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Relations :

Let A and B be non-empty sets. A relation R from A to B is a subset of A × B. If R, $R \subseteq A \times B$ and $(a, b) \in R$, we can say that a is related to b by R, we can write like aRb. If a is not related to b by R, we can write $a \not R b$.

e.g. i) Let $A = \{1, 2, 3\}$ and $B = \{r, s\}$
$R = \{(1, r), (2, s), (3, r)\}$ is a relation from A to B.

ii) R is a relation represents <
(less than) on A, $A = \{1, 2, 3, 4, 5\}$
aRb iff $a < b$
then $R = \{(1,2)(1,3)(1,4)(1,5)$
$(2,3)(2,4)(2,5)(3,4)$
$(3,5)(4,5)\}$

* Representation of Relation :-
2 methods of repreentation
1) Graphical &
2) tabular form.

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* Representation of relation -

Let $A = \{1, 2, 3\}$ and $B = \{r, s\}$

Let R is a relation from set A to set B

$R = \{(1,r)(2,s)(3,r)\}$

Then the matrix of R is

$$M_R = \begin{array}{c} 1 \\ 2 \\ 3 \end{array} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$$
$$\quad\quad\quad r \quad s$$

ex. Let $A = \{1, 2, 3, 4, 8\}$

$B = \{1, 2, 3, 4, 8\}$

aRb if and only if $a + b \leq 9$ find its relation matrix.

⇒ $R = \{(1,1)(1,2)(1,3)(1,4)(1,8)(2,1)$
$(2,2)(2,3)(2,4)(3,1)(3,2)(3,3)$
$(3,4)(4,1)(4,2)(4,3)(4,4)(8,1)\}$

$$\begin{array}{cc} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 8 \end{array} \\ M_R = \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 8 \end{array} & \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

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Ex. Let $A = \{a, b, c, d\}$ and let

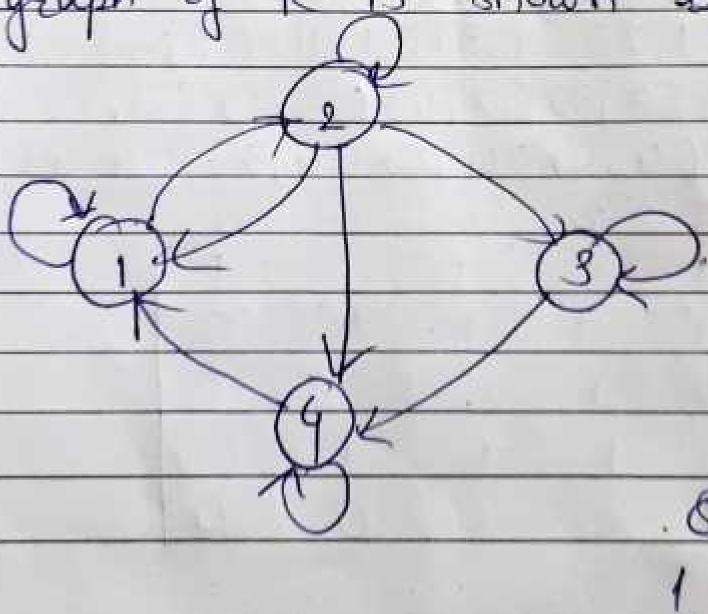
$$M_R = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & 1 & 1 & 0 & 0 \\ b & 0 & 0 & 1 & 1 \\ c & 0 & 0 & 1 & 1 \\ d & 1 & 0 & 1 & 0 \end{array}$$

Find R.

$\Rightarrow$ $R = \{(a,a)\ (a,b)\ (b,c)\ (b,d)\ (c,c)\ (c,d)\ (d,a)\ (d,c)\}$

Digraphs :-

Let $A = \{1, 2, 3, 4\}$ Let R is a relation from A to A. $R = \{(1,1)\ (1,2)\ (2,1)\ (2,2)\ (2,3)\ (2,4)\ (3,4)\ (4,1)\}$ Then the diagraph of R is shown below



