Week 02

MA1508E LINEAR ALGEBRA FOR ENGINEERING

Week 02

IVLE Quiz Discussion

Review of last week's content

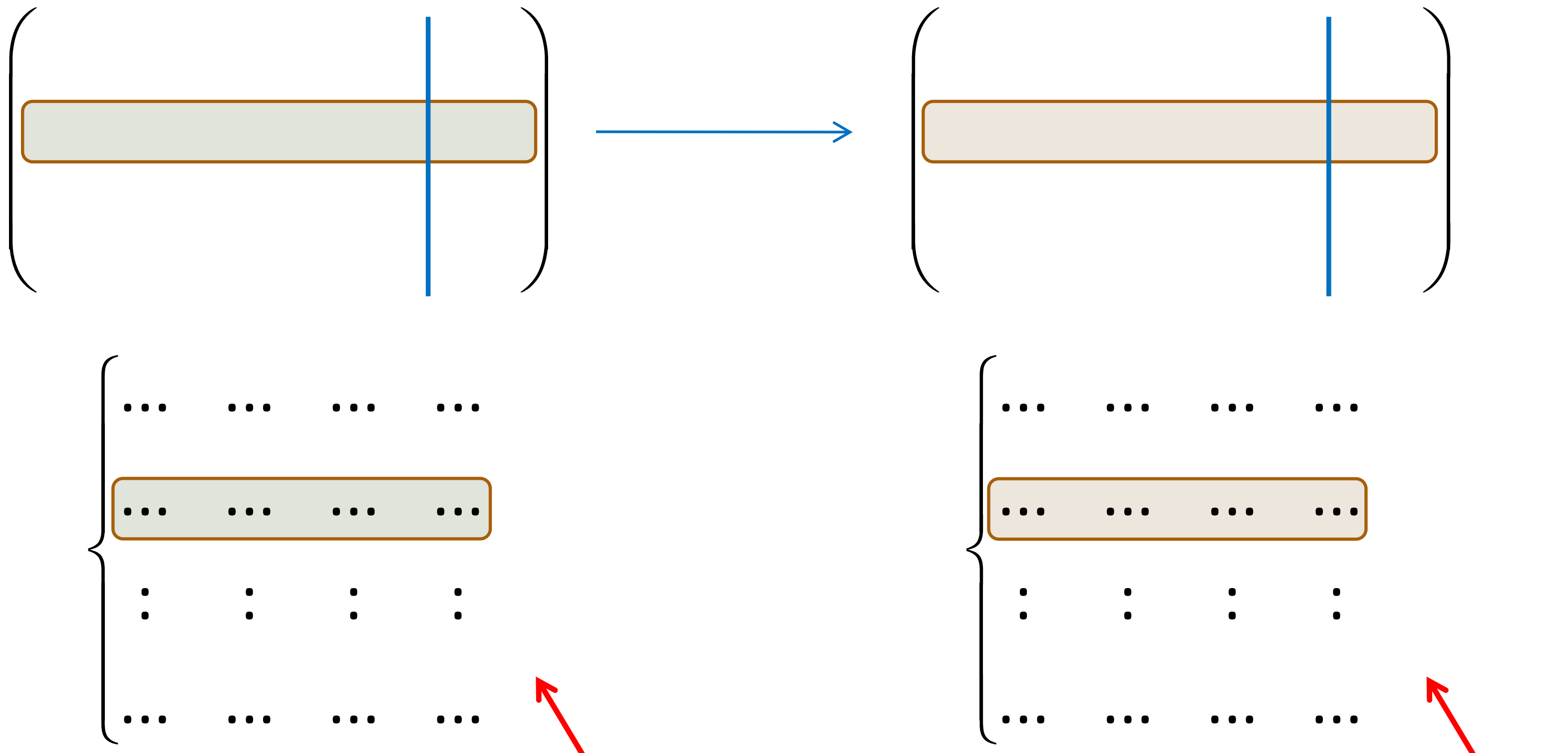
- What is a linear equation; what is a solution to a linear equation; what is a solution set
- What is a general solution
- Linear equation → linear system. What is a solution to a linear system
- (Geometrical interpretation) – linear equation in 2 variables. Solution set of the equation forms a line
- (Geometrical interpretation) – linear equation in 3 variables. Solution set of the equation forms a plane
- Augmented matrix
- Three types of elementary row operations

