

AVERAGE LINK CLUSTERING

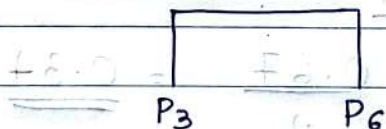
Compute clusters for given distance Matrix, Use Average Link Clustering.

P ₁	0					
P ₂	0.24	0				
P ₃	0.22	0.15	0			
P ₄	0.37	0.20	0.15	0		
P ₅	0.34	0.14	0.28	0.29	0	
P ₆	0.23	0.25	0.11	0.22	0.39	0

P₁ P₂ P₃ P₄ P₅ P₆

Step 1 : Merge two points with smallest distance as a cluster.

Here point P₃, P₆ are closest as can be observed from distance matrix.



Recomputing the distance matrix

$$d((P_3, P_6), P_1) = \frac{1}{2} [d(P_1, P_3) + d(P_1, P_6)]$$

$$= \frac{1}{2} [0.22 + 0.23]$$

$$= 0.45$$

$$= \underline{\underline{0.23}}$$

$$\begin{aligned}
 d((P_3, P_6), P_2) &= \frac{1}{2} [d(P_3, P_2) + d(P_6, P_2)] \\
 &= \frac{1}{2} [0.15 + 0.25] \\
 &= \frac{0.4}{2} \\
 &= \underline{\underline{0.2}}
 \end{aligned}$$

$$\begin{aligned}
 d((P_3, P_6), P_4) &= \frac{1}{2} [d(P_3, P_4) + d(P_6, P_4)] \\
 &= \frac{1}{2} [0.15 + 0.22] \\
 &= \frac{0.37}{2} = \underline{\underline{0.19}}
 \end{aligned}$$

$$\begin{aligned}
 d((P_3, P_6), P_5) &= \frac{1}{2} [d(P_3, P_5) + d(P_6, P_5)] \\
 &= \frac{1}{2} [0.28 + 0.39] \\
 &= \frac{0.67}{2} = \underline{\underline{0.34}}
 \end{aligned}$$

New Distance Matrix

P_1	0				
P_2	0.24	0			
(P_3, P_6)	0.23	0.2	0		
P_4	0.37	0.20	0.19	0	
P_5	0.34	0.14	0.34	0.29	0
	P_1	P_2	(P_3, P_6)	P_4	P_5

Merging P_2, P_5



Recomputing distance Matrix

$$d[(P_2, P_5), P_1] = \frac{1}{2} [d(P_2, P_1) + d(P_1, P_5)]$$

$$= \frac{0.24 + 0.34}{2}$$

$$= \frac{0.58}{2} = \underline{\underline{0.29}}$$

$$d[(P_2, P_5), (P_3, P_6)] = \frac{1}{4} [d(P_2, P_3) + d(P_2, P_6) + d(P_5, P_3) + d(P_5, P_6)]$$

$$= \frac{1}{4} [0.15 + 0.25 + 0.28 + 0.39]$$

$$= \frac{1}{4} [1.07]$$

$$= \underline{\underline{0.27}}$$

$$d[(P_2, P_5), P_4] = \frac{1}{2} [d(P_2, P_4) + d(P_5, P_4)]$$

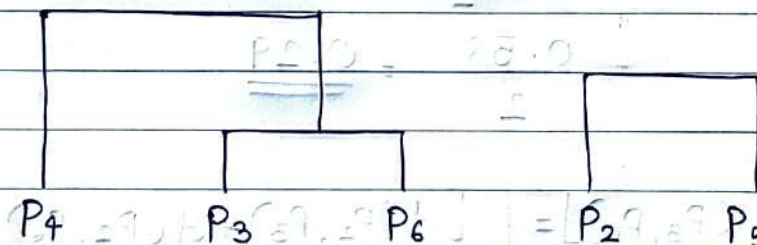
$$= \frac{1}{2} [0.14 + 0.29]$$

$$d[(P_2, P_5), P_4] = \frac{1}{2} [0.43]$$

$$= \underline{\underline{0.22}}$$

P_1	0	0.29	0.37	0
(P_2, P_5)	0.29	0		
(P_3, P_6)	0.22	0.27	0	
P_4	0.37	0.22	0.15	0

Merging $P_4, (P_3, P_6)$, as they have minimum distance



$$d[(P_3, P_6, P_4), (P_2, P_5)] = \frac{1}{6} [d(P_3, P_2) + d(P_3, P_5) + d(P_6, P_2) + d(P_6, P_5) + d(P_4, P_2) + d(P_4, P_5)]$$

