Types of Activation function

① Threshold function (binary or step function)

True or false

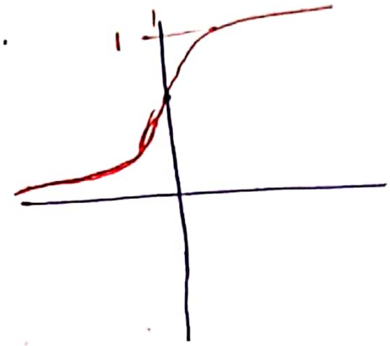
$$y = \begin{cases} 1, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

② Sigmoid function (Binary sigmoid) function

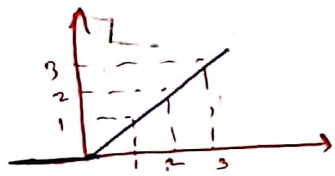
$$\sigma(x) = \frac{1}{1+e^{-x}}$$

$0 < 0.5 < 1$

Range $(0,1)$



③ Ramp function



~~$y = \max(0, x)$~~

$$y = \begin{cases} x, & x \geq 0 \\ 0, & x \leq 0 \end{cases}$$

→ o/p = i/p for +ve values

→ 0 for -ve i/p

④ Bipolar Sigmoid function

$$y = f(x) = \frac{1-e^{-x}}{1+e^{-x}}$$

Range $= (-1, +1)$

⑤ Continuous function

$f(x)$ is continuous at a point $x = a$ if

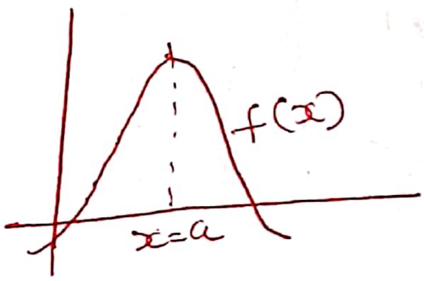
$f(a)$ exists;

$$\lim_{x \to a} f(x) = f(a)$$

eg. $f(x) = x^2$

$f(x) = x^3$

$f(x) = \sqrt{x}$



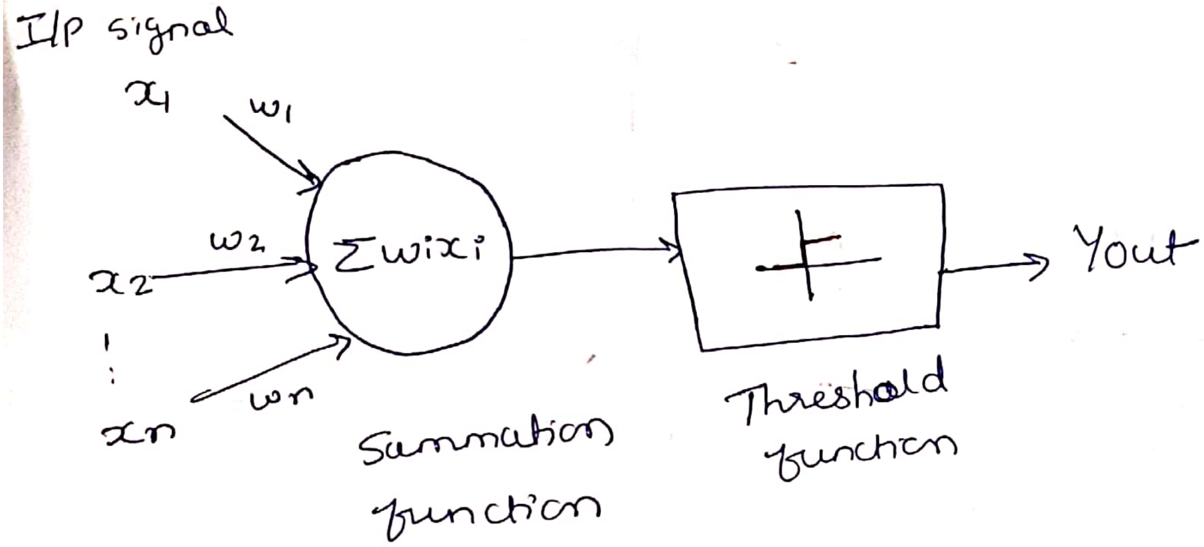
Neural Network ~~Architecture~~

Mc-Culloch pitts Model

Earliest neural network model. The input of this model could be either 0 or 1.

