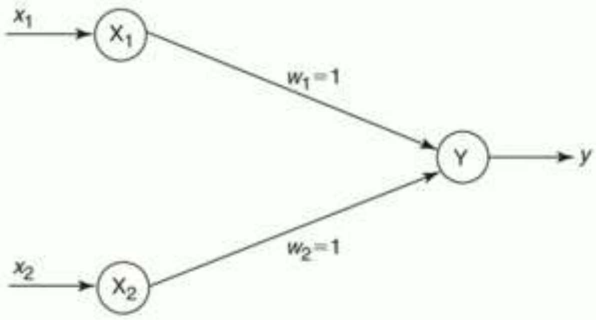
Implement AND function using McCulloch–Pitts Neuron

x_1	x_2	y
1	1	1
1	0	0
0	1	0
0	0	0

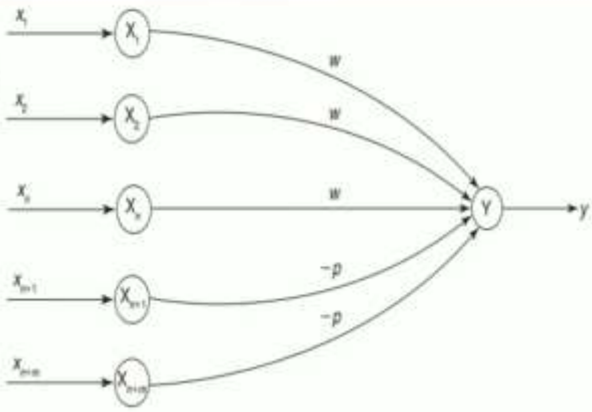


Implement AND function using McCulloch–Pitts Neuron

- The McCulloch–Pitts neuron was the earliest neural network discovered in 1943.
- It is usually called as M–P neuron.
- Since the firing of the output neuron is based upon the threshold, the activation function here is defined as

$$f(y_{in}) = \begin{cases} 1 & \text{if} \quad y_{in} \geq \theta \\ 0 & \text{if} \quad y_{in} < \theta \end{cases}$$

- The threshold value should satisfy the following condition: $\theta > nw - p$



Implement AND function using McCulloch–Pitts Neuron

- Consider the truth table for AND function
- The M–P neuron has no particular training algorithm
- In M-Pneuron, only analysis is being performed.
- Hence, assume the weights be w1 = 1 and w2 = 1.

x_1	x_2	y
1	1	1
1	0	0
0	1	0
0	0	0

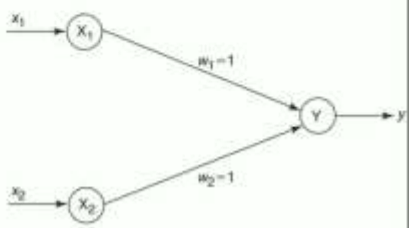
$(1, 1)$, $y_{in} = x_1w_1 + x_2w_2 = 1\times 1 + 1\times 1 = 2$

$(1, 0)$, $y_{in} = x_1w_1 + x_2w_2 = 1\times 1 + 0\times 1 = 1$

$(0, 1)$, $y_{in} = x_1w_1 + x_2w_2 = 0\times 1 + 1\times 1 = 1$

$(0, 0)$, $y_{in} = x_1w_1 + x_2w_2 = 0\times 1 + 0\times 1 = 0$

Threshold value is set equal to 2 ($\theta = 2$).



Implement AND function using McCulloch–Pitts Neuron

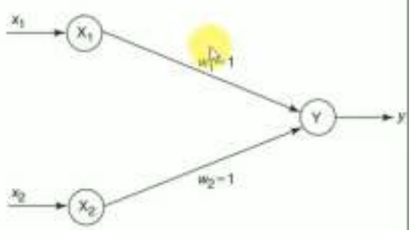
- This can also be obtained by

$$\theta \geq nw - p$$

- Here, n = 2, w = 1 (excitatory weights) and p = 0 (no inhibitory weights).
- Substituting these values in the above-mentioned equation we get $\theta \geq 2\times 1 - 0 \Rightarrow \theta \geq 2$
- Thus, the output of neuron Y can be written

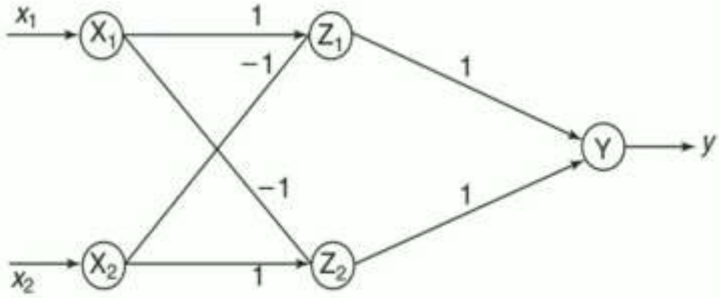
$$y = f(y_{in}) = \begin{cases} 1 & \text{if} \quad y_{in} \geq 2 \\ 0 & \text{if} \quad y_{in} < 2 \end{cases}$$

