NATIONAL UNIVERSITY OF SINGAPORE
Department of Mathematics

SEMESTER I, 2018/2019
MA1508E MID-TERM TEST

Full Name : ______________________________

Matric/Student Number : ______________

Tutorial Group : __________

INSTRUCTIONS
PLEASE READ CAREFULLY

- Write your **full name, matric number and tutorial group** clearly above on this cover page.
- There are **4** questions printed on **2** pages. Answer **all** questions.
- You must show all your working clearly, failure to do so will result in marks deducted.
- Use pen for this test.
- All answers and working have to be written on the answer book provided.
- Start on a new page for each question.
- Tie this cover page (and question paper) together with your answer book before submission.

Question 1 (12 marks)

(a) Solve the following linear system by Gaussian Elimination.

$$\begin{cases} x_1 - x_2 + 3x_3 + x_4 = 0 \\ x_1 - x_2 + x_3 - 2x_4 = 0 \\ -5x_1 + x_2 - 3x_3 - x_4 = 0 \end{cases}$$

(b) Consider the linear system below, where a is a real number.

$$\begin{cases} ax - y + z = 3 \\ 2x + (a+2)z = -1 \\ (a-1)y + 3z = 2 \end{cases}$$

(i) Find all values of a such that Cramer's Rule **cannot** be used to solve the system.

(ii) Solve the linear system when $a = 1$.

Question 2 (9 marks)

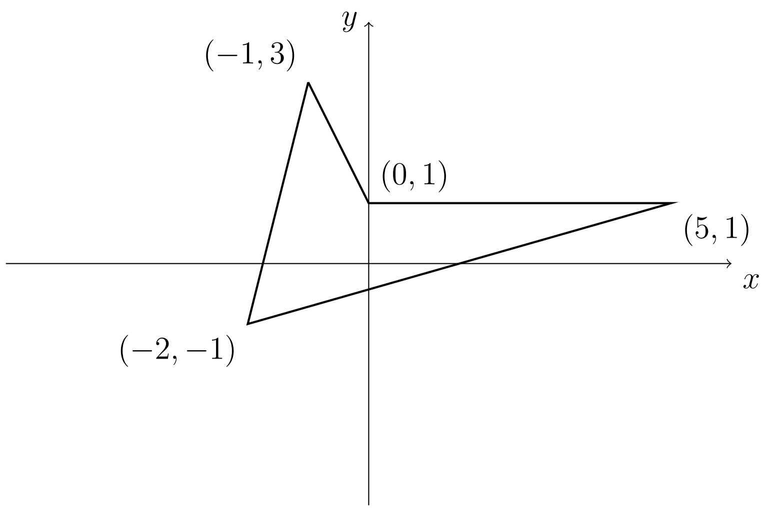
(a) For each of the statements below, determine if the statement is true or false. If it is true, provide justification. If it is false, provide a counterexample.

(i) If $\boldsymbol{A}$ and $\boldsymbol{B}$ are matrices of the same size such that $\boldsymbol{u}$ is a non trivial solution to both $\boldsymbol{Ax} = \boldsymbol{0}$ and $\boldsymbol{Bx} = \boldsymbol{0}$, then $\boldsymbol{u}$ is a non trivial solution to $(\boldsymbol{A} + \boldsymbol{B})\boldsymbol{x} = \boldsymbol{0}$.

(ii) If the reduced row-echelon form of a matrix $\boldsymbol{C}$ has a zero row, then $\boldsymbol{Cx} = \boldsymbol{0}$ has infinitely many solutions.

(iii) If $\boldsymbol{A}$ is row equivalent to $\boldsymbol{B}$, then $\boldsymbol{A}^T$ will be row equivalent to $\boldsymbol{B}^T$.

(b) Find the area of the following quadrilateral.



Question 3 (12 marks)

Let $\boldsymbol{A} = \begin{pmatrix} 1 & 1 & 1 \\ 2 & -1 & -1 \\ 1 & 2 & -3 \end{pmatrix}$.

(i) Find exactly 3 elementary matrices $\boldsymbol{E_1}, \boldsymbol{E_2}, \boldsymbol{E_3}$ such that $\boldsymbol{E_3E_2E_1A}$ is an upper triangular matrix.

(ii) Use your answer in (i) to find $\det(\boldsymbol{A})$. Explain why $\boldsymbol{A}$ is invertible.

(iii) Express $\boldsymbol{A}$ as $\boldsymbol{LU}$ where $\boldsymbol{L}$ and $\boldsymbol{U}$ are lower and upper triangular matrices respectively.

(iv) Use your answer in part (iii), solve the equation

$$\boldsymbol{A} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1 \\ 4 \\ 7 \end{pmatrix}.$$

(**Note:** You will not be given any marks if you solve the equation using other methods.)

Question 4 (7 marks)

The lecturer of a module needs to assign grades to the students taking the module. There are only 3 possible grades (A, B and C) that can be assigned to each student. Show that it is impossible for the lecturer to assign grades to his students such that the following conditions are **all satisfied**.

(1) The number of students receiving A grade plus twice the number of students receiving B grade is 300.

(2) The total number of students receving B or C grade is 300.

(3) There are 300 more students receiving A grade than twice the number of students receiving C grade.

If you are allowed to change the '300' in all the 3 conditions above to another value x (for all the 3 conditions), is it possible to choose x such that the lecturer is now able to assign grades to his students in such a way that satisfies all the 3 conditions? Justify your answer.

END OF TEST