Semester : IV Subject : Statistics for M.L.DS Academic Year: 2022-2024

MULTIPLE LINEAR REGRESSION IN MATRIX FORM :

$$Y_i = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + e_i$$

Find the coefficient of Regression in Matrix form from the data below:

Y	9	10	13	14	16
X_1	1	3	4	6	7
X_2	10	14	15	18	20

Solution:

$$\beta = (X^T X)^{-1} X^T Y$$

$$Y = \begin{bmatrix} 9 \\ 10 \\ 13 \\ 14 \\ 16 \end{bmatrix}, \quad X = \begin{bmatrix} 1 & 1 & 10 \\ 1 & 3 & 14 \\ 1 & 4 & 15 \\ 1 & 6 & 18 \\ 1 & 7 & 20 \end{bmatrix}$$

$$X^T = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 3 & 4 & 6 & 7 \\ 10 & 14 & 15 & 18 & 20 \end{bmatrix}$$

$$X^T X = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 3 & 4 & 6 & 7 \\ 10 & 14 & 15 & 18 & 20 \end{bmatrix} \begin{bmatrix} 1 & 1 & 10 \\ 1 & 3 & 14 \\ 1 & 4 & 15 \\ 1 & 6 & 18 \\ 1 & 7 & 20 \end{bmatrix}$$

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$$= \begin{bmatrix} 5 & 21 & 77 \\ 21 & 111 & 360 \\ 77 & 360 & 1245 \end{bmatrix} = X^T X.$$

$$X^T Y = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 3 & 4 & 6 & 7 \\ 10 & 19 & 15 & 13 & 20 \end{bmatrix} \begin{bmatrix} 9 \\ 10 \\ 13 \\ 14 \\ 16 \end{bmatrix} = \begin{bmatrix} 62 \\ 287 \\ 997 \end{bmatrix}$$

$$X^T X = \begin{bmatrix} 5 & 21 & 77 \\ 21 & 111 & 260 \\ 77 & 360 & 1245 \end{bmatrix}$$

$(X^T X)^{-1} = \frac{1}{\det(X^T X)}$. Adjunct of $X^T X$.

Adjunct of $X^T X$ = Cofactor of $[X^T X)$

$\det |X^T X| = 51.$

Cofactor of $X^T X = \begin{bmatrix} 8595 & 1575 & -987 \\ 1575 & 296 & -183 \\ -987 & -183 & 114 \end{bmatrix}$

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$$(X^TX)^{-1} = \frac{1}{51}\begin{bmatrix} 8595 & 1575 & -987 \\ 1575 & 296 & -183 \\ -987 & -183 & 114 \end{bmatrix}$$

$$B = (X^TX)^{-1} \cdot X^TY$$

$$\beta = \frac{1}{51}\begin{bmatrix} 8595 & 1575 & -987 \\ 1575 & 296 & -183 \\ -987 & -183 & 114 \end{bmatrix}\begin{bmatrix} 62 \\ 287 \\ 997 \end{bmatrix} = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{bmatrix}$$

$$\beta = \frac{1}{51}\begin{bmatrix} 876 \\ 151 \\ -57 \end{bmatrix} = \begin{bmatrix} 17.1765 \\ 2.9608 \\ -1.1176 \end{bmatrix} = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{bmatrix}$$

$\beta_0 = 17.17 \quad \beta_1 = 2.96 \quad \beta_2 = -1.11$

$Y = 17.17 + 2.96X_1 + -1.11X_2$