$$= L^{-1}\left[\frac{6s-12+12-4}{(s-2)^2+4^2}\right]$$

$$= L^{-1}\left[\frac{6(s-2)+8}{(s-2)^2+4^2}\right]$$

$$= e^{2t} \cdot L^{-1}\left[\frac{6s+8}{s^2+4^2}\right]$$

$$= e^{2t}\left[L^{-1}\left[\frac{6s}{s^2+4^2}\right] + 8 \cdot L^{-1}\left[\frac{1}{s^2+4^2}\right]\right]$$

$$= e^{2t}\left[6 \cdot \cos 4t + 8 \cdot \frac{1}{4}\sin 4t\right]$$

$$= e^{2t}\left[6\cos 4t + 2\sin 4t\right].$$

- We can use above method only when, we have factors of denominator as not in integer form. otherwise we will use method of partial fraction.

Method of Partial Fractions:

one can use method of partial fraction directly when degree of polynomial in numerator is less than the degree of polynomial in denominator.

How to apply partial fraction:

If $\phi(s)$ has following form then we can express as below,

1) $\phi(s) = \dfrac{F(s)}{(s+a)(s+b)} = \dfrac{A}{s+a} + \dfrac{B}{s+b}$.

2) $\phi(s) = \dfrac{F(s)}{(s+a)(s+b)^2} = \dfrac{A}{s+a} + \dfrac{B}{s+b} + \dfrac{C}{(s+b)^2}$.

3) $\phi(s) = \dfrac{F(s)}{(s+a)(s^2+b^2)} = \dfrac{A}{s+a} + \dfrac{Bs+C}{s^2+b^2}$.

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Examples :

1] Find $L^{-1}\left[\dfrac{3s+7}{s^2+2s-3}\right]$

Sol^n: consider, $\dfrac{3s+7}{s^2+2s-3} = \dfrac{3s+7}{(s-3)(s+1)}$.

Using partial fractions,

$$\frac{3s+7}{(s-3)(s+1)} = \frac{A}{(s-3)} + \frac{B}{s+1}$$

$$\therefore \quad 3s+7 = A(s+1) + B(s-3)$$

put $s=3$, $9+7 = 4A \Rightarrow 16 = 4A \Rightarrow A = 4$

put $s=-1$, $-3+7 = -4B \Rightarrow 4 = -4B \Rightarrow B = -1$

$$\therefore L^{-1}\left[\frac{3s+7}{s^2+2s-3}\right] = L^{-1}\left[\frac{4}{s-3} + \frac{(-1)}{s+1}\right]$$

$$= 4e^{3t} - e^{-t}.$$

2] Find $L^{-1}\left[\dfrac{3s-7}{s^2-6s+8}\right]$

Sol^n: consider, $\dfrac{3s-7}{s^2-6s+8} = \dfrac{3s-7}{(s-4)(s-2)}$

$$\therefore \frac{3s-7}{(s-4)(s-2)} = \frac{A}{(s-4)} + \frac{B}{(s-2)}$$

$$3s-7 = A(s-2) + B(s-4)$$

put $s=2 \Rightarrow 6-7 = -2B \Rightarrow B = 1/2$

put $s=4 \Rightarrow 12-7 = 2A \Rightarrow A = 5/2$.

$$\therefore L^{-1}\left[\frac{3s-7}{(s-4)(s-2)}\right] = L^{-1}\left[\frac{5/2}{s-4} + \frac{1/2}{s-2}\right]$$

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$$= L^{-1}\left[\frac{5}{2}\cdot\frac{1}{s-4} + \frac{1}{2}\cdot\frac{1}{s-2}\right]$$

$$= \frac{5}{2}\cdot e^{4t} + \frac{1}{2}\cdot e^{2t}.$$

3] $L^{-1}\left[\frac{1}{(s-2)(s+2)^2}\right]$

$\underline{\underline{sol^n}}$:
$$\frac{1}{(s-2)(s+2)^2} = \frac{A}{(s-2)} + \frac{B}{(s+2)} + \frac{C}{(s+2)^2}$$

