Semester: VIII Subject: AIFB AcademicYear: 2024-25

WISHART DISTRIBUTION:

The wishart distribution is widely used in statistics, finance and machine learning because it models random covariance matrices, which are essential for understanding relationships between multiple variables.

Covariance Matrix Estimation (Multivariate Statistics).

The Wishart distribution naturally arises when estimating the sample covariance matrix of a multivariate normal distribution:

If we have samples $X_1, X_2, \ldots, X_n$ from a multivariate normal distribution $N(\mu, \Sigma)$, the sample covariance matrix:

$$\boxed{S = \sum_{i=1}^{n} (X_i - \bar{X})(X_i - \bar{X})}.$$

follows a wishart distribution:

$$\boxed{W \sim W_p(n, \Sigma)}$$

where,

W is a $p \times p$ symmetric, positive-definite matrix.

n (degree of freedom) must be $n \geq p$.

Σ is a $p \times p$ symmetric, positive-definite scale matrix.

The distribution arises as the sample covariance matrix in multivariate normal distributions.

Example:

Suppose we have a 2-dimensional normal distribution with a population covariance matrix:

$$\Sigma = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$$

If we take a sample of size $n=5$, the wishart distributed sample covariance matrix would follow:

$$\boxed{W \sim W_2(5, \Sigma)}$$

We can generate samples from this distribution using Python:

```
import numpy as np
from scipy.stats import wishart.

#Define scale matrix (Σ)
Sigma = np.array([[2,1], [1,3]])

#Degree of Freedom (n)
n=5

#Generate a wishart - Distributed random matrix:
W = wishart.rvs(df=n, scale=Sigma, size=1)
print ("Sample Wishart Matrix :\n",W).
```

Output:

```
Sample Wishart Matrix:
[[11.59075027    -2.5134327
 -2.51348727    11.36798724]]
```

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Problem Statement:

A financial analyst is studying a portfolio of 3 assets. The true covariance matrix of the asset returns is given as:

$$\Sigma = \begin{bmatrix} 0.04 & 0.02 & 0.01 \\ 0.02 & 0.05 & 0.03 \\ 0.01 & 0.03 & 0.06 \end{bmatrix}$$

The analyst collected 10 daily return observations for each asset. Assuming the asset returns are multivariate normal, estimate a ~~normal~~ random sample covariance matrix following a Wishart distribution.

Solution:

Given:

Population covariance matrix Σ.

sample size $n = 10$ (degree of freedom).

Wishart-distributed covariance matrix follows:

$$\boxed{W \sim W_3(10, \Sigma)}$$

Use python to generate a sample covariance matrix using Wishart distribution.

```
import numpy as np
from scipy.stats import wishart

# Define the true covariance matrix (Σ)
sigma = np.array([[0.04, 0.02, 0.01],
                  [0.02, 0.05, 0.03],
                  [0.01, 0.03, 0.06]])
```

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#Degree of freedom (n)
n=5

#Generate a Wishart-Distributed random matrix:
W = wishart.rvs (df=n, scale=Sigma, size=1)
print ("Sample Wishart Matrix: \n", W)

Output:
Sample Wishart Distribution:
[[11.5907 -2.5134
-2.5134 11.36798]]

* Non-zero elements in the precision matrix indicate direct relationship between variables.
* Zero element implies the ~~element~~ variables are independent.
* This matrix is used in finance to estimate real-world risk and volatility when the true covariance matrix is unknown.
* As the sample size increases, the estimated covariance matrix converges to the true covariance Σ.
* It helps investors to do portfolio diversification.