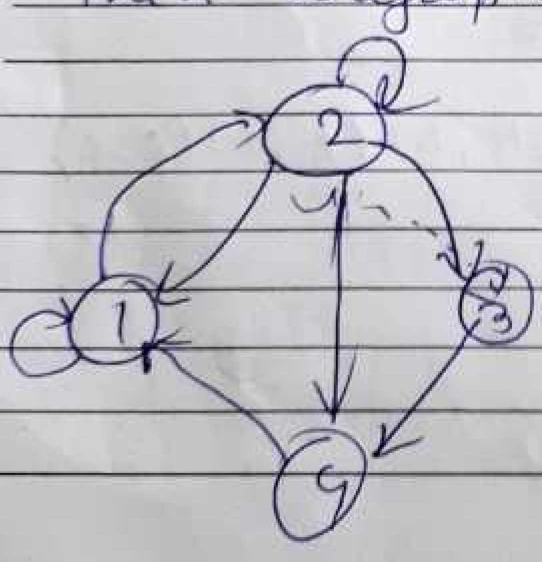
indegree -
1 = 3
2 = 2
3 = 2
4 = 3

Outdegree -
1 = 2, 3 = 2
2 = 4, 4 = 2

* Digraph-

Definition- If A is a finite set and R is a relation on A, then we can represent R as follows.
- Draw a small circle for each element of A and label the circle with the corresponding of A. These circles are vertices.
- Draw an arrow, called an edge, from vertex a_i to a_j iff $a_i R a_j$.

ex. let $A = \{1, 2, 3, 4\}$, let R be a relation from A to A.
$R = \{(1,1)(1,2)(2,1)(2,2)(2,3)(2,4)(3,4)(4,1)\}$
Then diagraph on R is shown below



ex ② Find the relation

⇒ $a_i \, R \, a_j$ iff there is an edge from a_i to a_j we have

$R = \{(1,1)(1,3)(2,3)(3,2)(3,3)(4,3)\}$

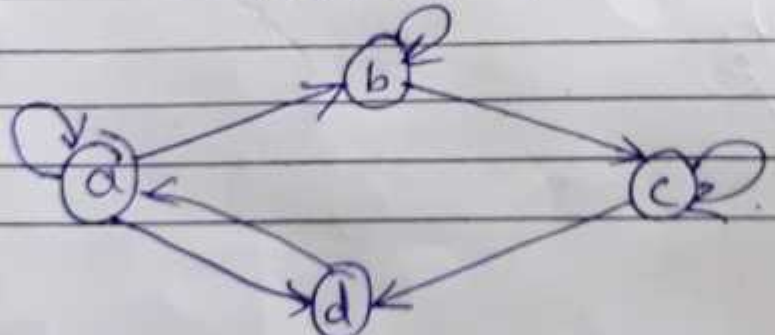
③ Let $A = \{a, b, c, d\}$ and

$$M_R = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Draw the digraph of R

⇒

$R = \{(a,b)(a,a)(a,d)(b,b)(b,c)(c,c)(c,d)(d,a)\}$



PARSHVANATH CHARITABLE TRUST'S
A.P. SHAH INSTITUTE OF TECHNOLOGY
Department of Computer Science and Engineering
Data Science

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Digraph-

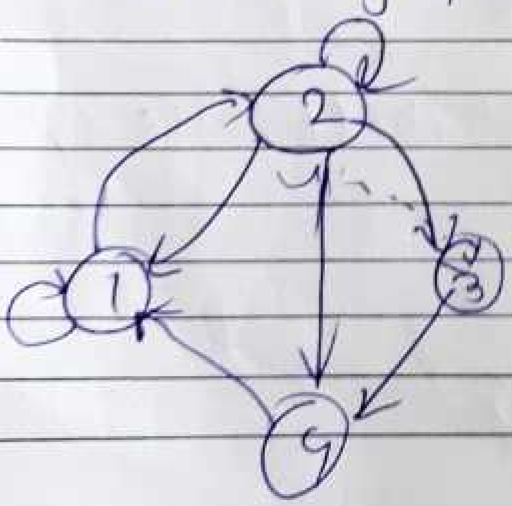
Definition- If A is a finite set and R is a relation on A, then we can represent R as follows.

- Draw a small circle for each element of A and label the circle with the corresponding of A. These circles are vertices.
- Draw an arrow, called an edge, from vertex a_i to a_j iff $a_i \, R \, a_j$.

ex. Let $A = \{1, 2, 3, 4\}$, let R be a relation from A to A.

