ELEMENTARY ROW OPERATIONS

CONSISTENT / INCONSISTENT

A linear system that has no solutions is inconsistent.

In this case, the solution set of the linear system is an empty set.

A linear system that has at least one solution is consistent.

In this case, the solution set of the linear system is non empty.

If a linear system has exactly one solution, we say that the linear system has a unique solution.

AUGMENTED MATRIX

A linear system

$$\begin{cases} a_{11}x_1 & + & a_{12}x_2 & + & \dots & + & a_{1n}x_n & = & b_1 \\ a_{21}x_1 & + & a_{22}x_2 & + & \dots & + & a_{2n}x_n & = & b_2 \\ & \vdots & & \vdots & & \vdots & & \vdots & \\ a_{m1}x_1 & + & a_{m2}x_2 & + & \dots & + & a_{mn}x_n & = & b_m \end{cases}$$

can be represented by a rectangular array of numbers:

$$\left(\begin{array}{ccccc|c} a_{11} & a_{12} & \dots & \dots & a_{1n} & b_1 \\ a_{21} & a_{22} & \dots & \dots & a_{2n} & b_2 \\ \vdots & \vdots & & & \vdots & \vdots \\ a_{m1} & a_{m2} & \dots & \dots & a_{mn} & b_m \end{array}\right)$$

AUGMENTED MATRIX

$$
\begin{array}{cc}
 & \text{columns} \\
\text{rows} & \left(\begin{array}{cccc|c}
a_{11} & a_{12} & \cdots\ \cdots & a_{1n} & b_1 \\
a_{21} & a_{22} & \cdots\ \cdots & a_{2n} & b_2 \\
\vdots & \vdots & & \vdots & \vdots \\
a_{m1} & a_{m2} & \cdots\ \cdots & a_{mn} & b_m
\end{array}\right)
\end{array}
$$

is called the **augmented matrix** of the linear system.

Note that if the linear system has n variables and m equations, then the augmented matrix will have m rows and $(n+1)$ columns.

HOW TO SOLVE THIS?

$$\begin{cases} 2x & + & y & = & 1 & (1) \\ x & - & 3y & = & -2 & (2) \end{cases}$$

multiply (2) by 2

$$\begin{cases} 2x & + & y & = & 1 & (1) \\ 2x & - & 6y & = & -4 & (3) \end{cases}$$

Subtract (3) from (1)

$$\begin{cases} 0x & + & 7y & = & 5 & (4) \\ 2x & - & 6y & = & -4 & (3) \end{cases}$$

Add (-1) times of (3) to (1)

$$7y = 5 \Rightarrow \underline{y = \frac{5}{7}}$$

Substitute $y = \frac{5}{7}$ into equation (3) $\Rightarrow \underline{x = \frac{1}{7}}$

CORRESPOND TO AUGMENTED MATRIX

What you do to equations in a linear system:

Multiply an equation by a non zero constant

What you do to rows of the augmented matrix:

Multiply a row by a non zero constant

CORRESPOND TO AUGMENTED MATRIX

What you do to equations in a linear system:

Interchange two equations

What you do to rows of the augmented matrix:

