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Subject: Mathematics for AI & ML

Optimality of A* seasch -
At has following properties >
D> The tree search version of A* is optimal if h(n) is admissible.
h(n) is admissible.
D) The graph search Version of A* is optimal if h(n) is consistent.
nd · ·
-> lets try to prove the 2nd claim:
If h(n) is consistent, then the values of f(n)
If h(n) is consistent, then the values of f(n) along any path are non-decreasing.
for any node n, and tany succuessor n' of n the following enequality should hold
$h(n) \leq c(n,q,n') + h(n') - A$
performing action a.
performing action a.



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		•
	lets say during our search	O start node
	process, me have reached	
	node n from start node.	
	and the cost for start node to	
	Nøden is gen).	
		(g(n)
	Now consider a node n'	
	which is a successor of	Node n
	noden. Let c(n,a,n) be	Z
		3 c(nain)
	the actual rost to go	
	from n to n'. n(m)	Node n'
	0	Nood
	we also have a goal wode	12
	& ut nen) be the	n(n')
	estimate of cost from	X
	rode n to goal node	Goal Node
	1 let h(n') be the	
	estimate of the cost	
	from node n' to goal rode.	
	V	
	Now, in case of At search we	e know, that the
	Now, in case of At search we evaluation function is	
	0	
	f(n) - g(n) + h(n)	(B)
	V	
Similar	$f(n') = g(n') + h(n') - \cdots$	- (i)
	from the above dig , g(n)=	g(n) + c(n,a,n) - (a)
		g(n) + c(n,a,n) - 2



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(Rychymath Charletta (Faust)

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	in eq" (1) septace value of g(n') by eq" (2)
	f(n') = g(n) + c(n,a,n') + h(n')' = 3
	In the above egh (c(ng,n')+h(n') is a part of enequality of egh (A).
ing"	B, so, now if we replace $C(n,a,n') + h(n')$ with h(n), it means we are replacing with a smaller value since eq (A) h(n) ≤ $C(n,a,n') + h(n')$
	Soing (3) become less than f(n'), mean
	f(n') > g(n) + h(n) G replacing this with eq" (B)
	$f(n') > f(n)$ \rightarrow so along any path if we have a node n of we have a successor n' of n the $f(n') > f(n)$.
	increare or semain une same means et es non-decreasing function,
	/



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,	Now we make another claim -	6
X V		-
**	Sellers a model of expension	-
	the optimal path to me that mode has been	-
ı	found,	
	Proof->	
	Start Node	C
	lets say we have 2 different	Ç
	and a form of a to	C
	Node n. optimie	c
		C
	Now out of these two	c
	parns lets consider our to	c
	be a optimal path of	_C
	other to be a sub-optimal Noden	-6
	Path 10 back 11	-C
	optimal means -> g-value along the path will be	-6
	less than the g-value along the sub-optimal pate	C
	lers than the Jevacue and Jevacue	<u>^</u> c
	Now lets say we have selected node n	(
	through the non-optimal path.	(
	BUT, this cannot happen. why?	C
	Start	0
	That's because there must be a	•
	node n' in the optimal path which is	6
	in the prontier list.	4.
	lets calculate f-values.	~
	D	4



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 f1 = f-value of node n dlong optimal path
f1 = f. value of node n dlong optimal path. f2 = f-value of node n along Sub-optimal path.
f3= f-value of node n'
we have seen
f3 (= f1 non-decreasing?
we will see that
$f_1 < f_2$
f(n) = g(n) + h(n)
hin) ralue doesn't depend on which path we
tollowed to reach upto node n. It is the
estimate to the goal node,
V V
since h-value is same, which f-value (through or through or through) is smaller, will depend on g-value.
or through is smaller, will depend on g-value.
path
Since g value of optimal path is less than the g value of non-optimal path, the f-value calculated for mode in along this optimal path(1) is less than the f-value calculated through sub-optimal path(1). So, f, <f2< th=""></f2<>
of non-optimal path, the f-value calculated for
mode in along this optimal path is less than
the traduce calculated through sub-ops was path.
80, f, <f2< th=""></f2<>
Now Based on, [f3 <= f, and [f, < f2]
We can say.
$f_3 < f_2$
which means, the finallie of mode n' is less than the fivalue of node n calculated along the sub-optima
path.
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Remember, Node n'is in the frontier list,
& A+ algo always selects node to expand
from frontier (open) list, it will select the
node which has the lowest f-value.
so as long as we have node n' in the frontier list, whose f-value is less than
tronlier list, whose f-value is less than &
the f-value of node-n through subophunds
path + then definetly node n' will be
selected instead of mode n. for the expansion
C
So, it can not happen that we have selected
the node on through the sub-optimal path
when we actually have a node n'in
the frontier list.
C C C C C C C C C C C C C C C C C C C
so, we have proved -
1) Et h(n) is consistent, then the values of t(n)
done and noth are non-decreasing.
along any path are non-decreasing. ii) whenever Ax selects a node n for expansion
the stimul path throughout to that node has
been found.
veen fand,
based on above turo claims, we can say
Dased on above to as as a series
The sequence of nodes expanded by At using
The sequence of nodes expanded by At being GRAPH-STARCH is in non-decreasing order, of f(n)
V



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3		Hence, the first goal mode selected for
		expansion must be the optimal solution
13		dans of in the true cost for and nodel
1		(which have h=0) and all later goal nodes
3		(which name h=0) and all later goal nodes will be at least as expensive.
3		
3		If ct is the cost of the optimal solution path, then
3		we can say the following:
		, , ,
		· A* expands all the nodes with f(n) < c*.
1		· At might then expand some of the nodes with
3		f(n) = c+ lectorese before selecting a goal nocle.
	¥	A* is also complete as long as there are only
		finite number of nodes with lost less than or
		eghod to c* (considering all step costs exceed some
		finite & and b is finite)
		b -> branching factor
	*	At is optimally efficient as no other optimal
		algorithm is guaranteed to expand fewer nodes
	,	than A*, A* expands only those nodes with f(n) <= c
		any algorithm that does not expand all nodes with
		At is optimally efficient as no other optimal algorithm is guaranteed to expand fewer nodes than At. At expands only those nodes with fin) <-c' any algorithm that does not expand all nodes with f(n) < c' ours the risk of missing the optimal
		espation.