Semester: III Subject: DSGT Academic Year: 2022-2023

ex: If $A \subseteq B$ then $P(A) \subseteq P(B)$.
$\Rightarrow$ Let $C \in P(A)$ Then $C \subseteq A$ which implies $C \subseteq B$
$C \in P(B)$
$P(A) \subseteq P(B)$

* Functions :-

Definition - Let A and B be non-empty sets. A function f from A to B, denoted as $f: A \rightarrow B$, is a relation from A to B such that for every $a \in A$, there exists a unique $b \in B$ such that $(a, b) \in f$.
Normally if $(a, b) \in f$, we write $f(a) = b$.

Property of function :
If $f(a) = b$ and $f(a) = c$
then $b = c$
This condition implies that to each element $a \in A$, a unique element $b \in B$ should be assigned by the relation f.
As f is a relation we may also express f as a set of ordered pairs, i.e.
$f = \{(a, f(a)) \mid a \in A, f(a) \in B\}$
domain of a function is uniquely defined, because function acts on every element of the domain.

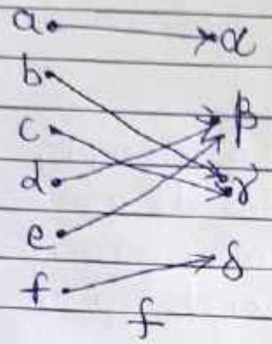
Semester : III Subject : DSGT Academic Year: 2022-2023

Function can be represented in graphical and tabular form.

e.g. Let A be a set of :

$A = \{a, b, c, d, e, f\}$

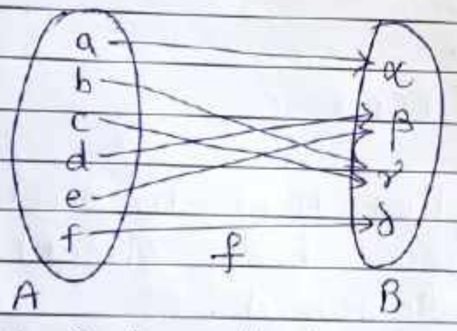
$B = \{\alpha, \beta, \gamma, \delta\}$

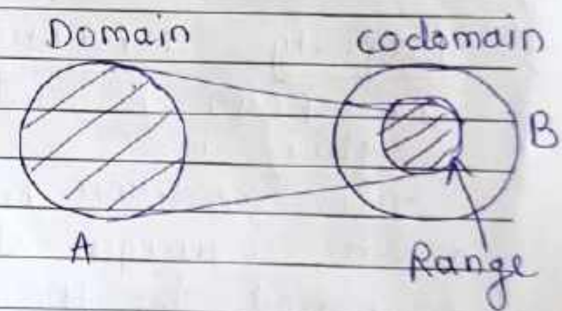


Graphical Repre^n

	α	β	γ	δ
a	✓			
b			✓	
c			✓	
d		✓		
e		✓		
f				✓

Tabular repres^n





The Set A is called as the domain of f, denoted by Dom(f). The set B is called as the codomain, and set $\{f(a) \mid a \in A\}$ which is a subset of B, is called as range of f and denoted by $Ran(f)$. And set B is called as codomain.

Pre-image and Image of a function -

A function $f: A \rightarrow B$ such that for each $a \in A$, there exists a unique $b \in B$ such that $(a, b) \in R$ then a is called the pre-image of f and b is called the image of f.

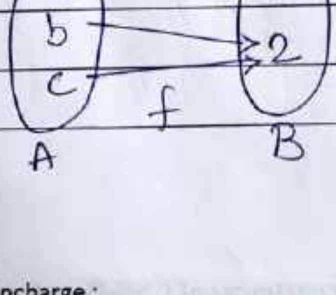
* Types of functions -

(1) Onto or Surjective function -

A function from A to B is said to be an onto function if every element of B is the image of one or more elements of A. Onto function is also called as surjective or 'f' is ONTO if $Ran(f) = B$.

$\therefore f: A \rightarrow B$ is onto if for each $b \in B$, there exists $a \in A$ such that $f(a) = b$

Let $A = \{a, b, c\}$ and $B = \{1, 2\}$ are two sets.



$\therefore f(a) = 1 \quad f(b) = 2$

$f(c) = 2$.

This is onto fun^n.

PARSHVANATH CHARITABLE TRUST'S

A.P. SHAH INSTITUTE OF TECHNOLOGY

Department of Computer Science and Engineering

Data Science

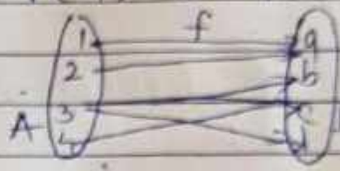
Semester : III   Subject : DSGT   Academic Year: 2022-2023

ex. Let $A = \{1,2,3,4\}$ and $B = \{a,b,c,d\}$ and $f = \{(1,a),(2,a),(3,d),(4,c),(3,b)\}$

$f(1) = a$ $f(2) = a$ $f(3) = d$ $f(4) = c$ $f(3) = b$

$Ran(f) = \{a,d,c,b\} = B$

So this funⁿ is onto function.



