CURVE FITTING

- Curve fitting using Least Square Method.

- Fitting of straight line

Suppose we have set of points (x, y). Then to fit a straight line to these values.

As $y = ax + b$ be a straight line then to find a & b we will solve following equations

$$\Sigma y = a\Sigma x + nb$$
$$\Sigma xy = a\Sigma x^2 + b\Sigma x$$

Here n = number of points given.

1) Fit a straight line $y = ax + b$ to the following data.

x:	1	2	3	4	5	6
y:	49	54	60	73	80	86

Also find y when $x = 8$

Sol^n Here, $n = 6$

We have equation of straight line as

$y = ax + b$

Then to find a & b we will solve following equations.

$$\Sigma y = a\Sigma x + nb \quad \text{—— (1)}$$
$$\Sigma xy = a\Sigma x^2 + b\Sigma x \quad \text{—— (2)}.$$

From given data $\Sigma y = 402$, $\Sigma x = 21$,

$\Sigma xy = 1545$, $\Sigma x^2 = 91$

∴ From (1) & (2) we get

$402 = 21a + 6b$ — ③
$1545 = 91a + 21b$ — ④

By solving ③ & ④ we get

$a = 7.8857$ & $b = 39.4$.

$\therefore$ Equation of line is $y = 7.8857x + 39.4$ — ⑤

To find y for $x = 8$.

$\therefore$ from ⑤ $y = 7.8857(8) + 39.4$
$= 102.4856$

2) Fit a straight line to the following data.

year x:	1951	1961	1971	1981	1991
production y: (tons)	10	12	8	10	15

Also estimate the production in 1987.

Solⁿ (Equation of straight line is $y = ax + b$ then one can solve a problem using direct method which we have used in problem 1). But here as values of x are bigger hence to make calculations easy we'll use following alternative method.)

From given data $\bar{x} = \frac{\sum x_i}{n}$.

Here $n = 5$ $\therefore \bar{x} = \frac{9855}{5} = 1971$

Scanned by CamScanner

Consider the following chart.

x	$X = x - \bar{x}$	y
1951	-20	10
1961	-10	12
1971	0	8
1981	10	10
1991	20	15

Clearly from chart $\Sigma X = 0$

$\Sigma y = 55$

$\Sigma X \cdot y = 80$

$\Sigma X^2 = 1000$

As Here we have replaced x by X

Our equation of line is

$$y = aX + b \quad \text{——— (1)}$$

Then to solve a & b we'll use following equations

$$\Sigma y = a\Sigma X + nb$$

$$\Sigma Xy = a\Sigma X^2 + b\Sigma X$$

by substituting the values from above eq^{n}s we get,

$$55 = a(0) + 5b \Rightarrow 5b = 55 \Rightarrow b = 11$$

$$80 = a(1000) + b(0)$$

$$\Rightarrow a = \frac{80}{1000} = 0.08$$

$\therefore$ From (1) we get, $y = 0.08X + 11$

but $X = x - \bar{x} = x - 1971$

$\therefore y = 0.08(x - 1971) + 11$

$\therefore y = 0.08x - 0.08 \times 1971 + 11$

$= 0.08x - 157.66 + 11$

$y = 0.08x - 146.68$

How to find y when $x = 1987$.

$y = 0.08(1987) - 146.68$

$y = 12.28$

Ex. 1) Fit a first degree curve (i.e straight line) for following data & estimate the value of y when $x = 73$

x :	10	20	30	40	50	60	70	80
y :	1	3	5	10	6	4	2	1

2) Fit a straight line to the following data:

x	100	120	140	160	180	200
y	0.45	0.55	0.60	0.70	0.80	0.85

- Fitting of Parabola (second degree curve)

We have equation of parabola as

$$y = ax^2 + bx + c$$

Suppose we need to fit parabola for n points.
Now to find a, b, c we need to solve following equations

$$\Sigma y = a\Sigma x^2 + b\Sigma x + nc$$
$$\Sigma xy = a\Sigma x^3 + b\Sigma x^2 + c\Sigma x$$
$$\Sigma x^2 y = a\Sigma x^4 + b\Sigma x^3 + c\Sigma x^2$$

1) Fit a parabola to the following data

x:	0	1	2	3	4
y:	1.0	1.5	1.5	2.5	3.5

Solⁿ Here $n = 5$

Equation of parabola is $y = ax^2 + bx + c$

Then to find a, b, c we'll use following equations

$$\Sigma y = a\Sigma x^2 + b\Sigma x + nc$$

$$\Sigma xy = a\Sigma x^3 + b\Sigma x^2 + c\Sigma x$$

$$\Sigma x^2 y = a\Sigma x^4 + b\Sigma x^3 + c\Sigma x^2$$

From given data.

$\Sigma y = 10$ $\Sigma x = 10$ $\Sigma x^2 = 30.$

$\Sigma xy = 26$ $\Sigma x^3 = 100$

$\Sigma x^2 y = 86$ $\Sigma x^4 = 354$

Substituting these values in above equations we get,

$$10 = 30a + 10b + 5c \longrightarrow 1)$$
$$26 = 100a + 30b + 10c \longrightarrow 2)$$
$$86 = 354a + 100b + 30c \longrightarrow 3)$$

Solving 1), 2) & 3) simultaneously we get

$$a = 0.1429 = \frac{1}{7}$$
$$b = 0.0286 = \frac{1}{35}$$
$$c = 1.0857 = \frac{38}{35}$$

$$\therefore y = \frac{1}{7}x^2 + \frac{1}{35}x + \frac{38}{35} \quad \text{or}$$

$$y = 0.1429x^2 + 0.0286x + 1.0857$$

Ex.2) Fit a second degree parabolic curve to the following data.