It has a threshold function as an activation fun.

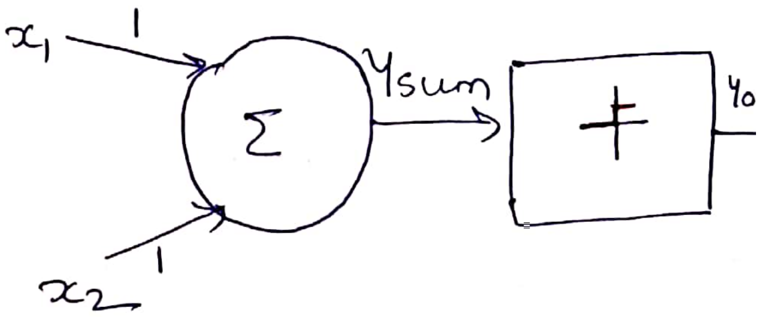
The o/p of the model is 1 if the input to the threshold function is greater than or equal to a given threshold value, else 0.



Simple Mc-culloch-pitts model can be used to design logical operation.

For OR gate

$w_1 = w_2 = 1$



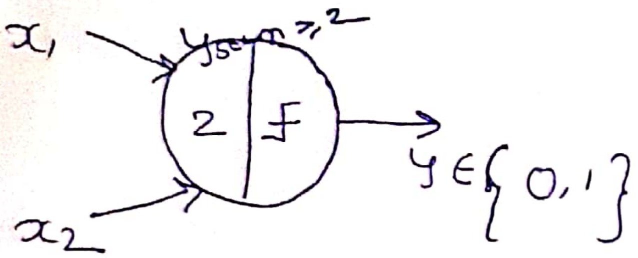
Truth Table

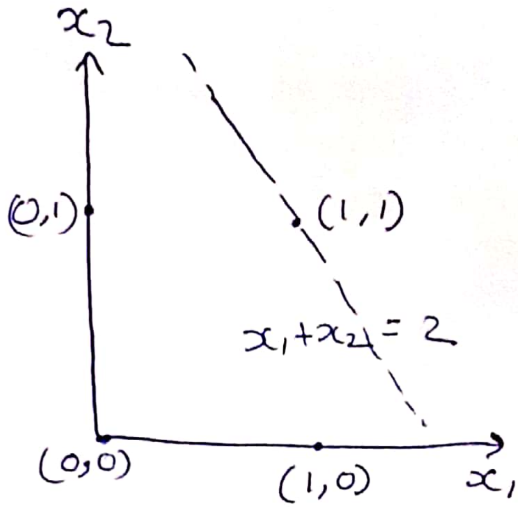
input			
x_1	x_2	Ysum	Yout
0	0	0	0
0	1	1	1
1	0	1	1
1	1	2	1