② One to One or Injective function -

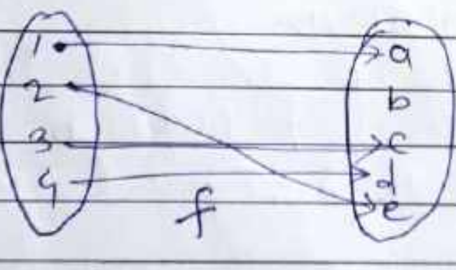
A function from A to B is said to be a one to one function if no two elements of A have the same image. One to one function is also called as injective function.

ex. Let $A = \{1,2,3,4\}$ and $B = \{a,b,c,d,e\}$ and $f = \{(1,a)(2,e)(3,c)(4,d)\}$

$f(1) = a$ $f(2) = e$

$f(3) = c$ $f(4) = d$

This is one to one or injective funⁿ

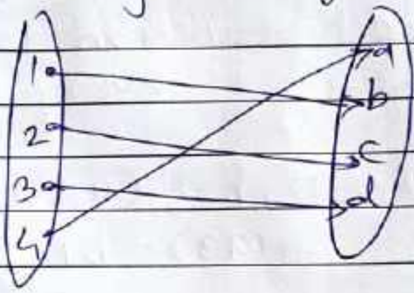


Semester : III Subject : DSGT Academic Year: 2022-2023

(3) One to one onto function or Bijective funⁿ.

A function from A to B is called as one to one onto function if it is both an onto and one to one function. One to one onto funⁿ is also called as bijective function.

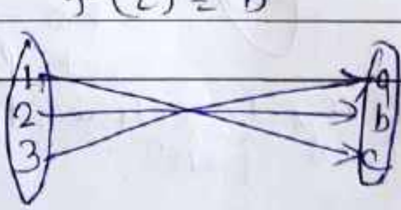
ex. Let $A = \{1, 2, 3, 4\}$
$B = \{a, b, c, d\}$
and $f = \{(1, b)(2, c)(3, d)(4, a)\}$
$f(1) = b$ $f(2) = c$ $f(3) = d$ $f(4) = a$
this funⁿ is bijective funⁿ



(4) Everywhere Defined Function -

A function from A to B is said to be everywhere defined if $Dom(f) = A$.

ex. Let $A = \{1, 2, 3\}$ $B = \{a, b, c\}$
and $f = \{(1, c)(2, b)(3, a)\}$
$f(1) = c$ $f(2) = b$
$f(3) = a$



This is everywhere defined function
$Dom(f) = \{1, 2, 3\}$

* Examples on Functions -

(1) Let $A = \{a_1, a_2, a_3\}$, $B = \{b_1, b_2, b_3\}$, $C = \{c_1, c_2\}$, $D = \{d_1, d_2, d_3, d_4\}$ consider the following four functions from A to B, A to D, B to C and D to B respectively.

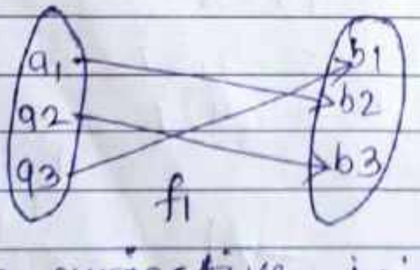
a> $f_1 = \{(a_1, b_2), (a_2, b_3), (a_3, b_1)\}$
b> $f_2 = \{(a_1, d_2), (a_2, d_1), (a_3, d_4)\}$
c> $f_3 = \{(b_1, c_2), (b_2, c_2), (b_3, c_1)\}$
d> $f_4 = \{(d_1, b_1) (d_2, b_2) (d_3, b_1)\}$

Determine whether each function is one to one, whether each function is onto and whether each funⁿ is everywhere defined.

$\Rightarrow$ a> $f_1 = \{(a_1, b_2), (a_2, b_3), (a_3, b_1)\}$

$f_1(a_1) = b_2$ $f_1(a_3) = b_1$
$f_1(a_2) = b_3$

f_1 is everywhere defined $Dom(f_1) = A$
f_1 is onto because $Range(f_1) = B$

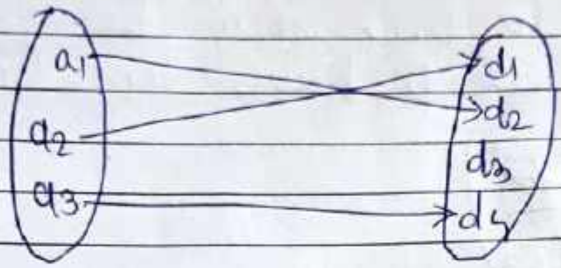


f_1 is also one to one funⁿ because no two elements of set B have same image.

