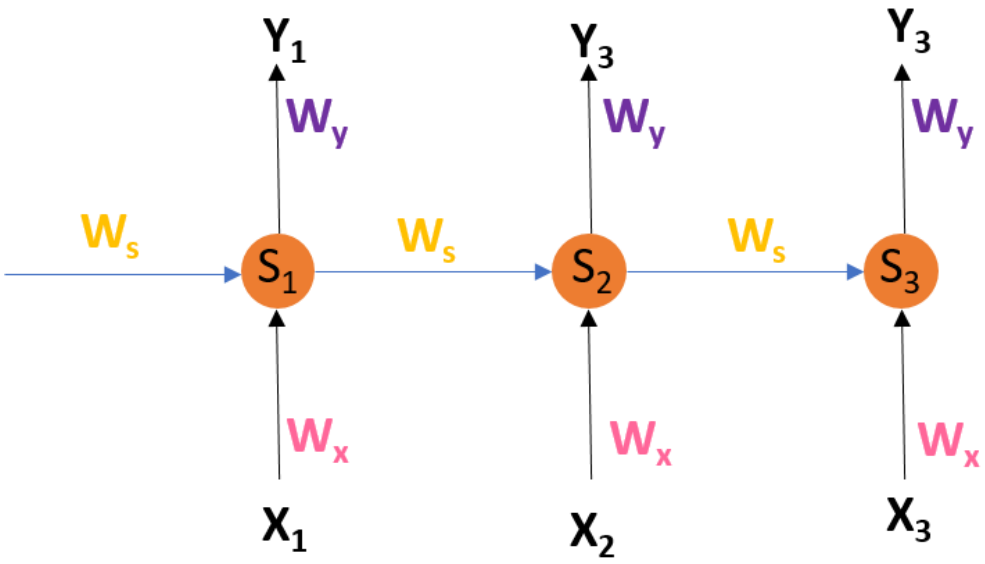
Module 5

Back Propagation through time – RNN

Last Updated : 04 May, 2020

Introduction: Recurrent Neural Networks are those networks that deal with sequential data. They predict outputs using not only the current inputs but also by taking into consideration those that occurred before it. In other words, the current output depends on current output as well as a memory element (which takes into account the past inputs). For training such networks, we use good old backpropagation but with a slight twist. We don't independently train the system at a specific time *"t"*. We train it at a specific time *"t"* as well as all that has happened before time *"t"* like t-1, t-2, t-3. Consider the following representation of a RNN:



RNN Architecture

S1, *S2*, *S3* are the hidden states or memory units at time *t1*, *t2*, *t3* respectively, and *Ws* is the weight matrix associated with it. *X1*, *X2*, *X3* are the inputs at time *t1*, *t2*, *t3* respectively, and *Wx* is the weight matrix associated with it. *Y1*, *Y2*, *Y3* are the outputs at time *t1*, *t2*, *t3* respectively, and *Wy* is the weight matrix associated with it. For any time, t, we have the following two equations:

$$S_t = g_1(W_x x_t + W_s S_{t-1})$$

$$Y_t = g_2(W_Y S_t)$$

Module 5

$$S_t = g_1(W_x x_t + W_s S_{t-1})$$

$$Y_t = g_2(W_Y S_t)$$

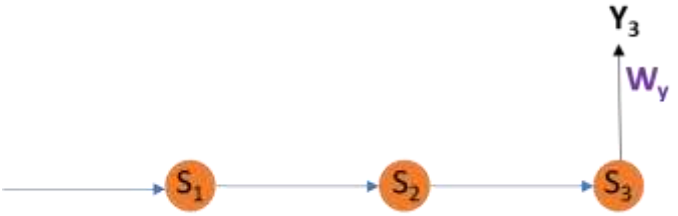
where g1 and g2 are activation functions. Let us now perform back propagation at time t = 3. Let the error function be:

$$E_t = (d_t - Y_t)^2$$

, so at t =3,

$$E_3 = (d_3 - Y_3)^2$$

*We are using the squared error here, where *d3* is the desired output at time *t = 3*. To perform back propagation, we have to adjust the weights associated with inputs, the memory units and the outputs. **Adjusting Wy** For better understanding, let us consider the following representation:

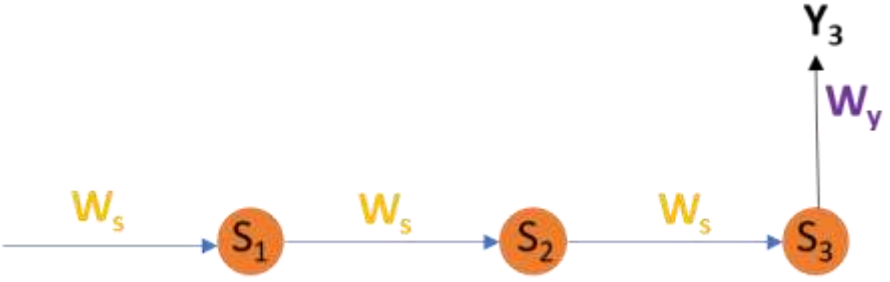


Adjusting Wy

Formula:

$$\frac{\partial E_3}{\partial W_y} = \frac{\partial E_3}{\partial Y_3} . \frac{\partial Y_3}{\partial W_Y}$$

