Unit 010 Matrices - definitions and special types

Slide 01: In this unit, we will introduce some basic terminologies for matrices and also some speical class of matrices.

Slide 02: A matrix is simply just a rectangular array of numbers, like $\boldsymbol{A}$ shown below.

(#)

The numbers in the array are called the entries of the matrix.

(#)

The size of the matrix is described by two integers m and n. We say the size of the matrix is $m \times n$ if it m rows and n columns.

(#)

Thus, $\boldsymbol{A}$ here is a 3×6 matrix and the $(1, 4)$-entry of $\boldsymbol{A}$ is the number you find in the first row, fourth column, which is -1.

Slide 03: If a matrix has only one column, we call it a column matrix.

(#)

Likewise, if a matrix has only one row, we call it a row matrix.

Slide 04: A matrix in generally can be denoted in the following manner.

(#)

Here $\boldsymbol{A}$ is a $m \times n$ matrix and a_{ij} is the entry in $\boldsymbol{A}$ found in the i-th row and j-th column.

(#)

Remember that i is the row index and j is the column index.

Slide 05: Matrices with the same number of rows and columns are called square matrices. We say a square matrix is of order n if it has n rows and n columns.

Slide 06: For a square matrix $\boldsymbol{A}$ of order n whose entries are represented by a_{ij}, the diagonal of $\boldsymbol{A}$ is the sequence of entries a_{11}, a_{22}, $\cdots$, a_{nn}.

(#)

These entries are called the diagonal entries of $\boldsymbol{A}$.

(#)

All other entries a_{ij} where $i \neq j$, are called the non-diagonal entries of $\boldsymbol{A}$.

Slide 07: If a square matrix is such that all its non-diagonal entries are equal to 0, we say that the square matrix is a diagonal matrix.

(#)

Thus, we say that a matrix $\boldsymbol{A}$ is diagonal if and only if $a_{ij} = 0$ whenever $i \neq j$.

Slide 08: If a diagonal matrix is such that all its diagonal entries are the same number, then such a matrix is called a scalar matrix.

(#)

Thus, we say that a matrix $\boldsymbol{A}$ is scalar if and only if $a_{ij} = 0$ whenever $i \neq j$ and $a_{ij} = c$ for some constant c whenever $i = j$.

Slide 09: A special type of scalar matrix is when the constant c is equal to 1. Such a matrix is called an identity matrix.

(#)

We have a notation for identity matrices. We use $\boldsymbol{I}_n$ to denote an identity matrix of order n.

(#)

In the event where the order of the identity matrix is not consequential, we simply write $\boldsymbol{I}$.

Slide 10: A zero matrix is one where all the entries are equal to zero.

(#)

A zero matrix with m rows and n columns is denoted as follows.

(#)

Again, if there is no danger of confusion, we can simply write zero to denote a zero matrix.

Slide 11: A symmetric matrix is a square matrix where $a_{ij} = a_{ji}$ for all i and j. Note that a_{ij} and a_{ji} are like mirror images of each other across the diagonal of the matrix.

(#)

Thus, a symmetric matrix is one where the bottom triangular part of the matrix is a mirror reflection of the upper triangular part of the matrix.

Slide 12: A square matrix is said to be an upper triangular matrix if $a_{ij} = 0$ for all $i > j$. Note that row index i bigger than column index j refers to the bottom triangular part of the matrix. This part is all made up of zeros and therefore only the upper triangular part of the matrix contains useful information. That's why we call such matrices upper triangular.

Slide 13: Likewise, a lower triangular matrix is a square matrix where $a_{ij} = 0$ for all $i < j$.

Slide 14: In this unit,

(#)

We introduce matrices, the size of a matrix, the entries, in particular the diagonal entries of a matrix.

(#)

Some special types of matrices like diagonal, scalar, identity and zero matrices were also introduced.

(#)

In particular, square matrices can be symmetric, upper triangular or lower triangular.