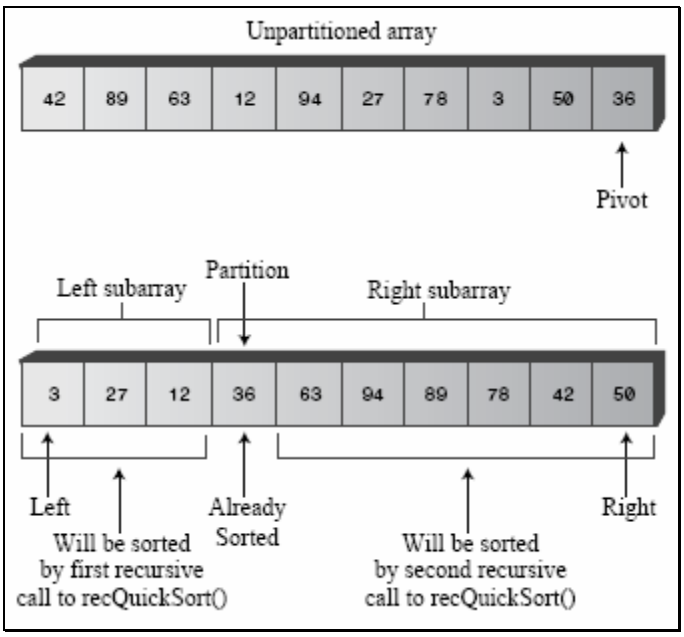
Introduction

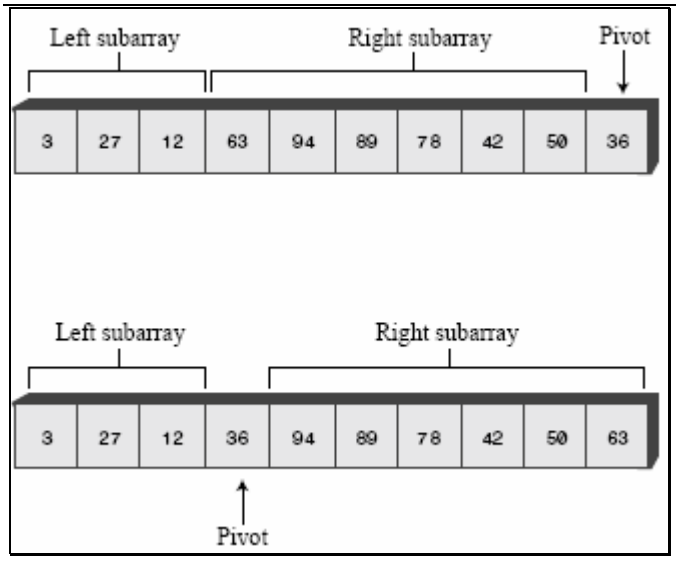
Quicksort is undoubtedly the most popular sorting algorithm, and for good reason: in the majority of situations, it's the fastest, operating in O(N*logN) time. (This is true only for in-memory sorting; for sorting data in disk files, other algorithms, such as mergesort, may be better.) Quicksort was discovered by C.A.R. Hoare in 1962.

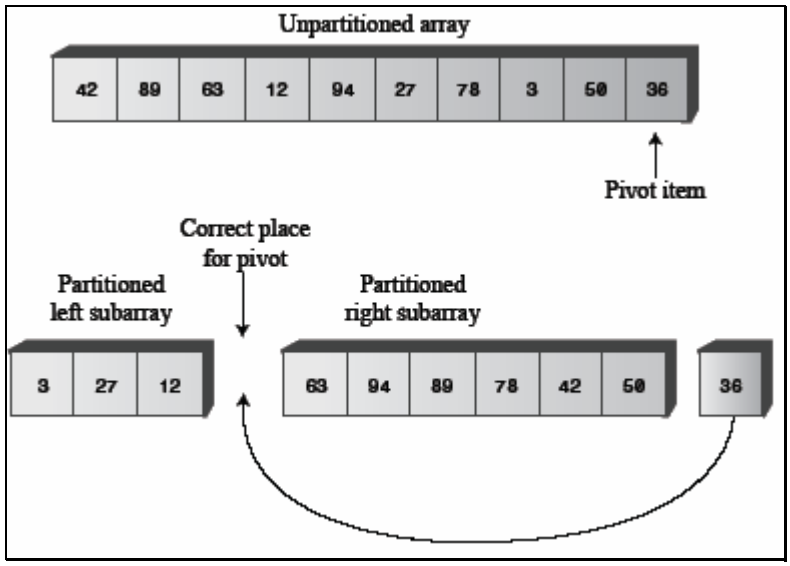
In the *quick sort* method, an array a[1],…..,a[n] is sorted by selecting some value in the array as a key element. We then swap the first element of the list with the key element so that the key will be in the first position. We then determine the key's proper place in the list. The proper place for the key is one in which all elements to the left of the key are smaller than the key, and all elements to the right are larger.

To obtain the key's proper position, we traverse the list in both directions using the indices `i` and `j`, respectively. We initialize `i` to that index that is one more than the index of the key element. That is, if the list to be sorted has the indices running from *m* to *n*, the key element is at index m, hence we initialize i to (m+1). The index i is incremented until we get an element at the i^{th} position that is greater than the key value. Similarly, we initialize j to n and go on decrementing j until we get an element with a value less than the key's value.

We then check to see whether the values of i and j have crossed each other. If not, we interchange the elements at the key (m^{th})position with the elements at the j^{th} position. This brings the key element to the j^{th} position, and we find that the elements to its left are less than it, and the elements to its right are greater than it. Therefore we can split the list into two sublists. The first sublist is composed of elements from the m^{th} position to the $(j–1)^{th}$ position, and the second sublist consists of elements from the $(j+1)^{th}$ position to the n^{th} position. We then repeat the same procedure on each of the sublists separately.







Choice of the key

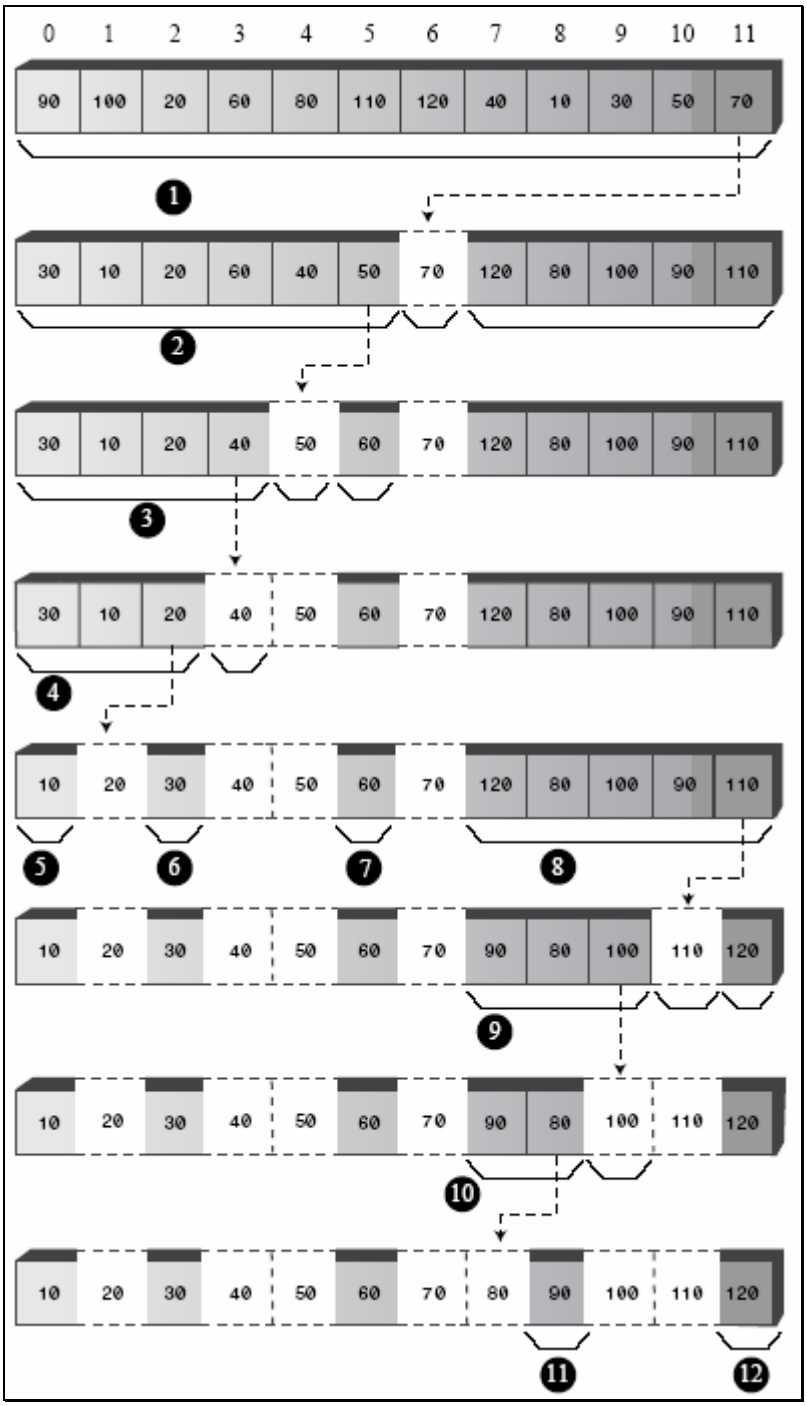
We can choose any entry in the list as the key. The choice of the first entry is often a poor choice for the key, since if the list has already been sorted, there will be no element less than the first element selected as the key. So, one of the sublists will be empty. So we choose a key near the center of the list in the hope that our choice will partition the list in such a manner that about half of the elements will end up on one side of the key, and half will end up on the other.

Therefore the function `getkeyposition` is

```
int getkeyposition(int i,j)
{
   return(( i+j )/ 2);
}
```

The choice of the key near the center is also arbitrary, so it is not necessary to always divide the list exactly in half. It may also happen that one sublist is much larger than the other. So some other method of selecting a key should be used. A good way to choose a key is to use a random number generator to choose the position of the next key in each activation of quick sort. Therefore, the function `getkeyposition` is:

```
int getkeyposition(int i,j)
{
   return(random number in the range of i to j);
}
```



Program

```
#include <stdio.h>
#define MAX 10
void swap(int *x,int *y)
{
   int temp;
   temp = *x;
```

```
   *x = *y;
   *y = temp;
}
int getkeyposition(int i,int j )
{
   return((i+j) /2);
}
void qsort(int list[],int m,int n)
{
   int key,i,j,k;
   if( m < n)
   {
      k = getkeyposition(m,n);
      swap(&list[m],&list[k]);
      key = list[m];
      i = m+1;
      j = n;
      while(i <= j)
      {
         while((i <= n) && (list[i] <= key))
                i++;
         while((j >= m) && (list[j] > key))
                j-;
         if( i < j)
                swap(&list[i],&list[j]);
      }
      swap(&list[m],&list[j]);
      qsort(list[],m,j-l);
      qsort(list[],j+1,n);
   }
}
void readlist(int list[],int n)
{
   int i;
   printf("Enter the elements\n");
   for(i=0;i<n;i++)
       scanf("%d",&list[i]);
}
void printlist(int list[],int n)
{
   int i;
   printf("The elements of the list are: \n");
   for(i=0;i<n;i++)
       printf("%d\t",list[i]);
}
```

```
void main()
{
   int list[MAX], n;
   printf("Enter the number of elements in the list max = 10\n");
   scanf("%d",&n);
   readlist(list,n);
   printf("The list before sorting is:\n");
   printlist(list,n);
   qsort(list,0,n-1);
   printf("\nThe list after sorting is:\n");
   printlist(list,n);
}
```

Example

Input

Enter the number of elements in the list, max = 10

```
10
```

Enter the elements

```
7
99
23
11
65
43
23
21
21
77
```

Output

The list before sorting is:

The elements of the list are:

```
7 99 23 11 65 43 23 21 21 77
```

The list after sorting is:

The elements of the list are:

```
7 11 21 21 23 23 43 65 77 99
```

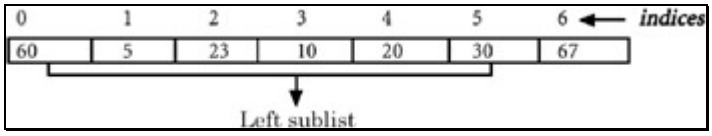
Explanation

Consider the following list:

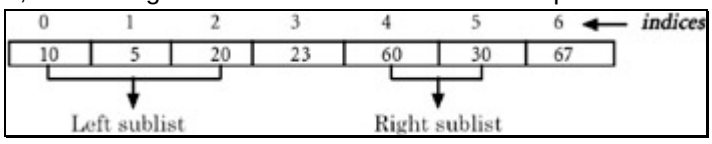
0	1	2	3	4	5	6 ← *indices*
10	5	23	67	20	30	60

1. When `qsort` is activated the first time, key = 67, `i` =1, and `j` =6. `i` is incremented until it becomes 7, because there is no element greater than the key. `j` is not decremented, because at position 6, the value that we have is less than the key. Since `i > j`, we interchange the key

element (the element at position 0) with the element at position 6, and call `qsort` recursively, with the left sublist made of elements from positions 0 to 5, and the right sublist empty as shown here:



2. When `qsort` is activated the second time on the left sublist as shown, key = 23, `i` =1, and `j` =5. `i` is incremented until it reaches 2. Because the element at position 2 is greater than the key, `j` is decremented to 4 because the value at position 4 is less than the key. Since `i < j`, the elements at positions 2 and 4 are swapped. `i` is then incremeneted to 4 and `j` is decremented to 3. Since `i > j`, we interchange the key element (the element at position 0), with the element at position 3, and call `qsort` recursively with the left sublist made of elements from position 0 to 2, and the right sublist made of elements from position 4 to 5, as shown here:



3. By continuing in this fashion, we eventually get the list sorted.
4. The average case-time complexity of the quick sort algorithm can be determined as follows:

   We assume that every time this is done, the list gets split into two approximately equal-sized sublists. If the size of a given list is n, it gets split into two sublists of size approximately *n*/2. Each of these sublists gets further split into two sublists of size n/4, and this is continued until the size equals 1. When the quick sort works with a list of size n, it places the key element (which takes the first element of the list under consideration) in its proper position in the list. This requires no more than n iterations. After placing the key element in its proper position in the list of size n, quick sort activates itself twice to work with the left and right sublists, each assumed to be of size *n*/2. Therefore T(n) is the time required to sort a list of size n. Since the time required to sort the list of size n is equal to the sum of the time required to place the key element in its proper position in the list of size *n*, and the time required to sort the left and right sublists, each assumed to be of size *n*/2. T(*n*) turns out to be:

   $\therefore T(n) = c*n + 2*T(n/2)$

   where c is a constant and T(*n*/2) is the time required to sort the list of size n/2.
5. Similarly, the time required to sort a list of size n/2 is equal to the sum of the time required to place the key element in its proper position in the list of size n/2 and the time required to sort the left and right sublists each assumed to be of size n/4. T(n/2) turns out to be:

   $T(n/2) = c*n/2 + 2*T(n/4)$

   where T(*n*/4) is the time required to sort the list of size *n*/4.

   - $\therefore T(n/4) = c*n/4 + 2*T(n/8)$, and so on. We eventually we get T(1) = 1.
   - $\therefore T(n) = c*n + 2(c*n(n/2) + 2T(n/4))$
   - $\therefore T(n) = c*n + c*n + 4T(n/4)) = 2*c*n + 4T(n/4) = 2*c*n + 4(c*(n/4) + 2T(n/8))$
   - $\therefore T(n) = 2*c*n + c*n + 8T(n/8) = 3*c*n + 8T(n/8)$
   - $\therefore T(n) = (\log n)*c*n + n\ T(n/n) = (\log n)*c*n + n\ T(1) = n + n*(\log n) *c$
   - $\therefore T(n)\ \mu\ n \log(n)$
6. Therefore, we conclude that the average complexity of the quick sort algorithm is O(nlog *n*). But the worst-case time complexity is of the $O(n^2)$. The reason for this is, in the worst case, one of the two sublists will always be empty and the other will be of size $(n-1)$, where n is the size of the original list. Therefore, in the worst case, T(*n*) turns out to be

$T(n) = c*n + T(n-1)$

$= c*n + c*(n-1) + T(n-2)$

$= 2*c*n - c + T(n-2)$

$= 2*c*n - c + c*(n-2) + T(n-3)$

$= 3*c*n - 3*c + T(n-3)$

...

...

$= n*c*n - n*c + T(1)$

$= n^2c - nc + 1$

Therefore $T(n)$ μ n^2, so the order is $O(n^2)$.

7. Space complexity: The average-case space complexity is $\log_2 n$, because the space complexity depends on the maximum number of activations that can exist. We find that if we assume that every time the list gets split into approximately two equal-sized lists, the maximum number of activations that will exist simultaneously will be $\log_2 n$.

   In the worst case, there exist n activations, because the depth of the recursion is n. So the worst-case space complexity is $O(n)$.

5. Exercises

E1. Giving an array: 9 3 2 1 7 6 12

Draw every step when sorting this array using: Bubble sort, Insertion sort, Selection sort, Quick sort algorithm.

E2. Input an integer array with random number.

Sort the array with: Bubble sort, Insertion sort, Selection sort, Quick sort algorithm.

Display the sorted array.

E2. Build an array with every element replace for a student have 3 fields: (Name: string,

ID:Integer, grade: float).

- Input 10 students into list
- Sort the list order by ID using: Bubble Sort, Quick sort, Selection Sort, Insertion Sort
- Find a student from ID using Linear search algorithm
- Find a student from ID using Binary search algorithm

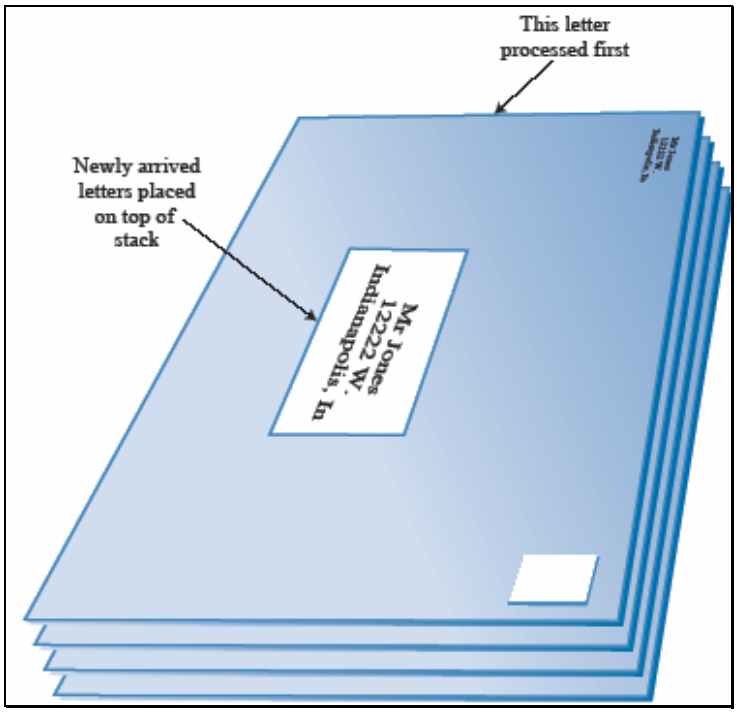
CHAPTER 5: STACKS AND QUEUES

1. THE CONCEPT OF STACKS AND QUEUES

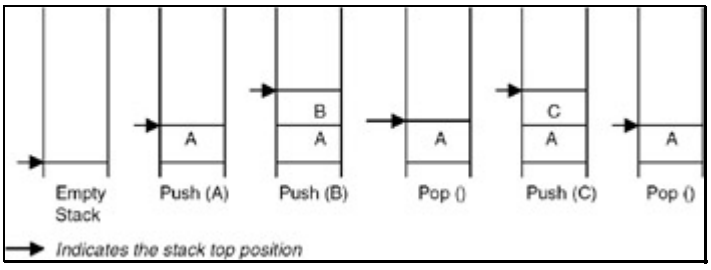
There are many applications requiring the use of the data structures *stacks* and *queues*. The most striking use of a data structure stack is the runtime stack that a programming language uses to implement a function call and return. Similarly, one of the important uses of a data structure queue is the process queue maintained by the scheduler. Both these data structures are modified versions of the list data structure, so they can be implemented using arrays or linked representation.

2. STACKS

A stack is simply a list of elements with insertions and deletions permitted at one end—called the stack top. That means that it is possible to remove elements from a stack in reverse order from the insertion of elements into the stack. Thus, a stack data structure exhibits the LIFO (last in first out) property.



Push and pop are the operations that are provided for insertion of an element into the stack and the removal of an element from the stack, respectively. Shown in Figure 5.1 are the effects of push and pop operations on the stack.



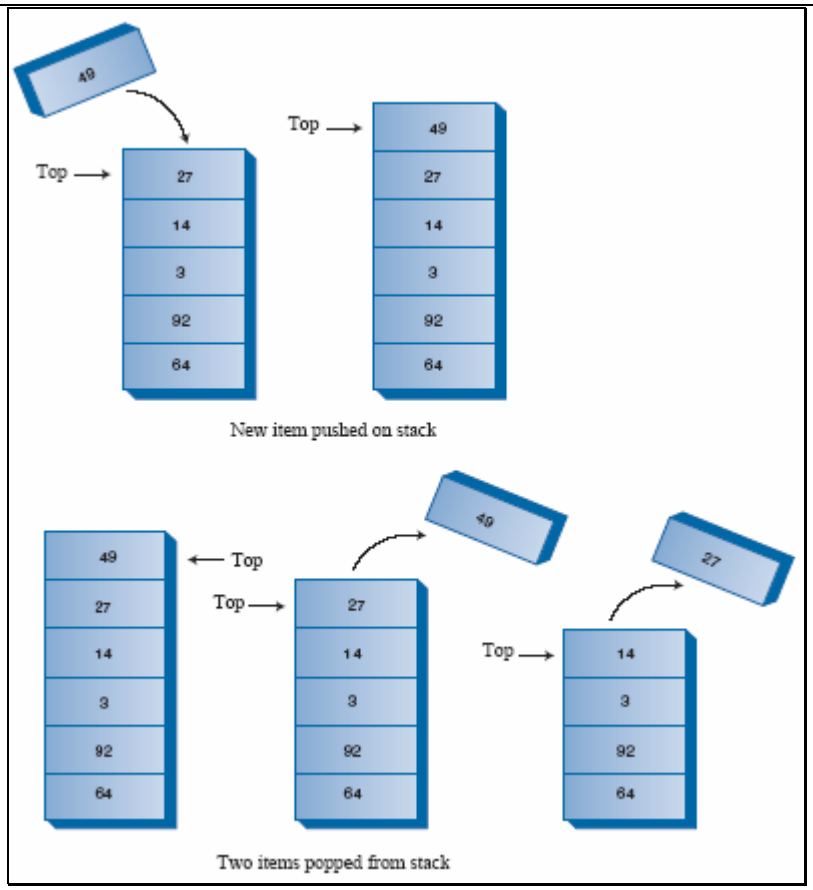


Figure 5.1: Stack operations.

Since a stack is basically a list, it can be implemented by using an array or by using a linked representation.

Array Implementation of a Stack

When an array is used to implement a stack, the push and pop operations are realized by using the operations available on an array. The limitation of an array implementation is that the stack cannot grow and shrink dynamically as per the requirement.

Program

A complete C program to implement a stack using an array appears here:

```
#include <stdio.h>
#define MAX 10 /* The maximum size of the stack */
#include <stdlib.h>

void push(int stack[], int *top, int value)
{
   if(*top < MAX )
   {
      *top = *top + 1;
      stack[*top] = value;
   }
  else
  {
      printf("The stack is full can not push a value\n");
```

```
      exit(0);
   }
}

void pop(int stack[], int *top, int * value)
{
   if(*top >= 0 )
   {
       *value = stack[*top];
      *top = *top - 1;
   }
   else
   {
      printf("The stack is empty can not pop a value\n");
      exit(0);
   }
}

void main()
{
   int stack[MAX];
   int top = -1;
   int n,value;
   do
   {
      do
      {
            printf("Enter the element to be pushed\n");
         scanf("%d",&value);
         push(stack,&top,value);
            printf("Enter 1 to continue\n");
         scanf("%d",&n);
      } while(n == 1);

      printf("Enter 1 to pop an element\n");
      scanf("%d",&n);
      while( n == 1)
      {
            pop(stack,&top,&value);
         printf("The value poped is %d\n",value);
            printf("Enter 1 to pop an element\n");
            scanf("%d",&n);
      }
      printf("Enter 1 to continue\n");
      scanf("%d",&n);
   } while(n == 1);
```

```
}
```

Example

Enter the element to be pushed

```
10
```

Enter 1 to continue

```
1
```

Enter the element to be pushed

```
20
```

Enter 1 to continue

```
0
```

Enter 1 to pop an element

```
1
```

The value popped is 20

Enter 1 to pop an element

```
0
```

Enter 1 to continue

```
1
```

Enter the element to be pushed

```
40
```

Enter 1 to continue

```
1
```

Enter the element to be pushed

```
50
```

Enter 1 to continue

```
0
```

Enter 1 to pop an element

```
1
```

The value popped is 50

Enter 1 to pop an element

```
1
```

The value popped is 40 Enter 1 to pop an element

```
1
```

The value popped is 10 Enter 1 to pop an element

```
0
```

Enter 1 to continue

```
0
```

Implementation of a Stack Using Linked Representation

Initially the list is empty, so the top pointer is NULL. The push function takes a pointer to an existing list as the first parameter and a data value to be pushed as the second parameter, creates a new node by using the data value, and adds it to the top of the existing list. A pop function takes a pointer to an existing list as the first parameter, and a pointer to a data object in which the popped value is to be returned as a second parameter. Thus it retrieves the value of the node pointed to by the top pointer, takes the top point to the next node, and destroys the node that was pointed to by the top.

If this strategy is used for creating a stack with the previously used four data values: 10, 20, 30, and 40, then the stack is created as shown in Figure 5.2.

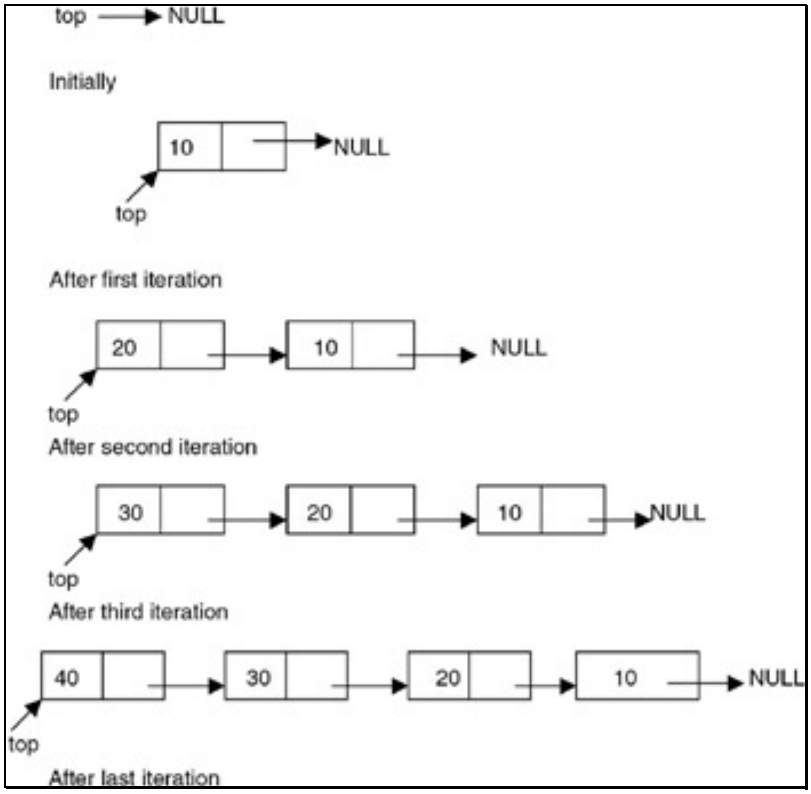


Figure 5.2: Linked stack.

