Semester : V   Subject : Statistics for AI &DS   Academic Year: 2023- 2024.

THE PROBLEM OF MULTIPLE COMPARISON:

BONFERRONI CORRECTION:-

Bonferroni correction is a method used to control the family- wise error rate.

What is family-wise error rate?

The family-wise error rate is the probability of incorrectly rejecting the true null hypothesis. In other words, it is about finding false positive result (Type 1 error).

The researchers and scientist start an experiment by stating α level or significance level. Most of the time the $\alpha = 0.05$. Based on this we calculate the Family Wise Error Rate (FWER).

$$FWER = 1-(1-\alpha)^m$$

where α : level of significance.
m = no. of tests (or) hypothesis being performed for a single experiment.

Usually $\alpha = 0.05$ and m in this case is 1.

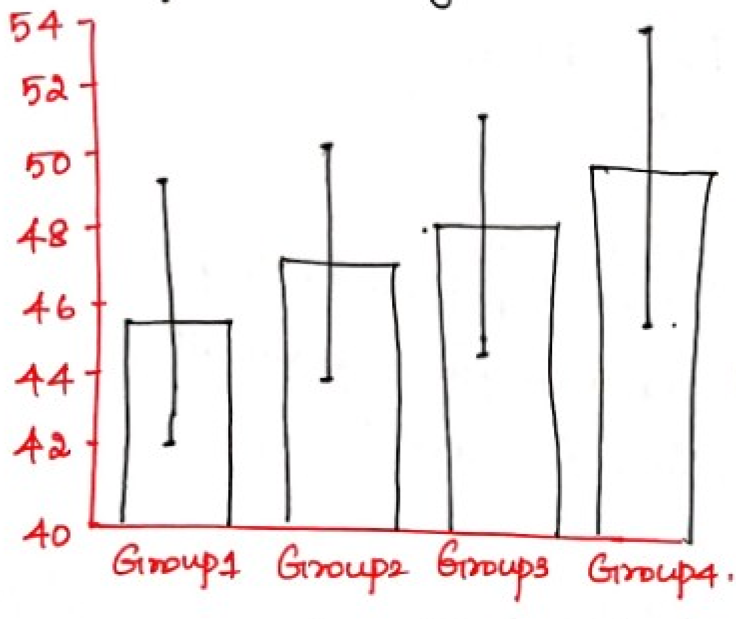
$$\boxed{\begin{aligned} FWER &= 1-(1-0.05)^1 \\ &= 0.05 = 5\% \end{aligned}}$$

We are accepting that there is 5% chance of obtaining a false- positive result.

Semester : IV   Subject : Statistics for AIDS   Academic Year: 2023-2024.

Consider the below example, say we have an experiment where we compared the means of 4 groups using a one-way Anova Test using an alpha level of 0.05. The p value for this test was 0.01, so ther is a statistically difference between the means of 4 groups.

Next step is to perform post hoc tests to see which group is different from the rest and so we usually perform a family of test comparing. each possible comparison. We compare the mean of various groups.



Comparison
G1 Vs G2
G1 Vs G3
G1 Vs G4
G2 Vs G3
G2 Vs G4
G3 Vs G4

In total we have to go 6 test. So let's calculate the family-wise Test.

Semester: IV Subject: Statistics for AI&DS Academic Year: 2023 2024.

$$FWER = 1-(1-\alpha)^m$$
$$= 1-(1-0.05)^6$$
$$= 1-0.95^6$$
$$= 0.265 = 26.5\%$$

Just by doing 6 test, there is a 26.5% chance of discovering one or more false-positive results. The more tests that are performed, the larges the family-wise error rate.

No. of Tests	FWER
10	40%
30	79%
60	95%

This is the issue of multiple comparisions. To account for this error, we can use a multiple comparisons correction method (eg. Bonferroni)

$$\text{Bonferroni-corrected } \alpha = \frac{\alpha \rightarrow \text{original alpha}}{k \rightarrow \text{number of tests}}$$

Consider the previous example $\alpha = 0.05$ and $k = 6$.

$$\text{Bonferroni-corrected } \alpha = \frac{0.05}{6} = 0.008$$

$$\boxed{\alpha = 0.008}$$

Now calculate the FWER:

$$FWER = 1 - (1 - 0.008)^6$$
$$= 1 - 0.992^6$$
$$= 0.047$$
$$\boxed{FWER = 4.7\%}$$

By correcting the alpha through the Bonferroni method we have reduced FWER back down to around 5%.

Regardless of no. of simultaneous tests performed the Bonferroni correction, it maintains FWER to 5%

TURKEY METHOD FOR ONE-FACTOR ANOVA

It is used to find the significant difference between the means.

This test is done after ANOVA. It is a post ANOVA Analysis.

Ho is not rejected (ie) we accept Ho $\longrightarrow$ No further analysis required.

Ho is rejected $\longrightarrow$ Further analysis required. (Turkey Test).

$H_0 : \mu_1 = \mu_2 = \mu_3$.

Semester : IV Subject : Statistics for AI&DS Academic Year: 2023-2024.

Example :-

Treatment			
	Type A	Type B	Type C
	77	83	80
	79	91	82
	87	94	86
	85	88	85
	78	85	80
Sample mean :	81.2	88.5	82.6

$H_0 : \mu_1 = \mu_2 = \mu_3$.

ANOVA :

$\alpha = 0.05$

Source of Variation	SS	df	MS	F	Pvalue	FCrit.
Between Groups. (Treatment)	137.20	2	68.60	4.3145	0.0387	3.8853
Within Groups. (Errors).	190.80	12	15.90			
Total	3.28	14			Reject H_0. (P-value $< \alpha$).	

As the result is reject H_0, we will do further analysis using Turkey method for pairwise comparison.

* Turkey method is also called the honestly significant difference (HSD) Test.

* Compare pairs of sample means, using their absolute differences:

Semester : 4 Subject : Statistics for AI&DS Academic Year: 2023-2024.

$|\bar{X}_1 - \bar{X}_2|$, $|\bar{X}_1 - \bar{X}_3|$ $|\bar{X}_2 - \bar{X}_3|$.

The Turkey criterion (T) is defined as :

$$\boxed{T = q_{\alpha(c,n-c)} \sqrt{\frac{MSE}{n_i}}} \quad \text{—— (1)}$$

$q_{\alpha}(c,n-c)$ = Studentized range distribution, based on c and n-c df.

c = No. of Treatments (i.e. number of coloumns)

n = Total sample size.

MSE = Mean square Error (from ANOVA Table)

n_i = sample size of the treatment group with the smallest no. of observations.

$\alpha = 0.05$

From the Studentized Range Distribution Table;

$$q_{\alpha}(c,n-c) = q_{0.05}(3,15-3) = q_{0.05}(3,12) \boxed{= 3.773}$$

Substitution in equation (1).

$$T = 3.773 \sqrt{\frac{15.9}{5}} = 6.73.$$

$$\boxed{T = 6.73}$$

Semester : 4 Subject : Statistics for AI&DS Academic Year: 2023- 2024.

Absolute values of the paired means are as follows:

$|\overline{x_1} - \overline{x_2}| = |81.2 - 88.2| = 7.0 > 6.73$

{difference between 1 and 2 is significant at $\alpha = 0.05$}

$|\overline{x_1} - \overline{x_3}| = |81.2 - 82.6| = 1.4 < 6.73$.

{difference between 1 and 3 is not significant}.

$|\overline{x_2} - \overline{x_3}| = |88.2 - 82.6| = 5.6 < 6.73$.

{difference between 2 and 3 is not significant}.