

(Approved by AICTE New Delhi & Cove, of Maharashara, Affiliated to University of Mambal) (Religious Jain Minerity)

	Date Page
	Complex Variables.
X	Complex function:
	If z=x+iy be a complex quantity—then co=f(z) which is again a complex quantity is called as
	complex function. (z sow both are complex.)
	As x=xtiy, one can write win the following form w= f(z) = f(x+iy) = u(x,y)+iv(x,y)
	$\omega = f(z) = f(x+iy) = u(x,y) + iv(x,y)$
	Here, us vare functions of xsy. Hence any complex function one can separate into real & img. parts.
	real la ima: male:
	paris

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Table 1	

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	Classm	ate s
Do	ite	$\overline{}$
Pa	ge	(

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A	Analytic functiont
M	each point of a domain D then it is called as
	and sout of a domain Dother it is called as
	analytic function or regular function or holomorphic
	function of Z in the Domain D.
*	Singular point:
	It a function f(z) is not analytic at a point in a
	Singular point: It a function $f(z)$ is not analytic at a point in a domain D. then the point is called as singular point eg. $f(z) = 1$ over $f(z) = 1$
	eq. f(z)= 1 over ¢
	Z-1
	at z=1 f(z) is not differentiable => f(z) is not analytic
	at pt. z=1 => z=1 is singular point.
×	Cauchy-Kiemann equations in Cartesian (o-ordinates.
	The necessary to sufficient condutions for a continuous
	Cauchy-Riemann equations in Cartesian (o-ordinates: The necessary to Sufficient conditions for a continuous function $\omega = f(z) = u(x, y) + iv(x, y)$ to be analytic in
	a givi k wa
	i) ux, uy, vx, vy are continuous functions of xby in
17	is) ux=vy & uy=-vx al each point of R
	The state of the s
#	by=-vx egns or C-R. egns.
	my=-vx egns or C-R. egns.
	The state of the s
	When +(z) is analytic, it's derivative is given by
	me of the following expression.
	$f'(z) = U_x + i V_x$ or $f'(z) = V_y + i V_x$. $f'(z) = U_x - i U_y$ or $f'(z) = V_y - i U_y$
	f(z) = Ux - ily 00 f(z) = Vy-ily
1	
	Generally we'll use 1'st expression i.e. of (z)=Ux+1Vx.
	7 1 Tot. Naticy Striothin

*	Note + Differentiated
1	Notet If f(z) is analytic then it can be differentiated in usual manner eg. f(z)=z² => f(z)=zz in usual manner eg. f(z)=z² is analytic function of f(z)=z² is analytic function
1/	in usual manner og - $f(z) = Z^2 \Rightarrow f(z) = 2Z$
	ou f(z)=z2 is analy HC function
2>	CR eques are only necessary conditions for a function to be analytic. This means even if C.R. eques are satisfied the function need not be analytic at that point
	to be analytic. This means even if C.R. eg's are
	entistied the function need not be analytic at that point
	Startspice star forces
3>	If C-R eans are not satisfied then f(z) is not analytic ine f(z) does not éxist.
	analytic in election does not éxist.
- 11	
*	Conjugate functions: It w= +(z) = u+iv & f(z) is analytic then the functions is by are called as conjugate functions.
	The first = utiv & f(z) is analytic then the
	functions is by are called as myugate functions.
	Dance of the same
IF.	soblems +
13	Show that the following functions are analytic & find their derivatives 22-y2+21xy
	find their derivatives
13	$x^2 - y^2 + 2ixy$
1	f(z) = x2-y2+ 1224 => U=x2-y2 & V=2x4
lu	x=2x, $4y=-2y$, $4x=2y$, $4y=2x$.
	· ux, uy, vx, vy are all confirmons.
11 .	HSO Ux = vy & uy = - vx i-e (-Regns au satisfied.
110	ence \$(z) = x2-42+1224 is analytic.
	#1(z) = Ux +iVx = 2x + izy = 2 (x+iy) = 2z
Ir	Note -
11 1	Here we are using above expression to find f'(2)
	s given f(z) is in the form of x by.
	her coise if the function is analytic one con
	nd the derivative in usual manner?
	nu in usua manner
	Prof. Nancy Sinollin



