Kathmandu University

Dhulikhel, Kavre

Environmental Modelling

ENVS 404

Lab Report – 4: Gaussian Dispersion Modelling

Submitted by:

Lakisha Shrestha

Roll no. 27

ENE 4th year

Submitted to:

Dr. Kundan Lal Shrestha

Department of Environmental Science and Engineering

2nd August, 2021

Gaussian Dispersion Modelling

Gaussian dispersion modeling is the mathematical simulation of how air pollutants disperse in the ambient atmosphere. It assumes that pollutant dispersion follows normal statistical distribution. These models are typically used to estimate the downwind ambient concentration of air pollutants emitted from sources like industrial plant

1. 1-D Gaussian Puff Model

Objectives

The one dimensional puff model is used to understand the concentration distribution of a pollutant in a fluid downstream when they occur in discrete internals (unsteady state) as opposed to continuously. These are also important for understanding how point sources of pollution spread in a fluid media.

Introduction

a. Background

Air pollution dispersion models have been around since the 1930s. Bosanquet and Pearson had an earlier model which was not completely accurate due to non-involvement of the Gaussian distribution or the ground water reflection. Later, Sir Graham Sutton developed a model that included both these parameters.

b. Principle

A one dimensional (1-D) model is a set of adjacent box models, stacked vertically or horizontally. Vertical 1-D models maybe used to study radiative transfer with photochemistry, gas and aerosol vertical transport, aerosol optical properties, aerosol sedimentation or cloud convection. When smoke is released from a stack (brick kiln or an industrial chimney), it gets mixed with the ambient air and travel along with the wind speed. While considering a puff model with discrete internals of the smoke being released, the first one gives a Gaussian curve that is tall and less deviated from mean. This means that the concentration is high at the beginning but it has not dispersed yet. As time passes, the curve is much flatter since there is now dispersion of pollutants and hence more deviation from mean position. This goes on which results in more deviated and flatter curves.

c. Application

- ➢ This model examines the concentration of a pollutant in air downstream to the area that it has been discharged to.
- ➢ It can be used to check whether air pollution standards are met or not.
- ➢ It can also predict future air pollution conditions.

Modelling methods

Gaussian distribution equation (normal distribution) is used to understand the dispersion of pollutant in air. Similar to all normal distribution equations, this equation also gives a graph that is symmetrical across the mean (μ) which is considered zero. Any deviation from this line is represented as ±σ in the two directions. The following equation is used:

$$C(x,t) = \frac{\mu}{\sqrt{2}\pi\sigma}\exp(-\frac{1}{2}\frac{(x-\mu)^2}{\sigma})$$

This equation is further modified into:

$$C(x,t) = \frac{M}{\sqrt{4\pi tD}}\exp(-\frac{(x-vt)^2}{4tD})$$

Where,

M = total mass per unit area of the fluid system

v = velocity in x-direction

D = dispersion coefficient

The concentration, C, is a solution of the transport equation, which account for diffusion and advection:

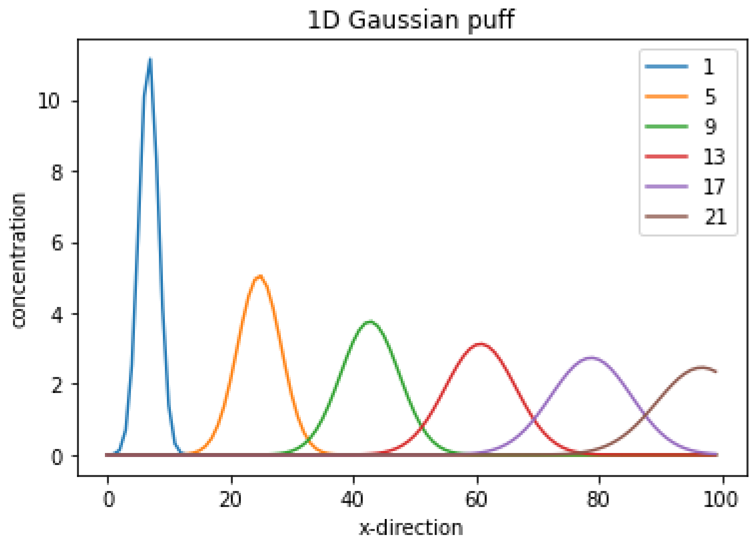
$$\frac{\partial C}{\partial t} = \frac{\partial}{\partial x}D\frac{\partial C}{\partial x} - v\frac{\partial C}{\partial x}$$

The first term of the above transport equation accounts for diffusion while the last term is the advection part which depends on wind speed.

