Extensive form game with perfect information

In game theory, an extensive-form game is a specification of a game allowing (as the name suggests) for the explicit representation of a number of key aspects, like the sequencing of players' possible moves, their choices at every decision point, the (possibly imperfect) information each player has about the other player's moves when they make a decision, and their payoffs for all possible game outcomes. Extensive-form games also allow for the representation of incomplete information in the form of chance events modeled as "moves by nature". Extensive-form representations differ from normal-form in that they provide a more complete description of the game in question, whereas normal-form simply boils down the game into a payoff matrix.

Finite extensive-form games

- A finite set of *n* (rational) players
- A rooted tree, called the *game tree*
- Each terminal (leaf) node of the game tree has an *n*-tuple of *payoffs*, meaning there is one payoff for each player at the end of every possible play
- A partition of the non-terminal nodes of the game tree in *n*+1 subsets, one for each (rational) player, and with a special subset for a fictitious player called Chance (or Nature). Each player's subset of nodes is referred to as the "nodes of the player". (A game of complete information thus has an empty set of Chance nodes.)
- Each node of the Chance player has a probability distribution over its outgoing edges.
- Each set of nodes of a rational player is further partitioned in information sets, which make certain choices indistinguishable for the player when making a move, in the sense that:
    - there is a one-to-one correspondence between outgoing edges of any two nodes of the same information set—thus the set of all outgoing edges of an information set is partitioned in equivalence classes, each class representing a possible choice for a player's move at some point—, and
    - every (directed) path in the tree from the root to a terminal node can cross each information set at most once
- the complete description of the game specified by the above parameters is common knowledge among the players

A play is thus a path through the tree from the root to a terminal node. At any given non-terminal node belonging to Chance, an outgoing branch is chosen according to the probability distribution. At any rational player's node, the player must choose one of the equivalence classes for the edges, which determines precisely one outgoing edge except (in general) the player doesn't know which one is being followed. (An outside observer knowing every other player's choices up to that point, and the realization of Nature's moves, can determine the edge precisely.) A pure strategy for a player thus consists of a selection—choosing precisely one class of outgoing edges for every information set (of his). In a game of perfect information, the information sets are singletons. It's less evident how payoffs should be interpreted in games with Chance nodes. It is assumed that each player has a von Neumann–

Morgenstern utility function defined for every game outcome; this assumption entails that every rational player will evaluate an a priori random outcome by its expected utility.

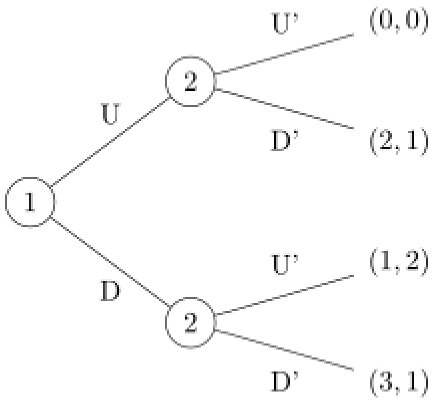
The above presentation, while precisely defining the mathematical structure over which the game is played, elides however the more technical discussion of formalizing statements about how the game is played like "a player cannot distinguish between nodes in the same information set when making a decision".

A perfect information two-player game over a game tree (as defined in combinatorial game theory and artificial intelligence) can be represented as an extensive form game with outcomes (i.e. win, lose, or draw). Examples of such games include tic-tac-toe, chess, and infinite chess. A game over an expect minimax tree, like that of backgammon, has no imperfect information (all information sets are singletons) but has moves of chance. For example, poker has both moves of chance (the cards being dealt) and imperfect information (the cards secretly held by other players).

Perfect and complete information

A complete extensive-form representation specifies:

1. the players of a game
2. for every player every opportunity they have to move
3. what each player can do at each of their moves
4. what each player knows for every move
5. the payoffs received by every player for every possible combination of moves



A game represented in extensive form

The game on the right has two players: 1 and 2. The numbers by every non-terminal node indicate to which player that decision node belongs. The numbers by every terminal node represent the payoffs to the players (e.g. 2,1 represents a payoff of 2 to player 1 and a payoff of 1 to player 2). The labels by every edge of the graph are the name of the action that edge represents.

The initial node belongs to player 1, indicating that player 1 moves first. Play according to the tree is as follows: player 1 chooses between *U* and *D*; player 2 observes player 1's choice and then chooses between *U'* and *D'* . The payoffs are as specified in the tree. There are four outcomes represented by the four terminal nodes of the tree: (U,U'), (U,D'), (D,U') and (D,D'). The payoffs associated with each outcome respectively are as follows (0,0), (2,1), (1,2) and (3,1).

If player 1 plays *D*, player 2 will play U' to maximise their payoff and so player 1 will only receive 1. However, if player 1 plays *U*, player 2 maximises their payoff by playing D' and player 1 receives 2. Player 1 prefers 2 to 1 and so will play *U* and player 2 will play D' . This is the subgame perfect equilibrium.