$R = \{(1,1)(1,2)(2,1)(2,2)\ (2,3)\ (2,4)\ (3,4)(4,1)\}$

Then diagraph on R is shown below



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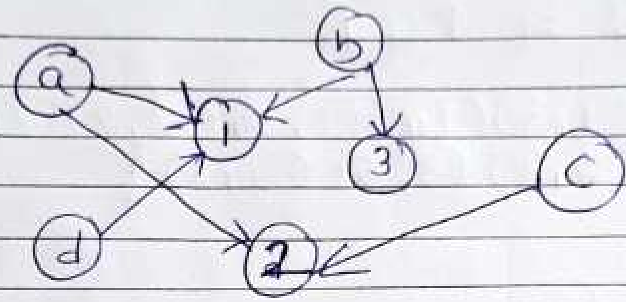
ex. 4 let $A = \{a, b, c, d\}$ $B = \{1, 2, 3\}$
$R = \{(a,1)(a,2)(b,1)(b,3)(c,2)(d,1)\}$

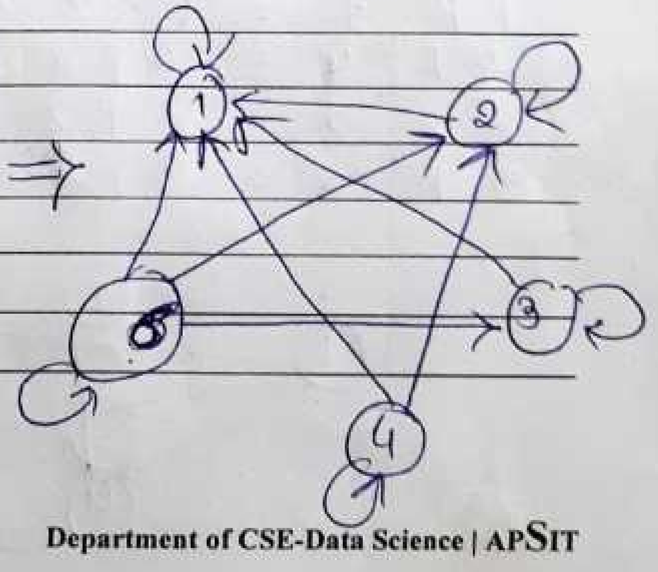


ex. 5 let $A = \{1, 2, 3, 4, 6\}$,
$B = \{1, 2, 3, 4, 6\}$
aRb if and only if a is multiple of b.

$\Rightarrow$

$R = \{(1,1)(2,1)(2,2)(3,1)(3,3)(4,1)$
$(4,2)(4,4)(6,1)(6,2)(6,3)(6,6)\}$

$$M_R = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 6 \\ \hline 1 & 1 & 0 & 0 & 0 & 0 \\ 2 & 1 & 1 & 0 & 0 & 0 \\ 3 & 1 & 0 & 1 & 0 & 0 \\ 4 & 1 & 1 & 0 & 1 & 0 \\ 6 & 1 & 1 & 1 & 0 & 1 \end{array} \Rightarrow$$



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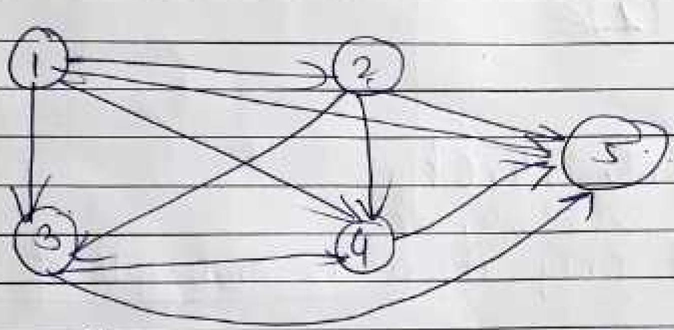
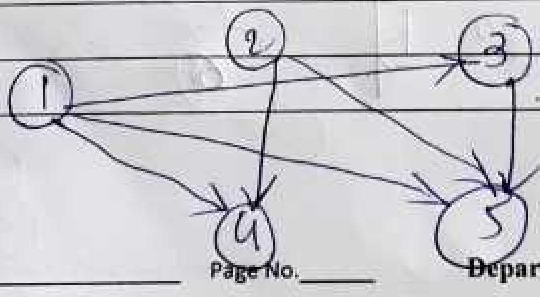
ex Let $A = \{1, 2, 3, 4, 5\}$ and R be the relation defined by a R b if and only if a < b compute R, R^2
Draw digraph of R, R^2.