AND fu

x_1

AND function



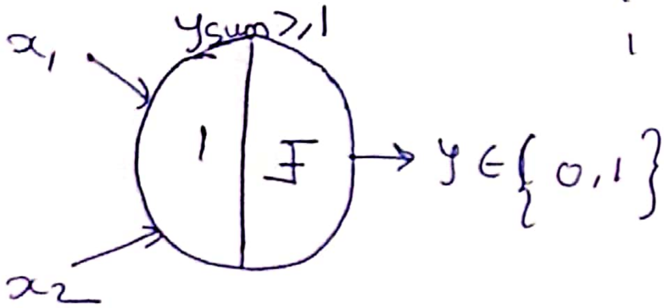


$$x_1 + x_2 = \sum_{i=1}^{2} w_i x_i \geq 2$$

$w_1 = w_2 = 1$

x_1	x_2	y_{sum}	y_{out}
0	0	0	0
0	1	1	0
1	0	1	0
1	1	2	1

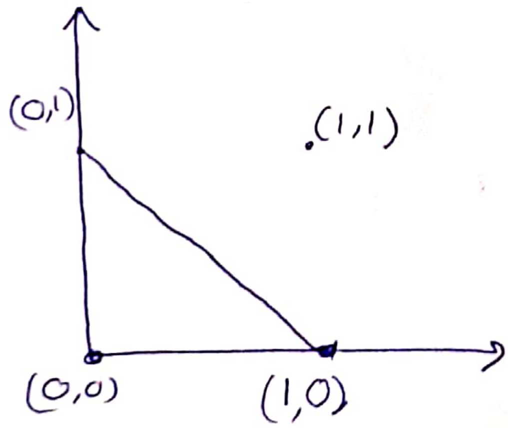
OR function



$w_1 = w_2 = 1$

$x_1 + x_2 = 1$

$$= w_1 x_1 + w_2 x_2 = \sum_{i=1}^{2} w_i x_i \geq 1$$



Truth Table

x_1	x_2	y_{sum}	y_{out}
0	0	0	0
0	1	1	1
1	0	1	1
1	1	2	1

XOR ~~gate~~ function with 3 inputs

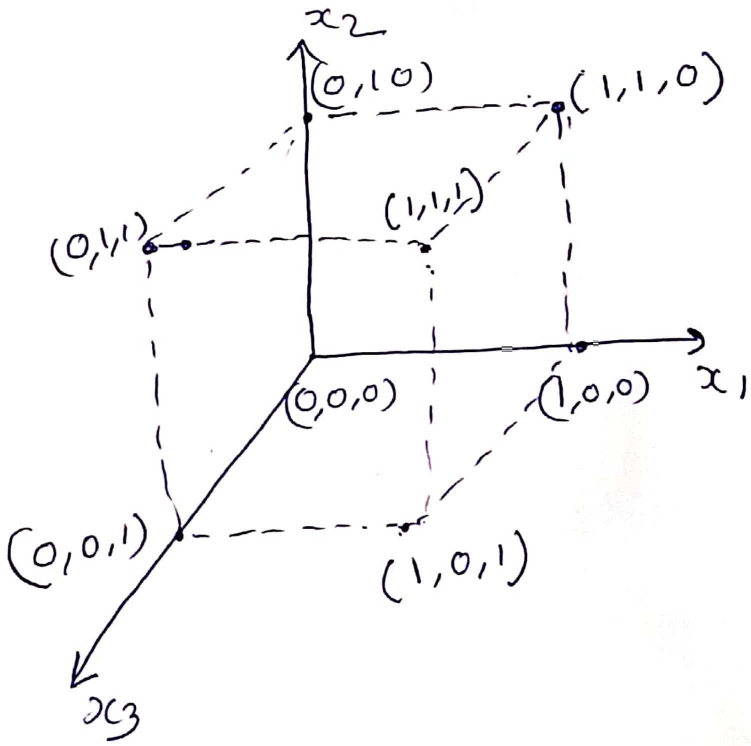
x_1	x_2	x_2	Yout	Actual O/P Ysum	Predicted O/P
0	0	0	0	0	0
0	0	1	1	1	1
0	1	0	1	2	1
0	1	1	1	3	1
1	0	0	1	2	1
1	0	1	1	3	1
1	1	0	1	3	1
1	1	1	1	4	1

$$\sum_{i=1}^{3} w_i x_i \geq 1$$

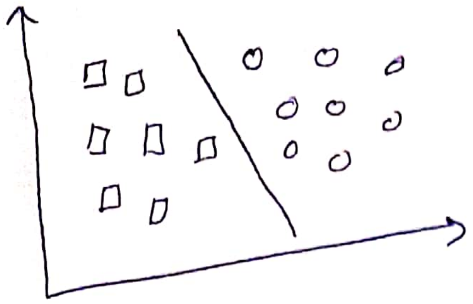
eqn. $x_1 + x_2 + x_3 = 1$

$$w_1 x_1 + w_2 x_2 + w_3 x_3 \geq 1$$

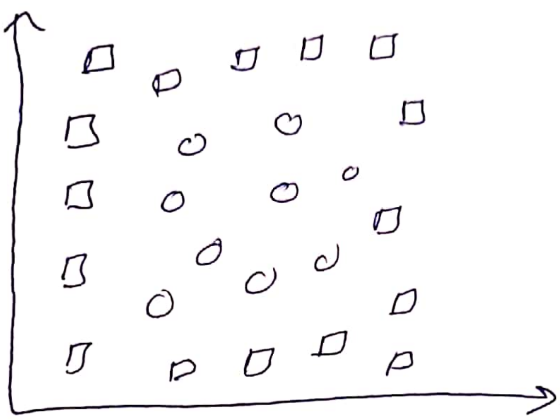
$$w_1 = w_2 = w_3 = 1$$



Linearly separable data



Linearly separable



x_1	x_2	~~x_1 OR~~ x_1 XOR x_2 $x_1 \oplus x_2$	class
0	0	0	C_2
0	1	1	C_1
1	0	1	C_1
1	1	0	C_2

Limitations of Mcculloch - pitts model

1) only binary inputs and outputs are allowed

2) No learning is possible

3) Manual adjustment of weights & threshold.

