This is in contradiction to our assumptions. Hence, for given m pigeonholes, one of these must contain at least $[(n-1)/m]+1$ pigeons.

examples -

(1) If eight people are chosen in anyway from some group, at least two of them will have been born on the same day of the week.

$\Rightarrow$ $n = 8$
$m = 7$ (days of week)

Here each person is assigned to the day of the week on which he or she was born. Since there are eight people and only seven days of the week, the pigeon hole principle tells us at least two people must be assigned to the same day of the week.

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(2) Show that 7 colors are used to paint 50 houses, at least 8 houses will be of same color.

⇒ If n pigeons are assigned to m pigeonholes and $m < n$, at least one pigeonhole contains two or more pigeons

By extended pigeonhole principle, at least

$\lfloor (50-1)/7 \rfloor + 1 = 8$

8 will be of the same color.

(3) What is the minimum number of students required in a discrete structures class to be sure that at least six will receive the same grade, if there are five possible grades A, B, C, D, E.

⇒ By extended pigeonhole principle.

$\lfloor (n-1)/m \rfloor + 1 = 6$

$\lfloor (n-1)/5 \rfloor + 1 = 6$

$\frac{(n-1)}{5} = 6 - 1$

$$\frac{n-1}{5} = 5$$

$$n-1 = 5 \times 5$$

$$n-1 = 25$$

$$n = 25+1$$

$$n = 26$$

$\therefore$ 26 students are required in a discrete structures class.

(4) How many friends must you have to guarantee that at least five of them will have birthdays in the same month.

$\Rightarrow$

Let n = no. of friends
if no. of months are to be pigeonhole then no. of friends will be pigeon.

$\therefore$ By extended pigeonhole principle

$$\left\lfloor \frac{(n-1)}{12} \right\rfloor + 1 = 5$$

$$\frac{n-1}{12} = 5-1$$

$$\frac{n-1}{12} = 4$$

$$n-1 = 12 \times 4$$

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$n - 1 = 48$

$n = 49$

Thus among 49 friends, at least five of them will have birthdays in the same month.

(5) Prove that among 100000 people, there are two who are born at exactly the same time (hour, minute, second).

$\Rightarrow$ Let A be the set of people
B the set of seconds of one day.

$|A| = 100000 = n$

$|B| = 24 \times 3600 = 86400 = m$

Then $k = \lfloor (n-1)/m \rfloor + 1$

$= \lfloor (100000-1)/86400 \rfloor + 1$

$= 1 + 1$

$k = 2$

Hence, at least two who are born on the same day.

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(6) There are 3000 students in a college which offers 7 distinct courses of 4 years duration. A student who has taken a course in Discrete Mathematics learns that the largest classroom can hold only 100 students. she at one realizes there is a problem. what is the problem?

⇒ Since, there are 7 distinct classes of 4 years duration,
we have $7 \times 4 = 28$ diff. classes.

By extended pigeonhole principle, each classroom must hold atleast

$$\lfloor (3000-1)/28 \rfloor + 1 = 107 + 1$$
$$= 108 \text{ students}$$

Since the capacity of the largest classroom is only 100,
This is exactly the problem.

(7) A bag contains 10 red marbles, 10 white marbles and 10 blue marbles. What is the minimum no. of marbles you have to choose randomly from the bag to ensure that we get 4 marbles of same color? Use pigeonhole principle.

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⇒ By pigeonhole principle,

no. of colors (pigeonholes) $n = 3$
no. of marbles (pigeons) $k+1 = 4$
∴ The minimum no. of marbles required $= kn+1$

by simplifying we get,
$kn+1 = 10$
Verification : ceil [Average] is
$[kn+1 / n] = 4$
$[10/3] = 4$

$kn+1 = 10$
i.e. 3 red, 3 white + 3 blue + 1 [red or white or blue] = 10.