

An Insight into Chemical Engineering

Through Instant Notes, Objective Types and Problems

(Useful for GATE and other Aptitude Tests)

m subbu



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An Insight into
Chemical Engineering
m subbu
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2003

To Almighty

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FOREWORD

Competitive examinations have become inevitable nowadays for admission to graduate courses and placement in chemical and allied industries, for undergraduate students in chemical engineering. For such examinations, apart from a wide knowledge of all aspects of unit operations of chemical engineering a quick wit to solve the questions/problems is absolutely essential. Exhaustive revision of all these operations studied in earlier semesters is a tedious procedure. A quick insight into the same along with solutions to short answer objective questions or problems is necessitated. With this objective in view, the author has made an attempt to prepare this monograph after going through many question papers of this type from several universities along with an intensive study of the operations covered.

I had the opportunity of revising the text and providing academic advice wherever necessary. I am confident this effort of the author will be well received by the budding chemical engineers taking their first degree from engineering colleges.

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Chennai
October 5, 2003

PREFACE

For competitive examinations to pursue higher studies or get jobs, depth of knowledge in problem solving and answering objective type questions should be the essential qualities of a successful Chemical Engineer.

A set of instant notes are given for various important topics in the core subjects of Chemical Engineering. I hope this set of topics detailed in this book will be of extreme use to the undergraduate students and practicing engineers for enhancing and checking their knowledge in the core concepts of the subject. In any competitive examination due weightage is being given to objective type questions. In this book I have also given objective type questions which will be useful for the chemical engineers and students for their preparation.

By lecturing for the past few years in core subjects of Chemical engineering like Fluid Mechanics, Thermodynamics and Process Control at Department of Chemical Engineering, Sri Venkateswara College of Engineering (SVCE), Sriperumbudur I have found that the students need additional input for applying theoretical knowledge for solving the problems. In this book a comprehensive set of problems in each of the nine subjects of Chemical Engineering are given with the steps in solving the same from the first principles which could be understood easily by a novice in the field also. Most of the problems are questions that were asked in GATE (Graduate Aptitude Test in Engineering) and in University examinations.

I extend my heartfelt thanks to Dr.N.Subramanian (Visiting Professor, SVCE — Former Professor in Indian Institute of Technology Madras, Chennai) for sparing his valuable time to review this book and provide technical guidance whenever needed.

I sincerely acknowledge the management of SVCE for their support.

I am grateful to Dr.V.Ravichandran (Assistant Professor, Dept. of Computer Applications, SVCE) for introducing and enthusing me to learn \LaTeX software, with which this book is typeset. Also he helped me in solving the objective type questions of Mathematics.

Mr.B.Nedumaran (Assistant Professor, Dept. of Chemical Engineering, SVCE) was always helpful in conducting GATE coaching to students of SVCE, that initiated me to prepare these materials.

Always my affection and thanks lie with my students who extended their interest to the fullest. Also I thank the Chemical Engineering community allover the world for their encouragement through my website <http://www.svce.ac.in/~msubbu> for the study materials provided in the website.

My thanks are due to my wife Mrs.S.Panchi for her suggestions in publishing this book and providing administrative assistance. I thank my dear parents and relatives for their emotional support during these years.

I thank the publisher for bringing out this edition at the earliest.

Awaiting for your valuable ideas, to be posted at <http://www.msubbu.com>

Chennai
October 15, 2003

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(M. SUBRAMANIAN)

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Chapter 1

Process Calculations

1.1 Instant Notes

1.1.1 Units and Standards

- S.I (International System of Units)
 - Mass in kg, Length in meter, Temperature in Kelvin, Number of gram-moles as mole, Force in Newton
 - The unit of temperature is Kelvin, K (not degree Kelvin, °K)
- Mole = $\frac{\text{mass in gram}}{\text{molecular weight}}$
- Composition of solids: expressed in weight % or in mole %. Whenever no specification is made about the composition i.e., whether it is mole % or weight %, it is taken as weight %.
- Very often, the impurities present in solid or liquid compounds are expressed in ppm (parts per million parts). This is expressed on a weight basis. When the solution is watery, 1 mg/lit = 1 ppm. In water treatment and effluent treatment problems, the analyses are given in ppm or mg/lit, which are both taken as similar.
- Molarity (M) is defined as the number of gmol of solute dissolved in 1 litre of solution.
- Normality (N) is defined as the number of gram equivalents dissolved in 1 litre of solution.
- Concentration in g/lit = Normality (N) × equivalent weight
- Molality is defined as the number of gmol of solute dissolved in 1 kilogram of solvent.
- The volumetric composition of a liquid solution will change with the temperature. The composition of a system expressed in mole percent will not vary with the temperature.

- For ideal gases, the composition in mole percent is exactly same as the composition in volumetric percent. It should be emphasized that this relation holds only for gases, and does not apply to liquid or solid systems.
- S.T.P: Pressure = 1.01325×10^5 N/m²; T = 273 K. At these conditions, for an ideal gas V = 22.4 m³/kmol.

1.1.2 Material Balance Calculations

- Without Chemical Reaction

— Under steady state conditions,

$$\text{Input} = \text{output}$$

— Under unsteady state conditions

$$\text{Input} = \text{output} + \text{accumulation}$$

- Atom balances

$$\text{Input (of some species of atom)} = \text{output (of the same species of atom)}$$

The chemical reactions that take place in a process which are very complicated can be solved by atom balances without knowing the chemical reactions taking place in that.

- **Inert substance:** In most oxidation processes air is used as the source of oxygen, and air is 79 mol percent nitrogen, an inert substance at the temperatures at which most substances are oxidized.
- **Tie substances:** Tie material is defined as a material, of necessarily inert or unreactive, which passes through a system from a single input stream to a single output stream and of course remains completely unchanged during this process.
- **Incomplete reactions:** Chemical reactions are generally not complete; that is, the limiting reactant is not completely used up. This idea is expressed as percent complete or percent completion, and it is important to note that this completion must be based on the limiting reactant.
- **Percent excess:** Percent excess of excess reactant is based on the 100 percent completion of the limiting reactant.
- **Recycle**
 - Recycle is the operation or a process step whereby a part or fraction of the products from the reactor is returned and mixed with the incoming fresh feed to the reactor.
 - Because of recycle, the overall conversion, which is based on the fresh feed, will be higher than the conversion per pass, which is based on the mixed feed to the reactor.

PROCESS CALCULATIONS

- The feed to the process itself is made up of two streams, the fresh feed and the recycled material. The gross product leaving the process is separated into two streams, the net product and the material to be recycled.

- Conversion

$$\text{Overall fractional conversion} = \frac{m_{rf} - m_{rp}}{m_{rf}} \quad (1.1)$$

where m_{rf} = mass of reactant in fresh feed

m_{rp} = mass of reactant in net product

$$\text{Single-pass (once-through) fractional conversion} = \frac{m_{rf} - m_{rx}}{m_{rf}} \quad (1.2)$$

where m_{rf} = mass of reactant fed into the reactor

m_{rx} = mass of reactant existing the reactor

- **Bypass stream:** one that skips one or more stages of the process and goes directly to another downstream stage.
- **Purge stream:** a stream to bleed off to remove an accumulation of inerts or unwanted material that might otherwise buildup in the recycle stream.

1.1.3 Energy Balance Calculations

- The distinction of constant volume and constant pressure processes is not so important in the case of liquids and solids, as the liquid and solid expands very little with increase in temperature.
- The specific heats of solids are lower than liquids.
- The *heat of formation* is a special type of heat of reaction from the elements. The reaction does not necessarily represent a real reaction but can be a fictitious process for the formation.
- All oxidation reactions are exothermic. Similarly, the dissolution of sulfuric acid or caustic soda is exothermic.
- Industrially important steam-hydrocarbon reforming reaction is endothermic.
- The dissolution of common salt in water is also endothermic.
- The temperature of the products under adiabatic conditions of reaction is called the *adiabatic reaction temperature*.

1.1.4 Saturation & Humidity

- Bone-dry air: Air that is bone dry contains absolutely no water vapor. Bone-dry implies completely dry.
- Molar humidity is also sometimes referred to as absolute humidity.

$$\text{Molar humidity} = \frac{\text{moles of water vapor}}{\text{bones of bone-dry air}}$$

- Humidity, or specific humidity = $\frac{\text{kg of water vapor}}{\text{kg bone-dry air}}$
- Relative humidity: is defined as the ratio of the partial pressure of water vapor in air to the vapor pressure of water at dry-bulb temperature. Relative humidity is widely used in weather reporting but finds little application in engineering where it has been replaced by percentage humidity.
- Percentage humidity or percentage saturation: is defined as the ratio of actual absolute humidity to the saturation humidity.
- Relative humidity is always greater than or equal to percentage humidity. Relative humidity is equal to percentage humidity only at saturation.
- Dew point
 - Partial pressure of water vapor in the mixture equals the vapor pressure of water at dew point.
 - Dew point is always lower than or equal to dry bulb temperature.
 - Dew point is always lower than or equal to wet-bulb temperature.
- At 100% saturation:

$$\text{Dry bulb temperature} = \text{Wet bulb temperature} = \text{Dew point}$$

- The difference between the wet-bulb temperature and the dry-bulb temperature is proportional to the amount of water vapor in the atmospheric air; it is zero if the air is saturated and highest if the air is bone-dry.

1.1.5 Combustion Calculations

- Calorific Value
 - The *calorific value* of a fuel is nothing but the heat of combustion of the fuel. It is defined as the total heat produced when a unit mass of fuel is completely burnt with pure oxygen. It is also said to be the heating value of the fuel.

PROCESS CALCULATIONS

- When a fuel is burnt, hydrogen combines with oxygen and gets converted into water. When water vapor is present in the flue gases, the latent heat of vaporization is lost. Hence this quantity of heat is not available for any useful purpose. Therefore, when the calorific value of a fuel is determined, considering that the water is present in the vapor form, it is said to be the net calorific value (NCV) or net heating value or lower heating value.

$$\text{NCV} = \text{GCV} - \text{weight \% of hydrogen} \times 9 \times \lambda$$

- Usually, NCV and GCV (Gross Calorific Value) of the fuels are reported at 25°C
- It is customary to specify the abbreviations NCV or GCV when the calorific values are reported, as the case may be. However, if no mention is made, the reported calorific value is taken as GCV.
- In comparison to all the three types of fuels, gaseous fuels offer higher heating values.
- Nitrogen reduces the flame temperature considerably and also accounts for high heat loss to the stack.
- Air requirements
 - The theoretical or stoichiometric amount of air is the minimum amount of air required to burn the fuel completely so that carbon, hydrogen and sulfur are converted to CO₂, H₂O and SO₂ respectively.
 - In actual practice, theoretical air is not sufficient to get complete combustion. Excess air supply is a must for complete combustion.
 - Normally, gaseous fuels require very less excess air. The effect of excess air is to reduce the flame temperature and to increase the heat losses through the flue gases.
 - The greater the moisture present in the flue gases, the higher is the dew point.
 - The major combustion reactions are all exothermic.

1.2 Objective Type Questions

1. For a steady state system

- (a) The rate of input is zero
- (b) The rate of generation is zero
- (c) The rate of consumption is zero
- (d) The rate of accumulation is zero

Answer: (d) At steady state, Input = Output (for nonreactive systems)

2. In a paper mill, a wash liquor containing 3% (by weight) solids is concentrated in an evaporator to yield a lye containing 30% (by weight) solids. The quantity of water evaporated per 100 kg of feed is:

(a) 30 kg (b) 70 kg (c) 60 kg (d) 90 kg

Answer: (d) Basis: 100 kg of feed containing 3% (by weight) solid.

$$\text{Solid in feed} = 0.03 \times 100 = 3 \text{ kg}$$

$$\text{Solid in lye (product)} = 3 \text{ kg}$$

$$\text{Total lye} = \frac{3}{0.3} = 10 \text{ kg}$$

$$\text{Water evaporated} = 100 - 10 = 90 \text{ kg}$$

3. The molar composition of a gas is 10% H₂, 10% O₂, 30% CO₂ and balance H₂O. If 50% H₂O condenses, the final mole percent of H₂ in the dry gas will be (GATE-2000-1.07).

(a) 10% (b) 5% (c) 18.18% (d) 20%

Answer: (d) For dry gas analysis H₂O amount should not be considered.

Therefore, H₂ in the dry gas = $100 \times [0.1/(0.1 + 0.1 + 0.3)] = 20\%$

4. A solution of specific gravity 1.0 consists of 35% A by weight and the remaining B. If the specific gravity of A is 0.7, the specific gravity of B is (GATE-1999-1.05)

(a) 1.25 (b) 1.3 (c) 1.35 (d) 1.2

Answer: (b) Basis: 1 ltr of solution.

Since sp.gr of pure A is 0.7 (i.e., $\rho = 0.7 \text{ kg/ltr}$),

$$0.7 \text{ kg of A} = 1 \text{ ltr}$$

$$0.35 \text{ kg of A} = 0.35 \times 1/0.7 = 0.5 \text{ ltr}$$

Therefore,

$$0.65 \text{ kg of B} = 1 - 0.5 = 0.5 \text{ ltr}$$

$$\rho \text{ of B} = \frac{\text{mass of B}}{\text{volume of B}} = \frac{0.65}{0.5} = 1.3$$

$$\text{i.e., Specific gravity of B} = 1.3$$

5. Pure A in gas phase enters a reactor. 50% of this A is converted to B through the reaction A → 3B. Mole fraction of A in the exit stream is (GATE-1999-1.06)

(a) 1/2 (b) 1/3 (c) 1/4 (d) 1/5

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Answer: (c) Basis: 1 mole of A in feed.

1 mole of A gives 3 moles of B for complete conversion.

Therefore for 50% conversion of A, product contains:

1.5 mole B and 0.5 mole A.

Mole fraction of A in product = $0.5/(1.5 + 0.5) = 1/4$

6. A liquid mixture contains 30% o-xylene, 60% p-xylene and 10% m-xylene (all percentages in w/w). Which of the following statements would be true in respect of this mixture? (GATE-1998-1.07)

(a) The mixture exhibits an azeotrope at 101.3 kPa.

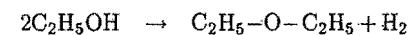
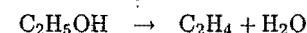
(b) The composition of the mixture, in percent by volume is: o-xylene 30, p-xylene 60, and m-xylene 10.

(c) The composition of the mixture in mole percent is: o-xylene 30, p-xylene 60, and m-xylene 10.

(d) The mixture contains optical isomers.

Answer: (c) For isomers, molecular weights are equal; hence mol% & weight% are same. The composition in volume% is equal to mole% for mixture of ideal gases.

7. Pure ethanol vapor is fed to a reactor packed with Alumina catalysts at the rate of 100 kmol/hr. The reactor product comprise: Ethylene = 95 kmol/hr, Water vapor = 97.5 kmol/hr and diethyl ether = 2.5 kmol/hr. The reactions occurring can be represented by:



The percent conversion of ethanol in the reactor is:

(GATE-1998-1.08)

(a) 100 (b) 97.5 (c) 95 (d) 2.5

Answer: (a) Basis: 1 hr.

Ethanol used:

for first reaction = 95 kmol (from ethylene rate)

for second reaction = $2.5 \times 2 = 5 \text{ kmol}$ (from diethylether rate)

Total = $95 + 5 = 100 \text{ kmol}$

Check for water vapor:

Water from first reaction = 95 kmol

Water from second reaction = 2.5 kmol

Total water produced = $95 + 2.5 = 97.5 \text{ kmol}$

(since ethanol used = ethanol in feed; conversion is 100%)

8. A sample of well water contains 140 g/m^3 Ca^{2+} ions and 345 g/m^3 Na^+ ions. The hardness of the sample of water, expressed in terms of equivalent CaCO_3 in g/m^3 [assuming atomic masses of $\text{Ca} = 40$, $\text{Na} = 23$, $\text{C} = 12$ and $\text{O} = 16$] is (GATE-1998-2.07)

(a) 350 (b) 485 (c) 140 (d) 345

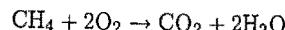
Answer: (a) Sodium ions does not contribute to hardness.
Molecular weight of $\text{CaCO}_3 = 40 + 12 + 3 \times 16 = 100$

$$\begin{aligned} 40 \text{ g of Ca} &\equiv 100 \text{ g of } \text{CaCO}_3 \\ 140 \text{ g of Ca} &\equiv \frac{100 \times 140}{40} = 350 \text{ g of } \text{CaCO}_3 \end{aligned}$$

9. Methane is completely burned with air. The maximum possible volume percent of carbon dioxide (on dry basis) in the flue-gas is (GATE-1993-8.b)

(a) 11.7 (b) 21.0 (c) 44.0 (d) 28.0

Answer: (a) Basis: 1 mole of CH_4



Flue gas contains:

$$\text{CO}_2 = 1 \text{ mole}$$

$$\text{N}_2 = 2 \times 79/21 = 7.524 \text{ mole}$$

$$\text{Volume \% of CO}_2 \text{ (on dry basis)} \text{ in flue gas} = 100 \times 1/(1 + 7.524) = 11.7$$

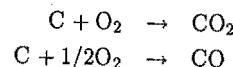
This amount is the maximum possible, since stoichiometric air is used.

10. Pure carbon is completely burnt in oxygen. The flue gas analysis is 70% CO_2 , 20% CO and 10% O_2 . The percent excess oxygen used is (GATE-1997-2.04)

(a) 20 (b) 12.5 (c) 0 (d) 10

Answer: (c) Basis: 1 mole of flue gas.

The possible reactions are:



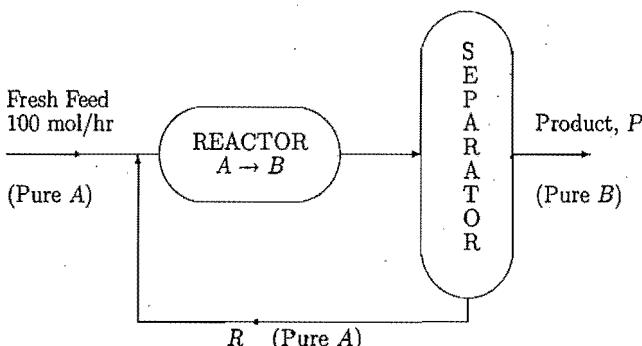
$$\begin{aligned} \text{O}_2 \text{ used up} &= \text{O}_2 \text{ used for } (\text{CO}_2 + \text{CO}) + \text{O}_2 \text{ in flue gas} \\ &= (0.7 + 0.2/2) + 0.1 = 0.9 \text{ mole} \end{aligned}$$

$$\text{O}_2 \text{ required} = 0.7 + 0.2 = 0.9 \text{ mole. (for complete conversion)}$$

$$\text{Excess O}_2 = 0.9 - 0.9 = 0 \text{ mole}$$

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11. The flowsheet is given in figure.



If the single-pass conversion (once-through conversion) of A to B is 20%, then the rate of recycle R (mol/hr) is (GATE-1997-2.05)

(a) 300 (b) 400 (c) 500 (d) 600

Answer: (b) $A \rightarrow B$. By overall balance, product $P = 100 \text{ mol/hr}$. Since conversion within the reactor is only 20%,

$$\begin{aligned} \text{mixed feed to the reactor} &= 100/0.2 = 500 \text{ mol/hr} \\ \text{recycle rate } R &= 500 - 100 = 400 \text{ mol/hr} \end{aligned}$$

12. For the case of fuel gas undergoing combustion with air, if the air/fuel ratio is increased, the adiabatic flame temperature will (GATE-2001-1.05)

(a) increase
(b) decrease
(c) increase or decrease depending on the fuel type
(d) not change

Answer: (b) If air/fuel ratio is increased, N_2 content is increased which does not take part in reaction. Some amount of heat will be used for raising the sensible heat content of N_2 , thereby reducing the adiabatic flame temperature.

13. An evaporator while concentrating an aqueous solution from 10 to 40% solids evaporates 30000 kg of water. The amount of solids handled by the system in kg is (GATE-1995-2.e)

(a) 4000 (b) 9000 (c) 4600 (d) 3000

Answer: (a) Overall mass balance: $F = V + P$. (where F = feed; V = vapor; P = product)

Balance on solid: $0.1F = 0.4P$. Therefore $P = (0.1/0.4)F = 0.25F$.
 $F = 30000 + 0.25F \Rightarrow F = 40000$.

Amount of solids in feed = $40000 \times 0.1 = 4000$ kg.

14. A multiple effect evaporator has a capacity to process 4000 kg of solid caustic soda per day when it is concentrating from 10% to 25% solids. The water evaporated in kg per day is
(GATE-1992-2.d)

(a) 600 (b) 24,000 (c) 60,000 (d) 48,000

Answer: (b) Basis: 1 day.

10% of solid weighs 4000 kg. Therefore,

$$\text{Feed} = \frac{4000}{0.1} = 40000 \text{ kg}$$

$$\text{Product} = \frac{4000}{0.25} = 16000 \text{ kg}$$

$$\text{Water evaporated} = 40000 - 16000 = 24000 \text{ kg}$$

15. 1000 kg of wet solids are to be dried from 60% to 20% moisture (by weight). The mass of moisture removed in kg is
(GATE-1995-2.f)

(a) 520 (b) 200 (c) 400 (d) 500

Answer: (d)

$$\text{Dry solid in feed} = 1000 \times (1 - 0.6) = 400 \text{ kg}$$

$$\text{Amount of dried product} = 400/(1 - 0.2) = 500 \text{ kg}$$

$$\text{Moisture evaporated} = 1000 - 500 = 500 \text{ kg}$$

16. 1000 kg of a solution containing 50% by weight of a salt dissolved in it is cooled. 400 kg of anhydrous salt is separated out. The solubility of the salt at the lower temperature in kg/100kg of water is
(GATE-1993-8.a)

(a) 80 (b) 50 (c) 40 (d) 20

Answer: (d)

$$\text{Water in feed} = 0.5 \times 1000 = 500 \text{ kg}$$

$$\text{Water in mother liquor} = \text{Water in feed} = 500 \text{ kg}$$

$$\text{Salt in mother liquor} = \text{salt in feed} - \text{crystals}$$

$$= (1000 - 500) - 400 = 100 \text{ kg}$$

$$\text{Solubility} = \frac{\text{kg salt}}{\text{kg water}} = \frac{100}{500} = 20 \text{ kg salt}/100 \text{ kg water}$$

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17. Assuming that CO₂ obeys perfect gas law, calculate the density of CO₂ (in kg/m³) at 263°C and 2 atm.
(GATE-1995-2.g)

(a) 1 (b) 2 (c) 3 (d) 4

Answer: (b) $PV = nRT$

$$\text{Molal density} = \frac{n}{V} = \frac{P}{RT}$$

$$\text{Mass density } \rho = \frac{n}{V} M = \frac{PM}{RT}$$

$$= \frac{(2 \times 1.01325 \times 10^5) \times 44}{8314 \times (273 + 263)} = 2 \text{ kg/m}^3$$

18. Pure O₂ is mixed with air to produce an enriched air containing 50 volume % of O₂. The ratio of moles of air to O₂ used is
(GATE-1995-2.h)

(a) 1.72 (b) 0.58 (c) 0.50 (d) 0.20

Answer: (a) Basis: Pure O₂ + Air = 1 mole. i.e., $P + A = 1$.

Balance on O₂: $P + 0.21A = 0.5(P + A)$.

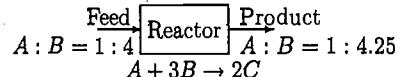
Solving these two equations, $A/P = 1.725$.

1.3 Problems with Solutions

1.3.1 Volume of Gas at STP

The gases entering a reactor contain A and B in the mole ratio of 1:4. The mole ratio of these gases in the exit stream from the reactor is 1:4.25. What volume of the gas in m³ at STP must enter the reactor to produce 100 kmol of product C per hour? The reaction is $A + 3B \rightarrow 2C$.
(GATE-1989-11ii)

Solution:



Let A in feed = 1 mole. Therefore, B in feed = 4 mole.

Let x be the fractional conversion of A. Then product contains: $A = 1 - x$; $B = 4 - 3x$; and $C = 2x$.

$$(1 - x) : (4 - 3x) = 1 : 4.25$$

$$4.25(1 - x) = 4 - 3x$$

$$4.25 - 4.25x = 4 - 3x$$

Solving, $x = 0.2$. Therefore, 1 mole of A in feed gives 0.2×2 mole of C .

$$0.4 \text{ mole of } C \equiv 1 \text{ mole of } A$$

$$100 \text{ mole of } C \equiv 100/0.4 = 250 \text{ mole of } A \text{ in feed.}$$

Feed contains:

$$A = 250 \text{ kmol}$$

$$B = 4 \times 250 = 1000 \text{ kmol}$$

$$\text{Total} = 1250 \text{ kmol}$$

At STP, 1 kmol of gas occupies 22.4 m^3 . Therefore

$$1250 \text{ kmol/hr} = 1250 \times 22.4 = 28000 \text{ m}^3/\text{hr}$$

1.3.2 Orsat Analysis of Flue Gas

The Orsat analysis of a flue gas is: $\text{CO}_2 = 12.7\%$; $\text{O}_2 = 7.1\%$; $\text{N}_2 = 80.2\%$. Determine the percent excess air used in combustion. The nitrogen present in the flue gas is contributed by air only.

(GATE-1995-11)

Solution:

Basis: 100 mol of flue gas.

$$80.2 \text{ mol of } \text{N}_2 \equiv 80.2 \times \frac{21}{79} = 21.32 \text{ mol of } \text{O}_2$$

$$\text{i.e., Total O}_2 \text{ entering} = 21.32 \text{ mol}$$

For the reaction $\text{C} + \text{O}_2 \rightarrow \text{CO}_2$, 1 mol of CO_2 requires 1 mol of O_2 . Therefore O_2 used for $\text{CO}_2 = 12.7 \text{ mol}$.

$$\begin{aligned} \text{O}_2 \text{ used for H}_2\text{O} &= \text{Total O}_2 \text{ entering} - (\text{O}_2 \text{ used for CO}_2 + \text{O}_2 \text{ leaving}) \\ &= 21.32 - (7.1 + 12.7) = 1.52 \text{ mol} \end{aligned}$$

$$\text{O}_2 \text{ needed for combustion} = 12.7 + 1.52 = 14.22 \text{ mol.}$$

$$\text{O}_2 \text{ used} = 21.32 \text{ mol}$$

$$\% \text{ excess air} = \frac{21.32 - 14.22}{14.22} \times 100 = 50\%$$

1.3.3 Concentration of SO_2 in Flue Gas

The concentration of SO_2 in the flue gases from a boiler was found to be 0.2 kg/m^3 at N.T.P. Determine the concentration of SO_2 in parts per million by volume, at N.T.P. Assume that

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the flue gases are perfect.

(GATE-1992-11)

Solution:

$$\text{Molecular weight of } \text{SO}_2 = 64 \text{ kg/kmol}$$

$$0.2 \text{ kg/m}^3 \text{ of } \text{SO}_2 = 0.2/64 = 3.125 \times 10^{-3} \text{ kmol/m}^3$$

$$\text{For a perfect gas at NTP, } 1 \text{ kmol} = 22.4 \text{ m}^3$$

$$\begin{aligned} \text{Therefore, } 3.125 \times 10^{-3} \text{ kmol/m}^3 &= 3.125 \times 10^{-3} \times 22.4 = 0.07 \text{ m}^3/\text{m}^3 \\ &= 70000 \times 10^{-6} \text{ m}^3/\text{m}^3 = 70000 \text{ ppm} \end{aligned}$$

1.3.4 Conversion of SO_2

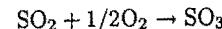
The analysis of the gas entering the secondary converter in a contact sulphuric acid plant is $4\% \text{ SO}_2$, $13\% \text{ O}_2$ and $83\% \text{ N}_2$ (volume %). In the converter SO_2 is oxidised to SO_3 . The gases leaving the converter contain $0.45\% \text{ SO}_2$ on an SO_3 -free basis (volume %). Calculate the percent conversion of SO_2 .

(GATE-1992-12a)

Solution:

Basis: 100 mol of gases entering the converter.

Reaction:



Let x be the fractional conversion of SO_2 in the converter. Then the gases leaving the converter will contain:

$$\text{SO}_3 = 4x \text{ mol}; \text{SO}_2 = 4 - 4x \text{ mol}; \text{O}_2 = 13 - 2x \text{ mol}; \text{N}_2 = 83 \text{ mol}$$

Given: SO_2 in converter outlet gases = 0.45% on SO_3 free basis. Therefore,

$$\frac{4 - 4x}{83 + (13 - 2x) + (4 - 4x)} = \frac{0.45}{100}$$

Solving, $x = 0.894$. Therefore, conversion of SO_2 in the converter = $0.894 \times 100 = 89.4\%$

1.3.5 Percentage Excess Air

Pure propane(C_3H_8) is burnt with an excess of air to give the following analysis of combustion products in volume percent:

$$\text{CO}_2 = 5.0, \text{CO} = 3.5, \text{H}_2\text{O} = 11.4, \text{O}_2 = 7.0 \text{ and } \text{N}_2 = 73.1$$

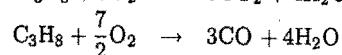
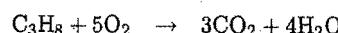
Calculate the percentage of excess air used.

(GATE-1990-11i)

Solution:

Basis: 100 mol of product gases.

The following reactions could be written for the combustion of C_3H_8 :



From stoichiometry, 3 mol of CO_2 is from 1 mol of C_3H_8 and also 3 mol of CO is from 1 mol of C_3H_8 . And for complete combustion of 1 mol of C_3H_8 , 5 mol of O_2 is required.

$$\begin{aligned} \text{Total propane in feed} &= \text{Propane for 5 mol of } CO_2 \text{ and 3.5 mol of CO} \\ &= \frac{5}{3} + \frac{3.5}{3} \\ &= 2.833 \text{ mol} \end{aligned}$$

$$\begin{aligned} \text{Theoretical air required} &= \text{Air required for complete combustion of } C_3H_8 \text{ to } CO_2 \\ &= 2.833 \times \frac{5}{0.21} = 67.46 \text{ mol} \end{aligned}$$

$$\text{Air actually used} = \frac{73.1}{0.79} = 92.53 \text{ mol}$$

$$\% \text{ excess air} = \frac{92.53 - 67.46}{67.46} \times 100 = 37.2\%$$

1.3.6 Calcination of Limestone

Limestone mixed with coke is being burnt in a kiln. An average analysis of limestone is $CaCO_3$: 84.5%, $MgCO_3$: 11.5% and the rest inert. The coke contains 76% carbon, 21% ash and 3% moisture. The calcination of $CaCO_3$ is only 95% complete and that of $MgCO_3$ 90%. The carbon in the coke is completely burnt to CO_2 . The kiln is fed with 1 kg of coke per 5 kg limestone. Calculate weight percent CaO in the product leaving the kiln. (GATE-1991-12ii)

Solution:

Basis: 1 kg of limestone in feed.

Feed contains:

$$CaCO_3 = 0.845 \text{ kg}$$

$$MgCO_3 = 0.115 \text{ kg}$$

$$\text{inerts} = 1 - 0.845 - 0.115 = 0.04 \text{ kg}$$

Molecular weight of

$$CaCO_3 = 40 + 12 + 3 \times 16 = 100$$

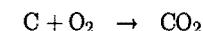
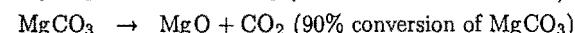
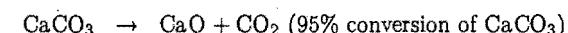
$$MgCO_3 = 24.3 + 12 + 3 \times 16 = 84.3$$

$$CaO = 40 + 16 = 56$$

$$MgO = 24.3 + 16 = 40.3$$

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Reactions:



Limestone : Coke = 5:1. Therefore,

$$\text{Feed coke} = 0.2 \text{ kg}$$

$$\text{Carbon in coke} = 0.76 \times 0.2 = 0.152 \text{ kg}$$

$$\text{Ash in coke} = 0.21 \times 0.2 = 0.042 \text{ kg}$$

$$\text{Moisture in coke} = 0.03 \times 0.2 = 0.006 \text{ kg}$$

Solid products leaving the kiln:

$$\text{Inerts} = 0.04 \text{ kg}$$

$$CaCO_3 = 0.845 \times 0.05 = 0.0423 \text{ kg}$$

$$MgCO_3 = 0.115 \times 0.1 = 0.0115 \text{ kg}$$

$$CaO = 0.845 \times 0.95 \times 56/100 = 0.4495 \text{ kg}$$

$$MgO = 0.115 \times 0.9 \times 40.3/84.3 = 0.0495 \text{ kg}$$

$$\text{Ash} = 0.042 \text{ kg}$$

Total solid products = $0.04 + 0.0423 + 0.0115 + 0.4495 + 0.0495 + 0.042 = 0.6348 \text{ kg}$.
Weight % of CaO in the products leaving the kiln = $(0.4495/0.6348) \times 100 = 70.81\%$

1.3.7 Percentage of CH_4 Burnt

In a laboratory experiment pure methane is burnt with theoretical quantity of air. Only part of the methane burns but the part that burns goes to CO_2 and H_2O . If the methane and air are initially at $25^\circ C$ and the products leave at $400^\circ C$ with water in vapor form, what % of the methane is burnt?

Data:

For the oxidation of CO_2 and H_2O (vapor) the standard heat of reaction $\Delta H_R^\circ = -802.9 \text{ kJ/mol}$ of CH_4 reacted.

Specific enthalpies at $400^\circ C$ in kJ/mol (Reference: Specific enthalpy at $25^\circ C = 0 \text{ kJ/mol}$)

$$O_2 = 11.64$$

$$N_2 = 11.13$$

$$CH_4 = 17.22$$

$$CO_2 = 16.43$$

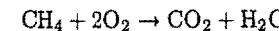
$$H_2O = 13.2$$

(GATE-1989-11iii)

Solution:

Basis: 1 mol of CH_4 in feed.

The following reactions could be written for the combustion of CH_4 :



Energy balance for adiabatic reaction:

$$\text{Heat content of products} = \text{Heat content of reactants} + \text{Heat released from reaction}$$

$$H_P = H_F + H_R$$

If reference temperature is chosen as 300 K, then $H_F = 0$.

Let x be the fraction of methane unburnt. Then,

$$(1-x) \times 802.9 = 1 \times (1-x) \times 16.43 + 2 \times (1-x) \times 13.2 + 2 \times \frac{79}{21} \times 11.13 + x \times 17.22 + 2 \times x \times 11.64$$

$$x = 0.845$$

$$\text{Percentage of methane burnt} = (1-x) \times 100 = (1-0.845) \times 100 = 15.5\%$$

1.3.8 Percentage Conversion of C_2H_4

Exit gases from an ethylene oxide reactor had the following analysis (mol% on dry basis): ethylene 2.3, ethylene oxide 0.9, nitrogen 79.0, oxygen 12.3 and carbon dioxide 5.5. Calculate

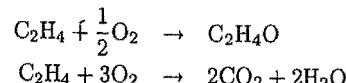
- (a) percent selectivity
- (b) percent overall conversion, and
- (c) air to ethylene mole ratio in the feed mixture.

(GATE-1988-11i)

Solution:

Basis: 100 mol of dry exit gases.

Reactions:



As per reaction 2,

$$\text{moles of } H_2O \text{ produced} \equiv \text{moles of } CO_2 \text{ produced} = 5.5$$

$$\text{Total number of moles leaving the reactor} = 100 + \text{moles of } H_2O = 105.5$$

Analysis of exit gases on wet basis:

$$\text{Ethylene} = \frac{2.3}{105.5} \times 100 = 2.18$$

$$\text{Similarly, ethylene oxide} = 0.853$$

$$N_2 = 74.88$$

$$O_2 = 11.66$$

$$CO_2 = 5.213$$

$$H_2O = 5.213$$

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$$\% \text{ Selectivity} = \frac{\text{C}_2\text{H}_2 \text{ used for the desirable reaction}}{\text{Total C}_2\text{H}_4 \text{ reacted}} \times 100$$

$$= \frac{0.853}{0.853 + 0.5 \times 5.213} \times 100 = 24.66\%$$

$$\% \text{ Overall conversion} = \frac{\text{Total C}_2\text{H}_4 \text{ reacted}}{\text{C}_2\text{H}_4 \text{ in feed}} \times 100$$

$$= \frac{0.853 + 0.5 \times 5.213}{0.853 + 0.5 \times 5.213 + 2.18} \times 100 = 61.34\%$$

$$\text{Ethylene in feed} = 0.853 + 0.5 \times 5.213 + 2.18 = 5.6395 \text{ mol}$$

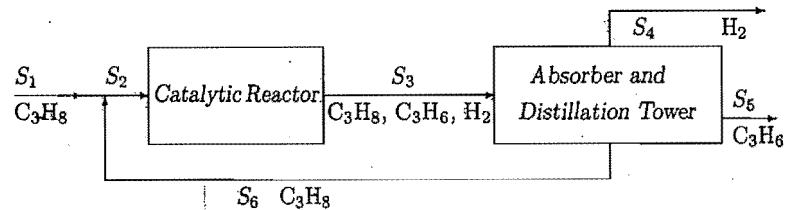
$$\text{Air in feed} = \frac{74.88}{0.79} = 94.785 \text{ mol}$$

$$\text{Air to ethylene mole ratio in feed} = \frac{94.785}{5.6395} : 1 = 16.81 : 1$$

1.3.9 Propane Dehydrogenation Plant

The process schematic of a propane dehydrogenation plant is shown below. It is desired to set up a simplified version of the material balance for this plant. Assume that the only reaction is the dehydrogenation of propane to propylene; there are no side reactions. The yield of propylene per pass is 30% (i.e., 30% of propane entering the reactor is converted to propylene). Assume that the amount of carbon formed on the catalyst is negligible. The product flow rate (stream S_5) is 50 kmol/hr. Calculate the flow rates of all the other streams. Notice that all streams except stream S_3 are pure.

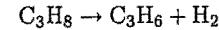
(GATE-2001-5)



Solution:

Basis: 1 hr of production time; 50 kmol/hr of product flow (C_3H_6)

Reaction:



1 mole of propylene is formed from 1 mole of propane. Therefore the stream S_1 is 50 kmol of C_3H_8 .

Stream S_4 contains hydrogen, and as per reaction the flow rate is 50 kmol/hr.

$$C_3H_6 \text{ in } S_3 = 50 \text{ kmol}$$

$$H_2 \text{ in } S_3 = 50 \text{ kmol}$$

As conversion of propane per pass is only 30%

$$\text{C}_3\text{H}_8 \text{ in } S_2 = \frac{50}{0.3} = 166.7 \text{ kmol}$$

$$\text{C}_3\text{H}_8 \text{ in } S_3 = 166.7 \times (1 - 0.3) = 116.7 \text{ kmol}$$

$$\text{C}_3\text{H}_8 \text{ in } S_6 = 116.7 \text{ kmol}$$

Summary:

$$S_1 = 50 \text{ kmol/hr}$$

$$S_2 = 166.7 \text{ kmol/hr}$$

$$S_3 = 116.7 + 50 + 50 = 216.7 \text{ kmol/hr}$$

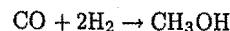
$$S_4 = 50 \text{ kmol/hr}$$

$$S_5 = 50 \text{ kmol/hr}$$

$$S_6 = 116.7 \text{ kmol/hr}$$

1.3.10 Methanol Production in Recycle Reactor

Methanol is produced by the reaction of CO with H₂ as

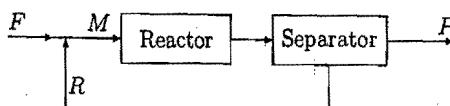


Only 15% of carbon monoxide entering the reactor is converted to methanol. The methanol formed is condensed and recovered completely. The unreacted CO and H₂ are recycled back to the reactor. The fresh feed will contain H₂ and CO in the molar ratio of 2:1. For 3200 kg/hr of methanol produced, calculate

(i) kmol/hr of fresh feed

(ii) kmol/hr of recycle gas

(GATE-1995-6)



$$\begin{aligned} \text{Molar flowrate of methanol}(P) &= \frac{\text{Mass flowrate}}{\text{Molecular weight}} \\ &= \frac{3200}{32} \\ &= 100 \text{ kmol/hr} \end{aligned}$$

PROCESS CALCULATIONS

Since the fresh feed is having the reactants in stoichiometric ratio, CO and H₂ in the mixed feed will also be in stoichiometric ratio.

From stoichiometry,

$$\begin{aligned} 1 \text{ mol of CH}_3\text{OH} &\equiv 1 \text{ mol of CO} \\ &\equiv 2 \text{ mol of H}_2 \end{aligned}$$

Therefore, to produce 100 kmol of CH₃OH, the fresh feed should have 100 kmol of CO and 200 kmol of H₂.

i.e., Fresh feed rate (*F*) = 100 + 200 = 300 kmol/hr.

Since the conversion is only 15%,

$$\text{CO in mixed feed should be} = 100/0.15 = 666.7 \text{ kmol}$$

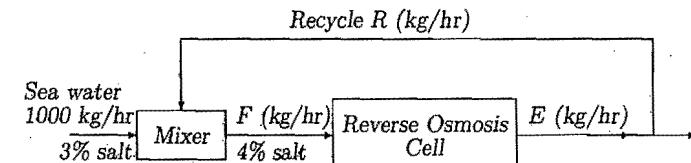
$$\text{Similarly, H}_2 \text{ in mixed feed} = 200/0.15 = 1333.4 \text{ kmol}$$

Therefore, mixed feed rate (*F* + *R*) = 666.7 + 1333.4 = 2000 kmol/hr.

Recycle flowrate = 2000 - 300 = 1700 kmol/hr.

1.3.11 Recycle Ratio in Reverse Osmosis Desalination

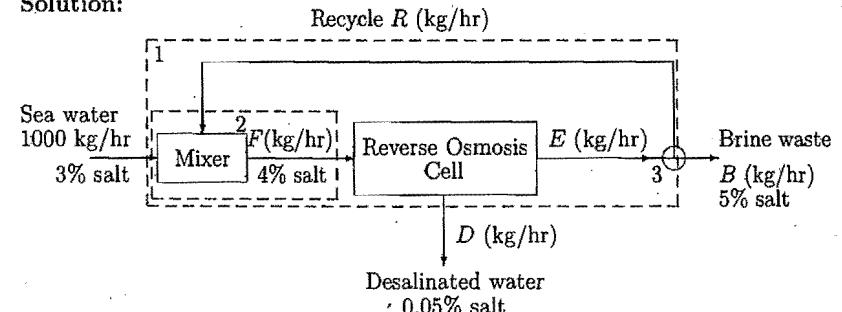
Sea water is desalinated by reverse osmosis as shown in figure.



All compositions are on mass basis. Calculate *R/E*.

(GATE-1997-12)

Solution:



Mass balance around boundary 1:

Overall:

$$1000 = B + D \quad (1.3)$$

Balance on salt:

$$1000 \times 0.03 = \frac{0.05}{100} \times D + 0.05 \times B \quad (1.4)$$

Solving Eqns.(1.3) and (1.4), $D = 404 \text{ kg/hr}$; $B = 596 \text{ kg/hr}$.

Mass balance around boundary 2:

Overall:

$$F = 1000 + R \quad (1.5)$$

Balance on salt:

$$0.03 \times 1000 + 0.05 \times R = 0.04 \times F \quad (1.6)$$

Solving Eqns.(1.5) and (1.6), $R = 1000 \text{ kg/hr}$.

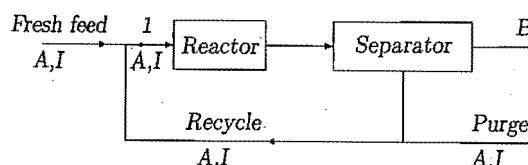
From overall mass balance at junction 3,

$$E = B + R = 596 + 1000 = 1596 \text{ kg/hr}$$

Therefore $R/E = 1000/1596 = 0.627$

1.3.12 Recycle to the Reactor

For the reaction $A \rightarrow B$, the process flow diagram is shown in figure. The fresh feed of A consists of 0.5% of inert by volume. 60% conversion per pass of A fed to the reactor is obtained. The concentration of inert going into the reactor at (1) must be held at 2% by volume. All streams are ideal gases and the process is at steady state. How many moles need to be recycled per mole of total feed to the reactor at (1). (GATE-1990-11ii)



Solution:

Basis: 1 mole of fresh feed

Reaction: $A \rightarrow B$

A in fresh feed $= 1 \times (1 - 0.005) = 0.995 \text{ mol}$

I in purge $= 1 \times 0.005 = 0.005 \text{ mol}$

Let y be the moles of B leaving separator. Then,

$$A \text{ to reactor at } 1 = y/0.6 = 1.67y \text{ mol}$$

$$A \text{ leaving the reactor} = 1.67y \times (1 - 0.6) = 0.67y \text{ mol}$$

Let x be the moles of I to the reactor at 1. Then,

$$\frac{x}{1.67y + x} = 0.02 \quad (1.7)$$

PROCESS CALCULATIONS

Ratio of A to I leaving the reactor (also in separator & in purge) $= 0.67y/x$

$$\text{Amount of } I \text{ in the purge} = 0.005$$

$$\text{Amount of } A \text{ in purge} = 0.005 \times 0.67y/x = 0.00335y/x$$

Overall mass balance:

$$\text{Fresh feed} = B + \text{Purge}$$

$$1 = y + 0.00335y/x + 0.005 \quad (1.8)$$

Solving Eqns.(1.7) and (1.8), $x = 0.031$; $y = 0.855$

$$A \text{ in purge} = 0.00335y/x = 0.00335 \times 0.855/0.031 = 0.092 \text{ mol}$$

$$A \text{ in recycle} = 0.67y - 0.092 = 0.67 \times 0.855 - 0.092 = 0.481 \text{ mol}$$

$$A/I \text{ in recycle} = 0.67y/x = 0.67 \times 0.855/0.031 = 18.5$$

$$I \text{ in recycle} = 0.481/18.5 = 0.026 \text{ mol}$$

$$\text{Recycle amount} = 0.481 + 0.026 = 0.507 \text{ mol}$$

$$\text{Total feed to reactor at 1} = 1 + 0.507 = 1.507 \text{ mol}$$

$$\text{Ratio of Recycle to total feed} = 0.507/1.507 = 0.336$$

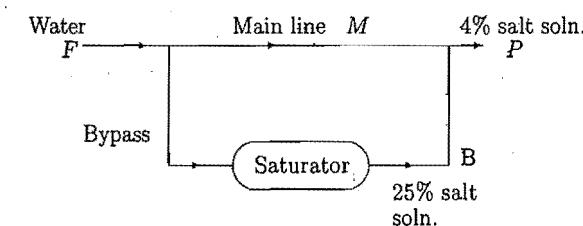
1.3.13 Saturator Bypass

A 4% solution of a salt in water is required in a continuous process. For this purpose, a part of the water is bypassed through a saturator from which a 25% solution is produced. If this stream is mixed with the main water stream to produce the 4% solution, what percentage of water must be bypassed? (GATE-1989-11i)

Solution:

Basis: 100 kg of 4% solution (P).

The streams are represented in the diagram as



Mass balance on water:

$$\text{Water in bypass} + \text{Water in mainline} = \text{Water in product}$$

$$B \times 0.75 + M = P \times 0.96 \quad (1.9)$$

Overall mass balance:

$$B + M = P = 100 \quad (1.10)$$

Solving Eqns.(1.9) and (1.10), $B = 16 \text{ kg}$; $M = 84 \text{ kg}$.
Salt in $B = 16 \times 0.25 = 4 \text{ kg}$.

Therefore, water entering the saturator = $16 - 4 = 12 \text{ kg}$.
Water in $F = 84 + 12 = 96 \text{ kg}$.

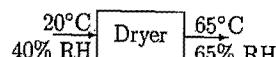
Fraction of water bypassed through the saturator = $12/96 = 0.125$

1.3.14 Drying of CaCO_3 Slurry

CaCO_3 slurry has to be dried. The dryer is designed to remove 100 kg moisture per hour. Air at 20°C and 40% relative humidity, enters the dryer and leaves at 65°C and 65% relative humidity. What is the weight (in kg) of bone-dry air required per hour? The atmospheric pressure is 103 kPa. If the humidity of the air entering can be varied, what is the minimum amount of dry air required? The constants for Antoine equation for vapor pressure of water in mm Hg may be taken as $A = 18.306$, $B = 3816.44$ and $C = -46.13$. (GATE-1999-7)

Solution:

Basis: 100 kg moisture removal per hour.



Antoine equation:

$$\ln P^{\text{sat}} = A - \frac{B}{t+C} = 18.306 - \frac{3816.44}{t - 46.13}$$

Check: at $t = 100^\circ\text{C} = 373 \text{ K}$,

$$P^{\text{sat}} = \exp \left(18.306 - \frac{3816.44}{373 - 46.13} \right) = 757.7 \text{ mm Hg}$$

Therefore, t in Antoine equation is in Kelvin. From Antoine equation, for inlet air at 20°C , $P^{\text{sat}} = 17.2 \text{ mm Hg}$; and for outlet air at 65°C , $P^{\text{sat}} = 186.3 \text{ mm Hg}$.

PROCESS CALCULATIONS

Given: Atmospheric pressure = 103 kPa = $(103/101.25) \times 760 = 773.1 \text{ mm Hg}$.

$$\begin{aligned} \text{Relative humidity} &= \frac{\text{partial pressure of water vapor}}{\text{vapor pressure of water}} \\ \text{Partial pressure of water vapor at inlet} &= 0.4 \times 17.2 = 6.88 \text{ mm Hg} \\ \text{Partial pressure of water vapor at outlet} &= 0.65 \times 186.3 = 121.1 \text{ mm Hg} \\ \text{Absolute molal humidity} &= \frac{\bar{P}_A}{P_t - \bar{P}_A} \\ \text{Absolute humidity of entering air} &= \frac{6.88}{773.1 - 6.88} \times \frac{18}{29} \\ &= 5.573 \times 10^{-3} \text{ kg water / kg air} \\ \text{Absolute humidity of leaving air} &= \frac{121.1}{773.1 - 121.1} \times \frac{18}{29} \\ &= 0.1153 \text{ kg water / kg air} \\ \text{Moisture taken by air} &= 0.1153 - 5.573 \times 10^{-3} \\ &= 0.1097 \text{ kg moisture / kg air} \end{aligned}$$

If the entering air is 0% relative humidity (bone-dry air), then

$$\begin{aligned} \text{moisture removed} &= \text{moisture out} - \text{moisture in} \\ &= 0.1153 - 0 = 0.1153 \text{ kg moisture / kg dry air} \\ \text{Minimum amount of air required} &= \frac{100}{0.1153} = 867.3 \text{ kg} \end{aligned}$$

1.3.15 Crystallization

1000 kg/hr of an aqueous solution of 20% Na_2CO_3 is cooled gradually to $T^\circ\text{C}$, to crystallize out $\text{Na}_2\text{CO}_3 \cdot 10\text{H}_2\text{O}$. The solubility of Na_2CO_3 at $T^\circ\text{C}$ is 2.1%. Calculate the percentage of Na_2CO_3 recovered in the form of crystals. (Assume no loss of Na_2CO_3 through the mother liquor adhering to the crystals and no carry over of crystals with the mother liquor). Draw a neat block diagram showing the inlet and exit compositions and flow rates. (Molecular weight of Na_2CO_3 can be assumed to be 106 and that of water to be 18). (GATE-1998-12)

Solution:

Basis: 1000 kg/hr of aqueous solution of 20% Na_2CO_3
Let us denote the flow rates of feed as F and mother liquor as M and crystals as C .
Composition of mother liquor:

$$\begin{aligned} \text{Weight fraction of } \text{Na}_2\text{CO}_3 &= 0.021 \\ \text{Weight fraction of } \text{H}_2\text{O} &= 1 - 0.021 = 0.979 \end{aligned}$$

Composition of crystals:

$$\text{Molecular weight of } \text{Na}_2\text{CO}_3 \cdot 10\text{H}_2\text{O} = 106 + 10 \times 18 = 286$$

$$\text{Weight fraction of } \text{Na}_2\text{CO}_3 \text{ in } \text{Na}_2\text{CO}_3 \cdot 10\text{H}_2\text{O} = \frac{106}{286} = 0.371$$

$$\text{Weight fraction of } \text{H}_2\text{O} \text{ in } \text{Na}_2\text{CO}_3 \cdot 10\text{H}_2\text{O} = 1 - 0.371 = 0.629$$

Balance for H_2O :

$$1000 \times 0.8 = 0.979M + 0.629C \quad (1.11)$$

Overall balance:

$$1000 = M + C \quad (1.12)$$

Solving Eqn.(1.11) and (1.12) simultaneously, $M = 488.6 \text{ kg/hr}$; and $C = 511.4 \text{ kg/hr}$

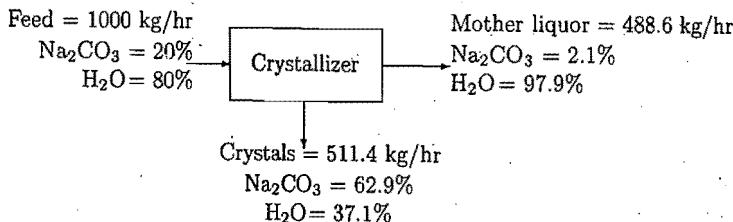
Recovery of Na_2CO_3 :

$$\text{Na}_2\text{CO}_3 \text{ in feed} = 1000 \times 0.2 = 200 \text{ kg}$$

$$\text{Na}_2\text{CO}_3 \text{ in crystals} = 511.4 \times 0.371 = 189.73 \text{ kg}$$

$$\% \text{ of Na}_2\text{CO}_3 \text{ recovered in the form of crystals} = \frac{189.73}{200} \times 100 = 94.87\%$$

Block Diagram:



1.3.16 Heat Load on Heater

1000 kg/hr of a thermic fluid, to be used as a heat transfer medium, is being indirectly heated in a heater, from 380 K to 550 K. Calculate the heat load on the heater, in kW. Also estimate the mean heat capacity of the thermic fluid over the temperature range of interest. The heat capacity equation for the thermic fluid is:

$$C_p = 1.436 + 0.00218T \text{ where } C_p \text{ is in kJ/kg.K, and } T \text{ is in K.} \quad (\text{GATE-1998-13})$$

Solution:

Basis: 1000 kg/hr of thermic fluid.

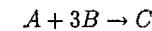
$$\begin{aligned} \text{Heat load} &= m \int_{380}^{550} C_p dT \\ &= 1000 \times \int_{380}^{550} (1.436 + 0.00218T) dT \\ &= 1000 \times 1.436 \times (550 - 380) + 1000 \times 0.00218 \times \left(\frac{550^2}{2} - \frac{380^2}{2} \right) \\ &= 416449 \text{ kJ/hr} \\ &= 115.68 \text{ kW} \end{aligned}$$

PROCESS CALCULATIONS

$$\begin{aligned} C_{p,\text{mean}} &= \frac{\int_{380}^{550} C_p dT}{550 - 380} \\ &= \frac{1.436 \times (550 - 380)}{550 - 380} + 0.00218 \times \frac{1}{2} \times \frac{550^2 - 380^2}{550 - 380} \\ &= 1.436 + 1.0137 \\ &= 2.4497 \text{ kJ/kg.K} \end{aligned}$$

1.3.17 Heat of Reaction

The heat of reaction at 300 K and at one atmosphere pressure for the following gas phase reaction:



is $-50,000$ calories per mole of A converted. Data on the molar heat capacity at constant pressure (cal/mol.K) of the various components are:

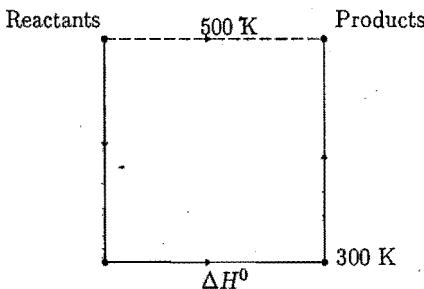
$$C_p \text{ for } A = -0.4 + 80 \times 10^{-3}T, \quad T \text{ in K}$$

$$C_p \text{ for } B = 7$$

$$C_p \text{ for } C = 26$$

Calculate the heat of reaction at 500 K and at one atmosphere pressure. (GATE-1994-8)

Solution:



Reference $T = 300 \text{ K}$. Reactant temperature = 500 K.

$$\begin{aligned} \text{Heat of reaction at 500 K} &= \text{Heat of reaction at 300 K} \\ &\quad + \text{Enthalpy of products} - \text{Enthalpy of reactants} \\ &= -50000 + 26 \times (500 - 300) - 3 \times 7 \times (500 - 300) \\ &\quad - [(-4) \times (500 - 300) + (80 \times 10^{-3}) \int_{300}^{500} T dT] \\ &= -50000 + 26 \times 200 - 21 \times 200 + 0.4 \times 200 \\ &\quad - (0.08/2) \times (500^2 - 300^2) \\ &= -55320 \text{ calorie per mole of } A \text{ converted.} \end{aligned}$$

1.3.18 Amount of Heat Required

Pure CO is mixed with 100% excess air and burnt. Only 80% of CO burns. The reactants are at 100°C and the products are at 300°C. Calculate the amount of heat added or removed per kmol of CO fed to the reactor.

Data: Mean molal specific heats between 25°C and T°C (given below in kJ/mol.K) are:

Gas	T = 100°C	T = 300°C
CO	29.22	30.61
CO ₂	-	43.77
O ₂	29.64	43.77
N ₂	29.17	29.66

Standard heat of formation at 25°C in kJ/kmol are:

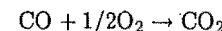
$$\begin{aligned} \text{CO} & -110524 \\ \text{CO}_2 & -393514 \end{aligned}$$

(GATE-1995-12)

Solution:

Basis: 1 kmol of CO in feed.

Reaction:



$$\text{O}_2 \text{ in feed} = 1 \text{ mol}$$

$$\text{N}_2 \text{ in feed} = \frac{79}{21} \text{ mol}$$

$$\text{For complete conversion 1 mol of CO requires} = 0.5 \text{ mol of O}_2$$

$$\text{unreacted O}_2 = 1 - 0.5 \times 0.8 = 0.6 \text{ mol}$$

Standard heat of reaction at 25°C:

$$\Delta H_R^o = (-393514 + 110524) \times 0.8 = -226392 \text{ kJ}$$

Heat content of reactants with reference to 25°C:

$$\begin{aligned} \Delta H_r &= \left[29.22 + 29.64 + \frac{79}{21} \times 29.17 \right] \times (100 - 25) \\ &= 12645 \text{ kJ} \end{aligned}$$

Heat content of products with reference to 25°C:

$$\begin{aligned} \Delta H_p &= \left[0.2 \times 30.61 + 0.8 \times 43.77 + 0.6 \times 30.99 + \frac{79}{21} \times 29.66 \right] \times (300 - 25) \\ &= 47110 \text{ kJ} \end{aligned}$$

PROCESS CALCULATIONS

Heat of reaction at 300°C = Heat content of products - Heat content of reactants
+ Standard heat of reaction

$$\begin{aligned} \text{i.e., } \Delta H_R &= \Delta H_p - \Delta H_r + \Delta H_R^o \\ &= 47110 - 12645 - 226392 \\ &= -191927 \text{ kJ} \end{aligned}$$

Heat released during the reaction = 191927 kJ.

1.3.19 Flame Temperature and Excess Air

Dry methane is burned with dry air. Both are initially at 300 K. The theoretical flame temperature is 1570 K. If complete combustion is assumed, how much excess air was used?

Data: Mean molar specific heat values in J/mol.K are:

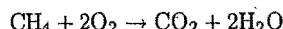
$$\text{CO}_2 = 51.8, \text{H}_2\text{O} = 40.2, \text{N}_2 = 32.2 \text{ and O}_2 = 32.4$$

$$\text{Standard heat of formation at } 300 \text{ K} = -8 \times 10^5 \text{ J/mol of CH}_4$$

(GATE-1988-11ii)

Solution:

Reaction:



Energy balance for adiabatic reaction:

Heat content of products = Heat content of reactants + Heat released from reaction

$$H_P = H_F + H_R$$

If reference temperature is chosen as 300 K, then $H_F = 0$.

If complete combustion is assumed, then the product gases will not contain CH₄. But it is possible to have N₂ and O₂ in the product gases. Let x be the moles of O₂ in the product gases. Then heat balance becomes,

$$(51.8 + 2 \times 40.2 + x \times 32.4 + x \times \underbrace{\frac{79}{21} \times 32.2}_{\text{due to N}_2} + 2 \times \frac{79}{21} \times 32.2)(1570 - 300) = 8 \times 10^5$$

Solving, $x = 1.664 \text{ mol}$

$$\text{excess O}_2 = 1.664 \text{ mol}$$

$$\text{O}_2 \text{ used in reaction} = 2 \text{ mol}$$

$$\% \text{ excess air} = \frac{1.669}{2} \times 100 = 83.2\%$$

1.3.20 Heat Released from Reaction

The reaction, $A(g) + 2B(g) \rightarrow 2C(g) + 3D(g)$, with a standard heat of reaction $\Delta H_R^o = -10^3 \text{ kJ/mol of A reacted}$, is carried out in a reactor in which the feed enters at 25°C and

the product stream emerges as a gas at 300°C. The feed consists of 100 mol A/hr and 250 mol B/hr. A is completely consumed during the reaction. Calculate the heat transferred to or from the reactor, assuming the operation at approximately 1 atm.

Data: Specific enthalpies at 300°C

(Datum: Specific enthalpies of all components at 25°C = 0 kJ/mol).

Component Sp.enthalpy (kJ/mol)

A	9
B	10
C	12
D	15

(GATE-1987-11iii)

Solution:

Basis: 100 mol of A as feed

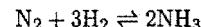
For complete conversion of A, and for the reaction $A(g) + 2B(g) \rightarrow 2C(g) + 3D(g)$, the product contains: C: 200 mol; D: 300 mol; B: 50 mol. Energy balance: Reference temperature = 25°C

$$\begin{aligned} \text{Heat of reaction at } 300^\circ\text{C} &= \text{Enthalpy of products} - \text{Enthalpy of reactants} + \\ &\quad \text{Std. heat of reaction} \\ &= (200 \times 12 + 300 \times 15 + 50 \times 10) - 0 - 100 \times 10^3 \\ &= -92600 \text{ kJ/hr} \end{aligned}$$

Heat transferred from the reactor = 92600 kJ/hr.

1.3.21 Conversion in Exothermic Reaction

Ammonia is produced by the following reaction



In a commercial process for ammonia production, the feed to an adiabatic reactor contains 1 kmol/sec of nitrogen and stoichiometric amount of hydrogen at 700 K. What is the maximum allowable conversion in the reactor, if the adiabatic temperature rise across the reactor should not exceed 100 K. Assume the feed and product streams to be ideal gas mixtures. The heat of reaction at 700 K for the above reaction is calculated to be -94.2 kJ/mol. Mean molar heat capacities (C_p), in the range 700–800 K, are 0.03, 0.0289 and 0.0492 kJ/mol.K for nitrogen, hydrogen and ammonia respectively. (GATE-2002-13)

Solution:

Energy balance:

$$\begin{aligned} \text{enthalpy of reactants} &+ \text{heat added by reaction} \\ &= \text{enthalpy of product stream} \end{aligned}$$

Also,

$$\begin{aligned} \text{Heat of reaction at } 700 \text{ K} &= \text{Enthalpy of products with reference to } 700 \text{ K} \\ &\quad - \text{Enthalpy of reactants with reference to } 700 \text{ K} \end{aligned}$$

PROCESS CALCULATIONS

If reference temperature is taken as 700 K then, enthalpy of reactants = 0. Therefore,
heat added by reaction = enthalpy of product stream

Let x be the fractional conversion per mol of N₂ entering the reactor. Then product stream contains: $2x$ moles of NH₃; $(1-x)$ moles of N₂; and $3(1-x)$ moles of H₂. Therefore, the heat balance ($\Delta H = \Sigma n C_p \Delta T$) becomes

$$94.2 \times 2x = [2x \times 0.0492 + (1-x) \times 0.03 + 3(1-x) \times 0.0289] \times 100$$

Solving, $x = 0.061$. Therefore, the maximum allowable conversion of N₂ in the reactor is 6.1%.

1.3.22 Maintaining Isothermal Operation of Reactor

A feed at 1298 K, consisting of flue gas (CO₂, O₂ and N₂) and air, is passed through a bed of pure carbon. The reactions that occur both go to completion.



The combustor is adiabatic and the product gases exit at 1298 K. Calculate the required moles of CO₂ per mole of O₂ in the feed stream, so that the net heat generated is zero and the bed temperature remains constant at 1298 K.

Data: Mean molar heat capacities, C_{pm}

Substance	C_{pm} , kJ/mol.K
C	0.02
O ₂	0.03
CO	0.03
CO ₂	0.05

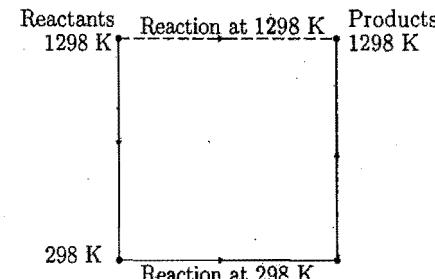
(GATE-1997-13)

Solution:

Reference T = 298 K. Feed & Product temperature = 1298 K

Let x be the moles of CO₂ per mole of O₂ in the feed stream. Then For the first reaction feed will be: x mole of C and x mole of CO₂; and product will be: $2x$ mole of CO.

For the second reaction, feed will be: 2 mole of C and 2 mole of O₂; and product will be: 1 mole of CO.



$$\begin{aligned} \text{Heat of reaction at } 1298 \text{ K} &= \text{Heat of reaction at } 298 \text{ K} \\ &\quad + \text{Enthalpy of products} - \text{Enthalpy of reactants} \end{aligned}$$

If heat generated is to be zero, heat of reaction at 1298 K should be zero. And since N₂ doesn't take part in reaction (also the amount of N₂ in outlet is equal to that in inlet),

$$\begin{aligned} 0 &= [x \times 2 \times 170 + 2 \times (-220.4)] \\ &\quad + [(x \times 2 \times 0.03 + 2 \times 0.03) \\ &\quad \quad -(x \times 0.05 + x \times 0.02 + 0.03 + 2 \times 0.02)] \times (1298 - 298) \end{aligned}$$

Solving, $x = 1.366$

Chapter 2

Fluid Mechanics

2.1 Instant Notes

2.1.1 Fluid Statics

- Manometer principle: pressure at the same level in a continuous body of fluid are equal.
- Buoyant force = weight of fluid displaced.

2.1.2 Fluid Flow

- At room temperature kinematic viscosity is 1 centiStoke for water and 15 centiStoke for air at atmospheric pressure.
- An ideal fluid has a viscosity of zero, and incompressible one. The flow of such an ideal fluid is called potential flow. Potential flow has two important characteristics: (i) Neither circulations nor eddies can form within the stream, so that potential flow is also called as irrotational flow, and (ii) friction cannot develop, so that there is no dissipation of mechanical energy into heat.
- Newtonian and Non-Newtonian fluids
 - Newtonian fluids: the slope of shear stress (τ) / strain rate (du/dy) = μ is a constant regardless of the strain rate. Eg: gases, water, liquid hydrocarbons, and most organic liquids of relatively low molecular weight.
 - Pseudoplastic (or shear-thinning) fluids: exhibit a viscosity that decreases with increasing strain rate. Eg: most polymers solutions and polymer melts.
 - Dilatant (or shear-thickening) fluids exhibit a viscosity that increases with increasing strain rate. They are less common than pseudoplastic fluids. Eg: sand or corn-starch suspension in water.
 - Bingham plastic: a minimum stress is required before the fluid flows. Once the fluid flows, the viscosity is constant. Eg: ketchup, toothpaste.

- Time-dependent behaviors:
 - * Thixotropic: viscosity decreases with time at constant strain rate. Eg: paint
 - * Rheopectic: viscosity increases with time at a constant strain rate.
- Visco-elastic: exhibit characteristics common to both viscous fluids and elastic solids. Eg: egg-white.
- Reynolds number = $\frac{\text{Inertial force}}{\text{Viscous force}}$
- For circular tubes the flow is laminar when Re is less than about 2100, although a stable sinuous motion sets in at a Reynolds number of about 1225. Above 2100 laminar motion may be maintained temporarily if the tubes are very smooth and free from vibrations; but if the system is disturbed or if there is any appreciable surface roughness the laminar motion will give way to the random motion that characterizes turbulent flow.
- The curve of v versus y (v = velocity; y = distance from the wall) is referred to as the *velocity distribution or profile*. At a sufficiently large distance from the tube entrance, the velocity profile assumes a constant shape; the flow is said to be fully developed. The velocity distribution for developed flow depends on whether the flow is laminar or turbulent.
- For ordinary laminar flow there is a parabolic variation of velocity from zero at the wall to v_{\max} at the center of the tube. For pseudo plastic fluids, a relatively flat profile is obtained, and in the limit of an infinitely pseudoplastic fluid there is rodlike flow. For dilatant fluids, the profile is sharper, and for the limiting case of an infinitely dilatant fluid, the velocity is a linear function of the radius. The velocity profile is conical.
- A fundamental principle of fluid mechanics, originally stated by Prandtl in 1904, is that, except for fluids moving at low velocities or possessing high viscosities, the effect of the solid boundary on the flow is confined to a layer of fluid immediately adjacent to the solid wall. This layer is called the boundary layer, and shear forces are confined to this part of the fluid.
- Boundary layer

- The boundary layer is the region near a solid where the fluid motion is affected by the solid boundary. Outside the boundary layer potential flow survives.
- If a fluid flowing with a uniform velocity enters a conduit, a boundary layer builds up and eventually fills the pipe. Thus, in fully developed laminar or turbulent flow, the entire radius of the pipe is in the boundary layer.
- If the boundary layer is turbulent and fills the pipe, as it usually does except very near the entrance to the pipe, a viscous sublayer persists near the walls.
- Boundary layer separation: Necessary condition $dP/dx > 0$, i.e., when pressure energy P increases in the direction of flow. This is the situation normally encountered in flow of fluid in the diverging section of venturi, if the angle of diverging section is more ($> 12^\circ$).

FLUID MECHANICS

- The thickness of the boundary layer varies with the distance from the leading edge as:

$$\frac{\delta}{x} = \frac{4.79}{\sqrt{Re_x}} \quad \text{for laminar flow}$$

$$\frac{\delta}{x} = \frac{0.376}{Re_x^{1/5}} \quad \text{for turbulent flow}$$

The transition from laminar to turbulent flow occurs in the boundary layer at $Re_x = 3.2 \times 10^5$.

• Friction factor

- Fanning Friction factor:

$$f = \frac{\tau_w}{\rho v^2 / 2}$$

- *Blasius or Darcy friction factor* is also commonly encountered in fluid mechanics literature, and is 4 times the Fanning friction factor. Care should be taken in reading friction factor charts, that whether the friction factor is Darcy or Fanning.

- Relation between Pressure drop due to skin friction (ΔP_s) and wall shear stress (τ_w) for flow through circular pipe:

$$\Delta P_s = \frac{4\tau_w L}{D}$$

- Skin friction and friction factor are related as:

$$\Delta P_s = \frac{2\mu f L v^2}{D}$$

- Laminar flow: Average velocity is precisely one-half the maximum velocity ($v_{\text{avg}} = v_{\max}/2$). Relation for velocity profile:

$$v_r = v_{\max} \left[1 - \left(\frac{r}{R} \right)^2 \right]$$

where v_{\max} is the centerline velocity.

- Hagen Poiseuille equation: for laminar flow the following relation is valid:

$$\Delta P_s = \frac{32L v \mu}{D^2}$$

One of its use is in the experimental measurement of viscosity; by measuring the pressure drop and volumetric flow rate through a tube of known length and diameter.

- Velocity profile in turbulent flow: In pipe flow, the following equation is valid up to $Re = 10^5$.

$$\frac{v_r}{v_c} = \left(\frac{y}{r} \right)^{1/7}$$

where v_x = velocity at a distance y from the walls

v_c = velocity at the centerline of pipe

r = radius of the pipe

This equation is referred to as the Prandtl one-seventh power law. It is also sometimes called as Blasius 1/7 power law. By using Prandtl one-seventh power law, the mean velocity of flow is found to be equal to 0.817 times the centerline velocity

- In turbulent flow a rough pipe leads to a larger friction factor for a given Reynolds number than a smooth pipe does. If a pipe is smoothed, the friction factor is reduced. When further smoothing brings about no reduction in friction factor for a given Reynolds number, the pipe is said to be hydraulically smooth.

• Roughness of pipes

- The relative roughness (k/D) affects the flow in several ways. In laminar flow through commercial pipe, in which k/D is usually somewhat less than 0.01, the effect of the wall roughness is negligible. In turbulent flow the wall roughness also has no effect if it is smaller in height than the thickness of the viscous sub-layer. In this case the pipe is said to be *hydraulically smooth*.
- However, if the irregularities enter into the main fluid stream, they increase the turbulence, change the velocity profile, and increase the flow resistance. Beyond a certain value of k , the effect of the roughness is so great that the inertia forces caused by the fluid flowing around the projections completely outweigh the viscous forces. In these circumstances the pipe is said to be relatively rough.
- The thickness of the viscous sub-layer is a function of the Reynolds number, so that the same pipe may be hydraulically smooth at one flow rate and completely rough at another.
- In laminar flow f is inversely proportional to the velocity. In completely rough turbulent flow, f is not a function of velocity.
- **Equivalent Diameter:** Friction factor versus Reynolds number chart of circular pipes can be used for turbulent flow through non-circular cross sections by replacing the diameter D by the equivalent diameter D_{eq} defined by the relation,

$$D_{eq} = 4r_H$$

In this relation r_H is the hydraulic radius, in turn defined by the equation,

$$r_H = \frac{\text{Flow cross sectional area}}{\text{Wetted perimeter of conduit}}$$

For a circular pipe it may be verified that this definition leads to the result that $D = D_{eq}$. The hydraulic radius can also be used with the friction factor charts for flow in open channels or partially filled pipes.

- Frictional loss coefficient for sudden expansion of cross section (K_e):

$$K_e = (1 - S_a/S_b)^2$$

Where S_a and S_b are the cross sectional areas upstream and downstream conduits, respectively.

- Frictional loss coefficient for sudden contraction of cross section (K_c):

$$K_c = 0.4(1 - S_b/S_a)$$

2.1.3 Flow Meters

• Venturi meter:

- The shape of the converging and diverging sections of venturi meter minimizes losses by eddy formation. Experimental measurements show that for $Re > 10000$ the frictional loss over the venturi meter is about 10 percent of ΔP . Where ΔP is pressure drop measured by the manometer connected between the upstream and throat sections of venturi meter. (Re is calculated based on the pipe diameter)
- For $Re > 10000$, C_d of venturi meter is constant at about 0.98. For smaller Reynolds numbers the coefficient decreases rapidly. This effect is partially caused by the non-uniform velocity distribution across the diameter in laminar flow (in the length of venturi meter). (Re is calculated based on the pipe diameter)

• Orifice meter

- The orifice plate can easily be changed to accommodate widely different flow rates, whereas the throat diameter of a venturi meter is fixed.
- The orifice meter has a large permanent loss of pressure because of the presence of eddies on the downstream side of the orifice plate. The shape of the venturi meter prevents the formation of these eddies and greatly reduces the permanent loss.
- There are a number of customary positions of the pressure taps which lead to the manometer such as pipe taps, flange taps and vena contracta taps out of which, flange tapes are the most common.
- The coefficient of discharge of orifice, C_d is dependent on the location of the pressure taps.
- For $Re > 10000$, C_d of orifice meter is constant at 0.61. At lower Reynolds numbers the orifice coefficient becomes a strong function of Re . (Re is calculated based on the diameter of orifice opening)
- The orifice discharge coefficient is significantly affected by flow disturbances which originates in valves, bends, and other fittings located upstream from the orifice. It is less affected by downstream disturbances. As a general rule, the meter should be placed 50 pipe diameters downstream and 10 pipe diameters upstream from any disturbances. The upstream distance can often be reduced by placing straightening vanes in the pipe.

- Rotameter: In the case of venturi and orifice meters the area of constriction remains constant and the pressure drop varies with flow rate; in the rotameter, the pressure drop remains nearly constant and the area of constriction varies.

2.1.4 Transportation of Fluids

- Check Valves are used when unidirectional flow is desired. They are automatic in operation and prevent flow in one direction but allow it in the other.
- Globe valves: The essential feature of these valves is a globular body with a horizontal internal partition having a circular passageway. The globe valve is ordinarily used in smaller sizes. It is generally considered a poor practice to use a globe valve in a size larger than 2 inch. Globe valves are widely used for controlling flow. The fluid passes through a restricted opening and changes direction several times. As a result the pressure drop in this kind of valve is large.
- Gate valves are universally used in larger sizes. Gate valves are not recommended for controlling flow and are usually left fully open or fully closed.
- Pipes

- Pipe is completely specified by giving its nominal diameter and schedule number. Lower the schedule numbers lower is the thickness. The commonly available schedule numbers are 10, 40, 80 and 120. The schedule 10 is mostly for stainless steel pipes. Nominal diameter is close to the actual outside diameter. Actual outside diameter of pipe is more than the nominal diameter up to a nominal diameter of 12 inch. Above 12 inch, outer diameter is equal to the nominal diameter.
- Regardless of wall thickness, the outside diameter of all pipe of a given nominal pipe size is the same, to ensure the interchangeability of the fittings.
- Tube is sold on the basis of actual outside diameter and wall thickness. The wall thickness of copper and brass tubing is often expressed in terms of Birmingham wire gauge(BWG). Higher the BWG lower is the thickness. BWG ranges from 24 (very light) to 7 (very heavy). The common BWG are 12, 14, 16 and 18.
- Pipes are rougher than tubes and produce more turbulence for equal Reynolds numbers.

Pumps

- Positive suction head: when it is required to pump boiling liquids or liquids at their saturation temperatures, a positive head at the eye of pump is essential, which means that the pump must be installed below the level of the liquid in the vessel from which it draws its supply.
- Priming: Before a centrifugal pump or similar type of non-positive displacement pump is started up, the pump casing and intake piping must be completely filled with water or whatever liquid is being handled, with the discharge valve being closed. This operation is known as priming. The objective of priming is to release the vapor held in the casing of the pump.

FLUID MECHANICS

2.1.5 Flow Past Immersed Bodies

- Drag: The force in the direction of flow exerted by the fluid on the solid is called drag.
- Drag coefficient:

$$C_D = \frac{F_D/A_p}{\rho u_0^2/2}$$

- Stokes law: $C_D = 24/Re_p$. Stokes law is valid only when $N_{Re,p}$ is considerably less than unity. The type of flow treated in this law is called creeping flow.
- For $1,000 < Re_p < 200,000$, the drag coefficient is approximately constant, $C_D \approx 0.4$.
- Fluidization:

- At the point of incipient fluidization, the pressure drop just suffices to support the downward weights per unit area of the bed. The void fraction at this point ranges between 0.4 and 0.5.
- There are two distinct modes of fluidization, depending mainly on the relative values of ρ_s , the density of solid particles, and ρ , the density of the fluid.
- Particulate fluidization: If ρ_s and ρ are of the same order of magnitude, the bed is usually a homogeneous mixture of fluid and particles. This situation occurs for liquid fluidized particles.
- Aggregative fluidization: When the density of fluid is considerably less than that of the particles, the bed is found to consist of two distinct phases: a particulate phase or emulsion, and a bubble phase. Aggregative fluidization occurs mainly in gas-fluidized beds; finds importance in catalytic reactors.

- Flow through bed of solids:

- Laminar flow ($Re_p = D_p v \rho / \mu < 1$) through bed of solids:

$$\frac{\Delta P}{\rho v^2} \frac{D_p}{L} \frac{\epsilon^3}{1-\epsilon} = \frac{150}{Re} \quad (\text{Blake-Kozeny equation})$$

where $Re = \frac{D_p v \rho}{\mu(1-\epsilon)}$

- Turbulent flow ($Re_p = D_p v \rho / \mu > 1000$) through bed of solids:

$$\frac{\Delta P}{\rho v^2} \frac{D_p}{L} \frac{\epsilon^3}{1-\epsilon} = 1.75 \quad (\text{Burke-Plummer equation})$$

- Ergun equation

$$\frac{\Delta P}{\rho v^2} \frac{D_p}{L} \frac{\epsilon^3}{1-\epsilon} = \frac{150}{Re} + 1.75 \quad (\text{applicable for all the regions})$$

2.2 Objective Type Questions

1. A plant has a water tank mounted on the top of a 27 m platform. The tank is 10 m high. The height of water in the tank if a pressure gauge on the second floor at a height of 5 m from the ground reads 2.7 bar is: (GATE-1991-9.ii)

(a) Full (b) 5.12 m (c) 3.12 m (d) 7.18 m

Answer: (b) 1 Bar = 10.19 m of water column. Therefore, 2.7 Bar \equiv 27.5 m. Let x be the height of water in the tank. The distance between pressure gauge and the platform is 22 m. Therefore, $22 + x = 27.5$ m. Hence, $x = 5.5$ m.

2. Bernoulli's equation for steady frictionless flow states that, along a streamline: (GATE-1990-2.ii)

(a) total pressure is constant
 (b) total mechanical energy is constant
 (c) velocity head is constant
 (d) none of the above

Answer: (b)

3. The condition that is not necessary for the applicability of Bernoulli equation is: (GATE-1989-2.i.b)

(a) steady state (b) incompressible (c) inviscid (d) irrotational

Answer: (d) The other conditions are necessary. Liquids are assumed to be incompressible. Inviscid refers to zero viscosity fluid.

4. A pipe of I.D. 4 m is bifurcated into two pipes of I.D. 2 m each. If the average velocity of water flowing through the main pipe is 5 m/s, the average velocity through the bifurcated pipes is (GATE-1999-1.09)

(a) 20 m/s (b) 10 m/s (c) 7.07 m/s (d) 5 m/s

Answer: (b) From equation of continuity, $\dot{m}_{in} = \dot{m}_{out}$. i.e., $\rho_1 A_1 v_1 = \rho_2 A_2 v_2$. For incompressible fluids $\rho_1 = \rho_2$. Therefore, $A_1 v_1 = A_2 v_2$. $4^2 \times 5 = 2(2^2 \times v)$

5. For flow through a horizontal system, the ratio of exit pressure to inlet pressure is found to be greater than one. (GATE-1988-2.iv)

(a) The system is a converging pipeline
 (b) The system is a diverging pipeline
 (c) The system is a pipeline of uniform diameter
 (d) The observation is not possible

Answer: (b) From Bernoulli equation (neglecting frictional losses),

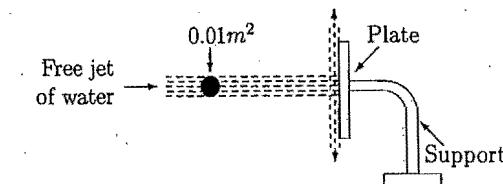
$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} = \frac{P_2}{\rho g} + \frac{v_2^2}{2g}$$

where 1 refers to inlet and 2 refers to outlet. Rearranging the above equation

$$P_2 - P_1 = \frac{\rho}{2}(v_1^2 - v_2^2)$$

Given: $P_2/P_1 > 1$, i.e., $P_2 - P_1 > 0$. This is possible if $v_1^2 - v_2^2 > 0$; i.e., if $v_2 < v_1$. This is the case with steady flow through a diverging section.

6. A free jet of water of cross-sectional area 0.01 m^2 and a velocity of 20 m/s strikes a plate and then flows in the plane parallel to the plate as shown in the figure. The horizontal component of the force on the support is (GATE-2000-2.10)



(a) 200 N (b) 400 N (c) 2000 N (d) 4000 N

Answer: (d) Force = Rate of transfer of momentum; Momentum in = $\dot{m}v$
 Mass flow rate $\dot{m} = 0.01 \times 20 \times 1000 = 200 \text{ kg/sec}$
 Therefore, momentum in = $200 \times 20 = 4000 \text{ kg.m/sec}^2 = 4000 \text{ N}$.

7. The shear stress-shear rate relationship for a liquid whose apparent viscosity decreases with increasing shear rate is given by (GATE-1994-1.j)

- (a) $\tau_{yx} = -m \left[\frac{dv_x}{dy} \right]^{n-1} \frac{dv_x}{dy}$ for $n < 1$
 (b) $\tau_{yx} = -m \left[\frac{dv_x}{dy} \right]^n \frac{dv_x}{dy}$ for $n = 1$
 (c) $\tau_{yx} = -m \left[\frac{dv_x}{dy} \right]^{n-1} \frac{dv_x}{dy}$ for $n > 1$
 (d) $\tau_{yx} = -m \frac{dv_x}{dy} + \tau_o$

Answer: (a) Pseudoplastic fluids have this behaviour. Slope of τ vs. shear rate is the apparent viscosity.

8. For an ideal fluid flow the Reynolds number is (GATE-1988-2.i)

(a) infinity (b) zero (c) one (d) 2100

Answer: (a) $Re = Dv\rho/\mu$. For ideal fluid $\mu = 0$. Therefore $Re = \infty$.

9. Toothpaste is (GATE-1996-1.04)

- (a) Bingham plastic (b) Pseudo plastic
(c) Newtonian fluid (d) Dilatant

Answer: (a)

10. For a dilatant fluid, the magnitude of the slope of the shear stress versus the velocity gradient curve —— with increasing velocity gradient. (GATE-1989-2.i.c)

- (a) increases (b) decreases
(c) remains unchanged (d) may decrease or increase

Answer: (a) For pseudoplastic fluids slope decreases, and for Newtonian fluids slope remains at a constant value.

11. For pseudoplastic fluids, increase in shear rate (GATE-1988-2.ii)

- (a) increases the apparent viscosity
(b) decreases the apparent viscosity
(c) has no effect on apparent viscosity
(d) has unspecified effect

Answer: (b) Apparent viscosity increases for dilatant fluids. For Newtonian fluids, apparent viscosity remains constant.

12. The velocity profile for a Bingham plastic fluid flowing (under laminar conditions) in a pipe is

- (a) parabolic
(b) flat
(c) flat near the wall and parabolic in the middle
(d) parabolic near the wall and flat in the middle

Answer: (d)

13. A Bingham fluid of viscosity $\mu = 10$ Pa.s, and yield stress $\tau_0 = 10$ kPa, is sheared between flat parallel plates separated by a distance 10^{-3} m. The top plate is moving with a velocity of 1 m/s. The shear stress on the plate is (GATE-2001-2.06)

- (a) 10 kPa (b) 20 kPa (c) 30 kPa (d) 40 kPa

Answer: (b) $\tau = \tau_0 + \mu dv/dy = 10 + 10 \times 1/1 = 20$ kPa.

14. A Newtonian liquid (ρ = density, μ = viscosity) is flowing with velocity v in a tube of diameter D . Let ΔP be the pressure drop across the length L . For a laminar flow, ΔP is proportional to (GATE-1994-1.k)

FLUID MECHANICS

- (a) $L\rho v^2/D$ (b) $D\rho v^2/L$ (c) $L\mu v/D^2$ (d) $\mu v/L$

Answer: (c) $\Delta P = \frac{2fL\rho v^2}{D}$. For laminar flow $f = \frac{16}{Re} = \frac{16\mu}{Dv\rho}$. Therefore, $\Delta P = \frac{32\mu Lv}{D^2}$

15. Consider a duct of square cross section of side ' b '. The hydraulic radius is given (GATE-1988-2.vi)

- (a) $b/8$ (b) $b/4$ (c) $b/2$ (d) b

Answer: (b) Hydraulic radius = $\frac{\text{flow cross sectional area}}{\text{wetted perimeter}} = \frac{b^2}{4b} = \frac{b}{4}$.
Equivalent diameter = $4 \times \text{hydraulic radius}$.

16. The hydraulic radius for flow in a rectangular duct of cross-sectional dimensions W is (GATE-1998-1.1)

- (a) $(HW/\pi)^{0.5}$ (b) $HW/[2(H+W)]$
(c) $(HW)^2/[4(H+W)^2]$ (d) $2HW/(H+W)$

Answer: (b)

17. The hydraulic diameter of an annulus of inner and outer radii R_i and R_o respectively is (GATE-1997-1.0)

- (a) $4(R_o - R_i)$ (b) $\sqrt{R_o \cdot R_i}$ (c) $2(R_o - R_i)$ (d) $R_o + R_i$

Answer: (c) Hydraulic diameter = $4r_H = \frac{4\pi(R_o^2 - R_i^2)}{2\pi(R_o + R_i)} = 2(R_o - R_i)$

18. For the turbulent flow through a smooth pipe, the following correlation for the friction factor is valid: $f = cRe^{-0.2}$; where c is a constant. Suppose that the velocity is increased by 100% the pressure drop: (GATE-1989-2.i)

- (a) increases by less than 100% (b) decreases by less than 100%
(c) increases by more than 100% (d) decreases by more than 100%

Answer: (c) $f = cRe^{-0.2} = c(Dv\rho/\mu)^{-0.2}$; $\Delta P = 2fL\rho v^2/D = c_1 v^{1.8}$

19. For the laminar flow of a fluid in a circular pipe of radius R , the Hagen-Poiseuille equation predicts that volumetric flowrate to be proportional to (GATE-1997-1.0)

- (a) R (b) R^2 (c) R^4 (d) $R^{0.5}$

Answer: (c) $\Delta P = \frac{2fL\rho v^2}{D}$; $f = 16/Re$. Therefore, $\Delta P = \frac{32L\mu v}{D^2}$; and hence $v \propto \frac{\Delta P D^2}{32L\mu}$. $Q = Av = \pi/4 D^2 v = \frac{\pi}{4} \frac{\Delta P D^4}{32L\mu} = \frac{\pi}{8} \frac{\Delta P R^4}{L\mu}$

20. Applying pressure drop across a capillary results in a volumetric flow rate Q under laminar flow conditions. The flow rate, for the same pressure drop, in a capillary of the same length but half the radius is (GATE-2001-1.08)

(a) $Q/2$ (b) $Q/4$ (c) $Q/8$ (d) $Q/16$

Answer: (d) $\Delta P = \frac{2fL\rho v^2}{D} = \frac{32L\mu v}{D^2} = \frac{128L\mu Q}{\pi D^4}$ since $f = 16/\text{Re} = 16\mu/(Dv\rho)$; and $Q = (\pi/4)D^2v$.

Therefore, for a given ΔP , $Q \propto D^4$.

21. In a fully turbulent flow ($\text{Re} > 10^5$) in a pipe of diameter d , for a constant pressure gradient, the dependence of volumetric flow rate of an incompressible fluid is (GATE-2000-1.09)

(a) d (b) d^2 (c) $d^{2.5}$ (d) d^4

Answer: (c) $\Delta P = 2fL\rho v^2/D = CL\rho Q^2/D^5$. (C - a constant, since f is a nearly constant at high Re . And $Q \propto D^2v$). Therefore, $Q \propto D^{2.5}$.

22. For laminar flow of a shear-thinning liquid in a pipe, if the volumetric flow rate is doubled, the pressure gradient will increase by a factor of (GATE-2000-1.11)

(a) 2 (b) < 2 (c) > 2 (d) $1/2$

Answer: (b) $\Delta P = \frac{128L\mu Q}{\pi D^4}$. For shear thinning fluids, μ decreases with increase in velocity (also Q).

23. Boundary layer separation is characterized by one of the following conditions given below, where Re is the Reynold's number for flow. Select the appropriate conditions. (GATE-1998-1.09)

(a) $\text{Re} \ll 1$, accelerating flow (b) $\text{Re} \gg 1$, accelerating flow
 (c) $\text{Re} \ll 1$, decelerating flow (d) $\text{Re} \gg 1$, decelerating flow

Answer: (d) Boundary layer separation occurs only in decelerating turbulent flow.

24. Indicate what will be the manometric reading if the venturi meter is replaced by an orifice meter of the same size:

(a) less than that for venturi meter
 (b) same as that for venturi meter
 (c) higher than that for venturi meter

Answer: (c) $Q \propto C_d A_1 \sqrt{\frac{\Delta P}{\rho}}$. Orifice meter has low C_d compared to venturi meter. $C_d \approx 0.6$ for orifice meter, and 0.98 for venturi meter.

