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Large Sample Test ($n > 30$)

(A) Interval Estimation

Single sample | Two sample

$\bar{X} \pm Z_\alpha \hat{\sigma}_{\bar{x}}$

$|\bar{x}_1 - \bar{x}_2| \pm Z_\alpha S.E.$

$\hat{\sigma}_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

$S.E. = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$

S.E. = Std Error

(B) Testing of claims

Single sample | Two sample

$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$

$Z = \frac{\bar{x}_1 - \bar{x}_2}{SE}$

(C) level of significance (flexibility)

$\alpha = 1\%$ $Z_\alpha = 2.58$

$\alpha = 5\%$ $Z_\alpha = 1.96$ (Assume)

$\alpha = 10\%$ $Z_\alpha = 1.64$

μ = Mean of population

$\bar{X}$ = Mean of sample

σ = Standard Deviation

$\sigma = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n}}$

Sum of squares of deviation from mean

$= \Sigma(x-\bar{x})^2 = \Sigma d^2 - \frac{(\Sigma d)^2}{n}$

Ex.1 Can it concluded that the average lifespan of an indian is more than 70 years. If a random sample of 100 indians has an average lifespan of 71.8 years with s.d. 8.9 years?

Soln:

Large

$n = 100$

$\bar{x} = 71.8$

$\mu = 70$

$\sigma = 8.9$

$\alpha = 5\%$

$\boxed{Z_\alpha = 1.96}$

× H_0 : Null Hypothesis: $\mu = 70$

✓ H_1 : Alternative Hypothesis: $\bar{x} = 71.8$

100

$\bar{x}$

71.8

Popn

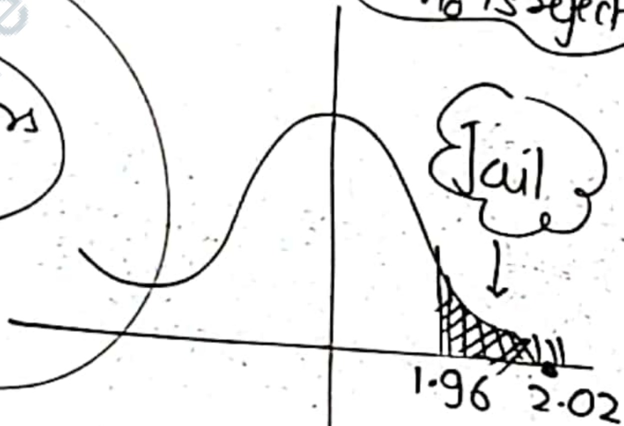
$\mu =$ 70 years

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

$$= \frac{71.8 - 70}{8.9/\sqrt{100}}$$

$$\boxed{Z = 2.02}$$

calculated value shaded portion mein आता है - H_0 is rejected



H_0 is rejected

Average lifespan 71.8 years

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Small sample Test [t-distribution]

$\boxed{n<30}$

(A) Interval Estimation:

Single sample | Two sample

$\bar{X} \pm t \cdot \hat{\sigma}_x$

$\hat{\sigma}_x = \frac{\sigma}{\sqrt{n-1}}$

$|\bar{x}_1 - \bar{x}_2| \pm t \; SE$

$S.E = SP \times \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$

Standard proportion

$SP = \sqrt{\frac{n_1\sigma_1^2 + n_2\sigma_2^2}{n_1+n_2-2}}$

(B) Testing of claims

Single sample | Two sample

$t = \frac{\bar{X} - \mu}{\sigma/\sqrt{n-1}}$

$t = \frac{\bar{x}_1 - \bar{x}_2}{SE}$

(C) Raw data (Actual samples)

i) $\bar{X} = A + \frac{\Sigma d}{n}$

ii) $\Sigma(x-\bar{x})^2 = \Sigma d^2 - \frac{(\Sigma d)^2}{n}$

iii) $\sigma = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n}}$

(D) Degree of freedom

Single sample: $df = n-1$

Two sample: $df = n_1+n_2-2$

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Ex. Ten individuals are chosen at random from a population & their heights are found as 63, 63, 64, 65, 66, 69, 69, 70, 71, 70 inches. Discuss the mean height of population is 65 inches.

