

# **MATRICES**

# DEFINITION

A **matrix** is a rectangular array of numbers.

The numbers in the array are called **entries**.

The **size** of a matrix is  $m \times n$  if it has  $m$  rows and  $n$  columns.

$$A = \begin{pmatrix} 3 & 2.4 & 1 & -1 & 0 & 11 \\ -5 & 2 & 2 & 0 & 0 & 1 \\ 4.1 & \pi & 20 & 10 & -2 & 1 \end{pmatrix}$$

$A$  is a  $3 \times 6$  matrix and the  $(1,4)$ -entry of  $A$  is  $-1$ .

# DEFINITION

A **column matrix** is a matrix with only one column.

$$\begin{pmatrix} 2 \\ 3 \\ -1 \\ 0 \end{pmatrix}$$


A **row matrix** is a matrix with only one row.

$$(1 \quad -1 \quad 0)$$

# NOTATION

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \text{ is a } m \times n \text{ matrix.}$$

We can also write  $\mathbf{A} = (a_{ij})_{m \times n}$  where  $a_{ij}$  is the  $(i, j)$ -entry of  $\mathbf{A}$ .

$i$  is the 'row index'   $a_{ij}$   $j$  is the 'column index'

We can also write  $\mathbf{A} = (a_{ij})$ .

# DEFINITION

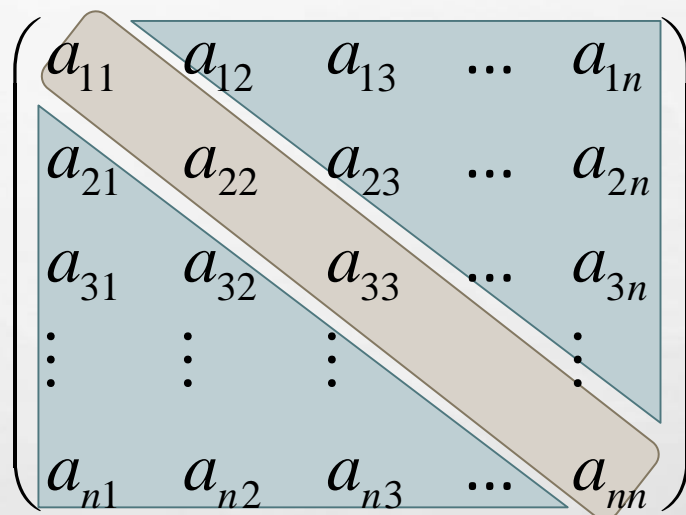
**Square matrices** are matrices with the same number of rows and columns.

$$(2) \quad \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

A square matrix with  $n$  rows and  $n$  columns is said to be of order  $n$ .

# DEFINITION

Given a square matrix  $A = (a_{ij})$  of order  $n$ ,

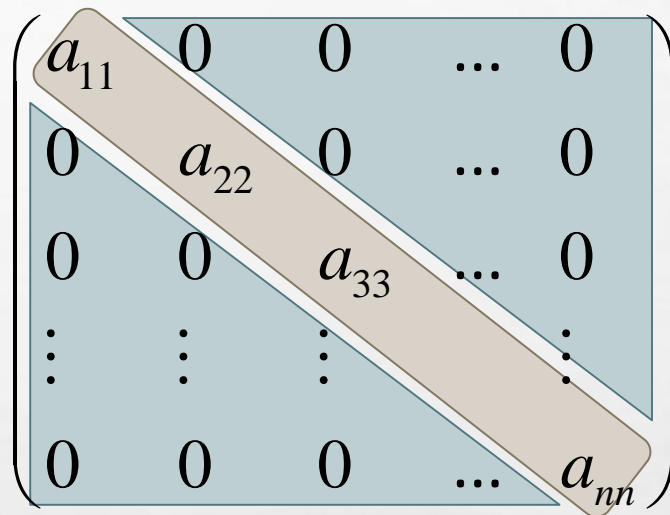

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \cdots & a_{nn} \end{pmatrix}$$

The **diagonal** of  $A$  is the sequence  $a_{11}, a_{22}, a_{33}, \dots, a_{nn}$ .

$a_{11}, a_{22}, a_{33}, \dots, a_{nn}$  are called the **diagonal entries** of  $A$ .

$a_{ij}, i \neq j$ , are called the **non-diagonal entries** of  $A$ .