The following assumption have been made for this model:

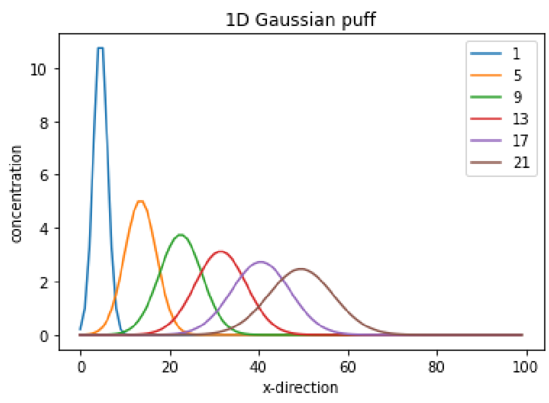
- Temperature has been neglected which can affect the concentration of pollutant.
- Secondary reaction between the pollutant and atmosphere has not been included.
- Height of stack (e.g. brick kiln) is not included in the equation
- Meteorological changes have not been considered.

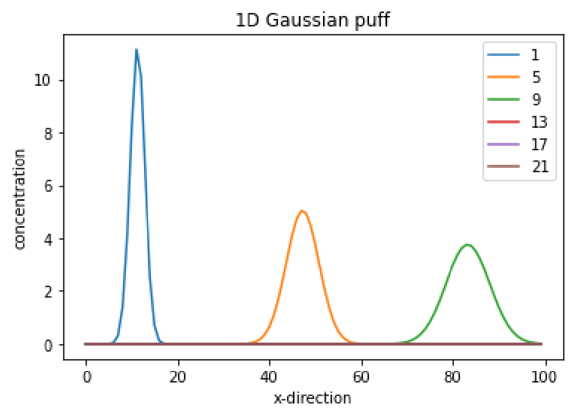
Results



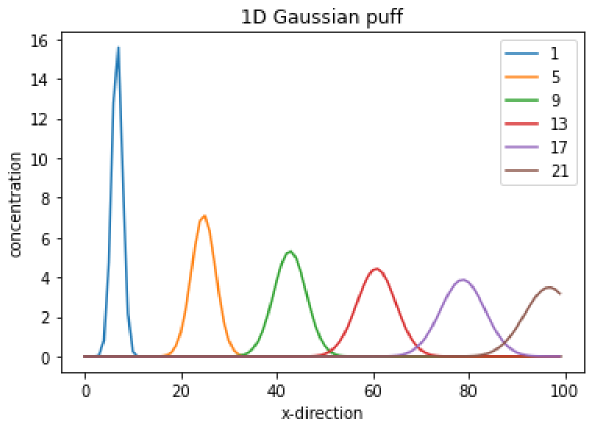
We can see from the above graph that the maximum concentration of the pollutant occurs at time (t) = 1, which is near the source of the pollution. At this point there is no dispersion of pollutants, so the concentration is very high. After some time, when the pollutants begin to disperse, the curve begins to flatten and we can see that the concentration decreases as we move away from the source.