25. The operation of a rotameter is based on (GATE-2001-1.07)

(a) variable flow area (b) rotation of a turbine
 (c) pressure drop across a nozzle (d) pressure at a stagnation point

Answer: (a) The operation of Venturi meter is based on pressure drop across a nozzle; and that of pitot's tube is pressure at a stagnation point.

26. A rotameter, through which air at room temperature and atmospheric pressure is flowing, gives a certain reading for a flow rate of 100 cc/s. If helium (Molecular weight 4) is used and the rotameter shows the same reading, the flow rate is (GATE-1996-2.02)

(a) 26 cc/s (b) 42 cc/s (c) 269 cc/s (d) 325 cc/s

Answer: (c) From Bernoulli's equation, $v \propto \sqrt{\Delta P/\rho}$. For variable area meters, $\Delta P = \text{const}$. Therefore $v \propto \sqrt{1/\rho}$. For gases $\rho \propto M$; where M is the molecular weight. Therefore

$$\frac{v_2}{v_1} = \sqrt{\frac{\rho_1}{\rho_2}} = \sqrt{\frac{29}{4}}$$

i.e., $v_2 = v_1 \sqrt{29/4} = 100 \times \sqrt{29/4} = 269 \text{ cc/s}$.

27. A pitot tube indicates 5 cm of water (manometer) when it is being used for measuring velocity of air. The velocity of air in m/s is

(a) 5 (b) 14.1 (c) 56.22 (d) 28.2

Answer: (d) $v = \sqrt{2gh}$ where h is the head of flowing fluid (i.e., air). $(\rho h)_{\text{air}} = (\rho h)_{\text{water}}$. Density of air $\approx 1.5 \text{ kg/m}^3$. Therefore, $h_{\text{air}} = 0.05 \times 1000/1.5 = 33.3 \text{ m}$. Hence $v = \sqrt{2 \times 9.812 \times 33.3} = 25.6 \text{ m/s}$. (which is the nearest answer)

28. Two iron pipes of the same nominal diameter but different schedule numbers will have (GATE-1997-1.28)

(a) the same inside diameter (b) the same outside diameter
 (c) the same wall thickness (d) none of the above

Answer: (b) According to the schedule number thickness of pipes of a given nominal dia, varies. Higher the schdule number higher the thickness for a given nominal diameter.

29. For a centrifugal pump the net positive suction head is defined as (GATE-1993-9.a)

(a) $h_{vs} + h_{ps}$ (b) $h_{vd} + h_{pd}$
 (c) $h_{vs} + h_{ps} - p_s$ (d) $h_{vd} + h_{pd} - p_d$

where h_{vs} = velocity head at suction;
 h_{vd} = velocity head at discharge;
 h_{ps} = pressure head at suction;
 h_{pd} = pressure head at discharge;
 p_s = vapor pressure of liquid at suction temperature;
 p_d = vapor pressure of liquid at discharge temperature.

Answer: (c) NPSH = pressure head at suction – vapor pressure of liquid at suction temperature.

30. A globe valve is most suitable for applications in which (GATE-1998-1.10)

- (a) the valve is required to be either fully open or fully closed
- (b) flow control is required
- (c) the fluid contains dispersed particles
- (d) one-way flow is required

Answer: (b)

31. In centrifugal pumps, cavitation occurs when pressure of the impeller eye or vane becomes (GATE-1999-1.10)

- (a) less than atmospheric pressure
- (b) more than liquid vapor pressure
- (c) less than liquid vapor pressure
- (d) more than atmospheric pressure

Answer: (c) When the surrounding pressure equals (or lesser than) the vapor pressure, the liquid boils; which leads to the formation of two-phase mixture.

32. A spherical particle is falling slowly in a viscous liquid such that Reynolds number is less than 1. Which statement is correct for this situation? (GATE-1992-3.b)

- (a) Inertial and drag forces are important
- (b) Drag, gravitational and buoyancy forces are important
- (c) Drag force and gravitational forces are important
- (d) None of the above

Answer: (b)

33. As the velocity V and thus the Reynolds number of a flow past a sphere increases from very low values, the drag force for $Re \ll 1$ (GATE-1992-3.c)

- (a) Increases linearly with V
- (b) Decreases linearly with V
- (c) Decreases as V^2
- (d) None of these

Answer: (a) $F_D = C_D(\rho u_o^2/2)A_p$ and $C_D = 24\mu/D u_o \rho$ for laminar flow ($Re \ll 1$).

34. A particle attains its terminal settling velocity when (GATE-1995-1.e)

- (a) gravity force + drag force = buoyancy force
- (b) gravity force – drag force = buoyancy force
- (c) buoyancy force + drag force = gravity force
- (d) drag force = buoyancy force

Answer: (c)

35. A bed consists of particles of density 2000 kg/m³. If the height of the bed is 1.5 m and its porosity 0.4, the pressure drop required to fluidize the bed is (GATE-1996-2.05)

- (a) 25.61 kPa
- (b) 11.77 kPa
- (c) 14.86 kPa
- (d) 21.13 kPa

Answer: (b) Pressure drop required = gravity force – buoyancy force
 $= (1 - \epsilon)\rho_s gh - (1 - \epsilon)\rho gh$
where $(1 - \epsilon)$ is the fraction of solid.

36. Hydraulic mean diameter for flow through packed bed of spherical particles of size D_p with porosity ϵ is (GATE-1995-2.1)

- (a) $D_p \frac{1}{6} \frac{\epsilon}{1-\epsilon}$
- (b) $D_p \frac{1}{6} \frac{1-\epsilon}{\epsilon}$
- (c) $D_p \frac{2}{3} \frac{1}{1-\epsilon}$
- (d) $D_p \frac{2}{3} \frac{\epsilon}{1-\epsilon}$

Answer: (a) for porous medium

$$r_H = \frac{\text{volume open to flow}}{\text{total wetted surface}}$$

i.e.,

$$r_H = \frac{\text{volume of bed} \times \epsilon}{\text{No. of spherical particles} \times \text{surface area of one particle}}$$

where

$$\text{No. of particles} = \frac{\text{volume of bed} \times (1 - \epsilon)}{\text{volume of one particle}}$$

Therefore

$$r_H = \frac{\text{volume of bed} \times \epsilon}{\text{volume of bed} \times (1 - \epsilon) \times \frac{\text{surface area}}{\text{volume}}} = \frac{\epsilon}{(1 - \epsilon) \left(\frac{\pi D_p^2}{(\pi/6) D_p^3} \right)} = \frac{D_p}{6} \frac{\epsilon}{1 - \epsilon}$$

37. In fluidization using perfect spherical particles the operating range of fluidization velocity is (GATE-1993-9.b)

- (a) independent of particle size
- (b) greater for bigger particle size
- (c) greater for smaller particle size
- (d) proportional to the square of particle size

Answer: (c) $u_t/\bar{V}_{OM} = 7.7$ for larger dia particles; and $u_t/\bar{V}_{OM} \approx 50$ for smaller dia particles. (Refer: McCabe-Smith)

38. For a fluidized bed, with the increase in expansion of the bed, upto solids carry over from the bed, the pressure drop across the bed (GATE-1988-2.v)

 - (a) increases rapidly
 - (b) decreases rapidly
 - (c) first increases then decreases
 - (d) remains essentially constant

Answer: (d) ΔP increases rapidly before the start of fluidization; and decreases rapidly once the velocity reaches terminal settling velocity of solids (as it leads to carryover of solid-bed).

39. For laminar flow through a packed bed the pressure drop is proportional to (V_s is superficial liquid velocity and D_p is the particle diameter) (GATE-1999-1.11)

Answer: (a) $\Delta P = 2fL\rho v^2/D$; for laminar flow, $f = 16/Re$. Therefore, $\Delta P = 32Lv^2/D^2$ (This equation is applicable for flow through pipe; but the equation for flow through packings is also similar.)

40. For laminar flow of a fluid through a packed bed of spheres of diameter d , the pressure drop per unit length of bed depends upon the sphere diameter as (GATE-1997-2.06)

Answer: (d) $\Delta P = 2fL\rho v^2/D$ and $f = 16/\text{Re} = 16\mu/(D\rho v)$

41. A 0.5 m high bed made up of a 1 mm diameter glass spheres (density 2500 kg/m^3) is to be fluidised by water (density 1000 kg/m^3). If at the point of incipient fluidisation, the bed voidage is 40%, the pressure drop across the bed is (GATE-1997-2.08)

(a) 4.4 kPa (b) 2.94 kPa (c) 3.7 kPa (d) none of these

42. Bed pressure drop in an air fluidised bed of catalyst particles ($\rho_p = 2000 \text{ kg/m}^3$, $D_p = 0.05 \text{ cm}$) of 60 cm bed depth and bed porosity of 0.5, expressed in cm of water.

Answer: (b) $\Delta P = (1 - \epsilon)\rho_s gh - (1 - \epsilon)\rho gh = 0.5 \times 2000 \times 9.812 \times 0.6 - 0 = 5887$
 N/m^2

$$\text{Manometer head } \Delta h = \frac{\Delta P}{\rho g} = \frac{5887.2}{1000 \times 9.812} = 0.6 \text{ m} = 60 \text{ cm.}$$

43. A 1 m high bed made up of 2 mm particles is to be fluidised by an oil (density 900 kg/m³, viscosity 0.01 Pa.s). If at the point of incipient fluidisation, the bed voidage is 39% and the pressure drop across the bed is 10 kPa, then the density of particles is (GATE-2000-2.09)

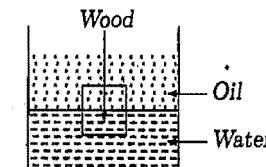
(a) 2571 kg/m^3 (b) 3514 kg/m^3 (c) 4000 kg/m^3 (d) 4350 kg/m^3

Answer: (a) $\Delta P = (1 - \epsilon)\rho_s gh - (1 - \epsilon)\rho qh$

2.3 Problems with Solutions

2.3.1 Two Layer Buoyancy

A piece of wood floats at the interface between layers of oil and water as shown in figure. The specific gravity of wood and oil are 0.95 and 0.75 respectively. Determine the volume fraction of wood in the water layer. (GATE-1993-19b)



Solution:

Let V be the volume of wood, and V_1 = volume of wood in oil; and V_2 = volume of wood in water. Therefore,

$$V_1 + V_2 = V \quad (2.1)$$

From the Archimedes principle, at floating, Weight of wood = weight of fluids displaced by wood. i.e.,

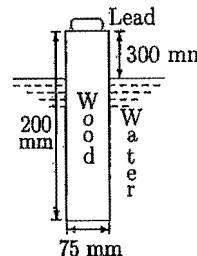
$$\begin{aligned} (V_1 + V_2)\rho_{\text{wood}}g &= V_1\rho_{\text{oil}}g + V_2\rho_{\text{water}}g \\ (V_1 + V_2) \times 0.95 &= V_1 \times 0.75 + V_2 \times 1 \end{aligned} \quad (2.2)$$

Solving Eqn.(2.1) and (2.2), $V_2/V = 0.8$

2.3.2 Floating of Wood on Water

A rectangular piece of wood (density = 650 kg/m^3) $75 \text{ mm} \times 75 \text{ mm} \times 2000 \text{ mm}$ in size floats on water (density = 1000 kg/m^3). How many kilograms of lead of density $11,200 \text{ kg/m}^3$ must be put on the top of the wooden piece so that it will float upright with 300 mm of its length above the water surface? (GATE-1997-15)

Solution:



$$\text{Mass of wood} = 0.075 \times 0.075 \times 2 \times 650 = 7.3125 \text{ kg}$$

$$\text{Mass of water displaced} = 0.075 \times 0.075 \times (2 - 0.3) \times 1000 = 9.5625 \text{ kg}$$

By Archimedes law,

$$\begin{aligned}\text{Mass of water displaced} &= \text{Mass of wood} + \text{Mass of lead} \\ 9.5625 &= 7.3125 + \text{Mass of lead}\end{aligned}$$

Therefore, mass of lead to be put on the top of wood = $9.5625 - 7.3125 = 2.25 \text{ kg}$

2.3.3 Force Required to Hold a Ball Immersed in Liquid

A jar is filled with two immiscible fluids A and B of densities 2000 kg/m^3 and 1000 kg/m^3 . A ball of $4 \times 10^{-6} \text{ m}^3$ volume and 0.01 kg mass is held submerged successively (a) in phase A (b) exactly at the interface of A and B, and (c) in phase B. Calculate the magnitude of the force required to hold the ball in position for each of the above cases. (GATE-1989-12i)

Solution:

From Archimedes principle,

$$F = F_g - F_b = m_s g - V_s \rho g \quad (2.3)$$

where F = net downward force

F_g = force of gravity

F_b = buoyancy force acting upwards

m_s = mass of ball

V_s = volume of ball

ρ = density of fluid

(a) Ball completely submerged in A:

From Eqn.(2.3),

$$F = 0.01 \times 9.812 - 4 \times 10^{-6} \times 2000 \times 9.812 = 0.0196 \text{ N}$$

(b) Ball exactly at the interface of A and B:

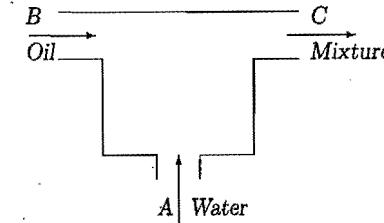
$$F = 0.01 \times 9.812 - (0.5 \times 4 \times 10^{-6} \times 2000 + 0.5 \times 4 \times 10^{-6} \times 1000) \times 9.812 = 0.0392 \text{ N}$$

(c) Ball completely submerged in B:

$$F = 0.01 \times 9.812 - 4 \times 10^{-6} \times 1000 \times 9.812 = 0.0589 \text{ N}$$

2.3.4 Average velocity of Mixture

Water is forced into the device shown in figure below at the rate of $0.15 \text{ m}^3/\text{sec}$ through pipe A, while oil of specific gravity 0.8 is forced in at the rate of $0.05 \text{ m}^3/\text{sec}$ through pipe B. If the liquids are incompressible and form a homogeneous mixture of oil globules in water, what is the average velocity and density of the mixture leaving through pipe C having a diameter of 0.564 m .
(GATE-1993-13a)



Solution:

Volumetric flow rate in pipe C (Q_c):

$$Q_c = Q_a + Q_b = 0.15 + 0.05 = 0.2 \text{ m}^3/\text{sec}$$

Average velocity in pipe C (\bar{v}_c):

$$\bar{v}_c = \frac{Q_c}{A_c} = \frac{0.2}{\frac{\pi}{4} \times 0.564^2} = 0.8 \text{ m/sec}$$

Mass flow rate at C (\dot{m}_c):

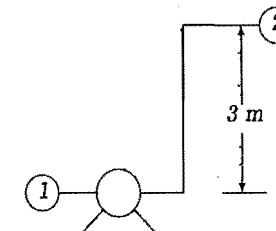
$$\dot{m}_c = \rho_a Q_a + \rho_b Q_b = 0.15 \times 1000 + 0.05 \times 800 = 190 \text{ kg/sec}$$

Average density of the mixture at C (ρ_c):

$$\rho_c = \frac{\dot{m}_c}{Q_c} = \frac{190}{0.2} = 950 \text{ kg/m}^3$$

2.3.5 Flow Direction

The following data were obtained on a section of piping through which an incompressible viscous fluid is flowing (See Figure)



Point 1:

$$\begin{aligned} \text{Pressure} &= 1.25 \times 10^5 \text{ Pa} \\ \text{Cross-sectional area} &= 15 \times 10^{-4} \text{ m}^2 \\ \text{Fluid Velocity} &= 1 \text{ m/s} \end{aligned}$$

Point 2:

$$\begin{aligned} \text{Pressure} &= 1.05 \times 10^5 \text{ Pa} \\ \text{Cross-sectional area} &= 5 \times 10^{-4} \text{ m}^2 \\ \text{Elevation above point 1} &= 3 \text{ m} \end{aligned}$$

Other Data:

$$\begin{aligned} \text{Density of fluid} &= 1000 \text{ kg/m}^3 \\ \text{Power delivered by the pump} &= 7.5 \text{ W} \\ \text{assume efficiency} &= 100\% \end{aligned}$$

Predict whether flow is taking place from point 1 to 2 or from 2 to 1. (GATE-1994-12)

Solution:

By equation of continuity, $A_1v_1 = A_2v_2$. Therefore, $v_2 = 15 \times 1/5 = 3 \text{ m/sec}$ Mass flow rate(\dot{m}) = $\rho Q = 1000 \times 15 \times 10^{-4} \times 1 = 1.5 \text{ kg/sec}$

$$\text{Pump head}(h_w) = 7.5 / (1.5 \times 9.812) = 0.51 \text{ m}$$

Assuming flow is from 1 to 2, we can write the Bernoulli equation between 1 and 2 as,

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + h_1 + h_w\eta = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + h_2 + h_f \quad (2.4)$$

Substituting for the known quantities,

$$\frac{1.25 \times 10^5}{1000 \times 9.812} + \frac{1^2}{2 \times 9.812} + 0 + 0.51 = \frac{1.05 \times 10^5}{1000 \times 9.812} + \frac{3^2}{2 \times 9.812} + 3 + h_f$$

Solving, $h_f = -0.858 \text{ m}$. For a possible flow, h_f can not be negative. Therefore, the assumed direction is not correct. Let us rework for flow from 2 to 1. Then,

$$\frac{1.05 \times 10^5}{1000 \times 9.812} + \frac{3^2}{2 \times 9.812} + 3 + 0.51 = \frac{1.25 \times 10^5}{1000 \times 9.812} + \frac{1^2}{2 \times 9.812} + 0 + h_f$$

Solving, $h_f = 1.419 \text{ m}$, which is a positive quantity. Therefore, flow is from 2 to 1.

2.3.6 Pressure Drop for Flow through Circular Pipe

Ammonia at atmospheric pressure and 300 K with a bulk stream velocity of 10 m/s flows in a pipe of i.d. 25 cm. Calculate the pressure drop per 100 m length of the pipe and the power consumed. Friction factor $f = 0.079Re^{-0.25}$ in the turbulent regime. Viscosity of ammonia may be taken as $10.2 \times 10^{-6} \text{ Kg/(m.s)}$. (GATE-1996-14)

Solution:

At room temperature and atmospheric pressure, ammonia is in vapor form. Density of ammonia vapor is calculated by assuming the validity of ideal gas law ($PV =$

$$nRT).$$

Molecular weight of ammonia (NH_3) = 17

17 kg of NH_3 at 273 K and 1 atm occupies 22.4 m^3 .

Therefore at 300 K and 1 bar, 1 kg $\text{NH}_3 \equiv 22.4 \times \frac{300}{273} \times \frac{1}{17} = 1.448 \text{ m}^3$

$$\text{Density} = 1/1.448 = 0.6906 \text{ kg/m}^3$$

$$\begin{aligned} \text{Re} &= \frac{DV\rho}{\mu} = \frac{0.25 \times 10 \times 0.6906}{10.2 \times 10^{-6}} = 169265 \\ f &= 0.079\text{Re}^{-0.25} = 0.079 \times 169265^{-0.25} = 0.0493 \end{aligned}$$

Pressure drop for 100 m length of pipe is,

$$\Delta P = \frac{2fL\rho v^2}{D} = \frac{2 \times 0.0493 \times 100 \times 0.6906 \times 10^2}{0.25} = 2723.7 \text{ N/m}^2$$

Power consumed :

$$\text{Power} = Q \times \Delta P = (\pi/4)D^2v \times \Delta P = (\pi/4) \times 0.25^2 \times 10 \times 2723.7 = 1337 \text{ Watt}$$

2.3.7 Velocity of Water in Circular Pipe

Water is pumped from a tank A to B using a 5 cm diameter pipe and a pump with the delivery head of 30 m. Both the tanks are uncovered and the water level in the tank B is 20 m above the water level in the tank A. The total pressure loss coefficient for the piping is 50. Calculate the velocity of water through the piping. Take $g = 10 \text{ m/s}^2$. (GATE-1989-12ii)

Solution:

Bernoulli's equation between A and B is given as,

$$\frac{P_A}{\rho g} + \frac{v_A^2}{2g} + h_A + h_{\text{pump}} = \frac{P_B}{\rho g} + \frac{v_B^2}{2g} + h_B + h_f \quad (2.5)$$

Given: $P_A = P_B = 0 \text{ atm(g)}$; $v_A = v_B$; $h_B - h_A = 20 \text{ m}$; and $h_{\text{pump}} = 30 \text{ m}$.

Substituting these in the above equation,

$$h_f = 10 \text{ m}$$

Also,

$$h_f = K \frac{v^2}{2g} \quad (2.6)$$

Given: Pressure loss coefficient (K) = 50.

Therefore,

$$10 = 50 \times \frac{v^2}{2 \times 10}$$

$$\text{i.e., } v^2 = 4$$

$$v = 2 \text{ m/sec}$$

Velocity of water through the piping (v) = 2 m/sec.

2.3.8 Flow Rate for a given Pressure Drop

Nikuradse developed a semi theoretical correlation for f vs. Re for steady turbulent flow in smooth pipes ($10^5 < Re < 10^7$): $1/\sqrt{f} = 1.75 \ln(Re\sqrt{f}) - 0.4$. Toluene ($\rho = 866 \text{ kg/m}^3$, $\mu = 0.0008 \text{ Ns/m}^2$) is to be conveyed through a 100 m pipe -line of diameter 0.2 m. What is the maximum flow rate of toluene in kg/sec that can be maintained, if the frictional pressure loss is not to exceed 10 kN/m²? (GATE-1990-12i)

Solution:

$$\Delta P = \frac{2fL\rho v^2}{D}$$

rearranging,

$$v^2 = \frac{\Delta PD}{2fLL\rho}$$

Substituting the known values,

$$v^2 = \frac{10000 \times 0.2}{2 \times f \times 100 \times 866} = \frac{0.01155}{f}$$

That is, from the pressure drop relation,

$$v^2 = \frac{0.01155}{f} \quad (2.7)$$

Nikuradse relation for f (given):

$$1/\sqrt{f} = 1.75 \ln(Re\sqrt{f}) - 0.4$$

Rearranging,

$$\begin{aligned} \ln(Re\sqrt{f}) &= \frac{0.4 + 1/\sqrt{f}}{1.75} \\ \ln Re &= \frac{0.4 + 1/\sqrt{f}}{1.75} - \ln \sqrt{f} \\ \ln(0.2 \times v \times 866/0.0008) &= \frac{0.4 + 1/\sqrt{f}}{1.75} - \ln \sqrt{f} \\ \ln v &= \frac{0.4 + 1/\sqrt{f}}{1.75} - \ln \sqrt{f} - 12.285 \end{aligned} \quad (2.8)$$

Equations (2.7) and (2.8) are solved by trial and error to get the value of v , and calculations are given below:

SINo	Assumed value of f	v from Eqn.(2.7)	v from Eqn.(2.8)
1	0.004	1.70	0.77
2	0.0035	1.817	1.537
3	0.0034	1.843	1.796
4	0.0033	1.871	2.112
5	0.00338	1.848	1.854

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Since for the value of $f = 0.00338$, v from the two equations are approximately equal (differing only in the third decimal), v shall be taken as 1.85 m/sec.

Volumetric flow rate(Q):

$$Q = (\pi/4) D^2 v = (\pi/4) \times 0.2^2 \times 1.85 = 58.1 \times 10^{-3} \text{ m/sec}$$

Mass flow rate(\dot{m}):

$$\dot{m} = Q\rho = 58.1 \times 10^{-3} \times 866 = 50.3 \text{ kg/sec}$$

2.3.9 Pressure Drop per Unit Length for a given Flow

What pressure drop per unit length is required in order to pump water at 25°C through a pipe 5 cm in diameter at a rate of 68 cm³/sec? Viscosity of water at 25°C is 1 cP. (GATE-1988-12i)

Solution:

Pressure drop is related to the flow parameters as,

$$\Delta P = \frac{2fL\rho v^2}{D}$$

Rearranging,

$$\frac{\Delta P}{L} = \frac{2f\rho v^2}{D}$$

Given: $D = 0.05 \text{ m}$; $\mu = 1 \text{ cP} = 0.001 \text{ kg/m.sec}$.

Velocity(v):

$$v = \frac{Q}{(\pi/4)D^2} = \frac{68 \times 10^{-6}}{(\pi/4) \times 0.05^2} = 0.0346 \text{ m/sec.}$$

$$Re = \frac{Dv\rho}{\mu} = \frac{0.05 \times 0.0346 \times 1000}{0.001} = 1732$$

Since $Re < 2100$, $f = 16/Re = 16/1732 = 0.00924$.

$$\frac{\Delta P}{L} = \frac{2 \times 0.00924 \times 1000 \times 0.0346^2}{0.05} = 0.442 \text{ Pa/m}$$

2.3.10 Pressure Drop for a given Flow

A hydrocarbon oil (viscosity 0.025 Pa.s and density 900 kg/m³) is transported using a 0.6 m diameter, 10 km long pipe. The maximum allowable pressure drop across the pipe length is 1 MPa. Due to a maintenance schedule on this pipeline, it is required to use a 0.4 m diameter, 10 km long pipe to pump the oil at the same volumetric flow rate as in the previous case. Estimate the pressure drop for the 0.4 m diameter pipe. Assume both pipes to be hydraulically smooth and in the range of operating conditions, the Fanning friction factor

is given by: $f = 0.079Re^{-0.25}$.

(GATE-2000-7)

Solution:

Pressure drop (ΔP) is related as,

$$\Delta P = \frac{2fL\rho v^2}{D}$$

Substituting for f ,

$$\Delta P = \frac{2 \times 0.079(Dv\rho/\mu)^{-0.25} L\rho v^2}{D}$$

Given: $\Delta P = 1 \times 10^6 \text{ N/m}^2$; $D = 0.6 \text{ m}$; $\rho = 900 \text{ kg/m}^3$; $\mu = 0.025 \text{ Pa.s}$; $L = 10000 \text{ m}$. Substituting these values in the above equation,

$$1 \times 10^6 = \frac{2 \times 0.079 \times (0.6 \times v \times 900/0.025)^{-0.25} \times 10000 \times 900 \times v^2}{0.6}$$

Solving, $v = 2.541 \text{ m/sec}$.

Flow rate (Q) = $(\pi/4)D^2v = (\pi/4) \times 0.6^2 \times 2.541 = 0.71845 \text{ m}^3/\text{sec}$.

Velocity in the 0.4 m dia pipe:

$$\begin{aligned} (\pi/4)0.4^2v &= 0.71845 \\ v &= 5.7173 \text{ m/sec} \end{aligned}$$

Friction factor (f):

$$f = 0.079Re^{-0.25} = 0.079 \times (0.4 \times 5.7173 \times 900/0.025)^{-0.25} = 0.0047$$

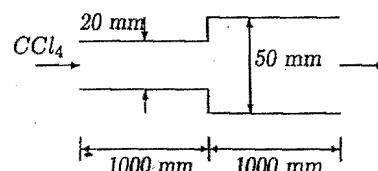
Pressure drop (ΔP):

$$\Delta P = \frac{2fL\rho v^2}{D} = \frac{2 \times 0.0047 \times 1000 \times 900 \times 5.7173^2}{0.4} = 6.86 \times 10^6 \text{ N/m}^2$$

Pressure drop for the 0.4 m dia pipe = 6.86 MPa.

2.3.11 Pressure Drop due to Friction

In a delivery line for carbon tetrachloride at the constant flowrate of $4 \times 10^{-5} \text{ m}^3/\text{sec}$, the first 1000 m long section is of 20 mm inside diameter smooth pipe followed by another 1000 m long section of 50 mm inside diameter smooth pipe, as shown in figure.



Estimate the pressure drop over the entire length of the delivery line. Neglect the minor losses due to sudden enlargement of pipe diameter.

For carbon tetrachloride, viscosity = 10^{-3} Pa.sec , and density = 1500 kg/m^3

For laminar flow, $f = 16/Re$

For transitional-turbulent flow, $f = 0.079Re^{-0.25}$ where f is the Fanning friction factor. (GATE-1997-14)

Solution:

For the smaller dia pipe:

$$\begin{aligned} \text{Velocity } (v_1) &= \frac{Q}{A_1} = \frac{4 \times 10^{-5}}{(\pi/4) \times (0.02)^2} = 0.127 \text{ m/s} \\ Re_1 &= \frac{D_1 v_1 \rho}{\mu} = \frac{0.02 \times 0.127 \times 1500}{10^{-3}} = 3810 \\ f_1 &= 0.079 \times (3810)^{-0.25} = 0.01 \end{aligned}$$

For the larger dia pipe:

$$\begin{aligned} \text{Velocity } (v_2) &= \frac{Q}{A_2} = \frac{4 \times 10^{-5}}{(\pi/4) \times (0.05)^2} = 0.0204 \text{ m/s} \\ Re_2 &= \frac{D_1 v_1 \rho}{\mu} = \frac{0.05 \times 0.0204 \times 1500}{10^{-3}} = 1530 \\ f_2 &= 16/1530 = 0.0105 \end{aligned}$$

Pressure drop due to friction (ΔP):

$$\begin{aligned} \Delta P &= \frac{2f_1 L_1 \rho v_1^2}{D_1} + \frac{2f_2 L_2 \rho v_2^2}{D_2} \\ &= \frac{2 \times 0.01 \times 1000 \times 1500 \times 0.127^2}{0.02} + \frac{2 \times 0.0105 \times 1000 \times 1500 \times 0.0204^2}{0.05} \\ &= 24193.5 + 262.2 = 24455.7 \text{ N/m}^2 = 24.5 \text{ kPa} \end{aligned}$$

2.3.12 Velocity of Water in a Drain Pipe

A liquid of specific gravity 1.25 is draining from the bottom of a large open tank through a 50 mm ID pipe. The drain pipe ends at a position 5 m below the surface of the liquid in the tank. Calculate the velocity of flow at the point of discharge from the pipe. Explain the significance of the result. (GATE-1987-12iv)

Solution:

Velocity (v) at the exit of pipe can be calculated from the Torricelli relation, given as

$$v = \sqrt{2gh} \quad (2.9)$$

Substituting for the values in Eqn.(2.9),

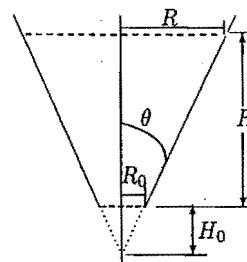
$$v = \sqrt{2 \times 9.812 \times 5} = 9.906 \text{ m/sec}$$

Significance of the Result:

- Velocity at the exit is independent of pipe diameter and fluid density.
- Coefficient of discharge is taken as 1.
- Increasing the length of the pipe, increases the velocity of discharge, because of increased potential head.

2.3.13 Time Required to Empty a Conical Vessel

Derive an expression to obtain the time required to empty the vessel containing water as shown in the following figure. The cross sectional area of the opening at the bottom of the tank is a . Assume $H_o \ll H$.
(GATE-1999-10)



Solution:

Since $H_o \ll H$, Volume of vessel (V) $\doteq \frac{1}{3}\pi R^2 H$; and, $\tan \theta = R/H$.
Therefore,

$$\begin{aligned} V &= \frac{1}{3}\pi R^2 H^3 \tan^2 \theta \\ \frac{dV}{dt} &= \pi H^2 \tan^2 \theta \frac{dH}{dt} \end{aligned}$$

Dia of opening $= 2R_o$; and, $R_o = H_o \tan \theta$.

Unsteady mass balance:

$$\text{Rate of mass in} - \text{Rate of mass out} = \text{Rate of mass accumulation}$$

$$0 - \rho a v_o = \rho \frac{dV}{dt}$$

$$-\rho(\pi/4)(2H_o \tan \theta)^2 v_o = \rho \pi H^2 \tan^2 \theta \frac{dH}{dt}$$

$$-H_o^2 v_o = \frac{dH}{dt}$$

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By energy balance, $v_o = \sqrt{2gH}$. Therefore,

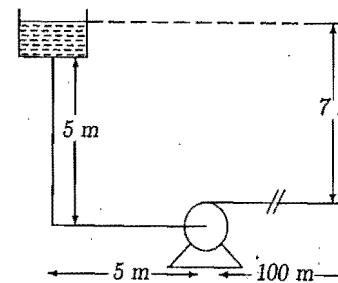
$$\begin{aligned} -H_o^2 \sqrt{2gH} &= H^2 \frac{dH}{dt} \\ -H_o^2 \sqrt{2g} dt &= H^{3/2} dH \\ H_o^2 \sqrt{2g} \int_0^t dt &= \int_0^H H^{3/2} dH \\ H_o^2 \sqrt{2g} t &= H^{5/2} \end{aligned}$$

Rearranging,

$$t = \frac{2}{5} \left(\frac{H}{H_o} \right)^2 \sqrt{\frac{H}{2g}}$$

2.3.14 Energy Required for Pumping

Oil of viscosity 100 cP is to be pumped as shown in the following figure. The pipe used everywhere is of 7 cm I.D. If the efficiency of the pump is 80%, find the energy required for pumping the oil at $20 \text{ m}^3/\text{hr}$. Density of the oil is 800 kg/m^3 .
(GATE-1999-11)



Solution:

Bernoulli's equation between section 1 and 2:

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + h_1 + h_w \eta = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + h_2 + h_f \quad (2.10)$$

Given: $Q = 20 \text{ m}^3/\text{hr}$ and $D = 7 \text{ cm}$.

Velocity:

$$v = Q/A = \frac{Q}{(\pi/4)D^2} = \frac{20}{3600 \times (\pi/4) \times 0.07^2} = 1.444 \text{ m/sec}$$

Reynolds Number:

$$Re = \frac{Dvp}{\mu} = \frac{0.07 \times 1.444 \times 800}{0.1} = 808.4$$

Frictional Pressure drop:

$$\Delta P_f = \frac{2f L \rho v^2}{D}$$

For $Re < 2100$, $f = 16/Re$. Therefore, $f = 16/808.4 = 0.0198$. And,

$$\Delta P_f = \frac{2 \times 0.0198 \times 110 \times 800 \times 1.444^2}{0.07} = 103702 \text{ N/m}^2$$

Frictional head:

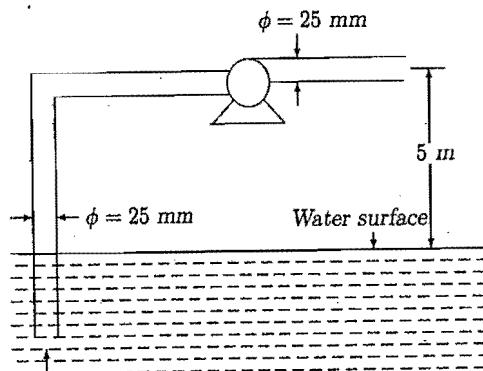
$$h_f = \frac{\Delta P_f}{\rho g} = \frac{103702}{800 \times 9.812} = 13.21 \text{ m}$$

Pressure head to be developed in the pump:

$$h_w = \frac{h_f - h_1}{\eta} = \frac{13.21 - 7}{0.8} = 338.5 \text{ Watt}$$

2.3.15 Power To Pump

In the figure given below, calculate the power required by the pump to deliver water at 3 m/sec from a pond. The inner diameter of the pipe is 25 mm. Neglect all losses in the pipe. Density of water is 1000 kg/m^3 .
(GATE-1997-4)



Solution:

By writing Bernoulli equation between the surface level of pond (1) to the discharge point of pipeline (2),

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + h_1 + h_w = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + h_2$$

Here, $P_1 = P_2 = 0$ (since they are open to atmosphere); $v_1 \approx 0$; and, h_w = head added by pump. Substituting these quantities,

$$0 + 0 + h_1 + h_w = 0 + \frac{3^2}{2 \times 9.812} + h_2$$

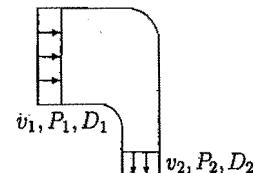
$$h_w = h_2 - h_1 + 0.459 = 5 + 0.459 = 5.459 \text{ m}$$

$$\text{Mass flow rate } (\dot{m}) = Q\rho = Av\rho = (\pi/4) \times (0.025)^2 \times 3 \times 1000 = 1.473 \text{ kg/s}$$

$$\text{Power} = \dot{m}gh_w = 1.473 \times 9.812 \times 5.459 = 78.9 \text{ Watt}$$

2.3.16 Friction Loss Factor of Bend-Reducer

The inlet velocity of water ($\rho = 1000 \text{ kg/m}^3$) in a right angled bend-reducer is $v_1 = 1 \text{ m/sec}$, as shown below. The inlet diameter is $D_1 = 0.8 \text{ m}$ and the outlet diameter is $D_2 = 0.4 \text{ m}$. The flow is turbulent and the velocity profiles at the inlet and outlet are flat (plug flow). Gravitational forces are negligible.



- (a) Find the pressure drop ($P_1 - P_2$) across the bend assuming negligible friction losses.
(b) If the actual pressure drop is $(P_1 - P_2) = 82.5 \text{ kPa}$, find the friction loss factor (K_f) based on the velocity v_1 .
(GATE-2001-8)

Solution:

(a) Bernoulli's equation between sections 1 and 2:

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} \quad (2.11)$$

From equation of continuity,

$$A_1 v_1 = A_2 v_2$$

Therefore,

$$v_2 = \frac{A_1 v_1}{A_2} = \frac{0.8^2}{0.4^2} v_1 = 4v_1$$

Substituting this in Eqn.(2.11)

$$\begin{aligned} \frac{P_1 - P_2}{\rho g} &= \frac{v_2^2 - v_1^2}{2g} = \frac{(4v_1)^2 - v_1^2}{2g} = \frac{15v_1^2}{2g} \\ P_1 - P_2 &= \frac{15v_1^2 \rho}{2} = \frac{15 \times 1 \times 1000}{2} = 7500 \text{ N/m}^2 \end{aligned}$$

(b) Bernoulli's equation between sections 1 and 2, modified for pressure losses:

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + h_f \quad (2.12)$$

Since $P_1 - P_2 = 8250 \text{ N/m}^2$,

$$h_f = \frac{8250}{\rho g} + \frac{v_1^2 - v_2^2}{2g} = \frac{8250}{1000g} + \frac{1^2 - (4 \times 1)^2}{2g} = \frac{0.75}{g}$$

By definition,

$$h_f = K_f \frac{v_1^2}{2g}$$

Substituting for h_f and v_1 ,

$$K_f = \frac{0.75 \times 2}{1^2} = 1.5$$

2.3.17 Flow through Tank Bottom

A tank full of water is open at the top with a hole near the bottom, the area of which is 0.25 m^2 and the cross-sectional area of the tank is 1 m^2 . If the height of the water above the hole is maintained at 10 m, what is the volumetric flow rate out of the hole? (GATE-1996-6)

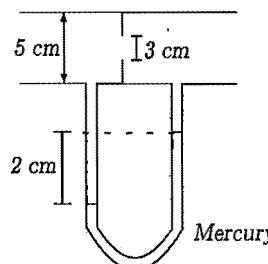
Solution:

$$\text{Velocity of flow through the hole} = \sqrt{2gh} = \sqrt{2 \times 9.812 \times 10} = 14 \text{ m/s}$$

$$\text{Volumetric flow rate through the hole} = A_o v = 0.25 \times 14 = 3.5 \text{ m}^3/\text{s}$$

2.3.18 Velocity Measurement by Orifice

Flow rate of water flowing through a pipe is being measured by using an orifice meter as shown in the figure.



- (a) What is the direction of flow in the pipe?
- (b) Derive an expression for velocity through the orifice. Determine the flow rate for an orifice coefficient of 0.8 (GATE-1999-9)

Solution:

(a) Flow is from left to right.

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(b) Bernoulli equation shall be written between sections 1 and 2 as,

$$\begin{aligned} \frac{P_1}{\rho g} + \frac{v_1^2}{2g} &= \frac{P_2}{\rho g} + \frac{v_2^2}{2g} \\ \text{i.e., } \frac{P_1 - P_2}{\rho g} &= \frac{v_2^2 - v_1^2}{2g} \\ \frac{\Delta P}{\rho g} &= \frac{v_2^2 - v_1^2}{2g} \end{aligned}$$

From equation of continuity, for incompressible flow: $A_1 v_1 = A_2 v_2$. Therefore,

$$\frac{\Delta P}{\rho g} = \frac{v_2^2 - (A_2/A_1)^2 v_1^2}{2g}$$

Rearranging,

$$\begin{aligned} v_2^2 &= \frac{1}{1 - (A_2/A_1)^2} \frac{2\Delta P}{\rho} \\ v_2 &= \frac{1}{\sqrt{1 - \left(\frac{A_2}{A_1}\right)^2}} \sqrt{\frac{2\Delta P}{\rho}} \end{aligned}$$

To account for the frictional losses and for the correction of A_2 , C_d is introduced in the above equation as

$$v_2 = \frac{C_d}{\sqrt{1 - \left(\frac{A_2}{A_1}\right)^2}} \sqrt{\frac{2\Delta P}{\rho}} \quad (2.13)$$

where v_2 = velocity of fluid at the orifice

C_d = coefficient of discharge of orifice

A_1 = flow cross-sectional area of pipe

A_2 = flow cross-sectional area of orifice

ΔP = pressure drop across orifice

ρ = density of flowing fluid

Given: $\Delta P = (\rho_m - \rho)gh = (13600 - 1000) \times 9.812 \times 0.02 = 2472.6 \text{ N/m}^2$

$D_1 = 0.05 \text{ m}; D_2 = 0.03 \text{ m}; C_d = 0.8$.

Substituting these in Eqn.(2.13),

$$v_2 = \frac{0.8}{\sqrt{1 - (0.03/0.05)^2}} \sqrt{\frac{2\Delta P}{\rho}} = 1.907 \text{ m/sec}$$

Flow rate(Q):

$$Q = A_2 v_2 = (\pi/4) \times 0.03^2 \times 1.907 = 1.348 \times 10^{-3} \text{ m}^3/\text{sec}$$

2.3.19 Orifice Size for a given Flow

What size of orifice would give a pressure difference of 41 cm water column for the flow of liquid styrene of specific gravity 0.9 at $0.055 \text{ m}^3/\text{s}$ in a 250 mm diameter pipe? Assume $C_d = 0.62$ (GATE-1991-13i)

Solution:

Flow through orifice meter is given by,

$$Q = \frac{C_d A_o}{\sqrt{1 - \left(\frac{A_o}{A_p}\right)^2}} \sqrt{\frac{2 \Delta P}{\rho}} \quad (2.14)$$

where C_d = coefficient of discharge of orifice

A_o = flow cross-sectional area of orifice

A_p = flow cross-sectional area of pipe

ΔP = pressure drop across orifice

ρ = density of flowing fluid

Given: $Q = 0.055 \text{ m}^3/\text{sec}$; $C_d = 0.62$; $D_p = 0.25 \text{ m}$; $\rho = 900 \text{ kg/m}^3$; $\Delta P = 0.41 \text{ m}$ of H_2O .

Pressure drop and Head are related as, $\Delta P = \rho g \Delta H$. Therefore,

$$\Delta P = 1000 \times 9.812 \times 0.41 = 4022.92 \text{ N/m}^2$$

Substituting the known quantities in Eqn.(2.14),

$$0.055 = \frac{0.62 \times (\pi/4) \times D_o^2}{\sqrt{1 - (D_o/0.25)^4}} \sqrt{\frac{2 \times 4022.92}{900}}$$

Solving, $D_o = 0.180 \text{ m}$. Diameter of orifice(D_o) = 180 mm.

2.3.20 Flow Measurement by Venturi Meter

Air is flowing in a 50 mm ID tube. There is a venturimeter in the line, and the manometric fluid is water. Calculate the volumetric flow rate of air for the following conditions:

Manometric reading = 100 mm water

Density of water = 1000 kg/m^3

Density of air = 1.185 kg/m^3

Coefficient of discharge = 0.98

Diameter of venturi throat = 25 mm

Pressure of air = 1 atm.

(GATE-1987-12ii)

Solution:

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Flow through Venturi meter is given by,

$$Q = \frac{C_d A_{th}}{\sqrt{1 - \left(\frac{A_{th}}{A_p}\right)^2}} \sqrt{\frac{2 \Delta P}{\rho}} \quad (2.15)$$

And, ΔP for simple U-tube manometer is given by

$$\Delta P = (\rho_m - \rho)gh \quad (2.16)$$

where C_d = coefficient of discharge of orifice;

A_{th} = flow cross-sectional area of throat;

A_p = flow cross-sectional area of pipe;

ΔP = pressure drop across venturi;

ρ = density of flowing fluid;

ρ_m = density of manometric fluid;

h = manometer reading.

Given: $C_d = 0.98$; $\rho = 1.185 \text{ kg/m}^3$; $\rho_m = 1000 \text{ kg/m}^3$; $h = 0.1 \text{ m}$; $D_p = 0.05 \text{ m}$; $D_{th} = 0.025 \text{ m}$.

From Eqn.(2.16)

$$\Delta P = (1000 - 1.185) \times 9.812 \times 0.1 = 980 \text{ N/m}^2$$

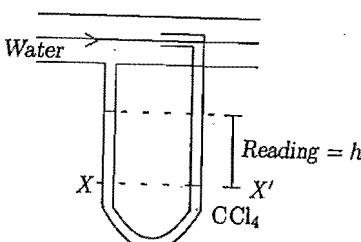
From Eqn.(2.15),

$$Q = \frac{0.98 \times (\pi/4) \times 0.025^2}{\sqrt{1 - (0.025/0.05)^4}} \sqrt{\frac{2 \times 980}{1.185}} \\ = 0.02 \text{ m}^3/\text{sec}$$

Flow rate of air through the venturi meter = $0.02 \text{ m}^3/\text{sec}$.

2.3.21 Flow Measurement by Pitot Tube

Water flows through a 30 mm i.d. pipe at atmospheric pressure. The velocity of water at the center of the pipe is measured by a pitot tube as shown in Figure. The pressure difference between the impact tube and the static tube is 20 cm of Carbon tetrachloride (density = 1600 kg/m^3). Calculate the volumetric flow rate through the pipe in cubic meter per hour. Viscosity of water is 1 cP. (GATE-1993-19a)



Solution:

Static pressure balance for the manometer shall be made at the level $X - X'$.

$$\begin{aligned} P_X \text{ in LHS} &= P_{X'} \text{ in RHS} \\ P_2 + \rho g x + \rho_m g h &= P_1 + \rho g (x + h) \\ \text{i.e., } P_1 - P_2 &= (\rho_m - \rho)gh \end{aligned}$$

Pressure difference can also be written in terms of head of flowing fluid as, $P_1 - P_2 = \rho g \Delta H$. Therefore,

$$\Delta H = \frac{h(\rho_m - \rho)}{\rho}$$

Given: $h = 0.2$ m of CCl_4 ; $\rho_m = 1600 \text{ kg/m}^3$; $\rho = 1000 \text{ kg/m}^3$. Therefore.

$$\Delta H = \frac{0.2 \times (1600 - 1000)}{1000} = 0.12 \text{ m of H}_2\text{O}$$

And from the formula for Pitot tube,

$$v = \sqrt{2g\Delta H} = \sqrt{2 \times 9.812 \times 0.12} = 1.535 \text{ m/sec}$$

Reynolds number corresponding to this velocity at the centerline is,

$$Re = \frac{Dv\rho}{\mu} = \frac{0.03 \times 1000}{0.001} = 46037$$

Since the flow is in turbulent regime, average velocity shall be taken as 0.82 times the centerline velocity. Therefore,

$$\begin{aligned} \bar{v} &= 0.82 \times v = 0.82 \times 1.535 = 1.258 \text{ m/sec} \\ \text{and, } Q &= A\bar{v} = \pi/4 D^2 \bar{v} \\ &= \pi/4 0.03^2 \times 1.258 = 0.9 \times 10^{-3} \text{ m}^3/\text{sec} \end{aligned}$$

2.3.22 Minimum fluidization Velocity

A tube of 0.05 m^2 cross-sectional area is packed with spherical particles up to a height of 0.25 m . The porosity of the bed is 0.35 . It is desired to fluidize the particles with water ($\rho = 1000 \text{ kg/m}^3$, $\mu = 10^{-3} \text{ Pa.s}$). Calculate the minimum fluidization velocity of fluidization given the Ergun's equation:

$$\frac{\Delta P}{\rho} = \frac{1.75V_s^2L(1-\varepsilon)}{D_p\varepsilon^3} + \frac{150\mu V_s(1-\varepsilon)^2 L}{D_p^2\varepsilon^3} \frac{L}{\rho}$$

Data:

Diameter of particles = 0.01 m

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Density of solid particles = 2600 kg/m^3

(GATE-1994-1)

Solution:

At minimum fluidization,

$$\Delta P = g(1-\varepsilon)(\rho_p - \rho)L$$

From the data given in the problem,

$$\begin{aligned} \Delta P &= 9.812 \times (1 - 0.35) \times (2600 - 1000) \times 0.25 \\ &= 2551.12 \text{ N/m}^2 \end{aligned}$$

Substituting the known values in the Ergun's equation,

$$\begin{aligned} \frac{2551.12}{1000} &= \frac{1.75 \times V_s^2 \times 0.25 \times (1 - 0.35)}{0.01 \times 0.35^3} + \frac{150 \times 10^{-3} \times V_s \times (1 - 0.35)^2 \times 0.25}{0.01^2 \times 0.35^3 \times 1000} \\ 2.5512 &= 663.3V_s^2 + 3.7V_s \end{aligned}$$

Solving the above equation, $V_s = 0.0593 \text{ m/sec}$

2.3.23 Water Trickling by Gravity through a Packed Bed

Water trickles by gravity over a bed of particles, each 1 mm dia in a bed of dia 6 cm at height 2 m . The water is fed from a reservoir whose diameter is much larger than that of the packed bed, with water maintained at a height of 0.1 m above the top of the bed. The bed has a porosity of 0.31 . Calculate the volumetric flow rate of water if its viscosity is 1 cP .

(GATE-1996-13)

Solution:

Available pressure drop for the water is given as,

$$\Delta P = \rho gh = 1000 \times 9.812 \times (2 + 0.1) = 20605.2 \text{ N/m}^2$$

Ergun's equation:

$$\frac{\Delta P}{\rho} = \frac{1.75V_s^2L(1-\varepsilon)}{D_p\varepsilon^3} + \frac{150\mu V_s(1-\varepsilon)^2 L}{D_p^2\varepsilon^3} \frac{L}{\rho}$$

Substituting the value of ΔP and the available data in Ergun's equation,

$$\begin{aligned} \frac{20605.2}{1000} &= \frac{1.75 \times V_s^2 \times 2 \times (1 - 0.31)}{1 \times 10^{-3} \times 0.31^3} + \frac{150 \times 0.001 \times V_s \times (1 - 0.31)^2 \times 2}{(1 \times 10^{-3})^2 \times 0.31^3 \times 1000} \\ 20.6052 &= 81065 V_s^2 + 4794.4 V_s \end{aligned}$$

Solving, $V_s = 4.021 \times 10^{-3} \text{ m/sec}$

Volumetric flow rate,

$$\begin{aligned} Q &= \pi/4 D^2 V_s \\ &= \pi/4 \times 0.06^2 \times 4.021 \times 10^{-3} \\ &= 11.38 \times 10^{-6} \text{ m}^3/\text{sec} \\ &= 11.38 \text{ cm}^3/\text{sec} \end{aligned}$$

2.3.24 Particle Size of Powder

Air flows through a packed bed of a powdery material of 1 cm depth at a superficial gas velocity of 1 cm/sec. A manometer connected to the unit registers a pressure drop of 1 cm of water. The bed has a porosity of 0.4. Assuming that Kozeny-Carman equation is valid for the range of study, estimate the particle size of the powder. $\rho_{\text{air}} = 1.23 \text{ kg/m}^3$; $\mu_{\text{air}} = 1.8 \times 10^{-5} \text{ kg/m.sec}$.
 (GATE-1995-18)

Solution:

Kozeny-Carman equation is given as

$$\frac{\Delta P}{\rho} = \frac{150\mu V_s(1-\varepsilon)^2 L}{D_p^2 \varepsilon^3 \rho}$$

Pressure drop measured by the manometer:

$$\Delta P = \rho_{\text{water}} gh = 1000 \times 9.812 \times 1 \times 10^{-2} = 98.12 \text{ N/m}^2$$

Therefore,

$$98.12 = \frac{150 \times 1.8 \times 10^{-5} \times 1 \times 10^{-2} \times (1 - 0.4)^2 \times 1 \times 10^{-2}}{D_p^2 \times 0.4^3}$$

Solving, D_p = particle size of powder = $1.244 \times 10^{-4} \text{ m} = 0.1244 \text{ mm}$.

Chapter 3

Mechanical Operations

3.1 Instant Notes

3.1.1 Size Analysis

- Sphericity Φ_s

$$\Phi_s = \frac{\text{Surface area of sphere of same volume as particle}}{\text{Surface area of particle}} = \frac{6v_p}{D_p s_p}$$

where D_p = equivalent diameter or nominal diameter of particle

s_p = surface area of one particle

v_p = volume of one particle

Equivalent diameter is defined as the diameter of sphere of equal volume as the particle.

$\Phi_s = 1$ corresponds to a sphere. Sphericities of all other particles must be less than one, because for a given volume a sphere has the minimum possible surface area.

- Microscopic analysis: ($1-100 \mu\text{m}$). Permits measurement of the projected area of the particle and also enables an assessment to be made of its two dimensional shape.
- Sedimentation/elutriation methods: ($> 1 \mu\text{m}$). These methods depend on the fact that the terminal falling velocity of a particle in a fluid increases with size.
- Permeability methods: ($> 1 \mu\text{m}$). These methods depend on the fact that at low flow rates the flow through a packed bed is directly proportional to the pressure difference, the proportionality constant being proportional to the square of the specific surface area of the powder.
- Sieving

- The efficiency of screening is defined as the ratio of the weight of material which passes the screen to that which is capable of passing.

- The effectiveness of a screen (often called the screen efficiency) is a measure of the success of a screen in closely separating materials A and B. If the screen functioned properly, all of material A would be in the overflow and all of material B would be in the underflow.
- Material balance¹:

$$\begin{aligned} F &= D + B \\ Fx_F &= Dx_D + Bx_B \end{aligned}$$

- Effectiveness:

- * Based on oversize:

$$E_A = \frac{Dx_D}{Fx_F}$$

- * Based on undersize:

$$E_B = \frac{B(1 - x_B)}{F(1 - x_F)}$$

- * Overall:

$$E = E_A E_B$$

3.1.2 Size Reduction

- Laws of crushing

- Kick's law

- * Energy required for crushing is proportional to the logarithm of the ratio between initial and final diameters.

$$\frac{P}{\dot{m}} = K_k \ln \left(\frac{D_f}{D_p} \right)$$

- * Applicable for coarse grinding, where the amount of surface produced is considerably less.

- Rittinger's law

- * Energy required for crushing is proportional to the surface area created.
- * Applicable for fine grinding, where the increase in surface per unit mass of material is large.

- Bond's law

- * Power required is proportional to the square root of the product size.

- * Work Index: Amount of energy required to reduce unit mass of material from an infinite particle size to a size of $100\mu\text{m}$.

- Wet grinding: Power consumption is considerably less by about 20–30% than dry grinding.

¹Terminologies are similar to binary distillation of A and B

- Size reduction is probably the most inefficient of all unit operations. Over 99% of energy goes to operating the equipment, producing undesirable heat and noise, leaving less than 1% for creating new surface.
- As the product becomes finer, the capacity of a given mill diminishes and the energy requirement increases.

- Principal forces in size reduction

- Impact — particle concussion by a single rigid force.
- Compression — particle disintegration by two rigid forces. Compression is the characteristic action of crushers.
- Shear — produced by fluid or particle-particle interaction.
- Attrition — arising from particles scraping against one another or against a rigid surface.

- The discharge from the *gyratory crusher* is continuous instead of intermittent as in a jaw crusher.

- Smooth roll crushers

$$\cos \alpha = \frac{r + d}{r + R}$$

where 2α = angle of nip

r = radius of crusher

$2d$ = smallest distance between the two rolls

R = radius of feed particle

- The maximum size of the product is approximately equal to $2d$.

- Normally $\alpha = 16^\circ$.

- To allow unbreakable material to pass through without damaging the machine, atleast one roll must be spring mounted.

- Toothed roll crushers

- Operate by compression, impact, and shear, not by compression alone, as do smooth-roll crushers.

- They are not limited by the problem of nip inherent with smooth rolls and can therefore reduce much larger particles.

- Some heavy-duty toothed double-roll crushers are used for the primary reduction of coal and similar materials.

- Hammer mills: impact and rubbing action.

- Tumbling mills

- A cylindrical shell slowly turning about a horizontal axis and filled to about half its volume with a solid grinding medium forms a tumbling mill.
- Little or no grinding is done when a mill is centrifuging, and operating speeds must be less than the critical speed.
- **Critical speed**

$$n_c = \frac{1}{2\pi} \sqrt{\frac{g}{R-r}}$$

where n_c = critical speed

r = radius of ball

R = radius of ball mill

- Tumbling mills run at 65 to 80% of the critical speed, with the lower values for wet grinding in viscous suspensions.
- **Ball mill:** or pebble mill. Impact force — principal size reduction force.
- **Tube mill** — continuous ball mill with long cylindrical shell.
- **Conical ball mill** — segregation of grinding units in a single chamber is possible.
- **Rod mill:** rolling, compression, and attrition.
- **Fluid energy mills:** Interparticle attrition (mostly); rubbing against the walls (less).

3.1.3 Size Separation

- Particles heavier than the suspending fluid may be removed from a gas or liquid in a settling tank, in which fluid velocity is low and the particles have ample time to settle-out. A settler which removes virtually all the particles from a liquid is known as a *clarifier*, whereas a device which separates the solid into two fractions is called a *classifier*. The same principles of sedimentation apply to both kinds of equipments.
- **Sorting Classifiers:** Devices which separate particles of different densities are known as sorting classifiers. They use one or the other of two principal separation methods — sink-and-float, and differential settling.
 - **Sink and float method:** A sink and float method uses a liquid sorting medium, the density of which is intermediate between that of light material and that of the heavy. This method has the advantage that, in principle, the separation depends only on the difference in the densities of the two substances and is independent of the particle size. This method is also called *heavy-fluid separation*.
 - **Differential settling methods:** Differential settling methods utilize the difference in terminal velocities that can exist between substances of different density. The density of medium is less than that of either substance. The disadvantage of this method is that since the mixture of materials to be separated covers a range of particle sizes, light particles settle at the same rate as the smaller, heavy ones and

a mixed fraction is obtained. For the same settling velocity the ratio of diameters of materials A and B (A - high denser; B - low denser) is given by:

$$\frac{d_B}{d_A} = \left[\frac{\rho_A - \rho}{\rho_B - \rho} \right]^3$$

where $S = 0.5$ for Stoke's law regime

$S = 1$ for Newton's law regime

- **Free settling**

— Laminar: $v_t = K_1 D^2$

— Turbulent: $v_t = K_2 D^{0.5}$

• **Hindered settling** — settling velocities are lower compared to free settling.

- **Richardson-Zaki correlation:**

$$\frac{v}{v_t} = \epsilon^n$$

where v = settling velocity of group of particles

v_t = terminal settling velocity of single particle

ϵ = void fraction

n = Richardson-Zaki index

$n = 4.65$ for $Re_p < 0.2$

$n = 2.39$ for $Re_p > 500$

- **Hydraulic Jigging:** If the particles are allowed to settle in a fluid for only a very short time, they will not attain their terminal falling velocity and a better degree of separation can be obtained. A particle of material A will have an initial acceleration $g(1 - \rho/\rho_A)$, because there is no fluid friction when the relative velocity is zero. Thus the initial velocity is a function of density only and is unaffected by size and shape. A very small particle of denser material will therefore always commence settling at a greater rate than a large particle of the less dense material. Hydraulic jig utilizes this feature.

- **Cyclones**

— Cyclone consists of a vertical cylinder with a conical bottom, a tangential inlet near the top, and an outlet for dust at the bottom of the cone. The inlet is usually rectangular.

— Separation factor:

$$\frac{F_c}{F_g} = \frac{u_{tan}^2}{rg}$$

where F_c = centrifugal force
 F_g = gravitational force
 u_{tan} = tangential velocity
 r = radius of cyclone

- A large-diameter cyclone has a much lower separation factor at the same velocity. Small-diameter cyclones may have separation factors as high as 2500.
- To handle large gas flows, a number of small diameter cyclones may be grouped in a single enclosure with common headers for the feed and product gas and a single dust hopper.

3.1.4 Filtration

- Constant pressure filtration

- The pressure drop is held constant and the flow rate is allowed to fall with time.
- If slurry is fed to a filter from a constant-head tank, the pressure at the upstream face of the cake is constant.

- Constant rate filtration

- Pressure drop is progressively increased and the flow rate is constant.
- Slurry is fed to filter by means of a positive displacement pump.

- Variable-rate – variable-pressure filtration: Slurry is fed by a centrifugal pump.
- For any filtration pressure, the rate of flow is greatest at the beginning of the process since the resistance is then a minimum.

- Compressibility of filter cakes

- For incompressible cakes specific cake resistance α is independent of the pressure drop and of position in the cake.
- In a compressible cake specific cake resistance α varies from place to place in the cake; it varies with the applied pressure drop; it may vary with time. In practice, however, the variations in α with time and location are ignored.
- Compressibility coefficient (s): If α is independent of ΔP , the sludge is incompressible. Ordinarily α varies with ΔP , as most sludges are at least to some extent compressible. For highly compressible sludges, α increases rapidly with ΔP .

$$\alpha = \alpha_0 (\Delta P)^s$$

Constant s is the compressibility coefficient of the cake. It is zero for incompressible sludges and positive for compressible ones. It usually falls between 0.2 and 0.8.

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- Filter medium resistance: Filter medium resistance varies with the pressure drop and with the age and cleanliness of the filter medium, but since it is important only during the early stages of filtration, it is nearly always satisfactory to assume that it is constant during any given filtration and to determine its magnitude from experimental data.

- Washing of filter cakes

- In most filters (except filter-press) the wash liquid follows the same path as that of the filtrate. The rate of flow of wash liquid is, in principle, equal to that of the last of the filtrate, provided the pressure drop remains unchanged in passing from filtration to washing.
- For plate-and-frame filter press with 3-button plates, the area through which the wash liquid flows is only half that used during filtration, while the thickness of the cake is twice that through which the filtrate passes. Hence the rate of flow during washing will be one-fourth the final rate of filtration.

- Filter press

- Simple washing: Plate and frame arrangements: 1-2-1-2-1. Wash liquid is fed in through the same channel as the slurry.
- Through washing: Plate and frame arrangements: 1-2-3-2-1. Thickness of the cake is twice that through which the filtrate passes; and area through which wash liquid flows is only half that used during filtration.

- Optimum time cycle: For new washing type plate-and-frame filter press operating at a constant pressure with negligible medium resistance, optimum time cycle occurs when the time of filtration equals the time lost in opening, dumping, cleaning, and reassembling the press.

- Rate of filtration

$$\frac{dV}{d\theta} = \frac{A^2(1-mw)\Delta P}{(V+V_f)\rho w \mu \alpha} = \frac{d(V+V_f)}{d\theta}$$

where θ = time

V = volume of filtrate

V_f = volume of filtrate held in filter medium

A = filtration area

α = specific cake resistance, m/kg

m = weight ratio of wet cake to dry cake

w = weight fraction of solids in the slurry

In simple form:

$$\frac{d\theta}{dV} = \frac{2V}{C} + \frac{2V_f}{C}$$

where C is proportional to $\Delta P/\alpha$.

In the above equation, $2V_f/C = 0$, if filter medium resistance can be neglected.

- **Rotary drum filters:** For constant pressure filtration, and where filter medium resistance can be neglected, rate of filtration is proportional to \sqrt{N} ; where N is the rotational speed of drum.

3.1.5 Agitation

- Though often confused, agitation and mixing are not synonymous.
 - *Agitation* refers to the induced motion of a material in a specified way, usually in a circulatory pattern inside some sort of container.
 - *Mixing* is the random distribution, into and through one another, of two or more initially separate phases.

- **Flow patterns in agitated vessels**

- In the usual case of a vertical shaft, the radial and tangential velocity components are in a horizontal plane, and the longitudinal components is vertical. The radial and longitudinal components are useful and provide the flow necessary for the mixing action. When the shaft is vertical and centrally located in the tank, the tangential component is generally dangerous.
- The tangential flow follows a circular path around the shaft, creates a *vortex* at the surface of the liquid. If solid particles are present, circulatory currents tend to throw the particles to the outside by centrifugal force, from where they move downward and to the centre of the tank at the bottom. Instead of mixing, its reverse — concentration occurs.
- In an unbaffled vessel circulatory flow is induced by all types of impellers, whether axial flow or radial flow.

- **Type of impellers**

- **Propellers:** A propeller is an axial flow, high speed impeller for liquids of low viscosity.
- **Turbines:** The principal currents are radial and tangential. The tangential components induce *vortexing* and *swirling*, which must be stopped by baffles or by a diffuser ring if the impeller is to be most effective.

- **Prevention of swirling**

- Impeller can be mounted off-centre. This is satisfactory for small tanks.
- In large tanks with vertical agitators, the preferable method of reducing swirling is to install baffles, which impede rotational flow without interfering with radial or tangential flow.

- **Power required for agitation**

$$Po = \phi(Re, Fr)$$

where $Po = \frac{P}{n^3 D_a^5 \rho} =$ Power number

$$Re = \frac{nD_a^2 \rho}{\mu} =$$
 Impeller Reynolds number

$$Fr = \frac{n^2 D_a}{g} =$$
 Froude number

P = Power required for agitation

n = Agitator rpm

D_a = Diameter of impeller

ρ = Density of fluid

μ = Viscosity of fluid

g = Acceleration due to gravity

- The shape of the tank has relatively little effect on Po. Circulation patterns, of course, are strongly affected by tank shape.
- **Froude number:** Froude number Fr is a measure of the ratio of the inertial stress to the gravitational force per unit area acting on the fluid. It appears in fluid-dynamic situations where there is significant wave motion on a liquid surface. It is significantly important in ship design. In unbaffled tanks, a vortex forms and the Froude number has an effect.
- For low Re, ($Re < 10$), Po versus Re is a straight line with a slope of -1 on logarithmic coordinates, for both baffled and unbaffled tanks.

$$Po \cdot Re = \frac{P}{n^2 D_a^3 \mu} = K_L \text{ (a constant)}$$

from which

$$P = K_L n^2 D_a^3 \mu$$

For Re less than 10, the flow is laminar and density is no longer a factor and power delivered to the liquid is proportional to D_a^3 for both baffled and unbaffled tanks.

- In baffled tanks at Reynolds number larger than about 10,000 the power number is independent of the Reynolds number, and viscosity is not a factor.

$$Po = K_T \text{ (a constant)}$$

from which

$$P = K_T n^3 D_a^5 \rho$$

- **Weber number**

$$We = \frac{\rho_c n^2 D_a^3}{\sigma} = \frac{\text{kinetic energy}}{\text{surface tension stress}}$$

where ρ_c = density of continuous liquid phase

σ = interfacial tension

For two-phase dispersions Weber number is a significant factor.

3.2 Objective Type Questions

1. The distribution given by microscopic analysis of powder is (GATE-1996-1.06)

- (a) Number (b) Length (c) Area (d) Volume

Answer: (c)

2. The sphericity of a solid particle of cubical shape is (GATE-2000-2.07)

- (a) π (b) $(\pi/6)^{1/3}$ (c) $(\pi/6)^{1/2}$ (d) $\pi/3$

Answer: (b) $\phi = \frac{\text{surface area of sphere of same volume as the particle}}{\text{surface area of particle}}$

Volume of particle = a^3

Volume of sphere = $a^3 = (4/3)\pi r^3$. Therefore, $r = \sqrt[3]{3/(4\pi)}$

Surface area of sphere = $4\pi r^2 = 4\pi [3/(4\pi)]^{2/3} a^2 = (4\pi)^{1/3} 3^{2/3} a^2$

Surface area of particle = $6a^2$

$$\phi = \frac{(4\pi)^{1/3} 3^{2/3} a^2}{6a^2} = \left(\frac{4\pi}{3}\right)^{1/3} \frac{3}{6} = (\pi/6)^{1/3}$$

3. The sphericity of a cylinder of 1 cm diameter and length 3 cm is (GATE-1996-2.06)

- (a) 0.9 (b) 0.78 (c) 0.6 (d) 0.5

Answer: (b) Volume of particle = $\pi r^2 h = \pi \times 0.5^2 \times 3 = 0.75\pi$.

Volume of sphere = $4/3 \pi r_s^3 = 0.75\pi$; from which $r_s = 0.825$ cm.

$$\phi = \frac{4\pi r_s^2}{2\pi r(h+r)} = \frac{4 \times \pi \times 0.825^2}{2 \times \pi \times 0.5 \times (3+0.5)} = 0.78$$

4. The angle formed by pouring a powder as a heap on a flat surface is known as: (GATE-1989-3.i.a)

- (a) Contact angle (b) Angle of nip (c) Angle of repose (d) Critical angle

Answer: (c) Angle of nip is related to roll crusher.

5. Weight mean diameter is given by: (GATE-1989-3.i.b)

- (a) $\frac{\sum n_i d_i^4}{\sum n_i d_i^3}$ (b) $\left(\frac{\sum n_i d_i^3}{\sum n_i}\right)^{1/3}$ (c) $\frac{\sum n_i d_i^3}{\sum n_i d_i^2}$ (d) $\frac{\sum n_i d_i^2}{\sum n_i d_i}$

Answer: (c) Weight mean dia = $\frac{\text{mass} \times \text{dia}}{\text{mass}}$

Mass $\propto 1/D^3$. Therefore, Weight mean or mass mean dia = $\frac{1/D^3 \times D}{1/D^3} = D^3/D^2$

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6. The work index in Bond's law for crushing of solids has the following dimension (GATE-1998-1.12)

- (a) No units (dimensionless) (b) kWh/ton
(c) kW/ton (d) $\text{kW m}^{1/2}/\text{ton}$

Answer: (b) Work index is defined as the energy required per ton of material, crush from a very large size to a size of $100\mu\text{m}$ (80% of material passing size).

7. In a gyratory crusher the size reduction is effected primarily by: (GATE-1990-3)

- (a) compression (b) impact (c) attrition (d) cutting action

Answer: (a) Impact is the primary action in hammer mill; attrition in ball mill; cutting action in cutters.

8. For crushing of solids, the Rittinger's law states that the work required for crushing proportional to (GATE-1997-1.1)

- (a) the new surface created
(b) the size reduction ratio
(c) the change in volume due to crushing
(d) none of these

Answer: (a)

9. The energy required per unit mass to grind limestone particles of very large size $100\mu\text{m}$ is 12.7 kWh/ton . An estimate (using Bond's Law) of the energy to grind the particles from a very large size to $50\mu\text{m}$ is (GATE-2001-2.1)

- (a) 6.35 kWh/ton (b) 9.0 kWh/ton (c) 18 kWh/ton (d) 25.4 kWh/ton

Answer: (c) $\frac{P}{\dot{m}} = \frac{K_b}{D_p}$

$$K_b = 12.7 \times \sqrt{100} \\ P = 12.7 \times \sqrt{100}/\sqrt{50} = 127/\sqrt{50} = 17.96\text{ kWh/ton.}$$

10. A fluid energy mill is used for (GATE-1995-1)

- (a) cutting (b) grinding (c) ultra grinding (d) crushing

Answer: (c)

11. Energy requirement (per unit mass of material crushed/ground) is highest for

- (a) Jaw crusher (b) Rod mill (c) Ball mill (d) Fluid energy mill

Answer: (d) Surface area produced is more for fine size reduction. Energy requirement is proportional to surface area created.

- (a) $Po \propto Re^{-1}$ (b) $Po \propto Re^0$ (c) $Po \propto Re^{0.5}$ (d) $Po \propto Re^1$

$Po = P/(\rho N^3 D^5)$: Power Number

$Re = \rho N D^2 / \mu$: Reynolds Number

N is the impeller rotational speed, and D is the impeller diameter.

Answer: (a) Power number is inversely proportional to Re at low Reynolds numbers.

23. At very low r.p.m (Re less than 5), the power required for agitation is proportional to (GATE-1993-10.c)

- (a) D (b) D^2 (c) D^3 (d) D^5

Answer: (c) $Po = \frac{P}{\rho N^3 D^5}$; and $Re = \frac{\rho N D^2}{\mu}$. At low Re , $Po \propto 1/Re$. Therefore, $P \propto D^3$.

24. In power correlations for agitated vessels the effect of Froude number appears: (GATE-1990-3.ii)

- (a) for baffled vessels when Reynolds number is less than 300
 (b) for unbaffled vessels when Reynolds number is greater than 300
 (c) when there is no vortex formation
 (d) when the Reynolds number is less than 300

Answer: (b) For baffled vessels or at low Re , Froude number has no effect.

25. The Power number for a stirred tank becomes constant at high Reynolds number. In this limit, the variation of power input with impeller rotational speed (N) is proportional to (GATE-2001-1.06)

- (a) N^0 (b) N^1 (c) N^2 (d) N^3

Answer: (d) $Po = \frac{P}{\rho N^3 D^5}$. At high Re , $Po \approx \text{constant}$.

3.3 Problems with Solutions

3.3.1 Surface-Volume Mean Diameter of Particles

Calculate the surface-volume mean diameter for the following particulate material. Show detailed calculations. (GATE-1995-7)

Size range, μm	Mass of particles in the range, gm
-704 + 352	25
-352 + 176	37.5
-176 + 88	62.5
-88 + 44	75
Pan	50

Solution:

Surface-volume mean diameter \bar{D}_s is given by

$$\bar{D}_s = \frac{1}{\sum_{i=1}^n (x_i / \bar{D}_{p,i})}$$

where x_i = mass fraction of material retained on the screen
 $\bar{D}_{p,i}$ = average size of particle on the screen

Total mass of particles = $25 + 37.5 + 62.5 + 75 + 50 = 250 \text{ gm}$

Mass fraction = mass / total mass

Size range, μm	Mass of particles	x_i	$\bar{D}_{p,i}$
-704 + 352	25	$25/250 = 0.1$	$(704+352)/2 = 528$
-352 + 176	37.5	0.15	264
-176 + 88	62.5	0.25	132
-88 + 44	75	0.3	66
Pan	50	0.2	$(44+0)/2 = 22$

$$\sum_{i=1}^n (x_i / \bar{D}_{p,i}) = \frac{0.1}{528} + \frac{0.15}{264} + \frac{0.25}{132} + \frac{0.3}{66} + \frac{0.2}{22} = 163 \times 10^{-4}$$

$$\bar{D}_s = \frac{1}{\sum_{i=1}^n (x_i / \bar{D}_{p,i})} = \frac{1}{163 \times 10^{-4}} = 61 \mu\text{m}$$

3.3.2 Work Index

- (a) The power required to crush 100 ton/hr of a material is 179.8 kW, if 80% of the feed passes through a 51 mm screen and 80% of the product passes through a 3.2 mm screen. What is the work index of the material?
- (b) What will be the power required for the same feed at 100 ton/hr to be crushed to a product such that 80% is to pass through a 1.6 mm screen? (GATE-1991-14i)

Solution:

Bond's law:

$$\frac{P}{\dot{m}} = K_b \left(\frac{1}{\sqrt{D_p}} - \frac{1}{\sqrt{D_f}} \right) \quad (3.1)$$

where P = power required for crushing

\dot{m} = mass flow rate of feed

K_b = Bond's law constant

D_f = mean diameter of feed

D_p = mean diameter of product

Work index W_i is defined as the power required per ton of feed to reduce from very large size to such a size that 80% of product passes a $100\mu\text{m}$ screen. i.e., $W_i = K_b/\sqrt{100}$. K_b is calculated from the given data as:

$$K_b = \frac{(179.8/100)}{(1/\sqrt{3200}) - (1/\sqrt{51000})} = 135.7 \text{ kWhr.}\mu\text{m}^{1/2}/\text{ton}$$

And,

$$W_i = K_b/\sqrt{100} = 135.7/10 = 13.57 \text{ kWhr/ton}$$

Power to make a product size of 1.6 mm:

$$P = 100 \times 135.7 \times \left(\frac{1}{\sqrt{1600}} - \frac{1}{\sqrt{51000}} \right) = 279.16 \text{ kW}$$

3.3.3 Energy Consumption in Crushing

A material is crushed in a jaw crusher and the average size of the particle reduced from 5 cm to 1 cm, with the consumption of energy of $1.32 \times 10^4 \text{ J/kg}$. What will be the consumption of energy to crush the same material of an average size of 7.5 cm to 2.5 cm, assuming

(a) Rittenger's law, and (b) Kick's law. (GATE-1989-13ii)

Solution:

Rittenger's law:

$$\frac{P}{m} = K_r \left(\frac{1}{D_p} - \frac{1}{D_f} \right) \quad (3.2)$$

Kick's law:

$$\frac{P}{m} = K_k \ln \frac{D_f}{D_p} \quad (3.3)$$

where P = energy required for crushing

m = mass of feed

D_f = mean diameter of feed

D_p = mean diameter of product

K_r = Rittenger's law constant

K_k = Kick's law constant

Rittenger's law constant:

$$K_r = \frac{P/m}{\left(\frac{1}{D_p} - \frac{1}{D_f} \right)} = \frac{1.32 \times 10^4}{(1/1 - 1/5)} = 1.65 \times 10^4 \text{ J.m/kg}$$

Kick's law constant:

$$K_k = \frac{P/m}{\ln \frac{D_f}{D_p}} = \frac{1.2 \times 10^4}{\ln(5/1)} = 8.202 \times 10^3 \text{ J/kg}$$

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Energy consumption, from Rittenger's law:

$$\frac{P}{m} = 1.65 \times 10^4 \left(\frac{1}{2.5} - \frac{1}{7.5} \right) = 4.4 \times 10^3 \text{ J/kg}$$

Energy consumption, from Kick's law:

$$\frac{P}{m} = 8.202 \times 10^3 \times \ln \frac{7.5}{2.5} = 9.01 \times 10^3 \text{ J/kg}$$

3.3.4 Specific Rate of Grinding

The rate of grinding of uniform sized particles is assumed to follow first order breakage of particles. 50 grams of powder of average diameter 215 microns was ground in a laboratory batch mill. The amount of unground material (215 microns) was measured at various times of grinding and the results are given in table. Estimate the specific rate of grinding. (GATE-1996-15)

Weight (gram)	50	17	12	8	6	2
Time (sec)	0	60	90	120	150	240

Solution:

Rate of grinding (dx/dt) can be expressed as

$$\frac{dx}{dt} = -kx \quad (3.4)$$

where k = specific rate of grinding

x = mass fraction of material unground at time t

Integrating Eqn.(3.4)

$$\ln x = -kt$$

x = weight/total weight.

k can be estimated for various time intervals as

$$k = \frac{\ln x_1 - \ln x_2}{t_2 - t_1}$$

Weight (gram)	Time (sec)	Mass fraction Retained (x)	$\ln x$	k
50	0	1	0	
17	60	0.34	-1.08	0.018
12	90	0.24	-1.43	0.012
8	120	0.16	-1.83	0.013
6	150	0.12	-2.12	0.010
2	240	0.04	-3.22	0.012

Average $k = (0.012 + 0.013 + 0.010 + 0.012)/4 = 0.0123 \text{ s}^{-1}$

(First value of k is not included in the average calculation as it deviates much from the other values).

3.3.5 Speed of Ball Mill

In a ball mill of diameter 2000 mm, 100 mm dia steel balls are being used for grinding. Presently, for the material being ground, the mill is run at 15 rpm. At what speed will the mill have to be run if the 100 mm balls are replaced by 50 mm balls, all the other conditions remaining the same? (GATE-1987-13i)

Solution:

The critical speed of ball mill is given by,

$$n_c = \frac{1}{2\pi} \sqrt{\frac{g}{R-r}}$$

where R = radius of ball mill

r = radius of ball

For $R = 1000$ mm and $r = 50$ mm,

$$n_c = \frac{1}{2\pi} \sqrt{\frac{9812 \times 60^2}{1000 - 50}} = 30.7 \text{ rpm}$$

But the mill is operated at a speed of 15 rpm. Therefore, the mill is operated at $100 \times 15/30.7 = 48.86\%$ of critical speed.

If 100 mm dia balls are replaced by 50 mm dia balls, and the other conditions are remaining the same,

$$\text{Speed of ball mill} = \frac{0.4886}{2\pi} \sqrt{\frac{9812 \times 60^2}{1000 - 25}} = 14.8 \text{ rpm}$$

3.3.6 Diameter of Roll Crusher

A pair of rolls is to take a feed equivalent to spheres of 3 cm in diameter and crush them to spheres having 1 cm diameter. If the coefficient of friction is 0.29, what would be the diameter of rolls? (GATE-1988-13i)

Solution:

The following formula relates the coefficient of friction (μ), radius of rolls (r), radius of product (d), and radius of feed (R):

$$\cos \alpha = \frac{r+d}{r+R} \quad (3.5)$$

where α is related to the coefficient of friction by the relation,

$$\mu = \tan \alpha$$

$$\text{Angle of nip} = 2\alpha$$

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We have, $\mu = 0.29$. Therefore, $\alpha = \tan^{-1}(0.29) = 16.17^\circ$. And we have, $d = 0.5$ cm; $R = 1.5$ cm. Substituting for the known quantities in Eqn.(3.5)

$$\begin{aligned} \cos(16.17) &= \frac{r+0.5}{r+1.5} \\ 0.9604 &= \frac{r+0.5}{r+1.5} \\ r &= 23.75 \text{ cm} \\ \text{Dia of rolls} &= 2 \times 23.75 = 47.5 \text{ cm.} \end{aligned}$$

3.3.7 Classification based on Terminal Settling Velocity-I

In a mixture of quartz (sp.gr = 2.65) and galena (sp.gr = 7.5), the size of the particles range from 0.0002 cm to 0.001 cm. On separation in a hydraulic classifier using water under free settling conditions, what are the size ranges of quartz and galena in the pure products? (Viscosity of water = 0.001 kg/m.s; density = 1000 kg/m³). (GATE-1991-14ii)

Solution:

Given:

$$\text{Density of quartz}(\rho_{sa}) = 2650 \text{ kg/m}^3$$

$$\text{Density of galena}(\rho_{sb}) = 7500 \text{ kg/m}^3$$

$$\text{Particle size range}(D_{\min} \text{ to } D_{\max}) = 0.0002 \text{ cm to } 0.001 \text{ cm}$$

$$\text{Viscosity of water}(\mu) = 0.001 \text{ kg/m.s}$$

$$\text{Density of water}(\rho) = 1000 \text{ kg/m}^3$$

Formulae / Theory:

For laminar settling regimes, terminal-settling velocity is given by Stoke's law: (valid for $Re_p < 0.1$)

$$v_t = \frac{D^2(\rho_s - \rho)g}{18\mu}$$

where D = diameter of particle

ρ_s = density of solid

$$Re_p = Dv_t\rho/\mu$$

For turbulent regime

$$v_t = \sqrt{\frac{4D(\rho_s - \rho)g}{3C_D\rho}}$$

where C_D is the drag coefficient.

If C_D is assumed to be same for the two different density particles of the same diameter (assumption is valid in high turbulent velocities and practically assumed to be correct for laminar regime also), for laminar flow, for equal settling velocities,

$$\frac{D_a}{D_b} = \sqrt{\frac{\rho_{sb} - \rho}{\rho_{sa} - \rho}} \quad (3.6)$$

Similarly for turbulent flow, for equal settling velocities,

$$\frac{D_a}{D_b} = \frac{\rho_{sb} - \rho}{\rho_{sa} - \rho} \quad (3.7)$$

Calculations:

First let us check for the flow regime for the given particle size range of particles:

For the largest diameter particle of higher density to follow laminar regime

$$Re_p = \frac{Dv_t\rho}{\mu} = \frac{D D^2(\rho_s - \rho)g\rho}{18\mu^2} \text{ should be less than } 0.1$$

i.e., the maximum size of particle is,

$$\begin{aligned} D^3 &= \frac{0.1 \times 18\mu^2}{(\rho_s - \rho)g\rho} \\ &= \frac{0.1 \times 18 \times 0.001^2}{(7500 - 1000) \times 9.81 \times 1000} \\ D &= 3.05 \times 10^{-5} \text{ m} = 0.00305 \text{ cm} \end{aligned}$$

which is well above the maximum size of the particles. Therefore, laminar flow equations can be used for this problem.

The fractions obtained after separation are: (i) Pure Galena only; (ii) Galena and quartz; and (iii) Quartz only.

The terminal settling velocity of minimum dia galena in fraction (i) corresponds to the settling velocity of maximum dia quartz particles.

From Eqn.(3.6),

$$\begin{aligned} \frac{D_a}{0.001} &= \sqrt{\frac{2.65 - 1}{7.5 - 1}} \\ D_a &= 0.0005 \text{ cm} \end{aligned}$$

The fraction (i) consists of pure galena in the size range 0.0005 cm to 0.001 cm

The terminal settling velocity of maximum dia quartz in fraction (ii) corresponds to the settling velocity of minimum dia galena particles.

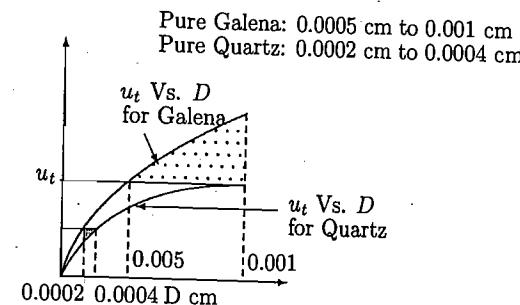
From Eqn.(3.6),

$$\begin{aligned} \frac{D_a}{0.0002} &= \sqrt{\frac{7.5 - 1}{2.65 - 1}} \\ D_a &= 0.0004 \text{ cm} \end{aligned}$$

The fraction (iii) consists of pure quartz in the size range 0.0002 cm to 0.0004 cm.

The fraction (ii) consists of quartz in the size range 0.0004 cm to 0.001 cm, and galena in the size range 0.0002 cm to 0.0005 cm.

This result is shown in the following diagram:



3.3.8 Range of Velocities for Classification

A binary mixture of 100 μm size having densities of 2 g/cm^3 and 4 g/cm^3 is to be classified by elutriation technique using water. Estimate the range of velocities that can do the job and recommend a suitable value.

(GATE-1995-16)

Solution:

Assuming the applicability of Stoke's law (valid for $Re_p < 1$)

$$v_t = \frac{D_p^2(\rho_s - \rho)g}{18\mu}$$

Therefore, Re_p of heavy density particles is

$$\begin{aligned} \frac{D_p v_t \rho}{\mu} &= \frac{D_p D_p^2 (\rho_s - \rho) g \rho}{18 \mu^2} \\ &= \frac{(100 \times 10^{-6})^3 \times (4000 - 1000) \times 9.812 \times 1000}{18 \times (0.001)^2} \\ &= 1.64 \text{ (which is nearly equal to 1; hence the assumption is valid)} \end{aligned}$$

Therefore, v_t is given by Stoke's law. Minimum v_t is obtained for low density particles; and Maximum v_t is obtained for heavy density particles.

$$\begin{aligned} v_{t,\min} &= \frac{(100 \times 10^{-6})^2 \times (2000 - 1000) \times 9.812}{18 \times 0.001} = 5.45 \times 10^{-3} \text{ m/s} \\ v_{t,\max} &= \frac{(100 \times 10^{-6})^2 \times (4000 - 1000) \times 9.812}{18 \times 0.001} = 16.35 \times 10^{-3} \text{ m/s} \end{aligned}$$

The recommended range of velocities is between 5.45×10^{-3} and 16.35×10^{-3} m/s.

3.3.9 Classification based on Terminal Settling Velocity-II

In a mixture of quartz, specific gravity 2.65, and galena, specific gravity 7.5, the size of the particles range from 0.00052 cm to 0.0025 cm. On separation in a hydraulic classifier with

water as the medium, under free settling streamline conditions, three fractions are obtained: one consisting of only quartz, one of only galena and one a mixture of quartz and galena. What are the ranges of particle sizes of the two substances in the mixed portion? (GATE-1989-13iii)

Solution:

Let us assume that the settling velocity is in Stoke's regime. Then, terminal settling velocity (v_t) of the largest size particle of galena is

$$\begin{aligned} v_t &= \frac{(\rho_p - \rho)gD_p^2}{18\mu} \\ &= \frac{(7500 - 1000) \times 9.812 \times (0.0025 \times 10^{-2})^2}{18 \times 0.001} \\ &= 2.214 \times 10^{-3} \text{ m/sec} \end{aligned}$$

$$\begin{aligned} Re_p &= \frac{D_p v_t \rho}{\mu} \\ &= \frac{(0.0025 \times 10^{-2})(2.214 \times 10^{-3})(1000)}{0.001} \\ &= 0.055 < 0.1 \end{aligned}$$

Since the Re_p for the maximum settling velocity particle is less than 0.1 (Stoke's law regime), all the particles will settle at the rate, that calculated from Stoke's law.

According to Stoke's law

$$v_t = \frac{(\rho_p - \rho)gD_p^2}{18\mu}$$

For equal terminal settling velocity

$$\begin{aligned} \frac{(\rho_{p,gal} - \rho)gD_{p,gal}^2}{18\mu} &= \frac{(\rho_{p,qua} - \rho)gD_{p,qua}^2}{18\mu} \\ \frac{\rho_{p,gal} - \rho}{\rho_{p,qua} - \rho} &= \frac{D_{p,qua}^2}{D_{p,gal}^2} \end{aligned}$$

From which

$$\frac{D_{p,qua}}{D_{p,gal}} = \sqrt{\frac{\rho_{p,gal} - \rho}{\rho_{p,qua} - \rho}}$$

For getting pure galena product, $D_{p,qua} = 0.0025$ cm. Therefore,

$$\frac{0.0025}{D_{p,gal}} = \sqrt{\frac{7.5 - 1}{2.65 - 1}}$$

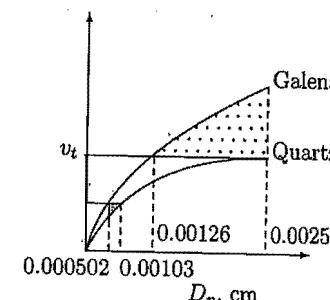
From which, $D_{p,gal} = 0.00126$ cm.

For getting pure quartz product, $D_{p,gal} = 0.00052$ cm. Therefore,

$$\frac{D_{p,qua}}{0.00052} = \sqrt{\frac{7.5 - 1}{2.65 - 1}}$$

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From which, $D_{p,qua} = 0.00103$ cm.



Mixed portion will have

quartz of size between 0.00103 cm to 0.0025 cm, and
galena of size between 0.000502 cm to 0.00126 cm.

3.3.10 Screening and Classification

A mixture of coal and sand particles having sizes smaller than 1×10^{-4} m in diameter to be separated by screening and subsequent elutriation with water. Recommend a screen aperture such that the oversize from the screen can be separated completely into sand and coal particles by elutriation. Calculate also the required water velocity. Assume that Stoke's law is applicable. Density of sand = 2650 kg/m^3 ; density of coal = 1350 kg/m^3 ; density of water = 1000 kg/m^3 ; viscosity of water = $1 \times 10^{-3} \text{ kg/m.s}$; $g = 9.812 \text{ m/s}^2$. (GATE-1990-13i)

Solution:

For laminar settling regimes, terminal-settling velocity is given by Stokes law: (valid for $Re_p < 0.1$)

$$v_t = \frac{D^2(\rho_s - \rho)g}{18\mu}$$

where D = diameter of particle

ρ_s = density of solid

$$Re_p = Dv_t\rho/\mu$$

$$\begin{aligned} v_t \text{ of larger dia coal particle} &= \frac{(1 \times 10^{-4})^2 \times (1350 - 1000) \times 9.812}{18 \times 0.001} \\ &= 1.9079 \times 10^{-3} \text{ m/sec} \end{aligned}$$

Dia of sand particle corresponding to the u_t of larger dia coal particle:

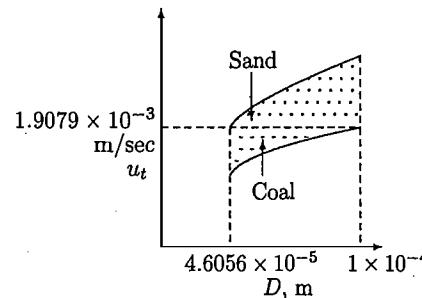
$$1.9079 \times 10^{-3} = \frac{D^2 \times (2650 - 1000) \times 9.812}{18 \times 0.001}$$

$$D = 4.6056 \times 10^{-5} \text{ m}$$

The size of screen aperture needed so that the oversize particles can be separated completely into sand and coal by elutriation = $4.6056 \times 10^{-5} \text{ m}$.

