2) In a normal distribution, 10.03% of the items are under 25 Kilogram weight & 89.97% of the items are under 70 kg weight. Find the mean & SD of the distribution.

→ Let X : the weight (in kg) of the items)

$X \sim N(\mu, \sigma)$

Given : $P(X < 25) = 10.03\% = 0.1003$

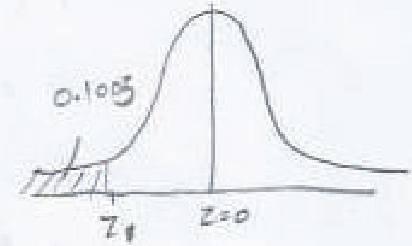
$P(X < 70) = 89.97\% = 0.8997$

Let $P(X<25) = 0.1003$

$\Rightarrow P\left(\frac{X-\mu}{\sigma} < \frac{25-\mu}{\sigma}\right) = 0.1003$

$\Rightarrow P(Z < z_1) = 0.1003$

where $z_1 = \frac{25-\mu}{\sigma}$ — (1)




$\therefore P(z_1 < z < 0) = 0.5 - 0.1003 = 0.3997$

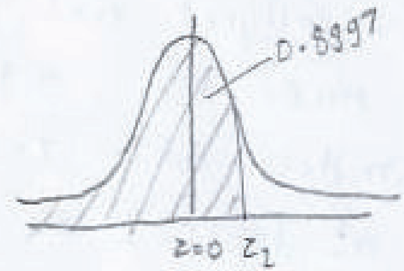
$\therefore$ from table, $\boxed{z_1 = -1.28}$

Now, $P(X < 70) = 0.8997$

$\Rightarrow P\left(\frac{X-\mu}{\sigma} < \frac{70-\mu}{\sigma}\right) = 0.8997$

$\Rightarrow P(Z < z_2) = 0.8997$

where $z_2 = \frac{70-\mu}{\sigma}$ — (2)




$\therefore P(0 < z < z_2) = 0.8997 - 0.5 = 0.3997$

$\therefore$ from table, $\boxed{z_2 = 1.28}$

(1) $\Rightarrow -1.28 = \frac{25-\mu}{\sigma} \Rightarrow -1.28\sigma = 25-\mu$

$\Rightarrow \boxed{\mu = 47.5}$

(2) $\Rightarrow 1.28 = \frac{70-\mu}{\sigma} \Rightarrow 1.28\sigma = 70-\mu$

& $\boxed{\sigma = 17.578}$

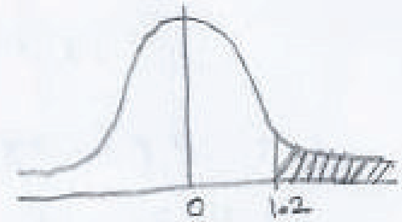
3) In a male population of 1000, the mean height is 68.16 inches & SD 3.2 inches. How many men may be more than 6 feet (72 inches)?

→ Let X : height of men in inches

$X \sim N(\mu = 68.16, \sigma = 3.2)$

first find the probability that their height is more than 72 inches.

$\therefore P(X > 72) = P\left(\frac{X-\mu}{\sigma} > \frac{72-68.16}{3.2}\right)$




$= P(Z > 1.2)$

$= 0.5 - P(0 \leq Z \leq 1.2)$

$= 0.5 - 0.3849$

$\doteq 0.1151$

$\therefore$ Number of men having height more than 6 feet

$= 1000 \times 0.1151 = 115.1 \approx \underline{115}$

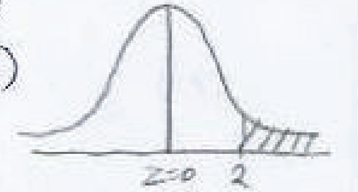
4) The marks obtained by a number of students in a certain subject are approximately normally distributed with mean 65, S.D. 5. If 3 students are selected at random from this group, what is the probability that at least 1 of them would have scored above 75?

