Unit 004 Row equivalent matrices

Slide 01: In this unit, we will learn about row equivalent matrices.

Slide 02: Recall that augmented matrices are used to represent linear systems and there are three types of elementary row operations that can be performed on an augmented matrix. We say that two augmented matrices are row equivalent if one can be obtained from the other by a series of elementary row operations.

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Note that the concept of row equivalent matrices can be applied to all matrices in general.

Slide 03: Consider this augmented matrix. It is clear that it corresponds to a linear system with 3 equations involving 3 unknowns.

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Suppose we add -2 times of row 1 to row 2. This results in the following augmented matrix.

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We perform another elementary row operation, namely adding -3 times of row 1 to row 3. This gives us the following augmented matrix.

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Lastly, we perform one more elementary row operation by adding $\frac{6}{4}$ times of row 2 to row 3, giving us this augmented matrix.

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The four augmented matrices are all row equivalent matrices. We also say that the matrices are row equivalent to each other.

Slide 04: Think about the following remark. If we perform a single elementary row operation on augmented matrix $\boldsymbol{A}$, resulting in another augmented matrix $\boldsymbol{B}$, then we can perform another elementary row operation on $\boldsymbol{B}$ to return back to $\boldsymbol{A}$.

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To see this, consider the two augmented matrices shown, where we performed one elementary row operation on the matrix on the left to obtain the new matrix on the right. The elementary row operation performed was adding -2 times of row 1 to row 2. What elementary row operation can be performed on $\boldsymbol{B}$ to return back to $\boldsymbol{A}$.

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Your are right. We simply add 2 times of row 1 to row 2 and we will be back with augmented matrix $\boldsymbol{A}$.

Slide 05: What about this type of elementary row operation? Is there a single elementary row operation that can be performed on the matrix on right to return to the original augmented matrix?

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It is easy to see that multiplying row 1 by $\frac{1}{3}$ does exactly that.

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Finally consider the following row swap performed on the augmented matrix. Interestingly performing the same row swap to the matrix on the right

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gets us back to the augemented matrix on the left. This remark illustrates that when two augmented matrices are row equivalent, we can perform a series of elementary row operations from either one of them to obtain the other matrix.

Slide 06: The following theorem is one of the key result in our discussion on linear systems. The theorem states that if augmented matrices of two linear systems are row equivalent, then the two linear systems will have the same solution set. For example, if linear system X and linear system Y have their respective augmented matrices

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and suppose the two augmented matrices are row equivalent.

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Then the theorem tells us that linear systems X and Y will have exactly the same solution set.

Slide 07: For example, there are four linear systems here.

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Each system's augmented matrix can be found quite easily.

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By the previous theorem, once we know that the 4 augmented matrices are row equivalent,

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Slide 08: then all the linear systems here have the solution set.

Slide 09: Consider the two linear systems shown here. Which system do you think is easier for us to obtain the solutions to? It wouldnt take you too long to figure out that the linear system with equations (1) (4) and (6) can be solved easily by first finding the value of z from equation (6), followed by finding the value of y from equation (4) after substituting the value of z into the equation. Lastly, once we have found the solutions to y and z, we can use equation (1) to solve for x.

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Furthermore, suppose the corresponding augmented matrices are row equivalent. Then by the previous theorem, the solution set obtained for the second linear system is also the solution set for the original linear system comprising of equations (1) (2) and (3)

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The main reason why the second linear system can be solved easily is because the linear system is in a 'nice form' which allows us to solve for the unknowns by performing some substitutions. The corresponding augmented matrix also has this nice form, which we will formally define in the next unit.

Slide 10: We are now able to describe a strategy for us to solve linear systems. Using the theorem we have seen in this unit,

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our strategy is to perform a series of carefully selected elementary row operations on the starting augmented matrix with the intention of changing the augmented into a nice form.

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So with the starting augmented matrix representing the linear system we want to solve,

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we will perform elementary row operations on it

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until it is changed into a nice form. In the next unit, we will discuss how this series of elementary row operations can be chosen and performed systematically.

Slide 11: To summarise, in this unit,

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We first saw the definition of row equivalent matrices.

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This was followed by an interesting remark which indicates that the 'reverse' of an elementary row operation is also an elementary row operation.

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Lastly we have a theorem which says that if two augmented matrices are row equivalent, then their corresponding linear systems will have the same solution set.