$t_{0.05,9} = 2.262$

65

10

Soln a) Raw data $(\bar{x}, \sigma)$

x	63	63	64	65	66	69	69	70	71	70	
$d = x - \overset{A}{65}$	-2	-2	-1	0	1	4	4	5	6	5	$\Sigma d = 20$
$d^2 = (x-65)^2$	4	4	1	0	1	16	16	25	36	25	$\Sigma d^2 = 128$

$\bar{X} = A + \frac{\Sigma d}{n} = 65 + \frac{20}{10} = 67$

$\Sigma(x-\bar{x})^2 = \Sigma d^2 - \frac{(\Sigma d)^2}{n} = 128 - \frac{(20)^2}{10} = 88$

$\sigma = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n}} = \sqrt{\frac{88}{10}} = 2.96$

$n = 10$

$\bar{X} = 67$

$\mu = 65$

$\sigma = 2.96$

$\alpha = 5\% = 0.05$

$^{*}df = n-1 = 10-1 = 9$

$t_\alpha = 2.262$ (Table)

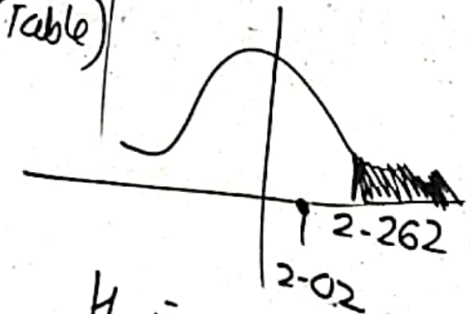
$H_0 : \mu = 65$

$H_1 : \bar{X} = 67$

$t = \frac{\bar{X} - \mu}{\sigma / \sqrt{n-1}}$

$= \frac{67-65}{2.96/\sqrt{9}}$

$t = 2.02$



H_0 is accepted

Mean is 65 inches

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Ex2 A certain injection administered to 12 patients resulted in the following changes in B.P.

5, 2, 8, -1, 3, 0, 6, -2, 1, 5, 0, 4.

can it be concluded that the injection will be in general accompanied by an increase in B.P.

$t_{\alpha, df} = 2.201$

Sol: $n = 12$

x	5	2	8	-1	3	0	6	-2	1	5	0	4	
$d = x - 0$	5	2	8	-1	3	0	6	-2	1	5	0	4	$\Sigma d = 31$
$d^2 = (x-0)^2$	25	4	64	1	9	0	36	4	1	25	0	16	$\Sigma d^2 = 185$

$$\bar{X} = A + \frac{\Sigma d}{n} = 0 + \frac{31}{12} = 2.58$$

$$\Sigma(x-\bar{x})^2 = \Sigma d^2 - \frac{(\Sigma d)^2}{n} = 185 - \frac{(31)^2}{12} = 104.91$$

$$s = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n}} = \sqrt{\frac{104.91}{12}} = 2.96$$

$n = 12$

$\bar{X} = 2.58$

$\mu = 0$ (Assume)

$s = 2.96$

$\alpha = 5\% = 0.05$

$df = n-1 = 12-1 = 11$

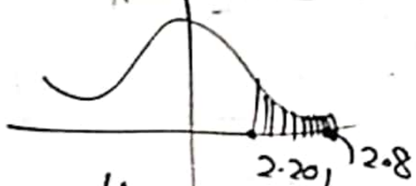
$\times$ $H_0: \mu = 0$

$\checkmark$ $H_1: \bar{X} = 2.58$

$$t = \frac{\bar{X} - \mu}{s/\sqrt{n-1}}$$

$$= \frac{2.58 - 0}{2.96/\sqrt{11}}$$

$$\boxed{t = 2.8}$$



H_0 is rejected

There is increase in B.P.

