

Semester : IIISubject : DSGT

Academic Year: 2022-2023

* Coset :- A subgroup H of G is said to be a normal subgroup of G if for every $a \in G$,
 $aH = Ha$.

A subgroup of an Abelian group is normal.
 $aH \rightarrow$ left coset
 $Ha \rightarrow$ right coset.

① Let $H = \{[0]_6, [3]_6\}$. Find the left & right cosets in group Z_6 . Is H a normal subgroup of group Z_6 .

\Rightarrow The addition modulo 6 group, table of Z_6 is.

$+_6$	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	2	3	4	5	0
2	2	3	4	5	0	1
3	3	4	5	0	1	2
4	4	5	0	1	2	3
5	5	0	1	2	3	4

This is Abelian group since for all $a, b \in Z_6$
 $a +_6 b = b +_6 a$.



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Left coset of H with respect to a in the set

$$aH = \{a * h \mid h \in H\}$$

$$0H = \{0 +_6 0, 0 +_6 3\} = \{0, 3\}$$

$$1H = \{1 +_6 0, 1 +_6 3\} = \{1, 4\}$$

$$2H = \{2 +_6 0, 2 +_6 3\} = \{2, 5\}$$

$$3H = \{3 +_6 0, 3 +_6 3\} = \{3, 0\}$$

$$4H = \{4 +_6 0, 4 +_6 3\} = \{4, 1\}$$

$$5H = \{5 +_6 0, 5 +_6 3\} = \{5, 2\}$$

Right coset of H with respect to a in the list is. $Ha = \{h * a \mid h \in H\}$

$$H0 = \{0 +_6 0, 3 +_6 0\} = \{0, 3\}$$

$$H1 = \{0 +_6 1, 3 +_6 1\} = \{1, 4\}$$

$$H2 = \{0 +_6 2, 3 +_6 2\} = \{2, 5\}$$

$$H3 = \{0 +_6 3, 3 +_6 3\} = \{3, 0\}$$

$$H4 = \{0 +_6 4, 3 +_6 4\} = \{4, 1\}$$

$$H5 = \{0 +_6 5, 3 +_6 5\} = \{5, 2\}$$

Here,

$$0H = H0$$

$$3H = H3$$

$$1H = H1$$

$$4H = H4$$

$$2H = H2$$

$$5H = H5$$

$\therefore H$ is a normal subgroup of Z_6 .



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EX. Let $G = \mathbb{Z}_8$. Determine all left cosets of $H = \{[0], [4]\}$ in G .

⇒ The addition modulo 8 table is

$+_8$	0	1	2	3	4	5	6	7
0	0	1	2	3	4	5	6	7
1	1	2	3	4	5	6	7	0
2	2	3	4	5	6	7	0	1
3	3	4	5	6	7	0	1	2
4	4	5	6	7	0	1	2	3
5	5	6	7	0	1	2	3	4
6	6	7	0	1	2	3	4	5
7	7	0	1	2	3	4	5	6

Left coset of H with respect to a is.

$$aH = \{a * h \mid h \in H\}$$

$$0H = \{0 +_8 0, 0 +_8 4\} = \{0, 4\}$$

$$1H = \{1 +_8 0, 1 +_8 4\} = \{1, 5\}$$

$$2H = \{2 +_8 0, 2 +_8 4\} = \{2, 6\}$$

$$3H = \{3 +_8 0, 3 +_8 4\} = \{3, 7\}$$

$$4H = \{4 +_8 0, 4 +_8 4\} = \{4, 0\}$$

$$5H = \{5 +_8 0, 5 +_8 4\} = \{5, 1\}$$

$$6H = \{6 +_8 0, 6 +_8 4\} = \{6, 2\}$$

$$7H = \{7 +_8 0, 7 +_8 4\} = \{7, 3\}$$