DSGT

Q1.
- A recurrence relation is an equation which represents a sequence based on some rule.
- It helps in finding the subsequent term (next term) dependent upon the preceding term (previous term).
- If we know the previous term in a given series, then we can easily determine the next term.
- The sequence or series generated by recurrence relⁿ is called Recurrence ~~relation~~ sequence.
- Recurrence Relation formula

Let us assume x_n is n^{th} term of series. Then the recurrence relⁿ is shown in the form of

$$x_n + 1 = f(x_n) \quad n > 0$$

where $f(x_n)$ is the function.

To write recurrence relation of first order, say order k

$$x_n = f(n, x_{n-1}, x_{n-2}, \ldots x_{n-k}) ; n-k > 0.$$

- Solving Recurrence relations.

Solve the recurrence relation $a_n = a_{n-1} - n$ with initial term $a_0 = 4$.

→Sol:

Let us write the sequence based on the equation given starting with initial number

The sequence will be 4, 5, 7, 10, 14, 19, ...

Now the difference between each term

$a_1 - a_0 = 1$

$a_2 - a_1 = 2$

$a_3 - a_2 = 3$

⋮

$a_n - a_{n-1} = n.$

Adding all these ~~eqs.~~ equations

$$1 + 2 + 3 + 4 + \ldots n = \frac{1}{2}(n(n+1))$$

Page No.
Date

$$(a_1 - a_0) + (a_2 - a_1) + (a_3 - a_2) + \ldots + (a_{n-1} - a_{n-2}) = \frac{1}{2} n(n+1)$$

$$a_n - a_0 = \frac{1}{2}(n(n+1))$$

$$a_n = \frac{1}{2} n(n+1) + a_0$$

Hence the sol^n to recurrence rel^n with $a_0 = 4$ is
$$a_n = \frac{1}{2}(n(n+1)) + 4.$$

2.] $n(S)$ = no. of integers $= 100$
$n(A)$ = no. of integers divisible by 2
$= \frac{100}{2} = 50$

$n(B)$ = no. of integers divisible by 3
$= \frac{100}{3} = 33.$

$n(C)$ = no. of integers divisible by 5
$= \frac{100}{5} = 20$

$n(A \cap B)$ = no. of integers divisible by 2 and 3
$= \frac{100}{2 \times 3} = 16.$

$n(B \cap C) = \frac{100}{3 \times 5} = 6.$

$n(A \cap C) = \frac{100}{2 \times 5} = 10.$

$n(A \cap B \cap C) = \frac{100}{2 \times 3 \times 5} = 3.$

$$n(A\cup B\cup C) = n(A) + n(B) + n(C) - n(A\cap B) - n(A\cap C) - n(B\cap C) + n(A\cap B\cap C)$$

$$= 50 + \cancel{20}^{33} + 20 - 16 - 10 - 6 + 3$$

$$= 74.$$

Not divisible by 2, 3 or 5

$$= n(\overline{A\cup B\cup C})$$

$$= n(S) - n(A\cup B\cup C)$$

$$= 100 - 74$$

$$= 26$$

3] Suppose we have n people in a room.

- The first person shakes hands with everybody in the room except for himself. His total no. of handshakes is therefore one lower than the total no. of people.
- The second person has now shaken hands with the first person but still needs to shake hands with everybody else. The no. of people left is therefore, two lower than the total no. of people in a room.
- This continues with each person having one less handshake to make until we get the penultimate person who has to shake hands with the last person.

Pigeonhole principle

Theorem – If n pigeons are assigned to m pigeonholes, & $m<n$ then at least one pigeonhole contains two or more pigeons.

Proof –

- Consider labelling m pigeonholes with the numbers 1 to m & n pigeons with numbers 1 to n.

