**Academic Year: 2022-2023**

**Question Bank**

**Subject: DSGT**

**Sem: III**

Q. 1 Solve the given equation using mathematical induction.

2+5+8+..+(3n-1)=n(3n+1)/2

Q. 2 Solve/Prove by induction that the sum of the cubes of the three consecutive numbers is divisible by 9

Q. 3 Use the laws of logic to show that following statements are tautology

i) [(p->q)^~q]->~p

ii) (p^(p->q))->q.

Q. 4 Show that for any positive integer n>1, $\frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}+..+\frac{1}{\sqrt{n}} > \sqrt{n}$

Q.5 Prove that 1.1! +2.2! + 3.3! + ..... + n.n! = (n+1)!- 1, where n is a positive integer.

Q. 6 Solve : Let A={a1,a2,a3,a4,a5} and let R be a relation on A whose matrix is

$$M_R = \begin{vmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{vmatrix}$$

Find $M_R^*$ by Warshall's algorithm.

Let R is a binary relation. Let S={(a,b)|(a,c) ∈ R and (c,b)∈ R for some c}. Show that if R is an equivalence relation then S is also an equivalence relation.

Q. 7 Define equivalence relation with example. let 'T' be a set of triangles in a plane and define R as the R = {(a, b) a,b ∈ T and a is congruent to b } then show that R is an equivalence relation.

Q. 8 Prove the following (use laws of set theory)

A x (X ∩ Y) = (A x X) ∩ (A x Y)

Q. 9 Let f(x)=x+2, g(x)=x-2 and h(x)=3x for x ∈ R, where R=set of real numbers. Find (gof),(fog),(fof),(gog),(foh),(hog),(hof),(fohog)

Q.10

i)Let A={1,2,3,4} and Let R={(1,1)(1,2)(1,4)(2,4)(3,1)(3,2)(4,2)(4,3)(4,4)}. Find transitive closure by Warshall's algorithm.

ii) Define reflexive closure and symmetric closure of a relation. Also find reflexive and symmetric closure of R

A={1,2,3,4}

R={(1,1),(1,2),(1,4),(2,4),(3,1),(3,2),(4,2),(4,3),(4,4)}

Q. 11 Find the greatest lower bound and least upper bound of the set{3,9,12} and {1,2,4,5,10} if they exists in the poset (z+,/). Where / is the relation of divisibility.

Q. 12 Determine the matrix of the partial order of divisibility on the set A. Draw the Hasse diagram of the Poset. Indicate those which are chains.

1.A={1,2,3,5,6,10,15,30}

2.A={3,6,12,36,72}

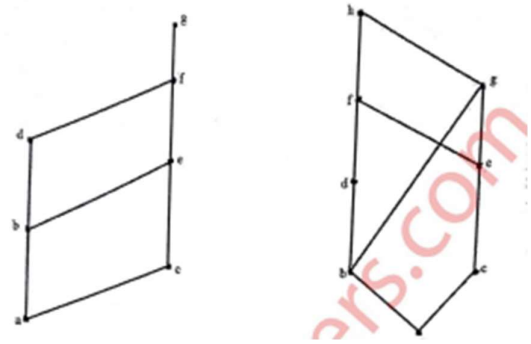
Q.13 For the set X = {2,23,6,12,24,36} , is a relation $\leq\leq$ is defined as x $\leq\leq$ y if x divides y, Draw the Hasse diagram for (X, $\leq\leq$) . Answer the following:

i) What are the maximal and minimal elements?

ii) give one example of a chain and antichain?

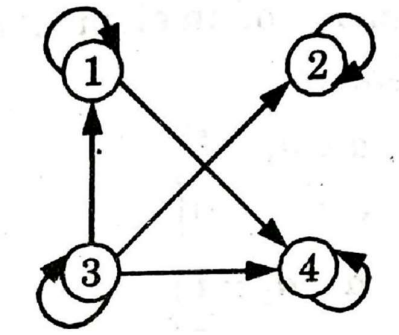
iii) Is the Poset a lattice?

Q. 14 Determine whether the Poset with the following Hasse diagrams are lattices or not. Justify your answer.



Q. 15 Determine the number of positive integers n where $1 \leq n \leq 100$ and n is not divisible by 2,3, or 5

Q. 16 The directed graph G for a relation R on set A = {1, 2, 3, 4} is shown below:




Verify that (A, R) is a poset and find its Hasse diagram.

