Problem 1: Two firms are deciding whether to enter or stay out of a market. Each firm believes the market is favorable with probability 0.5 and unfavorable with probability 0.5. The payoffs for the firms are:

- Both Enter: Each earns 0.
- One Enters, One Stays Out: The entering firm earns 3 if the market is favorable, and 1 if the market is unfavorable. The staying-out firm earns 0.
- Both Stay Out: Each earns 1.

Assuming each firm believes the market is favorable with probability 0.5, construct a Bayesian game model and identify the Bayesian Nash Equilibrium**.**

1. Constructing the Bayesian Game Model

Players:

- Two firms: Firm A and Firm B.

Types:

- Each firm can believe that the market is either favorable (F) or unfavorable (U). Each firm has a 0.5 probability of believing the market is favorable or unfavorable.

Strategies:

- Each firm has two strategies: Enter (E) or Stay Out (S).

Payoffs: Given the market conditions, the payoffs are as follows:

- **Both Enter (E, E):**
  - Each firm earns 0 regardless of the market condition.
- **One Enters, One Stays Out (E, S) or (S, E):**
  - If the market is favorable (F):
    - The entering firm earns 3, and the staying-out firm earns 0.

- If the market is unfavorable (U):
  - The entering firm earns 1, and the staying-out firm earns 0.
- **Both Stay Out (S, S):**
  - Each firm earns 1 regardless of the market condition.

Beliefs:

- Each firm believes with probability 0.5 that the market is favorable or unfavorable.

2. Construct the Payoff Matrix

To simplify, we will construct a matrix for each possible market condition and then average the payoffs considering each firm's belief.

Payoff Matrix for the Market Being Favorable (F):

Firm B / Firm A	**Enter (E)**	**Stay Out (S)**
Enter (E)	(0, 0)	(3, 0)
Stay Out (S)	(0, 3)	(1, 1)

Payoff Matrix for the Market Being Unfavorable (U):

Firm B / Firm A	**Enter (E)**	**Stay Out (S)**
Enter (E)	(0, 0)	(1, 0)
Stay Out (S)	(0, 1)	(1, 1)

3. Compute Expected Payoffs

Since each firm believes the market is favorable with probability 0.5 and unfavorable with probability 0.5, the expected payoffs for each strategy profile need to be calculated.

Expected Payoffs for Firm A:

1. **If Firm A chooses Enter (E):**
   - **If Firm B chooses Enter (E):**
     - Payoff = 0 (regardless of market condition)
   - **If Firm B chooses Stay Out (S):**
     - Payoff = 0.5 * 3 + 0.5 * 1 = 2
2. **If Firm A chooses Stay Out (S):**
   - **If Firm B chooses Enter (E):**
     - Payoff = 0.5 * 0 + 0.5 * 1 = 0.5
   - **If Firm B chooses Stay Out (S):**
     - Payoff = 1 (regardless of market condition)

Expected Payoffs for Firm B:

1. **If Firm B chooses Enter (E):**
   - **If Firm A chooses Enter (E):**
     - Payoff = 0 (regardless of market condition)
   - **If Firm A chooses Stay Out (S):**
     - Payoff = 0.5 * 0 + 0.5 * 1 = 0.5
2. **If Firm B chooses Stay Out (S):**
   - **If Firm A chooses Enter (E):**
     - Payoff = 0.5 * 0 + 0.5 * 1 = 0.5
   - **If Firm A chooses Stay Out (S):**

- Payoff = 1 (regardless of market condition)

4. Construct the Expected Payoff Matrix

For each firm, the expected payoffs can be summarized as follows:

Firm B / Firm A	Enter (E)	Stay Out (S)
Enter (E)	(0, 0)	(2, 0.5)
Stay Out (S)	(0.5, 2)	(1, 1)

5. Identify Bayesian Nash Equilibrium

We need to find the strategies where neither firm can improve their expected payoff given the strategy of the other firm.

1. **If Firm A chooses Enter (E):**
   - Firm B's expected payoff is 0 against Enter (E) or 0.5 against Stay Out (S). Firm B prefers Stay Out (S) with a payoff of 0.5.
2. **If Firm A chooses Stay Out (S):**
   - Firm B's expected payoff is 0.5 against Enter (E) or 1 against Stay Out (S). Firm B prefers Stay Out (S) with a payoff of 1.

From Firm A's perspective:

- If Firm B chooses Enter (E), Firm A prefers Stay Out (S) because 0.5 > 0.
- If Firm B chooses Stay Out (S), Firm A prefers Stay Out (S) because 1 > 2.

From Firm B's perspective:

- If Firm A chooses Enter (E), Firm B prefers Stay Out (S) because 0.5 > 0.
- If Firm A chooses Stay Out (S), Firm B prefers Stay Out (S) because 1 > 0.5.