Neural NAND gate

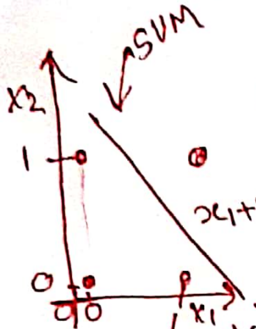
x_1	x_2	y

Feature vector

$$X' = \begin{bmatrix} 1 \\ x_1 \\ x_2 \end{bmatrix}$$

Feature vector

$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$



weight vector

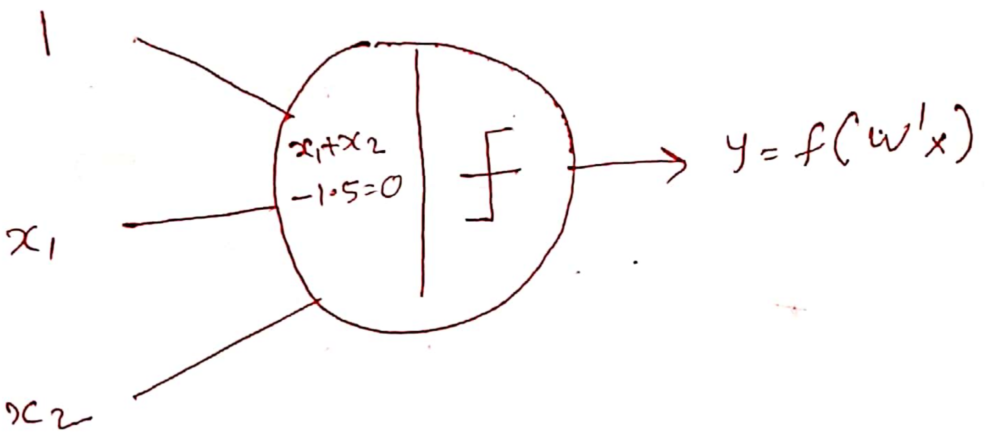
$x_1+x_2-1.5=0$

best line fit

$$W = \begin{bmatrix} -1.5 \\ 1 \\ 1 \end{bmatrix}$$

$$X^T = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}$$

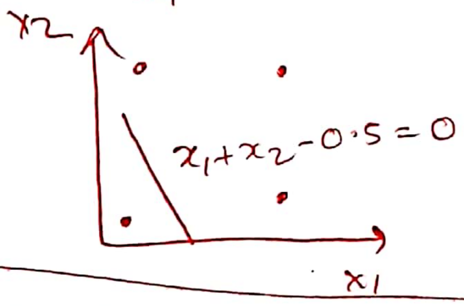
$$X^T W = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -1.5 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1.5 \\ -0.5 \\ -0.5 \\ 0.5 \end{bmatrix} \Rightarrow f \Rightarrow \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$



OR gate

x_1	x_2	y

$x_1+x_2-0.5=0$

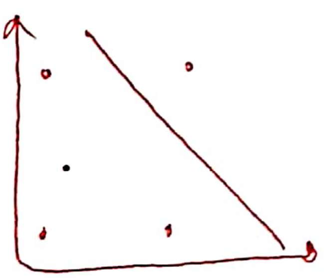


$$X^T W \quad \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -0.5 \\ 1 \\ 1 \end{bmatrix}$$

