Practice Problems

Determine the nature of following propositions-

1. $p \wedge \sim p$
2. $(p \wedge (p \rightarrow q)) \rightarrow \sim q$
3. $[(p \rightarrow q) \wedge (q \rightarrow r)] \wedge (p \wedge \sim r)$
4. $\sim(p \rightarrow q) \vee (\sim p \vee (p \wedge q))$
5. $(p \leftrightarrow r) \rightarrow (\sim q \rightarrow (p \wedge r))$

Solution-1

1. p ⋀ ~p

Last column of the truth table contains only F.

Therefore, given proposition is-

- Contradiction
- Invalid
- Falsifiable
- Unsatisfiable

p	~p	p ∧ ~p
F	T	F
T	F	F

Solution-2

p	q	$p \rightarrow q$	$p \wedge (p \rightarrow q)$	$\sim q$	$(p \wedge (p \rightarrow q)) \rightarrow \sim q$
F	F	T	F	T	T
F	T	T	F	F	T
T	F	F	F	T	T
T	T	T	T	F	F

2. $(p \wedge (p \rightarrow q)) \rightarrow \sim q$

Last column of the truth table contains both T and F.

Therefore, given proposition is-

- Contingency
- Invalid
- Falsifiable
- Satisfiable

Solution-3

3. [(p → q) ∧ (q → r)] ∧ (p ∧ ~r)

Last column of the truth table contains only F.

Therefore, given proposition is-

- Contradiction
- Invalid
- Falsifiable
- Unsatisfiable

p	q	r	p → q	q → r	(p → q) ∧ (q → r)	p ∧ ~r	R
F	F	F	T	T	T	F	F
F	F	T	T	T	T	F	F
F	T	F	T	F	F	F	F
F	T	T	T	T	T	F	F
T	F	F	F	T	F	T	F
T	F	T	F	T	F	F	F
T	T	F	T	F	F	T	F
T	T	T	T	T	T	F	F

Solution-4

p	q	~p	p → q	~(p → q)	p ∧ q	~p ∨ (p ∧ q)	R
F	F	T	T	F	F	T	T
F	T	T	T	F	F	T	T
T	F	F	F	T	F	F	T
T	T	F	T	F	T	T	T

4. ~(p → q) ∨ (~p ∨ (p ∧ q))

Last column of the truth table contains only T.

Therefore, given proposition is-

- Tautology
- Valid
- Unfalsifiable
- Satisfiable

Solution-5

5. $(p \leftrightarrow r) \rightarrow (\sim q \rightarrow (p \wedge r))$

Last column of the truth table contains both T and F.

Therefore, given proposition is-

- Contingency
- Invalid
- Falsifiable
- Satisfiable

p	q	r	~q	p→r	r→p	p↔r	p∧r	~q→(p∧r)	R
F	F	F	T	T	T	T	F	F	F
F	F	T	T	T	F	F	F	F	T
F	T	F	F	T	T	T	F	T	T
F	T	T	F	T	F	F	F	T	T
T	F	F	T	F	T	F	F	F	T
T	F	T	T	T	T	T	T	T	T
T	T	F	F	F	T	F	F	T	T
T	T	T	F	T	T	T	T	T	T

Use the truth table method to verify whether the following formulas are valid, satisfiable or unsatisfiable:

- $(p \to q) \land \neg q \to \neg p$
- $(p \to q) \to (p \to \neg q)$
- $(p \lor q \to r) \lor p \lor q$
- $(p \lor q) \land (p \to r \land q) \land (q \to \neg r \land p)$
- $(p \to (q \to r)) \to ((p \to q) \to (p \to r))$
- $(p \lor q) \land (\neg q \land \neg p)$
- $(\neg p \to q) \lor ((p \land \neg r) \leftrightarrow q)$
- $(p \to q) \land (p \to \neg q)$
- $(p \to (q \lor r)) \lor (r \to \neg p)$