3. In a lab experiment two samples are given,

Sample	Size	mean	Sum of squares of deviation from mean
1	10	15	90
2	13	14	108

Test the hypothes that the samples are from same population

$t_{\alpha, df} = 2.08$

Sol:

$n_1 = 10 \quad n_2 = 13$

$\bar{x}_1 = 15 \quad \bar{x}_2 = 14$

$\Sigma(x-\bar{x})^2 = 90 \quad \Sigma(x-\bar{x})^2 = 108$

$s_1 = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n_1}} \quad s_2 = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n_2}}$

$\boxed{s_1 = 3} \quad \boxed{s_2 = 2.88}$

$\alpha = 5\% = 0.05$

$df = n_1 + n_2 - 2 = 23 - 2 = 21$

$H_0 : \mu_1 = \mu_2$

$H_1 : \mu_1 \neq \mu_2$

$t = \frac{\bar{x}_1 - \bar{x}_2}{SE}$

$SE = Sp \times \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$

$\sqrt{\frac{1}{n_1} + \frac{1}{n_2}} = 0.42$

$Sp = \sqrt{\frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}} = 3.06$

$\boxed{SE = 1.28}$

$t = \frac{15-14}{1.28} = 0.77$

H_0 is accepted

Samples are from same Pop^n

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Ex 4. Two independent samples of size 8 & 7 gave following results.

Sample 1 : 19 17 15 21 16 18 16 14

Sample 2 : 15 14 15 19 15 18 16

Is the difference between sample mean significant

Sol $n_1 = 8$

x_1	19	17	15	21	16	18	16	14	
$d = x-15$	4	2	0	6	1	3	1	-1	$\Sigma d = 16$
$d^2 = (x-15)^2$	16	4	0	36	1	9	1	1	$\Sigma d^2 = 68$

$$\bar{x}_1 = A + \frac{\Sigma d}{n_1} = 15 + \frac{16}{8} = 17$$

$$\Sigma(x-\bar{x})^2 = \Sigma d^2 - \frac{(\Sigma d)^2}{n_1} = 68 - \frac{(16)^2}{8} = 36$$

$$\sigma_1 = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n_1}} = 2.12$$

$n_2 = 7$

x	15	14	15	19	15	18	16	
$d = x-15$	0	-1	0	4	0	3	1	$\Sigma d = 7$
$d^2 = (x-15)^2$	0	1	0	16	0	9	1	$\Sigma d^2 = 27$

$$\bar{x}_2 = A + \frac{\Sigma d}{n_2} = 15 + \frac{7}{7} = 16$$

$$\Sigma(x-\bar{x})^2 = (\Sigma d)^2 - \frac{(\Sigma d)^2}{n_2} = 20$$

$$\sigma_2 = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n_2}} = \sqrt{\frac{20}{7}} = 1.69$$

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Ex 4. Two independent samples of size 8 & 7 gave following results.

Sample 1 : 19 17 15 21 16 18 16 14

Sample 2 : 15 14 15 19 15 18 16

Is the difference between sample mean significant

Sol

$n_1 = 8 \qquad n_2 = 7$

$\bar{x}_1 = 17 \qquad \bar{x}_2 = 16$

$s_1 = 2.12 \qquad s_2 = 1.69$

$\alpha = 5\% = 0.05$

$df = n_1 + n_2 - 2 = 13$

$\boxed{t = 2.16}$

$H_0 : \mu_1 = \mu_2$

$H_1 : \mu_1 \neq \mu_2$

$t_{\alpha, df} = 2.16$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{S.E}$$

