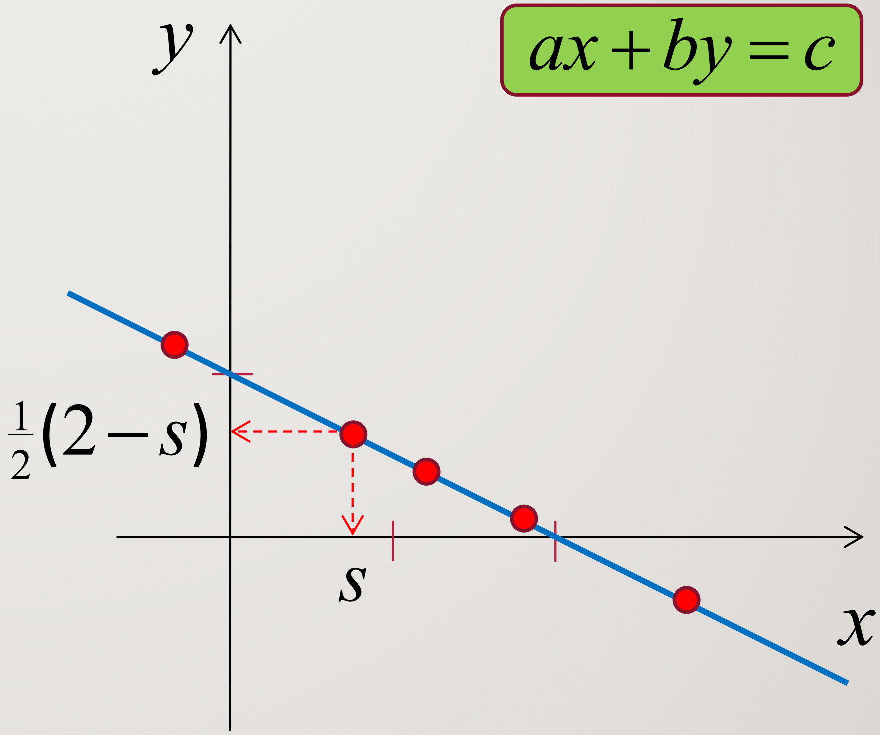
GEOMETRICAL INTERPRETATION

LINEAR EQUATION IN TWO VARIABLES

$$x + 2y = 2$$

$$\boxed{ax + by = c}$$

$$\begin{cases} x = s \\ y = \frac{1}{2}(2 - s), \; s \in \mathbb{R} \end{cases}$$



The solution set of the equation $x + 2y = 2$

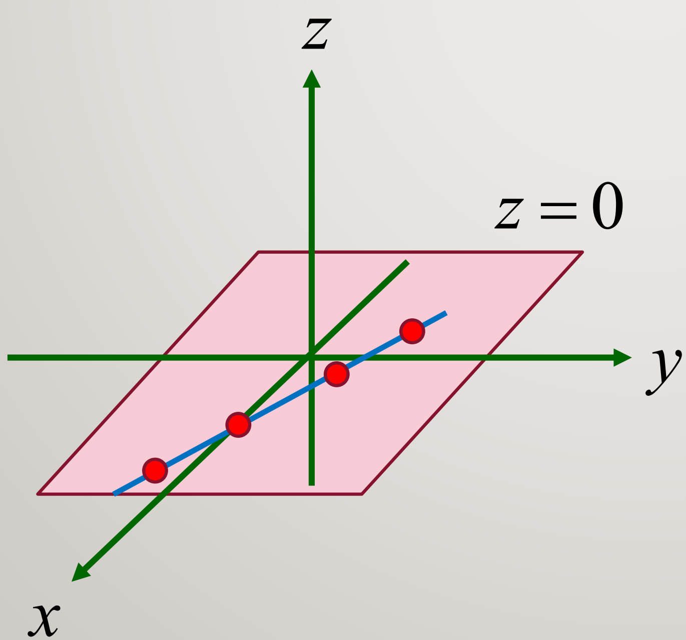
contains all the points $(x, y) = (s, \frac{1}{2}(2 - s)), \; s \in \mathbb{R}$.

These points form the line $x + 2y = 2$.

