Properties of Operators

- **Commutativity:**
  - $P \wedge Q = Q \wedge P$, or
  - $P \vee Q = Q \vee P$.
- **Associativity:**
  - $(P \wedge Q) \wedge R = P \wedge (Q \wedge R)$,
  - $(P \vee Q) \vee R = P \vee (Q \vee R)$
- **Associativity:**
  - $(P \wedge Q) \wedge R = P \wedge (Q \wedge R)$,
  - $(P \vee Q) \vee R = P \vee (Q \vee R)$
- **Distributive:**
  - $P \wedge (Q \vee R) = (P \wedge Q) \vee (P \wedge R)$.
  - $P \vee (Q \wedge R) = (P \vee Q) \wedge (P \vee R)$.
- **DE Morgan's Law:**
  - $\neg (P \wedge Q) = (\neg P) \vee (\neg Q)$
  - $\neg (P \vee Q) = (\neg P) \wedge (\neg Q)$.
- **Double-negation elimination:**
  - $\neg (\neg P) = P$.
- **Identity element:**
  - $P \wedge True = P$,
  - $P \vee True = True$.

INFERENCE

1. Generating the conclusions from evidence and facts is termed as Inference.
2. Rules

   2.1. **Implication:** represented as $P \rightarrow Q$

   2.2. **Converse:** The converse of implication, which means the right-hand side proposition goes to the left-hand side and vice-versa. It can be written as $Q \rightarrow P$.

   2.3. **Contrapositive:** The negation of converse is termed as contrapositive : $\neg Q \rightarrow \neg P$.

   2.4. **Inverse:** The negation of implication is called inverse : $\neg P \rightarrow \neg Q$.

P	Q	$P \rightarrow Q$	$Q \rightarrow P$	$\neg Q \rightarrow \neg P$	$\neg P \rightarrow \neg Q$.
T	T	T	T	T	T
T	F	F	T	F	T
F	T	T	F	T	F
F	F	T	T	T	T

Types of Inference Rules

1. Modus Ponens
2. Modus Tollens
3. Hypothetical Syllogism
4. Disjunctive Syllogism
5. Addition
6. Simplification
7. Resolution

Modus Ponens

It states that if P and P → Q is true, then we can infer that Q will be true. It can be represented as:

Notation for Modus ponens: $$\frac{P \rightarrow Q, \quad P}{\therefore Q}$$

Example:

Statement-1: "If I am sleepy then I go to bed" ==> P→ Q
Statement-2: "I am sleepy" ==> P
Conclusion: "I go to bed." ==> Q.
Hence, we can say that, if P→ Q is true and P is true then Q will be true.

Proof by Truth table:

P	Q	$P \rightarrow Q$
0	0	0
0	1	1
1	0	0
1	1	1

Modus Tollens

It state that if P→ Q is true and ¬ **Q is true, then ¬ P** will also true. It can be represented as:

Notation for Modus Tollens: $\dfrac{P \to Q,\ \sim Q}{\sim P}$

Statement-1: "If I am sleepy then I go to bed" ==> P→ Q

Statement-2: "I do not go to the bed."==> ~Q

Statement-3: Which infers that "**I am not sleepy**" => ~P

Proof by Truth table:

P	Q	$\sim P$	$\sim Q$	$P \to Q$
0	0	1	1	1
0	1	1	0	1
1	0	0	1	0
1	1	0	0	1

Hypothetical Syllogism

It states that if P→R is true whenever P→Q is true, and Q→R is true.

Statement-1: If you have my home key then you can unlock my home. **P→Q**

Statement-2: If you can unlock my home then you can take my money. **Q→R**

Conclusion: If you unlock my home key then you can take my money. **P→R**

Proof by truth table:

P	Q	R	$P \to Q$	$Q \to R$	$P \to R$
0	0	0	1	1	1
0	0	1	1	1	1
0	1	0	1	0	1
0	1	1	1	1	1
1	0	0	0	1	1
1	0	1	0	1	1
1	1	0	1	0	0
1	1	1	1	1	1

Disjunctive Syllogism

It states that if P∨Q is true, and ¬P is true, then Q will be true

Notation of Disjunctive syllogism: $\dfrac{P \lor Q, \quad \neg P}{Q}$

Example:

Statement-1: Today is Sunday or Monday. ==>P∨Q
Statement-2: Today is not Sunday. ==> ¬P
Conclusion: Today is Monday. ==> Q

Proof by truth-table:

P	Q	$\neg P$	$P \lor Q$
0	0	1	0
0	1	1	1
1	0	0	1
1	1	0	1

Addition

It states that If P is true, then $P \vee Q$ will be true

Notation of Addition: $\dfrac{P}{P \vee Q}$

Example:

Statement: I have a vanilla ice-cream. ==> P
Statement-2: I have Chocolate ice-cream.
Conclusion: I have vanilla or chocolate ice-cream. ==> $(P \vee Q)$

Proof by Truth-Table:

P	Q	$P \vee Q$
0	0	0
1	0	1
0	1	1
1	1	1

Simplification

It states that if **P** $\wedge$ **Q** is true, then **Q or P** will also be true

Notation of Simplification rule: $\frac{P \wedge Q}{Q}$ Or $\frac{P \wedge Q}{P}$

Proof by Truth-Table:

P	Q	$P \wedge Q$
0	0	0
1	0	0
0	1	0
1	1	1

Resolution

It states that if $P \vee Q$ and $\neg P \wedge R$ is true, then $Q \vee R$ will also be true

Notation of Resolution $\dfrac{P \vee Q, \ \neg P \wedge R}{Q \vee R}$

Proof by Truth-Table:

P	¬P	Q	R	$P \vee Q$	$\neg P \wedge R$	$Q \vee R$
0	1	0	0	0	0	0
0	1	0	1	0	0	1
0	1	1	0	1	1	1
0	1	1	1	1	1	1
1	0	0	0	1	0	0
1	0	0	1	1	0	1
1	0	1	0	1	0	1
1	0	1	1	1	0	1