The required water velocity = $1.9079 \times 10^{-3} \text{ m/sec}$. By operating at the water velocity of $1.9079 \times 10^{-3} \text{ m/sec}$, coal particles will be carried along with water, and sand particles will settle down.

This calculation is aided with the following diagram:



3.3.11 Filtration Equation

The volumetric flow rate during constant pressure filtration is

$$\frac{dV}{dt} = \frac{1}{K_c V + 1/q_0}$$

where V is the total volume of filtrate collected in time t , and K_c and q_0 are constants.

- (a) Integrate the above equation to obtain a relation between V and t .
- (b) Make a sketch of t/V versus V from your results.
- (c) Given $V = 1.0 \text{ litre}$ at $t = 41.3 \text{ sec}$ and $V = 2.0 \text{ litre}$ at $t = 108.3 \text{ sec}$, find K_c .
(GATE-2001-9)

Solution:

Given:

$$\frac{dV}{dt} = \frac{1}{K_c V + 1/q_0}$$

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Let $Q_o = 1/q_0$ Rearranging

$$(K_c V + Q_o) dV = dt$$

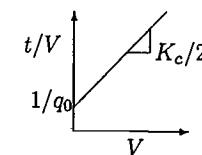
Integrating,

$$\int_Q^V K_c V + Q_o dV = \int_0^t dt$$

$$K_c V^2 / 2 + Q_o V = t$$

$$K_c \frac{V}{2} + Q_o = \frac{t}{V} \quad (3.8)$$

The above equation is represented in the sketch as given below:



Given: $V = 1.0 \text{ litre}$ at $t = 41.3 \text{ sec}$ and $V = 2.0 \text{ litre}$ at $t = 108.3 \text{ sec}$ Substituting these in the Eqn.(3.8)

$$K_c \frac{1}{2} + Q_o = \frac{41.3}{1}$$

$$K_c \frac{2}{2} + Q_o = \frac{108.3}{2}$$

Solving the above equations, $K_c = 25.7 \text{ sec/ltr}^2$.

3.3.12 Constant Pressure Filtration-I

A filtration is carried out for 10 min at a constant rate in a leaf filter and thereafter it is continued at constant pressure. This pressure is that attained at the end of the constant rate period. If one quarter of the total volume of the filtrate is collected during the constant rate period, what is the total filtration time? Assume that the cake is incompressible and the filter medium resistance is negligible. (GATE-1990-13ii)

Solution:

For filtration operation,

$$\frac{d\theta}{dV} = \frac{2V}{C} + \frac{2V_f}{C} \quad (3.9)$$

where C is a function of ΔP and mean specific cake resistance (α). i.e.,

$$C = C_1 \frac{\Delta P}{\alpha} \quad (3.10)$$

where C_1 is a constant.

Specific cake resistance α is related to the compressibility coefficient s by the relation

$$\alpha = \alpha_0(\Delta P)^s$$

where α_0 is a constant. For an incompressible cake, $s = 0$; in other words, α is a constant. Therefore, Eqn.(3.10) can be written as $C = C_2\Delta P$ where C_2 is a constant.

If filter medium resistance is negligible, then $V_f = 0$. Now Eqn.(3.9) can be written in the form,

$$\frac{d\theta}{dV} = \frac{2V}{C} = \frac{2V}{C_2\Delta P} \quad (3.11)$$

For constant rate filtration, $d\theta/dV = \text{Constant} = \theta/V$.

Given: $\theta = 10$ min for $V = 0.25V_T$, where V_T = total volume of filtrate (filtrate collected in constant rate operation and constant pressure operation)

From Eqn.(3.11),

$$\frac{2V}{C_2\Delta P} = \frac{10}{0.25V_T} = 40/V_T$$

i.e.,

$$\Delta P = \frac{2V}{40C_2/V_T} = VV_T/(20C_2)$$

ΔP at the end of constant rate operation = $0.25V_T^2/(20C_2) = 0.0125V_T^2/C_2$

Constant Pressure filtration:

For the given problem conditions $C_2 = \text{Constant}$, and ΔP for the constant pressure operation is that obtained at the end of the constant rate operation.

From Eqn.(3.11),

$$\frac{d\theta}{dV} = \frac{2V}{C_2\Delta P} = \frac{2V}{0.0125V_T^2} \quad (3.12)$$

Integrating Eqn.(3.12) between the limits $V = 0.25V_T$ to V_T and $\theta = 10$ min to θ_T

$$\begin{aligned} \int_{10}^{\theta_T} &= \frac{2}{0.0125V_T^2} \int_{0.25V_T}^{V_T} V dV \\ \theta_T - 10 &= (80/V_T^2) \times [V_T^2 - (0.25V_T)^2] \\ \theta_T &= 85 \text{ minutes} \end{aligned}$$

Total filtration time = 85 minutes.

3.3.13 Constant Pressure Filtration-II

A constant pressure filtration test gave data that can fit an expression, $dt/dV = 9.3V + 8.5$; (t in seconds; V in litres). If the resistance of the filter medium is assumed unaffected with pressure drop and the compressibility coefficient of the filter cake is 0.3, what will be the time taken for the collection of 3.5 litres of filtrate at a filtration pressure twice that used in the test?

(GATE-1993-20)

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Solution:

Given:

$$\frac{dt}{dV} = 9.3V + 8.5 \quad (3.13)$$

For filtration operation, we know

$$\frac{dt}{dV} = \frac{2V}{C} + \frac{2V_f}{C}$$

$$\text{where } C = \frac{C_1\Delta P}{\alpha}$$

$C_1 = \text{constant}$

$\alpha = \text{specific cake resistance} = \alpha_0(\Delta P)^s$

$s = \text{compressibility coefficient}$

Given that V_f is constant with changes in ΔP . Therefore

$$C = \frac{C_1\Delta P}{\alpha_0(\Delta P)^{0.3}} = C_2(\Delta P)^{0.7}$$

where $C_2 = \text{constant}$.

Hence

$$\frac{dt}{dV} = \frac{2V}{C_2(\Delta P)^{0.7}} + \frac{2V_f}{C_2(\Delta P)^{0.7}} \quad (3.14)$$

On comparison of Eqns.(3.13) and (3.14)

$$\frac{2V_f}{C_2(\Delta P)^{0.7}} = 8.5 \text{ or } \frac{2V_f}{C_2} = 8.5(\Delta P)^{0.7}$$

and

$$\frac{2}{C_2(\Delta P)^{0.3}} \text{ or } \frac{2}{C_2} = 9.3(\Delta P)^{0.7}$$

If pressure is doubled, $\Delta P_{\text{new}} = 2\Delta P$. Therefore

$$\begin{aligned} \frac{2V_f}{C_2} &= 8.5(2\Delta P)^{0.7} \\ &= 13.81(\Delta P)^{0.7} \\ \frac{2V_f}{C_2(\Delta P)^{0.7}} &= 13.81 \end{aligned} \quad (3.15)$$

Similarly

$$\begin{aligned} \frac{2}{C_2} &= 9.3(2\Delta P)^{0.7} \\ &= 15.11(\Delta P)^{0.7} \\ \frac{2}{C_2(\Delta P)^{0.3}} &= 15.11 \end{aligned} \quad (3.16)$$

From Eqns.(3.15) & (3.16), Eqn.(3.14) becomes

$$\frac{dt}{dV} = 15.11V + 13.81$$

Integrating, and with the initial condition of $V = 0$ at $t = 0$,

$$t = 15.11 \frac{V^2}{2} + 13.81V$$

Given: $V = 3.5$ litres. Therefore

$$t = 15.11 \times \frac{3.5^2}{2} + 13.81 \times 3.5 = 140.9 \text{ seconds}$$

3.3.14 Washing Time

A leaf filter filtering a slurry, gave a total of 8 m^3 filtrate in 30 minutes. Filtration was continued till 11.3 m^3 of filtrate was collected. Estimate the washing time in minutes, if 11.3 m^3 of wash water are used. The resistance of the cloth can be neglected and a constant pressure is used throughout. (GATE-1989-13i)

Solution:

Filtration equation:

$$\frac{d\theta}{dV} = \frac{2V}{C} + \frac{2V_f}{C}$$

Since filter-medium resistance is negligible, $V_f = 0$. Therefore

$$\frac{d\theta}{dV} = \frac{2V}{C}$$

Integrating with the initial condition of $V = 0$ at $t = 0$,

$$\theta = V^2/C$$

Given: when $\theta = 30 \text{ min}$, $V = 8 \text{ m}^3$. Therefore,

$$C = 8^2/30 = 64/30$$

Final rate of filtration = $(dV/d\theta)_{\text{final}} = \frac{C}{2V} = \frac{(64/30)}{2 \times 11.3} = 0.09 \text{ m}^3/\text{min}$.

Rate of washing = final rate of filtration

Therefore

$$\text{washing time} = \frac{\text{volume of wash water}}{\text{rate of washing}} = \frac{11.3}{0.09} = 119.7 \text{ min.}$$

3.3.15 Washing Time in Filter Press

A filter press contains 20 frames, each of 0.6 m by 0.6 m inside dimensions. The frames are 0.025 m thick. The press is equipped with 1 and 3 button plates for washing. The volume of wash water used is 10% of the filtrate per cycle. The time required for filtering at constant pressure is 2 hours by which time the frames are full. Washing is done at the same pressure as filtering and the viscosity of wash water is nearly the same as that of the filtrate. What is the time for washing? There is 0.05 m^3 of final cake per m^3 of filtrate. Neglect the resistance of the filter medium. (GATE-1992-14b)

Solution:

$$\text{Volume of cake} = 20 \times 0.6 \times 0.6 \times 0.025 = 0.18 \text{ m}^3$$

$$\text{Volume of filtrate} = 0.18 \times \frac{1}{0.05} = 3.6 \text{ m}^3$$

Filtration equation is

$$\frac{d\theta}{dV} = \frac{2V}{C} + \frac{2V_f}{C}$$

Neglecting filter-medium resistance, the above equation becomes

$$\frac{d\theta}{dV} = \frac{2V}{C}$$

where V = volume of filtrate at time t

C = a constant for constant pressure filtration of incompressible cake

Integrating with the initial condition that $V = 0$ at $\theta = 0$,

$$\theta = \frac{V^2}{C}$$

Given: $\theta = 2 \text{ hours}$, and from the above calculations $V = 3.6 \text{ m}^3$. Therefore

$$C = \frac{V^2}{\theta} = \frac{3.6^2}{2}$$

For washing with 3-button plates, the rate of washing is one-fourth the final rate of filtration. (Refer: Badger & Bancherro, Introduction to Chemical Engineering, Mc-Graw Hill)

i.e.,

$$\left(\frac{dV}{d\theta} \right)_{\text{washing}} = \frac{1}{4} \left(\frac{dV}{d\theta} \right)_{\text{final filtration rate}}$$

$$\text{Final filtration rate} = \frac{C}{2V} = \frac{(3.6^2/2)}{2 \times 3.6} = 0.9 \text{ m}^3/\text{hr}$$

$$\text{Washing rate} = \frac{1}{4} \times 0.9 \text{ m}^3/\text{hr}$$

$$\text{Washing time} = \frac{\text{volume of wash water}}{\text{washing rate}} = \frac{3.6 \times 0.1}{1/4 \times 0.9} = 1.6 \text{ hr}$$

(Note: Washing in 3-button plates is also referred to as through washing. If only 1 & 2 button plates are available, rate of washing equals the final rate of filtration.)

3.3.16 Optimal Time in Filter Press

Show that for a nonwashing plate and frame filter press, operating at constant pressure with negligible filter medium resistance, the optimum cycle occurs when the time for filtering equals the time lost in opening, dumping, cleaning and reassembling the press. (GATE-1988-13ii)

Solution:

For a filtration process

$$\frac{d\theta}{dV} = \frac{2V}{C} + \frac{2V_f}{C}$$

If filter medium resistance is negligible $2V_f/C$ term becomes zero. Therefore

$$\frac{d\theta}{dV} = \frac{2V}{C}$$

Integrating

$$\theta = \frac{V^2}{C} = BV^2 \quad (3.17)$$

where $B = 1/C$. Let θ' be the total time required in opening, dumping, cleaning and reassembling the press, per cycle. Then

$$\text{overall rate of filtration } (W) = \frac{V}{\theta + \theta'}$$

where V is the volume of filtrate collected per cycle.

$$W = \frac{V}{BV^2 + \theta'} \quad (3.18)$$

W is maximum when $dW/dV = 0$. Hence from Eqn.(3.18)

$$\frac{dW}{dV} = \frac{(BV^2 + \theta') - V(2BV)}{(BV^2 + \theta')^2} = 0$$

Therefore

$$BV^2 + \theta' - 2BV^2 = 0$$

or

$$\theta' = BV^2 \quad (3.19)$$

Comparing Eqns.(3.17) & (3.19)

$$\theta = \theta'$$

Therefore, optimum time cycle occurs while the time during which filtration is carried-out, is exactly equal to the time the filter press out of service.

In practice, in order to obtain the maximum overall rate of filtration, the filtration time must always be somewhat greater in order to allow for the resistance of the cloths.

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3.3.17 Compressibility of Filter Cake

In a filtration process, if V is the volume of filtrate collected in time t , a general relationship can be given as

$$\frac{dt}{dV} = \frac{\mu}{A(\Delta P)} \left(\frac{\alpha cV}{A} + R_m \right)$$

where α is the specific cake resistance, R_m is the filter medium resistance, A is the filter area, c is the concentration of solids in the slurry, μ is the viscosity of the filtrate and ΔP is the overall pressure drop.

- (a) Filtration experiments were carried out at a constant pressure drop on a slurry containing 20 kg/m^3 of CaCO_3 in water. The data obtained from the plots of t/V vs. V at two different pressure drops are given in the table below:

Pressure drop (N/m^2)	Slope (s/litre^2)	Intercept (s/litre)
5×10^4	12.5	26.5
35×10^4	3.5	6.9

If the filter area is 0.09 m^2 and the viscosity of the filtrate is 0.001 kg/m.s , determine the specific cake resistance and the filter medium resistance corresponding to each pressure drop.

- (b) Determine from the above data whether the cake is compressible. (GATE-2002-2)

Solution:

Given:

$$\frac{dt}{dV} = \frac{\mu}{A(\Delta P)} \left(\frac{\alpha cV}{A} + R_m \right)$$

Substituting for the known quantities,

$$\frac{dt}{dV} = \frac{0.001}{0.09 \Delta P} \left(\frac{\alpha \times 20 \times V}{0.09} + R_m \right) = \frac{0.011}{\Delta P} (222.2V\alpha + R_m)$$

Rearranging and integrating the above equation,

$$t = \frac{0.011}{\Delta P} (111.1V^2\alpha + R_m V)$$

Rearranging,

$$\frac{t}{V} = \frac{1.22}{\Delta P} \alpha V + \frac{0.011}{\Delta P} R_m \quad (3.20)$$

If $\Delta P = 5 \times 10^4 \text{ N/m}^2$, Eqn.(3.20) becomes

$$\begin{aligned} \frac{t}{V} &= \frac{1.22}{5 \times 10^4} \alpha V + \frac{0.011}{5 \times 10^4} R_m \\ &= 2.44 \times 10^{-5} \alpha V + 2.2 \times 10^{-7} R_m \end{aligned} \quad (3.21)$$

From Eqn.(3.21), slope = $2.44 \times 10^{-5} \alpha$.

Given: slope = $12.5 \text{ s/ltr}^2 = 12.5 \times 10^6 \text{ s/m}^6$. Therefore

$$2.44 \times 10^{-5} \alpha = 12.5 \times 10^6$$

Solving, $\alpha = 5.123 \times 10^{11} \text{ m/kg}$. From Eqn.(3.21), intercept = $2.2 \times 10^{-7} R_m$.

Given: intercept = $26.5 \text{ s/ltr} = 26.5 \times 10^3 \text{ s/m}^3$. Therefore

$$2.2 \times 10^{-7} R_m = 26.5 \times 10^3$$

Solving, $R_m = 12.05 \times 10^{10} \text{ m}^{-1}$.

If $\Delta P = 35 \times 10^4 \text{ N/m}^2$, Eqn.(3.20) becomes

$$\begin{aligned} \frac{t}{V} &= \frac{1.22}{35 \times 10^4} \alpha V + \frac{0.011}{35 \times 10^4} R_m \\ &= 34.86 \times 10^{-7} \alpha V + 0.314 \times 10^{-7} R_m \end{aligned} \quad (3.22)$$

From Eqn.(3.22), slope = $34.86 \times 10^{-7} \alpha$.

Given: slope = $3.5 \text{ s/ltr}^2 = 3.5 \times 10^6 \text{ s/m}^6$. Therefore

$$34.86 \times 10^{-7} \alpha = 3.5 \times 10^6$$

Solving, $\alpha = 10 \times 10^{11} \text{ m/kg}$. From Eqn.(3.22), intercept = $0.314 \times 10^{-7} R_m$.

Given: intercept = $6.9 \text{ s/ltr} = 6.9 \times 10^3 \text{ s/m}^3$. Therefore

$$0.314 \times 10^{-7} R_m = 6.9 \times 10^3$$

Solving, $R_m = 21.97 \times 10^{10} \text{ m}^{-1}$.

Since α changes with ΔP , the cake is compressible.

3.3.18 Speed of Agitator

An agitated baffle vessel is being used to prepare a uniform solution of viscosity 2 cP, running the agitator at 100 rpm, so as to obtain a Reynolds number of 50,000. If the contents of the vessel are replaced by a solution of viscosity 4 cP, and the agitator rpm is increased to 200, by how much will the power requirement change? (GATE-1987-13ii)

Solution:

Theory:

For agitated vessel, the following dimensional relationship is applicable.

$$Po = \psi(Re, Fr, S_1, S_2, \dots, S_n)$$

$$\text{where } Po = \text{Power Number} = \frac{P}{n^3 D_a^5 \rho}$$

$$Re = \text{Reynolds number} = \frac{n D_a^2 \rho}{\mu}$$

$$Fr = \text{Froude Number} = \frac{n^2 D_a}{g}$$

S_1, S_2, \dots, S_n = shape factors

n = rotation per unit time of agitator

D_a = dia of impeller

ρ = density of fluid

μ = viscosity of fluid

If the shape factors are remaining constant, then

$$Po = \psi(Re, Fr)$$

For baffled vessel, Po is a function of only Re provided the shape factors are remaining at constant value. i.e.,

$$Po = \psi(Re)$$

For various impeller configurations, and system geometry, Po vs. Re chart is available to estimate the power required.

From the charts available, it could be seen that, for $Re > 10000$, Po is independent of Re, and remains at a constant value. i.e., Po = constant, (for $Re > 10000$), and viscosity is not a factor.

Calculations:

For the given problem, for the case 1:

$$\mu_1 = 2 \text{ cP}; n_1 = 100; Re_1 = 50000$$

$$Re_1 = \frac{n_1 D_a^2 \rho}{\mu_1} = 50000$$

Therefore, $D_a^2 \rho = 50000 \times 2/100 = 1000$.

For the case 2:

$$\mu_2 = 4 \text{ cP}; n_2 = 200$$

$$Re_2 = \frac{(D_a^2 \rho) n_2}{\mu_2} = \frac{1000 \times 200}{4} = 50000$$

Here Re is more than 10000. Therefore, Po = constant = K

$$Po = \frac{P}{n^3 D_a^5 \rho} = K$$

i.e., $P = Kn^3 D_a^5 \rho$.

Since D_a and ρ are same for the two cases, we can group $KD_a^5 \rho$ as a constant, say M. i.e., $P = Mn^3$.

The ratio of power required for case 2 to 1 is,

$$\frac{P_2}{P_1} = \frac{n_2^3}{n_1^3} = \frac{200^3}{100^3} = 8$$

The power required for the second case will be 8 times that of the first case. In other words, the power requirement will rise by, $100 \times (8 - 1)/1 = 700\%$.

3.3.19 Dimensional Analysis of Agitation

For geometrically similar baffled stirred tanks, the Power number is known to remain constant at high Reynolds number.

- (a) Let P be the power supplied per unit volume of the fluid, N the revolutions per second of the agitator, ρ the density of the fluid, μ the viscosity of the fluid, and D the diameter of the impeller. Then determine α, β, γ , and δ in the following equation:

$$P = N^\alpha \rho^\beta \mu^\gamma D^\delta$$

- (b) What is the effect of Froude number on P ? (GATE-1994-14)

Solution:

Let us write the Dimensions of various quantities:

Symbol	Unit	Dimension
ρ	$\text{W/m}^3 = \text{kg}/(\text{m.s}^3)$	$ML^{-1}t^{-3}$
N	s^{-1}	t^{-1}
ρ	kg/m^3	ML^{-3}
μ	$\text{kg}/(\text{m.s})$	$ML^{-1}t^{-1}$
D	m	L

Given:

$$P = N^\alpha \rho^\beta \mu^\gamma D^\delta$$

Substituting the dimensions of various quantities:

$$ML^{-1}t^{-3} = t^{-\alpha}(ML^{-3})^\beta(ML^{-1}t^{-1})^\gamma L^\delta$$

Equating the exponents of various dimensions:

For M :

$$1 = \beta + \gamma \quad (3.23)$$

For L :

$$-1 = -3\beta - \gamma + \delta \quad (3.24)$$

For t :

$$-3 = -\alpha - \delta \quad (3.25)$$

From Eqn.(3.23)

$$\beta = 1 - \gamma \quad (3.26)$$

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and, from Eqn.(3.25)

$$\alpha = 3 - \gamma \quad (3.27)$$

Substituting these in Eqn.(3.24),

$$\begin{aligned} -1 &= -3(1 - \gamma) - \gamma + \delta \\ -1 &= -3 + 3\gamma - \gamma + \delta \\ \delta &= 2 - 2\gamma \end{aligned} \quad (3.28)$$

In Eqns.(3.26),(3.27) and (3.28), by substituting $\gamma = 0$ we get, $\alpha = 3$; $\beta = 1$; and $\delta = 2$. Therefore

$$P = N^3 \rho \mu^0 D^2 = \text{const.}$$

i.e.,

$$\frac{P}{N^3 \rho D^2} = P_o = \text{const.}$$

where P_o is the power number.

(b) For baffled vessels, power (P) is not a function of Froude number (Fr).

3.3.20 Scale-up Criteria of Agitation

A small model reactor is to be built for scale up studies of the behavior of a proposed large industrial stirred tank reactor having 1000 times capacity. The bigger unit of 2 m diameter will have a liquid depth of 2 m. This will be fitted with a four bladed Rushton turbine of 0.6 m diameter.

- (a) Estimate the dimensions of the smaller unit.
 (b) For the optimum stirrer speed of 330 rpm observed in the smaller model, what will be the recommended speed in the industrial unit under the following conditions:
 (i) Power per unit volume is kept constant.
 (ii) Reynolds number does not change.
 (c) What design criteria would you recommend for this type of study. (GATE-1995-17)

Solution:

(a) Dimensions of model (smaller unit):

$$\begin{aligned} \text{Capacity of bigger unit} &= (\pi/4)D_B^2 L_B = (\pi/4) \times 2^2 \times 2 = 6.283 \text{ m}^3 \\ \text{Capacity of model} &= 6.283 / 1000 \text{ m}^3 = 6.283 \times 10^{-3} \text{ m}^3 \end{aligned}$$

Diameter of model:

$$\begin{aligned} \pi/4 D_M^3 &= 6.283 \times 10^{-3} \text{ m}^3 \\ D_M &= 0.2 \text{ m} \end{aligned}$$

For geometric similarity between the model and bigger unit,

$$\frac{D_B}{D_M} = \frac{D_{aB}}{D_{aM}}$$

Here

$$\frac{D_B}{D_M} = \frac{2}{0.2} = 10$$

Therefore

$$D_{aM} = \frac{D_{aB}}{10} = \frac{0.6}{10} = 0.06\text{m}$$

i.e., Diameter of impeller of model unit = 0.06 m.

(b-i)

$$\text{Power per unit volume} \propto \frac{n^3 D_a^5}{D_a^3} \propto n^3 D_a^2$$

Therefore, if power per unit volume is to be kept constant,

$$\begin{aligned} [n^3 D_a^2]_B &= [n^3 D_a^2]_M \\ n_B^2 &= \frac{330^3 \times 0.06^2}{0.6^2} \\ n_B &= 71\text{rpm} \end{aligned}$$

Recommended speed of bigger unit = 71 rpm.

(b-ii)

$$\text{Reynolds number} = \frac{n D_a^2 \rho}{\mu}$$

If Reynolds number should not change (and if fluid is not changed),

$$\begin{aligned} \text{Re}_B &= \text{Re}_M \\ [n D_a^2]_B &= [n D_a^2]_M \\ n_B &= \frac{330 \times 0.6^2}{0.06^2} = 3.3\text{rpm} \end{aligned}$$

Tip speed = $\pi D_a n = \pi \times 0.06 \times 3.3 = 6.22 \text{ m/min}$. This tip-speed will give very low degree of agitation. Hence the design criteria should be based on "Power per unit is to be kept constant".

Chapter 4

Thermodynamics

4.1 Instant Notes

4.1.1 Introduction

- Thermodynamics tells us the amount of minimum amount of work necessary for a certain process, but the amount which will actually be used will depend upon many circumstances. Likewise thermodynamics shows us whether a certain reaction may proceed, and what maximum yield may be obtained, but gives no information as to the time required.

Open system mass and energy transfers across the boundary of the system.

Eg: Open vessel having a liquid.

Closed system energy transfers across the boundary of the system.

Eg: A gas confined in a piston-cylinder arrangement the walls of cylinder is a good thermal conductor.

Isolated system no transfer of mass or energy across the boundary of the system.

Eg: A gas confined in a piston-cylinder arrangement the walls of which are made of insulating materials.

Point property The change in property values depend only on the initial and final state of the them.

Eg: internal energy, enthalpy, entropy.

Path property The change in property values depend also on the path by which the process is carried-out, in addition to the initial and final state of the system.
Eg: work, heat.

Extensive properties : depend on the quantity (or extent) of matter specified in the system. Mass and volume are extensive properties. The total value of any extensive property is the sum of the values of the property of individual components into which the system can be subdivided.

Intensive properties : independent of the size of the system. Pressure, temperature, specific volume, density, etc., are intensive properties. Some intensive properties are derived from the extensive properties by specifying the unit amount of the substance concerned. Examples are specific volume, specific heat, and density.

4.1.2 Energy and its Transformations

- Energy transformed as mechanical work:

- Energy possessed by a gas at a high pressure can be transformed into mechanical work by allowing it to expand against a resisting pressure. If the gas is confined in an insulated cylinder, the energy stored in the gas after the expansion is less than that before the expansion because a certain amount of work has been done against the resisting surroundings. The amount of energy given up by the gas can be either be transformed fully into $\int_{P_1}^{P_2} PdV$ work or partially transformed between the limits of $\int_{P_1}^{P_2} PdV$ and 0.
- Work is measurable by the product of an extensive product times the change in an extensive property (i.e., $\int PdV$)

- Energy transformed as heat:

- The temperature integrated with respect to the extensive property, "Entropy" or S , is also used to represent the transfer of thermal energy. The increase in thermal energy stored in a system is $\int_{S_1}^{S_2} TdS$. It is important to remember that an increase in thermal energy in the system may not be exclusively due to heat effect. In other words, the change of thermal energy stored in the system may not be equal to q . For example, if one kg of water confined in an insulated vessel is agitated by a stirrer until enough frictional dissipation occurs to raise the temperature from 80 to 90°C, its is evident that the thermal energy stored in the system is increased. Since the vessel is insulated, the heat transfer q is equal to zero. Therefore, it would appear that

$$q \leq \int_{S_1}^{S_2} TdS$$

- The value of q is between the maximum, $\int_{S_1}^{S_2} TdS$, and the minimum zero.

4.1.3 Reversible and Irreversible Processes

- **Spontaneous process:** A process is said to be "spontaneous" when at least one intensive property of the system is greater than that of the surroundings. Under this condition the process can be conducted at least for one variation at a finite rate without any external aid or, in other words, is spontaneous.

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- The thermodynamic criteria for irreversible processes are:
(Δ = final - initial)

$$\Delta G_{T,P} < 0 \quad (4.1)$$

$$\Delta A_{T,V} < 0 \quad (4.2)$$

$$\Delta U_{S,V} < 0 \quad (4.3)$$

$$\Delta H_{S,P} < 0 \quad (4.4)$$

Since, from fundamental property relations for a system of constant composition:

$$dG = -SdT + VdP$$

$$dA = -PdV - SdT$$

$$dU = TdS - PdV$$

$$dH = VdP + TdS$$

4.1.4 First Law of Thermodynamics

- Work done in reversible process:

$$W = - \int PdV$$

- Work done in irreversible process:

$$W = -P_r \Delta V$$

where P_r is the resisting pressure; and $\Delta V = V_2 - V_1$. V refers to total volume.

- **First law of thermodynamics**
Closed system

$$\Delta U = Q + W$$

Sign conventions: Q and W coming to the system (i.e., done on the system) are taken as positive; and leaving the system (i.e., done by the system) are taken as negative.

- **Open system**

$$\Delta H + \frac{\Delta u^2}{2} + g\Delta z = Q + W_s$$

where u = velocity

W_s = shaft work

If kinetic and potential energy changes are negligible, then

$$\Delta H = Q + W_s$$

- Heat capacity dQ/dT

$$C_v = \left(\frac{\partial U}{\partial T} \right)_v$$

$$C_p = \left(\frac{\partial H}{\partial T} \right)_p$$

- Internal Energy U

For ideal gas U is a function of T only, and

$$\Delta U = \int C_v dT$$

- Enthalpy H : $H = U + PV$. For ideal gas H is a function of T only.

- Polytropic exponent δ , in $PV^\delta = \text{const.}$

Isobaric process (const. P) $\delta = 0$

Isochoric process (const. V) $\delta = \infty$

Isothermal process (const. T) $\delta = 1$

Isentropic process (const. S) $\delta = k = C_p/C_v$

- Work in isothermal process:

$$W = nRT \ln \left(\frac{P_2}{P_1} \right)$$

- Work in polytropic process:

$$W = \frac{P_2 V_2 - P_1 V_1}{\delta - 1}$$

where δ is the polytropic exponent. If $\delta = \gamma$ the above expression gives the work of adiabatic process.

4.1.5 PVT Relationships

- Ideal gas law: $Pv = RT$. Any gas can be considered as ideal under low P and high T . v refers to molar volume.

- van der Waals gas:

$$\left(P + \frac{a}{v^2} \right) (v - b) = RT$$

At critical conditions ($T = T_c$ and $P = P_c$)

$$\left(\frac{\partial P}{\partial V} \right)_T = \left(\frac{\partial^2 P}{\partial V^2} \right)_T = 0$$

- Compressibility factor:

$$Z = \frac{Pv}{RT}$$

- Virial equations:

$$Z = 1 + \frac{B}{v} + \frac{C}{v^2}$$

Another form:

$$Z = 1 + B'P + C'P^2$$

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4.1.6 Second Law of Thermodynamics

- Carnot engine: 'A heat engine operating in a completely reversible manner'.

$$\eta = \frac{W_{\text{net}}}{Q_H} = \frac{|Q_H| - |Q_C|}{|Q_H|} = 1 - \frac{T_C}{T_H}$$

- Entropy: a state function.

$$dS = \frac{dQ_{\text{rev}}}{T}$$

- Entropy change for an isothermal process:

$$\Delta S = \frac{\int dQ}{T}$$

- Entropy changes of an ideal gas:

$$\Delta S = nC_p \ln \frac{T_2}{T_1} - nR \ln \frac{P_2}{P_1}$$

also

$$\Delta S = nC_v \ln \frac{T_2}{T_1} + nR \ln \frac{V_2}{V_1}$$

- Entropy change of mixing of two gases at same T and P :

$$\Delta S = - \sum n_i R \ln x_i$$

- Mathematical statement of the second law:

$$\Delta S_{\text{Total}} \geq 0$$

where $\Delta S_{\text{Total}} = \Delta S_{\text{system}} + \Delta S_{\text{surrounding}}$

$\Delta S_{\text{Total}} = 0$ for a reversible process.

No process is possible for which the total entropy decreases.

- Third law of thermodynamics: Absolute entropy is zero for all perfectly crystalline substances at absolute zero temperature. (i.e., $S = 0$ at $T = 0$ K)

4.1.7 Refrigeration

- Refrigeration:

$$\text{Coefficient of Performance} = \frac{\text{cooling effect produced}}{\text{net work required}}$$

- 1 ton of refrigeration: energy required to make 1 ton of ice per day from water at 0°C .

$$1 \text{ ton of refrigeration} = 50 \text{ kcal/min}$$

- Heat pump:

$$\text{Coefficient of Performance} = \frac{\text{heating effect produced}}{\text{net work required}}$$

4.1.8 Thermodynamic Relations

- From the definition of Gibbs free energy

$$G = H - TS$$

and from the definition of Helmholtz free energy

$$A = U - TS$$

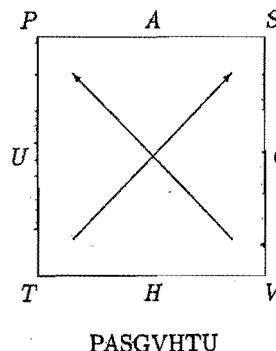
- Fundamental property relations:

$$dU = TdS - PdV$$

$$dH = TdS + VdP$$

$$dA = -PdV - SdT$$

$$dG = VdP - SdT$$



- If $F = F(x, y)$ then

$$dF = Mdx + Ndy$$

Exactness Criteria:

$$\left(\frac{\partial M}{\partial y}\right)_x = \left(\frac{\partial N}{\partial x}\right)_y$$

- If $F = F(x, y, z)$ then

$$dF = Mdx + Ndy + Pdz$$

Exactness Criteria:

$$\left(\frac{\partial M}{\partial y}\right)_{x,z} = \left(\frac{\partial N}{\partial x}\right)_{y,z}$$

$$\left(\frac{\partial M}{\partial z}\right)_{x,y} = \left(\frac{\partial P}{\partial x}\right)_{y,z}$$

$$\left(\frac{\partial N}{\partial z}\right)_{x,y} = \left(\frac{\partial P}{\partial y}\right)_{x,z}$$

- Maxwell Relations:

$$\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial P}{\partial S}\right)_V$$

$$\left(\frac{\partial P}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T$$

$$\left(\frac{\partial S}{\partial P}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_P$$

$$\left(\frac{\partial V}{\partial S}\right)_P = \left(\frac{\partial T}{\partial P}\right)_S$$

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- For a closed system, $d(nG) = (nV)dP - (nS)dT$.
- For open system of constant or variable composition

$$d(nG) = (nv)dP - (nS)dT + \sum \mu_i dn_i$$

where

$$\mu_i = \left[\frac{\partial(nG)}{\partial n_i} \right]_{P,T,n_j}$$

- Phase change: for a pure component T , P and G are constant.

- Clapeyron equation:

$$\frac{dP^{\text{sat}}}{dT} = \frac{\Delta H^{\text{lv}}}{T \Delta V^{\text{lv}}}$$

- Clausius-Clapeyron equation:

$$-\text{Assumptions: } \Delta V^{\text{lv}} = V^v = \frac{RT}{P^{\text{sat}}}$$

$$\frac{dP^{\text{sat}}/P^{\text{sat}}}{dT/T^2} = \frac{\Delta H^{\text{lv}}}{R}$$

i.e.,

$$\ln \frac{P_2^{\text{sat}}}{P_1^{\text{sat}}} = \frac{\Delta H}{R} \left(\frac{1}{T_1} - \frac{1}{T_2} \right)$$

- According to the Clausius-Clapeyron equation ΔH^{lv} is almost constant, virtually independent of T . This is not true; ΔH^{lv} decreases monotonically with increasing temperature from the triple point temperature to the critical point, where it becomes zero. The assumptions on which the Clausius-Clapeyron equation are based have approximate validity only at low pressures.

- Antoine equation:

$$\ln P^{\text{sat}} = A - \frac{B}{T+C}$$

- Mollier diagram: $H - S$ diagram for water.

- Residual Properties: Any extensive property M is given by:

$$M = M^{\text{ig}} + M^R$$

where M^{ig} is ideal gas value of the property, and M^R is the residual value of the property.

For example for the extensive property V :

$$V = V^{\text{ig}} + V^R = \frac{RT}{P} + V^R$$

Since $V = ZRT/P$

$$V^R = \frac{RT}{P}(Z - 1)$$

4.1.9 Flow Processes

- Mach number = $\frac{\text{velocity of fluid}}{\text{velocity of sound in fluid}}$

- In a pipe of constant cross-section for the flow of compressible flow, the maximum velocity that could be reached is the velocity of sound.

- Joule-Thomson coefficient μ_J

$$\mu_J = \left(\frac{\partial T}{\partial P} \right)_H$$

- For an ideal gas $\mu = 0$. For a real gas μ_J can take any values including zero.
- For a real gas, $\mu_J = 0$ at the inversion point.
- If μ_J is positive, the substance cools upon being throttled to a lower pressure. If μ_J is negative, temperature increases upon throttling to low pressure.
- Any gas for which volume is linear with temperature along an isobar will have a zero Joule-Thomson coefficient. i.e., if $V/T = \text{constant} = \phi(P)$, $\mu_J = 0$.
- Inversion curve: $T - P$ diagram. The points in the curve correspond to $\mu_J = 0$. In the region inside the curve μ_J is positive.

- Compression

For reversible-adiabatic compression

$$W = \frac{\gamma P_1 V_1}{\gamma - 1} \left[\left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right]$$

- Multistage compression:

$$\text{Optimum compression ratio per stage} = \left(\frac{P_2}{P_1} \right)^{1/n}$$

- For a given pressure rise isothermal compression requires less work compared to adiabatic compression. To approach isothermal compression, compression may be carried out stages.

4.1.10 Solution Thermodynamics

- Partial properties: P , T and moles other than the species under consideration are constant.
- Partial molal volume:

$$\bar{V}_i = \left[\frac{\partial(nV)}{\partial n_i} \right]_{P,T,n_j}$$

$$V = \sum x_i \bar{V}_i$$

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- Partial molal quantities are intensive properties. They depend not upon the total amount of each constituent but only upon the composition.

- Fugacity coefficient $\phi = f/P$

$$\ln \phi = \int_0^P (Z - 1) \frac{dP}{P} \quad (\text{at const. } T)$$

- Fugacity of an ideal gas is equal to its pressure. $f = P$.
- Deviation from ideal gas behavior - residual properties.
- Deviation from ideal solution behavior - excess properties.
- Gibbs/Duhem equation:

$$\sum x_i d\bar{M}_i = 0 \quad (\text{const. } T, P)$$

- Other forms of Gibbs/Duhem equation:

$$x_1 \frac{d \ln \gamma_1}{dx_1} + x_2 \frac{d \ln \gamma_2}{dx_1} = 0 \quad (\text{const. } T \text{ and } P)$$

- Positive deviations from ideality: $\gamma_i > 1$ and $\ln \gamma_i > 0$.
- For ideal gases, enthalpy change of mixing is zero.

4.1.11 Phase Equilibrium

- All irreversible processes occurring at constant T and P proceed in such a direction as to cause a decrease in Gibbs energy of the system

$$(dG)_{T,P} \leq 0$$

The equilibrium state of a closed system is that state for which the total Gibbs energy is a minimum with respect to all possible changes at a given T and P .

- For phase equilibrium $\mu_i^\alpha = \mu_i^\beta$.
- Raoult's law:

$$y_i P = x_i P_i^{\text{sat}}$$

4.1.12 Reaction Equilibrium

- If a mixture of chemical species is not in chemical equilibrium, any reaction that occurs must be irreversible and, if the system is maintained at constant T and P , the total Gibbs energy of the system must decrease.
- Feasibility of a chemical reaction:

- When $\Delta G_{T,P} = 0$, the reaction is in equilibrium and no further change may occur.
- When $\Delta G_{T,P} < 0$, the reaction is promising.
- When $\Delta G_{T,P} > 0$, the reaction is unfavorable.
- For an impossible reaction, the value of K should be equal to zero or ΔG° should be infinite.

• Equilibrium constant K :

- unaffected by the pressure of the system, by the presence or absence of inert, or by the kinetics of the system, but is affected by the temperature of the system.
- though K is unaffected by pressure or inert, the equilibrium concentration of materials and equilibrium conversion of reactants can be influenced by these variables.
- The value $K \gg 1$ indicates that practically complete conversion is possible and that the reaction can be considered to be irreversible. $K \ll 1$ indicates that reaction will not proceed to any appreciable extent.
- For an increase in temperature, equilibrium conversion rises for endothermic reaction and drops for exothermic reaction.
- For an increase in pressure in gas reactions, conversion rises when the number of moles decreases with reaction; conversion drops when the number of moles increases with reaction.
- A decrease of inert for all reactions acts in the way that an increase in pressure acts for gas reactions.

4.2 Objective Type Questions

1. It is desired to bring about a certain change in the state of a system by performing work on the system under adiabatic conditions. (GATE-1998-2.17)

- (a) The amount of work needed is path-dependent.
- (b) Work alone cannot bring about such a change of state
- (c) The amount of work needed is independent of path
- (d) More information is needed to conclude anything about the path dependence or otherwise the work needed

Answer: (c) From first law of thermodynamics, $dU = dQ + dW$. For adiabatic process $dQ = 0$. Therefore $dU = dW$. Since U is a point function, W is also point function in this case.

2. For an ideal gas the slope of the pressure-volume curve at a given point, will be (GATE-1991-7.ii)

- (a) steeper for an isothermal than for an adiabatic process
- (b) steeper for an adiabatic than for an isothermal process
- (c) identical for both the processes
- (d) of opposite signs

Answer: (b) For isothermal process $PV = \text{constant}$; or $\ln P \propto -\ln V$. For adiabatic process $PV^\gamma = \text{constant}$; or $\ln P \propto -\gamma \ln V$. Since $\gamma > 1$, adiabatic curve is more steeper than isothermal curve.

3. The first law of thermodynamics takes the form $W = \Delta H$ when applied to: (GATE-1993-5.c)

- (a) A closed system undergoing a reversible adiabatic process
- (b) An open system undergoing an adiabatic process with negligible changes in kinetic and potential energies
- (c) A closed system undergoing a reversible constant volume process
- (d) A closed system undergoing a reversible constant pressure process

Answer: (b) First law for open system is $Q + W = \Delta H$; and for closed system is $Q + W = \Delta U$. For a closed system undergoing a reversible adiabatic process, $Q = 0$; hence $\Delta U = W$. For a closed system undergoing a reversible constant volume process $W = 0$; hence $\Delta U = Q$ for such process. For a closed system undergoing a reversible constant pressure process $Q + W = \Delta U$.

4. Air enters an adiabatic compressor at 300 K. The exit temperature for a compression ratio of 3, assuming air to be an ideal gas ($\gamma = C_P/C_V = 7/5$) and the process to be reversible, is (GATE-2001-2.07)

- (a) $300(3^{2/7})$
- (b) $300(3^{3/5})$
- (c) $300(3^{3/7})$
- (d) $300(3^{5/7})$

Answer: (a) For reversible adiabatic process $PV^\gamma = \text{constant}$. From ideal gas law $PV = RT$. Therefore, $P^{1-\gamma}T^\gamma = \text{constant}$. Given: $P_2/P_1 = 3$. $T_2/T_1 = \frac{P_2^{(\gamma-1)/\gamma}}{P_1^{(\gamma-1)/\gamma}}$ which gives $T_2 = 300(3^{2/7})$

5. The kinetic energy of gas molecules is zero at (GATE-1995-1.n)

- (a) 0°C
- (b) 273°C
- (c) 100°C
- (d) -273°C

Answer: (d)

6. A Carnot cycle consists of the following steps: (GATE-1998-1.22)

- (a) two isothermal and two isentropics
- (b) two isobarics and two isotherms
- (c) two isochorics and two isobars
- (d) two isotherms and two isochorics

Answer: (a)

7. The shape of $T - S$ diagram for Carnot cycle is (GATE-1991-7.v)

- (a) a rectangle (b) a rhombus (c) a trapezoid (d) a circle

Answer: (a) Carnot cycle: Two isotherms (const. T), and Two isentropes (const. S).

8. Mixing of two fluids is a :

- | | |
|------------------------|--------------------------|
| (a) Reversible process | (b) Irreversible process |
| (c) Isothermal process | (d) None of these |

Answer: (b)

9. Entropy change for an irreversible process, taking into account both the system and surroundings together, is:

- (a) Positive (b) Negative (c) Zero (d) None of these

Answer: (a) For any possible process $\Delta S \geq 0$. $\Delta S = 0$ for reversible changes in the system and surroundings and for reversible adiabatic process.

10. Entropy is : (GATE-1989-6.ii.b)

- | | |
|------------------------|-----------------------|
| (a) Intensive property | (b) Derived property |
| (c) Extensive property | (d) None of the above |

Answer: (c) Molar or specific entropy is intensive property. $P, V, T, U \& S$ are primary properties; H, A, G and other are derived properties.

11. A reversible heat transfer demands: (GATE-1993-5.d)

- (a) The temperature difference causing heat transfer tends to zero
- (b) The system receiving heat must be at a constant temperature
- (c) The system transferring out heat must be at a constant temperature
- (d) Both interacting systems must be at constant temperatures

Answer: (a)

12. Which among the following relations is/are valid ONLY for reversible process undergone by a pure substance? (GATE-1993-5.e)

- | | |
|--------------------------------|---------------------------|
| (a) $\delta Q = dU + \delta W$ | (b) $TdS = dU + \delta W$ |
| (c) $TdS = dU + pdV$ | (d) $\delta Q = pdV + dU$ |

Answer: (c) $\delta Q = dU + \delta W$ applies to any process in closed system. For reversible process $\delta Q = TdS$, and $\delta W = pdV$. (here, work done by the system is taken as positive).

13. When a system undergoes an irreversible cycle: (GATE-1993-5.f)

- (a) $\oint dQ/T < 0$ (b) $\oint dS > 0$ (c) $\oint dS = 0$ (d) $\oint dQ/T > 0$

Answer: (a) Clausius inequality is $\oint dQ/T \leq 0$ which is the mathematical statement of second law. $\oint dQ/T = 0$ for any reversible process; $\oint dQ/T > 0$ for any impossible process.

$\oint dS = 0$ for any process (reversible or irreversible) as S is a point function.

14. The second law of thermodynamics states that (GATE-1994-1.t)

- (a) the energy change of a system undergoing any reversible process is zero
- (b) it is not possible to transfer heat from a lower temperature to a higher temperature
- (c) the total energy of the system and surroundings remains constant
- (d) none of the above

Answer: (b) Statement of first law: "total energy of the system and surroundings remains at constant".

15. On a $P - V$ diagram of an ideal gas, suppose a reversible adiabatic line intersects a reversible isothermal line at point A. Then at point A, the slope of the reversible adiabatic line $(\partial P/\partial V)_S$ and the slope of the reversible isothermal line $(\partial P/\partial V)_T$ are related as (GATE-2000-1.06)

- | | |
|---|---|
| (a) $(\partial P/\partial V)_S = (\partial P/\partial V)_T$ | (b) $(\partial P/\partial V)_S = [(\partial P/\partial V)_T]^\gamma$ |
| (c) $(\partial P/\partial V)_S = \gamma(\partial P/\partial V)_T$ | (d) $(\partial P/\partial V)_S = (1/\gamma)(\partial P/\partial V)_T$ |

where $\gamma = C_P/C_V$

Answer: (c) For a isothermal ($T = \text{const.}$) process $PV = \text{const.} = P_1V_1$; $P = P_1V_1/V$; $\partial P/\partial V = -P_1V_1/V^2$; $(\partial P/\partial V)_1 = -P_1V_1/V_1^2 = -P_1/V_1$.

For reversible adiabatic ($S = \text{const.}$) process $PV^\gamma = \text{const.} = P_1V_1^\gamma$; $(\partial P/\partial V)_1 = -\gamma P_1/V_1$. Therefore $(\partial P/\partial V)_S = \gamma(\partial P/\partial V)_T$

16. A system undergoes a change from a given initial state to a given final state either by an irreversible process or by a reversible process. Then (GATE-1997-1.19)

- | | |
|---|--|
| (a) ΔS_I is always $> \Delta S_R$ | (b) ΔS_I is sometimes $> \Delta S_R$ |
| (c) ΔS_I is always $< \Delta S_R$ | (d) ΔS_I is always $= \Delta S_R$ |

where ΔS_I and ΔS_R are the entropy changes of the system for the irreversible and reversible processes, respectively.

Answer: (d) as S is a state function.

17. A closed system is cooled reversibly from 100°C to 50°C. If no work is done on the system
(GATE-1995-2.r)

- (a) its internal energy (U) decreases and its entropy (S) increases
- (b) U and S both decreases
- (c) U decreases but S is constant
- (d) U is constant but S decreases

Answer: (b) S and U decreases with decrease in T . $S = 0$ when $T = 0$ K (from third law of thermodynamics).

18. The thermal efficiency of a reversible heat engine operating between two given thermal reservoirs is 0.4. The device is used either as a refrigerator or as a heat pump between the same reservoirs. Then the coefficient of performance as a refrigerator (COP_R) and the coefficient of performance as a heat pump (COP_{HP}) are
(GATE-2000-2.05)

- (a) $(COP)_R = (COP)_{HP} = 0.6$
- (b) $(COP)_R = 2.5$; $(COP)_{HP} = 1.5$
- (c) $(COP)_R = 1.5$; $(COP)_{HP} = 2.5$
- (d) $(COP)_R = (COP)_{HP} = 2.5$

Answer: (c) For $T_1 > T_2$, $\eta = \frac{T_1 - T_2}{T_1} = 0.4$

$$COP_{HP} = \frac{T_1}{T_1 - T_2} = 1/\eta = 2.5$$

$$COP_R = \frac{T_2}{T_1 - T_2} = \frac{T_1}{T_1 - T_2} - 1 = 2.5 - 1 = 1.5$$

19. Which of the following is true for Virial equation of state
(GATE-1999-1.07)

- (a) Virial coefficients are universal constants
- (b) Virial coefficient B represents three body interactions
- (c) Virial coefficients are functions of temperature only
- (d) For some gases, Virial equations and ideal gas equations are the same

Answer: (c) Virial coefficient B represents two body interactions; C represents three body interactions, and D represents four body interactions. Normally two body and two body interactions are more common than higher body interactions. Hence virial equations are truncated with terms containing C .

Virial equations and ideal gas equations are same for all gases in ideal gas state (at low P and high T).

20. A solid is transformed into its vapor state without passing through the liquid state at:

- (a) Triple point
- (b) Boiling point
- (c) Always
- (d) Below triple point

Answer: (d) This process is called as sublimation.

21. For a system in equilibrium, at a given temperature and pressure:
(GATE-1991-7.iii)

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- (a) the entropy must be a minimum
- (b) the enthalpy must be a minimum
- (c) the internal energy must be a minimum
- (d) the Gibbs-free energy must be a minimum

Answer: (d) Any process proceeds in the direction of decreasing free energy. At equilibrium $dG = 0$.

22. To obtain the integrated form of Clausius-Clapeyron equation

$$\ln \frac{P_2}{P_1} = \frac{\Delta H_v}{R} \left(\frac{1}{T_1} - \frac{1}{T_2} \right)$$

from the exact Clapeyron equation, it is assumed that:

(GATE-1991-7.iv)

- (a) the volume of the liquid phase is negligible compared to that of the vapor phase
- (b) the vapor phase behaves as an ideal gas
- (c) the heat of vaporization is independent of temperature
- (d) all of the above are applicable

Answer: (d) Clapeyron equation is

$$\frac{dP}{dT} = \frac{\Delta H_v}{T \Delta V}$$

From the first assumption $\Delta V \approx V^v$. From the second assumption $V^v = RT/P$.

23. Ideal gas law is applicable at

(GATE-1994-1.s)

- (a) low T , low P
- (b) high T , high P
- (c) low T , high P
- (d) high T , low P

Answer: (d) Any gas can be considered as ideal if it is at $T \gg T_r$ and $P \ll P_r$; where r refers to reduced conditions. i.e., $T_r = T/T_c$ and $P_r = P/P_c$.

24. Maxwell's relation corresponding to the identity, $dH = TdS + VdP + \sum \mu_i dn_i$ is
(GATE-1999-2.05)

- (a) $(\partial T/\partial V)_{S,ni} = -(\partial P/\partial S)_{V,ni}$
- (b) $(\partial S/\partial P)_{T,ni} = (\partial V/\partial T)_{P,ni}$
- (c) $(\partial S/\partial V)_{T,ni} = (\partial P/\partial T)_{V,ni}$
- (d) $(\partial T/\partial P)_{S,ni} = (\partial V/\partial S)_{P,ni}$

Answer: (d)

25. A change in state involving a decrease in entropy can be spontaneous only if
(GATE-1998-1.21)

- (a) it is exothermic
- (b) it is isenthalpic
- (c) it takes place isothermally
- (d) it takes place at constant volume

37. During the change of phase of a pure substance: (GATE-1993-5.h)

- (a) $dG = 0$ (b) $dp = 0$ (c) $dH = 0$ (d) $dU = 0$

Answer: (a, b)

38. At the triple point of a pure substance, the number of degrees of freedom is: (GATE-1993-5.i)

- (a) 0 (b) 1 (c) 2 (d) 3

Answer: (a)

39. The number of degrees of freedom for a mixture of ice and water (liquid) are (GATE-1996-1.02)

- (a) 2 (b) 3 (c) 1 (d) 0

Answer: (c) $F = C - P + 2 = 1 - 2 + 2 = 1$.

40. For gas and liquid phases in equilibrium, a component in a non-ideal system follows the relation, (GATE-1989-6.ii.a)

- (a) $p_i = x_i P_i^s$ (b) $f_i^L = f_i^V$ (c) $p_i = y_i \pi$ (d) $f_i^V = p_i$

Answer: (b) The relation (a) is statement of Raoult's law; and it is valid for ideal solution. (c) is the relation for partial pressure to total pressure; and (d) is valid for ideal vapor phase.

41. The necessary and sufficient condition for equilibrium between two phases is (GATE-1992-7.c)

- (a) concentration of each component should be same in the two components
- (b) the temperature of each phase should be same
- (c) the pressure should be same in two phases
- (d) the chemical potential of each component should be same in two phases

Answer: (d)

42. The theoretical minimum work required to separate one mole of a liquid mixture at 1 atm, containing 50 mole % each of n-heptane and n-octane into pure compounds each at 1 atm is (GATE-1996-2.09)

- (a) $-2RT \ln 0.5$ (b) $-RT \ln 0.5$ (c) $0.5RT$ (d) $2RT$

Answer: (b) Minimum work required = $T\Delta S = T(-\sum n_i R \ln x_i) = -RT \ln 0.5$.

43. Criterion of a chemical equilibrium is that the total Gibbs free energy change is: (GATE-1988-6.b.ii)

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- | | |
|---------------------|------------------------------|
| (a) always positive | (b) always negative |
| (c) zero | (d) not specifically defined |

Answer: (c)

44. The equilibrium constant K for a chemical reaction depends on (GATE-1997-1.2)

- | | |
|------------------------------|------------------------|
| (a) temperature only | (b) pressure only |
| (c) temperature and pressure | (d) ratio of reactants |

Answer: (a)

45. The decomposition of A into B is represented by the exothermic reaction $A \rightleftharpoons B$. To achieve maximum decomposition, it is desirable to carry out the reaction (GATE-1997-1.22)

- | | |
|------------------------------|-----------------------------|
| (a) at high P and high T | (b) at low P and high T |
| (c) at low P and low T | (d) at high P and low T |

Answer: (d)

4.3 Problems with Solutions

4.3.1 Rise in Temperature of Air

A vertical cylinder with a freely floating piston contains 0.1 kg air at 1.2 bar and a small electric resistor. The resistor is wired to an external 12 Volt battery. When a current of 1.5 Amps is passed through the resistor for 90 sec, the piston sweeps a volume of 0.01 m³. Assume (i) piston and the cylinder are insulated and (ii) air behaves as an ideal gas with $C_v = 700 \text{ J}/(\text{kg.K})$. Find the rise in temperature of air. (GATE-1993-6.2)

Solution:

Heat flow into the system:

$$Q = VIt = 12 \times 1.5 \times 90 = 1620 \text{ J.}$$

Work done by system:

$$W = \int PdV = -P\Delta V = -1.2 \times 10^5 \times 0.01 = -1200 \text{ J.}$$

From first law of thermodynamics

$$\Delta U = Q + W = 1620 - 1200 = 420 \text{ J}$$

For an ideal gas

$$\Delta U = mC_v\Delta T$$

Therefore,

$$\Delta T = \frac{\Delta U}{mC_v} = \frac{420}{0.1 \times 700} = 6^\circ\text{C}$$

Rise in temperature of air = 6°C .

4.3.2 Work Required to Compress an Ideal Gas

100 m³ of carbon dioxide initially at 423 K and 50 bar (50×10^5 Pa) is to be isothermally compressed in a frictionless piston and cylinder device to a final pressure of 300 bar (300×10^5 Pa). Assuming ideal gas behavior ($R = 8.314 \times 10^{-2}$ bar.m³/kmol.K),

- (a) Write a general expression for the energy balance for the gas within the piston and cylinder device as the system, and define all the terms.
- (b) Calculate the volume of the compressed carbon dioxide gas at 300 bar.
- (c) Calculate the work done to compress the carbon dioxide gas.
- (d) Calculate the heat flow on compression.

(GATE-2001-6)

Solution:

For the system given

$$\Delta U = Q + W \quad (4.5)$$

where ΔU = Internal energy changes in the gas

Q = Amount of heat transferred to or from the gas

W = Amount of work transferred to or from the gas

For an ideal gas

$$PV = nRT$$

where V is total volume.

For isothermal compression, $T_1 = T_2$. Therefore

$$P_1V_1 = P_2V_2$$

From which

$$V_2 = \frac{P_1V_1}{P_2} = \frac{50 \times 100}{300} = 16.7 \text{ m}^3$$

$$W = - \int P dV \quad (4.6)$$

For an ideal gas ($P = nRT/V$). Therefore

$$\begin{aligned} W &= - \int_{V_1}^{V_2} nRT \frac{dV}{V} \\ &= nRT \ln \frac{V_1}{V_2} \\ &= P_1V_1 \ln \frac{V_1}{V_2} \\ &= 50 \times 10^5 \times 100 \times \ln \frac{100}{16.7} \\ &= 896 \times 10^6 \text{ J} = 896 \text{ MJ} \end{aligned}$$

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For isothermal process $\Delta T = T_2 - T_1 = 0$. Since for ideal gas $\Delta U = nC_v\Delta T$, and from Eqn.(4.5),

$$\begin{aligned} \Delta U &= Q + W \\ 0 &= Q + W \\ Q &= -W = -896 \text{ MJ} \end{aligned}$$

Heat removed from the system = 896 MJ.

4.3.3 Triple Point Temperature and Pressure

In the vicinity of the triple point, the vapor pressures of liquid and solid ammonia are respectively given by

$$\ln p = 15.16 - 3063/T \text{ and } \ln p = 18.70 - 3754/T$$

where p is in atmospheres and T is in Kelvin. What is the temperature at the triple point? Also calculate the triple point pressure. (GATE-1993-6.8)

Solution:

At the triple point, all the phases (solid, liquid and vapor) exist in equilibrium. Therefore,

$$\text{vapor pressure of liquid ammonia} = \text{vapor pressure of solid ammonia}$$

$$15.16 - 3063/T = 18.7 - 3754/T$$

$$3754/T - 3063/T = 18.7 - 15.16$$

$$3754 - 3063 = T(18.7 - 15.16)$$

$$691 = 3.54T$$

$$T = 691/3.54 = 195.2 \text{ K}$$

Triple point temperature = 195.2 K

Triple point pressure can be obtained from any of the vapor pressure to temperature relation. By the vapor pressure to temperature relation of liquid ammonia,

$$\ln p = 15.16 - 3063/195.2$$

$$p = 0.588 \text{ atm}$$

Similarly, by the vapor pressure to temperature relation of solid ammonia,

$$\ln p = 18.7 - 3754/195.2$$

$$p = 0.588 \text{ atm}$$

Triple point pressure = 0.588 atm

4.3.4 Thermal Efficiency of Carnot Cycle

10^6 Joules of heat are transferred from a reservoir at 327°C to an engine that operates on the Carnot cycle. The engine rejects heat to a reservoir at 27°C. Determine the thermal

efficiency of the cycle and the work done by the engine.

(GATE-1992-17a)

Solution:

Thermal efficiency η , work done W , and, heat transferred Q , are related to reservoir temperatures at T_1 and T_2 as

$$\eta = \frac{T_1 - T_2}{T_1} = \frac{W}{Q}$$

$$\eta = \frac{(327 + 273) - (27 + 273)}{(327 + 273)} = 0.5 = 50\%$$

Therefore, $W/Q = 0.5$. i.e.,

$$\frac{W}{10^6} = 0.5$$

$$W = 0.5 \times 10^6 \text{ Joules}$$

4.3.5 Evaluating a Claim

An inventor claims to have developed a refrigeration unit which maintains the refrigerated space at -3°C while operating in a room where the temperature is 27°C , and which has a coefficient of performance of 9.5. How do you evaluate his claim? (GATE-1992-17b)

Solution:

Given: $T_C = -3^\circ\text{C} = -3 + 273 \text{ K} = 270 \text{ K}$; and $T_H = 27^\circ\text{C} = 300 \text{ K}$.

For refrigeration cycle

$$\text{Coefficient of Performance (COP)} = \frac{\text{Refrigerating effect}}{\text{Work supplied}}$$

For a Carnot cycle working between the temperature limits of T_C and T_H ,

$$COP = \frac{T_C}{T_H - T_C} = \frac{270}{300 - 270} = 9$$

COP of Carnot cycle (fully reversible cycle) is the maximum. And COP more than this is impossible, as this will lead to violation of second law of thermodynamics.

Since the COP claimed by the inventor (9.5) is more than the COP of carnot cycle (9), the claim is a false one.

4.3.6 Entropy Change in Mixing of Two Gases

A rigid insulated cylinder has two compartments separated by a thin membrane. While one compartment contains 1 kmol nitrogen at a certain temperature and pressure, the other contains 1 kmol of CO_2 at the same temperature and pressure. The membrane is ruptured and the two gases are allowed to mix. Assume that the gases behave as ideal gases. Calculate the increase in entropy of the contents of the cylinder. Universal gas constant $R =$

8314 J/kmol.K

(GATE-1993-6.7)

Solution:

Entropy change (ΔS) in mixing of two gases at same T and P is given by

$$\Delta S = - \sum n_i R \ln x_i$$

where n_i = number of moles of component i

x_i = mole fraction of i in the mixture

Here

$$\begin{aligned} \Delta S &= - \left[\underbrace{(1 \times R \times \ln 0.5)}_{\text{for } N_2} + \underbrace{(1 \times R \times \ln 0.5)}_{\text{for } CO_2} \right] \\ &= -2R \ln 0.5 \\ &= 11526 \text{ J/K} \end{aligned}$$

4.3.7 Irreversibility

Air expands steadily through a turbine from 6 bar, 800 K to 1 bar, 520 K. During the expansion, heat transfer from air to the surroundings at 300 K is 10 kJ/kg air. Neglect the changes in kinetic and potential energies and evaluate the irreversibility per kg air. Assume air to behave as an ideal gas with $C_p = 1.0 \text{ kJ}/(\text{kg.K})$ and $R = 0.3 \text{ kJ}/(\text{kg.K})$ (GATE-1993-6.4)

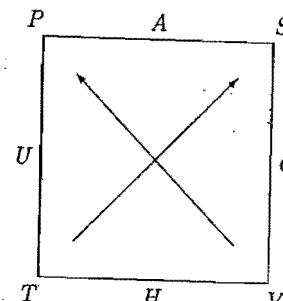
Solution:

Considering entropy (S) as a function of T and P ,

$$S = S(T, P)$$

Differentiating,

$$dS = \left(\frac{\partial S}{\partial T} \right)_P dT + \left(\frac{\partial S}{\partial P} \right)_T dP$$



From the diagram and from the relation for entropy variation and C_p ,

$$dS = \frac{C_p}{T} dT - \left(\frac{\partial V}{\partial T} \right)_P dP$$

For an ideal gas $PV = RT$. Therefore,

$$\left(\frac{\partial V}{\partial T} \right)_P = \frac{R}{P}$$

With this substitution

$$dS = \frac{C_p}{T} dT - \frac{R}{P} dP$$

In integral form

$$\Delta S = C_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$$

Substituting for the variables,

$$\Delta S = 1 \times \ln(520/800) - 0.3 \times \ln(1/6) = 0.107 \text{ kJ/kg.K}$$

Heat transferred from air to surrounding at 300 K = 10 kJ/kg

$$\text{Corresponding entropy change} = \frac{-10}{300} = -0.033 \text{ kJ/kg.K}$$

Total entropy change = $\Delta S_{\text{Total}} = 0.107 - 0.033 = 0.074 \text{ kJ/kg.K}$

Irreversibility per kg of air = 0.074 kJ/K.

4.3.8 COP of Refrigeration Cycle

In a vapor-compression refrigeration system, designed to maintain an enclosure at -3°C , the equipment are so sized that all heat transfer units realize an approach of 5°C . Cooling water is available at 32°C . Given that:

Enthalpy of saturated refrigerant vapor = 180 kJ/kg

Enthalpy of refrigerant vapor leaving the compressor = 206 kJ/kg

Enthalpy of saturated liquid leaving the throttle valve = 61.0 kJ/kg

Estimate:

(a) the maximum possible C.O.P.

(b) The C.O.P. obtained in the cycle.

(GATE-1987-16ii)

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Solution:

Given: approach of heat transfer units = 5°C . Therefore,

$$\begin{aligned} T_C \text{ of refrigeration cycle} &= -3 - 5 = -8^{\circ}\text{C} = 2650 \text{ K} \\ T_H \text{ of refrigeration cycle} &= 32 + 5 = 37^{\circ}\text{C} = 310 \text{ K} \\ \text{Maximum possible COP} &= \text{COP of Carnot cycle} \\ &= \frac{T_C}{T_H - T_C} \\ &= \frac{265}{310 - 265} = 5.89 \end{aligned}$$

For the given refrigeration (shown in the diagram) $H_4 = H_1$, as expansion is through a throttle valve (isenthalpic expansion). Therefore,

$$\begin{aligned} Q_C &= 180 - 61 = 119 \text{ kJ/kg} \\ Q_H &= 206 - 61 = 145 \text{ kJ/kg} \\ \text{COP} &= \frac{Q_C}{W} = \frac{Q_C}{Q_H - Q_C} = \frac{119}{145 - 119} = 4.58 \end{aligned}$$

4.3.9 Change in Entropy for Ice to Steam Transformation

Calculate the change in entropy when one gram of ice at 0°C is converted into steam at 100°C . The latent heat of fusion of ice = 80 cal/g. Latent heat of vaporization = 540 cal/g and the mean specific heat of water between 0°C and 100°C = 1 cal/(g. $^{\circ}\text{C}$). (GATE-1995-15)

Solution:

Entropy change:

$$\Delta S = \int \frac{dQ}{T}$$

During phase change T remains constant, and $\int dQ = \text{latent heat of phase change}$. And during sensible heat changes, $\int dQ/T = \int C_p dT/T = C_p \ln(T_2/T_1)$ Therefore,

$$\begin{aligned} \Delta S = &\quad \text{Entropy change for melting of ice} \\ &+ \text{Entropy change for change of } T \text{ of water from } 0^{\circ}\text{C to } 100^{\circ}\text{C} \\ &+ \text{Entropy change for vaporization of water} \end{aligned}$$

i.e.,

$$\Delta S = \frac{80}{273} + 1 \times \ln \frac{373}{273} + \frac{540}{373} = 2.053 \text{ cal/(g.}^{\circ}\text{C)}$$

4.3.10 Change in Internal Energy of Non-ideal Gas

Calculate the change in internal energy of 25 kmol of CO₂ gas when it is isothermally expanded from 10132 kPa to 101.32 kPa at 373 K, the corresponding molar volumes being 0.215 m³/kmol and 30.53 m³/kmol. Assume CO₂ to obey $(P + 365/v^2)(v - 0.043) = RT$. (GATE-1996-27)

Solution:

Writing Internal energy(U) as a function of Temperature(T) and Volume(V),

$$\begin{aligned} U &= U(T, V) \\ dU &= \left(\frac{\partial U}{\partial T}\right)_V dT + \left(\frac{\partial U}{\partial V}\right)_T dV \\ &= C_v dT + \left(\frac{\partial U}{\partial V}\right)_T dV \end{aligned}$$

From thermodynamic property relation,

$$dU = TdS - pdV$$

differentiating with respect to V at constant T

$$\left(\frac{\partial U}{\partial V}\right)_T = T \left(\frac{\partial S}{\partial V}\right)_T - P$$

From Maxwell's relation,

$$\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V$$

Therefore,

$$dU = C_v dT + \left[T \left(\frac{\partial P}{\partial T}\right)_V - P\right] dV \quad (4.7)$$

Given,

$$(P + 365/v^2)(v - 0.043) = RT$$

Rearranging,

$$P = \frac{RT}{v - 0.043} - \frac{365}{v^2}$$

Differentiating w.r.to T at const. V,

$$\left(\frac{\partial P}{\partial T}\right)_V = \frac{R}{v - 0.043}$$

Substituting in Eq. (4.7),

$$\begin{aligned} dU &= C_v dT + \left[T \left(\frac{R}{v - 0.043}\right) - \frac{RT}{v - 0.043} + \frac{365}{v^2}\right] dV \\ &= C_v dT + \frac{365}{v^2} dV \end{aligned}$$

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Integrating the above between the initial molal volume v_1 (=0.125 m³/kmol) to the final molal volume v_2 (=30.53 m³/kmol),

$$\begin{aligned} \Delta U &= 0 + 365 \int_{v_1}^{v_2} \frac{dv}{v^2} \\ &= -365 \left[\frac{1}{v}\right]_{v_1}^{v_2} \\ &= 365 \left[\frac{1}{v}\right]_{v_2}^{v_1} \\ &= 365 \left[\frac{1}{0.215} - \frac{1}{30.53}\right] \\ &= 1685.7 \end{aligned}$$

Units of van der Waals equation constant 'a' (i.e., units of 365):

Let us substitute the known data in van der Waals equation as using R value of 8.314 kJ/kmol.K. Then,

$$\left(10132 + \frac{365}{0.215^2}\right) (0.215 - 0.043) = 8.314 \times 373 \\ 3101 = 3101$$

This calculation indicates that ΔU calculated above is in kJ/kmol.
Therefore,

$$\begin{aligned} \text{Change in internal energy for 25 kmol of CO}_2 &= 25 \times 1685.7 \\ &= 42142.5 \text{ kJ} \end{aligned}$$

4.3.11 Thermodynamic Relations for a van der Waals gas

For 1 gm mole of a van der Waal gas, evaluate the following coefficients:

$$(a) \left(\frac{\partial P}{\partial T}\right)_V \quad (b) \left(\frac{\partial^2 P}{\partial T^2}\right)_V \quad (c) \left(\frac{\partial V}{\partial P}\right)_T \quad (d) \left(\frac{\partial C_p}{\partial P}\right)_T$$

Solution:

van der Waals equation:

$$\left(P + \frac{a}{v^2}\right)(v - b) = RT \quad (4.8)$$

Rearranging,

$$P = \frac{RT}{v - b} - \frac{a}{v^2} \quad (4.9)$$

Differentiating Eqn.(4.9) with respect to T at constant V

$$\left(\frac{\partial P}{\partial T}\right)_V = \frac{RT}{v - b} \quad (4.10)$$

Differentiating Eqn.(4.10) once again with respect to T at constant V

$$\left(\frac{\partial^2 P}{\partial T^2}\right)_V = 0$$

Differentiating Eqn.(4.9) with respect to V at constant T ,

$$\begin{aligned} \left(\frac{\partial P}{\partial V}\right)_T &= \frac{-RT}{(v-b)^2} + \frac{2a}{v^3} \\ &= \frac{2a(v-b)^2 - RTv^3}{v^3(v-b)^2} \end{aligned}$$

Therefore,

$$\left(\frac{\partial V}{\partial P}\right)_T = \frac{v^3(v-b)^2}{2a(v-b)^2 - RTv^3}$$

From the relation

$$\left(\frac{\partial C_p}{\partial P}\right)_T = -T \left(\frac{\partial^2 V}{\partial T^2}\right)_P \quad (4.11)$$

Rearranging Eqn.(4.8)

$$RT = Pv - Pb + \frac{a}{v} - \frac{ab}{v^2}$$

Differentiating with respect to v at constant P ,

$$\left(\frac{\partial T}{\partial V}\right)_P = \frac{1}{R} \left(P - \frac{a}{v^2} + \frac{2ab}{v^3}\right)$$

Therefore,

$$\left(\frac{\partial V}{\partial T}\right)_P = \frac{R}{P - (a/v^2) + (2ab/v^3)} \quad (4.12)$$

Differentiating Eqn.(4.12) once again with respect to T at constant P

$$\left(\frac{\partial^2 V}{\partial T^2}\right)_P = -R \left[\frac{(2a/v^3) - (6ab/v^4)}{(P - (a/v^2) + (2ab/v^3))^2} \right] \left(\frac{\partial V}{\partial T}\right)_P$$

Substituting for $\left(\frac{\partial V}{\partial T}\right)_P$ from Eqn.(4.12) and from Eqn.(4.11)

$$\left(\frac{\partial C_p}{\partial P}\right)_T = TR^2 \left[\frac{(2a/v^3) - (6ab/v^4)}{(P - (a/v^2) + (2ab/v^3))^3} \right]$$

4.3.12 Joule-Thomson Expansion

A pure gas flows at a low rate through a well insulated horizontal pipe at high pressure and is throttled to a slightly lower pressure. The equation of state for the system is given as $P(V-c) = RT$, where c is a positive constant. Kinetic energy changes are negligible. Prove whether or not the gas temperature rises or falls due to throttling by using the following equation for Joule-Thomson coefficient, μ_H ,

(GATE-1989-16i)

$$\mu_H = \left(\frac{\partial T}{\partial P}\right)_H = \frac{T \left(\frac{\partial V}{\partial T}\right)_P - V}{C_P}$$

Solution:

For the equation of state $P(V-c) = RT$,

$$\begin{aligned} V - c &= \frac{RT}{P} \\ V &= \frac{RT}{P} + c \\ \left(\frac{\partial V}{\partial T}\right)_P &= \frac{R}{P} \end{aligned}$$

Therefore

$$\begin{aligned} \mu_H &= \frac{T \left(\frac{\partial V}{\partial T}\right)_P - V}{C_P} \\ &= \frac{T(R/P) - V}{C_P} = \frac{-c}{P} \end{aligned}$$

Since c is positive, μ_H is negative.

For the pressure difference ΔP at constant H ,

$$\frac{\Delta T}{\Delta P} = \mu_H$$

Therefore

$$\Delta T = \mu_H \Delta P$$

During throttling, pressure is reduced. i.e., $\Delta P (= P_2 - P_1)$ is negative. Therefore $\Delta T (= T_2 - T_1)$ is positive. In other words, $(T_1 - T_2)$ is negative. Therefore temperature increases during expansion. (1 refers to inlet and 2 refers to outlet conditions)

4.3.13 Work of Compression

Consider the compression of air from 10^5 Pa at 27°C to 3×10^6 Pa in an ideal two-stage compressor with intercooling. Assume that the temperature of the air leaving the intercooler

is also 27°C, and the optimum interstage pressure is used. The compressor is water-jacketed, and the polytropic exponent δ is 1.3 for both stages. Determine the work of compression per kg of air.
(GATE-1992-17c)

Solution:

For optimum intersatge pressure, compression ratio per stage (r) is

$$r = \left(\frac{P_2}{P_1} \right)^{1/N}$$

where N = number of stages

P_1 = inlet pressure

P_2 = outlet pressure

Here

$$r = \left(\frac{3 \times 10^6}{10^5} \right)^{1/2} = \sqrt{30}$$

Work of compression per mole (W) is given by

$$W = \frac{N\delta RT_1}{\delta - 1} \left[1 - r^{\frac{\delta-1}{\delta}} \right]$$

where δ = polytropic exponent

T_1 = temperature at inlet

R = universal gas constant

Here

$$W = \frac{2 \times 1.3 \times 8.314 \times (273 + 27)}{1.3 - 1} \left[1 - (30)^{\frac{1.3-1}{2 \times 1.3}} \right] = -10388.8 \text{ kJ/kmol}$$

Work of compression per kg of air = $10388.8/29 = 358.2 \text{ kJ}$.

4.3.14 Fugacity of van der Waals Gas

Calculate the fugacity of CO_2 gas at 100 atm pressure and 75°C assuming that the gas obeys van der Waals equation of state. The constants for this equation of state are:

$$a = 3.606 \times 10^6 \text{ atm.cc/gmol}^2 \quad b = 42.88 \text{ cc/gmol}$$

Solution:

Fugacity (f) is defined by the equation

$$\ln \frac{f}{f^\circ} = \frac{1}{RT} \int_{P^\circ}^P v dP \quad (4.13)$$

and $f^\circ = P^\circ$ as $P^\circ \rightarrow 0$, the integration being along an isotherm.

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van der Waals equation of state:

$$(P + a/v^2)(v - b) = RT$$

rearranging,

$$P = \frac{RT}{v - b} - \frac{a}{v^2} \quad (4.14)$$

Differentiating Eqn.(4.14) at constant T ,

$$dP = \frac{-RTdv}{(v - b)^2} + \frac{2adv}{v^3}$$

Then,

$$\begin{aligned} vdP &= \frac{-RTvdv}{(v - b)^2} + \frac{2adv}{v^2} \\ \int_{P^\circ}^P vdP &= \int_{v^\infty}^v \left[\frac{-RTv}{(v - b)^2} + \frac{2a}{v^2} \right] dv \end{aligned}$$

v and v^∞ are the volume limits corresponding to P and P° respectively. Substituting in Eqn.(4.13) and integrating,

$$\begin{aligned} \ln f - \ln f^\circ &= \left[\frac{b}{v - b} - \ln(v - b) - \frac{2a}{RTv} \right]_{v^\infty}^v \\ &= \frac{b}{v - b} - \frac{b}{v^\infty - b} - \ln(v - b) + \ln(v^\infty - b) - \frac{2a}{RTv} + \frac{2a}{RTv^\infty} \end{aligned}$$

As $P \rightarrow 0$, $v^\infty \rightarrow \infty$, and we can write

$$\frac{1}{v^\infty} = 0; \quad v^\infty - b = v^\infty = \frac{RT}{P^\circ}$$

Making these substitutions,

$$\begin{aligned} \ln f - \ln f^\circ &= \frac{b}{v - b} - \ln(v - b) + \ln \frac{RT}{P^\circ} - \frac{2a}{RTv} \\ &= \ln \frac{RT}{v - b} + \frac{b}{v - b} - \ln P^\circ - \frac{2a}{RTv} \end{aligned}$$

But since, from the definition of fugacity, $f^\circ = P^\circ$ as $P^\circ \rightarrow 0$, we finally get

$$\ln f = \ln \frac{RT}{v - b} + \frac{b}{v - b} - \frac{2a}{RTv} \quad (4.15)$$

The volume corresponding to $P = 100$ atm and $T = 75^\circ\text{C} = 348 \text{ K}$ is (obtained from van der Waals equation by a trial solution) = 169.5 cc.
Substituting in Eqn.(4.15),

$$\ln f = 3.68 \quad f = 39.64 \text{ atm}$$

4.3.15 Fugacity and Fugacity Coefficient

At high temperature and pressure N₂ obeys the equation of state $P(V - b) = RT$. Calculate the fugacity of N₂ at 1000°C and 1000 atm, if $b = 39.1 \text{ ml/mol}$. Calculate the fugacity coefficient also.

Solution:

Fugacity coefficient (ϕ) is related to fugacity (f) as,

$$\phi = \frac{f}{P} \quad (4.16)$$

Fugacity coefficient (ϕ) is related to compressibility factor (Z) as,

$$\ln \phi = \int_0^P (Z - 1) \frac{dP}{P} \quad (4.17)$$

Given: Equation of state of gas,

$$\begin{aligned} P(V - b) &= RT \\ PV - Pb &= RT \end{aligned}$$

dividing the above equation by RT , and from the definition of Z (i.e., $Z = \frac{PV}{RT}$),

$$Z = 1 + \frac{Pb}{RT} \quad (4.18)$$

Substituting for Z from Eqn.(4.18) in Eqn.(4.17),

$$\ln \phi = \int_0^{1000} \frac{b}{RT} dP$$

Substituting the values with proper system of units in the above equation, at $T = 1273 \text{ K}$,

$$\begin{aligned} \ln \phi &= \frac{39.1 \times 10^{-3}}{8314 \times 1273} (1000 \times 1.01325 \times 10^5 - 0) \\ &= 0.373 \end{aligned}$$

Therefore, $\phi = 1.453$.

From Eqn.(4.16), $f = 1.453 \times 1000 = 1453 \text{ atm}$.

4.3.16 Partial Molal Volume

At 30°C and 1 atm, the volumetric data for a liquid mixture of benzene(b) and cyclo-benzene(c) are represented by the equation

$$V (\text{cm}^3/\text{gmol}) = 109.4 - 16.8x_b - 2.64x_b^2$$

where x_b is the mole fraction of benzene. Find an expression for the partial molal volumes of the two components \bar{V}_b and \bar{V}_c . (GATE-1988-16ii)

Solution:

For any extensive thermodynamic property

$$M = x_1 \bar{M}_1 + x_2 \bar{M}_2 \quad (4.19)$$

where

$$\bar{M}_1 = M + x_2 \frac{dM}{dx_1}$$

$$\bar{M}_2 = M - x_1 \frac{dM}{dx_1}$$

Since volume is an extensive property similar equations can be written. Therefore,

$$\bar{V}_b = V + x_c \frac{dV}{dx_b}$$

$$\begin{aligned} \text{i.e., } \bar{V}_b &= 109.4 - 16.8x_b - 2.64x_b^2 + x_c(-16.8 - 2.64 \times 2x_b) \\ &= 109.4 - 16.8x_b - 2.64x_b^2 - 16.8x_c - 5.28x_bx_c \\ &= 109.4 - 16.8(x_b + x_c) - 2.64x_b^2 - 5.28x_b(1 - x_c) \\ &= 109.4 - 16.8 - 2.64x_b^2 - 5.28x_b + 5.28x_b^2 \end{aligned}$$

$$\text{i.e., } \bar{V}_b = 92.6 - 5.28x_b + 2.64x_b^2$$

And

$$\bar{V}_c = V - x_b \frac{dV}{dx_b}$$

$$\begin{aligned} \text{i.e., } \bar{V}_c &= 109.4 - 16.8x_b - 2.64x_b^2 - x_b(-16.8 - 5.28x_b) \\ &= 109.4 - 16.8x_b - 2.64x_b^2 + 16.8x_b + 5.28x_b^2 \\ \text{i.e., } \bar{V}_c &= 109.4 + 2.64x_b^2 \end{aligned}$$

Check:

$$\begin{aligned} V &= x_b \bar{V}_b + x_c \bar{V}_c \\ &= x_b \bar{V}_b + (1 - x_b) \bar{V}_c \\ &= 92.6x_b - 5.28x_b^2 + 2.64x_b^3 + 109.4 + 2.64x_b^2 - 109.4x_b - 2.64x_b^3 \\ &= 109.4 - 16.8x_b - 2.64x_b^2 \end{aligned}$$

Hence verified.

4.3.17 Bubble Point Vapor

A mixture of A and B conforms closely to Raoult's law. The pure component vapor pressures P_A^{sat} and P_B^{sat} in kPa are given by (t in °C)

$$\ln P_A^{\text{sat}} = 14.27 - 2945/(t + 224)$$

$$\ln P_B^{\text{sat}} = 14.20 - 2973/(t + 209)$$

If the bubble point of a certain mixture of A and B is 76°C at a total pressure of 80 kPa, find the composition of the first vapor that forms. (GATE-1998-22)

Solution:

For a two-component vapor-liquid-equilibrium system, degree of freedom is 2. ($F = C - P + 2$). That is, it is enough to fix any two thermodynamic coordinates in P, T, x_A, y_A to define the state of a system.

In this problem, T and P are specified, and we have to find x_A , and y_A .

At bubble point $\sum_{i=1}^n y_i = 1$, for the liquid composition of $\sum_{i=1}^n x_i = 1$.

And

$$y_i = \frac{P_i^{\text{sat}}}{P} x_i$$

where P_i^{sat} is the vapor pressure of component i , and P is the total pressure.

At the given temperature of 76°C, vapor pressures of A and B are calculated as,

$$\ln P_A^{\text{sat}} = 14.27 - 2945/(76 + 224)$$

That is, $P_A^{\text{sat}} = 85.91$ kPa.

By similar calculations, $P_B^{\text{sat}} = 43.31$ kPa.

To find the bubble point x_A and y_A compositions, we shall assume a series of values to x_A and find the corresponding y_A values. The value of x_A which satisfy the condition of $\sum y_i = 1$ is the bubble point liquid composition and the corresponding y_A is the bubble point vapor composition.

x_A	y_A	$y_B = (1 - y_A)$	$y_A + y_B$
0.1	0.107	0.487	0.594
0.3	0.322	0.379	0.701
0.5	0.537	0.271	0.808
0.7	0.752	0.162	0.917
0.8	0.859	0.108	0.967
0.9	0.966	0.054	1.020
0.85	0.913	0.081	0.994
0.86	0.924	0.076	1.000

Therefore, bubble point composition at 76°C and 80 kPa is $x_A = 0.860$ and $y_A = 0.924$.

An alternative method:

For vapor liquid equilibrium of two component systems,

$$P = P_A^{\text{sat}} x_A + P_B^{\text{sat}} x_B$$

$$= P_A^{\text{sat}} x_A + P_B^{\text{sat}} (1 - x_A)$$

$$P - P_B^{\text{sat}} = P_A^{\text{sat}} x_A + P_B^{\text{sat}} x_A$$

$$\frac{P - P_B^{\text{sat}}}{P_A^{\text{sat}} + P_B^{\text{sat}}} = x_A$$

Therefore,

$$x_A = \frac{80 - 43.31}{85.91 - 43.31} = 0.861$$

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And composition in vapor y_A is given by

$$y_A = \frac{P_A^{\text{sat}} x_A}{P} = \frac{85.91 \times 0.861}{80} = 0.924$$

4.3.18 VLE of Non-ideal Solution

The activity coefficients of benzene (A) - cyclohexane (B) mixtures at 40°C are given by $RT \ln \gamma_A = bx_B^2$ and $RT \ln \gamma_B = bx_A^2$. At 40°C, A and B form an azeotrope containing 49.4 mol % A at a total pressure of 202.6 mm Hg. If the vapor pressures of pure A and B are 182.6 and 183.5 mm Hg respectively, calculate the total pressure of the vapor at 40°C in equilibrium with a liquid mixture containing 12.6 mol % A. (GATE-1996-26)

Solution:

At azeotropic composition, $x_A = y_A$.

The vapor-liquid equilibrium relationship is given by,

$$\begin{aligned} x_A \gamma_A P_A^{\text{sat}} &= y_A P \\ 0.494 \times 182.6 \times \gamma_A &= 0.494 \times 202.6 \\ \text{i.e., } \gamma_A &= 1.1095 \end{aligned}$$

Similarly

$$\begin{aligned} x_B \gamma_B P_B^{\text{sat}} &= y_B P \\ 0.506 \times 183.5 \times \gamma_B &= (1 - 0.494) \times 202.6 \\ \text{i.e., } \gamma_B &= 1.1041 \end{aligned}$$

From the given relation, $RT \ln \gamma_A = bx_B^2$

$$\begin{aligned} b &= \frac{RT \ln \gamma_A}{x_B^2} \\ &= \frac{RT \ln(1.1095)}{0.506^2} \\ &= 0.4058 RT \end{aligned}$$

Therefore, $\ln \gamma_A = 0.4058 x_B^2$, and $\ln \gamma_B = 0.4058 x_A^2$.
When $x_A = 0.126$,

$$\begin{aligned} \ln \gamma_A &= 0.4058 \times (1 - 0.126)^2 \\ \gamma_A &= 1.3634 \end{aligned}$$

And,

$$\begin{aligned} \ln \gamma_B &= 0.4058 \times 0.126^2 \\ \gamma_B &= 1.0065 \end{aligned}$$

Total pressure (P) is given by,

$$\begin{aligned} P &= x_A \gamma_A P_A^{\text{sat}} + x_B \gamma_B P_B^{\text{sat}} \\ &= 0.126 \times 1.3634 \times 182.6 + (1 - 0.126) \times 1.0065 \times 183.5 \\ &= 31.37 + 161.42 \\ &= 192.79 \text{ mm Hg} \end{aligned}$$

4.3.19 Activity Coefficient and Excess Gibbs Free Energy

At 318 K at total pressure of 24.4 kPa, the composition of the system ethanol(1) and toluene(2) at equilibrium is $x_1 = 0.3$ and $y_1 = 0.634$. The saturation pressures at the given temperature for pure components are $P_1 = 23.06 \text{ kPa}$ and $P_2 = 10.05 \text{ kPa}$ respectively. Calculate:

(a) the liquid phase activity coefficients

(b) the value of G^E/RT for the liquid phase.

(GATE-1991-17ii)

Solution:

Vapor-liquid equilibrium relationship is given by,

$$y_i P = x_i \gamma_i P_i^{\text{sat}}$$

For component 1,

$$\begin{aligned} y_1 P &= x_1 \gamma_1 P_1^{\text{sat}} \\ 0.634 \times 24.4 &= 0.3 \times 23.06 \times \gamma_1 \\ \gamma_1 &= 2.23 \end{aligned}$$

For component 2,

$$\begin{aligned} y_2 P &= x_2 \gamma_2 P_2^{\text{sat}} \\ (1 - 0.634) \times 24.4 &= (1 - 0.3) \times 10.05 \times \gamma_2 \\ \gamma_2 &= 1.269 \end{aligned}$$

And,

$$\begin{aligned} \frac{G^E}{RT} &= x_1 \ln \gamma_1 + x_2 \ln \gamma_2 \\ &= 0.3 \ln(2.236) + (1 - 0.3) \ln(1.269) \\ &= 0.2414 + 0.1668 \\ &= 0.4082 \end{aligned}$$

4.3.20 Activity Coefficient

The binary mixture $A - B$ forms an azeotrope with a boiling point of 71.8°C at 1 bar pressure, the azeotropic composition being 55 mol percent A. The pure component vapor pressures at 71.8°C are: $A = 0.710 \text{ bar}$; $B = 0.743 \text{ bar}$. Estimate the activity coefficient of A in the liquid, assuming the vapor to be an ideal. (GATE-1987-16i)

Solution:

For phase equilibrium

$$\phi_i y_i P = \gamma_i x_i P_i^{\text{sat}}$$

Assuming ideal vapor mixture, $\phi_i = 1$. Therefore for A

$$y_A P = \gamma_A x_A P_A^{\text{sat}}$$

At azeotropic composition, $x_A = y_A$. Therefore,

$$0.55 \times 1 = \gamma_A \times 0.55 \times 0.71$$

Solving, $\gamma_A = 1.408$.

4.3.21 Activity Coefficients as a Function of Compositions

The Excess Gibbs free energy for cyclohexanone(1)/phenol(2) is given by

$$\frac{G^E}{RT} = -2.1x_1 x_2$$

where x_1 and x_2 are the mole fractions of components 1 and 2 in the liquid phase. The vapor pressures of components at 417 K are $P_1^{\text{sat}} = 75.2 \text{ kPa}$ and $P_2^{\text{sat}} = 31.66 \text{ kPa}$.