Bayesian Nash Equilibrium

The Bayesian Nash Equilibrium for this game is (Stay Out, Stay Out).

- In this equilibrium, both firms stay out of the market, resulting in a payoff of 1 for each firm. Neither firm has an incentive to deviate given the other firm's strategy.

Problem 1: Investment Decision

Scenario: Two firms are deciding whether to invest in a new technology. Each firm believes the market conditions are either good (G) or bad (B). The probability that the market is good is 0.6. If both firms invest, each earns a payoff of 1 if the market is good and -1 if the market is bad. If one firm invests and the other does not, the investing firm earns 3 if the market is good and 0 if the market is bad. The non-investing firm earns 0 regardless of the market condition. If neither firm invests, each earns 2.

Solution:

1. **Construct Payoff Matrix:**

   For each firm, the payoffs can be summarized based on the market conditions:

   - **If Market is Good (G):**
     - Both Invest (I, I): Payoff = (1, 1)
     - One Invests, One Does Not (I, N) or (N, I): Payoff = (3, 0) or (0, 3)
     - Neither Invest (N, N): Payoff = (2, 2)
   - **If Market is Bad (B):**
     - Both Invest (I, I): Payoff = (-1, -1)
     - One Invests, One Does Not (I, N) or (N, I): Payoff = (0, 0) or (0, 0)

- Neither Invest (N, N): Payoff = (2, 2)

2. **Expected Payoffs:**

   Each firm believes the market is good with probability 0.6 and bad with probability 0.4.

   - **Expected Payoff for Firm A:**
     - **If Firm A chooses Invest (I):**
       - **If Firm B chooses Invest (I):**
         - Payoff = 0.6 * 1 + 0.4 * (-1) = 0.6 - 0.4 = 0.2
       - **If Firm B chooses Not Invest (N):**
         - Payoff = 0.6 * 3 + 0.4 * 0 = 1.8
     - **If Firm A chooses Not Invest (N):**
       - **If Firm B chooses Invest (I):**
         - Payoff = 0.6 * 0 + 0.4 * 0 = 0
       - **If Firm B chooses Not Invest (N):**
         - Payoff = 0.6 * 2 + 0.4 * 2 = 2
   - **Expected Payoff for Firm B:**
     - **If Firm B chooses Invest (I):**
       - **If Firm A chooses Invest (I):**
         - Payoff = 0.6 * 1 + 0.4 * (-1) = 0.2
       - **If Firm A chooses Not Invest (N):**
         - Payoff = 0.6 * 3 + 0.4 * 0 = 1.8
     - **If Firm B chooses Not Invest (N):**
       - **If Firm A chooses Invest (I):**
         - Payoff = 0.6 * 0 + 0.4 * 0 = 0
       - **If Firm A chooses Not Invest (N):**
         - Payoff = 0.6 * 2 + 0.4 * 2 = 2

3. **Bayesian Nash Equilibrium:**

- If both firms choose Invest (I), the expected payoff for both is 0.2.
- If one firm invests and the other does not, the investing firm earns 1.8.
- If neither invests, each earns 2.

Both firms prefer not investing when the other does not invest, given their expected payoffs are higher.

Bayesian Nash Equilibrium: (N, N) – Both firms choose not to invest.

Problem 2: Public Goods Contribution

Scenario: Two individuals, Alice and Bob, decide whether to contribute to a public good. Each has private information about their valuation of the public good. Alice values the public good at $10 with probability 0.4 and $5 with probability 0.6. Bob has the same probabilities. If both contribute, each pays a cost of $3. If only one contributes, that person pays $3 and the other pays nothing. If neither contributes, the cost is $0. The public good is valued at $15 if at least one person contributes.

Solution:

1. **Construct Payoff Matrix:**
   - **If Both Contribute (C, C):**
     - Cost: -$3 (Alice) + -$3 (Bob) = -$6
     - Benefit: $15, shared equally: $7.50 each
     - Net Payoff: $7.50 - $3 = $4.50 each
   - **If One Contributes and One Does Not (C, N) or (N, C):**
     - Contributor:
       - Cost: -$3
       - Benefit: $15
       - Net Payoff: $15 - $3 = $12

- Non-Contributor:
  - Benefit: $15
  - Net Payoff: $15
- **If Neither Contributes (N, N):**
  - Benefit: $0
  - Net Payoff: $0

2. **Expected Payoffs:**

Alice's Expected Payoff:

- **If Alice contributes:**
  - **If Bob contributes:** $4.50
  - **If Bob does not contribute:** $12

  Expected payoff for Alice = 0.4 * 4.50 + 0.6 * 12 = 1.8 + 7.2 = 9

- **If Alice does not contribute:**
  - **If Bob contributes:** $15
  - **If Bob does not contribute:** $0

  Expected payoff for Alice = 0.4 * 15 + 0.6 * 0 = 6

Bob's Expected Payoff:

- **If Bob contributes:**
  - **If Alice contributes:** $4.50
  - **If Alice does not contribute:** $12

  Expected payoff for Bob = 0.4 * 4.50 + 0.6 * 12 = 1.8 + 7.2 = 9

- **If Bob does not contribute:**

- **If Alice contributes:** $15
- **If Alice does not contribute:** $0

Expected payoff for Bob = 0.4 * 15 + 0.6 * 0 = 6

3. **Bayesian Nash Equilibrium:**

Each individual prefers to contribute if the other is not contributing (since 12 > 6), and both prefer not to contribute if the other is also not contributing.