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2>	A(E) COSZ
\sim	+(z)= cosz - cos(x+14) = cosx cosiy - 81mx 81niy
	= cosx cosky - i snnx snhy
	> U = rosx roshy V = - 8172 sinhy
	Un = - Sinx costy, uy = cosx sinhy, Vx = - cosx sinhy,
	Vy sinx coshy
	> ux, uy, vx, vy are all continuous functions.
	Also Ux= Vy & Uy=-vx.i.e. (-R eq's au satisfied
	=) +(z) = cosz 15 analytic.
	$\frac{1}{2} \cdot \frac{1}{2} (z) = -8in z$
	for Al(z) = Ux +ivx = -sina coshy-i cosa sinhy.
	= - (8mx cosiy + cosx simiy);
	$= -\left(8n\left(x+iy\right)^{\circ}\right) = -8nz.$
3>	ze^{2Z} .
\sim	f(z)=ze2z = cx+iy) e2(x+iy)
	= (x+iy) e2x+2iy = (x+iy) e2x. e12y
l,	- e2x (xtix) (roszy + isin zy)
	= e 2 2 COSZY +12 Sm24 + 14 COSW - 481724
	= e ²² [x cos 2y - y sm2y] + i & [x sin2y + y cos2y
	$-: U = e^{2x} \left[x \cos 2y - y \sin 2y \right] V = e^{2x} \left[x \sin 2y + y \cos 2y \right]$
	$u_x = e^{2x} [\cos 2y] + [x\cos 2y - y\sin 2y] 2e^{2x}$ = $e^{2x} [\cos 2y + 2x\cos 2y - 2y\sin 2y]$
	= e2x [cos 24 + 2 x ros 24 - 248m24]
	ly = e2x -2 x sin 2y - (24 cos24 + sin24)
-	uy = e2x [2x8m 2y - (2y cos2y + sin2y)] = -e2x [2x8m 2y + 2y cos2y + sin2y]
	22 [SIM 24] + [x8m24+y10524] 2e2
	= e [sm2y + 2x8m2y + 2y cos 2y]
	$V_{x} = e^{2x} [s_{1} x_{2}] + [x_{3} x_{1} x_{2} + y_{1} x_{3} x_{2}] + 2e^{2x} $ $= e^{2x} [s_{1} x_{2} x_{1} + 2x_{3} x_{2} x_{2}] + 2y_{1} x_{3} x_{2} + 2y_{1} x_{3} x_{3} x_{2} + 2y_{1} x_{3} x_{3} x_{3} x_{3} + 2y_{1} x_{3} x_{3} x_{3} x_{3} x_{3} + 2y_{1} x_{3} x_{3}$
1	U

⇒ Ux, Uy, Vx, Vy are all continuous.

Also Ux = Vy & uy = -vx i.e (-R eqns are satisfied)

⇒ f(z) = z e²z is analytic. Syl(2) = 22 et + 82

H. W. Show that following functions are analytic & also find their derivatives. ez, smhz, sinz 1 pZ.

f(z)=ez = extiy - ex eiy = ex (cosy + isiny)

ercosy v=ersiny ux, uy, vx, vy are all continuous funct

Also un= vy by =vx iec-R egns are satisfied.

=) f'(z)=0Z.

2) Sinhz

f(z) = Simhz = Simh(x+iy) = simhx coshiy + roshx smliy = 8mhx cosy + i coshx siny.

) Ux, uy, vx, vy are all continuous functions.

In uy = - Vx i.e C-R egns are satisfied. > f(2)=isinhz is analytic

: 1(z) = (oshz.

3> Sinz

f(z) = sin2 = sin(x+iy) = sinx cosiy + cosx siniy = sin x costy + i cosx sinhy.