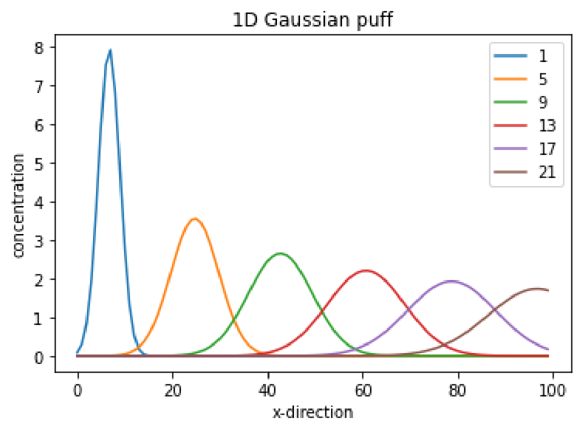
Sensitivity analysis





In the first graph, the velocity was halved and then the model was run. We can see that peaks are much closer to each other as the particles are dispersed slowly along the x-direction due to smaller velocity. In the second graph, the velocity was doubled, which results in particles being dispersed much faster due to higher velocity.





In the first graph, diffusivity was halved and then the model was run. We can see the peaks are narrower as low difussivity results in slow dispersion of the pollutants. In the second graph, diffusivity was doubled, which results is peaks being slightly broader as particles are being dispersed at a much faster rate.

Therefore, from the above results, we can conclude that the model is much more sensitive to velocity than diffusivity as change in velocity results in much more significant changes in the model.

Source code

```
import numpy as np

import matplotlib.pylab as plt

Dx = 0.000625       # diffusivity

v = 0.1             # velocity

M = 1               # mass

xmin = -0.05

xmax = 2.15         # x-axis interval

t = range(1,24,4)   # time

x = np.linspace(xmin,xmax,100)

ctot = []

for i in t:

    xx = x - v*i

    c = (M/np.sqrt(4*np.pi*Dx*i))*np.exp(-(xx*xx)/(4*Dx*i))

    #print(i,c)

    ctot.append(c)

    plt.plot(c,label=str(i))

plt.xlabel('x-direction')

plt.ylabel('concentration')

plt.title('1D Gaussian puff')

plt.legend()

plt.show()
```

2. 2-D Gaussian Puff Model

Objectives

Unlike a 1-D model, this 2-D model helps us to understand how the pollutant travels in two direction from a central point of pollution (source). This presents us with a circular graph that changes its shape based on the amount of dispersion.

Introduction

a. Background

Air pollution dispersion is quite extensive and dates back to the 1930s. One of the early air pollutant dispersion equations was derived by Bosanquet and Pearson but they did not assume Gaussian distribution nor include the effect of ground reflection of the pollutant. Later in 1947, Sir Graham Sutton derived an equation including the assumption of Gaussian distribution and also included the effect of ground reflection.

b. Principle

When smoke is introduced into the air from a point source it disperses out from that point in a concentric manner with decrease in concentration as we move outside. This gives us both the x and y direction of dispersion. The center is most polluted. As the pollutant mixes with the surrounding air, it gets diluted and the circle increases. While the pollutant is spreading out it is also moving in the x direction (direction of wind) due to advection. This is a Langrangian model since our view moves along with pollutant particles.

c. Application

- To check the dispersion of pollutant in the air from its source.
- To understand how pollutant move in different direction after it has been introduced.
- To monitor air pollution standards
- To predict future air pollution concentrations.

Modelling methods

Like in 1-D model, we also use a normal distribution here with a few modifications. Due to the point of view of the dispersion being from above, the curve shown is not a Gaussian curve rather concentric circles that are dependent on the values of x, y, and time (t). The graph shows that, the smaller circle/ ellipse has greater concentration of pollutant. There is more advection in x-direction due to greater values placed.

The equation is given as follows:

$$C(x, y, t) = \frac{M}{4\pi\sqrt{D_x D_y}} \exp(-\frac{1}{4t}\left(\frac{(x - vt)^2}{D_x} + \frac{y^2}{D_y}\right) - \lambda t)$$

