First Order Logic

- known as Predicate logic or First-order predicate logic
- **Objects:** A, B, people, numbers, colors, wars, theories, squares, pits, wumpus
- **Relations: It can be unary relation such as:** red, round, is adjacent, **or n-any relation such as:** the sister of, brother of, has color, comes between
- **Function:** Father of, best friend, third inning of, end of

Basic Elements of FOPL

Constant	1, 2, A, John, Mumbai, cat,....
Variables	x, y, z, a, b,....
Predicates	Brother, Father, >,....
Function	sqrt, LeftLegOf, ...
Connectives	∧, ∨, ¬, ⇒, ⇔
Equality	==
Quantifier	∀, ∃

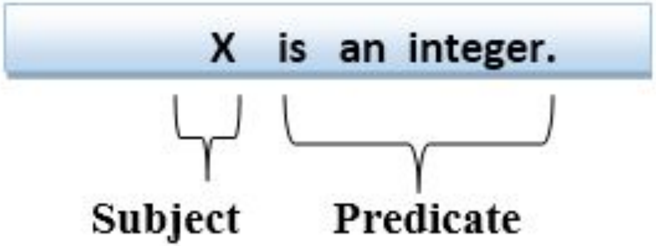
Atomic Sentences and Complex Sentences

- formed from a predicate symbol followed by a parenthesis with a sequence of terms.
- Predicate (term1, term2, ..., term n)
- Example:
  - Ravi and Ajay are brothers: => Brothers(Ravi, Ajay).
  - Chinky is a cat: => cat (Chinky).
- Complex sentences are made by combining atomic sentences using connectives

First Order Logic Statements

First-order logic statements can be divided into two parts:

- Subject: Subject is the main part of the statement.
- Predicate: A predicate can be defined as a relation, which binds two atoms together in a statement.



Quantifiers

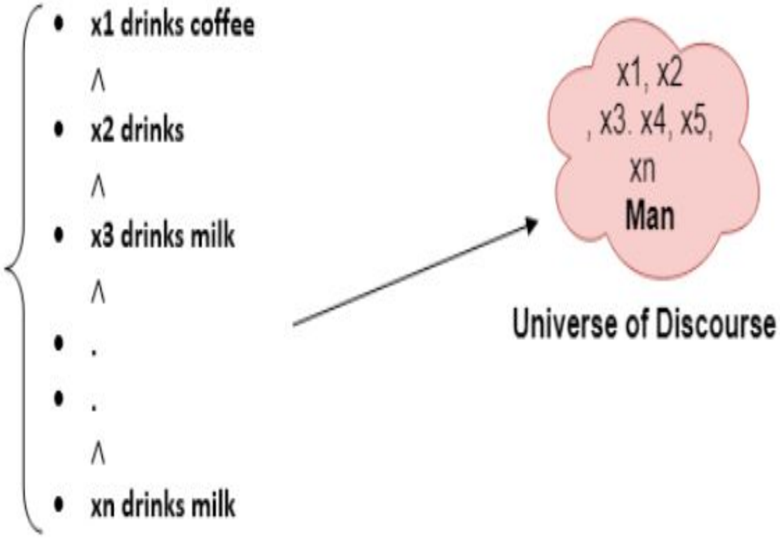
a. Universal Quantifier, (for all, everyone, everything)
b. Existential quantifier, (for some, at least one).

Universal Quantifiers

- The Universal quantifier is represented by a symbol $\forall$
- If x is a variable, then $\forall$x is read as:
    - **For all x**
    - **For each x**
    - **For every x.**
- Example: **All man drink coffee**

  $\forall$x man(x) → drink (x, coffee)

There are all x where x is a man who drink coffee.

