Small sample tests

students t- distribution

It is used when i) the sample size is 30 or less ii) σ is unknown

$$t = \frac{\bar{x} - \mu}{s / \sqrt{n-1}}$$

$$s = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n-1}}$$

with $(n-1)$ degrees of freedom.

Remark:-

* If σ is known then $z = \dfrac{\bar{x} - \mu}{\sigma / \sqrt{n}}$

* If σ is not known & the parent population is normal, then

$$t = \frac{\bar{x} - \mu}{s / \sqrt{n}} \qquad s^2 = \frac{\Sigma(x_i - \bar{x})^2}{n-1}$$

* If σ is unknown then

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n-1}} \qquad s^2 = \frac{\Sigma (x_i - \bar{x})^2}{n}$$

① Nine items of a sample had the following values 45, 47, 50, 52, 48, 47, 49, 53, 51. Does the mean of 9 items differ significantly from the assumed population mean 47.5.

Soln:-

X :	45	47	50	52	48	47	49	53	51	
d_i $x_i - 48$:	-3	-1	2	4	0	-1	1	5	3	10
d_i^2	9	1	4	16	0	1	1	25	9	66

$$\bar{x} = a + \frac{\Sigma d_i}{n}$$

$$= 48 + \frac{10}{9}$$

$$= 49.11$$

$$\Sigma(x_i-\bar{x})^2 = \Sigma d_i^2 - \frac{(\Sigma d_i)^2}{n}$$

$$= 66 - \frac{100}{9} = 54.89.$$

$$s^2 = \frac{\Sigma(x_i-\bar{x})^2}{n}$$

$$= \frac{54.89}{9} = 6.099.$$

(1) Null hypothesis : $\mu_1 = \cancel{\mu}\ 47.5$

Alternative hypothesis :- $\mu \neq 47.5$

(2) Test statistic.

$$t = \frac{\bar{x}-\mu}{s/\sqrt{n-1}} \quad (n<30)$$

$$= \frac{49.11 - 47.5}{\sqrt{6.099}/\sqrt{8}}$$

$$|t| = 1.84$$

③ Level of significance

$\alpha = 0.05$

④ Critical value :-

The value of z_α at 5% level of significance for $n-1 = 9-1 = 8$ degrees of freedom

$|z_\alpha| =$

⑤ Decision :-

$|t| < |t_\alpha|$.

$\therefore$ We accept the hypothesis.

$\therefore$ The items does not differ significantly from mean.

HW

② Tests made on breaking strength of 10 pieces of a metal wire gave the follg. results

578, 572, 570, 568, 572, 570, 570, 572, 596 &

5&6. Test if the breaking strength of the metal in (kgs) wire can be assumed to be 577 kg.
(Soln :- Yes ; $|t| = 0.65$

Testing the Difference b/w means

(a) Independent samples

$$S_p = \sqrt{\frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}}.$$

$$SE = S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{SE}.$$

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① A sample of 8 students of 16 yrs each shown up a mean systolic blood pressure of 118.4 mm of Hg with sd of 12.17 mm while a sample of 10 students of 17 yrs each showed the mean systolic BP of 121.0 mm with sd of 12.88 during an investigation. The investigator feels that the systolic BP is related to age. Do you think that the data provides enough reasons to support investigators feeling at 5% LOS.

Soln:- $n_1 = 8$ $\bar{x}_1 = 118.4$ $s_1 = 12.17$

$n_2 = 10$ $\bar{x}_2 = 121.0$ $s_2 = 12.88$

(1) Null hypothesis: $\mu_1 = \mu_2$

Alternative hypothesis:- $\mu_1 \neq \mu_2$.

(2) Test statistic

$$S_p = \sqrt{\frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}}$$

$$= \sqrt{\frac{8(12.17)^2 + 10(12.88)^2}{10 + 8 - 2}}$$

$$= 13.33$$

$$SE = S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$= 13.33 \sqrt{\frac{1}{8} + \frac{1}{10}} = 6.32$$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{SE}$$

$$= \frac{118.4 - 121.0}{6.32} = -0.41$$

$$|t| = 0.41$$

(3) Level of Significance

$\alpha = 0.05$.

(4) Critical value:-

$|t_\alpha|$ at $\alpha = 0.05$ for $n-1 = n_1 + n_2 - 2$

$= 16$ dof is $|t_\alpha| = 2.12$.

(5) Decision:-

$|t| < |t_\alpha|$

$\therefore$ We accept the hypothesis.