GER1000 2018 Sem 2
Quiz 7 and solutions

A standard deck of playing cards consists of four suits: Spades, Hearts, Clubs and Diamonds. Each suit contains 13 cards: which are 2, 3, …, 10, Jack, Queen, King, Ace. So there are a total of 52 cards.

1. Three sensors, operating independently, are set to detect intruders moving through a certain area. Each sensor has a probability of 0.9 of detecting an intruder in this area. If an intruder enters the area, what is the probability it goes undetected?
   (a) 0.3
   (b) 0.1
   (c) 0.001
   (d) 0.729

   Explanation: The probability of any sensor failing to detect an intruder in this area is 1-0.9=0.1 (See Unit 2 Slide 7). Since the sensors are operating independently, the probability of an intruder going undetected is 0.1x0.1x0.1=0.001 (See Unit 2 Slide 10).

2. A card is drawn at random, and you win $100 if it is a Heart or a King. What is the probability of winning $100?
   (a) 13/52
   (b) 14/52
   (c) 16/52
   (d) 17/52

   Explanation: As described in the question, there are 13 cards in a standard deck which are Hearts and 4 cards which are Kings (one from each suit). Of these cards, the King of Hearts belongs to the Hearts suit and is also a King. Hence there are 16 cards that could win you $100, giving a probability of 16/52 (See Unit 2 Slide 5).

   Alternatively, we can also arrive at the same conclusion in the following way: There are 13 Hearts, of which one is a King. Of the remaining 39 cards, there are 3 Kings. So there are a total of 13+3=16 cards that will win $100. This gives a probability of 16/52.

3. A particular insurance policy charges every subscriber a premium of $1,000. Suppose 0.2% of subscribers make claims of $100,000 each, and 1% of subscribers make claims of $10,000 each. The rest of the subscribers do not make any claims. What is the average gain of the insurance company per subscriber?
   (a) $400
   (b) $500
   (c) $600
   (d) $700
   (e) $790

   Explanation: Following the discussion on Unit 3 Slide 12, the insurance company makes a "gain" of $(1,000-100,000) on 0.2% of policies, a "gain" of $(1,000-10,000) on 1% of policies, and a gain of \$1,000 on the rest of the 98.8% of policies. This gives an average gain of \$(-99,000x0.2% -9,000x1% +1,000x98.8%) = \$700.

We can also compute this in another way: Average claim per person is given by \$100,000*0.2% + \$10,000*1% = \$300. This gives the average gain per customer = \$1000 - \$300 = \$700.

4. For which of the following pairs of events are independent?
   (a) Obtaining Kings on two random draws without replacement from a standard deck of 52 cards.
   (b) Obtaining Heads on two tosses of a coin.

   Explanation: For (a), getting a King on your first draw leaves 3 Kings out of 51 cards. Hence P(second is King | first is King) = 3/51. By a similar reasoning, P(second is King | first is not King) = 4/51. So the events in are dependent. (b) The outcome of any coin toss does not affect the outcome of future coin tosses. (See Unit 2, Slide 9)

5. A device has two LED lights. When a button is pressed, the red light flashes with probability 0.6 and the blue light flashes with probability 0.6. The probability both lights will flash is 0.3. What is the probability that only the red light will flash?
   (a) 0.2
   (b) 0.3
   (c) 0.4
   (d) 0.5
   (e) 0.6

   Explanation: Using the addition rule (See Unit 2 Slide 6), we have P(red light flashes) = P(only red light flashes) + P(both lights flash). So, P(only red light flashes) = 0.6-0.3 =0.3.