Review of last week’s content (cont’d)

- Row equivalent matrices. If one can perform one ERO from matrix $\boldsymbol{A}$ to get matrix $\boldsymbol{B}$, then one can also perform ERO from $\boldsymbol{B}$ to get $\boldsymbol{A}$
- If two augmented matrices are row equivalent, then their corresponding linear systems have the same solution set
- Augmented matrices in ‘nice-forms’ corresponds to linear systems that are easier to solve, leading to the definition of row-echelon forms.
- Definition of row-echelon form; definition of pivot point/leading entry; definition of pivot column
- Definition of reduced row-echelon form
- What a row-echelon form can tell us (about consistency of the system)
- How to write down a general solution by looking at a row-echelon form

Do you know why?

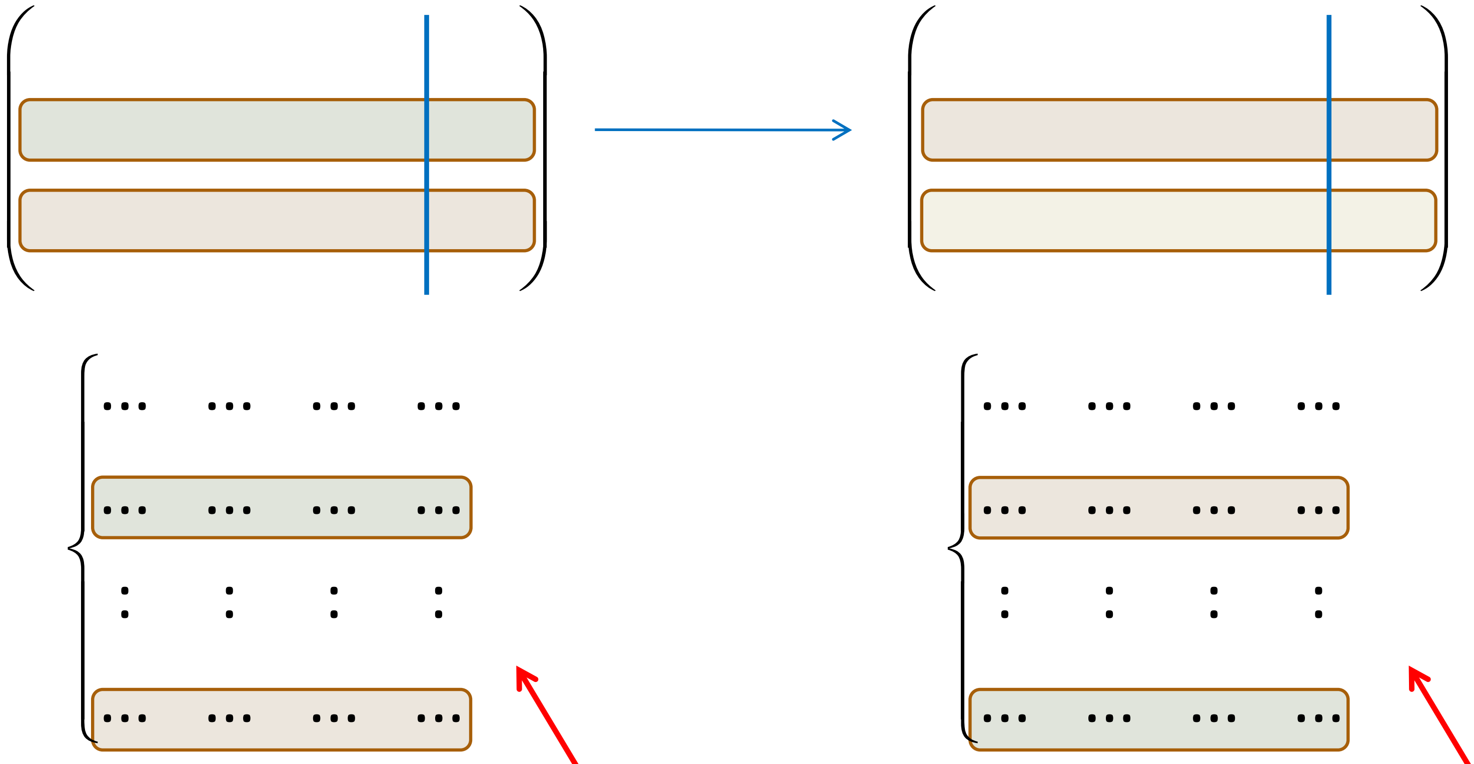
1) Multiply a row by a non zero constant



(...,...,...) is a solution of if and only if it is a solution of

Do you know why?