i)

x :	−2	−1	0	1	2
y :	1.0	1.8	1.3	2.5	6.3

ii)

x :	1	2	3	4	5	6	7	8	9
y :	2	6	7	8	10	11	11	10	9

3) Fit a second degree parabolic curve to the following data & estimate the production in 1982

Year(x):	1974	1975	1976	1977	1978	1979	1980	1981
Production: (Y) tons	12	14	26	42	40	50	52	53

Solⁿ (We can also solve the problem using above method) Now we'll solve this problem by finding mean of x & hence we can reduce the values of x by considering $X = x - \bar{x}$

$n = 8$,

$$\therefore \bar{x} = \frac{\sum x_i}{n} = \frac{15820}{8} = 1977.5$$

x	$X = x - \bar{x}$	y
1974	-3.5	12
1975	-2.5	14
1976	-1.5	26
1977	-0.5	42
1978	0.5	40
1979	1.5	50
1980	2.5	52
1981	3.5	53

As $X = x - \bar{x}$

Hence, parabolic equation is $y = aX^2 + bX + c$

Then to find $a, b, \& c$ we'll solve

$$\sum y = a\sum X^2 + b\sum X + nc$$
$$\sum Xy = a\sum X^3 + b\sum X^2 + c\sum X$$
$$\sum X^2 y = a\sum X^4 + b\sum X^3 + c\sum X^2$$

$\sum y = 289$, $\sum X^2 = 42$, $\sum X = 0$

$\sum Xy = 273.5$, $\sum X^3 = 0$, $\sum X^2 y = 1400.25$, $\sum X^4 = 388.5$

Hence from above equations

$$289 = 42a + 0b + 8c \quad \text{——— 1>}$$

$$273.5 = 0a + 42b + 0c \quad \text{——— 2>}$$

$$1400.25 = 3885a + 0b + 42c \quad \text{——— 3>}$$

by Solving 1>, 2> & 3> we get

$$a = -0.6964 \ , \ b = 6.5119 \ , \ x = 39.7813$$

Hence we'll have parabolic equation as

$$y = (-0.6964)X^2 + 6.5119X + 39.7813$$

$$= (-0.6964)(x-\bar{x})^2 + 6.5119(x-\bar{x}) + 39.7813$$

$$= (-0.6964)(x^2 - 2x\bar{x} + \bar{x}^2) + 6.5119x - 6.5119\bar{x} + 39.7813$$

$$= -0.6964x^2 + 2 \times 0.6964 \times 1977.5x - 0.6964 \times (1977.5)^2$$

$$+ 6.5119x - 6.5119(1977.5) + 39.7813$$

$$= -0.6964x^2 + (2 \times 0.6964 \times 1977.5 + 6.5119)x$$

$$- 0.6964 \times (1977.5)^2 - 6.5119(1977.5) + 39.7813$$

$$y = -0.6964x^2 + 2760.7739x - 2736114.053.$$

Now to fing production for year 1982

i.e. to find y for $x = 1982$

$$y = -0.6964(1982)^2 + 2760.7739(1982) - 2736114.053$$

$$= 54.9828$$

- Fitting of exponential curve.

$y = ae^{bx}$

Taking log on both side,

$\log y = \log(ae^{bx})$

$= \log(a) + \log e^{bx}$

$= \log(a) + bx(\log e)$

$\log y = \log(a) + bx$.

$Y = A + bx$, Here $Y = \log y$ & $A = \log a$

$Y = bx + A$ ——— (1)

which is a straight line & then we can find constants b & A, we'll use following equations

$$\Sigma Y = b\Sigma x + nA$$
$$\Sigma xY = b\Sigma x^2 + A\Sigma x.$$

where n is given number of points

After solving above equations we'll get value of b & A then, from $A = \log a$ & $Y = \log y$ we can write $y = ae^{bx}$

1) Fit a exponential curve $y = ae^{bx}$ for the following data.

x	1	2	3	4	5	6
y	120	90	60	20	11	5

Solⁿ To fit a curve $y = ae^{bx}$

Taking log on both side

$\log y = \log(ae^{bx})$

$\log y = \log a + \log e^{bx}$

$\log y = \log a + bx(\log e)$

$\Rightarrow \log y = \log a + bx$.

Consider $\log y = Y$ & $\log a = A$
then $Y = A + bx$
i.e $Y = bx + A$ is a straight line
& then to find b & A we'll solve following eqns.

$$\Sigma Y = b\Sigma x + nA \quad \text{— 1)}$$
$$\Sigma xY = b\Sigma x^2 + A\Sigma x \quad \text{— 2)}$$

From given data first to fing $Y = \log y = \ln y$

x	y	$Y = \log y$ (or $\ln y$)
1	120	4.7875
2	90	4.4998
3	60	4.0943
4	20	2.9957.
5	11	2.3979
6.	5	1.6094

Here $n = 6$.
$\Sigma Y = 20.3846$, $\Sigma x = 21$,
$\Sigma xY = 59.6987$, $\Sigma x^2 = 91$

Hence from 1) & 2) we have
$20.3846 = 21b + 6A$
$59.6987 = 91b + 21A$
$b = -0.6656$
$A = 5.7269$

as $A = \log a$

$\Rightarrow 5.7269 = \log a$

$\Rightarrow a = e^{5.7269} = 307.016$

$\therefore y = ae^{bx}$ becomes

$y = (307.016)\, e^{-0.6656x}$

Ex 2) Fit a curve $y = ae^{bx}$ for the following data.

x	1	2	3	4	5	6	7	8
y	1	1.2	1.8	2.5	3.6	4.7	6.6	9.1