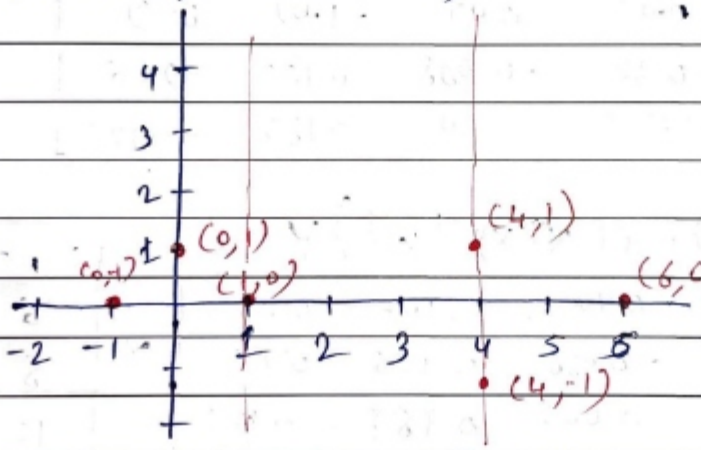
Support Vector Machine – Solved Example

Suppose,

- Points (4,1), (4,-1) and (6,0) belong to class positive &
- Points (1,0), (0,1) and (0,-1) belong to negative class

Find hyperplane using SVM.

Ans:- It can be observed that the support vectors are (1,0), (4,1) and (4,1)



$$S_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad S_2 = \begin{pmatrix} 4 \\ 1 \end{pmatrix}, \quad S_3 = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$$

→ The augmented vector can be obtained by adding the bias given as follows:

$$\tilde{S}_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \quad \tilde{S}_2 = \begin{pmatrix} 4 \\ 1 \\ 1 \end{pmatrix}, \quad \tilde{S}_3 = \begin{pmatrix} 4 \\ -1 \\ 1 \end{pmatrix}.$$

– from these, a set of three equations can be obtained based on three support vector as follows:

Semester : VI | Subject :Machine Learning | Academic Year: 2023 - 2024

$$\alpha_1 \tilde{s}_1 \tilde{s}_1 + \alpha_2 \tilde{s}_2 \tilde{s}_1 + \alpha_3 \tilde{s}_3 \tilde{s}_1 = -1$$
$$\alpha_1 \tilde{s}_1 \tilde{s}_2 + \alpha_2 \tilde{s}_2 \tilde{s}_2 + \alpha_3 \tilde{s}_3 \tilde{s}_2 = +1$$
$$\alpha_1 \tilde{s}_1 \tilde{s}_3 + \alpha_2 \tilde{s}_2 \tilde{s}_3 + \alpha_3 \tilde{s}_3 \tilde{s}_3 = +1$$

$$\alpha_1 \begin{pmatrix}1\\0\\1\end{pmatrix}\begin{pmatrix}1\\0\\1\end{pmatrix} + \alpha_2 \begin{pmatrix}4\\1\\1\end{pmatrix}\begin{pmatrix}1\\0\\1\end{pmatrix} + \alpha_3 \begin{pmatrix}4\\-1\\1\end{pmatrix}\begin{pmatrix}1\\0\\1\end{pmatrix}$$
$$= \boxed{2\alpha_1 + 5\alpha_2 + 5\alpha_3 = -1}$$

$$\alpha_1 \begin{pmatrix}1\\0\\1\end{pmatrix}\begin{pmatrix}4\\1\\1\end{pmatrix} + \alpha_2 \begin{pmatrix}4\\1\\1\end{pmatrix}\begin{pmatrix}4\\1\\1\end{pmatrix} + \alpha_3 \begin{pmatrix}4\\-1\\1\end{pmatrix}\begin{pmatrix}4\\1\\1\end{pmatrix}$$
$$= \boxed{5\alpha_1 + 18\alpha_2 + 16\alpha_3 = +1}$$

$$\alpha_1 \begin{pmatrix}1\\0\\1\end{pmatrix}\begin{pmatrix}4\\-1\\1\end{pmatrix} + \alpha_2 \begin{pmatrix}4\\1\\1\end{pmatrix}\begin{pmatrix}4\\-1\\1\end{pmatrix} + \alpha_3 \begin{pmatrix}4\\-1\\1\end{pmatrix}\begin{pmatrix}4\\-1\\1\end{pmatrix}$$
$$= \boxed{5\alpha_1 + 16\alpha_2 + 18\alpha_3 = +1}$$

→ Solving these three Simultaneous equation with three unknowns yields the values:

$\alpha_1 = -2.44$
$\alpha_2 = 0.3889$
$\alpha_3 = 0.3889$

The optimal hyperplane is given as:-

$$w = \sum_{1}^{3} \alpha_i \times \tilde{S}_i$$

$$= -2.44 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + 0.388 \begin{pmatrix} 4 \\ 1 \\ 1 \end{pmatrix} + 0.388 \begin{pmatrix} 4 \\ -1 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} -2.44 \\ 0 \\ -2.44 \end{pmatrix} + \begin{pmatrix} 1.552 \\ 0.388 \\ 0.388 \end{pmatrix} + \begin{pmatrix} 1.552 \\ -0.388 \\ 0.388 \end{pmatrix}$$

$$w = \begin{pmatrix} 0.66 \\ 0 \\ -1.68 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix}$$

if $w \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ line is parallel to y-axis

if $w \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ line is parallel to x-axis

if $w \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ line will make 45° with respect to x-axis and y-axis.

So now $w = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ So line is parallel to y axis.

Semester : VI | Subject :Machine Learning | Academic Year: 2023 - 2024

→ Finally remembering that our vectors are augmented with a bias.

→ We can equate the last entry in $\tilde{w}$ as the hyperplane offset b and write the seperating Hyperplane equation = $y = mx + b$

with $w = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $b = -2$