Where,

M is total mass per unit length in 2-D

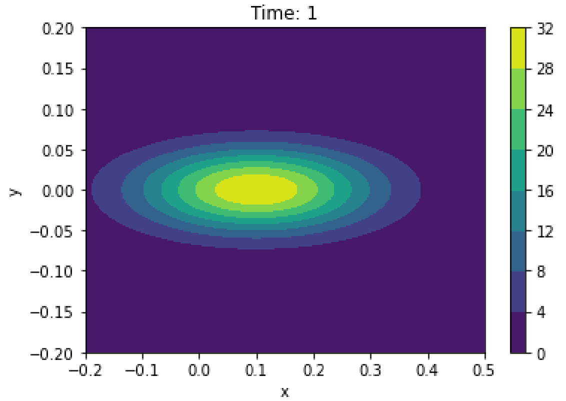
D_x is diffusivity along x-direction

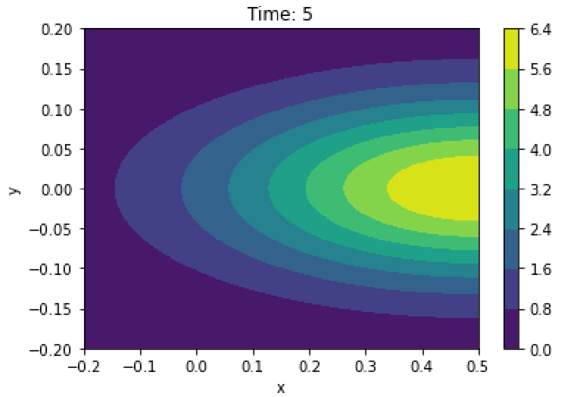
D_y is diffusivity along y-direction

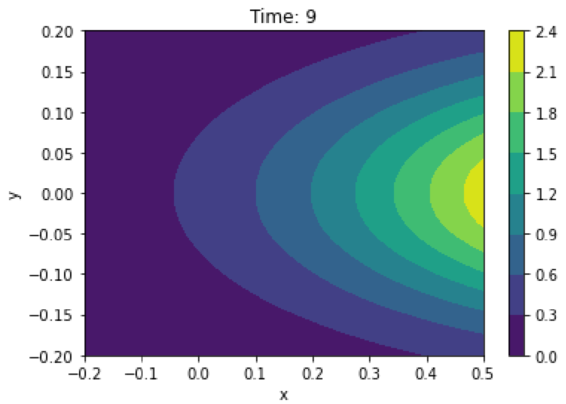
The following assumption have been made for this model:

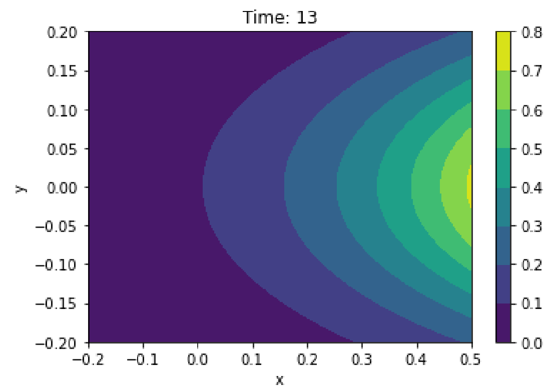
- Temperature has been neglected which can affect the concentration of pollutant.
- Secondary reaction between the pollutant and atmosphere has not been included.
- Height of stack is not included in the equation.
- Meteorological changes have not been considered.
- Wind speed is considered equal in both directions.