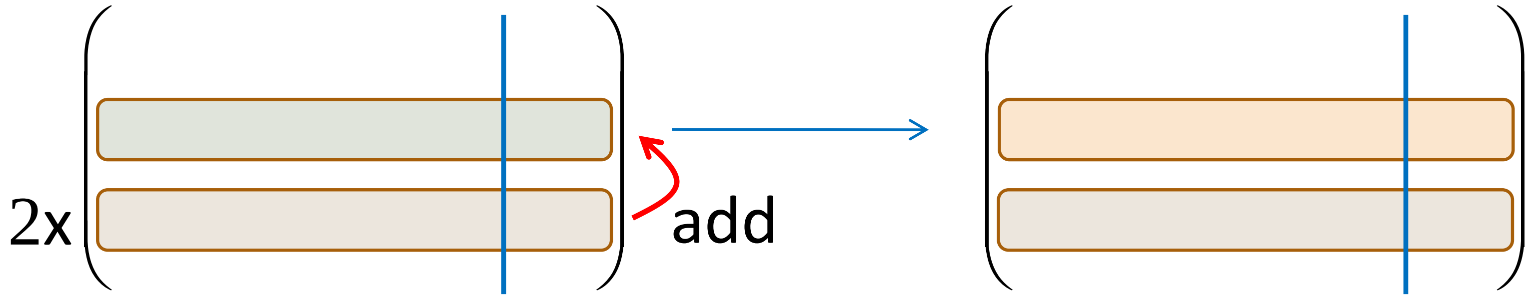
2) Interchanging two rows

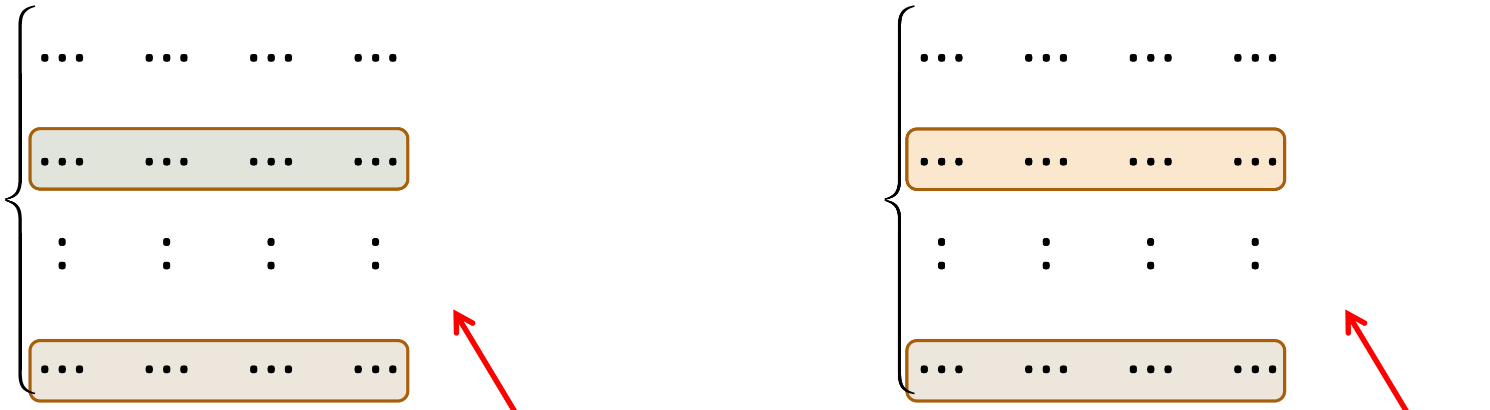


(…,…,…) is a solution of if and only if it is a solution of

Do you know why?

