**GER1000 QUANTITATIVE REASONING**
**TUTORIAL 4**
*Please work on the problems before coming to class. In class, you will engage in group work.*

**Question 1**

Suppose you want to settle a dispute with your friend by tossing a coin. Unfortunately both of you do not carry any coin, but there happens to be a bottle cap on the floor. So you suggest to toss the bottle cap, and you both agree to call the event that it lands with the top facing up “H”, and the other event “T”. Furthermore, both of you agree that the events are not equally likely, i.e., the respective chances, h and t, are not equal.

Which of the following way of tossing the cap twice should be used to guarantee fairness in settling the dispute?

(i) Your friend wins if both caps land with the same side up. That is, he wins if the outcome is HH or TT. Otherwise, you win.
(ii) Your friend wins if the outcome is HT (first H, then T); you win if the outcome is TH. Otherwise, you repeat the experiment, and go through the previous rules to decide who wins.

**Question 2**

A game about the impact of climate change on human life uses ten independent rolls of a fair die to simulate the weather for ten years. If the die shows one spot or six spots, then there is a crisis.

a) Calculate the probability that there is no crisis in a ten-year period, to 2 significant digits.
b) The quality of life increases by 10 points if there is no crisis in a ten-year period, and decreases by 2 points otherwise. Calculate the average amount by which the quality of life increases after ten years. Interpret this number in terms of a large number of worlds independently controlled by this game.

**Question 3**

According to the Myers-Briggs Foundation, about 1% of people have a personality type called “advocate”. Such people are ``capable of taking concrete steps to realise their goals and make a lasting positive impact.’’

Suppose there is a test that asks some questions, and based on the answer given by a person, declares the person is an advocate (positive), or not (negative). Among the advocates, 90% will test positive. Among the non-advocates, 20% will test positive.

A person is randomly chosen from a city of 10,000 people. Find the probability that this person is an advocate, given that the test is positive. Does it seem too low, compared to (i) the sensitivity and specificity; (ii) the base rate?

**Question 4**

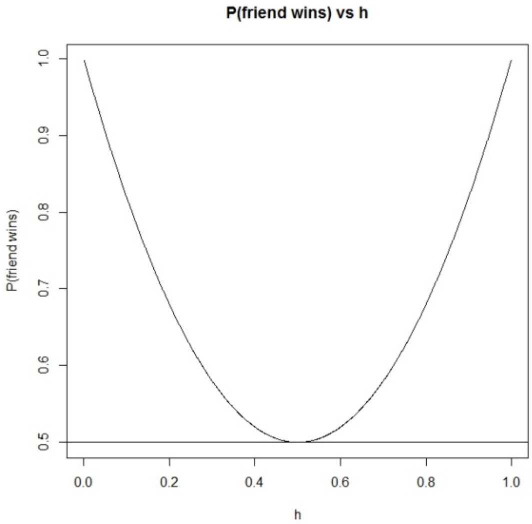
On Alan’s first visit to Macau, he decided to try his luck in at the Venetian Casino Resort. He played a game that is supposed to offer a 50% chance of winning. Out of seven independent plays, he won the first 6 games and lost the last game. His friend Brad who always bets against Alan is unhappy and thinks the game has been rigged. Brad approaches this issue by testing a hypothesis.

(a) State Brad’s null and alternative hypotheses.
(b) If the null hypothesis is true, what is the chance of winning 6 games in a row and losing the next one?
(c) Calculate the P value based on Alan’s data, to one significant figure.
(d) What can Brad conclude, based on 5% level of statistical significance?

Solutions:

Question 1

i. By independence, $P(HH) = h \times h = h^2$ and $P(TT) = t \times t = t^2$. Since HH and TT are mutually exclusive, $P(\text{friend wins}) = h^2 + t^2$, which is $h^2 + (1-h)^2$, because $t = 1 - h$.




The graph suggests that for any h value not equal to 0.5, your friend has an edge, so (i) is not fair.

[Extra] A more convincing argument is to write $h = ½ + e$, where e represents how far h is away from ½. Then $P(\text{friend wins}) = (1/2+e)^2 + (1/2-e)^2 = (1/4 + e + e^2) + (1/4 - e + e^2) = ½ + 2e^2$, which is larger than ½, since e is not 0.

ii. By independence, $P(TH) = t \times h$ and $P(HT) = h \times t$, which are equal. So in every experiment, it is equally likely that your friend wins or you win. Hence, in the entire game, it is equally likely that your friend wins or you win, i.e. (ii) is fair. Furthermore, (ii) settles the dispute unless the probability that the bottle cap lands heads is 0 or 1.

Question 2

(a) In any year, there is no crisis if the die shows 2, 3, 4, or 5 spots. So P(no crisis in a year) = 4/6. Since years are independent, by the multiplication rule P(no crisis in 10 years) = $(4/6)^{10} \approx 0.017$.

(b) The quality of life increases by 10 with probability 0.017, and by -2 with probability 1 – 0.017 = 0.983 (at least one crisis in 10 years). The average value is 0.017 x 10 + 0.983 x (-2) = -1.80. Imagine a large number of worlds, independently experiencing climate as stipulated. Some lucky ones will have an increase in quality of life, though most will suffer. On average, they will experience a decrease of about 1.8 points.

Question 3

We are given P(advocate) = 0.01 (base rate), P(+|advocate) = 0.90 (sensitivity), and P(+|not advocate) = 2/10, so the specificity is P(-|not advocate) = 8/10. The city can be presented in this table:

| | Positive (+) | Negative (-) | sum |
|---|---|---|---|
| Advocate | 90 | 10 | 100 |
| Not | 1,980 | 7,920 | 9,900 |
| sum | 2,070 | 7,930 | 10,000 |

From the table, P(advocate|+) = 90/2,070 ≈ 0.04. It is very low compared to the sensitivity and specificity, but higher than the base rate. The test helps improve prediction, though not by much. If the base rate were higher, then P(advocate|+) will be more impressive. For example, if P(advocate) = 0.1, then P(advocate|+) = 900/2700 ≈ 0.33.

Question 4

(a) Null: The chance of Alan winning the game is 0.5. Alternative: The chance of Alan winning the game is more than 0.5.

(b) Under the null hypothesis, the chance of winning is 0.5, so the chance of losing is also 0.5. By independence, P(6 wins in a row) = 0.5 x 0.5 x 0.5 x 0.5 x 0.5 x 0.5 = $0.5^6$. Again by independence, P(6 wins in a row followed by a loss)= $0.5^6$ x 0.5 = $0.5^7$ ≈ 0.0078.

(c) The P value is the chance of getting exactly 6 wins or 7 wins. The event of getting exactly 6 wins and the event of getting exactly 7 wins are mutually exclusive, so the P value is obtained by adding these two chances. The event "exactly 6 wins" can occur in 7 mutually exclusive ways, each with chance given in (b): P(exactly 6 wins) ≈ 0.0078 x 7 ≈ 0.055. P(exactly 7 wins) = $0.5^7$ ≈ 0.0078. So the P value is roughly 0.055 + 0.0078 ≈ 0.06 = 6%.

(d) If Brad follows the threshold of 5% strictly, he will declare that the observed event is insufficient evidence to reject the null hypothesis. However, the choice of threshold is a convention adopted by experimenters. If the threshold was adopted as 10%, then Brad would reject the null hypothesis.