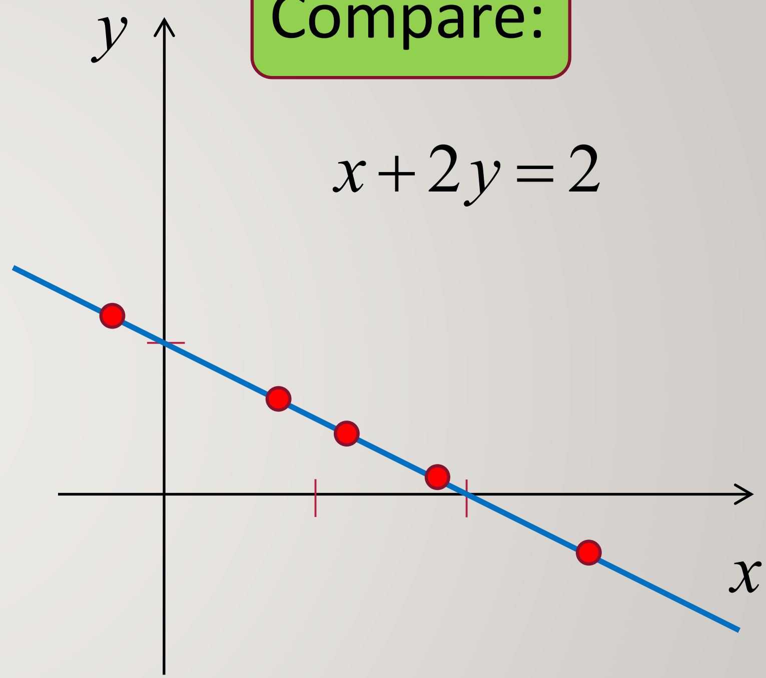
LINEAR EQUATION IN THREE VARIABLES

$$x + 2y + 0z = 2$$

$$\begin{cases} x = 2 - 2s \\ y = s \\ z = t \qquad s,t \in \mathbb{R} \end{cases}$$



Compare:



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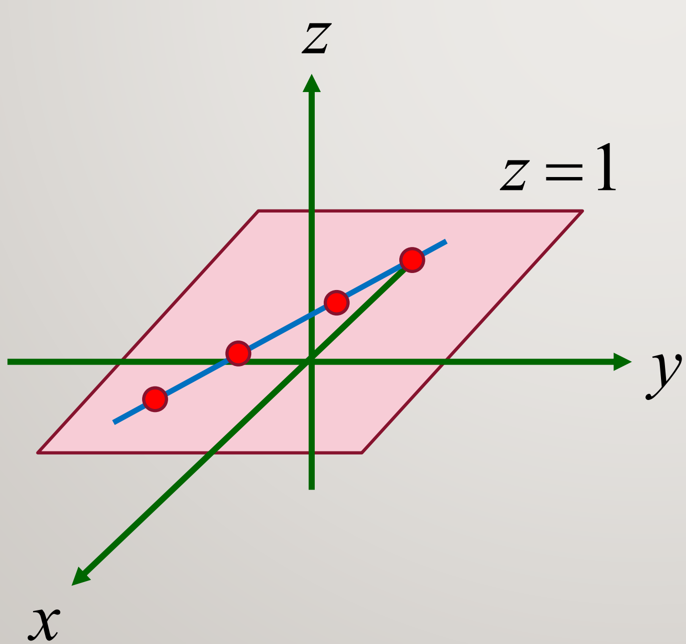

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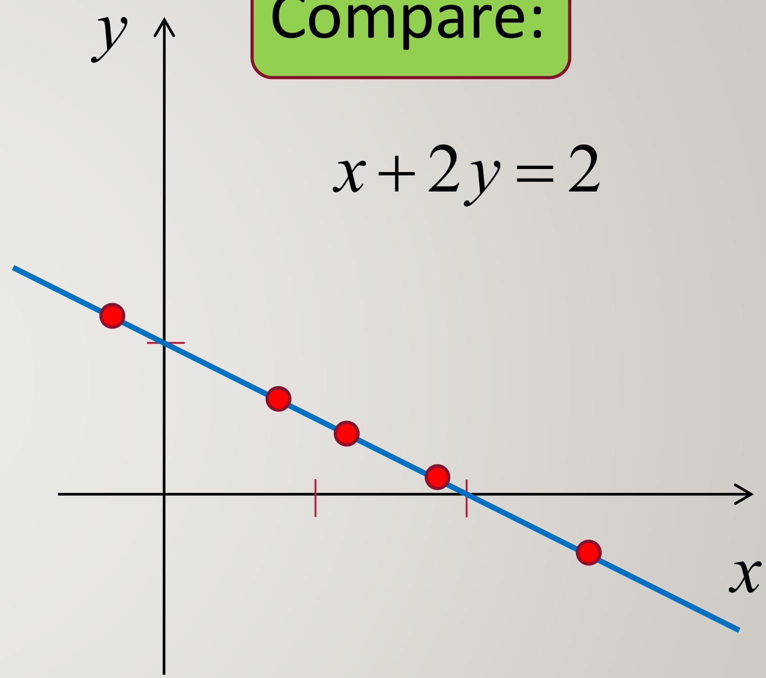

