Rational Choices in Game Theory – Summary

Basics of Games

- **Definition**: A game consists of a set of players, a set of strategies available to each player, and a payoff function that assigns a payoff to each player for every combination of strategies.
- **Types of Games**:
    - **Static vs. Dynamic**: Static games are played once with simultaneous moves. Dynamic games are played over several periods with possible sequential moves.
    - **Complete vs. Incomplete Information**: In complete information games, all players know the payoff functions and strategies available. In incomplete information games, some information is private.
    - **Zero-Sum vs. Non-Zero-Sum**: In zero-sum games, one player's gain is another's loss. In non-zero-sum games, all players can gain or lose.

Strategy

- **Pure Strategy**: A specific predetermined action a player will take in every possible situation.
- **Mixed Strategy**: A probability distribution over possible actions, allowing for randomization.
- **Dominant Strategy**: A strategy that is always the best regardless of what the opponents do.
- **Nash Equilibrium**: A set of strategies where no player can benefit by unilaterally changing their strategy.

Preferences and Payoffs

- **Utility Function**: A representation of a player's preferences over outcomes, often represented numerically.
- **Payoff Matrix**: A table that shows the payoffs for each player for every possible combination of strategies.

Mathematical Basics

- **Probability Theory**: Essential for understanding mixed strategies and expected payoffs.
- **Linear Algebra**: Used in solving game matrices and finding equilibria.
- **Optimization**: Techniques like linear programming are used to find optimal strategies

Game Theory and Rational Choice

- **Rational Choice**: Assumes players are rational and will strive to maximize their utility.
- **Solution Concepts**:
    - **Nash Equilibrium**: No player can improve their payoff by changing their strategy unilaterally.

    - **Subgame Perfect Equilibrium**: Used in dynamic games, requiring strategies to be optimal in every subgame.
    - **Bayesian Equilibrium**: In games with incomplete information, players maximize expected utility based on beliefs about other players' types.

Non-Cooperative vs. Cooperative Games

- **Non-Cooperative Games**: Players make decisions independently, without collaboration.
- **Cooperative Games**: Players can form coalitions and make binding agreements to improve their payoffs.

Computational Issues

- **Finding Equilibria**:
    - **Algorithmic Approaches**: Includes the Lemke-Howson algorithm for Nash equilibria and the simplex method for linear programming problems.
    - **Complexity**: Computing Nash equilibria can be computationally intensive (PPAD-complete).
- **Learning in Games**:
    - **Repeated Games**: Players adjust strategies based on past outcomes.
    - **Reinforcement Learning**: Players learn optimal strategies through trial and error.

Application Areas for Game Theory

- **Google's Sponsored Search**: Auction mechanism where advertisers bid for ad placement, modeled using auction theory and Nash equilibria.
- **eBay Auctions**: Online auctions modeled using dynamic games and Bayesian equilibria.
- **Electricity Trading Markets**: Participants bid for electricity supply, modeled using game theory to optimize bidding strategies and ensure market efficiency.