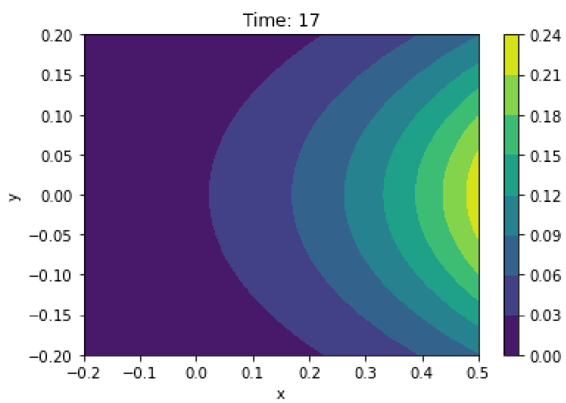
Results

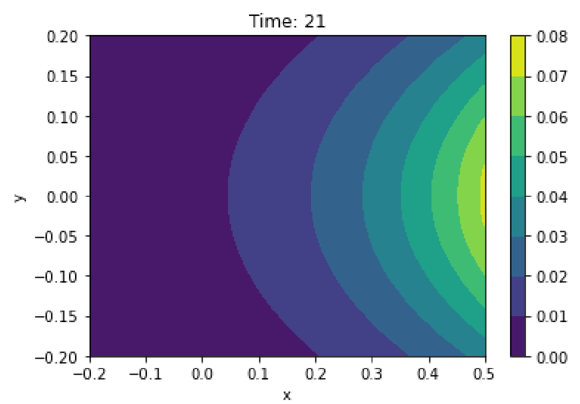








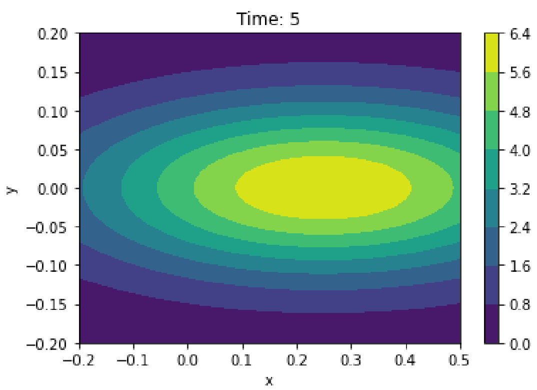


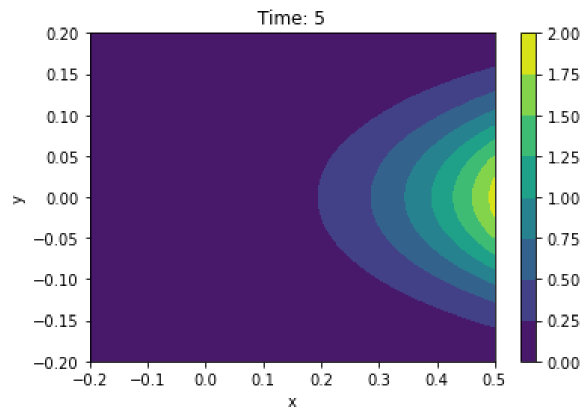


We can see from the above graphs that initially at time (t) =1, the concentration of pollutant is high near the source of the pollutant i.e. near x = 0 and y = 0. As time passes and the pollutant is dispersed in both the direction, the concentration of the pollutants decreases near the source and its surrounding.

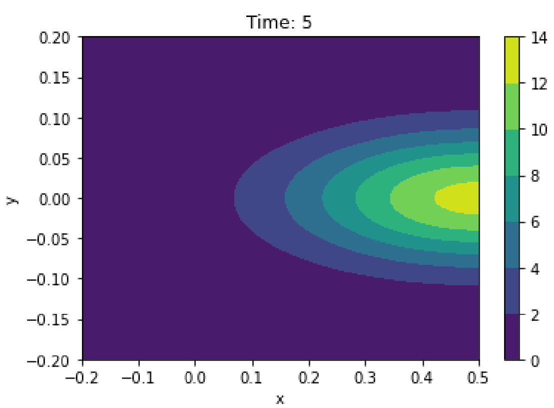
Sensitivity analysis

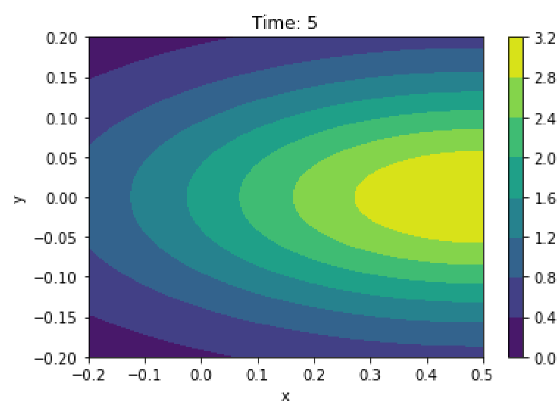
For sensitivity analysis, we have taken time (t) = 5 as our reference.





