MA1512 TUTORIAL 3

1. Solve the following differential equations:

(a) $y'' + 6y' + 9y = 0, \qquad y(0) = 1, \qquad y'(0) = -1$

(b) $y'' - 2y' + (1 + 4\pi^2)y = 0, \qquad y(0) = -2, \qquad y'(0) = 2(3\pi - 1)$

2. Find particular solutions of the following:

(a) $y'' + 2y' + 10y = 25x^2 + 3$ (b) $y'' - 6y' + 8y = x^2 e^{3x}$

(c) $y'' - y = 2xsin(x)$ (d) $y'' + 4y = sin^2(x)$

3. Use the method of variation of parameters to find particular solutions of

(a) $y'' + 4y = sin^2(x)$ (b) $y'' + y = sec(x)$

Question 4

A simple pendulum of length 1 metre is at rest at its stable equilibrium position. At time $t = 0$, it is given an initial angular velocity of 1 radian per second. Neglecting friction, determine the angular displacement of the pendulum at time $t = 0.8$ second. Take the value of the gravitational constant $g = 9.8$ metre per second square. Give your answer correct to two decimal places.

Question 5

A fully loaded large oil tanker can be modelled as a solid object with perfectly vertical sides and a perfectly horizontal bottom, so all horizontal cross-sections have the same area, equal to A. Archimedes' principle [http://en.wikipedia.org/wiki/Buoyancy] states that the upward force exerted on a ship by the sea is equal to the weight of the water pushed aside by the ship. Let ρ be the mass density of seawater, and let M be the mass of the ship, so that its weight is Mg, where g is 9.8 m/sec^2. When the ship is at rest, find the distance d from sea level to the bottom of the ship. This is called the **draught** of the ship.

Suppose now that the ship is *not* at rest; instead it is moving in the vertical direction. Let d + x(t) be the distance from sea level to the bottom of the ship, where d is the draught as above. Show that, if gravity and buoyancy are the only forces acting on the ship, it will bob up and down with an angular frequency given by $\omega = \sqrt{\rho \mathrm{A}\, \mathrm{g}/\mathrm{M}}$.

Next, suppose that waves from a storm strike the ship [which is initially at rest with $\mathrm{x}(0) = 0$] and exert a vertical force $\mathrm{F}_0 \cos(\omega \mathrm{t})$ on the ship, where F_0 is the amplitude of the wave force. Let H be the height of the deck of the ship above sea level when the ship is at rest. [We assume that the ship is heavily loaded, so H is much less than d.] Write down a formula which allows you to compute when the ship sinks. [That is, find an equation satisfied by t_{sink}, the time at which the ship's deck first goes under water. You don't need to solve this equation.]

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Application of ODE in Civil Engineering in the Design of Beams

Context

Buildings and bridges are made up of various elements such as beams, columns, walls and foundations which are put together to ensure that the load they support (the dead-load of the construction materials and the live-load of vehicles, people, furnishings, wind load, seismic load etc.) can be safely transferred to the supporting foundation, piled deep into the ground.

Girders or beams, oriented horizontally in construction are made from steel, reinforced concrete or timber and are have cross sections (typically “H” or ‘T’ shape) which provide maximum flexural rigidity values. The design of the beams requires careful analysis of the loads they are required to support. Examples of beams are shown below:

http://www.syntapa.com/wp-content/uploads/2012/04/MP9003993091.jpg http://landtransportguru.net/teck-lee-station/

One of the most important design criteria in the construction of large civil infrastructure is to ensure minimal deflection of the beam under load. A large deflection of the beam (vertical displacement due to dead- and live-loads) may lead to severe cracking of the beam and consequently collapse of the structure. Beam design is carried out according to principles outlined in the Code of Practice and the amount of deflection needs to be calculated, using principle of mathematics and compare this figure to the acceptable value recommended in the codes.

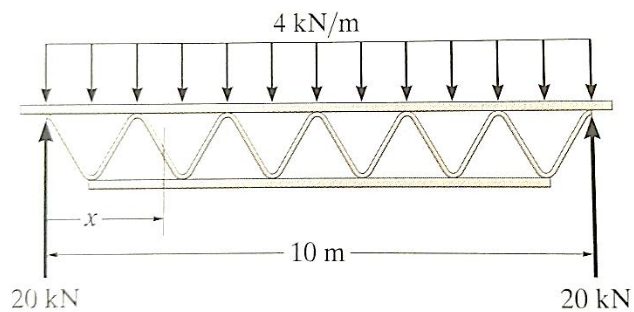
Engineering Background- Determining deflection by ODE

In a given beam set-up, the amount of deflection can be mathematically related to the load acting on the beam through an equation called the *Moment Equation*. Given some simplification, the moment equation is expressed as

$$EI\frac{d^2v}{dx^2} = M$$

Where v is the deflection, M is the moment function for a given beam set-up, E is the Young's Modulus, I is the beam's moment of inertia about the neutral axis. EI, taken together represents the beam flexural rigidity or resistance to bending and assume a constant for the length of the beam, while x, is the distance from the origin (selected arbitrarily). The first derivative of v is known as the slope of the bend at any point along the beam- used as a boundary condition.

***Question 6**- "Simply Supported Beam Uniformly Distributed Load"*



Each simply supported floor joist shown in the photo is subjected to a uniform design loading of 4 kN/m. Determine the maximum deflection of the joist. EI is constant.

Given the moment function is as follows,

$$M = 20x - 4x\left(\frac{x}{2}\right) = 20x - 2x^2.$$