Duality :-

Definition :-

The phenomenon occcuring in linear programming that given a problem there exists another closely related problem with the same set of data. and with the same solution is called the principle of duality.

Procedure :-

* Write the objective function in maximisation type, if not.
* Write all the constraints in $\leq$ type. If any constraint is in $\geq$ type multiply the inequality by '-1' and change the inequality $\leq$.
* If the constraint is of equality type, change it into inequality type,

for example $3x_1+6x_2=7$ is equivalen to

$3x_1+6x_2 \le 7$ & $3x_1+6x_2 \ge 7$

Now, $3x_1+6x_2 \ge 7 \Rightarrow -3x_1-6x_2 \le -7$.

$\therefore$ $3x_1+6x_2=7$ is now equivalent to

$3x_1+6x_2 \le 7$ &

$-3x_1-6x_2 \le -7$.

* All decision variables must be '$\ge$' type. If any variable is unrestricted say x_2, then write x_2 as

$x_2 = x_2' - x_2''$

* In the primal in the standard form

(i) the objective function is of maximisation type with the constraints in '$\le$' type &

(ii) the objective function is of minimisation type with the constraints in '$\ge$' type.

Now write the dual by

* Write the objective function from maximisation to minimisation

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(1) Construct the dual of the following problem

Minimise $Z = x_2 + 3x_3$

Subject to $2x_1 + x_2 \leq 3$

$x_1 + 2x_2 + 6x_3 \geq 5$

$-x_1 + x_2 + 2x_3 = 2$

$x_1, x_2, x_3 \geq 0.$

Soln:-

Since the objective function is of minimisation type, the constraints should be '$\geq$' type

$\therefore 2x_1 + x_2 \leq 3 \Rightarrow -2x_1 - x_2 \geq -3$

$-x_1 + x_2 + 2x_3 = 2$ can be written as

$\Rightarrow -x_1 + x_2 + 2x_3 \leq 2$ & $-x_1 + x_2 + 2x_3 \geq 2$

$\Rightarrow x_1 - x_2 - 2x_3 \geq -2$ & $-x_1 + x_2 + 2x_3 \geq 2.$

∴ The ~~dual~~ Primal is

Minimise $Z = 0x_1 + x_2 + 3x_3$

subject to $-2x_1 - x_2 + 0x_3 \ge -3$

$x_1 + 2x_2 + 6x_3 \ge 5$

$-x_1 + x_2 + 2x_3 \ge 2$

$x_1 - x_2 - 2x_3 \ge -2$

$x_1, x_2, x_3 \ge 0$.

Since the last constraint is now expressed as 2 constraints, we have $y_3 = y_3' - y_3''$.

∴ the dual is

Maximise $w = -3y_1 + 5y_2 + 2y_3' - 2y_3''$

subject to $-2y_1 + y_2 - y_3' + y_3'' \le 0$

$-y_1 + 2y_2 + y_3' - y_3'' \le 1$

$0y_1 + 6y_2 + 2y_3' - 2y_3'' \le 3$.

Since $y_3 = y_3' - y_3''$, y_3 is unrestricted,

∴ the dual is

Maximise $w = -3y_1 + 5y_2 + 2y_3$

subject to $-2y_1 + y_2 - y_3 \le 0$

$-y_1 + 2y_2 + y_3 \le 1$

$6y_2 + 2y_3 \le 3$

$y_1, y_2 \ge 0$, y_3 is unrestricted.

(2) Find the dual of the follg: LPP.

Maximise $Z = 2x_1 - x_2 + 3x_3$

subject to $x_1 - 2x_2 + x_3 \ge 4$

$2x_1 + x_3 \le 10$

$x_1 + x_2 + 3x_3 = 20$

$x_1, x_3 \ge 0$, x_2 unrestricted.

Soln:-

x_2 is unrestricted, $\therefore x_2 = x_2' - x_2''$.

Maximise $Z = 2x_1 - (x_2' - x_2'') + 3x_3$

subject to $-[x_1 - 2(x_2' - x_2'') + x_3] \le -4$.

$2x_1 + 0x_2 + x_3 \le 10$

$x_1 + (x_2' - x_2'') + 3x_3 \le 20$ &

$-[x_1 + (x_2' - x_2'') + 3x_3] \le -20$

$\therefore$ The Primal is

Maximise $Z = 2x_1 - x_2' + x_2'' + 3x_3$

subject to $-x_1 + 2x_2' - 2x_2'' - x_3 \leq -4$

$2x_1 + 0x_2' - 0x_2'' + x_3 \leq 10$

$x_1 + x_2' - x_2'' + 3x_3 \leq 20$

$-x_1 - x_2' + x_2'' - 3x_3 \leq -20$

$x_1, x_2', x_2'', x_3 \geq 0$.

The dual is

Minimise $w = -4y_1 + 10y_2 + 20y_3' - 20y_3''$

subject to $-y_1 + 2y_2 + y_3' - y_3'' \geq 2$

$2y_1 + 0y_2 + y_3' - y_3'' \geq -1$

$-2y_1 + 0y_2 - y_3' + y_3'' \geq 1$

$-y_1 + y_2 + 3y_3' - 3y_3'' \geq 3$.

(i) Minimise $w = -4y_1 + 10y_2 + 20y_3$

subject to $-y_1 + 2y_2 + y_3 \geq 2$

$2y_1 + 0y_2 + y_3 = -1$

$-y_1 + y_2 + 3y_3 \geq 3$

$y_1, y_2 \geq 0$

y_3 unrestricted.