Semester : VI Subject :Machine Learning Academic Year: 2023 - 2024

Inner product space (Example)

Question :- Show that $V_n(C)$ is an inner product space with inner product define on $\alpha = (a_1, a_2, \dots a_n)$ $\beta = (b_1, b_2, \dots b_n) \in V_n(C)$ by $(\alpha, \beta) = a_1\bar{b}_1 + a_2\bar{b}_2 + \dots + a_n\bar{b}_n$

Solution :-

Let $\alpha = (a_1, a_2, \dots a_n)$, $\beta = (b_1, b_2, \dots, b_n)$ & $\gamma = (c_1, c_2, \dots c_n) \in V_n(F)$ and $a, b \in C$

1) Non-Negativity :- $(\alpha, \alpha) = a_1 \cdot \bar{a}_1 + a_2 \cdot \bar{a}_2 + \dots + a_3 \cdot \bar{a}_3$

$= |a_1|^2 + |a_2|^2 + \dots + |a_n|^2$

Since each $|a_1|^2 \geq 0, |a_2|^2 \geq 0 \dots |a_n|^2 \geq 0$

$\Rightarrow (\alpha, \alpha) \geq 0$

If $(\alpha, \alpha) = 0 \Leftrightarrow |a_1|^2 + |a_2|^2 + \dots + |a_n|^2 = 0$

$\Leftrightarrow |a_1|^2 = 0, |a_2|^2 \dots |a_n|^2 = 0$

$\Leftrightarrow a_1 = 0 \quad a_2 = 0, \dots a_n = 0$

for each $a_i = 0 \Rightarrow \alpha = 0$

ii) Conjugate Symmetry :-

$(\alpha, \beta) = \cancel{(\beta, \gamma)} \; \overline{(\beta, \alpha)}$

$(\beta, \alpha) \; \cancel{(\beta, \beta)} = b_1\bar{a}_1 + b_2\bar{a}_2 + \dots + b_n\bar{a}_n$

$\overline{(\beta, \alpha)} \; \cancel{(\beta, \beta)} \Rightarrow \overline{(b_1\bar{a}_1 + b_2\bar{a}_2 + \dots + b_n\bar{a}_n)}$

$= (\bar{b}_1\bar{\bar{a}}_1 + \bar{b}_2\bar{\bar{a}}_2 + \dots + \bar{b}_n\bar{\bar{a}}_n)$

$= (\bar{b}_1 \cdot a_1) + (\bar{b}_2 \cdot a_2) + \dots + (\bar{b}_n \cdot a_n)$

$\overline{(\beta, \alpha)} \; \cancel{(\beta, \beta)} = a_1 \cdot \bar{b}_1 + a_2\bar{b}_2 + \dots (a_n \cdot \bar{b}_n)$

$= (\alpha, \beta)$

iii) Linearity :- $a\alpha + b\beta = a(a_1, a_2, \ldots a_n) + b(b_1, b_2 \ldots b_n)$

$= (aa_1 + bb_1, a_1a_2 + b_1b_2, \ldots aa_n + bb_n)$

No. $(a\alpha + b\beta, \gamma) = (aa_1 + bb_1)\bar{c}_1 + (aa_2 + bb_2)\bar{c}_2 + \cdots + (aa_n + bb_n)\bar{c}_n$

$= a(a_1\bar{c}_1 + a_2c_2 + \cdots + a_n\bar{c}_n) + b(b_1\bar{c}_1 + b_2\bar{c}_2 + \cdots + b_n\bar{c}_n)$

$= a(\alpha, \gamma), b(\beta, \gamma)$

Here inner product define by 1 satisfy all 3 Conditions.