- (a) Derive expressions for activity coefficients of each component as a function of composition.
- (b) Verify whether the expressions derived in (a) satisfy the Gibbs-Duhem equation.
- (c) Determine the equilibrium pressure P and vapor composition for a liquid composition $x_1 = 0.8$ and 417 K. Assume vapor phase to be ideal gas. (GATE-2002-10)

Solution:

Activity coefficient is related to G^E/RT as

$$\ln \gamma_i = \left[\frac{\partial (nG^E/RT)}{\partial n_i} \right]_{P,T,n}, \quad (4.20)$$

By the definition of mole fraction, $x_i = n_i/n$. Therefore,

$$\frac{G^E}{RT} = -2.1x_1 x_2 = -2.1 \frac{n_1 n_2}{(n_1 + n_2)^2}$$

and

$$\frac{nG^E}{RT} = -2.1n_2 \frac{n_1}{n_1 + n_2}$$

From Eqn.(4.20)

$$\begin{aligned}\ln \gamma_1 &= -2.1n_2 \left(\frac{(n_1 + n_2) - n_1}{n^2} \right) \quad (\text{since, } d(u/v) = (uv' - vu')/v^2) \\ &= -2.1 \frac{n_2^2}{n^2} = -2.1x_2^2\end{aligned}$$

By similar calculations,

$$\ln \gamma_2 = -2.1x_1^2$$

Gibbs-Duhem equation:

$$\sum_i x_i d \ln \gamma_i = 0 \quad (4.21)$$

Dividing by dx_1 ,

$$x_1 \frac{d \ln \gamma_1}{dx_1} + x_2 \frac{d \ln \gamma_2}{dx_1} = 0 \quad (4.22)$$

$$\ln \gamma_1 = -2.1x_2^2 = -2.1(1 - x_1)^2$$

$$\frac{d \ln \gamma_1}{dx_1} = 4.2(1 - x_1)$$

$$\ln \gamma_2 = -2.1x_1^2$$

$$\frac{d \ln \gamma_2}{dx_1} = -4.2x_1$$

Substituting these in Eqn.(4.22),

$$\begin{aligned}4.2(1 - x_1)x_1 + x_2(-4.2x_1) &= 0 \\ 4.2x_2x_1 - 4.2x_1x_2 &= 0 \\ 0 &= 0\end{aligned}$$

Hence the derived equations for γ_i satisfy Gibbs-Duhem equation.

At $x_1 = 0.8$,

$$\begin{aligned}\ln \gamma_1 &= -2.1 \times (1 - 0.8)^2 = -0.084; \quad \text{and } \gamma_1 = 0.919 \\ \ln \gamma_2 &= -2.1 \times (0.8)^2 = -1.344; \quad \text{and } \gamma_2 = 0.261\end{aligned}$$

For the ideal vapor phase, the equilibrium relation is given by

$$y_i P = \gamma_i x_i P_i^{\text{sat}}$$

Therefore

$$\begin{aligned}P = y_1 P + y_2 P &= \gamma_1 x_1 P_1^{\text{sat}} + \gamma_2 x_2 P_2^{\text{sat}} \\ &= 0.919 \times 0.8 \times 75.2 + 0.261 \times 0.2 \times 31.66 \\ &= 56.95 \text{ kPa}\end{aligned}$$

4.3.22 Activity Coefficients and G^E/RT

An experimental determination of a vapor-liquid equilibrium state of ether(1) and acetone(2) binary system gave the following result:

$$\begin{aligned}x_1 &= 0.3 \quad T = 40^\circ\text{C} \\ y_2 &= 0.42 \quad P = 10^5 \text{ Pa}\end{aligned}$$

The saturation vapor pressures of the pure components at 40°C are:

$$\begin{aligned}\text{ether}(1) &= 1.21 \times 10^5 \text{ Pa} \\ \text{acetone}(2) &= 0.56 \times 10^5 \text{ Pa}\end{aligned}$$

The vapor phase can be assumed to be ideal.

(a) Calculate the liquid phase activity coefficients.

(b) What is the value of excess Gibbs free energy G^E/RT , for the liquid phase? (GATE-1994-11)

Solution:

At equilibrium

$$y_1 P = \gamma_1 x_1 P_1^{\text{sat}}$$

Therefore

$$(1 - 0.42) \times 10^5 = \gamma_1 \times 0.3 \times 1.21 \times 10^5$$

Solving, $\gamma_1 = 1.6$. Similarly,

$$y_2 P = \gamma_2 x_2 P_2^{\text{sat}}$$

i.e.,

$$0.42 \times 10^5 = \gamma_2 \times (1 - 0.3) \times 0.56 \times 10^5$$

Solving, $\gamma_2 = 1.07$.

$$\begin{aligned}\frac{G^E}{RT} &= \sum_i x_i \ln \gamma_i \\ &= x_1 \ln \gamma_1 + x_2 \ln \gamma_2 \\ &= 0.3 \times \ln(1.6) + 0.7 \times \ln(1.07) \\ &= 0.188\end{aligned}$$

4.3.23 Dewpoint Pressure

An equimolar mixture of benzene and toluene is contained in a piston/cylinder arrangement at a temperature T . What is the maximum pressure below which this mixture will exist as a vapor phase alone? At the given T , the vapor pressures of benzene and toluene are 1530 and 640 mm Hg, respectively. Assume that Raoult's law is valid. (GATE-1997-7)

Solution:

Below the dewpoint pressure, the mixture will exist as a vapor phase. At dewpoint pressure

$$\sum_i x_i = 1$$

From Raoult's law

$$y_i P = x_i P_i^{\text{sat}}$$

Therefore

$$x_1 = \frac{y_1 P}{P_1^{\text{sat}}}; \text{ and } x_2 = \frac{y_2 P}{P_2^{\text{sat}}}$$

Hence,

$$\frac{y_1 P}{P_1^{\text{sat}}} + \frac{y_2 P}{P_2^{\text{sat}}} = 1$$

i.e.,

$$\begin{aligned} P &= \frac{1}{\frac{y_1}{P_1^{\text{sat}}} + \frac{y_2}{P_2^{\text{sat}}}} \\ &= \frac{1}{0.5/1530 + 0.5/640} = 902.5 \text{ mm Hg} \end{aligned}$$

Dewpoint pressure = 902.5 mm Hg

4.3.24 Composition of Vapor

A binary system consists of *n*-pentanol (1) and *n*-hexane (2) at 30°C. Determine the composition of the vapor of the component (1) which is in equilibrium with a liquid containing 20 mole % *n*-pentanol (1) at 30°C. The following data are given:

Vapor pressure of *n*-pentanol (1) at 30°C, $P_1^S = 4.306 \times 10^{-3}$ bar

Vapor pressure of *n*-hexane (2) at 30°C, $P_2^S = 249.5 \times 10^{-3}$ bar

The activity coefficients of (1) and (2) at $x_1 = 0.2$ at 30°C: $\gamma_1 = 2.831$, $\gamma_2 = 1.1716$
Assume that at low pressures, gases are in ideal state, but the liquid phase is not. Use the following equation.

$$y_i \phi_i^v \pi = x_i \gamma_i P_i^S \phi_i^s$$

where π = total pressure and ϕ_i^v and ϕ_i^s are the fugacity coefficients of vapor and saturated vapor respectively. (GATE-1989-16iii)

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Solution:

At ideal gas state $\phi_i^v = \phi_i^s = 1$. Therefore,

$$y_i \pi = x_i \gamma_i P_i^S$$

$$\begin{aligned} \pi &= y_1 + y_2 \\ &= x_1 \gamma_1 P_1^S + x_2 \gamma_2 P_2^S \\ &= 0.2 \times 2.831 \times 4.306 \times 10^{-3} + (1 - 0.2) \times 1.1716 \times 249.5 \times 10^{-3} \\ &= 0.2363 \text{ bar} \end{aligned}$$

$$\begin{aligned} y_1 &= \frac{x_1 \gamma_1 P_1^S}{\pi} \\ &= \frac{0.2 \times 2.831 \times 4.306 \times 10^{-3}}{0.2363} = 0.01 = 1 \text{ mole \%} \end{aligned}$$

4.3.25 $P - x - y$ Data

Generate $P - x - y$ data for a binary system at a temperature at which $P_1^{\text{sat}} = 84.562 \text{ kPa}$, and $P_2^{\text{sat}} = 19.953 \text{ kPa}$. Assume system to follow Raoult's law. (Determine P and y_1 values for $x_1 = 0.2, 0.4, 0.6$, and 0.8 .)

Solution:

For a system obeying Raoult's law,

$$y_1 P = x_1 P_1^{\text{sat}}$$

where P is the total pressure, and y_1 is mole fraction of component 1 in equilibrium with the liquid of composition x_1 .

For two component system, $y_1 + y_2 = 1$. Therefore, total pressure(P) is given by

$$P = x_1 P_1^{\text{sat}} + x_2 P_2^{\text{sat}}$$

Since $x_2 = 1 - x_1$,

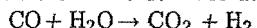
$$P = x_1 P_1^{\text{sat}} + (1 - x_1) P_2^{\text{sat}}$$

Given: $P_1^{\text{sat}} = 84.562 \text{ kPa}$; $P_2^{\text{sat}} = 19.953 \text{ kPa}$. Using the above relations and from the data given, P and y values are calculated for the various x values and tabulated.

x_1	$P, \text{ kPa}$	y_1
0.2	32.875	0.514
0.4	45.797	0.739
0.6	58.718	0.864
0.8	71.640	0.944

4.3.26 Variation of Equilibrium Constant with Temperature

Calculate the values of K at 25°C and 800°C for the water gas reaction:



using the following data:

Substance	H_2	CO	H_2O	CO_2
ΔH_f° (kJ)	0	-110.52	-241.83	-392.51
ΔG_f° (kJ)	0	-137.27	-228.60	-394.40

where ΔH_f° and ΔG_f° refer to standard state enthalpy and free energy change respectively at 25°C .

Solution:

$$\text{Heat of reaction} = \text{Heat of formation of products} - \text{Heat of formation of reactants}$$

Therefore,

$$\begin{aligned}\Delta H^\circ &= -392.51 - (-110.52 - 241.83) \\ &= -40.16 \text{ kJ/kg}\end{aligned}$$

Standard free energy change of the reaction:

$$\begin{aligned}\Delta G^\circ &= -394.40 - (-137.27 - 228.6) \\ &= -28.53 \text{ kJ/kg}\end{aligned}$$

Equilibrium constant and standard state (K) is related to ΔG° as,

$$\frac{\Delta G^\circ}{RT} = -\ln K \quad (4.23)$$

ΔG° per kmol of $\text{CO}_2 = -28.53 \times \text{MW} = -28.53 \times 44 = -1255.32 \text{ kJ/kmol}$.

Similarly, ΔH° per kmol of $\text{CO}_2 = -40.16 \times 44 = -1767.04 \text{ kJ/kmol}$.

From Eqn.(4.23),

$$\frac{-1255.32}{8.314 \times 298} = -\ln K$$

Therefore at 25°C , $K = 1.66$.

If the heat of reaction(ΔH°) is assumed to remain constant with temperature, then

$$\ln \frac{K}{K_1} = \frac{-\Delta H^\circ}{R} \left(\frac{1}{T} - \frac{1}{T_1} \right) \quad (4.24)$$

From Eqn.(4.24), K at any temperature T can be found from known values of K_1 and T_1 . If T_1 is taken as 298 K then, $K_1 = 1.66$.

Therefore,

$$\ln \frac{K}{1.66} = \frac{1767.04}{8.314} \left(\frac{1}{273+800} - \frac{1}{298} \right)$$

Solving, $K = 0.992$ (at 800°C).

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4.3.27 Partial Pressures at Reaction Equilibrium

The reaction $\text{N}_2 + \text{O}_2 \leftrightarrow 2\text{NO}$ takes place in the gas phase at 2700°C and 2025 kPa. The reaction mixture initially comprises 1 mole % oxygen, 17 mole % nitrogen and the rest inert. The standard Gibb's free energy change for the reaction is 113.83 kJ/mol at this temperature. Assuming ideal gas behaviour, calculate the partial pressures of all species at equilibrium. (GATE-1998-23)

Solution:

Equilibrium constant(K) and free energy change of reaction(ΔG) are related as

$$\frac{\Delta G}{RT} = -\ln K \quad (4.25)$$

For the given reaction, from Eqn.(4.25)

$$\frac{113.83 \times 10^3}{8.314 \times (2700 + 273)} = -\ln K$$

Therefore, $K = 0.0105$.

For ideal gas mixture, equilibrium constant K is related to mole fraction of the species as,

$$\prod (y_i)^{\nu_i} = P^{-\nu} K \quad (4.26)$$

where

y_i = mole fraction of species i at equilibrium condition;

ν_i = stoichiometric coefficient of species i , with proper sign (+ve for products; -ve for reactants);

P = total pressure of the system;

ν = net stoichiometric coefficient of the reaction. ($\nu = \sum \nu_i$).

For the given reaction, $\nu = 2 - (1+1) = 0$.

Therefore, Eqn.(4.26) reduces to,

$$K = \prod (y_i)^{\nu_i} \quad (4.27)$$

The mole fraction y_i of the species present are related to the conversion ϵ by

$$y_i = \frac{n_i}{n} = \frac{n_{i0} + \nu_i \epsilon}{n_o + \nu \epsilon} \quad (4.28)$$

where

n_{i0} = initial moles of species of i ;

n_o = total moles before reaction; and

ϵ = extent of reaction.

From Eqn.(4.28), mole fractions of the various species present are given as

$$y_{\text{N}_2} = \frac{0.77 - \epsilon}{1} = 0.77 - \epsilon$$

$$y_{\text{O}_2} = 0.15 - \epsilon$$

$$y_{\text{NO}} = 0 + 2\epsilon = 2\epsilon$$

From Eqn.(4.27),

$$K = \frac{y_{\text{NO}}^2}{y_{\text{N}_2} y_{\text{O}_2}}$$

$$0.0105 = \frac{(2\epsilon)^2}{(0.77 - \epsilon)(0.15 - \epsilon)}$$

Solving, $\epsilon = 0.0159$.

Mole fractions at equilibrium:

$$y_{\text{N}_2} = 0.77 - 0.0159 = 0.7451$$

$$y_{\text{O}_2} = 0.15 - 0.0159 = 0.1341$$

$$y_{\text{NO}} = 2 \times 0.0159 = 0.0318$$

$$\text{inerts} = 1 - (0.7451 + 0.1341 + 0.0318) = 0.08$$

Partial pressures at equilibrium:

$$P_{\text{N}_2} = 0.7451 \times 2025 = 1572 \text{ kPa}$$

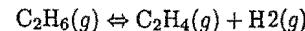
$$P_{\text{O}_2} = 0.1341 \times 2025 = 271.6 \text{ kPa}$$

$$P_{\text{NO}} = 0.0318 \times 2025 = 64.4 \text{ kPa}$$

$$P_{\text{inerts}} = 0.08 \times 2025 = 162 \text{ kPa}$$

4.3.28 Dehydrogenation of Ethane

Calculate the fraction of pure ethane that would dehydrogenate at 750 K and 5 atm, if the following reaction goes to equilibrium



ΔG° for the reaction at 750 K = 42.576 kJ. Assume ideal gas behaviour. (GATE-1991-17i)

Solution:

$$\Delta G = -RT \ln K$$

Therefore,

$$K = \exp\left(\frac{-\Delta G}{RT}\right) = \exp\left(\frac{-42.576}{8.314 \times 750}\right) = 0.9932$$

$$K = \frac{y_{\text{C}_2\text{H}_4} \cdot y_{\text{H}_2}}{y_{\text{C}_2\text{H}_6}} P^\nu$$

Here $\nu = (1+1) - 1 = 1$.

$$y_i = \frac{n_i}{n} = \frac{n_{i0} + \nu_i \epsilon}{n_o + \nu \epsilon}$$

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And

$$y_{\text{C}_2\text{H}_4} = \frac{0 + \epsilon}{1 + \epsilon} = \frac{\epsilon}{1 + \epsilon}$$

$$y_{\text{H}_2} = \frac{0 + \epsilon}{1 + \epsilon} = \frac{\epsilon}{1 + \epsilon}$$

$$y_{\text{C}_2\text{H}_6} = \frac{1 - \epsilon}{1 + \epsilon}$$

Therefore,

$$0.9932 = \frac{\left(\frac{\epsilon}{1+\epsilon}\right) \left(\frac{\epsilon}{1+\epsilon}\right)}{\left(\frac{1-\epsilon}{1+\epsilon}\right)} \times 5$$

Solving, $\epsilon = 0.407$. i.e., fraction of pure ethane dehydrogenated = 0.407.

4.3.29 Equilibrium Composition of Non-ideal Gas phase Reaction

Compute the equilibrium mole fraction of each of the species in the gas phase reaction $\text{CO}_2 + \text{H}_2 \rightleftharpoons \text{CO} + \text{H}_2\text{O}$ at 1000 K and at (a) 1 atm. total pressure, and (b) at 500 atm. total pressure. The reaction equilibrium constant $K_a = 0.693$ at 1000 K. Standard state is pure gases at 1000 K and 1 atm. pressure. Initially there are equal amounts of CO_2 and H_2 present. The fugacity coefficients at 500 atm. pressure for the species are:

For CO_2 — 0.99

H_2 — 1.15

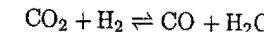
CO — 1.08

H_2O — 0.86.

Solution:

Basis: 1 mole of feed gases.

Reaction:



(a) at 1 atm. total pressure:

$$K_a = \frac{y_{\text{CO}} y_{\text{H}_2\text{O}}}{y_{\text{CO}_2} y_{\text{H}_2}} \quad (4.29)$$

where

$$y_{\text{CO}} = \frac{\epsilon}{1} = \epsilon$$

and

$$y_{\text{H}_2\text{O}} = \epsilon$$

$$y_{\text{CO}_2} = 0.5 - \epsilon$$

$$y_{\text{H}_2} = 0.5 - \epsilon$$

Substituting these in Eqn.(4.29) and from the value of K_a at 1000 K,

$$0.693 = \frac{(\epsilon)(\epsilon)}{(0.5 - \epsilon)(0.5 - \epsilon)}$$

Solving the above equation, $\epsilon = 0.227$.

Therefore, at equilibrium the mole fractions of various species are:

$$y_{CO} = \epsilon = 0.227$$

$$y_{H_2O} = \epsilon = 0.227$$

$$y_{CO_2} = 0.5 - \epsilon = 0.5 - 0.227 = 0.273$$

$$y_{H_2} = 0.5 - \epsilon = 0.5 - 0.227 = 0.273$$

(b) at 500 atm. total pressure:

$$K_a = \frac{y_{CO}\hat{\phi}_{CO} \cdot y_{H_2O}\hat{\phi}_{H_2O}}{y_{CO_2}\hat{\phi}_{CO_2} \cdot y_{H_2}\hat{\phi}_{H_2}} \quad (4.30)$$

Given: $\hat{\phi}_{CO} = 1.08$; $\hat{\phi}_{H_2O} = 0.86$; $\hat{\phi}_{CO_2} = 0.99$; and, $\hat{\phi}_{H_2} = 1.15$.

Substituting for $\hat{\phi}_i$ and y_i in equation Eqn.(4.30)

$$0.693 = \frac{(\epsilon \times 1.08)(\epsilon \times 0.86)}{[(0.5 - \epsilon) \times 0.99][(0.5 - \epsilon) \times 1.15]}$$

Solving the above equation, $\epsilon = 0.24$.

The equilibrium compositions at 500 atm. pressure are:

$$y_{CO} = \epsilon = 0.24$$

$$y_{H_2O} = \epsilon = 0.24$$

$$y_{CO_2} = 0.5 - \epsilon = 0.5 - 0.24 = 0.26$$

$$y_{H_2} = 0.5 - \epsilon = 0.5 - 0.24 = 0.26$$

Chapter 5

Heat Transfer

5.1 Instant Notes

5.1.1 Conduction

- Thermal conductivity, k

- unit: W/(m.K)

- At very low temperatures k varies rapidly with temperatures.

- Thermal conductivity increases in the order:
gases → water → ice → bricks → alloy steels → low-carbon steels → aluminium
→ copper.

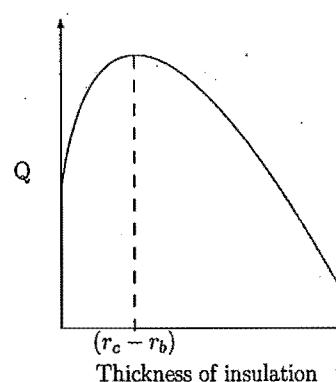
- Thermal diffusivity, α

- Definition: $\alpha = \frac{k}{\rho C_p}$

- Higher the thermal diffusivity, the less time is required for heat to penetrate into the solid.

- Critical radius of insulation, r_c

- If the radius (for cylindrical and spherical surfaces) is greater than the critical radius, any addition of insulation on the tube surface decreases the heat loss. But if the radius is less than the critical radius, as in small-diameter tubes, cables, or wires, the heat loss will increase continuously with the addition of insulation until the radius of the outer surface of the insulation equals the critical radius. The heat loss becomes maximum at the critical thickness of insulation and begins to decrease with the addition of insulation beyond the critical thickness.



- If the insulation of a steam pipe is wetted, the thermal conductivity of the insulation increases, which in turn increases the critical radius. Then it is possible that with the resulting critical radius the heat loss from the pipe will become larger with wet insulation than with no insulation.

• Fins

- Heat transfer by convection between a surface and the fluid surrounding it can be increased by attaching to the surface thin strips of metal, called fins.
- When heat transfer takes place by convection from both interior and exterior surfaces of a tube or a plate, generally fins are used on the surface where the heat transfer coefficient is low.

• Biot number

$$Bi = \frac{h(V/A)}{k}$$

- Ratio of conductive resistance to heat transfer ($[V/A]/k$), to the convective resistance to heat transfer ($1/h$).
- A small value of Biot number represents the case where internal resistance is negligibly small and there is more capacity to transfer heat by conduction than there is by convection. In this case, the controlling heat transfer phenomenon is convection, and temperature gradients within the metal are quite small.
- An extremely small internal temperature gradient is the basic assumption in lumped-system analysis. A commonly used rule of thumb is that the error inherent in a lumped-parameter analysis will be less than 5% for a value of Bi less than 0.1.

5.1.2 Convection

• Boundary layers

- The relative thickness of the thermal boundary layer $\delta_{t(x)}$ and velocity boundary layer δ_x depend on the magnitude of the Prandtl number for the fluid. For fluids having Prandtl number equal to unity, such as gases, $\delta_{t(x)} = \delta_x$.
- The thermal boundary layer is much thicker than the velocity boundary layer for fluids having $Pr \ll 1$, such as liquid metals, and is much thinner than velocity boundary layer for fluids having $Pr \gg 1$.

• Dimensionless numbers

• Reynolds number

$$Re = \frac{Dv\rho}{\mu} = \frac{\text{Inertia force}}{\text{Viscous force}}$$

• Prandtl number

$$Pr = \frac{C_p\mu}{k} = \frac{\text{Molecular diffusivity of momentum}}{\text{Molecular diffusivity of heat}}$$

• Nusselt number

$$Nu = \frac{hL}{k} = \frac{h\Delta T}{k\Delta T/L}$$

Nusselt number is the ratio of heat transfer by convection to conduction across the fluid layer of thickness L . A larger value of Nusselt number implies enhanced heat transfer by convection.

$$- \frac{Gr}{Re^2} = \frac{\text{Buoyant force}}{\text{Inertial force}}$$

$$- \text{Stanton number: } St = \frac{Nu}{Re \cdot Pr}$$

$$- \text{Rayleigh number: } Ra = Gr \cdot Pr$$

$$- \text{Peclat number: } Pe = Re \cdot Pr$$

- Graetz number: $Gz = \frac{Re \cdot Pr}{L/D}$, is finding applications in thermally developing flow. In the entry to any cooling or heating sections, it needs some length of pipeline for the thermal boundary layer to develop and occupy the full radius of pipe. In the sections where the boundary layer is not fully developed, for heat transfer coefficient estimations we have to use the relations which is having Gz as a parameter.

• Heat transfer enhancement

- Increasing the friction factor increases the heat transfer coefficient. Roughness of the surfaces increases the friction factor.
- Coiled tubes can serve as a heat transfer enhancement device because the secondary flow produced by the curvature causes an increase in heat transfer coefficient.

• Free convection

- For free convection, in general $Nu = \phi(Gr, Pr)$
- For gases $Pr \approx 1$, hence the Nusselt number for free convection is a function of Grashof number only. $Nu = \phi(Gr)$
- If $Gr/Re^2 \ll 1$, flow is primarily by forced convection.
- If $Gr/Re^2 \gg 1$, free convection becomes dominant.
- If $Gr/Re^2 \approx 1$, mixed convection.

- Forced convection

$$Nu = \phi(Re, Pr)$$

Dittus-Boelter equation is of the form

$$Nu = 0.023 Re^{0.8} Pr^{0.33}$$

This is a simplified form of the relation given by Seider-Tate, given as

$$Nu = 0.023 Re^{0.8} Pr^{0.33} (\mu/\mu_w)^{0.14}$$

where μ = viscosity of fluid at bulk temperature

μ_w = viscosity of fluid at wall temperature

The viscosity correction factor (μ/μ_w) is needed for highly viscous fluids. Because for this high viscous fluids there will be a marked difference in viscosity at bulk temperature and at wall temperature.

- Analogies of heat and momentum transfer

- Reynolds analogy

$$St = \frac{f}{2}$$

where f is Fanning friction factor. The following restrictions apply:

- (i) $Pr = 1$.
- (ii) No form drag.

- Colburn analogy

$$St \cdot Pr^{2/3} = \frac{f}{2}$$

or

$$j_H = \frac{f}{2}$$

$$\text{where } j_H = St \cdot Pr^{2/3}$$

The following restrictions apply:

- (i) $0.5 < Pr < 50$.
- (ii) No form drag.

5.1.3 Boiling

- The number of points at which bubbles originate is dependent upon the texture of the tube surface, the number increasing with the roughness. If the surface tension is higher, it is necessary to form larger bubbles before the buoyant force can free it from the surface.
- When vaporization takes place directly at the heating surface, it is called nucleate boiling and when it takes place through a blanketing film of gas, it is film boiling.
- The *film boiling* reduces the heat transfer coefficient. Film boiling will occur for nearly all fluids, if the temperature difference between the metal surface and the liquid is greater than about 50°F . If the temperature drop from the tube surface to the liquid is less than this *nucleate boiling* is maintained.
- In practical applications the nucleate boiling regime is most desirable, because large heat fluxes are obtainable with small temperature differences.

5.1.4 Condensation

- Condensers are coolers whose primary purpose is the removal of latent heat instead of sensible heat.
- The condensing film coefficient is influenced by the texture of the surface on which condensation occurs and also by whether the condensing surface is mounted vertically or horizontally.
- The coefficient of a film condensing on a horizontal tube is considerably larger than that on a vertical tube under similar conditions, unless the tubes are very short or there are many horizontal tubes in each stack.
- Condensation on horizontal tube banks

- If it is assumed that the drainage from one tube flows smoothly on to the tube below, then for a vertical tier of N tubes each of diameter D , the average heat transfer coefficient for the N tubes is given by

$$h_m|_{N\text{tubes}} = \frac{1}{N^{1/4}} h_m|_{1\text{tube}}$$

- Horizontal tube arrangements are generally preferred to vertical tube arrangements in condenser design. Heat transfer on horizontal tube is higher than that of vertical tube.

• Dropwise condensation

- If traces of oil are present in steam and the condensing surface is highly polished, the condensate film breaks into droplets. This type of condensation is called dropwise condensation. The droplets grow, coalesce, and runoff the surface, leaving a greater portion of the condensing surface freely exposed incoming steam.

- Since the entire condensing surface is not covered with a continuous layer of liquid film, the heat transfer for ideal dropwise condensation of steam is much higher than that filmwise condensation of steam. The heat transfer coefficients may be 5 to 10 times greater.
- Steam is the only pure vapor known to condense in a dropwise manner, and special conditions are required for its occurrence.
- For subcooling, vertical condensers are often used.
- Typical calculations show that the condensing heat transfer coefficient for steam may be reduced as much as 50% when air in amounts as low as 1% by mass is present. Consequently, it is customary practice in steam condenser design to provide means of removing as much of the noncondensable gases as possible through venting or the use of ejectors.
- In a horizontal condenser using conventional segmental baffles it is imperative that they be arranged for *side to side flow* and not *up and down flow*. Failure to provide side to side flow causes pools of condensate to form between each pair of baffles whose cut-out areas are on the top of the shell, thereby impeding the passage of vapor.
- When the condensation range over which the condensation of a mixture occurs is small, perhaps not exceeding 10 to 20°F, it may be treated as a pure compound with the true temperature being LMTD for 1-1 condensers or $F_T \times$ LMTD for 1-2 condensers.

5.1.5 Heat Exchangers

- Specific heat for unit mass is very low for solids, high for liquids, and usually intermediate for gases.
- The smaller heat transfer coefficient is the *controlling film coefficient*. Because it offers more resistance
- The thickness of a fluid film is greatly reduced by increasing the velocity of a fluid through a conduit, and to a certain extent by decreasing the conduit diameter. It is approximately true that for fluids passing through conduits at a fairly rapid rate, the coefficient of heat transfer varies as the 0.8 power of the mass flow rate of the fluid, and inversely as the 0.2 power of the conduit diameter.
- When a pound of water is vaporized or condensed, the energy change in either process is identical. The rates at which either process can be made to progress with an independent source or receiver, however are inherently very different. Vaporization is generally a much more rapid phenomenon than condensation.
- In the indirect contact type or surface heat exchangers there is no mixing of fluid.
- Compact heat exchangers

- The ratio of the heat transfer surface area on one side of the heat exchanger to the volume can be used as a measure of the compactness of heat exchangers. A heat exchanger having a surface area density on any one side greater than about $700 \text{ m}^2/\text{m}^3$ quite arbitrarily is referred to as a compact heat exchanger regardless of its structural design.
- Shell and tube heat exchangers are not compact heat exchangers.
- To increase the effectiveness or compactness fins are used.

• Regenerators

- Regenerative type rotary heat exchangers are suitable only for gas to gas exchange, because only for gases is the heat capacity of the heat transfer matrix much greater than the heat capacity of the gas contained in the heat flow passages.
- It is not suitable for liquid-to-liquid heat exchange, because the heat capacity of the heat transfer matrix is much less than that of the liquid.

• Parallel flow exchangers

- In a parallel flow heat exchanger, the outlet temperature of the cold fluid can not exceed that of the hot fluid. Therefore, the temperature effectiveness of the parallel flow exchangers is limited. Because of this limitation, generally they are not considered for heat recovery. However, since the metal temperature lies approximately midway between the hot and cold fluid temperatures, the wall is almost at a uniform temperature.
- Parallel flow is used for cold viscous fluids, since the arrangement may enable a high value of U to be obtained.

• Counter flow exchangers

- In a counterflow exchanger, the exit temperature of the cold fluid can be higher than that of the hot fluid. Theoretically, the exit temperature of one fluid may approach the inlet temperature of the other. Therefore, the thermal capacity of the counterflow exchanger can be twice that of the parallel flow exchanger.
- In a pure counter flow heat exchanger, the temperature rise in the cold fluid is equal to temperature drop in the hot fluid; thus the temperature difference ΔT between the hot and cold fluids is constant throughout.
- The high heat recovery and temperature effectiveness of this exchanger makes it preferable to the parallel flow exchanger whenever the design requirements permit such a choice. The metal temperature in the counterflow exchanger, in contrast to that of the parallel flow one, involves a step gradient along the path of flow.

- The fluid velocity and the fluid temperature appear to be among the factors that affect the rate of *fouling* on a given surface. An increase in the velocity decreases both the rate of deposit and the ultimate amount of deposit on the surface. Increasing the fluid bulk temperature increases both the rate of buildup of fouling and its ultimate stable level.

- Log mean temperature difference: LMTD

$$\text{LMTD} = \frac{\Delta T_0 - \Delta T_L}{\ln(\Delta T_0 / \Delta T_L)}$$

where $\Delta T_0 = T_{h,\text{out}} - T_{c,\text{in}}$ for counterflow

$\Delta T_L = T_{h,\text{in}} - T_{c,\text{out}}$ for counterflow

- When $\Delta T_0 = \Delta T_L$, LMTD = $\Delta T_0 = \Delta T_L$
- When one of the fluid proceeds through the apparatus isothermally (e.g. condensing steam), parallel flow and counterflow yield identical temperature differences.
- Correction factor for LMTD, (FT) for multipass and cross-flow exchangers:

$$\Delta T_{\text{corrected}} = FT \times \text{LMTD} \text{ for counterflow}$$

- Generally FT is less than unity for cross flow and multipass arrangements; it is unity for true counterflow heat exchanger. It represents the degree of departure of the true mean temperature difference from LMTD for counterflow.

- Overall heat transfer coefficient, U

$$\text{Resistance } R = \frac{1}{A_i h_i} + \frac{t}{A_m k} + \frac{1}{A_o h_o}$$

$$\text{where } A_m = \frac{A_o - A_i}{\ln(A_o/A_i)}$$

t = thickness of heat exchanger tube

k = thermal conductivity of tube metal

$$U_o = \frac{1}{A_o R}$$

Overall heat transfer coefficient is generally low for fluids having low thermal conductivity, such as gases or oils.

- Less heat transfer area is required for counterflow arrangement.

- Effectiveness of an exchanger

$$\epsilon = \frac{Q}{Q_{\max}} = \frac{\text{actual heat transfer rate}}{\text{maximum possible heat transfer rate from one stream to the other}}$$

- In order to make removal of the tube bundle possible and to allow for considerable expansion of the tubes, a *floating head exchanger* is used.
- Shell and Tube Exchangers - Fluid Allocation

- *Corrosion*: Fewer costly alloy components are needed if the corrosive fluid is inside the tubes. Corrosive fluid cannot be sent in the shell side, since the shell side fluid will affect both shell and tubes.
- *Fouling*: Placing the fouling fluid inside the tubes allow better velocity control; increased velocities tend to reduce fouling. Straight tubes allow mechanical cleaning without removing the tube bundle.
- *Temperature & Pressure*: For high temperature / pressure services requiring special or expensive alloy materials, fewer alloy components are needed when hot fluid is placed within the tubes
- *Flow rate*: Placing the fluid with the lower flow rate on the shell side usually results in a more economical design. Turbulence exists on the shell side at much lower velocities than within the tubes.

5.1.6 Evaporation

- Evaporators are employed for the concentration of a solution by the evaporation of water. If any other fluid besides water, the unit is a *vaporizer*.
- The feed to a vaporizer should not be vaporized completely. If less than 100% of the feed is vaporized in a exchanger, the residual liquid can be counted on to prevent the accumulation of dirt directly on the heating element. A maximum amount of 80% vaporization appears to provide favorable operation in 1-2 exchangers, although higher percentages may be obtained in vessels having internal disengagement space.
- Calandria evaporator is referred to as standard evaporator.
- Horizontal-tube evaporators are suited to process industries in which the final product is a liquor instead of a solid. Here steam is used in the tube side.

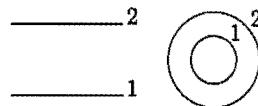
• Feeding arrangements in evaporators

- In forward feed evaporation no pumps are required and liquor removal pump is required only on the last effect. Vapor flashing is there.
- The steam cost will be less for backward feed if the feed is cold and less for forward feed if the feed liquor is at approximately the operating temperature of the first effect or higher.
- Need for maintaining the last effect in vacuum: In multiple effect evaporation, steam is fed only to the first effect. For the subsequent effects, the vapor from the prior effects are used. And because of this there is a need to maintain pressure drops in each effect. In other words, the operating pressure will decrease from first effect to last effect. So if the last effect is in vacuum, we can very well make use of low pressure steam to the first effect (low pressure steam has high latent heat). If the last effect is not in vacuum, then for the first effect there is a need for high pressure steam. This creates the problems such as high vessel thickness, high pressure steam production units etc.

5.1.7 Radiation

- Highly polished and white surfaces generally have lower emissivities than rough or black surfaces.
- A *gray body* is defined as one which has a constant value of emissivity, so that for any temperature range, it radiates the same proportion of energy radiated by a black body. Similarly it will have a constant absorptivity.
- Values of emissivity ϵ have been measured for many materials and it is found that for most industrial non-metallic surfaces and for non-polished metals ϵ may be taken as 0.9; for highly polished surfaces such as copper or aluminium values of ϵ as low as 0.03 are obtained. A small cavity in a body has an effective emissivity of unity and therefore behaves as a black body.

• View factors



- $F_{11} + F_{12} = 1$, and $F_{21} + F_{22} = 1$.

- For flat surfaces

$$F_{11} = 0 \quad F_{12} = 1$$

$$F_{21} = 1 \quad F_{22} = 0$$

- For cylinders & spheres

$$F_{11} = 0 \quad F_{12} = 1$$

$$F_{21} = A_1/A_2 \quad F_{22} = 1 - A_1/A_2$$

- For transfer between two large parallel plates, each of which has a gray surface, the heat transfer rate per unit area is given by:

$$q = \frac{\sigma(T_1^4 - T_2^4)}{1/\epsilon_1 + 1/\epsilon_2 - 1}$$

- For long concentric cylinders or concentric spheres, the heat transfer rate per unit area of surface 1 is given by:

$$q = \frac{\sigma(T_1^4 - T_2^4)}{1/\epsilon_1 + (A_1/A_2)(1/\epsilon_2 - 1)}$$

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5.2 Objective Type Questions

1. The variation of thermal conductivity of a metal with temperature is often correlated using an expression of the form

$$k = k_o + aT$$

Where k is the thermal conductivity, and T is the temperature in K. The units of ' a ' in the SI system will be
(GATE-1998-1.06)

- | | |
|------------------------|------------------------------------|
| (a) W/m.K | (b) W/m |
| (c) W/m.K ² | (d) None; ' a ' is just a number |

Answer: (c) k is having the units of W/m.K

2. Walls of a cubical oven are of thickness L , and they are made of material of thermal conductivity k . The temperature inside the oven is 100°C and the inside heat transfer coefficient is $3k/L$. If the wall temperature on the outside is held at 25°C what is the inside wall temperature in degrees C?
(GATE-1999-2.10)

- | | | | |
|----------|-----------|-----------|-----------|
| (a) 35.5 | (b) 43.75 | (c) 81.25 | (d) 48.25 |
|----------|-----------|-----------|-----------|

Answer: (c) $q = \frac{75}{L/(3k) + L/k} = \frac{100 - T_w}{L/(3k)}$
i.e., $\frac{75}{4L/(3k)} = \frac{100 - T_w}{L/(3k)}$
 $75/4 = 100 - T_w; T_w = 81.25^\circ\text{C}$

3. The heat flux (from outside to inside) across an insulating wall with thermal conductivity $k = 0.04 \text{ W/m.K}$ and thickness 0.16 m is 10 W/m^2 . The temperature of the inside wall is -5°C . The outside wall temperature is
(GATE-2001-2.11)

- | | | | |
|----------|----------|----------|----------|
| (a) 25°C | (b) 30°C | (c) 35°C | (d) 40°C |
|----------|----------|----------|----------|

Answer: (c) $q = k\Delta T/\Delta L; \Delta T = 10 \times 0.16/0.04 = 40$. Therefore outside temperature $= -5 + 40 = 35^\circ\text{C}$.

4. A composite flat wall of a furnace is made of two materials *A* and *B*. The thermal conductivity of *A* is twice of that of material *B*, while the thickness of layer *A* is half of that of *B*. If the temperature at the two sides of the wall are 400 and 1200 K, then the temperature drop (in K) across the layer of material *A* is
(GATE-2000-2.12)

- | | | | |
|---------|---------|---------|---------|
| (a) 125 | (b) 133 | (c) 150 | (d) 160 |
|---------|---------|---------|---------|

Answer: (d) $\frac{800}{(L/2)/(2k) + L/k} = \frac{\Delta T_1}{(L/2)/(2k)}$
 $\frac{800}{5L/(4k)} = \frac{\Delta T_1}{L/(4k)}$
 $\Delta T_1 = 800/5 = 160 \text{ K}$.

5. The critical radius r of insulation on a pipe is given by (GATE-1996-1.10)

- (a) $r = 2k/h$ (b) $r = k/h$ (c) $r = k/(2h)$ (d) $r = h/k$

where k is the thermal conductivity of the insulation and h the heat transfer coefficient with the ambient.

Answer: (b) $r = 2k/h$ for a sphere.

6. The thermal conductivity is minimum for: (GATE-1989-4.I.a)

- | | |
|---------------|-------------------------|
| (a) Silver | (b) Chrome-nickel steel |
| (c) Aluminium | (d) Carbon steel |

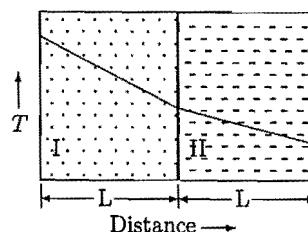
Answer: (b) k is low for high-alloys; and high for pure metals.

7. A layer of insulation is applied on a spherical metallic tank containing a cryogenic liquid. The rate of heat leakage into the tank

- (a) always decreases with increase in insulation thickness
- (b) increases with insulation thickness and remains constant beyond a critical thickness
- (c) may increase upto a thickness and then decreases, depending upon the thermal conductivity of the insulating material
- (d) may increase upto a thickness and then decrease depending upon the thermal conductivity of the metal

Answer: (c) Rate of heat leakage increases till r_{ins} equals r_c , if $r_c > r_o$; where $r_c = 2k/h$ = critical radius of insulation, and r_o = outer radius of spherical tank, and r_{ins} = radius of insulation. If $r_c < r_o$, any addition of insulation decreases heat loss.

8. At steady state, the temperature variation in a plane wall, made of two different solids I and II is shown below:



Then, the thermal conductivity of material I

(GATE-1997-2.09)

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- (a) is smaller than that of II
- (b) is greater than that of II
- (c) is equal to that of II
- (d) can be greater than or smaller than that of II

Answer: (a) $q = kdT/dx = k \times \text{slope} = \text{const.}$ Here $\text{slope}_I > \text{slope}_{II}$. Therefore $k_I < k_{II}$

9. A metal wire of 0.01 m dia and thermal conductivity 200 W/m.K is exposed to a fluid stream with a convective heat transfer coefficient 1000 W/m².K. The Biot number is (GATE-1994-1.n)

- (a) 5.6 (b) 0.0125 (c) 3.5 (d) 0.0035

Answer: (b) $Bi = hL/k$; $L = \frac{V}{A} = \frac{\pi r^2 L}{2\pi r L} = \frac{r}{2} = \frac{0.005}{2} = 0.0025$
 $Bi = \frac{1000 \times 0.0025}{200} = 0.0125$

10. In pipe flow, heat is transferred from hot wall to the liquid by (GATE-1999-1.15)

- (a) conduction only
- (b) forced convection only
- (c) forced convection and conduction
- (d) free and forced convection

Answer: (c) Conduction through the wall and forced convection in the liquid.

11. Heat transfer occurs by natural convection because change in temperature causes differences in (GATE-1999-1.16)

- (a) viscosity
- (b) density
- (c) thermal conductivity
- (d) heat capacity

Answer: (b)

12. The Grashof number is defined as the ratio of

(GATE-2000-1.12)

- (a) buoyancy to inertial forces
- (b) buoyancy to viscous forces
- (c) inertial to viscous forces
- (d) buoyancy to surface tension forces

Answer: (b) $Gr = \frac{g\beta L^3(T_w - T_\infty)}{\nu^2}$. Inertial to viscous forces is Reynolds number.

13. For turbulent flow in a tube, the heat transfer coefficient is obtained from Dittus-Boelter correlation. If the tube diameter is halved and the flow rate is doubled, then the heat transfer coefficient will change by a factor of (GATE-2000-2.13)

- (a) 1 (b) 1.74 (c) 6.1 (d) 37

Answer: (c) $Nu = 0.023 Re^{0.8} Pr^{0.33}$
 $hD/k = (Dv\rho/\mu)^{0.8} (C_p\mu/k)^{0.33}$

$$h \propto v^{0.8} D^{-0.2}$$

Given: $Q_2 = 2Q_1$ and $D_2 = D_1/2$.

From equation of continuity: $Q \propto D^2 v$.

$$Q_1 \propto D_1^2 v_1; Q_2 \propto D_2^2 v_2$$

$$2D_1^2 v_1 = D_2^2 v_2 = (D_1/2)^2 v_2. \text{ Therefore } v_2 = 8v_1.$$

$$\text{Hence } \frac{h_2}{h_1} = \frac{v_2^{0.8} D_2^{-0.2}}{v_1^{0.8} D_1^{-0.2}} = \frac{(8v_1)^{0.8} (D_1/2)^{-0.2}}{v_1^{0.8} D_1^{-0.2}} = 8^{0.8} \cdot 0.5^{-0.2} = 6.06$$

14. In natural convection heat transfer the correlating parameter is (GATE-1996-1.09)

- | | |
|--------------------|-------------------|
| (a) Graetz number | (b) Eckert number |
| (c) Grashof number | (d) Bond number |

Answer: (c) Graetz number is the correlating parameter in thermally developing flow.

15. The hydrodynamic and thermal boundary layers will merge when (GATE-1996-2.07)

- (a) Prandtl number is one
- (b) Schmidt number tends to infinity
- (c) Nusselt number tends to infinity
- (d) Archimedes number is greater than 10,000

Answer: (a) If $Pr < 1$, thermal boundary layer is thicker than hydrodynamic boundary layer (as with liquid metals).

16. When the ratio of the Grashof number to the square of the Reynolds number is one, the dominant mechanism of heat transfer is (GATE-1996-2.08)

- (a) free convection
- (b) entry length problem in laminar forced convection (developing thermal boundary layer)
- (c) mixed convection (both free and forced)
- (d) forced convection

Answer: (c)

17. Prandtl number is the ratio of (GATE-1994-1.l)

- (a) mass diffusivity to thermal diffusivity
- (b) momentum diffusivity to thermal diffusivity
- (c) thermal diffusivity to mass diffusivity
- (d) thermal diffusivity to momentum diffusivity

$$\text{Answer: (b)} \quad Pr = \frac{C_p\mu}{k} = \frac{\mu/\rho}{k/(\rho C_p)} = \frac{\nu}{\alpha}$$

18. The widely used Dittus-Boelter equation is valid provided (GATE-1991-5.ii)

- (a) $2100 < Re < 10,000$ and the properties of the fluid are evaluated at the average film temperature
- (b) $Re < 2100$ and the properties of the fluid are evaluated at the bulk temperature
- (c) $10,000 < Re < 120,000$ and the fluid properties are evaluated at the bulk temperature
- (d) none of the above

Answer: (c)

19. Grashof number is associated with: (GATE-1990-4.iii)

- | | |
|-----------------------|---------------------------------|
| (a) buoyancy effects | (b) free convection |
| (c) forced convection | (d) high temperature difference |

Answer: (b)

20. The rate of diffusion of momentum relative to the rate of diffusion of heat is: (GATE-1989-4.i.b)

- (a) proportional to Prandtl number
- (b) inversely proportional to Prandtl number
- (c) Proprtional to the Colburn's j_H factor
- (d) proportional to Stanton number

Answer: (a) $Pr = \nu/\alpha$

21. Heat transfer coefficient in a helical coil compared to that in a straight pipe is: (GATE-1989-4.i.c)

- (a) Lower
- (b) Higher
- (c) Same

Answer: (b)

22. To determine the heat transfer coefficient Dittus-Boelter equation is (GATE-1989-4.i.d) valid for:

- | | |
|--------------------|------------------------|
| (a) Laminar flow | (b) For liquid metals |
| (c) Turbulent flow | (d) Natural convection |

Answer: (c)

23. Heat transfer by natural convection is enhanced in systems with (GATE-2001-1.10)

- (a) high viscosity
- (b) high coefficient of thermal expansion
- (c) low temperature gradients
- (d) low density change with temperature

Answer: (b)

24. The Sieder-Tate correlation for heat transfer in turbulent flow in a pipe gives $\text{Nu} \propto \text{Re}^{0.8}$, where Nu is the Nusselt number and Re is the Reynolds number for the flow. Assuming that this relation is valid, the heat transfer coefficient varies with the pipe diameter (D) as (GATE-2001-2.09)

- (a) $D^{-1.8}$ (b) $D^{-0.2}$ (c) $D^{0.2}$ (d) $D^{1.8}$

Answer: (b) $\text{Nu} \propto \text{Re}^{0.8}$

$$hD/k \propto (Dv\rho/\mu)^{0.8}$$

$$h \propto D^{-0.2}$$

25. As the difference between the wall temperature and bulk temperature increases, the boiling heat transfer coefficient (GATE-1999-1.14)

- (a) continues to increase (b) continues to decrease
 (c) goes through a minimum (d) goes through a maximum

Answer: (d) Heat flux corresponding to maximum is called as critical heat flux; and the corresponding temperature difference is called as critical temperature difference.

26. Gibbs phase rule finds application when heat transfer occurs by (GATE-1999-1.13)

- (a) conduction (b) convection (c) radiation (d) condensation

Answer: (d) For condensation of multicomponent systems.

27. Saturated steam is condensed as a film on a vertical plate of height 50 cm. If the local heat transfer coefficient at a distance of 50 cm from the top is $7200 \text{ W/m}^2 \cdot ^\circ\text{C}$ the average heat transfer coefficient over the entire plate in $\text{W/m}^2 \cdot ^\circ\text{C}$ is (GATE-1993-11.b.i)

- (a) 5400 (b) 7200 (c) 9600 (d) 14400

Answer: (c) For condensation over vertical plate, from Nusselt equation $h_x \propto (1/x)^{1/4}$, from which $h_m = 4/3 h_x|_L = (4/3) \times 7200 = 9600$

28. For condensation of pure vapours, if the heat transfer coefficients in filmwise and dropwise condensation are respectively h_f and h_d , then (GATE-1997-2.11)

- (a) $h_f = h_d$ (b) $h_f > h_d$
 (c) $h_f < h_d$ (d) h_f could be greater or smaller than h_d

Answer: (c)

29. In a heat exchanger, floating head is provided to (GATE-1999-1.31)

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- (a) facilitate cleaning of the exchanger
- (b) increase the heat transfer area
- (c) relieve stresses caused by thermal expansion
- (d) increase the log mean temperature gradient

Answer: (c)

30. The advantage of using a 1-2 shell and tube heat exchanger over a 1-1 shell and tube heat exchanger is (GATE-1998-1.13)

- (a) Lower tube side pressure drop
- (b) Lower shell side pressure drop
- (c) Higher tube side heat transfer coefficient
- (d) Higher shell side heat transfer coefficient

Answer: (c) Because of increased velocity in tube side.

31. For a counter-current heat exchanger with $T_h^i = 80^\circ\text{C}$, $T_c^o = 60^\circ\text{C}$, $T_h^o = 50^\circ\text{C}$ and $T_c^i = 30^\circ\text{C}$ and the temperature difference between the two streams being the same everywhere along z , the direction of flow of the hot fluid, the temperature profile should satisfy (GATE-1999-2.09)

- (a) $d^2T/dz^2 > 0$ (b) $d^2T/dz^2 = 0$ (c) $d^2T/dz^2 < 0$ (d) $dT/dz = 0$

Answer: (b) Here $dT/dz = \text{constant}$. Therefore $d^2T/dz^2 = 0$.

32. In a heat exchanger with steam outside the tubes, a liquid gets heated to 45°C when its flow velocity in the tube is 2 m/s. If the flow velocity is reduced to 1 m/s, other things remaining the same, the temperature of the exit liquid will be (GATE-1992-5.a)

- (a) less than 45°C
- (b) equal to 45°C
- (c) greater than 45°C
- (d) initially decreases and remains constant thereafter.

Answer: (a) Tube side heat transfer coefficient reduces as $h \propto v^{0.8}$. Therefore overall heat transfer coefficient U decreases.

$$Q = UA\Delta T = ni_s \lambda = \text{constant}$$

Therefore ΔT increases; i.e., the exit temperature of liquid decreases.

33. In a finned tube heat exchanger:

(GATE-1990-4.ii)

- (a) only heat transfer area is augmented
- (b) only film coefficient is augmented
- (c) both heat transfer area and film coefficient are augmented
- (d) none of the above

Answer: (a) Because of increased heat transfer area. Film coefficient is a function of temperature and fluid properties.

34. The overall heat transfer coefficient for a shell and tube heat exchanger for clean surface is $U_o = 400 \text{ W/m}^2\text{K}$. The fouling factor after one year of operation is found to be $h_{do} = 2000 \text{ W/m}^2\text{K}$. The overall heat transfer coefficient at this time is (GATE-2001-2.10)

(a) $1200 \text{ W/m}^2\text{K}$ (b) $894 \text{ W/m}^2\text{K}$ (c) $333 \text{ W/m}^2\text{K}$ (d) $287 \text{ W/m}^2\text{K}$

Answer: (c) Overall resistance to heat transfer ($1/U$) = film resistance + resistance to heat transfer due to fouling

$$1/U = 1/400 + 1/2000$$

$$U = 333.3 \text{ W/m}^2\text{K}$$

35. In thermal radiation, for a black body (GATE-1997-1.12)

(a) $\alpha = 1; \epsilon \neq 1$ (b) $\alpha \neq 1; \epsilon = 1$ (c) $\alpha \neq 1; \epsilon \neq 1$ (d) $\alpha = 1; \epsilon = 1$

Answer: (d) For gray body $\alpha = \epsilon$ under all wavelengths of incident radiation, but they need not be equal to 1.

36. The thermal radiative flux from a surface of emissivity 0.4 is 22.68 kW/m^2 . The approximate surface temperature(K) is (GATE-1997-2.10)

(a) 1000 (b) 727 (c) 800 (d) 1200

$$\text{Stefan-Boltzmann's constant} = 5.67 \times 10^{-8} \text{ W/m}^2\text{K}^4$$

Answer: (a) $q = \sigma \epsilon T^4$

$$T = \left(\frac{22680}{5.67 \times 10^{-8} \times 0.4} \right)^{1/4} = 1000$$

37. The heat transfer by radiation from a mild steel surface is to be reduced by reducing the emissivity of the surface. This can be best achieved by (GATE-2001-1.09)

(a) painting the surface black (b) painting the surface white
(c) giving the surface a mirror finish (d) roughening the surface

Answer: (c) Roughening the surface increases the emissivity. For wavelengths of visible light region, painting the surface black increases the emissivity, and painting the surface white decreases the emissivity.

38. A multiple effect evaporator as compared to a single effect evaporator of the same capacity has (GATE-1998-1.14)

(a) Lower heat transfer area
(b) Lower steam economy
(c) Higher steam economy
(d) Higher solute concentrations in the product

Answer: (c)

39. The capacity of a multiple-effect evaporator when compared with a single-effect evaporator both operating with significant boiling point rise at the same terminal temperatures and surface area in each effect equal to surface area of single-effect evaporator (GATE-1993-11.b.ii)

(a) decreases (b) increases (c) remains the same

Answer: (a) If boiling point elevation is zero, then capacity of multiple effect evaporator is equal to the capacity of single effect.

40. The advantage of backward-feed multiple effect evaporators over forward-feed units is that (GATE-1992-5.d)

(a) heat sensitive materials can be handled
(b) there is no additional cost of pumping
(c) most concentrated liquor is at highest temperature
(d) equal heat transfer coefficients exist in various effects

Answer: (c) This increases the heat transfer coefficient in the effect where most concentrated liquid is there. There is no additional cost of pumping with forward feed.

41. Evaporators are normally operated at reduced pressures in order to

(a) reduce the wall thickness of the evaporator body
(b) enable the use of low pressure steam as heating medium
(c) increase the overall heat transfer coefficients
(d) prevent thermal degradation of the solute

Answer: (a, b, d)

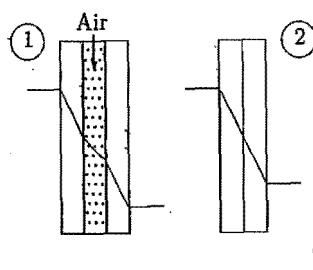
5.3 Problems with Solutions

5.3.1 Conduction through Thermopane

A thermopane window consists of two sheets of glass each 6 mm thick, separated by a layer of stagnant air also 6 mm thick. Find the percentage reduction in heat loss from this pane as compared to that of a single sheet of glass 6 mm thickness. The temperature drop between inside and outside remains same at 15°C . Thermal conductivity of glass is 30 times that of air.

(GATE-1988-14iii)

Solution:



$$q = \frac{\Delta T}{L/k}$$

Let k be the thermal conductivity of glass. Then, thermal conductivity of air = $k/30$.

$$q_1 = \frac{15}{\frac{6}{k} + \frac{6 \times 30}{k} + \frac{6}{k}} = \frac{15}{192} k$$

$$q_2 = \frac{15}{6/k} = \frac{15}{6} k$$

$$\text{Reduction in heat loss} = \frac{q_2 - q_1}{q_2} \times 100 = \frac{(15/6) - (15/192)}{(15/6)} \times 100 = 96.9\%$$

5.3.2 Conduction through Two Layers

The wall of a cold storage unit comprises a brick layer (thickness $\delta_B = 0.1$ m, thermal conductivity $k_B = 1.4$ W/m.K) and an inner layer of polyurethane foam (thickness $\delta_P = 0.05$ m, thermal conductivity $k_P = 0.015$ W/m.K). Assume one dimensional heat transfer by conduction through the composite wall, and that the inner surface of the polyurethane layer is at a temperature T_c and the outer surface of the brick layer is at temperature T_h .

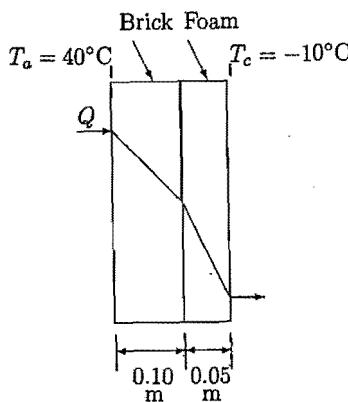
(a) Derive an expression for the heat flux through the wall.

(b) Calculate the rate of heat gain when $T_c = -10^\circ\text{C}$ and $T_h = 40^\circ\text{C}$.

The surface area for heat transfer is 260 m².

(GATE-1998-16)

Solution:



Let T_i be the temperature at the contact of the two layers. Then

$$Q/A = q = \frac{k_B(T_h - T_i)}{\delta_B} = \frac{k_P(T_i - T_c)}{\delta_P}$$

By summing the numerators and denominators, heat flux q is obtained as

$$q = \frac{T_h - T_c}{(\delta_B/k_B) + (\delta_P/k_P)}$$

$$\begin{aligned} \text{Rate of heat gain } Q &= qA \\ &= \frac{A(T_h - T_c)}{(\delta_B/k_B) + (\delta_P/k_P)} \\ &= \frac{260(40 - (-10))}{(0.1/1.4) + (0.05/0.015)} \\ &= 3818 \text{ W} = 3818 \text{ J/sec} \end{aligned}$$

5.3.3 Critical Thickness of Insulation

A pipe of 20 mm inner diameter and 30 mm outer diameter is insulated with 35 mm thick insulation. The thermal conductivity of insulating material is 0.15 W/m.K and the convective heat transfer coefficient of outside air is 3 W/m².K. The temperature of bare pipe is 200°C and the ambient air temperature is 30°C. The heat transfer resistance of the pipe metal can be neglected.

- (a) Comment with reasoning about the heat transfer rates with and without insulation.
- (b) If the same insulating material is used, what is the minimum thickness above which there is a reduction in heat loss as compared to the bare pipe?
- (c) For optimum design, what conductivity of insulating material do you suggest for the conditions given in the problem?

(GATE-1993-21b)

Solution:

Critical radius of insulation r_c :

For cylindrical heat transfer surface, $r_c = \frac{k}{h}$

where k = thermal conductivity of insulation, and
 h = heat transfer coefficient of surrounding air.

Given: $k = 0.15 \text{ W/m.K}$, and $h = 3 \text{ W/m}^2.\text{K}$. Therefore,

$$r_c = \frac{0.15}{3} = 0.05 \text{ m} = 50 \text{ mm}$$

Since the radius of insulation ($15 + 35 = 50 \text{ mm}$) is equal to the critical radius of insulation (50 mm), rate of heat loss is increased by adding insulation. Heat loss per meter length of bare pipe Q_b :

$$Q_b = \frac{hA\Delta T}{L} = h2\pi r_b \Delta T = 3 \times 2 \times \pi \times 0.015 \times (200 - 30) = 48.07 \text{ W} \quad (5.1)$$

Heat loss per meter length of insulated pipe Q_{ins} :

$$Q_{ins} = \frac{\Delta T}{\frac{1}{2\pi k} \ln \frac{r_o}{r_b} + \frac{1}{2\pi r_o h}} \quad (5.2)$$

where r_o = outer radius of insulation, and
 r_b = outer radius of bare pipe.

The minimum thickness of insulating material is calculated by equating Eqn.(5.1) and (5.2):

$$\begin{aligned} \frac{\Delta T}{\frac{1}{2\pi k} \ln \frac{r_o}{r_b} + \frac{1}{2\pi r_o h}} &= 48.07 \\ \frac{200 - 30}{\frac{1}{(2)(\pi)(0.15)} \ln \frac{r_o}{15} + \frac{1}{(2\pi r_o)(2)}} &= 48.07 \end{aligned}$$

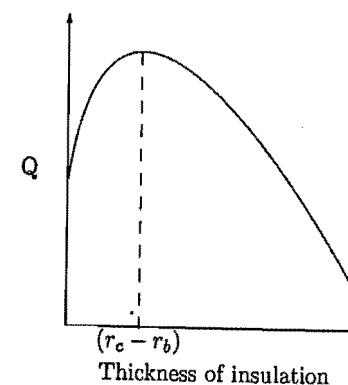
Solving iteratively, $r_o = 0.367 \text{ m} = 367 \text{ mm}$.

Therefore, minimum thickness of insulation, required = $367 - 15 = 352 \text{ mm}$. The thickness more than this value reduces the heat loss compared with the bare pipe.

Thermal conductivity of insulating material for optimum design:

$$k = r_o h = 0.015 \times 3 = 0.045 \text{ W/m.K}$$

Insulating materials with thermal conductivity values of 0.045 W/m.K or less has to be chosen, so that addition of insulating material in any thickness leads to reduction of heat loss.



5.3.4 Reduction in Heat Loss by Insulation

The outside surface temperature of a pipe (radius = 0.1m) is 400 K. The pipe is losing heat to atmosphere, which is at 300 K. The film heat transfer coefficient is 10 W/(m².K). To reduce the rate of heat loss, the pipe is insulated by a 50 mm thick layer of asbestos ($k = 0.5 \text{ W}/(\text{m.K})$). Calculate the percentage reduction in the rate of heat loss. (GATE-2000-20)

Solution:

$$\begin{aligned} \text{Rate of heat loss without insulation}(Q_1) &= hA\Delta T \\ &= 10 \times \pi D \times (400 - 300) \\ &= 200\pi \text{ W/m length} \end{aligned}$$



$$\text{Rate of heat loss with insulation}(Q_2) = \frac{\Delta T}{R_a + R_b}$$

where

$$R_a = \frac{1}{2\pi k} \ln \frac{r_1}{r_0} = \frac{1}{2\pi \times 0.5} \ln \left(\frac{0.1 + 0.05}{0.1} \right) = 0.4055/\pi$$

and

$$R_b = \frac{1}{2\pi r_1 h} = \frac{1}{2\pi \times (0.1 + 0.05) \times 10} = 0.3333/\pi$$

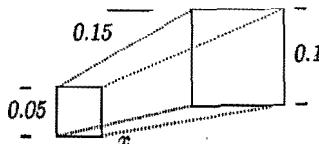
Therefore

$$Q_2 = \frac{400 - 300}{(0.4055 + 0.3333)/\pi} = 135.4\pi \text{ W/m length}$$

$$\text{Reduction in heat loss} = \frac{Q_1 - Q_2}{Q_1} \times 100 = 32.3\%$$

5.3.5 Heat Conduction through Increasing Cross-Section

An asbestos pad, square in cross-section, measures 0.05 m on a side and increases linearly to 0.1 m on the side at the other end (see the figure). The length of the pad is 0.15 m.

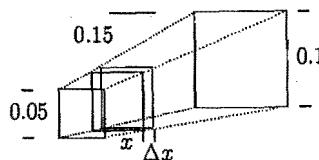


If the small end is held at 600 K and the larger end at 300 K. What will be the heat flow rate if the other four sides are insulated. Assume one directional heat flow. Thermal conductivity of asbestos is 0.173 W/m.K.
(GATE-1994-18)

Solution:

Variation of area with respect to x :

$$\begin{aligned} A &= 0.05 \times 0.05 + x \left[\frac{0.1 \times 0.1 - 0.05 \times 0.05}{0.15} \right] \\ &= 0.0025 + 0.05x \\ &= 0.05(0.05 + x) \end{aligned}$$



Heat balance for the pad at a distance x from the small end, with thickness Δx :

$$q_x A|_x - q_x A|_{x+\Delta x} = 0$$

Dividing by Δx , and in the limit as $\Delta x \rightarrow 0$,

$$\frac{-d(Aq)}{dx} = 0; \quad \text{i.e.,} \quad Aq = \text{const.} = A_o q_o$$

where $A_o = 0.05 \times 0.05 = 0.0025 \text{ m}^2$.

Writing the heat conduction equation,

$$Aq = -kA \frac{dT}{dx} = A_o q_o$$

Substituting for A ,

$$-k(0.05(0.05 + x)) \frac{dT}{dx} = 0.0025 q_o$$

HEAT TRANSFER

Rearranging and integrating,

$$\begin{aligned} - \int_{600}^{300} dT &= \frac{q_o 0.0025}{0.05 \times 0.173} \int_0^{0.15} \frac{dx}{0.05 + x} \\ -600 - 300 &= \frac{q_o (0.05)}{0.173} \ln \left[\frac{0.05 + 0.15}{0.05} \right] \end{aligned}$$

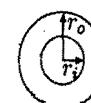
Solving, $q_o = 1439 \text{ W/m}^2$.

Heat flow rate $= A_o q_o = 0.0025 \times 1439 = 3.6 \text{ Watt}$.

5.3.6 Expression for Steady State Temperature Profile

Obtain expressions for steady-state temperature profile and heat transfer rate for a hollow spherical container. The inner surface (at $r = r_i$) is maintained at $T = T_i$ and the outer surface (at $r = r_o$) is maintained at $T = T_o$.
(GATE-1999-12)

Solution:



One dimensional heat conduction equation:

$$\frac{1}{r^n} \frac{\partial}{\partial r} \left(r^n k \frac{\partial T}{\partial r} \right) + g = \rho C_p \frac{\partial T}{\partial t} \quad (5.3)$$

where $n = 0$ for rectangular geometry

$n = 1$ for cylindrical geometry

$n = 2$ for spherical geometry

g = rate of heat generation per unit volume

For the given problem, $g = 0$; $n = 2$. Therefore

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 k \frac{\partial T}{\partial r} \right) = \rho C_p \frac{\partial T}{\partial t}$$

For steady state $\partial T / \partial t = 0$. Therefore

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 k \frac{\partial T}{\partial r} \right) = 0$$

i.e.,

$$\frac{\partial}{\partial r} \left(r^2 k \frac{\partial T}{\partial r} \right) = 0$$

For constant k ,

$$\frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) = 0$$

or,

$$\frac{d}{dr} \left(r^2 \frac{dT}{dr} \right) = 0$$

Integrating,

$$r^2 \frac{dT}{dr} = C_1$$

i.e.,

$$\frac{dT}{dr} = \frac{C_1}{r^2} \quad (5.4)$$

Integrating,

$$T = \frac{-C_1}{r} + C_2 \quad (5.5)$$

Boundary conditions:

$$\text{at } r = r_i \quad T = T_i$$

$$\text{at } r = r_o \quad T = T_o$$

Substituting the boundary conditions in Eqn.(5.5),

$$T_i = \frac{-C_1}{r_i} + C_2 \quad (5.6)$$

$$T_o = \frac{C_1}{r_o} + C_2 \quad (5.7)$$

From Eqns.(5.6) and (5.7),

$$T_i - T_o = \frac{-C_1}{r_i} + \frac{C_1}{r_o}$$

Solving for C_1 ,

$$C_1 = \frac{-r_o r_i}{(r_o - r_i)} (T_i - T_o)$$

Substituting for C_1 in Eqn.(5.7)

$$T_o = \frac{r_i r_o}{r_o - r_i} \frac{(T_i - T_o)}{r_o} + C_2$$

Solving for C_2 ,

$$C_2 = \frac{T_o r_o - T_i r_i}{r_o - r_i}$$

Substituting for C_1 and C_2 in Eqn.(5.5),

$$T = \frac{r_o r_i}{(r_o - r_i)r} (T_i - T_o) + \frac{T_o r_o - T_i r_i}{r_o - r_i}$$

Heat flux

$$q = -k \frac{dT}{dr}$$

HEAT TRANSFER

Heat transfer rate

$$Q = qA = -k(4\pi r^2) \frac{dT}{dr}$$

From Eqn.(5.4),

$$Q = -k(4\pi r^2) \frac{C_1}{r^2} = -4\pi k C_1$$

Substituting for C_1 ,

$$Q = 4\pi k \frac{r_o r_i}{(r_o - r_i)} (T_i - T_o)$$

5.3.7 Time Required for Cooling of Steel Ball

A steel ball of 5 cm diameter at 200°C is suddenly placed in a controlled environment maintained at 100°C. How much time is required for the ball to attain a temperature of 150°C. $C_p = 450 \text{ J/kg.K}$, $k = 35 \text{ W/m.K}$, $h = 10 \text{ W/m}^2.\text{K}$ and $\rho = 8000 \text{ kg/m}^3$. Internal resistance can be neglected.

(GATE-1989-14i)

Solution:

Unsteady heat balance for steel ball:

$$Ah(T_\infty - T) = \rho V C_p \frac{dT}{dt} \quad (5.8)$$

where A = heat transfer area

T = temperature of steel ball at time t

T_∞ = temperature of environment surrounding the steel ball

Rearranging Eqn.(5.8),

$$\frac{Ah}{\rho V C_p} dt = \frac{dT}{T_\infty - T}$$

Integrating,

$$\frac{Ah}{\rho V C_p} t = -\ln(T_\infty - T) + C \quad (5.9)$$

where C is integration constant.

From the initial condition: when $t = 0$; $T = 200^\circ\text{C}$

$$C = \ln(T_\infty - 200)$$

Therefore, Eqn.(5.9) becomes,

$$\frac{Ah}{\rho V C_p} t = \ln \left(\frac{T_\infty - 200}{T_\infty - T} \right) \quad (5.10)$$

Given:

Final temperature of steel (T) = 150°C

$$T_{\infty} = 100^{\circ}\text{C}$$

$$C_p = 450 \text{ J/kg.K}$$

$$h = 10 \text{ W/m}^2.\text{K}$$

$$\rho = 8000 \text{ kg/m}^3 \text{ and}$$

$$V/A = \frac{4/3\pi r^3}{4\pi r^2} = r/3 = D/6 = 0.05/6 = 8.33 \times 10^{-3} \text{ m.}$$

Substituting these data in Eqn.(5.10):

$$\begin{aligned} \frac{10}{8000 \times 8.33 \times 10^{-3} \times 450} t &= \ln \left(\frac{100 - 200}{100 - 150} \right) \\ t &= 2079 \text{ sec} \\ &= 34.7 \text{ min} \end{aligned}$$

5.3.8 Unsteady Heat Transfer

A thermocouple junction may be approximated as a sphere of diameter 2 mm with thermal conductivity $30 \text{ W}/(\text{m} \cdot ^{\circ}\text{C})$, density 8600 kg/m^3 and specific heat $0.4 \text{ kJ}/(\text{kg} \cdot ^{\circ}\text{C})$. The heat transfer coefficient between the gas stream and the junction is $280 \text{ W}/(\text{m}^2 \cdot ^{\circ}\text{C})$. How long will it take for the thermocouple to record 98 percent of the applied temperature difference? (GATE-1996-20)

Solution:

$$\begin{aligned} \text{Characteristic dimension}(L) &= \frac{\text{volume}}{\text{surface area}} \\ L &= \frac{V}{A} = \frac{(4/3)\pi r^3}{4\pi r^2} \\ &= \frac{r}{3} = \frac{D}{6} = \frac{2 \times 10^{-3}}{6} = 0.333 \times 10^{-3} \end{aligned}$$

Biot number(Bi):

$$Bi = \frac{hL}{k} = \frac{280 \times 0.333 \times 10^{-3}}{30} = 0.0031$$

Since $Bi \ll 0.1$, it may be assumed that whole of thermocouple sphere is at uniform temperature.

Unsteady heat balance for the thermocouple junction may be written as

$$\begin{aligned} \text{Heat in} - \text{Heat out} &= \text{Heat accumulation} \\ Ah(T_{\infty} - T) &= \rho C_p V \frac{dT}{dt} \end{aligned} \quad (5.11)$$

HEAT TRANSFER

where T_{∞} = temperature of material surrounding the thermocouple....

T = temperature of thermocouple junction at time t

ρ = density of thermocouple junction

C_p = specific heat of thermocouple junction

V = volume of thermocouple junction

Let $\theta = T - T_{\infty}$. Then $d\theta = dT$. Making these substitutions in Eqn.(5.11) and simplifying,

$$\frac{Ah}{\rho C_p V} dt = -\frac{d\theta}{\theta}$$

Integrating,

$$\ln \theta = -\frac{Ah}{\rho C_p V} t + C$$

where C is integration constant.

When $t = 0$, $\theta = T_0 - T_{\infty} = \theta_0$. Therefore,

$$\ln \frac{\theta}{\theta_0} = -\frac{Ah}{\rho C_p V} t = -\frac{h}{C_p L} t$$

When $\theta = 0.98 \theta_0$,

$$\ln 0.98 = t \frac{-280}{8600 \times 400 \times 0.333 \times 10^{-3}}$$

Solving, $t = 0.083 \text{ sec}$.

5.3.9 Expression for Unsteady Heat Transfer

A metallic slab of thickness $2R$, initially at a uniform temperature T_i throughout, is immersed in

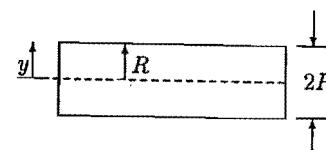
(a) large pool of fluid, at a temperature T_f , flowing at a very large velocity.

(b) a large pool of fluid, at a temperature T_f , flowing at a small velocity.

It is required to obtain the temperature distribution in the slab, in each of the above cases. Write down the governing equation that needs to be solved, together with all relevant initial and or boundary conditions necessary for the solution, in each of the above cases.

Consider unidirectional conduction only and neglect radiation effects. (GATE-1987-14i)

Solution:



(a) Pool of fluid at T_f flowing at very large velocity:

Since the fluid is flowing at high velocity, resistance offered by the liquid film in contact with the solid is negligible. So the problem is equivalent to suddenly maintaining the surface of the solid at temperature T_f at time $t = 0$.

One dimensional unsteady heat conduction:

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial y^2}$$

where $\alpha = k/(\rho C_p)$. Expressing $\theta = \frac{T_f - T}{T_f - T_i}$, $\eta = \frac{y}{R}$, and $\tau = \frac{\alpha t}{R^2}$

$$\frac{\partial T}{\partial t} = \frac{-(T_f - T_i)}{(R^2/\alpha)} \frac{\partial \theta}{\partial \tau}$$

and

$$\frac{\partial^2 T}{\partial y^2} = \frac{\alpha[-(T_f - T_i)]}{R^2} \frac{\partial^2 \theta}{\partial \eta^2}$$

Therefore,

$$\frac{-\alpha(T_f - T_i)}{R^2} \frac{\partial \theta}{\partial \tau} = \frac{-\alpha(T_f - T_i)}{R^2} \frac{\partial^2 \theta}{\partial \eta^2}$$

or

$$\frac{\partial \theta}{\partial \tau} = \frac{\partial^2 \theta}{\partial \eta^2}$$

This is a dimensionless equation with boundary conditions:

(i) at $\tau = 0$ $\theta = 1$

(ii) at $\eta = \pm 1$ $\theta = 0$

(b) Pool of fluid at T_f flowing at a small velocity:

Here the fluid film offers a resistance, with rate of heat transfer from the surface per unit area as given by

$$q = h(T - T_f) \text{ or } -k \frac{\partial T}{\partial y} = q_1 \text{ (a constant)}$$

Equation to be solved is, once again

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial y^2}$$

which is simplified with dimensionless quantities

$$\theta_1 = \frac{T - T_i}{q_1(R/k)}, \quad \eta = \frac{y}{R}, \quad \text{and} \quad \tau_1 = \frac{\alpha t}{R^2}$$

as

$$\frac{\partial T}{\partial t} = \frac{q_1 R}{k(R^2/\alpha)} \frac{\partial \theta_1}{\partial \tau_1}$$

and

$$\frac{\partial^2 T}{\partial y^2} = \frac{q_1 R}{k R^2} \frac{\partial^2 \theta_1}{\partial \eta^2}$$

From the above the dimensionless equation is obtained as

$$\frac{\partial \theta_1}{\partial \tau_1} = \frac{\partial^2 \theta_1}{\partial \eta^2}$$

with the boundary conditions:

(i) at $\tau_1 = 0$ $\theta_1 = 0$

(ii) at $\eta = \pm 1$ $-\partial \theta_1 / \partial \eta = 1$

5.3.10 Time Required for Freezing the Tank Contents

A vertical, cylindrical tank is filled with water at 65°F . The tank is insulated at the top and bottom but is exposed on its vertical sides to cold 10°F night air. The diameter of the tank is 2 ft and its height is 3 ft. The overall heat transfer coefficient is $20 \text{ BTU/hr.ft}^2.\text{°F}$. Neglect the resistance of metal wall of the tank and assume that water in the tank is perfectly mixed.

(a) Calculate how many minutes it will be until the first crystal of ice is formed.

(b) How long will it take to completely freeze the water in the tank?

The heat of fusion of water is 144 BTU/lb .

Solution:

For water $C_p = 1 \text{ BTU/lb.}^\circ\text{F}$; and $\rho = 1000 \text{ kg/m}^3 = 62.4 \text{ lb/ft}^3$.

Heat transfer area(A):

$$A = \pi D L = \pi \times 2 \times 3 = 6\pi \text{ ft}^2$$

Energy balance:

$$\text{Energy in} - \text{Energy out} = \text{Energy accumulation}$$

$$0 - UA(T - 10) = m C_p \frac{dT}{dt}$$

$$-20 \times 6\pi(T - 10) = (\pi \times 1^2 \times 3) \times 62.4 \times 1 \times \frac{dT}{dt}$$

$$-120(T - 10) = 3 \times 62.4 \times \frac{dT}{dt}$$

Grouping the terms and integrating,

$$-0.641 \int_0^t dt = \int_{65}^{32} \frac{dT}{T - 10}$$

$$-0.641t = [\ln(T - 10)]_{65}^{32}$$

$$-0.641t = \ln\left(\frac{22}{55}\right)$$

$$t = 1.4295 \text{ hr} = 85.8 \text{ min}$$

Once temperature of water reaches 32°F there will be no further change in water temperature till the entire water is converted to ice. Heat balance during this condition becomes,

$$UA(32 - 10) = \dot{m}_f \lambda$$

where \dot{m}_f = mass of water freezing per time
 λ = latent heat of freezing

Therefore,

$$\begin{aligned}\dot{m}_f &= \frac{UA(32 - 10)}{\lambda} \\ &= \frac{20 \times 6 \times \pi \times 22}{144} \\ &= 57.6 \text{ lb/hr}\end{aligned}$$

Mass of water in the tank = $\pi \times 1^2 \times 3 \times 62.4 = 588 \text{ lb}$.

Time needed for freezing the entire content = $588/57.6 = 10.21 \text{ hr}$.

5.3.11 Heat Transfer Area for Constant Heat Flux

A liquid metal flows at a rate of 5 kg/sec through a 5 cm diameter stainless steel tube. It enters at 425°C and is heated to 450°C as it passes through the tube. If a constant heat flux is maintained along the tube and the tube wall is at a temperature 20°C higher than the liquid metal bulk temperature, calculate the area required to effect the heat transfer. At constant heat flux,

$$Nu = 4.82 + 0.0185Pe^{0.827}$$

relation holds good.

Properties of the compound: $\mu = 1.34 \times 10^{-3} \text{ kg/m.sec}$; $C_P = 0.149 \text{ kJ/kg.K}$; $k = 15.6 \text{ W/m.K}$; $Pr = 0.013$.
(GATE-1989-14iii)

Solution:

Heat transfer rate Q:

$$Q = \dot{m}C_P\Delta T = (5)(0.149)(450 - 425) = 18.625 \text{ kW}$$

Estimation of heat transfer coefficient h:

$$N_{Re} = \frac{Dv\rho}{\mu}$$

Since,

$$v = \frac{(\dot{m}/\rho)}{(\pi/4)D^2}$$

Therefore,

$$v\rho = \frac{\dot{m}}{(\pi/4)D^2}$$

HEAT TRANSFER

Hence,

$$Re = \frac{(0.05)(5)}{(1.34 \times 10^{-3})(\pi/4)(0.05)^2} = 95018$$

$$\text{Peclet number } Pe = Re \cdot Pr = 95018 \times 0.013 = 1235.2$$

Therefore from the given relation,

$$Nu = 4.82 + 0.0185Pe^{0.827} = 4.82 + 0.0185 \times (1235.2)^{0.827} = 11.49$$

Since $Nu = hD/k$,

$$h = \frac{kNu}{D} = \frac{15.6 \times 11.49}{0.05} = 3585 \text{ W/m}^2\text{K}$$

Estimation of heat transfer area A:

$$\text{Rate of heat transfer by convection } Q = hA\Delta T$$

Therefore,

$$A = \frac{Q}{h\Delta T} = \frac{18625}{3585 \times 20} = 0.26 \text{ m}^2$$

Area required to effect the heat transfer = 0.26 m².

5.3.12 Convective Heat Transfer

Water, flowing in a steel pipe of diameter 0.02 m, is to be cooled from 40°C to 30°C. The velocity of water in the steel pipe is 1.5 m/s. The inside surface temperature of the steel pipe is maintained at 25°C. The physical properties of water at mean bulk temperature of the fluid are:

Specific heat = 4.174 kJ/kg.° K

Density = 995 kg/m³

Thermal conductivity = 0.623 W/m.K

Viscosity = 7.65×10^{-4} Pa.s

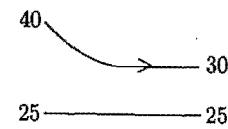
Calculate:

(a) The convective heat transfer coefficient for water

(b) The length of the tube required.

(GATE-1994-17)

Solution:



$$\Delta T_{\ln} = \frac{(40 - 25) - (30 - 25)}{\ln \left(\frac{40-25}{30-25} \right)} = 9.1^\circ\text{C}$$

From Dittus-Boelter relation,

$$\begin{aligned} \text{Nu} &= 0.023 \text{Re}^{0.8} \text{Pr}^{0.33} \\ \frac{hD}{k} &= 0.023 \left(\frac{Dv\rho}{\mu} \right)^{0.8} \left(\frac{C_p \mu}{k} \right)^{0.33} \\ \frac{h \times 0.02}{0.623} &= 0.023 \left(\frac{0.02 \times 1.5 \times 995}{7.65 \times 10^{-4}} \right)^{0.8} \left(\frac{4174 \times 7.65 \times 10^{-4}}{0.623} \right)^{0.33} \\ 0.0321h &= 0.023 \times 4710 \times 1.715 \\ h &= 5124 \text{ W/m}^2\text{K} \end{aligned}$$

Heat balance:

$$\begin{aligned} Q &= \dot{m} C_p \Delta T \\ &= (\pi/4 D^2 v \rho) C_p \Delta T \\ &= (\pi/4) \times 0.02^2 \times 1.5 \times 995 \times 4174 \times (40 - 30) \\ &= 19571 \text{ W} \end{aligned}$$

Also

$$Q = hA\Delta T_{\ln}$$

Therefore

$$A = \frac{Q}{h\Delta T_{\ln}} = \frac{19571}{5124 \times 9.1} = 0.42 \text{ m}^2$$

Since $A = \pi DL$,

$$L = \frac{A}{\pi D} = \frac{0.42}{\pi \times 0.02} = 6.7 \text{ m}$$

i.e., Length of tube required = 6.7 m.

5.3.13 Emissivity Measurement

A horizontal steam pipe 20 m long, 50 mm internal diameter, 60 mm outside diameter losses 13.5 kW heat to the surroundings at 310 K. The pipe carries steam at 500 K. Given that the convective heat transfer coefficient $h_c = 1.65(\Delta T)^{0.25} \text{ W/m}^2\text{K}$ and the Stefan-Boltzmann constant $= 5.67 \times 10^{-8} \text{ W/m}^2\text{K}^4$. Find the emissivity of the bare surface of the pipe. (GATE-1991-15ii)

Solution:

HEAT TRANSFER

Heat transfer rate Q is related as,

$$\begin{aligned} \frac{Q/A}{13.5 \times 10^3 / \pi \times 0.06 \times 20} &= h_c A (T_1 - T_2) + \sigma \epsilon (T_1^4 - T_2^4) \\ 3581 &= 1.65(\Delta T)^{0.25} (T_1 - T_2) + \sigma \epsilon (T_1^4 - T_2^4) \\ &= 1.65 \times (500 - 310)^{1.25} + 5.67 \times 10^{-8} \epsilon (500^4 - 310^4) \\ \epsilon &= 0.8 \end{aligned}$$

Emissivity (ϵ) of the bare surface of pipe = 0.8.

5.3.14 Momentum & Heat Transfer Analogy

Air flows through a smooth tube, 2.5 cm diameter and 10 m long, at 37°C. If the pressure drop through the tube is 10000 Pa, estimate

- (a) the air velocity through the tube and the friction factor
- (b) the heat transfer coefficient using Colburn Analogy [$j_H = (St)(Pr)^{0.67}$], where St is the Stanton Number and Pr is the Prandtl Number.

Gas constant, $R = 82.06 \text{ cm}^3\text{atm/mol.K}$. Darcy friction factor, $f = 0.184/\text{Re}^{0.2}$. Other relevant properties of air under the given conditions: viscosity = $1.8 \times 10^{-5} \text{ kg/m.s}$, density = 1.134 kg/m^3 , specific heat capacity, $C_p = 1.046 \text{ kJ/kg}^\circ\text{C}$, thermal conductivity = $0.028 \text{ W/m}^\circ\text{C}$.

(GATE-2002-8)

Solution:

Pressure drop due to friction is related to velocity as

$$\Delta P = \frac{2f L \rho v^2}{D} \quad (5.12)$$

Given: $f = \text{Darcy friction factor} = 0.184/\text{Re}^{0.2}$

Darcy friction factor = $4 \times$ Fanning friction factor

In Eqn.(5.12), f denotes Fanning friction factor. Therefore,

$$f = 0.25 \times 0.184/\text{Re}^{0.2} = 0.046/\text{Re}^{0.2}$$

Expanding,

$$f = \frac{0.046}{(Dv\rho/\mu)^{0.2}} = \frac{0.046\mu^{0.2}}{(Dv\rho)^{0.2}} \quad (5.13)$$

Substituting this in Eqn.(5.12),

$$\begin{aligned} \Delta P &= \frac{2 \times 0.046 \times \mu^{0.2} L \rho v^2}{D^{1.2} v^{0.2} \rho^{0.2}} \\ &= \frac{0.092 \mu^{0.2} L \rho^{0.8} v^{1.8}}{D^{1.2}} \end{aligned}$$

Substituting for the known quantities,

$$10000 = \frac{0.092 \times (1.8 \times 10^{-5})^{0.2} \times 10 \times (1.134)^{0.8} \times v^{1.8}}{(2.5 \times 10^{-2})^{1.2}}$$

Solving, v = air velocity through the tube = 47.6 m/s.

From Eqn.(5.13),

$$f = \frac{0.046 \times (1.8 \times 10^{-5})^{0.2}}{(2.5 \times 10^{-2} \times 22.02 \times 1.134)^{0.2}} = 0.0049$$

By Calburn analogy,

$$j_H = (\text{St})(\text{Pr})^{0.67} = \frac{f}{2}$$

where St = Stanton number = $\text{Nu}/(\text{Re} \cdot \text{Pr}) = h/(\rho C_p v)$

Pr = Prandtl number = $C_p \mu / k$

f = Fanning friction factor

Therefore,

$$\frac{h}{1.134 \times 1046 \times 47.6} \times \left(\frac{1046 \times 1.8 \times 10^{-5}}{0.028} \right)^{0.67} = \frac{0.0049}{2}$$

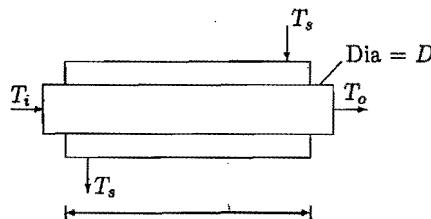
Solving, h = heat transfer coefficient = 180.5 W/m².K

5.3.15 Expression for Optimum Length of Heat Exchanger

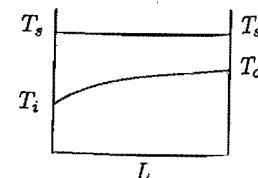
A fluid is heated from a temperature T_i to T_o in a double pipe heat exchanger with steam condensing in the outer pipe at a temperature T_s . The flow rate of fluid in the inner pipe (inside diameter D) is Q , and the heat transfer coefficient is h_i . The film heat transfer coefficient for the condensing steam is h_o , and the wall resistance and fouling are negligible.

- (a) Obtain an expression for the length of the heat exchanger required to carry out the heating operation. Assume that the outer diameter of the inner pipe is nearly equal to its inside diameter. The specific heat capacity of the fluid is C_p and its density is ρ .
- (b) Obtain an expression for the optimum diameter at which the heat exchanger length is minimum assuming $h_i = CD^{-1.8}$ where C is a constant. (GATE-1998-17)

Solution:



HEAT TRANSFER



Rate of heat transfer is given by

$$Q = UA\Delta T_{ln} \quad (5.14)$$

where

$$\Delta T_{ln} = \frac{(T_s - T_i) - (T_s - T_o)}{\ln \left(\frac{T_s - T_i}{T_s - T_o} \right)} = \frac{T_o - T_i}{\ln \left(\frac{T_s - T_i}{T_s - T_o} \right)}$$

and

$$\frac{1}{U} = \frac{1}{h_i} + \frac{1}{h_o} = \frac{h_o + h_i}{h_i h_o}$$

or

$$U = \frac{h_i h_o}{h_i + h_o}$$

Also $Q = \dot{m}C_p\Delta T$ of fluid which is getting heated. i.e.,

$$Q = (V\rho)C_p(T_o - T_i) = \pi/4 D^2 v \rho C_p (T_o - T_i)$$

where V is the volumetric flow rate of fluid in the inner pipe, and v is the velocity of fluid in the inner pipe.

Substituting in Eqn.(5.14)

$$\pi/4 D^2 v \rho C_p (T_o - T_i) = \left(\frac{h_i h_o}{h_i + h_o} \right) A \frac{T_o - T_i}{\ln \left(\frac{T_s - T_i}{T_s - T_o} \right)}$$

Therefore

$$A = \frac{\pi/4 D^2 v \rho C_p \ln \left(\frac{T_s - T_i}{T_s - T_o} \right)}{\left(\frac{h_i h_o}{h_i + h_o} \right)} = \frac{V \rho C_p \ln \left(\frac{T_s - T_i}{T_s - T_o} \right)}{\left(\frac{h_i h_o}{h_i + h_o} \right)}$$

Since $A = \pi D L$,

$$L = \frac{V \rho C_p \ln \left(\frac{T_s - T_i}{T_s - T_o} \right)}{\pi D \left(\frac{h_i h_o}{h_i + h_o} \right)}$$

For a given volumetric flow rate V , and terminal temperatures T_i, T_o , and steam temperature T_s ,

$$L = \frac{B}{D \left(\frac{h_i h_o}{h_i + h_o} \right)}$$

where

$$B = \frac{V\rho C_p \ln \left(\frac{T_s - T_i}{T_s - T_o} \right)}{\pi} = \text{a constant}$$

$$\begin{aligned} L &= \frac{B}{D \left(\frac{CD^{-1.8}h_o}{CD^{-1.8} + h_o} \right)} \\ &= \frac{B}{\left(\frac{CD^{-0.8}h_o}{CD^{-1.8} + h_o} \right)} \\ &= \frac{BCD^{-1.8} + Bh_o}{Ch_o D^{-0.8}} \\ &= \frac{C_1 D^{-1.8} + C_2}{C_3 D^{-0.8}} \end{aligned}$$

where $C_1 = BC$, $C_2 = Bh_o$, and $C_3 = Ch_o$ are constants.
For L to be minimum, $dL/dD = 0$. Therefore

$$\frac{dL}{dD} = \frac{C_3 D^{-0.8} [-1.8C_1 D^{-2.8}] - (C_1 D^{-1.8} + C_2) [-0.8C_3 D^{-1.8}]}{[C_3 D^{-0.8}]^2} = 0$$

Since

$$d(u/v) = \frac{vu' - uv'}{v^2}$$

Solving the above equation, and substituting for the constants

$$D = \left[\frac{C}{0.8h_o} \right]^{1/1.8}$$

5.3.16 Heat Transfer Area for Counterflow & Parallel flow

In a 1 - 1 counter flow shell and tube heat exchanger, a process stream ($CP = 4.2 \text{ kJ}/(\text{kg.K})$) is cooled from 450 to 350 K using water ($CP = 4.2 \text{ kJ}/(\text{kg.K})$) at 300 K. The process stream flows on the shell-side at a rate of 1 kg/s and the water on the tube-side at a rate of 5 kg/s. If the heat transfer coefficients on the shell and tube sides are $1000 \text{ W}/(\text{m}^2.\text{K})$ and $1500 \text{ W}/(\text{m}^2.\text{K})$, respectively, determine

- (a) the required heat transfer area.
- (b) by what factor will the required area change if the flow is concurrent?