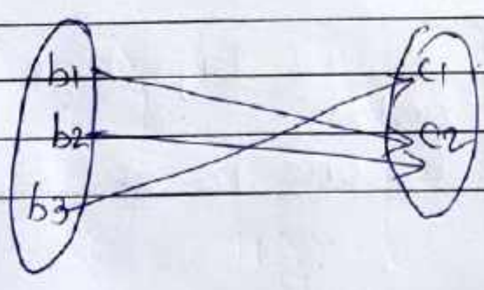
So f_1 is surjective, injective, bijective and everywhere defined funⁿ.

Semester: III Subject: DSGT Academic Year: 2022-2023

b) $f_2 = \{(a_1, d_2), (a_2, d_1), (a_3, d_4)\}$
$f_2(a_1) = d_2$ $f_2(a_2) = d_1$
$f_2(a_3) = d_4$
f_2 is everywhere defined as $dom(f_2) = A$
f_2 is not onto fun^n $Ran(f_2) \neq D$.
f_2 is one to one fun^n.



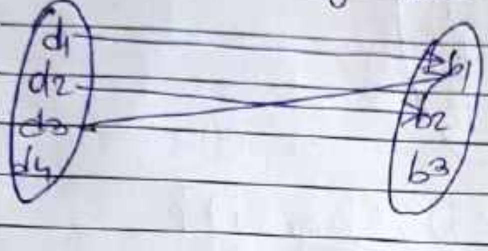
c) $f_3 = \{(b_1, c_2), (b_2, c_2), (b_3, c_1)\}$
$f_3(b_1) = c_2$ $f_3(b_2) = c_2$
$f_3(b_3) = c_1$
f_3 is everywhere defined fun^n because.
$Dom(f_3) = B$
f_3 is onto surjective fun^n because
$Ran(f_3) = C$
f_3 is not one to one fu^n.



Semester : III Subject : DSGT Academic Year: 2022 - 2023

d) $f_4 = \{(d_1, b_1) (d_2, b_2) (d_3, b_1)\}$
$f_4(d_1) = b_1$ $f_4(d_2) = b_2$
$f_4(d_3) = b_1$

f_4 is not onto funⁿ because $Ran(f_4) \neq B$
f_4 is not everywhere defined funⁿ because $Dom(f_4) \neq D$.
f_4 is not one to one funⁿ because two elements of set D have same image



ex

Let $A = B = R$, the set of real numbers. Let $f: A \to B$ be given by the formula $f(x) = 2x^3 - 1$ and let $g: B \to A$ be given by $g(y) = \sqrt[3]{\frac{1}{2}y + \frac{1}{2}}$

Show that f is bijection between A and B and g is a bijection between B and A.

$\Rightarrow$ A function from A to B is Bijection if it is one to one and onto.
$\therefore$ for $f(x) = 2x^3 - 1$ to be one to one & onto.

Semester: III Subject: DSGT Academic Year: 2022-2023

If $a, b \in A$
such that $f(a) = f(b)$
$\Rightarrow$

$2a^3 - \cancel{1} = 2b^3 - \cancel{1}$ $\quad \cancel{2}a^3 = \cancel{2}b^3 \quad = a^3 = b^3$

$a = b$

$\therefore$ f is one to one.

Now for $y = 2x^3 - 1$

$1 + y = 2x^3$

$x^3 = \frac{1}{2} + \frac{y}{2}$

$x = \sqrt[3]{\frac{1}{2} + \frac{y}{2}}$

$\therefore$ for each $y \in B$, There is a unique x in A such that $f(x) = y$.

f is onto.

$\therefore$ f is bijective funⁿ between A and B.

Similarly for $g: B \to A$ to be one to one & onto

$g(a) = g(b) = \sqrt[3]{\frac{1}{2} + \frac{a}{2}} = \sqrt[3]{\frac{1}{2} + \frac{b}{2}}$

$\cancel{\frac{1}{2}} + \frac{a}{2} = \cancel{\frac{1}{2}} + \frac{b}{2}$

$\frac{a}{2} = \frac{b}{2} \quad = \cancel{\frac{1}{2}} a = \cancel{\frac{1}{2}} b$

$\therefore a = b$

g is one to one.

also for $x = \sqrt[3]{\frac{1}{2} + \frac{y}{2}}$

$x^3 = \frac{1}{2} + \frac{y}{2}$ $\qquad x^3 = \frac{1+y}{2}$

$2x^3 = 1 + y$

$y = 2x^3 - 1$

for each x in A. There is a corresponding y in B.

such that $g(y) = x$.

$\therefore$ g is onto function

So g is bijective funⁿ betⁿ B and A.

May 18

ex.

Test whether the following function is one-to-one onto or both.

$f: Z \to Z$, $f(x) = x^2 + x + 1$.

$\Rightarrow$ A function from A to B is one to one if no two elements of A have the same image.

Let, $x = -2$, $f(x) = x^2 + x + 1$

$f(-2) = (-2)^2 + (-2) + 1$

$= 4 - 2 + 1$

$= 3$

Let $x = 1$

$f(1) = 1^2 + 1 + 1 = 3$ Elements 1 and -2 have same image so the function is not one to one.