$$SE = Sp \times \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

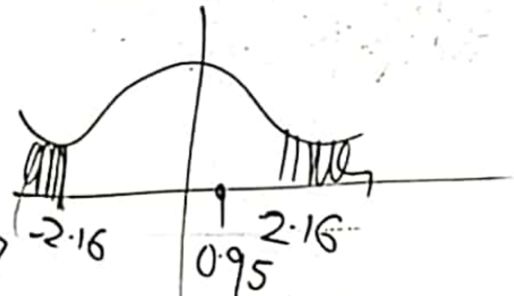
$$\sqrt{\frac{1}{n_1} + \frac{1}{n_2}} = 0.51$$

$$Sp = \sqrt{\frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}} = 2.07$$

$\boxed{SE = 1.05}$

$$t = \frac{17 - 16}{1.05}$$

$\boxed{t = 0.95}$



H_0 is accepted

There is no difference in Mean

Chi-square test (χ^2)

(I) $$\chi^2 = \sum \frac{(O-E)^2}{E}$$

O : observed frequency : Given

E : Expected frequency : To be calculated

Raw data: $E = \frac{\sum f}{n}$

Tabular Data: $E = \frac{R_T \times C_T}{T_F}$

(II) Degree of freedom.

Raw data: $df = n-1$

Tabular Data: $df = (r-1)(c-1)$ (r = Raw, c = Column)

(III)

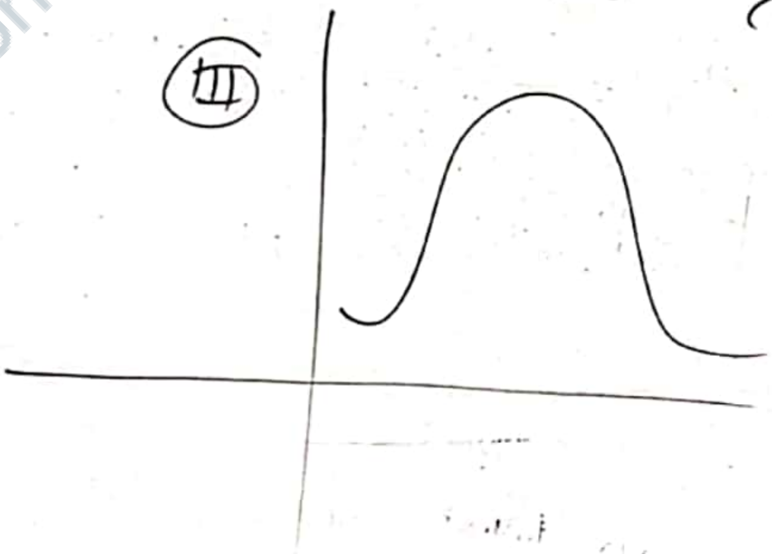


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Table shows the performances of students in mathematics & physics. Test the hypothesis that the performance in maths is independent of performance in physics.

		Grade in maths			
		High	Medium	Low	Total
Grades in physics	High	56	71	$\boxed{12}$	139
	Medium	47	163	38	248
	Low	14	42	81	137
	Total	117	276	131	$\boxed{524}$

Sol^n: (I) Expected frequency $= \dfrac{R_T \times C_T}{T_F}$

E.F. for 56 $= \dfrac{139 \times 117}{524} = 31.03 \approx 31$

EF for 71 $= \dfrac{139 \times 276}{524} = 73.21 \approx 73$

EF for 12 $= \dfrac{139 \times 131}{524} = 34.75 \approx 35$

EF for 47 $= \dfrac{248 \times 117}{524} = 55.37 \approx 55$

EF for 163 $= \dfrac{248 \times 276}{524} = 130.6 \approx 131$

EF for 38 $= \dfrac{248 \times 131}{524} = 61.7 \approx 62$

EF for 14 $= \dfrac{137 \times 117}{524} = 30.58 \approx 31$

EF for 42 $= \dfrac{137 \times 276}{524} = 72.16 \approx 72$

EF for 81 $= \dfrac{137 \times 131}{524} = 34.25 \approx 34$

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Table shows the performances of students in mathematics & physics. Test the hypothesis that the performance in maths is independent of performance in physics.

