MA1512 TUTORIAL 5

1. The oil tanker in Tutorial 3 is at rest in an almost calm sea. Suddenly, at time $t = T > 0$, it is hit by a single rogue wave [http://en.wikipedia.org/wiki/Rogue_wave] which imparts to it a vertical [upward] momentum P, doing so almost instantaneously. Neglecting friction, solve for $x(t)$, the downward displacement of the ship. How far down does the ship go [if it doesn't sink!]? [Hint: according to Newton's second law, momentum is the time integral of force. So to get the force as a function of time in this problem, you have to find a function which is zero except at $t = T$, and which has an integral equal to P. Note that the delta function has units of 1/time.]

Question 2

Billionaire engineer Tan Ah Lian attributes her enormous success to the fact that she never talked in class when she was an Engineering student at NUS. One day in the lecture the prof announces that a certain gadget contains an electrical circuit with a resistance, capacitance, and inductance, with values of R, C, L which were all stated. [For those who don't know the physics: the current in an electrical circuit is a function of time, I(t), which satisfies the equation

$$V(t) = RI + L\dot{I} + \frac{1}{C}\int_0^t I\ dt.$$

Here V(t) is a given function of time called the voltage, and R, L, and C are certain constants called the resistance, inductance, and capacitance; as usual $\dot{I}$ is the time derivative of I. You *do not* need to know the physics to solve this problem.] Sadly, Ah Lian could not hear all of the numbers mentioned due to the incessant babbling of a talkative minority; all she could hear was that the resistance R is 2 ohms. Undeterred, she steals back into the room after class and quickly switches the gadget on and off at t = 2, thus firing a short burst of voltage into it, and finds that the current is $I(t) = u(t-2)\left[e^{-(t-2)}\cos(t-2) - e^{-(t-2)}\sin(t-2)\right]$ amperes, where $u(t)$ is the step function. She then deduces what the prof must have said about the inductance and the capacitance, L and C. What are her answers? [Hint: recall the formula for the Laplace transform of an integral.]

Question 3

Using the method of separation of variables, solve the following partial differential equations:

(a) $yu_x - xu_y \;=\; 0$;

(b) $u_x = yu_y, y > 0$;

(c) $u_{xy} = u$;

(d) $xu_{xy} + 2yu \;=\; 0, x > 0$.

Ans: (a) $u(x,y) = ke^{c(x^2+y^2)}$; (b) $u(x,y) = ky^c e^{cx}$; (c) $u(x,y) = ke^{cx+y/c}$; (d) $u(x,y) = kx^c e^{-y^2/c}$.

Question 4

Show carefully that the d'Alembert solution of the wave equation, given in lectures, does satisfy the equation and the boundary and initial conditions.

Question 5

Solve

$$u_t = 2u_{xx}, \qquad 0 < x < 3, \quad t > 0,$$

given boundary conditions $u(0,t) = 0, \quad u(3,t) = 0$, and initial condition

$$u(x,0) \;=\; \sin^5 \pi x.$$

(Hint: Use the trigonometric identity $\sin^5 \pi x = \frac{5}{8}\sin \pi x - \frac{5}{16}\sin 3\pi x + \frac{1}{16}\sin 5\pi x$.)

Question 6 (Suggested by FoE)

Consider a solid object with mass m immersed in water. A pulse horizontal force is applied initially. The dynamic equation is

$$m\frac{d^2x}{dt^2} = -A\frac{dx}{dt} + B\delta(t)$$

where the 1st term on the right is the hydrodynamic resistance force, and A and B are constants. The initial conditions are $x(0)=0$ and $x'(0)=0$. Apply Laplace transform to solve the above ODE to determine the location *x* as a function of time.