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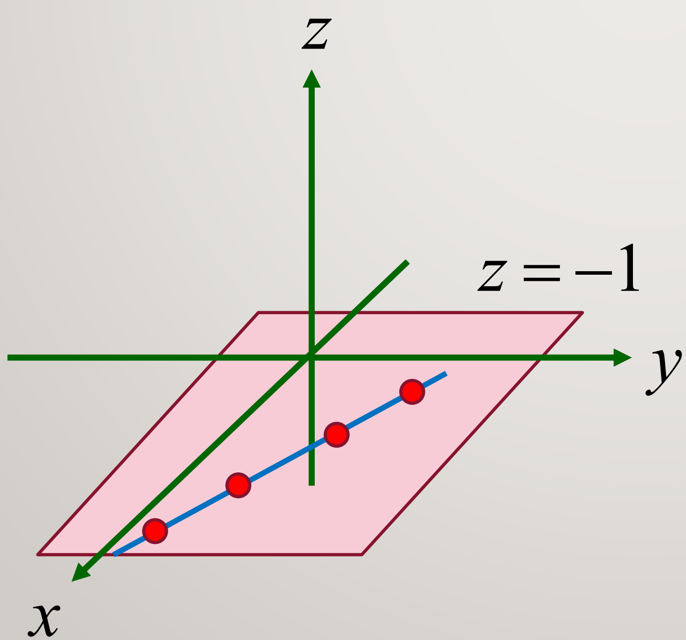

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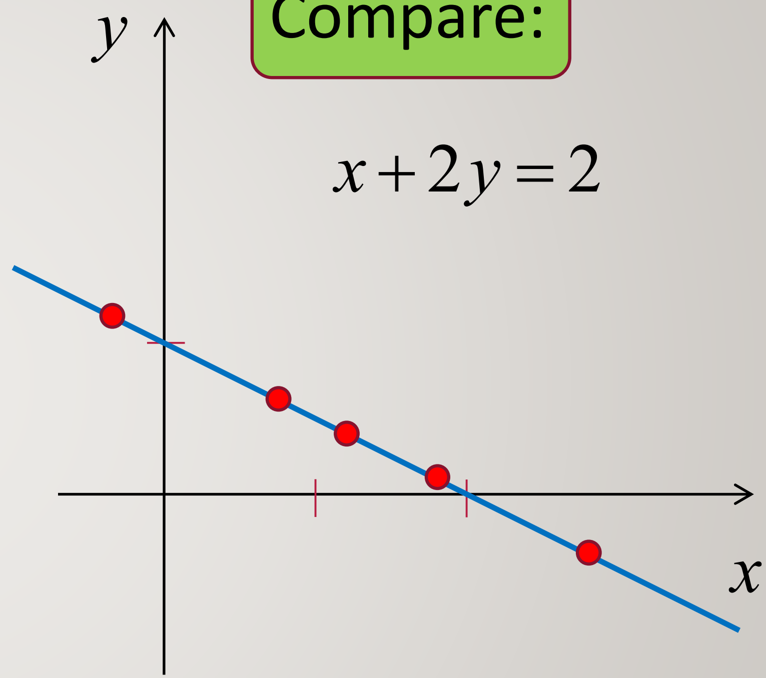

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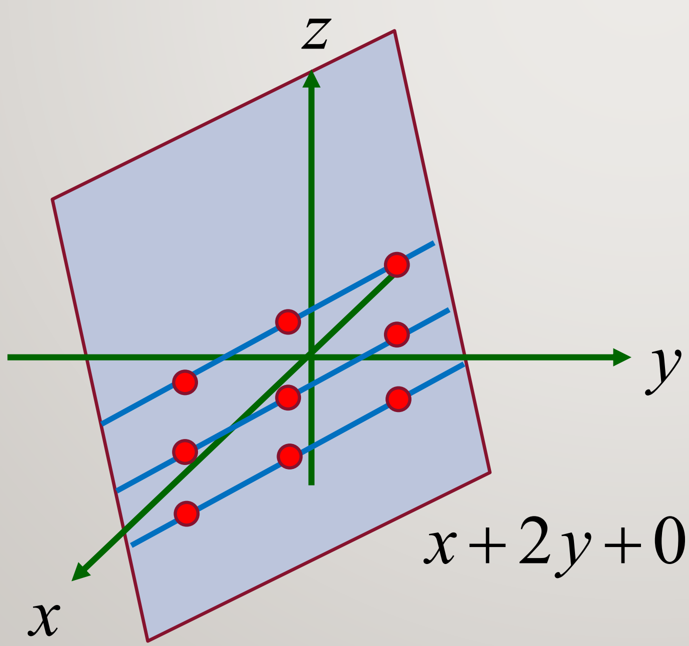