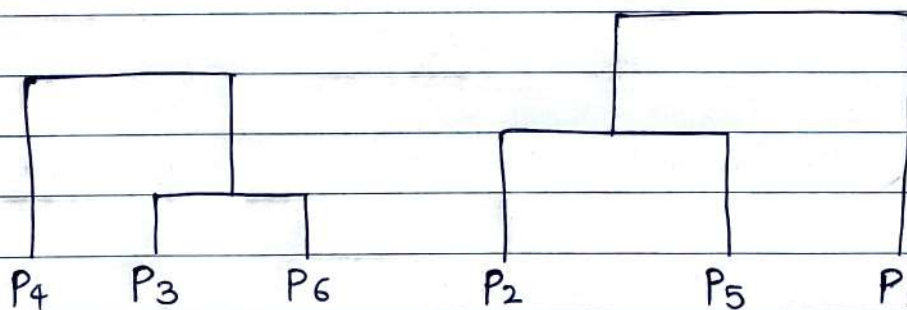
$$= \frac{1}{6} [0.15 + 0.28 + 0.25 + 0.39 + 0.20 + 0.29]$$

$$= \frac{1}{6} [1.56]$$

$$= \underline{\underline{0.26}}$$

$$\begin{aligned}
 d[(P_3, P_6, P_4), P_1] &= \frac{1}{3} [d(P_3, P_1) + d(P_6, P_1) + d(P_4, P_1)] \\
 &= \frac{1}{3} [0.22 + 0.23 + 0.37] \\
 &= \frac{1}{3} [0.82] \\
 &= \underline{\underline{0.27}}
 \end{aligned}$$

P_1	0		
P_2, P_5	0.24	0	
P_3, P_6, P_4	0.27	0.26	0
	P_1	(P_2, P_5)	(P_3, P_6, P_4)

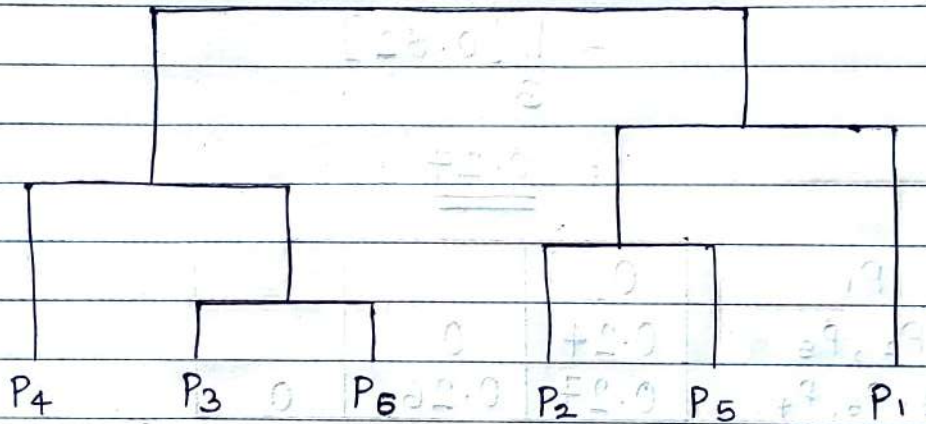


Merging (P_2, P_5) with P_1 as per the distance Matrix

$(P_1, (P_2, P_5))$	0	
(P_3, P_6, P_4)	0.26	0
	$(P_1, (P_2, P_5))$	(P_3, P_6, P_4)

$$= \frac{(-9, 9, 0) + (-9, 9, 0)}{2} = (-9, 9, 0)$$

$$= \frac{(-2, 0) + (-2, 0) + (-2, 0)}{3} = (-2, 0)$$



$$(-9, 9, 0) - (-9, 9, 0) = 0$$

19 21 23 25 27 29

Let us say we have a set of numbers $(-9, 9, 0)$ and we want to find the median.

$$\begin{aligned} & \frac{(-9, 9, 0) + (-9, 9, 0)}{2} = (-9, 9, 0) \\ & \frac{(-2, 0) + (-2, 0) + (-2, 0)}{3} = (-2, 0) \end{aligned}$$