x_1	x_2	y
1	1	1
1	0	0
0	1	0
0	0	0



Implement XOR function using McCulloch–Pitts Neuron

x_1	x_2	y
0	0	0
0	1	1
1	0	1
1	1	0



Implement XOR function using McCulloch–Pitts Neuron

- Consider the truth table for XOR function
- The M–P neuron has no particular training algorithm
- In M-P neuron, only analysis is being performed.
- XOR function cannot be represented by simple and single logic function; it is represented as

x_1	x_2	y
0	0	0
0	1	1
1	0	1
1	1	0

$$y = x_1\overline{x_2} + \overline{x_1}x_2$$

Implement XOR function using McCulloch–Pitts Neuron

$$y = x_1 \overline{x_2} + \overline{x_1} x_2$$

$$y = z_1 + z_2$$

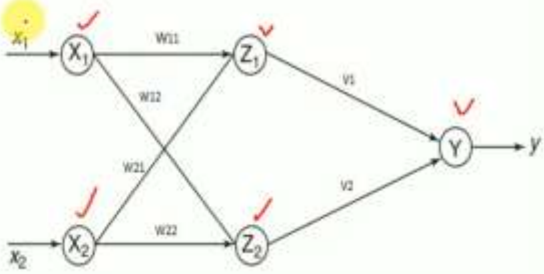
where

$z_1 = x_1 \overline{x_2}$ (function 1)

$z_2 = \overline{x_1} x_2$ (function 2)

$y = z_1 \text{ (OR) } z_2$ (function 3)

x_1	x_2	y
0	0	0
0	1	1
1	0	1
1	1	0



- A single-layer net is not sufficient to represent the XOR function. We need to add an intermediate layer is necessary.

Implement XOR function using McCulloch–Pitts Neuron

- First function $z_1 = x_1 \overline{x_2}$
- The truth table for function z_1
- Assume the weights are initialized to

$$w_{11} = w_{21} = 1$$

- Calculate the net inputs,

$(0, 0), z_{1in} = 0 \times 1 + 0 \times 1 = 0$

$(0, 1), z_{1in} = 0 \times 1 + 1 \times 1 = 1$

$(1, 0), z_{1in} = 1 \times 1 + 0 \times 1 = 1$

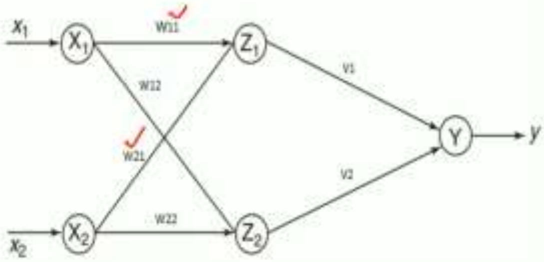
$(1, 1), z_{1in} = 1 \times 1 + 1 \times 1 = 2$

- Hence, it is not possible to obtain function z_1 using these weights.

$\theta = 1, 2$

$$f(y_{in}) = \begin{cases} 1 & \text{if } y_{in} \geq \theta \\ 0 & \text{if } y_{in} < \theta \end{cases}$$

x_1	x_2	z_1
0	0	0
0	1	0
1	0	1
1	1	0



Implement XOR function using McCulloch–Pitts Neuron

- First function $z_1 = x_1 \overline{x_2}$
- The truth table for function z_1
- Assume the weights are initialized to

$$w_{11} = 1; \quad w_{21} = -1$$

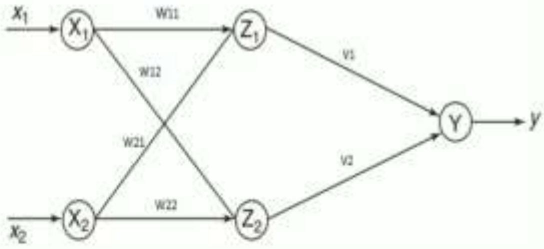
- Calculate the net inputs,

$$(0, 0), z_{1in} = 0 \times 1 + 0 \times -1 = 0 \quad ✗$$
$$(0, 1), z_{1in} = 0 \times 1 + 1 \times -1 = -1 \quad ✗$$
$$(1, 0), z_{1in} = 1 \times 1 + 0 \times -1 = 1 \quad ✓$$
$$(1, 1), z_{1in} = 1 \times 1 + 1 \times -1 = 0 \quad ✗$$

