



Semester: IV

Subject: Statistics for A18DS

Academic Year: 2023-2024

MATRIX APPROACH FOR LINEAR LEAST SQUARE:-

Example:

Find Linear Regression of the data of week and product sales (in thousands) given in Table:

x_i (Week)	y_i (Sales in thousands).
1	1
2	8
3	4
4	8

Use Linear Regression in matrix form. Predict the 5th week sales.

Solution:

Here the independent variable X is given as:

$$X^T = [1 \ 2 \ 3 \ 4]$$

The dependent variables is given as follows:

$$Y^T = [1 \ 3 \ 4 \ 8]$$

The data can be given in matrix form as follows.

$$X = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \end{bmatrix} \quad Y = \begin{bmatrix} 1 \\ 3 \\ 4 \\ 8 \end{bmatrix}$$

↓

The first column is used for setting bias.

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The linear regression is given as:

$$a = ((X^T X)^{-1} X^T) y$$

The computation order of this equation is shown step by step as:

(1) Computation of $(X^T X)$.

$$= \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \end{pmatrix} \times \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \end{bmatrix}$$

$$= \begin{pmatrix} 4 & 10 \\ 10 & 30 \end{pmatrix}$$

(2) Computation of matrix inverse of $(X^T X)^{-1}$

$$= \begin{pmatrix} 4 & 10 \\ 10 & 30 \end{pmatrix}^{-1} = \begin{bmatrix} 30 & -10 \\ -10 & 4 \end{bmatrix} \begin{matrix} \rightarrow (1, 1) \\ \rightarrow (-1, 1) \end{matrix}$$

→ Divide this with Determinant of matrix

$$= \frac{1}{20} \begin{bmatrix} 30 & -10 \\ -10 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1.5 & -0.5 \\ -0.5 & 0.2 \end{bmatrix}$$



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(3) Computation of $(X^T X)^{-1} X^T$

$$= \begin{bmatrix} 1.5 & 1 & -0.5 \\ 0.5 & 0.2 \end{bmatrix} \times \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0.5 & 0 & -0.5 \\ -0.3 & -0.1 & 0.1 & 0.3 \end{bmatrix}$$

(4) Finally $(X^T X)^{-1} X^T y$

$$= \begin{bmatrix} 1 & 0.5 & 0 & -0.5 \\ -0.3 & -0.1 & 0.1 & 0.3 \end{bmatrix} \times \begin{bmatrix} 1 \\ 3 \\ 4 \\ 8 \end{bmatrix}$$

$$= \begin{bmatrix} (1 \times 1) + (0.5 \times 3) + (0 \times 4) + (-0.5 \times 8) \\ (-0.3 \times 1) + (-0.1 \times 3) + (0.1 \times 4) + (0.3 \times 8) \end{bmatrix} = \begin{bmatrix} -1.5 \\ 2.2 \end{bmatrix} \rightarrow \begin{bmatrix} \text{Intercept} \\ \text{slope} \end{bmatrix}$$

Regression equation is

$$y = a_0 + a_1 * x$$

$$y = -1.5 + 2.2 * x$$

The predicted 5th week sale (when $x=5$) is,

$$y = -1.5 + 2.2 * 5 = 9.5$$

$$\boxed{y = 9.5}$$