Neglect tube wall resistance and fouling resistances.

(GATE-2000-21)

Solution:

Heat balance:

$$\begin{aligned} \text{Heat lost by shell side fluid} &= \text{Heat gained by tube side fluid} \\ 1 \times 4.2 \times (450 - 350) &= 5 \times 4.2 \times (T_{ot} - 300) \\ T_{ot} &= 320 \text{ K} \end{aligned}$$

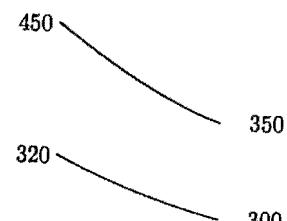
Rate of heat transfer Q :

$$Q = \dot{m}_s C_{ps} (T_{ts} - T_{os}) = 1 \times 4.2 \times (450 - 350) = 420 \text{ kW}$$

Overall heat transfer coefficient U :

$$\begin{aligned} U &= \frac{1}{(1/h_i) + (1/h_o)} \\ &= \frac{1}{(1/1500) + (1/1000)} \\ &= 600 \text{ W/m}^2.\text{K} \end{aligned}$$

ΔT_{ln} for countercurrent flow:

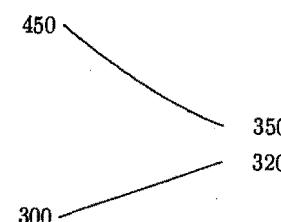


$$\Delta T_{ln} = \frac{(450 - 320) - (350 - 300)}{\ln \frac{450 - 320}{350 - 300}} = 83.7^\circ\text{C}$$

Heat transfer area for countercurrent flow A_c :

$$A_c = \frac{Q}{U \Delta T_{ln}} = \frac{420 \times 1000}{600 \times 83.7} = 8.363 \text{ m}^2$$

ΔT_{ln} for parallel (or cocurrent) flow:



$$\Delta T_{ln} = \frac{(450 - 300) - (350 - 320)}{\ln \frac{450-300}{350-320}} = 74.6^\circ\text{C}$$

Heat transfer area for countercurrent flow A_p :

$$A_p = \frac{Q}{U \Delta T_{ln}} = \frac{420 \times 1000}{600 \times 74.6} = 9.383 \text{ m}^2$$

$$\frac{A_p}{A_c} = \frac{9.383}{8.363} = 1.122$$

Therefore, area for heat transfer in parallel flow is 1.122 times that of countercurrent flow.

5.3.17 Heat Transfer Area of Jacketted Vessel

A hot fluid flows through a well mixed stirred tank which is provided with a cooling jacket. The fluid in the cooling jacket can also be assumed to be well mixed. Calculate the heat transfer area of the jacket required given the following data:

Hot fluid:

Flow rate, $W_h = 50 \text{ kg/sec}$; $T_{hi} = 205^\circ\text{C}$; $C_{ph} = 2 \text{ kJ/kg} \cdot ^\circ\text{C}$.

Cold fluid:

Flow rate, $W_c = 100 \text{ kg/sec}$; $T_{ci} = 25^\circ\text{C}$; $T_{co} = 45^\circ\text{C}$; $C_{pc} = 4 \text{ kJ/kg} \cdot ^\circ\text{C}$.

$U = 2.5 \text{ kW/m}^2 \cdot ^\circ\text{C}$ (GATE-1990-14ii)

Solution:

Rate of heat transfer = Rate of heat gained by cold fluid

i.e.,

$$\begin{aligned} Q &= W_c C_{pc} (T_{co} - T_{ci}) \\ &= 100 \times 4 \times (45 - 25) = 8000 \text{ kW} \end{aligned}$$

Also,

Rate of heat lost by fluid = Rate of heat transfer

i.e.,

$$W_h C_{ph} (T_{hi} - T_{ho}) = 50 \times 2 \times (205 - T_{ho})$$

Therefore, $T_{ho} = 125^\circ\text{C}$.

Heat transfer rate Q is also given by

$$Q = U A \Delta T_{ln} \quad (5.15)$$

Since the contents in the tank are well mixed, the temperature of contents is equal to the outlet temperature. Similar reasoning is applicable for the fluid in the jacket.

Therefore the ΔT_{ln} between the tank fluid and jacket fluid is $= T_{ho} - T_{co} = 125 - 45 = 80^\circ\text{C}$. From Eqn.(5.15)

$$A = \frac{Q}{U \Delta T_{ln}} = \frac{8000}{2.5 \times 80} = 40 \text{ m}^2$$

Required heat transfer area of jacket $A = 40 \text{ m}^2$.

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5.3.18 Heat Transfer Area & Rate of Condensation of Steam

A shell and tube steam condenser is to be constructed of 2.5 cm O.D., 2.2 cm I.D., single pass horizontal tubes with steam condensing at 54°C on the outside of the tubes. The cooling water enters at 20°C and leaves at 36°C at a flow rate of 1 kg/sec. The heat transfer coefficient for the condensation of steam is $7900 \text{ W}/(\text{m}^2 \cdot ^\circ\text{C})$. Calculate the tube length. If the latent heat of condensation is 2454 kJ/kg , calculate the condensation rate per tube. The properties of water are as follows:

Specific heat $4180 \text{ J}/(\text{kg} \cdot ^\circ\text{C})$

Viscosity $0.86 \times 10^{-3} \text{ kg}/(\text{m.sec})$

Thermal conductivity $0.61 \text{ W}/(\text{m} \cdot ^\circ\text{C})$

The heat transfer coefficient for turbulent flow in a pipe may be determined by

$$Nu = 0.023 Re^{0.8} Pr^{0.4}$$

(GATE-1990-51)

Solution:

Estimation of inside heat transfer coefficient h_i :

$$Re = \frac{Dv\rho}{\mu} = \frac{D\dot{m}}{(\pi/4)D^2\mu} = \frac{1}{(\pi/4)(0.022)(0.86 \times 10^{-3})} = 67296$$

$$Pr = \frac{C_p\mu}{k} = \frac{(4180)(0.86 \times 10^{-3})}{0.61} = 5.893$$

$$\begin{aligned} Nu &= 0.023 Re^{0.8} Pr^{0.4} \\ &= 0.023(67296)^{0.8}(5.893)^{0.4} \\ &= 340.6 \end{aligned}$$

Since $Nu = h_i D_i / k$,

$$h_i = \frac{Nu \cdot k}{D_i} = \frac{340.6 \times 0.61}{0.022} = 9444 \text{ W/m}^2 \cdot ^\circ\text{C}$$

Overall heat transfer coefficient based on outside heat transfer area of pipe U_o is given by

$$U_o = \frac{1}{\frac{D_o}{D_i} \frac{1}{h_i} + \frac{1}{h_o}}$$

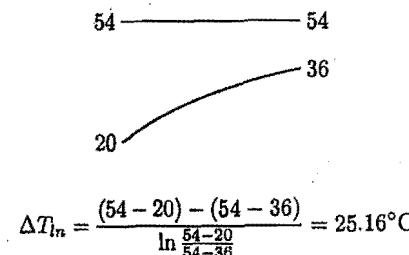
Therefore,

$$U_o = \frac{1}{\frac{0.025}{0.022} \frac{1}{9444} + \frac{1}{7900}} = 4050 \text{ W/m}^2 \cdot ^\circ\text{C}$$

Rate of heat transfer Q :

$$\begin{aligned} Q &= \text{Rate of heat gained by cooling water} \\ &= \dot{m} C_p (T_{co} - T_{ci}) \\ &= 1 \times 4180 \times (36 - 20) \\ &= 66880 \text{ W} \end{aligned}$$

Log mean temperature difference ΔT_{ln} :



Outside heat transfer area A_o :

$$A_o = \frac{Q}{U_o \Delta T_{ln}} = \frac{66880}{4050 \times 25.16} = 0.6563 \text{ m}^2$$

Tube length L :

$$L = \frac{A_o}{\pi D_o} = \frac{0.6563}{\pi \times 0.025} = 8.36 \text{ m}$$

Rate of condensation of steam \dot{m}_s :

$$\begin{aligned} \dot{m}_s &= \frac{Q}{\lambda_s} \\ &= \frac{66880}{2454 \times 1000} \\ &= 0.0273 \text{ kg/sec} \\ &= 98.1 \text{ kg/hr} \end{aligned}$$

5.3.19 Condensation of Steam

Saturated steam at $6.9 \times 10^4 \text{ Pa}$ pressure and 90°C saturation temperature condenses on a vertical pipe of 0.025 m outside diameter and 0.3 m length. The average condensing heat transfer coefficient on the tube is $12000 \text{ W/m}^2\text{.K}$. Other data:

Outside surface temperature of the pipe = 86°C

Enthalpy of saturated steam = 2659 kJ/kg

Enthalpy of condensate = 375 kJ/kg

Viscosity of condensate at the film temperature = $3.24 \times 10^{-4} \text{ Pa.s}$

Assume the flow of the condensate is laminar:

(a) Calculate the rate of steam condensation

(b) Check whether the flow is laminar.

(GATE-1994-16)

Solution:

$$Q = hA\Delta T = 12000 \times \pi \times 0.025 \times 0.3 \times (90 - 86) = 1131 \text{ watt}$$

Also, $Q = \dot{m}_s \lambda$

Therefore

$$\dot{m}_s = \frac{1131}{(2659 - 375) \times 10^3} = 4.952 \times 10^{-4}$$

i.e., rate of condensation of steam = $4.952 \times 10^{-4} \text{ kg/sec}$.

$$Re = \frac{D_h v \rho}{\mu}$$

Since D_h = hydraulic diameter = $4A/P$,

$$Re = \frac{(4A/P)v\rho}{\mu}$$

Mass flow rate (\dot{m}) is related as

$$\dot{m} = Av\rho$$

Therefore

$$\begin{aligned} Re &= \frac{4\dot{m}}{P\mu} \\ &= \frac{4 \times 4.952 \times 10^{-4}}{\pi \times 0.025 \times 3.24 \times 10^{-4}} \\ &= 77.84 < 2000 \end{aligned}$$

Hence the flow is laminar.

5.3.20 Time Required for Heating the Contents of Jacketted Vessel

150 kg of water is to be heated in a steam-jacketed vessel from 25°C to 80°C . Steam is condensing at 120°C , and the heat transfer area is 0.25 m^2 . The heat transfer coefficients for condensation of steam and heating of water by convection are $1000 \text{ W/m}^2\text{.}^\circ\text{C}$ and $500 \text{ W/m}^2\text{.}^\circ\text{C}$ respectively. Write appropriate unsteady state balance equations and find the time required for heating the water. Assume that the specific heat of water in the temperature range of interest is $4.18 \times 10^3 \text{ J/kg.}^\circ\text{C}$.

(GATE-1999-13)

Solution:

Overall heat transfer coefficient(U):

$$\begin{aligned} \frac{1}{U} &= \frac{1}{h_i} + \frac{1}{h_o} \\ &= \frac{1}{500} + \frac{1}{1000} \end{aligned}$$

Therefore, $U = 333.3 \text{ W/m}^2\cdot^\circ\text{C}$.

Unsteady heat balance:

Rate of heat in by steam = Rate of accumulation of energy in the jacket contents

i.e.,

$$UA(T_s - T) = mC_p dT/dt \quad (5.16)$$

where U = overall heat transfer coefficient

A = heat transfer area

m = mass of jacket contents(water)

C_p = specific heat of water

T_s = temperature of steam

T = temperature of jacket contents

Rearranging Eqn.(5.16),

$$\begin{aligned} \frac{dT}{T_s - T} &= \frac{UA}{mC_p} dt \\ \int_{T_1}^{T_2} \frac{dT}{T_s - T} &= \frac{UA t}{mC_p} \\ \ln \left(\frac{T_s - T_1}{T_s - T_2} \right) &= \frac{UA t}{mC_p} \\ \text{i.e., } t &= \frac{mC_p}{UA} \ln \left(\frac{T_s - T_1}{T_s - T_2} \right) \end{aligned}$$

Substituting the given data,

$$\begin{aligned} t &= \frac{120 \times 4180}{333.3 \times 0.25} \ln \left(\frac{120 - 25}{120 - 80} \right) \\ &= 5207 \text{ sec} \end{aligned}$$

Time required for heating the jacket contents = 5207 sec = 1 hr 27 min.

5.3.21 Estimation of Fouling Factor

In a counter-current heat exchanger which has been in service for some time, due to formation of scale, the heat transfer rate is reduced to 85% of its original value based on clean surface. Assuming that the terminal temperatures of fluids are same in both cases, and the effective heat transfer area does not change appreciably due to scale formation, determine the overall fouling factor if clean overall heat transfer coefficient is $500 \text{ W/m}^2\cdot\text{K}$ (GATE-1993-21aii)

Solution:

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Let U_c be the clean overall heat transfer coefficient, and U_d be the value of overall heat transfer coefficient measured after some service time t .

$$\begin{aligned} \text{Heat transfer resistance after time } t &= \text{Heat transfer resistance at } t = 0 \\ &\quad + \text{Resistance due to fouling} \end{aligned}$$

i.e., the fouling factor R is related as:

$$\frac{1}{U_d} = \frac{1}{U_c} + R$$

Given: $U_d = 0.85U_c$, and $U_c = 500 \text{ W/m}^2\cdot\text{K}$.

Therefore,

$$\begin{aligned} \frac{1}{0.85U_c} &= \frac{1}{U_c} + R \\ \frac{1}{0.85 \times 500} &= \frac{1}{500} + R \\ R &= 3.53 \times 10^{-4} \text{ m}^2\cdot\text{K/W} \end{aligned}$$

Fouling factor = $3.53 \times 10^{-4} \text{ m}^2\cdot\text{K/W}$.

5.3.22 Heat Transfer Area for 1-2 Exchanger

Estimate the heat transfer area for an exchanger to cool an organic liquid from 105°C to 50°C . The hot liquid will flow at a rate of 10,000 kg/hr and will be cooled by circulating foul water containing some salt. The cooling water will leave at 40°C . It is proposed to use one shell pass and two tube pass exchanger for the above duty.

Cooling water inlet temperature = 25°C

Heat capacity for water = $4.2 \text{ kJ/kg}\cdot^\circ\text{C}$

Heat capacity for hot liquid = $2.84 \text{ kJ/kg}\cdot^\circ\text{C}$

F_t , the correction factor for the design will be 0.85

The recommended overall heat transfer coefficient U will be $600 \text{ W/m}^2\cdot^\circ\text{C}$.

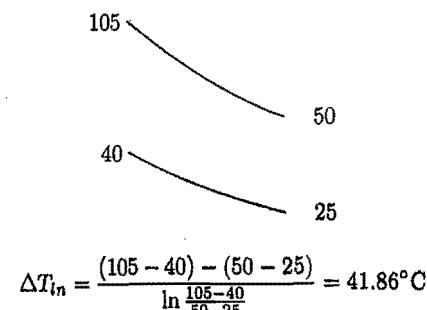
(GATE-1995-8)

Solution:

Heat balance:

$$\begin{aligned} \text{Rate of heat transfer } Q &= \text{Rate of heat lost organic fluid} \\ &= 10000 \times 2.84 \times (105 - 50) \\ &= 1562000 \text{ kJ/hr} \\ &= 433.9 \text{ kW} \end{aligned}$$

ΔT_{in} for countercurrent flow:



True temperature difference $\Delta T_{corr} = F_t \Delta T_{ln} = 0.85 \times 41.86 = 35.58^\circ\text{C}$

Heat transfer area for 1-2 exchanger A :

$$A = \frac{Q}{U \Delta T_{corr}} = \frac{433.9 \times 10^3}{600 \times 35.58} = 20.32 \text{ m}^2$$

5.3.23 Condenser Load in Evaporation

10,000 kg/hr of an aqueous feed containing 1% dissolved solids is to be concentrated to 20% solids, in a single effect evaporator. The feed enters at 25°C. The steam chest is fed with saturated steam at 110°C. The absolute pressure maintained in the evaporator is such that the water will boil at 55°C. The heat transfer area available is 50 m². The boiling point elevations are as follows:

Feed: 0.2°C; 20% solution: 15°C

The overall heat transfer coefficient, under normal operating conditions would be 2500 W/m².°C. Estimate the heat load on the condenser, assuming no sub cooling of condensate. (GATE-1987-14iii)

Solution:

Rate of heat transfer Q :

$$Q = UA\Delta T = 2500 \times 50 \times (110 - 70) = 5000 \text{ kW} = 18000000 \text{ kJ/hr}$$

Heat balance (Reference Temperature = 70°C):

$$\text{Heat content of feed} + \text{heat given by steam} = \text{heat content of vapor}$$

Since,

$$\text{heat given by steam} = \text{rate of heat transfer}$$

and

heat content of vapor with respect to 70°C = heat load on condenser
the above heat balance becomes,

$$\text{Heat content of feed} + \text{rate of heat transfer} = \text{heat load on condenser}$$

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Assuming specific heat of feed is same as that of water,

$$1000 \times 4.184(25 - 70) + 18000000 = \text{heat load on condenser}$$

Therefore, heat load on condenser = $16.1172 \times 10^6 \text{ kJ/hr} = 4477 \text{ kW}$.

5.3.24 Steam Economy & Heat Transfer Area

An aqueous solution of a solute is concentrated from 5% to 20% (mass basis) in a single-effect short-tube evaporator. The feed enters the evaporator at a rate of 10 kg/s and at a temperature of 300 K. Steam is available at a saturation pressure of 1.3 bar. The pressure in the vapor space of the evaporator is 0.13 bar and the corresponding saturation temperature of steam is 320 K. If the overall heat transfer coefficient is 5000 W/(m².K), calculate the

(a) steam economy

(b) heat transfer surface area.

	Enthalpy (kJ/kg)	Heat of vaporization (kJ/kg)
Saturated steam (1.3 bar; 380 K)	2000	
Saturated steam (0.13 bar; 320 K)	2200	
Feed (5%; 300 K)	80	
Concentrated liquor (20%; 325 K)	400	
Boiling point elevation is 5 K.		

(GATE-2000-22)

Solution:

Mass balance:

Let us denote flow rates of feed as F , vapor as V , concentrated product as P , steam as S and the mass fraction of solute as x .

Overall mass balance:

$$F = V + P \quad (5.17)$$

Balance on solute:

$$Fx_F = Px_P \quad (5.18)$$

From Eqs.(5.17) and (5.18)

$$P = \frac{10 \times 0.05}{0.2} = 2.5 \text{ kg/sec}$$

And

$$V = F - P = 10 - 2.5 = 7.5 \text{ kg/sec}$$

Energy balance:

$$FH_F + S\lambda_S = VH_V + PH_P \quad (5.19)$$

Given:

$$H_F = 80 \text{ kJ/kg}; \lambda_S = 2000 \text{ kJ/kg}$$

$$H_V = 2200 \text{ kJ/kg}; H_P = 400 \text{ kJ/kg}$$

Therefore,

$$\begin{aligned} 10 \times 80 + 2000S &= 7.5 \times 2200 + 2.5 \times 400 \\ S &= 8.35 \text{ kg/sec} \end{aligned}$$

$$\text{Steam Economy} = \frac{V}{S} = \frac{7.5}{8.35} = 0.898 = 89.8\%$$

Estimation of heat transfer area:

$$\text{Rate of heat transfer } Q = S\lambda_S = 8.35 \times 2000 = 16700 \text{ kJ/sec}$$

Also

$$Q = UA\Delta T$$

Therefore,

$$A = \frac{16700 \times 1000}{5000 \times (380 - 325)} = 60.73 \text{ m}^2$$

5.3.25 Temperature of Radiation Shield

Consider three infinite parallel plates. Plate 1 is maintained at 1227°C and plate 3 is maintained at -174°C . Emissivities are equal to that of a black body. Plate 2 is placed between plates 1 and 3, and receives no heat from external sources. What is the temperature of plate 2?
(GATE-1992-15a)

Solution:

Rate of heat transfer between plate 1 and 3, before placing plate 2, Q_o :

$$Q_o = A\sigma(T_1^4 - T_3^4) \quad (5.20)$$

Rate of heat transfer after placing plate 2, and for equal rate of heat transfer,

$$Q_1 = A\sigma(T_1^4 - T_2^4) = A\sigma(T_2^4 - T_3^4) \quad (5.21)$$

From Eqn.(5.21),

$$T_1^4 - T_2^4 = T_2^4 - T_3^4 \quad (5.22)$$

Given: $T_1 = 1227 + 273 = 1500$ K, and $T_3 = -174 + 273 = 99$ K. Therefore, from Eqn.(5.22)

$$\begin{aligned} 1500^4 - T_2^4 &= T_2^4 - 99^4 \\ T_2 &= 1261.4 \text{ K} \end{aligned}$$

Temperature of plate 2 = 1261.4 K.

Chapter 6

Mass Transfer

6.1 Instant Notes

6.1.1 Diffusion

- The flux of a constituent A relative to the average velocity of all constituents J_A is given by *Fick's law* as

$$J_A = -D_{AB} \frac{\partial C_A}{\partial z}$$

- Total molar flux of A i.e., N_A is made up of two parts, namely, that resulting from the bulk motion N of constituent A i.e., Nx_A and that resulting from diffusion J_A , i.e.,

$$N_A = Nx_A + J_A$$

- If $C_A + C_B = \text{constant}$, then $D_{AB} = D_{BA}$.
- Steady-state diffusion of A thorough nondiffusing B :**
 - Here $N_B = 0$, and, $N_A = \text{constant}$.
 - For example, in absorption of ammonia into water from a mixture of air-ammonia gas mixture, diffusion of ammonia in the gas-film is of this type.
- Steady-state equimolar counter diffusion:**
 - Here $N_A = -N_B = \text{constant}$
 - This is a situation which frequently pertains in distillation operations.
- Steady-state diffusion in multicomponent mixtures:**
 - Diffusivity of A in mixture $D_{A,m}$ is given by

$$D_{A,m} = \frac{1 - y_A}{\sum_{i=B}^n \frac{y_i}{D_{A,i}}}$$

- Fick's second law: the basic equation for one dimensional unsteady state diffusion.

$$\frac{\partial C_A}{\partial T} = D \frac{\partial^2 C_A}{\partial z^2}$$

- Diffusion coefficients or diffusivities D in gases:

- At 1 atmosphere and near room temperature D lies between 0.1 and 1 cm²/sec.
- $D \propto \frac{1}{P}$
- $D \propto T^n$, n ranges from 1.5 to 1.8.
- $D \propto \frac{1}{\sqrt{M}}$, where M is molecular weight.

- Diffusion coefficients in liquids:

- $D \approx 10^{-5}$ cm²/sec.
- Diffusion often limits the rate of processes occurring in liquids. Diffusion in liquids is important because it is slow.
- $D \propto T$
- $D \propto \frac{1}{\mu}$. In extremely high viscous solutions, diffusion becomes independent of viscosity.

6.1.2 Convective Mass Transfer

- Molar flux N is given by,

$$N = k \Delta C$$

where k is the mass transfer coefficient.

- Theories of mass transfer coefficient:

- Film theory:

$$k = \frac{D}{l}$$

where l is the film thickness.

- Penetration theory:

$$k = 2 \sqrt{\frac{D}{\pi \theta}}$$

where θ is the exposure time = v/l .

- Surface renewal theory:

$$k = \sqrt{\frac{D}{\tau}}$$

where τ = residence time.

- Mass transfer coefficient varies with the diffusion coefficient to the 0.5 to 0.7 power; and with the fluid velocity to the 0.7 power.

- Two-film theory:

$$- N_A = K_y (y_{A,G} - y_A^*) = K_x (x_A^* - x_{A,L})$$

$$- \frac{1}{K_x} = \frac{1}{mk_y} + \frac{1}{k_x}$$

$$- \frac{1}{K_y} = \frac{1}{k_y} + \frac{m}{k_x}$$

$$- \frac{\text{Resistance in gas phase}}{\text{Total resistance}} = \frac{1/k_y}{1/K_y}$$

$$- \frac{\text{Resistance in liquid phase}}{\text{Total resistance}} = \frac{1/k_x}{1/K_x}$$

- Liquid-side controls: low k_x . A consequence of slow diffusion in the liquid state. Or solute is relatively insoluble in liquid.

- Gas-side controls: $K_y = k_y$. Solute is very soluble in the liquid.

- Dimensionless Numbers:

- Sherwood number:

$$Sh = \frac{kL}{D_{AB}} = \frac{\text{mass transfer velocity}}{\text{diffusion velocity}}$$

where L is the characteristic length.

- Schmidt number:

$$Sc = \frac{\mu}{\rho D_{AB}} = \frac{\text{diffusivity of momentum}}{\text{diffusivity of mass}}$$

- Prandtl number:

$$\begin{aligned} Pr &= \frac{C_p \mu}{k} = \frac{\rho C_p (\mu/\rho)}{k} = \frac{\nu}{\alpha} \\ &= \frac{\text{diffusivity of momentum}}{\text{diffusivity of energy}} \end{aligned}$$

where $\nu = \mu/\rho$, kinematic viscosity

$\alpha = k/(\rho C_p)$, thermal diffusivity

- Lewis number

$$Le = \frac{Sc}{Pr} = \frac{\text{diffusivity of energy}}{\text{diffusivity of mass}}$$

- If heat and mass transfer are occurring simultaneously in boundary layer flow, the identity of the differential energy and diffusion equations and their boundary conditions leads to identical temperature and composition profiles when Sc/Pr is unity.

6.1.3 Gas-Liquid Contactors

- Packed towers

- Flooding usually occurs at a pressure drop of 2—3 inches of water per foot of packed height.
- Loading usually occurs at a pressure drop in the range of 0.5—1.0 inch of water per foot of height.
- Packed columns are often designed to operate at their loading velocities; they often have their lowest HETP's at such conditions.

- Redistributors

- Maldistribution of the liquid may develop in very tall towers in which the height of packing is more than 3 times the tower diameter. For tall towers, therefore, packing may be divided into several sections with liquid redistributors between the section.
- For packed towers, flooding determines the minimum possible diameter, and the usual design is for 50 to 70% of the flooding velocity.

- Tray towers

- In oversize columns, vapor pressure drops through the tray openings near the tray inlet may not be great enough to prevent liquid from passing through them. When this occurs on bubble cap trays, it is called *dumping*; on sieve trays it is called *weeping*.
- Spray towers: Resistance to transfer within the gas phase is reduced by the swirling motion of the falling liquid droplets. Spray towers are used for mass transfer of highly soluble gases where the gas-phase resistance normally controls the rate of mass transfer.
- Bubble towers: Gas is dispersed into the liquid phase in the form of fine droplets. Bubble towers are used with systems in which the liquid phase controls the rate of mass transfer; that is, the absorption of relatively insoluble gases.

- Equipment selection

- Packed towers usually will be less expensive than plate towers when the tower diameter is less than about 0.75 m. For larger diameter towers, the choice must be justified on the basis of detailed cost comparisons.
- In packed towers at any time, small amount of liquid is retained. This is generally an advantage, since it decreases the load on the tower and removes liquid from the tower as rapidly as possible. In some cases it may be a disadvantage, especially where the reaction between the gas and liquid is slow, or where the solubility of the gas in the liquid is not great.
- Packed towers offer low pressure drop compared to tray column.

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6.1.4 Tray Efficiencies

- Because a liquid changes in composition in passing across a tray, the tray efficiency of any type of cross flow tray increases somewhat with the diameter of the column, and efficiencies exceeding 100% have been observed in larger-diameter columns. However, the tray efficiencies of distillation columns are in the range of 50–100%.
- In multicomponent distillation, unless explicitly stated otherwise, tray efficiency is always expressed in terms of the key components.
- *Murphree efficiency* is the efficiency determined for a single tray.
 - In small diameter columns, the liquid on a plate is sufficiently agitated by vapor flow through the perforations for there to be no measurable concentration gradients in the liquid as it flows across the plate. Since the concentration of the liquid on the plate is constant, that of the vapor from the plate is also constant and no gradients exist in the vapor streams across the cross-section. So for small diameter columns, the local (point) and Murphree efficiencies are equal.
 - In larger columns, liquid mixing in the direction of flow is not complete, and a concentration gradient does exist in the liquid on the plate. For these conditions the local efficiency is considerably lower than the Murphree efficiency. Murphree efficiency can exceed 100%, but the local efficiency can not be greater than 100%.

6.1.5 Absorption

- Usually but not always, the solution of a gas results in an evolution of heat, and it follows that in most cases the solubility of gas decreases with increasing temperature.
- Operating line
 - The operating line indicates that the relation between the gas and liquid concentration at any level in the tower.
 - For an absorber the operating line always lies above the equilibrium solubility curve, while for a stripper the line is always below.
 - The operating line is straight only when plotted in terms of the mole-ratio units.
- Absorption factor
 - The absorption factor $A = L/(mG)$ is the slope of operating line to that of the equilibrium curve.
 - The most economical A will be in the range from 1.25 to 2.0
 - The reciprocal of absorption factor is called the stripping factor.
- For absorption, an exothermic reaction, the temperature tends to rise; and for stripping, temperature tends to fall (endothermic).

- In general, absorption is gas-film controlling if essentially all resistance to mass transfer is in the gas film. This happens when the gas is quite soluble in, or reactive with, the liquid.

- Absorber operating conditions**

- Operating pressure: As a general rule, an absorber should be operated at the highest possible pressure consistent with the requirements of other steps in the process.
- Gas velocity: The design velocity is normally specified at about 60% of the flooding value.
- Temperature of lean solvent: Cooling the liquid stream increases the solubility of the solute (decreases m) and thereby decreases the required liquid circulation rate. Normally there will be little justification for cooling the solvent below the solute-rich inlet-gas temperature.

- Stripper operating conditions**

- When the solvent from an absorption operation must be regenerated for recycling back to the absorber, one may employ either a *pressure swing* concept, a *temperature swing* concept or a combination of both in specifying stripping conditions. In pressure swing operation, the temperature of the stripper is about the same that of the absorber, but the stripping pressure is much lower.
- In the normal case involving stripping without chemical reactions, the liquid-phase resistance will dominate. So that $H_{OL} = H_L$.

- Continuous-contact equipment**

- NTU or N_{tOG} is a measure of difficulty of the separation. If the NTU's are large, the separation is hard.

$$N_{tOG} = \int_{y_2}^{y_1} \frac{dy}{y - y^*} + \frac{1}{2} \ln \frac{1 - y_2}{1 - y_1}$$

- HTU or H_{tOG} on the other hand give an idea of efficiency of the equipment.

$$H_{tOG} = \frac{G}{K_y a(1 - y)_{*M}}$$

A small HTU is a sign of good tower, implying a large surface area per unit volume. Because the overall mass transfer coefficient often depends on the velocity, the HTU can be largely independent of flow over the practical range. It tends to be between 0.3 meter and 1 meter.

- Height of tower $z = \text{NTU} \times \text{HTU}$
- Relation between overall and individual transfer units

$$\frac{1}{N_{tOG}} = \frac{1}{N_{tG}} + \frac{mG}{L} \frac{1}{N_{tL}}$$

$$H_{tOG} = H_{tG} + \frac{mG}{L} H_{tL}$$

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6.1.6 Distillation

- Separation by distillation is accomplished by partial vaporization and partial condensation.

- Vapor-liquid equilibrium relationship**

- $y = Kx$, where K is the equilibrium ratio.
- The equilibrium ratio for any component depends on temperature, on pressure and on compositions of the liquid and vapor. Because higher temperatures favor vaporization and higher pressure retard it, equilibrium ratios generally become larger as the temperature is raised or as pressure is reduced.

- First drop of liquid vaporizes at bubble point and last drop at dew point.

- Relative Volatility (α)**

- This is the ratio of A and B in one phase to that in the other and is a measure of the separability.

$$\alpha = \frac{y^*/(1 - y^*)}{x/(1 - x)} = \frac{P_{satA}}{P_{satB}}$$

- If $\alpha = 1$, no separation is possible.
- The relative volatility and hence the separability usually becomes less at high pressures.
- Volatility of different materials generally approach each other as temperature is raised. Because increasing the pressure on any system raises its boiling temperature, relative volatilities become smaller as pressures are raised.

- Equilibrium relation**

$$y^* = \frac{\alpha x}{1 + x(\alpha - 1)}$$

- Relative volatility varies with the temperature.
- As the pressure is increased, the relative volatility decreases.

- Ideality of solutions**

- A liquid or vapor mixture constitutes an ideal solution if there are no interactions when its components are mixed, no heat is evolved and volumes are additive. By this definition, vapor mixtures that obey perfect gas law must constitute ideal solutions.
- In ideal solutions, volatilities are independent of composition.

- Because distillation conditions are usually close to isobaric, equilibrium diagrams are drawn for constant-pressure conditions.
- Positive deviations from ideality**

- A mixture whose total pressure is greater than that computed for ideality is said to show positive deviations from Raoult's law. Most mixtures fall in this category. In these cases, the partial pressures of each component are larger than the ideal.
- Since the activity coefficient γ is greater than unity in these cases $\log \gamma$ is positive, and hence the name positive deviations from ideality.

- Minimum boiling azeotropes

- When positive deviations from ideality are sufficiently large and the vapor pressures of the components are not too apart, then minimum boiling azeotropes will form.
- Azeotropic mixtures of this sort are very common. One of the most important is ethanol-water azeotrope which at 1 atm occurs at 89.4 mole percent of ethanol and 78.2°C. Azeotropism disappears in this system at pressures below 70 mm Hg.

- Partial liquid miscibility: *Heteroazeotropes*

- Example: Isobutanol-water system. In this azeotropic composition lies inside the limits of solubility at the bubble point.
- In relatively few instances the azeotropic composition lies outside the limits of solubility, as in the systems of methyl ethyl ketone-water and phenol-water.
- Heterogeneous azeotropes are always minimum boiling mixtures because activity coefficients must be significantly greater than 1 to cause splitting into two phases.
- When the components have a very large difference in their boiling points, no azeotrope can form, as for ammonia-toluene, and carbon dioxide-water.

- Insoluble liquids: *Steam distillation*

- Mutual solubility of liquids is so small.
- If the liquids are completely insoluble, the vapor pressure of either component cannot be influenced by the presence of the other and exerts its true vapor pressure at the prevailing temperature. When the sum of the separate vapor pressure equals the total pressure, the mixture boils, and the vapor composition is readily computed, assuming the applicability of ideal gas law.

$$\begin{aligned} P_A + P_B &= P_t \\ y^* &= \frac{P_A}{P_t} \end{aligned}$$

- By this method of distillation with steam, so long as liquid water is present, the high boiling organic liquid can be made to vaporize at a temperature much lower than its normal boiling point without the necessity of a vacuum pump.
- For the greatest economy in steam distillation, the still should be heated from an external source of energy to the highest allowable temperature, and should be operated under as high a vacuum as the cooling water will permit.

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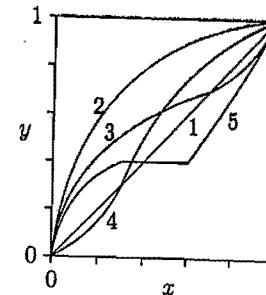
- Negative deviations from ideality

- When the total pressure of a system at equilibrium is less than the ideal value the system is said to deviate negatively from Raoult's law.

- Maximum boiling azeotropes

- Maximum boiling azeotropes are less common than the minimum type.
- Examples: acetone-chloroform, HCl-water systems.

- $x - y$ diagrams:



1. Stereoisomers, $\alpha = 1$
2. Ideal solutions
3. Minimum boiling azeotropes
4. Maximum boiling azeotropes
5. Heteroazeotropes

- With systems of type 1, ($y = x$) no separation is possible by distillation. Examples: stereoisomers, $\text{H}_2\text{O}-\text{D}_2\text{O}$ system.
- Those systems having curves of type 3, which cross the diagonal, possess minimum boiling points at the intersection, while the systems of type 4 (curves with steeper slopes than the diagonal at the point of intersection) are mixtures that possess maximum boiling points at the point where $y = x$.

- Differential or simple distillation

- *Rayleigh's Equation:*

$$\ln \frac{F}{W} = \int_{x_W}^{x_F} \frac{dx}{y^* - x}$$

- For constant α :

$$\ln \frac{Fx_F}{Wx_W} = \alpha \ln \frac{F(1-x_F)}{W(1-x_W)}$$

- Reflux ratio R

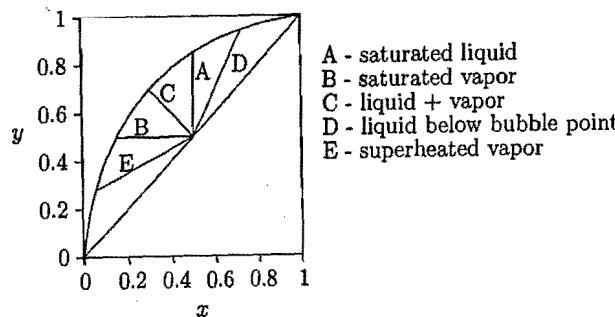
$$R = \frac{L}{D}$$

- At the minimum reflux ratio, column requires an infinite number of trays, and consequently the fixed cost is infinite; but the operating costs (heat for reboiler, condenser cooling water, power for reflux pump) are least.

- Optimum reflux ratio is about $1.2R_m$ to $1.5R_m$, where R_m is the minimum reflux ratio.
- q -lines
 - $q = \text{heat required to convert 1 mole of feed from its condition } h_f \text{ to a saturated vapor } h_g \text{ divided by molar latent heat.}$

$$q = \frac{h_g - h_f}{h_g - h_l}$$

- Slope of q -line = $\frac{q}{q-1}$



• Fenske's equation

- To find minimum number of trays N_m at total reflux, for systems of constant relative volatility.

$$N_m + 1 = \frac{\log \left[\frac{x_D}{1-x_D} \cdot \frac{1-x_W}{x_W} \right]}{\log \alpha}$$

- The optimal reflux ratio to minimum reflux ratio is usually in the range of 1.1 to 1.5.

• Use of open steam

- When a water solution in which the nonaqueous component is the more volatile, is fractionated, so that the water is removed as the residue product, the heat required can be provided by admission of steam directly to the bottom of the tower. The reboiler is dispensed with.
- For a given reflux ratio and distillate composition, more trays will usually be required in the tower.
- In trays calculation, the enriching section of the tower is unaffected by the use of open steam.

- It is the condensing temperature in a condenser which determines the operating pressure of the distillation column, since the saturation temperature of a vapor varies with its pressure. If the condensing temperature is very close to cooling water range at atmospheric pressure, the distillation pressure must be elevated to permit a larger ΔT .

• Reboilers

- Kettle type reboiler — a theoretical stage.
- Thermosyphon reboilers: It is safest not to assume that theoretical stage's worth of fractionation will occur with thermosyphon reboilers but instead to provide the necessary stages in the tower itself.

• Heat losses in the tower

- Heat losses increases the internal reflux ratio L_n/G_{n+1} , and for a given condenser heat load, fewer trays for a given separation are required.
- For a given reboiler heat load, fewer trays are required for a given separation if the heat losses are eliminated.

• Use of HETP data for packed-bed distillation

- Distillation design methods normally involve determination of the number of theoretical equilibrium stages or plates N . Thus when packed columns are employed in distillation applications, it is common practice to rate the efficiency of tower packings in terms of the height of packing equivalent to one theoretical stage (HETP).

6.1.7 Drying

- Equilibrium moisture content: If a material is exposed to air at a given temperature and humidity, the material will either loss or gain water until an equilibrium condition is established.
- Bound moisture: This is water retained in such a way that it exerts a vapor pressure less than that of free water at the same temperature.
- Free moisture: This is water in excess of the equilibrium moisture content.
- Humidity: mass of water per unit mass of dry air.
- Percent humidity = $\frac{\text{humidity of air}}{\text{humidity of saturated air}} \times 100$
- Relative humidity = $\frac{\text{partial pressure of water vapor in air}}{\text{vapor pressure of water}} \times 100$
- The distinction between percentage humidity and relative humidity should be noted. The difference in values of the two quantities does not usually exceed 7 to 8 percent.

- Constant rate drying period

- Drying takes place from a saturated surface of the material by diffusion of the water vapor through a stationary air film into the air stream.
- Rate of drying of a variety of materials in this stage are substantially the same and about 2—2.7 kg water evaporated/m².hr.

- Falling rate period

- rate of drying $\propto 1/l^2$

where l is the thickness of material.

If drying is from one side, l is total thickness, and if from two sides, l is half the thickness.

- Dryers selection

- Tray or shelf dryers - for granular materials
- Tunnel dryers - for drying paraffin wax, gelatine, soap, pottery ware, etc.
- Spray dryers - used in a variety of applications where a fairly high grade product is to be made in granular form.
- Freeze drying - for food materials.

6.2 Objective Type Questions

1. The units of diffusivity are:

- (a) m/s (b) m²/s (c) kmol/(m².s) (d) none of these

Answer: (b) Molar flux = Diffusivity × Concentration gradient. Units of molar flux is kmol/(m².s); and that of concentration gradient is kmol/(m³.m).

2. Diffusion coefficient in a binary gas mixture at low pressures varies with pressure as (GATE-1994-1.0)

- (a) P (b) P^2 (c) $1/P$ (d) independent of P

Answer: (c) From kinetic theory of low density gases

$$D_{AB} \propto \frac{T^{3/2} \left[\frac{1}{M_A} + \frac{1}{M_B} \right]^{1/2}}{P}$$

where M refers to molecular weight; T is absolute temperature; P is pressure.

3. Molecular diffusivity of a liquid

(GATE-1997-1.15)

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- increases with temperature
- decreases with temperature
- may increase or decrease with temperature
- is independent of temperature

Answer: (a) For diffusion in liquids, $D_{AB} \propto TM_B^{1/2}/\mu_B$

4. The mass transfer coefficient for a solid sphere of radius ' a ', dissolving in a large volume of quiescent liquid, in which D is the diffusivity of the solute, is (GATE-1998-1.16)

- | | |
|-------------------------------|---------------------------------------|
| (a) D/a | (b) $D/(2a)$ |
| (c) proportional to $D^{0.5}$ | (d) dependent on the Reynold's number |

Answer: (a) $Sh = kL/D$. For sphere $Sh = 2$ and $L = \text{diameter} = 2a$. Therefore $k = 2D/(2a) = D/a$,

5. In an interphase mass transfer process, the lesser the solubility of a given solute in a liquid, the higher are the chances that the transfer process will be (GATE-1998-1.17)

- liquid phase resistance controlled
- gas phase resistance controlled
- impossible
- driven by a nonlinear driving force

Answer: (a) If the solute is highly soluble in the liquid, the process may be liquid phase resistance controlled.

6. In an interphase heat transfer process, the equilibrium state corresponds to equality of temperatures in the two phases, while the condition for equilibrium in an interphase mass transfer process is (GATE-1998-2.19)

- equality of concentrations
- equality of chemical potentials
- equality of activity coefficients
- equality of mass transfer coefficients

Answer: (b)

7. Penetration theory states that the mass transfer coefficient is equal to (where D_e is diffusivity and t is time) (GATE-1999-1.19)

- (a) $(D_e t)^{1/2}$ (b) $(D_e/t)^{1/2}$ (c) $(4D_e/(\pi t))^{1/2}$ (d) $(4D_e t)^{1/2}$

Answer: (c)

8. The individual mass transfer coefficients (mol/m².s) for absorption of a solute from a gas mixture into a liquid solvent are $k_L = 4.5$ and $k_G = 1.5$. The slope of the equilibrium line is 3. Which of the following resistance(s) is (are) controlling? (GATE-2000-2.14)

- (a) liquid-side
 (b) gas-side
 (c) interfacial
 (d) both liquid and gas side

Answer: (d) Overall resistance = resistance in gas phase + resistance in liquid phase.

$$\frac{1}{K_L} = \frac{1}{k_L} + \frac{m}{k_a} = \frac{1}{1.5} + \frac{3}{4.5}$$

9. Mass transfer coefficient, k , according to penetration theory varies with mass diffusivity as (GATE-1994-1.p)

- (a) $D^{0.5}$ (b) D (c) $1/D$ (d) $D^{1.2}$

Answer: (a)

10. In the absorption of a solute gas from a mixture containing inert in a solvent, it has been found that the overall gas transfer coefficient is nearly equal to the individual gas film transfer coefficient. It may therefore be concluded that: (GATE-1990-5.i)

- (a) the process is liquid film controlled
 (b) the gas is sparingly soluble in the solvent
 (c) the transfer rate can be increased substantially by reducing the thickness of the liquid film
 (d) the transfer rate can be increased substantially by reducing the thickness of the gas film

Answer: (d)

11. The surface renewal frequency in Danckwert's model of mass transfer is given by (k_L : mass transfer coefficient, m/s) (GATE-2001-1.11)

- (a) $\sqrt{k_L^2 D_A}$ (b) $k_L^2 D_A$ (c) k_L^2 / D_A (d) k_L / D_A^2

Answer: (c) $k_L = \sqrt{D_A s}$; where s is the surface renewal frequency. Rearranging $s = k_L^2 / D_A$

12. The interfacial area per unit volume of dispersion, in a gas-liquid contactor, for fractional hold-up of gas = 0.1 and gas bubble diameter = 0.5 mm is given by (in m^2/m^3) (GATE-2001-2.12)

- (a) 500 (b) 1200 (c) 900 (d) 800

Answer: (b) Basis: Total volume = 1 m^3 . Volume of bubble = holdup fraction \times total volume = $0.1 \times 1 = 0.1 \text{ m}^3$.

Let n be the number of gas bubbles. $n4/3 \pi r^3 = 0.1 \text{ m}^3$. Given: $r = 0.5/2 = 0.25 \text{ mm}$. Therefore,

$$n = \frac{3}{4\pi} \frac{0.1}{(0.25 \times 10^{-3})^3} = 1.528 \times 10^9$$

$$\text{Surface area} = n4\pi r^2 = 1200 \times 4 \times \pi \times (0.25 \times 10^{-3})^2 = 1200 \text{ m}^2$$

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13. If the Prandtl number is greater than the Schmidt number, (GATE-1998-2.14)

- (a) the thermal boundary layer lies inside the concentration boundary layer
 (b) the concentration boundary layer lies inside the thermal boundary layer
 (c) the thermal and concentration boundary layers are of equal thickness
 (d) the hydrodynamic (i.e., momentum) boundary layer is thicker than the other two

Answer: (a) Given: $\text{Sc}/\text{Pr} < 1$.

$$\frac{\text{Sc}}{\text{Pr}} = \frac{\frac{\text{momentum diffusivity}}{\text{mass diffusivity}}}{\frac{\text{momentum diffusivity}}{\text{thermal diffusivity}}} = \frac{\text{thermal diffusivity}}{\text{mass diffusivity}}$$

If $\text{Pr} < 1$, thermal boundary layer is thicker than momentum boundary layer. i.e.,

$$\text{Pr} = \frac{\text{thickness of momentum boundary layer}}{\text{thickness of thermal boundary layer}}$$

By similar analogy

$$\frac{\text{Sc}}{\text{Pr}} = \frac{\text{thickness of thermal boundary layer}}{\text{thickness of mass boundary layer}}$$

Since $\text{Sc}/\text{Pr} < 1$, mass boundary layer is thicker than thermal boundary layer.

14. Sherwood number in mass transfer is analogous to the following dimensionless group in heat transfer

- (a) Graetz number (b) Grashoff number
 (c) Nusselt number (d) Prandtl number

Answer: (c) Schmidt number is analogous to Prandtl number.

15. Lewis number = 1 signifies:

- (a) $\text{Pr} = \text{Sc}$ (b) $\text{Pr} = \text{Re}$ (c) $\text{Sc} = \text{Re}$ (d) $\text{Nu} = \text{Sh}$

Answer: (a) $\text{Le} = \text{Sc}/\text{Pr}$. Lewis number finds application in combined mass and heat transfer as with humidification.

16. For turbulent mass transfer in pipes, the Sherwood number depends upon the Reynolds number(Re) as (GATE-1997-1.16)

- (a) $\text{Re}^{0.33}$ (b) $\text{Re}^{0.53}$ (c) $\text{Re}^{0.83}$ (d) Re

Answer: (c) Similar relation as that of Dittus-Boelter relation in heat transfer ($\text{Nu} \propto \text{Re}^{0.8} \text{Pr}^{0.33}$). For laminar flow $\text{Sh} \propto \text{Re}^{0.5} \text{Sc}^{0.3}$

17. The absorption factor is defined as

27. For absorbing sparingly soluble gas in a liquid (GATE-1995-2.p)

- (a) gas side mass transfer coefficient should be increased
- (b) liquid side mass transfer coefficient should be increased
- (c) liquid side mass transfer coefficient should be decreased
- (d) mass transfer coefficient must be kept constant

Answer: (b) This can be achieved by using bubble towers. Here liquid is the continuous phase, and gas is the dispersed phase.

28. An alkaline solution is used to reduce the concentration of carbon dioxide in a stream from 10% to 0.1% by absorption with irreversible chemical reaction. The overall number of transfer units based on gas phase is (GATE-1999-1.20)

- (a) 9.21 (b) 4.605 (c) 100 (d) 0.001

Answer: (b) $N_t = \int_{y_1}^{y_2} \frac{dy}{y - y^*}$. For mass transfer with irreversible gas reactions $y^* \approx 0$. Therefore $N_t = \ln \frac{y_2}{y_1} = \ln \frac{0.1}{0.001} = 4.605$.

29. In binary distillation, the separation of the components is easier if the relative volatility (α) is (GATE-1997-1.14)

- (a) $\alpha \gg 1$ (b) $\alpha \ll 1$ (c) $\alpha = 1$ (d) none of these

Answer: (a) Higher the relative volatility greater the ease of separation.

30. The vapor pressures of benzene and toluene are 3 and 4/3 atmospheres respectively. A liquid feed of 0.4 moles of benzene and 0.6 moles of toluene is vaporized. Assuming that the products are in equilibrium, the vapor phase mole fraction of benzene is (GATE-1999-2.12)

- (a) 0.4 (b) 0.6 (c) 0.8 (d) 0.2

Answer: (b) $P = x_A P_A^{\text{sat}} + x_B P_B^{\text{sat}} = 0.4 \times 3 + 0.6 \times 4/3 = 1.2 + 0.8 = 2$; $y_A = \bar{P}_A / P = 1.2/2 = 0.6$.

31. At the boiling point of a completely immiscible system of two liquids, the partial pressure of one component in the vapor phase is (GATE-1988-6.b.i)

- (a) always less than its vapor pressure at that temperature
- (b) always greater than its vapor pressure at that temperature
- (c) equal to its vapor pressure at that temperature
- (d) dependent on the nature and amount of the other component in the liquid phase

Answer: (c) For completely immiscible system $P_t = P_A^{\text{sat}} + P_B^{\text{sat}}$. This is the case with steam distillation.

32. Assume that benzene is insoluble in water. The normal boiling points of benzene and water are 80.1°C and 100°C respectively. At a pressure of 1 atm, the boiling point of a mixture of benzene and water is (GATE-2000-1.05)

- (a) 80.1°C
- (b) less than 80.1°C
- (c) 100°C
- (d) greater than 80.1°C but less than 100°C

Answer: (b)

33. In steam distillation of nitrobenzene (bp. 210.6°C) at a total pressure of one atmosphere, the boiling point of mixture is

- (a) less than 100°C
- (b) 100°C
- (c) between 100°C and 210.6°C
- (d) 210.6°C

Answer: (a)

34. In distillation column design, the McCabe-Thiele procedure is inadequate and Ponchon-Savarit procedure is needed when (GATE-1998-1.18)

- (a) saturated feed is not used
- (b) an azeotrope forms
- (c) the latent heats of vaporization of the more and less volatile components are greatly different
- (d) a total condenser is used

Answer: (c) Feed condition only affects the q -line. If molar latent heats of vaporization of more volatile and less volatile components are nearly equal, then McCabe-Thiele method can be used. If a partial condenser is used it is also counted as a tray.

35. In a binary distillation column, if the feed contains 40 mol% vapor, the q line will have a slope of (GATE-1997-2.12)

- (a) 1.5 (b) -0.6 (c) -1.5 (d) 0.6

Answer: (c) Slope of q -line = $q/(q-1)$; where q is the amount of heat required to vaporize 1 mole of feed / latent heat. Here $q = (1 - 0.4)\lambda/\lambda = 0.6$

$$\frac{q}{q-1} = \frac{0.6}{0.6-1} = -1.5$$

36. In distillation where q is defined as the moles of liquid flow in the stripping section per mole of feed introduced, for saturated liquid feed (GATE-1995-2.n)

- (a) $q > 1$ (b) $q < 1$ (c) $q = 1$ (d) $q = 0$

Answer: (c)

37. Solvent used in extractive distillation (GATE-1999-1.17)

- (a) is of low volatility
- (b) forms a low-boiling azeotrope
- (c) forms a high-boiling azeotrope
- (d) does not alter the relative volatility of the original components

Answer: (a) Because of low volatility it goes with the bottoms. In azeotropic distillation, solvent used forms a low-boiling azeotrope and goes along with the distillate.

38. For a fixed number of ideal stages in a distillation column, as the reflux ratio is increased, the difference in composition between the product streams (GATE-1998-1.15)

- (a) increases
- (b) decreases
- (c) remains unchanged
- (d) passes through a maximum

Answer: (a) With increase in reflux ratio for a given number of stages, the product purity increases. Hence the difference in composition between the distillate and residue increases.

39. Starting at the minimum reflux ratio, as the reflux ratio is increased for a given separation, the fixed cost of a fractionating column (GATE-1988-5.b.i)

- (a) increases
- (b) decreases
- (c) first decreases and then increases
- (d) first increases and then decreases

Answer: (c) Fixed cost first decreases as the number of trays reduces, latter increases as the tower diameter increases.

40. In a particular distillation operation, the feed is to be changed from saturated vapors to saturated liquid (composition unchanged), the number of plates required in enriching section would

- (a) decrease
- (b) remain same
- (c) increase
- (d) none of these

(Distillate and residue compositions remain unchanged) **Answer:** (a) as saturated liquid feed enriches the vapors coming from the stripping section. With saturated vapor feed, the liquid coming from the enriching section will be stripped of more volatile component, and this will lead to decrease in number of trays in the stripping section.

41. When a multistage tray tower uses a minimum reflux ratio it implies (GATE-1996-1.13)

- (a) infinite trays and maximum reboiler heat load
- (b) infinite trays and minimum reboiler heat load
- (c) minimum trays and minimum reboiler heat load
- (d) minimum trays and maximum reboiler heat load

Answer: (b) As reflux is minimum, it also requires a minimum condenser load.

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42. In distillation under minimum reflux conditions, number of theoretical stages would be (GATE-1993-12.ii)

- (a) one
- (b) minimum
- (c) infinite
- (d) none of these

Answer: (c)

43. Minimum number of ideal stages are required in a fractionating column when the reflux ratio is equal to: (GATE-1990-5.iii)

- (a) minimum reflux ratio
- (b) optimum reflux ratio
- (c) zero
- (d) infinity

Answer: (d)

44. By increasing the reflux ratio in a binary fractionating column from minimum reflux ratio onwards, the: (GATE-1989-5.ii.d)

- (a) operating cost increases but the fixed cost decreases continuously
- (b) operating cost decrease but the fixed cost increases continuously
- (c) operating cost first decreases and then increases but the fixed cost decreases
- (d) operating cost increases but the fixed cost first decreases and then increases

Answer: (d)

45. By increasing the feed rate to a fractionating column, separating a binary mixture at a fixed reflux ratio and separation, the required number of ideal stages: (GATE-1989-5.ii.c)

- (a) Remains unaltered
- (b) Decreases
- (c) Increases
- (d) Will increase or decrease

Answer: (a) Feed rate decides the column diameter required.

46. The type of tray which gives greatest flexibility in distillation columns is:

- (a) Sieve tray
- (b) Bubble cap tray
- (c) Valve tray
- (d) None of the above

Answer: (c) Valve trays can handle a wide range of vapor velocities.

47. In distillation columns, the number of bubble caps per tray primarily depends on the (GATE-1997-1.30)

- (a) allowable liquid velocity
- (b) allowable gas velocity
- (c) allowable gas and liquid velocities
- (d) feed composition

Answer: (b) Allowable liquid velocity decides the downcomer diameter.

48. The reason for preferring packed towers over plate towers in distillation practice is that the packed tower operation gives
 (GATE-1996-1.12)

- (a) low pressure drop and high hold up
- (b) high pressure drop and low hold up
- (c) low pressure drop and low hold up
- (d) high pressure drop and high hold up

Answer: (c) Because of low ΔP packed towers are preferred for vacuum distillation. In tray columns the vapor need to pass through the column of liquid held in the trays which leads to high pressure drops.

49. In the layout plan for a vacuum distillation unit, operating at 60 mm Hg, supported by a barometric condenser, the appropriate place for the location of vacuum drum for collecting the distillate will be
 (GATE-1992-10.c)

- (a) at ground level
- (b) 2 m above ground
- (c) 5 m above ground
- (d) 10 m above ground

Answer: (d) $760 \text{ mm Hg} = 10.33 \text{ m Water column}$. $60 \text{ mm Hg} = 10.33 \times 60/760 = 0.82 \text{ m}$. Therefore, to collect the condensate from the barometric condenser, a column of water to a height of $10.33 - 0.82 = 9.51 \text{ m}$ should be there in the pipeline connecting the condenser to the barometric seal (opened to the atmosphere). Barometric seal is normally provided at the ground level.

50. For the Air-Water system under ambient conditions, the adiabatic saturation temperature and the wet-bulb temperature are nearly equal, because
 (GATE-1998-2.13)

- (a) water has a high latent heat of evaporation
- (b) Lewis number is close to unity
- (c) they are always equal under all circumstances
- (d) solubility of the components of air in water is very small

Answer: (b)

51. Air, initially at 101.3 kPa and 40°C , and with a relative humidity of 50%, is cooled at constant pressure to 30°C . The cooled air has
 (GATE-1998-2.08)

- (a) A higher dew point
- (b) A higher absolute (specific) humidity
- (c) A higher relative humidity
- (d) A higher wet bulb temperature

Answer: (c) Dew point and absolute humidity remains constant during constant pressure heating or cooling. Wet bulb temperature decreases for cooling and increases for heating.

52. If the specific heats of a gas and vapor are 0.2 kJ/kg.K and 1.5 kJ/kg.K respectively, and the humidity is 0.01, the humid heat in kJ/kg.K is
 (GATE-1999-2.11)

- (a) 0.31
- (b) 0.107
- (c) 0.017
- (d) 0.215

Answer: (d) Humid heat (C_s) is defined as the amount of heat required to raise the temperature of unit mass of gas and its accompanying vapor by one degree at constant pressure. $C_s = C_B + Y'C_A = 0.2 + 0.01 \times 1.5 = 0.215$.

53. In a laboratory test run, the rate of drying was found to be $0.5 \times 10^{-3} \text{ kg/m}^2.\text{s}$ when the moisture content reduced from 0.4 to 0.1 on a dry basis. The critical moisture content of the material is 0.08 on a dry basis. A tray drier is used to dry 100 kg (dry basis) of the same material under identical conditions. The surface area of the material is $0.04 \text{ m}^2/\text{kg}$ of dry solid. The time required (in seconds) to reduce the moisture content of the solids from 0.3 to 0.2 (dry basis) is
 (GATE-2000-2.15)

- (a) 2000
- (b) 4000
- (c) 5000
- (d) 6000

Answer: (c) Drying takes place at constant rate period. For constant rate period, $N_A = -\frac{S}{A} \frac{dX}{d\theta}$. Given: $N_A = 0.5 \times 10^{-3} \text{ kg/m}^2.\text{s}$; $S/A = 1/0.04 \text{ kg of dry solid/m}^2$. Therefore

$$0.5 \times 10^{-3} = -\frac{1}{0.04} \frac{\Delta X}{\Delta \theta}$$

Given: $\Delta X = 0.2 - 0.3 = -0.1$. Therefore, $\Delta \theta = 0.1/(0.04 \times 0.5 \times 10^{-3}) = 5000 \text{ s}$.

54. Air with a dry bulb temperature t_{d1} and a wet bulb temperature t_{w1} is passed through a heater and then through an adiabatic cooler from which it emerges with dry and wet bulb temperatures of t_{d2} and t_{w2} respectively. Then
 (GATE-1988-5.b.vi)

- (a) always $t_{w1} > t_{w2}$ and $t_{d1} > t_{d2}$
- (b) always $t_{w1} < t_{w2}$ and $t_{d1} > t_{d2}$
- (c) always $t_{w1} > t_{w2}$ and $t_{d1} < t_{d2}$
- (d) $t_{w1} < t_{w2}$ and t_{d1} can be greater than, equal to or less than t_{d2}

Answer: (d) Heating increases the wet bulb temperature; and the adiabatic cooling does not make any change in wet bulb temperature. Therefore $t_{w1} < t_{w2}$. The value of t_{d2} depends on the amount of saturation effected during adiabatic cooling.

55. If the temperature of atmosphere increases at constant absolute humidity, the percentage saturation would

- (a) decrease
- (b) remain constant
- (c) increase
- (d) none of these

Answer: (a) Also the relative humidity decreases.

56. If the temperature of atmosphere increases at constant absolute humidity, the wet-bulb temperature would

- (a) decrease
- (b) remain constant
- (c) increase
- (d) none of these

Answer: (c)

57. In a mixture of benzene vapor and nitrogen gas at a total pressure of 900 mm Hg, if the absolute humidity of benzene is 0.2 kg benzene / kg nitrogen, the partial pressure of benzene in mm Hg is
(GATE-1996-2.17)

(a) 180 (b) 60.3 (c) 720 (d) 200

Answer: (b) Let B denotes Benzene and N denotes Nitrogen.

$$\frac{P_B M_B}{P_N M_N} = 0.2. \text{ i.e., } \frac{P_B \times 78}{(900 - P_B) \times 28} = 0.2$$

Solving $P_B = 60.3$ mm Hg.

58. A batch of material is dried under constant drying conditions. When drying is taking place from all the surfaces, the rate of drying during the constant rate period is:
(GATE-1990-5.iv)

- (a) directly proportional to the solid thickness
- (b) independent of the solid thickness
- (c) inversely proportional to the solid thickness
- (d) directly proportional to the square of solid thickness

Answer: (b)

59. While drying a material from a moisture content above the critical moisture content to a moisture content very close to the equilibrium moisture content, the surface temperature of the solid (ignoring initial adjustment period):
(GATE-1989-5.ii.b)

- (a) Remains a constant
- (b) At first remains a constant and then increases
- (c) Increases continuously
- (d) At first remains a constant and then decreases

Answer: (b) T remains at constant during constant rate period, and increases during falling rate period.

60. Air is passed through a heater at constant pressure. The inlet and outlet conditions of the air are as follows:

Inlet:

Dry bulb temperature = 31.7°C
Wet bulb temperature = 23.9°C
Humidity = 30%

Outlet: Dry bulb temperature = 70°C
The inlet air will have a dew point

(GATE-1989-5.ii.e)

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- (a) Equal to 23.9°C
- (b) Between 23.9°C and 31.7°C
- (c) Greater than 31.7°C
- (d) Less than 23.9°C

Answer: (d) Dew point will be less than dry bulb and wet bulb temperatures. They are equal at 100% saturation. $t_{Dew} < t_{WB} < t_{DB}$

61. In the above problem, the air from the heater will have a humidity: (GATE-1989-5.ii.f)

- (a) Equal to 30%
- (b) Greater than 30%
- (c) Less than 30%
- (d) None of the above

Answer: (c) Heating decreases percentage humidity and relative humidity.

62. The Lewis relation for air-water humidification is given by (k_Y : mass transfer coefficient of moisture in air; h_G : heat transfer coefficient; C_S : heat capacity of vapor-gas mixture)
(GATE-2001-1.13)

- (a) $\frac{h_G^2}{k_Y C_S} = 1$
- (b) $\frac{k_Y C_S^2}{h_G} = 1$
- (c) $\frac{h_G}{k_Y C_S} = 1$
- (d) $\frac{k_Y^2 h_G}{C_S} = 1$

Answer: (c) $Le = 1$.

$$Le = \frac{Sc}{Pr} = \frac{\frac{\mu}{\rho D_{AB}}}{\frac{C_p \mu}{k}} = \frac{k}{\rho D_{AB} C_p} = \frac{k/L}{\rho (D_{AB}/L) C_p} = \frac{h}{k_y C_p}$$

6.3 Problems with Solutions

6.3.1 Molecular Diffusivity

In a gas mixture of hydrogen and oxygen, steady state equimolar counter diffusion is occurring at a total pressure of 100 kPa and temperature of 20°C. If the partial pressures of oxygen at two planes 0.01 m apart, and perpendicular to the direction of diffusion are 15 kPa and 5 kPa, respectively and the mass diffusion flux of oxygen in the mixture is 1.6×10^{-5} kmol/m².sec, calculate the molecular diffusivity for the system.
(GATE-1994-19)

Solution:

For equimolar countercurrent diffusion:

$$N_A = \frac{D_{AB}}{RT_z} (\bar{p}_{A1} - \bar{p}_{A2}) \quad (6.1)$$

where N_A = molar flux of A (1.6×10^{-5} kmol/m².sec);

D_{AB} = molecular diffusivity of A in B;

R = Universal gas constant (8.314 kJ/kmol.K);

T = Temperature in absolute scale ($273 + 20 = 293$ K);

z = distance between two measurement planes 1 and 2 (0.01 m);

\bar{P}_{A1} = partial pressure of A at plane 1 (15 kPa); and

\bar{P}_{A2} = partial pressure of A at plane 2 (5 kPa).

Substituting these in Eqn.(6.1),

$$1.6 \times 10^{-5} = \frac{D_{AB}}{(8.314)(293)(0.01)} (15 - 5)$$

$$\text{Therefore, } D_{AB} = 3.898 \times 10^{-5} \text{ m}^2/\text{sec}$$

6.3.2 Diffusion of A through Non-diffusing B

Oxygen is diffusing in a mixture of oxygen-nitrogen at 1 std atm, 25°C. Concentration of oxygen at planes 2 mm apart are 10 and 20 volume % respectively. Nitrogen is non-diffusing.

(a) Derive the appropriate expression to calculate the flux of oxygen. Define units of each term clearly.

(b) Calculate the flux of oxygen. Diffusivity of oxygen in nitrogen = 1.89×10^{-5} m²/sec.

(GATE-1993-22a)

Solution:

Let us denote oxygen as A and nitrogen as B. Flux of A i.e., N_A is made up of two components, namely that resulting from the bulk motion of A i.e., Nx_A and that resulting from molecular diffusion J_A :

$$N_A = Nx_A + J_A \quad (6.2)$$

From Fick's law of diffusion,

$$J_A = -D_{AB} \frac{\partial C_A}{\partial z} \quad (6.3)$$

For one dimensional diffusion Eqn.(6.3) becomes

$$J_A = -D_{AB} \frac{dC_A}{dz}$$

Substituting this Eqn.(6.2)

$$N_A = Nx_A - D_{AB} \frac{dC_A}{dz} \quad (6.4)$$

Since $N = N_A + N_B$, and $x_A = C_A/C$ Eqn.(6.4) becomes

$$N_A = (N_A + N_B) \frac{C_A}{C} - D_{AB} \frac{dC_A}{dz}$$

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Rearranging the terms and integrating between the planes between 1 and 2,

$$\int \frac{dz}{cD_{AB}} = - \int_{C_{A1}}^{C_{A2}} \frac{dC_A}{N_A C - C_A (N_A + N_B)} \quad (6.5)$$

Since B is nondiffusing $N_B = 0$. Also, the total concentration C remains constant. Therefore, Eqn.(6.5) becomes

$$\begin{aligned} \frac{z}{cD_{AB}} &= - \int_{C_{A1}}^{C_{A2}} \frac{dC_A}{N_A C - N_A C_A} \\ &\approx \frac{1}{N_A} \ln \frac{C - C_{A2}}{C - C_{A1}} \end{aligned}$$

Therefore,

$$N_A = \frac{CD_{AB}}{z} \ln \frac{C - C_{A2}}{C - C_{A1}} \quad (6.6)$$

Replacing concentration in terms of pressures using Ideal gas law, Eqn.(6.6) becomes

$$N_A = \frac{D_{AB} P_t}{RTz} \ln \frac{P_t - P_{A2}}{P_t - P_{A1}} \quad (6.7)$$

where D_{AB} = molecular diffusivity of A in B

P_t = total pressure of system

R = universal gas constant

T = temperature of system in absolute scale

z = distance between two planes across the direction of diffusion

P_{A1} = partial pressure of A at plane 1, and

P_{A2} = partial pressure of A at plane 2.

Given:

$D_{AB} = 1.89 \times 10^{-5}$ m²/sec

$P_t = 1$ atm = 1.01325×10^5 N/m²

T = 25°C = $273 + 25 = 298$ K

z = 2 mm = 0.002 m

$P_{A1} = 0.2 \times 1 = 0.2$ atm (From Ideal gas law and additive pressure rule)

$P_{A2} = 0.1 \times 1 = 0.1$ atm

Substituting these in Eqn.(6.7)

$$\begin{aligned} N_A &= \frac{(1.89 \times 10^{-5})(1.01325 \times 10^5)}{(8314)(298)(0.002)} \ln \left(\frac{1 - 0.1}{1 - 0.2} \right) \\ &= 4.552 \times 10^{-5} \text{ kmol/m}^2.\text{sec} \end{aligned}$$

6.3.3 Interfacial Gas Concentration

Consider a system in which component A is being transferred from a gas phase to a liquid phase. The equilibrium relation is given by $y_A = 0.75x_A$ where y_A and x_A are mole fractions

of A in gas and liquid phase respectively. At one point in the equipment, the gas contains 10 mole % A and liquid 2 mole % A . Gas film mass transfer coefficient k_y at this point is $10 \text{ kmol}/(\text{hr} \cdot \text{m}^2 \cdot \Delta y_A)$ and 60% of the resistance is in the gas film. Calculate:

(a) the overall mass transfer coefficient in $\text{kmol}/(\text{hr} \cdot \text{m}^2 \cdot \Delta y_A)$.

(b) mass flux of A in $\text{kmol}/(\text{hr} \cdot \text{m}^2)$.

(c) the interfacial gas concentration of A in mole fraction.

(GATE-1990-15iii)

Solution:

Equilibrium composition of gas y_A^* corresponding to $x_A = 0.02$ (i.e., 2 mole %):

$$y_A^* = 0.75x_A = 0.75 \times 0.02 = 0.015$$

Since individual gas film resistance $1/k_y$ is 60% of overall resistance $1/K_y$,

$$\begin{aligned} 0.6 \frac{1}{K_y} &= \frac{1}{k_y} \\ K_y &= 0.6 \times 10 = 6 \text{ kmol}/(\text{hr} \cdot \text{m}^2 \cdot \Delta y_A) \end{aligned}$$

Mass flux in terms of overall mass transfer coefficient:

$$\begin{aligned} \text{Mass flux} &= K_y(y_A - y_A^*) \\ &= 6(0.1 - 0.015) = 0.51 \text{ kmol}/(\text{hr} \cdot \text{m}^2) \end{aligned} \quad (6.8)$$

Mass flux in terms of individual mass transfer coefficient:

$$\text{Mass flux} = k_y(y_A - y_{Ai}) \quad (6.9)$$

where y_{Ai} = interfacial gas concentration.

Substituting for mass flux, k_y and y_A in Eqn.(6.9),

$$\begin{aligned} 10(0.1 - y_{Ai}) &= 0.51 \\ y_{Ai} &= 0.049 \end{aligned}$$

6.3.4 Heat and Mass Transfer Analogy

A stream of air at 100 kPa pressure and 300 K is flowing on the top surface of a thin flat sheet of solid naphthalene of length 0.2 m with a velocity of 20 m/sec. The other data are:

Mass diffusivity of naphthalene vapor in air = $6 \times 10^{-6} \text{ m}^2/\text{sec}$

Kinematic viscosity of air = $1.5 \times 10^{-5} \text{ m}^2/\text{sec}$

Concentration of naphthalene at the air-solid naphthalene interface = $1 \times 10^{-5} \text{ kmol/m}^3$

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Calculate:

(a) the overage mass transfer coefficient over the flat plate

(b) the rate of loss of naphthalene from the surface per unit width

Note: For heat transfer over a flat plate, convective heat transfer coefficient for laminar flow can be calculated by the equation:

$$Nu = 0.664 Re_L^{1/2} Pr^{1/3}$$

You may use analogy between mass and heat transfer.

(GATE-1994-20)

Solution:

Given: Correlation for heat transfer

$$Nu = 0.664 Re_L^{1/2} Pr^{1/3}$$

The analogous relation for mass transfer is

$$Sh = 0.664 Re_L^{1/2} Sc^{1/3} \quad (6.10)$$

where Sh = Sherwood number = kL/D_{AB}

Re_L = Reynolds number = $Lv\rho/\mu$

Sc = Schmidt number = $\mu/(\rho D_{AB})$

k = overall mass transfer coefficient

L = length of sheet

D_{AB} = diffusivity of A in B

v = velocity of air

μ = viscosity of air

ρ = density of air, and

μ/ρ = kinematic viscosity of air.

Substituting for the known quantities in Eqn.(6.10),

$$\begin{aligned} \frac{k(0.2)}{6 \times 10^{-6}} &= 0.664 \left(\frac{(0.2)(20)}{1.5 \times 10^{-5}} \right)^{1/2} \left(\frac{1.5 \times 10^{-5}}{6 \times 10^{-6}} \right)^{1/3} \\ k &= 0.014 \text{ m/sec} \end{aligned}$$

$$\text{Rate of loss of naphthalene} = k(C_{Ai} - C_{A\infty})$$

$$= 0.014(1 \times 10^{-5} - 0) = 1.4024 \times 10^{-7} \frac{\text{kmol}}{\text{m}^2 \cdot \text{sec}}$$

$$\text{Rate of loss per meter width} = (1.4024 \times 10^{-7})(0.2) = 2.8048 \times 10^{-8} \frac{\text{kmol}}{\text{m} \cdot \text{sec}}$$

$$= 0.101 \frac{\text{gmol}}{\text{m} \cdot \text{hr}}$$

6.3.5 Sherwood Number for Equimolar Counter Diffusion

For equimolar counterdiffusion from a sphere to a surrounding stationary infinite medium, the mass flux N_{Ai} of the diffusing component A at the interface is given by $N_{Ai} = D_A(C_{Ai} - C_{Ab})/R$ where D_A is the diffusivity, R the radius of the sphere and C_{Ai} and C_{Ab} the molar concentrations of A at the interface and at a point far away from the sphere. Show that the Sherwood number, based on the diameter of the sphere, is equal to 2. (GATE-1988-15i)

Solution:

$$\text{Sherwood number } Sh = \frac{k_c D}{D_A}$$

Here $k_c = D_A/R$ (since $N_{Ai} = D_A(C_{Ai} - C_{Ab})/R = k_c(C_{Ai} - C_{Ab})$). Therefore

$$\begin{aligned} Sh &= \frac{D_A D}{R D_A} \\ &= \frac{D}{R} \\ &= \frac{2R}{R} = 2 \end{aligned}$$

Hence proved.

6.3.6 Relative Humidity of Air

Air at 30°C and 150 kPa in a closed container is compressed and cooled. It is found that the first droplet of water condenses at 200 kPa and 15°C. Calculate the percent relative humidity of the original air. The vapor pressures of water at 15°C and 30°C are 1.7051 kPa and 4.246 kPa respectively. (GATE-1994-9)

Solution:

Since the first droplet of water condenses at 15°C, and 200 kPa, at this conditions the air is 100% humidity.

$$\text{i.e., Percentage humidity or percentage absolute humidity} = 100 \times \frac{p_A(p_t - p_S)}{p_S(p_t - p_A)} = 100$$

Therefore,

$$\frac{p_A}{p_t - p_A} = \frac{p_S}{p_t - p_S}$$

where p_A = partial pressure of water vapor

p_S = vapor pressure of water

p_t = total pressure of system

Assuming water-vapor air mixture as an ideal gas,

$$\text{No of moles of water vapor per mole of dry air} = \frac{1.7051}{200 - 1.7051} = 0.0086$$

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This ratio (moles of water vapor / mole of dry air) is not going to change for a closed system and before condensation. Therefore, partial pressure of A at 30°C and 150 kPa is found by equating this mole ratio as

$$\begin{aligned} \frac{p_A}{p_t - p_A} &= 0.0086 \\ \frac{p_A}{150 - p_A} &= 0.0086 \\ p_A &= 1.279 \text{ kPa} \end{aligned}$$

Percentage relative humidity of the original air:

$$\text{percentage relative humidity} = 100 \times \frac{p_A/p_S}{1.279/4.246} = 100 \times (1.279/4.246) = 30.12\%$$

6.3.7 Dry-bulb Temperature of Air

Air at 1 atm is blown past the bulb of a mercury thermometer. The bulb is covered with a wick. The wick is immersed in an organic liquid (molecular weight = 58). The reading of the thermometer is 7.6°C. At this temperature, the vapor pressure of the liquid is 5 kPa. Find the air temperature, given that the ratio of heat transfer coefficient to the mass transfer coefficient (psychrometric ratio) is 2 kJ/kg.K and the latent heat of vaporization of the liquid is 360 kJ/kg. Assume that the air, which is blown, is free from the organic vapor (GATE-2000-11)

Solution:

For simultaneous mass and heat transfer, heat flux q and mass flux N_A are related as

$$q = N_A \lambda \quad (6.11)$$

where λ is the latent heat of vaporization. Mass flux is given by

$$N_A = k_Y (Y'_w - Y') \quad (6.12)$$

where k_Y = mass transfer coefficient;

Y'_w = mass ratio of vapor in surrounding air at saturation; and

Y' = mass ratio of vapor in surrounding air.

Convective heat flux is given by

$$q = h(T - T_w) \quad (6.13)$$

where h = heat transfer coefficient;

T_w = wet bulb temperature of air; and

T = dry bulb temperature of air.

Substituting for N_A and q from Eqn.(6.12) and Eqn.(6.13) in Eqn.(6.11),

$$\begin{aligned} h(T - T_w) &= k_Y (Y'_w - Y') \lambda \\ T - T_w &= \frac{\lambda (Y'_w - Y')}{h/k_Y} \end{aligned} \quad (6.14)$$

Given: $Y' = 0$; $\lambda = 360 \text{ kJ/kg}$; $h/k_Y = 2 \text{ kJ/kg.K}$; and $T_w = 7.6^\circ\text{C}$.

$$\begin{aligned} Y'_w &= \frac{\text{kg organic vapor at saturation}}{\text{kg dry air}} \\ &= \frac{5}{101.3 - 5} \frac{58}{29} = 0.1038 \end{aligned}$$

Substituting these in Eqn.(6.14)

$$\begin{aligned} T - 7.6 &= \frac{(360)(0.1038 - 0)}{2} = 18.69 \\ T &= 18.69 + 7.6 = 26.29^\circ\text{C} \end{aligned}$$

Temperature of air = 26.29°C .

6.3.8 Minimum Liquid Rate for Absorption

The carbon dioxide issuing out of a fermenter contains 0.01 mole fraction of ethanol, which has to be reduced to 0.0001 mole fraction by scrubbing with water in a countercurrent packed tower. The gas flow rate is 227.3 kmol/hr and may be assumed constant throughout the tower. The equilibrium mole fraction of ethanol in the gas phase y^* is related to that in the liquid x as

$$y^* = 1.07x$$

Determine the minimum liquid rate needed, and the number of overall gas-side transfer units needed at 1.5 times the minimum liquid rate. The entering liquid may be assumed to be free of ethanol.

(GATE-1998-19)

Solution:

Mole fractions x , y and mole ratios X , Y are related as

$$y = \frac{Y}{1+Y} \text{ and } x = \frac{X}{1+X}$$

In reverse,

$$Y = \frac{y}{1-y} \text{ and } X = \frac{x}{1-x}$$

Therefore, equilibrium relation becomes

$$\frac{Y^*}{1+Y^*} = 1.07 \frac{X}{1+X}$$

Inverting the above equation,

$$\begin{aligned} \frac{1+Y^*}{Y^*} &= \frac{1}{1.07} \frac{1+X}{X} \\ \frac{1}{Y^*} &= \frac{1}{1.07X} + \frac{1}{1.07} - 1 \\ &= \frac{0.9346}{X} - 0.0654 \\ &= \frac{0.9346 - 0.0654X}{X} \end{aligned}$$

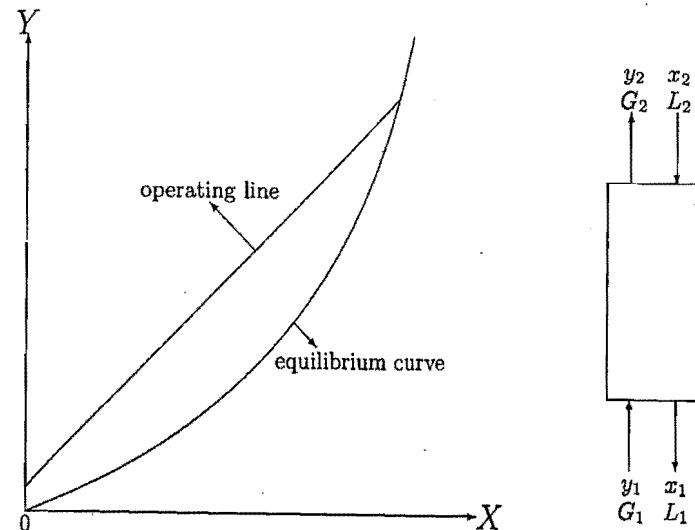
MASS TRANSFER

Inverting again,

$$Y^* = \frac{X}{0.9346 - 0.0654X} \quad (6.15)$$

Nature of curve of Eqn.(6.15):

X	Y^*
0.1	0.108
0.2	0.217
0.3	0.328



These values indicate that, the equilibrium curve in terms of mole ratios Y^* vs. X is concave upward.

The operating line Y vs. X is straight line. The minimum liquid-gas ratio then corresponds to an exit liquid concentration in equilibrium with the entering gas. (i.e., the point of intersection of operating and equilibrium lines.)

$$\text{Entering mole ratio of gas } Y_1 = \frac{0.01}{1 - 0.01} = 0.0101$$

Exit liquid concentration X_2^* in equilibrium with Y_1 is obtained from Eqn.(6.15) as:

$$\begin{aligned} 0.0101 &= \frac{X_2^*}{0.9346 - 0.0654X_2^*} \\ 0.00944 - 0.00066X_2^* &= X_2^* \\ X_2^* &= 0.00943 \end{aligned}$$

Minimum liquid rate L_{\min} :

$$L_2 = \frac{0.0001}{1 - 0.0001} = 0.0001$$

$$G(Y_1 - Y_2) = L_{\min}(X_1 - X_2^*)$$

$$L_{\min} = 227.3 \frac{0.0101 - 0.0001}{0.00943 - 0} = 241.04 \text{ kmol/hr}$$

Operating liquid rate:

$$L = 1.5L_{\min} = (1.5)(241.04) = 361.56 \text{ kmol/hr}$$

Absorption factor A:

$$A = \frac{\text{slope of operating line}}{\text{slope of equilibrium curve}}$$

$$= \frac{L}{mG}$$

$$= \frac{361.56}{(1.07)(227.3)} = 1.4455$$

$$1/A = 0.692$$

For absorption of dilute solutions, overall number of transfer units N_{tOG} is given by

$$N_{tOG} = \frac{\ln \left[\frac{y_1 - mx_2}{y_2 - mx_2} \left(1 - \frac{1}{A} \right) + \frac{1}{A} \right]}{1 - \frac{1}{A}} \quad (6.16)$$

Substituting for the values in Eqn.(6.16),

$$N_{tOG} = \frac{\ln \left[\left(\frac{0.01}{0.0001} \right) (1 - 0.692) + 0.692 \right]}{1 - 0.692} = 11.2$$

Number of overall gas-side transfer units = 11.2

6.3.9 Height of Absorption Column

Equilibrium relationship for the system heptane-oil-air is given by $Y = 2X$ (Y and X are kg-heptane / kg-air and kg-heptane / kg-oil respectively). Oil containing 0.005 kg-heptane/kg-oil is being used as solvent for reducing the heptane content of air from 0.10 to 0.02 kg-heptane/kg-air in a continuous countercurrent packed bed absorber. What column height is required to treat 1400 kg/(hr.m² of empty tower cross section) of pure air containing heptane if the overall gas mass transfer coefficient is 320 kg/(hr.m³.ΔY). The oil rate employed is 3100 kg/(hr.m²).

Solve analytically.

(GATE-1992-16a)

Solution:

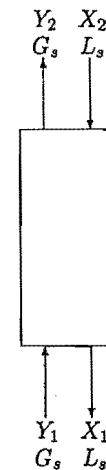
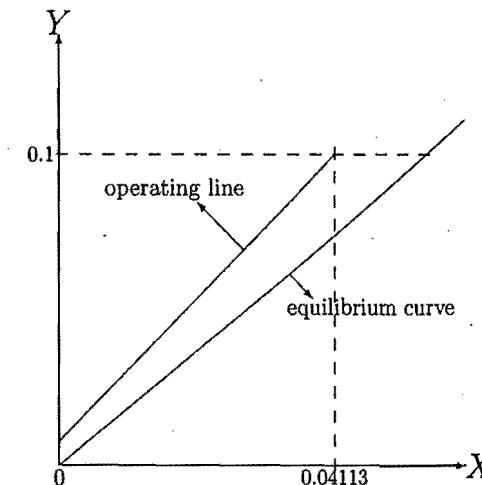
Material balance for absorber:

$$L_s(X_1 - X_2) = G_s(Y_1 - Y_2)$$

$$3100(X_1 - 0.005) = 1400(0.1 - 0.02)$$

$$X_1 - 0.005 = 0.03613$$

$$X_1 = 0.04113$$



Equation of operating line:

$$Y - Y_2 = \frac{L_s}{G_s}(X - X_2)$$

$$Y - 0.02 = \frac{3100}{1400}(X - 0.005)$$

$$Y = 2.2143X + 0.0089 \quad (6.17)$$

Number of overall gas transfer units N_{tOG} is related to gas-phase mole ratios as

$$N_{tOG} = \int_{Y_2}^{Y_1} \frac{dY}{Y - Y^*} + \frac{1}{2} \ln \frac{1 + Y_2}{1 + Y_1} \quad (6.18)$$

where Y^* = equilibrium gas composition in mole ratio. Given:

$$Y^* = 2X$$

Substituting for Y^* in Eqn.(6.18) and from Eqn.(6.17), Eqn.(6.18) becomes

$$N_{tOG} = \int_{Y_2}^{Y_1} \frac{dY}{0.2143X + 0.0089} + \frac{1}{2} \ln \frac{1 + Y_2}{1 + Y_1} \quad (6.19)$$

From Eqn.(6.17),

$$Y = 2.2143X + 0.0089$$

differentiating,

$$dY = 2.2143dX$$

Using this and changing the integration limits in terms of X , Eqn.(6.19) becomes

$$N_{tOG} = \int_{X_2}^{X_1} \frac{2.2143dX}{0.2143X + 0.0089} + \frac{1}{2} \ln \frac{1+X_2}{1+X_1}$$

Integrating

$$N_{tOG} = \frac{2.2143}{0.2143} \ln \left(\frac{0.2143X_1 + 0.0089}{0.2143X_2 + 0.0089} \right) + \frac{1}{2} \ln \frac{1+X_2}{1+X_1} \quad (6.20)$$

Substituting for $X_1 = 0.04113$ and $X_2 = 0.005$, in Eqn.(6.20),

$$\begin{aligned} N_{tOG} &= \frac{2.2143}{0.2143} \ln \left(\frac{(0.2143)(0.04113) + 0.0089}{(0.2143)(0.005) + 0.0089} \right) + \frac{1}{2} \ln \frac{1+0.005}{1+0.04113} \\ &= 5.9375 - 0.0177 = 5.9198 \end{aligned}$$

Height of overall gas transfer unit H_{tOG} is given by

$$H_{tOG} = \frac{G_s}{K_y a (1-Y)_{*M}} \quad (6.21)$$

where

$K_y a$ = overall gas phase mass transfer coefficient, and

$(1-Y)_{*M}$ = mean of logarithmic averages of $(1-Y)$ and $(1-Y^*)$ between terminals of tower, given as

$$(1-Y)_{*M} = \frac{1}{2} \left(\frac{(1-Y)_1 - (1-Y^*)_1}{\ln[(1-Y)_1/(1-Y^*)_1]} + \frac{(1-Y)_2 - (1-Y^*)_2}{\ln[(1-Y)_2/(1-Y^*)_2]} \right) \quad (6.22)$$

From the diagram shown, it can be visualized that when

$$\begin{aligned} Y_1 &= 0.02 & Y_1^* &= 0 \\ \text{and when } Y_2 &= 0.1 & Y_2^* &= 2 \times 0.04113 = 0.08226 \end{aligned}$$

Substituting these in Eqn.(6.22),

$$\begin{aligned} (1-Y)_{*M} &= \frac{1}{2} \left(\frac{(1-0.02)-1}{\ln[(1-0.02)/(1)]} + \frac{(1-0.1)-(1-0.08226)}{\ln[(1-0.1)/(1-0.08226)]} \right) \\ &= (0.5)(0.99 + 0.9088) = 0.9494 \end{aligned}$$

Then, from Eqn.(6.21),

$$H_{tOG} = \frac{1400}{(320)(0.9494)} = 4.6081 \text{ m}$$

Height of column = $N_{tOG} H_{tOG} = 5.9198 \times 4.6081 = 27.29 \text{ m}$.

6.3.10 Flash Vaporization

A binary liquid mixture containing 50 mole % of the more volatile component is fed to a heater where 40% of the feed is flash vaporized. If the liquid and vapor produced are in equilibrium, calculate the composition of the liquid product obtained (in mole %). Relative volatility may be taken as 2.0
(GATE-1989-15ii)

Solution:

Given: Fraction of feed vaporized = 0.6; and composition of feed (z_F) = 0.5.
If feed (F) is taken as 1 mole, then distillate (D) is 0.4 mole.

From overall material balance:

$$F = D + B$$

Therefore, bottoms (B) = 0.6 mole.

Component material balance:

$$F z_F = D x_D + B x_B \quad (6.23)$$

From equilibrium relationship:

$$y^* = \frac{\alpha x}{1 + x(\alpha - 1)} \quad (6.24)$$

Since vapor leaving the flash column is in equilibrium with the liquid, Eqn.(6.24) shall be written as,

$$x_D = \frac{\alpha x_B}{1 + x_B(\alpha - 1)} \quad (6.25)$$

Given: $\alpha = 2$. Therefore Eqn.(6.25) becomes,

$$x_D = \frac{2x_B}{1 + x_B} \quad (6.26)$$

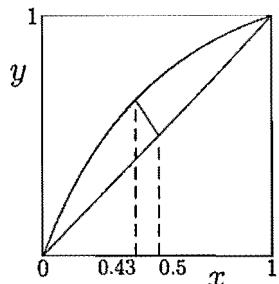
Substituting for x_D from Eqn.(6.26) in Eqn.(6.23),

$$\begin{aligned} F z_F &= \frac{2Dx_B}{1+x_B} + Bx_B \\ 1 \times 0.5 &= \frac{(2)(0.4)x_B}{1+x_B} + 0.6x_B \\ 0.5 &= \frac{0.8x_B + 0.6x_B + 0.6x_B^2}{1+x_B} \end{aligned}$$

Solving, $x_B = 0.43$.