# DEFINITION



The diagram shows a square matrix enclosed in large parentheses. A light blue shaded triangular region covers the upper-left portion of the matrix, starting from the top-left corner and extending towards the bottom-right. A light tan shaded diagonal band runs from the top-left to the bottom-right, containing the diagonal elements. The matrix is filled with zeros in the off-diagonal positions. The diagonal elements are labeled as follows: the top-left element is  $a_{11}$ , the second row second column is  $a_{22}$ , the third row third column is  $a_{33}$ , and the bottom-right element is  $a_{nn}$ . Ellipses ( $\dots$ ) are used to indicate the continuation of the matrix between the third row and the final row. Vertical ellipses ( $\vdots$ ) are used in the first and last columns to indicate the continuation of rows.

$$\begin{pmatrix} a_{11} & 0 & 0 & \dots & 0 \\ 0 & a_{22} & 0 & \dots & 0 \\ 0 & 0 & a_{33} & \dots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \dots & a_{nn} \end{pmatrix}$$

A square matrix is a **diagonal matrix** if all its non-diagonal entries are zero.

$$\mathbf{A} = (a_{ij}) \text{ is diagonal} \Leftrightarrow a_{ij} = 0 \text{ whenever } i \neq j.$$

# DEFINITION

$$\begin{pmatrix} c & 0 & 0 & \dots & 0 \\ 0 & c & 0 & \dots & 0 \\ 0 & 0 & c & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & c \end{pmatrix}$$

A diagonal matrix is a **scalar matrix** if all its diagonal entries are the same.

$$\begin{aligned} A = (a_{ij}) \text{ is scalar} &\Leftrightarrow a_{ij} = 0 \text{ whenever } i \neq j \text{ and} \\ &a_{ij} = c \text{ whenever } i = j \\ &(c \text{ is a constant}). \end{aligned}$$



# DEFINITION

$$\begin{pmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \dots & 1 \end{pmatrix}$$

A diagonal matrix is an identity matrix if all its diagonal entries are equal to 1.

$\mathbf{I}_n$  is an identity matrix of order  $n$ .

If there is no danger of confusion, we simply write  $\mathbf{I}$ .

# DEFINITION

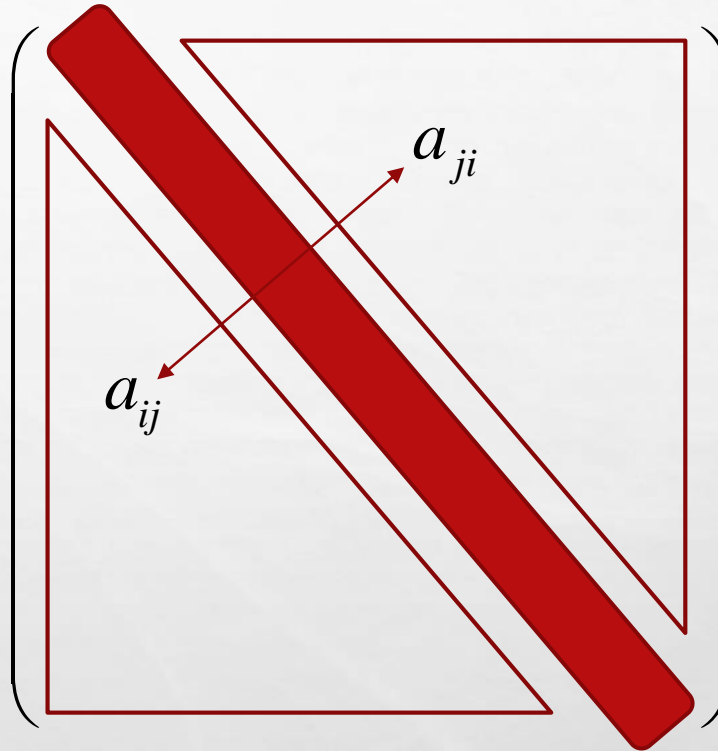
$$\begin{pmatrix} 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \dots & 0 \end{pmatrix}$$

A **zero matrix** is a matrix with all entries equal to zero.

$\mathbf{0}_{m \times n}$  is a  $m \times n$  zero matrix.

If there is no danger of confusion, we simply write  $\mathbf{0}$ .

