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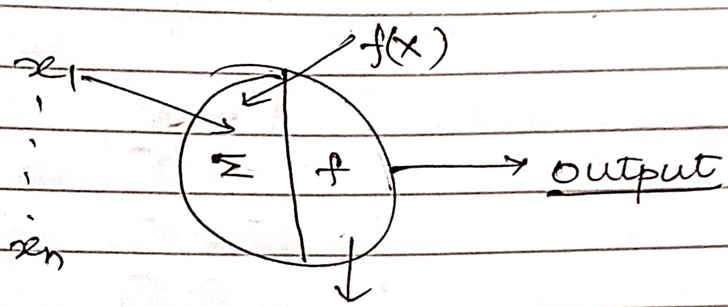
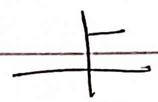
ML

Neural Network

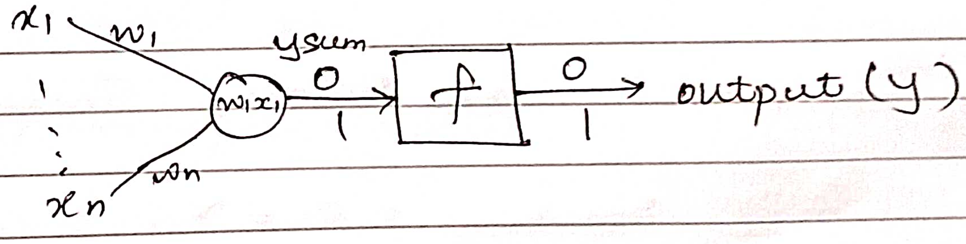
MC - culloch pits model





Step fun

sigma fun



AND gate

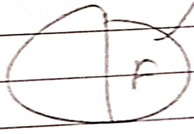
x_1	x_2	y
0	0	0
0	1	0
1	0	0
1	1	1

$\Sigma w_1 x_1 = w_1 x_1 + w_2 x_2 = 1 \times 0 + 1 \times 0 = \underline{\underline{0}}$

input vector $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}$

$$\begin{array}{ccc} \text{Bias} & x_1 & x_2 \end{array}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -1.5 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1.5 \\ -0.5 \\ -0.5 \\ 0.5 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$



$w_1 = 1.2$ $\quad w_2 = 0.6$ $\quad$ Threshold $= 1$

$\Sigma w_i x_i = 1.2 \times 0 + 0.6 \times 0 = 0$ ✓ $\Rightarrow 0$

$\quad = 1.2 \times 0 + 0.6 \times 1 = 0.6 \Rightarrow 0$

$\Sigma w_i x_i = 1.2 \times 0 + 0.6 \times 0 = 1.2 > 1 \Rightarrow 0$

$\quad = 1.2 \times 1 + 0.6 \times 0$

Change weights $n = 0.5$

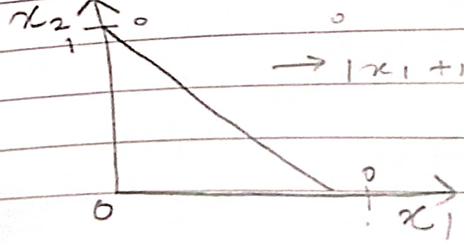
$w_i = w_i + n(t-o)x_i$

w_1 new $= 1.2 + 0.5(0-1) = 1.2 - 0.5 = 0.7$

w_2 new $= 0.6 + 0.5(0-1)\,0 = 0.6$

~~w_3 new =~~

~~w_4 new =~~



$\rightarrow 1x_1 + 1x_2 = -0.5$ (1)

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -0.5 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -0.5 \\ -0.5+1 \\ -0.5+1 \\ -0.5+1+1 \end{bmatrix} = \begin{bmatrix} -0.5 \\ 0.5 \\ 0.5 \\ 1.5 \end{bmatrix} \Rightarrow f$$

$$\begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

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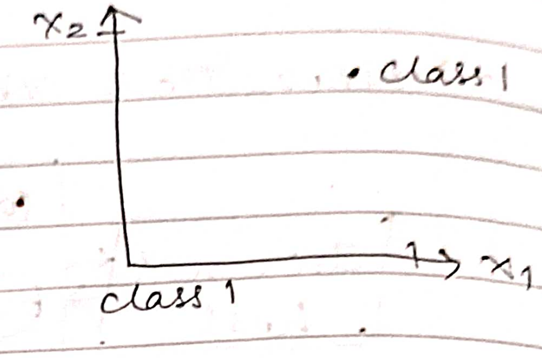
N/W for XOR