Is this a lattice? What are the maximal and minimal elements?

Q. 17 Find the complete solution of the recurrence relation

$a_n = -a_{n-1} + 4a_{n-2} + 4a_{n-3}$ with $a_0=8$, $a_1=6$ and $a_2=26$?

Q. 18 Determine the number of positive integers n where 1<=n<=100 and n is not divisible by 2,3 or 5

Q. 19 Show that in any room of people who have been doing some handshaking there will always be at least two people who have shaken hands the same number of times. Also explain Pigeonhole principle and Extended Pigeonhole principle.

Q. 20 Prove that among 100000 people, there are two who are born at exactly the same time(HH:MM:SS).

Q. 21 What is the minimum no. of students required in a discrete structures class to be sure that at least six will receive the same grade, if there are 5 possible grades A,B,C,D and E.

Q. 22 Prove that set G={1,2,3,4,5,6} is a finite abelian group of order 6 with respect to multiplication modulo 7. Also discuss the terms of Groups and Subgroups.

Q. 23 Solve following:

Given the parity check matrix

$$H=\begin{vmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{vmatrix}$$

Find the minimum distance of the code generated by H. how many errors it can detect and correct.

Q. 24 Prove that the set G=(0,1,2,3,4,5) is an abelian group of order 6 with respect to addition modulo 6.

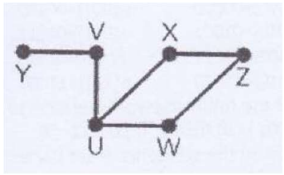
Q. 25 Let H={$[0]_6$ ,$[3]_6$} find the left and right cosets in group $Z_6$. Is H a normal subgroup of group of $Z_6$.

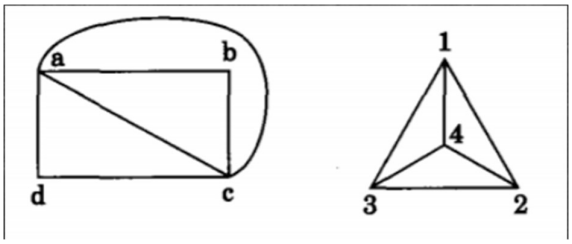
Q. 26 Consider the (3,6) encoding function e:$B^3$ -> $B^6$ defined by

e(000)=000000 e(001)=001100 e(010)= 010011 e(011)=011111
e(100)=100101 e(101)=101001
e(110)=110110 e(111)=111010

Decode the following words relative to a maximum likelihood decoding function
i) 000101 ii) 010101

Q. 27 Show whether the following graphs are isomorphic or not. Also define isomorphic graphs and homomorphic graphs.




**Q. 28**

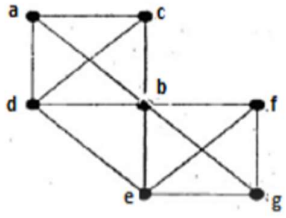



Fig. a

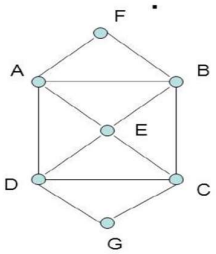



Fig. c

Determine Euler Cycle and path in graph shown in fig a,c.

Q. 29

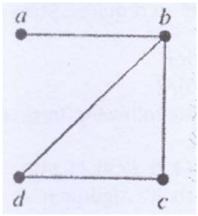



Fig. b

Determine Hamiltonian Cycle and path in graph shown in fig b.

Q. 30 Let $G=Z_8$, determine all left cosets of $H=\{[0],[4]\}$ in G.

Q. 31 In a group of 6 boys and 4 girls, four children are to be selected. In how many different ways can they be selected such that at least one boy should be there?

Q. 32 A box contains 6 white balls and 5 red balls. In many ways 4 balls can be drawn from the box if i) they are to be of any color ii) all the balls to be of the same color.

Q. 33 how many friends must you have to guarantee that at least five of them will have birthdays in the same month.

Q. 34 In how many ways a committee of three faculty members and 2 students can be formed from 7 faculty members and 8 students.