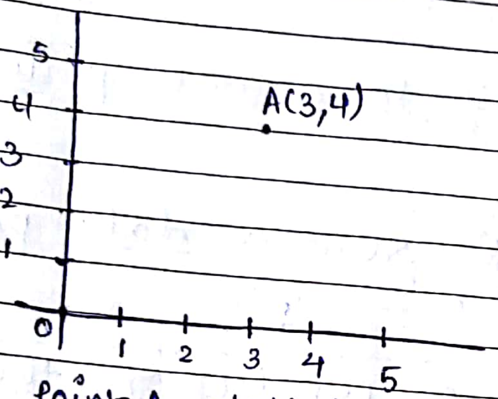
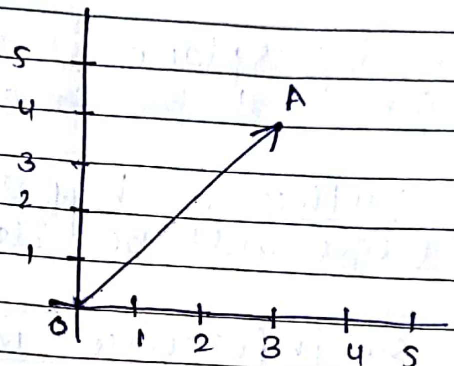
Basics of Vectors →

① Vector →

→ A vector is an object that has both magnitude and direction.



Point A plotted in R^2 plane



Vector $\overrightarrow{OA}$.

Any point $x = (x_1, x_2)$, $x \neq 0$ in R^2 specifies a vector in the plane. {vector starting at origin and ending at x}.

② Norm of Vector → (Magnitude) or (length)

The magnitude or length of vector x is written as $\|x\|$ and is called its norm.

For vector $\overrightarrow{OA}$, $\|OA\|$ is the length of the segment $\overline{OA}$.

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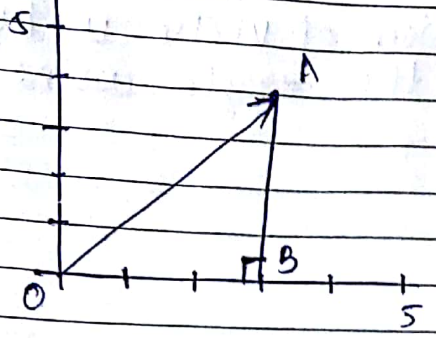


Using Pythagoras theorem

$OA^2 = OB^2 + AB^2$
$= 3^2 + 4^2$
$OA = \sqrt{25}$
$\|OA\| = 5$

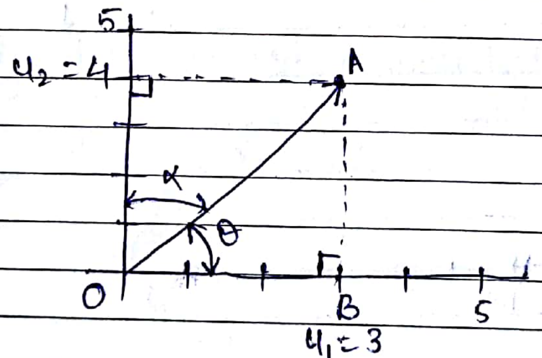
③ Direction of vector →

Direction is the second component of a vector.

→ Direction of a vector $u(u_1, u_2)$ is the vector $w\left(\frac{u_1}{\|u\|}, \frac{u_2}{\|u\|}\right)$.

→ To find the direction of a vector, we need to use its angles.

The direction of the ~~angle~~ vector is defined by angle θ with respect to the horizontal axis and angle α with respect to the vertical axis.



→ The cosine of the angles is used to compute the direction.

In a right angle triangle, the cosine of an angle β is defined by

$$\cos(\beta) = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$

So, we can define the direction of vector u by the cosine of angle θ and the ~~angle~~ cosine of angle α as:

$$\cos(\theta) = \frac{u_1}{\|u\|}$$

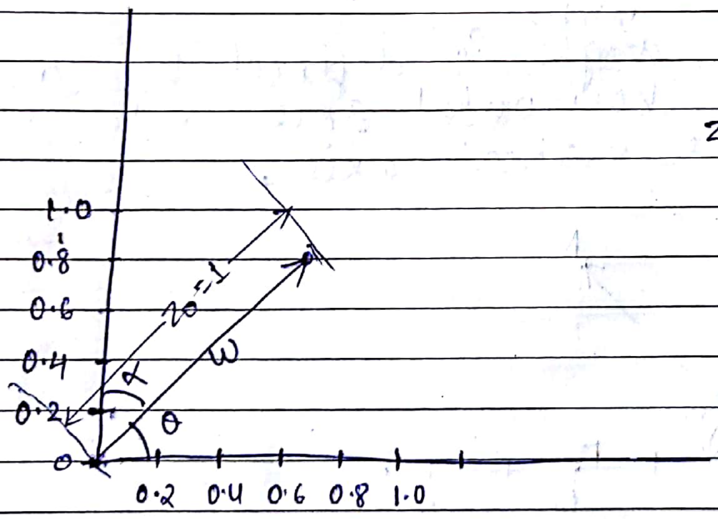
$$\cos(\alpha) = \frac{u_2}{\|u\|}$$

Hence the original definition of ~~the~~ vector w. That is why its coordinates are also called direction cosines.

