# MATRIX OPERATIONS

# MATRIX EQUALITY

For real numbers, we can add, subtract, multiply and (to some extent) divide. What about for matrices with real number entries?

Before we can write the 'equivalent' of an equation like

$$x = y,$$

we need to define what is meant by '$=$'

in terms of matrices

# MATRIX EQUALITY

Two matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ are said to be equal, written

$$\boldsymbol{A} = \boldsymbol{B},$$

if they have the same size (rows and columns) and

$$a_{ij} = b_{ij} \quad \text{for all } i, j.$$

$$\boldsymbol{A} = \begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix} \qquad \boldsymbol{B} = \begin{pmatrix} 1 & y \\ x & z \end{pmatrix} \qquad \boldsymbol{C} = \begin{pmatrix} 0 & y \\ x & z \end{pmatrix}$$

$$\boldsymbol{D} = \begin{pmatrix} 1 & 1 & x \\ 0 & -1 & 0 \end{pmatrix}$$

# MATRIX OPERATIONS

If $\boldsymbol{A}$ and $\boldsymbol{B}$ are two $m \times n$ matrices, then

1) $\boldsymbol{A} + \boldsymbol{B}$ is a $m \times n$ matrix such that

$$\boldsymbol{A} + \boldsymbol{B} = (a_{ij} + b_{ij}).$$

2) $\boldsymbol{A} - \boldsymbol{B}$ is a $m \times n$ matrix such that

$$\boldsymbol{A} - \boldsymbol{B} = (a_{ij} - b_{ij}).$$

If $\boldsymbol{A}$ is a $m \times n$ matrix and $c$ is a real number, then

3) $c\boldsymbol{A}$ is a $m \times n$ matrix such that

$$c\boldsymbol{A} = (ca_{ij}).$$

# REMARK

1) You cannot add or subtract matrices that are not of the same size.

2) $-\boldsymbol{A}$ is the same as $(-1)\boldsymbol{A}$.

3) $\boldsymbol{A}-\boldsymbol{B}$ is the same as $\boldsymbol{A}+(-1)\boldsymbol{B}$.

4) Just like for real numbers, there are laws for matrix operation which we can use.

# MATRIX OPERATION LAWS

(Commutative Law for Matrix Addition)

For real numbers $x, y$, we have

$$x + y = y + x.$$

For matrices $\boldsymbol{A}, \boldsymbol{B}$ (of the same size), we have

$$\boldsymbol{A} + \boldsymbol{B} = \boldsymbol{B} + \boldsymbol{A}.$$

# MATRIX OPERATION LAWS

(Associative Law for Matrix Addition)

For real numbers $x, y, z$ we have

$$x+(y+z)=(x+y)+z.$$

For matrices $\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}$ (of the same size), we have

$$\boldsymbol{A}+(\boldsymbol{B}+\boldsymbol{C})=(\boldsymbol{A}+\boldsymbol{B})+\boldsymbol{C}.$$

# MATRIX OPERATION LAWS

For real numbers $x, y, z$ we have

$$x(y+z)=xy+xz \quad \text{and} \quad (xy)z=x(yz).$$

For matrices $\boldsymbol{A}, \boldsymbol{B}$ (of the same size) and real numbers $a, b$, we have

$$a(\boldsymbol{A}+\boldsymbol{B})=a\boldsymbol{A}+a\boldsymbol{B}; \quad (a+b)\boldsymbol{A}=a\boldsymbol{A}+b\boldsymbol{A};$$

and

$$a(b\boldsymbol{A})=(ab)\boldsymbol{A}=b(a\boldsymbol{A}).$$

# REMARK

1) Since $(\boldsymbol{A}+\boldsymbol{B})+\boldsymbol{C}=\boldsymbol{A}+(\boldsymbol{B}+\boldsymbol{C})$, we may simply write

$$\boldsymbol{A}_1+\boldsymbol{A}_2+\ldots+\boldsymbol{A}_k$$

without any parentheses.

2) Let $\boldsymbol{A}$ be a matrix.

$$\boldsymbol{A}+\mathbf{0}=\mathbf{0}+\boldsymbol{A}=\boldsymbol{A}. \qquad \boldsymbol{A}-\boldsymbol{A}=\boldsymbol{A}+(-\boldsymbol{A})=\mathbf{0}.$$

$$0\boldsymbol{A}=\mathbf{0}.$$

# SUMMARY

1) Matrix equality.

2) Matrix operations and some laws.