NATIONAL UNIVERSITY OF SINGAPORE
Department of Mathematics

SEMESTER II, 2017/2018
MA1508E MID-TERM TEST

Full Name : ________________________________

Matric/Student Number : _______________

Tutorial Group : ___________

INSTRUCTIONS
PLEASE READ CAREFULLY

- Write your **full name, matric number and tutorial group** clearly above on this cover page.
- There are **4** questions printed on **3** pages. Answer **all** questions.
- You must show all your working clearly, failure to do so will result in marks deducted.
- Use pen for this test.
- All answers and working have to be written on the answer book provided.
- Start on a new page for each question.
- Tie this cover page (and question paper) together with your answer book before submission.

Question 1

(i) Let

$$\boldsymbol{A} = \begin{pmatrix} 1 & 0 & -2 & -1 \\ -2 & 1 & 6 & 7 \\ 3 & -2 & -5 & -3 \end{pmatrix}.$$

Find three elementary matrices $\boldsymbol{E_1}, \boldsymbol{E_2}, \boldsymbol{E_3}$ such that $\boldsymbol{E_3E_2E_1A}$ is a matrix in row-echelon form.

(ii) Hence, solve the following linear system

$$\begin{cases} x & & & - & 2z & = & -1 \\ -2x & + & y & + & 6z & = & 7 \\ 3x & - & 2y & - & 5z & = & -3 \end{cases}$$

(iii) **Use your answer in (ii)** to solve the following linear system

$$\begin{cases} 2a & & & + & 4c & = & -1 \\ -4a & + & 3b & - & 12c & = & 7 \\ 6a & - & 6b & + & 10c & = & -3 \end{cases}$$

Warning: You should not solve the linear system directly (that is, perform any further elementary row operations).

(iv) Use your answer in (i) to write down the $\boldsymbol{LU}$ factorisation of $\begin{pmatrix} 1 & 0 & -2 \\ -2 & 1 & 6 \\ 3 & -2 & -5 \end{pmatrix}$. Hence solve the following linear system:

$$\begin{pmatrix} 1 & 0 & -2 \\ -2 & 1 & 6 \\ 3 & -2 & -5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \\ -5 \end{pmatrix}.$$

Question 2

(i) Let $\boldsymbol{A}$ and $\boldsymbol{B}$ be row equivalent, square matrices of order 3 such that

$$\boldsymbol{A} \xrightarrow{R_3 - 3R_1} \xrightarrow{R_2 \leftrightarrow R_3} \xrightarrow{\frac{1}{4}R_2} \xrightarrow{R_3 - R_2} \begin{pmatrix} 1 & -3 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & -5 \end{pmatrix} = \boldsymbol{B}.$$

Find $\det(\boldsymbol{A})$ and $\boldsymbol{A}$.

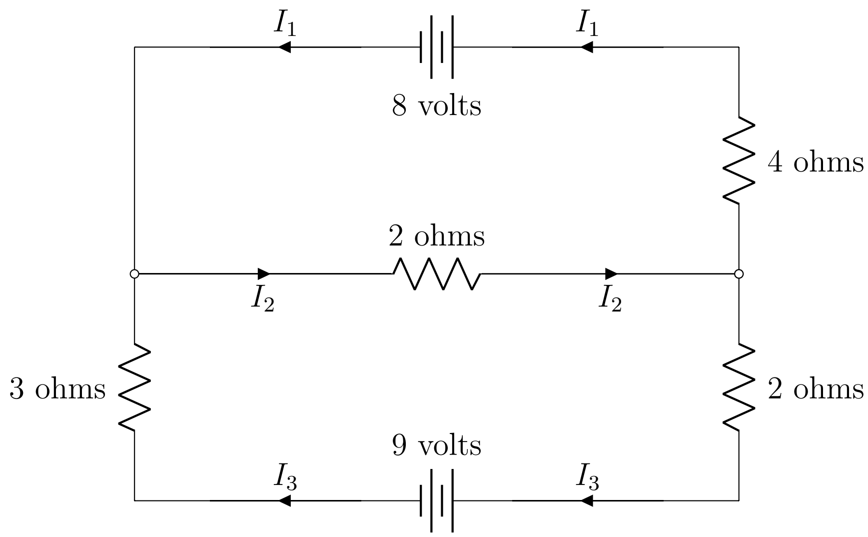
(ii) Find $\boldsymbol{B}^{-1}$ and write down $\operatorname{adj}(\boldsymbol{B})$.

(iii) If $\boldsymbol{C}$ is row equivalent to $\boldsymbol{A}$, then $\boldsymbol{C}$ must be invertible. Is this statement true or false? Justify your answer.

(iv) If $\boldsymbol{D}$ is row equivalent to $\boldsymbol{B}$ and $\det(\boldsymbol{D}) = 1$, then $\boldsymbol{D} = \boldsymbol{I_3}$. Is this statement true or false? Justify your answer.

Question 3

Consider the following electrical network.



(i) Write down a linear system with 3 equations involving unknowns I_1, I_2 and I_3 using KCL and KVL.

(ii) Solve the linear system using Gauss-Jordan Elimination.

Question 4

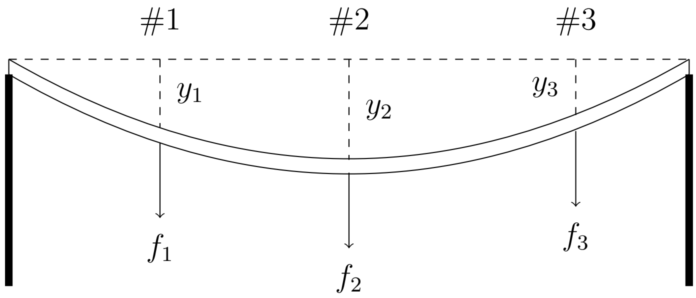
Let

$$\boldsymbol{u} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \quad \boldsymbol{v} = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}, \quad \boldsymbol{w} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}.$$

(i) Find $a_1, a_2, a_3, b_1, b_2, b_3, c_1, c_2, c_3$ such that

$$a_1\boldsymbol{u} + a_2\boldsymbol{v} + a_3\boldsymbol{w} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad b_1\boldsymbol{u} + b_2\boldsymbol{v} + b_3\boldsymbol{w} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad c_1\boldsymbol{u} + c_2\boldsymbol{v} + c_3\boldsymbol{w} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}.$$

(ii) Recall the definition of a 3×3 *flexibility matrix* $\boldsymbol{D}$ used to study the flexibility of a horizontal elastic beam shown in the figure below.



We know that, by Hooke's law,

$$\boldsymbol{y} = \boldsymbol{D}\boldsymbol{f}$$

where $\boldsymbol{f} = \begin{pmatrix} f_1 \\ f_2 \\ f_3 \end{pmatrix}$ represents the forces applied at the 3 points #1, #2 and #3 and $\boldsymbol{y} = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$ represents the amount of deflection of the beam at the 3 points when it is subjected to $\boldsymbol{f}$.

Suppose we wish to determine the flexibility matrix $\boldsymbol{D}$ of an elastic beam. Three sets of experiments were conducted, where different units of force were applied at the 3 points and each time, the deflections were measured. The results of the experiment are given in the table below.

	Force applied			Deflection observed		
	f_1	f_2	f_3	y_1	y_2	y_3
Experiment 1	1	0	1	0.5	0.3	0.5
Experiment 2	0	1	2	0.1	0.3	0.7
Experiment 3	2	1	0	0.7	0.3	0.1

Use your answer in part (i) to find the flexibility matrix $\boldsymbol{D}$.

END OF TEST