U = SIMU coshy, V = cosx 8mhy

Ux = cosx costy, Uy = smx &nly

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	at the METERS School
	Vx = - smx sinhy, Vy = cosx roshy;
	=> Un, uy, Vx, Vy are all confirmers functions
	Also Ux= vy b uy=-vx le C-R eqns are satisfied.
	=) f(z)= Sinz is analytic.
	:. A'(z)= (OSZ.
	Dalamin II A h . a a a h i (a a a
/	Defermine the fun (x3-3xy2+3x) +i (3x2y-y3+3y).
	1(2) = (x3-3xy2+3x) + i (3x2y-y3+3y)
	$\sqrt{-2}\sqrt{4} - 3$
	$Ux = 3x^2 - 3y^2 + 3$, $Uy = -6xy$. $Vx = 6xy$, $vy = 3x^2 - 3y^2 + 3$
	$v_x = 6xy$, $v_y = 3x^2 - 3y^2 + 3$
	The state of the s
	Also ux = vy & uy = -vx i.e (-R eg3 are salisfied.
	=) f(z)= (x3-3x42+3x) +i (3x24-43+34) is analytic
	Also $u_x = v_y$ & $u_y = -v_x$ i.e (-R egg an satisfied. =) $f(z) = (x^3 - 3xy^2 + 3x) + i (3x^2y - y^3 + 3y)$ is analytic $f(z) = u_x + i v_x = (3x^2 - 3y^2 + 3) + i 6xy = 1$
	Note + Suppose we want to find derivative in terms of Z, then we use Milne-Thompson method. (we'll see this later). Using this we put x=z byzo. in 1)
	of Z then we use Milne Thompson well al
	(we'll see this later) using this
	in 1)]
	By Milne - Thompson method put 2-2 0 11-2
	1(z) = 3z2+3+1(8z2) (3z2+1)(12)
7.5	
3/	Show that w= 2 iy is analytic & find.
	du in terms of z.
	(1) 2/
	$\frac{\omega - \chi}{\chi^2 + y^2} = \frac{iy}{\chi^2 + y^2} = \frac{\omega - \chi}{\chi^2 + y^2} = \frac{1}{\chi^2 + y^2} = \frac{1}{\chi^2 + y^2}$
	11. Cm2.122
	$Ux = (x^2 + y^2) \cdot - x \cdot 2x = -x^2 + y^2$ $(x^2 + y^2)^2 = (x^2 + y^2)^2$
	$(\chi'+\gamma')'$

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1. e (2=0,4=0)

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H.W. C) $f(z) = 2x + ixy^2 \Rightarrow U = 2x, v = xy^2$ $Ux = 2, Uy = 0, vx = y^2, vy = 2xy$ $\Rightarrow Ux + vy & uy + -vx$. $\Rightarrow (-k \text{ equs are not satisfied.} +) f(z) \text{ does not exist.}$



141	Bud och and a it.
1.5/	Find a,b, c,d,e if 1 \$\f(z)=(ax^4+bx^2y^2+cy^4+dx^2-2y^2)+i(4x^3y-exy^3+4xy) is analytically all the control of the control o
	11 1000 + 0600 + 1000
	10 - 10 x24 - 003 + 44 Vy = 4x3 - 3 exy2 + 4x.
	$\sqrt{x} = 12 x^{2}y - ey^{3} + 4y$ As $f(z)$ is analytic firstly $x = 4x^{3} - 3exy^{2} + 4x$. As $f(z)$ is analytic firstly $x = 4x^{3} - 3exy^{2} + 4x$.
	as ux = vy => 4ax3+2bxy2+2dx = 4x3-3exy2+4x
	Comparing the coefficients we get.
- I	$4a=4 \Rightarrow a=1$
	2b=-3e i>
	90-4
	also $uy = -Vx = 2bx^2y + 4uy^3 - 4y = -12x^2y + ey^3 - 4y$ Company - the coefficients we get, $2b = -12 \implies b = -6$: from $1 > -12 = -3e \implies e = 4$
	Company - the coefficients we get,
	$2bz-12 \Rightarrow b=-6$: from 1) -12 z -3e = ez4
	⇒ 4C=4 ⇒ [(=1).
	:. a=1, b=-6, C=1, d=2, e=4.
H V. V.	10. 1 9 0 0 10 10 0
1 W 27	$f(z) = (ax^{3} + bxy^{2} + 3x^{2} + cy^{2} + 3c) + i(dx^{2}y - 2y^{3} + exy + y)$ is analytic. Here, $u = ax^{3} + bxy^{2} + 3x^{2} + cy^{2} + x$ $v = dx^{2}y - 2y^{3} + exy + y$
	is analytic.
	Here, U = ax3+6xy +3x2+ cy2+x V = dx4-2y3+exy+y
	Cinen +(2) is analytic. => +(2) Satisfine GR. eqn
	$\frac{1}{2} = \frac{1}{2} \cdot \frac{1}$
	$U_{x} = 3ax^{2} + by^{2} + 6x + 1$ $U_{y} = 2bxy + 2cy$
	$\sqrt{x} - 2dxy + ey$ $\sqrt{y} = dx^2 - 6y^2 + ex + 1$
	As ux= Vy => 3ax2+by2+6x+1=dx2-6y2+ex+1
	Comparing the coefficients we get,
	30=d1>, b=-6 , e=6
	Also, ly=-1/x => 2bxy +2cy = -2dxy -ey
	Comparing the coefficients we get,
	$2b=-2d$ \Rightarrow $b=-d$. $as b=-6 \Rightarrow -d=-6 \Rightarrow d=6$
	$2C=-e$, as $e=6 \Rightarrow 2C=-6 \Rightarrow C=-3$ rom 1) $3a=d$ as $d=6 \Rightarrow 3a=6 \Rightarrow a=2 $
	a=2, $b=-6$, $C=-3$, $d=6$, $e=6$. Prof. Nancy Sinollin