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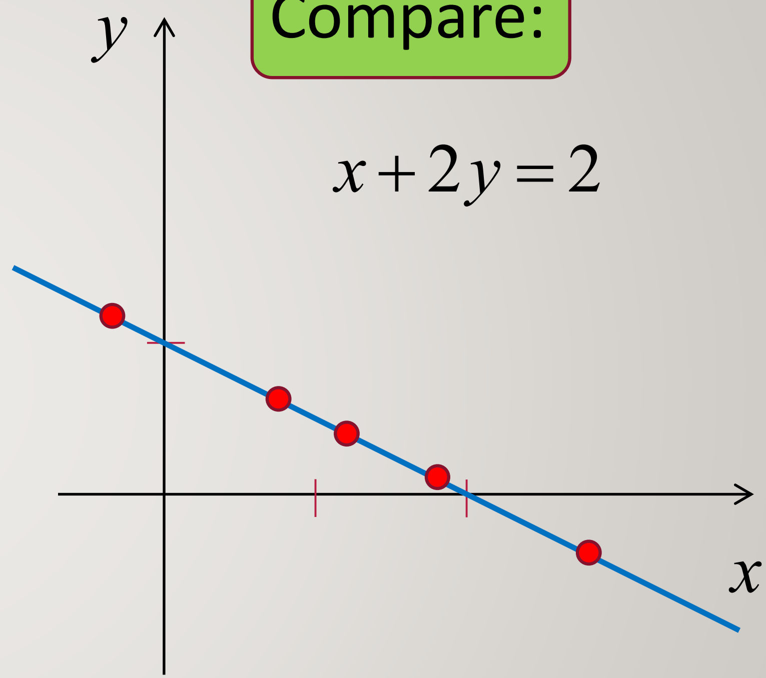

$$x+2y=2$$

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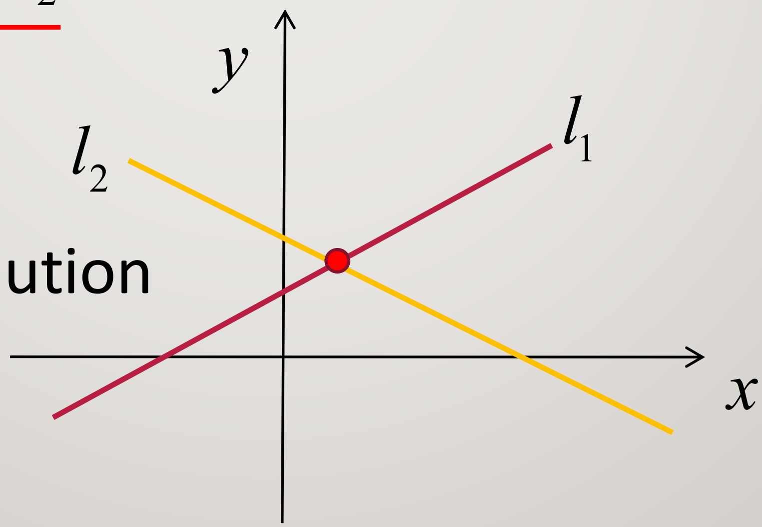
LINEAR SYSTEM IN TWO VARIABLES

l_1 and l_2 are two lines in the xy plane.

$$\begin{cases} a_1x + b_1y = c_1 & (l_1) \\ a_2x + b_2y = c_2 & (l_2) \end{cases}$$

A solution to the linear system is a point (x, y) that lies on both l_1 and l_2.

exactly one solution



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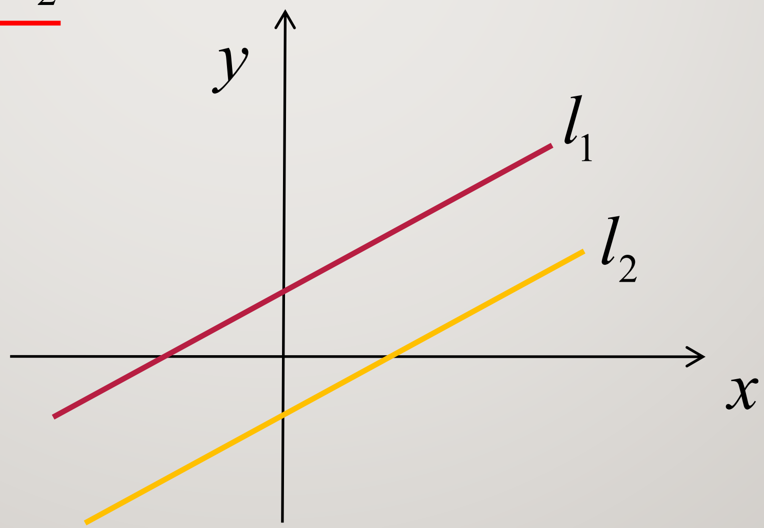
no solution