$$1 = A(s+2)^2 + B(s-2)(s+2) + C(s-2)$$

put $s=-2$, $1 = C(-4) \Rightarrow \underline{C = -1/4}$

put $s=2$, $1 = A(4)^2 \Rightarrow \underline{\underline{A = 1/16}}$

put $s=0$, $1 = A(4) + B(-2)(2) + C(-2)$

$$1 = 4A - 4B - 2C$$

$$1 = 4\cdot\frac{1}{16_4} - 4\cdot B + 2\cdot\frac{1}{4_2}$$

$$1 = \frac{1}{4} - 4B + \frac{1}{2}$$

$$-1 + \frac{1}{4} + \frac{1}{2} = 4B$$

$$\frac{-4+1+2}{4} = 4B$$

$$\underline{\underline{\frac{-1}{16} = B}}.$$

$$\therefore L^{-1}\left[\frac{1}{(s-2)(s+2)^2}\right] = L^{-1}\left[\frac{1/16}{s-2} - \frac{1/16}{(s+2)} - \frac{1/4}{(s+2)^2}\right]$$

$$L^{-1}\left[\frac{1}{(s-2)(s+2)^2}\right] = \frac{1}{16}e^{2t} - \frac{1}{16}e^{-2t} - \frac{1}{4}e^{-2t}\cdot t$$

4] $L^{-1}\left[\frac{1}{(s-1)(s^2+4)}\right]$

$\underline{sol^n}$: consider,
$$\frac{1}{(s-1)(s^2+4)} = \frac{A}{s-1} + \frac{Bs+C}{s^2+4}$$

$$1 = \frac{A}{s-1} + \frac{Bs}{s^2+4} + \frac{C}{s^2+4}.$$

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$\therefore \quad 1 = A(s^2+4) + (Bs+C)(s-1)$

put $s=1 \quad \therefore \quad 1 = A(5) + (B+C)(0)$

$$\underline{A = 1/5}$$

put $s=0 \quad \therefore \quad 1 = A(4) + C(-1)$

$$\underline{C = -1/5}$$

put $s=2 \quad \therefore \quad 1 = (4+4)A + (B(2)+C)(1)$

$$1 = 8A + (2B+C)$$

$$\underline{\underline{B = -1/5}}$$

$$\therefore L^{-1}\left[\frac{1}{(s^2+4)(s-1)}\right] = L^{-1}\left[\frac{1/5}{(s-1)} - \frac{1/5(s) - 1/5}{s^2+4}\right]$$

$$= L^{-1}\left[\frac{1/5}{s-1}\right] - \frac{1}{5}L^{-1}\left[\frac{s}{s^2+4}\right] + \frac{1}{5}L^{-1}\left[\frac{1}{s^2+4}\right]$$

$$= \frac{1}{5}e^t - \frac{1}{5}\cos 2t + \frac{1}{5}\cdot\frac{1}{2}\sin 2t$$

$$= \frac{e^t}{5} - \frac{\cos 2t}{5} + \frac{\sin 2t}{10}$$

5) $L^{-1}\left[\frac{2s^2-1}{(s^2+1)(s^2+4)}\right]$

Sol^n: consider, $\frac{2s^2-1}{(s^2+1)(s^2+4)} = \frac{As+B}{s^2+1} + \frac{Cs+D}{s^2+4}$

$$2s^2-1 = (As+B)(s^2+4) + (Cs+D)(s^2+1)$$

$$= As^3 + 4As + Bs^2 + 4B + Cs^3 + Cs + Ds^2 + D$$

$$2s^2-1 = (A+C)s^3 + (4A+C)s + (B+D)s^2 + (4B+D).$$

$\therefore \quad A+C = 0$

$4A + C = 0 \qquad \therefore \quad A = 0$

$B + D = 2 \qquad C = 0$

$4B + D = -1 \qquad B = -1$

$D = 3$

$$\therefore L^{-1}\left[\frac{2s^2-1}{(s^2+1)(s^2+4)}\right] = L^{-1}\left[\frac{-1}{s^2+1}\right] + L^{-1}\left[\frac{3}{s^2+4}\right]$$

$$= -\sin t + \frac{3}{2}\sin 2t$$

6] $L^{-1}\left[\frac{5s+3}{(s-1)(s^2+2s+5)}\right]$

Consider, $\frac{5s+3}{(s-1)(s^2+2s+5)} = \frac{A}{s-1} + \frac{Bs+C}{s^2+2s+5}$

$$5s+3 = A(s^2+2s+5) + (Bs+C)(s-1)$$

$$5s+3 = As^2+2As+5A+Bs^2-Bs+Cs-C$$

$$5s+3 = (A+B)s^2 + (2A-B+C)s + (5A-C)$$

$\therefore$ Comparing coefficients of s on both sides

$\therefore$ $A+B=0$, $2A-B+C=5$, $5A-C=3$

$\therefore$ $A=1$, $B=-1$, $C=2$.

