Semester : III Subject : DSGT Academic Year: 2022 - 2023

Module No :- 5

Algebraic Structures

Algebraic structure -

A non empty set S is called algebraic structure with respect to binary operation * if $(a*b) \in S$ where $\forall (a,b) \in S$.

'*' is closure operation on S.

Let A be a non-empty set. A function $f : A \times A \rightarrow A$ is called a binary operation.

Binary operation is a function, one and only one element of A is assigned to one ordered pair of $A \times A$, we denote binary operations by * or (+) instead of f. Since a binary operation is a function to each $(a, b) \in A \times A$, there exists a unique element $a * b \in A$.

This property is called as A is closed under *.

e.g. $(Z, +)$

set ↑ operation

N - Natural no = $\{1, 2, 3, ...\}$
Z - Integers no = $\{-2, -1, 0, 1, 2, ..\}$
Q - Rational no = $\{\frac{1}{2}, \frac{3}{2}, -\frac{3}{2}\}$
R - Real no

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(1) Let $A = Z$ and $a * b = a + b$

Then $*$ is a binary operation on Z.

if $Z = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, \ldots\}$

$* = +$ binary operation

$a = 3$ & $b = 6$

then $a + b = 3 + 6$

$= 9$

here $9 \in Z$.

hence Z is closed under $*$.

as $(a, b) \in Z$ and $(a+b) \in Z$

(2) $A = Z$, and $a * b$ be $a - b$

Then $*$ is not binary operation on Z

since $a - b$ may not be an element of A for some a, b in A.

$3 * 7 = 3 - 7 = \underline{\underline{-4}}$

$-4 \notin Z$

(3) $A = Z$, $a * b$ be a/b

$*$ is not binary operation on Z

since a/b may not be an element of Z for some a, b in A.

$5 * 0 = 5/0 \notin A$.

if Z is a set of non zero positive integers

then $a * b$, a/b is binary operation.

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(4) if L be a lattice, $a * b$ be $a \wedge b$ (GLB of a, b) meet

Then $a * b$ is a binary operation because for every ordered pair a, b of L, there exists a unique $a \wedge b$.

(5) Let L be a lattice, $a * b$ be $a \vee b$ (LUB of a, b) join

Then $a * b$ is a binary operation because for every ordered pair a, b of L, there exist a unique $a \vee b$.

(*) Identity and Inverse :-

Identity - Given a non-empty set A and a binary operation (*) if there is an element $e \in A$, such that for every $a \in A$,
$a * e = e * a = a$

here e is called as identity element for the operation *.

- e.g. in a set of real numbers
$R = \{$ set of real numbers $\}$
0 is identity element for the operation addition

because $a+0 = 0+a = a$ for every $a \in R$.

unity or 1 is identity element for multiplication because
$a \times 1 = 1 \times a = a$ for every $a \in R$.

* <u>Inverse</u> -

Given a non-empty set A and a binary operation * if A has an identity element e and if for any two element $a, b \in S$
$a * b = b * a = e$
then b is called the inverse of a &
is denoted by a^{-1}.

e.g.