For $\vec{OA}$ = $\cos(\theta) = \frac{u_1}{\|u\|} = \frac{3}{5} = 0.6$

$\cos(\alpha) = \frac{u_2}{\|u\|} = \frac{4}{5} = 0.8$

→ The direction of u(3,4) is the vector w(0.6, 0.8).



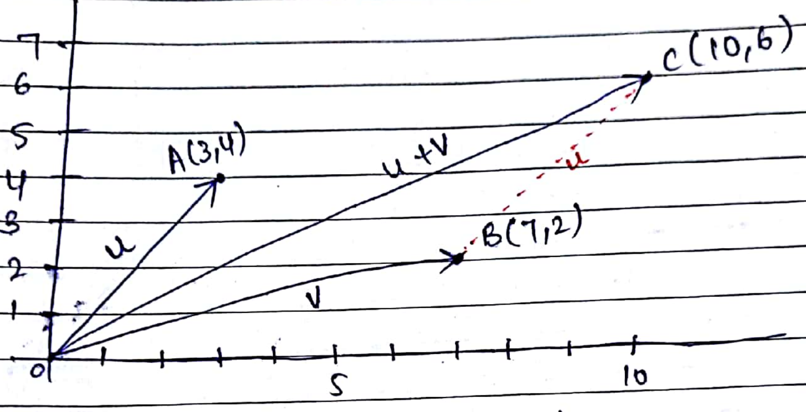
$$z_0 = \sqrt{(0.6)^2 + (0.8)^2}$$
$$= 1$$

w has same look as u except that it is smaller. Something interesting about the direction vector like w is that their norm is equal to 1. That's why they are also called unit vectors.

④ Sum of Two vectors →

$u(u_1, u_2)$ $\quad$ $v(v_1, v_2)$

$u+v = (u_1+v_1, \; u_2+v_2)$



sum of two vectors

⑤ Difference between two vectors →

$u(u_1, u_2)$ $\quad$ $v(v_1, v_2)$

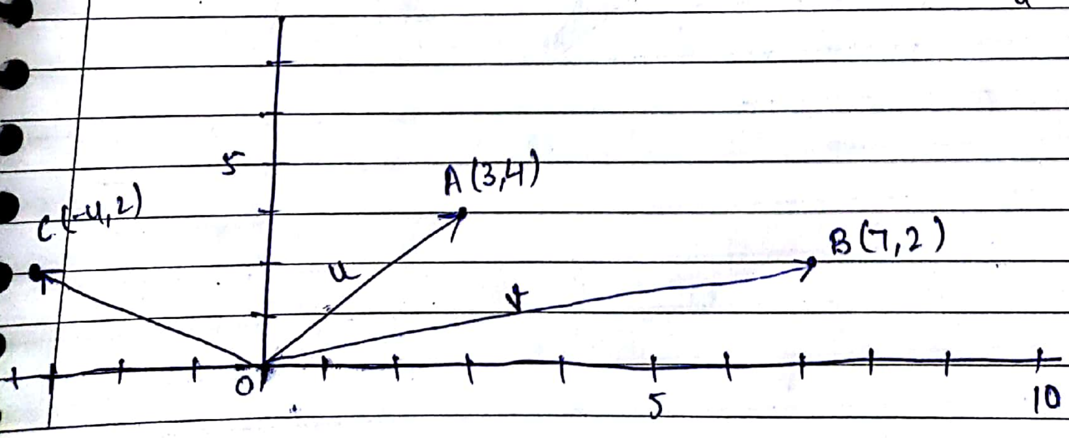
$(u-v) = (u_1-v_1, \; u_2-v_2)$

Since the subtraction is not commutative, we can also consider the case:

$v-u = (v_1-u_1, \; v_2-u_2)$

$u = A(3,4)$ $\quad$ $v = B(7,2)$

$u-v = (-4,2)$



⑥ Dot Product →

If we have two vectors x and y and there is an angle θ between them, then their dot product is given by :

$$x \cdot y = \|x\| \, \|y\| \cos(\theta)$$