Week 08 F2F Example Solutions

1. **Example 7.1**

$$\left(\begin{array}{cccc|c} 1 & 3 & -1 & 2 & 0 \\ 0 & -3 & 1 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 \end{array}\right) \longrightarrow \left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -\frac{1}{3} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right)$$

   So a general solution is

$$\begin{cases} x_1 &= 0 \\ x_2 &= \frac{s}{3} \\ x_3 &= s \\ x_4 &= 0, \qquad s \in \mathbb{R} \end{cases}$$

   A basis for the solution space is $\{(0,1,3,0)\}$ and the dimension is 1.

2. **Example 7.2**

   (a)

$$\begin{aligned} & c_1\boldsymbol{u_1} + c_2(\boldsymbol{u_1} + \boldsymbol{u_2}) + c_3(\boldsymbol{u_1} + \boldsymbol{u_2} + \boldsymbol{u_3}) = \boldsymbol{0} \\ \Rightarrow\ & (c_1 + c_2 + c_3)\boldsymbol{u_1} + (c_2 + c_3)\boldsymbol{u_2} + c_3\boldsymbol{u_3} = \boldsymbol{0} \\ \Rightarrow\ & c_1 = c_2 = c_3 = 0. \end{aligned}$$

   So $\{\boldsymbol{v_1}, \boldsymbol{v_2}, \boldsymbol{v_3}\}$ is a linearly indepenent set too and thus is a basis for V.

   (b) No, the vectors $\boldsymbol{v_1}, \boldsymbol{v_2}, \boldsymbol{v_3}$ are linearly dependent since $\boldsymbol{v_1} + \boldsymbol{v_2} + \boldsymbol{v_3} = \boldsymbol{0}$.

3. **Example 7.3**

   (a) For example, $a = -2$, $b = -1$, $c = 1$, $d = 0$.

   (b) $\boldsymbol{u_3} = 2\boldsymbol{u_1} + \boldsymbol{u_2}$ and $\boldsymbol{u_4} = -2\boldsymbol{u_1} + \boldsymbol{u_2}$.

   (c) $\{\boldsymbol{u_1},\ \boldsymbol{u_2}\}$ is a basis for V and $\dim(V) = 2$.

   (d) For example, let $W = \mathrm{span}\{\boldsymbol{u_1},\ \boldsymbol{u_2},\ (0,0,0,1)\}$. Then $\dim(W) = 3$. Since $W \cap V = V$, $\dim(W \cap V) = \dim(V) = 2$.

4. **Example 7.4**

   (a) Since the reduced row-echelon form of $\boldsymbol{A}$ is $\boldsymbol{B}$, they have the same row space.

   (b) $\{(1,0,0,1,2), (0,1,0,-1,-1), (0,0,1,0,1)\}$ is a basis for the row space of $\boldsymbol{A}$. Dimension of row space is 3.