NAND

x_1	x_2	y
0	0	1
0	1	1
1	0	1
1	1	0

$-x_1-x_2+1.5=0$



$w_1 = 1.2$ $w_2 = 0.6$ Threshold $= 1$

$A = 0$, $B = 0$ & Target $= 0$

$w_i x_i = 0 \times 1.2 + 0 \times 0.6 = 0$

This is not greater than the threshold ≈ 1

So the o/p is $= 0$

2) $A = 0$, $B = 1$ Target 0

$w_i x_i = 0 \times 1.2 + 1 \times 0.6 = 0.6$

not greater than threshold . so o/p $= 0$

3) $A = 1$; $B = 0$ Target $= 0$

$w_i x_i = 1 \times 1.2 + 0.0.6 = 1.2$

1.2 is greater than the threshold ≈ 1

so o/p is $= 1$ but target is 0

target $\neq$ o/p $\therefore$ weights are modified

$w_i = w_i + n(\overset{target}{t} - \underset{o/p}{o})\, x_i$

$w_1 = 1.2 + 0.5(0 - 1)\, 1 = 0.7$

$w_2 = 0.6 + 0.5(0 - 1)\, 0 = 0.6$

new weights $w_1 = 0.7$

$w_2 = 0.6$

rain
Group of student
Give individual responsibility   NO   may   definitely yes
                                        $0 \to 0.5 \to 1$
Train student to a specific part of body

A → eyes   Result goalo   S
B → nose → use formula as
C → ear → face = eye × 0.2 + nose × 0.5 + ears × 0.3
                                   + more weights
D → leg → T → O
E → tail
F →

Cata = face × 0.6 + body × 0.4

eye   nose
line → 0.03   0.1

Single neural network

Given an age of person and predict if person will buy insurance or not

binary classification problem

0 → will not buy insurance
1 → buy insurance

$$\text{Sigmoid}(z) = \frac{1}{1+e^{-z}}$$

$e = \sim 2.71828$

Step I

$y = mx + ~~\$~~ c$

↓ age

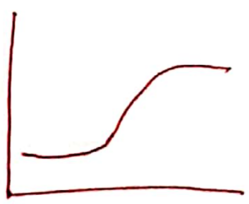
$$z = \frac{1}{1+e^{-y}}$$

↓ if person will buy insurance

by linear mode

m = coeff

c = intercept



Age = 35 → 0.48 will not by insurance

age = 45 → 0.57 buy insurance

$y = m_1 x_1 + m_2 x_2 + m_3 x_3 + c$

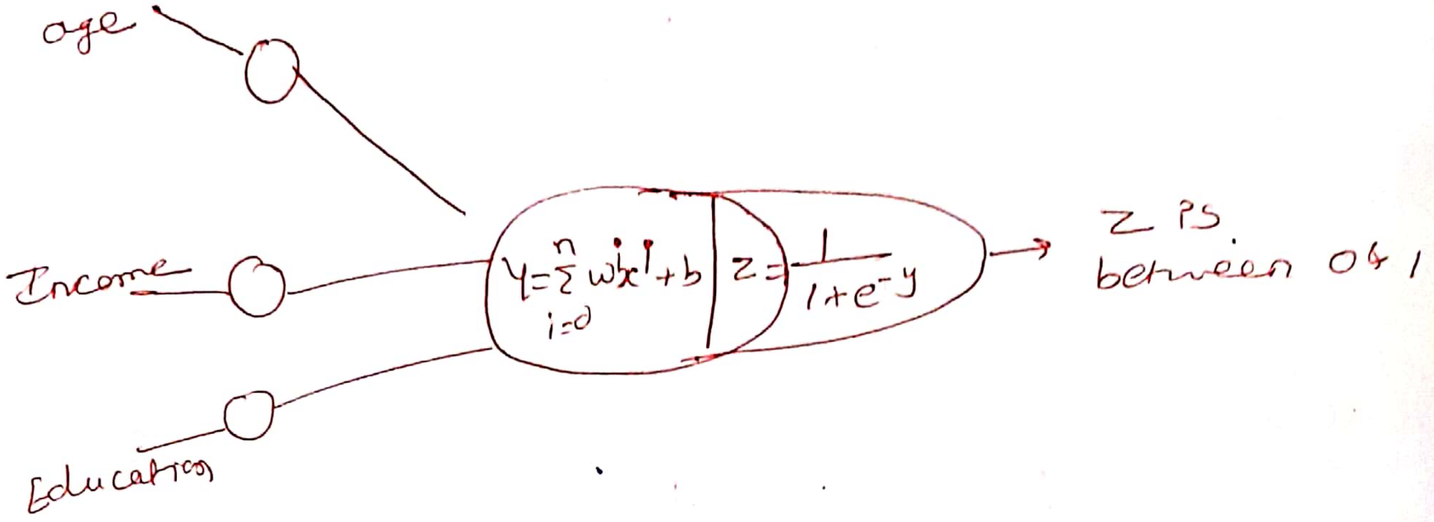
x_1 → age, x_2 → Income, x_3 → education