- If the θ=1 then the neuron fires.
- Hence $w_{11} = 1; \quad w_{21} = -1$

$$f(y_{in}) = \begin{cases} 1 & \text{if } y_{in} \ge \theta \\ 0 & \text{if } y_{in} < \theta \end{cases}$$

x_1	x_2	z_1
0	0	0
0	1	0
1	0	1
1	1	0



Implement XOR function using McCulloch–Pitts Neuron

- Second function $z_2 = \overline{x_1} x_2$
- The truth table for function z_2
- Assume the weights are initialized to

$$w_{12} = w_{22} = 1$$

- Calculate the net inputs,

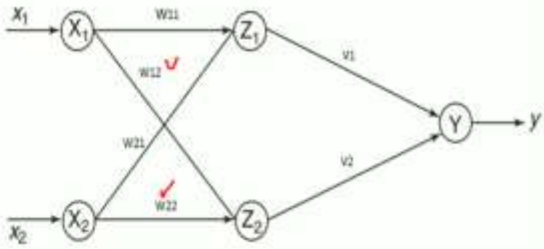
$$(0, 0), z_{2in} = 0 \times 1 + 0 \times 1 = 0 \quad ✗ \quad ✓$$
$$(0, 1), z_{2in} = 0 \times 1 + 1 \times 1 = 1 \quad ✓ \quad ✗$$
$$(1, 0), z_{2in} = 1 \times 1 + 0 \times 1 = 1 \quad ✓ \quad ✗$$
$$(1, 1), z_{2in} = 1 \times 1 + 1 \times 1 = 2 \quad ✓ \quad ✓$$

- Hence, it is not possible to obtain function z_2 using these weights.

θ=1, 2

$$f(y_{in}) = \begin{cases} 1 & \text{if } y_{in} \ge \theta \\ 0 & \text{if } y_{in} < \theta \end{cases}$$

x_1	x_2	z_2
0	0	0
✓0	1	1
1	0	0
1	1	0



Implement XOR function using McCulloch–Pitts Neuron

- Second function $z_2 = \overline{x_1} x_2$
- The truth table for function z_2

$\theta=1$

x_1	x_2	z_2
0	0	0
0	1	1
1	0	0
1	1	0

- Assume the weights are initialized to

$$w_{12} = -1; \quad w_{22} = 1$$

$$f(y_{in}) = \begin{cases} 1 & \text{if } y_{in} \ge \theta \\ 0 & \text{if } y_{in} < \theta \end{cases}$$

- Calculate the net inputs,

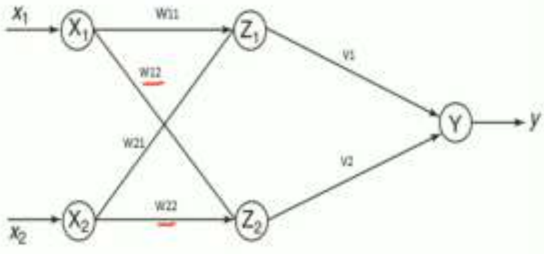
$(0, 0), z_{2in} = 0 \times -1 + 0 \times 1 = 0$ ✗

$(0, 1), z_{2in} = 0 \times -1 + 1 \times 1 = 1$ ✓

$(1, 0), z_{2in} = 1 \times -1 + 0 \times 1 = -1$ ✗

$(1, 1), z_{2in} = 1 \times -1 + 1 \times 1 = 0$ ✗

- If the θ=1 then the neuron fires.
- Hence $w_{12} = -1; \quad w_{22} = 1$



Implement XOR function using McCulloch–Pitts Neuron

- Third function $y = z_1 \text{ (OR) } z_2$
- The truth table for function y

x_1	x_2	y	z_1	z_2
0	0	0	0	0
0	1	1	0	1
1	0	1	1	0
1	1	0	0	0

$$y_{in} = z_1 v_1 + z_2 v_2$$

$\theta = 0, 1$

- Assume the weights are initialized to

$$v_1 = v_2 = 1$$

- Calculate the net inputs,

$$f(y_{in}) = \begin{cases} 1 & \text{if } y_{in} \ge \theta \\ 0 & \text{if } y_{in} < \theta \end{cases}$$

$(0, 0), y_{in} = 0 \times 1 + 0 \times 1 = 0$ ✓ ✗

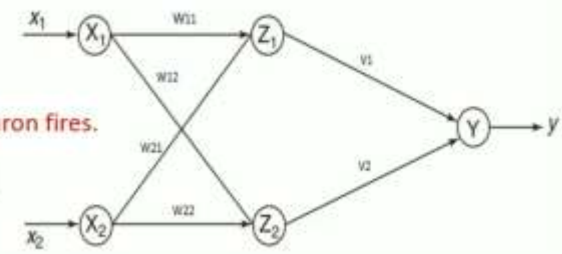
$(0, 1), y_{in} = 0 \times 1 + 1 \times 1 = 1$ ✓

$(1, 0), y_{in} = 1 \times 1 + 0 \times 1 = 1$ ✓

$(0, 0), y_{in} = 0 \times 1 + 0 \times 1 = 0$ ✓ ✗

If the θ=1 then the neuron fires.

Hence $v_1 = v_2 = 1$



Implement XOR function using McCulloch–Pitts Neuron

$$w_{11} = 1; \quad w_{21} = -1$$

$$w_{12} = -1; \quad w_{22} = 1$$

$$v_1 = v_2 = 1$$