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* Least Upper Bound and Greatest Lower Bound :-

Let say we have a poset (S, R) such that S is an arbitrary set and R is a partial order defined on set S.

Also, let say $T \subseteq S$.

(Join) **Least Upper Bound** - (LUB) → denoted by $\vee$

Let U is the set of all upper bounds of set T. Then, an element $x \in U$ is called the least upper bound if $\forall y \in V (x, y) \in R$.

$LUB(T) = \text{minimum}\{UB(T)\}$

(Meet) **Greatest Lower Bound** (GLB) → denoted by $\wedge$

Let L is the set of all lower bounds of set T. Then, an element $x \in L$ is called the greatest lower bound if $\forall y \in L (y, x) \in R$.

$GLB(T) = \text{maximum}\{LB(T)\}$

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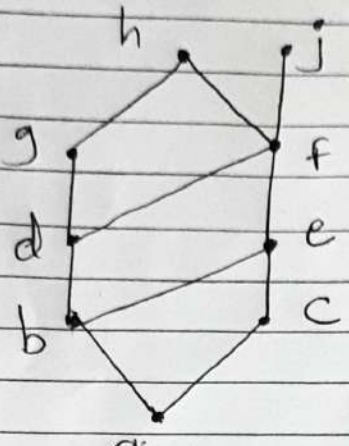
Ex: (1) Find GLB and LUB of {b, d, g} if they exist in the poset with the Hasse diagram shown below.



LUB =
Upper bounds of {b, d, g} = g, h

Least Upper Bound of {b, d, g} = g

because: g and h are upper bounds of {b, d, g} where bRg, dRg, gRg bRh, dRh, gRh.
we can say gRh but h ~~R~~ g.
hence g is LUB.

GLB = lower bound of {b, d, g}
= a and b.
Greatest lower bound of {b, d, g}
= b.

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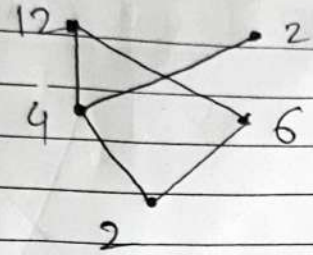
* More examples on minimal elements, maximal elements.

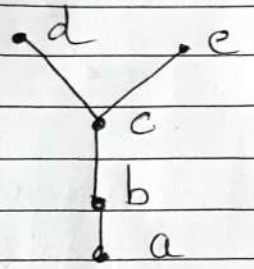
(1) $A = \{x \mid x \text{ is a real no. and } 0 < x \leq 2\}$
maximal element = 2
minimal element = ϕ [$\because 0 \notin A$]

(2)



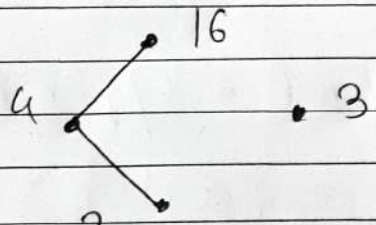
Minimal = 2
Maximal = 12, 20

(3)



Minimal = a
Maximal = d, e.

(4)



Maximal = 3, 16
Minimal = 3, 2

3 is not comparable.

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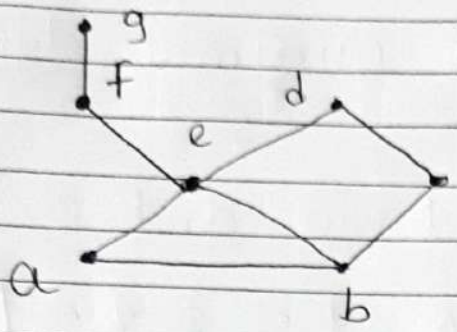
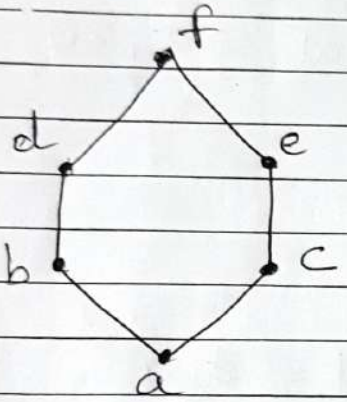
(5)



Maximal = d, g
Minimal = a, b.

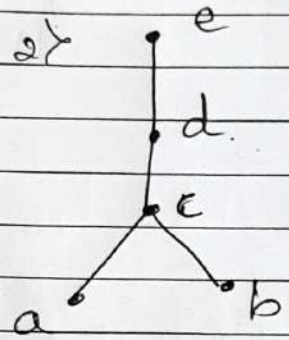
* Examples on Greatest & Least element.

1)



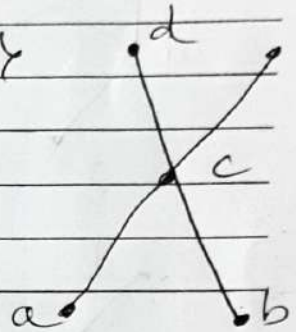
Greatest = f
least = a

2)



Greatest = e
least = ϕ

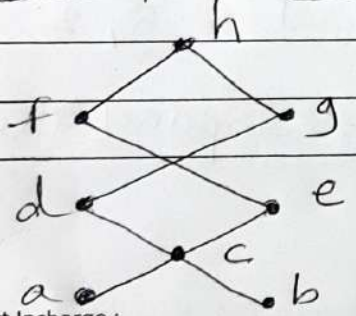
3)



Greatest = ϕ
least = ϕ

* Examples on lower bound & upper bound.