- Now beginning with pigeon 1, assign each pigeon in order to pigeonhole with same number.
- This assigns as many pigeons as possible to individual pigeon holes but because $m < n$, there are $n - m$ pigeons that have not yet been assigned to a pigeonhole.
- At least one pigeonhole will be assigned a second pigeon.

n = pigeons
m = pigeonhole
$m < n$

~~Suppose~~ Extended pigeonhole principle

- If there are m pigeonhole and $2m^n$ pigeons, then three or more pigeons will have to be assigned to at least of the pigeonholes.
- If n and m are positive integers, then $\lfloor n/m \rfloor$ stands for the largest integer less than or equal to rational number n/m.

Thus $\lfloor 3/2 \rfloor$ is 1.
$\lfloor 9/4 \rfloor$ is 2
$\lfloor 6/3 \rfloor$ is 2.

Theorem:
If n pigeons are assigned to m pigeonholes then one of the pigeonhole must contain atleast $\lfloor (n-1)/m \rfloor + 1$ pigeons.

Proof:
Assume that each pigeonhole does not contain more than $\lceil (n-1)/m \rceil$ pigeons.
Then there will be at must $m \lceil (n-1)/m \rceil \leq m(n-1)/m = n-1$ pigeons in all.

- Suppose there are n people in a room.
- In the given case, the pigeonhole is hands shaken & pigeons are people.
- Since you can never shake hands with yourself, you only shake $(n-1)$ other people's hand and for a total of at most $(n-1)$ handshakes.
- So there are $(n-1)$ possible numbers of handshakes for a given person and n possible people i.e. more pigeons than pigeonholes.
- Acc. to pigeonhole principle, Average value $\frac{\text{pigeons}}{\text{pigeonholes}} = \frac{n}{n-1}$ is greater than 1 but smaller than 2, so maximum must be atleast 2
- Therefore atleast two people have the same handshake numbers.

4.) let A be set of people
B be the set of seconds of one day.

$|A| = 100000 = n$
$|B| = 24 \times 3600 = 86400 = m.$

Then $k = \lfloor (n-1)/m \rfloor + 1$

$= \lfloor (100000-1)/86400 \rfloor + 1$

$= 1 + 1$

$= 2.$

Hence atleast 2 are born ~~a~~ on same day.

5.] By extended pigeonhole principle

$\lfloor (n-1)/m \rfloor + 1 = 6$

$\lfloor (n-1)/5 \rfloor + 1 = 6$

$\frac{(n-1)}{5} = 5$

$n-1 = 25$

$n = 26.$

$\therefore$ 26 students ~~are required to~~ will receive same grades

Q6] $G = \{1, 2, 3, 4, 5, 6\}$.

order = 6

multiplication mod 7 with order 6.

* mod 7	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	1	3	5
3	3	6	2	5	1	4
4	4	1	5	2	6	3
5	5	3	1	6	4	2
6	6	5	4	3	2	1

(i) closure property

All entries in composition table exists in set G.

$a = 5$, $b = 2$

$a * b \in G$

$5 \times_7 2 = 3 \in G$

$\therefore$ It is algebraic structure.

(ii) Associative

$a * (b * c) = (a * b) * c$

e.g. $a = 3$, $b = 2$, $c = 4$

$3 \times_7 (2 \times_7 4) = (3 \times_7 2) \times_7 4$

$3 \times_7 1 = 6 \times_7 4$

$3 = 3$

L.H.S. = R.H.S.

∴ It is semigroup

(iii) Identity

$a * e = a$

let $a = 3$

$3 \times_7 e = 3$

$e = 1$.

∴ It is monoid

(iv) Inverse

$a * b = e$ & $b * a = e$.

$a = 3$

$3 \times_7 b = 1$

$b = \cancel{\frac{1}{3}}\ 5$

Inverse of 1, 2, 3, 4, 5, 6 is 1, 4, 5, 2, 3, 6.

∴ It is group.

(v) Commutative

$a * b = b * a$

$a = 3$, $b = 2$

$3 \times_7 2 = 2 \times_7 3$

$6 = 6$

~~Hence Proved~~.

∵ operation is commutative

It is abelian grp.