In the first graph, the velocity was halved then the model was run. We can see that the rate of dispersion is very low in this case and the concentration of the pollutants is high. In the second graph, the velocity was doubled and then the model was run. In this case, the concentration is low as higher velocity has resulted in faster dispersion of the pollutants.





In the first graph, diffusivities in both the directions was halved and then the model was run. We can see that the rate of dispersion is low which has resulted in higher concentration of the pollutants. In the second graph, the diffusivities in both the directions was halved then the model was run. In this case, the rate of dispersion is high as we can see that the concentration of pollutants has decreased in the same interval of time.

Therefore, we can conclude that the model is sensitive to both the change in velocity and change in diffusivity.

Source code

```
import numpy as np
import matplotlib.pylab as plt
Dx = 0.000625      # diffusivity
v = 0.1            # velocity
M = 1              # mass
xmin = -0.05
xmax = 2.15        # x-axis interval
t = range(1,24,4)  # time
x = np.linspace(xmin,xmax,100)
ctot = []
for i in t:
    xx = x - v*i
    c = (M/np.sqrt(4*np.pi*Dx*i))*np.exp(-(xx*xx)/(4*Dx*i))
    #print(i,c)
    ctot.append(c)
    plt.plot(c,label=str(i))
plt.xlabel('x-direction')
plt.ylabel('concentration')
plt.title('1D Gaussian puff')
plt.legend()
plt.show()
```

3. Gaussian Plume Model

Objectives

Gaussian plume models are used in air quality modeling and environmental consultancy. This model is used to calculate maximum ground level impacts of plumes and also to calculate the distance of maximum impact from the source

Introduction

a. Background

Air pollution dispersion is quite extensive and dates back to the 1930s. One of the early air pollutant dispersion equations was derived by Bosanquet and Pearson but they did not assume Gaussian distribution nor include the effect of ground reflection of the pollutant. Later in 1947, Sir Graham Sutton derived an equation including the assumption of Gaussian distribution and also included the effect of ground reflection.

b. Principle

Gaussian Plume model uses a realistic description of dispersion. It represents an analytical solution to the diffusion equation for idealized circumstances. This model assumes that the atmospheric turbulence is both stationary and homogeneous. In reality, none of these conditions are fully satisfied. However, Gaussian plume model has been successfully used for rural configurations.

It uses steady state advection-diffusion equation. It follows the following modeling principles:

- In the stable atmosphere case (producing a fanning plume), there is horizontal dispersion at a right angle to the wind due to turbulence and diffusion. In the vertical, dispersion is suppressed by the stability of the atmosphere, so pollution does not spread toward the ground. This results in very low pollution concentration at the ground.
- In unstable air, the plume will whip up and down as the atmosphere mixes around (whenever an air parcel goes up, there must be air going down someplace else to maintain continuity, and the plume follows these air currents). This gives the plume the appearance that it is looping around.

- An inversion aloft will trap pollutants underneath it, since the stable inversion prevents vertical dispersion. Pollution released underneath the inversion layer will fumigate the mixed layer.
- In the neutral atmosphere case, the horizontal dispersion at a right angle to the wind is due to turbulence and diffusion, which occurs at the same rate as the vertical dispersion, which is not being opposed nor encouraged by the stability (or lack of it) in the atmosphere. So, the plume spreads equally in the vertical and horizontal as it propagates downstream, forming a coming plume.

In the lofting case, pollution dilutes upward. This produces much lower pollution concentrations at the ground at a distance downstream than the straight stable case (fanning plume), because molecular diffusion and some turbulence allow smoke to reach the ground eventually, and the fanning plume does not have the upward dispersion that the lofting plume has.

c. Application

This model can be used to illustrate the following phenomena:

- Effect of wind fluctuations/ speed on pollutant concentrations
- Effects of vertical stability on mixing concentrations at the ground
- Effects of multiple stacks emitting pollutants
- Demonstration of an annual cycle in stability on concentrations at the ground
- The effect of humidity on particulate matter
- The effect of aerosol chemistry on particulate matter
- Short and long term exceedances.