Program

A complete C program for implementation of a stack using the linked list is given here:

```
# include <stdio.h>
# include <stdlib.h>
struct node
{
   int data;
   struct node *link;
};
struct node *push(struct node *p, int value)
{
   struct node *temp;
   temp=(struct node *)malloc(sizeof(struct node));
       /* creates new node
       using data value
       passed as parameter */
   if(temp==NULL)
```

```
   {
      printf("No Memory available Error\n");
      exit(0);
   }
   temp->data = value;
   temp->link = p;
   p = temp;
   return(p);
}

struct node *pop(struct node *p, int *value)
{
   struct node *temp;
   if(p==NULL)
   {
      printf(" The stack is empty can not pop Error\n");
      exit(0);
   }
   *value = p->data;
   temp = p;
   p = p->link;
   free(temp);
   return(p);
}

void main()
{
   struct node *top = NULL;
   int n,value;
   do
   {
      do
      {
            printf("Enter the element to be pushed\n");
         scanf("%d",&value);
         top = push(top,value);
         printf("Enter 1 to continue\n");
         scanf("%d",&n);
      } while(n == 1);

      printf("Enter 1 to pop an element\n");
      scanf("%d",&n);
      while( n == 1)
      {
            top = pop(top,&value);
         printf("The value poped is %d\n",value);
```

```
            printf("Enter 1 to pop an element\n");
            scanf("%d",&n);
      }
      printf("Enter 1 to continue\n");
      scanf("%d",&n);
   } while(n == 1);
}
```

Example

Input and Output

Enter the element to be pushed

10

Enter 1 to continue

1

Enter the element to be pushed

20

Enter 1 to continue

0

Enter 1 to pop an element

1

The value popped is 20

Enter 1 to pop an element

1

The value poped is 10

Enter 1 to pop an element

0

Enter 1 to continue

1

Enter the element to be pushed

30

Enter 1 to continue

1

Enter the element to be pushed

40

Enter 1 to continue

0

Enter 1 to pop an element

1

```
The value popped is 40

Enter 1 to pop an element
0

Enter 1 to continue
1

Enter the element to be pushed
50

Enter 1 to continue
0

Enter 1 to pop an element
1

The value popped is 50

Enter 1 to pop an element
1

The value popped is 30

Enter 1 to pop an element
0

Enter 1 to continue
0
```

3. APPLICATIONS OF STACKS

Introduction

One of the applications of the stack is in expression evaluation. A complex assignment statement such as a = b + c*d/e–f may be interpreted in many different ways. Therefore, to give a unique meaning, the precedence and associativity rules are used. But still it is difficult to evaluate an expression by computer in its present form, called the infix notation. In *infix notation*, the binary operator comes in between the operands. A unary operator comes before the operand. To get it evaluated, it is first converted to the postfix form, where the operator comes after the operands. For example, the postfix form for the expression a*(b–c)/d is abc–*d/. A good thing about postfix expressions is that they do not require any precedence rules or parentheses for unique definition. So, evaluation of a postfix expression is possible using a stack-based algorithm.

Program

Convert an infix expression to prefix form.

```
#include <stdio.h>
#include <string.h>
#include <ctype.h>

#define N 80

typedef enum {FALSE, TRUE} bool;
```

```
#include "stack.h"
#include "queue.h"

#define NOPS 7

char operators [] = "()^/*+-";
int priorities[] = {4,4,3,2,2,1,1};
char associates[] = " RLLLL";

char t[N]; char *tptr = t; // this is where prefix will be saved.
int getIndex( char op ) {
    /*
     * returns index of op in operators.
     */
    int i;
    for( i=0; i<NOPS; ++i )
        if( operators[i] == op )
                return i;
    return -1;
}
int getPriority( char op ) {
    /*
     * returns priority of op.
     */
    return priorities[ getIndex(op) ];
}

char getAssociativity( char op ) {
    /*
     * returns associativity of op.
     */
    return associates[ getIndex(op) ];
}

void processOp( char op, queue *q, stack *s ) {
    /*
     * performs processing of op.
     */
    switch(op) {
       case ')':
              printf( "\t S pushing )...\n" );
              sPush( s, op );
              break;
       case '(':
              while( !qEmpty(q) ) {
```

```
                        *tptr++ = qPop(q);
                        printf( "\tQ popping %c...\n", *(tptr-1) );
                }
                while( !sEmpty(s) ) {
                        char popop = sPop(s);
                        printf( "\tS popping %c...\n", popop );
                        if( popop == ')' )
                                break;
                        *tptr++ = popop;
                }
                break;
        default: {
                int priop;    // priority of op.
                char topop;   // operator on stack top.
                int pritop;   // priority of topop.
                char asstop;  // associativity of topop.
                while( !sEmpty(s) ) {
                        priop = getPriority(op);
                        topop = sTop(s);
                        pritop = getPriority(topop);
                        asstop = getAssociativity(topop);
if( pritop < priop || (pritop == priop && asstop == 'L')
    || topop == ')' ) // IMP.
                         break;
                   while( !qEmpty(q) ) {
                          *tptr++ = qPop(q);
                   printf( "\tQ popping %c...\n", *(tptr-1) );
                   }
                   *tptr++ = sPop(s);
                   printf( "\tS popping %c...\n", *(tptr-1) );
          }
          printf( "\tS pushing %c...\n", op );
          sPush( s, op );
          break;
       }
   }
}
bool isop( char op ) {
    /*
     * is op an operator?
     */
    return (getIndex(op) != -1);
}

char *in2pre( char *str ) { /*
     * returns valid infix expr in str to prefix.
```

```
     */
    char *sptr;
    queue q = {NULL};
    stack s = NULL;
    char *res = (char *)malloc( N*sizeof(char) );
    char *resptr = res;
    tptr = t;
    for( sptr=str+strlen(str)-1; sptr!=str-1; -sptr ) {
       printf( "processing %c tptr-t=%d...\n", *sptr, tptr-t );
       if( isalpha(*sptr) ) // if operand.
              qPush( &q, *sptr );
       else if( isop(*sptr) )      // if valid operator.
              processOp( *sptr, &q, &s );
       else if( isspace(*sptr) )    // if whitespace.
              ;
       else {
              fprintf( stderr, "ERROR:invalid char %c.\n", *sptr );
              return "";
       }
   }
   while( !qEmpty(&q) ) {
       *tptr++ = qPop(&q);
       printf( "\tQ popping %c...\n", *(tptr-1) );
   }
   while( !sEmpty(&s) ) {
       *tptr++ = sPop(&s);
       printf( "\tS popping %c...\n", *(tptr-1) );
   }
   *tptr = 0;
   printf( "t=%s.\n", t );
   for( -tptr; tptr!=t-1; -tptr ) {
      *resptr++ = *tptr;
   }
   *resptr = 0;

   return res;
}

int main() {
    char s[N];

    puts( "enter infix freespaces max 80." );
    gets(s);
    while(*s) {
       puts( in2pre(s) );
       gets(s);
```

```
    }

   return 0;
}
```

Explanation

1. In an infix expression, a binary operator separates its operands (a unary operator precedes its operand). In a postfix expression, the operands of an operator precede the operator. In a prefix expression, the operator precedes its operands. Like postfix, a prefix expression is parenthesis-free, that is, any infix expression can be unambiguously written in its prefix equivalent without the need for parentheses.
2. To convert an infix expression to reverse-prefix, it is scanned from right to left. A queue of operands is maintained noting that the order of operands in infix and prefix remains the same. Thus, while scanning the infix expression, whenever an operand is encountered, it is pushed in a queue. If the scanned element is a right parenthesis (‘)’), it is pushed in a stack of operators. If the scanned element is a left parenthesis (‘(‘), the queue of operands is emptied to the prefix output, followed by the popping of all the operators up to, but excluding, a right parenthesis in the operator stack.
3. If the scanned element is an arbitrary operator o, then the stack of operators is checked for operators with a greater priority then o. Such operators are popped and written to the prefix output after emptying the operand queue. The operator o is finally pushed to the stack.
4. When the scanning of the infix expression is complete, first the operand queue, and then the operator stack, are emptied to the prefix output. Any whitespace in the infix input is ignored. Thus the prefix output can be reversed to get the required prefix expression of the infix input.

Example

If the infix expression is a*b + c/d, then different snapshots of the algorithm, while scanning the expression from right to left, are shown in Table 5.1.

Table 5.1: Scanning the infex expression a*b+c/d from right to left

STEP	REMAINING EXPRESSION	SCANNED ELEMENT	QUEUE OF OPERANDS	STACK OF OPERATORS	PREFIX OUTPUT
0	a*b+c/d	nil	empty	empty	nil
1	a*b+c/	d	d	empty	nil
2	a*b+c	/	d	/	nil
3	a*b+	c	d c	/	nil
4	a*b	+	empty	+	dc/
5	a*	b	b	+	dc/
6	a	*	b	* +	dc/
7	nil	a	b a	* +	dc/
8	nil	nil	empty	empty	dc/ba*+

The final prefix output that we get is dc/ba*+ whose reverse is +*ab/cd, which is the prefix equivalent of the input infix expression a*b+c*d. Note that all the operands are simply pushed to the queue in steps 1, 3, 5, and 7. In step 2, the operator / is pushed to the empty stack of operators.

In step 4, the operator + is checked against the elements in the stack. Since / (division) has higher priority than + (addition), the queue is emptied to the prefix output (thus we get ‘dc’ as the output) and then the operator / is written (thus we get ‘dc/’ as the output). The operator + is then pushed to the stack. In step 6, the operator * is checked against the stack elements. Since * (multiplication) has a higher priority than + (addition), * is pushed to the stack. Step 8 signifies that all of the infix expression

is scanned. Thus, the queue of operands is emptied to the prefix output (to get 'dc/ba'), followed by the emptying of the stack of operators (to get 'dc/ba*+').

Points to remember

1. A prefix expression is parenthesis-free.
2. To convert an infix expression to the postfix equivalent, it is scanned from right to left. The prefix expression we get is the reverse of the required prefix equivalent.
3. Conversion of infix to prefix requires a queue of operands and a stack, as in the conversion of an infix to postfix.
4. The order of operands in a prefix expression is the same as that in its infix equivalent.
5. If the scanned operator o1 and the operator o2 at the stack top have the same priority, then the associativity of o2 is checked. If o2 is right-associative, it is popped from the stack.

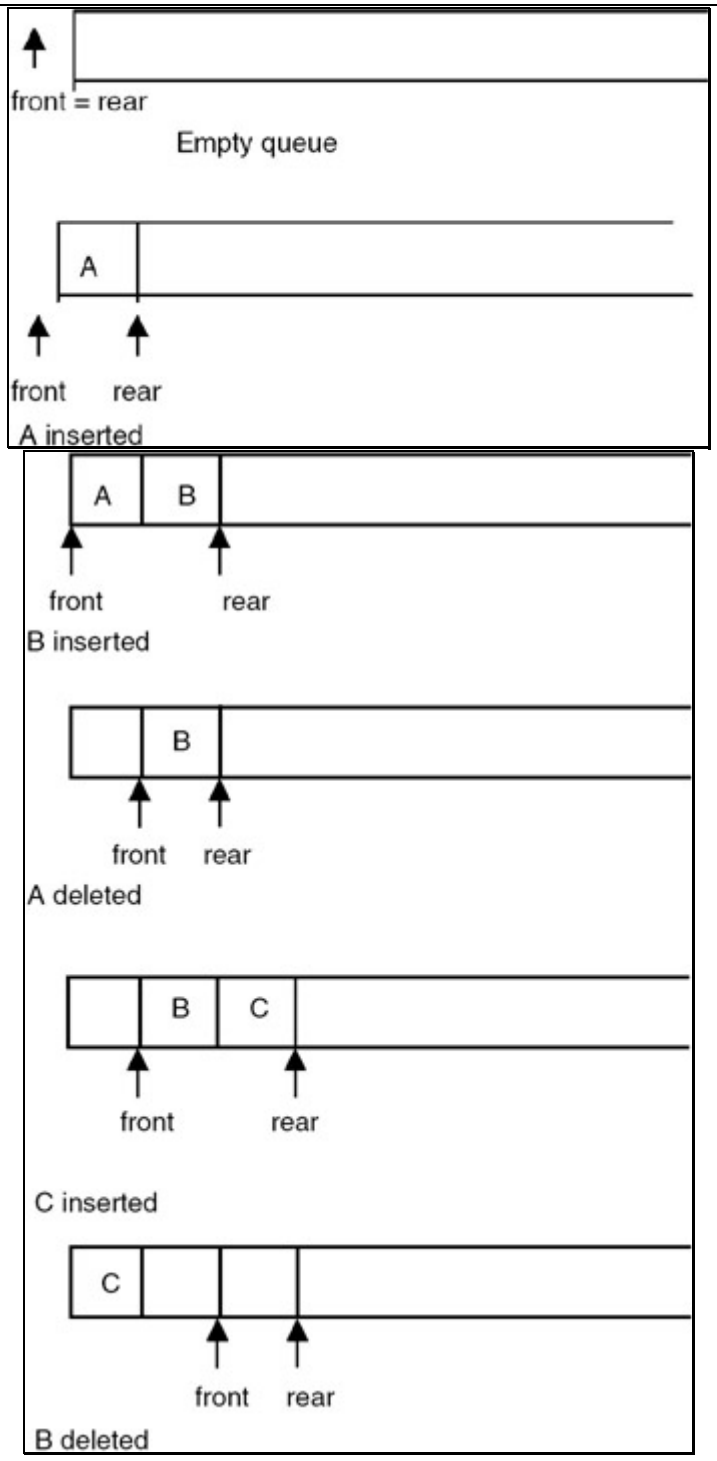
4. QUEUES

Introduction

A *queue* is also a list of elements with insertions permitted at one end—called the rear, and deletions permitted from the other end—called the front. This means that the removal of elements from a queue is possible in the same order in which the insertion of elements is made into the queue. Thus, a queue data structure exhibits the *FIFO* (*first in first out*) property.

A FIFO queue

`insert` and `delete` are the operations that are provided for insertion of elements into the queue and the removal of elements from the queue, respectively. Shown in Figure 5.3 are the effects of `insert` and `delete` operations on the queue.


front = rear
Empty queue
A
front
rear
A inserted
A
B
front
rear
B inserted
B
front
rear
A deleted
B
C
front
rear
C inserted
C
front
rear
B deleted

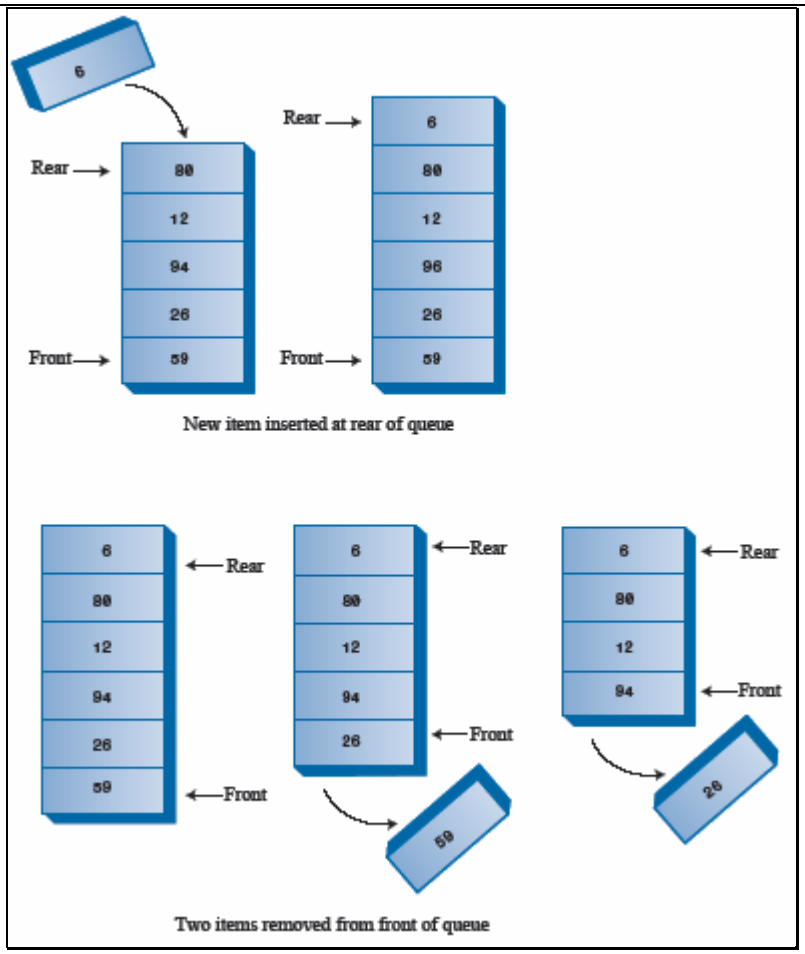


Figure 5.3: Operations on a queue.

5. IMPLEMENTATION OF QUEUES

Introduction

Since a queue is also a list, it can be implemented using an array or it can be implemented using a linked representation.

Array Implementation of a Stack

When an array is used to implement a queue, then the insert and delete operations are realized using the operations available on an array. The limitation of an array implementation is that the queue cannot grow and shrink dynamically as per the requirement.

Program

A complete C program to implement a queue by using an array is shown here:

```
#include <stdio.h>
#define MAX 10 /* The maximum size of the queue */
#include <stdlib.h>

void insert(int queue[], int *rear, int value)
{
   if(*rear < MAX-1)
   {
```

```
      *rear= *rear +1;
      queue[*rear] = value;
   }
   else
   {
      printf("The queue is full can not insert a value\n");
      exit(0);
   }
}

void delete(int queue[], int *front, int rear, int * value)
{
   if(*front == rear)
   {
      printf("The queue is empty can not delete a value\n");
      exit(0);
   }
   *front = *front + 1;
   *value = queue[*front];
}

void main()
{
   int queue[MAX];
   int front,rear;
   int n,value;
   front=rear=(-1);
   do
   {
      do
      {
            printf("Enter the element to be inserted\n");
         scanf("%d",&value);
         insert(queue,&rear,value);
         printf("Enter 1 to continue\n");
            scanf("%d",&n);
      } while(n == 1);

      printf("Enter 1 to delete an element\n");
      scanf("%d",&n);
      while( n == 1)
      {
            delete(queue,&front,rear,&value);
         printf("The value deleted is %d\n",value);
            printf("Enter 1 to delete an element\n");
         scanf("%d",&n);
```

```
      }
      printf("Enter 1 to continue\n");
      scanf("%d",&n);
   } while(n == 1);
}
```

Example

Input and Output

Enter the element to be inserted

```
10
```

Enter 1 to continue

```
1
```

Enter the element to be inserted

```
20
```

Enter 1 to continue

```
1
```

Enter the element to be inserted

```
30
```

Enter 1 to continue

```
0
```

Enter 1 to delete an element

```
1
```

The value deleted is 10

Enter 1 to delete an element

```
1
```

The value deleted is 20

Enter 1 to delete an element

```
0
```

Enter 1 to continue

```
1
```

Enter the element to be inserted

```
40
```

Enter 1 to continue

```
1
```

Enter the element to be inserted

```
50
```

Enter 1 to continue

```
0
```

Enter 1 to delete an element

```
1
```

The value deleted is 30

Enter 1 to delete an element

```
1
```

The value deleted is 40

Enter 1 to delete an element

```
0
```

Enter 1 to continue

```
0
```

6. IMPLEMENTATION OF A QUEUE USING LINKED REPRESENTATION

Introduction

Initially, the list is empty, so both the front and rear pointers are `NULL`. The `insert` function creates a new node, puts the new data value in it, appends it to an existing list, and makes the rear pointer point to it. A `delete` function checks whether the queue is empty, and if not, retrieves the data value of the node pointed to by the front, advances the front, and frees the storage of the node whose data value has been retrieved.

If the above strategy is used for creating a queue with four data values —10, 20, 30, and 40, the queue gets created as shown in Figure 5.5.

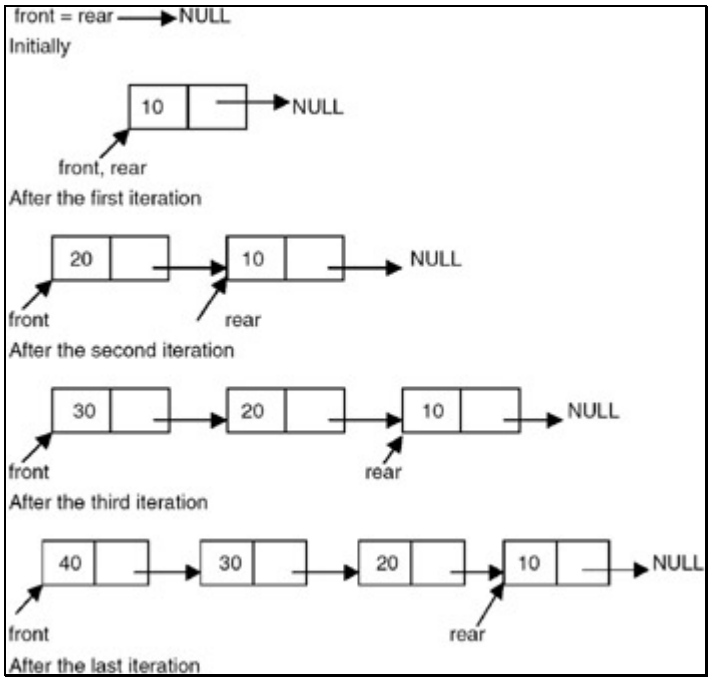


Figure 5.5: Linked queue.

Program

A complete C program for implementation of a stack using the linked list is shown here:

```
# include <stdio.h>
```

```
# include <stdlib.h>
struct node
{
   int data;
   struct node *link;
};

void insert(struct node **front, struct node **rear, int value)
{
   struct node *temp;
   temp=(struct node *)malloc(sizeof(struct node));
       /* creates new node
       using data value
       passed as parameter */
   if(temp==NULL)
   {
      printf("No Memory available Error\n");
      exit(0);
   }
   temp->data = value;
   temp->link=NULL;
   if(*rear == NULL)
   {
      *rear = temp;
      *front = *rear;
   }
   else
   {
      (*rear)->link = temp;
      *rear = temp;
   }
}

void delete(struct node **front, struct node **rear, int *value)
{
   struct node *temp;
   if((*front == *rear) && (*rear == NULL))
   {
      printf(" The queue is empty cannot delete Error\n");
      exit(0);
   }
   *value = (*front)->data;
   temp = *front;
   *front = (*front)->link;
   if(*rear == temp)
   *rear = (*rear)->link;
```

```
   free(temp);
}

void main()
{
   struct node *front=NULL,*rear = NULL;
   int n,value;
   do
   {
      do
      {
            printf("Enter the element to be inserted\n");
         scanf("%d",&value);
         insert(&front,&rear,value);
         printf("Enter 1 to continue\n");
         scanf("%d",&n);
      } while(n == 1);

      printf("Enter 1 to delete an element\n");
      scanf("%d",&n);
      while( n == 1)
      {
            delete(&front,&rear,&value);
            printf("The value deleted is %d\n",value);
            printf("Enter 1 to delete an element\n");
            scanf("%d",&n);
      }
      printf("Enter 1 to continue\n");
      scanf("%d",&n);
   } while(n == 1);
}
```

Example

Input and Output

Enter the element to be inserted

10

Enter 1 to continue

1

Enter the element to be inserted

20

Enter 1 to continue

1

Enter the element to be inserted

```
30
```

Enter 1 to continue

```
0
```

Enter 1 to delete an element

```
1
```

The value deleted is 10

Enter 1 to delete an element

```
1
```

The value deleted is 20

Enter 1 to delete an element

```
0
```

Enter 1 to continue

```
1
```

Enter the element to be inserted

```
40
```

Enter 1 to continue

```
1
```

Enter the element to be inserted

```
50
```

Enter 1 to continue

```
0
```

Enter 1 to delete an element

```
1
```

The value deleted is 30

Enter 1 to pop an element

```
1
```

The value deleted is 40

Enter 1 to delete an element

```
1
```

The value deleted is 50

Enter 1 to delete an element

```
1
```

The queue is empty, cannot delete Error

7. APPLICATIONS OF QUEUES

Introduction

One application of the queue data structure is in the implementation of priority queues required to be maintained by the scheduler of an operating system. It is a queue in which each element has a priority value and the elements are required to be inserted in the queue in decreasing order of priority. This requires a change in the function that is used for insertion of an element into the queue. No change is required in the `delete` function.