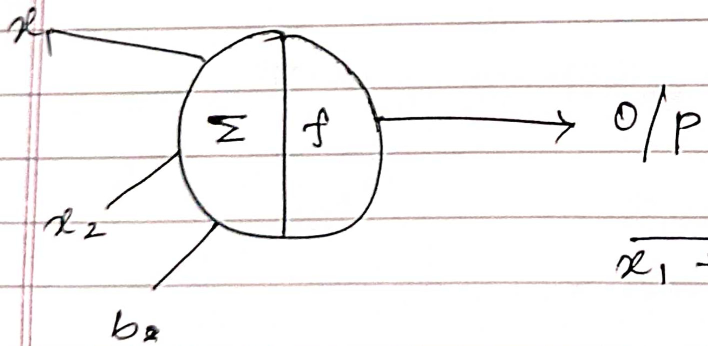
x_1	x_2	y	
0	0	0	class 1
0	1	1	class 2
1	0	1	
1	0	0	



$$x_1 \oplus x_2 = (x_1 + x_2) + (\bar{x}_1 + \bar{x}_2)$$

$$h_1 = x_1 + x_2 \qquad h_2 = \bar{x}_1 + \bar{x}_2$$

x_1	x_2	$h_1 = x_1 + x_2$	$h_2 = \bar{x}_1 + \bar{x}_2$	$h_1 . h_2$
0	0	0	1	0
0	1	1	1	1
1	0	1	1	1
1	1	1	0	0



$$\overline{x_1 + x_2} = \bar{x}_1 . \bar{x}_2$$

I/p vector NAND $= \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}$

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$$bias = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$$x^T W = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -0.5 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -0.5 \\ 0.5 \\ 0.5 \\ 1.5 \end{bmatrix} \Rightarrow f = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

AND gate

$$x^T W = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -1.5 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1.5 \\ -0.5 \\ -0.5 \\ +0.5 \end{bmatrix} \Rightarrow f \begin{bmatrix} \\ \end{bmatrix}$$

NAND

$$x^T W = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1.5 \\ -1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1.5 \\ 0.5 \\ 0.5 \\ -1.5 \end{bmatrix} \Rightarrow f = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

$$x^T W = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -0.5 & 1.5 \\ 1 & -1 \\ 1 & -1 \end{bmatrix}_{2\times 2}$$

$$= \begin{bmatrix} -0.5 & 1.5 \\ 0.5 & 0.5 \\ 0.5 & 0.5 \\ 0.5 & -1.5 \end{bmatrix}_{4\times 2} \Rightarrow f = \begin{array}{cc} h_1 & h_2 \end{array} \begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 1 & 1 \\ 1 & 0 \end{bmatrix} \Rightarrow h$$

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$$h^T W = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} -1.5 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -0.5 \\ 0.5 \\ 0.5 \\ -0.5 \end{bmatrix} \Rightarrow f = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

o/p of XOR

	Mc-culloch pits Algo	Single layer perceptron with bias
AND	$w_1 = w_2 = 1$ $T = 2$	$w_1 = w_2 = 1$ bias = 1 weight (bias) = −1.5
OR	$w_1 = w_2 = 1$ $T = 1$	$w_1 = w_2 = 1$ bias = 1 weight (bias) = 0.5
NAND	$w_1 = w_2 = -1$	$w_1 = w_2 = -1$ bias = 1 weight (bias) = 1.5
NOR	$w_1 = w_2 = -1$ $T = 0$	$w_1 = w_2 = -1$ bias = 1 weight (bias = 0.5)
XOR		Calculate h_1 & h_2 $w_1 = w_2 = 1$ bias = 1 weight = −1.5