7] $H = \begin{vmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{vmatrix}$

- In given parity check matrix all columns are distinct and non-zero, $d \geq 3$
- Use the property that minimum distance of a binary linear code is equal to the smallest number of columns of parity check matrix H that sum upto 0.
- Sum of first 3 columns is zero.

  so minimum distance $d_{min} = 3$.

  It can detect

  $d_{min} - 1 = 3 - 1 = 2$ errors

  It can correct

  $(d_{min} - 1)/2 = 1$ error.

7) $H = \begin{vmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{vmatrix}$

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$(d_{min} - 1)/2 = 1$ error.

Prove that set G ~~is~~ = $\{0, 1, 2, 3, 4, 5\}$ is an Abelian grp of order 6 with respect to addition modulo 6.

Composition table

$+_6$	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	2	3	4	5	0
2	2	3	4	5	0	1
3	3	4	5	0	1	2
4	4	5	0	1	2	3
5	5	0	1	2	3	4

$-\frac{6}{2}$

Page No.
Date

i) $0+0 \bmod 6$

$0 \bmod 6$

0

Closure property
i) all entries in composition table belongs to or exists in set G.

if $a = 1 \quad b = 2$

$* = +_6$

$1 +_6 2 = 3$

$1, 2, 3 \in G$ (Hence it is closed w.r.t. op^n $+_6$)

Associative
ii) $a * (b * c) = (a * b) * c$

$a = 2 \quad b = 3 \quad c = 4$

$2 +_6 (3 +_6 4) = (2 +_6 3) +_6 4$

$2 +_6 1 = 5 +_6 4$

$3 = 3$

L.H.S = R.H.S.

Hence op^n is associative

Hence it is semigroup

Identity
iii) $a * e = a$

$a = 3$

$3 +_6 e = 3$

$\boxed{e = 0}$

$a = 5 \quad 5 +_6 e = 5$

$\boxed{e = 0}$

Hence it is monoid

Inverse

(iv) $a * b = e$ & $b * a = e$

$a = 5$

0, 3, 2 4, 1 5
inverse of each other

$5 * b = 0$

$5 +_6 b = 0$

inverse of 0, 1, 2, 3, 4, 5   0, 5, 4, 3, 2, 1

Hence it is group

Commutative

(v) $a * b = b * a$

$a = 5, b = 2$

$5 +_6 2 = 2 +_6 5$

$1 = 1$

$a = 3, b = 0$

$3 +_6 0 = 0 +_6 3$

$3 = 3$

wrt opn raddian mod 6 is operation is commutative

Hence it is an Abelian grp.

Let $H = \{[0]_6, [3]_6\}$ find left and right coset in group Z_6.

Is H a normal subgroup of group Z_6.

$+_6$	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	2	3	4	5	0
2	2	3	4	5	0	1
3	3	4	5	0	1	2
4	4	5	0	1	2	3
5	5	0	1	2	3	4

left coset

aH

$0H = \{0 +_6 0, 0 +_6 3\} = \{0, 3\}$

$1H = \{1 +_6 0, 1 +_6 3\} = \{1, 4\}$

$2H = \{2 +_6 0, 2 +_6 3\} = \{2, 5\}$

$3H = \{3 +_6 0, 3 +_6 3\} = \{3, 0\}$

$4H = \{4 +_6 0, 4 +_6 3\} = \{4, 1\}$

$5H = \{5 +_6 0, 5 +_6 3\} = \{5, 2\}$

Right coset

Ha

$H0 = \{0 +_6 0, 3 +_6 0\} = \{0, 3\}$

$H1 = \{0 +_6 1, 3 +_6 1\} = \{1, 4\}$

$H2 = \{0 +_6 2, 3 +_6 2\} = \{2, 5\}$

$H3 = \{0 +_6 3, 3 +_6 3\} = \{3, 0\}$

$H4 = \{0 +_6 4, 3 +_6 4\} = \{4, 1\}$

$H5 = \{0 +_6 5, 3 +_6 5\} = \{5, 2\}$

Hence proved $aH = Ha$