Therefore, composition of liquid product = 43 mole % of more volatile component.

The calculations are represented in the diagram as below:



6.3.11 Differential Distillation

100 moles of an Acetonitrile-Nitromethane mixture is differentially distilled in a batch still at a pressure of 70 kPa. The feed contains 74 mole % acetonitrile. Distillation is continued till the liquid left behind in the still contains 32 mole % acetonitrile. The vapor-liquid equilibria for the system at this pressure are correlated as follows:

$$\begin{aligned} y^* &= 1.05x + 0.13 \quad \text{for } 0.3 < x < 0.52 \text{ and} \\ y^* &= 0.77x + 0.28 \quad \text{for } 0.52 < x < 0.80 \end{aligned}$$

where x and y^* refer to the mole fractions of acetonitrile in the liquid and equilibrium vapor respectively.

Find the average composition of the distillate collected. (GATE-1998-6)

Solution:

For differential distillation, from Rayleigh equation:

$$\ln \frac{F}{W} = \int_{x_w}^{x_F} \frac{dx}{y^* - x} \quad (6.27)$$

where F = moles of feed;

W = moles of residue;

x_F = composition of feed;

x_w = composition of residue;

x = mole fraction of more volatile component in liquid; and

y^* = mole fraction of more volatile component in vapor which is in equilibrium with x .

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Substituting for y^* , x_F , x_w , and F in Eqn.(6.27)

$$\begin{aligned} \ln \frac{100}{W} &= \int_{0.32}^{0.52} \frac{dx}{1.05x + 0.13 - x} + \int_{0.52}^{0.74} \frac{dx}{0.77x + 0.28 - x} \\ &= \int_{0.32}^{0.52} \frac{dx}{0.05x + 0.13} + \int_{0.52}^{0.74} \frac{dx}{-0.23x + 0.28} \\ &= \left[\frac{1}{0.05} \ln(0.05x + 0.13) \right]_{0.32}^{0.52} + \left[\frac{1}{-0.23} \ln(-0.23x + 0.28) \right]_{0.52}^{0.74} \\ &= 1.325 + 1.6479 = 2.9729 \end{aligned}$$

$$\text{Therefore, } W = 5.1155$$

Material balance for the differential distillation still:

$$F = D + W$$

$$\text{Therefore, } D = 100 - 5.1155 = 94.8845 \text{ moles.}$$

Component balance:

$$Fx_F = Dy_{D,\text{av}} + Wx_W \quad (6.28)$$

Substituting for the known data in Eqn.(6.28)

$$(100)(0.74) = (94.8845)y_{D,\text{av}} + (5.1155)(0.32)$$

$$\text{Therefore, } y_{D,\text{av}} = 0.7626.$$

Average composition of distillate ($y_{D,\text{av}}$) = 0.7626 = 76.26 mole % of Acetonitrile.

6.3.12 Equation of q -Line

Obtain the equation for the q -line given that the operating lines are:

$$y = \frac{L}{L+D}x + \frac{D}{L+D}x_D \quad (\text{enriching section})$$

$$y = \frac{\bar{L}}{\bar{L}-B}x + \frac{B}{\bar{L}-B}\bar{x}_B \quad (\text{stripping section})$$

where L and \bar{L} are the liquid flow rates in the enriching and stripping sections, D and B are the top and bottom product flow rates, and x_D and \bar{x}_B are the mole fractions of top and bottom products respectively. (GATE-2000-9)

Solution:

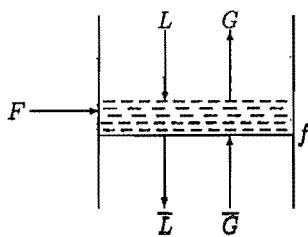
Derivation for definition of ' q ':

Material balance around feed tray:

$$F + L + \bar{G} = G + \bar{L} \quad (6.29)$$

Energy balance around feed tray:

$$FH_F + LH_{L,-1} + \bar{G}H_{G,+1} = GH_{G,f} + \bar{L}H_{L,f} \quad (6.30)$$



For a system of components with equimolar latent heats, $H_{G,i} = H_{G,i+1}$ and $H_{L,i+1} = H_{L,i}$. Therefore, Eqn.(6.30) becomes

$$(\bar{L} - L)L = (\bar{G} - G)H_G + FH_F \quad (6.31)$$

From Eqn.(6.29), $(\bar{G} - G) = (\bar{L} - L) + F$. Substituting this in Eqn.(6.31) gives,

$$\frac{\bar{L} - L}{F} = \frac{H_G - H_F}{H_G - H_L} = q \quad (6.32)$$

Derivation for 'q'-line:

Given:

Operating line for enriching section:

$$y = \frac{L}{L+D}x + \frac{D}{L+D}x_D \quad (6.33)$$

Operating line for stripping section:

$$y = \frac{\bar{L}}{\bar{L}-B}x + \frac{B}{\bar{L}-B}x_B \quad (6.34)$$

From Eqn.(6.33)

$$(L+D)y = Lx + Dx_D \quad (6.35)$$

From Eqn.(6.34)

$$(\bar{L}-B)y = \bar{L}x - Bx_B \quad (6.36)$$

Subtracting Eqn.(6.36) from Eqn.(6.35)

$$(\bar{L} - L)y + (D + B)y = (\bar{L} - L)x + (Dx_D + Bx_B) \quad (6.37)$$

From overall material balance,

$$F = D + B$$

And from component balance,

$$Fz_F = Dx_D + Bx_B$$

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Substituting these in Eqn.(6.37)

$$(\bar{L} - \bar{L})y + Fy = (\bar{L} - \bar{L})x + Fz_F$$

Dividing throughout by F and rearranging,

$$\left(\frac{\bar{L} - \bar{L}}{F} + 1 \right) y = \left(\frac{\bar{L} - \bar{L}}{F} \right) x + z_F$$

From the definition $(\bar{L} - \bar{L})/F$ from Eqn.(6.32), the above equation becomes

$$(-q + 1)y = -qx + z_F$$

Rearranging

$$y = \frac{q}{q-1}x - \frac{z_F}{q-1} \quad (6.38)$$

Eqn.(6.38) is the equation of q -line.

6.3.13 Reflux Ratio

A continuous rectifying column is separating a binary mixture containing A and B (A more volatile than B) into an overhead distillate product containing 95 mole % A. The liquid overflowing from two adjacent plates in the enriching section contain 58.6 mole % and 50 mole % A respectively. The plate efficiency is 100%. Reflux is at its bubble point. Relative volatility may be taken as 2.5. Calculate the reflux ratio.
(Solve analytically).

(GATE-1988-15iii)

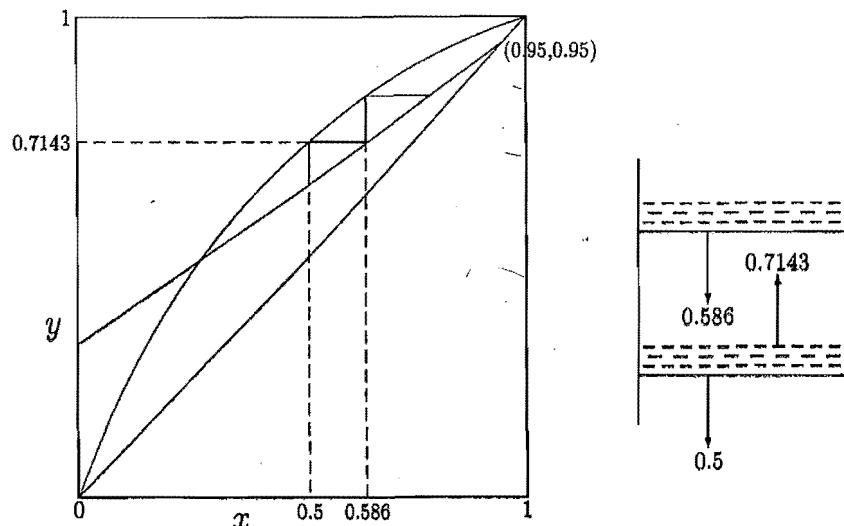
Solution:

The compositions of liquid(x) leaving two adjacent ideal plates are represented in the x -diagram.

The leaving-vapor composition corresponding to the leaving-liquid composition of 0.5 for the tray is calculated from the equilibrium relationship:

$$\begin{aligned} y^* &= \frac{\alpha x}{1 + x(\alpha - 1)} \\ &= \frac{2.5 \times 0.5}{1 + 0.5(2.5 - 1)} = 0.7143 \end{aligned} \quad (6.39)$$

The (x, y) composition (0.586, 0.7143) is a point in enriching-section operating line.



From figure,

$$\begin{aligned}\text{Slope of enriching section operating line } (L/G) &= \frac{0.95 - 0.7143}{0.95 - 0.586} \\ &= 0.6476\end{aligned}$$

L/G is related to reflux ratio(R) as:

$$\frac{L}{G} = \frac{R}{R+1}$$

Therefore,

$$\begin{aligned}\frac{R}{R+1} &= 0.6476 \\ R &= 1.838\end{aligned}$$

6.3.14 Quality of Feed to Distillation Column

A binary distillation column is operating under the conditions given below:

Feed rate = 350 kmol/hr

Overhead product rate = 150 kmol/hr

Mole fraction of more volatile component in overhead product = 0.97

Mole fraction of more volatile component in bottom product = 0.02

Bottom product rate = 200 kmol/hr

Reflux ratio = 3.5

In the stripping section it was found that the mole fraction of the volatile component in the vapor leaving a plate is 0.33 while its mole fraction in the liquid coming to the same plate is 0.25. Assuming constant molal counter flow, determine whether the feed is vapor, liquid or partially vaporized.
(GATE-1994-22)

Solution:

Given:

$$\begin{aligned}\text{Feed rate}(F) &= 350 \text{ kmol/hr} \\ \text{Distillate rate}(D) &= 150 \text{ kmol/hr} \\ \text{Bottom product rate}(B) &= 200 \text{ kmol/hr} \\ \text{Reflux ratio}(R) &= 3.5\end{aligned}$$

Slope of enriching section operating-line (L/G):

$$\begin{aligned}\frac{L}{G} &= \frac{R}{R+1} \\ &= \frac{3.5}{3.5+1} = 0.778\end{aligned}$$

Equation of enriching section operating-line:

$$\begin{aligned}y &= \frac{L}{G}x + \frac{x_D}{R+1} \\ &= 0.778x + \frac{0.97}{3.5+1} \\ &= 0.778x + 0.216\end{aligned}\quad (6.40)$$

Given: A point on stripping section operating-line is (0.25, 0.33).

$$\begin{aligned}\text{Slope of stripping operating-line}(m) &= \frac{0.33 - 0.02}{0.25 - 0.02} \\ &= 1.348\end{aligned}$$

Equation of straight line:

$$\frac{y - y_1}{x - x_1} = m$$

Equation of stripping section operating-line:

$$\begin{aligned}y - 0.33 &= 1.348(x - 0.25) \\ y &= 1.348x - 0.007\end{aligned}\quad (6.41)$$

Point of intersection of stripping and enriching section operating-lines is obtained by equating Eqns.(6.40) and (6.41) as below:

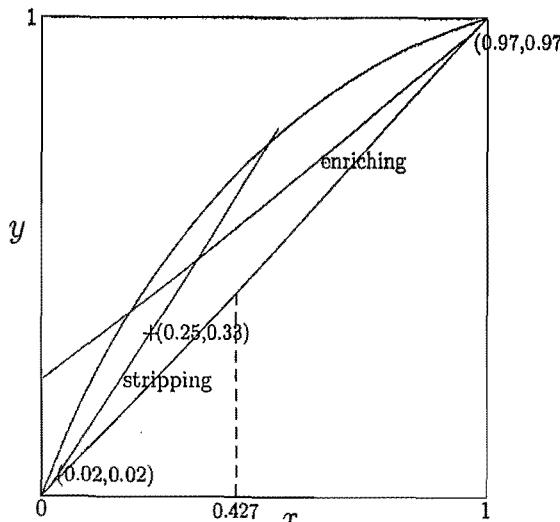
$$\begin{aligned}0.778x + 0.216 &= 1.348x - 0.007 \\ x &= 0.391 \\ \text{and, } y &= 1.348 \times 0.391 - 0.007 = 0.520\end{aligned}$$

Composition of feed (z_F) is obtained from material balance for the distillation column as given below:

$$\begin{aligned} Fz_F &= Dx_D + Bx_B \\ z_F &= \frac{(150)(0.97) + (200)(0.02)}{350} = 0.427 \end{aligned} \quad (6.42)$$

Since the line connecting intersection point of operating lines ($x = 0.391, y = 0.520$) and the feed composition ($z_F = 0.427$) is making a negative slope, feed is partially vaporized.

From the following diagram, the above calculations shall be best understood.



6.3.15 Reflux Ratio and Composition of Vapor

The following information is available from the record of a binary fractionating column:

Feed = 180 kmol/hr and 60% vaporised;

Distillate = 100 kmol/hr with 0.98 mole fraction of more volatile component;

Reboiler steam demand = 420 kg/hr;

Latent heat of column liquid = 3×10^4 J/mol; and

Latent heat of steam used in reboiler = 2200 J/g

Calculate:

- (a) the operating reflux ratio.

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- (b) the composition of vapor entering that plate from which liquid overflow contains 0. mole fraction of the more volatile component. (GATE-1992-16)

Solution:

$$\begin{aligned} \text{Reboiler load}(Q_B) &= \text{Reboiler steam demand} \times \text{Latent heat of steam} \\ &= (420)(2200) = 924 \times 10^3 \text{ kJ/hr} \end{aligned}$$

Heat balance over distillation column:

(Reference temperature = average boiling temperatures of top and bottom products)

Heat removed in condenser = Heat input from feed + Heat input in reboiler

$$G(3 \times 10^4) = (0.6)(180)(3 \times 10^4) + 924 \times 10^3$$

$$G = 138.8 \text{ kmol/hr}$$

For the enriching section,

$$G = L + D$$

Reflux ratio(R) is related to L and D as,

$$R = \frac{L}{D}$$

Therefore,

$$\begin{aligned} G &= RD + D \\ &= D(R + 1) \\ 138.8 &= 100(R + 1) \\ R &= 0.388 \end{aligned}$$

Slope of enriching section operating-line(L/G):

$$\frac{L}{G} = \frac{R}{R+1} = \frac{0.388}{1+0.388} = 0.2795$$

Intercept of enriching section operating-line:

$$\text{intercept} = \frac{x_D}{R+1} = \frac{0.98}{1+0.388} = 0.706$$

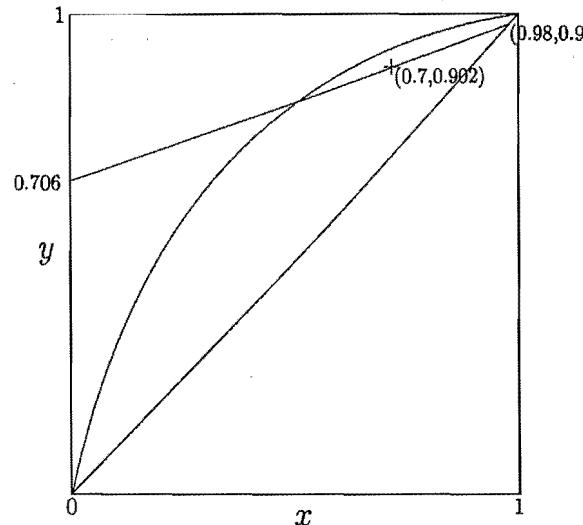
Equation of enriching section operating-line:

$$\begin{aligned} y &= \frac{L}{G}x + \frac{x_D}{R+1} \\ &= 0.2795x + 0.706 \end{aligned}$$

Since the composition of feed is not given, (and also not possible to estimate from the given data) it shall be assumed that the composition $x = 0.7$, is in the enriching section. Composition of vapor entering the plate, for which the overflow liquid composition x is 0.7, is obtained from the enriching section operating-line as:

$$y = (0.2975)(0.7) + 0.706 = 0.902$$

The calculations are represented in the diagram as below:



6.3.16 Reflux Ratio and Vapor Rate

A continuous rectification column is used to separate a binary mixture of A and B. Distillate is produced at 100 kmol/hr containing 98 mole % A. The mole fractions of A in the liquid and in the vapor, x and y respectively, from two adjacent ideal plates in the enriching section are as follows:

x	y
0.65	0.82
0.56	0.76

If the latent heat of vaporization is the same for all mixtures and if the feed is a saturated liquid, calculate:

(a) the reflux ratio

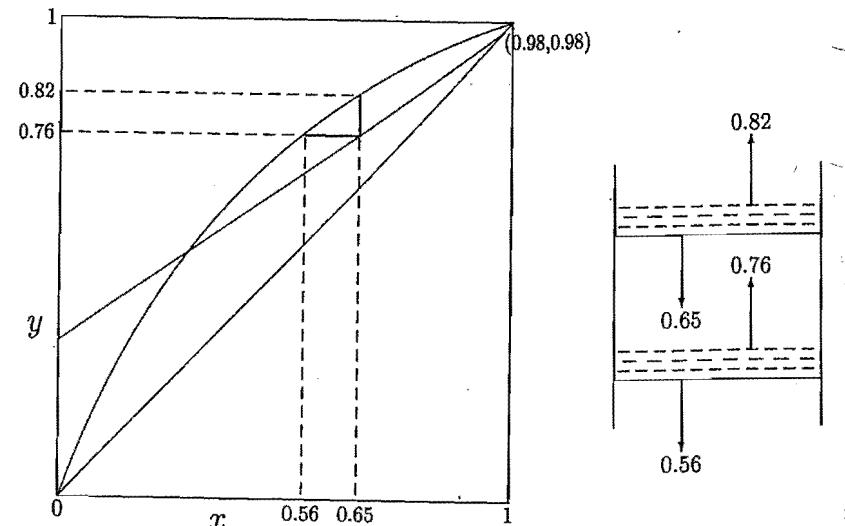
(b) vapor rate in the stripping section in kmol/hr.

(GATE-1990-15i)

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Solution:

The compositions of x and y leaving two adjacent ideal plates are represented in the $x - y$ diagram as below:



From figure,

$$\begin{aligned} \text{Slope of enriching section operating line}(L/G) &= \frac{0.98 - 0.79}{0.98 - 0.65} \\ &= \frac{0.22}{0.33} = \frac{2}{3} \end{aligned}$$

L/G is related to reflux ratio(R) as:

$$\frac{L}{G} = \frac{R}{R+1}$$

Therefore,

$$\begin{aligned} \frac{R}{R+1} &= \frac{2}{3} \\ R &= 2 \end{aligned}$$

And,

$$\begin{aligned}\text{Vapor rate}(G) &= L + D \\ &= \frac{2}{3}G + D \\ \frac{G}{3} &= D \\ G &= 3D \\ &= 3 \times 100 = 300 \text{ kmol/hr}\end{aligned}$$

Since feed is a saturated liquid, vapor rate in stripping section (\bar{G}) is equal to vapor rate in enriching section (G). i.e., $\bar{G} = 300 \text{ kmol/hr}$.

6.3.17 Distillation with Partial Condenser

A feed of known composition (binary mixture of constant relative volatility) is to be distilled in a continuous fractionating column consisting of a partial condenser, one plate, and a reboiler. The feed enters the reboiler from which a bottom product is continuously withdrawn. The liquid reflux from the partial condenser is returned to the plate. The distillate composition (x_D) is 0.8 and the reflux ratio is 2.

(a) What is the slope of the operating line on $x - y$ plot and its intercept on the y -axis?

(b) Using the McCabe-Thiele method, qualitatively locate on the $x - y$ plot:

- (i) the composition of the streams leaving the plate, and
- (ii) the bottom product composition.

(Do not use graph paper)

(GATE-1997-20)

Solution:

Given: $R = 2$

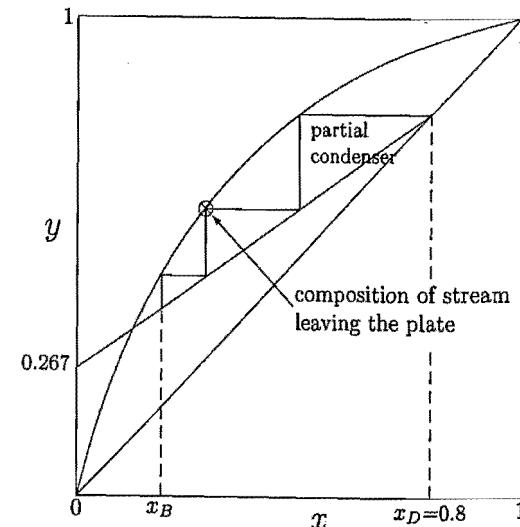
Slope of operating-line (L/G):

$$\begin{aligned}\frac{L}{G} &= \frac{R}{R+1} \\ &= \frac{2}{3+1} = 0.667\end{aligned}$$

Intercept of operating line:

$$\begin{aligned}\text{Intercept} &= \frac{x_D}{R+1} \\ &= \frac{0.8}{2+1} = 0.267\end{aligned}$$

Distillation trays and compositions (x_D and x_B) are qualitatively located on the $x - y$ plot as below:



6.3.18 Adsorption of Moisture by Silica Gel

The equilibrium water adsorbed by a certain silica gel in contact with moist air varies linearly with the humidity of air as,

$$Y = 3.4435 \times 10^{-2} X$$

where

$$\begin{aligned}X &= \text{kg water adsorbed / kg dry gel, and} \\ Y &= \text{humidity of air in, kg moisture / kg dry air.}\end{aligned}$$

0.5 kg of silica gel containing 5% (dry basis) adsorbed water is placed in a collapsible vessel in which there are 10 m³ of moist air, partial pressure of water being 15 mm Hg. The total pressure and temperature are kept at 1 std atm and 25°C respectively.

- (a) What is the amount of water picked up from the moist air in the vessel by silica gel?
- (b) What is the final partial pressure of water vapor in the vessel? (GATE-1993-22b)

Solution:

Basis: 0.5 kg of silica gel

$$\begin{aligned}\text{moisture in silica gel} &= 5\% \text{ on dry basis} \\ &= \frac{0.05}{1+0.05} \times 100 = 4.762\% \text{ on wet basis}\end{aligned}$$

$$\begin{aligned}\text{Initial moisture content of silica gel} &= 0.04762 \times 0.5 = 0.0238 \text{ kg} \\ \text{Amount of dry gel} &= 0.5 - 0.0238 = 0.4762 \text{ kg}\end{aligned}$$

Given: Partial pressure of water vapor in air at initial condition = 15 mm Hg.

$$\begin{aligned}\text{Moisture content in air} &= \frac{15}{760-15} \times \frac{18}{29} \\ &= 0.0125 \frac{\text{kg moisture}}{\text{kg dry air}}\end{aligned}$$

Given: Volume of vessel at initial condition = 10 m³.

By Dalton's law of partial pressures and from Ideal gas law, the initial conditions of moist air are:

$$\begin{aligned}\text{kmol of dry air} &= \frac{[(760-15)/760] \times 1.01525 \times 10^5 \times 10}{8314 \times 298} = 0.4009 \text{ kmol} \\ \text{kg of dry air} &= 0.4009 \times 29 = 11.626 \text{ kg} \\ \text{kg of moisture in air} &= \frac{(15/760) \times 1.01525 \times 10^5 \times 10}{8314 \times 298} \times 18 = 0.1453 \text{ kg}\end{aligned}$$

Mass balance for adsorption:

$$\begin{aligned}\text{moisture adsorbed by silica gel} &= \text{moisture given up by air} \\ 0.4762(X-0.05) &= 11.626(0.0125-Y) \quad (6.43)\end{aligned}$$

Silica gel adsorbs moisture till X reaches to the equilibrium value, given by

$$X = \frac{Y}{3.4435 \times 10^{-2}}$$

Therefore, Eqn.(6.43) becomes

$$\begin{aligned}0.4762 \left(\frac{Y}{3.4435 \times 10^{-2}} - 0.05 \right) &= 11.626(0.0125 - Y) \\ Y - 1.72175 \times 10^{-3} &= 10.50875 \times 10^{-3} - 0.8407Y \\ Y &= 0.00572 \text{ kg moisture/kg dry air.} \\ \text{The corresponding equilibrium value of } X &= \frac{0.00572}{3.4435 \times 10^{-2}} \\ &= 0.1661 \text{ kg water/kg dry silica gel} \\ \text{Water picked from moist air by silica gel} &= 0.4762(0.1661 - 0.05) = 0.0553 \text{ kg}\end{aligned}$$

MASS TRANSFER

At final conditions:

$$\begin{aligned}\text{Amount of water vapor in air} &= 0.00572 \times 11.626 = 0.0665 \text{ kg} \\ \text{Mole ratio of water vapor in air} &= \frac{0.0665}{11.626} \times \frac{29}{18} = 0.00922 \\ \text{Mole fraction of water vapor in air} &= \frac{0.00922}{1+0.00922} = 0.00913 \\ \text{Partial pressure of water vapor in air} &= 0.00913 \times 760 = 6.94 \text{ mm Hg}\end{aligned}$$

6.3.19 Leaching of Sulfur

A mineral containing 20% elemental sulfur is to be leached with hot gas oil in which sulfur is soluble to the extent of 10 parts in 100 parts of oil by weight. The oil is recycled over the batch of ground mineral till no further dissolution of sulfur takes place. On drainage, the solid minerals retain the solution to the extent of one-tenth the weight of undissolved solid (sulfur and gauge). No preferential adsorption takes place. Calculate the equilibrium compositions and quantities of the solid and liquid phases if 50 kg of oil is used for leaching 100 kg of fresh mineral. (GATE-1993-23)

Solution:

Let us denote the various streams entering and leaving the leaching chamber as below:

$$\begin{aligned}R_1 &= \text{mineral input} \\ R_2 &= \text{mineral output} \\ E_1 &= \text{gas oil input} \\ E_2 &= \text{gas oil output}\end{aligned}$$

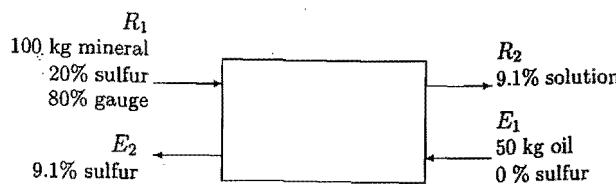
It is given that sulfur is soluble to the extent of 10 parts in 100 parts of oil. Therefore,

$$\text{weight fraction of sulfur in } E_2 = \frac{10}{100+10} = 0.091$$

Given: Solution in R_2 = one-tenth the weight of undissolved solid.

$$\text{weight fraction of solution in solid} = \frac{1}{10+1} = 0.091$$

The solution in the solid will contain sulfur corresponding to the equilibrium solubility of 0.091 weight fraction.



Overall mass balance:

$$\begin{aligned} E_2 + R_2 &= E_1 + R_1 \\ E_2 + R_2 &= 100 + 50 = 150 \end{aligned} \quad (6.44)$$

Balance for oil:

$$\begin{aligned} (1 - 0.091)E_2 + 0.091 \times (1 - 0.091)R_2 &= 50 \\ 0.909E_2 + 0.0827R_2 &= 50 \end{aligned} \quad (6.45)$$

Solving Eqn.(6.44) and (6.45) simultaneously,

$$E_2 = 45.5 \text{ kg, and } R_2 = 104.5 \text{ kg}$$

Composition of E_2 :

$$\begin{aligned} \text{sulfur} &= 45.5 \times 0.091 = 4.095 \text{ kg} \\ \text{oil} &= 45.5 - 4.095 = 40.905 \text{ kg} \end{aligned}$$

Composition of R_2 :

$$\begin{aligned} \text{gauge} &= 100 \times 0.8 = 80 \text{ kg} \\ \text{sulfur} &= 20 - 4.095 = 15.905 \text{ kg} \\ \text{oil} &= 104.5 - (80 + 15.905) = 8.595 \text{ kg} \end{aligned}$$

6.3.20 Time Required for Drying

Drying of a food product is carried out in an insulated tray. The drying air has a partial pressure of water equal to 2360 Pa and a wet bulb temperature of 30°C. The product has a drying surface of $0.05 \text{ m}^2/\text{kg dry solid}$. The material has a critical moisture content of 0.12 (dry basis) and negligible equilibrium moisture content. The drying rate in the falling rate period is proportional to the moisture content and the mass transfer coefficient is $5.34 \times 10^{-4} \text{ kg/m}^2.\text{hr.Pa}$.

Calculate:

- the drying rate in the constant rate period in $\text{kg/m}^2 \text{ hr}$
- the time required to dry the material from a moisture of 0.22 to 0.06 (both on dry basis).

Vapor pressure of water at 30°C = 4232 Pa. (GATE-1990-15ii)

Solution:

Rate of drying at constant rate period N_c :

$$N_c = k_G \Delta \bar{p}_A = k_G (\bar{p}_A^* - \bar{p}_A) \quad (6.46)$$

where \bar{p}_A^* = partial pressure of water vapor at equilibrium, or vapor pressure of water (4232 Pa)

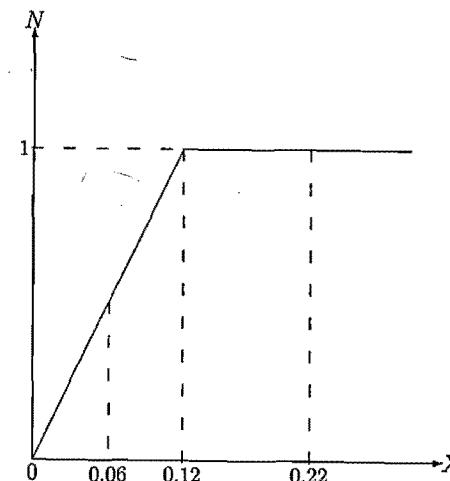
\bar{p}_A = partial pressure of water vapor in drying air (2360 Pa)

k_G = mass transfer coefficient ($5.34 \times 10^{-4} \text{ kg/m}^2.\text{hr.Pa.}$)

MASS TRANSFER

Therefore, from Eqn.(6.46)

$$N_c = 5.34 \times 10^{-4} (4232 - 2360) = 1 \text{ kg/hr.m}^2$$



Falling rate period:

Given: drying rate is proportional to moisture content. Therefore,

$$\text{Slope of falling rate period curve} = \frac{1}{0.12} = 8.333$$

$$\text{Rate of drying in falling rate period } N_f = 8.333X$$

From the definition of drying rate

$$N = \frac{-S_s dX}{A d\theta} \quad (6.47)$$

where S_s = mass of dry solid;
 A = drying surface area; and
 θ = time of drying.

Given: $A/S_s = 0.05 \text{ m}^2/\text{kg dry solid}$.

For constant rate period Eqn.(6.47) becomes

$$\frac{d\theta}{dX} = \frac{1}{N_c} \frac{-S_s}{A} = (1/1)(-1/0.05) = -20$$

Integrating between $X_1 = 0.22$ and $X_2 = 0.12$,

$$\theta_c = -20(0.12 - 0.22) = 2 \text{ hr.}$$

Similarly for falling rate period,

$$\frac{d\theta}{dX} = \frac{-S_s}{A} \frac{1}{N_f} = -20 \left(\frac{1}{8.333X} \right)$$

Integrating between $X_2 = 0.06$ and $X_1 = 0.12$,

$$\theta_f = -20 \frac{1}{8.333} \ln \frac{0.06}{0.12} = 1.664 \text{ hr.}$$

Total drying time $\theta = \theta_c + \theta_f = 2 + 1.664 = 3.664 \text{ hr.}$

6.3.21 Equilibrium Moisture Content

In a laboratory drying test with a solid material the following relation for the falling rate period was obtained,

$$\frac{dX}{d\theta} = -0.8(X - 0.05)$$

where X is the moisture content on dry basis and θ is the time in hours. The critical moisture content is 1.4 kg moisture per kg of dry material.

Calculate:

- (a) the time required for drying the material from $X_1 = 4.0$ to $X_2 = 0.1$
- (b) the equilibrium moisture content. (GATE-1988-15iv)

Solution:

Given:

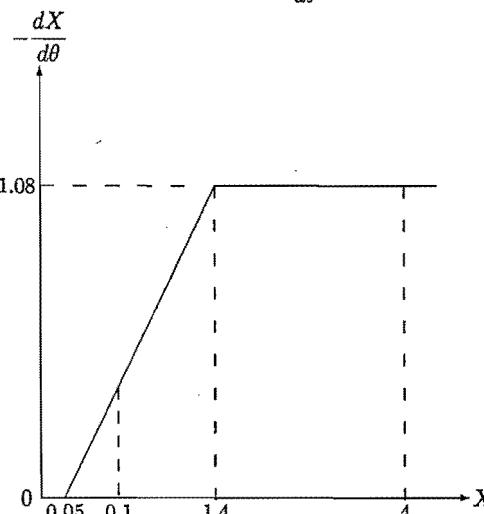
Relation for falling rate period:

$$\frac{dX}{d\theta} = -0.8(X - 0.05) = -0.8X + 0.04 \quad (6.48)$$

Relation for constant rate period:

Substituting for $X = 1.4$ from critical moisture content data,

$$\frac{dX}{d\theta} = -0.8 \times 1.4 + 0.04 = -1.08 \quad (6.49)$$



Time required for drying in constant rate period θ_c :

Integrating Eqn.(6.49) between $X_1 = 4.0$ and $X_2 = X_c = 1.4$,

$$\theta_c = \frac{1.4 - 4.0}{-1.08} = 2.407 \text{ hr.}$$

Time required for drying in falling rate period θ_f :

Integrating Eqn.(6.48) between $X_1 = X_c = 1.4$ and $X_2 = 0.1$,

$$\theta_f = \frac{1}{-0.8} [\ln(-0.8X + 0.04)]_{1.4}^{0.1} = 4.12 \text{ hr.}$$

Total time required for drying $\theta = \theta_c + \theta_f = 2.407 + 4.12 = 6.527 \text{ hr.}$

Equilibrium moisture content X^* is obtained by equating $dX/d\theta$ in Eqn.(6.48) to zero.

$$\text{i.e., } -0.8X^* + 0.04 = 0$$

$$\text{Therefore, } X^* = 0.05$$

6.3.22 Drying of Solid

160 kg of wet solid is to be dried from an initial moisture content of 25% to a final value of 6%. Drying test shows that the rate of drying is constant at $3 \times 10^{-4} \text{ kg H}_2\text{O} / \text{m}^2 \cdot \text{s}$ in the region 0.2 – 0.44 kg H₂O / kg solid. The drying rate falls linearly in the range 0.01 – 0.2 kg H₂O / kg solid. If the equilibrium moisture content is 0.01 kg H₂O / kg solid, calculate the time of drying. The drying surface is 1 m² / 30 kg dry weight. (GATE-1996-7)

Solution:

$$\text{Wet solid} = 160 \text{ kg}$$

$$\text{Dry solid} = 160 \times (1 - 0.25) = 120 \text{ kg}$$

$$\text{Initial moisture content} = 25\% = \frac{0.25}{1 - 0.25} = 33.3\% \text{ on dry basis}$$

$$\text{Final moisture content} = 6\% = \frac{0.06}{1 - 0.06} = 6.4\% \text{ on dry basis}$$

Drying rate (N) in falling rate period:

$$N = \frac{3 \times 10^{-4}}{0.2 - 0.01} X + C$$

Given: at $X = 0.01$, $N = 0$. Therefore,

$$0 = \frac{3 \times 10^{-4}}{0.2 - 0.01} \times 0.01 + C$$

Solving, $C = -1.579 \times 10^{-5} \text{ kg H}_2\text{O/m}^2\text{sec}$. Therefore

$$N = \frac{3 \times 10^{-4}}{0.19} X - 1.579 \times 10^{-5} = 1.579 \times 10^{-3} X - 1.579 \times 10^{-5}$$

By formula

$$N = \frac{-S_s}{A} \frac{dX}{d\theta} = \frac{-30}{1} \frac{dX}{d\theta} = -30 \frac{dX}{d\theta}$$

Time of drying in falling rate period:

$$\begin{aligned} N &= -30 \frac{dX}{d\theta} \\ 1.579 \times 10^{-3} X - 1.579 \times 10^{-5} &= -30 \frac{dX}{d\theta} \\ \frac{d\theta}{30} &= \frac{-dX}{1.579 \times 10^{-3} X - 1.579 \times 10^{-5}} \end{aligned}$$

Integrating,

$$\begin{aligned} \frac{\theta_f}{30} &= \frac{-1}{1.579 \times 10^{-3}} \int_{0.2}^{0.064} \frac{dX}{X - 0.01} \\ \theta_f &= \frac{30}{1.579 \times 10^{-3}} [\ln(X - 0.01)]_{0.064}^{0.2} \\ \theta_f &= 23902 \text{ sec} = 6.64 \text{ hr.} \end{aligned}$$

Time of drying in constant rate period:

$$\begin{aligned} \theta_c &= \frac{S_s}{A} \frac{X_1 - X_c}{N_c} \\ &= 30 \times \left(\frac{0.333 - 0.2}{3 \times 10^{-4}} \right) \\ &= 30 \times \left(\frac{0.333 - 0.2}{3 \times 10^{-4}} \right) = 13300 \text{ sec} = 3.69 \text{ hr} \end{aligned}$$

Total drying time = $6.64 + 3.69 = 10.33 \text{ hr} = 10 \text{ hours \& 20 minutes.}$

6.3.23 Drying of Slabs

A material having a critical moisture content of 24% is dried under constant drying conditions in slabs of dimensions $20\text{cm} \times 20\text{cm} \times 2\text{cm}$ from a moisture content of 40% to 25% in 8 hours. Drying takes place from the two larger surfaces only. The same material is dried in slabs of $40\text{cm} \times 40\text{cm} \times 4\text{cm}$ under similar drying conditions from 50% to 25% moisture content. All moisture content are on wet basis. If the critical moisture content and other physical properties remain unchanged, how long will the drying take? (GATE-1989-15)

Solution:

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For drying in constant rate period, there is no change in drying rate per unit area, with the thickness of the material.

For the First Case:

$$\text{Drying area (from 2 sides)} A_1 = 20 \times 20 \times 2 = 800 \text{ cm}^2$$

$$\text{Weight of dry solid } S_{s1} = V_1 \rho (1 - 0.4) = 2 \times 20 \times 20 \times 0.6 \rho = 480 \rho$$

$$\begin{aligned} \text{Drying rate } N_1 &= \frac{-S_{s1}}{A_1} \frac{dX}{d\theta} \\ &= \frac{480\rho}{800} \times \frac{X_1 - X_2}{\theta} \end{aligned}$$

On dry basis,

$$X_1 = \frac{0.4}{1 - 0.4} = 0.667; \quad X_2 = \frac{0.25}{1 - 0.25} = 0.333$$

Therefore,

$$N_1 = \frac{480\rho}{800} \times \frac{0.667 - 0.333}{8} = 0.02505\rho = \text{constant}$$

For the Second Case:

$$\text{Drying area (from 2 sides)} A_2 = 40 \times 40 \times 2 = 3200 \text{ cm}^2$$

$$\text{Weight of dry solid } S_{s2} = V_2 \rho (1 - 0.5) = 4 \times 40 \times 40 \times 0.5 \rho = 3200 \rho$$

$$\text{Drying rate } N_2 = \frac{-S_{s2}}{A_2} \frac{dX}{d\theta}$$

As drying rate per unit area does not change with thickness of material,

$$N_2 = N_1 = 0.02505\rho$$

Also

$$N_2 = \frac{-S_{s2}}{A_2} \frac{dX}{d\theta} = \frac{3200\rho}{3200} \frac{(X_1 - X_2)}{\theta}$$

On dry basis,

$$X_1 = \frac{0.5}{1 - 0.5} = 1; \quad X_2 = \frac{0.25}{1 - 0.25} = 0.333$$

Therefore,

$$0.02505\rho = \frac{3200\rho (1 - 0.333)}{3200}$$

Solving, $\theta = 26.63$ hours. Therefore the total drying time for the second case is = 26.63 hour.

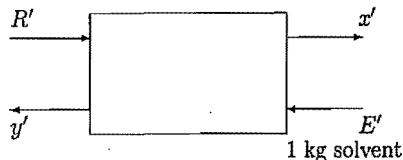
6.3.24 Extraction of Acetone

Acetone from 1 kg of an aqueous solution containing 20% acetone (by weight) is to be extracted using 1 kg of an organic solvent trichloroethane. Trichloroethane and water are immiscible. The distribution coefficient,

$$\frac{\text{kg acetone}}{\text{kg trichloroethane}} = 1.65 \frac{\text{kg acetone}}{\text{kg water}}$$

Estimate the amounts of acetone in extract phase and aqueous phase at equilibrium. (GATE-1987-15ii)

Solution:



Let

$$y' = \frac{\text{kg acetone}}{\text{kg trichloroethane}}$$

$$x' = \frac{\text{kg acetone}}{\text{kg water}}$$

Given: $y' = 1.65x'$. On solute free basis, let R' be the amount of raffinate and E' be the amount of extract.

From mass balance:

$$R'(0.25 - x') = E'(y' - 0)$$

$$0.8(0.25 - x') = 1(1.65x' - 0)$$

$$0.2 - 0.8x' = 1.65x'$$

$$x' = 0.0816 \text{ kg acetone/kg water}$$

$$y' = 1.65 \times 0.0816 = 0.1346 \text{ kg acetone/kg trichloroethane}$$

$$\text{Amount of acetone in extract phase} = E'y' = 0.1346 \text{ kg}$$

$$\text{Amount of acetone in aqueous phase} = R'x' = 0.8 \times 0.0816 = 0.0653 \text{ kg}$$

Chapter 7

Reaction Engineering

7.1 Instant Notes

7.1.1 Reaction Kinetics

Theories of Rate constant:

Arrhenius law

$$k \propto e^{-E/RT}$$

Collision theory

$$k \propto T^{1/2} e^{-E/RT}$$

Transition state theory

$$k \propto Te^{-E/RT}$$

To remember as ACT (Arrhenius, Collision, Transition); T is in the increasing power (0, 1/2, 1).



Collision theory views the first step of Eqn.(7.1) is slow and rate controlling, whereas the transition state theory views the second step to be the rate controlling.

- No elementary reaction with molecularity greater than *three* have been observed.

7.1.2 Design of Reactors

- Design Equations for Ideal Reactors:

— Batch Reactor:

$$t = N_{A0} \int_0^{X_A} \frac{dX_A}{(-r_A)V}$$

— Constant flow Stirred Tank Reactor (CSTR):

$$\frac{V}{F_{A0}} = \frac{\tau}{C_{A0}} = \frac{X_A}{-r_A}$$

- Plug Flow Reactor (PFR):

$$\frac{V}{F_{A0}} = \frac{\tau}{C_{A0}} = \int_0^{X_A} \frac{dX_A}{-\tau_A}$$

where F_{A0} = molar flow rate of A

N_{A0} = number of moles of A at $t = 0$

X_A = conversion of A at time $t = (N_{A0} - N_A)/N_{A0}$

C_{A0} = feed concentration of A

V = volume of reactor

τ = space time

• Arrangement of Reactors:

- Two equal size CSTRs in series will give a higher conversion than two CSTRs in parallel of the same size, when the reaction order is greater than zero. i.e., conversion in the series arrangement is greater than in parallel for CSTRs.
- For a reaction whose rate-concentration curve rises monotonically (any n^{th} order reaction, $n > 0$) the reactors should be connected in series. They should be ordered so as to keep the concentration of reactant as high as possible if the rate-concentration curve is concave ($n > 1$), as low as possible if the curve is convex ($n < 1$). When $n > 1$, the ordering of the units should be plug → small mixed → large mixed.
- Reactors for multiple reactions in parallel: For reactions in parallel the concentration level of reactants is the key to proper control of product distribution. A high reactant concentration favors the reaction of higher order, a low concentration favors the reaction of lower order, while the concentration level has no effect on the product distribution for reactions of the same order.

7.1.3 Non-isothermal Reactors

- Optimum-temperature progression in adiabatic reactors:

- Reversible exothermic reactions: For reversible exothermic reactions, the optimum temperature progression is a changing temperature; a high starting temperature which decreases as conversion rises. This can be achieved for adiabatic operation by connecting reactors in series with interstage cooling.
- Reversible endothermic reactions: A rise in temperature increases both equilibrium composition and the rate of reaction. Use interstage heating for higher conversions.
- Irreversible reactions: Rate always increases with temperature at any composition, so the highest rate occurs at the highest allowable temperature.

REACTION ENGINEERING

7.1.4 Heterogeneous Reactions

- Thiele Modulus ϕ

$$\phi^2 = \frac{\text{surface reaction rate}}{\text{diffusion rate}}$$

$$\text{For first order reaction } \phi = L \sqrt{\frac{k}{D_{AB}}}.$$

If ϕ is small, surface reaction is usually rate limiting. For conditions of diffusion control ϕ will be large.

– Effectiveness factor (ζ):

$$\zeta = \frac{\text{actual reaction rate within the pore rate, if not slowed by pore diffusion}}{\text{rate, if not slowed by pore diffusion}}$$

– For first order reaction:

$$\begin{aligned} \zeta &= \frac{\tanh \phi}{\phi} \\ &= \frac{1}{\phi} \text{ (for } \phi > 5) \\ &= 1 \text{ (for } \phi < 0.5) \end{aligned}$$

- Rate limiting step

– Diffusion and mass transfer controlled processes tend to occur in relatively high temperature regions, whereas chemically controlled processes occur at lower temperatures. This is because the rate of a chemical reaction (also chemisorption and desorption) depends according to the Arrhenius equation, on the factor $e^{-E/RT}$ while rates of diffusion and mass transfer are not strongly influenced by the temperature.

- Knudsen diffusion: When the mean free path of the gas molecules is considerably greater than the pore dia, collisions between molecules in the gas are much less numerous than between molecules and pore-walls. Under these conditions the mode of transport is Knudsen diffusion.

When the mean free path of the gas molecules is much smaller than the pore diameter, gaseous collisions will be more frequent than collisions of molecules with pore walls, and under these conditions ordinary bulk diffusion occurs.

7.1.5 Catalytic Reactions

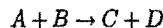
- Catalyst can not change the ultimate conversion dictated by thermodynamics but accelerates the rate of approach to equilibrium. Catalytic reaction requires lesser activation energy than homogeneous reaction.

• Catalysts:

Groups	Functions	Examples
Supported metal catalysts	Hydrogenation, dehydrogenation, hydrocracking	Fe, Ni, Pd, Pt, Ag
Metal compounds: Semiconducting oxides and sulfides	Oxidation, dehydrogenation, desulfurisation	NiO, ZnO, MnO ₂ , Cr ₂ O ₃ , Bi ₂ O ₃ , MoO ₃ , WS ₂
Insulator oxides	Dehydrogenation	Al ₂ O ₃ , SiO ₂ , MgO
Acid & Zeolite	Cracking.	SiO ₂ , Al ₂ O ₃

7.2 Objective Type Questions

1. The experimentally determined overall order for the reaction



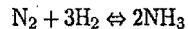
is two. Then the

(GATE-2000-1.18)

- (a) reaction is elementary with a molecularity of 2
- (b) molecularity of the reaction is 2, but the reaction may not be elementary
- (c) reaction may be elementary with a molecularity of 2
- (d) reaction is elementary but the molecularity may not be 2

Answer: (b) If it is given that $-r_A = kC_A C_B$, then the reaction is elementary. Otherwise we cannot say about the reaction type just from the reaction order.

2. From the stoichiometry, one can say that the following reaction is non-elementary.



The reason is

(GATE-1989-7.i.a)

- (a) No elementary reaction has an order greater than 3
- (b) The order of the forward reaction is not the same as that of the backward reaction
- (c) The reaction takes place in gas phase only
- (d) None of the above

Answer: (a)

3. Overall order of reaction for which the rate constant has units of $(mol/L)^{-3/2}sec^{-1}$ is (GATE-1999-1.21)

- (a) -3/2 (b) 1/2 (c) 3/2 (d) 5/2

Answer: (d) $-r_A = kC_A^n$. r_A has units of mol/(ltr.sec)

4. For the reaction $A + B \rightarrow 2B + C$, (GATE-1999-1.22)

- (a) $r_A = r_B$ (b) $r_A = -r_B$ (c) $r_A = 2r_B$ (d) $r_A = r_B/2$

Answer: (b) The reaction can be written as $A \rightarrow B + C$.

5. For the reaction $P + 2Q \rightarrow 3R$, molar rate of consumption of P is (GATE-1998-1.19)

- (a) double of that of Q (b) same as that of Q
(c) half of that of Q (d) 2/3 of that of Q

Answer: (c) $2Q$ is utilized for $1P$.

6. In a homogeneous gas phase reaction $A + 2B \rightarrow R + S$ what is the relationship between (GATE-1990-7.iii) r_A and r_B :

- (a) $2r_A = r_B$ (b) $r_A = 2r_B$ (c) $r_A = r_B$ (d) None of the above

Answer: (a) When 1 mole of A is consumed, 2 mole of B is utilized.

7. Molecularity of an elementary reaction $P + Q \rightarrow R + S$ is (GATE-1998-1.20)

- (a) 1 (b) 2 (c) 3 (d) 4

Answer: (b)

8. From collision theory, the reaction rate constant is proportional to (GATE-1996-1.03)

- (a) $\exp\left(-\frac{E}{RT}\right)$ (b) $\exp\left(-\frac{E}{2RT}\right)$
(c) \sqrt{T} (d) $T^m \exp\left(-\frac{E}{RT}\right)$

Answer: (d) $m = 1/2$.

9. The units of frequency factor in Arrhenius equation (GATE-1992-8.b)

- (a) are the same as those of the rate constant
- (b) depend on the order of the reaction
- (c) depend on temperature, pressure etc. of the reaction
- (d) are cycles per unit time

Answer: (a) $k = k_0 e^{-E/RT}$

- (a) 0.5 (b) 1 (c) 1.5 (d) 2

Answer: (c) Slope of $\ln t_{1/2}$ vs. $\ln C_{A0}$ curve = $1 - n$.

$$\frac{\ln t_{1/2,1} - \ln t_{1/2,2}}{\ln C_{A0,1} - \ln C_{A0,2}} = \frac{\ln 2 - \ln 1}{\ln 2 - \ln 8} = -0.5$$

i.e., $1 - n = 0.5$. Therefore $n = 1.5$.

19. The conversion of a reactant, undergoing a first order reaction, at a time equal to three times the half life of the reaction is (GATE-1992-8.a)

- (a) 0.875
 (b) 0.5
 (c) 0.425
 (d) not possible to calculate because of insufficient data

Answer: (a) For first order reaction $C_A/C_{A0} = e^{-kt}$.

For half-life $C_A/C_{A0} = 0.5 = e^{-kt_{1/2}}$

$$t_{1/2} = (\ln 2)/k$$

$$At t = 3t_{1/2} = 3(\ln 2)/k = (\ln 8)/k$$

$$\ln 8 = \ln C_{A0}/C_A$$

$$C_A/C_{A0} = 0.125. \text{ Therefore conversion} = 0.875.$$

20. A gaseous reaction $A \rightarrow 2B+C$ takes place isothermally in a constant pressure reactor. Starting with a gaseous mixture containing 50% A (rest inert), the ratio of final to initial volume is found to be 1.6. The percentage conversion of A is (GATE-1992-2.c)

- (a) 30 (b) 50 (c) 60 (d) 74

Answer: (c) $\epsilon_A = \frac{V_{X_A=1} - V_{X_A=0}}{V_{X_A=0}} = \frac{3 - 1}{1} = 2$

$$\frac{V}{V_0} = 1.6 = \frac{\text{volume of inert} + \text{volume of } A}{\text{volume at initial}} = \frac{0.5V_0 + 0.5V_0(1 + \epsilon_A X_A)}{V_0}$$

$$\text{Solving } X_A = 0.6.$$

21. The reaction $A \rightarrow B$ is conducted in an isothermal batch reactor. If the conversion of A increases linearly with holding time, then the order of the reaction is (GATE-2000-1.19)

- (a) 0 (b) 1 (c) 1.5 (d) 2

Answer: (a) For the zero order reaction $-r_A = kC_A^0 = -dC_A/dt$.

Integrating $C_{A0} - C_A = kt$. i.e., $C_{A0} - C_{A0}(1 - X_A) = kt$. Hence $X_A = kt/C_{A0}$.

22. The conversion for a first order liquid-phase reaction $A \rightarrow B$ in a CSTR is 50%. If another CSTR of the same volume is connected in series, then the % conversion at the exit of the second reactor will be (GATE-2000-2.16)

- (a) 60 (b) 75 (c) 90 (d) 100

Answer: (b) 50% conversion in the first reaction. Another 50% of A is converted by 50% in the second reactor. Therefore the overall conversion = $50 + 0.5 \times 50 = 75\%$.

23. For the liquid phase zero-order reaction $A \rightarrow B$, the conversion of A in a CSTR is found to be 0.3 at a space velocity of 0.1 min^{-1} . What will be the conversion for a PFR with a space velocity of 0.2 min^{-1} ? Assume that all the other operating conditions are the same for CSTR and PFR. (GATE-1999-2.13)

- (a) 0.15 (b) 0.3 (c) 0.6 (d) 0.9

Answer: (a) CSTR: $\tau = \frac{C_{A0}X_A}{-r_A}$

$$\frac{1}{0.1} = \frac{0.3}{k}. \text{ Therefore } k = 0.03 \text{ ltr/(mol.min)}$$

PFR: $\tau = C_{A0} \int_0^{X_{Af}} \frac{dX_A}{-r_A}$

$$\frac{1}{0.2} = \frac{X_{Af}}{0.03}. \text{ Therefore } X_{Af} = 0.15.$$

24. The sequence in which three CSTR's of volumes 5, 10 and 15 cu.m. will be connected in series to obtain the maximum production in a second order irreversible reaction is (GATE-1996-1.14)

- (a) 15, 10, 5 (b) 5, 10, 15 (c) 10, 5, 15 (d) 10, 15, 5

Answer: (b)

25. The energy balance equation over a tubular reactor under transient conditions is (GATE-1996-1.01)

- (a) an ordinary nonlinear differential equation
 (b) an algebraic equation
 (c) a linear partial differential equation
 (d) a nonlinear partial differential equation

Answer: (d) T varies with t and L (also with r); T varies in a nonlinear manner with t . In CSTR T varies with t only, which is represented by ordinary nonlinear differential equation.

26. For a mixed flow reactor operating at steady state, the rate of reaction is given by (GATE-1996-1.15)

- (a) $\frac{F_{A0}}{V} - \frac{dC_A}{dt}$ (b) $\frac{F_{A0}}{V} + \frac{dC_A}{dt}$ (c) $\frac{F_{A0}X_A}{V}$ (d) $-\frac{dC_A}{dt}$

Answer: (c)

27. For a tubular reactor with space time τ and residence time θ , the following statement holds (GATE-1996-1.16)

- (a) τ and θ are always equal
- (b) $\tau = \theta$ when the fluid density changes in the reactor
- (c) $\tau = \theta$ for an isothermal tubular reactor in which the density of the process fluid is constant
- (d) $\tau = \theta$ for a non-isothermal reactor

Answer: (c) τ and θ are different for variable volume gas phase reactions or for non-isothermal gas reactions.

τ — the time required to process one reactor volume of feed material at specified conditions.

θ = volume / volumetric flow rate at reactor conditions.

28. For an isothermal variable volume batch reactor, the following reaction is applicable for a first order irreversible reaction (GATE-1996-2.03)

- (a) $X_A = kt$
- (b) $\frac{C_{A0}}{1 + \epsilon_A X_A} \left(\frac{dX_A}{dt} \right) = k$
- (c) $-\ln(1 - X_A) = kt$
- (d) $\epsilon_A \ln(1 - X_A) = kt$

Answer: (c) Mass balance:

Input = output + disappearance + accumulation

For batch reactor, input = output. Therefore $(-r_A)V = -dN_A/dt$. $N_A = N_{A0}(1 - X_A)$. Therefore

$$-r_A V = \frac{-d[N_{A0}(1 - X_A)]}{dt} = N_{A0} \frac{dX_A}{dt}$$

Integrating

$$t = N_{A0} \int_0^{X_A} \frac{dX_A}{-r_A V} = N_{A0} \int_0^{X_A} \frac{dX_A}{-r_A V_0(1 + \epsilon_A X_A)}$$

Since $C_{A0} = N_{A0}/V_0$; and for first order reaction $-r_A = kC_A$; and for variable volume reaction

$$C_A = C_{A0} \frac{1 - X_A}{1 + \epsilon_A X_A}$$

Therefore

$$t = \int \frac{dX_A}{k(1 - X_A)}$$

i.e., $-\ln(1 - X_A) = kt$.

29. For the gaseous reaction $2A \rightarrow B$ where the feed consists of 50 mol % A and 50 mol % inert, the expansion factor is (GATE-1994-1.q)

- (a) 1
- (b) -0.5
- (c) -0.25
- (d) 0

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Answer: (c) $\epsilon_A = \frac{V_{X_A=1} - V_{X_A=0}}{V_{X_A=0}} = \frac{1.5 - 2}{2} = -0.25$

30. What is the exit conversion of reactant A for a zero order reaction taking place in CSTR with the following data (rate constant = 1 mol/(min.litre); feed concentration = 1 mol/litre; feed flow rate = 0.5 litre/min and reactor volume = 1 litre): (GATE-1990-7.ii)

- (a) 50%
- (b) 75%
- (c) 100%
- (d) 200%

Answer: (d) $\frac{V}{F_{A0}} = \frac{X_A}{-r_A}$

$F_{A0} = 1 \times 0.5 = 0.5 \text{ mol/min}$. $-r_A = k = 1 \text{ mol}/(\text{min.ltr})$ Therefore, $\frac{1}{0.5} = \frac{X_A}{1}$
 $X_A = 200\%$.

31. Liquid A decomposes by first order kinetics in an isothermal batch reactor. Fifteen percent of A is converted in 300 seconds. The time required for a conversion of 7 percent is (GATE-1988-7.b.i)

- (a) $300 \times 0.75/0.25$ seconds
- (b) $300 \ln 1$ seconds
- (c) $300 \ln(0.5/0.25)$ seconds
- (d) $300[\ln(0.25)/\ln(0.5)]$ seconds

Answer: (d) For first order reaction $C_A = C_{A0}e^{-kt}$ or $\ln(C_A/C_{A0}) = -kt$. Since $C_A = C_{A0}(1 - X_A)$, $\ln(1 - X_A) = -kt$

Given: when $t = 300$; $X_A = 0.5$. Hence, $-300k = \ln 0.5$. and $-tk = \ln 0.25$. Therefore $t = 300[\ln(0.25)/\ln(0.5)]$.

32. For a given duty and for all positive reaction orders, the size of a mixed reactor is:

- (a) smaller than that of a plug flow reactor
- (b) same as that of a plug flow reactor
- (c) larger than that of a plug flow reactor
- (d) none of the above

Answer: (c)

33. The gas phase reaction $2A \rightarrow B$ is carried out in an isothermal plug flow reactor. The feed consists of 80 mol% A and 20 mol% inert. If the conversion of A at the reactor exit is 50%, then C_A/C_{A0} at the outlet of the reactor is (GATE-1997-2.14)

- (a) 2/3
- (b) 5/8
- (c) 1/3
- (d) 3/8

Answer: (b) $C_A = \frac{N_A}{V} = \frac{N_{A0}(1 - X_A)}{V_0(1 + \epsilon_A X_A)} = C_{A0} \frac{1 - X_A}{1 + \epsilon_A X_A}$

$$\epsilon_A = \frac{V_{X_A=1} - V_{X_A=0}}{V_{X_A=0}} = \frac{(0.8 + 0.4) - 2}{2} = -0.4$$

$$\frac{C_A}{C_{A0}} = \frac{1 - 0.5}{1 - 0.4 \times 0.5} = \frac{5}{8}$$

34. The conversion for a second order, irreversible reaction (constant volume), $A \xrightarrow{k_2} B$, in batch mode is given by
 (GATE-2001-1.14)

$$(a) \frac{1}{1 + k_2 C_{A0} t} \quad (b) \frac{k_2 C_{A0} t}{1 + k_2 C_{A0} t} \quad (c) \frac{(k_2 C_{A0} t)^2}{1 + k_2 C_{A0} t} \quad (d) \frac{k_2 C_{A0} t}{(1 + k_2 C_{A0} t)^2}$$

Answer: (b) $-\frac{dC_A}{dt} = k_2 C_A^2$

$$-\frac{dC_A}{C_A^2} = k_2 dt$$

$$\frac{1}{C_A} - \frac{1}{C_{A0}} = k_2 t; \frac{1}{C_A} = k_2 t + \frac{1}{C_{A0}}$$

$$\text{or } \frac{C_A}{C_{A0}} = \frac{1}{1 + k_2 C_{A0} t}$$

$$\left(1 - \frac{C_A}{C_{A0}}\right) = X_A = \frac{k_2 C_{A0} t}{1 + k_2 C_{A0} t}$$

35. The first-order, gas phase reaction $A \xrightarrow{k_1} 2B$ is conducted isothermally in batch mode. The rate of change of conversion with time is given by
 (GATE-2001-2.17)

$$(a) dX_A/dt = k_1(1 - X_A)^2(1 + 2X_A) \quad (b) dX_A/dt = k_1(1 - X_A)(1 + 0.5X_A)$$

$$(c) dX_A/dt = k_1(1 - X_A) \quad (d) dX_A/dt = k_1(1 - X_A)/(1 + X_A)$$

Answer: (c) $\varepsilon_A = \frac{2-1}{1} = 1$

$$-\frac{dC_A}{dt} = k_1 C_A$$

$$\begin{aligned} k_1 C_A &= -\frac{1}{V} \frac{dN_A}{dt} \\ &= -\frac{1}{V_0(1 + \varepsilon_A X_A)} \frac{N_{A0} d(1 - X_A)}{dt} \\ &= \frac{C_{A0}}{1 + X_A} \frac{dX_A}{dt} \end{aligned}$$

Since $C_A = C_{A0} \frac{1 - X_A}{1 + \varepsilon_A X_A}$,

$$\frac{C_{A0}}{1 + X_A} \frac{dX_A}{dt} = k_1 C_A = k_1 \frac{C_{A0}(1 - X_A)}{1 + X_A}$$

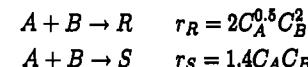
$$\text{Therefore, } \frac{dX_A}{dt} = k_1(1 - X_A).$$

36. The conversion X_A and residence time τ data are collected for zero order liquid phase reaction in a stirred tank reactor. Which of the following will be a straight line.
 (GATE-1995-1.i)

$$\begin{array}{ll} (a) X_A \text{ vs. } \tau & (b) X_A \text{ vs. } \ln \tau \\ (c) X_A/(1 - X_A) \text{ vs. } \tau & (d) X_A(1 - X_A) \text{ vs. } \tau \end{array}$$

Answer: (a) For CSTR, $\tau = C_{A0} X_A / (-r_A)$. For zeroth order reaction $-r_A = k$. Therefore, $\tau = C_{A0} X_A / k$.

37. To maximize the formation of R in the simultaneous reactions

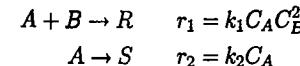


we should have

$$\begin{array}{ll} (a) \text{low } C_A, \text{ low } C_B & (b) \text{low } C_A, \text{ high } C_B \\ (c) \text{high } C_A, \text{ low } C_B & (d) \text{high } C_A, \text{ high } C_B \end{array}$$

Answer: (b) $\frac{r_R}{r_S} = \frac{2C_B}{1.4C_A^{0.5}} = 1.43C_B C_A^{-0.5}$. For positive orders have high concentration (use PFR); negative orders have low concentration (use CSTR).

38. The reaction between A and B produces the desired product R as well as the unwanted product S . What level of reactant concentrations (high, medium, low) should we use for the following reaction scheme in order to maximize the conversion of A to R . Reaction scheme:

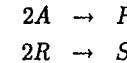


(GATE-1991-8.i.a)

$$\begin{array}{ll} (a) \text{low } C_A, \text{ low } C_B & (b) \text{high } C_B, \text{ any } C_A \\ (c) \text{high } C_A, \text{ low } C_B & (d) \text{high } C_A, \text{ any } C_B \end{array}$$

$$\text{Answer: (b) } \frac{r_1}{r_2} = \frac{k_1}{k_2} C_B^2$$

39. For multiple reactions



the number of moles of S present when the number of moles of A and R are 0.3 and 0.5 respectively (Initially 2 moles of A are only present) are
 (GATE-1993-13.b)

$$(a) 0.125 \quad (b) 0.175 \quad (c) 0.535 \quad (d) 0.350$$

Answer: (b) $N_{A0} = 2$ moles. Number of moles of A consumed = $2 - 0.3 = 1.7$ moles. 1.7 mole of A should have given 0.85 mole of R , of which 0.5 mole of R are present. So 0.35 mole of R are consumed, which should have given 0.175 mole of S .

40. For the liquid phase parallel reactions

$$\begin{array}{ll} A \rightarrow R & r_R = k_1 C_A^2 \\ A \rightarrow S & r_S = k_2 C_A; \end{array} \quad \begin{array}{l} E_1 = 80 \text{ kJ/mol} \\ E_2 = 120 \text{ kJ/mol} \end{array}$$

the desired product is R . A higher selectivity of R will be achieved if the reaction is conducted at
(GATE-2000-1.20)

- (a) low temperature in a CSTR (b) high temperature in a CSTR
(c) low temperature in a PFR (d) high temperature in a PFR

Answer: (a) $\frac{r_R}{r_S} = \frac{k_1}{k_2} C_A$. Since $E_1 < E_2$, at low T , k_1/k_2 is more. To maintain high C_A , PFR is required.

41. The reaction $A \rightarrow B$ is conducted in an adiabatic plug flow reactor (PFR). Pure A at a concentration of 2 kmol/m^3 is fed to the reactor at the rate of $0.01 \text{ m}^3/\text{s}$ and at a temperature of 500 K . If the exit conversion is 20% , then the exit temperature (in K) is
(GATE-2000-2.19)

- (a) 400 (b) 500 (c) 600 (d) 1000

Data: Heat of reaction at $298 \text{ K} = -50,000 \text{ kJ/kmol}$ of A reacted
Heat capacities, $C_{p,A} = C_{p,B} = 100 \text{ kJ/kmol.K}$ (may be assumed independent of temperature).

Answer: (c) Molal flow rate of $A = 0.02 \times 0.2 = 0.004 \text{ kmol/s}$.

Heat released from reaction = $0.004 \times 50000 = 200 \text{ kJ}$.

Heat balance: $mC_p\Delta T = 200$

i.e., $2 \times 10 \times (T - 500) = 200$. Therefore $T = 600 \text{ K}$.

42. When an exothermic reversible reaction is conducted adiabatically the rate of reaction
(GATE-1996-2.13)

- (a) continuously increases (b) continuously decreases
(c) passes through a maximum (d) passes through a minimum

Answer: (c) Equilibrium conversion decreases with increase in temperature. Rate constant increases with increase in temperature.

43. An exothermic gas phase reaction proceeds according to the equation $3A + 2B \rightleftharpoons 2R$. The equilibrium conversion for this reaction:
(GATE-1990-7.i)

- (a) increases with increase in temperature
(b) decreases on dilution with an inert gas
(c) decreases with an increase in pressure
(d) is unaffected by the presence of a catalyst

Answer: (d)

44. A reversible liquid phase endothermic reaction is to be carried out in a plug flow reactor. For minimum reactor volume, it should be operated such that the temperature along the length
(GATE-1997-1.17)

- (a) decreases
(b) increases
(c) is at the highest allowable temperature throughout
(d) first increases and then decreases

Answer: (c) Higher the temperature higher is the equilibrium conversion for endothermic reaction. Rate constant k increases with increase in temperature.
end of cre

45. In solid catalysed reactions the diffusional effects are more likely to affect the overall rate of reaction for
(GATE-2000-1.21)

- (a) fast reactions in catalysts of small pore diameter
(b) fast reactions in catalysts of large pore diameter
(c) slow reactions in catalysts of small pore diameter
(d) slow reactions in catalysts of large pore diameter

Answer: (b) For fast reactions, mass transfer may control; and diffusional resistances are more with large pore diameter.

46. The Knudsen diffusivity is dependent on
(GATE-1996-1.17)

- (a) the molecular velocity only
(b) the pore radius of the catalyst only
(c) the molecular mean free path only
(d) the molecular velocity and pore radius of the catalyst

Answer: (d)

47. For a first order chemical reaction in a porous catalyst, the Thiele modulus is 10. The effectiveness factor is approximately equal to
(GATE-1996-1.21)

- (a) 1 (b) 0.5 (c) 0.1 (d) 0

Answer: (c) For Thiele modulus (mL) > 5 , effectiveness factor is inversely proportional to Thiele modulus. Therefore, effectiveness factor = $1/10 = 0.1$.

48. The rate expression for a heterogeneous catalytic reaction is given by

$$-r_A = k k_A P_A / (1 + k_A P_A + k_R P_R)$$

where k is the surface reaction rate constant and k_A and k_R are adsorption equilibrium constants of A and R respectively. If $k_R P_R \gg (1 + k_A P_A)$ the apparent activation energy E_A is equal to (given E is the activation energy for the reaction, and ΔH_R and ΔH_A are the activation energies of adsorption of R and A)
(GATE-1996-2.11)

- (a) E
 (c) $E + \Delta H_A - \Delta H_R$
- (b) $E + \Delta H_A$
 (d) $\Delta H_A + \Delta H_R$

Answer: (c) If $k_R P_R \gg (1 + k_A P_A)$, $-r_A = k k_A P_A / (k_R P_R)$.
 $k_0 e^{-E/RT} = k_0 e^{-E/RT} \cdot k_A e^{-\Delta H_A/RT} / (k_R e^{-\Delta H_R/RT})$
 $-E_A = -E - \Delta H_A + \Delta H_R$
 Therefore, $E_A = E + \Delta H_A - \Delta H_R$.

49. The reaction $A \rightarrow B$ occurs in an isothermal catalyst pellet under steady state conditions. If the diffusion of A into the pellet is the rate controlling step, the rate of diffusion of A is:
 (GATE-1988-7.b.iii)

- (a) faster than the rate of reaction
 (b) equal to the rate of reaction
 (c) slower than the rate of reaction
 (d) may be faster or slower, depending on the kinetics

Answer: (c) Rate controlling step is one which is having a slower rate.

50. Thiele modulus is defined as

- (a) D/k
 (b) k/D
 (c) $L(k/D)^{0.5}$
 (d) Lk/D

Answer: (c)

51. For a first-order isothermal chemical reaction in a porous catalyst, the effectiveness factor is 0.3. The effectiveness factor will increase if the
 (GATE-1997-2.15)

- (a) catalyst size is reduced or the catalyst diffusivity is reduced
 (b) catalyst size is reduced or the catalyst diffusivity is increased
 (c) catalyst size is increased or the catalyst diffusivity is reduced
 (d) catalyst size is increased or the catalyst diffusivity is increased

Answer: (b) Effectiveness factor increases with decrease in Thiele modulus ($L\sqrt{k/D}$). Thiele modulus decreases if L decreases and/or D increases.

52. A first order reaction $A \rightarrow B$ occurs in an isothermal porous catalyst pellet of spherical shape. If the concentration of A at the centre of the pellet is much less than that at the external surface, the process is limited by
 (GATE-1995-1.k)

- (a) diffusion within the pellet
 (b) reaction
 (c) external mass transfer
 (d) none of the above

Answer: (a) $C_s \gg C_{\text{centre}}$. ΔC is large. Otherwise diffusional resistance is large.

7.3 Problems with Solutions

7.3.1 Activation Energy

For an elementary reaction $A + B \rightarrow \text{Products}$, the reaction rate at 500 K is ten times that at 400 K. Calculate the activation energy for this reaction.
 (GATE-1990-17.ii)

Solution:

From Arrhenius law

$$k \propto e^{-E/(RT)} \quad (7.2)$$

where k = rate constant

E = activation energy

R = universal gas constant

T = reaction temperature in absolute scale

Rate constant at 400 K (k_1):

$$k_1 \propto e^{-E/(400R)} \quad (7.3)$$

Rate constant at 500 K (k_2):

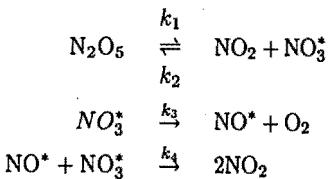
$$k_2 \propto e^{-E/(500R)} \quad (7.4)$$

Since it is given that $k_2 = 10k_1$, from Eqns.(7.3) and (7.4)

$$\begin{aligned} \frac{e^{-E/(500R)}}{e^{-E/(400R)}} &= 10 \\ e^{\frac{E}{2000R}} &= 10 \\ \frac{E}{2000R} &= \ln 10 \\ E &= 2000 \times 8.314 \times \ln 10 \\ &= 38287 \text{ kJ/kmol} = 38.287 \text{ kJ/gmol} \end{aligned}$$

7.3.2 Expression for Rate for the given Mechanism

The decomposition of N_2O_5 is postulated to occur by the following mechanism:



Using the steady state approximation, derive an expression for the rate of decomposition of N_2O_5 .
 (GATE-1988-17.i)

Solution:

Rate of decomposition of N_2O_5 is given by

$$-\frac{d[\text{N}_2\text{O}_5]}{dt} = k_1[\text{N}_2\text{O}_5] - k_2[\text{NO}_2][\text{NO}_3^*] \quad (7.5)$$

By steady state hypothesis:

$$\frac{d[\text{NO}_3^*]}{dt} = 0 = k_1[\text{N}_2\text{O}_5] - k_2[\text{NO}_2][\text{NO}_3^*] - k_3[\text{NO}_3^*] - k_4[\text{NO}^*][\text{NO}_3^*] \quad (7.6)$$

$$\text{and } \frac{d[\text{NO}^*]}{dt} = 0 = k_3[\text{NO}_3^*] - k_4[\text{NO}^*][\text{NO}_3^*] \quad (7.7)$$

From Eqn.(7.7)

$$[\text{NO}^*] = \frac{k_3[\text{NO}_3^*]}{k_4[\text{NO}_3^*]} = \frac{k_3}{k_4} \quad (7.8)$$

Substituting (7.8) in (7.6)

$$k_1[\text{N}_2\text{O}_5] = k_2[\text{NO}_2][\text{NO}_3^*] + k_3[\text{NO}_3^*] + k_4[\text{NO}^*][\text{NO}_3^*]$$

i.e.,

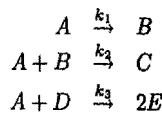
$$[\text{NO}_3^*] = \frac{k_1[\text{N}_2\text{O}_5]}{k_2[\text{NO}_2] + 2k_3} \quad (7.9)$$

Thus Eqn.(7.5) becomes

$$\begin{aligned} -\frac{d[\text{N}_2\text{O}_5]}{dt} &= k_1[\text{N}_2\text{O}_5] - k_2[\text{NO}_2] \left(\frac{k_1[\text{N}_2\text{O}_5]}{k_2[\text{NO}_2] + 2k_3} \right) \\ &= k_1[\text{N}_2\text{O}_5] \left(1 - \frac{k_2[\text{NO}_2]}{k_2[\text{NO}_2] + 2k_3} \right) \end{aligned}$$

7.3.3 Relation for Concentrations in Multiple Reactions

Consider the set of elementary reactions:



At time $t = 0$, a batch reactor is filled with a mixture of A and D . What is the relation between the concentrations of B and D after a time t ? (GATE-1992-18.c)

Solution:

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Given: At $t = 0$; $C_A = C_{A0}$; $C_B = 0$; $C_C = 0$; and $C_D = C_{D0}$. Rate of change of the reactants can be written as:

$$\frac{dC_A}{dt} = -(k_1 C_A + k_2 C_A C_B + k_3 C_A C_D) \quad (7.10)$$

$$\frac{dC_B}{dt} = k_1 C_A - k_2 C_A C_B \quad (7.11)$$

$$\frac{dC_C}{dt} = k_2 C_A C_B \quad (7.12)$$

$$\frac{dC_D}{dt} = -k_3 C_A C_D \quad (7.13)$$

From Eqns.(7.11) and (7.13)

$$\frac{dC_B}{dC_D} = \frac{k_1 C_A - k_2 C_A C_B}{-k_3 C_A C_D} = \frac{k_1 - k_2 C_B}{-k_3 C_D}$$

$$\text{Rearranging, } \frac{dC_B}{k_1 - k_2 C_B} = -\frac{dC_D}{k_3 C_D}$$

$$\text{Integrating, } \frac{1}{k_1} \int_{C_{B0}}^{C_B} \frac{dC_B}{1 - (k_2/k_1)C_B} = -\frac{1}{k_3} \int_{C_{D0}}^{C_D} \frac{dC_D}{C_D}$$

$$\text{i.e., } \frac{1}{k_1} \left[-\ln \left(1 - \frac{k_2}{k_1} C_B \right) \frac{k_1}{k_2} \right]_0^{C_B} = -\frac{1}{k_3} \ln \frac{C_D}{C_{D0}}$$

$$\text{Rearranging, } -\frac{1}{k_2} \ln \left(1 - \frac{k_2}{k_1} C_B \right) = \frac{1}{k_3} \ln \frac{C_{D0}}{C_D}$$

7.3.4 Material Balance Equations for Isothermal CSTR

The reactions $A \rightleftharpoons B \rightarrow C$ are conducted in a steady state isothermal, continuous stirred tank reactor. All reactions are of first order, with identical rate constants(k). The reactor volume(V) and volumetric flow rate(v_0) are constant. If the feed consists of pure A at a concentration C_{Af} , formulate material balance equations for A , B and C . (GATE-1988-17.ii)

Solution:

Let the feed concentration of A be C_{Af} . The rates of change of A , B and C can be written as

$$\frac{dC_A}{dt} = -kC_A + kC_B = k(C_B - C_A) \quad (7.14)$$

$$\frac{dC_B}{dt} = kC_A - kC_B - kC_B = k(C_A - 2C_B) \quad (7.15)$$

$$\frac{dC_C}{dt} = kC_B \quad (7.16)$$

$$\text{Residence time } (\tau) = \frac{V}{V_0} = \text{const.}$$

Residence time based on A: from Eqn.(7.14)

$$\begin{aligned} \tau &= \frac{C_{Af} - C_A}{-r_A} = \frac{C_{Af} - C_A}{k(C_A - C_B)} \\ \text{i.e., } C_A - C_B &= \frac{C_{Af} - C_A}{k\tau} \\ \text{Rearranging, } C_B &= \frac{C_A(1 + k\tau) - C_{Af}}{k\tau} \end{aligned} \quad (7.17)$$

Residence time based on B: from Eqn.(7.15)

$$\begin{aligned} \tau &= \frac{C_{Bf} - C_B}{-r_B} = \frac{-C_B}{-dC_B/dt} = \frac{C_B}{k(C_A - 2C_B)} \\ \text{i.e., } C_B &= k\tau(C_A - 2C_B) \\ \text{Rearranging, } C_B &= \frac{k\tau C_A}{1 + 2k\tau} \end{aligned} \quad (7.18)$$

Similarly based on C,

$$\begin{aligned} \tau &= \frac{C_C}{kC_B} \\ \text{i.e., } C_C &= k\tau C_B \end{aligned} \quad (7.19)$$

From Eqn.(7.17) and (7.18)

$$\frac{C_A(1 + k\tau) - C_{Af}}{k\tau} = \frac{k\tau C_A}{1 + 2k\tau}$$

which gives

$$C_A = \frac{C_{Af}(1 + 2k\tau)}{1 + 3k\tau + k^2\tau^2} \quad (7.20)$$

Substituting this in Eqn.(7.18)

$$C_B = \frac{k\tau C_{Af}}{1 + 3k\tau + k^2\tau^2} \quad (7.21)$$

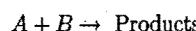
Substituting this in Eqn.(7.19)

$$C_C = \frac{C_{Af}k^2\tau^2}{1 + 3k\tau + k^2\tau^2} \quad (7.22)$$

From Eqns.(7.20), (7.21) and (7.22), checking for $C_A + C_B + C_C$ gives C_{Af} .

7.3.5 Conversion in CSTR for Zeroth Order Reaction

The rate of the liquid phase reaction of the type



is found to be independent of concentration of A and B, and equal to $1 \text{ kmol/m}^3 \cdot \text{min}$ at 300 K. Find the conversion in a mixed flow reactor having volume equal to 2 m^3 with feed concentration of A and B equal to 5 kmol/m^3 , feed flow rate equal to $1 \text{ m}^3/\text{min}$, and the reactor temperature equal to 300 K. (GATE-1991-18.ii)

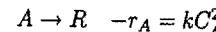
Solution:

The design equation for mixed flow reactor (CSTR) is given by,

$$\begin{aligned} \frac{V}{V_o} &= \tau = \frac{C_{Ao}X_{Af}}{-r_A} \\ \text{i.e., } \frac{2}{1} &= \frac{5X_{Af}}{1} \quad (\text{since } -r_A = k = 1) \\ \text{Therefore, } X_{Af} &= \frac{2}{5} = 0.4 \end{aligned}$$

7.3.6 Conversion in Equal Volume CSTR & PFR

A homogeneous liquid phase reaction



takes place with 50% conversion in a well mixed reactor operating isothermally. What will be the conversion if the reactor is replaced by a plug flow type of equal size—all else remaining the same? (GATE-1992-18.b)

Solution:

CSTR:

$$\begin{aligned} \frac{V}{F_{Ao}} &= \frac{X_{Af}}{-r_A} \\ &= \frac{0.5}{kC_A^2} \\ &= \frac{0.5}{kC_{Ao}^2(1 - X_A^2)} \\ \text{i.e., } \frac{V}{F_{Ao}}kC_{Ao}^2 &= 2 \end{aligned}$$

PFR:

$$\begin{aligned} \frac{V}{F_{Ao}} &= \int_0^{X_{Af}} \frac{dX_A}{-r_A} = \int_0^{X_{Af}} \frac{dX_A}{kC_A^2} = \frac{1}{kC_{Ao}^2} \int_0^{X_{Af}} \frac{dX_A}{(1 - X_A)^2} \\ \text{i.e., } \frac{V}{F_{Ao}}kC_{Ao}^2 &= \left[\frac{1}{1 - X_A} \right]_0^{X_{Af}} = \frac{X_{Af}}{1 - X_{Af}} \end{aligned}$$

Since the volume of PFR is equal to CSTR, and since C_{A0} and F_{A0} remains unchanged, $\frac{V}{F_{A0}}kC_{A0}^2 = 2$ for both the cases. (2 is obtained from the calculations for CSTR); Therefore,

$$\begin{aligned}\frac{X_{Af}}{1-X_{Af}} &= 2 \\ \text{i.e., } X_{Af} &= 2(1-X_{Af}) \\ X_{Af} &= \frac{2}{3} = 0.67\end{aligned}$$

7.3.7 Conversion in Constant Pressure Batch-Reactor

The irreversible gas-phase reaction $A \rightarrow 3B$ will be carried out isothermally. The reaction is zero order, the initial concentration of A is 2 mol/lit and the system contains 40% inert. The specific reaction gas constant is 0.1 mol/lit.min. Calculate the time required to achieve 80% conversion in a constant pressure batch reactor. (GATE-1993-24.a)

Solution:

Time required for conversion in a batch reactor is given by,

$$\begin{aligned}t &= N_{A0} \int_0^{X_{Af}} \frac{dX_A}{-r_A V} \\ \text{and, } V &= V_0(1 + \varepsilon_A X_A) \\ \text{and } \varepsilon_A &= \frac{V_{X_A=1} - V_{X_A=0}}{V_{X_A=0}} = \frac{3 \times 0.6 + 0.4 - 1}{1} = 1.2 \\ \text{i.e., } V &= V_0(1 + 1.2 X_A) \\ \text{Therefore, } t &= \frac{N_{A0}}{V_0} \int_0^{X_{Af}} \frac{dX_A}{k(1 + 1.2 X_A)} \\ &= \frac{C_{A0}}{k} \left[\frac{1}{1.2} \ln(1 + 1.2 X_A) \right]_0^{X_{Af}} \\ &= \frac{2}{0.1} \left[\frac{1}{1.2} \ln(1 + 1.2 \times 0.8) \right] \\ &= 11.22 \text{ min.}\end{aligned}$$

7.3.8 Conversion in a Batch Reactor

In a batch reactor an irreversible first order reaction $A \rightarrow R$ takes place. $C_{A0} = 0.1 \text{ mol/m}^3$. Find the conversion of the reactant after 2 seconds. (GATE-1994-6)

Solution:

For a first order irreversible reaction with rate constant k ,

$$\begin{aligned}\frac{-dC_A}{dt} &= kC_A \\ \frac{dC_A}{C_A} &= -k dt \\ \int_{C_{A0}}^{C_A} \frac{dC_A}{C_A} &= -k \int_0^t dt \\ \ln\left(\frac{C_A}{C_{A0}}\right) &= -kt \\ \ln(1 - X_A) &= -kt \quad (\text{Since } C_A = C_{A0}(1 - X_A))\end{aligned}$$

7.3.9 Order of Reaction from PFR Data

Steady state plug flow reactor (PFR) data for isothermal irreversible reaction $A \rightarrow B$ is shown in the table. Reactor space time was 10 seconds in both the cases. Other things such as feed and product density, reactor temperature etc. are same in both the cases.

	Concentration of A (kmol/m ³)	
	In Feed	In Product
Case I	1	0.5
Case II	2	0.555

If the reaction is known to be of non-zero integer order, find the reaction order and the rate constant. (GATE-1998-20.ii)

Solution:

Design equation for plug flow reactor (PFR):

$$\tau = - \int_{C_{A0}}^{C_{Af}} \frac{dC_A}{-r_A} \quad (7.23)$$

For first order reaction, $-r_A = kC_A$.
Therefore,

$$\begin{aligned}\tau &= - \frac{1}{k} \int_{C_{A0}}^{C_{Af}} \frac{dC_A}{C_A} \\ &= \frac{1}{k} \ln \frac{C_{A0}}{C_{Af}} \\ k &= \frac{1}{\tau} \ln \frac{C_{A0}}{C_{Af}}\end{aligned}$$

For case-I:

$$k = \frac{\ln(1/0.5)}{10} = 0.0693$$

For case-II:

$$k = \frac{\ln(2/0.55)}{10} = 0.1282$$

Since the two k values are not equal, the reaction is not first order.

For the second order reaction, $-r_A = kC_A^2$.

Therefore, Eqn.(7.23) becomes

$$\begin{aligned}\tau &= -\frac{1}{k} \int_{C_{A0}}^{C_{Af}} \frac{dC_A}{C_A^2} \\ &= -\frac{1}{k} \left[\frac{-1}{C_A} \right]_{C_{A0}}^{C_{Af}} \\ &= \frac{1}{k} \left(\frac{1}{C_{Af}} - \frac{1}{C_{A0}} \right) \\ k &= \frac{1}{\tau} \left(\frac{1}{C_{Af}} - \frac{1}{C_{A0}} \right)\end{aligned}$$

For case-I:

$$k = \frac{1}{10} (1/0.5 - 1/1) = 0.1$$

For case-II:

$$k = \frac{1}{10} (1/0.55 - 1/2) = 0.1318$$

Since the two k values are not equal, the reaction is not second order.

For third order reaction, $-r_A = kC_A^3$.

Therefore, Eqn.(7.23) becomes

$$\begin{aligned}\tau &= -\frac{1}{k} \int_{C_{A0}}^{C_{Af}} \frac{dC_A}{C_A^3} \\ &= -\frac{1}{k} \left[\frac{-1}{2C_A^2} \right]_{C_{A0}}^{C_{Af}} \\ &= \frac{1}{2k} \left(\frac{1}{C_{Af}^2} - \frac{1}{C_{A0}^2} \right) \\ k &= \frac{1}{\tau} \left(\frac{1}{C_{Af}^2} - \frac{1}{C_{A0}^2} \right)\end{aligned}$$

For case-I:

$$k = \frac{1}{10} (1/(0.5^2) - 1/(1^2)) = 0.3$$

For case-II:

$$k = \frac{1}{10} (1/(0.55^2) - 1/(2^2)) = 0.306$$

Since the two k values are nearly equal, the reaction is third order.

7.3.10 Conversion of Second Order Reaction in CSTR

50% conversion is obtained in a CSTR for a homogeneous, isothermal, liquid phase, irreversible second order reaction. What is the conversion if the reactor volume is five times the original - all else remaining unchanged? (GATE-1994-24)

Solution:

Design equation for CSTR:

$$\frac{V}{F_{A0}} = \frac{X_{Af}}{-r_A} \quad (7.24)$$

For second order reaction

$$-r_A = kC_A^2 = kC_{A0}^2(1 - X_{Af})^2 \quad (7.25)$$

Therefore,

$$\frac{VkC_{A0}^2}{F_{A0}} = \frac{X_{Af}}{(1 - X_{Af})^2} \quad (7.26)$$

Given: when $V = V_1$; $X_{Af} = 0.5$. Therefore

$$\frac{V_1 k C_{A0}^2}{F_{A0}} = \frac{0.5}{0.5^2} = 2$$

Therefore, when $V = 5V_1$ Eqn.(7.26) becomes

$$5 \times 2 = \frac{X_{Af}}{(1 - X_{Af})^2}$$

Solving, $X_{Af} = 0.73$.

Therefore, conversion in the reactor with five times the original volume is 73%.

7.3.11 Size of PFR for Gas-phase Reaction-I

A homogeneous gas phase decomposition reaction $4A \rightarrow B + 7S$ takes place in an isothermal plug flow reactor. The reaction rate is, $-r_A = kC_A$ with $k = 0.17 \text{ sec}^{-1}$; feed concentration of A (C_{A0}) = 0.1 mol/m^3 ; feed flow rate (F_{A0}) = 0.17 mol/sec . Determine the size of the reactor in order to achieve 50% conversion. (GATE-1994-25)

Solution:

Design equation for plug flow reactor (PFR):

$$\frac{V}{F_{A0}} = \int_0^{X_{Af}} \frac{dX_A}{-r_A} \quad (7.27)$$

Substituting the known data,

$$\begin{aligned}\frac{V}{0.17} &= \int_0^{0.5} \frac{dX_A}{0.17 C_A} \\ V &= \int_0^{0.5} \frac{dX_A}{C_A}\end{aligned}$$

Conversion X_A and concentration C_A for gas-phase reactions are related as

$$C_A = C_{A0} \frac{1 - X_A}{1 + \varepsilon_A X_A}$$

where ε_A is the fractional change in volume, given as

$$\varepsilon_A = \frac{V_{X_A=1} - V_{X_A=0}}{V_{X_A=0}}$$

For the reaction $4A \rightarrow B + 7S$

$$\varepsilon_A = \frac{8 - 4}{4} = 1$$

Therefore,

$$C_A = C_{A0} \frac{1 - X_A}{1 + X_A}$$

Hence

$$\begin{aligned} V &= \frac{1}{0.1} \int_0^{0.5} \frac{1 + X_A}{1 - X_A} dX_A \\ &= 10 \int_0^{0.5} \left(\frac{2}{1 - X_A} - 1 \right) dX_A \\ &= 10 \left[2[-\ln(1 - X_A)]_0^{0.5} - 0.5 \right] \\ &= 10[-2\ln(1 - 0.5) - 0.5] \\ &= 8.863 \text{ m}^3 \end{aligned}$$

Volume of reactor to achieve 50% conversion = 8.863 m³.

