Matrix representation and homogeneous co-ordinates

- Many graphics application involves sequence of geometric transformations.
- For example in design and picture construction application we perform Translation, Rotation and Scaling to fit the picture components into their proper positions.
- For efficient processing we will reformulate transformation sequences.
- we have matrix representation of basic transformation ~~is~~ and we can express it in the general matrix form as:

$$P' = M_1 \cdot P + M_2$$

where P & P' are initial and final point position, M_1 contains rotation and scaling term and M_2 contains translation

- For efficient utilization we must calculate all sequence of transformation in one step and for that reason we reformulate above equation to eliminate the matrix addition associated with translation term in matrix M_2.

- we can combine that thing by expanding 2×2 matrix representation into 3×3 matrix.
- It will allows us to convert all transformation into matrix multiplication but we need to represent vertex position (x, y) with homogeneous co-ordinate triple (x_h, y_h, h) where $x = \frac{x_h}{h}$, $y = \frac{y_h}{h}$
thus we can also write triple as $(h \cdot x, h \cdot y, h)$
- For two dimensional geometric transformations we can take value of h is any positive number so we can get infinite homogeneous representation for co-ordinate value (x, y)
- But convenient choice is set $h=1$ as it is multiplicative identity. then (x, y) is represented as $(x, y, 1)$.
- Expressing co-ordinates in homogeneous coordinates form allows us to represent all geometric transformation equations as matrix multiplication.
- Lets see each representation with $h=1$

Translation

$$P' = T(t_x, t_y) \cdot P$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

* ~~Inverses~~
* Inverse of translation matrix is obtain by putting $-t_x$ & $-t_y$ instead of t_x & t_y.

Rotation

$$P' = R(\theta) \cdot P$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

* Inverse of rotation matrix is obtained by replacing θ by $-\theta$.

Scaling

$$P' = S(s_x, s_y) \cdot P$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

* Inverse of scaling matrix is obtained by replacing s_x & s_y by $1/s_x$ & $1/s_y$ respectively.