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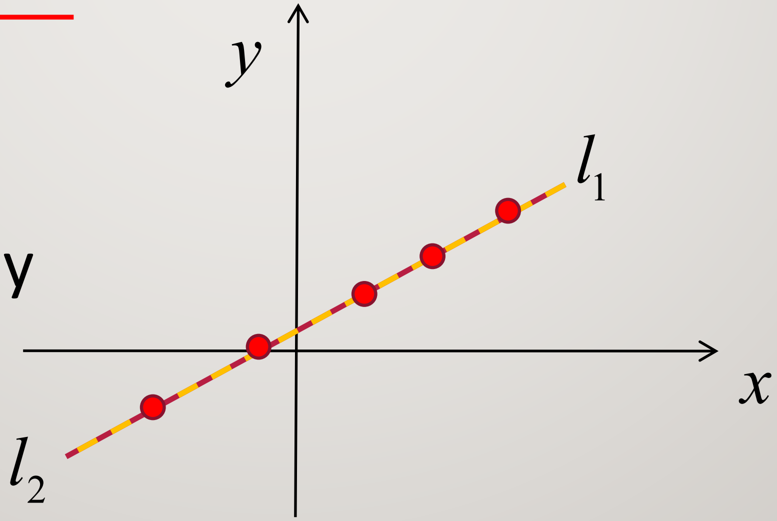
infinitely many solutions

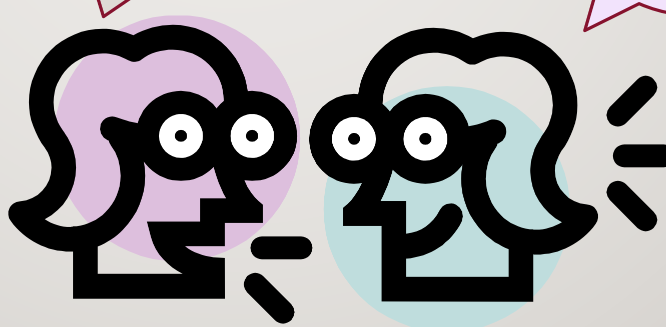


HOW MANY SOLUTIONS?

It turns out, every linear system has either no solution, exactly one solution or infinitely many solutions.

Is there a linear system with exactly 3 solutions?

No!



HOW MANY SOLUTIONS?

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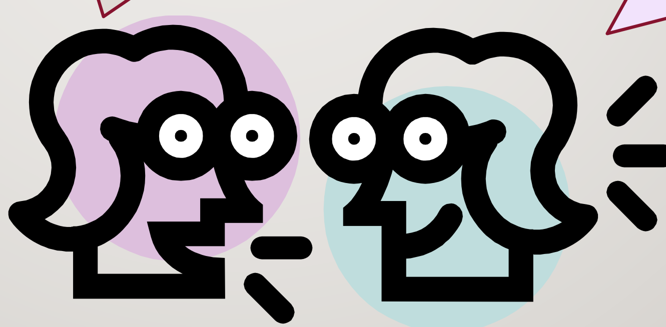
So what if a linear system has at least 3 solutions?

Then it will have infinitely many!



LINEAR SYSTEM IN THREE VARIABLES

p_1, p_2 and p_3 are three planes in the three dimensional space.

$$\begin{cases} a_1x & + & b_1y & + & c_1z & = & d_1 & \qquad (p_1) \\ a_2x & + & b_2y & + & c_2z & = & d_2 & \qquad (p_2) \\ a_3x & + & b_3y & + & c_3z & = & d_3 & \qquad (p_3) \end{cases}$$

A solution to the linear system is a point (x, y, z) that lies on p_1, p_2 and p_3.

No solution?

Exactly one solution?

Infinitely many solutions?

SUMMARY

1) Solutions to a linear equation in two variables
(forms a line)

2) Solutions to a linear equation in three variables
(forms a plane)

3) Solutions to a linear system in two variables
(none? exactly one? infinitely many?)

4) Solutions to a linear equation in three variables
(none? exactly one? infinitely many?)