(4)



i) B = {a, b}
lower bound = ϕ
upper bound = c, d, e, f, g, h.

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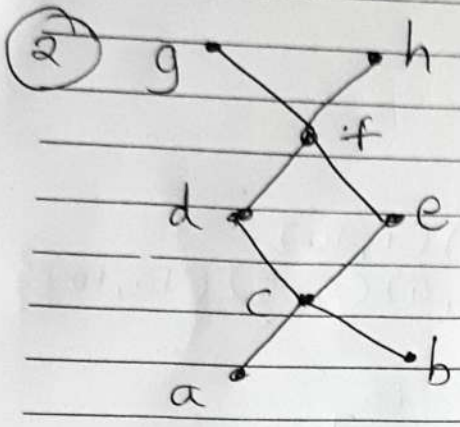
ii) $B = \{d, e\}$
lower bound = a, b, c
upper bound = f, g, h.

(2)



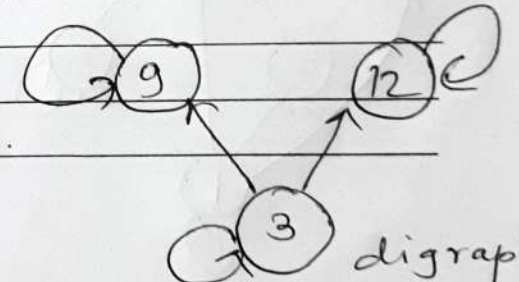
$A = \{c, d, e\}$
lower bound = a, b
upper bound = g, h.

(3) Find greatest lower bound & least upper bound of the sets $\{3, 9, 12\}$ and $\{1, 2, 4, 5, 10\}$ if they exist in the poset $(Z, /)$ where / is a relation of divisibility.

i) $\{3, 9, 12\}$
$R = \{(3,3)(3,9)(3,12)(9,9)(12,12)\}$

$M_R =$

	3	9	12
3	1	1	1
9	0	1	0
12	0	0	1



digraph of R

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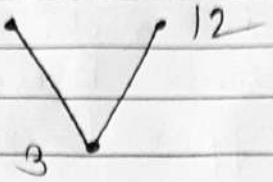
Hasse diagram =



lower bound = 3
GLB = 3
upper bound = ϕ

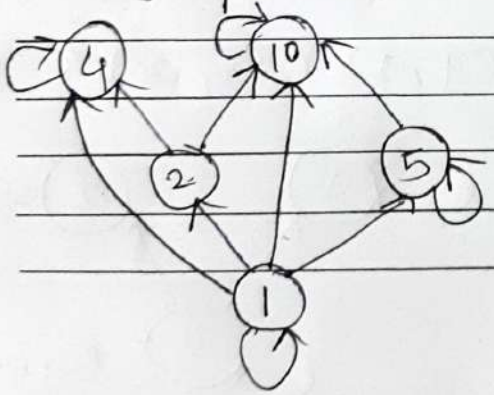
ii) {1, 2, 4, 5, 10}

R = {(1,1) (1,2) (1,4) (1,5) (1,10) (2,2) (2,4) (2,10) (4,4) (5,5) (10,10) (5,10)}

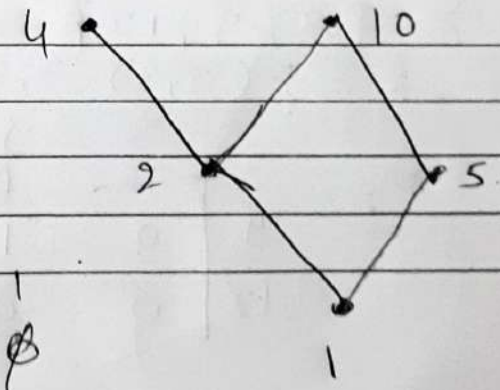
M_R =

	1	2	4	5	10
1	1	1	1	1	1
2	0	1	1	0	1
4	0	0	1	0	0
5	0	0	0	1	1
10	0	0	0	0	1

digraph of R



Hasse dia.



GLB = 1
LUB = ϕ

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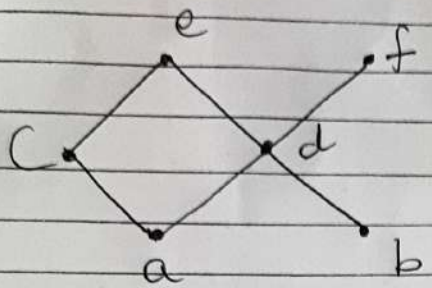
X.



Find GLB, LUB

i) B = {c, d}
UB = {e}
LB = a
LUB = e
GLB = a

ii) A = {a, b}
UB = d, e, f
LUB = d
LB = ∅
GLB = ∅

iii) C = {e, f}
UB = ∅
LUB = ∅
LB = a, d, b
GLB = d.