$\Rightarrow$ $R = \{(1,2)(1,3)(1,4)(1,5)(2,3)(2,4)$
$(2,5)(3,4)(3,5)(4,5)\}$



$R^2 = 1 R^2 3 = 1 R 2 \; \& \; 2 R 3 = 1 R 3$
$1 R^2 4 = 1 R 2 \; \& \; 2 R 4 = 1 R 4$
$1 R^2 5 = 1 R 2 \; \& \; 2 R 5 = 1 R 5$
$2 R^2 4 = 2 R 2 \; \& \; 2 R 4 = 2 R 4$
$2 R^2 5 = 2 R 2 \; \& \; 2 R 5 = 2 R 5$
$3 R^2 5 = 3 R 2 \; \& \; 2 R 5 = 3 R 5$



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* Examples on types of relations -

(1) write relations R on $A = \{1, 2, 3\}$ having the stated property.
i> R is reflexive
ii> R is transitive but not symmetric.
iii> R is symmetric but not transitive.
iv> R is both symmetric & anti symmetric
v> R is neither symmetric nor anti-symmetric.

$\Rightarrow$ $A = \{1, 2, 3\}$

i> $R = \{(1,1)(2,2)(3,3)\}$

ii> $R = \{(1,2)(2,3)(1,3)\}$

iii> $R = \{(1,2)(2,1)\}$

iv> $R = \{(1,1)(2,2)\}$ antisymmetric
$(a,b)(b,a) \in R, a = b$

v> $R = \{(1,2)(2,3)(3,2)\}$

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ex (2) Define a relation on the set $A = \{a, b, c, d\}$ that is
i) reflexive, transitive and symmetric
ii) symmetric & transitive

⇒ i) reflexive, transitive & symmetric

$A = \{a, b, c, d\}$

$R = \{(a,a)(b,b)(c,c)(d,d)(a,b)(b,a)(a,c)(c,a)(a,d)(d,a)(b,c)(c,b)(b,d)(d,b)(c,d)(d,c)\}$

ii) symmetric and transitive.

$A = \{a, b, c, d\}$

$R = \{(a,b)(b,a)(a,a)(c,d)(d,c)(c,c)\}$

~~(3) A = {1, 2, 3, 4} check whether given relⁿ shows as below~~

(3) $A = \{1, 2, 3, 4\}$ check whether given relation is reflexive, symmetric, transitive or anti-symmetric

i) $\{(1,1)(1,2)(2,1)(2,2)(3,3)(4,4)\}$

ii) $\{(1,1)(2,2)(3,3)(4,4)\}$

iii) $\{(1,3)(1,4)(2,3)(2,4)(3,1)(3,4)\}$

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⇒ i) $A = \{1, 2, 3, 4\}$
$R = \{(1,1) (2,2) (3,3) (4,4) (1,2) (2,1)\}$.
it is reflexive as
(1,1) (2,2) (3,3)
$R_1 = (1,1) (2,2) (3,3) (4,4)\}$ it is reflexive.
~~(1,1) (2,2) (3,3) (4,4)~~
R = (1,2) ∈ R and (2,1) ∈ R
it is symmetric
R = (1 R 2) and (2 R 1), it is transitive.
(1,1)

ii) $A = \{1, 2, 3, 4\}$
$R = \{(1,1) (2,2) (3,3) (4,4)\}$ it is reflexive.

iii) $A = \{1, 2, 3, 4\}$
$R = \{(1,3) (1,4) (2,3) (2,4) (3,1) (3,4)\}$
R is not reflexive
$R = \{(1,3) (3,1)$ but $(1 \not R 1)$
$(2,3) (3,1)$ but $(2 \not R 1)$
$(3,1) (1,3)$ but $(3 \not R 3)$
R is not symmetric, not transitive.

Q. Let $A = \{1, 2, 3, 4, 5\}$ Determine whether the relation R whose digraph is given is reflexive irreflexive, symmetric, asymmetric, antisymmetric or transitive.