MA1512 TUTORIAL 1

1. Solve the following differential equations:

(a) $x(x+1)y' = 1$ (b) $(sec(x))y' = cos(5x)$

(c) $y' = e^{(x-3y)}$ (d) $(1+y)y' + (1-2x)y^2 = 0$

2. Experiments show that the rate of change of the temperature of a small iron ball is proportional to the difference between its temperature $T(t)$ and that of its environment, T_{env} (which is constant). Write down a differential equation describing this situation. Show that $T = T_{env}$ is a solution. Does this make sense? The ball is heated to $300°F$ and then left to cool in a room at $75°F$. Its temperature falls to $200°F$ in half an hour. Show that its temperature will be $81.6°F$ after 3 hours of cooling.

3. In very dry regions, the phenomenon called **Virga** is very important because it can endanger aeroplanes. [See http://en.wikipedia.org/wiki/Virga]. Virga is rain in air that is so dry that the raindrops evaporate before they can reach the ground. Suppose that the volume of a raindrop is proportional to the 3/2 power of its surface area. [Why is this reasonable? Note: raindrops are not spherical, but let's assume that they always have the same shape, no matter what their size may be.] Suppose that the rate of reduction of the volume of a raindrop is proportional to its surface area. [Why is this reasonable?] Find a formula for the amount of time it takes for a virga raindrop to evaporate completely, expressed in terms of the constants you introduced and the initial surface area of a raindrop. Check that the units of your formula are correct. Suppose somebody suggests that the rate of reduction of the volume of a raindrop is proportional to the **square** of the surface area. Argue that this cannot be correct.

4. One theory about the behaviour of moths states that they navigate at night by keeping a fixed angle between their velocity vector and the direction of the Moon. A certain moth flies near to a candle and mistakes it for the Moon.

(i) Prove that in polar coordinates (r, θ) with the candle at the origin, the formula for the angle ψ between the radius vector and the velocity vector is given by $\tan(\psi) = r\frac{d\theta}{dr}$.

(ii)Use this formula to solve for r as a function of θ and discuss what will happen to the moth.

5. Solve the following equations:

(a) $y' = \frac{1-2y-4x}{1+y+2x}$ (b) $y' = \left(\frac{x+y+1}{x+y+3}\right)^2$

MA1512 Tutorial 1 (Questions suggested by the Engineering Faculty)

Question 6. When a cake is removed from an oven, its temperature is measured at 130 C. Three minutes later its temperature is 90 C. How long will it take for the cake to cool off to 26 C, with room temperature at 25 C?

Question 7. (Context: Persistence of a radioactive contaminant)

If we start with an initial concentration, Ci, of radioactive radon-222, what would its concentration be after 5 days? The half-life for radon is 3.8 days.

Question 8. (Context: Historical world population growth rate)

It took the world about 300 years to increase in population from 0.5 billion to 4.0 billion. If we assume exponential growth at a constant rate over that period of time, what would that growth rate be?