* Naive Bayesian Classification *

→ Classify unlabeled sample $x = \langle$ Red, SUV, Domestic $\rangle$ by using Bayesian classification (Naive Bayes classifies) for following data set.

→ {We have a Red Domestic SUV and need to predict that whether it is stolen or not}.

carno.	colour	type	origin	stolen ← class attribute.
1	Red	sport	Domestic	Yes.
2	Red	sport	Domestic	No.
3	Red	sport	Domestic	Yes.
4	Yellow	sport	Imported	No.
5	Yellow	sport	Imported	Yes.
6	Yellow	SUV	Imported	No.
7	Yellow	SUV	Domestic	Yes.
8	Yellow	SUV	Imported	No.
9	Red	SUV	Imported	No.
10	Red	sport	Imported	Yes.

Solution :- Calculate the probability of each class. (It can be computed based on training tuples).

$$P(\text{stolen} = \text{Yes}) = \frac{5}{10} = 0.5$$

$$P(\text{Stolen} = \text{No}) = \frac{5}{10} = 0.5.$$

Step 2 :-

Now compute the probability of each column with respect to class attribute.

{ we compute the conditional probabilities }.

$P(\text{colour} = \text{Red} \mid \text{stolen} = \text{Yes}) = \frac{3}{5}$

$P(\text{colour} = \text{Red} \mid \text{stolen} = \text{No}) = \frac{2}{5}$.

$P(\text{colour} = \text{Yellow} \mid \text{stolen} = \text{Yes}) = \frac{2}{5}$

$P(\text{colour} = \text{Yellow} \mid \text{stolen} = \text{No}) = \frac{3}{5}$

$P(\text{type} = \text{sport} \mid \text{stolen} = \text{Yes}) = \frac{4}{5}$

$P(\text{type} = \text{sport} \mid \text{stolen} = \text{No}) = \frac{2}{5}$.

$P(\text{type} = \text{suv} \mid \text{stolen} = \text{Yes}) = \frac{1}{5}$

$P(\text{type} = \text{suv} \mid \text{stolen} = \text{No}) = \frac{3}{5}$.

$P(\text{origin} = \text{domestic} \mid \text{stolen} = \text{Yes}) = \frac{2}{5}$.

$P(\text{origin} = \text{domestic} \mid \text{stolen} = \text{No}) = \frac{3}{5}$

$P(\text{origin} = \text{imported} \mid \text{stolen} = \text{Yes}) = \frac{3}{5}$

$P(\text{origin} = \text{imported} \mid \text{stolen} = \text{No}) = \frac{2}{5}$.

Using these probabilities, we obtain,

~~P(X | stolen = Red) = P(Red | Yes)~~

Given, $X = (\text{colour} = \text{Red}, \text{type} = \text{SUV}, \text{origin} = \text{domestic})$

$$P(X \mid \text{stolen} = \text{Yes}) = P(\text{Red} \mid \text{Yes}) \times P(\text{SUV} \mid \text{Yes}) \times P(\text{domestic} \mid \text{Yes})$$

$$= \frac{3}{5} \times \frac{1}{5} \times \frac{2}{5}$$

$$= 0.6 \times 0.2 \times 0.4$$

$$\boxed{P(X \mid \text{Yes}) = 0.048}$$

Similarly,

$$P(X \mid \text{stolen} = \text{No}) = P(\text{Red} \mid \text{No}) \times P(\text{SUV} \mid \text{No}) \times P(\text{domestic} \mid \text{No})$$

$$= \frac{2}{5} \times \frac{3}{5} \times \frac{3}{5}$$

$$= 0.4 \times 0.6 \times 0.6$$

$$= 0.144$$

$$\boxed{P(X \mid \text{No}) = 0.144}$$

To find the class C_i, that maximize $P(X \mid C_i) \cdot P(C_i)$, we compute,

$$P(X \mid \text{stolen} = \text{Yes}) \times P(\text{stolen} = \text{Yes}) = 0.048 \times 0.5 = \underline{0.024}$$

$$P(X \mid \text{stolen} = \text{No}) \times P(\text{stolen} = \text{No}) = 0.144 \times 0.5 = 0.072$$

$P(X \mid \text{No}) > P(X \mid \text{Yes})$ Hence the Bayesian classifies predicts stolen = No for the tuple X.

{ Ans for the given tuple will be "No" }.