Preferences, Payoffs, and Utility

Utility Functions

How to Define and Interpret Utility

- **Utility**: A numerical representation of a player's preferences over a set of outcomes. It allows for the ranking of these outcomes based on the player's level of satisfaction.
- **Utility Function**: A function $U(x)$ that assigns a real number to each possible outcome x. Higher utility values indicate more preferred outcomes.
    - **Example**: For an outcome x, $U(x) = x^2$ might represent the utility.

Interpretation of Utility

- **Ordinal Utility**: The order of preferences matters, but the magnitude of differences between utilities does not.
    - **Example**: If $U(A) = 2$ and $U(B) = 4$, B is preferred over A, but we cannot say B is twice as preferred as A.
- **Cardinal Utility**: The actual numerical values of utility are meaningful, and the differences between them can be interpreted.
    - **Example**: If $U(A) = 2$ and $U(B) = 4$, B is not only preferred over A but is considered twice as preferable.

Examples of Utility Functions

1. **Linear Utility**:
    - $U(x) = ax + b$
    - Represents constant marginal utility.
    - Example: $U(x) = 2x + 3$

2. **Quadratic Utility**:
   - $U(x) = ax^2 + bx + c$
   - Represents diminishing or increasing marginal utility.
   - Example: $U(x) = -x^2 + 4x$
3. **Logarithmic Utility**:
   - $U(x) = \log(x)$
   - Used to model risk-averse behavior.
   - Example: $U(x) = \log(x)$, where x must be positive.

Payoff Matrices (20 minutes)

Constructing Payoff Matrices

- **Definition**: A payoff matrix is a table that shows the payoffs for each player for every possible combination of strategies.
- **Steps to Construct a Payoff Matrix**:
  1. **Identify the Players**: Determine who the decision-makers are.
  2. **List Possible Strategies**: Enumerate the strategies available to each player.
  3. **Determine Payoffs**: Assign payoffs for each combination of strategies.

Examples and Case Studies

- **Example 1: Prisoner's Dilemma**:
  - Two players: Prisoner 1 and Prisoner 2.
  - Strategies: Confess (C) or Stay Silent (S).
  - Payoff matrix:

	Confess (C)	Silent (S)
Confess (C)	(-1, -1)	(0, -3)
Silent (S)	(-3, 0)	(1, 1)

- **Example 2: Battle of the Sexes**:
  - Players: Husband (H) and Wife (W).
  - Strategies: Opera (O) or Football (F).
  - Payoff matrix:

	Opera (O)	Football (F)
Opera (O)	(2, 1)	(0, 0)
Football (F)	(0, 0)	(1, 2)

Expected Utility and Risk

Decision-Making Under Uncertainty

- **Expected Utility Theory**: A framework for making decisions under uncertainty, where outcomes are uncertain and each has a probability associated with it.
- **Expected Utility**: The weighted average of utilities, where the weights are the probabilities of each outcome.

- **Formula**: $E(U) = \sum_i p_i U(x_i)$
- **Example**: If there are two outcomes, x_1 and x_2, with probabilities p_1 and p_2 and utilities $U(x_1)$ and $U(x_2)$:

$$E(U) = p_1 U(x_1) + p_2 U(x_2)$$

Risk Aversion and Expected Utility Theory

- **Risk Aversion**: Preference for certain outcomes over uncertain ones with the same expected value.
  - **Utility Function for Risk-Averse Individuals**: Typically concave, e.g., $U(x) = \log(x)$ or $U(x) = x^{0.5}$.
- **Risk-Neutral**: Indifference to risk, focusing solely on expected value.
  - **Utility Function for Risk-Neutral Individuals**: Typically linear, e.g., $U(x) = x$.
- **Risk-Seeking**: Preference for uncertain outcomes over certain ones with the same expected value.
  - **Utility Function for Risk-Seeking Individuals**: Typically convex, e.g., $U(x) = x^2$.

Examples and Case Studies

1. **Example 1: Lottery Ticket**:
   - Two outcomes: Win $1000 with probability 0.1, and win nothing with probability 0.9.
   - Risk-averse utility function: $U(x) = \sqrt{x}$.
   - Expected Utility: $E(U) = 0.1\sqrt{1000} + 0.9\sqrt{0} = 0.1 \times 31.62 + 0 = 3.162$.
2. **Example 2: Insurance**:
   - A person has a 1% chance of losing $10,000 and a 99% chance of losing nothing.
   - Without insurance, expected loss: $0.01 \times 10000 + 0.99 \times 0 = 100$.
   - Utility function: $U(x) = \log(x)$.
   - With insurance, person pays $100 for coverage.
   - Expected Utility without insurance: $0.01 \log(9000) + 0.99 \log(10000)$.
   - Expected Utility with insurance: $\log(9900)$.

Summary

- **Utility Functions**: Represent preferences and help in making decisions.
- **Payoff Matrices**: Show the payoffs for each combination of strategies in a game.

- **Expected Utility Theory**: Framework for decision-making under uncertainty, considering risk preferences.