Scanned with CamScanner

$$y = \sum_{i=0}^{n} \cancel{\omega x}$$

$$y = w_1x_1 + w_2x_2 + w_3x_3 + \cancel{b}\, c \ldots$$

$$y = \sum_{i=0}^{n} w^i x^i + c$$



Neural Network XOR

Perceptrons → perceptrons are linear classifiers

A very simple neural unit

- Binary output (0 or 1)
- Non linear activation function

$$y = \begin{cases} 0, & \text{if } w\cdot x + b \le 0 \\ 1, & \text{if } w\cdot x + b >= 0 \end{cases}$$

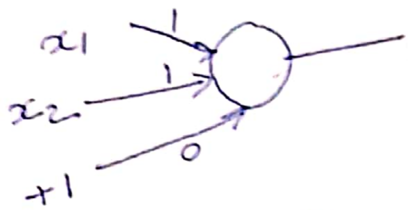
Build AND or OR with perceptrons

$$y = \begin{cases} 0, & \text{if } w\cdot x + b \le 0 \\ 1, & \text{if } w\cdot x + b > 0 \end{cases}$$

AND

x_1	x_2	y
0	0	0
0	1	0
1	0	0
1	1	1

OR



OR

x_1	x_2	y
0	0	0
0	1	1
1	0	1
1	1	1

XOR with perceptrons

Not possible to comput XOR with perceptrons is the equation given x_1 & x_2 perceptron equation of line

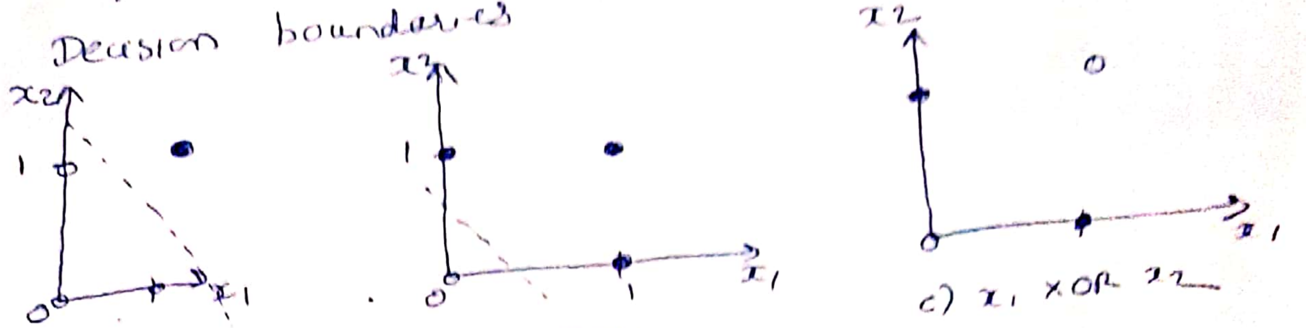
$$w_1x_1 + w_2x_2 + b = 0$$

in standard linear format

$$x_2 = \underbrace{(-w_1/w_2)}_{\text{Slope}} x_1 + (-b/w_2)$$

This line acts a a decision boundary
- 0 if i/p is on one side of the line
- 1 if on the other side of the line

Decision boundaries



a) x_1 and x_2

b) x_1 OR x_2

c) x_1 XOR x_2

XOR is not a linearly separable function

XOR can't be calculated by a single perceptron

XOR can be calculated by a layered n/w of units

XOR

x_1	x_2	y
0	0	0
0	1	1
1	0	1
1	1	0

both the inputs are same then o/p will be zero

$y = x_1\bar{x}_2 + \bar{x}_1 x_2$ $z_1 = x_1\bar{x}_2$

$y = z_1 + z_2$ $z_2 = \bar{x}_1 x_2$

$y = z_1 \text{ (OR) } z_2$

4 (-1.5)

calculate $z_1 = x_1\bar{x}_2$ & $z_2 = \bar{x}_1 x_2$

x_1	x_2	$z_1 = x_1\bar{x}_2$	$z_2 = \bar{x}_1 x_2$	$y = z_1 + z_2$
0	0	0	0	0
0	1	0	1	1
1	0	1	0	1
1	1	0	0	0