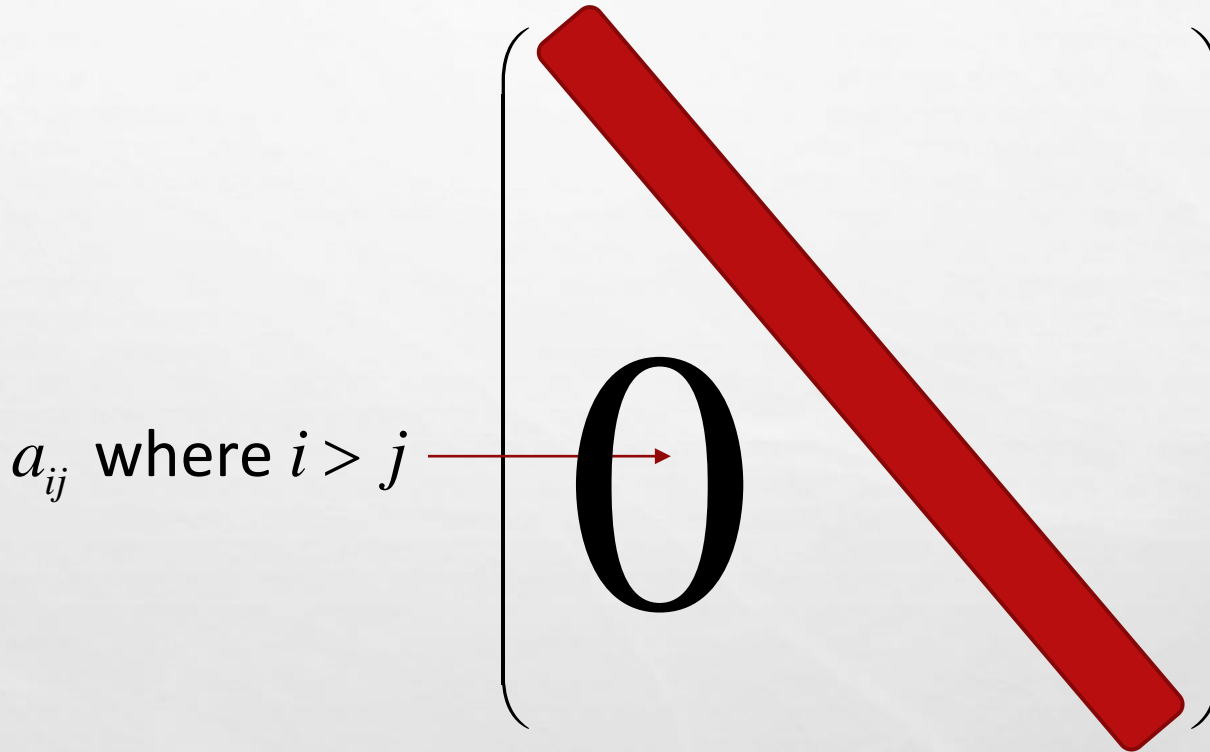
# DEFINITION



A symmetric matrix  $(a_{ij})$  is a square matrix where

$$a_{ij} = a_{ji} \text{ for all } i, j.$$

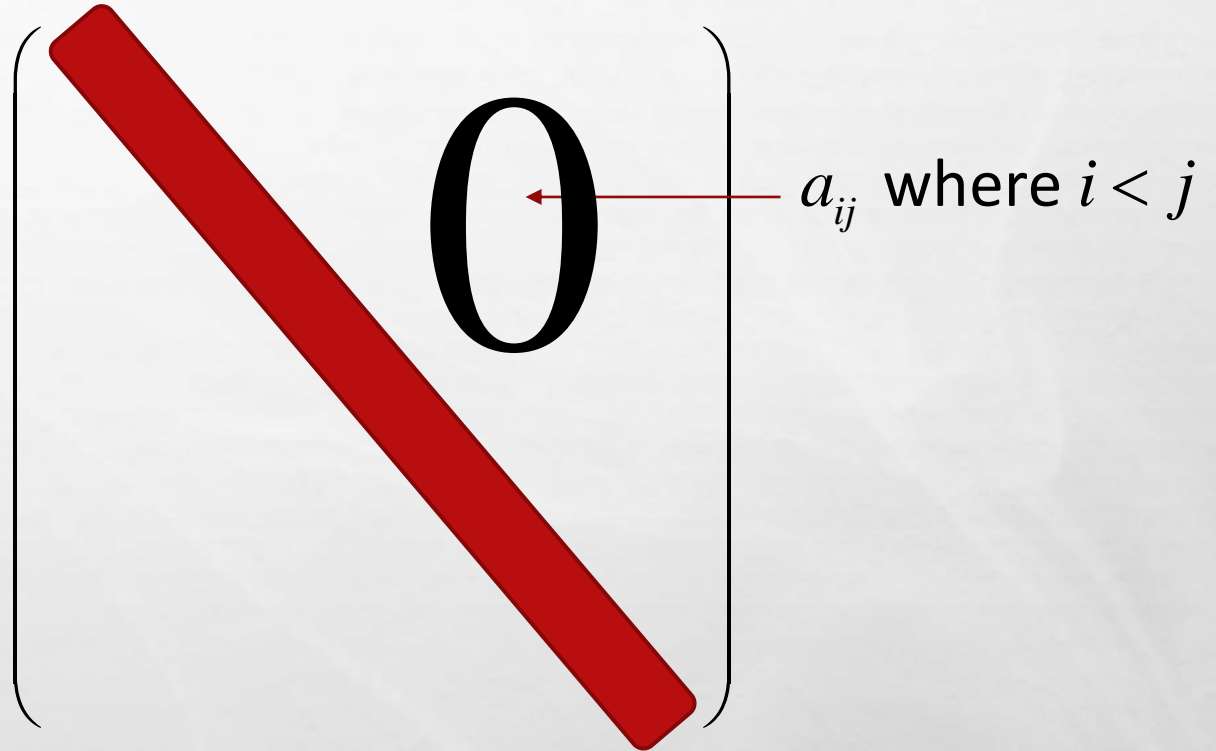
# DEFINITION



A square matrix  $(a_{ij})$  is an upper triangular matrix if

$$a_{ij} = 0 \text{ for all } i > j.$$

# DEFINITION



A square matrix  $(a_{ij})$  is an **lower triangular matrix** if

$$a_{ij} = 0 \text{ for all } i < j.$$

# SUMMARY

- 1) Matrices, entries, size, diagonal entries.
- 2) Diagonal matrix, scalar matrix, identity matrix, zero matrix.
- 3) Symmetric matrix, upper triangular matrix, lower triangular matrix.