Program

A complete C program implementing a priority queue is shown here:

```
# include <stdio.h>
# include <stdlib.h>
struct node
{
   int data;
   int priority;
   struct node *link;
};
void insert(struct node **front, struct node **rear, int value, int
priority)
{
   struct node *temp,*temp1;
   temp=(struct node *)malloc(sizeof(struct node));
      /* creates new node using data value
   passed as parameter */
   if(temp==NULL)
   {
      printf("No Memory available Error\n");
      exit(0);
   }
   temp->data = value;
   temp->priority = priority;
   temp->link=NULL;
   if(*rear == NULL) /* This is the first node */
   {
      *rear = temp;
      *front = *rear;
   }
   else
   {
      if((*front)->priority < priority)
             /* the element to be inserted has
      highest priority hence should
             be the first element*/
      {
```

```
            temp->link = *front;
            *front = temp;
      }
      else
            if( (*rear)->priority > priority)
                  /* the element to be inserted has
                  lowest priority hence should
                  be the last element*/
            {
                  (*rear)->link = temp;
                  *rear = temp;
            }

      else
      {
                  temp1 = *front;
                  while((temp1->link)->priority >= priority)
     /* find the position and insert the new element */
                           temp1=temp1->link;
             temp->link = temp1->link;
             temp1->link = temp;
             }
}
void delete(struct node **front, struct node **rear, int *value, int
*priority)
{
   struct node *temp;
   if((*front == *rear) && (*rear == NULL))
   {
      printf(" The queue is empty cannot delete Error\n");
      exit(0);
   }
   *value = (*front)->data;
   *priority = (*front)->priority;
   temp = *front;
   *front = (*front)->link;
   if(*rear == temp)
       *rear = (*rear)->link;
   free(temp);
}

void main()
{
   struct node *front=NULL,*rear = NULL;
   int n,value, priority;
   do
```

```
   {
   do
   {
         printf("Enter the element to be inserted and its priority\n");
     scanf("%d %d",&value,&priority);
     insert(&front,&rear,value,priority);
     printf("Enter 1 to continue\n");
     scanf("%d",&n);
   } while(n == 1);

   printf("Enter 1 to delete an element\n");
   scanf("%d",&n);
   while( n == 1)
   {
         delete(&front,&rear,&value,&priority);
         printf("The value deleted is %d\ and its priority is %d \n",
                value,priority);
         printf("Enter 1 to delete an element\n");
      scanf("%d",&n);
    }
    printf("Enter 1 to delete an element\n");
    scanf("%d",&n);
  }  while( n == 1)
}
```

Example

Input and Output

Enter the element to be inserted and its priority

```
10 90
```

Enter 1 to continue

```
1
```

Enter the element to be inserted and its priority

```
5 8
```

Enter 1 to continue

```
1
```

Enter the element to be inserted and its priority

```
11 60
```

Enter 1 to continue

```
1
```

Enter the element to be inserted and its priority

```
12 75
```

Enter 1 to continue

```
1
```

Enter the element to be inserted and its priority

```
13 10
```

Enter 1 to continue

```
1
```

Enter the element to be inserted and its priority

```
14 6
```

Enter 1 to continue

```
0
```

Enter 1 to delete an element

```
1
```

The value deleted is 10 and its priority is 90

Enter 1 to delete an element

```
1
```

The value deleted is 12 and its priority is 75

Enter 1 to delete an element

```
1
```

The value deleted is 11 and its priority is 60 Enter 1 to delete an element

```
1
```

The value deleted is 13 and its priority is 10 Enter 1 to delete an element

```
1
```

The value deleted is 5 and its priority is 8

Enter 1 to delete an element

```
1
```

The value deleted is 14 and its priority is 6

Enter 1 to delete an element

```
1
```

The queue is empty cannot delete Error

Points to Remember

1. A stack is basically a list with insertions and deletions permitted from only one end, called the stack-top, so it is a data structure that exhibits the LIFO property.
2. The operations that are permitted to manipulate a stack are push and pop.
3. One of the important applications of a stack is in the implementation of recursion in the programming language.
4. A queue is also a list with insertions permitted from one end, called rear, and deletions permitted from the other end, called front. So it is a data structure that exhibits the FIFO property.
5. The operations that are permitted on a queue are `insert` and `delete`.
6. A circular queue is a queue in which the element next to the last element is the first element.

7. When the size of the stack/queue is known beforehand, the array implementation can be used and is more efficient.
8. When the size of the stack/queue is not known beforehand, then the linked representation is used. It provides more flexibility.

8. Exercises

E1. Write a C program to implement a stack of integer numbers. Write push, pop, isEmpty function.

- Push values into stack: 9 8 1 3 7 9 4
- Pop 3 numbers.
- Display number of elements in stack.
- Display stack.
- Remove all items in stack

E2. Build a queue to store integer numbers. Write push, pop, isEmpty function.

- Push values into queue: 9 8 1 3 7 9 4
- Pop 3 numbers.
- Display number of elements in queue.
- Display queue.
- Remove all items in queue

E3. Using queue to invert a input number. Example: N=12345, the result is: 54321

CHAPTER 6: LINKED LISTS

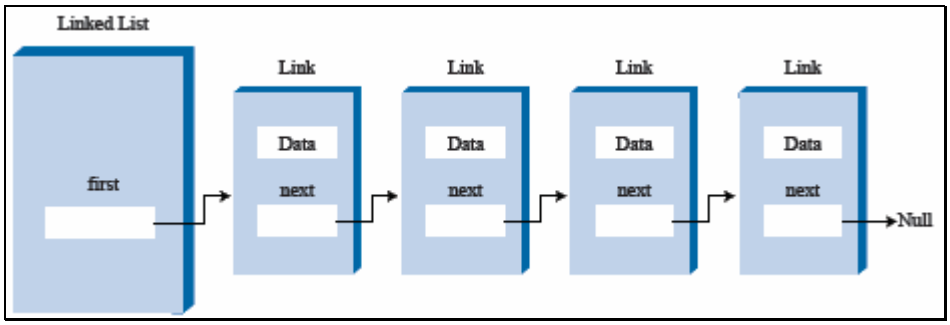
1. THE CONCEPT OF THE LINKED LIST

Introduction

When dealing with many problems we need a dynamic list, dynamic in the sense that the size requirement need not be known at compile time. Thus, the list may grow or shrink during runtime. A *linked list* is a data structure that is used to model such a dynamic list of data items, so the study of the linked lists as one of the data structures is important.

Concept

An array is represented in memory using sequential mapping, which has the property that elements are fixed distance apart. But this has the following disadvantage: It makes insertion or deletion at any arbitrary position in an array a costly operation, because this involves the movement of some of the existing elements.



When we want to represent several lists by using arrays of varying size, either we have to represent each list using a separate array of maximum size or we have to represent each of the lists using one single array. The first one will lead to wastage of storage, and the second will involve a lot of data movement.

So we have to use an alternative representation to overcome these disadvantages. One alternative is a linked representation. In a linked representation, it is not necessary that the elements be at a fixed distance apart. Instead, we can place elements anywhere in memory, but to make it a part of the same list, an element is required to be linked with a previous element of the list. This can be done by storing the address of the next element in the previous element itself. This requires that every element be capable of holding the data as well as the address of the next element. Thus every element must be a structure with a minimum of two fields, one for holding the data value, which we call a data field, and the other for holding the address of the next element, which we call link field.

Therefore, a linked list is a list of elements in which the elements of the list can be placed anywhere in memory, and these elements are linked with each other using an explicit link field, that is, by storing the address of the next element in the link field of the previous element.

Program

Here is a program for building and printing the elements of the linked list:

```
# include <stdio.h>
# include <stdlib.h>
struct node
{
```

```
   int data;
   struct node *link;
   };
   struct node *insert(struct node *p, int n)
   {
   struct node *temp;
   /* if the existing list is empty then insert a new node as the
starting node */
   if(p==NULL)
   {
      p=(struct node *)malloc(sizeof(struct node)); /* creates new node
data value passes
   as parameter */

         if(p==NULL)
         {
      printf("Error\n");
             exit(0);
         }
         p-> data = n;
         p-> link = p; /* makes the pointer pointing to itself because it
is a circular list*/
      }
      else
      {
      temp = p;
   /* traverses the existing list to get the pointer to the last node of
   it */
      while (temp-> link != p)
         temp = temp-> link;
            temp-> link = (struct node *)malloc(sizeof(struct node)); /*
   creates new node using
              data value passes as
                parameter and puts its
              address in the link field
              of last node of the
              existing list*/
           if(temp -> link == NULL)
           {
         printf("Error\n");
              exit(0);
           }
           temp = temp-> link;
           temp-> data = n;
           temp-> link = p;
          }
```

```
        return (p);
}
void printlist ( struct node *p )
{
struct node *temp;
 temp = p;
printf("The data values in the list are\n");
   if(p!= NULL)
   {
   do
         {
         printf("%d\t",temp->data);
         temp=temp->link;
         } while (temp!= p);
   }
   else
      printf("The list is empty\n");
}

void main()
{
   int n;
   int x;
   struct node *start = NULL ;
   printf("Enter the nodes to be created \n");
   scanf("%d",&n);
   while ( n -- > 0 )
   {
printf( "Enter the data values to be placed in a node\n");
      scanf("%d",&x);
      start = insert ( start, x );
   }
   printf("The created list is\n");
   printlist ( start );
}
```

Explanation

1. This program uses a strategy of inserting a node in an existing list to get the list created. An `insert` function is used for this.
2. The `insert` function takes a pointer to an existing list as the first parameter, and a data value with which the new node is to be created as a second parameter, creates a new node by using the data value, appends it to the end of the list, and returns a pointer to the first node of the list.
3. Initially the list is empty, so the pointer to the starting node is `NULL`. Therefore, when `insert` is called first time, the new node created by the `insert` becomes the start node.
4. Subsequently, the insert traverses the list to get the pointer to the last node of the existing list, and puts the address of the newly created node in the link field of the last node, thereby appending the new node to the existing list.

5. The main function reads the value of the number of nodes in the list. Calls iterate that many times by going in a `while` loop to create the links with the specified number of nodes.

Points to Remember

1. Linked lists are used when the quantity of data is not known prior to execution.
2. In linked lists, data is stored in the form of nodes and at runtime, memory is allocated for creating nodes.
3. Due to overhead in memory allocation and deallocation, the speed of the program is lower.
4. The data is accessed using the starting pointer of the list.

2. INSERTING A NODE BY USING RECURSIVE PROGRAMS

Introduction

A linked list is a recursive data structure. A *recursive data structure* is a data structure that has the same form regardless of the size of the data. You can easily write recursive programs for such data structures.

Program

```
# include <stdio.h>
# include <stdlib.h>
struct node
{
int data;
struct node *link;
};
struct node *insert(struct node *p, int n)
{
   struct node *temp;
   if(p==NULL)
   {
      p=(struct node *)malloc(sizeof(struct node));
      if(p==NULL)
      {
   printf("Error\n");
         exit(0);
   }
   p-> data = n;
   p-> link = NULL;
}
else
      p->link = insert(p->link,n);/* the while loop replaced by
recursive call */
   return (p);
}
void printlist ( struct node *p )
{
      printf("The data values in the list are\n");
```

```
        while (p!= NULL)
        {
    printf("%d\t",p-> data);
           p = p-> link;
        }
}
void main()
{
        int n;
        int x;
        struct node *start = NULL ;
        printf("Enter the nodes to be created \n");
        scanf("%d",&n);
        while ( n- > 0 )
        {
    printf( "Enter the data values to be placed in a node\n");
           scanf("%d",&x);
           start = insert ( start, x );
        }
        printf("The created list is\n");
        printlist ( start );
}
```

Explanation

1. This recursive version also uses a strategy of inserting a node in an existing list to create the list.
2. An `insert` function is used to create the list. The `insert` function takes a pointer to an existing list as the first parameter, and a data value with which the new node is to be created as the second parameter. It creates the new node by using the data value, then appends it to the end of the list. It then returns a pointer to the first node of the list.
3. Initially, the list is empty, so the pointer to the starting node is `NULL`. Therefore, when `insert` is called the first time, the new node created by the `insert` function becomes the start node.
4. Subsequently, the `insert` function traverses the list by recursively calling itself.
5. The recursion terminates when it creates a new node with the supplied data value and appends it to the end of the list.

Points to Remember

1. A linked list has a recursive data structure.
2. Writing recursive programs for such structures is programmatically convenient.

3. SORTING AND REVERSING A LINKED LIST

Introduction

To sort a linked list, first we traverse the list searching for the node with a minimum data value. Then we remove that node and append it to another list which is initially empty. We repeat this process with the remaining list until the list becomes empty, and at the end, we return a pointer to the beginning of the list to which all the nodes are moved, as shown in Figure 6.1.

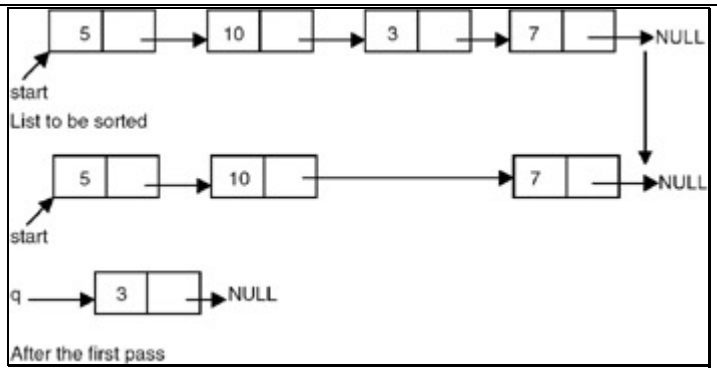


Figure 6.1: Sorting of a linked list.

To reverse a list, we maintain a pointer each to the previous and the next node, then we make the link field of the current node point to the previous, make the previous equal to the current, and the current equal to the next, as shown in Figure 6.2.

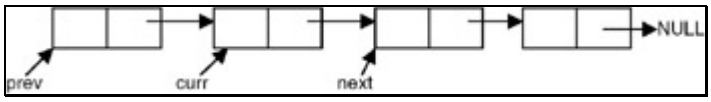


Figure 6.2: A linked list showing the previous, current, and next nodes at some point during reversal process.

Therefore, the code needed to reverse the list is

```
Prev = NULL;
While (curr != NULL)
{
   Next = curr->link;
   Curr -> link = prev;
   Prev = curr;
   Curr = next;
}
```

Program

```
# include <stdio.h>
# include <stdlib.h>
struct node
{
   int data;
   struct node *link;
};
struct node *insert(struct node *p, int n)
{
   struct node *temp;
   if(p==NULL)
   {
      p=(struct node *)malloc(sizeof(struct node));
      if(p==NULL)
      {
            printf("Error\n");
```

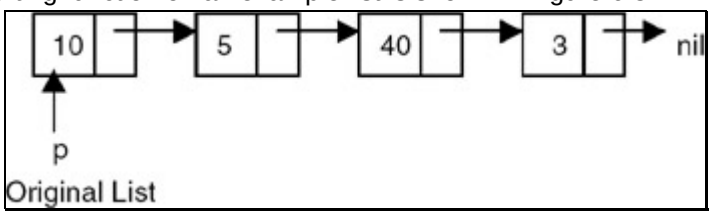
```
         exit(0);
      }
      p-> data = n;
      p-> link = NULL;
   }
   else
   {
      temp = p;
      while (temp-> link!= NULL)
      temp = temp-> link;
      temp-> link = (struct node *)malloc(sizeof(struct node));
      if(temp -> link == NULL)
      {
            printf("Error\n");
         exit(0);
      }
       temp = temp-> link;
       temp-> data = n;
       temp-> link = null;
      }
      return(p);
}

void printlist ( struct node *p )
{
      printf("The data values in the list are\n");
      while (p!= NULL)
      {
            printf("%d\t",p-> data);
         p = p-> link;
      }
}

/* a function to sort reverse list */
struct node *reverse(struct node *p)
{
   struct node *prev, *curr;
   prev = NULL;
   curr = p;
   while (curr != NULL)
   {
      p = p-> link;
      curr-> link = prev;
      prev = curr;
      curr = p;
   }
```

```
   return(prev);
}
/* a function to sort a list */
struct node *sortlist(struct node *p)
{
   struct node *temp1,*temp2,*min,*prev,*q;
   q = NULL;
   while(p != NULL)
   {
         prev = NULL;
      min = temp1 = p;
      temp2 = p -> link;
      while ( temp2 != NULL )
      {
               if(min -> data > temp2 -> data)
         {
                     min = temp2;
                     prev = temp1;
         }
         temp1 = temp2;
         temp2 = temp2-> link;
      }
      if(prev == NULL)
             p = min -> link;
      else
             prev -> link = min -> link;
      min -> link = NULL;
      if( q == NULL)
      q = min; /* moves the node with lowest data value in the list
pointed to by p to the list
   pointed to by q as a first node*/
         else
         {
            temp1 = q;
            /* traverses the list pointed to by q to get pointer to its
last node */
            while( temp1 -> link != NULL)
                        temp1 = temp1 -> link;
            temp1 -> link = min; /* moves the node with lowest data value
in the list pointed to
   by p to the list pointed to by q at the end of list pointed by
   q*/
      }
   }
   return (q);
}
```

```
void main()
{
      int n;
      int x;
      struct node *start = NULL ;
      printf("Enter the nodes to be created \n");
      scanf("%d",&n);
      while ( n- > 0 )
      {
              printf( "Enter the data values to be placed in a
node\n");
         scanf("%d",&x);
         start = insert ( start,x);
      }
      printf("The created list is\n");
      printlist ( start );
      start = sortlist(start);
      printf("The sorted list is\n");
      printlist ( start );
      start = reverse(start);
      printf("The reversed list is\n");
      printlist ( start );
}
```

Explanation

The working of the sorting function on an example list is shown in Figure 6.3.



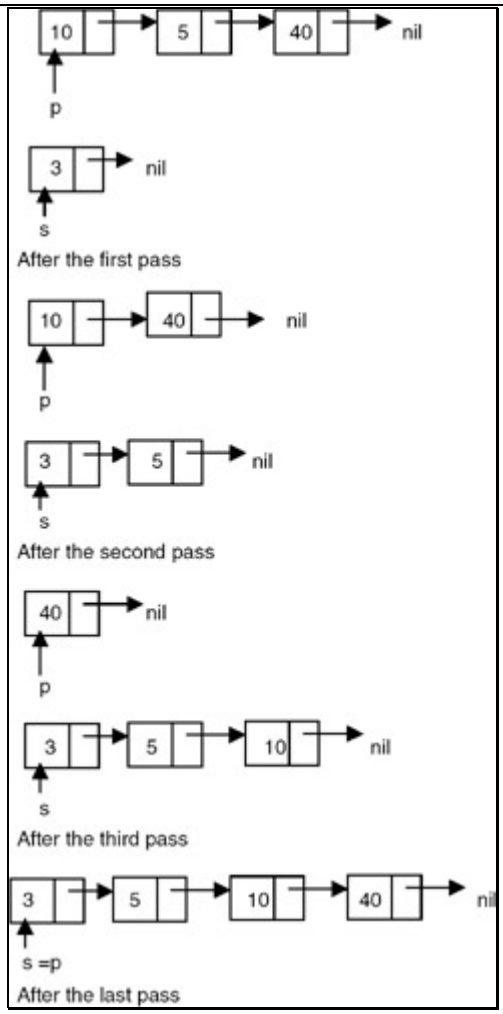


Figure 6.3: Sorting of a linked list.

The working of a reverse function is shown in Figure 6.4.

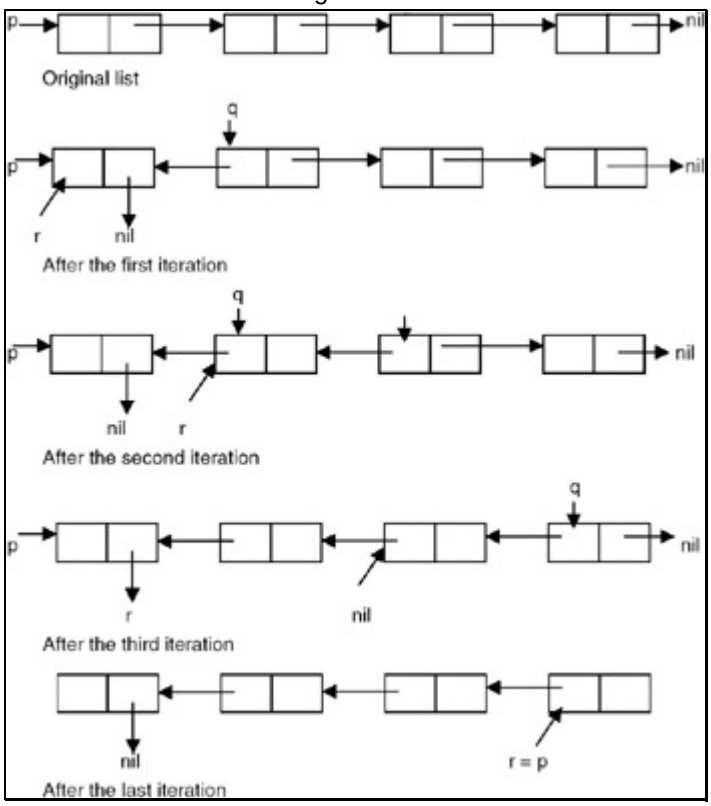
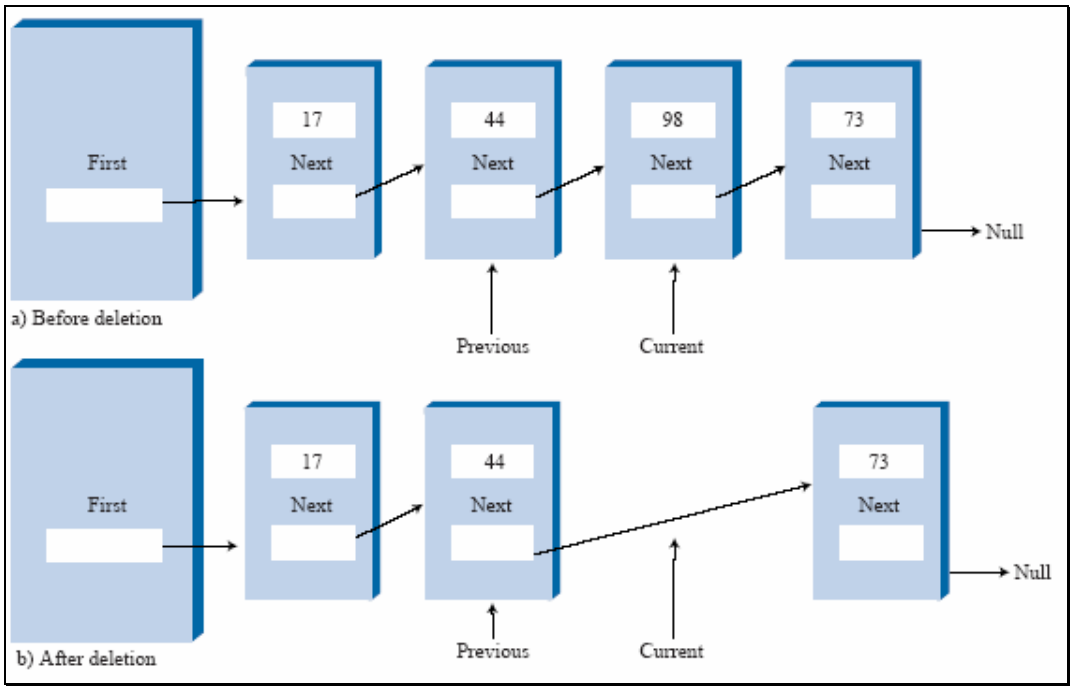


Figure 6.4: Reversal of a list.

4. DELETING THE SPECIFIED NODE IN A SINGLY LINKED LIST

Introduction

To delete a node, first we determine the node number to be deleted (this is based on the assumption that the nodes of the list are numbered serially from 1 to n). The list is then traversed to get a pointer to the node whose number is given, as well as a pointer to a node that appears before the node to be deleted. Then the link field of the node that appears before the node to be deleted is made to point to the node that appears after the node to be deleted, and the node to be deleted is freed. Figures 20.5 and 20.6 show the list before and after deletion, respectively.



Program

```
# include <stdio.h>
# include <stdlib.h>
struct node *delet ( struct node *, int );
int length ( struct node * );
struct node
{
   int data;
   struct node *link;
};
struct node *insert(struct node *p, int n)
{
   struct node *temp;
   if(p==NULL)
   {
      p=(struct node *)malloc(sizeof(struct node));
      if(p==NULL)
      {
            printf("Error\n");
```

```
         exit(0);
      }
      p-> data = n;
      p-> link = NULL;
   }
   else
   {
      temp = p;
      while (temp-> link != NULL)
       temp = temp-> link;
       temp-> link = (struct node *)malloc(sizeof(struct node));
      if(temp -> link == NULL)
      {
            printf("Error\n");
         exit(0);
      }
      temp = temp-> link;
      temp-> data = n;
      temp-> link = NULL;
      }
      return (p);
}

void printlist ( struct node *p )
{
      printf("The data values in the list are\n");
      while (p!= NULL)
      {
            printf("%d\t",p-> data);
         p = p-> link;
      }
}

void main()
{
   int n;
   int x;
   struct node *start = NULL;
   printf("Enter the nodes to be created \n");
   scanf("%d",&n);
   while ( n- > 0 )
   {
       printf( "Enter the data values to be placed in a node\n");
      scanf("%d",&x);
      start = insert ( start, x );
   }
```

```
   printf(" The list before deletion id\n");
   printlist ( start );
   printf("% \n Enter the node no \n");
   scanf ( " %d",&n);
   start = delet (start , n );
   printf(" The list after deletion is\n");
   printlist ( start );
}

   /* a function to delete the specified node*/
struct node *delet ( struct node *p, int node_no )
{

   struct node *prev, *curr ;
   int i;

   if (p == NULL )
   {
      printf("There is no node to be deleted \n");
   }
   else
   {
      if ( node_no > length (p))
      {
            printf("Error\n");
      }
      else
      {
            prev = NULL;
            curr = p;
            i = 1 ;
            while ( i < node_no )
            {
                  prev = curr;
                  curr = curr-> link;
                  i = i+1;
            }
            if ( prev == NULL )
            {
                  p = curr -> link;
                  free ( curr );
            }
            else
            {
               prev -> link = curr -> link ;
               free ( curr );
```

```
            }
      }
   }
   return(p);
}
/* a function to compute the length of a linked list */
int length ( struct node *p )
{
   int count = 0 ;
   while ( p != NULL )
   {
      count++;
      p = p->link;
   }
   return ( count ) ;
}
```

Explanation

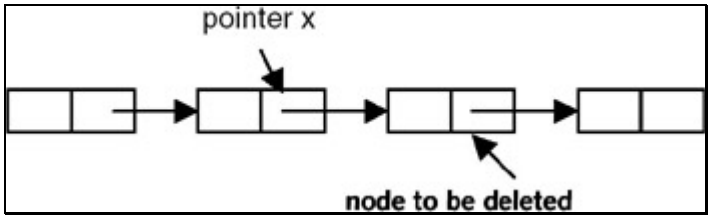


Figure 6.5: Before deletion.

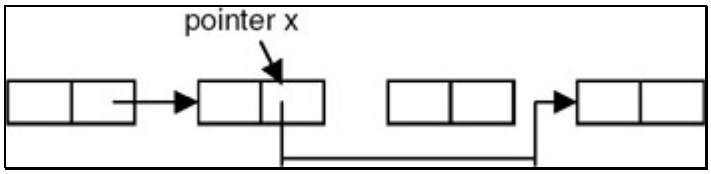
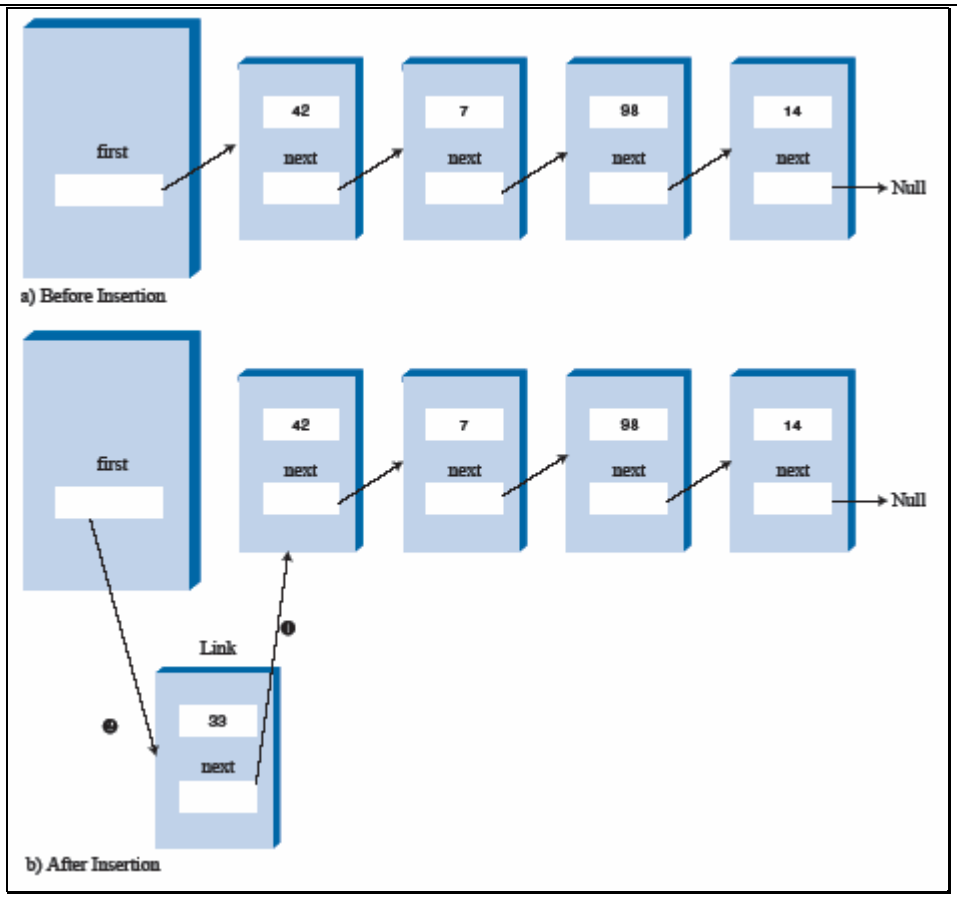


Figure 6.6: After deletion.

