SVM cont. →

Orthogonal projection of a vector →

Given two vectors x and y, we would find orthogonal projection of x onto y.

To do this, we project x onto y as shown:

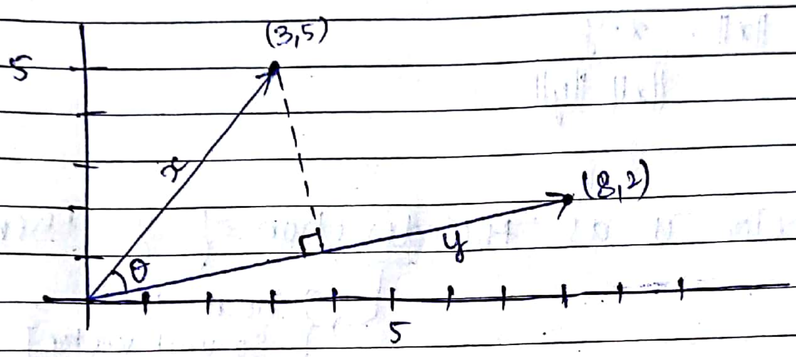

Fig ①

This gives us vector z as shown below:

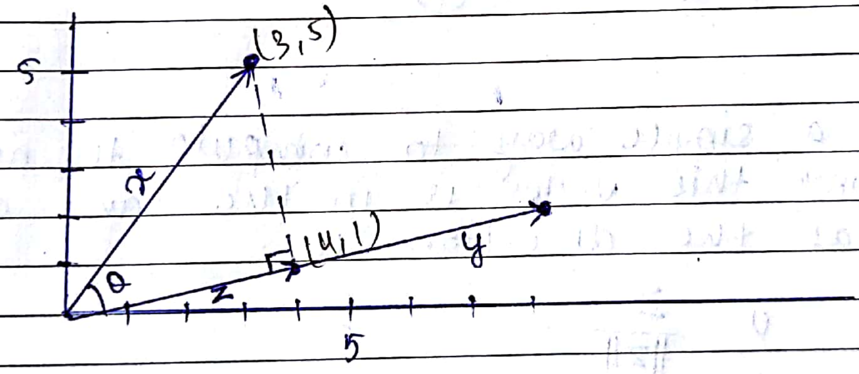

Fig ②

To find the orthogonal projection of a vector, first we need to calculate cos(θ).

$$\cos(\theta) = \frac{\text{Adjacent}}{\text{Hypotaneous}} = \frac{\|z\|}{\|x\|}$$

$$\|z\| = \|x\| \cos\theta \quad \text{—— (1)}$$

By the definition of dot product

$$x \cdot y = \|x\| \|y\| \cos(\theta)$$

$$\Rightarrow \quad \cos(\theta) = \frac{x \cdot y}{\|x\| \|y\|} \quad -(2)$$

Substitute (2) in (1).

$$\|z\| = \|x\| \cdot \frac{x \cdot y}{\|x\| \|y\|}$$

If we define vector u as the direction of y then

$$u = \frac{y}{\|y\|}$$

{ norm = 1 so unit vector }

then $\|z\| = x \cdot u \quad -(3)$

We now have a simple way to compute the norm of vector z. Since this vector is in the same direction as y, it has the direction u.

$$u = \frac{z}{\|z\|}$$

$$z = \|z\| \cdot u \quad -(4)$$

Substitute (3) in (4)

$$\boxed{z = (u \cdot x) \cdot u}$$

So the vector $z = (u \cdot x) \cdot u$ is the orthogonal projection of x onto y.

This orthogonal projection allows us to compute the distance between x and the line which goes through y.

So, for the points given in fig (2)

norm of vector $y = \sqrt{8^2 + 2^2} = \sqrt{68}$

direction of vector $y = \left(\frac{8}{\sqrt{68}}, \frac{2}{\sqrt{68}}\right)$
Lets say, vector(u).

norm of vector $u = \sqrt{\left(\frac{8}{\sqrt{68}}\right)^2 + \left(\frac{2}{\sqrt{68}}\right)^2} = 1$ (unit vector)

norm of vector $x = \sqrt{3^2 + 5^2} = 4$

$z = (u \cdot x) \cdot u$

$z = \left[\left(\frac{8}{\sqrt{68}}, \frac{2}{\sqrt{68}}\right) \cdot (3,5)\right] \cdot u$

$= \left[\frac{24}{\sqrt{68}} + \frac{10}{\sqrt{68}}\right] \cdot u = \left(\frac{34}{\sqrt{68}}\right) \cdot u$

$= \left(\frac{34}{\sqrt{68}}\right)\left(\frac{8}{\sqrt{68}}, \frac{2}{\sqrt{68}}\right)$

$= \left(\frac{264}{68}, \frac{68}{68}\right) = (3.88, 1) \simeq (4, 1)$