Bayesian Nash Equilibrium: (C, C) or (N, N) – Both contribute or both do not contribute.

Problem 3: Job Offer Decision

Scenario: Two job applicants, John and Mary, are applying for a job where they can either accept an offer (A) or reject it (R). Each believes there's a 70% chance they will receive an offer from another company and a 30% chance they will not. The payoff for each depends on whether they receive an offer:

- **If both accept (A, A):**
  - Both get $4 each.
- **If one accepts and the other rejects (A, R) or (R, A):**
  - The acceptor gets $6 and the rejector gets $2.
- **If both reject (R, R):**
  - Both get $3.

Solution:

1. **Construct Payoff Matrix:**
   - **If Both Accept (A, A):**

- Payoff: (\$4, \$4)
- **If One Accepts and One Rejects (A, R) or (R, A):**
  - Payoff: (\$6, \$2) or (\$2, \$6)
- **If Both Reject (R, R):**
  - Payoff: (\$3, \$3)

2. **Expected Payoffs:**

   John's Expected Payoff:

   - **If John accepts:**
     - **If Mary accepts:** \$4
     - **If Mary rejects:** \$6

     Expected payoff for John = 0.7 * 4 + 0.3 * 6 = 2.8 + 1.8 = 4.6

   - **If John rejects:**
     - **If Mary accepts:** \$2
     - **If Mary rejects:** \$3

     Expected payoff for John = 0.7 * 2 + 0.3 * 3 = 1.4 + 0.9 = 2.3

   Mary's Expected Payoff:

   - **If Mary accepts:**
     - **If John accepts:** \$4
     - **If John rejects:** \$6

     Expected payoff for Mary = 0.7 * 4 + 0.3 * 6 = 2.8 + 1.8 = 4.6

   - **If Mary rejects:**
     - **If John accepts:** \$2

- **If John rejects:** $3

Expected payoff for Mary = 0.7 * 2 + 0.3 * 3 = 1.4 + 0.9 = 2.3

3. **Bayesian Nash Equilibrium:**

   Each person prefers to accept if they believe the other will accept due to higher payoffs.

   Bayesian Nash Equilibrium: (A, A) – Both accept.

Q. Two parties are involved in bargaining over a resource. Party 1 is uncertain about Party 2's valuation, which could be either high or low. Party 1 makes an offer, and Party 2 can either accept or reject it. If Party 2 accepts, the resource is split according to the offer. If Party 2 rejects, both parties receive nothing.

a) Model this situation as a game of incomplete information..

b) Identify the role of beliefs in this bargaining scenario and how they influence the outcome.

Ans,

a) Modeling the Situation as a Game of Incomplete Information

Players:

- Party 1 (P1)
- Party 2 (P2)

Types of Party 2:

- Party 2 can have two types:
  - **High Valuation (H):** Values the resource highly.
  - **Low Valuation (L):** Values the resource less.

Assume Party 2's valuation is high with probability ppp and low with probability 1−p1-p1−p.

Strategies:

- **Party 1's Strategy:** Make an offer OOO for how to split the resource.
- **Party 2's Strategy:** Accept or reject the offer.

Payoffs:

- If Party 2 accepts an offer OOO from Party 1, the resource is split according to OOO. Suppose OOO is a fraction α\alphaα of the resource going to Party 1, and 1−α1 - \alpha1−α going to Party 2.
- If Party 2 rejects the offer, both parties get nothing.

Types and Beliefs:

- Party 2's valuation (high or low) is private information.
- Party 1 knows the probability distribution of Party 2's valuation but not the exact type.

c) Role of Beliefs in the Bargaining Scenario

Beliefs:

- Party 1 has beliefs about Party 2's type based on the probability distribution.
- Party 1's beliefs about Party 2's valuation influence the offer they make.
- The beliefs affect the decision of Party 2 whether to accept or reject the offer.

Influence on the Outcome:

- Party 1's offer is based on the expectation that Party 2 will accept if the offer meets or exceeds their reservation value.

- The more Party 1 believes Party 2 is likely to have a high valuation, the more Party 1 can afford to offer less and still expect acceptance.
- Party 2's acceptance depends on their private valuation and whether the offer meets their reservation value given their belief about the probability distribution.