5. INSERTING A NODE AFTER THE SPECIFIED NODE IN A SINGLY LINKED LIST

Introduction

To insert a new node after the specified node, first we get the number of the node in an existing list after which the new node is to be inserted. This is based on the assumption that the nodes of the list are numbered serially from 1 to n. The list is then traversed to get a pointer to the node, whose number is given. If this pointer is x, then the link field of the new node is made to point to the node pointed to by x, and the link field of the node pointed to by x is made to point to the new node. Figures 20.7 and 20.8 show the list before and after the insertion of the node, respectively.



Program

```
   # include <stdio.h>
   # include <stdlib.h>
   int length ( struct node * );
   struct node
   {
      int data;
      struct node *link;
   };

   /* a function which appends a new node to an existing list used for
building a list */
   struct node *insert(struct node *p, int n)
   {
      struct node *temp;
      if(p==NULL)
      {
          p=(struct node *)malloc(sizeof(struct node));
         if(p==NULL)
         {
                 printf("Error\n");
            exit(0);
         }
         p-> data = n;
```

```
         p-> link = NULL;
      }
      else
      {
         temp = p;
         while (temp-> link != NULL)
                temp = temp-> link;
         temp-> link = (struct node *)malloc(sizeof(struct node));
        if(temp -> link == NULL)
         {
                 printf("Error\n");
            exit(0);
         }
         temp = temp-> link;
         temp-> data = n;
         temp-> link= NULL;
      }
      return (p);
   }
   /* a function which inserts a newly created node after the specified
node */
   struct node * newinsert ( struct node *p, int node_no, int value )
   {
      struct node *temp, * temp1;
      int i;
      if ( node_no <= 0 || node_no > length (p))
      {
              printf("Error! the specified node does not exist\n");
         exit(0);
      }
      if ( node_no == 0)
      {
         temp = ( struct node * )malloc ( sizeof ( struct node ));
         if ( temp == NULL )
         {
                 printf( " Cannot allocate \n");
                 exit (0);
         }
         temp -> data = value;
         temp -> link = p;
         p = temp ;
   }
   else
   {
       temp = p ;
      i = 1;
```

```
      while ( i < node_no )
      {
            i = i+1;
            temp = temp-> link ;
      }
      temp1 = ( struct node * )malloc ( sizeof(struct node));
      if ( temp == NULL )
      {
            printf ("Cannot allocate \n");
            exit(0)
      }
      temp1 -> data = value ;
      temp1 -> link = temp -> link;
      temp -> link = temp1;
   }
   return (p);
   }

   void printlist ( struct node *p )
   {
      printf("The data values in the list are\n");
      while (p!= NULL)
      {
         printf("%d\t",p-> data);
         p = p-> link;
      }
   }
   void main ()
   {
      int n;
      int x;
      struct node *start = NULL;
      printf("Enter the nodes to be created \n");
      scanf("%d",&n);
      while ( n- > 0 )
      {
         printf( "Enter the data values to be placed in a node\n");
         scanf("%d",&x);
         start = insert ( start, x );
      }
      printf(" The list before deletion is\n");
      printlist ( start );
      printf(" \n Enter the node no after which the insertion is to be
done\n");
      scanf ( " %d",&n);
      printf("Enter the value of the node\n");
```

```
      scanf("%d",&x);
      start = newinsert(start,n,x);
      printf("The list after insertion is \n");
      printlist(start);
   }
```

Explanation

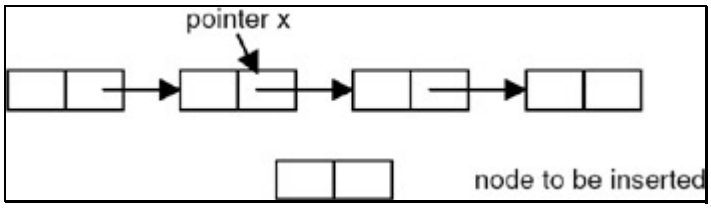


Figure 6.7: Before insertion.

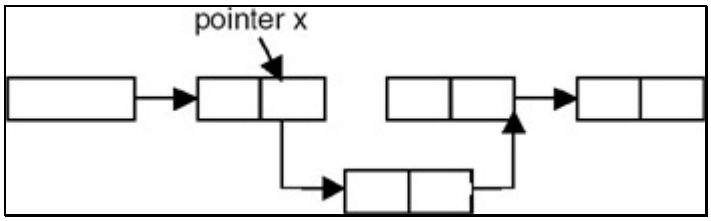


Figure 6.8: After insertion.

6. INSERTING A NEW NODE IN A SORTED LIST

Introduction

To insert a new node into an already sorted list, we compare the data value of the node to be inserted with the data values of the nodes in the list starting from the first node. This is continued until we get a pointer to the node that appears immediately before the node in the list whose data value is greater than the data value of the node to be inserted.

Program

Here is a complete program to insert an element in a sorted list of elements using the linked list representation so that after insertion, it will remain a sorted list.

```
# include <stdio.h>
# include <stdlib.h>
struct node
{
int data;
struct node *link;
};
struct node *insert(struct node *, int);
struct node *sinsert(struct node*, int );
void printlist ( struct node * );
struct node *sortlist(struct node *);

struct node *insert(struct node *p, int n)
{
   struct node *temp;
```

```
   if(p==NULL)
   {
      p=(struct node *)malloc(sizeof(struct node));
      if(p==NULL)
      {
   printf("Error\n");
         exit(0);
      }
      p-> data = n;
      p-> link = NULL;
   }
   else
   {
      temp = p;
      while (temp-> link!= NULL)
     temp = temp-> link;
      temp-> link = (struct node *)malloc(sizeof(struct node));
      if(temp -> link == NULL)
      {
   printf("Error\n");
        exit(0);
       }
     temp = temp-> link;
      temp-> data = n;
      temp-> link = NULL;
      }
      return (p);
}

void printlist ( struct node *p )
{
      printf("The data values in the list are\n");
      while (p!= NULL)
      {
   printf("%d\t",p-> data);
        p = p-> link;
      }
   }

/* a function to sort a list */
struct node *sortlist(struct node *p)
{
   struct node *temp1,*temp2,*min,*prev,*q;
   q = NULL;
   while(p != NULL)
   {
```

```
   prev = NULL;
      min = temp1 = p;
      temp2 = p -> link;
      while ( temp2 != NULL )
      {
         if(min -> data > temp2 -> data)
              {
         min = temp2;
                 prev = temp1;
               }
               temp1 = temp2;
               temp2 = temp2-> link;
            }
      if(prev == NULL)
            p = min -> link;
            else
            prev -> link = min -> link;
            min -> link = NULL;
            if( q == NULL)
            q = min; /* moves the node with lowest data value in the list
pointed to by p to the list
   pointed to by q as a first node*/
            else
            {
               temp1 = q;
               /* traverses the list pointed to by q to get pointer to its
last node */
               while( temp1 -> link != NULL)
               temp1 = temp1 -> link;
               temp1 -> link = min; /* moves the node with lowest data
value
in the list pointed to
   by p to the list pointed to by q at the end of list pointed by
   q*/
         }
      }
      return (q);
   }

   /* a function to insert a node with data value n in a sorted list
pointed to by p*/
   struct node *sinsert(struct node *p, int n)
   {
      struct node *curr, *prev;
      curr =p;
      prev = NULL;
```

```
     while(curr ->data < n)
          {
                      prev = curr;
                curr = curr->link;
          }
   if ( prev == NULL) /* the element is to be inserted at the start of
the list because
                  it is less than the data value of the first node*/
       {
            curr = (struct node *) malloc(sizeof(struct node));
            if( curr == NULL)
              {
                printf("error cannot allocate\n");
                exit(0);
              }
           curr->data = n;
           curr->link = p;
           p = curr;
       }
       else
        {
            curr->data = n;
            curr->link = prev->link;
            prev->link = curr;
        }
      return(p);
      }

      void main()
      {
         int n;
         int x;
         struct node *start = NULL ;
         printf("Enter the nodes to be created \n");
         scanf("%d",&n);
         while ( n-- > 0 )
         {
      printf( "Enter the data values to be placed in a node\n");
            scanf("%d",&x);
            start = insert ( start,x);
         }
         printf("The created list is\n");
         printlist ( start );
         start = sortlist(start);
         printf("The sorted list is\n");
         printlist ( start );
```

```
        printf("Enter the value to be inserted\n");
        scanf("%d",&n);
        start = sinsert(start,n);
        printf("The list after insertion is\n");
        printlist ( start );
}
```

Explanation

1. If this pointer is `prev`, then `prev` is checked for a `NULL` value.
2. If `prev` is `NULL`, then the new node is created and inserted as the first node in the list.
3. When `prev` is not `NULL`, then a new node is created and inserted after the node pointed by `prev`, as shown in Figure 6.9.

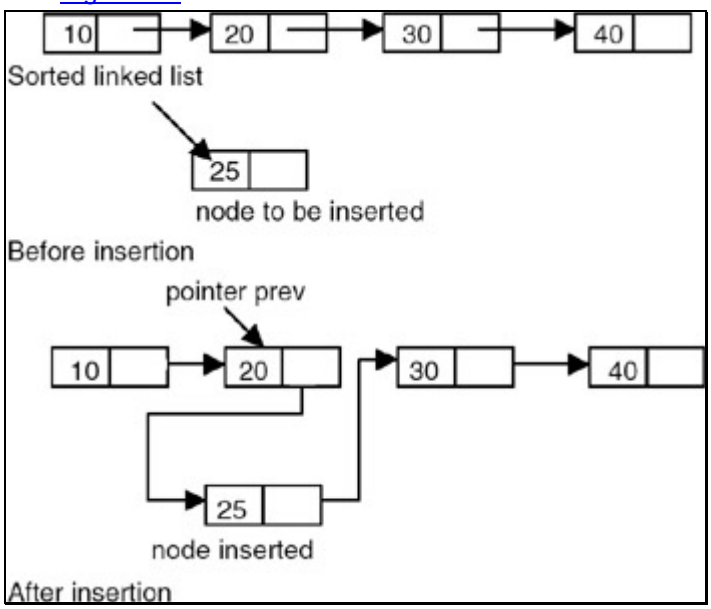


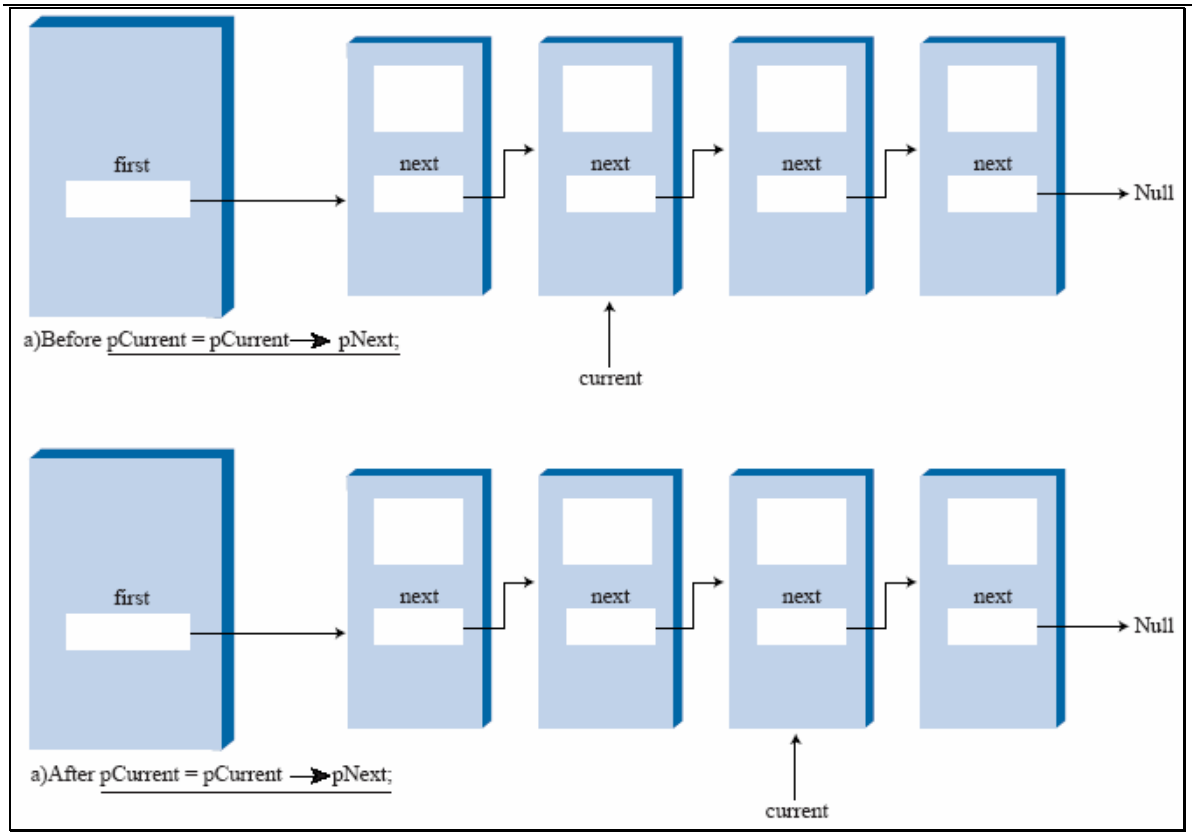
Figure 6.9: Insertion in a sorted list.

7. COUNTING THE NUMBER OF NODES OF A LINKED LIST

Introduction

Counting the number of nodes of a singly linked list requires maintaining a counter that is initialized to 0 and incremented by 1 each time a node is encountered in the process of traversing a list from the start.

Here is a complete program that counts the number of nodes in a singly linked chain p, where p is a pointer to the first node in the list.



Program

```
# include <stdio.h>
# include <stdlib.h>
struct node
{
int data;
struct node *link;
};
struct node *insert(struct node *, int);
int nodecount(struct node*);
void printlist ( struct node * );

struct node *insert(struct node *p, int n)
{
   struct node *temp;
   if(p==NULL)
   {
      p=(struct node *)malloc(sizeof(struct node));
      if(p==NULL)
      {
   printf("Error\n");
         exit(0);
      }
      p-> data = n;
      p-> link = NULL;
```

```
   }
   else
   {
      temp = p;
      while (temp-> link!= NULL)
     temp = temp-> link;
      temp-> link = (struct node *)malloc(sizeof(struct node));
      if(temp -> link == NULL)
      {
   printf("Error\n");
         exit(0);
      }
     temp = temp-> link;
      temp-> data = n;
      temp-> link = NULL;
      }
      return (p);
}

void printlist ( struct node *p )
{
      printf("The data values in the list are\n");
      while (p!= NULL)
      {
   printf("%d\t",p-> data);
         p = p-> link;
      }
}

/* A function to count the number of nodes in a singly linked list */
int nodecount (struct node *p )
{
   int count=0;
   while (p != NULL)
   {
      count ++;
      p = p->link;
   }
      return(count);
}

void main()
{
   int n;
   int x;
   struct node *start = NULL ;
```

```
   printf("Enter the nodes to be created \n");
   scanf("%d",&n);
   while ( n-- > 0 )
   {
printf( "Enter the data values to be placed in a node\n");
      scanf("%d",&x);
      start = insert ( start,x);
   }
   printf("The created list is\n");
   printlist ( start );
   n = nodecount(start);
   printf("The number of nodes in a list are: %d\n",n);

}
```

8. ERASING A LINKED LIST

Introduction

Erasing a linked list involves traversing the list starting from the first node, freeing the storage allocated to the nodes, and then setting the pointer to the list to NULL. If p is a pointer to the start of the list, the actions specified through the following code will erase the list:

```
while(p != NULL)
   {
           temp = p;
           p = p->link;
           free(t);
   }
```

But a better strategy of erasing a list is to mark all the nodes of the list to be erased as free nodes without actually freeing the storage of these nodes. That means to maintain this list, a list of free nodes, so that if a new node is required it can be obtained from this list of free nodes.

Program

Here is a complete program that erases a list pointed to by p by adding the nodes of a list pointed by p to the free list.

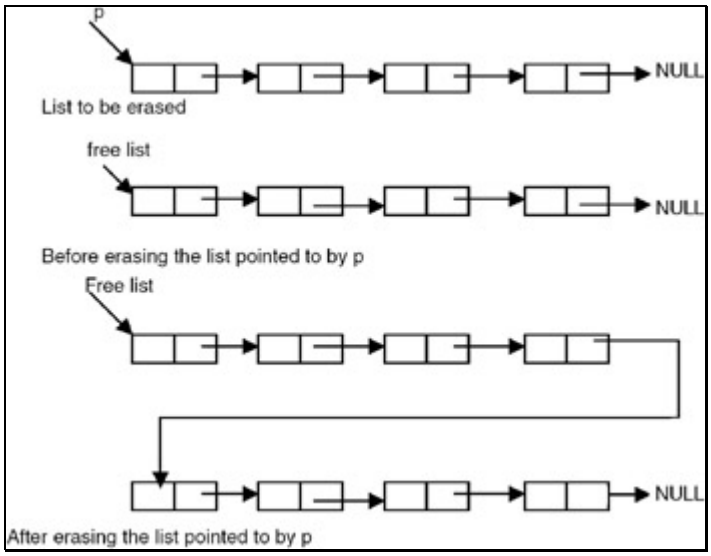
```
# include <stdio.h>
# include <stdlib.h>
struct node
{
int data;
struct node *link;
};
struct node *insert(struct node *, int);
void erase(struct node **,struct node **);
void printlist ( struct node * );
```

```
void erase (struct node **p, struct node **free)
{
   struct node *temp;
   temp = *p;
   while (temp->link != NULL)
         temp = temp ->link;
   temp->link = (*free);
   *free = *p;
   *p = NULL;
}

struct node *insert(struct node *p, int n)
{
   struct node *temp;
   if(p==NULL)
   {
      p=(struct node *)malloc(sizeof(struct node));
      if(p==NULL)
      {
   printf("Error\n");
          exit(0);
      }
      p-> data = n;
      p-> link = NULL;
   }
   else
   {
     temp = p;
     while (temp-> link!= NULL)
    temp = temp-> link;
    temp-> link = (struct node *)malloc(sizeof(struct node));
    if(temp -> link == NULL)
    {
  printf("Error\n");
         exit(0);
      }
    temp = temp-> link;
      temp-> data = n;
      temp-> link = NULL;
      }
      return (p);
}

void printlist ( struct node *p )
{
      printf("The data values in the list are\n");
```

```
      while (p!= NULL)
      {
   printf("%d\t",p-> data);
         p = p-> link;
      }
   }

   void main()
   {
         int n;
         int x;
         struct node *start = NULL ;
         struct node *free=NULL;

         /* this code will create a free list for the test purpose*/
        printf("Enter the number of nodes in the initial free list \n");
         scanf("%d",&n);
         while ( n-- > 0 )
         {
            printf( "Enter the data values to be placed in a node\n");
            scanf("%d",&x);
            free = insert ( free,x);
         }

            /* this code will create a list to be erased*/
           printf("Enter the number of nodes in the list to be created for
erasing \n");
            scanf("%d",&n);
            while ( n-- > 0 )
            {
               printf( "Enter the data values to be placed in a node\n");
               scanf("%d",&x);
               start = insert ( start,x);
            }
               printf("The free list islist is:\n");
               printlist ( free );
               printf("The list to be erased is:\n");
               printlist ( start);
               erase(&start,&free);
               printf("The free list after adding all the nodes from the
list to
be erased is:\n");
               printlist ( free );
   }
```

Explanation

The method of erasing a list requires adding all the nodes of the list to be erased to the list of free nodes, as shown here.



9. CIRCULAR LINKED LISTS

Introduction

A circular list is a list in which the link field of the last node is made to point to the start/first node of the list, as shown in Figure 6.14.

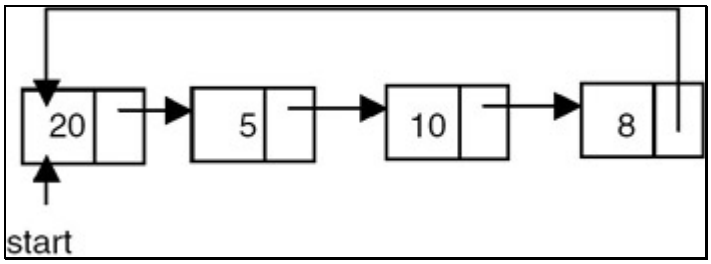


Figure 6.14: A circular list.

In the case of circular lists, the empty list also should be circular. So to represent a circular list that is empty, it is required to use a header node or a head-node whose data field contents are irrelevant, as shown in Figure 6.15.

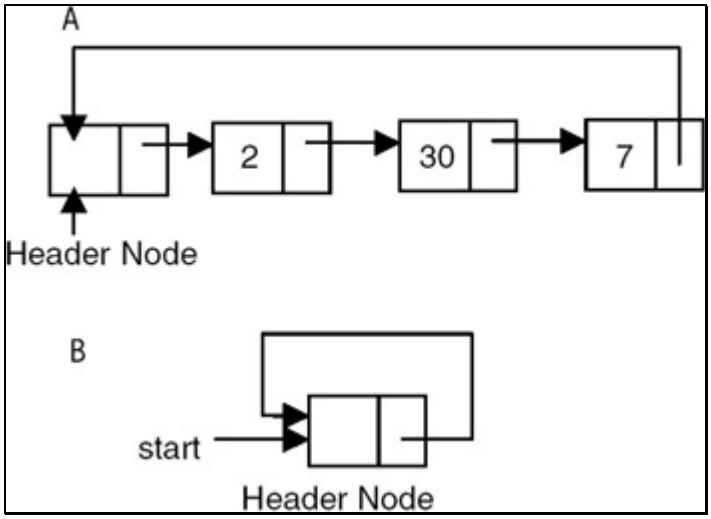


Figure 6.15: (A) A circular list with head node, (B) an empty circular list.

Program

Here is a program for building and printing the elements of the circular linked list.

```
# include <stdio.h>
# include <stdlib.h>
struct node
{
   int data;
   struct node *link;
};
struct node *insert(struct node *p, int n)
{
   struct node *temp;
   /* if the existing list is empty then insert a new node as the
starting node */
       if(p==NULL)
       {
         p=(struct node *)malloc(sizeof(struct node)); /* creates new
node data value passes
   as parameter */
        if(p==NULL)
        {
                 printf("Error\n");
                 exit(0);
         }
         p-> data = n;
          p-> link = p; /* makes the pointer pointing to itself because it
is a circular list*/
      }
      else
      {
         temp = p;
   /* traverses the existing list to get the pointer to the last node of
it */
         while (temp-> link != p)
                temp = temp-> link;
         temp-> link = (struct node *)malloc(sizeof(struct node)); /*
creates new node using
         data value passes as
         parameter and puts its
         address in the link field
         of last node of the
         existing list*/
         if(temp -> link == NULL)
         {
                  printf("Error\n");
```

```
         }
            exit(0);
            temp = temp-> link;
            temp-> data = n;
            temp-> link = p;
         }
         return (p);
   }
   void printlist ( struct node *p )
   {
      struct node *temp;
      temp = p;
      printf("The data values in the list are\n");
      if(p!= NULL)
      {
      do
      {
                printf(%d\t",temp->data);
                temp=temp->link;
      } while (temp!= p)
   }
   else
      printf("The list is empty\n");
   }

   void main()
   {
      int n;
      int x;
      struct node *start = NULL ;
      printf("Enter the nodes to be created \n");
      scanf("%d",&n);
      while ( n- > 0 )
      {
              printf( "Enter the data values to be placed in a
node\n");
         scanf("%d",&x);
         start = insert ( start, x );
      }
      printf("The created list is\n");
      printlist ( start );
   }
```

Explanation

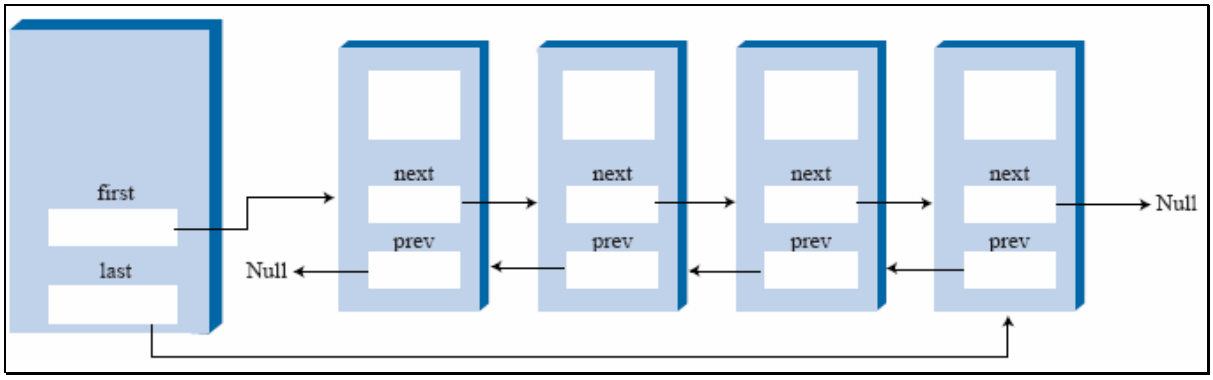
The program appends a new node to the existing list (that is, it inserts a new node in the existing list at the end), and it makes the link field of the newly inserted node point to the start or first node of the list. This ensures that the link field of the last node always points to the starting node of the list.

10. DOUBLY LINKED LISTS

Introduction

The following are problems with singly linked lists:

1. A singly linked list allows traversal of the list in only one direction.
2. Deleting a node from a list requires keeping track of the previous node, that is, the node whose link points to the node to be deleted.
3. If the link in any node gets corrupted, the remaining nodes of the list become unusable.



These problems of singly linked lists can be overcome by adding one more link to each node, which points to the previous node. When such a link is added to every node of a list, the corresponding linked list is called a doubly linked list. Therefore, a doubly linked list is a linked list in which every node contains two links, called left link and right link, respectively. The left link of the node points to the previous node, whereas the right points to the next node. Like a singly linked list, a doubly linked list can also be a chain or it may be circular with or without a header node. If it is a chain, the left link of the first node and the right link of the last node will be NULL, as shown in Figure 6.18.