Explanation: *E3* is a function of *Y3*. Hence, we differentiate *E3* w.r.t *Y3*. *Y3* is a function of *WY*. Hence, we differentiate *Y3* w.r.t *WY*. **Adjusting Ws** For better understanding, let us consider the following representation:



Adjusting Ws

Module 5

Formula:

$$\frac{\partial E_3}{\partial W_S} = (\frac{\partial E_3}{\partial Y_3}.\frac{\partial Y_3}{\partial S_3}.\frac{\partial S_3}{\partial W_S})+$$

$$(\frac{\partial E_3}{\partial Y_3}.\frac{\partial Y_3}{\partial S_3}.\frac{\partial S_3}{\partial S_2}.\frac{\partial S_2}{\partial W_S})+$$

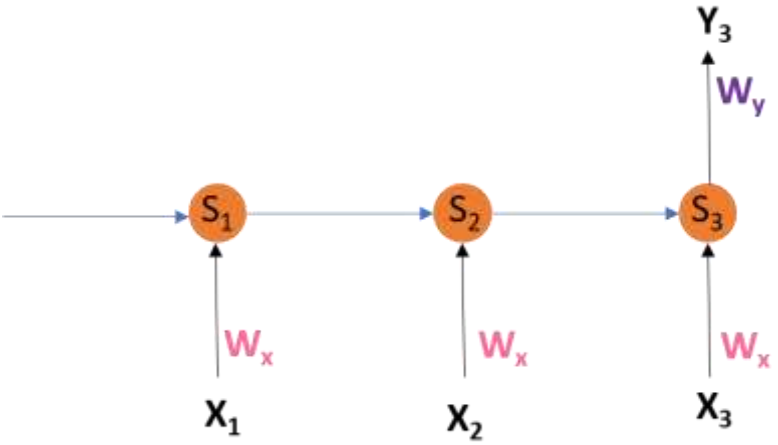
$$(\frac{\partial E_3}{\partial Y_3}.\frac{\partial Y_3}{\partial S_3}.\frac{\partial S_3}{\partial S_2}.\frac{\partial S_2}{\partial S_1}.\frac{\partial S_1}{\partial W_S})$$

Explanation: *E3* is a function of *Y3*. Hence, we differentiate *E3* w.r.t *Y3*. *Y3* is a function of *S3*. Hence, we differentiate *Y3* w.r.t *S3*. *S3* is a function of *WS*. Hence, we differentiate *S3* w.r.t *WS*. But we can't stop with this; we also have to take into consideration, the previous time steps. So, we differentiate (partially) the Error function with respect to memory units *S2* as well as *S1* taking into consideration the weight matrix *WS*. We have to keep in mind that a memory unit, say S_t is a function of its previous memory unit S_{t-1}. Hence, we differentiate *S3* with *S2* and *S2* with *S1*. Generally, we can express this formula as:

$$\frac{\partial E_N}{\partial W_S} = \sum_{i=1}^{N} \frac{\partial E_N}{\partial Y_N}.\frac{\partial Y_N}{\partial S_i}.\frac{\partial S_i}{\partial W_S}$$

Adjusting WX: For better understanding, let us consider the following representation:

Module 5



Adjusting Wx

Formula:

$$\frac{\partial E_3}{\partial W_X} = (\frac{\partial E_3}{\partial Y_3} . \frac{\partial Y_3}{\partial S_3} . \frac{\partial S_3}{\partial W_X}) +$$

$$(\frac{\partial E_3}{\partial Y_3} . \frac{\partial Y_3}{\partial S_3} . \frac{\partial S_3}{\partial S_2} . \frac{\partial S_2}{\partial W_X}) +$$

$$(\frac{\partial E_3}{\partial Y_3} . \frac{\partial Y_3}{\partial S_3} . \frac{\partial S_3}{\partial S_2} . \frac{\partial S_2}{\partial S_1} . \frac{\partial S_1}{\partial W_X})$$

Explanation: *E3* is a function of *Y3*. Hence, we differentiate *E3* w.r.t *Y3*. *Y3* is a function of *S3*. Hence, we differentiate *Y3* w.r.t *S3*. *S3* is a function of *WX*. Hence, we differentiate *S3* w.r.t *WX*. Again we can't stop with this; we also have to take into consideration, the previous time steps. So, we differentiate (partially) the Error function with respect to memory units *S2* as well as *S1* taking into consideration the weight matrix WX. Generally, we can express this formula as:

$$\frac{\partial E_N}{\partial W_S} = \sum_{i=1}^{N} \frac{\partial E_N}{\partial Y_N} . \frac{\partial Y_N}{\partial S_i} . \frac{\partial S_i}{\partial W_X}$$

Limitations: This method of Back Propagation through time (BPTT) can be used up to a limited number of time steps like 8 or 10. If we back propagate further, the gradient

becomes too small. This problem is called the "Vanishing gradient" problem. The problem is that the contribution of information decays geometrically over time. So, if the number of time steps is >10 (Let's say), that information will effectively be discarded. **Going Beyond RNNs:** One of the famous solutions to this problem is by using what is called Long Short-Term Memory (LSTM for short) cells instead of the traditional RNN cells. But there might arise yet another problem here, called the **exploding gradient** problem, where the gradient grows uncontrollably large. **Solution:** A popular method called gradient clipping can be used where in each time step, we can check if the gradient > threshold. If yes, then normalize it.