	A PASSIBLE INSINGUISHED OF INDICTINOLOGY (Approved by AICTE New Dolln & Gave, of Maharenbura, Affiliated to University of Membra) Chassmate (Kellgloon Jain Minority) Date Page
HWIS	Find a,b, c,d if f(z) = x2+ 2axy+by2+i((x2+2dxy+y2) is analytic
	Here, U= x2+ 20xy+ by2; V= 22+2dxy+y2
	Find a,b, c,d if $f(z) = x^2 + 2axy + by^2 + i((x^2 + 2dxy + y^2))$ is analytic. Here, $u = x^2 + 2axy + by^2$; $v = x^2 + 2dxy + y^2$. As $f(z)$ is analytic it satisfies $c - k$ egns.
	1-l Ux=Vy & Uy=-Vx
	:Uz = 2x +2ay, Uy = 2ax + 2by,
	$V_X = 2CX + 2dy$, $V_Y = 2dx + 2y$.
	$as(u_x = v_y =)$ $2x + 2ay - adx + 2y$
- P	Comparing the coefficients we get, $2d=2 \Rightarrow d=1 $ & $2a=2 \Rightarrow a=1 $
	$2d=2 \Rightarrow a=1 $ & $2a=2 \Rightarrow a=1 $
X	Also, $Uy = -vx \Rightarrow 2ax + 2by = -2(x - 2dy)$.
	Comparing the coefficients we get,
	$2a = -2c \Rightarrow a = -c \Rightarrow 1 = -c \Rightarrow 1 = -c$
	$2b = -2d \Rightarrow b = -d \Rightarrow \boxed{b} = -1$
	a=1, b=-1, c=-1, d=1.
HW 16	Find k such that I log (22+42) + i tan kx is analytic.
7	2 log (2+42) + 1 Fan k2 13 analytic
	Tut, f(z) = 2 log (x+42) + 1 tan kx
Park	:. U= /2 log (x2+42) V = tan(kx)
1	as f(z) is analytic, it satisfies C-R. egns.
	· Uy = Vy & Uy Va
	$U_{\chi} = \chi \chi$ = χ $U_{\chi} = \chi$
	$1/\chi = \frac{2\chi}{2(\chi^2 + y^2)} = \frac{\dot{\chi}}{\chi^2 + y^2}$, $1/\chi = \frac{2y}{2(\chi^2 + y^2)} = \frac{y}{\chi^2 + y^2}$
	Vx= 1 = /k) - 42
	$\frac{1+(kx)^2}{(4)} \left(\frac{4}{y}\right) = \frac{k^2+1^2+2^2}{4^2+1^2+2^2}$
	Vy = 1 - (-1x) 1x
	1+(kx)2 y2) 42+12x2 y2 = -2x
	as Ux = Vy & Uy = -Vx
	=> 2 - kx 0 4
	72-fu2 K4