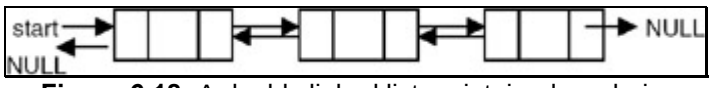


Figure 6.18: A doubly linked list maintained as chain.

If it is a circular list without a header node, the right link of the last node points to the first node. The left link of the first node points to the last node, as shown in Figure 6.19.

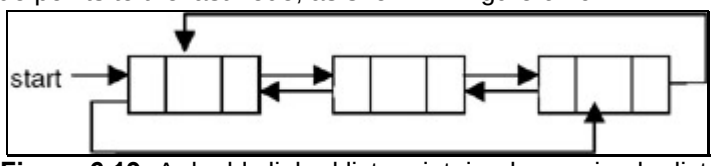


Figure 6.19: A doubly linked list maintained as a circular list.

If it is a circular list with a header node, the left link of the first node and the right link of the last node point to the header node. The right link of the header node points to the first node and the left link of the header node points to the last node of the list, as shown in Figure 6.20.

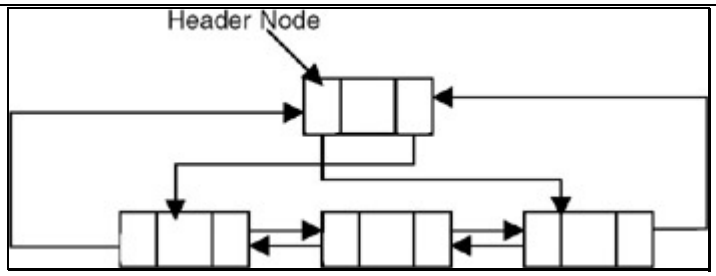


Figure 6.20: A doubly linked list maintained as a circular list with a header node.

Therefore, the following representation is required to be used for the nodes of a doubly linked list.

```
struct dnode
   {
      int data;
      struct dnode *left,*right;
   };
```

Program

A program for building and printing the elements of a doubly linked list follows:

```
   # include <stdio.h>
   # include <stdlib.h>
   struct dnode
   {
   int data;
   struct dnode *left, *right;
   };
   struct dnode *insert(struct dnode *p, struct dnode **q, int n)
   {
   struct dnode *temp;
   /* if the existing list is empty then insert a new node as the
starting node */
   if(p==NULL)
     {
         p=(struct dnode *)malloc(sizeof(struct dnode)); /* creates new
node data value
      passed as parameter */

         if(p==NULL)
         {
      printf("Error\n");
             exit(0);
         }
         p->data = n;
         p-> left = p->right =NULL;
         *q =p;
      }
      else
      {
```

```
          temp = (struct dnode *)malloc(sizeof(struct dnode)); /* creates
new node using
          data value passed as
            parameter and puts its
          address in the temp
   */
       if(temp == NULL)
       {
     printf("Error\n");
            exit(0);
       }
       temp->data = n;
       temp->left = (*q);
       temp->right = NULL;
       (*q)->right = temp;
       (*q) = temp;
    }
      return (p);
   }
   void printfor( struct dnode *p )
   {
      printf("The data values in the list in the forward order are:\n");
      while (p!= NULL)
      {
         printf("%d\t",p-> data);
         p = p->right;
      }
   }
   void printrev( struct dnode *p )
   {
   printf("The data values in the list in the reverse order are:\n");
   while (p!= NULL)
      {
         printf("%d\t",p->data);
         p = p->left;
      }
   }
   void main()
   {
         int n;
         int x;
         struct dnode *start = NULL ;
         struct dnode *end = NULL;
         printf("Enter the nodes to be created \n");
         scanf("%d",&n);
         while ( n-- > 0 )
```

```
        {
    printf( "Enter the data values to be placed in a node\n");
          scanf("%d",&x);
          start = insert ( start, &end,x );
        }
        printf("The created list is\n");
        printfor ( start );
        printrev(end);
}
```

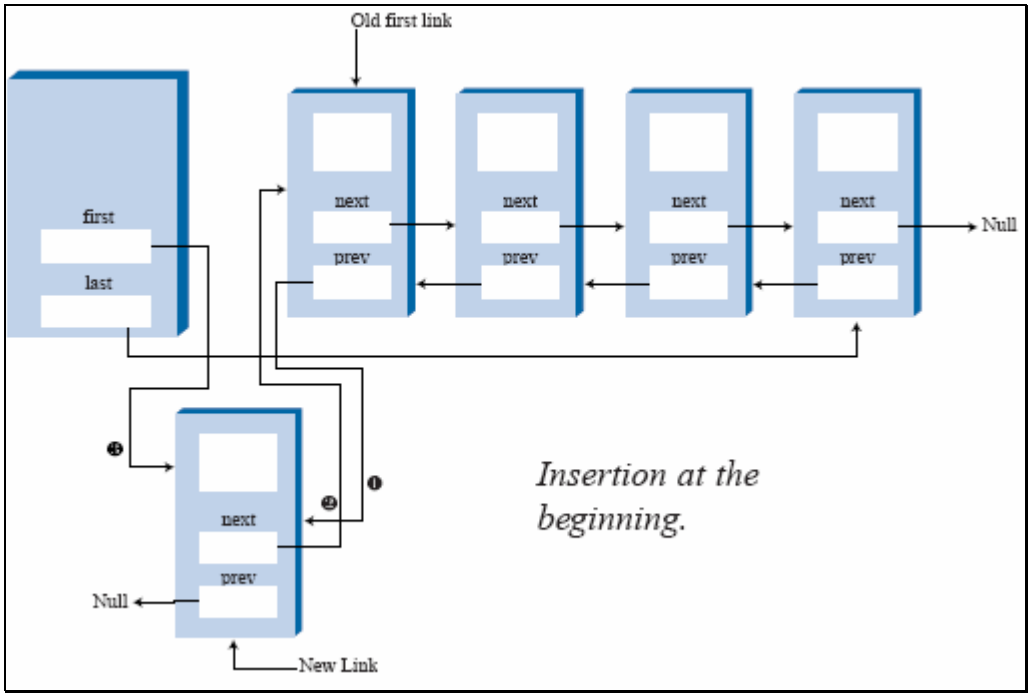
Explanation

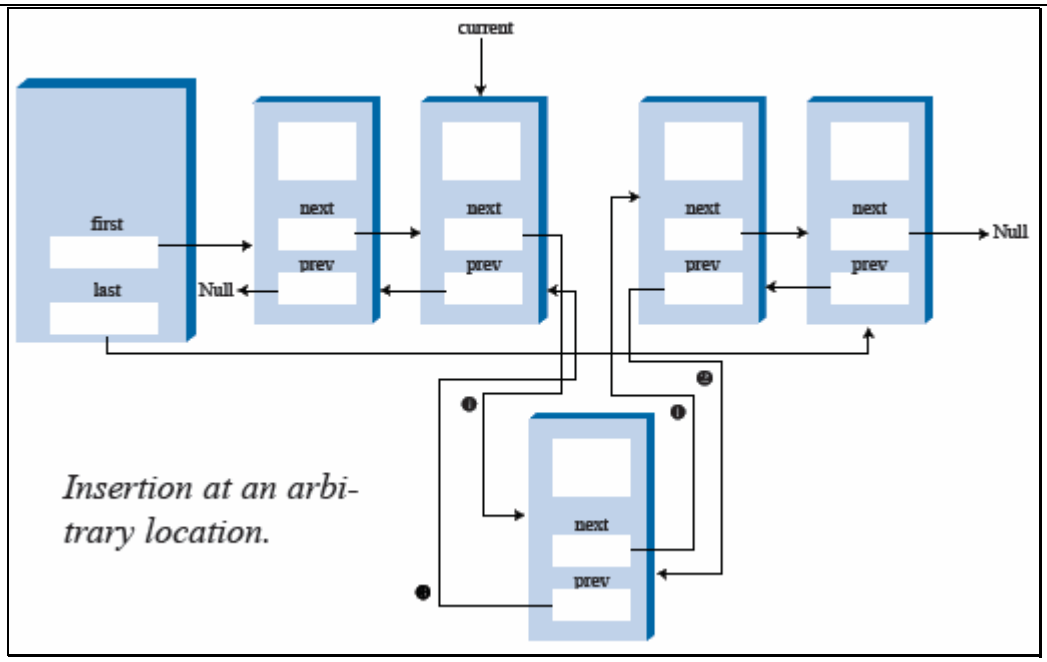
1. This program uses a strategy of inserting a node in an existing list to create it. For this, an `insert` function is used. The `insert` function takes a pointer to an existing list as the first parameter.
2. The pointer to the last node of a list is the second parameter. A data value with which the new node is to be created is the third parameter. This creates a new node using the data value, appends it to the end of the list, and returns a pointer to the first node of the list. Initially, the list is empty, so the pointer to the start node is `NULL`. When `insert` is called the first time, the new node created by the `insert` becomes the start node.
3. Subsequently, `insert` creates a new node that stores the pointer to the created node in a temporary pointer. Then the left link of the node pointed to by the temporary pointer becomes the last node of the existing list, and the right link points to `NULL`. After that, it updates the value of the end pointer to make it point to this newly appended node.
4. The main function reads the value of the number of nodes in the list, and calls `insert` that many times by going in a `while` loop, in order to get a doubly linked list with the specified number of nodes created.

11. INSERTION OF A NODE IN A DOUBLY LINKED LIST

Introduction

The following program inserts the data in a doubly linked list.





Program

```
   # include <stdio.h>
   # include <stdlib.h>
   struct dnode
   {
   int data;
   struct node *left, *right;
   };
   struct dnode *insert(struct dnode *p, struct dnode **q, int n)
   {
   struct dnode *temp;
   /* if the existing list is empty then insert a new node as the
starting node */
   if(p==NULL)
     {
         p=(struct dnode *)malloc(sizeof(struct dnode)); /* creates new
node data value
      passed as parameter */

         if(p==NULL)
         {
      printf("Error\n");
              exit(0);
         }
         p-> data = n;
         p-> left = p->right =NULL;
         *q =p
      }
```

```
      else
      {
         temp = (struct dnode *)malloc(sizeof(struct dnode)); /* creates
new node using
         data value passed as
         parameter and puts its
        address in the temp
   */
      if(temp == NULL)
      {
   printf("Error\n");
        exit(0);
      }
      temp-> data = n;
      temp->left = (*q);
      temp->right = NULL;
      (*q) = temp;
    }
      return (p);
   }
   void printfor( struct dnode *p )
   {
      printf("The data values in the list in the forward order are:\n");
      while (p!= NULL)
         {
            printf("%d\t",p-> data);
            p = p-> right;
         }
   }
   /* A function to count the number of nodes in a doubly linked list */
   int nodecount (struct dnode *p )
   {
      int count=0;
      while (p != NULL)
      {
         count ++;
         p = p->right;
      }
         return(count);
   }

   /* a function which inserts a newly created node after the specified
node in a doubly
   linked list */
struct node * newinsert ( struct dnode *p, int node_no, int value )
      {
```

```
    struct dnode *temp, * temp1;
    int i;
    if ( node_no <= 0 || node_no > nodecount (p))
    {
    printf("Error! the specified node does not exist\n");
    exit(0);
    }
    if ( node_no == 0)
    {
    temp = ( struct dnode * )malloc ( sizeof ( struct dnode ));
    if ( temp == NULL )
    {
    printf( " Cannot allocate \n");
    exit (0);
    }
    temp -> data = value;
    temp -> right = p;
    temp->left = NULL
    p = temp ;
    }
    else
    {
    temp = p ;
    i = 1;
    while ( i < node_no )
    {
    i = i+1;
    temp = temp-> right ;
    }
    temp1 = ( struct dnode * )malloc ( sizeof(struct dnode));
    if ( temp == NULL )
    {
    printf("Cannot allocate \n");
    exit(0);
    }
    temp1 -> data = value ;
    temp1 -> right = temp -> right;
    temp1 -> left = temp;
    temp1->right->left = temp1;
    temp1->left->right = temp1
    }
    return (p);
}
void main()
{
      int n;
```

```
        int x;
        struct dnode *start = NULL ;
        struct dnode *end = NULL;
        printf("Enter the nodes to be created \n");
        scanf("%d",&n);
        while ( n- > 0 )
        {
     printf( "Enter the data values to be placed in a node\n");
           scanf("%d",&x);
           start = insert ( start, &end,x );
        }
        printf("The created list is\n");
        printfor ( start );
        printf("enter the node number after which the new node is to be
inserted\n");
        scanf("%d",&n);
        printf("enter the data value to be placed in the new node\n");
        scanf("%d",&x);
        start=newinsert(start,n,x);
        printfor(start);
  }
```

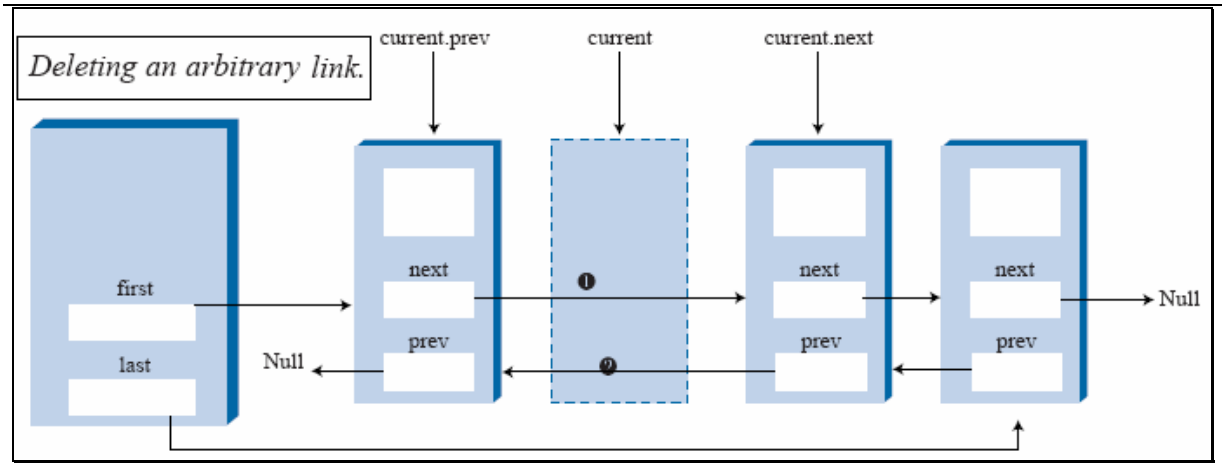
Explanation

1. To insert a new node in a doubly linked chain, it is required to obtain a pointer to the node in the existing list after which a new node is to be inserted.
2. To obtain this pointer, the node number after which the new node is to be inserted is given as input. The nodes are assumed to be numbered as 1,2,3,…, etc., starting from the first node.
3. The list is then traversed starting from the start node to obtain the pointer to the specified node. Let this pointer be x. A new node is then created with the required data value, and the right link of this node is made to point to the node to the right of the node pointed to by x. And the left link of the newly created node is made to point to the node pointed to by x. The left link of the node which was to the right of the node pointed to by x is made to point to the newly created node. The right link of the node pointed to by x is made to point to the newly created node.

12. DELETING A NODE FROM A DOUBLY LINKED LIST

Introduction

The following program deletes a specific node from the linked list.



Program

```
   # include <stdio.h>
   # include <stdlib.h>
   struct dnode
   {
   int data;
   struct dnode *left, *right;
   };
   struct dnode *insert(struct dnode *p, struct dnode **q, int n)
   {
   struct dnode *temp;
   /* if the existing list is empty then insert a new node as the
starting node */
    if(p==NULL)
      {
         p=(struct dnode *)malloc(sizeof(struct dnode)); /* creates new
node
data value
      passed as parameter */

         if(p==NULL)
         {
      printf("Error\n");
            exit(0);
         }
         p-> data = n;
         p-> left = p->right =NULL;
         *q =p;
      }
      else
      {
         temp = (struct dnode *)malloc(sizeof(struct dnode)); /* creates
new node using
         data value passed as
```

```
          parameter and puts its
         address in the temp
   */
       if(temp == NULL)
       {
    printf("Error\n");
         exit(0);
      }
      temp-> data = n;
      temp->left = (*q);
      temp->right = NULL;
      (*q)->right = temp;
      (*q) = temp;
    }
      return (p);
   }
   void printfor( struct dnode *p )
   {
      printf("The data values in the list in the forward order are:\n");
      while (p!= NULL)
         {
            printf("%d\t",p-> data);
            p = p-> right;
         }
   }
   /* A function to count the number of nodes in a doubly linked list */
   int nodecount (struct dnode *p )
   {
      int count=0;
      while (p != NULL)
      {
         count ++;
         p = p->right;
      }
        return(count);
   }

   /* a function which inserts a newly created node after the specified
node in a doubly
          linked list */
   struct dnode * delete( struct dnode *p, int node_no, int *val)
   {
   struct dnode *temp ,*prev=NULL;
   int i;
   if ( node_no <= 0 || node_no > nodecount (p))
   {
```

```
printf("Error! the specified node does not exist\n");
exit(0);
}
if ( node_no == 0)
{
 temp = p;
 p = temp->right;
 p->left = NULL;
 *val = temp->data;
 return(p);
}
else
{
temp = p ;
 i = 1;
while ( i < node_no )
{
i = i+1;
 prev = temp;
temp = temp-> right ;
}
 prev->right = temp->right;
 if(temp->right != NULL)
  temp->right->left = prev;
 *val = temp->data;
 free(temp);
}
   return (p);
}

void main()
{
      int n;
      int x;
      struct dnode *start = NULL ;
      struct dnode *end = NULL;
      printf("Enter the nodes to be created \n");
      scanf("%d",&n);
      while ( n-- > 0 )
      {
   printf( "Enter the data values to be placed in a node\n");
        scanf("%d",&x);
        start = insert ( start, &end,x );
      }
      printf("The created list is\n");
      printfor ( start );
```

```
        printf("enter the number of the node which is to be deleted\n");
        scanf("%d",&n);
        start=delete(start,n,&x);
        printf("The data value of the node deleted from list is :
%d\n",x);
        printf("The list after deletion of the specified node is :\n");
        printfor(start);
   }
```

Explanation

1. To delete a node from a doubly linked chain, it is required to obtain a pointer to the node in the existing list that appears to the left of the node which is to be deleted.
2. To obtain this pointer, the node number which is to be deleted is given as input. The nodes are assumed to be numbered 1,2,3,…, etc., starting from the first node.
3. The list is then traversed starting from the start node to obtain the pointer to the specified node. Let this pointer be x. A pointer to the node to the right of the node x is also obtained. Let this be pointer y (this is a pointer to the node to be deleted). The right link of the node pointed to by x is the node pointing to the node to which the right link of the node pointed to by y points. The left link of the node to the right of the node pointed to by y is made to point to x. The node pointed to by y is then freed.

13. APPLICATION OF DOUBLY LINKED LISTS TO MEMORY MANAGEMENT

Introduction

A doubly linked list is used to maintain both the list of allocated blocks and the list of free blocks by the memory manager of the operating system. To keep track of the allocated and free portions of memory, the memory manager is required to maintain a linked list of allocated and free segments. Each node of this list contains a starting address, size, and status of the segment. This list is kept sorted by the starting address field to facilitate the updating, because when a process terminates, the memory segment allocated to it becomes free, and so if any of the segments are freed, then they can be merged with the adjacent segment, if the adjacent segment is already free. This requires traversal of the list both ways to find out whether any of the adjacent segments are free. So this list is required to be maintained as a doubly linked list. For example, at a particular point in time, the list may be as shown in Figure 6.21.

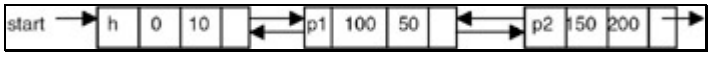


Figure 6.21: Before termination of process p1.

If the process p1 terminates, it is required to be modified as shown in Figure 6.22.

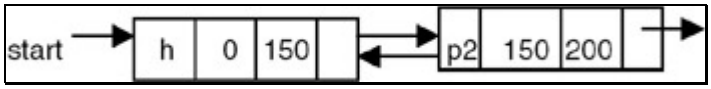


Figure 6.22: After termination of process p1.

General Comments on Linked Lists

1. A linked list is a dynamic data structure that can grow and shrink based on need.
2. The elements are not necessarily at a fixed distance apart.
3. In a linked list, the elements are placed in non-contiguous blocks of memory, and each block is linked to its previous block.
4. To link the next element to the previous element, the address of the next element is stored in the previous element itself.

5. Insertion or deletion at any arbitrary position in the linked list can be done easily, since it requires adjustment of only a few pointers.
6. Linked lists can be used for manipulation of symbolic polynomials.
7. A linked list is suitable for representation of sparse matrices.
8. A circular list is a list in which the link field of the last node is made to point to the start/first node of the list
9. A doubly linked list (DLL) is a linked list in which every node contains two links, called the left link and right link, respectively.
10. The left link of the node in a DLL is made to point to the previous node, whereas the right link is made to point to the next node.
11. A DLL can be traversed in both directions.
12. Having two pointers in a DLL provides safety, because even if one of the pointers get corrupted, the node still remains linked.
13. Deleting a particular node from a list, therefore, does not require keeping track of the previous node in a DLL.

Exercises

1. Write a C program to delete a node with the minimum value from a singly linked list.
2. Write a C program that will remove a specified node from a given doubly linked list and insert it at the end of the list.
3. Write a C program to transform a circular list into a chain.
4. Write a C program to merge two given lists A and B to form C in the following manner:

   The first element of C is the first element of A and the second element of C is the first element of B. The second elements of A and B become the third and fourth elements of C, and so on. If either A or B gets exhausted, the remaining elements of the other are to be copied to C.
5. Write a C program to delete all occurrences of x from a given singly linked list.

14. Exercises

E1. Build a single linked list with every node replace for a student have 3 fields: (Name: string,

ID:Integer, grade: float).

- Input 10 students into list
- Sort the list order by ID using: Bubble Sort, Quick sort, Selection Sort, Insertion Sort
- Find a student from ID using Binary search
- Print all student have grade > 8.
- Remove a student from imput ID
- Insert new student in order. Display error message if this student's ID existed in List
- Remove first student from list
- Remove last student from list
- Display all student's information from the list.
- Display number of students in the list.

E2. Make the program the same to E1, but using double linked list.

E3. Make a program to build a circular linked list with 15 integer numbers.

- Display all item in circular linked list.
- Insert a new item into circular linked list.
- Remove a item in circular linked list (user enter item value)
- Display number of items in circular linked list
- Find Sum of all value in circular linked list

CHAPTER 7: TREES

1. THE CONCEPT OF TREES

Introduction

Trees are used to impose a hierarchical structure on a collection of data items. For example, we need to impose a hierarchical structure on a collection of data items while preparing organizational charts and geneologies to represent the syntactic structure of a source program in compilers. So the study of trees as one of the data structures is important.

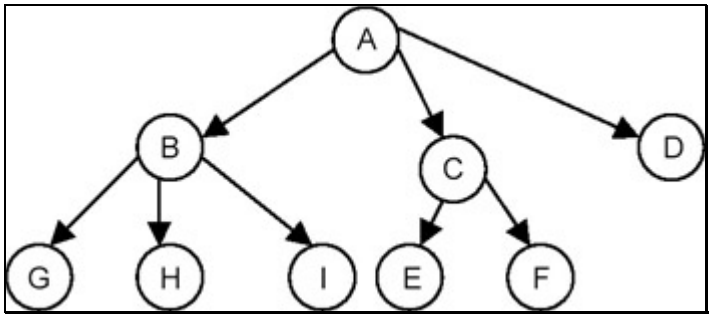
Definition of a Tree

A tree is a set of one or more nodes T such that:

i. there is a specially designated node called a root

ii. The remaining nodes are partitioned into *n disjointed* set of nodes T_1, T_2,…,T_n, each of which is a tree.

A tree strucutre is shown in Figure 7.1.



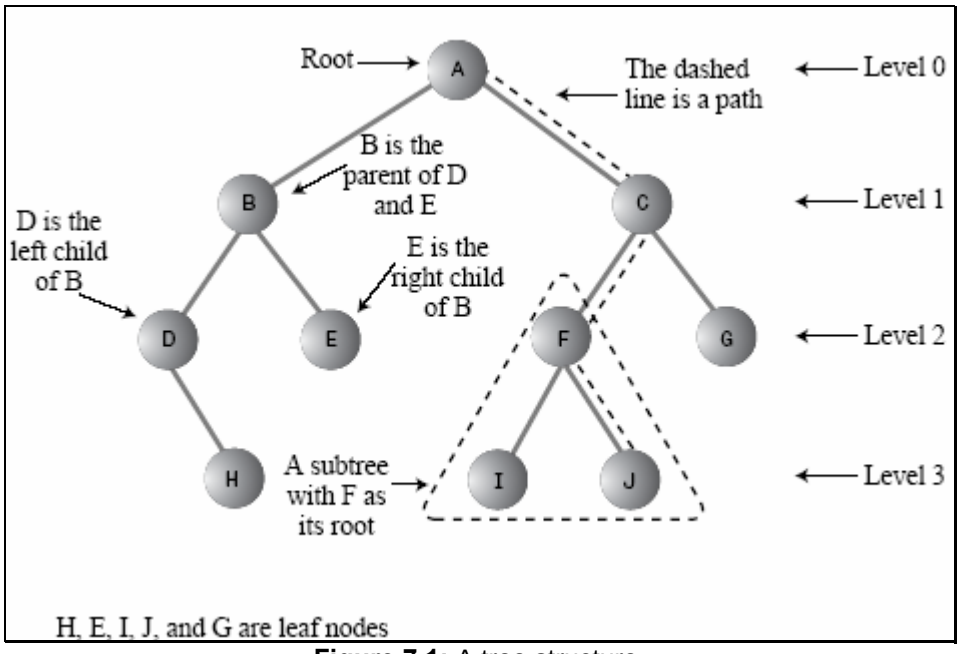


Figure 7.1: A tree structure.

This is a tree because it is a set of nodes {A,B,C,D,E,F,G,H,I}, with node A as a root node and the remaining nodes partitioned into three disjointed sets {B,G,H,I}, { C,E,F} and {D}, respectively. Each of these sets is a tree because each satisfies the aforementioned definition properly.

Shown in Figure 7.2 is a structure that is not a tree.

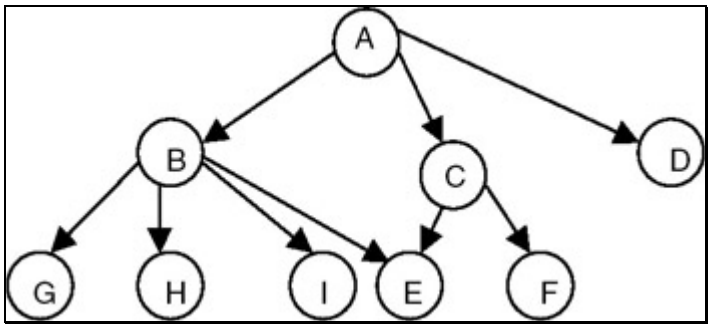


Figure 7.2: A non-tree structure.

Even though this is a set of nodes {A,B,C,D,E,F,G,H,I}, with node A as a root node, this is not a tree because the fact that node E is shared makes it impossible to partition nodes B through I into disjointed sets.

