

Assembly line scheduling

Assembly line scheduling is a manufacturing problem. In automobile industries assembly lines are used to transfer parts from one station to another station.

– Manufacturing of large items like car, trucks etc. generally undergoes through multiple stations, where each station is responsible for assembling particular part only. Entire product be ready after it goes through predefined n stations in sequence.

– Manufacturing of car may be done through several stages like engine fitting, coloring, light fitting, fixing of controlling system, gates, seats and many other things.

-The particular task is carried out at the station dedicated to that task only. Based on the requirement there may be more than one assembly line.

-In case of two assembly lines if the load at station j at assembly 1 is very high, then components are transfer to station of assembly line 2 the converse is also true. This technique helps to speed ups the manufacturing process.

-The time to transfer partial product from one station to next station on the same assembly line is negligible. During rush factory may transfer partially completed auto from one assembly line to another, complete the manufacturing as quickly as possible.

Assembly line scheduling is a problem in operations management that involves determining the optimal sequence of tasks or operations on an assembly line to minimize production costs or maximize efficiency. This problem can be solved using various data structures and algorithms. One common approach is dynamic programming, which involves breaking the problem down into smaller sub-problems and solving them recursively.

The following is an overview of the steps involved in solving an assembly line scheduling problem using dynamic programming:

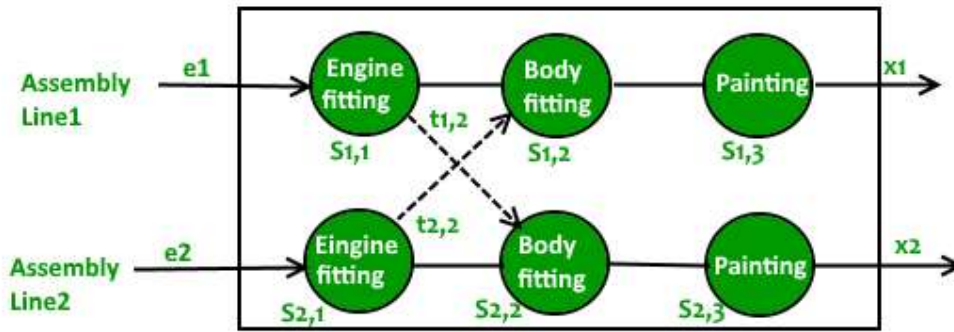
- **Define the problem:** The first step is to define the problem, including the number of tasks or operations involved, the time required to perform each task on each assembly line, and the cost or efficiency associated with each task.
- **Define the sub-problems:** Next, we need to define the sub-problems by breaking down the problem into smaller pieces. In

assembly line scheduling, this involves determining the optimal sequence of tasks for each station along the assembly line.

- **Define the recurrence relation:** The recurrence relation defines the relationship between the sub-problems and the overall problem. In assembly line scheduling, the recurrence relation involves computing the minimum cost or maximum efficiency of the assembly line by considering the cost or efficiency of the previous station and the time required to transition to the next station.
- **Solve the sub-problems:** To solve the sub-problems, we can use a table or matrix to store the minimum cost or maximum efficiency of each station. We can then use this table to determine the optimal sequence of tasks for the entire assembly line.
- **Trace the optimal path:** Finally, we can trace the optimal path through the table or matrix to determine the sequence of tasks that minimizes production costs or maximizes efficiency.

A car factory has two assembly lines, each with n stations. A station is denoted by $S_{i,j}$ where i is either 1 or 2 and indicates the assembly line the station is on, and j indicates the number of the station. The time taken per station is denoted by $a_{i,j}$. Each station is dedicated to some sort of work like engine fitting, body fitting, painting, and so on. So, a car chassis must pass through each of the n stations in order before exiting the factory. The parallel stations of the two assembly lines perform the same task. After it passes through station $S_{i,j}$, it will continue to station $S_{i,j+1}$ unless it decides to transfer to the other line. Continuing on the same line incurs no extra cost, but transferring from line i at station $j - 1$ to station j on the other line takes time $t_{i,j}$. Each assembly line takes an entry time e_i and exit time x_i which may be different for the two lines. Give an algorithm for computing the minimum time it will take to build a car chassis.

The below figure presents the problem in a clear picture:



The following information can be extracted from the problem statement to make it simpler:

- Two assembly lines, 1 and 2, each with stations from 1 to n.
- A car chassis must pass through all stations from 1 to n in order (in any of the two assembly lines). i.e. it cannot jump from station i to station j if they are not at one move distance.
- The car chassis can move one station forward in the same line, or one station diagonally in the other line. It incurs an extra cost $t_{i,j}$ to move to station j from line i. No cost is incurred for movement in same line.
- The time taken in station j on line i is $a_{i,j}$.
- $S_{i,j}$ represents a station j on line i.

Breaking the problem into smaller sub-problems:

We can easily find the i th factorial if $(i-1)$ th factorial is known. Can we apply the similar funda here?

If the minimum time taken by the chassis to leave station $S_{i,j-1}$ is known, the minimum time taken to leave station $S_{i,j}$ can be calculated quickly by combining $a_{i,j}$ and $t_{i,j}$.

T1(j) indicates the minimum time taken by the car chassis to leave station j on assembly line 1.

T2(j) indicates the minimum time taken by the car chassis to leave station j on assembly line 2.

Base cases:

The entry time e_i comes into picture only when the car chassis enters the car factory.

Time taken to leave the first station in line 1 is given by:

$T1(1) = \text{Entry time in Line 1} + \text{Time spent in station } S_{1,1}$

$T1(1) = e_1 + a_{1,1}$

Similarly, time taken to leave the first station in line 2 is given by:

$$T2(1) = e_2 + a_{2,1}$$

Recursive Relations:

If we look at the problem statement, it quickly boils down to the below observations:

The car chassis at station $S_{1,j}$ can come either from station $S_{1,j-1}$ or station $S_{2,j-1}$.

Case #1: Its previous station is $S_{1,j-1}$

The minimum time to leave station $S_{1,j}$ is given by:

$T1(j)$ = Minimum time taken to leave station $S_{1,j-1}$ + Time spent in station $S_{1,j}$

$$T1(j) = T1(j-1) + a_{1,j}$$

Case #2: Its previous station is $S_{2,j-1}$

The minimum time to leave station $S_{1,j}$ is given by:

$T1(j)$ = Minimum time taken to leave station $S_{2,j-1}$ + Extra cost incurred to change the assembly line + Time spent in station $S_{1,j}$

$$T1(j) = T2(j-1) + t_{2,j} + a_{1,j}$$

The minimum time $T1(j)$ is given by the minimum of the two obtained in cases #1 and #2.

$$T1(j) = \min((T1(j-1) + a_{1,j}), (T2(j-1) + t_{2,j} + a_{1,j}))$$

Similarly, the minimum time to reach station $S_{2,j}$ is given by:

$$T2(j) = \min((T2(j-1) + a_{2,j}), (T1(j-1) + t_{1,j} + a_{2,j}))$$

The total minimum time taken by the car chassis to come out of the factory is given by:

$Tmin = \min(\text{Time taken to leave station } S_{1,n} + \text{Time taken to exit the car factory})$

$$Tmin = \min(T1(n) + x_1, T2(n) + x_2)$$