



Subject: Applied Mathematics IV

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Penalty (Big-M or charne's) Method :-

If any one or some of the constraints are of greater than or equal type to convert the inequality to equality we have to subtract the surplus variables i.e. $-s_1, -s_2, \dots$. But we would not get a unit matrix. To overcome this difficulty in addition to surplus variables we introduce artificial variables A_1, A_2, \dots with the sign to the constraints. In the objective function we assign big penalty by subtracting MA_1, MA_2, \dots if the objective function is of maximisation type.

Remark:-

* The artificial variable leave the process and the optimality condition is satisfied by the basic variables.



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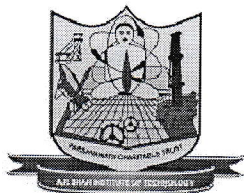
* Atleast one of the artificial variables remains in the basis with zero value and the optimality condition is satisfied. This the optimal basic feasible solution.

* Atleast one of the artificial variables remains in the basis with non-zero value and the optimality condition is satisfied. This is not an optimal solution since it contains large penalty M . This is not a solution but a pseudo-solution.

① Using Penalty (Big-M) method solve the following

LPP. Maximise $Z = 3x_1 - x_2$
subject to $8x_1 + x_2 \leq 2$
 $x_1 + 3x_2 \geq 3$
 $x_2 \leq 4$
 $x_1, x_2 \geq 0$.

Soln:- Since the second constraint is ' \geq ' type we introduce Artificial variable A_2 and big



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penalty in the objective function.

$$\text{Maximize } z = 3x_1 - x_2 - 0s_1 - 0s_2 - 0s_3 - M A_2 \rightarrow (1)$$

$$\text{Subject to } 2x_1 + x_2 + s_1 + 0s_2 + 0s_3 + 0A_2 = 2 \rightarrow (2)$$

$$x_1 + 3x_2 + 0s_1 - s_2 + 0s_3 + A_2 = 3 \rightarrow (3)$$

$$0x_1 + x_2 + 0s_1 + 0s_2 + s_3 + 0A_2 = 4 \rightarrow (4)$$

$$x_1, x_2, s_1, s_2, A_2 \geq 0$$

We eliminate MA_2 from the objective function by adding $M \times 2$ constraint to it.

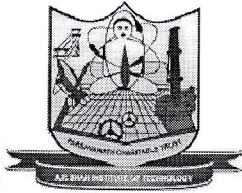
$$(1) + (2)M \Rightarrow z = (3+M)x_1 - (1-3M)x_2 - 0s_1 - Ms_2 - 0s_3 - 0A_2 - 3M$$

$$z - (3+M)x_1 + (1-3M)x_2 + 0s_1 + Ms_2 + 0s_3 + 0A_2 = -3M$$

Simplex table

Iteration no	Basic Var.	co-effts of						RHS soln	Ratio
		x_1	x_2	s_1	s_2	s_3	A_2		
0	Z	-3-M	1-3M	0	M	0	0	-3M	
A_2 leaves	s_1	2	1	1	0	0	0	2	$2/1 = 2$
x_2 enters	A_2	1	3*	0	-1	0	1	3	$3/3 = 1$ ←
	s_3	0	1	0	0	1	0	4	$4/1 = 4$

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$$1 \quad Z \quad -10/3 \quad 0 \quad 0 \quad 1/3 \quad 0 \quad - \quad -1$$

$$\begin{array}{l}
 S_1 \text{ leaves } S_1 \\
 x_1 \text{ enters } x_2 \\
 S_3
 \end{array}
 \begin{array}{c}
 \boxed{
 \begin{array}{ccccccc}
 5/3^* & 0 & 1 & 1/3 & 0 & - & 1 \\
 1/3 & 1 & 0 & -1/3 & 0 & - & 1 \\
 -1/3 & 0 & 0 & 1/3 & 1 & - & 3
 \end{array}
 }
 \end{array}
 \begin{array}{l}
 3/5 \leftarrow \\
 3 \\
 -9
 \end{array}$$

↑↑

$$\begin{array}{l}
 2 \quad Z \\
 x_1 \\
 x_2 \\
 S_3
 \end{array}
 \begin{array}{ccccccc}
 0 & 0 & 2 & 1 & 0 & - & 1 \\
 1 & 0 & 3/5 & 1/5 & 0 & - & 3/5 \\
 0 & 1 & -1/5 & -2/5 & 0 & - & 4/5 \\
 0 & 0 & 1/5 & 2/5 & 1 & - & 16/5
 \end{array}$$

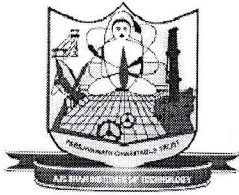
$$\therefore x_1 = 3/5 \quad x_2 = 4/5 \quad Z_{\max} = 1.$$

2) Using the penalty (Big M) method solve the following LPP Maximise $Z = 3x_1 + 2x_2$

subject to $2x_1 + x_2 \leq 2$

$$3x_1 + 4x_2 \geq 12$$

$$x_1, x_2 \geq 0.$$



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Soln:-

$$\text{Maximise } Z = 3x_1 + 2x_2 - 0s_1 - 0s_2 - MA_2 \rightarrow (1)$$

$$\text{subject to } 2x_1 + x_2 + s_1 + 0s_2 + 0A_2 = 2 \rightarrow (2)$$

$$3x_1 + 4x_2 + 0s_1 - s_2 + A_2 = 12 \rightarrow (3)$$

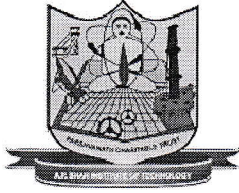
Eliminate M in (1) by $(3) \times M + (1)$.

$$\therefore Z = (3+3M)x_1 + (2+4M)x_2 - 0s_1 - Ms_2 - 0A_2 = 12M$$

$$\therefore Z - (3+3M)x_1 - (2+4M)x_2 + 0s_1 + Ms_2 + 0A_2 = -12M$$

Simplex table

Iteration no	Basic Variables	co-effts of					RHS soln	Ratio
		x_1	x_2	s_1	s_2	A_2		
0	Z	$-3-3M$	$-2-4M$	0	M	0	$-12M$	
	s_1	2	1*	1	0	0	2	2 \leftarrow
	A_2	3	4	0	-1	1	12	3
1	Z	$1+5M$	0	$2+4M$	M	0	$4-4M$	
	x_2	2	1	1	0	0	2	
	A_2	-5	0	-4	-1	1	4	



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Since the artificial variable A_2 appears not at zero level and all entries in the row of z have M with positive co-efficient, feasible solution does not exist. The solution is called pseudo-optimum basic feasible solution.
