$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \qquad A = XDX^{-1}$$

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 1-\lambda & 1 & 1 \\ 1 & 1-\lambda & 1 \\ 1 & 1 & 1-\lambda \end{vmatrix} = 0$$

$$\lambda^3 - 3\lambda^2 + 0\lambda - 0 = 0$$

$$\lambda^3 - 3\lambda^2$$

$$\lambda^2(\lambda - 3) = 0$$

$$\lambda = \{3, 0, 0\}$$

$$0 \quad 0 \quad 3 \qquad \alpha$$

$\lambda = 3$

$$\left\{ \begin{bmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right.$$

$$\begin{matrix} 1 & -2 & 1 \\ 1 & 1 & -2 \end{matrix}$$

$$\frac{x}{3} = \frac{-y}{-3} = \frac{z}{3}$$

$$\lambda_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$\lambda = 0, 0$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x + y + z = 0$

calculated assumed

x	y	z	
-1	0	1	$\leftarrow \lambda_2$
-1	1	0	$\leftarrow \lambda_3$

$$\lambda_2 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \quad \& \quad \lambda_3 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$$

$$P \begin{bmatrix} 1 & -1 & -1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \qquad |P| \neq 0$$

$$P^{-1} = \frac{\text{Adj}[P]}{|P|}$$

$\boxed{\text{from calci}}$

$$P^{-1} = \frac{1}{(-3)} \begin{bmatrix} -1 & -1 & -1 \\ 1 & 1 & -2 \\ 1 & -2 & 1 \end{bmatrix}$$

$$P^{-1} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ -1 & -1 & 2 \\ -1 & 2 & -1 \end{bmatrix}$$

$$D = P^{-1} A P$$

$$= \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ -1 & -1 & 2 \\ -1 & 2 & -1 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 & -1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} 3 & 3 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & -1 & -1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} 9 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & -3 \\ 2 & 4 & -6 \\ -1 & -2 & 3 \end{bmatrix}$$

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 1-\lambda & 2 & -3 \\ 2 & 4-\lambda & -6 \\ -1 & -2 & 3-\lambda \end{vmatrix} = 0$$

$$\lambda^3 - 8\lambda^2 + 0\lambda - 0 = 0$$

$$\lambda^3 - 8\lambda^2 = 0$$

$$\lambda^2(\lambda - 8) = 0$$

$$\lambda = \{8, 0, 0\}$$

for $\lambda = 8$ $\quad |A - \lambda I| = 0$

$$\left\{ \begin{bmatrix} -7 & 2 & -3 \\ 2 & -4 & -6 \\ -1 & -2 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right.$$

$$\begin{matrix} 2 & -4 & -6 \\ -1 & -2 & -5 \end{matrix}$$

$$\frac{x}{8} = \frac{-y}{-16} = \frac{z}{0}$$

$$\lambda_1 = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix} \qquad \begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix}$$

$$\lambda = 0, 0$$

$$\begin{bmatrix} 1 & 2 & -3 \\ 2 & 4 & -6 \\ -1 & -2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x + 2y - 3z = 0$$

calculated	assumed		
x	y	z	
3	0	1	$\leftarrow \lambda_1$
-2	1	0	$\leftarrow \lambda_2$

$$\lambda_2 = \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix} \quad \& \quad \lambda_3 = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$$

Rules

1. Write eigen values in decreasing order

2. for cramers rule take 2 equations with different ratio

3. for single equation

   assume values of y & z

   and calculate x

least square method
& RMSE

$$Y = b_0 + b_1 X$$

$$b_0 = \bar{y} - b_1 \bar{x}$$

$$b_1 = \frac{\Sigma dx\, dy}{\Sigma dx^2}$$

$dx = (x - \bar{x})$
$dy = (y - \bar{y})$

$$RMSE = \sqrt{\frac{\Sigma(\hat{y} - y)^2}{n}}$$

orthogonal

given — to find

$$[x \;\; y \;\; z] \perp [a \;\; b \;\; c]$$

or

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \perp \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$$ax + by + cz = 0$$

assume values of a & b and calculate value of c

$$c = \frac{-(ax+by)}{z}$$

3 linear equations by cramers rule

(i) write as $AX = B$

$$\begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$$D = \begin{vmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix}$$

$$D_x = \begin{vmatrix} b_1 & a_2 & a_3 \\ b_2 & a_5 & a_6 \\ b_3 & a_8 & a_9 \end{vmatrix}$$

$$D_y = \begin{vmatrix} a_1 & b_1 & a_3 \\ a_4 & b_2 & a_6 \\ a_7 & b_3 & a_9 \end{vmatrix}$$

$$D_z = \begin{vmatrix} a_1 & a_2 & b_1 \\ a_4 & a_5 & b_2 \\ a_7 & a_8 & b_3 \end{vmatrix}$$

$$x = \frac{D_x}{D}$$

$$y = \frac{D_y}{D}$$

$$z = \frac{D_z}{D}$$

By Cramer's rule

SVD (omit)

too lengthy.