Degree of a Node of a Tree

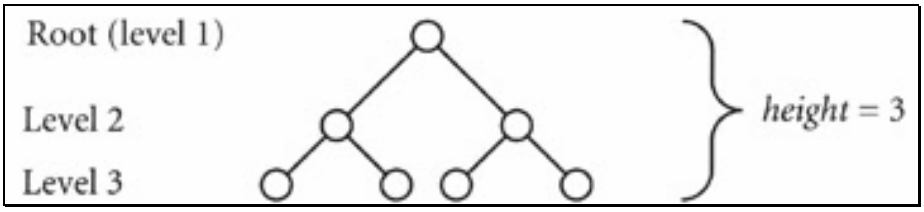
The *degree of a node of a tree* is the number of subtrees having this node as a root. In other words, the degree is the number of descendants of a node. If the degree is zero, it is called a terminal or leaf node of a tree.

Degree of a Tree

The *degree of a tree* is defined as the maximum of degree of the nodes of the tree, that is, degree of tree = max (degree(node i) for I = 1 to n)

Level of a Node

We define the level of the node by taking the level of the root node as 1, and incrementing it by 1 as we move from the root towards the subtrees. So the level of all the descendants of the root nodes will be 2. The level of their descendants will be 3, and so on. We then define the depth of the tree to be the maximum value of the level of the node of the tree.



2. BINARY TREE AND ITS REPRESENTATION

Introduction

A *binary tree* is a special case of tree as defined in the preceding section, in which no node of a tree can have a degree of more than 2. Therefore, a binary tree is a set of zero or more nodes T such that:

i. there is a specially designated node called the root of the tree
ii. the remaining nodes are partitioned into two disjointed sets, T_1 and T_2, each of which is a binary tree. T_1 is called the left subtree and T_2 is called right subtree, or vice-versa.

A binary tree is shown in Figure 7.3.

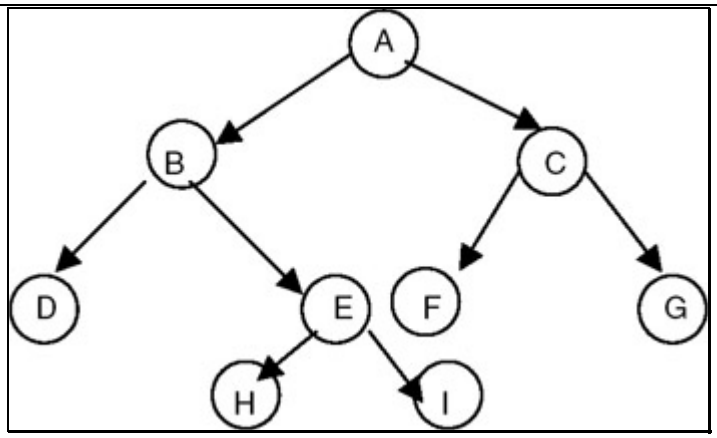


Figure 7.3: Binary tree structure.

So, for a binary tree we find that:

i. The maximum number of nodes at level i will be 2^{i-1}

ii. If k is the depth of the tree then the maximum number of nodes that the tree can have is

$$2^k - 1 = 2^{k-1} + 2^{k-2} + \ldots + 2^0$$

Also, there are skewed binary trees, such as the one shown in Figure 7.4.

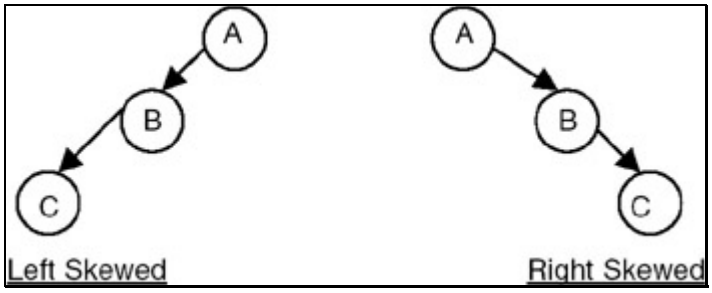


Figure 7.4: Skewed trees.

A full binary tree is a binary of depth k having $2^k - 1$ nodes. If it has $< 2^k - 1$, it is not a full binary tree. For example, for k = 3, the number of nodes = $2^k - 1 = 2^3 - 1 = 8 - 1 = 7$. A full binary tree with depth k = 3 is shown in Figure 7.5.

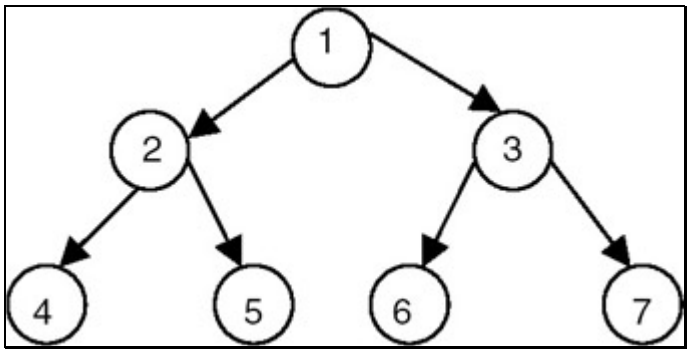


Figure 7.5: A full binary tree.

We use numbers from 1 to $2^k - 1$ as labels of the nodes of the tree.

If a binary tree is full, then we can number its nodes sequentially from 1 to 2^{k-1}, starting from the root node, and at every level numbering the nodes from left to right.

A complete binary tree of depth *k* is a tree with *n* nodes in which these *n* nodes can be numbered sequentially from 1 to *n*, as if it would have been the first *n* nodes in a full binary tree of depth *k*.

A complete binary tree with depth *k* = 3 is shown in Figure 7.6.

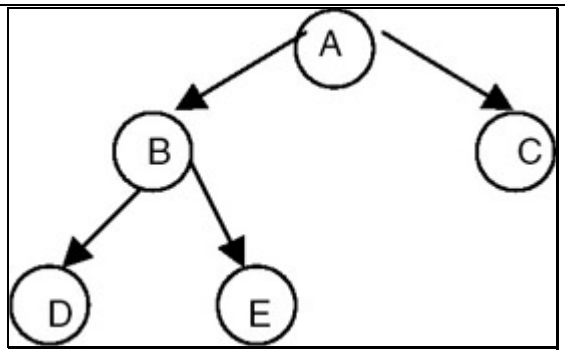


Figure 7.6: A complete binary tree.

Representation of a Binary Tree

If a binary tree is a *complete binary tree*, it can be represented using an array capable of holding *n* elements where n is the number of nodes in a complete binary tree. If the tree is an array of *n* elements, we can store the data values of the i^{th} node of a complete binary tree with *n* nodes at an index i in an array tree. That means we can map node i to the i^{th} index in the array, and the parent of node i will get mapped at an index i/2, whereas the left child of node i gets mapped at an index 2i and the right child gets mapped at an index 2i + 1. For example, a complete binary tree with depth *k* = 3, having the number of nodes *n* = 5, can be represented using an array of 5 as shown in Figure 7.7.

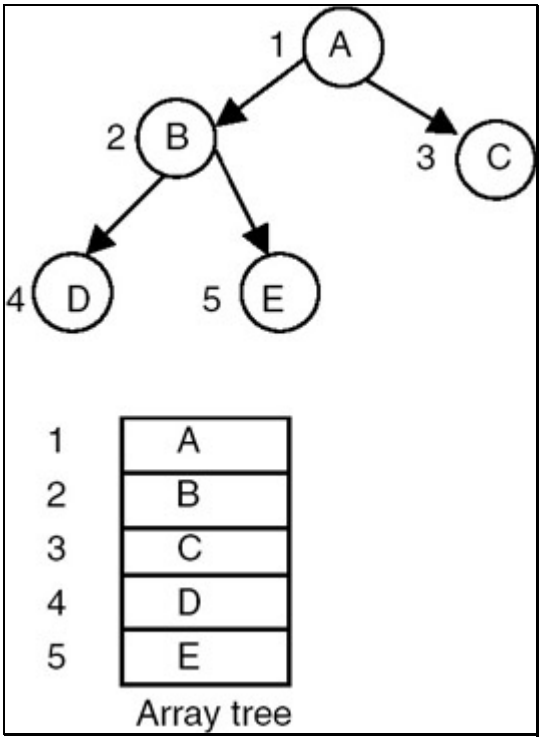


Figure 7.7: An array representation of a complete binary tree having 5 nodes and depth 3.

Shown in Figure 7.8 is another example of an array representation of a complete binary tree with depth *k* = 3, with the number of nodes *n* = 4.

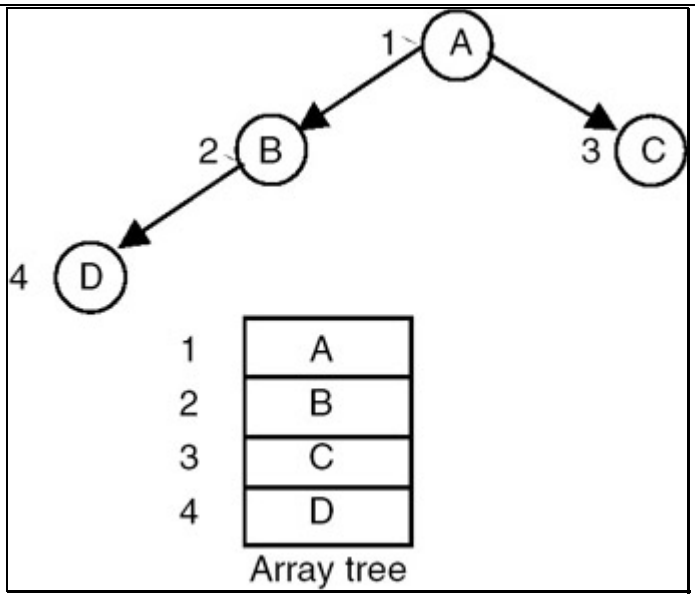


Figure 7.8: An array representation of a complete binary tree with 4 nodes and depth 3.

In general, any binary tree can be represented using an array. We see that an array representation of a complete binary tree does not lead to the waste of any storage. But if you want to represent a binary tree that is not a complete binary tree using an array representation, then it leads to the waste of storage as shown in Figure 7.9.

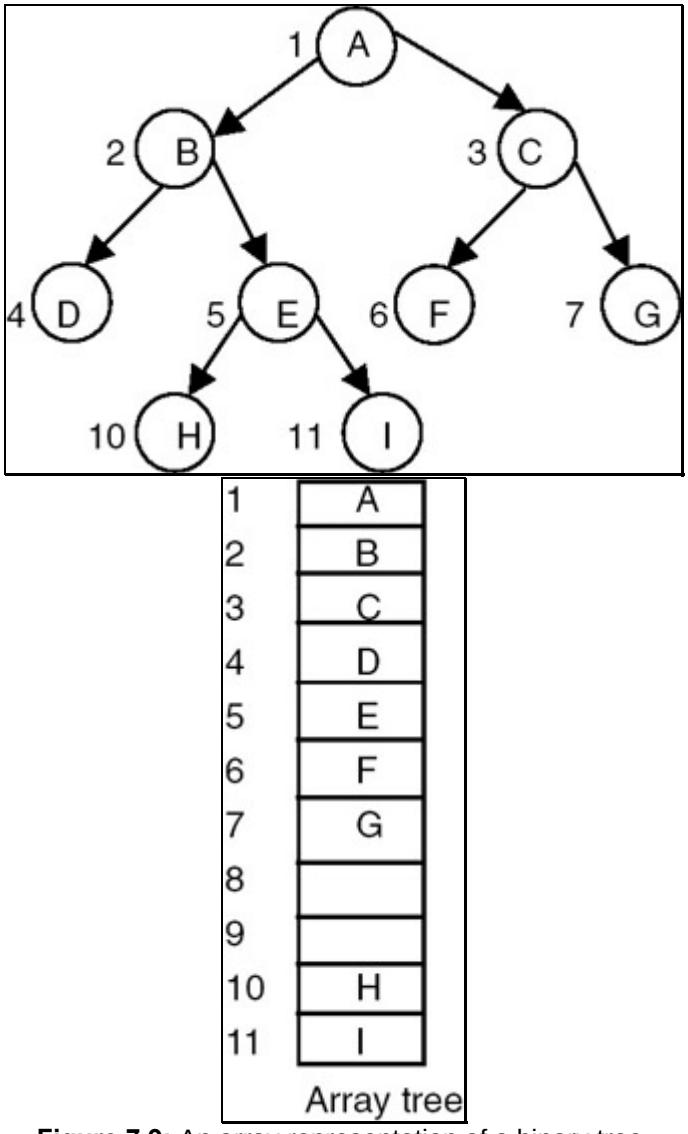


Figure 7.9: An array representation of a binary tree.

An array representation of a binary tree is not suitable for frequent insertions and deletions, even though no storage is wasted if the binary tree is a complete binary tree. It makes insertion and deletion in a tree costly. Therefore, instead of using an array representation, we can use a linked representation, in which every node is represented as a structure with three fields: one for holding data, one for linking it with the left subtree, and the third for linking it with right subtree as shown here:

leftchild	data	rightchild

We can create such a structure using the following C declaration:

```
struct tnode
{
   int data
   struct tnode *lchild,*rchild;
};
```

A tree representation that uses this node structure is shown in Figure 7.10.

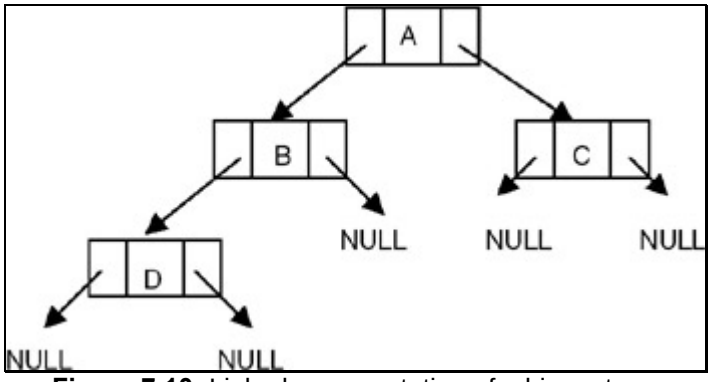


Figure 7.10: Linked representation of a binary tree.

3. BINARY TREE TRAVERSAL

Introduction

Order of Traversal of Binary Tree

The following are the possible orders in which a binary tree can be traversed: LDR, LRD, DLR, RDL, RLD, DRL, where L stands for traversing the left subtree, R stands for traversing the right subtree, and D stands for processing the data of the node. Therefore, the order LDR is the order of traversal in which we start with the root node, visit the left subtree, process the data of the root node, and then visit the right subtree. Since the left and right subtrees are also the binary trees, the same procedure is used recursively while visiting the left and right subtrees.

The order LDR is called as inorder; the order LRD is called as postorder; and the order DLR is called as preorder. The remaining three orders are not used. If the processing that we do with the data in the node of tree during the traversal is simply printing the data value, then the output generated for a tree is given in Figure 7.11, using inorder, preorder and postorder as shown.

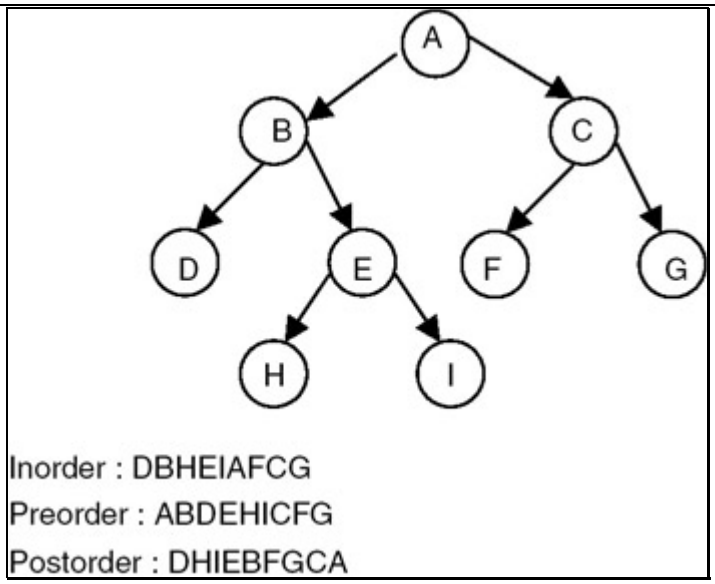


Figure 7.11: A binary tree along with its inorder, preorder and postorder.

If an expression is represented as a binary tree, the inorder traversal of the tree gives us an infix expression, whereas the postorder traversal gives us a postfix expression as shown in Figure 7.12.

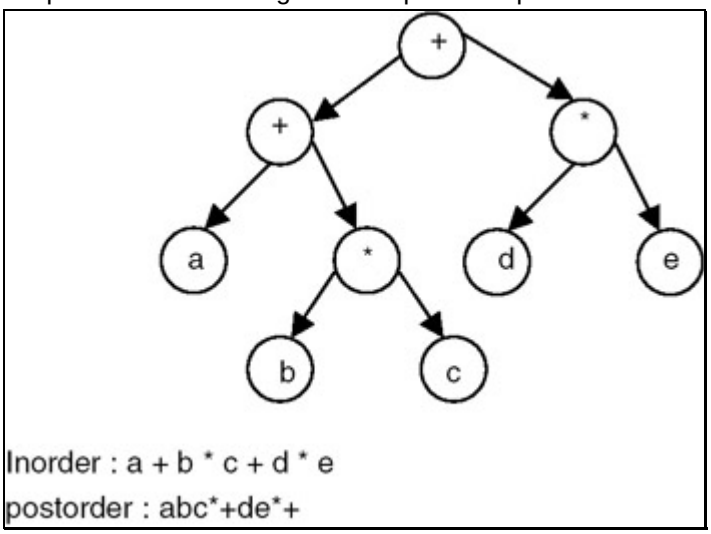


Figure 7.12: A binary tree of an expression along with its inorder and postorder.

Given an order of traversal of a tree, it is possible to construct a tree; for example, consider the folowing order:

Inorder = DBEAC

We can construct the binary trees shown in Figure 7.13 by using this order of traversal:

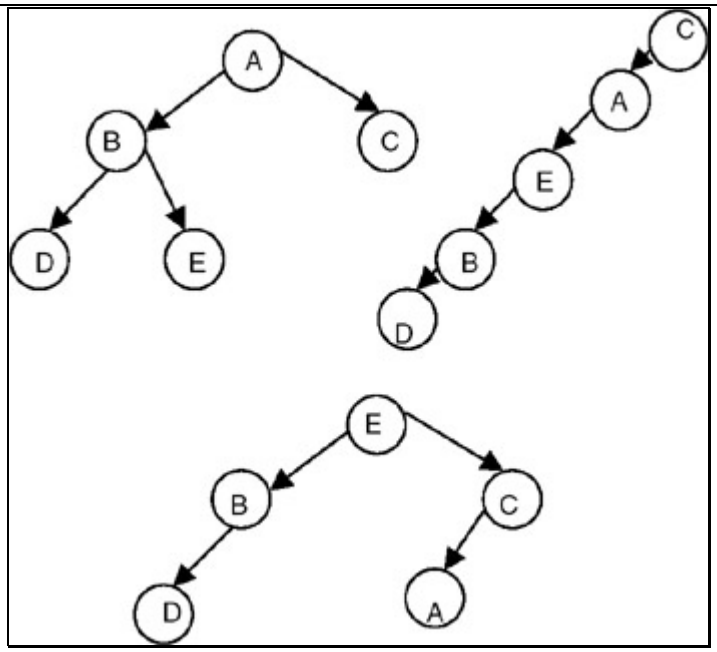


Figure 7.13: Binary trees constructed using the given inorder.

Therefore, we conclude that given only one order of traversal of a tree, it is possible to construct a number of binary trees; a unique binary tree cannot be constructed with only one order of traversal. For construction of a unique binary tree, we require two orders, in which one has to be inorder; the other can be preorder or postorder. For example, consider the following orders:

Inorder = DBEAC

Postorder = DEBCA

We can construct the unique binary tree shown in Figure 7.14 by using these orders of traversal:

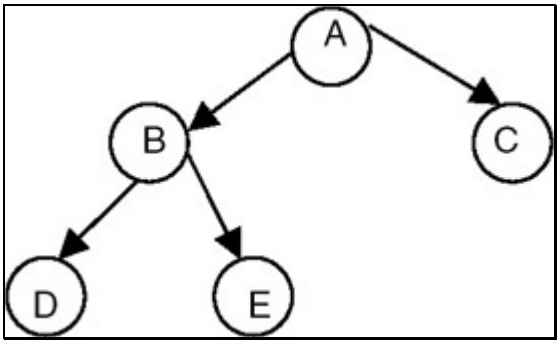


Figure 7.14: A unique binary tree constructed using its inorder and postorder.

4. BINARY SEARCH TREE

Introduction

A *binary search tree* is a binary tree that may be empty, and every node must contain an identifier. An identifier of any node in the left subtree is less than the identifier of the root. An identifier of any node in the right subtree is greater than the identifier of the root. Both the left subtree and right subtree are binary search trees.

A binary search tree is shown in Figure 7.15.

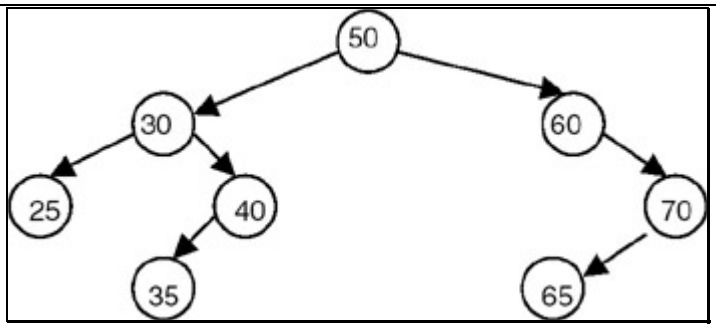


Figure 7.15: The binary search tree.

The binary search tree is basically a binary tree, and therefore it can be traversed in inorder, preorder, and postorder. If we traverse a binary search tree in inorder and print the identifiers contained in the nodes of the tree, we get a sorted list of identifiers in ascending order.

A binary search tree is an important search structure. For example, consider the problem of searching a list. If a list is ordered, searching becomes faster if we use a contiguous list and perform a binary search. But if we need to make changes in the list, such as inserting new entries and deleting old entries, using a contiguous list would be much slower, because insertion and deletion in a contiguous list requires moving many of the entries every time. So we may think of using a linked list because it permits insertions and deletions to be carried out by adjusting only a few pointers. But in an n-linked list, there is no way to move through the list other than one node at a time, permitting only sequential access. Binary trees provide an excellent solution to this problem. By making the entries of an ordered list into the nodes of a binary search tree, we find that we can search for a key in O(*n* log*n*) steps.

Program: Creating a Binary Search Tree

We assume that every node of a binary search tree is capable of holding an integer data item and that the links can be made to point to the root of the left subtree and the right subtree, respectively. Therefore, the structure of the node can be defined using the following declaration:

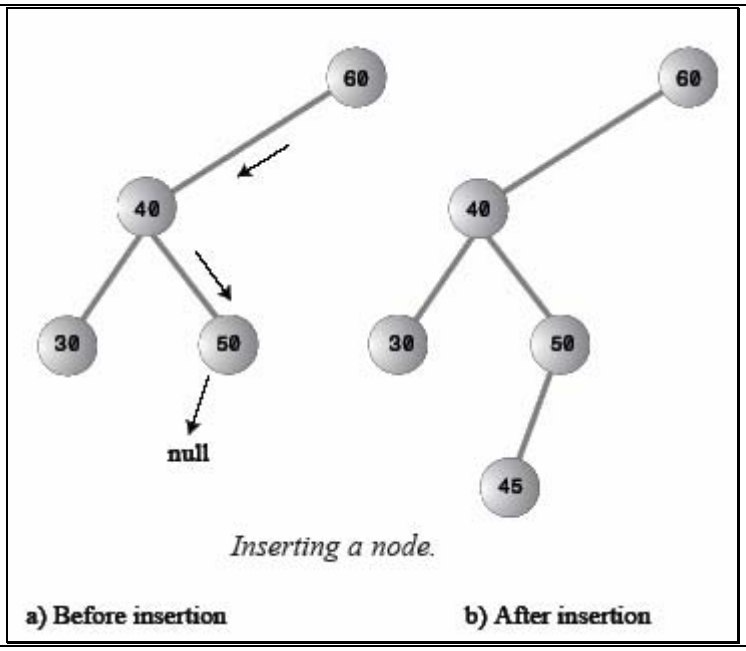
```
struct tnode
    {
        int data;
        struct tnode *lchild,*rchild;
    };

    A complete C program to create a binary search tree follows:
#include <stdio.h>
#include <stdlib.h>
struct tnode
{
   int data;
   struct tnode *lchild, *rchild;
};

struct tnode *insert(struct tnode *p,int val)
{
   struct tnode *temp1,*temp2;
   if(p == NULL)
   {
      p = (struct tnode *) malloc(sizeof(struct tnode)); /* insert the new
node as root node*/
```

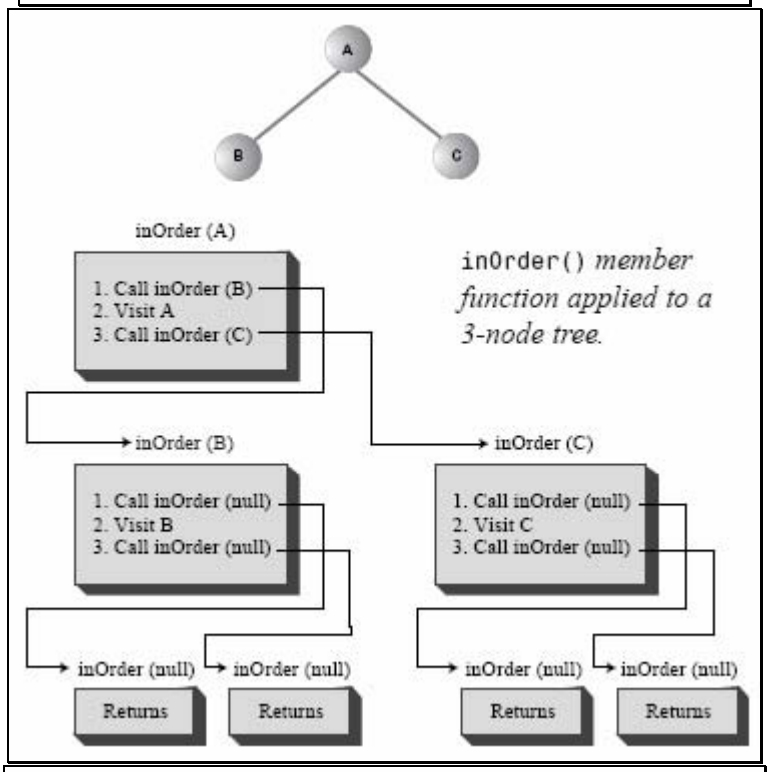
```
         if(p == NULL)
           {
      printf("Cannot allocate\n");
      exit(0);
          }
       p->data = val;
       p->lchild=p->rchild=NULL;
   }
  else
  {
   temp1 = p;
   /* traverse the tree to get a pointer to that node whose child will be
the newly created node*/
  while(temp1 != NULL)
  {
     temp2 = temp1;
     if( temp1 ->data > val)
          temp1 = temp1->lchild;
     else
          temp1 = temp1->rchild;
  }
  if( temp2->data > val)
  {
  temp2->lchild = (struct tnode*)malloc(sizeof(struct tnode));/*inserts the
newly created node as left child*/
          temp2 = temp2->lchild;
          if(temp2 == NULL)
               {
          printf("Cannot allocate\n");
          exit(0);
              }
          temp2->data = val;
          temp2->lchild=temp2->rchild = NULL;
  }
  else
  {
     temp2->rchild = (struct tnode*)malloc(sizeof(struct tnode));/ *inserts
the newly created node
   as left child*/
      temp2 = temp2->rchild;
      if(temp2 == NULL)
           {
      printf("Cannot allocate\n");
      exit(0);
          }
     temp2->data = val;
     temp2->lchild=temp2->rchild = NULL;
```