Modelling methods

One of the key assumption of this model is that over short period of time, steady state conditions exist with regard to air pollutant emissions and meteorological changes. Air pollution is represented by an idealized plume coming from the top of a stack (e.g. brick kiln) of some height and diameter. One of the primary calculation is the effective stack height. As the gases are heated in the plant, the hot plume will be thrust upward some distance above the top of the stack i.e. the effective stack height. We need to be able to calculate this vertical displacement, which depends on the stack gas exit velocity and temperature, and the temperature of the surrounding air.

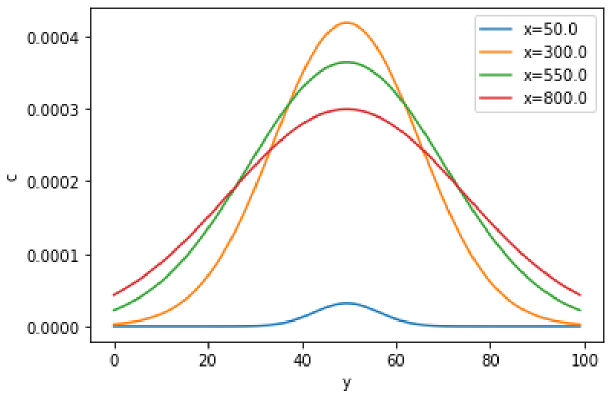
Once the plume has reached its effective stack height, dispersion will begin in three dimensions. Dispersion in the downwind direction is a function of the mean wind speed blowing across the plume. Dispersion in the cross-wind direction and in the vertical direction will be governed by the Gaussian plume equations of lateral dispersion. Lateral dispersion value depends on a value known as the atmospheric condition, which is a measure of the relative stability of surrounding sir. The model assumes that the dispersion in these two dimensions will take the form of a normal Gaussian curve, with the maximum concentration in the center of the plume.

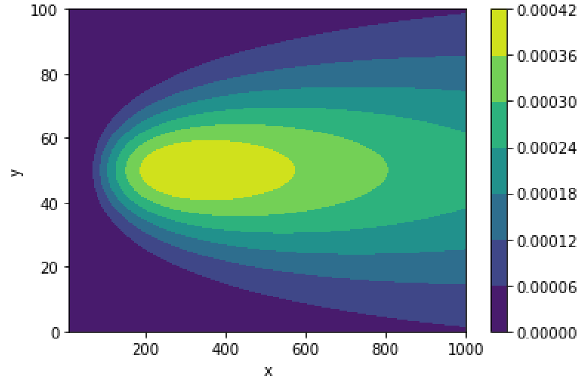
$$C(x, y, z) = \frac{Q}{4\pi x\sqrt{D_y D_z}} \exp\left(-\frac{v y^2}{4xD_y}\right)\left(\exp\left(-\frac{v(z-H)^2}{4xD_z}\right) + \exp\left(-\frac{v(z+H)^2}{4xD_z}\right)\right)\exp\left(\frac{\lambda x}{v}\right)$$

The following assumption have been made for this model:

- Emission is constant and continuous
- Terrain is flat
- Wind speed is constant
- No settling velocity
- No reaction of pollutants
- Pollutants only reflect from the ground