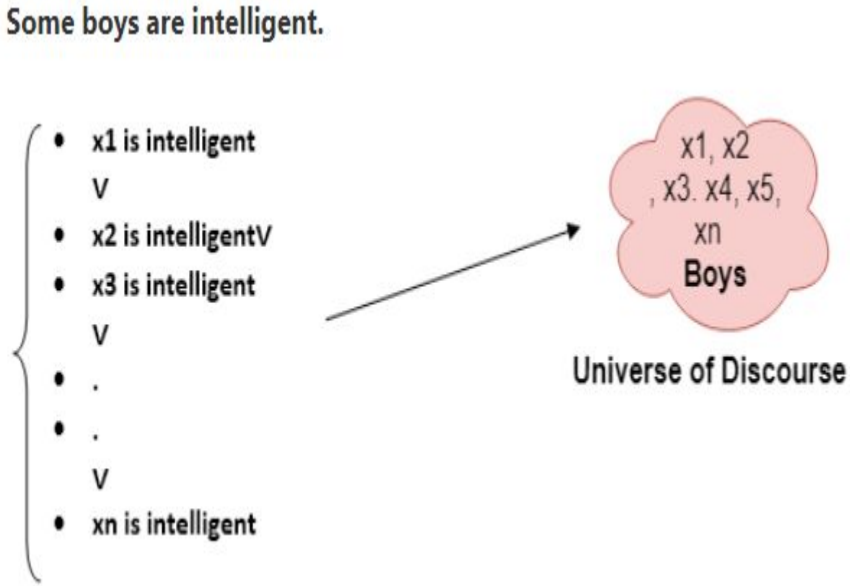


Existential Quantifier

- denoted by the logical operator ∃
- If x is a variable, then existential quantifier will be ∃x or ∃(x). And it will be read as:
  - **There exists a 'x.'**
  - **For some 'x.'**
  - **For at least one 'x.'**
- Example: **Some boys are intelligent.**

  ∃x: boys(x) ∧ intelligent(x)

There are some x where x is a boy who is intelligent.



Points to remember:

- The main connective for universal quantifier $\forall$ is implication $\rightarrow$.
- The main connective for existential quantifier $\exists$ is and $\wedge$.

Properties of Quantifiers:

- In universal quantifier, $\forall x \forall y$ is similar to $\forall y \forall x$.
- In Existential quantifier, $\exists x \exists y$ is similar to $\exists y \exists x$.
- $\exists x \forall y$ is not similar to $\forall y \exists x$.

Examples

1. All birds fly

$\forall x\ bird(x) \rightarrow fly(x)$

2. Every man respects his parent.

$\forall x\ man(x) \rightarrow respects\ (x, parent).$

3. Some boys play cricket.

$\exists x\ boys(x) \land play(x, cricket).$

4. All students are smart.

$\forall x\ (Student\ (x) \Rightarrow Smart\ (x))$

5. There exists a student.

$\exists x\ Student(x)$

Examples

6. There exists a smart student.

$\exists x\, (Student(x)\ \wedge\ Smart\,(x))$

7. All grass is green

$\forall (y)(Grass(y) \rightarrow Green(y))$

8. There exist some friends which are not perfect

$\exists x\, (F(x)\ \wedge\ \neg P(x))$

9. There are some people who are not my friend and are perfect

$\exists x\, (\neg F(x)\ \wedge\ P(x))$

Examples

10. There exist some people who are not my friend and are not perfect.

$\exists x\ (\neg F(x) \land \neg P(x))$

11. There doesn't exist any person who is my friend and perfect.

$\neg \exists x\ (F(x) \land P(x))$

12. "There exist some numbers which are either real OR rational"

$\exists x\ (real(x) \lor rational(x))$

13. "All real numbers are rational"

$\forall x\ (real(x) \rightarrow rational(x))$

Examples

14. There exist some numbers which are both real AND rational

$\exists x\ (real(x) \land rational(x))$

CNF (Conjunctive Normal Form)

A formula which is equivalent to a given formula and which consists of a product of elementary sums is called a conjunctive normal form of given formula.

$(P{\sim} \vee Q) \wedge (Q \vee R) \wedge ({\sim} P \vee Q \vee {\sim} R)$