

Subject: Applied Mathematics IV

SEM:IV

Small sample tests

students t-distribution

It is used when i) the sample size is 30 or less ii) σ is unknown

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n-1}}$$

$$s = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}}$$

with $(n-1)$ degrees of freedom.

Remark:-

* If σ is known then $z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$

* If σ is not known & the parent population is normal, then

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

$$s^2 = \frac{\sum (x_i - \bar{x})^2}{n-1}$$



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* If σ is unknown then

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n-1}} \quad s^2 = \frac{\sum (x_i - \bar{x})^2}{n}$$

① Nine items of a sample had the following values 45, 47, 50, 52, 48, 47, 49, 53, 51. Does the mean of 9 items differ significantly from the assumed population mean 47.5.

Soln:-

$X :$ 45 47 50 52 48 47 49 53 51

$d_i x_i - 48 :$ -3 -1 2 4 0 -1 1 5 3/10

d_i^2 9 1 4 16 0 1 1 25 9/66

$$\bar{x} = a + \frac{\sum d_i}{n}$$

$$= 48 + \frac{10}{9}$$

$$= 49.11$$



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$$\sum (x_i - \bar{x})^2 = \sum d_i^2 - \frac{(\sum d_i)^2}{n}$$
$$= 66 - \frac{100}{9} = 54.89.$$

$$s^2 = \frac{\sum (x_i - \bar{x})^2}{n}$$
$$= \frac{54.89}{9} = 6.099.$$

① Null hypothesis : $\mu = 47.5$

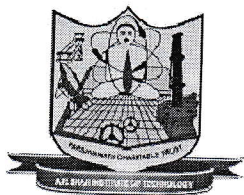
Alternative hypothesis :- $\mu \neq 47.5$

② Test statistic.

$$t = \frac{\bar{x} - \mu}{s / \sqrt{n-1}} \quad (n < 30)$$

$$= \frac{49.11 - 47.5}{\sqrt{6.099} / \sqrt{8}}$$

$$|t| = 1.84$$



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③ Level of significance

$$\alpha = 0.05$$

④ Critical value:-

The value of z_{α} at 5% level of significance for $n-1 = 9-1 = 8$ degrees of freedom

$$|t_{\alpha}| =$$

⑤ Decision:-

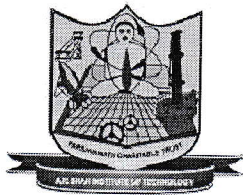
$$|t| < |t_{\alpha}|.$$

\therefore We accept the hypothesis.

\therefore The items does not differ significantly from mean.

② ^{HW} Tests made on breaking strength of 10 pieces of a metal wire gave the follg. results

578, 572, 570, 568, 570, 570, 570, 572, 596 &



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584. Test if the breaking strength of the metal in (kg) wire can be assumed to be 577 kg.

(Soln: - $\mu = 577$ | $H = 0.05$)

Testing the Difference b/w means

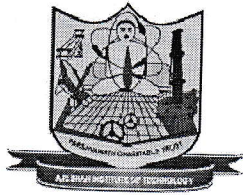
(a) Independent samples

$$SP = \sqrt{\frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}}$$

$$SE = SP \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{SE}$$

① A sample of 8 students of 16 yrs each shown up a mean systolic blood pressure of 118.4 mm Hg with sd of 12.17 mm while a sample of 10 students of 17 yrs each showed the mean systolic BP of 121.0 mm with sd of 12.88 during an



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investigation. The investigator feels that the systolic BP is related to age. Do you think that the data provides enough reasons to support investigators feeling at 5% LOS.

Soln:-

| | | |
|------------|---------------------|---------------|
| $n_1 = 8$ | $\bar{x}_1 = 118.4$ | $S_1 = 12.17$ |
| $n_2 = 10$ | $\bar{x}_2 = 121.0$ | $S_2 = 12.88$ |

① Null hypothesis : $\mu_1 = \mu_2$

• Alternative hypothesis :- $\mu_1 \neq \mu_2$

② Test statistic

$$S_p = \sqrt{\frac{n_1 S_1^2 + n_2 S_2^2}{n_1 + n_2 - 2}}$$

$$= \sqrt{\frac{8(12.17)^2 + 10(12.88)^2}{10 + 8 - 2}}$$

$$= 13.33$$



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$$SE = SP \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$
$$= 13.33 \sqrt{\frac{1}{8} + \frac{1}{10}} = 6.32$$

$$t = \frac{\bar{X}_1 - \bar{X}_2}{SE}$$
$$= \frac{-118.4 - 121.0}{6.32} = -0.41$$

$$|t| = 0.41$$

③ Level of Significance

$$\alpha = 0.05$$

④ Critical value:-

$$|t_{\alpha}| \text{ at } \alpha = 0.05 \text{ for } n-1 = n_1 + n_2 - 2$$
$$= 16 \text{ dof is } |t_{\alpha}| = 2.12$$

⑤ Decision:-

$$|t| < |t_{\alpha}|$$

\therefore We accept the hypothesis.