Graph Theory

A graph is a structure amounting to a set of objects in which some pairs of the objects are in some sense "related". The objects of the graph correspond to vertices and the relations between them correspond to edges. A graph is depicted diagrammatically as a set of dots depicting vertices connected by lines or curves depicting edges.

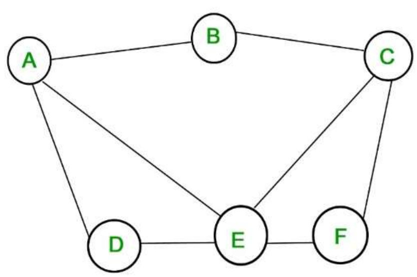
A graph is a set of points, called nodes or vertices, which are interconnected by a set of lines called edges. The study of graphs, or **graph theory** is an important part of a number of disciplines in the fields of mathematics, engineering and computer science.

Formally,

"A graph G = (V, E) consists of V , a non-empty set of vertices (or nodes) and E , a set of edges. Each edge has either one or two vertices associated with it, called its endpoints."

There are several types of graphs distinguished on the basis of edges, their direction, their weight etc.

1. **Simple graph** – A graph in which each edge connects two different vertices and where no two edges connect the same pair of vertices is called a simple graph. For example, Consider the following graph –

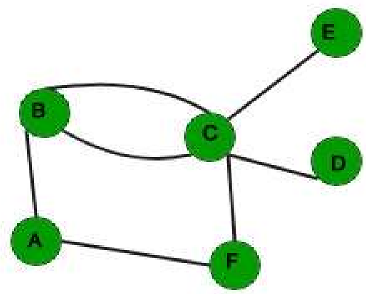


   The above graph is a simple graph, since no vertex has a self-loop and no two vertices have more than one edge connecting them.

   The edges are denoted by the vertices that they connect- {A,B} is the edge connecting vertices A and B .

2. **Multigraph** – A graph in which multiple edges may connect the same pair of vertices is called a multigraph.

   Since there can be multiple edges between the same pair of vertices, the multiplicity of edge tells the number of edges between two vertices.



The above graph is a multigraph since there are multiple edges between B and C . The multiplicity of the edge {B, C} is 2.

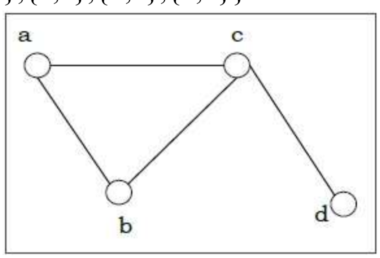
In some graphs, unlike the one's shown above, the edges are directed. This means that the relation between the objects is one-way only and not two-way. The direction of the edges may be important in some applications.

Based on whether the edges are directed or not we can have directed graphs and undirected graphs. This property can be extended to simple graphs and multigraphs to get simple directed or undirected simple graphs and directed or undirected multigraphs.

What is a Graph?

Definition − A graph (denoted as G=(V,E)G=(V,E)) consists of a non-empty set of vertices or nodes V and a set of edges E.

Example − Let us consider, a Graph is G=(V,E)G=(V,E) where V={a,b,c,d}V={a,b,c,d} and E={{a,b},{a,c},{b,c},{c,d}}E={{a,b},{a,c},{b,c},{c,d}}



Degree of a Vertex − The degree of a vertex V of a graph G (denoted by deg (V)) is the number of edges incident with the vertex V.

Vertex	Degree	Even / Odd
a	2	even
b	2	even
c	3	odd
d	1	odd

Even and Odd Vertex − If the degree of a vertex is even, the vertex is called an even vertex and if the degree of a vertex is odd, the vertex is called an odd vertex.

Degree of a Graph − The degree of a graph is the largest vertex degree of that graph. For the above graph the degree of the graph is 3.

The Handshaking Lemma − In a graph, the sum of all the degrees of all the vertices is equal to twice the number of edges.

$\sum_{v \in V} \deg(v) = 2|E|$

Properties of a Graph

- The root can be described as a starting point of the network.
- A graph will be known as the assortative graph if nodes of the same types are connected to one another. The graph will be known as the disassortative graph in all the other cases.
- If a cycle graph contains a single cycle, then that type of cycle graph will be known as a graph.
- If a single edge is used to connect all the pairs of vertices, then that type of graph will be known as the complete graph.
- If there is the same direction or reverse direction in which each pair of vertices are connected, then that type of graph will be known as the symmetry graph.
- If there is a graph which has a single graph, then that type of graph will be a path graph.