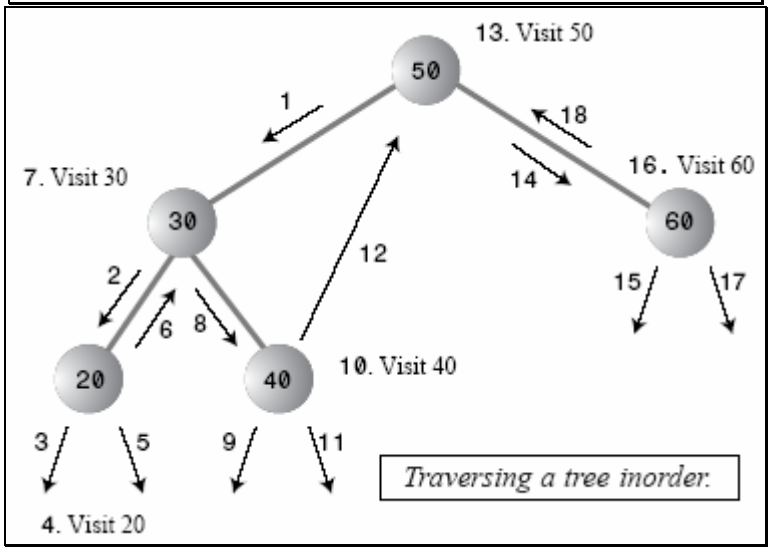
```
}
}
return(p);
}
/* a function to binary tree in inorder */
void inorder(struct tnode *p)
   {
      if(p != NULL)
        {
           inorder(p->lchild);
          printf("%d\t",p->data);
         inorder(p->rchild);
         }
   }
void main()
{
   struct tnode *root = NULL;
   int n,x;
   printf("Enter the number of nodes\n");
   scanf("%d",&n);
   while( n - > 0)
     {
         printf("Enter the data value\n");
         scanf("%d",&x);
         root = insert(root,x);
    }
     inorder(root);
 }
```

Explanation

1. To create a binary search tree, we use a function called `insert`, which creates a new node with the data value supplied as a parameter to it, and inserts it into an already existing tree whose root pointer is also passed as a parameter.
2. The function accomplishes this by checking whether the tree whose root pointer is passed as a parameter is empty. If it is empty, then the newly created node is inserted as a root node. If it is not empty, then it copies the root pointer into a variable `temp1`. It then stores the value of `temp1` in another variable, `temp2`, and compares the data value of the node pointed to by `temp1` with the data value supplied as a parameter. If the data value supplied as a parameter is smaller than the data value of the node pointed to by `temp1`, it copies the left link of the node pointed to by `temp1` into `temp1` (goes to the left); otherwise it copies the right link of the node pointed to by `temp1` into `temp1` (goes to the right).
3. It repeats this process until `temp1` reaches 0. When `temp1` becomes 0, the new node is inserted as a left child of the node pointed to by `temp2`, if the data value of the node pointed to by `temp2` is greater than the data value supplied as a parameter. Otherwise, the new node is inserted as a right child of the node pointed to by `temp2`. Therefore the insert procedure is:
   - Input: 1. The number of nodes that the tree to be created should have
     2. The data values of each node in the tree to be created
   - Output: The data value of the nodes of the tree in inorder


60
40
30
50
null
60
40
30
50
45
Inserting a node.
a) Before insertion
b) After insertion


A
B
C
inOrder (A)
1. Call inOrder (B)
2. Visit A
3. Call inOrder (C)
inOrder() member function applied to a 3-node tree.
inOrder (B)
inOrder (C)
1. Call inOrder (null)
2. Visit B
3. Call inOrder (null)
1. Call inOrder (null)
2. Visit C
3. Call inOrder (null)
inOrder (null)
inOrder (null)
inOrder (null)
inOrder (null)
Returns
Returns
Returns
Returns


13. Visit 50
50
1
18
7. Visit 30
14
16. Visit 60
30
60
12
2
15
17
6
8
20
40
10. Visit 40
3
5
9
11
Traversing a tree inorder.
4. Visit 20

Example

- Input: 1. The number of nodes that the created tree should have = 5
  2. The data values of the nodes in the tree to be created are: 10, 20, 5, 9, 8
- Output : 5 8 9 10 20

Program

A function for inorder traversal of a binary tree:

```
void inorder(struct tnode *p)
    {
       if(p != NULL)
         {
 inorder(p->lchild);
 printf("%d\t",p->data);
    inorder(p->rchild);
}
```

A non-recursive/iterative function for traversing a binary tree in inorder is given here for the purpose of doing the analysis.

```
void inorder(struct tnode *p)
{
 struct tnode *stack[100];
 int top;
 top = −1;
if(p != NULL)
 {
    top++;
    stack[top] = p;
    p = p->lchild;
    while(top >= 0)
       {
          while ( p!= NULL)/* push the left child onto stack*/
           {
      top++;
                stack[top] =p;
     p = p->lchild;
   }
        p = stack[top];
   top-;
printf("%d\t",p->data);
   p = p->rchild;
   if ( p != NULL) /* push right child*/
      {
         top++;
                        stack[top] = p;
          p = p->lchild;
      }
```

```
   }
 }
}
```

A function for preorder traversal of a binary tree:

```
void preorder(struct tnode *p)
    {
       if(p != NULL)
         {
printf("%d\t",p->data);
            preorder(p->lchild);
preorder(p->rchild);
          }
```

A function for postorder traversal of a binary tree:

```
void postorder(struct node *p)
    {
       if(p != NULL)
         {
    postorder(p->lchild);
       postorder(p->rchild);
printf("%d\t",p->data);
 }
```

Explanation

Consider the iterative version of the inorder just given. If the binary tree to be traversed has *n* nodes, the number of `NULL` links are *n*+1. Since every node is placed on the stack once, the statements `stack[top]:=p` and `p:=stack[top]` are executed *n* times. The test for `NULL` links will be done exactly *n*+1 times. So every step will be executed no more than some small constant times *n*. So the order of the algorithm is O(*n*). A similar analysis can be done to obtain the estimate of the computation time for preorder and postorder.

Constructing a Binary Tree Using the Preorder and Inorder Traversals

To obtain the binary tree, we reverse the preorder traversal and take the first node that is a root node. We then search for this node in the inorder traversal. In the inorder traversal, all the nodes to the left of this node will be the part of the left subtree, and all the nodes to the right of this node will be the part of the right subtree. We then consider the next node in the reversed preorder. If it is a part of the left subtree, then we make it the left child of the root; if it is part of the right subtree, we make it part of right subtree. This procedure is repeated recursively to get the tree as shown in Figure 7.16.

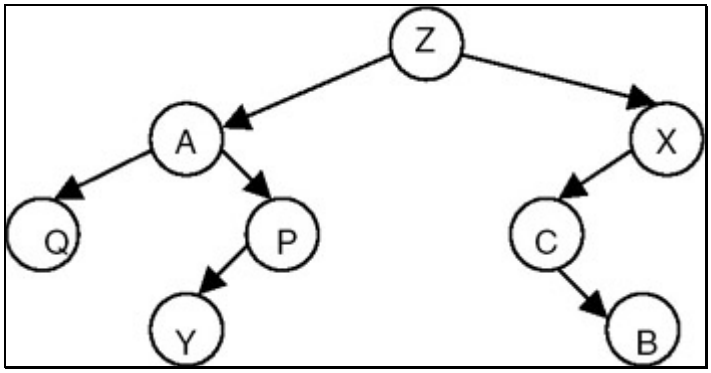


Figure 7.16: A unique binary tree constructed using the inorder and postorder.

For example, for the preorder and inorder traversals of a binary tree, the binary tree and its postorder traversal are as follows:

- Z,A,Q,P,Y,X,C,B = Preorder
- Q,A,Z,Y,P,C,X,B = Inorder

The postorder for this tree is:

Z,A,P,X,B,C,Y,Q

5. COUNTING THE NUMBER OF NODES IN A BINARY SEARCH TREE

Introduction

To count the number of nodes in a given binary tree, the tree is required to be traversed recursively until a leaf node is encountered. When a leaf node is encountered, a count of 1 is returned to its previous activation (which is an activation for its parent), which takes the count returned from both the children's activation, adds 1 to it, and returns this value to the activation of its parent. This way, when the activation for the root of the tree returns, it returns the count of the total number of the nodes in the tree.

Program

A complete C program to count the number of nodes is as follows:

```
#include <stdio.h>
#include <stdlib.h>
struct tnode
{
   int data;
   struct tnode *lchild, *rchild;
};
int count(struct tnode *p)
    {
               if( p == NULL)
      return(0);
              else
      if( p->lchild == NULL && p->rchild == NULL)
                return(1);
      else
                return(1 + (count(p->lchild) + count(p->rchild)));

    }

struct tnode *insert(struct tnode *p,int val)
   {
      struct tnode *temp1,*temp2;
      if(p == NULL)
      {
         p = (struct tnode *) malloc(sizeof(struct tnode)); /* insert the
new node as root node*/
```

```
            if(p == NULL)
              {
          printf("Cannot allocate\n");
          exit(0);
               }
           p->data = val;
           p->lchild=p->rchild=NULL;
      }
     else
     {
      temp1 = p;
      /* traverse the tree to get a pointer to that node whose child will
be the newly created node*/
     while(temp1 != NULL)
     {
       temp2 = temp1;
       if( temp1 ->data > val)
            temp1 = temp1->lchild;
       else
            temp1 = temp1->rchild;
     }
     if( temp2->data > val)
     {
        temp2->lchild = (struct tnode*)malloc(sizeof(struct tnode)); /
*inserts the newly created node
      as left child*/
         temp2 = temp2->lchild;
         if(temp2 == NULL)
              {
         printf("Cannot allocate\n");
         exit(0);
             }
        temp2->data = val;
        temp2->lchild=temp2->rchild = NULL;
   }
   else
   {
      temp2->rchild = (struct tnode*)malloc(sizeof(struct tnode));/
*inserts the newly created node
  as left child*/
     temp2 = temp2->rchild;
     if(temp2 == NULL)
          {
     printf("Cannot allocate\n");
     exit(0);
         }
```

```
      temp2->data = val;
      temp2->lchild=temp2->rchild = NULL;
   }
   }
   return(p);
   }
   /* a function to binary tree in inorder */
   void inorder(struct tnode *p)
       {
          if(p != NULL)
            {
              inorder(p->lchild);
             printf("%d\t",p->data);
            inorder(p->rchild);
            }
       }
void main()
{
   struct tnode *root = NULL;
   int n,x;
   printf("Enter the number of nodes\n");
   scanf("%d",&n);
   while( n --- > 0)
      {
         printf("Enter the data value\n");
         scanf("%d",&x);
         root = insert(root,x);
     }
      inorder(root);
     printf("\nThe number of nodes in tree are :%d\n",count(root));
}
```

Explanation

- Input: 1. The number of nodes that the tree to be created should have
  2. The data values of each node in the tree to be created
- Output: 1. The data value of the nodes of the tree in inorder
  2. The count of number of node in a tree.

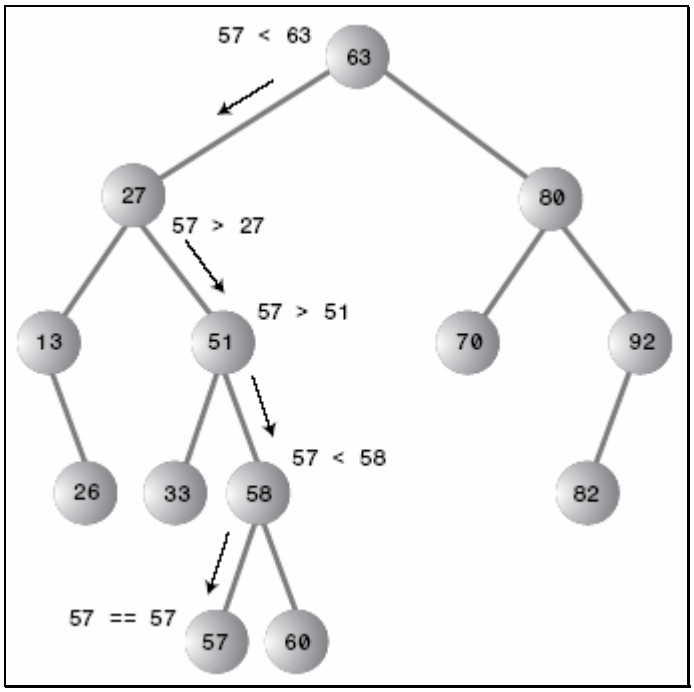
Example

- Input: 1. The number of nodes the created tree should have = 5
- 2. The data values of the nodes in the tree to be created are: 10, 20, 5, 9, 8
- Output: 1. 5 8 9 10 20
- 2. The number of nodes in the tree is 5

6. SEARCHING FOR A TARGET KEY IN A BINARY SEARCH TREE

Introduction

Data values are given which we call a key and a binary search tree. To search for the key in the given binary search tree, start with the root node and compare the key with the data value of the root node. If they match, return the root pointer. If the key is less than the data value of the root node, repeat the process by using the left subtree. Otherwise, repeat the same process with the right subtree until either a match is found or the subtree under consideration becomes an empty tree.



Program

A complete C program for this search is as follows:

```
#include <stdio.h>
#include <stdlib.h>
struct tnode
{
  int data;
  struct tnode *lchild, *rchild;
};
/* A function to serch for a given data value in a binary search tree*/
struct tnode *search( struct tnode *p,int key)
   {
     struct tnode *temp;
      temp = p;
     while( temp != NULL)
       {
         if(temp->data == key)
                return(temp);
         else
```

```
   if(temp->data > key)
      temp = temp->lchild;
   else
      temp = temp->rchild;
        }
return(NULL);
}

/*an iterative function to print the binary tree in inorder*/
void inorder1(struct tnode *p)
{
  struct tnode *stack[100];
  int top;
  top = -1;
 if(p != NULL)
 {
    top++;
    stack[top] = p;
    p = p->lchild;
    while(top >= 0)
       {
          while ( p!= NULL)/* push the left child onto stack*/
           {
                       top++;
                 stack[top] =p;
                 p = p->lchild;
         }
         p = stack[top];
         top-;
         printf("%d\t",p->data);
         p = p->rchild;
         if ( p != NULL) /* push right child*/
           {
               top++;
                              stack[top] = p;
               p = p->lchild;
                 }

         }
    }
}
/* A function to insert a new node in binary search tree to
get a tree created*/
struct tnode *insert(struct tnode *p,int val)
{
```

```
   struct tnode *temp1,*temp2;
   if(p == NULL)
   {
      p = (struct tnode *) malloc(sizeof(struct tnode)); /* insert the new
node as root node*/
      if(p == NULL)
        {
   printf("Cannot allocate\n");
   exit(0);
       }
    p->data = val;
    p->lchild=p->rchild=NULL;
   }
  else
  {
    temp1 = p;
   /* traverse the tree to get a pointer to that node whose child will be
the newly created node*/
  while(temp1 != NULL)
  {
    temp2 = temp1;
    if( temp1 ->data > val)
         temp1 = temp1->lchild;
    else
         temp1 = temp1->rchild;
  }
  if( temp2->data > val)
  {
     temp2->lchild = (struct tnode*)malloc(sizeof(struct tnode));/
*inserts the newly created node
  as left child*/
     temp2 = temp2->lchild;
     if(temp2 == NULL)
          {
     printf("Cannot allocate\n");
     exit(0);
          }
    temp2->data = val;
    temp2->lchild=temp2->rchild = NULL;
  }
  else
  {
     temp2->rchild = (struct tnode*)malloc(sizeof(struct tnode));/ *inserts
the newly created node
  as left child*/
     temp2 = temp2->rchild;
     if(temp2 == NULL)
```

```
          {
     printf("Cannot allocate\n");
     exit(0);
         }
     temp2->data = val;
     temp2->lchild=temp2->rchild = NULL;
  }
}
return(p);
}
void main()
{
   struct tnode *root = NULL, *temp = NULL;
   int n,x;
   printf("Enter the number of nodes in the tree\n");
   scanf("%d",&n);
   while( n - > 0)
       {
         printf("Enter the data value\n");
         scanf("%d",&x);
         root = insert(root,x);
       }
      printf("The created tree is :\n");
      inorder1(root);
      printf("\n Enter the value of the node to be searched\n");
      scanf("%d",&n);
      temp=search(root,n);
      if(temp != NULL)
          printf("The data value is present in the tree \n");
      else
          printf("The data value is not present in the tree \n");
}
```

Explanation

- Input: 1. The number of nodes that the tree to be created should have
  2. The data values of each node in the tree to be created
  3. The key value
- Output: If the key is present and appears in the created tree, then a message "The data value is present in the tree" appears. Otherwise the message "The data value is not present in the tree" appears.

Example

- Input: 1. The number of nodes that the created tree should have = 5
  2. The data values of the nodes in the tree to be created are: 10, 20, 5, 9, 8
  3. The key value = 9
- Output: The data is present in the tree

7. DELETION OF A NODE FROM BINARY SEARCH TREE

Introduction

To delete a node from a binary search tree, the method to be used depends on whether a node to be deleted has one child, two children, or no children.

Deletion of a node with two children

Consider the binary search tree shown in Figure 7.17.

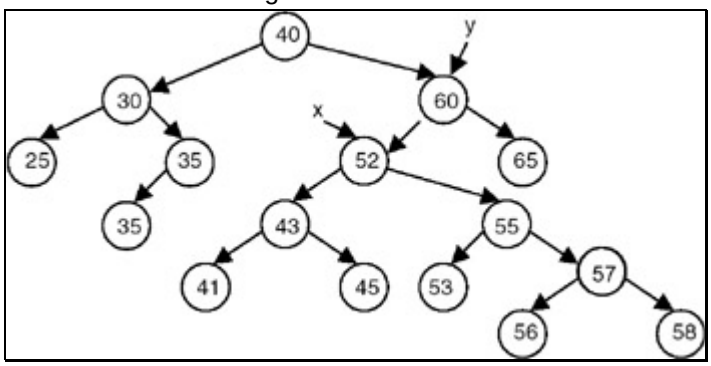


Figure 7.17: A binary tree before deletion of a node pointed to by x.

To delete a node printed to by *x*, we start by letting *y* be a pointer to the node that is the root of the node pointed to by *x*. We store the pointer to the left child of the node pointed to by *x* in a temporary pointer temp. We then make the left child of the node pointed to by *y* the left child of the node pointed to by *x*. We then traverse the tree with the root as the node pointed to by temp to get its right leaf, and make the right child of this right leaf the right child of the node pointed to by x, as shown in Figure 7.18.

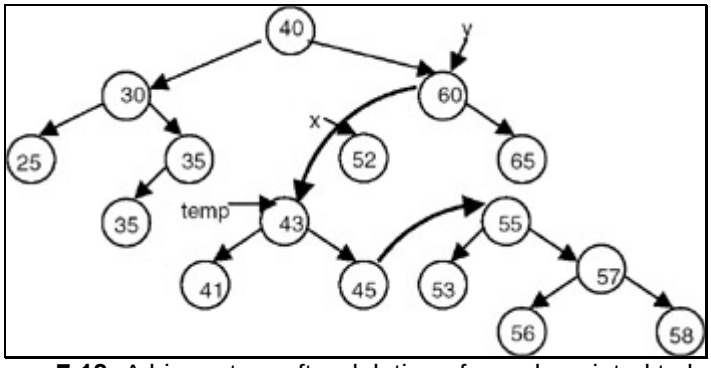


Figure 7.18: A binary tree after deletion of a node pointed to by x.

Another method is to store the pointer to the right child of the node pointed to by *x* in a temporary pointer temp. We then make the left child of the node pointed by *y* to be the right child of the node pointed to by *x*. We then traverse the tree with the root as the node pointed to by temp to get its left leaf, and make the left child of this left leaf the left child of the node pointed to by *x*, as shown in Figure 7.19.

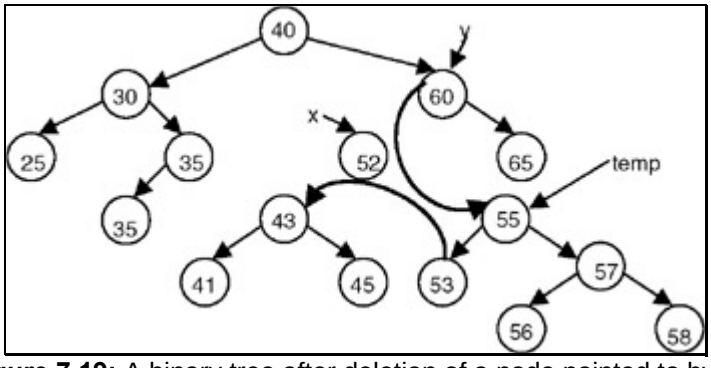


Figure 7.19: A binary tree after deletion of a node pointed to by x.

Deletion of a Node with One Child

Consider the binary search tree shown in Figure 7.20.

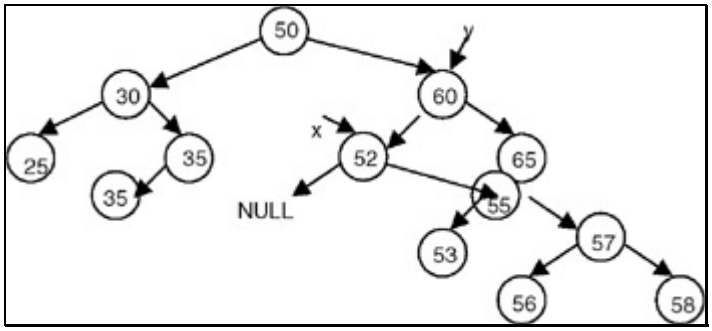


Figure 7.20: A binary tree before deletion of a node pointed to by x.

If we want to delete a node pointed to by *x*, we can do that by letting *y* be a pointer to the node that is the root of the node pointed to by *x*. Make the left child of the node pointed to by *y* the right child of the node pointed to by *x*, and dispose of the node pointed to by *x*, as shown in Figure 7.21.

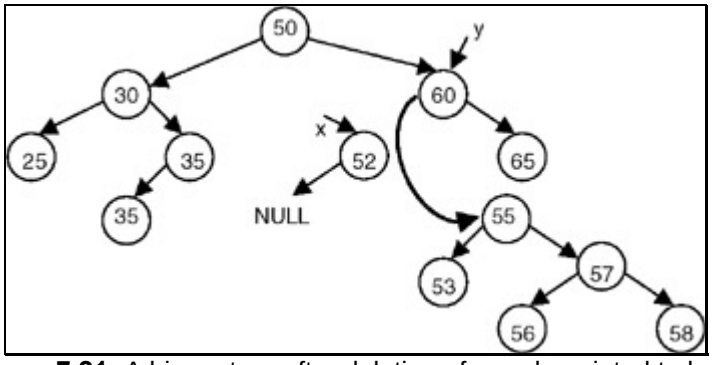


Figure 7.21: A binary tree after deletion of a node pointed to by x.

Deletion of a Node with No Child

Consider the binary search tree shown in Figure 7.22.

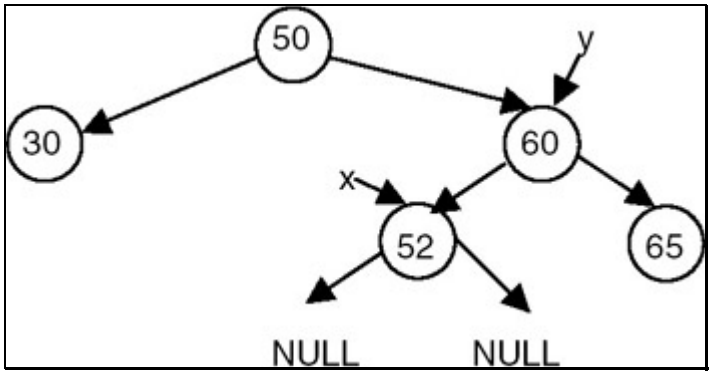


Figure 7.22: A binary tree before deletion of a node pointed to by x.

Set the left child of the node pointed to by y to NULL, and dispose of the node pointed to by x, as shown in Figure 7.23.

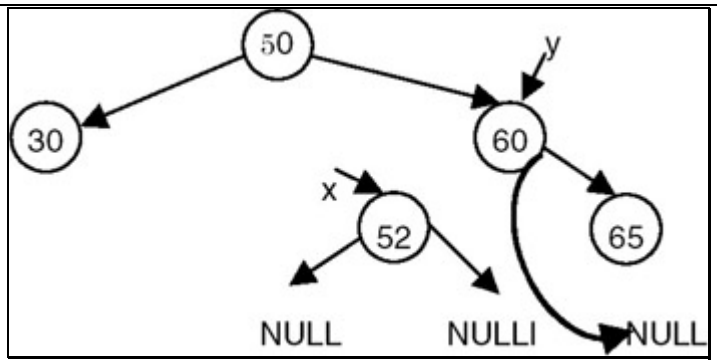


Figure 7.23: A binary tree after deletion of a node pointed to by x.