Sol: (II) Table

O	E	O-E	$(O-E)^2$	$\frac{(O-E)^2}{E}$
56	31	25	625	20.16
71	73	-2	4	0.05
12	35	-23	529	15.11
47	55	-8	64	1.16
163	131	32	1024	7.81
38	62	-24	576	9.29
14	31	-17	289	9.32
42	72	-30	900	12.5
81	34	47	2209	64.97

(III) H_0: Performance is independent

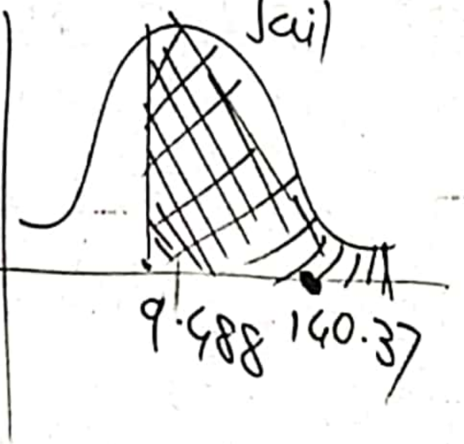
H_1: Performance is dependent

$$\chi^2 = \sum \frac{(O-E)^2}{E} = 140.37$$

$$df = (r-1)(c-1) = (3-1)(3-1)$$
$$= 4$$

$\alpha = 5\% = 0.05$

$\chi^2_{Table} = 9.488$



H_0 is rejected

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IMP

2. Based on the data, can you say that there is no relation between smoking & literacy.

	Smokers	non-smokers	Total
Literates	83	57	140
Illiterates	45	68	113
Total	128	125	253

Sol: (I) Expected frequency

$$E = \frac{R_T \times C_T}{T_F}$$

E.F. for 83 $= \frac{140 \times 128}{253} = 70.83 \approx 71$

EF for 57 $= \frac{140 \times 125}{253} = 69.16 \approx 69$

EF for 45 $= \frac{113 \times 128}{253} = 57.16 \approx 57$

EF for 68 $= \frac{113 \times 125}{253} = 55.83 \approx 56$

(II) Table

O	E	O-E	$(O-E)^2$	$\frac{(O-E)^2}{E}$
83	71	12	144	2.02
57	69	-12	144	2.08
45	57	-12	144	2.52
68	56	12	144	2.57

(III) H_0: No relation

H_1: There is Relation

$$\chi^2 = \frac{\Sigma (O-E)^2}{E}$$

$$= 9.19$$

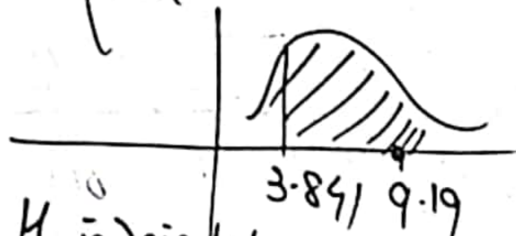
$df = (r-1)(c-1)$

$= (2-1)(2-1)$

$\boxed{df = 1}$

$\alpha = 5\% = 0.05$

$\chi^2_{table} = 3.841$



H_0 is rejected

Hint for Raw data

n=10

Raw: 71 72 69 53 11 29 30 68 71 26

Sol^n (I) Expected frequency

$$E = \frac{\Sigma f}{n} = \frac{500}{10} = 50$$

(II) Table

O	E	(O-E)	$(O-E)^2$
71	50		
72	50		
69	50		
53	50		
11	50		
29	50		
30	50		
68	50		
71	50		
26	50		

$$\chi^2 = \Sigma \frac{(O-E)^2}{E}$$

$$= \frac{\quad}{50}$$

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