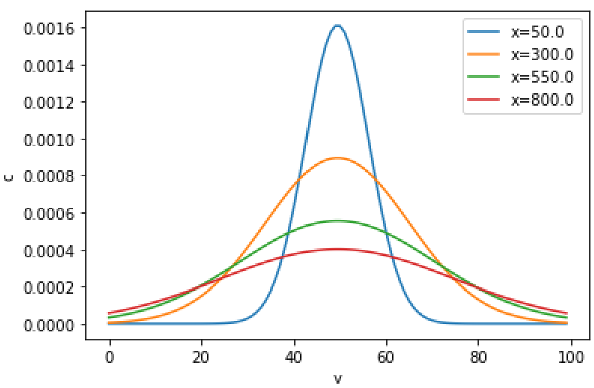
Results

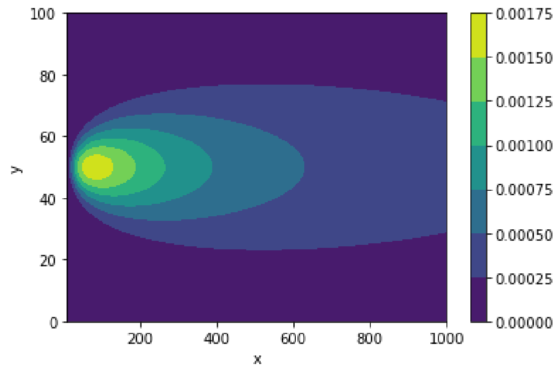




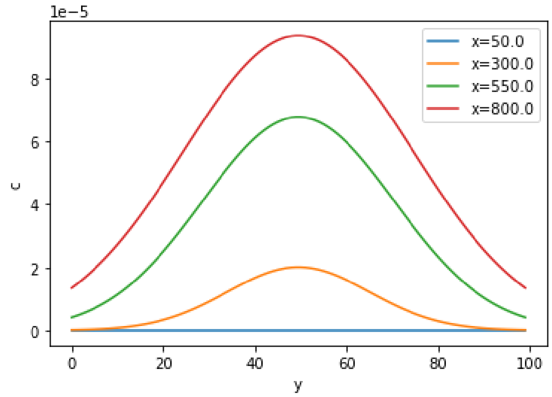
From the above graphs, we can observe that the maximum concentration of pollutants is when the plume is x = 300units away from the source of the pollutant. The maximum concentration in y-direction is at y = 50units as the height of the stack is 50 units.

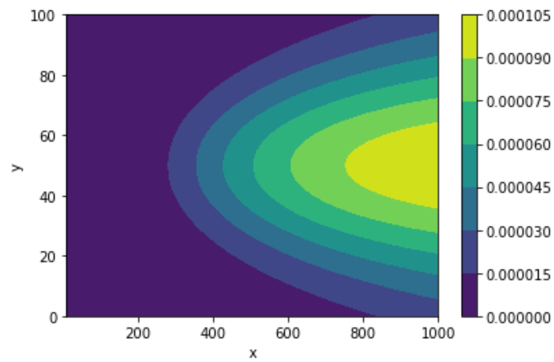
Sensitivity analysis





The height of the stack was halved and the model was run. We can see from the above graphs that after decreasing the stack height pollution has increased the concentration of pollution near the source.





Now, the model was run after doubling the effective height of the stack. From the above two graphs, we can see that the concentration of the pollutants in and around the source has decreased drastically.

Therefore, we can conclude that the effective height of the stack should be increased for decreasing the concentration.

Source code

```
Dy = 0.2; Dz = 1          # diffusivities

v = 0.5                   # velocity

#k = 0.01                 # decay rate

k = 0                     # decay rate (lambda in equation)

Q = 1                     # emission rate

xstack = 0; ystack = 50   # stack location

xmin = 10; xmax = 1000    # x-axis interval

ymin = 0; ymax = 100      # y-axis interval

H = 50                    # effective stack height(s)

z = 0                     # height of observation (=0 for ground surface)

#---------------------------------execution------------------------------

x,y = np.meshgrid(np.linspace(xmin,xmax,200),np.linspace(ymin,ymax,100))
```

```
xx = x - xstack
yy = y - ystack
c = Q/(4*np.pi*xx*np.sqrt(Dy*Dz))\
    *np.exp(-v*yy*yy/(4*Dy*xx))\
    *(np.exp(-v*(z-H)*(z-H)/(4*Dz*xx))\
    +np.exp(-v*(z+H)*(z+H)/(4*Dz*xx)))\
    *np.exp(-k*xx/v)
#----------------------------------output----------------------------------
print(c.shape)
plt.figure()
for i in range(10,200,50):
    plt.plot(range(100),c[:,i],label="x="+str(i*xmax/200))
    plt.xlabel('y')
    plt.ylabel('c')
    plt.legend()
plt.figure()
plt.contourf(x,y,c)
plt.colorbar()
plt.xlabel('x')
plt.ylabel('y')
plt.show()
```