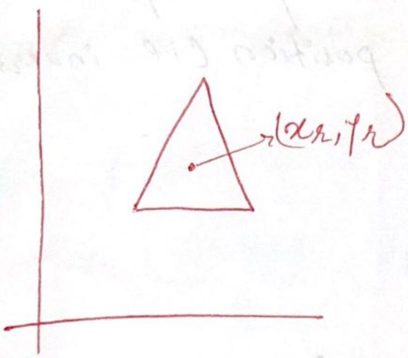
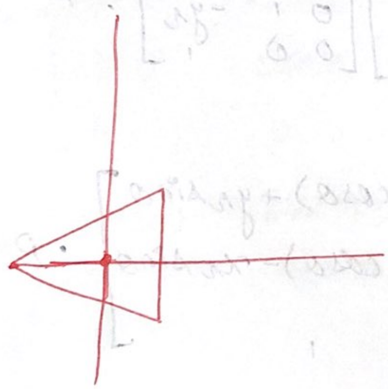
General Pivot-Point Rotation.

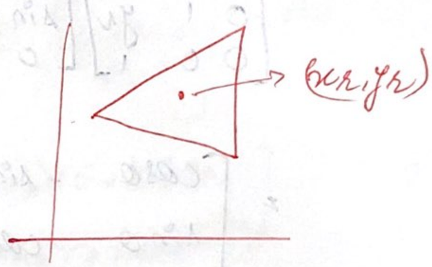


original position of object and pivot point

Translation of object so that pivot point (x_r, y_r) is at origin



Rotation about the origin



Translation of object so that pivot point is return to position (x_r, y_r)

- For rotating object about arbitory point called pivot point we need to apply following sequence of transfo-rmations.

1. Translate the object so that the pivot point coincides with the co-ordinate origin.

2. Rotate the object about the co-ordinate origin with specified angle.
3. Translate the object so that the pivot point is returned to its original position (i.e. inverse of step 1).

- Matrix equation for this is:

$$P' = T(x_r, y_r) \cdot [R(\theta) \cdot \{T(-x_r, -y_r) \cdot P\}]$$

$$P' = \{T(x_r, y_r) \cdot R(\theta) \cdot T(-x_r, y_r)\} \cdot P$$

$$= \begin{bmatrix} 1 & 0 & x_r \\ 0 & 1 & y_r \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -x_r \\ 0 & 1 & -y_r \\ 0 & 0 & 1 \end{bmatrix} \cdot P$$

$$= \begin{bmatrix} \cos\theta & -\sin\theta & x_r(1-\cos\theta)+y_r\sin\theta \\ \sin\theta & \cos\theta & y_r(1-\cos\theta)-x_r\sin\theta \\ 0 & 0 & 1 \end{bmatrix} \cdot P$$

$$P' = R(x_r, y_r\, \theta) \cdot P$$

Here P' & P are column vector of final and initial point co-ordinate respectively and (x_r, y_r) are the co-ordinates of pivot-point.

Example: - Locate new position of the triangle A(5,4) B(8,3) C(8,8) after its rotation by 90° clockwise about the centroid.

Sol.ⁿ pivot point i.e. centroid is calculated as:

$$x_r = \frac{5+8+8}{3} = 7, \quad y_r = \frac{4+3+8}{3} = 5$$

$(x_r, y_r) = (7, 5)$

rotation is clockwise i.e. $\theta = -90°$

$$P' = \begin{bmatrix} \cos\theta & -\sin\theta & x_r(1-\cos\theta) + y_r\sin\theta \\ \sin\theta & \cos\theta & y_r(1-\cos\theta) - x_r\sin\theta \\ 0 & 0 & 1 \end{bmatrix} \cdot P$$

$$= \begin{bmatrix} \cos(-90) & -\sin(-90) & 7(1-\cos(-90)) + 5\sin(-90) \\ \sin(-90) & \cos(-90) & 5(1-\cos(-90)) + 7\sin(-90) \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 5 & 8 & 8 \\ 4 & 3 & 8 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 7(1-0) - 5(1) \\ -1 & 0 & 5(1-0) + 7(1) \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 5 & 8 & 8 \\ 4 & 3 & 8 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & 12 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 5 & 8 & 8 \\ 4 & 3 & 8 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 13 & 18 \\ 7 & 4 & 4 \\ 1 & 1 & 1 \end{bmatrix}$$

Final co-ordinates after rotations are A'(11,7)

B'(13,4)

C'(18,4)