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ex. Examples on Multiplication modulo 'P'.

May 18, May 19 ** * ① Prove that the set $G = \{1, 2, 3, 4, 5, 6\}$ is a finite Abelian group of order 6 with respect to multiplication modulo 7.

⇒ Composition Table -

$\times_7$	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	1	3	5
3	3	6	2	5	1	4
4	4	1	5	2	6	3
5	5	3	1	6	4	2
6	6	5	4	3	2	1

$$\left.\begin{aligned} 2\times1 \bmod 7 &= 2 \\ 2\times2 \bmod 7 &= 4 \\ 2\times3 \bmod 7 &= 6 \\ 2\times4 \bmod 7 &= 1 \\ 2\times5 \bmod 7 &= 3 \\ 2\times6 \bmod 7 &= 5 \end{aligned}\right\} 2^{nd} \text{ row}$$

similarly we can compute for all the rows.

i) All the entries in the composition table are elements of G. Here G is closed with respect to multiplication modulo 7. $(\times_7)$

ii) The composition of $\times_7$ is associative. Let a, b, c are any three elements of G, then

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$a * (b * c) = (a * b) * c$

$a \times_7 (b \times_7 c) = (a \times_7 b) \times_7 c$.

Let $a = 1$, $b = 2$, $c = 3$.

$1 \times_7 (2 \times_7 3) = (1 \times_7 2) \times_7 3$

$1 \times_7 6 = 2 \times_7 3$

$6 = 6$

hence it is an associative operation. since it is satisfying for all $a, b, c \in G$.

iii) We have $1 \in G$.

If a is any element of G, then by identity property.

$a * e = e * a = a$

$1 \times_7 1 = 1 \times_7 1 = 1$

$3 \times_7 1 = 1 \times_7 3 = 3$

$6 \times_7 1 = 1 \times_7 6 = 6$

1 is an identity element.

iv) By inverse property,

$a * b = e$ ∴ e is an identity

$1 \times_7 1 = 1$	$4 \times_7 2 = 1$
$2 \times_7 4 = 1$	$5 \times_7 3 = 1$
$3 \times_7 5 = 1$	$6 \times_7 6 = 1$

Hence inverses of 1, 2, 3, 4, 5, 6 are 1, 4, 5, 2, 3, 6 respectively.

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v> The composition $\times_7$ is commutative as the corresponding rows and columns in the table are identical. $a * b = b * a$

$\therefore 2 \times_7 3 = 3 \times_7 2$
$6 = 6$
$4 \times_7 5 = 5 \times_7 4$
$6 = 6$

$2 \times_7 5 = 5 \times_7 2$
$3 = 3$

Hence it is commutative.

vi> The set has 6 elements hence group $(G, \times_7)$ is a finite Abelian group of order 6.

ex. 2
Let Z_4 i.e. $G = \{0, 1, 2, 3\}$
i> prepare its composition table with respect to $\times_4$.
ii> Is it a group?

$\Rightarrow$ Let $G = \{0, 1, 2, 3\}$

composition table -

$\times_4$	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	0	2
3	0	3	2	1