VARIANCE:

Definition: Suppose X is a r.v., then the variance of X is defined to be

$$Var(X) = \sigma^2 = \sigma_X^2 = E\left(X - E(X)\right)^2$$
$$= E(X)^2 - \{E(X)\}^2 \; (after\ simplification)$$

STANDARD DEVIATION:

The positive square root of the variance of X is defined to be the standard deviation of X and is denoted as σ or σ_X .

Examples

1. Suppose a continuous r.v. X has the pdf $f(x) = \begin{cases} \frac{1}{4} e^{-x/4}, & x > 0 \\ 0 & elsewhere \end{cases}$

Find the mean and variance.

Solution: We have,

$$Mean = E(X) = \int_{-\infty}^{\infty} x f(x) dx$$
$$= \int_0^{\infty} x(\frac{1}{4} e^{-x/4}) dx$$
$$= \frac{1}{4}\left(x(\frac{e^{-x/4}}{(-1/4)}) - (1(\frac{e^{-x/4}}{(-1/4)^2}) \right)_0^{\infty}$$
$$= \frac{1}{4}(16)$$
$$= 4$$

We have,

$$E(X^2) = \int_{-\infty}^{\infty} x^2 f(x)dx$$

$$= \int_0^{\infty} x^2 (\frac{1}{4} e^{-x/4})dx$$

$$= \frac{1}{4}\left(x^2 (\frac{e^{-x/4}}{(-1/4)}) - (2x(\frac{e^{-x/4}}{(-1/4)^2}) + (2(\frac{e^{-x/4}}{(-1/4)^3}) \right)_0^{\infty}$$

$$= \frac{1}{4}(2(64))$$

$$= 32$$

Therefore

$$Variance = \sigma^2 = E(X^2) - (E(X))^2$$

$$= 32 - 16$$

$$= 16$$

2. A continuous r.v. *X* has the pdf $f(x) = ke^{-x}x^2,\ x \geq 0$ Find k and the mean and variance of *X*.

Solution: To find k:

Since $f(x)$ is a pdf, we have,

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\Rightarrow \int_0^{\infty} ke^{-x}x^2\,dx = 1$$

$$\Rightarrow k\left[x^2 \frac{e^{-x}}{-1} - 2x\left(\frac{e^{-x}}{(-1)^2}\right) + 2\left(\frac{e^{-x}}{(-1)^3}\right)\right]_0^{\infty} = 1$$

$$\Rightarrow k = \frac{1}{2}$$

Therefore we have $f(x) = \frac{1}{2} e^{-x} x^2,\ x \geq 0$

$$Mean = E(X) = \int_{-\infty}^{\infty} xf(x)dx$$

$$= \int_{0}^{\infty} x(\frac{1}{2}e^{-x}x^2)dx$$

$$= \frac{1}{2}\left(x^3(\frac{e^{-x}}{(-1)}) - (3x^2(\frac{e^{-x}}{(-1)^2})) + (6x(\frac{e^{-x}}{(-1)^3})) - (6(\frac{e^{-x}}{(-1)^4})) \right)_0^{\infty}$$

$$= \frac{1}{2}(6)$$

$$= 3$$

We have,

$$E(X^2) = \int_{-\infty}^{\infty} x^2 f(x)dx$$

$$= \int_{0}^{\infty} x^2(\frac{1}{2}e^{-x}x^2)dx$$

$$= \frac{1}{2}\left(x^4(\frac{e^{-x}}{(-1)}) - (4x^3(\frac{e^{-x}}{(-1)^2})) + (12x^2(\frac{e^{-x}}{(-1)^3})) - (24x(\frac{e^{-x}}{(-1)^4})) + (24(\frac{e^{-x}}{(-1)^5})) \right)_0^{\infty}$$

$$= \frac{1}{2}(24)$$

$$= 12$$

Therefore

$$Variance = \sigma^2 = E(X^2) - \left(E(X)\right)^2$$

$$= 12 - 9$$

$$= 3$$