Semester : III    Subject : DSGT    Academic Year: 2022-2023

# * Pigeonhole Principle -

**Theorem** - If n pigeons are assigned to m pigeonholes, and $m < n$, then at least one pigeonhole contains two or more pigeons.

**Proof** -

Consider labelling the m pigeonholes with the numbers 1 through m and the n pigeons with the numbers 1 through n.

Now beginning with pigeon 1, assign each pigeon in order to the pigeonhole with the same number. This assigns as many pigeons as possible to individual pigeon holes, but because $m < n$, there are $n - m$ pigeons that have not yet been assigned to a pigeonhole.

At least one pigeonhole will be assigned a second pigeon.

n = pigeons

m = pigeonhole    $m < n$

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## Extended pigeonhole principle -

If there are $m$ pigeonholes and more than $2m$ pigeons, then three or more pigeons will have to be assigned to at least one of the pigeonholes.

If $n$ and $m$ are positive integers then $\lfloor n/m \rfloor$ stands for the largest integer less than or equal to the rational number $n/m$.

Thus $\lfloor 3/2 \rfloor$ is 1,

$\lfloor 9/4 \rfloor$ is 2,

$\lfloor 6/3 \rfloor$ is 2.

**Theorem** :

If $n$ pigeons are assigned to $m$, pigeonholes, then one of the pigeonhole must contain at least $\lfloor (n-1)/m \rfloor + 1$ pigeons.

**Proof** :

Assume that each pigeonhole does not contain more than $\lfloor (n-1)/m \rfloor$ pigeons. Then there will be at most

$$m \lfloor (n-1)/m \rfloor \leq m(n-1)/m = n-1 \text{ pigeons}$$

in all.