Interchange two rows

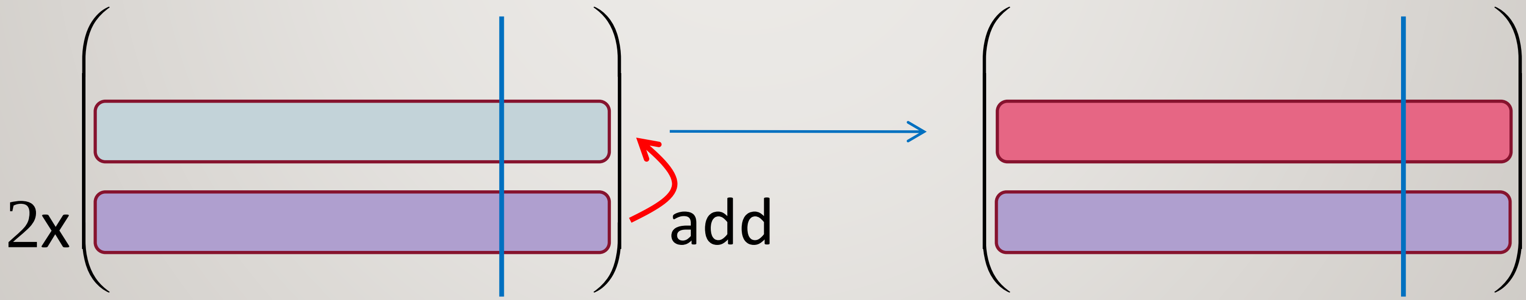
CORRESPOND TO AUGMENTED MATRIX

What you do to equations in a linear system:

Add a multiple of one equation to another equation

What you do to rows of the augmented matrix:

Add a multiple of one row to another row



ELEMENTARY ROW OPERATIONS

The three operations

1) Multiply a row by a non zero constant
2) Interchanging two rows
3) Adding a multiple of one row to another row

performed on an augmented matrix are called elementary row operations.

Remark: Elementary row operations can be performed on any matrix in general (not just augmented matrices).

EXAMPLE

$$\begin{cases} x + y + 3z = 0 & (1) \\ 2x - 2y + 2z = 4 & (2) \\ 3x + 9y = 3 & (3) \end{cases} \qquad \left(\begin{array}{ccc|c} 1 & 1 & 3 & 0 \\ 2 & -2 & 2 & 4 \\ 3 & 9 & 0 & 3 \end{array}\right)$$

multiply equation (1) by 2

multiply row 1 by 2

$$\begin{cases} 2x + 2y + 6z = 0 & (4) \\ 2x - 2y + 2z = 4 & (2) \\ 3x + 9y = 3 & (3) \end{cases} \qquad \left(\begin{array}{ccc|c} 2 & 2 & 6 & 0 \\ 2 & -2 & 2 & 4 \\ 3 & 9 & 0 & 3 \end{array}\right)$$

EXAMPLE

$$\begin{cases} x + y + 3z = 0 & (1) \\ 2x - 2y + 2z = 4 & (2) \\ 3x + 9y = 3 & (3) \end{cases} \qquad \left(\begin{array}{ccc|c} 1 & 1 & 3 & 0 \\ 2 & -2 & 2 & 4 \\ 3 & 9 & 0 & 3 \end{array}\right)$$

swap equations (2) and (3)

swap rows 2 and 3

$$\begin{cases} x + y + 3z = 0 & (1) \\ 3x + 9y = 3 & (2) \\ 2x - 2y + 2z = 4 & (3) \end{cases} \qquad \left(\begin{array}{ccc|c} 1 & 1 & 3 & 0 \\ 3 & 9 & 0 & 3 \\ 2 & -2 & 2 & 4 \end{array}\right)$$

EXAMPLE

$$\begin{cases} x + y + 3z = 0 & (1) \\ 2x - 2y + 2z = 4 & (2) \\ 3x + 9y = 3 & (3) \end{cases} \qquad \left(\begin{array}{ccc|c} 1 & 1 & 3 & 0 \\ 2 & -2 & 2 & 4 \\ 3 & 9 & 0 & 3 \end{array}\right)$$

add -2 times of equation (1) to equation (2)

add -2 times of row 1 to row 2

$$\begin{cases} x + y + 3z = 0 & (1) \\ \quad -4y - 4z = 4 & (4) \\ 3x + 9y = 3 & (3) \end{cases} \qquad \left(\begin{array}{ccc|c} 1 & 1 & 3 & 0 \\ 0 & -4 & -4 & 4 \\ 3 & 9 & 0 & 3 \end{array}\right)$$

SUMMARY

1) Consistent and inconsistent linear systems. What is a unique solution.

2) Using an augemented matrix to represent a linear system.

3) Three types of elementary row operations that can be performed on an augmented matrix.