Semester : **III**   Subject : **DSGT**   Academic Year: 2022- 2023

* Warshalls algorithm -

More efficient method for computing the transitive closure of a relation.

e.g. Find transitive closure of the relation using warshalls algorithm

$$R = \{ (1,4)(2,1)(2,3)(3,1)(3,4)(4,3) \}$$
on set $A = \{1,2,3,4\}$

$\Rightarrow$ Given $R = \{ (1,4)(2,1)(2,3)(3,1)(3,4)(4,3) \}$

$$P_0 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 0 & 0 & 1 \\ 2 & 1 & 0 & 1 & 0 \\ 3 & 1 & 0 & 0 & 1 \\ 4 & 0 & 0 & 1 & 0 \end{array}$$

① First iteration, look 1st column & 1st row

column	row
C	R
$\{ 2,3 \}$	$\{ 4 \}$

now $C \times R = \{ (2,4)(3,4) \}$

$$P_1 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 0 & 0 & 1 \\ 2 & 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 0 & 1 \\ 4 & 0 & 0 & 1 & 0 \end{array}$$

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(ii) second iteration, look 2nd column & 2nd row

column   row

C   R

$\{\ \}$ $\{1,3,4\}$

now $C \times R = \{\ \}$

	1	2	3	4
$P_2 = 1$	0	0	0	1
2	1	0	1	1
3	1	0	0	1
4	0	0	1	0

(iii) Third iteration, look 3rd col & 3rd row

column C   row R

$\{2,4\}$ $\{1,4\}$

$C \times R = \{(2,1)(2,4)(4,1)(4,4)\}$

	1	2	3	4
$P_3 = 1$	0	0	0	1
2	1	0	1	1
3	1	0	0	1
4	1	0	1	1

(iv) fourth iteration, look 4th col & 4th row

Column C   row R

$\{1,2,3,4\}$ $\{1,3,4\}$

$C \times R = \{(1,1)(1,3)(1,4)(2,1)(2,3)(2,4)\\(3,1)(3,3)(3,4)(4,1)(4,3)(4,4)\}$

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$$P_4 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 0 & 1 & 1 \\ 2 & 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 1 & 1 \\ 4 & 1 & 0 & 1 & 1 \end{array}$$

This is transitive closure matrix.

$R = \{ (1,1)(1,3)(1,4)(2,1)(2,3)(2,4),$
$(3,1)(3,3)(3,4)(4,1)(4,3)(4,4) \}$

R^∞ or R^*