Program

A complete C program to delete a node, where the data value of the node to be deleted is known, is as follows:

```
#include <stdio.h>
#include <stdlib.h>
struct tnode
{
  int data;
  struct tnode *lchild, *rchild;
};
/* A function to get a pointer to the node whose data value is given
    as well as the pointer to its root */
struct tnode *getptr(struct tnode *p, int key, struct tnode **y)
{
  struct tnode *temp;
     if( p == NULL)
       return(NULL);
     temp = p;
     *y = NULL;
      while( temp != NULL)
       {
         if(temp->data == key)
             return(temp);
         else
     {
             *y = temp; /*store this pointer as root */
     if(temp->data > key)
        temp = temp->lchild;
     else
        temp = temp->rchild;
        }
       }
      return(NULL);
}
```

```
/* A function to delete the node whose data value is given */
struct tnode *delete(struct tnode *p,int val)
  {
    struct tnode *x, *y, *temp;
    x = getptr(p,val,&y);
    if( x == NULL)
    {
      printf("The node does not exists\n");
      return(p);
    }
    else
    {
    /* this code is for deleting root node*/
    if( x == p)
      {
        temp = x->lchild;
        y = x->rchild;
        p = temp;
        while(temp->rchild != NULL)
      temp = temp->rchild;
      temp->rchild=y;
      free(x);
      return(p);
    }
/* this code is for deleting node having both children */
  if( x->lchild != NULL && x->rchild != NULL)
     {

       if(y->lchild == x)
       {
temp = x->lchild;
y->lchild = x->lchild;
while(temp->rchild != NULL)
    temp = temp->rchild;
temp->rchild=x->rchild;
x->lchild=NULL;
x->rchild=NULL;
        }
        else
        {
          temp = x->rchild;
          y->rchild = x->rchild;
           while(temp->lchild != NULL)
      temp = temp->lchild;
    temp->lchild=x->lchild;
x->lchild=NULL;
```

```
 x->rchild=NULL;
         }

free(x);
   return(p);
  }
   /* this code is for deleting a node with on child*/
   if(x->lchild == NULL && x->rchild !== NULL)
     {
       if(y->lchild == x)
  y->lchild = x->rchild;
          else
            y->rchild = x->rchild;
          x->rchild; = NULL;
          free(x);
          return(p);
       }
       if( x->lchild != NULL && x->rchild == NULL)
         {
           if(y->lchild == x)
              y->lchild = x->lchild ;
           else
              y->rchild = x->lchild;
           x->lchild = NULL;
           free(x);
           return(p);
         }
       /* this code is for deleting a node with no child*/
       if(x->lchild == NULL && x->rchild == NULL)
        {
           if(y->lchild == x)
              y->lchild = NULL ;
           else
              y->rchild = NULL;
           free(x);
           return(p);
        }
    }
}
/*an iterative function to print the binary tree in inorder*/
void inorder1(struct tnode *p)
{
 struct tnode *stack[100];
 int top;
 top = -1;
```

```
if(p != NULL)
 {
    top++;
    stack[top] = p;
    p = p->lchild;
    while(top >= 0)
       {
          while ( p!= NULL)/* push the left child onto stack*/
           {
                       top++;
                 stack[top] =p;
                 p = p->lchild;
 }
      p = stack[top];
      top-;
      printf("%d\t",p->data);
      p = p->rchild;
      if ( p != NULL) /* push right child*/
         {
             top++;
         stack[top] = p;
             p = p->lchild;
      }
   }
     }
   }
   /* A function to insert a new node in binary search tree to get a tree
created*/
  struct tnode *insert(struct tnode *p,int val)
  {
     struct tnode *temp1,*temp2;
     if(p == NULL)
     {
        p = (struct tnode *) malloc(sizeof(struct tnode)); /* insert the
new node as root node*/
        if(p == NULL)
          {
     printf("Cannot allocate\n");
     exit(0);
          }
       p->data = val;
       p->lchild=p->rchild=NULL;
  }
  else
  {
    temp1 = p;
```

```
   /* traverse the tree to get a pointer to that node whose child will be
the newly created node*/
  while(temp1 != NULL)
  {
    temp2 = temp1;
    if( temp1 ->data > val)
         temp1 = temp1->lchild;
    else
         temp1 = temp1->rchild;
  }
  if( temp2->data > val)
  {
     temp2->lchild = (struct tnode*)malloc(sizeof(struct tnode));/ *inserts
the newly created node
  as left child*/
     temp2 = temp2->lchild;
     if(temp2 == NULL)
          {
     printf("Cannot allocate\n");
     exit(0);
         }
    temp2->data = val;
    temp2->lchild=temp2->rchild = NULL;
  }
  else
  {
      temp2->rchild = (struct tnode*)malloc(sizeof(struct tnode));/
*inserts the newly created node
  as left child*/
     temp2 = temp2->rchild;
     if(temp2 == NULL)
          {
     printf("Cannot allocate\n");
     exit(0);
         }
    temp2->data = val;
    temp2->lchild=temp2->rchild = NULL;
  }
  }
  return(p);
  }


  void main()
  {
    struct tnode *root = NULL;
    int n,x;
    printf("Enter the number of nodes in the tree\n");
```

```
    scanf("%d",&n);
    while( n - > 0)
        {
          printf("Enter the data value\n");
          scanf("%d",&x);
          root = insert(root,x);
        }
       printf("The created tree is :\n");
       inorder1(root);
       printf("\n Enter the value of the node to be deleted\n");
       scanf("%d",&n);
       root=delete(root,n);
       printf("The tree after deletion is \n");
       inorder1(root);
}
```

Explanation

This program first creates a binary tree with a specified number of nodes with their respective data values. It then takes the data value of the node to be deleted, obtains a pointer to the node containing that data value, and obtains another pointer to the root of the node to be deleted. Depending on whether the node to be deleted is a root node, a node with two children a node with only one child, or a node with no children, it carries out the manipulations as discussed in the section on deleting a node. After deleting the specified node, it returns the pointer to the root of the tree.

- Input: 1. The number of nodes that the tree to be created should have
  2. The data values of each node in the tree to be created
  3. The data value in the node to be deleted
- Output: 1. The data values of the nodes in the tree in inorder before deletion
  2. The data values of the nodes in the tree in inorder after deletion

Example

- Input: 1. The number of nodes taht the created tree should have = 5
  2. The data values of the nodes in the tree to be created are: 10, 20, 5, 9, 8
  3. The data value in the node to be deleted = 9
- Output: 1.5 8 9 10 20
  2 5 8 10 20

Applications of Binary Search Trees

One of the applications of a binary search tree is the implementation of a dynamic dictionary. This application is appropriate because a dictionary is an ordered list that is required to be searched frequently, and is also required to be updated (insertion and deletion mode) frequently. So it can be implemented by making the entries in a dictionary into the nodes of a binary search tree. A more efficient implementation of a dynamic dictionary involves considering a key to be a sequence of characters, and instead of searching by comparison of entire keys, we use these characters to determine a multi-way branch at each step. This will allow us to make a 26-way branch according to the first letter, followed by another branch according to the second letter and so on.

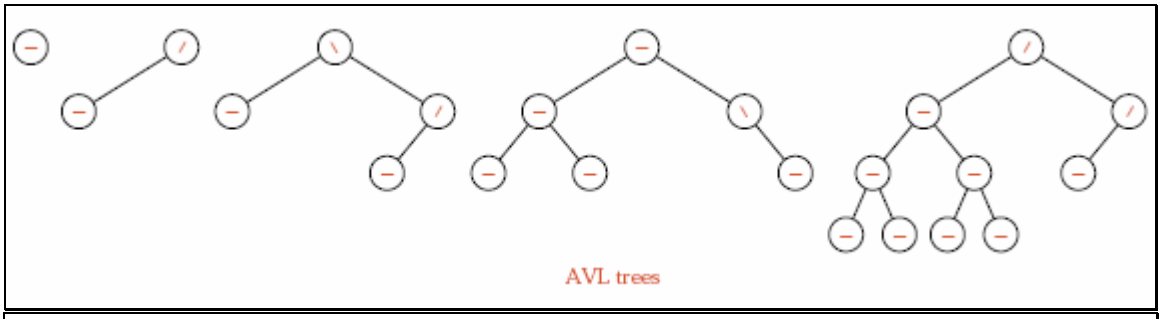
General Comments on Binary Trees

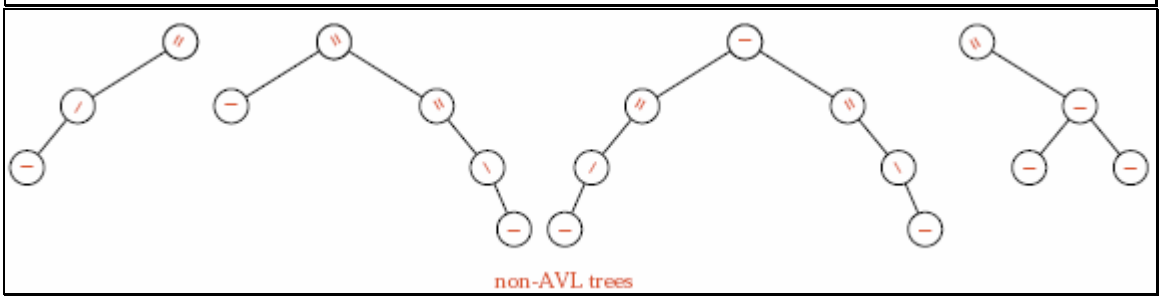
1. Trees are used to organize a collection of data items into a hierarchical structure.
2. A tree is a collection of elements called nodes, one of which is distinguished as the root, along with a relation that places a hierarchical structure on the node.
3. The degree of a node of a tree is the number of descendants that node has.
4. A leaf node of a tree is a node with a degree equal to 0.

5. The degree of a tree is the maximum of the degree of the nodes of the tree.
6. The level of the root node is 1, and as we descend the tree, we increment the level of each node by 1.
7. Depth of a tree is the maximum value of the level for the nodes in the tree.
8. A binary tree is a special case of tree, in which no node can have degree greater than 2.
9. The maximum number of nodes at level i in a binary tree is 2^{i-1}.
10. The maximum number of nodes in a binary tree of depth *k* is 2^{k-1}.
11. A complete binary tree of depth *k* is a tree with n nodes in which these n nodes can be numbered sequentially from 1 to *n*.
12. If a binary tree is a complete binary tree, it can be represented by an array capable of holding *n* elements where n is the number of nodes in a complete binary tree.
13. Inorder, preorder, and postorder are the three commonly used traversals that are used to traverse a binary tree.
14. In inorder traversal, we start with the root node, visit the left subtree first, then process the data of the root node, followed by that of the right subtree.
15. In preorder traversal, we start with the root node. First we process the data of the root node, then visit the left subtree, then the right subtree.
16. In postorder traversal, we start with the root node, visit the left subtree first, then visit the right subtree, and then process the data of the root node.
17. To construct a unique binary tree, we require two orders of traversal, in which one has to be inorder; the other could be preorder or postorder.
18. A binary search tree is an important search structure that is dynamic and allows a search by using $O(\log_2^n)$ steps.

8. AVL Tree

An AVL tree is a binary search tree in which the heights of the left and right subtrees of the root differ by at most 1 and in which the left and right subtrees are again AVL trees. With each node of an AVL tree is associated a balance factor that is lefthigher, equal-height, or right-higher according, respectively, as the left subtree has height greater than, equal to, or less than that of the right subtree.





An AVL Tree is a form of binary tree, however unlike a binary tree, the worst case scenario for a search is O(log n). The AVL data structure achieves this property by placing restrictions on the difference in height between the sub-trees of a given node, and re-balancing the tree if it violates these restrictions.

AVL Tree Balance Requirements

The complexity of an AVL Tree comes from the balance requirements it enforces on each node. A node is only allowed to possess one of three possible states (or balance factors):

Left-High (balance factor -1)

The left-sub tree is one level taller than the right-sub tree

Balanced (balance factor 0)

The left and right sub-trees are both the same heights

Right-High (balance factor +1)

The right sub-tree is one level taller than the left-sub tree.

If the balance of a node becomes -2 (it was left high and a level was lost from the left sub-tree) or +2 (it was right high and a level was lost from the right sub-tree) it will require re-balancing. This is achieved by performing a rotation about this node (see the section below on rotations).

Inserting in an AVL Tree

Nodes are initially inserted into AVL Trees in the same manner as an ordinary binary search tree (that is, they are always inserted as leaf nodes). After insertion, however, the insertion algorithm for an AVL Tree travels back along the path it took to find the point of insertion, and checks the balance at each node on the path. If a node is found that is unbalanced (that is, it has a balance factor of either -2 or +2), then a rotation is performed (see the section below on rotations) based on the inserted nodes position relative to the node being examined (the unbalanced node).

NB. There will only ever be at most one rotation required after an insert operation.

Deleting from an AVL Tree

The deletion algorithm for AVL Trees is a little more complex, as there are several extra steps involved in the deletion of a node. If the node is not a leaf node (that is, is has at least one child), then the node must be swapped with either it's in-order successor or predecessor (based on availability). Once the node has been swapped we can delete it (and have its parent pick up any children it may have - bear in mind that it will only ever have at most one child). If a deletion node was originally a leaf node, then it can simply be removed.

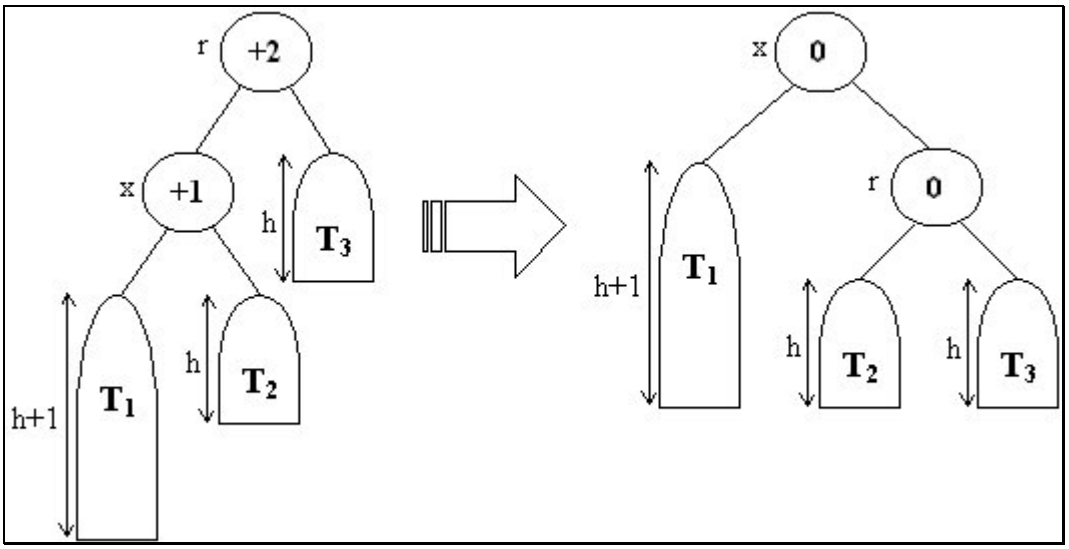
Now, as with the insertion algorithm, we traverse back up the path to the root node, checking the balance of all nodes along the path. If we encounter an unbalanced node we perform an appropriate rotation to balance the node (see the section below on rotations).

NB. Unlike the insertion algorithm, more than one rotation may be required after a delete operation, so in some cases we will have to continue back up the tree after a rotation.

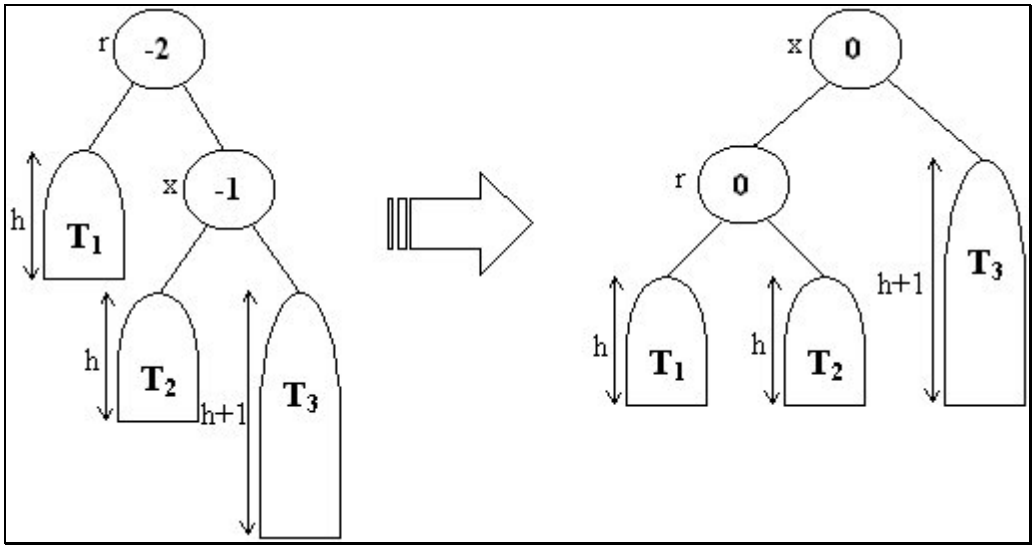
AVL Tree Rotations

As mentioned previously, an AVL Tree and the nodes it contains must meet strict balance requirements to maintain is O(log n) search capabilities. These balance restrictions are maintained using various rotation functions. Below is a diagrammatic overview of the four possible rotations that can be performed on an unbalanced AVL Tree, illustrating the before and after states of an AVL Tree requiring the rotation.

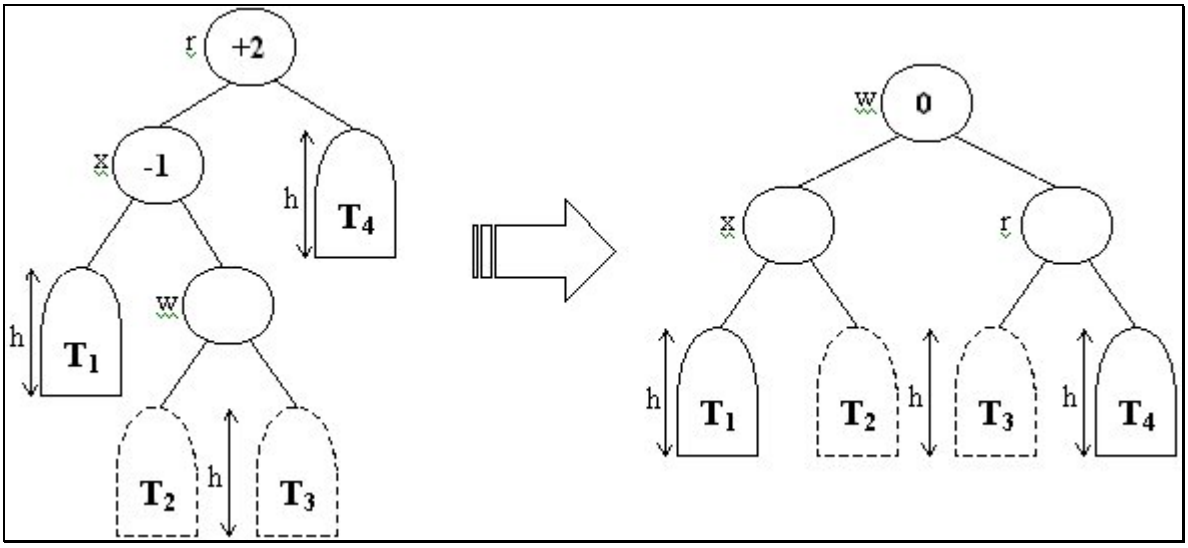
LL Rotation



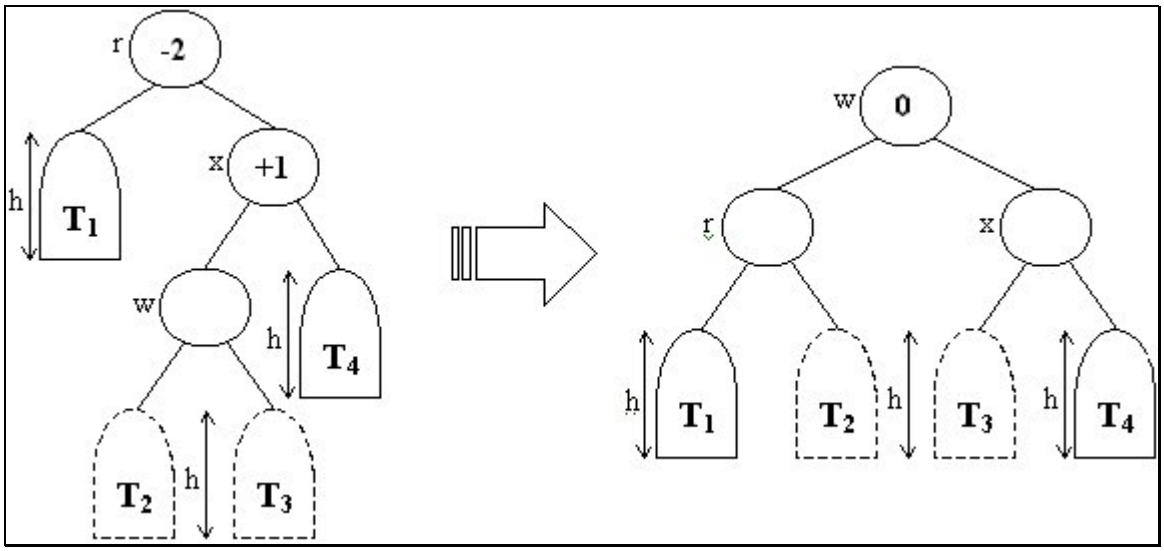
RR Rotation



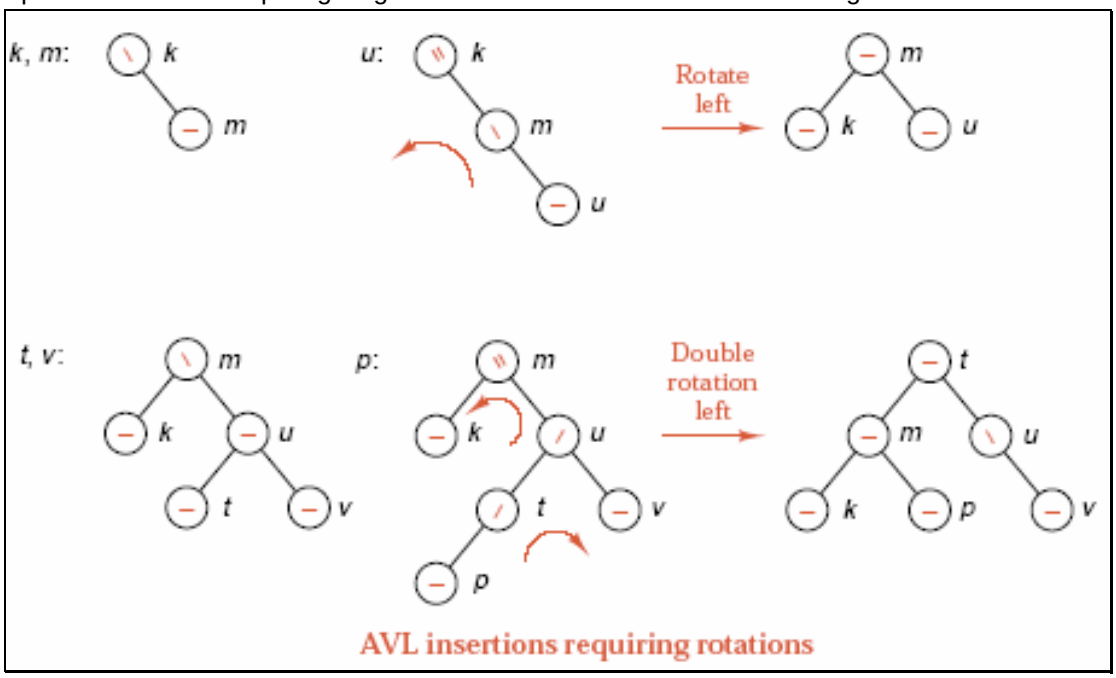
LR Rotation

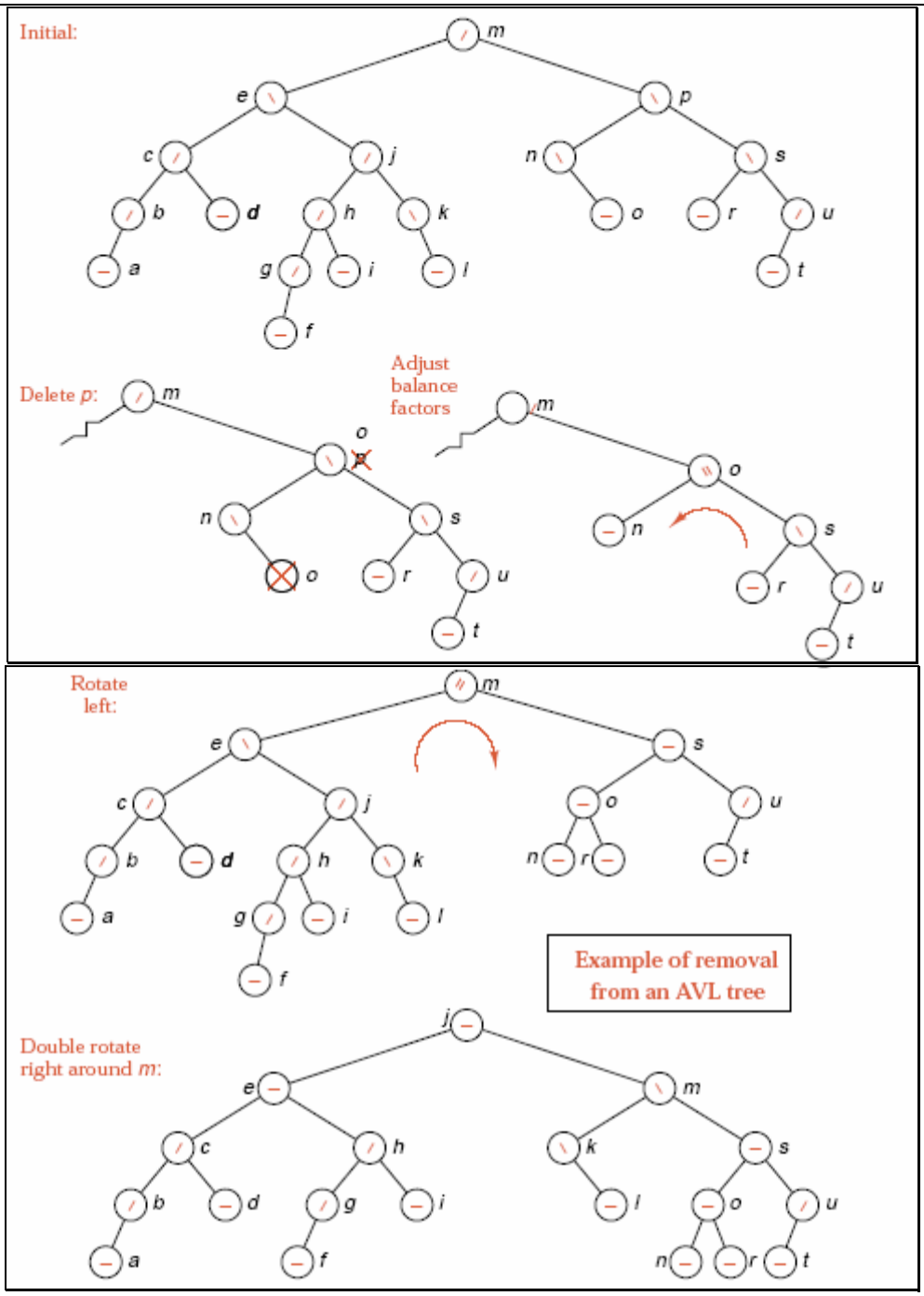


RL Rotation



Examples of insertions requiring single and double rotations are shown in Figure.





9. Exercises

E1. Write a program to construct a binary tree with inorder and preorder traversals. Test it for the following inorder and preorder traversals. Each node (info) is a integeer number. Enter -1 to finish input value.

+Inorder: 5, 1, 3, 11, 6, 8, 4, 2, 7, -1
+Preorder: 6, 1, 5, 11, 3, 4, 8, 7, 2, -1

- Display result when travesal tree using: NLR, LNR, LRN.
- Count the number of nodes in tree
- Count the number of leaf nodes in the tree.
- Count the number of non-leaf nodes of the binary tree.

- Find the sum of all node values in the tree.
- Find a node in the tree with input value. Replace the info number with new number.
- Adding a new node to the tree.
- Remove a node from the tree. The node is found from input value.
- Find the height of the tree.
- Input a X value. Display all node which have value is bigger than X.
- Write a C program to Write a C program to delete all the leaf nodes of a binary tree.