→ Let X : marks obtained by the students

$X \sim N(\mu = 65, \sigma = 5)$

$\therefore P(\text{students scores above } 75) = P(X > 75)$

$= P\left(\frac{X-\mu}{\sigma} > \frac{75-65}{5}\right) = P(Z > 2)$




$= 0.5 - P(0 < Z < 2)$

$= 0.5 - 0.4772$ ... (from table)

$P(X>75) = 0.0228$

$\therefore$ Let $p = 0.0228 \Rightarrow q = 1 - p = 0.9772$ & $n = 3$

Since $p$ is same for all the students, the number $Y$ of (successes) students scoring above 75, follows a binomial distribution.

$\therefore$ P(atleast 1 student score above 75)

$= P(Y \geq 1) = 1 - P(Y = 0)$

$= 1 - {}^nC_0\, p^0 q^n$

$= 1 - {}^3C_0 (0.0228)^0 (0.9772)^3 = \underline{0.0667}$

* <u>Additive Property of Normal Distribution</u> :-

If $X_i$ $(i = 1, \ldots, n)$ be $n$ independent normal RVs with mean $\mu_i$ & variance $\sigma_i^2$ then $\sum_{i=1}^{n} a_i X_i$ is also a normal RV with mean $\sum_{i=1}^{n} a_i \mu_i$ & variance $\sum_{i=1}^{n} a_i^2 \sigma_i^2$

<u>Note</u>: If $\bar{x}$ is the mean of the sample of size $n$ drawn from the population with mean $\mu$ & S.D. $\sigma$ then $\bar{x}$ is normally distributed with mean $\mu$ & S.D. $\sigma/\sqrt{n}$ i.e. $z = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}}$ is standard normal variate as $n \to \infty$

5) In an examination marks obtained by students in Mathematics, Physics & chemistry are normally distributed with means 51, 53, 46 with S.D. 15, 12, 16 resp. Find the probability of securing total marks

(i) 180 or above

(ii) 80 or below

→ Let $X_1$ : marks obtained in Mathematics

$X_2$ : —"— Physics

$X_3$ : —"— Chemistry

$X_1 \sim N(\mu_1 = 51, \sigma_1^2 = 15^2)$, $X_2 \sim N(\mu_2 = 53, \sigma_2^2 = 12^2)$,

$X_3 \sim N(\mu_3 = 46, \sigma_3^2 = 16^2)$

$\therefore$ $Y = X_1 + X_2 + X_3$ is normally distributed with mean

$\mu = \mu_1 + \mu_2 + \mu_3 = 51 + 53 + 46 = 150$ & variance

$\sigma^2 = \sigma_1^2 + \sigma_2^2 + \sigma_3^2 = 625 = 25^2$

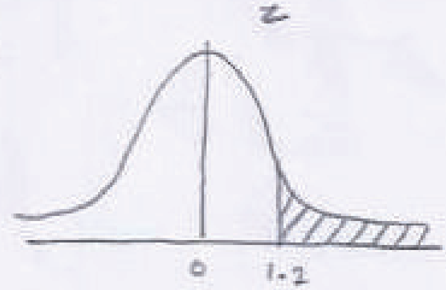
(i) $P(Y \geq 180) = P\left(\frac{Y-\mu}{\sigma} \geq \frac{180-150}{25}\right)$

$= P(Z \geq 1.2)$

$= 0.5 - P(0 < Z < 1.2)$

$= 0.5 - 0.3849$ ... from table

$\therefore P(Y \geq 180) = 0.1151$



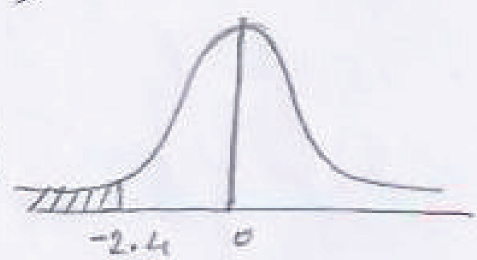

(ii) $P(Y \leq 80) = P\left(\frac{Y-\mu}{\sigma} \leq \frac{80-150}{25}\right)$

$= P(Z \leq -2.4)$

$= 0.5 - P(0 < Z < 2.4)$

$= 0.5 - 0.4918$

$P(Y \leq 80) = 0.0082$



6) A normal population has a mean of 0.1 & s.d. of 2.1. Find the probability that mean of a sample of size 900 will be negative.