7.3.12 Size of PFR for Gas-phase Reaction-II

Acetaldehyde (*A*) decomposes to methane (*B*) and carbon monoxide (*C*) according to the irreversible gas phase reaction $A \rightarrow B + C$. 1 kmol/sec of *A* is to be decomposed at 527°C and 1 atmosphere in a plug flow reactor. The first order rate constant *k* was 0.5/sec. Calculate the volume of reactor for 40% decomposition of *A*. (GATE-1996-24)

Solution:

Design equation for plug flow reactor (PFR):

$$\frac{V}{F_{A0}} = \int_0^{X_{A_f}} \frac{dX_A}{-r_A} \quad (7.28)$$

Substituting the known data,

$$\begin{aligned} \frac{V}{1} &= \int_0^{0.4} \frac{dX_A}{0.5C_A} \\ V &= \int_0^{0.4} \frac{dX_A}{0.5C_A} \end{aligned}$$

Conversion X_A and concentration C_A for gas-phase reactions are related as

$$C_A = C_{A0} \frac{1 - X_A}{1 + \varepsilon_A X_A}$$

where ε_A is the fractional change in volume, given as

$$\varepsilon_A = \frac{V_{X_A=1} - V_{X_A=0}}{V_{X_A=0}}$$

For the reaction $A \rightarrow B + C$

$$\varepsilon_A = \frac{2 - 1}{1} = 1$$

Therefore,

$$C_A = C_{A0} \frac{1 - X_A}{1 + X_A}$$

Feed concentration of *A* i.e., C_{A0} is calculated by assuming ideal gas law as:

$$C_{A0} = \frac{P_{A0}}{RT} = \frac{1.01325 \times 10^5}{8314 \times (273 + 527)} = 15.234 \times 10^{-3} \text{ kmol/m}^3$$

Hence

$$\begin{aligned} V &= \frac{1}{0.5 \times 15.234 \times 10^{-3}} \int_0^{0.4} \frac{1 + X_A}{1 - X_A} dX_A \\ &= 131.3 \int_0^{0.4} \left(\frac{2}{1 - X_A} - 1 \right) dX_A \\ &= 131.3 \left[2[-\ln(1 - X_A)]_0^{0.4} - 0.4 \right] \\ &= 131.3[-2\ln(1 - 0.4) - 0.4] = 81.61 \text{ m}^3 \end{aligned}$$

Volume of reactor to achieve 40% conversion = 81.61 m³.

7.3.13 Reactors in Series

An irreversible homogeneous liquid phase reaction $A \rightarrow B + C$ is carried out in two isothermal flow reactors of 100 litres capacity each operating at 60°C. Find the exit conversion if both the reactors are operated in series, when

(a) both the reactors are ideal plug flow reactors.

(b) an ideal plug flow reactor is followed by an ideal back-mix reactor.

Additional Data:

Feed rate = 20 litres/min

Feed concentration = 1 gmol/litre

Rate constant = 0.5 min⁻¹

(GATE-1990-17.iii)

Solution:

(a) PFR in series:

Design equation for PFR:

$$\frac{V}{v_o} = \tau = C_{A0} \int_0^{X_{Af}} \frac{dX_A}{-r_A} \quad (7.29)$$

For first order reaction

$$-r_A = kC_A = kC_{A0}(1 - X_A)$$

Therefore, Eqn.(7.29) becomes

$$\frac{V}{v_o} = \tau = \frac{1}{k} \int_0^{X_{Af}} \frac{dX_A}{1 - X_A} \quad (7.30)$$

Two PFRs in series is equivalent to a single PFR with volume equal to the sum of the two.

Therefore,

$$V = V_1 + V_2 = 100 + 100 = 200 \text{ litre}$$

Substituting the known data in Eqn.(7.30)

$$\begin{aligned} \frac{200}{20} &= \frac{1}{0.5} \int_0^{X_{Af}} \frac{dX_A}{1 - X_A} \\ 5 &= \int_0^{X_{Af}} \frac{dX_A}{1 - X_A} \\ &= [-\ln(1 - X_A)]_0^{X_{Af}} = -\ln(1 - X_{Af}) \end{aligned}$$

Solving, $X_{Af} = 0.993$.

Exit conversion when two reactors are in plug-flow = 99.3%.

(b) PFR is followed by CSTR:

For the first reactor (PFR),

$$\begin{aligned} \frac{100}{20} &= \frac{1}{0.5} \int_0^{X_{Af}} \frac{dX_A}{1 - X_A} \\ 2.5 &= -\ln(1 - X_{Af}) \end{aligned}$$

Solving, $X_{Af} = 0.918$.

Exit concentration C_{Af} of reactants from PFR:

$$C_{Af} = C_{A0}(1 - X_{Af}) = 1 \times (1 - 0.918) = 0.082 \text{ gmol/litre}$$

This is the entry concentration C_{A0} to CSTR.

Design equation for CSTR:

$$\frac{V}{v_o} = \frac{C_{A0}X_{Af}}{-r_A} \quad (7.31)$$

For the first order reaction, Eqn.(7.31) becomes

$$\frac{V}{v_o} = \frac{X_{Af}}{k(1 - X_{Af})} \quad (7.32)$$

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Substituting the known data in Eqn.(7.32)

$$\frac{100}{20} = \frac{X_{Af}}{0.5(1 - X_{Af})}$$

Solving, $X_{Af} = 0.714$.

Exit concentration C_{Af} of reactants from CSTR:

$$C_{Af} = C_{A0}(1 - X_{Af}) = 0.082 \times (1 - 0.714) = 0.02345 \text{ gmol/litre}$$

$$\text{Total conversion for the reactor schemes} = \frac{1 - 0.02345}{1} \times 100 = 97.7\%$$

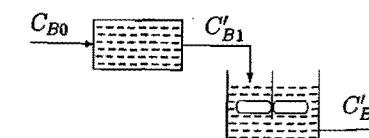
7.3.14 Reactors Arrangement

The constant density isothermal elementary reaction $A + B \rightarrow C + D$ is conducted in a set-up consisting of a plug flow reactor followed by a mixed reactor. A is in excess and hence the reaction may be considered first order in B. Does reversing the order of the two units increase the production? Justify your answer. (GATE-1996-8)

Solution:

As the volume of reactors are not given, it is assumed that the reactors are of equal volumes with τ (residence time) equal.

Case I: PFR followed by a CSTR.



PFR:

$$C'_B1 = C_B0 \exp(-k\tau)$$

Since

$$\frac{-dC_B}{dt} = kC_B$$

and

$$-\ln \frac{C_B}{C_{B0}} = k\tau$$

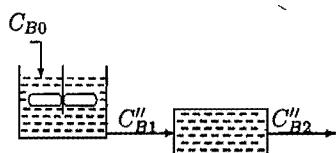
CSTR:

$$\tau = \frac{C'_B1 - C'_B2}{-r_B} = \frac{C'_B1 - C'_B2}{kC'_B2}$$

Therefore

$$\begin{aligned} C'_B2 &= \frac{C'_B1}{1 + k\tau} \\ &= \frac{C_B0 \exp(-k\tau)}{1 + k\tau} \end{aligned}$$

Case II: CSTR followed by PFR.



$$C''_{B1} = \frac{C_{B0}}{1 + k\tau}$$

and

$$C''_{B2} = C'_{B1} \exp(-k\tau)$$

Therefore

$$C''_{B2} = \frac{C_{B0} \exp(-k\tau)}{1 + k\tau}$$

Both arrangements give the same outlet concentrations for *B*, and hence no increase in production for particular feed rate.

7.3.15 Increase in Production Rate by PFR

At present a first order, isothermal, liquid phase reaction is being conducted in a cascade of two equal sized mixed flow reactors to obtain 95% conversion. If this system is replaced by a plug flow reactor of the same total volume, what is the increase in the production rate for the same conversion? (GATE-1993-24.b)

Solution:

CSTR

$$\frac{C_o}{C_N} = (1 + k\tau_i)^N = \frac{1}{1 - X_N}$$

$$\text{for 2 reactors in series, } (1 + k\tau_i)^2 = \frac{1}{1 - 0.95}$$

$$\text{i.e., } (1 + k\tau_i)^2 = 20$$

$$\text{solving the above equation, } \tau_i = \frac{3.47}{k}$$

Since there are two reactors in series, total $\tau = 2 \times \tau_i = 6.94/k$
For CSTR, the design equation is

$$\frac{VC_{A0}}{F_{A0}} = \tau$$

$$\text{Therefore, } F_{A0} = \frac{VkC_{A0}}{6.94} \quad (7.33)$$

PFR

$$\begin{aligned} \frac{V}{F_{A0}} &= \int_0^{X_A} \frac{dX_A}{-r_A} \\ &= \int_0^{0.95} \frac{dX_A}{kC_{A0}(1 - X_A)} \\ &= \frac{1}{kC_{A0}} [-\ln(1 - X_A)]_0^{0.95} \\ &= \frac{1}{kC_{A0}} [\ln 0.05 - \ln 1] \\ &= \frac{3}{kC_{A0}} \end{aligned}$$

Therefore,

$$F_{A0} = \frac{VkC_{A0}}{3} \quad (7.34)$$

Dividing Eqn.(7.34) by Eqn.(7.33)

$$\frac{F_{A0}\text{PFR}}{F_{A0}\text{CSTR}} = 2.315$$

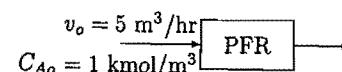
Therefore, production rate increases by 131.5% in PFR.

7.3.16 Actual Conversion in the Reactor

An isothermal plug flow reactor is designed to give 80% conversion of *A* for a second-order liquid phase reaction $A \rightarrow B$. Pure *A* at concentration 1 kmol/m³ is fed to the reactor at a flow rate of 5 m³/hr. The rate constant for the reaction at a specified operating temperature is 0.5 m³/kmol.hr. When the reactor is actually operated based on this design, it was found that 30% of the initial reactor behaved as a well-mixed reactor while the remaining behaved as a plug flow reactor. Calculate the conversion obtained in such a reactor. (GATE-1999-17)

Solution:

From the units of rate constant, it is found that the order of the reaction is 2.



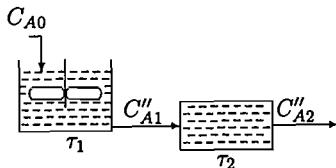
Design equation for plug flow reactor (PFR) is

$$\frac{V}{F_{A0}} = \frac{\tau}{C_{A0}} = \int \frac{dX_A}{-r_A}$$

since $F_{A0} = C_{A0}v_0$, and $-r_A = kC_A^2 = kC_{A0}^2(1 - X_A)^2$

$$\begin{aligned}\frac{V}{C_{A0}v_0} &= \int_0^{X_A} \frac{dX_A}{kC_{A0}^2(1 - X_A)^2} \\ V &= \frac{v_0 C_{A0}}{kC_{A0}^2} \int_0^{X_A} \frac{dX_A}{(1 - X_A)^2} \\ &= \frac{v_0}{kC_{A0}} \left[\frac{1}{1 - X_A} \right]_0^{X_A} \\ &= \frac{5}{(0.5)(1)} \left[\frac{1}{1 - X_A} - 1 \right] = 10 \left[\frac{X_A}{1 - X_A} \right] \\ &= 10 \left[\frac{0.8}{1 - 0.8} \right] = 40 \text{ m}^3\end{aligned}\quad (7.35)$$

Actual Configuration within the Reactor:



Volume of CSTR (V_1) = $0.3 \times 40 = 12 \text{ m}^3$

Volume of PFR (V_2) = $0.7 \times 40 = 28 \text{ m}^3$.

Conversion in CSTR:

$$\frac{V_1}{F_{A0}} = \frac{C_{A0}X_{A1}}{-r_A} = \frac{X_{A1}}{-r_A}$$

Rearranging,

$$\begin{aligned}\tau_1 &= \frac{C_{A0}X_{A1}}{-r_A} = \frac{C_{A0}X_{A1}}{kC_{A0}^2(1 - X_A)^2} \\ \tau_1 k C_{A0} &= \frac{X_{A1}}{(1 - X_{A1})^2} \\ (12/5)(0.5)(1) &= \frac{X_{A1}}{(1 - X_{A1})^2} \\ 1.2 &= \frac{X_{A1}}{(1 - X_{A1})^2}\end{aligned}$$

Solving, $X_{A1} = 0.413$. And $C_{A1} = C_{A0}(1 - X_{A1}) = 1 \times (1 - 0.413) = 0.587 \text{ kmol/m}^3$.

Conversion in PFR:

From Eqn.(7.35)

$$\frac{V_2 k C_{A1}}{v_0} = \frac{X_{A2}}{1 - X_{A2}}$$

Substituting the known data,

$$\begin{aligned}\frac{(28)(0.5)(0.587)}{5} &= \frac{X_{A2}}{1 - X_{A2}} \\ 1.6436 &= \frac{X_{A2}}{1 - X_{A2}}\end{aligned}$$

Solving, $X_{A2} = 0.622$. And $C_{A2} = C_{A1}(1 - X_{A2}) = 0.587 \times (1 - 0.622) = 0.222 \text{ kmol/m}^3$.

Therefore, conversion in the reactor = $\frac{1 - 0.222}{1} \times 100 = 77.8\%$

7.3.17 Expected Conversion in the Reactor System

The elementary, second order, liquid phase reaction $A + B \rightarrow C + D$ is conducted in an isothermal plug flow reactor of 1 m^3 capacity. The inlet volumetric flow rate is $10 \text{ m}^3/\text{h}$ and $C_{A0} = C_{B0} = 2 \text{ kmol/m}^3$. At these conditions, conversion of A is 50%. Now, if a stirred tank reactor of 2 m^3 capacity is installed in series, upstream of the plug flow reactor, then what conversion can be expected in the new system of reactors? (GATE-2000-16)

Solution:

Given: $A + B \rightarrow C + D$

Since $C_{A0} = C_{B0}$, at any instant of time $C_A = C_B$.

For a second order reaction, $-r_A = kC_A C_B = kC_A^2$.

For PFR:

$$\begin{aligned}\frac{V}{F_{A0}} &= \frac{\tau}{C_{A0}} = \int \frac{dX_A}{-r_A} = \int \frac{dX_A}{kC_{A0}^2(1 - X_A)^2} \\ &= \frac{1}{kC_{A0}^2} \int \frac{dX_A}{(1 - X_A)^2} \\ \tau k C_{A0} &= \int \frac{dX_A}{(1 - X_A)^2} = \int_0^{X_A} \frac{-d(1 - X_A)}{(1 - X_A)^2} \\ &= \left[\frac{1}{1 - X_A} \right]_0^{X_A} = \frac{1}{1 - X_A} - 1 = \frac{X_A}{1 - X_A}\end{aligned}$$

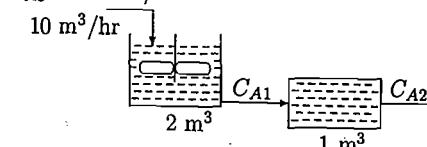
Given: $X_A = 0.5$; $\tau = 1/10 = 0.1 \text{ hr}$; $C_{A0} = 2 \text{ kmol/m}^3$. Therefore

$$0.1 \times k \times 2 = \frac{0.5}{1 - 0.5}$$

Solving, $k = 5 \text{ m}^2/(\text{kmol.hr})$.

New Reactor Scheme

$$C_{A0} = 2 \text{ kmol/m}^3$$



For CSTR:

$$\frac{V}{F_{A0}} = \frac{\tau}{C_{A0}} = \frac{X_A}{-r_A} = \frac{X_A}{kC_{A0}^2(1-X_A)^2}$$

Rearranging and substituting the known data,

$$\begin{aligned}\tau k C_{A0} &= \frac{X_A}{(1-X_A)^2} \\ (0.2)(5)(2) &= \frac{X_A}{(1-X_A)^2} \\ 2 &= \frac{X_A}{(1-X_A)^2}\end{aligned}$$

Solving, $X_A = 0.5$. Therefore $C_{A1} = C_{A0}(1-X_A) = 2(1-0.5) = 1 \text{ kmol/m}^3$.

For PFR:

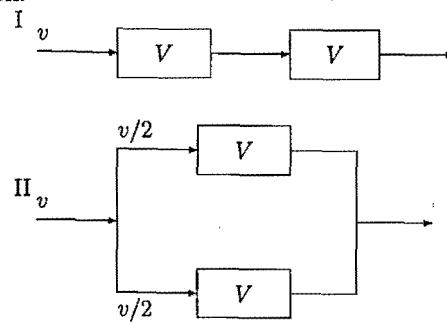
$$\begin{aligned}\tau k C_{A1} &= \frac{X_A}{1-X_A} \\ (0.1)(5)(1) &= \frac{X_A}{1-X_A}\end{aligned}$$

Solving, $X_A = 0.333$. Therefore $C_{A2} = C_{A1}(1-X_A) = 1(1-0.333) = 0.667$, and conversion in the new reactor scheme = $100 \times (2 - 0.667)/2 = 66.7\%$

7.3.18 Reactors Arrangement for Higher Conversion

A reaction $A \rightarrow B$ of unknown kinetics is to be carried out isothermally using two plug flow reactors of equal volume arranged either in series or in parallel. In the parallel arrangement, the feed is split equally between the two reactors. The total feed rate and the inlet concentration of the reactant respectively are the same for both the arrangements. Which arrangement does give higher overall conversion? Why? (GATE-1989-17.i)

Solution:



REACTION ENGINEERING

Conversion in PFR is given by

$$\frac{\tau}{C_{A0}} = \int_{X_{in}}^{X_{out}} \frac{dX_A}{-r_A}$$

where $\tau = \frac{\text{volume of reactor}}{\text{volumetric flow rate}} = \frac{V}{v}$

Case I:

$$\frac{\tau}{C_{A0}} = \frac{2V}{vC_{A0}} = \int_{X_{in}}^{X_{out}} \frac{dX_A}{-r_A} \quad (7.36)$$

Case II:

$$\frac{\tau}{C_{A0}} = \frac{V}{v/2 C_{A0}} = \int_{X_{in}}^{X_{out}} \frac{dX_A}{-r_A} \quad (7.37)$$

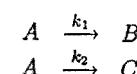
On comparing Eqns.(7.36) and (7.37) it is observed that conversion is same in both the cases as τ is same in both the cases.

7.3.19 Rate Constants of Parallel Reactions

Two parallel first order reactions $A \xrightarrow{k_1} B$ and $A \xrightarrow{k_2} C$ are taking place in liquid phase in a well mixed batch reactor. After 60 min of operation, 80% of A has reacted awhile 2 moles of B per mole of C was detected in the reactor. Calculate the rate constants k_1 and k_2 for the two reactions. Assume that no B and C were initially present in the reactor. (GATE-1999-16)

Solution:

For the reactions



rates of formation of B and C are denoted as r_B and r_C are

$$r_B = \frac{dC_B}{dt} = k_1 C_A \quad (7.38)$$

$$r_C = \frac{dC_C}{dt} = k_2 C_A \quad (7.39)$$

From Eqns.(7.38) and (7.39)

$$\frac{dC_B}{dC_C} = \frac{k_1}{k_2} \quad (7.40)$$

Given: when conversion of A i.e., $X_A = 0.8$:

$$C_B = 2C_C$$

differentiating,

$$\frac{dC_B}{dC_C} = 2 \quad (7.41)$$

From Eqns.(7.40) and (7.41)

$$\frac{k_1}{k_2} = 2$$

Rate of conversion of A is given by

$$\begin{aligned}-r_A &= k_1 C_A + k_2 C_A \\ &= k_1 C_A + 0.5 k_1 C_A \\ &= 1.5 k_1 C_{A0}(1 - X_A)\end{aligned}$$

Design equation for Batch reactor:

$$t = C_{A0} \int_0^{X_A f} \frac{dX_A}{-r_A}$$

Substituting for r_A

$$t = \frac{1}{1.5 k_1} \int_0^{X_A f} \frac{dX_A}{1 - X_A}$$

Integrating,

$$t = \frac{1}{1.5 k_1} [-\ln(1 - X_A)]_0^{X_A f} = \frac{-1}{1.5 k_1} \ln(1 - X_A)$$

Given: when $t = 60$ min; $X_A = 0.8$. Therefore

$$60 = \frac{-1}{1.5 k_1} \ln(1 - 0.8)$$

Solving, $k_1 = 0.0179 \text{ min}^{-1}$. And, $k_2 = k_1/2 = 0.00895 \text{ min}^{-1}$.

7.3.20 Catalyst Effectiveness Factor

The Thiele modulus for a first order isothermal reaction for a flat plate geometry catalyst is found to be 2. Calculate the catalyst effectiveness factor. (GATE-1994-23a)

Solution:

$$\begin{aligned}\text{Catalyst effectiveness factor } (\eta) &= \frac{1}{\phi_s} \left[\frac{1}{\tanh(3\phi_s)} - \frac{1}{3\phi_s} \right] \\ &= \frac{1}{2} \left[\frac{1}{\tanh(3 \times 2)} - \frac{1}{3 \times 2} \right] \\ &= \frac{1}{2} [1 - 1/6] \\ &= 0.417\end{aligned}$$

7.3.21 Rate Controlling Step

A particular metal reacts with a certain liquid and the product passes into solution. Three non-porous solid spheres of same metal and of diameters 10, 20 and 30 mm respectively were placed in a very large liquid pool of reactive liquid at the same time. After an hour, it was found that the pool had only two spheres of diameter 10 and 20 mm, respectively. After another hour, the pool had only one sphere of diameter 10 mm. This sphere also disappeared after another hour. Explain these observations through appropriate derivation using a more likely rate controlling step out of the following two.

(a) Film mass transfer

(b) Surface reaction

(GATE-1998-21)

Solution:

At t=0: $D_1 = 10 \text{ mm}$, $D_2 = 20 \text{ mm}$, $D_3 = 30 \text{ mm}$

After 1 hr: $D'_1 = 0 \text{ mm}$, $D'_2 = 10 \text{ mm}$, $D'_3 = 20 \text{ mm}$

After 2 hrs: $D''_2 = 0 \text{ mm}$, $D''_3 = 10 \text{ mm}$

After 3 hrs: $D'''_3 = 0 \text{ mm}$

Let rate controlling step be surface reaction. Then

$$\frac{-dN_A}{dt} = \pi D^2 k C_{AS}$$

where D = instantaneous diameter of the particle

C_{AS} = concentration in the surface of the metal (as it is non-porous)

C_{AS} = constant

k = reaction rate constant based on unit surface area (I order)

$$\begin{aligned}N_A &= \text{volume} \times \text{molar density} \\ &= \frac{\pi D^3}{6} \rho_m\end{aligned}$$

Therefore

$$\begin{aligned}\frac{-\pi}{6} \rho_m \frac{dD^3}{dt} &= \pi D^2 k C_{AS} \\ \frac{-\pi}{6} \rho_m 3D^2 \frac{dD}{dt} &= \pi D^2 k C_{AS} \\ \frac{-dD}{dt} &= \frac{2k C_{AS}}{\rho_m}\end{aligned}$$

Integrating,

$$-\int dD = \frac{2k C_{AS}}{\rho_m} \int dt$$

For particle 1,

$$(10) \text{mm} = \frac{2kC_{AS}}{\rho_m} (1) \text{ hr}$$

$$\Rightarrow \frac{2kC_{AS}}{\rho_m} = 10 \text{ mm/hr}$$

For particle 2,

$$(20) \text{mm} = \frac{2kC_{AS}}{\rho_m} (2) \text{ hr}$$

$$\Rightarrow \frac{2kC_{AS}}{\rho_m} = 10 \text{ mm/hr}$$

For particle 3,

$$(30) \text{mm} = \frac{2kC_{AS}}{\rho_m} (3) \text{ hr}$$

$$\Rightarrow \frac{2kC_{AS}}{\rho_m} = 10 \text{ mm/hr}$$

Therefore surface reaction is rate controlling.

7.3.22 Expression for Catalytic Reaction

A gaseous reactant diffuses through a gas film and reacts on the surface of a non-porous spherical catalyst particle. The rate of surface reaction is $k_1 C_s$, where C_s is the reactant concentration on the catalyst surface. The reaction rate constant (k_1) = $0.83 \times 10^{-4} \text{ m/s}$ and the gas film mass transfer coefficient (k_m) = $1.66 \times 10^{-4} \text{ m/s}$. Derive the reaction rate expression in terms of bulk gas phase concentration (C_o). (GATE-1994-23b)

Solution:

$$\text{Rate of surface reaction} = k_1 C_s \quad (7.42)$$

$$\text{Rate of diffusion} = k_m(C_o - C_s) \quad (7.43)$$

At steady state rate of diffusion should equal the rate of surface reaction. Therefore,

$$k_1 C_s = k_m(C_o - C_s)$$

Rearranging,

$$C_s = \frac{k_m C_o}{k_1 + k_m}$$

Substituting this in Eqn.(7.42),

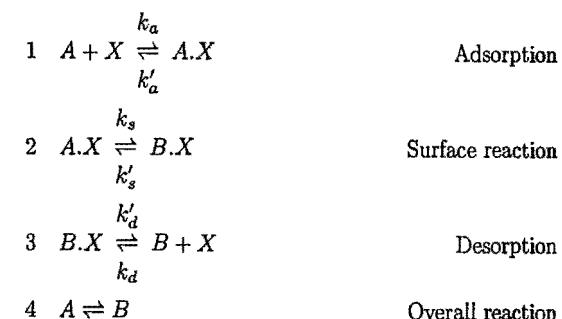
$$\begin{aligned} \text{Rate of reaction} &= k_1 k_m \frac{C_o}{k_1 + k_m} = \frac{C_o}{1/k_m + 1/k_1} \\ &= \frac{C_o}{\frac{1}{1.66 \times 10^{-4}} + \frac{1}{0.83 \times 10^{-4}}} = 5.53 \times 10^{-5} \text{ m/sec} \end{aligned}$$

7.3.23 Expression for Rate of Catalytic Reaction

Cis-2-butene (*A*) isomerizes to trans-2-butene (*B*) on a solid catalyst under isothermal conditions according to the reaction $A \rightleftharpoons B$. Assuming desorption of *B* from the surface of the catalyst to be rate controlling, derive an expression for the intrinsic rate of reaction per unit mass of catalyst. Sketch rate of reaction vs total pressure (at constant composition) for the above mechanism. (GATE-1996-25)

Solution:

Sequence of steps:



Let

$$\begin{aligned} \bar{C}_V &= \text{concentration of uncovered surface on the catalyst} \\ \bar{C}_A &= \text{concentration of adsorbed } A \\ \bar{C}_B &= \text{concentration of adsorbed } B \\ C_A &= \text{concentration of } A \text{ in the gas phase} \\ C_B &= \text{concentration of } B \text{ in the gas phase} \end{aligned}$$

1) Rate of adsorption:

$$r_{\text{ads}} = k_a C_A (\bar{C}_V - k'_a \bar{C}_A) = k_a \left[C_A \bar{C}_V - \frac{1}{K_A} \bar{C}_A \right]$$

where $K_A = k_a/k'_a$.

Simplifying the above expression,

$$r_{\text{ads}} = k_a \left[C_A \bar{C}_V - \frac{1}{K_A} \bar{C}_A \right]$$

2) Surface reaction:

$$r_{\text{surf}} = k_s \bar{C}_A - k'_s \bar{C}_B = k_s \left[\bar{C}_A - \frac{1}{K_S} \bar{C}_B \right]$$

where $K_S = k_s/k'_s$.

3) Desorption:

$$r_{\text{des}} = k'_d \bar{C}_B - k_d C_B \bar{C}_V = -k_d \left[C_B \bar{C}_V - \frac{1}{K_D} \bar{C}_B \right]$$

where $\bar{K}_D = k_d/k'_d$.

If surface reaction is controlling, both adsorption and surface reaction occur at equilibrium.
So,

$$C_A \bar{C}_V = \frac{1}{K_A} \bar{C}_A$$

and

$$\bar{C}_A = \frac{1}{K_S} \bar{C}_B$$

Therefore

$$\begin{aligned} r &= -k_d \left[C_B \bar{C}_V - \frac{1}{K_D} \bar{C}_B \right] \\ &= k_d \left[\frac{1}{K_D} \bar{C}_B - C_B \bar{C}_V \right] \\ &= k_d \left[\frac{K_S}{K_B} \bar{C}_A - C_B \bar{C}_V \right] \\ &= k_d \left[\frac{K_S K_A}{K_D} C_A \bar{C}_V - C_B \bar{C}_V \right] \\ &= k_d \bar{C}_V \left[\frac{K_S K_A}{K_D} C_A - C_B \right] \\ &= k_d \bar{C}_V [K C_A - C_B] \quad (\text{where } K = K_S K_A / K_D) \\ &= k_d \bar{C}_V K \left[C_A - \frac{1}{K} C_B \right] \end{aligned}$$

Concentration of vacant sites \bar{C}_V is given by

$$\bar{C}_V = \frac{1}{1 + K_{aA} + K_{aB}}$$

And

$$C_A = \frac{P_A}{RT}, \quad C_B = \frac{P_B}{RT}$$

Also

$$P_A = P_B, \quad \text{and} \quad P_t = P_A + P_B$$

Therefore

$$\begin{aligned} r &= \frac{k_d}{1 + K_{aA} + K_{aB}} K \left(\frac{1}{2} \frac{P_t}{RT} - \frac{1}{K} \frac{1}{2} \frac{P_t}{RT} \right) \\ &= \frac{k_d}{1 + (K_{aA} + K_{aB}) \frac{1}{2} P_t RT} K \left(\frac{1}{2} P_t RT - \frac{1}{K} \frac{1}{2} \frac{P_t}{RT} \right) \\ &= \frac{a' P_t}{1 + b' P_t} - \frac{c' P_t}{1 + b' P_t} = \frac{d' P_t}{1 + b' P_t} \end{aligned}$$

where a', b', c' are constants.

Chapter 8

Process Control

8.1 Instant Notes

8.1.1 Laplace Transforms

- Laplace Transforms of simple functions

$$\text{Unit step} \quad \frac{1}{s}$$

$$\text{Unit Ramp} \quad \frac{1}{s^2}$$

$$\sin(\omega t) \quad \frac{\omega}{s^2 + \omega^2}$$

- Initial value theorem $\lim_{t \rightarrow 0} f(t) = \lim_{s \rightarrow \infty} s \tilde{f}(s)$

- Final value theorem $\lim_{t \rightarrow \infty} f(t) = \lim_{s \rightarrow 0} s \tilde{f}(s)$

8.1.2 Qualitative Response of Systems

- Poles of Transfer functions

- Distinct poles on the negative real axis produce terms that decay to zero with time, while real positive poles makes response of the system to grow toward infinity with time.
- A real, multiple pole gives rise to terms which either grow to infinity, if the pole is positive or zero, or decay to zero if the pole is negative.
- Complex conjugate poles:

* Complex poles always appear in conjugate pairs (+ & -) and never alone.

- * A pair of complex conjugate poles give rise to oscillatory behaviour, whose amplitude may grow continuously if the real part of the complex poles is positive, decay to zero if it is negative, or remains unchanged, if the real part of the poles is zero.

8.1.3 Open-loop Response of Dynamic Systems

- Dynamic behaviour of First-order systems

- First order lag, linear lag, or exponential transfer lag system:

$$* G(s) = \frac{K_p}{\tau_p s + 1}$$

where K_p = steady-state gain, or static gain, or simply, gain
 $=$
 τ_p = time constant of the process

- * Time constant = Capacitance \times Resistance

- Pure capacitive process

$$* G(s) = \frac{K_p}{s}$$

- Dynamic response of a pure capacitive process (pure integrator)

- A pure capacitive process will cause series control problems, because it cannot balance itself.

- Dynamic response of first order lag system

- A first order process is self regulating. Unlike a pure capacitive process, it reaches a new steady state.
- The time constant τ_p of a process is a measure of the time necessary for the process to adjust to a change in its input. The value of the response $y(t)$ reaches 63.2% of its final value when the time elapsed is equal to one time constant, τ_p .
- The ultimate value of the response (i.e., its value at the new steady state) is equal to K_p for a unit step change in input.
- The larger the static gain of a process, the larger is the steady state value of its output.

- Dynamic behaviour of second order systems

- (i) Overdamped response, $\zeta > 1$

- Response is sluggish.
- Overdamped are the responses of multicapacity processes, which result from the combination of first order systems in series.

- (ii) Critically damped response, $\zeta = 1$

- * Critical damping approaches its ultimate value faster than does an over-damped system.

- (iii) Underdamped response, $\zeta < 1$

- Although the response is initially faster and reaches its ultimate value quickly, it does not stay there, but it starts oscillating with progressively decreasing amplitude.

- The oscillatory behaviour becomes more pronounced with smaller values of the damping factor, ζ .

- Almost all the underdamped responses in a chemical plant are caused by the interaction of the controllers, with the process unit they control.

- Characteristics of an underdamped response

$$(i) \text{Overshoot} = \exp\left(\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}\right)$$

$$(ii) \text{Decay ratio} = (\text{Overshoot})^2$$

$$(iii) \text{Period of oscillation (time elapsed between two successive peaks)}$$

$$T = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}}$$

- Non-interacting capacities always result in an overdamped or critically damped second-order system and never in an underdamped system.

- Interacting capacities are more sluggish than non-interacting. Response of interacting capacities is always overdamped.

- Processes with dead-time are difficult to control because the output does not contain information about current events.

- When a system posses an inverse response, its transfer function has a positive zero. System with inverse response are particularly difficult to control and requires special attention.

- Inherently second-order processes

- Such a process can exhibit underdamped behaviour, and consequently it cannot be decomposed into two first-order systems.

- They are characterized by their inertia to motion.

- Examples: simple manometers, externally mounted level-indicators, pneumatic valves.

8.1.4 Dynamics of Controllers

- Types of controllers

- Proportional controller

- * The *proportional band PB* characterizes the range over which the error must change in order to drive the actuating signal of the controller over its full range. Usually $1 \leq PB \leq 500$.

$$* PB = \frac{100}{K_c}$$

- * The larger the gain K_c , or equivalently, the smaller the proportional band, the higher the sensitivity of the controller's actuating signal to deviations ϵ .
- * A proportional controller with a very narrow proportional band (large gain) is similar to a *on-off* controller.

- Proportional-Integral controller

- * Can eliminate even small errors.

- Derivative action: (Anticipatory control)

- * For a response with constant non-zero error, it gives no control action.

- The dominant dynamic feature of composition analyzers is the time-delay (dead time) in their response, which can be quite large.

- Dynamic behaviour of pneumatic transmission line can be neglected.

- Pneumatic valve:

- The most commercial valves move from fully open to fully closed as the air pressure at the top of the diaphragm changes from 3 to 15 psig.

- Types of control problems

- Servo problem: The disturbances does not change, while the setpoint undergoes a change.

- Regulator problem: The setpoint remains the same, while the load changes.

- Comparison of Controller Modes

<u>Mode</u>	<u>Advantages</u>	<u>Disadvantages</u>
On-off	Simple, inexpensive	Constant cycling
Proportional	Does not add lag	Almost, always has offset
Integral	Eliminates offset	Adds time-lag to system
Derivative	Speeds up response	Responds to noise

- Effect of proportional control on the response of a controlled process

- (i) Closed-loop response of first-order system:

- * It remains first-order with respect to load and setpoint changes.

- * Time constant has been reduced, which means the closed-loop response becomes faster than the open-loop response, to changes in setpoint or load.

- (ii) Closed-loop response of second-order system:

- * It remains second order.

- * Static gain decreases.

- * Both the *natural period* and *damping factor* decrease. This implies that an overdamped process with proportional control and appropriate value of K_c , may become underdamped (oscillatory).

- Liquid-level can be effectively controlled with proportional control.

- Effect of integral control action

- It increases the dynamics for the closed-loop response (By increasing the order, its response become more sluggish).

- Integral control action eliminates any offset.

- We can improve the speed of response of the closed-loop response at the expense of higher deviations and longer oscillations.

- Increasing the integral control action (i.e., increasing K_c and decreasing τ_I) makes the response of the closed-loop system more sensitive.

- Effect of Derivative control action

- The derivative control does not change the order of the response.

- The response of the controlled process is slower than the original first order process.

- Pure Integrating Processes: Processes having the term $1/s$ in their transfer function, when they are controlled with proportional controller, do not exhibit offset for setpoint changes but they do for sustained load changes (eg. step changes).

8.1.5 Stability Analysis of Feedback Systems

- A dynamic system is said to be stable if for every bounded input it produces a bounded output, regardless of its initial state. *Bounded* is an input that always remains between an upper and a lower limit (eg: sinusoidal, step, but not the ramp).

- If the transfer function of a dynamic system has even one pole with positive real root, the system is unstable.

- Frequency response analysis of linear processes

- When a linear system is subjected to a sinusoidal input, its ultimate response (after a long time) is also a sustained sinusoidal wave.

- Frequency response of a pure capacitive process

- The ultimate sinusoidal response of the system lags behind the input wave by 90° .

• Frequency response of feedback controllers

- Proportional controller: phase shift or lag (ϕ) = 0.
- PI controller: $\phi < 0$.
- PD controller: $\phi > 0$.

8.2 Objective Type Questions

1. A control system has the following transfer function,

$$F(s) = \frac{(s-1)(s+1)}{s(s-2)(s+4)}$$

The initial value of the corresponding time function is (GATE-1999-2.16)

- (a) 1 (b) 1/8 (c) 7/8 (d) -1

Answer: (a) From initial value theorem

$$\lim_{t \rightarrow 0} F(t) = \lim_{s \rightarrow \infty} sF(s) = \frac{(s-1)(s+1)}{(s-2)(s+4)} = \frac{s^2-1}{s^2+2s-8}$$

By L-Hospital rule

$$\lim_{s \rightarrow \infty} sF(s) = \frac{(s-1)(s+1)}{(s-2)(s+4)} = \frac{s^2-1}{s^2+2s-8} = \frac{2s}{2s+2} = \frac{2}{2} = 1$$

2. The initial value ($t = 0^+$) of the unit step response of the transfer function $[(s+1)/(2s+1)]$ is (GATE-2000-2.22)

- (a) 0 (b) 1/2 (c) 1 (d) 2

Answer: (b) $\lim_{t \rightarrow 0} F(t) = \lim_{s \rightarrow \infty} sF(s) = \lim_{s \rightarrow \infty} \frac{1}{s} \frac{s+1}{2s+1}$

$$\text{By L-Hospital rule, } \lim_{s \rightarrow \infty} \frac{s+1}{2s+1} = 1/2$$

3. The transfer function of a system is given by, $Y/X = 1/(s^2 + 5s + 6)$. The roots of the characteristic equation are located, (GATE-1988-8.c.ii)

- (a) to the left of the imaginary axis and on the real axis
- (b) on the imaginary axis
- (c) right of the imaginary axis and on the real axis
- (d) at the origin

$$\text{Answer: (a) } s = \frac{-5 \pm \sqrt{5^2 - 4 \times 1 \times 6}}{2 \times 1} = \frac{-5 \pm 1}{2} = -3, -2$$

4. For an input forcing function, $X(t) = 2t^2$, the Laplace transform of this function is (GATE-1997-1.26)

(a) $\frac{2}{s^2}$ (b) $\frac{4}{s^2}$ (c) $\frac{2}{s^3}$ (d) $\frac{4}{s^3}$

Answer: (d) $\mathcal{L}(t^n) = \frac{n!}{s^{n+1}}$

5. A process is initially at steady state with its output $y = 1$ for an input $u = 1$. The input is suddenly changed to 2 at $t = 0$. The output response is $y(t) = 1 + 2t$. The transfer function of the process is (GATE-2001-1.18)

(a) $\frac{2}{s}$ (b) $1 + \frac{2}{s^2}$ (c) $1 + \frac{2}{s}$ (d) $\frac{1}{s} \left(1 + \frac{2}{s}\right)$

Answer: (c) $G_p(s) = Y(s)/U(s)$. Given: $y(t) = 1 + 2t$ and $u(t) = 2 - 1 = 1$. $Y(s) = \mathcal{L}[y(t)] = 1/s + 2/s^2$; and $U(s) = \mathcal{L}[u(t)] = 1/s$. Therefore

$$G_p(s) = \frac{1/s + 2/s^2}{1/s} = 1 + 2/s$$

6. The transfer function of a pure dead time system with dead time τ_d is (GATE-1995-1.v)

(a) $\frac{1}{\tau_d s + 1}$ (b) $\tau_d s + 1$ (c) $e^{-\tau_d s}$ (d) $e^{\tau_d s}$

Answer: (c)

7. A rectangular tank is filled with a valve at the bottom and is used for storing a liquid. The area of cross-section of the tank is 10 m^2 and the flow resistance of the valve (assumed constant) is 0.1 s/m^2 . The time constant of the tank will be: (GATE-1988-8.c.i)

- (a) 1 (b) 100 (c) 10.1 (d) 9.9

Answer: (a) $\tau = \text{storage capacitance} \times \text{resistance to flow} = A \times R = 10 \times 0.1 = 1$

8. A certain thermocouple has a specific time constant of 2 sec. If the process temperature changes abruptly from 800 to 900°C , the temperature reading in an indicator attached to the thermocouple after 6 sec will be approximately, (GATE-1991-9.i)

- (a) 860°C (b) 900°C (c) 890°C (d) 895°C

Answer: (d) $800 + (900 - 800) \times (1 - e^{-t/\tau}) = 895^\circ\text{C}$.

9. The open loop transfer function of a control system is $KR/(1+Ts)$. This represents (GATE-1996-1.24)

- (a) a first order system
- (b) dead time system
- (c) a first order timelag
- (d) a second order system

Answer: (c)

10. The transfer function for a first-order process with time delay is (GATE-1997-1.24)

(a) $\frac{e^{\tau_d s}}{(\tau s + 1)}$
 (c) $\frac{1}{(\tau s + 1)(\tau_d s + 1)}$

(b) $\frac{e^{-\tau_d s}}{(\tau s + 1)}$
 (d) $\frac{\tau_d s}{(\tau s + 1)}$

Answer: (b)

11. The time constant of a first order system with resistance R and capacitance C is (GATE-1995-1.w)

(a) $R + C$ (b) $R - C$ (c) RC (d) $1/(RC)$

Answer: (c)

12. The second order system with the transfer function $4/(s^2 + 2s + 4)$ has a damping ratio of (GATE-1998-2.22)

(a) 2.0 (b) 0.5 (c) 1.0 (d) 4.0

Answer: (b) $s^2 + 2s + 4 = \tau^2 s^2 + 2\zeta\tau + 1$
 $\tau^2 = 1/4; \tau = 1/2; 2\zeta\tau = 2/4 = 1/2$
 $\zeta = 2/(2 \times 2) = 1/2 = 0.5$

13. A typical example of a physical system with under-damped characteristics is a (GATE-1999-1.28)

- (a) U-tube manometer
 (b) spring-loaded diaphragm valve
 (c) CSTR with first-order reaction
 (d) thermocouple kept immersed in a liquid-filled thermowell

Answer: (a)

14. The unit step response of the transfer function $1/(s^2 + 2s + 3)$ (GATE-2000-1.26)

- (a) has a non-zero slope at the origin
 (b) has a damped oscillatory characteristic
 (c) is overdamped
 (d) is unstable

Answer: (b) Roots of $(s^2 + 2s + 3) = 0$

$$s = \frac{-2 \pm \sqrt{4 - 12}}{2}$$

Since the roots are complex numbers with negative real parts, the system is stable.

15. When a bare thermocouple is covered by a protective sheath, the response becomes: (GATE-1992-9.a)

- (a) Faster and oscillatory (b) Faster and non-oscillatory
 (c) Slower and oscillatory (d) Slower and non-oscillatory

Answer: (d) Order of the system increases (because of the two first orders in series), and damping factor increases.

16. The transfer function of a process is $\frac{1}{16s^2 + 8s + 4}$. If a step change is introduced into the system, then the response will be (GATE-1997-2.19)

- (a) underdamped (b) critically damped
 (c) overdamped (d) none of the above

Answer: (a) $16s^2 + 8s + 4 = \tau^2 s^2 + 2\zeta\tau + 1$
 $\tau^2 = 16/4 = 4; \tau = 2; 2\zeta\tau = 8/4 = 2; \zeta = 1/2 = 0.5$

17. A system has the transfer function $Y/X = 10/(s^2 + 1.6s + 4)$. A step change of 4 units magnitude is introduced in this system. The percent overshoot is: (GATE-1991-9.iii)

- (a) 20 (b) 30 (c) 25 (d) 35

Answer: (c) $s^2 + 1.6s + 4 = \tau^2 s^2 + 2\zeta\tau + 1$
 $\tau^2 = 1/4$ and $2\zeta\tau = 1.6/4$. Solving $\tau = 0.5$ and $\zeta = 0.4$.

$$\text{Overshoot} = \exp\left(\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}\right) = \exp\left(\frac{-\pi \times 0.4}{\sqrt{1-0.4^2}}\right) = 0.254 = 25.4\%$$

18. A second order system can be obtained by connecting two first order systems $1/(\tau_1 s + 1)$ and $1/(\tau_2 s + 1)$ in series. The damping ratio of the resultant second order system for the case $\tau_1 \neq \tau_2$ will be (GATE-2001-2.20)

- (a) > 1 (b) $= 1$ (c) < 1 (d) $= \tau_2/\tau_1$

Answer: (d) Two first order systems in series make the system to be overdamped. ($\zeta > 1$)

19. A proportional plus integral controller with proportional sensitivity $K_c = 1 \text{ mA/mA}$ and integral time $T_I = 1 \text{ min}$ is initially at steady state with error signal $e = 0$ and controller output I at 10 mA . At time $t = 0$, a step change of 1 mA is given in the error signal. The output will show one of the following responses. Identify the correct one (GATE-1998-2.20)

- (a) I jumps instantaneously to 11 mA and remains at that value thereafter
 (b) I increases at a linear rate of 1 mA/min

- (c) I jumps to 11 mA and then falls exponentially at the rate of $e^{-t/T}$
 (d) I jumps instantaneously to 11 mA and then increases linearly at the rate of 1 mA/min

Answer: (d)

$$c(t) = K_c \epsilon(t) + \frac{K_c}{\tau_I} \int_0^t \epsilon(t) dt + c_s$$

Given: $c_s = 10$ mA. $\epsilon(t) = 1$. Therefore $c(t) = 1 \times 1 + 1/1 \int_0^t dt + 10 = 11 + t$

20. Oscillations of a system may be reduced by one of the following controllers (GATE-1989-8.i.a)

- (a) Proportional controller (b) Proportional-Integral controller
 (c) Proportional-Derivative controller (d) No controller can reduce

for one of the following reasons

- (e) Proportional part of the controller (f) Derivative part of the controller
 (g) Integral part of the controller (h) Oscillations cannot be reduced

Answer: (c,f)

21. The response of a first order system with a proportional controller K_c has an offset to a step input. The offset can be eliminated by which one of the following actions? (GATE-1989-8.i.b)

- (a) Decreasing K_c
 (b) Adding an integral mode
 (c) Adding a derivative mode
 (d) No change needs to be done because offset cannot be eliminated

By this action, what happens to the overall order of the system?

- (e) Increased
 (f) Decreased
 (g) Does not change

Answer: (b,e) Decreasing K_c increases offset.

22. For microprocessor based automatic process control, name the most suitable instrument for temperature measurement. If more than one is suitable, name them all. (GATE-1989-8.i.c)

- (a) Gas thermometer (b) Mercury filled thermometer
 (c) Resistance thermometer (d) Thermocouple

Answer: (c)

23. The transfer function for an ideal proportional plus reset controller (Reset time T) is (GATE-1996-2.15)

- (a) $K_c(1 + 1/Ts)$ (b) $K_c(1 + Ts)$ (c) $K_c/(1 + Ts)$ (d) $K_c/(1 + s/T)$

Answer: (a) Reset time is also called as integral time.

24. The transfer function of a PID controller is (GATE-1997-1.23)

- | | |
|--|--|
| (a) $K_c(1 + \tau_I s + \tau_D s)$ | (b) $K_c \left(1 + \frac{1}{\tau_I s} + \tau_D s \right)$ |
| (c) $K_c \left(1 + \tau_I s + \frac{1}{\tau_D s} \right)$ | (d) $K_c \left(1 + \frac{1}{\tau_I s} + \frac{1}{\tau_D s} \right)$ |

Answer: (b)

25. A proportional controller with a gain of K_c is used to control a first-order process. The offset will increase if (GATE-1997-1.27)

- (a) K_c is reduced
 (b) K_c is increased
 (c) integral control action is introduced
 (d) derivative control action is introduced

Answer: (a)

26. An ideal PID controller has the transfer function $[1+1/(0.5s) + 0.2s]$. The frequency at which the magnitude ratio of the controller is 1, is (GATE-2001-2.18)

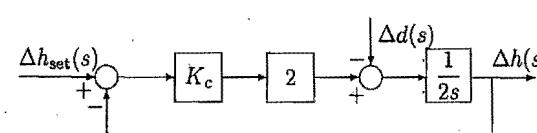
- (a) 0.5/0.2 (b) 0.2/0.5 (c) 0.2×0.5 (d) $1/\sqrt{0.2 \times 0.5}$

Answer: (d) $AR = K_c \sqrt{(\tau_D \omega - 1/(\tau_I \omega))^2 + 1}$

$$\tau_D \omega = \frac{1}{\tau_I \omega}$$

$$\omega^2 = 1/(\tau_I \tau_D)$$

27. The block diagram of an integrating level process is given below. For unit step change in the set point $\Delta h_{set}(s) = 1$ with $\Delta d = 0$, the offset exhibited by the system is



(GATE-2001-2.19)

- (a) $\frac{K_c}{1 + K_c}$ (b) $\frac{1}{1 + K_c}$ (c) 0 (d) $\frac{2K_c}{1 + 2K_c}$

Answer: (c) Processes having the term $1/s$ in their transfer function, when they are controlled with proportional controller, do not exhibit offset for setpoint changes.

28. The offset introduced by proportional controller with gain K_c in response of first order system can be reduced by
 (GATE-1995-1.p)

- (a) reducing value of K_c (b) introducing integral control
 (c) introducing derivative control (d) none of the above

Answer: (b)

29. Gas chromatography is used for measurement of
 (GATE-1995-1.s)
 (a) temperature (b) pressure (c) concentration (d) flow rate

Answer: (c)

30. The characteristic equation of a process is:

$$C = \frac{G_1 G_2}{1 + G_1 G_2 H} R + \frac{G_2}{1 + G_1 G_2 H} U$$

The open-loop transfer function is:

- (a) $G_1 G_2 H$ (b) $1 + G_1 G_2 H$ (c) $\frac{G_2}{1 + G_1 G_2 H}$ (d) $\frac{G_1 G_2}{1 + G_1 G_2 H}$

Answer: (a) Characteristic equation:

$$1 + \text{open loop transfer function} = 0$$

31. Which of the systems having the following transfer functions is stable? (GATE-1999-2.17)

- (a) $1/(s^2 + 2)$ (b) $1/(s^2 - 2s + 3)$
 (c) $1/(s^2 + 2s + 2)$ (d) $e^{-20s}/(s^2 + 2s - 1)$

Answer: (c)

32. A first order system with a time constant of 1 min is subjected to frequency response analysis. At an input frequency of 1 radian/min, the phase shift is (GATE-1998-1.24)

- (a) 45° (b) -90° (c) -180° (d) -45°

Answer: (d) Phase lag $\phi = \tan^{-1}(-\omega\tau) = \tan^{-1}(-1) = 45^\circ$.
 i.e., phase shift = -45° .

33. A sinusoidal variation in the input passing through a linear first-order system (GATE-1999-1.27)

- (a) becomes more oscillatory (frequency increases)
 (b) becomes less oscillatory (frequency decreases)
 (c) gets amplified (magnitude increases)
 (d) gets attenuated (magnitude decrease)

Answer: (d)

34. Select the correct statement from the following:
 (GATE-2000-1.27)

- (a) The frequency response of a pure capacity process is unbounded
 (b) The phase lag of a pure time delay system decreases with increasing frequency
 (c) The amplitude ratio of a pure capacity process is inversely proportional to the frequency
 (d) The amplitude ratio of a pure time delay system increases with frequency

Answer: (c) For a pure capacitive process $AR = K_p/\omega$; $\phi = -90^\circ$.

35. The time constant of a unity gain, first order plus time delay process is 5 min. If the phase lag at a frequency of 0.2 rad/min is 60° then the dead time (in minutes) is (GATE-2000-2.23)

- (a) $5\pi/12$ (b) $\pi/6$ (c) $\pi/12$ (d) $\pi/3$

Answer: (a) Transfer function of a first order plus time delay system is $\frac{K_p e^{\tau_d s}}{\tau_p s + 1}$

$$\text{Phase lag } \phi = \tan^{-1}(-\omega\tau_p) + (-\tau_d\omega)$$

$$\text{Given: } \phi = -60^\circ = -\pi/3 \text{ radians}$$

$$\text{Therefore, } -\pi/3 = \tan^{-1}(-2 \times 5) - \tau_d \times 0.2$$

$$\tau_d = 1.309 = 5\pi/12. \text{ (In using the calculator, mode is to be changed to RADIANS.)}$$

36. For a feedback control system to be stable, the

(GATE-2000-1.28)

- (a) roots of the characteristics equation should be real
 (b) poles of the closed loop transfer function should lie in the left half of the complex plane
 (c) Bode plots of the corresponding open loop transfer function should monotonically decrease
 (d) poles of the closed loop transfer function should lie in the right half of the complex plane

Answer: (b)

37. Bode diagrams are generated from output response of the system subjected to which of the following input
 (GATE-1995-1.q)

- (a) impulse (b) step (c) ramp (d) sinusoidal

Answer: (d)

38. According to Bode stability criterion, a system is unstable if the open loop frequency response exhibits an amplitude ratio exceeding unity at frequency for which phase lag is
 (GATE-1995-1.t)

- (a) 0° (b) 45° (c) 90° (d) 180°

Answer: (d) Bode stability criterion: A feedback control system is unstable if the Amplitude Ratio of the corresponding openloop transfer function is larger than 1 at the cross-over frequency (frequency at which phase lag is 180°).

8.3 Problems with Solutions

8.3.1 Linearization of Single Variable

Linearize the following single-input, single-output nonlinear dynamic model

$$\frac{dy}{dt} = \alpha y + \beta y^2 + \gamma \ln y$$

where α , β , and γ are constants.

Solution:

The given equation can be written as

$$\frac{dy}{dt} = \alpha y + \beta f_1(y) + \gamma f_2(y) \quad (8.1)$$

where $f_1(y) = y^2$ and $f_2(y) = \ln y$.

$f_1(y)$ can be expanded in Taylor series as:

$$f_1(y) = f_1(y_o) + \left(\frac{\partial f_1}{\partial y} \right)_{y_o} (y - y_o)$$

Since $f_1(y) = y^2$,

$$\left(\frac{\partial f_1}{\partial y} \right)_{y_o} = 2y_o$$

Therefore,

$$f_1(y) = y_o^2 + 2y_o(y - y_o)$$

Similarly $f_2(y) = \ln y$, and

$$\left(\frac{\partial f_2}{\partial y} \right)_{y_o} = \frac{1}{y_o}$$

Therefore,

$$f_2(y) = \ln y_o + \frac{1}{y_o}(y - y_o)$$

Hence Eqn.(8.1) becomes,

$$\begin{aligned} \frac{dy}{dt} &= \alpha y + \beta[y_o^2 + 2y_o(y - y_o)] + \gamma[\ln y_o + \frac{1}{y_o}(y - y_o)] \\ &= \alpha y + \beta y^2 + 2\beta y_o y - 2y_o^2 \beta + \gamma \ln y_o + \frac{\gamma}{y_o} y - \gamma \\ &= y \left(\alpha + 2\beta y_o + \frac{\gamma}{y_o} \right) + (\gamma \ln y_o - \gamma - \beta y_o^2) \end{aligned}$$

where y_o is the value of y when $t = 0$.

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8.3.2 Temperature of Solution Leaving a Stirred Tank

An aqueous solution (density = 1000 kg/m^3 , specific heat = $4 \text{ kJ/kg}\cdot\text{C}$) at 300 K is continuously fed at a flow rate of $1 \text{ m}^3/\text{min}$ to a continuous flow stirred tank of volume 1 m^3 containing a heater having a heating capacity of 1000 kW . If the liquid in the tank is also at 300 K to start with, find the equation which predicts the exit temperature of the solution as a function of time after the heater is switched on. (GATE-1991-19.ii)

Solution:

Energy balance:

$$\text{Rate of energy in} - \text{Rate of energy out} = \text{Rate of energy accumulation}$$

$$\begin{aligned} 60 \times 1000 + 1 \times 1000 \times 4 \times 300 - 1 \times 1000 \times 4 \times T &= 1000 \times 4 \times \frac{dT}{dt} \\ 60 + 4 \times 300 - 4T &= 4 \frac{dT}{dt} \\ \frac{dT}{dt} + T &= 315 \end{aligned}$$

This equation is of the form

$$\frac{dy}{dx} + Py = Q$$

Solution to this type of equation is

$$ye^{\int P dx} = \int Q e^{\int P dx} dx + C$$

Here $P = 1$. And $\int P dt = \int dt = t$.

Therefore

$$Te^t = 315 \int e^t dt + C$$

$$Te^t = 315e^t + C$$

Given: when $t = 0$; $T = 300^\circ\text{C}$. With this initial condition, $C = -15$. Therefore

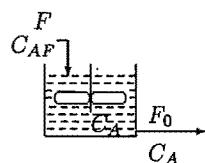
$$T = 315 - 15e^{-t}$$

Here, T is in $^\circ\text{C}$ and t in minutes.

8.3.3 Transfer Function for First Order Reaction

A first order reaction $A \rightarrow B$ with the rate constant k is taking place in CSTR fed with A at concentration C_{AF} which remains unchanged. There are likely to be some deviations in feed rate (F) of A . Derive linearized transfer function between concentration of A in the outlet and feed rate of A assuming that volume V of reacting mixture remains unchanged. (GATE-1993-26a)

Solution:



Mass balance around the reactor:

$$\begin{aligned} \text{moles of } A \text{ in} & - \text{moles of } A \text{ out} + & \text{rate of generation of } A \\ & = & \text{moles of } A \text{ accumulated} \\ FC_{AF} - F_o C_A + r_A V & = & \frac{dN_A}{dt} \end{aligned}$$

Since $-r_A = kC_A$,

$$\frac{dN_A}{dt} + F_o C_A + kV C_A = F_{CF} \quad (8.2)$$

We know that $C_A = N_A/V$. Therefore $N_A = C_A V$, and

$$\frac{dN_A}{dt} = \frac{dC_A V}{dt} = V \frac{dC_A}{dt}$$

Substituting these in Eqn.(8.2)

$$\begin{aligned} V \frac{dC_A}{dt} + F_o C_A + kV C_A & = FC_{AF} \\ V \frac{dC_A}{dt} + (F_o + kV) C_A & = FC_{AF} \\ \left(\frac{V}{F_o + kV} \right) \frac{dC_A}{dt} + C_A & = \left(\frac{C_{AF}}{F_o + kV} \right) F \end{aligned}$$

Therefore transfer function is:

$$\frac{C_A(s)}{F(s)} = \frac{K_p}{\tau_p s + 1}$$

where

$$K_p = \frac{C_{AF}}{F_o + kV} \text{ and } \tau_p = \frac{V}{F_o + kV}$$

8.3.4 Dynamics of Thermometer

A thermometer follows first-order dynamics with a time constant of 0.2 min. It is placed in a temperature bath at 100°C and is allowed to reach steady state. It is suddenly transferred to another bath at 150°C at time $t = 0$ and is left there for 0.2 min. It is immediately returned to the original bath at 100°C. Calculate the readings at:

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(i) $t = 0.1 \text{ min}$

(ii) $t = 0.4 \text{ min}$

(GATE-1992-19a)

Solution:

Given: At $t = 0$, $\Delta T = 50^\circ\text{C}$.

For first order system, fractional change in response $= 1 - e^{-t/\tau}$.
Therefore, at $t = 0.1$

$$\begin{aligned} T &= \Delta T(1 - e^{-0.1/0.2}) + T_{\text{initial}} \\ &= 50 \times (1 - e^{-0.5}) + 100 \\ &= 119.7^\circ\text{C} \end{aligned}$$

at $t = 0.2$

$$\begin{aligned} T &= 50 \times (1 - e^{-0.2/0.2}) + 100 \\ &= 131.6^\circ\text{C} \end{aligned}$$

At $t = 0.2$ again a step change in T is introduced. Therefore at $t = 0.4$,

$$T = (100 - 131.6) \times (1 - e^{-0.2/0.2}) + 131.6 = 111.6^\circ\text{C}$$

8.3.5 Response of Thermometer to Step Input

A thermometer of time constant 10 seconds, initially at 30°C, is suddenly immersed into water at 100°C. How long will it take for the thermometer reading to reach 90°C? (GATE-1990-18.i)

Solution:

The response of first order system to step input is given by

$$y = 1 - e^{-t/\tau} \quad (8.3)$$

where y = change in output/change in input at time t
 τ = time constant

Here,

$$\begin{aligned} \text{change in input} &= 100 - 30 = 70^\circ\text{C} \\ \text{change in output} &= 90 - 30 = 60^\circ\text{C} \\ \tau &= 10 \text{ sec} \end{aligned}$$

Therefore, from Eqn.(8.3)

$$\begin{aligned} \frac{60}{70} &= 1 - e^{-t/10} \\ t &= 19.46 \text{ sec} \end{aligned}$$

Time to reach to the reading of 90°C = 19.46 sec.

8.3.6 Time Constant of Thermocouple

The response of a thermocouple can be modelled as a first order process to changes in the temperature of the environment. If such a thermocouple at 25°C is immersed suddenly in a fluid at 80°C and held there, it is found that the thermocouple reading (in °C) reaches 63.2% of the final steady value in 40 seconds. Find the time constant of the thermocouple. (GATE-2000-13)

Solution:

The response of first order system to step input is given by

$$y = 1 - e^{-t/\tau} \quad (8.4)$$

where y = change in output/change in input at time t

τ = time constant

Here,

$$\text{change in input} = 80 - 25 = 55^\circ\text{C}$$

$$\text{change in output} = 0.632 \times 80 - 25 = 25.56^\circ\text{C}$$

$$t = 40 \text{ sec}$$

Therefore, from Eqn.(8.4)

$$\frac{25.56}{55} = 1 - e^{-40/\tau}$$

$$\tau = 64 \text{ sec}$$

Time constant of thermocouple $\tau = 64$ sec.

8.3.7 Dynamics of Temperature Alarm Unit

A temperature alarm unit, a unity gain first order system with a time constant of two minutes, is subjected to a sudden 100 K rise because of fire. If an increase of 50 K is required to activate the alarm, what will be the delay in signalling the temperature change? (GATE-1989-18ii)

Solution:

For first order system, fractional change in response = $1 - e^{-t/\tau}$.

Here,

$$50 = 100 \times 1 - e^{-t/\tau}$$

$$0.5 = 1 - e^{-t/2}$$

Solving, $t = 1.386$ min. i.e., delay in signalling the temperature change = 1.386 min.

8.3.8 Analytical Expression for Unit Impulse Response

Derive an analytical expression for a unit impulse response of a system whose transfer function is given by

$$\frac{Y(s)}{X(s)} = \frac{1.5}{s^2 + 3s + 2}$$

(GATE-1994-26)

Solution:

For unit impulse input $X(s) = 1$. Therefore,

$$\begin{aligned} Y(s) &= \frac{1.5}{s^2 + 3s + 2} \\ &= \frac{1.5}{(s+1)(s+2)} \\ &= \frac{1.5}{s+1} - \frac{1.5}{s+2} \end{aligned}$$

Since

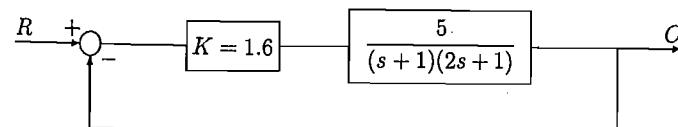
$$\mathcal{L}(e^{-at}) = \frac{1}{s+a}$$

$$\begin{aligned} Y(t) &= 1.5e^{-t} - 1.5e^{-2t} \\ &= 1.5(e^{-t} - e^{-2t}) \end{aligned}$$

8.3.9 Step Input to Setpoint

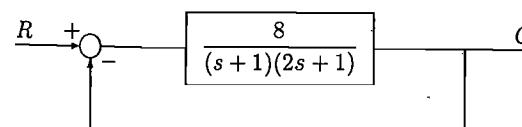
The setpoint of the control system shown in figure is given a step change of 0.1 unit. Determine:

- (a) the offset
- (b) period of oscillation

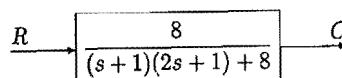


Solution:

The given block diagram is reduced to:



i.e.,

Value of C for $R = 0.1/s$:

$$C(s) = \frac{8}{(s+1)(2s+1)+8} \cdot \frac{0.1}{s}$$

Ultimate value of C is obtained from the final value theorem:

$$\lim_{t \rightarrow \infty} C(t) = \lim_{s \rightarrow 0} sC(s)$$

$$sC(s) = \frac{0.8}{(s+1)(2s+1)+8}$$

when $s \rightarrow 0$,

$$sC(s) = \frac{0.8}{9} = 0.089$$

Therefore, offset = input - steady state output = $0.1 - 0.089 = 0.011$.For $R = 0.1/s$,

$$\begin{aligned} C(s) &= \frac{0.8}{(s+1)(2s+1)+8} \\ &= \frac{0.8}{2s^2 + 3s + 9} \\ &= \frac{0.8/9}{(2/9)s^2 + (1/3)s + 1} \end{aligned}$$

 $C(s)$ can be written as:

$$C(s) = \frac{K_p}{\tau^2 s^2 + 2\zeta\tau s + 1} \quad (8.5)$$

where $K_p = 0.8/9$; $\tau^2 = 2/9$; and $2\zeta\tau = 1/3$. Therefore, $\tau = 0.471$ and $\zeta = 0.3536$.Period of oscillation(T):

$$T = \frac{2\pi\tau}{\sqrt{1-\zeta^2}} \quad (8.6)$$

Substituting for τ and ζ in Eqn.(8.6),

$$T = \frac{2 \times \pi \times 0.471}{\sqrt{1 - 0.3536^2}} = 3.164 \text{ sec}$$

8.3.10 Output from Control Valve

A 5% equal percentage pneumatic valve has an input range 20–100 kN/m². When the pressure input is 70 kN/m² the corresponding flow rate is 2000 kg/hour. If the pressure changes to 80 kN/m², what will be the flow rate? (GATE-1988-18ii)

Solution:

1% change in input pressure results in 5% change in valve opening. Therefore, $100/(100 - 20) = 12.5\%$ change in input pressure results in $12.5 \times 5 = 62.5\%$ changes in valve opening.

Therefore if the valve is of air-to-open type, the new flow rate will be $= 2000 + 2000 \times 0.625 = 3250 \text{ kg/hour}$.

If the valve is of air-to-close type, the new flow rate will be $= 2000 - 2000 \times 0.625 = 750 \text{ kg/hour}$.

8.3.11 Output of Proportional Controller

An electronic proportional controller has an input range of 4 to 20 mA and output range of 1 to 5 V. For an input change of 4 mA, what will be the output change, if the proportional band of the controller is 40%? (GATE-1988-18i)

Solution:

For an input change of $[(20 - 4) \times 0.4] \text{ mA}$ the output change is $(5 - 1) \text{ V}$. i.e., for an input change of 6.4 mA, output change = 4 V.

Therefore, for an input change of 4 mA, output change = $4 \times 4/6.4 = 2.5 \text{ V}$.

8.3.12 Output from PID Controller

A PID controller has a steady state output pressure of 0.4 kgf/cm². The setpoint is now increased at the rate of 1 cm/min.

(a) Obtain the nature of the output pressure. (Assume $K = 0.1 \text{ kgf/cm}^2/\text{cm}$; $\tau_I = 1.2 \text{ min}$; and $\tau_D = 0.5 \text{ min}$)

(b) If the set point is raised suddenly by 1 cm what would be the output pressure?

Solution:

Output pressure(P) of the PID controller is given by

$$P = K\epsilon + K\tau_D \frac{d\epsilon}{dt} + \frac{K}{\tau_I} \int_0^t \epsilon dt + P_s \quad (8.7)$$

where

ϵ = error;

K = controller gain;

τ_I = reset time;

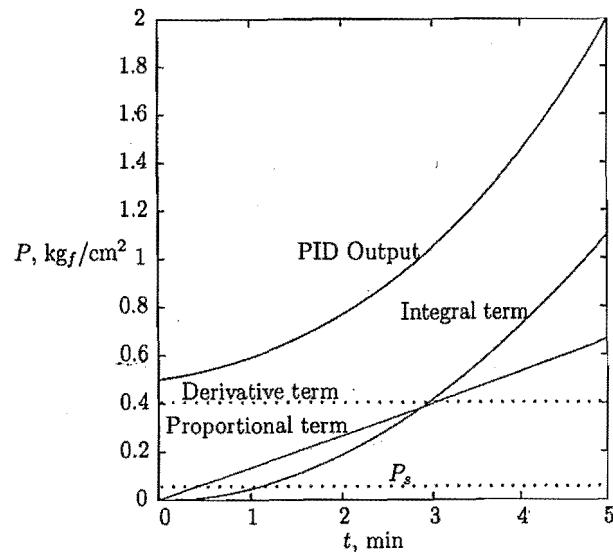
τ_D = derivative time; and

P_s = steady state output pressure.

(a) Given: $d\epsilon/dt = 1 \text{ cm/min}$. Therefore $\epsilon = t \text{ cm}$.

Hence, Eqn.(8.7) becomes,

$$\begin{aligned} P &= Kt + K\tau_D + \frac{K}{\tau_I} \int_0^t t dt + P_s \\ &= Kt + K\tau_D + \frac{Kt^2}{\tau_I \cdot 2} + P_s \\ &= 0.1t + 0.1 \times 0.5 + \frac{0.1t^2}{1.2 \times 2} + 0.4 \\ &= 0.0417t^2 + 0.1t + 0.45 \end{aligned}$$



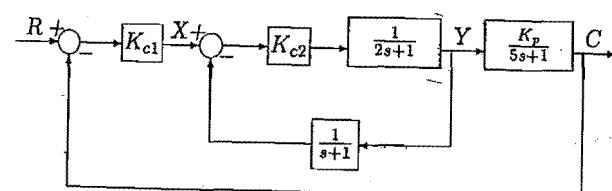
(b) Given: $\epsilon = 1 \text{ cm}$.

For step inputs the derivative part of the controller will not respond. Therefore Eqn.(8.7) becomes,

$$\begin{aligned} P &= K \times 1 + \frac{K}{\tau_I} t + P_s \\ &= 0.5 + 0.0833t \end{aligned}$$

8.3.13 Block Diagram Reduction and Offset Calculation

Given the following block diagram,



(a) Express the transfer function (Y/X) as a ratio of polynomials in s .

(b) Calculate the offset in C due to a unit step change in the reference signal (set-point) R .

(GATE-2000-15)

Solution:

For the inner-loop

$$\left[x - Y \left(\frac{1}{s+1} \right) \right] K_{c2} \frac{1}{2s+1} = Y$$

Rearranging,

$$X \left(\frac{K_{c2}}{2s+1} \right) = Y \left(1 + \frac{K_{c2}}{(s+1)(2s+1)} \right)$$

i.e.,

$$\begin{aligned} \frac{Y}{X} &= \frac{K_{c2}[1/(2s+1)]}{1 + K_{c2}[(1/(2s+1)][1/(s+1)]]} \\ &= \frac{K_{c2}(s+1)}{(2s+1)(s+1) + K_{c2}} \end{aligned}$$

For the outer loop,

$$(R - C) \left(K_{c1} \frac{K_{c2}(s+1)}{(2s+1)(s+1) - K_{c2}} \right) \left(\frac{K_p}{5s+1} \right) = C$$

i.e.,

$$R \left(K_{c1} \frac{K_{c2}(s+1)}{(2s+1)(s+1) - K_{c2}} \right) \left(\frac{K_p}{5s+1} \right) = C \left[1 + \left(K_{c1} \frac{K_{c2}(s+1)}{(2s+1)(s+1) - K_{c2}} \right) \left(\frac{K_p}{5s+1} \right) \right]$$

Rearranging,

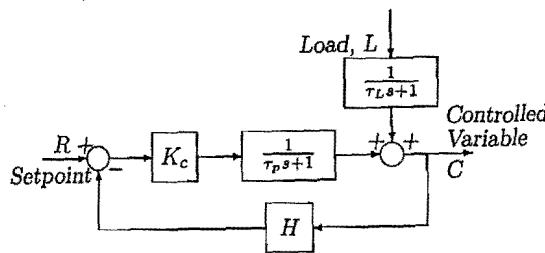
$$\frac{C}{R} = \frac{K_{c1} K_{c2} K_p (s+1)}{(5s+1)[(2s+1)(s+1) + K_{c2}] + K_{c1} K_{c2} K_p (s+1)}$$

For unit step input $R = 1/s$.

$$\begin{aligned}\text{Offset} &= 1 - \text{final value of } C \\ &= 1 - \lim_{s \rightarrow 0} sC(s) \\ &= 1 - \frac{K_{c1}K_{c2}K_p}{1 + K_{c2} + K_{c1}K_{c2}K_p} \\ &= \frac{1 + K_{c2} + K_{c1}K_{c2}K_p - K_{c1}K_{c2}K_p}{1 + K_{c2} + K_{c1}K_{c2}K_p}\end{aligned}$$

8.3.14 Servo and Regulator Transfer Functions

A control system is represented by the following block diagram:



Derive the servo and regulator transfer functions.

(GATE-1988-18iii)

Solution:

Servo transfer function is obtained by setting Load changes = 0. i.e., Servo transfer function = C/R .

$$\frac{C}{R} = \frac{\text{multiplication of transfer functions in the feedforward path}}{1 + \text{multiplication of transfer functions in the loop}}$$

i.e.,

$$\begin{aligned}\frac{C}{R} &= \frac{K_c \frac{1}{\tau_p s + 1}}{1 + K_c \frac{1}{\tau_p s + 1} H} \\ &= \frac{K_c}{\tau_p s + 1 + K_c H}\end{aligned}$$

Regulator transfer function is obtained by setting setpoint changes = 0. i.e., Regulator transfer function = C/L .

$$\begin{aligned}\frac{C}{L} &= \frac{\frac{1}{\tau_L s + 1}}{1 + K_c \frac{1}{\tau_p s + 1} H} \\ &= \frac{(\tau_p s + 1)/(\tau_L s + 1)}{\tau_p s + 1 + K_c H}\end{aligned}$$

Another Way:

By block-diagram algebra

$$C = L \frac{1}{\tau_L s + 1} + K_c \frac{1}{\tau_p s + 1} \epsilon$$

where $\epsilon = R - CH$. Therefore

$$C = L \frac{1}{\tau_L s + 1} + \frac{K_c R}{\tau_p s + 1} - \frac{K_c H C}{\tau_p s + 1}$$

Rearranging

$$C \left(1 + \frac{K_c H}{\tau_p s + 1} \right) = \frac{1}{\tau_L s + 1} L + \frac{K_c}{\tau_p s + 1} R$$

i.e.,

$$C = \underbrace{\frac{\frac{1}{\tau_L s + 1}}{1 + \frac{K_c H}{\tau_p s + 1}}}_\text{Load transfer function} L + \underbrace{\frac{\frac{K_c}{\tau_p s + 1}}{1 + \frac{K_c H}{\tau_p s + 1}}}_\text{Servo transfer function} R$$

8.3.15 Steady State Error in a Closed Loop System

A unity feedback system has a forward loop transfer function

$$\frac{4}{s(s+5)}$$

Calculate the steady state error (offset) of the closed loop system for unit step change in the setpoint.

Solution:

$$\begin{aligned}\text{Closed loop transfer function} &= \frac{G(s)}{1 + G(s)} \\ &= \frac{4/[s(s+5)]}{1 + 4/[s(s+5)]} \\ &= \frac{4}{s(s+5) + 4}\end{aligned}$$

$$\text{Response to step change in input} = \frac{4}{s(s+5) + 4} \cdot \frac{1}{s}$$

Steady state value of output is obtained from the final value theorem:

$$\lim_{t \rightarrow \infty} F(t) = \lim_{s \rightarrow 0} sF(s)$$

i.e.,

$$sF(s) = \frac{4}{s(s+5) + 4}$$

when $s = 0$,

$$sF(s) = 1$$

Offset = input - steady state output = $1 - 1 = 0$.

8.3.16 Proportional Band of Pneumatic Controller

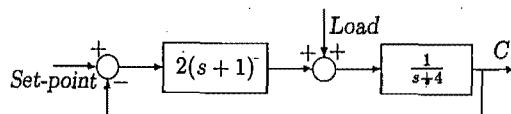
Determine the sensitivity and proportional band of a pneumatic controller with a scale range of 0-120°C when the output changes from 20-100 kN/m² as the temperature rises from 95-110°C.
(GATE-1987-20i)

Solution:

$$\begin{aligned} \text{Sensitivity} &= \frac{\text{fractional change in output}}{\text{fractional change in input}} \\ &= \frac{100 - 20}{110 - 95} = \frac{80}{15} \\ &= 5.33 \text{ (kN/m}^2\text{)}/^\circ\text{C.} \\ \text{Proportional band} &= \frac{110 - 95}{120 - 0} \times 100 = 12.5\% \end{aligned}$$

8.3.17 Offset in Feedback Control System

A control system is shown below.



(a) Determine the variation of C with time for a unit step change in the set-point.

(b) What is the offset?

(GATE-1997-8)

Solution:

$$\begin{aligned} C(s) &= \frac{2(1+s)\frac{1}{s+4}}{1+2(1+s)\frac{1}{s+4}} \bar{y}_{sp}(s) \\ &= \frac{2(1+s)}{(s+4)+2(1+s)} \bar{y}_{sp}(s) \\ &= \frac{2(1+s)}{3(s+2)} \bar{y}_{sp}(s) \end{aligned}$$

PROCESS CONTROL

For unit step change in setpoint, $\bar{y}_{sp}(s) = 1/s$. Therefore

$$\begin{aligned} C(s) &= \frac{2(1+s)}{3(s+2)} \\ &= \frac{2}{3} \left[\frac{1}{s+2} + \frac{1}{s+2} \right] \\ &= \frac{2}{3} \left[\frac{1/2}{s} - \frac{1/2}{s+2} + \frac{1}{s+2} \right] \\ &= \frac{1}{3} \left[\frac{1}{s} + \frac{1}{s+2} \right] \end{aligned}$$

Using inverse Laplace transform,

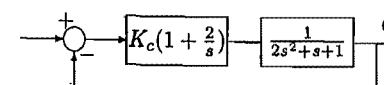
$$C(t) = (1/3) [1 + e^{-2t}]$$

This expression gives the variation of C with time for unit step change in setpoint.

$$\begin{aligned} \text{Offset} &= 1 - \lim_{t \rightarrow \infty} C(t) \\ &= 1 - 1/3 = 2/3 \end{aligned}$$

8.3.18 Value of K_c from Routh Test

A control system is shown below.



Using the Routh test, determine the value of K_c , at which the system just becomes unstable. (GATE-1997-27)

Solution:

Open-loop transfer function is

$$G_{OL} = K_c \left(1 + \frac{2}{s} \right) \left(\frac{1}{2s^2 + s + 1} \right)$$

Characteristic equation is

$$1 + G_{OL} = 0$$

Therefore

$$\begin{aligned} 1 + K_c \left(1 + \frac{2}{s} \right) \left(\frac{1}{2s^2 + s + 1} \right) &= 0 \\ (2s^3 + s^2 + s) + K_c(s+2) &= 0 \\ 2s^3 + s^2 + (1 + K_c)s + 2K_c &= 0 \end{aligned}$$

Routh Array:

Row			
1	2		$1 + K_c$
2	1		2
3		$1 \times (1 + K_c) - 2 \times 2$	
4		1	

For stability,

$$\frac{1 \times (1 + K_c) - 2 \times 2}{1} \geq 0$$

$$1 + K_c - 4 \geq 0$$

$$K_c \geq 3$$

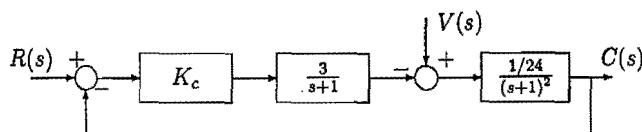
At the value of $K_c = 3$ the system just becomes unstable.

8.3.19 Maximum Gain for Stable Operation

Determine:

- (i) The maximum gain for stable operation.
- (ii) The corresponding frequency of oscillation.

(GATE-1992-19b)



Solution:

Open-loop transfer function G_{OL} is given by

$$G_{OL} = K_c \frac{3}{s+1} \frac{1/24}{(s+1)^2} = \frac{0.125K_c}{(s+1)^3}$$

Amplitude ratio (AR) is given by

$$AR = \frac{0.125K_c}{(\sqrt{\omega^2 + 1})^3} = \frac{0.125K_c}{(\omega^2 + 1)^{3/2}}$$

And phase angle (ϕ) is

$$\phi = 3 \tan^{-1}(-\omega)$$

Cross-over frequency:

At cross-over frequency, phase lag is 180° . Therefore,

$$-180^\circ = 3 \tan^{-1}(-\omega)$$

$$-\pi \text{ rad} = 3 \tan^{-1}(-\omega)$$

PROCESS CONTROL

Solving, $\omega = 1.732 \text{ rad/min}$.

According to Bode stability criterion, for a stable system Amplitude Ratio of open-loop transfer function at cross-over frequency should be ≤ 1 . Therefore,

$$\frac{0.125K_c}{(\omega^2 + 1)^{3/2}} \leq 1$$

$$\frac{0.125K_c}{(1.732^2 + 1)^{3/2}} \leq 1$$

$$K_c \leq 64$$

Therefore the maximum gain for stable operation is 64, and the corresponding frequency of oscillation is 1.732 rad/min.

8.3.20 Maximum Controller Gain for Stable Closed Loop System-I

The characteristic equation of a closed loop control system is

$$0.25s^3 + 0.8s^2 + 5.6s + 1 + 0.35K = 0$$

Find the limiting value of K , above which the closed loop system will be unstable. (GATE-1998-24)

Solution:

Characteristic equation:

$$0.25s^3 + 0.8s^2 + 5.6s + 1 + 0.35K = 0$$

Routh array:

0.25	5.6
0.8	$1 + 0.35K$
$0.8 \times 5.6 - 0.25(1 + 0.35K)$	0.8

Limiting value of K is obtained equating $\frac{0.8 \times 5.6 - 0.25(1 + 0.35K)}{0.8}$ to zero.

$$\frac{0.8 \times 5.6 - 0.25(1 + 0.35K)}{0.8} = 0$$

$$0.8 \times 5.6 - 0.25(1 + 0.35K) = 0$$

$$4.23 - 0.0875K = 0$$

$$K = 48.34$$

8.3.21 Maximum Controller Gain for Stable Closed Loop System-II

Find the maximum value of K such that a closed loop system having the following characteristic equation is stable:

$$s^3 + Ks^2 + (K+1)s + 4 = 0$$

(GATE-1991-19.i)

Solution:

Characteristic equation:

$$s^3 + Ks^2 + (K+1)s + 4 = 0$$

Routh array:

1	$K+1$
K	4
$\frac{K \times (K+1) - 1 \times 4}{K}$	

Limiting value of K is obtained equating $[K(K+1) - 4]$ to zero.

$$\begin{aligned} K(K+1) - 4 &= 0 \\ K^2 + K - 4 &= 0 \end{aligned}$$

Solving for K :

$$\begin{aligned} K &= \frac{-1 \pm \sqrt{1+16}}{2} \\ K &= 1.56, -2.56 \\ K &= 1.56 \text{ (since } K \text{ can not be negative)} \end{aligned}$$

Maximum value of $K = 1.56$.

8.3.22 Limits on Controller Gain

A closed loop feedback control system consists of a second order process

$$G_p(s) = \frac{K_p}{(\tau_1 s + 1)(\tau_2 s + 1)}$$

and a proportional controller $G_c(s) = K_c$. The roots of characteristic equation of the closed loop system are -2 and -1 in absence of controller and roots are $-1.5 \pm 0.5i$ when $K_c = 4$.

- (i) Determine K_p, τ_1, τ_2
- (ii) Determine limits on K_s so that the response of the system to a step input is non-oscillatory. (GATE-1993-26b)

Solution:

Characteristic equation in the absence of controller:

$$1 + G_p(s) = 0 \quad (8.8)$$

$$\begin{aligned} 1 + \frac{K_p}{(\tau_1 s + 1)(\tau_2 s + 1)} &= 0 \\ \tau_1 \tau_2 s^2 + \tau_1 s + \tau_2 s + 1 + K_p &= 0 \\ \tau_1 \tau_2 s^2 + (\tau_1 + \tau_2)s + (1 + K_p) &= 0 \end{aligned} \quad (8.9)$$

Given: roots $= -2, -1$. Therefore characteristic equation is

$$(s+2)(s+1) = 0 \quad (8.10)$$

$$s^2 + 3s + 2 = 0 \quad (8.11)$$

Comparing Eqns.(8.9) and (8.11)

$$1 + K_p = 2$$

from which $K_p = 1$ and $\tau_1 \tau_2 = 1$. Also $(\tau_1 + \tau_2) = 3$.
Therefore

$$\tau_1 + \frac{1}{\tau_1} = 3$$

Solving, $\tau_1 = 0.382$, and $\tau_2 = 2.618$.

With Controller:

The characteristic equation is

$$\begin{aligned} 1 + \frac{K_p K_c}{(\tau_1 s + 1)(\tau_2 s + 1)} &= 0 \\ 1 + \frac{K_p K_c}{(0.382 s + 1)(2.618 s + 1)} &= 0 \\ s^2 + 3s + 1 + K_c &= 0 \\ s = \frac{-3 \pm \sqrt{9 - 4(1 + K_c)}}{2} \end{aligned}$$

For non-oscillatory response, there should be no complex roots.
Therefore

$$9 - 4(1 + K_c) \geq 0$$

$$9 - 4(1 + K_c) = 0$$

$$K_c = 1.25$$

Maximum value of $K_c = 1.25$.

8.3.23 Stability of Closed Loop System

The characteristic equation of a closed loop control system is

$$s^4 + 4s^3 + 6s^2 + 2s + 3 = 0$$

Check whether the system is stable or not. (GATE-1994-28)

Solution:

Characteristic equation:

$$s^4 + 4s^3 + 6s^2 + 2s + 3 = 0$$

Routh array:

$$\begin{array}{ccc} 1 & 6 & 3 \\ 4 & 2 & \\ \frac{4 \times 6 - 1 \times 2}{5.5} = 5.5 & \frac{4 \times 3 - 1 \times 0}{4} = 3 \\ \frac{5.5 \times 2 - 4 \times 3}{5.5} = -1/5.5 & \end{array}$$

Since the first column of Routh array is having negative element (i.e., $-1/5.5$), the given system is unstable.

8.3.24 Steady State Error in the Closed Loop System

The transfer function of a process, a measuring element and a control valve is given respectively by

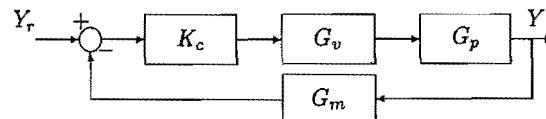
$$G_p = \frac{2}{2s+1}, \quad G_m = \frac{1}{5s+1}, \quad G_v = \frac{1.5}{3s+1}$$

A proportional controller with a gain of $K_c = 1$ is used.

- (a) Write the closed loop transfer function relating the output (Y) to the set point (Y_r)
- (b) What is the steady state error in the output for a unit step change in the set point. (GATE-1994-27)

Solution:

The given system is represented in the block diagram as given below:



Laplace transform of output is related as

$$Y(s) = \frac{K_c G_v G_p}{1 + K_c G_v G_p G_m} Y_r(s)$$

Substituting for the transfer functions,

$$\begin{aligned} Y(s) &= \frac{\left(\frac{1.5}{3s+1}\right) \left(\frac{2}{2s+1}\right)}{1 + \left(\frac{1.5}{3s+1}\right) \left(\frac{2}{2s+1}\right) \left(\frac{1}{5s+1}\right)} Y_r(s) \\ &= \frac{3(5s+1)}{(3s+1)(2s+1)(5s+1)+3} Y_r(s) \end{aligned}$$

Closed loop transfer function is obtained as

$$\frac{3(5s+1)}{(3s+1)(2s+1)(5s+1)+3}$$

PROCESS CONTROL

For step change in set point,

$$Y(s) = \frac{3(5s+1)}{(3s+1)(2s+1)(5s+1)+3} \frac{1}{s}$$

$$\begin{aligned} \lim_{t \rightarrow \infty} Y(t) &= \lim_{s \rightarrow 0} [sY(s)] \\ \text{i.e., } Y_{t,\text{final}} &= \frac{3}{1+3} = \frac{3}{4} \end{aligned}$$

$$\text{Steady state error} = 1 - \frac{3}{4} = 0.25$$

8.3.25 Crossover Frequency & Ultimate Controller Gain - I

A feedback control loop with a proportional controller has an open loop transfer function

$$G_L(s) = \frac{K_c}{s(5s+1)^2}$$

where time is in minutes. Obtain the crossover frequency and the ultimate controller gain. (GATE-2000-14)

Solution:

Given:

$$G_L(s) = \frac{K_c}{s(5s+1)^2}$$

Rewriting the above equation as

$$G_L = \frac{K_c}{s(5s+1)(5s+1)}$$

$$\begin{aligned} \text{Phase shift } \phi &= -90 + \tan^{-1}(-5\omega) + \tan^{-1}(-5\omega) \\ &= -90 + 2\tan^{-1}(-5\omega) \end{aligned}$$

Crossover frequency is the frequency at which $\phi = -180$. Therefore,

$$180 = -90 + 2\tan^{-1}(-5\omega)$$

$$270/2 = \tan^{-1}(-5\omega)$$

$$\omega = 0.2 \text{ rad/min}$$

Ultimate controller gain is the gain corresponding to the amplitude ratio (AR) of 1.

$$AR = \frac{K_c}{\omega} \frac{1}{\sqrt{25\omega^2 + 1}} \frac{1}{\sqrt{25\omega^2 + 1}} = 1$$

$$\text{i.e., } 1 = \frac{K_c}{0.2 \times (\sqrt{25 \times 0.2^2 + 1})^2}$$

$$K_c = 0.4$$

8.3.26 Crossover frequency & Ultimate Controller Gain - II

The open loop transfer function for a process is $\frac{1}{4(3s+1)^4}$, where the time constant is in minutes. Determine (a) the crossover frequency, and (b) the ultimate gain. (GATE-1997-26)

Solution:

With proportional controller, the open-loop transfer function is,

$$G_{OL} = \frac{K_c}{4(3s+1)^4}$$

Amplitude ratio of this transfer function is

$$AR = \frac{0.25K_c}{(9\omega^2 + 1)^2}$$

Phase angle of this transfer function is

$$\phi = 4 \tan^{-1}(-3\omega)$$

At cross-over frequency (ω_{co}) phase angle is -180° . Therefore

$$-180^\circ = 4 \tan^{-1}(-3\omega_{co})$$

$$-45^\circ = \tan^{-1}(-3\omega_{co})$$

$$\tan(-45^\circ) = -3\omega_{co}$$

$$-1 = -3\omega_{co}\omega_{co} = 1/3 = 0.333 \text{ rad/time}$$

From Bode stability criterion, AR of the open-loop transfer function at the cross-over frequency should be less than or equal to 1.

Therefore,

$$\frac{0.25K_c}{(9\omega_{co}^2 + 1)^2} = 1$$

$$\frac{0.25K_c}{9 \times 0.333^2 + 1)^2} = 1$$

$$K_c = 16$$

Ultimate gain = 16.

Chapter 9

Process Economics & Design

9.1 Instant Notes

9.1.1 Cost Estimation

- Total capital investment = Fixed capital investment + working capital
- Most chemical plants use an working capital accounting to 10 to 20% of the total capital investment.
- Cost indexes: for updating cost data from cost data for a particular year.

$$\text{Present cost} = \text{original cost} \times \left(\frac{\text{index value at present time}}{\text{index value at time original cost was obtained}} \right)$$

- Cost scaling: cost data for a particular size. Normally *six-tenths factor rule* is used. According to this rule, if the cost of a given unit at one capacity is known, the cost of a similar unit with X times the capacity of the first is approximately $X^{0.6}$ times the cost of the initial unit.

$$\text{Cost of equipment of } a = \text{cost of equipment } b \times \left(\frac{\text{capacity of equipment } a}{\text{capacity of equipment } b} \right)^{0.6}$$

However, the application of the 0.6 rule of thumb for most purchased equipment is an oversimplification of a valuable cost concept, since the actual values of the cost-capacity factor vary from less than 0.2 to greater than 1.0. Because of this, 0.6 factor should only be used in the absence of other information. In general, the cost-capacity concept should not be used beyond a tenfold range of capacity.