$$\therefore L^{-1}\left[\frac{5s+3}{(s-1)(s^2+2s+5)}\right] = L^{-1}\left[\frac{1}{s-1} + \frac{-s+2}{s^2+2s+5}\right]$$

$$= L^{-1}\left[\frac{1}{s-1}\right] + L^{-1}\left[\frac{-s-1+3}{s^2+2s+1+4}\right]$$

$$= e^t + L^{-1}\left[\frac{-(s+1)+3}{(s+1)^2+2^2}\right]$$

$$= e^t + L^{-1}\left[\frac{-(s+1)}{(s+1)^2+2^2}\right] + L^{-1}\left[\frac{3}{(s+1)^2+2^2}\right]$$

$$= e^t - e^{-t}\cos 2t + e^{-t}\, 3\cdot\frac{1}{2}\cdot\sin 2t$$

$$= e^t - e^{-t}\cos 2t + \frac{3e^{-t}}{2}\sin 2t.$$

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Use of change of scale property

We know that if $L[f(t)] = \phi(s)$ then

$$L[f(at)] = \frac{1}{a}\phi\left(\frac{s}{a}\right)$$

Taking inverse laplace on both sides

$$\therefore L^{-1}\left[\frac{1}{a}\phi\left(\frac{s}{a}\right)\right] = f(at)$$

$$\therefore L^{-1}\left[\phi(s/a)\right] = a \cdot f(at).$$

Examples: If $L^{-1}\left[\frac{s}{(s^2+4)^2}\right] = \frac{t}{4}\sin 2t$ then find $L^{-1}\left[\frac{s}{(s^2+1)^2}\right]$.

Sol^n: Replace s by s/a and t by at.

$$\therefore L^{-1}\left[\frac{1}{a} \cdot \frac{s/a}{((s/a)^2+4)^2}\right] = \frac{at}{4}\sin 2at$$

$$\therefore L^{-1}\left[\frac{1}{a} \cdot \frac{s/a}{\left(\frac{s^2+4a^2}{a^2}\right)^2}\right] = \frac{at}{4}\sin 2at.$$

$$L^{-1}\left[\frac{1}{a} \cdot a^4 \cdot \frac{s/a}{(s^2+4a^2)^2}\right] = \frac{at}{4}\sin 2at$$

$$L^{-1}\left[a^2 \cdot \frac{s}{(s^2+4a^2)^2}\right] = \frac{at}{4}\sin 2at$$

$$L^{-1}\left[\frac{s}{(s^2+4a^2)^2}\right] = \frac{t}{4a} \cdot \sin 2at$$

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Now $4a^2=1 \Rightarrow a=\frac{1}{2}$.

$$\therefore L^{-1}\left[\frac{s}{(s^2+4\cdot\frac{1}{4})^2}\right]=\frac{t}{4\times\frac{1}{2}}\sin\left(2\cdot\frac{1}{2}t\right)$$

$$L^{-1}\left[\frac{s}{(s^2+1)^2}\right]=\frac{t}{2}\sin t.$$

Examples for practice :

1] Find L^{-1} by using partial fraction.

1) $\dfrac{s+29}{(s+4)(s^2+9)}$ 2) $\dfrac{s}{(s^2+16)(s^2+4)}$

3) $\dfrac{1}{(s^2+1)(s^2+36)}$ 4) $\dfrac{s^2+2s+3}{(s^2+2s+5)(s^2+2s+2)}$

5) $\dfrac{s}{s^4+4a^4}$ 6) $\dfrac{(s^2+2a)^2\cdot a}{s^4+4a^4}$.

2] Find $L^{-1}\left[\dfrac{s^2+4}{(s^2-4)^2}\right]$, if $L^{-1}\left[\dfrac{s^2+1}{(s^2-1)^2}\right]=t\cosh t$.

3] If $L^{-1}\left[\dfrac{s}{(s^2+1)^2}\right]=\dfrac{t}{2}\sin t$, find $L^{-1}\left[\dfrac{s}{(s^2+a^2)^2}\right]$.

4] If $L^{-1}\left[\dfrac{s^2+4}{(s^2-4)^2}\right]=t\cosh 2t$, find $L^{-1}\left[\dfrac{s^2+9}{(s^2-9)^2}\right]$.