→ Given: $\mu = 0.1$, $\sigma = 2.1$, $n = 900$

The standard normal variate corresponding to $\bar{x}$ is

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{\bar{x} - 0.1}{2.1/\sqrt{900}} = \frac{\bar{x} - 0.1}{0.07}$$

$$\Rightarrow \bar{x} = 0.1 + 0.07 Z \quad \text{where} \quad Z \sim N(0,1)$$

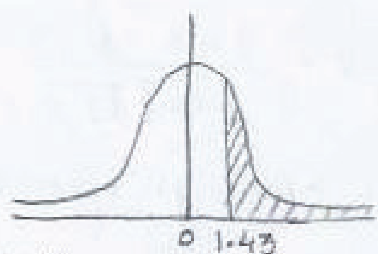
The probability that the sample mean is negative,

$$P(\bar{x} < 0) = P(0.1 + 0.07 Z < 0)$$

$$= P\left(Z < \frac{-0.1}{0.07}\right)$$

$$= P(Z < -1.43)$$

$$= P(Z \geq 1.43)$$




$$= 0.5 - P(0 < Z < 1.43)$$

$$= 0.5 - 0.4236$$

$$= \underline{0.0764}$$

7) In a competitive examination, the top 15% of the student appeared will get grade 'A', while the bottom 20% will be declared fail. If the grades are normally distributed with mean of marks 75 & S.D. 10, determine the lowest % of ~~the~~ marks to receive grade A & the lowest % of marks that passes.

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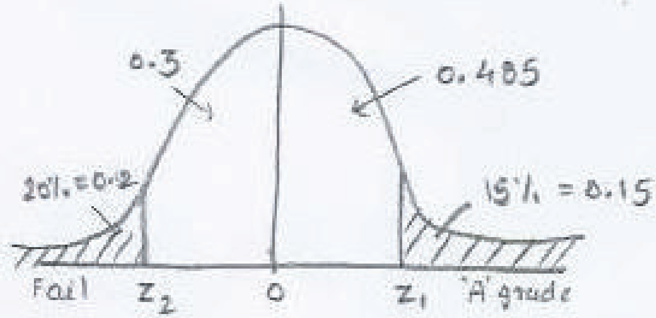
→ Given: $\mu = 75$, $\sigma = 10$

Let X denote the marks of students.

$$X \sim N(75, 10)$$

Standard variate $z \sim N(0,1)$

$$z = \frac{x-\mu}{\sigma} = \frac{x-75}{10}$$




$P(z \geq z_1) = 0.15$

$\Rightarrow P(0 \leq z \leq z_1) = 0.5 - 0.15$

$= 0.35$

From the table $z_1 = 1.04$

But $z_1 = \frac{x_1 - \mu}{\sigma} \Rightarrow 1.04 = \frac{x_1 - 75}{10} \Rightarrow x_1 = 10.4 + 75 = \underline{85.4}$

$\therefore$ Minimum marks for grade 'A' $= 85.4$

Also, $P(Z \leq z_2) = 0.2$

$\Rightarrow P(0 \geq z \geq z_2) = 0.5 - 0.2 = 0.3$

From the table, $z_2 = -0.84$

But $z_2 = \frac{x_2 - \mu}{\sigma} \Rightarrow -0.84 = \frac{x_2 - 75}{10} \Rightarrow x_2 = 10(-0.84) + 75 = \underline{66.6}$

$\therefore$ Minimum marks for passing $= 66.6$

8) A group of 625 students has a mean age of 15.8 years with a S.D. of 0.6 years. The ages are normally distributed. How many students are younger than 16.2 years?

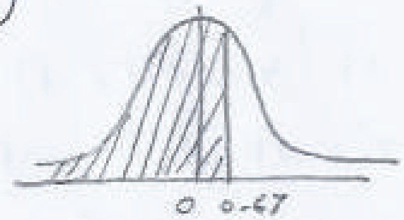
→ Let X : age of the students.

$X \sim N(\mu = 15.8,\ \sigma = 0.6)$

$\therefore P(X \le 16.2) = P\left(\frac{X-\mu}{\sigma} \le \frac{16.2 - 15.8}{0.6}\right)$

$= P(Z \le 0.67)$

$= 0.5 + P(0 \le Z \le 0.67)$




$= 0.5 + 0.2486$ .... from table

$= 0.7486$

$\therefore$ Number of students younger than 16.2 years

$= 625 \times 0.7486 = 467.875 \approx \underline{468}$ students.