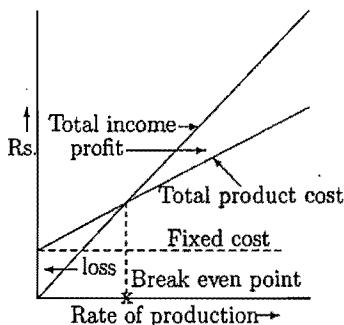
- Lang¹ multiplication factors for estimation of fixed-capital investment or total capital investment

¹Ref: Plant Design and Economics for Chemical Engineers - Peters & Timmerhaus, McGraw-Hill

Factor \times delivered equipment cost = fixed-capital investment or total capital investment.

Type of plant	Factor for	
	Fixed-capital investment	Total capital investment
Solid-processing plant	3.9	4.6
Solid-fluid-processing plant	4.1	4.9
Fluid-processing plant	4.8	5.7

- Break-even point



The fixed cost remains constant, and the total production cost increases as the rate of production increases. The point where the total product cost equals the total income is known as break-even point.

- Turnover ratio = $\frac{\text{gross annual sales}}{\text{fixed capital investment}}$

The reciprocal of turnover ratio is sometimes defined as the capital ratio or investment ratio. For the chemical industry, as a very rough rule of thumb, the turnover ratio can be approximated as 1.

9.1.2 Depreciation

- Amortization or depreciation are often used interchangeably.
- Book value or unamortized cost: The difference between the original cost of a property, and all the depreciation charges made to date is defined as the *book value* (some times called as unamortized cost).
- Straight line method

$$d = \frac{V - V_s}{n}$$

where d = annual depreciation amount

V = original value of the property at the start of the service period

V_s = salvage value of the property at the end of service life

n = service life, years

The asset value (or book value) V_a of the equipment at any time

$$V_a = V - ad$$

where a is the number of years in actual use.

- Declining balance or fixed percentage method

$$V_a = V(1 - f)^a$$

where

$$f = 1 - \left(\frac{V_s}{V}\right)^{1/n}$$

Comparison with the straight line method shows that declining balance method shows that declining-balance depreciation permits the investment to be paid off more rapidly during the early years of life.

- Double declining balance method: f is two times the depreciation rate (d/V) of straight line method. If salvage value is zero, $f = 2/n$.

- Sum of years digits method

$$\text{Depreciation for year } a = \frac{n - a + 1}{\sum_{i=1}^n a} (V - V_s)$$

For example, for $n = 5$ years, depreciation for the first year is

$$d_1 = \frac{5}{15} (V - V_s)$$

and, depreciation for the second year is

$$d_2 = \frac{4}{15} (V - V_s)$$

- Sinking fund method and present worth method are rarely used for depreciation cost accounting.

9.1.3 Interest and Investment Costs

- Simple interest

$$S = P(1 + i n)$$

where S = the amount after n interest periods

P = principal

i = interest rate

n = number of interest periods

- Compound interest

$$S = P(1+i)^n$$

- Nominal and effective interest rates: In common industrial practice, the length of interest period is assumed to be 1 year and the interest rate i is based on 1 year. However, there are cases where other time units are employed. Even though the actual interest period is not 1 year, the interest rate is often expressed on an annual basis. Consider an example in which the interest rate is 3 percent per period and the interest is compounded at half-year periods. A rate of this type would be referred to as "6 percent compounded semiannually." Interest rates stated in this form are known as *nominal interest rates*. The actual annual return on the principal would not be exactly 6 percent but would be somewhat larger because of the compounding effect at the end of the semianual period.

the exact or effective annual interest rate $= i_{\text{eff}} = \left(1 + \frac{r}{m}\right)^m - 1$

where r = nominal interest rate

m = interest periods per year

- Present worth

$$\text{Present worth} = P = S \frac{1}{(1+i)^n}$$

- Annuities: An *annuity* is a series of equal payments occurring at equal time intervals. A common type of annuity involves payments which occur at the end of each interest period. This is known as an *ordinary annuity*.

- Relation between amount of ordinary annuity and the periodic payments

$$S = R \frac{(1+i)^n - 1}{i}$$

where S = amount of annuity, i.e., the amount available at the end of n payment periods

R = uniform periodic payment

- Perpetuity: A *perpetuity* is an annuity in which the periodic payments continue indefinitely.

- Present worth of an annuity

$$P = R \frac{(1+i)^n - 1}{i(1+i)^n}$$

- Capitalized cost, K

$$K = C_V + \frac{C_R}{(1+i)^n - 1}$$

where C_V = original cost of equipment

C_R = replacement cost

9.1.4 Mechanical Design of Process Equipments

Thickness of Shell

- Cylindrical vessel

— Longitudinal stress:

$$t = \frac{pd}{4fJ}$$

— Circumferential stress:

$$t = \frac{pd}{2fJ}$$

where t = thickness of shell

d = diameter of cylinder

p = design pressure

f = allowable stress

J = joint efficiency

Circumferential stress is the controlling stress; and cylindrical shell is designed based on the circumferential stress formula.

- Spherical vessel

$$t = \frac{pd}{4fJ}$$

- If cost of fabrication were not a prime consideration, the most economical shape for a vessel would be a sphere. However, the fabrication cost of spherical vessels are so great that their use is limited to special considerations. Cylindrical vessels are more easily fabricated, in the majority of cases are considerably simpler to erect, are readily shipped, and are therefore more readily used in the process industries.

- From the viewpoint of material savings and uniform distribution of stresses in the material of walls from loads, a spherical shape is most favorable. A spherical geometry provides minimum surface area per unit volume, and its wall thickness is minimum for a given pressure.

- In shells operating under external pressure, compressive stresses arise instead of tensile.

- Allowable stress decreases with increase in temperature.

- The criteria used in establishing the allowable stresses in the ASME² Code
Lowest value of stresses obtained from:
 - 25% of the specified minimum tensile strength at room temperature, or
 - 25% of the minimum expected tensile strength at operating temperature, or
 - 62.5% of the minimum expected yield strength for 0.2% offset at operating temperature.
- Many engineers still use a "factor of safety" of 3 for structural steel and a factor of safety of 6 for gray cast iron, based upon the ultimate strength, when designing structural parts.
- Joint Efficiency**
No radiography : 70%
Spot radiography : 85%
100% radiography : 100%
Joint efficiency is 100% for seamless heads.
- A cylindrical vessel under internal pressure tends to retain its shape in that any out of roundness or dents resulting from shop fabrication or erection tend to be removed when the vessel is placed under internal pressure. Thus *any deformation resulting from internal pressure tends to make an imperfect cylinder more cylindrical*. However, the opposite is true for imperfect cylindrical vessels under external pressure, and any imperfection will tend to be aggravated with the result of possible collapse of the vessel. For this reason, *a given vessel under external pressure in general has a pressure rating only 60% as high as it would have under internal pressure*.

Closures for Vessels

- Inside corner radius - referred to as the "knuckle radius".
- Crown radius - radius of dish.
- If the radius of dish is greater than the shell outside diameter, the head is known as a "flanged and shallow dished head". If the radius of dish is equal to or less than the outside diameter, the head is known as "flanged and standard dished".
- Vessels with *flanged and shallow dished heads* are primarily used for horizontal storage tanks.
- Torispherical head:** Inside corner radius is atleast equal to 3 times the metal thickness; and in no case less than 6% of the inside diameter. Also the radius of the dish may be made equal or less than the diameter of the head. Used for 15 to 200 psig pressure vessels.
- Elliptical flanged & dished head**
 - For pressures more than 200 psig.

²American Society of Mechanical Engineers, Boiler and Pressure Vessel Code

- If the ratio of major to minor axis is 2:1, the strength of the head is approximately equal to the strength of seamless cylindrical shell having the corresponding inside and outside diameters. For this reason, most manufacturers have standardized on elliptical dished heads having a 2:1 ratio of axis. The inside depth of the dish is half of the minor axis and is equal to one fourth of the inside diameter of the head.
- The diameter of a circular plate required for forming an ellipsoidal head is approximately 22% greater than the internal diameter of the finished vessel.

Hemispherical head

- For a given thickness, hemispherical heads are the strongest of the formed heads.
- These heads can be used to resist approximately twice the pressure rating of an elliptical dished head or cylindrical shell of the same thickness and diameter.
- If large access openings for manways or nozzles for piping are to be cut into formed closures, the hemispherical head may be used to advantage because its greater strength will make less reinforcement required.
- During spinning of the heads, thinning out of the plate occurs at the corner radius. Therefore, for heads having an outside diameter of under 150 inch, plate thickness must be increased by 1/16 inch for plates upto 1 inch thickness and 1/8 inch for plates 1 to 2 inch thickness, if the minimum thickness is to be maintained throughout the corners.

Stiffening and Reinforcement

- The openings in the cylindrical shell operating under external pressure should be reinforced.
- Vertical vessels may be stiffened by the addition of internal or external members attached in either the longitudinal or circumferential direction, and in some cases in both directions.

Reinforcement of top-course of shell for large open tanks

- Open vessels of large diameter may not have the necessary inherent rigidity to withstand wind loads without deforming and excessively deforming the structure.
- Two methods of stiffening are available: shell plates may be made thicker, or suitable stiffening girders may be added to the structure. The use of thicker shell plates usually is more costly than the use of stiffening girders. The stiffening ring is placed preferably on the outside of the shell rather than the inside. Structural steel can be used for stiffeners.
- If the tanks are closed with a roof, the roof provides additional structural rigidity to the upper course of shell plates. As a result, smaller stiffening rings are used for closed vessels.

Supports

- Skirt supports

- For tall vertical vessels.
- Use mainly for installation on foundation outside buildings, especially if the ratio of apparatus height to diameter is greater than 5.

- Lug supports

- For vertical vessels under internal pressure.
- Lug supported vessels are usually of much smaller height than skirt supported vessels.
- Lug supports are very suitable for thick-walled vessels.

- Saddle supports

- For horizontal vessels.
- More than two saddles is undesirable.
- Many horizontal cylindrical apparatus such as storage vessels, heat exchangers, etc., are commonly supported by saddle supports or cradles.

Tall Vertical Vessels

- Stresses in tall vertical vessels

- (i) The axial and circumferential stresses resulting from internal pressure or vacuum in the vessel.
 - (ii) The compressive stresses resulting from dead loads including the weight of the vessel itself plus its contents and the weight of insulation and attached equipments.
 - (iii) Stresses resulting from bending moments caused by wind loads acting on the vessel and its attachments.
 - (iv) Stresses caused by any eccentricity resulting from irregular load distribution.
 - (v) Stresses resulting from seismic (earthquake) forces.
 - (vi) In addition, stresses during fabrication, welding, and cold working, etc.
- The height of columns should not exceed 175 feet, because of foundation and wind-loading problems.

Process Design

- Distillation column trays are usually 18 or 24 inch apart.
- For columns less than 3 feet (1 m) in diameter, packed towers are usually used (in preference to tray columns) because of high cost involved in fabricating small trays.

9.2 Objective Type Questions

1. In a manufacturing industry, break-even point occurs when (GATE-1999-1.29)

- (a) the annual rate of production equals the assigned value
- (b) the total annual product cost equals the total annual sales
- (c) the annual profit equals the expected value
- (d) the annual sales equals the fixed costs

Answer: (b)

2. A standard type of heat exchanger with a negligible scrap value costs Rs.40,000 and will have a useful life of 6 years. Assuming an effective compound interest of 10% per year, the capitalized cost of a heat exchanger will be (GATE-1991-10.ii.a)

- (a) Rs.60,880 (b) Rs.90,880 (c) Rs.91,880 (d) Rs.81,880

$$\text{Answer: (c)} \quad K = C_V + \frac{C_R}{(1+i)^n - 1} = 40000 + \frac{40000}{1.1^6 - 1} = 91843.$$

3. The total capital investment for a chemical plant is Rs.1,000,000 and the working capital is Rs.100,000. If a turnover ratio is 1, the gross annual sales will be: (GATE-1991-10.ii.c)

- (a) Rs.800,000 (b) Rs.900,000 (c) Rs.1,000,000 (d) Rs.1,100,000

$$\text{Answer: (b)} \quad \text{Turn over ratio} = \frac{\text{gross annual sales}}{\text{fixed capital investment}}$$

$$1 = \frac{\text{gross annual sales}}{1000000 - 100000}$$

4. An investment of Rs.100 lakhs is to be made for construction of a plant which will take two years to start production. The annual profit from operation of the plant is Rs.20 lakhs. What will be the payback time? (GATE-2001-1.22)

- (a) 5 years (b) 7 years (c) 12 years (d) 10 years

Answer: (b)

5. An investment of Rs.1000 is carrying an interest of 10% compounded quarterly. The value of the investment at the end of five years will be (GATE-2001-2.21)

- (a) $1000(1 + 0.1/4)^{20}$ (b) $1000(1 + 0.1)^{20}$
(c) $1000(1 + 0.1/4)^5$ (d) $1000(1 + 0.1/2)^5$

$$\text{Answer: (a)} \quad \text{Effective interest rate } i_{\text{eff}} = (1 + r/m)^m - 1 = (1 + 0.1/4)^4 - 1$$

$$S = P(1 + i_{\text{eff}})^n = 1000(1 + 0.1/4)^{4 \times 5} = 1000(1 + 0.1/4)^{20}$$

6. P is the investment made on an equipment, S is its salvage value and n is the life of the equipment in years. The depreciation for the m^{th} year by the Sum-of-Years-Digits method will be
 (GATE-2001-1.22)

(a) $\frac{P-S}{n}$

(c) $\frac{m}{n}(P-S)$

(b) $1 - \left(\frac{P}{S}\right)^{\frac{1}{m}}$

(d) $\frac{2(n-m+1)}{n(n+1)}(P-S)$

Answer: (d) Depreciation for m^{th} year = $\frac{n-m+1}{1+2+\dots+n}(P-S) = \frac{2(n-m+1)}{n(n+1)}(P-S)$

7. The ends of a cylindrical vessel can be closed by a head, which can be one of the four shapes. For the same thickness, choose the one which can withstand the highest pressure.
 (GATE-2001-1.21)

- (a) flat plate (b) hemispherical (c) torispherical (d) ellipsoidal

Answer: (b)

8. In a cylindrical vessel subjected internal pressure, the longitudinal stress σ_l , and the circumferential stress σ_h are related by
 (GATE-2001-2.23)

- (a) $\sigma_h = 2\sigma_l$ (b) $\sigma_h = \sigma_l$ (c) $\sigma_h = \sigma_l/2$ (d) no relation exists

Answer: (a) $\sigma_h = \frac{pd}{2t}; \sigma_l = \frac{pd}{4t}$

9. A cylindrical shell A closed at the ends have been provided with 3 circumferential ring stiffeners. Shell B is identical to A but is provided with 6 circumferential stiffeners. Which vessel can withstand more internal pressure?
 (GATE-1988-9.b.iv)

- (a) A (b) B
 (c) cannot be predicted (d) same pressure in both

Answer: (b) Addition of stiffeners increases the strength.

10. Two 30 cm diameter steel pipes are connected by:
 (GATE-1988-9.b.ii)

- (a) threaded joints (b) union coupling (c) flanges (d) sleeves

Answer: (c) Threaded joints and union couplings are for pipes of sizes less than 5 cm.

11. For a cylindrical internally pressurized vessel, which of the following closure types would withstand highest pressure if each closure is of the same material and thickness?
 (GATE-1998-1.25)

- (a) Hemispherical (b) Ellipsoidal (2:1) (c) Conical (d) Flat plate

Answer: (a)

12. For a cylindrical vessel of moderate height, the weld joint efficiency for joints parallel to the cylinder axis is given as 1.0 while for joints along the girth (circumference) its is given as 0.8. In calculating the shell wall thickness using code formula, for an internally pressurized cylindrical vessel, what value of weld joint efficiency should be used?
 (GATE-1998-2.25)

- (a) 0.8 (b) 0.9 (c) 1.0 (d) $(0.8)^{0.5}$

Answer: (a)

9.3 Problems with Solutions

9.3.1 Cost Index & Capacity Factor

A heat exchanger of area 10 m^2 costed Rs.50,000 in the year 1985. What is the estimated cost of a 15 m^2 exchanger in 1987. Assume that the cost index in 1985 was 270 and in 1987 it is 320.
 (GATE-1988-19i)

Solution:

From six-tenths-factor rule,

$$\text{Cost of equipment of } a = \text{cost of equipment } b \times \left(\frac{\text{capacity of equipment } a}{\text{capacity of equipment } b} \right)^{0.6}$$

Therefore,

$$\begin{aligned} \text{cost of heat exchanger of size } 15 \text{ m}^2 \text{ in 1985} &= 50000 \times (15/10)^{0.6} \\ &= \text{Rs.}63771.23 \end{aligned}$$

Relation for cost index:

$$\text{Present cost} = \text{original cost} \times \left(\frac{\text{index value at present time}}{\text{index value at time original cost was obtained}} \right)$$

Therefore,

$$\begin{aligned} \text{cost of heat exchanger of size } 15 \text{ m}^2 \text{ in 1987} &= 63771.23 \times (320/270) \\ &= \text{Rs.}75580.71 \end{aligned}$$

9.3.2 Capital Cost of Fluid-processing Plant

If the delivered costs of equipments of a fluid processing plant is Rs. 4×10^6 , what is the capital cost of the plant?
(GATE-1994-30)

Solution:

Table 9.1: Lang multiplication factors for estimation of fixed-capital investment or total capital investment.

Factor \times delivered equipment cost = fixed-capital investment or total capital investment.

Type of plant	Factor for	
	Fixed-capital investment	Total capital investment
Solid-processing plant	3.9	4.6
Solid-fluid-processing plant	4.1	4.9
Fluid-processing plant	4.8	5.7

From the above Table,

$$\begin{aligned} \text{Total capital cost of the plant} &= 5.7 \times (4 \times 10^6) \\ &= \text{Rs. } 22.8 \times 10^6. \end{aligned}$$

9.3.3 Depreciation

The plant of a chemical company has an initial worth of Rs.50 lakhs, and an estimated salvage value of Rs.2 lakhs in a service life of 8 years.

- (a) Given a choice between the straight-line and declining-balance methods of depreciation. Which method would you recommend to save tax and why?
- (b) Estimate the book value of the plant at the end of 4 years for each of the two methods of depreciation.
(GATE-1992-20c)

Solution:

(a) Declining-balance method of depreciation is to be chosen, as this method permits greater depreciation allowances in the early life of the property than in the latter life.

(b) Book value at the end of 4 years:

(i) Straight line method:

The annual depreciation cost d is given as:

$$d = \frac{V - V_s}{n} \quad (9.1)$$

where V = original value of the property at the start of the service-life period

V_s = salvage value of property at the end of service life
 n = service life, years

The asset value V_a (or book value) of the equipment at any time during the service life is given as:

$$V_a = V - ad \quad (9.2)$$

where a = number of years in actual use.

Given: V = Rs. 50 lakhs; V_s = Rs. 2 lakhs; n = 8 years; and a = 4 years.

Therefore,

$$d = \frac{50 - 2}{8} = \text{Rs. } 6 \text{ lakhs}$$

And

$$V_a = 50 - 4 \times 6 = \text{Rs. } 26 \text{ lakhs}$$

(ii) Declining balance method:

The asset value V_a is given by the relation:

$$V_a = V(1 - f)^a \quad (9.3)$$

where f represents the fixed-percentage factor, related to V and V_s as:

$$f = 1 - \left(\frac{V_s}{V} \right)^{1/n} \quad (9.4)$$

Therefore,

$$f = 1 - \left(\frac{2}{50} \right)^{1/8} = 0.3313$$

And

$$V_a = 50 \times (1 - 0.3313)^4 = \text{Rs. } 10 \text{ lakhs}$$

9.3.4 Present Worth of a Future Amount

A bond matures after five years and has a maturity value of Rs.1000. If the interest rate is 12%, what is the present worth of the bond?
(GATE-1988-19iii)

Solution:

The present worth (or present value) P of a future amount S after n interest periods can be determined from

$$P = S \frac{1}{(1+i)^n} \quad (9.5)$$

where i = compound interest rate.

Therefore,

$$\text{Present worth of the bond} = 1000 \frac{1}{(1+0.12)^5} = \text{Rs.}567.43$$

9.3.5 Capitalized Cost

An equipment costs Rs.1,70,000 and will have a scrap value of Rs.25,000 at the end of its useful life of 10 years. If the interest is compounded at 10% per year, what are the cost of replacement, the present worth and the capitalised cost? (GATE-1989-19i)

Solution:

If cost of equipment remains at constant value, then

$$\text{cost of replacement } C_R = \text{equipment cost at present time} - \text{scrap value} \quad (9.6)$$

Therefore

$$C_R = 170000 - 25000 = \text{Rs.}1,45,000.$$

Present worth P is related to C_R as

$$P = \frac{C_R}{(1+i)^n - 1} \quad (9.7)$$

Therefore

$$P = \frac{145000}{(1+0.1)^{10} - 1} = \text{Rs.}90,981.$$

Capitalized cost K is related to C_R and original cost of equipment C_V as

$$K = C_V + \frac{C_R}{(1+i)^n - 1} \quad (9.8)$$

Therefore

$$K = 170000 + 90981 = \text{Rs.}2,60,981.$$

9.3.6 Annuity & Periodic Payment

A piece of equipment has an initial installed value of Rs.240,000. It is estimated that its useful life period will be 12 years and its scrap value at the end of 12 years will be Rs.40,000. The depreciation will be charged as a cost by making equal charges each year, the first payment being made at the end of first year. The depreciation fund will be accumulated at an annual interest rate of 12%. At the end of the life period, enough money must have been

accumulated to account for the decrease in equipment value. Determine the yearly cost due to depreciation under these conditions. (GATE-1987-18iii)

Solution:

The amount of annuity S (i.e., future worth of the accumulated amount) and the periodic payment R required are related as

$$S = R \frac{(1+i)^n - 1}{i} \quad (9.9)$$

where n = number of interest periods

i = interest rate

Given: $S = 240000 - 40000 = \text{Rs.}200,000$; $n = 12$ years; and $i = 0.12$. Therefore,

$$200000 = R \frac{(1+0.12)^{12} - 1}{0.12}$$

Solving, $R = \text{Rs.}8288/\text{year}$; i.e., the yearly cost due to depreciation = Rs.8288.

9.3.7 Net Profit & Payout Period

A plant is designed to produce 1.2×10^8 kg/yr of an agrochemical. The estimated fixed capital investment is Rs. 1.5×10^9 . The working capital is Rs. 2×10^8 and start-up cost (only in the first year of commissioning and to be accounted for in the first year) is Rs. 1.5×10^8 .

The following cost data are available:

Raw materials: Rs. 0.8/kg product

Labour and utilities, etc: Rs. 0.27/kg product

Selling price of product: Rs. 10/kg

Other costs (on per year basis) including maintenance, insurance, etc. @10% of fixed capital.

Indirect cost of administration, R& D, marketing, etc. 20% of sale proceeds.

The plant will be fully depreciated over a period of 5 years using the straight-line method.

The rate of income tax is 40%.

Calculate

(a) the net profit at the end of first year

(b) the payout period

(GATE-2000-12)

Solution:

Basis: 1 year

Total sales	$= 10 \times 1.2 \times 10^8 = \text{Rs. } 12 \times 10^8$
Cost of raw materials	$= 0.8 \times 1.2 \times 10^8 = \text{Rs. } 0.96 \times 10^8$
Cost of labour & utilities	$= 0.27 \times 1.2 \times 10^8 = \text{Rs. } 0.324 \times 10^8$
Other costs including maintenance	$= 0.1 \times 1.5 \times 10^9 = \text{Rs. } 1.5 \times 10^8$
Indirect cost of administration	$= 0.2 \times 12 \times 10^8 = \text{Rs. } 2.4 \times 10^8$
Startup cost in the first year	$= \text{Rs. } 1.5 \times 10^8$
Depreciation	$= \frac{1.5 \times 10^9}{5} = \text{Rs. } 3 \times 10^8$

For the first year:

Total expenses	$= (0.96 + 0.324 + 1.5 + 2.4 + 1.5 + 3) \times 10^8$
	$= \text{Rs. } 9.684 \times 10^8$
Income tax	$= (\text{revenue} - \text{total expenses}) \times \text{tax rate}$
	$= (12 \times 10^8 - 9.684 \times 10^8) \times 0.4$
	$= \text{Rs. } 2.316 \times 10^8$
Net profit at the end of first year	$= \text{revenue} - \text{total expenses} - \text{income tax}$
	$= (12 - 9.684 - 2.316) \times 10^8 = \text{Rs. } 0$

Net profit for the subsequent years:

Total expenses	$= 9.684 \times 10^8 - 1.5 \times 10^8 = \text{Rs. } 8.184 \times 10^8$
Income tax	$= (12 \times 10^8 - 8.184 \times 10^8) \times 0.4 = \text{Rs. } 1.5264 \times 10^8$
Net profit per year	$= (12 - 8.184 - 1.5264) \times 10^8 = \text{Rs. } 2.2896 \times 10^8$

Payout period:

$$\begin{aligned} \text{Payout period in years} &= \frac{\text{depreciable fixed-capital investment}}{\text{avg profit/yr} + \text{avg depreciation/yr}} \\ &= \frac{1.5 \times 10^9}{2.2896 \times 10^8 + 3 \times 10^8} = 2.84 = 3 \end{aligned}$$

9.3.8 Capitalized Cost - Useful Life for the Alternative

A new equipment made of material A costs, post installation, Rs.3,00,000 and is expected to have a scrap value of 10% of this cost at the end of a useful life of 10 years. Similar equipment made of material B costs Rs.1,50,000, but is likely to have no scrap value. Assume that both types of equipments could be replaced at a cost that is 20% more than the original value. On the basis of equal capitalized costs for both types of equipments, estimate what should be the useful life for equipment made of material B. The company has to pay an annual interest on the investment at a rate of 15%. (GATE-1999-21)

Solution:

Capitalized cost K is related to cost of replacement C_R and original cost of equipment C_V as

$$K = C_V + \frac{C_R}{(1+i)^n - 1} \quad (9.10)$$

Therefore capitalized cost of equipment made of material A

$$K_A = 300000 + \frac{1.2 \times 300000 - \overbrace{0.1 \times 300000}^{\text{scrap value}}}{(1+0.15)^{10} - 1} = \text{Rs. } 4,08,355.$$

Capitalized cost of equipment made of material B is given by

$$K_B = 150000 + \frac{1.2 \times 150000}{(1+0.15)^n - 1}$$

Since it is given that K_B and K_A are to be equal,

$$408355 = 150000 + \frac{1.2 \times 150000}{(1+0.15)^n - 1}$$

Solving, $n = 3.78$ years.

Therefore, the useful life of equipment made of material B should be 3.87 years.

9.3.9 Capitalized Cost as a Criteria

A heat exchanger with an initial investment of Rs.300,000 has a 6 years life. How much can be spent on an improved design which has a life of 12 years and is expected to save Rs.10,000 per year?

Annual compound interest rate = 8%.

(GATE-1991-20ii)

Solution:

$$\text{Capitalized cost} = \frac{C_R(1+i)^n}{(1+i)^n - 1} + V_s \quad (9.11)$$

where C_R = cost of replacement

V_s = salvage value at the end of useful life

n = estimated useful life of equipment

i = interest rate

For the exchanger with 6 years life:

$$\text{Capitalized cost} = \frac{300000(1+0.08)^6}{(1+0.08)^6 - 1} = \text{Rs. } 811183$$

Present worth (P) of savings from the exchanger with 12 years life:

$$\begin{aligned} P &= R \frac{(1+i)^n - 1}{i(1+i)^n} \\ &= 10000 \frac{(1+0.08)^{12} - 1}{0.08(1+0.08)^{12}} = \text{Rs. } 75361 \end{aligned} \quad (9.12)$$

Capitalized cost of the improved exchanger with 12 years life shall be equal to the sum of the present worth of its savings and the capitalized cost of exchanger with 6 years life.

Therefore, for the improved exchanger

$$75361 + 811183 = \frac{C_R(1+0.08)^{12}}{(1+0.08)^{12} - 1}$$

Solving, $C_R = \text{Rs. } 534485$.

Therefore an amount of Rs. 534485 shall be invested for the improved design.

9.3.10 Discounted-Cash-Flow Rate of Return

For a project having a life of ten years the following cash flow pattern is expected:

End of year	Net cash flow (Rs.)
0	-50,00,000
1-10	20,00,000
10	-1,50,00,000

If the expected interest rate is 20 percent, what is your recommendation about implementing the project?
(GATE-1990-19ii)

Solution:

At the end of 10 years, the net cash flow S to the project, computed on the basis of end-of-year income, will be

$$\begin{aligned} S &= -5 \times 10^6 (1+0.2)^{10} \\ &\quad + 2 \times 10^6 [(1+0.2)^9 + (1+0.2)^8 + (1+0.2)^7 + (1+0.2)^6 \\ &\quad + (1+0.2)^5 + (1+0.2)^4 + (1+0.2)^3 + (1+0.2)^2 + (1+0.2)] \\ &\quad + 2 \times 10^6 - 15 \times 10^6 \\ &= -30.96 \times 10^6 + (2 \times 10^6)(24.96) + 2 \times 10^6 - 15 \times 10^6 \\ &= \text{Rs. } 5.96 \times 10^6 \end{aligned}$$

Since there is a positive net-cash flow, it is recommended to implement the project.

9.3.11 Alternative Investments

A single effect evaporator costs Rs. 100,000 and a two effect evaporator costs Rs. 200,000 for identical duty. The annual steam cost for single effect is Rs. 25,000, and for double effect it is Rs. 5,000. Depreciation is calculated on a straight line basis, assuming a ten-year service life and zero salvage value. Should one effect or two effects be used if the value of money is 15%?
(GATE-1988-19ii)

Solution:

Annual charge for capital recovery (R) is related to the present worth of a capital (P) as

$$R = P \frac{i(1+i)^n}{(1+i)^n - 1} \quad (9.13)$$

For single-effect evaporator:

$$\begin{aligned} \text{Annual charge for capital recovery} &= 100000 \frac{0.15(1+0.15)^{10}}{(1+0.15)^{10} - 1} = \text{Rs. } 19925 \\ \text{Annual steam cost} &= \text{Rs. } 25000 \\ \text{Total annual cost} &= 19925 + 25000 = \text{Rs. } 44925 \end{aligned}$$

For two-effect evaporator:

$$\begin{aligned} \text{Annual charge for capital recovery} &= 200000 \frac{0.15(1+0.15)^{10}}{(1+0.15)^{10} - 1} = \text{Rs. } 39850 \\ \text{Annual steam cost} &= \text{Rs. } 5000 \\ \text{Total annual cost} &= 39850 + 5000 = \text{Rs. } 44850 \end{aligned}$$

Since the total annual cost of two-effects evaporator is less than that of single effect, two-effects evaporator is to be choosed.

9.3.12 Rate of Return as a Profitability Criteria

A plant is producing 1000 tonnes/year of a product. The overall yield is 70% on mass basis. The raw material costs Rs.100/t, and the product sells at Rs.350/t. A process modification has been devised that will increase the yield to 75%. The additional investment required is Rs.400,000. Is the modification worth making if the minimum acceptable rate of return is 20%?
(GATE-1987-18i)

Solution:

$$\begin{aligned} \text{Increase in production rate} &= 1000 \times (75/70) - 1000 \\ &= 71.4 \text{ t/year} \\ \text{Additional profit per year} &= 71.4 \times (350 - 100) \\ &= \text{Rs. } 17850 \\ \text{Rate of return for the additional investment} &= \frac{17850}{400000} \times 100 = 4.5\% \end{aligned}$$

Since this is less than the minimum acceptable rate of return of 20%, it is not worth to make process modification.

9.3.13 Break-Even Point

The annual production costs for a plant are Rs.36.4 lakhs, while the sum of the annual fixed charges, overhead costs and general expenses are Rs.26.0 lakhs. What is the break-even point, in units of production per year if the total annual sales are Rs.72.8 lakhs and the product sells at Rs.520 per unit?
(GATE-1989-19ii)

Solution:

$$\text{Number of units produced} = \frac{72.8 \times 10^5}{520} = 14000$$

$$\text{Production cost per unit} = \frac{36.4 \times 10^5}{14000} = \text{Rs.}260$$

At break-even point,

$$\text{total annual sales} = \text{total product cost}$$

If n is the number of units at break-even point, then

$$520 n = 260 n + 26 \times 10^5$$

Solving, $n = 10000$.

9.3.14 Economic Pipe Diameter

Capital cost of a pipeline of exotic material is estimated as $3 D^{1.5}$ Rs/m. where D is the pipe diameter in mm. The annual maintenance cost is estimated as 10% of the total capital cost. Annual operating cost of the pipe is given as follows:

$$\text{Annual operating cost} = 3 \times 10^{15}/D^5 \text{ Rs/(m.year)}$$

Estimate the most economic pipe diameter based on the least annual cost approach to the nearest multiple of 10 mm. Estimated amortization period is 10 years. (GATE-1998-27)

Solution:

Total annual cost C_T of pipeline is related to the pipe diameter as

$$\begin{aligned} C_T &= \frac{3D^{1.5}}{10} + 0.1 \times 3D^{1.5} + \frac{3 \times 10^{15}}{D^5} \\ &= 0.6D^{1.5} + \frac{3 \times 10^{15}}{D^5} \end{aligned}$$

Differentiating this expression with respect to D ,

$$\frac{dC_T}{dD} = 0.9D^{0.5} - \frac{15 \times 10^{15}}{D^6}$$

Minimum total annual cost is obtained by equating dC_T/dD to zero. Therefore,

$$0.9D^{0.5} - \frac{15 \times 10^{15}}{D^6} = 0$$

Solving, $D = 313$ mm.

Diameter to the nearest multiple of 10 mm:

$$\text{Total annual cost of pipeline of } D = 310 \text{ mm} = 0.6 \times 310^{1.5} + \frac{3 \times 10^{15}}{310^5} = \text{Rs. } 4323$$

$$\text{Total annual cost of pipeline of } D = 320 \text{ mm} = 0.6 \times 320^{1.5} + \frac{3 \times 10^{15}}{320^5} = \text{Rs. } 4329$$

Therefore, most economic pipe diameter = 310 mm

Chapter 10

Chemical Technology

10.1 Instant Notes

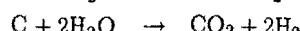
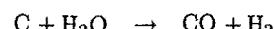
10.1.1 Water Conditioning

- Hardness: expressed in CaCO_3 equivalent.
 - Carbonate hardness - temporary hardness.
 - * Caused by bicarbonates of calcium and magnesium.
 - * Temporary hardness can usually be reduced by heating.
 - Non-carbonate hardness - permanent hardness.
 - * Due to sulfates and chlorides of calcium and magnesium.
- Softening: by cation exchange process.
 - Sodium-cation-exchange process is the most widely employed method for softening water.
 - If demineralization is required, water from cation exchangers is passed through anion-exchange material.

10.1.2 Fuel Gases

- Water gas: or blue gas (because of the color of the flame when it is burned).

– Produced by the reaction of steam on coal/coke.



– Composition:

CO 43%

CO₂ 3%

H₂ 50%

N₂ 3%

- Natural gas

CH ₄	96%
N ₂	3%
CO ₂	1%
- Coke-oven gas

CO	6%
CO ₂	2%
H ₂	53%
CH ₄	32%
N ₂	3%
- Producer gas
 - obtained by passing air and steam through a bed of hot coal or coke.
 - Producer gas has about 15% of the heating value of natural gas.
- Heating value of various gaseous fuels

Natural gas	36–46 MJ/m ³
Coke-oven gas	21 MJ/m ³
Blue-water gas	11 MJ/m ³
Carburated water gas (obtained by atomized oil spraying in water gas)	20 MJ/m ³
Producer gas	5.5 MJ/m ³

- Heat of combustion of common gaseous constituents

CH ₄	37.56 MJ/m ³
C ₂ H ₆	65.80 MJ/m ³
C ₃ H ₈	93.65 MJ/m ³
Butane	121.18 MJ/m ³
Pentane	149 MJ/m ³
- LPG
 - Liquified petroleum gas.
 - Propane-butane mixture.
 - Butane does not vaporize below 0°C whereas propane vaporizes down to –42°C.
- Reactions involving coal gasification to produce substitute natural gas (SNG)

C + H ₂ O → CO + H ₂	Gasification
CO + H ₂ O → CO ₂ + H ₂	Water gas shift reaction
C + CO ₂ → 2CO	Bourouard reaction
CO + 3H ₂ → CH ₄ + H ₂ O	Methanation

10.1.3 Sulfuric Acid

- One of the disadvantages which led to the demise of lead-chamber process is that it can produce acid of only 78% strength.
- Sulfuric acid is a strong dibasic acid.
- Solutions of sulfuric acid may be concentrated economically to about 93% by weight of H₂SO₄. Stronger acids may be made by dissolving sulfur trioxide in 98 to 99% acid.
- The strength of oleum is expressed as percent of free sulfur trioxide. 20% oleum means that, in 100 kg, there are 20 kg of SO₃ and 80 kg of H₂SO₄.
- Sulfuric acid production
 - Vanadium pentoxide (V₂O₅) catalyst
 - SO₂ + 1/2O₂ ⇌ SO₃ equilibrium reaction
SO₃ + H₂O → H₂SO₄
 - The high conversion accomplished in double-absorption plants are not the result of increased catalyst efficiency, but are accomplished by passing the gas through the converter a second time after a majority of the sulfur trioxide has been scrubbed out of the gas, thus equilibrium is disturbed.
 - It has been long known that a concentration of acid between 98.5 and 99% sulfuric acid is the most efficient agent for the absorption of sulfur trioxide, probably because acid of this strength has a lower vapor pressure than any other concentration. Water cannot be used because direct contact of water results in an acid mist that is almost impossible to absorb.

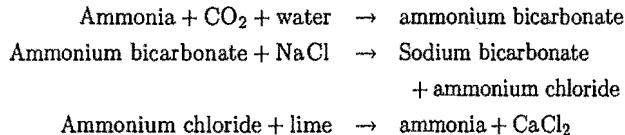
• Materials of construction

- Because of the corrosive effect of hot acid on steel, the drying and absorbing towers and the acid pump tanks associated with these towers are constructed of brick-lined steel.
- Good quality cast iron is the material normally used as piping for the acid circulating over the drying and 98 to 99% absorbing towers.
- Cast iron is not suitable for use in oleum systems.
- In gas purification system, steel is used for handling SO₂ gases having temperatures above the acid dew point. For temperatures below the acid dew point and for liquids, lead or lead-lined steel are used.
- The weak acid coolers in the gas purification system cannot be fabricated of cast iron or steel which would corrode rapidly.

10.1.4 Chlor-Alkali Industries

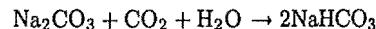
- Soda Ash (Na_2CO_3)

- The present synthetic method of production - Solvay process. Before this was developed, LeBlanc process was in practical use.
- Ernest Solvay developed this ammonia-soda process in 1861.
- Sold on the basis of Na_2O (Sodium oxide) content, which is generally about 58%.
- **Solvay process**
 - * Salt, lime stone and coke or natural gas are the raw materials.



* CaCl_2 is the major pollutant evolved from this process.

- Sodium bicarbonate (NaHCO_3)

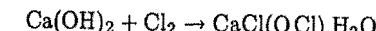


- Also known as Baking soda.
- Mainly used in food industry.
- Causticized soda ash: Soda ash with 10 to 50% caustic.
- Sal soda or washing soda: $\text{Na}_2\text{CO}_3 \cdot 10\text{H}_2\text{O}$
- Caustic Soda NaOH
 - Na_2O content - 76%
 - The term caustic soda is widely used because this compound is corrosive to the skin.
 - Sea water contains about 3.4% salt with $\text{pH} = 8$.
 - Manufacture
 - * Brine electrolysis produces chlorine at the anode and hydrogen along with the alkali hydroxide at the cathode.
 - * Brine electrolysis: 3.0 – 4.5 V per cell.
 - * Evaporation is in nickel tubed evaporators.
 - * Three type of process: Membrane cell, mercury cell and diaphragm cell.
 - * Japan outlawed mercury cell use by 1984.
 - * Diaphragm cell: separates the anode from the cathode. This allows ions to pass through by electrical migration but reduces the diffusion of products.

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- * A major advantage of the diaphragm cell is that it can run on dilute (20%), fairly impure brine.
- * Diaphragm cells and membrane cells use about the same amount of electrical energy, mercury cells somewhat more. The ratio is approximately 3:4.
- * NaCl content in product: More in diaphragm cell product, and comparatively less in membrane cell. Mercury cell product has least chloride content.
- * 50% NaOH is for direct use. Mercury cell produces caustic of this concentration with out concentration.

- Bleaching powder: $\text{CaCl}(\text{OCl}) \cdot \text{H}_2\text{O}$



Contains 49.6% of available chlorine.

- Chlorine dioxide: ClO_2 . Used in water purification, for odor control, and pulp bleaching.

10.1.5 Nitrogen Industries

- Ammonia

- Ammonia is the base from which virtually all nitrogen-containing products are derived.
- Commercial aqueous ammonia - 28% NH_3 .
- **Ammonia production**

- * Iron catalyst with added promoters of oxides - oxides of aluminum, zirconium, or silicon at about 3% concentration and potassium oxide at about 1%.
- * 13-14% conversion per pass.
- * 20 MPa pressure, 500°C.

- Ammonium phosphate: as a fire-retarding agent for wood, paper and cloth.

- Urea

- Urea (NH_2CONH_2) is the most convenient form for nitrogen.
- Production

$$\text{CO}_2 + 2\text{NH}_3 \rightarrow \text{NH}_2\text{COONH}_4$$
 at 14 MPa and 180°C, exothermic reaction.

$$\text{NH}_2\text{COONH}_4 \rightleftharpoons \text{NH}_2\text{CONH}_2 + \text{H}_2\text{O}$$
, endothermic reaction

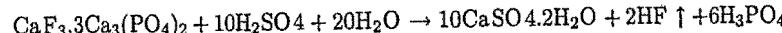
- * Undesirable side reaction: Biuret ($\text{NH}_2\text{CONHCONH}_2 \cdot \text{H}_2\text{O}$) formation, which must be kept low because it adversely affects the growth of some plants.
- * Carbamate ($\text{NH}_2\text{COONH}_4$) is highly corrosive to both ordinary and stainless steels but with oxygen present, 300 series stainless steels resist it very well, so some air is introduced along with the CO_2 reagent to reduce system corrosion.

- Nitric acid

- Ammonia can be burned in air in the presence of a platinum - 10% rhodium catalyst, and nitrogen oxides thus formed can be oxidized further and absorbed in water to form nitric acid.
- NO absorber: Chlorine impurity presents a unique problem in the absorber. Chlorides cannot leave through the bottom because the strong nitric acid oxidizes them to chlorine, which is swept up the column. They cannot leave in the top gas because here NO reduces them to HCl which goes down the column in the aqueous phase. The combination of HCl and HNO₃ is highly corrosive. Chlorides must therefore be excluded from entry or provision made for their purging.

10.1.6 Phosphorus Industries

- Wet process for the production of phosphoric acid: (from fluorapatite ore)



- Single super phosphate: Ca(H₂PO₄)₂.H₂O

- Made from reacting phosphate rock [CaF₃.3Ca₃(PO₄)₂] with sulfuric acid.
- Contains 16-20% of available P₂O₅.

- Triple super phosphate: Ca(H₂PO₄)₂.H₂O.

- Made by the action of phosphoric acid on phosphate rock; no diluent calcium sulfate is formed.
- This material is a much more concentrated fertilizer than ordinary super phosphate, containing from 45 to 46% of available P₂O₅, or nearly 3 times the amount in regular phosphate.

10.1.7 Fertilizers

- NPK: % N - % P₂O₅ - % K₂O
- If the fertilizer sample is urea, then it is expressed as 46 - 0 - 0. If on the other hand, it is normal super phosphate, it is reported as 0 - 20 - 0.
- Of the entire requirement of plant nutrients, two-thirds is in the form of nitrogen.
- Potassium chloride: 97% pure KCl is commonly referred to as *muriate of potash*.
- Plant Nutrients
 - Nitrogen: required to promote development of stems and leaves.
 - Phosphorus: stimulates growth and accelerates fruit and seed formation.
 - Potash: which is essential to the development of starches, sugars and fibers and helps to prevent disease and lessens the effect of excessive nitrogen applications.

10.1.8 Cement Industries

- Wet process: The wet process, though the original one, is being displaced by the dry process, especially for new plants, because of saving in heat, accurate control, and mixing of the raw material it affords.
- Raw Materials: limestone, clay, sand, gypsum, fly-ash, iron-bearing material.
- Dry process kilns may be as short as 45 m, but in the wet process 90 to 180 m kilns are not uncommon.
- Gypsum (4 to 5%) is added as a raw material in cement manufacture, to regulate the settling time of cement. Gypsum acts as a settling retarder. The raw materials are finely ground, mixed, and heated (burned) in a rotary kiln to form cement clinker.
- Regular cement: CaO - 61%, SiO₂ - 21%, Al₂O₃ - 5.8%, Fe₂O₃ - 2.9%, MgO - 2.5%.
- High alumina cement: High alumina cement, essentially a calcium aluminate cement, is manufactured by fusing mixture of limestone and bauxite, that latter usually containing iron oxide, silica, magnesia, and other impurities. It is characterized by a very rapid rate of development of strength and superior resistance to sea water and sulfate-bearing water.

10.1.9 Sugar Industries

- Sugarcane contains 11 to 15% sucrose by weight.
- Bagasse - for burning, for paper & paper board.
- Triple or quadruple effect evaporators are used for the concentration of juice.
- Molasses contains 48 to 55% sugar mainly sucrose. It is the principal raw material for distilleries.
- °Brix is the percentage by weight of sucrose in a pure sugar solution.
- Sucrose (C₁₂H₂₂O₁₁) is a disaccharide, consisting of two monosaccharides: D-glucose and D-fructose.

10.1.10 Alcohol Industries

- In industrial nomenclature 'alcohol' means ethyl alcohol. It contains 95% ethanol and 5% water both by volume at 15.5°C.
- Alcohol by fermentation process: The liquors in the fermentors, after fermentation action are called beer. The beer containing 6.5 to 11% alcohol by volume is separated by distillation.
- Alcohol is second only to water in solvent value.

- Denatured alcohol: The completely denatured formulas are admixtures of substances which are difficult to separate from the alcohol and which smell and taste bad, all this being designed to render the alcohol non-potable. Factories find it an essential raw material.
- One of the chief difficulties in using alcohol in gasoline is that the usual 95% gasoline is not miscible with gasoline. Only absolute or 99.5% alcohol will mix with gasoline for gasohol.
- Beer: 2 to 7% alcohol content.
- Wine: natural - 7 to 14% alcohol, fortified - 14 to 30% alcohol.

10.1.11 Leather Industries

- Vegetable tanning & Chrome tanning.
- More than 90 percent of the world's output of leather is chrome tanned
- Preparation of leather by vegetable tanning requires 2 to 4 months, while chrome tanning requires only 1 to 3 weeks.

10.1.12 Oils and Fats

- Hydrogenation
 - Improves the color, flavor, and odor of the original crude product as well as its keeping power
 - Hydrogenation or hardening, as applied to fats and oils, may be defined as the conversion of various unsaturated radicals of glycerides into more highly or completely saturated glycerides by the addition of hydrogen in the presence of a catalyst.
 - The catalyst is commercially nickel.
 - The reaction is exothermic.
 - It is frequently accompanied by isomerization by cis to trans.
 - The amount of hydrogen necessary is a function of the degree of reduction of unsaturation required, as measured by the decrease in the iodine number of the oil during hydrogenation. The theoretical quantity needed to reduce the iodine number one unit is 0.95 m^3 of hydrogen per metric ton of oil.
 - The objective of hydrogenation is not only to raise the melting point but also to greatly improve the keeping qualities, taste and odor of many oils.
 - Solvent extraction of vegetable oils uses hexane as the solvent.

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10.1.13 Pulp & Paper Industries

- Kraft or sulfate (Na_2SO_4) pulping: alkaline process, by which most pulp is presently made.
 - Cooking solution: Na_2S , NaOH and Na_2CO_3 .

10.1.14 Petroleum Processing

- A petroleum crude which gives diesel oil of high cetane number, gives gasoline of low octane number and vice versa.
- Cetane number increases in the order of

Aromatics → Isoparaffins → Naphthenes → Olefins → n-paraffins

Where as, octane number decreases in the same order.

- Cracking, or pyrolysis
 - Cracking means heating of higher boiling petroleum fractions like heavy fuel oil at high temperature (above decomposition temperature) and pressure to produce lower boiling lighter fractions.
 - Zeolite catalysts are common.
 - Cracking of heavy fuel oils is done to produce mainly petrol. Cracking is also done to viscosity of fuel oil (visbreaking), to produce coke (coking), and to produce olefins which is used for petrochemical industry.
- Reforming (or aromatization)
 - Reforming means rearrangement of molecules without much affecting the average molecular weight of feed which is generally naphtha of gasoline boiling range.
 - Reforming is carried out to produce high quality (octane number) gasoline by heating with or without catalyst naphtha. Low octane number thermally cracked gasoline is also subjected to reforming to improve its quality.
 - The main reaction in the reforming process is dehydrogenation of naphthene to produce aromatics.
- Platforming: The name comes from the fact that this catalytic reforming process uses Al_2O_3 catalyst containing 0.25% platinum.
- Isomerization
 - Alteration of the arrangement of the atoms in a molecule without changing the number of atoms.
 - Present isomerization applications in petroleum refining are to provide additional feedstock for alkylation units or high-octane fractions for gasoline blending.

- Straight-chain paraffins (n-butane, n-pentane, and n-hexane) are converted to respective iso-compounds by continuous catalytic (aluminum chloride and noble metals) processes.
- Non-regenerable aluminum chloride catalyst is employed with various carriers in a fixed-bed or liquid contactor.
- Platinum or other metal catalyst processes utilize a fixed bed operation and can be regenerable or non-regenerable.

• Alkylation

- The combination of olefins with paraffins or aromatics to form higher isoparaffins is termed alkylation.
- Although alkylation is possible without catalysts, commercial processes use aluminium chloride, sulfuric acid, or hydrogen fluoride as catalysts when the reactions can take place at low temperatures, minimizing undesirable side reactions such as polymerization of olefins.
- In refinery practice, only iso-butane is alkylated by reaction with iso- or normal butane and iso-octane is the product.
- Polymerization: In the petroleum industry polymerization is the process by which olefin gases are converted to liquid condensation products that may be suitable for gasoline (hence polymer gasoline) or other liquid fuels.

• Dewaxing

- Removal of wax from petroleum product.
- Methyl ethyl ketone (MEK) is used as a dewaxing solvent

• Sweetening

- Removal of sulfur and its compounds like sulfides, mercaptants etc, from petroleum products is called sweetening. Presence of these compounds makes the oil sour. Sour stocks have bad odor. The processes by which foul smelling petroleum products are converted into sweet smelling or odorless products are called sweetening processes.
- Several processes are available for the alteration of objectionable sulfur and the consequent improvement in odor. The doctor treatment was the original method of "doctoring" of oil to reduce the odor and utilized sodium plumbite (Na_2PbO_2) to convert mercaptants to less objectionable disulfides.
- Octane boosters: Methyl tertiary butyl ether (MTBE), methanol, ethanol etc.

10.1.15 Polymer Industries

• Polyamides

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- Nylon 6,6: (Nylon 66) obtained by polymerization reaction of adipic acid and hexamethylene diamine. The number 6,6 is because of each of the raw material chain contains 6 carbon atoms.
- Nylon 6: is the homopolymer of caprolactum. It has similar properties to nylon 66, but posses better abrasion resistance
- Polyesters: The commonest polyester fibers are polymers of the ester formed from dimethyl terephthalate and ethylene glycol
- Thermosetting resins: The most important thermosetting resins are formaldehyde condensation products with phenol, or with urea or melamine. Other thermosetting types include the epoxy resins, unsaturated polyester resins, urethane foams etc.
- Polymerization procedures

— Bulk polymerization

- * Bulk polymerization are those in which only the monomer (or reactants) plus possibly an initiator or a catalyst are added to a reactor vessel.
- * In other types of polymerization, a solvent or water is also added.
- * Bulk polymerization often produce higher purity polymers and in general require simpler separation procedures.
- * e.g: for the production of polystyrene, polyethylene, polypropylene, polyethylene terephthalate (PET) and nylon

— Solution or solvent polymerization

- * Solution polymerization is often resorted to for those applications in which the polymer is required in solution form.
- * Surface coatings and impregnated compositions are among the commonest examples where the polymer is applied in solution.

— Emulsion polymerization

- * Emulsion polymerizations generally produce polymers with very high molecular weights. As a consequence, emulsion processes are often used for the production of polymers requiring high molecular weights such as Styrene-Butadiene elastomer (SBR) and Teflon.

— Suspension polymerization

- * Suspension polymerization processes are used for about 90% of all PVC produced. In addition, major amounts of polymers are produced using other olefins, including styrene and methyl methacrylate.

10.1.16 Rubbers or Elastomers

- About 60% of the world market is held by styrene-butadiene rubber (SBR), which is based on a copolymer of about 75% butadiene and 25% styrene.
- Polybutadiene rubber (BR) itself holds almost 20% of the market.

- Natural rubber: obtained from Hevea Brasiliensis tree. Natural rubber is cis-1,4-polyisoprene.
- Vulcanization leads to a cross-linked thermoset polymer, that cannot be softened or melted by heating to the original melting point.

10.1.17 Common Names

Gypsum	$\text{CaSO}_4 \cdot 2\text{H}_2\text{O}$
Epsom salt	$\text{MgSO}_4 \cdot 7\text{H}_2\text{O}$
Lime or Quick lime	CaO
Lime stone	CaCO_3
Hydrated or slaked lime	$\text{Ca}(\text{OH})_2$
Caustic soda	NaOH
Soda ash	Na_2CO_3
Bleaching powder	CaClOCl
Salt or common salt	NaCl
Sal soda or Washing soda	$\text{Na}_2\text{CO}_3 \cdot 10\text{H}_2\text{O}$
Baking soda	Sodium bicarbonate (NaHCO_3)
Hypo	Sodium thiosulfate ($\text{Na}_2\text{S}_2\text{O}_3$)
Salt cake	Crude Sodium sulfate (Na_2SO_4)
Glauber's salt	$\text{Na}_2\text{SO}_4 \cdot 10\text{H}_2\text{O}$
Niter cake	NaHSO_4 (sodium bisulfate)
Synthesis gas	A mixture of CO and H ₂ for synthesis of organic compounds
Ammonia synthesis gas	$3\text{H}_2 + 1\text{N}_2$
Naphtha	Materials with boiling ranges between gasoline and kerosene

10.1.18 Safety Tips

- For diluting the acid: Don't add water to acid. Add acid to water.
- Caustic burns: wash with 5% ammonium chloride solution.
- Limits of Flammability : There are two composition limits of flammability for air and a gaseous fuel under specified conditions. The lower limit corresponds to the minimum concentration of combustible gas that supports combustion, the higher limit the maximum concentration.

10.1.19 Materials of Construction for the Process Industries

- Mild steel
 - low carbon steel contains 0.002 to 0.2% carbon

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- Ordinary steel is widely used for sulfuric acid in concentrations over 70%. Lead and steel complement each other. Lead takes over below 70% acid where steel is attacked, and vice versa.
- Lead-chamber process generally produces acid up to 78% concentration. The contact plants produce strong acid and oleum. This accounts for the extensive use of lead in the former and steel in the latter.
- Ordinary carbon steels and cast irons are never used for hydrochloric acid.

• Cast iron

- Usually they contain 2.5 to 4.2% carbon and some amount of silica. It has the ability to take good impression during casting. Because of the ease of casting it has been a traditional material of construction for pumps, valves etc.
- Gray cast iron: The presence of flake graphite in gray cast iron accounts for its good machinability as graphite acts as a lubricant. This cast iron accounts for nearly three fourth of the castings.
- Duriron - High silicon cast iron contains 14.5% silicon, 0.95% carbon
- Durichlor - 14.5% silicon, 0.95% carbon & 3% molybdenum.

• Stainless steels: Stainless steels are ferrous alloys containing minimum of 12% chromium. Other alloying elements, principally nickel may be added for improving corrosion resistance and fabricability.

SS 304	18 - 20 % Cr, 8 - 12 % Ni, 0.08% (max) C	18-8 stainless steel, most commonly used material for process equipments
SS 304L	18 - 20 % Cr, 8 - 12 % Ni, 0.03% (max) C	Low carbon version of SS 304
SS 316	16 - 18 % Cr, 10 - 14% Ni, 2 - 3 % Mo, 0.1% (max) C	Addition of molybdenum improves resistance to chloride environments.
SS 316L	16 - 18 % Cr, 10 - 14% Ni, 2 - 3 % Mo, 0.03% (max) C	Low carbon version of SS 316
SS 430	14 - 18 % Cr, 0.5% Ni	Tableware. The first chemical plant application of stainless steel was SS 430 tank-car for shipping nitric acid.

• Intergranular corrosion

- When an austenitic stainless steel is held in the temperature range of 430°C to 900°C the carbon in solution has a tendency to migrate to grain boundaries and react with the chromium to form its carbide, which precipitates there. As a consequence, the areas around the grain boundaries are depleted of chromium.

The reduction of chromium introduces heterogeneity and concentration gradient which are fertile grounds for corrosive attack. The local depletion of chromium can also take place to the extent of a non-protective or non-passive level. During welding operation, a zone just outside the actual weld about 3 mm from it, is in such corrosion susceptible condition. As the carbide precipitation occurs along the grain boundaries and the corrosion takes place around them and parallel to the boundaries it is usually termed "intergranular corrosion".

- The intergranular corrosion can be eliminated by taking such measures as:
 - (i) reduction in the amount of carbon to a level as low as 0.03 % (max) as in AISI grades 304L and 316L.
 - (ii) making stainless steel alloys that contain titanium or columbium which have greater affinity for carbon than chromium and will therefore preferentially combine with it during welding.
- Nickel: Most tough corrosion problems involving caustic and caustic solutions are handled with nickel. Corrosion resistance to caustic is almost directly proportional to the nickel content of an alloy.
- Nickel alloys:

Monel	66% Ni, 31% Cu, 1.4% Fe
Inconel	76% Ni, 18% Cr, 8% Fe
Hastelloy C	56% Ni, 15% Cr, 17% Mo, 5% Fe
- Monel is a natural for hydrofluoric acid
- Zinc - as sacrificial metal for cathodic protection of steel
- Galvanized - zinc coated
- Copper: copper is highly resistant to atmospheric gases and contaminants in rural and marine environments.
- Brass: these are alloys of copper with zinc and the amount of zinc generally varies from 15 to 45%.
- Bronze: originally copper-tin alloys were called bronzes, now they are called tin bronzes. Similarly copper-aluminum alloys are called aluminum bronzes and so on. Tin bronze is superior to brass in corrosion resistance and is less liable to stress corrosion cracking. It can withstand the action of alkalies better than brass or aluminum bronze.
- Titanium - wet chlorine: The industries like paper, textile, plastics and detergents which use wet chlorine and bleaching agents have started using titanium equipment for extended life of their plant and equipment.
 - Should not be used with dry chlorine
- Aluminum in nitric acid shows somewhat the same general pattern as steel in sulfuric acid. Aluminum and its alloys show excellent resistance to strong nitric acid, but they are readily attacked by concentrations below 80%.

- Ammonia and ammoniacal solutions generally do not present difficult corrosion problems. In manufacture and handling, steel and cast iron are satisfactory except for high temperatures, where types 430 and 304 stainless steels are required. Aluminum is often adopted in refrigeration systems and storage tanks. The major warning is not to use copper and copper-base alloys because even traces of ammonia can cause *stress corrosion*.
- Glass: has excellent resistance to most acids, both inorganic and organic. The exceptions are hydrofluoric acid of all concentrations and hot concentrated phosphoric acid. Acids primarily attack the alkali in the glass, and the remaining silica then provides an inert barrier for further attack. But in the case of hydrofluoric acid, the silica itself reacts with the acid.
 - Alkaline solutions attack the silica matrix of glass and the rate of attack increases with the strength of alkali and rise of temperature.
 - Abrasion resistance of different glasses varies, e.g. borosilicate glass has three times abrasion resistance of soda-lime glass.
- HCl Production: HCl gas is absorbed in water in a tantalum or impervious/impregnated graphite absorber. Karbate (resin bonded graphite) is normally used as a material of construction for HCl absorber.

10.2 Objective Type Questions

1. The most widely used coagulant for removing suspended impurities from water is (GATE-1994-1.i)

(a) Bleaching powder	(b) Chlorine
(c) Calcium sulfate	(d) Alum

Answer: (d)
2. Synthesis gas is a mixture of

(a) CO and H ₂	(b) N ₂ and H ₂
(c) H ₂ , CH ₄ and CO	(d) CO ₂ and H ₂

Answer: (a) Ammonia synthesis gas: 3H₂ + 1N₂
3. Which of the following gaseous fuels is likely to have the highest Gross Calorific value?

(a) Sewage Gas	(b) LPG	(c) Producer Gas	(d) Natural Gas
----------------	---------	------------------	-----------------

Answer: (b) On per m³ basis, LPG is having highest calorific value. But on the basis of per kg, natural gas is having highest calorific value.
4. The gas which contributes the maximum to the heating value of natural gas is (GATE-1999-1.35)

- (a) CO (b) CO₂ (c) H₂ (d) CH₄

Answer: (d)

5. Which of the following fuels has the highest calorific value per unit mass? (GATE-1994-1.h)

- (a) Coal (b) Kerosene (c) Natural gas (d) Furnace oil

Answer: (c)

6. Triple super phosphate is manufactured by heating (GATE-2000-1.24)

- (a) phosphate rock with phosphoric acid
- (b) phosphate rock with sulphuric acid
- (c) phosphate rock with nitric acid
- (d) ammonium phosphate with phosphoric acid

Answer: (a) Super phosphate: 16–25% P₂O₅; made from phosphate rock + phosphoric acid.

Triple super phosphate contains: 42–50% P₂O₅

7. The ammonia synthesis catalyst is

- | | |
|----------------|------------------------|
| (a) Alumina | (b) Vanadium Pentoxide |
| (c) Iron Oxide | (d) Kiselguhr |

Answer: (c)

8. For the reaction SO₂(g) + 1/2O₂(g) ⇌ SO₃(g), increase in pressure causes

- (a) increase in rate of reaction
- (b) lower equilibrium conversion
- (c) higher equilibrium conversion
- (d) no change in conversion

Answer: (c)

9. For handling concentrated hydrochloric acid a suitable material of construction is: (GATE-1988-9.b.i)

- (a) aluminium (b) hastelloy (c) stainless steel (d) cast iron

Answer: (b) HCl is the most corrosive acid. Corrosive to mild steel and stainless steels.

10. Solvay process is used for the manufacture of (GATE-1994-1.f)

- | | |
|--------------------|---------------|
| (a) Caustic soda | (b) Soda ash |
| (c) Caustic potash | (d) Soda lime |

Answer: (b)

11. For SO₂/SO₃ service at 400°C the recommended material of construction is: (GATE-1990-9.i)

- (a) Stainless steel (b) Carbon steel (c) Cast steel (d) Monel

Answer: (a) For low T, carbon steel is enough.

12. Catalyst used in the contact process of sulfuric acid manufacture is: (GATE-1989-10.i)

- | | |
|----------------|------------------------|
| (a) Alumina | (b) Vanadium pentoxide |
| (c) Iron oxide | (d) Silicon dioxide |

Answer: (b)

13. In the converter of the contact process for the manufacture of H₂SO₄, the equilibrium conversion of SO₂ —(i)— with increase in the temperature and —(ii)— with increase in the mole ratio of SO₂ to air. (GATE-2001-2.24)

- (a) (i) increases (ii) decreases
- (b) (i) decreases (ii) increases
- (c) (i) increases (ii) increases
- (d) (i) decreases (ii) decreases

Answer: (b)

14. The ethyl alcohol content in the fermented liquor from molasses, is

- (a) 50 - 55% (b) 20 - 22% (c) 8 - 10% (d) 3 - 5%

Answer: (c)

15. Sucrose is a disaccharide consisting of

(GATE-1999-1.32)

- | | |
|---------------------------|----------------------------|
| (a) glucose and glucose | (b) glucose and fructose |
| (c) glucose and galactose | (d) fructose and galactose |

Answer: (b)

16. Which one of the following is not likely to be constituent of vegetable oils? (GATE-1999-1.34)

- (a) citric acid (b) oleic acid (c) stearic acid (d) glycerol

Answer: (a)

17. A bio-degradable detergent is one which

(GATE-1993-7.a.i)

- (a) is manufactured using bio-technology
- (b) contains straight chain alkyl benzenes
- (c) contains branch chain alkyl benzenes
- (d) is easily decomposed by micro-organisms

Answer: (d)

18. Hydrogenation of edible oils is done to

(GATE-1993-7.a.ii)

- (a) decrease the number of unsaturated bonds
- (b) lower the melting point of oil
- (c) increase the thermal conductivity of oil
- (d) enable the oil to be packed in tin containers

Answer: (a)

19. For the hydrogenation of oils, —(i)— is commonly used as catalyst, and —(ii)— is a catalyst poison.

(GATE-2001-2.25)

- | | |
|------------------------------|-------------------------------|
| (a) (i) Platinum (ii) Sulfur | (b) (i) Palladium (ii) Oxygen |
| (c) (i) Nickel (ii) Sulfur | (d) (i) Nickel (ii) Oxygen |

Answer: (c)

20. In petroleum refining, the process used for conversion of hydrocarbons to aromatics is

(GATE-2000-1.22)

- | | |
|------------------------|-------------------------|
| (a) catalytic cracking | (b) catalytic reforming |
| (c) hydrotreating | (d) alkylation |

Answer: (b)

21. In the fluid catalytic cracker (FCC), the cracking reaction is —(i)— and the regeneration is —(ii)—

(GATE-2001-1.25)

- | | |
|--------------------------------------|-------------------------------------|
| (a) (i) Exothermic (ii) Endothermic | (b) (i) Exothermic (ii) Exothermic |
| (c) (i) Endothermic (ii) Endothermic | (d) (i) Endothermic (ii) Exothermic |

Answer: (a)

22. Commercially, ethylene is produced from naphtha by

(GATE-2000-1.23)

- | | |
|------------------------|-------------------------------|
| (a) catalytic cracking | (b) catalytic dehydrogenation |
| (c) pyrolysis | (d) hydrocracking |

Answer: (c)

23. Epoxy resins come under the category of:

(GATE-1988-9.b.iii)

- | | |
|---------------------------------|------------------------|
| (a) thermoplastic | (b) thermosetting |
| (c) oil soluble or oil modified | (d) protein substances |

Answer: (b)

24. The organic acid monomer in Nylon 66 is

(GATE-1999-1.33)

- | | |
|------------------|-----------------------|
| (a) sebacic acid | (b) terephthalic acid |
| (c) adipic acid | (d) benzoic acid |

Answer: (c)

25. Phthalic anhydride is produced by the oxidation of

(GATE-1994-1.g)

- | | | | |
|----------------|-------------|-------------|-------------|
| (a) Napthalene | (b) Benzene | (c) Toluene | (d) Aniline |
|----------------|-------------|-------------|-------------|

Answer: (a) Also from o-xylene.

26. A typical example of an exothermic reaction conducted at high pressures in industry is

(GATE-1996-2.14)

- | | |
|----------------------------|--------------------------------|
| (a) dehydration of ethanol | (b) methanol synthesis |
| (c) reformation of methane | (d) polymerisation of ethylene |

Answer: (d)

27. Styrene-Butadiene rubber is commercially manufactured by

(GATE-1996-1.22)

- | | |
|-----------------------------|-------------------------------|
| (a) Bulk polymerisation | (b) Suspension polymerisation |
| (c) Solution polymerisation | (d) Emulsion polymerisation |

Answer: (d)

28. Styrene is produced from ethylbenzene by the process of

(GATE-2001-1.24)

- | | |
|---------------------|-----------------|
| (a) Dehydrogenation | (b) Oxidation |
| (c) Alkylation | (d) Dehydration |

Answer: (c) $C_2H_6 + C_2H_4 \rightarrow C_6H_5CHCH_2 + H_2$

29. The material preferred for the storage tanks for 98% sulfuric acid is

(GATE-1998-1.04)

- | | |
|-------------------------|----------------|
| (a) Aluminium | (b) Lead |
| (c) Stainless steel 316 | (d) Mild steel |

Answer: (d) For high temperature service, SS is required.

30. For NH_3 reactors the material of construction is: (GATE-1990-9.ii)

- (a) Carbon steel (b) 5 Cr 1/2 Mo Steel
 (c) Cast steel (d) Stainless steel

Answer: (a)

31. Dilute sulfuric acid is handled in vessels made of: (GATE-1989-9.i.a)

- (a) Stainless steel (b) Brass (c) Lead (d) Cast iron

Answer: (c) Lead-chamber process produces < 70% H_2O_4 , as lead is not suitable for high-concentrated sulfuric acid. But at concentrations less than 70% carbon steel is not suitable. In other words lead is suitable for dilute sulfuric acid and carbon steel for concentrated acid and vice versa.

32. Phosphoric acid is handled in: (GATE-1989-9.i.b)

- (a) Glass-lined vessels (b) Aluminium vessels
 (c) Brass vessels (d) Stainless steel vessels

Answer: (d) Also handled in Hastelloy-C

33. A suitable material of construction to use with fuming sulfuric acid is (GATE-1997-1.29)

- (a) Carbon steel (b) Stainless steel type 304
 (c) Nickel (d) Monel

Answer: (a)

3. For an even function $f(x)$, (GATE-2000-1.02)

- (a) $\int_{-a}^a f(x)dx = 0$ (b) $\int_{-a}^a f(-x)dx = 0$
 (c) $f(x) = -f(-x)$ (d) $f(x) = f(-x)$

Answer: (d) A function defined on $[-a, a]$ is

- (i) even if $f(-x) = f(x)$
- (ii) odd if $f(-x) = -f(x)$

For eg, $x, x^3, \sin x$ are odd on $[-\pi, \pi]$, and $x^2, \cos x$ are even functions.

For odd function on $[-a, a]$,

$$\int_{-a}^a f(x)dx = 0$$

For even function on $[-a, a]$,

$$\int_{-a}^a f(x)dx = 2 \int_0^a f(x)dx$$

4. The integrating factor for the differential equation: $(\cos^2 x) \frac{dy}{dx} + y = \tan x$, is (GATE-2000-1.03)

- (a) $e^{\tan x}$
- (b) $\cos 2x$
- (c) $e^{-\tan x}$
- (d) $\sin 2x$

Answer: (a) For the equation

$$\frac{dy}{dx} + Py = Q$$

the integrating factor is $e^{\int P dx}$.

For this given equation after dividing by $\cos^2 x$, $P = 1/\cos^2 x = \sec^2 x$. Integrating factor = $e^{\int \sec^2 x dx} = e^{\tan x}$

5. The system of equations,

$$\begin{aligned} 2x + 4y &= 10 \\ 5x + 10y &= 25 \end{aligned}$$

(GATE-1999-1.01)

- (a) has no unique solution
- (b) has only one solution
- (c) has only two solutions
- (d) has infinite solutions

Answer: (d) A system of equations $AX = B$, has

- (i) unique solution if $r(A) = r(A|B) = \text{number of unknowns}$.
- (ii) no solution if $r(A) \neq r(A|B)$
- (iii) infinite number of solutions if $r(A) = r(A|B) < \text{number of unknowns}$.

6. The rank of the matrix $\begin{pmatrix} 3 & 0 & 1 & 2 \\ 4 & 7 & 3 & 3 \\ 1 & 7 & 2 & 1 \end{pmatrix}$ is (GATE-1999-1.03)

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- (a) 0
- (b) 1
- (c) 2
- (d) 3

Answer: (c) Note that $R_1 + R_3 = R_2$ here. In this case, we have 2 independent rows. Hence rank ≤ 2 .

But, for eg. $\begin{vmatrix} 3 & 0 \\ 4 & 7 \end{vmatrix} = 21 \neq 0$. Therefore rank = 2.

7. The harmonic series $\sum_{n=1}^{\infty} \frac{1}{n^p}$

(GATE-1999-1.04)

- (a) converges for $p > 1$
- (b) diverges for $p > 1$
- (c) converges for $p < 1$
- (d) diverges for $p < 1$

Answer: (a) $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges for $p > 1$ and diverges for $p \leq 1$.

8. The Laplace transform of the function e^{-at} has the form:

(GATE-1998-1.01)

- (a) $\frac{1}{s+1}$
- (b) $\frac{1}{s(s+a)}$
- (c) $\frac{a}{s}$
- (d) $\frac{1}{s+a}$

Answer: (d)

$$\mathcal{L}[e^{-at}] = \int_0^{\infty} e^{-st} e^{-at} dt = \int_0^{\infty} e^{-(s+a)t} dt = \frac{1}{s+a}$$

$$\text{Also } \mathcal{L}[e^{at}] = \frac{1}{s-a}$$

9. The unit normal to the plane $2x + y + 2z = 6$ can be expressed in the vector form: (GATE-1998-1.02)

- (a) $3i + 2j + 2k$
- (b) $\frac{2}{3}i + \frac{1}{3}j + \frac{2}{3}k$
- (c) $\frac{1}{3}i + \frac{1}{2}j + \frac{1}{2}k$
- (d) $-\frac{2}{3}i + \frac{1}{3}j - \frac{2}{3}k$

Answer: (b) The unit normal to any surface $\phi = 0$ is given by $\frac{\nabla \phi}{|\nabla \phi|}$. Here

$$\phi = 2x + y + 2z - 6$$

$$\nabla \phi = 2i + j + 2k$$

and

$$|\nabla \phi| = \sqrt{2^2 + 1^2 + 2^2} = 3$$

Therefore unit normal = $\frac{2}{3}i + \frac{1}{3}j + \frac{2}{3}k$

10. $\lim_{x \rightarrow 0} \frac{x - \sin 2x}{x + \sin 3x}$ has the value:

(GATE-1998-1.03)

- (a) 1 (b) -1/4 (c) 0 (d) ∞

Answer: (b) Since $\lim_{x \rightarrow 0} \frac{\sin nx}{x} = n$,

$$\lim_{x \rightarrow 0} \frac{1 - \frac{\sin 2x}{x}}{1 + \frac{\sin 3x}{x}} = \frac{1 - 2}{1 + 3} = \frac{-1}{4}$$

11. The sum of the infinite series $3 + 1 + \frac{1}{3} + \left(\frac{1}{3}\right)^2 + \dots + \left(\frac{1}{3}\right)^n$ is (GATE-1997-1.01)

- (a) 9 (b) $\frac{9}{2}$ (c) $\frac{15}{2}$ (d) infinity

Answer: (b) Since $1 + x + \dots + x^n = \frac{1 - x^{n+1}}{1 - x}$, we have

$$3 + \left(1 + \frac{1}{3} + \dots + \left(\frac{1}{3}\right)^n\right) = 3 + \frac{1 - (1/3)^{n+1}}{1 - 1/3} = 3 + [1 - (1/3)^{n+1}] \frac{3}{2}$$

If $n \rightarrow \infty$, $(1/3)^{n+1} \rightarrow 0$. Therefore, sum of the given series = $3 + 3/2 = 9/2$

Another simple method: If $n \rightarrow \infty$, $1 + x + x^2 + \dots + x^n = 1/(1-x)$.
Therefore,

$$3 + \left(1 + \frac{1}{3} + \dots + \left(\frac{1}{3}\right)^n\right) = 3 + \frac{1}{1 - 1/3} = 3 + \frac{1}{2/3} = 3 + 3/2 = 9/2$$

12. $\lim_{x \rightarrow \infty} \frac{x^3 + 1}{2x^2 + 80x + 1}$ is (GATE-1997-1.02)

- (a) 0 (b) $\frac{1}{2}$ (c) 1 (d) infinity

Answer: (d) Divide numerator and denominator by x^3 to get

$$\lim_{x \rightarrow \infty} \frac{1 + 1/x^3}{(2/x) + (80/x^2) + (1/x^3)} = \frac{1 + 1/\infty}{0 + 0 + 0} = 1/0 = \infty$$

13. The value of $\int_0^{5\pi} (2 - \sin x) dx$ is (GATE-1997-1.03)

- (a) > 0 (b) < 0 (c) 0 (d) undefined

Answer: (a) Note that $-1 \leq \sin x \leq 1$. Therefore, $1 \leq (2 - \sin x) \leq 3$.

$$\int_0^{5\pi} 1 dx \leq \int_0^{5\pi} (2 - \sin x) dx \leq 3 \int_0^{5\pi} dx$$

i.e.,

$$5\pi \leq \int_0^{5\pi} (2 - \sin x) \leq 15\pi$$

14. Given $f(x, y) = x^2 + y^2$, $\nabla^2 f$ is

- (a) 4 (b) 2 (c) 0 (d) $4(x + y)^2$

Answer: (a) $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$, $\frac{\partial f}{\partial x} = 2x$, $\frac{\partial^2 f}{\partial x^2} = 2$; similarly $\frac{\partial^2 f}{\partial y^2} = 2$. Therefore, $\nabla^2 f = 4$

15. The rank of matrix $\begin{pmatrix} 0 & 2 & 0 \\ 3 & 0 & 0 \end{pmatrix}$ (GATE-1995-1.a)

- (a) 0 (b) 1 (c) 2 (d) 3

Answer: (c) $\begin{vmatrix} 0 & 2 \\ 3 & 0 \end{vmatrix} = -6 \neq 0$. Therefore, $r = 2$.

16. The angle between two vectors $2\vec{i} - \vec{j} + \vec{k}$ and $\vec{i} + \vec{j} + 2\vec{k}$ is (GATE-1995-1.b)

- (a) 0° (b) 30° (c) 45° (d) 60°

Answer: (d) $\cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} = \frac{2 - 1 + 2}{\sqrt{4+1+1}\sqrt{1+1+4}} = \frac{3}{6} = \frac{1}{2}$
 $\theta = \cos^{-1} 1/2 = 60^\circ$

17. The inverse of a matrix $\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$ is (GATE-1994-1.a)

- (a) $\begin{pmatrix} ab & 0 \\ 0 & 1 \end{pmatrix}$ (b) $\begin{pmatrix} b & 0 \\ a & b \end{pmatrix}$
(c) $\begin{pmatrix} 1/a & 0 \\ 0 & 1/b \end{pmatrix}$ (d) $\begin{pmatrix} a & 0 \\ 0 & 1/b \end{pmatrix}$

Answer: (c) In general $\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

Here $\begin{vmatrix} a & 0 \\ 0 & b \end{vmatrix} = ab$.

Therefore $\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}^{-1} = \frac{1}{ab} \begin{pmatrix} b & 0 \\ 0 & a \end{pmatrix} = \begin{pmatrix} 1/a & 0 \\ 0 & 1/b \end{pmatrix}$

18. The limit of $f(x) = x/\sin x$ as $x \rightarrow 0$ is (GATE-1994-1.b)
- (a) 0 (b) 1 (c) 2 (d) ∞

Answer: (b) $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$. Therefore, $\lim_{x \rightarrow 0} \frac{x}{\sin x} = \lim_{x \rightarrow 0} \frac{1}{(\sin x)/x} = \frac{1}{1} = 1$

19. Integrating factor for the differential equation $\frac{dy}{dx} + P(x)y = Q(x)$ is (GATE-1994-1.c)
- (a) $e^{\int P dx}$ (b) $e^{-\int P dx}$ (c) $\int P dx$ (d) dP/dx

Answer: (a)

20. If $\vec{i}, \vec{j}, \vec{k}$ are the unit vectors in rectangular coordinates, then the curl of the vector $\vec{i}y + \vec{j}y + \vec{k}z$ is (GATE-1994-1.d)
- (a) \vec{k} (b) $-\vec{k}$ (c) $\vec{j} + \vec{k}$ (d) $\vec{i} + \vec{k}$

Answer: (b) $\text{curl } \vec{f} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ f_1 & f_2 & f_3 \end{vmatrix} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y & y & z \end{vmatrix} = \vec{i}0 - \vec{j}0 + \vec{k}(0-1) = -\vec{k}$

21. The solution for the differential equation $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = 0$ is (GATE-1994-1.e)
- (a) $C_1 e^{-2x} + C_2 e^{3x}$ (b) $C_1 \sin 2x + C_2 \cos 2x$
 (c) $C_1 e^{2x} + C_2 e^{-3x}$ (d) $C_1 e^{-2x} + C_2 e^{-3x}$

Answer: (d) Auxiliary equation: $m^2 + 5m + 6 = 0$. Roots $m = -3, -2$. Solution = $C_1 e^{-2x} + C_2 e^{-3x}$

22. The function $f(x, y) = x^2 + y^2 - xy - x - y + 5$ has the (GATE-2001-2.01)
- (a) Maximum at $(1, 1)$
 (b) Saddle point at $(1, 1)$
 (c) Minimum at $(1, 1)$
 (d) None of the above at $(1, 1)$

Answer: (c)

$$f_x = 2x - y - 1$$

$$f_{xx} = 2$$

$$f_y = 2y - x - 1$$

$$f_{yy} = -1$$

$$f_{xx}f_{yy} - f_{xy}^2 = 4 - 1 = 3 > 0$$

Therefore the function is minimum.

At minimum, $f_x = 0, f_y = 0$. Therefore, $2x - y = 1$ and $2y - x = 1$; giving $x = 1, y = 1$.

Details:

Let $f_x = 0, f_y = 0$ at (a, b) . Then

- (i) f has relative maximum at (a, b) if

$$f_{xx}(a, b) < 0, f_{xx}f_{yy} - f_{xy}^2 > 0$$

- (ii) f has relative minimum if

$$f_{xx} > 0, f_{xx}f_{yy} - f_{xy}^2 > 0$$

(iii) Saddle point, if $f_{xx}f_{yy} - f_{xy}^2 < 0$

(iv) Inconclusive, if $f_{xx}f_{yy} - f_{xy}^2 = 0$

23. The line integral of $\int_C \left\{ \frac{y}{x^2+y^2} dx - \frac{x}{x^2+y^2} dy \right\}$, where C is the unit circle around the origin traversed once in the counter-clockwise direction, is (GATE-2000-2.01)

- (a) -2π (b) 0 (c) 2π (d) π

Answer: (a) Circle C : $x = \cos \theta, y = \sin \theta, \theta \in [0, 2\pi]$

Integral

$$\begin{aligned} I &= \int_0^{2\pi} \frac{\sin \theta}{\cos^2 \theta + \sin^2 \theta} d\cos \theta - \frac{\cos \theta}{\cos^2 \theta + \sin^2 \theta} d\sin \theta \\ &= \int_0^{2\pi} (-\sin^2 \theta - \cos^2 \theta) d\theta = \int_0^{2\pi} -d\theta = -2\pi \end{aligned}$$

24. The inverse of the matrix $\begin{pmatrix} 1 & -1 \\ -1 & -1 \end{pmatrix}$ (GATE-2000-2.02)

- (a) does not exist (b) is $\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

- (c) is $\begin{pmatrix} 0.5 & -0.5 \\ -0.5 & -0.5 \end{pmatrix}$ (d) is $\begin{pmatrix} -0.5 & 0.5 \\ 0.5 & 0.5 \end{pmatrix}$

Answer: (c)

25. The complex conjugate of $\frac{1}{1+i}$ is (GATE-2000-2.03)

- (a) $\frac{1}{1-i}$ (b) $(1-i)$

- (c) $0.5(1-i)$ (d) in the first quadrant of the complex plane

Answer: (a) For any rational function $\frac{P}{Q}$, $\overline{\left(\frac{P}{Q}\right)} = \frac{\bar{P}}{\bar{Q}}$

Therefore, $\overline{\left(\frac{1}{1+i}\right)} = \frac{\bar{i}}{1+\bar{i}} = \frac{1}{1-i}$

26. The general solution of $\frac{d^4y}{dx^4} + 2\frac{d^2y}{dx^2} + y = 0$ is (GATE-2000-2.04)

- (a) $(C_1x + C_2)e^x + (C_3 + C_4x)e^{-x}$
 (b) $C_1 \cos x + C_2 \sin x + C_3 e^x + C_4 e^{-x}$
 (c) $C_1 e^{ix} + C_2 e^{-ix}$
 (d) $(C_1 + C_2x) \cos x + (C_3 + C_4x) \sin x$

where C_1, C_2, C_3 , and C_4 are constants.

Answer: (d) AE: $m^4 + 2m^2 + 1 = 0$

$$(m^2 + 1)^2 = 0$$

$$m = i, -i, -1, -i.$$

$$\text{CF: } y = (C_1 + C_2x) \cos x + (C_3 + C_4x) \sin x$$

27. The gradient of $xy^2 + yz^3$ at the point $(-1, 2, 1)$ is (GATE-1999-2.02)

- (a) $3i - 3j + 3k$ (b) $3i - 3j + 6k$ (c) $4i - j + 3k$ (d) $4i - 3j + 6k$

Answer: (d) grad = $\nabla\phi$

$$\begin{aligned}\nabla\phi &= i\frac{\partial\phi}{\partial x} + j\frac{\partial\phi}{\partial y} + k\frac{\partial\phi}{\partial z} \\ &= y^2i + (2xy + z^3)j + 3yz^2k = 4i + (-4 + 1)j + 6k \\ &= 4i - 3j + 6k\end{aligned}$$

28. The function $z = (x - 1)^2 - 2y^2$ has (GATE-1998-2.01)

- (a) a stationary point which is a minimum
 (b) a stationary point which is a maximum
 (c) a stationary point at which no extremum exists
 (d) no stationary point

Answer: (c) $z_x = 2(x - 1) = 0 \Rightarrow x = 1$
 $z_y = -4y = 0 \Rightarrow y = 0$ (1, 0) is a stationary point.

$$z_{xx} = 2 > 0, z_{yy} = -4 < 0, z_{xy} = 0$$

$$z_{xx}z_{yy} - z_{xy}^2 = 2 \times (-4) - 0 = -8 < 0$$

Therefore, (1, 0) is a saddle point.

29. The differential equation $\frac{d^2x}{dt^2} + 3\frac{dx}{dt} + 2x = 0$ will have a solution of the form (GATE-1998-2.02)

- (a) $C_1 e^{3t} + C_2 e^{2t}$
 (b) $C_1 e^{-2t} + C_2 e^{-t}$
 (c) $C_1 e^{-3t} + C_2 e^{-2t}$
 (d) $C_1 e^{-5t}$

Answer: (b) AE: $m^2 + 3m + 2 = 0$; $m = -1, -2$. Solution = $C_1 e^{-2t} + C_2 e^{-t}$

30. The integral $\int \frac{dx}{x^p}$ is convergent for (GATE-1998-2.03)

- (a) no value of p (b) $p > 1$
 (c) $p < 1$ (d) all values of p

Answer: (b)

$$\begin{aligned}\int \frac{dx}{x^p} &= \int x^{-p} dx = \frac{x^{-p+1}}{-p+1} \\ \int_1^\infty \frac{dx}{x^p} &= \lim_{p \rightarrow \infty} \frac{x^{1-p}}{1-p} = \frac{1}{p-1} \text{ for } p > 1\end{aligned}$$

For $p \leq 1$, the integral diverges.

31. The cubic equation $x^3 - x + 10 = 0$ has a root in the interval (GATE-1997-2.02)

- (a) $(-1, 0)$ (b) $(0, 1)$ (c) $(-3, -1)$ (d) $(3, 4)$

Answer: (c) Let $f(x) = x^3 - x + 10$.

$$f(-3) = -27 + 3 + 10 < 0; f(-1) = -1 + 1 + 10 = 10 > 0.$$

Therefore, a root is between -3 & -1 .

32. The Fourier series of the function

$$f(x) = \begin{cases} 1 & 0 \leq x < \pi \\ -1 & -\pi < x < 0 \end{cases}$$

extended periodically, $f(x + 2\pi) = f(x)$, is (GATE-1997-2.03)

- (a) a sine series (b) a cosine series
 (c) a mixed series (d) a power series

Answer: (a) $f(x) = \frac{x}{|x|}$; $f(-x) = -\frac{x}{|x|} = -f(x)$. Therefore, $f(x)$ is odd, and cosine terms will not come in Fourier series. It will contain only sine terms.

33. $\lim_{x \rightarrow 0} \frac{\tanh x}{x} =$ (GATE-1995-2.b)

- (a) ∞ (b) 1 (c) 0 (d) -1

Answer: (b)

$$\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \dots$$

$$\cosh x = 1 + \frac{x^2}{2!} + \dots$$

$$\lim_{x \rightarrow 0} \frac{\tanh x}{x} = \lim_{x \rightarrow 0} \frac{1 + x^2/(3!) + \dots}{1 + x^2/(2!) + \dots} = 1$$

34. The second order Taylor series expansion for a function $f(x) = x^2$ at $x = 1$ is (GATE-1995-2.c)

(a) x^2 (b) $1+x^2$ (c) $1+x+x^2$ (d) $1-x+x^2$

Answer: (a)

35. The average value of function $f(x) = x^3$ in the interval $0 \leq x \leq 2$ is (GATE-1995-2.d)

(a) 1 (b) 2 (c) 4 (d) 8

Answer: (b) Average = $\frac{1}{2-0} \int_0^2 x^3 dx = \frac{1}{2} \left[\frac{x^4}{4} \right]_0^2 = \frac{1}{2} \frac{2^4}{4} = \frac{4}{2} = 2$

Average of $f(x)$ on (a, b) is $\frac{1}{b-a} \int_a^b f(x) dx$

Recommended Books for Detailed Study

Process Calculations

- Bhatt & Vora, Stoichiometry, Tata McGraw Hill.
- Houghen & Watson, Chemical Process Principles - Part I, CBS Publishers.
- Himmelblau, Basic Principles and Calculations in Chemical Engineering, Prentice-Hall India.
- Ghosal, Introduction to Chemical Engineering, Tata McGraw-Hill.

Fluid Mechanics

- James O. Wilkes, Fluid Mechanics for Chemical Engineers, Prentice Hall.
- Noel De Nevers, Fluid Mechanics for Chemical Engineers, McGraw-Hill.
- McCabe & Smith, Unit Operations of Chemical Engineering, McGraw-Hill.
- Bird, Transport Phenomena, McGraw-Hill.

Mechanical Operations

- Foust, Unit Operations, John Wiley.
- McCabe & Smith, Unit Operations of Chemical Engineering, McGraw-Hill.
- Badger & Banchero, Introduction to Chemical Engineering, McGraw-Hill.

Thermodynamics

- Smith & Van Ness, Introduction to Chemical Engineering, McGraw-Hill.
- K.V.Narayanan, Chemical Engineering Thermodynamics, Prentice-Hall India.
- J.P.Holman, Thermodynamics, McGraw-Hill.

Heat Transfer

- Necati Ozisik, Heat Transfer – a Basic Approach, McGraw-Hill.
- Kern, Process Heat Transfer, McGraw-Hill.

Mass Transfer

- Treybal, Mass Transfer Operations, McGraw-Hill.
- Welty, Wicks, & Wilson, Fundamentals of Momentum, Heat, and Mass Transfer, John Wiley.
- Bennett & Myers, Momentum, Heat and Mass Transfer, McGraw-Hill.
- Cussler, Diffusion in Mass Transfer Systems, Cambridge University Press.

Reaction Engineering

- Levenspiel, Chemical Reaction Engineering, Wiley Eastern.
- Smith, Chemical Engineering Kinetics, McGraw-Hill.
- Fogler, Elements of Chemical Reaction Engineering, Prentice-Hall India.

Process Control

- Stephanopoulos, Chemical Process Control, Prentice-Hall India.
- Coughanowr & Koppel, Process Systems Analysis and Control, McGraw-Hill.

Process Economics & Design

- Peter & Timmerhaus, Plant Design and Economics for Chemical Engineers, McGraw-Hill.
- Brownell & Young, Process Equipment Design - Vessel Design, Wiley Eastern.

Chemical Technology

- Austin, Shreve's Chemical Process Industries, McGraw-Hill.
 - Gopala Rao & Marshall Sitting, Dryden's Outlines of Chemical Technology, East-West Press.
 - Fontana, Corrosion Engineering, McGraw-Hill.
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