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	A	Cauchy-Riemann Egns in Polar Co-ordinales.
	*	Prove the C-R egns in polar form.
		Let f(z) be analytic function.
1		Consider the polar coordinates 1.e. x= rcoso & y= rsino
		Consider the polar conordinates i.e. x=rcoso & y=rsino as z=x+iy = rcoso +irsino = r(coso+ismo) = reio
		As f(2) = u + iv i.e. + (re10) = u+iv1>.
	The second	Diff. 1) partially w.r.f. r: we get, f'(reib) e'o = du + i dv
		$f'(re^{i\theta})e^{i\theta} = \partial u + i \partial v = 2$
		dr dr-
		Diff 1) partially wirto we get, flore 10) réo : i = du + i dv do do do.
		floreio) relo i = du + i dv
		20 20.
		#(reio) e'o r.i = du + 1200
		30 30
		using 2) we get, Qu + i dv r. i = du + i dv
	,	100 Ja 27 30 Ja
	18	$\Rightarrow i r \frac{\partial u}{\partial r} - r \frac{\partial v}{\partial r} = \frac{\partial u}{\partial 0} + i \frac{\partial v}{\partial 0}.$
		92 92 90, 90.
	1	=> -rdv +irdu = du +idv
-		2r 2r 20 20
	9	Comparing we get -rdv = du & rdu - dv
	15	9x 90 9x 90.
e ^{30 °}		=> du - I dv & durdv
		1 9x x.90 90 3x
10		i.e. Ur= 1 vo & U0 = - ~ vr
		which are C-R. egns in polar form.
		71. 10(010 00011
	A	From 2) we get . # (reio) = = 10 (ur+ivr)
	*	1.e. f(z) = e-io (ux fivx).
3	#	1.6.1(2) - 6 (ASTIVY)
	24	Prof. Nancy Sinollin
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	O. College
. 17	Defermine whether wis analytic & find dw for follow
	1 1 10 2 10 0 1
	w-1092
	Consider the polar co-ordinates x=rcoso, y=rsno
100	1 21111 >1) - 1007 = 1091 1111
	as $z = x + ig = \omega z \times ig = log (r cos 0 + ir sin 0)$
	100 (x((000+180MQ)) = 209(xe)=209x+209e
	= logr + iologe = logr +io.
	3 4 = 109 V V = 0
1	$-U_{\gamma} = \frac{1}{2} U_{0} = 0 , V_{\gamma} = 0 V_{0} = 1$
	· Ur = I Vo & Uo = O = - TVr = Uo = - TVr
	=> C-R ears are satisfied by two.
	also un, uo, vr, vo are confinuous.
	Hence, to is analytic.
~	$du = e^{io}(ur + ivr) = e^{io}\left(\frac{1}{r}\right) = \frac{1}{\gamma e^{io}} = \frac{1}{z}$
MW2	wzzn.
	Ensider the potar co-ordinales x=xcoso, y=xsino.
	$\alpha x = x + iy$
	as $z=x+iy$ $\omega=z^n=(x+iy)^n=(r(os0+irsn0)^n=r^n(ros0+isn0)^n$ $=r^n(ros0+isnn0)^n=r^n(ros0+isnn0)^n$
	by De Mons thm
	u= r cosno v= r sinno.
	: Ux=nxn-1 cosno, uo=-mn sinno
	$V_r = n \gamma^n s n n 0$ $V_0 = n \gamma^n c s n 0$
	$f: \frac{1}{2} v_0 = n \gamma^{n-1} \cos n \theta = U \gamma = \frac{1}{2} v_0$
	311110 - 000 2) U0 = - 7 Vy
	=> C-R egns are sab's fied by w
, t	elso, Ur, Uo, Vr, Vo are Continuous.
	tence, wis analytic dw - nzn-
. 11.	
	dz Prof. Nancy Sinollin



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2	Find 3 if \$(z)=2 cos20 + 1 x2 8mg0 is analytic.
1	Given , f(z)= r2(0520 +122 8190
- 11	
	Act(2) is analytic It south the Compount
	ie ur= Ivo & up=, - vv
	il Ur= I vo & Up=, - r vr. Here, ur= 2 r. cos 20; Up= -2 r281 n 20
	1/2-27 Synla 1/2- 27 (1880)
	As $U_{x} = \frac{1}{7} V_{0} \Rightarrow 2 r \cos 20 = \frac{1}{7} 8 r^{2} \cos 90 \Rightarrow 2 r \cos 20 = \frac{9}{7} r \cos 90$
	⇒ 8=2.
	Also $u_0 = -\gamma V_{\gamma} \Rightarrow -2\gamma^2 \sin 2\theta = -2\gamma^2 \sin 9\theta \Rightarrow \sin 2\theta = \sin 9\theta$ $\Rightarrow S = 2$. Hence, we get $S = 2$ Here.
	=> S=2, Hence, we get S=2 Here.
	, 0
3>	Is, f(z)= 7/2 15 analytic. &
-	mel z=x+iy
	By polar co-ordinates x=rcoso, y=rsino.
	$\frac{1}{2} = r(\cos \theta + i \sin n\theta) = re^{i\theta}$
	$z = x - iy = \gamma(\cos \theta - i\sin \theta) = re$ $z = x - iy = \gamma(\cos \theta - i\sin \theta) = re^{-i\theta}$.
	$z = x - iy = \gamma(\cos 0 - i\sin 0) = \tau e^{it}$ $f(z) = \frac{z}{z} = \frac{\pi e^{i0}}{\pi e^{-i0}} = e^{i20} = \cos 20 + i\sin 20 = utiv$
	=) U= (0520 1 V=8m20.
	200-20
	Here $u_{\gamma} = 0$, $u_0 = -281n20$. $v_{\gamma} = 0$, $v_0 = 2\cos 20$.
	V ~ = U , V = = 1
	ur = - vo. & vo = - v v
	> C-R egns are not sabsfied.
	=) wif(z) is not analytic.
- 6.	Prof. Nancy Sinollin
	And the second of the second o