9) Heights of swedish men follow a normal distribution with mean 72 in & S.D. 5 in. How high must a doorway be so that 90% of swedish men can go through without having to bend?

→ Let X : height of the men.

$X \sim N(\mu = 72,\ \sigma = 5)$

Let $x_1$ : height of the doorway

Given : $P(X \le x_1) = 0.9$

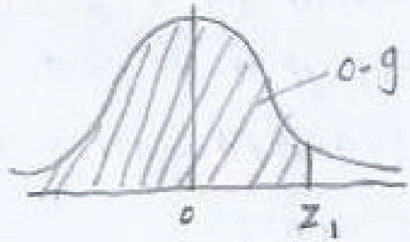
(since 90% men can go through the door, height of the doorway must be greater than height of the men)

$\therefore P\left(\frac{X-\mu}{\sigma} \le \frac{x_1 - 72}{5}\right) = 0.9$

$\therefore P(Z \le z_1) = 0.9$

$\therefore P(Z \le Z_1) = 0.9$

where $Z_1 = \dfrac{X_1 - 72}{5}$




$\therefore P(0 < Z < Z_1) = 0.9 - 0.5 = 0.4$

$\therefore$ from table, $Z_1 = 1.28$

but $Z_1 = \dfrac{X_1 - 72}{5} \Rightarrow 1.28 = \dfrac{X_1 - 72}{5} \Rightarrow \underline{\underline{X_1 = 78.4}}$

$\therefore$ The height of the doorway must be 78.4 inches.

10) If $X \sim N(8, 2)$, $Y \sim N(12, 4\sqrt{3})$. Find the value of $\lambda$ such that $P(2X - Y \le 2\lambda) = P(X + 2Y \ge \lambda)$

$\rightarrow$ Let $U = 2X - Y$, $\therefore$ by additive property of normal distr

mean $= 2 \times 8 - 12 = 4$

$S.D = \sqrt{2^2 \times 2^2 + 1^2 (4\sqrt{3})^2} = \sqrt{64} = 8 \quad \therefore U \sim N(4, 8)$

Let $V = X + 2Y$

$\therefore$ mean $= 8 + 2 \times 12 = 32$

$S.D = \sqrt{(2)^2 + (2)^2 (4\sqrt{3})^2} = 14 \quad \therefore V \sim N(32, 14)$

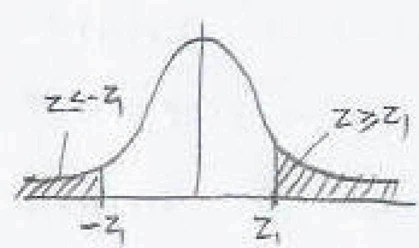
Now, $P(2X - Y \le 2\lambda) = P(X + 2Y \ge \lambda)$

$\therefore P(U \le 2\lambda) = P(V \ge \lambda)$

$\therefore P\left(\dfrac{U - \mu}{\sigma} \le \dfrac{2\lambda - 4}{8}\right) = P\left(\dfrac{V - \mu}{\sigma} \ge \dfrac{\lambda - 32}{14}\right)$

$\therefore P\left(Z \le \dfrac{2\lambda - 4}{8}\right) = P\left(Z \ge \dfrac{\lambda - 32}{14}\right)$

(If $P(Z \ge Z_1) = \alpha$ then $P(Z \le -Z_1) = \alpha$) by symmetry




$\therefore P\left(Z \le \dfrac{2\lambda - 4}{8}\right) = P\left(Z \le -\left(\dfrac{\lambda - 32}{14}\right)\right)$

$\therefore \dfrac{2\lambda - 4}{8} = \dfrac{-\lambda + 32}{14} \Rightarrow \dfrac{\lambda - 2}{4} = \dfrac{-\lambda + 32}{14}$

$\therefore 14\lambda - 28 = -4\lambda + 128 \Rightarrow \lambda = \dfrac{156}{18} = \dfrac{26}{3} \Rightarrow \boxed{\lambda = \dfrac{26}{3}}$