Week 04 IVLE Quiz

1. $\boldsymbol{C}$ is a 3×3 matrix. Consider the 3×6 matrix $(\boldsymbol{C} \mid \boldsymbol{I_3})$. Suppose we know that the reduced row-echelon form of $(\boldsymbol{C} \mid \boldsymbol{I_3})$ has 3 pivot columns. How many of the following statement(s) is/are definitely true?

   (I) $\boldsymbol{C}$ is invertible.

   (II) $\boldsymbol{Cx} = \boldsymbol{0}$ has only the trivial solution.

   (III) $\boldsymbol{C} = \boldsymbol{I_3}$.

   (A) None

   (B) One

   (C) Two

   (D) All three.

   Answer: (A)

2. Let a linear system with 4 equations and 4 unknowns be represented by $\boldsymbol{Ax} = \boldsymbol{b}$. Suppose we know that $\boldsymbol{Ax} = \boldsymbol{b}$ has a unique solution. Which of the following statements are definitely true?

   (I) $\boldsymbol{A}$ can be written as a product of elementary matrices.

   (II) $\boldsymbol{Ax} = \boldsymbol{0}$ has infinitely many solutions.

   (III) $\boldsymbol{A}$ is invertible.

   (A) All three

   (B) (I) and (III) only

   (C) (II) and (III) only

   (D) Cannot be determined. More information is needed.

   Answer: (B)

3. Consider the following matrix $\boldsymbol{B}$. What is the $(2, 3)$-cofactor of $\boldsymbol{B}$?

$$\boldsymbol{B} = \begin{pmatrix} 1 & 0 & -1 \\ 2 & 1 & 2 \\ 3 & 1 & 2 \end{pmatrix}.$$

   (A) 1

   (B) -1

   (C) 2

   (D) -2

Answer: (B)

4. Consider the following matrix

$$\begin{pmatrix} a & b & c & d \\ 1 & 2 & 3 & 1 \\ 2 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{pmatrix},$$

where a, b, c, d are real numbers. Which of the following will result in the matrix being singular?

(A) $-b + c - d = 0$

(B) $-c + 3d - 3b = 0$

(C) $3b - c - 3d = 0$

(D) $c + 3d - 3b = 0$

Answer: (C) or (D) (both are correct)

5. Let $\boldsymbol{A}$ be a square matrix such that $\boldsymbol{Ax} = \boldsymbol{0}$ has infinitely many solutions. Consider the following statements:

(I) The reduced row-echelon form of $\boldsymbol{A}$ is not the identity matrix.

(II) $\boldsymbol{A}$ cannot be expressed as a product of elementary matrices.

(III) For any $\boldsymbol{b}$, the non-homogeneous system $\boldsymbol{Ax} = \boldsymbol{b}$ is always inconsistent.

How many of the above statements is/are always true?

(A) None of the above statements is always true.

(B) Exactly one of the above statements is always true.

(C) Exactly two of the above statements are always true.

(D) All of the above statements are always true.

Answer: (C)