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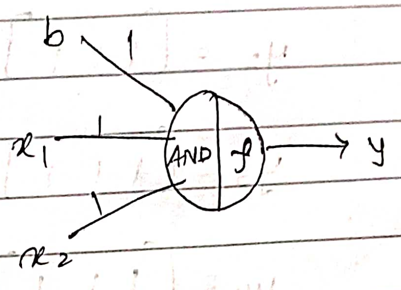
ML

→ Weight updation rule

AND function

Input		Output (y)
x_1	x_2	y
0	0	0
0	1	0
1	0	0
1	1	1



$$W_{new} = w - n \sum_{i=1}^{n} (\hat{y}_i - y_i)\, x_i$$ weight change formula

Input vector with bias

$$x = \begin{bmatrix} 1 \\ x_1 \\ x_2 \end{bmatrix}, \quad x_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad x_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \quad x_3 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$x_4 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$y = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}$$

$$W = \begin{bmatrix} w_0 & w_1 & w_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$$

$$W = \begin{bmatrix} w_0 \\ w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$\hat{y}$ = y predicted
y_1 = Actual y

$\hat{y_1} = w^T . x = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = 1 \xrightarrow{\text{filter}} 1$

$y_2 = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \Rightarrow 2 = 1$

$y_3 = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \Rightarrow 2 = 1$

$y_4 = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 3 \Rightarrow 1$

$(\hat{y_i} - y) x_i$

$(\hat{y}_1 - y_1) x_1 = (1 - 0) \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$(\hat{y}_2 - y_2) x_2 = (1 - 0) \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

$(\hat{y}_3 - y_3) x_3 = (1 - 0) \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

$(\hat{y}_4 - y_4) x_4 = (1 - 0) \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

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$$\begin{bmatrix}1\\0\\0\end{bmatrix} + \begin{bmatrix}1\\0\\1\end{bmatrix} + \begin{bmatrix}1\\1\\0\end{bmatrix} + \begin{bmatrix}0\\0\\0\end{bmatrix} = \begin{bmatrix}3\\1\\1\end{bmatrix}$$

$$n = 0.3$$

$$w_{new} \begin{bmatrix}1\\1\\1\end{bmatrix} - 0.3 \times \begin{bmatrix}3\\1\\1\end{bmatrix} = \begin{bmatrix}1\\1\\1\end{bmatrix} - \begin{bmatrix}0.9\\0.3\\0.3\end{bmatrix}$$

$$w_{new} = \begin{bmatrix}0.1\\0.7\\0.7\end{bmatrix}$$

$$\Rightarrow \hat{y} = w^T x = \begin{bmatrix}0.1 & 0.7 & 0.7\end{bmatrix} \begin{array}{c} \begin{bmatrix}1 & 1 & 1 & 1\\0 & 0 & 1 & 1\\0 & 1 & 0 & 1\end{bmatrix} \end{array} \begin{array}{l} \rightarrow \text{bias} \\ \rightarrow x_1 \text{ AND} \\ \rightarrow x_2 \text{ i/p} \end{array}$$

$$= \begin{bmatrix}0.1 & 0.8 & 0.8 & \overset{1.5}{\cancel{0.8}}\end{bmatrix} \rightarrow \text{filter} \rightarrow \begin{bmatrix}1 & 1 & 1 & 1\end{bmatrix}$$

$$\Rightarrow \Sigma(\hat{y}_i - y_i)x_i = \begin{bmatrix}3\\1\\1\end{bmatrix}$$

$$\Rightarrow w_{new} = w_{old} - n\,\Sigma(\hat{y}_i - y_i)x_i$$

$$= \begin{bmatrix}0.1\\0.7\\0.7\end{bmatrix} - 0.3 \times \begin{bmatrix}3\\1\\1\end{bmatrix} = \begin{bmatrix}-0.8\\0.4\\0.4\end{bmatrix}$$

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$$\hat{y} = w^T x = \begin{bmatrix} -0.8 & 0.4 & 0.4 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -0.8 & -0.4 & -0.4 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}$$

= Stop iteration {does not match last one]