3) Adding a multiple of one row to another row





(…,…,…) is a solution of if and only if it is a solution of

Week 02 content (motivation)

- We saw that row-echelon forms are useful in the solving of linear systems.
- Since we know that linear systems with row equivalent augmented matrices have the same solution set, we need to develop a systematic procedure to change an augmented matrix into row-echelon form that is row equivalent to the initial augmented matrix.
- While performing various elementary row operations, it would be good to have a standard set of notations to avoid confusion.
- A special kind of linear systems (homogeneous)
- A matrix is a very useful way of arranging information. Such an arrangement and representation of information can be useful in numerical computations.
- Matrix operations – can we do to matrices what we do to numbers?

Week 02 (units 007-012) overview

007 Gaussian elimination and Gauss-Jordan elimination

- Gaussian elimination
- Gauss-Jordan elimination
- Standard notations to use while performing Gaussian elimination

008 Examples (Gaussian and Gauss-Jordan elimination)

009 Homogeneous linear systems

- What are homogeneous linear systems
- Trivial and non-trivial solutions
- How many solutions can a homogeneous linear system have?
- Homogeneous systems with more variables than equations (under-determined linear systems)

Week 02 (units 007-012) overview

010 Matrices – definitions and special types

- Matrices, entries, size and diagonal entries
- Diagonal matrix, scalar matrix, identity matrix, zero matrix
- Symmetric matrix, upper and lower triangular matrices

011 Matrix operations

- Matrix equality
- Matrix operations and some laws

012 Matrix multiplication

- How matrices can be multiplied
- Matrix multiplication laws
- Instances where real number multiplication and matrix multiplication differ

Example 2.1

(a) Solve the following linear system by Gaussian elimination

$$\begin{cases} \phantom{2a - {}} b - 3c + 4d = 1 \\ 2a - 2b + c = -1 \\ 2a - b - 2c + 4d = 0 \\ -6a + 4b + 3c - 8d = 1 \end{cases}$$

(b) Solve the following linear system by Gauss-Jordan elimination

$$\begin{cases} x + 2y + 3z = 14 \\ 3x + 2y + z = 10 \\ 3x + y + 2z = 11 \end{cases}$$

Example 2.2

(a) Find the conditions on a and b such that the following linear system has

(i) no solution

(ii) exactly one solution

(iii) infinitely many solutions

$$\begin{cases} x + y + 3z = 2 \\ x + 2y + 4z = 3 \\ x + 3y + az = b \end{cases}$$

(b) Find the conditions on a and b such that the following linear system has

(i) no solution

(ii) exactly one solution

(iii) infinitely many solutions

$$\begin{cases} ax + y + = a \\ x + y + z = 1 \\ y + az = b \end{cases}$$

Example 2.3

The following is the reduced row-echelon form of the augmented matrix of a linear system

$$\left(\begin{array}{ccc|c} a & b & c & d \\ 0 & e & f & g \\ 0 & 0 & h & k \end{array}\right)$$

where $a, b, c, d, e, f, g, h, k$ are constants. Suppose the solution set of this system is represented by a line that passes through the origin and the point $(1,1,1)$. Find the values of $a, b, c, d, e, f, g, h, k$. Justify your answers.

Example 2.4

Determine which of the following statements are true. Justify your answer.

(a) A homogeneous system can have a non-trivial solution.

(b) A non-homogeneous system can have a trivial solution.

(c) If a homogeneous system has the trivial solution, then it cannot have a non-trivial solution.

(d) If a homogeneous system has a non-trivial solution, then it cannot have a trivial solution.

(e) If a homogeneous system has a unique solution, then the solution has to be trivial.

Example 2.4

Determine which of the following statements are true. Justify your answer.

(g) If a homogeneous system has the trivial solution, then the solution has to be unique.

(h) If a homogeneous system has a non-trivial solution, then there are infinitely many solutions to the system.

Example 2.5

Given $\boldsymbol{A} = (a_{ij})_{n\times p}, \boldsymbol{B} = (b_{ij})_{p\times n}$ and $\boldsymbol{C} = (c_{ij})_{p\times p}$.

Identify which entry (and in which matrix) does each of the following sums represent?

(a) $\sum_{k=1}^{p} a_{3k} b_{k4}$ (b) $\sum_{r=1}^{n} a_{r2} b_{3r}$

Write down the (i, j) entry of

(a) $\boldsymbol{CB}$ (b) $\boldsymbol{BAC}$

Finally…

THE END