~~Addition moduto operation:-~~

Semester : III   Subject : DSGT   Academic Year: 20 22 20 23

* Prove that the set $G = \{0, 1, 2, 3, 4, 5\}$ is an abelian group of order 6 with respect to addition modulo 6.

Composition table ⇒

$+_6$	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	2	3	4	5	0
2	2	3	4	5	0	1
3	3	4	5	0	1	2
4	4	5	0	1	2	3
5	5	0	1	2	3	4

$$\left.\begin{array}{l} 0+0 \bmod 6 = 0 \\ 0+1 \bmod 6 = 1 \\ 0+2 \bmod 6 = 2 \\ 0+3 \bmod 6 = 3 \\ 0+4 \bmod 6 = 4 \\ 0+5 \bmod 6 = 5 \end{array}\right\} 1^{st} \text{ row}$$

similarly we can calculate other rows

i) All the entries in the composition table are elements of the set G.
Hence G is closed to the operation; ~~to~~ i.e. we can say that G is closed with respect to addition modulo 6. $(+_6)$

ii) The composition table is associative. If a, b, c are three elements of G, then
$a +_6 (b +_6 c) = (a +_6 b) +_6 c$.
$a = 1, \; b = 2, \; c = 3$
then by associativity property,

Semester: III Subject: DSGT Academic Year: 2022-2023

$1 +_6 (2 +_6 3) = (1 +_6 2) +_6 3$

$1 +_6 5 = 3 +_6 3$

$0 = 0$

hence $+_6$ is an associative operation.

iii) If a is any element of G, then by identity property.

$a * e = e * a = a$

$2 +_6 0 = 0 +_6 2 = 2$

like $4 +_6 0 = 0 +_6 4 = 4$

hence 0 is an identity element.

iv) by inverse property, $a * b = \underline{e}$

$0 +_6 0 = 0$ | $3 +_6 3 = 0$
$1 +_6 5 = 0$ | $4 +_6 2 = 0$
$2 +_6 4 = 0$ | $5 +_6 1 = 0$

$\because$ e is an identity element

here 0 is an identity element

hence 0, 1, 2, 3, 4, 5 having their inverses as 0, 5, 4, 3, 2, 1 respectively.

v) The composition table is commutative as the corresponding rows and columns in the position are identical.

Semester : III Subject : DSGT Academic Year: 2022-2023

$a * b = b * a$

here, $0 +_6 3 = 3 +_6 0$

$3 = 3$

$4 +_6 2 = 2 +_6 4$

$0 = 0$

hence it is commutative.

vi) The no. of elements in the set $G = 6$.

$\therefore$ $(G, +_6)$ is a finite Abelian group of order 6.

Multiplication Modulo 'p' operation -

A type of multiplication is called as "multiplication modulo p" and it is written as $a \times_p b$ where a and b are any integers and p is fixed positive integer is defined as :

$a \times_p b = r \quad 0 \leq r \leq p$