$\underline{z=3}$

$2 = 4B$

$\Rightarrow \boxed{B = 1/2}$

$\underline{z=-1}$

$-2 = -4A$

$\Rightarrow \boxed{A = 1/2}$

$$\frac{z-1}{z^2-2z+3} = \frac{1}{2(z+1)} + \frac{1}{2(z-3)}$$

$f(z)$ is not analytic in $z=-1$ & $z=3$.

$\therefore$ The possibilities are $|z|<1$, $|z|>1$, $|z|<3$ & $|z|>3$.

$\Rightarrow$ (i) $|z|<1$

(ii) $1<|z|<3$

iii) $|z|>3$.

(i) $|z|<1$

$\Rightarrow |z|<3$

$$f(z) = \frac{1}{2(1+z)} + \frac{1}{2\times 3\,(z/3 - 1)}$$

$$= \frac{1}{2(1+z)} - \frac{1}{6(1-z/3)}$$

$$= \frac{1}{2}(1+z)^{-1} - \frac{1}{6}(1-z/3)^{-1}$$

$$= \frac{1}{2}(1 - z + z^2 - z^3 + \cdots) - \frac{1}{6}(1 + z/3 + (z/3)^2 + (z/3)^3 + \cdots)$$

ii) $1 < |z| < 3$

$\Rightarrow |z| > 1$ & $|z| < 3$

$$f(z) = \frac{1}{2z(1+\frac{1}{z})} + \frac{1}{2\times 3\,(z/3 - 1)}$$

$$= \frac{1}{2z}\left(1+\frac{1}{z}\right)^{-1} - \frac{1}{6}(1 - z/3)^{-1}$$

$$= \frac{1}{2z}\left(1 - \frac{1}{z} + \left(\frac{1}{z}\right)^2 - \left(\frac{1}{z}\right)^3 + \cdots\right)$$
$$- \frac{1}{6}\left(1 + z/3 + (z/3)^2 + (z/3)^3 + \cdots\right)$$

iii) $|z|>3$

$\Rightarrow |z|>1$

$$\therefore f(z) = \frac{1}{2(z+1)} + \frac{1}{2(z-3)}$$

$$= \frac{1}{2z\left(1+\frac{1}{z}\right)} + \frac{1}{2\times 3(1-3/z)}$$

$$= \frac{1}{2z}(1+1/z)^{-1} + \frac{1}{6}(1-3/z)^{-1}$$

$$= \frac{1}{2z}\left(1-\frac{1}{z}+\left(\frac{1}{z}\right)^2-\left(\frac{1}{z}\right)^3+\cdots\right)$$

$$+\frac{1}{2z}\left(1+\frac{3}{z}+\left(\frac{3}{z}\right)^2+\left(\frac{3}{z}\right)^3+\cdots\right).$$

HW

1) Expand $f(z) = \dfrac{1}{z^2-3z^2+2z}$ as Laurent's series about $z=0$ for i) $|z|<1$ ii) $1<|z|<2$ iii) $|z|>2$.