Ranking systems

Ranking systems are methods used to order a set of items, alternatives, or choices based on some criteria or input from multiple sources, such as individuals' preferences, performance, or evaluations. In various fields—such as social choice theory, decision-making, search engines, sports, and academia—ranking systems are used to provide a consistent and fair way of organizing items or making decisions.

1. Types of Ranking Systems

1.1 Ordinal vs. Cardinal Ranking

- **Ordinal Ranking**: In ordinal ranking, the alternatives are arranged in a strict order of preference or importance. The distances between the ranks do not matter. For example, in a sports competition, first, second, and third places represent an ordinal ranking, but we don't know how much better the first place is than the second.
- **Cardinal Ranking**: In cardinal ranking, not only the order but also the magnitude of differences between the ranks matter. For example, grading students where one student scores 95 and another scores 85 gives us a sense of how much better one is than the other.

1.2 Positional Voting Methods

In voting, different positional ranking systems are used to aggregate the preferences of individuals:

- **Borda Count**: Each voter ranks the alternatives, assigning points to each alternative based on its position in their ranking. The alternative with the highest total score wins. For example, in a 3-alternative election, a voter's first choice may receive 2 points, the second choice 1 point, and the third choice 0 points.
- **Plurality Voting**: Each voter selects their top choice, and the alternative with the most votes wins. This system ignores all rankings beyond the first.
- **Ranked Choice Voting (Instant Runoff Voting)**: Voters rank alternatives, and if no alternative has a majority in the first round, the least-preferred option is eliminated, and the process repeats with the remaining alternatives until one achieves a majority.

1.3 Pairwise Comparison Systems

In these systems, alternatives are compared pairwise (head-to-head) to determine the ranking:

- **Condorcet Method**: This method finds the alternative that wins the most pairwise comparisons against other alternatives. If there is a Condorcet winner (an option that beats every other option in direct comparisons), that option is chosen.
- **Copeland's Method**: Each alternative receives a point for every head-to-head victory and loses a point for every head-to-head defeat. The alternative with the most points is ranked the highest.

1.4 Score-Based Ranking

In score-based systems, each option is given a score or rating, and the alternatives are ranked based on these scores:

- **Approval Voting**: Voters approve or disapprove of each alternative, and the alternative with the most approvals is ranked highest. This system can be seen as a simple score-based method where voters can express approval for more than one option.
- **Range Voting**: Voters assign a numerical score to each alternative within a given range (e.g., 0-10). The alternative with the highest average score wins. This system takes into account the intensity of voters' preferences.

2. Ranking Algorithms in Computational Contexts

2.1 Search Engines (e.g., Google's PageRank)

In search engines, ranking systems are used to order search results based on relevance. The PageRank algorithm is one of the most well-known systems used by Google. It ranks websites based on the number and quality of links pointing to them, with the idea that a website linked to by many high-quality sites is more important.

2.2 Recommender Systems

Recommender systems, such as those used by Netflix or Amazon, use ranking algorithms to suggest items (movies, products) to users based on their past behavior, preferences, and sometimes the behavior of similar users. These systems rely on collaborative filtering, content-based filtering, or hybrid methods to rank and suggest items.

3. Applications of Ranking Systems

3.1 Sports

In sports, teams and individuals are ranked based on their performance. Some common ranking systems include:

- **Elo Rating System**: Used in chess and other competitive games, the Elo rating system ranks players based on their match results against others, adjusting scores dynamically after each match.
- **Round-robin Tournaments**: In some tournaments, every player competes against every other player, and the player with the most victories or points is ranked highest.

3.2 Academic Rankings

In academia, ranking systems are used to evaluate universities, researchers, and research papers:

- **University Rankings**: Universities are ranked based on various criteria such as research output, faculty quality, student satisfaction, and funding. Examples include the **QS World University Rankings** and **Times Higher Education Rankings**.
- **h-index**: A researcher's h-index is a measure of their academic productivity and impact, based on the number of citations of their papers. A higher h-index indicates a higher rank in terms of influence.

3.3 Election and Political Systems

Many political elections use ranking systems to determine winners, especially in cases where there are multiple candidates. Examples include:

- **Majority Runoff Elections**: If no candidate gets more than 50% of the votes, the top two candidates proceed to a second round where the winner is determined by majority vote.
- **Ranked-Choice Voting**: This system allows voters to rank candidates, and rounds of counting are used to eliminate the weakest candidates until one has a majority.

4. Ranking Systems in Social Choice Theory

Ranking systems are central to social choice theory, which explores how to aggregate individual preferences into a collective decision:

Arrow's Impossibility Theorem

In the context of ranking, Arrow's theorem (as discussed in the previous sections) shows that it is impossible to design a ranking system that satisfies all desirable properties (unrestricted domain, Pareto efficiency, non-dictatorship, independence of irrelevant alternatives, and transitivity) for more than two alternatives.

Condorcet Paradox

The Condorcet paradox illustrates how collective preferences can be cyclic, even if individual preferences are not. For example, in a group of voters, A may be preferred to B, B to C, but C may be preferred to A, leading to no clear ranking of alternatives.

5. Challenges in Ranking Systems

- **Manipulability**: Many ranking systems can be manipulated by strategic voting. For instance, in the Borda count, voters may rank a weaker alternative higher to increase the chances of their true preference winning.
- **Fairness and Bias**: Ranking systems can introduce bias, either through the criteria they use (such as search algorithms favoring popular content) or through the way scores are aggregated (such as using averages, which may not capture diversity in preferences).
- **Complexity and Implementation**: More sophisticated ranking systems, such as those involving pairwise comparisons or scoring methods, may be harder to implement and understand, especially when applied in large-scale elections or competitions.

Ranking systems are essential tools in decision-making processes, used in everything from elections to sports to search engines. Each system has strengths and weaknesses, and the choice of ranking method depends on the specific goals and constraints of the situation. However, the existence of paradoxes and impossibility theorems in social choice theory reminds us that no perfect ranking system exists, and trade-offs are inevitable.