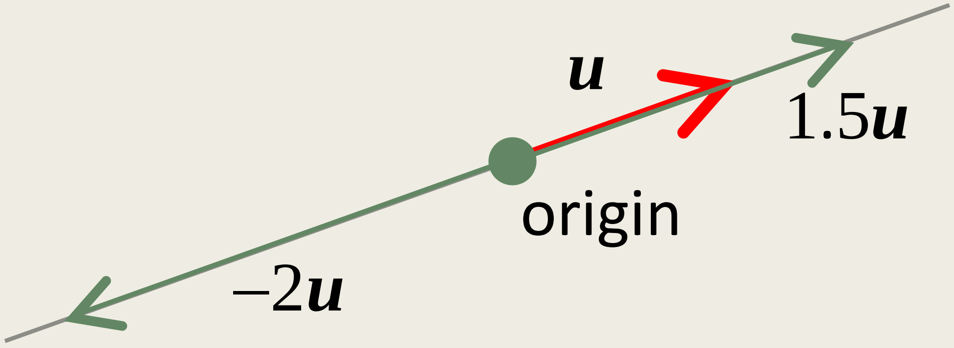
SUBSPACES IN R^2 AND R^3

Linear span of one vector

Let $\boldsymbol{u}$ be a nonzero vector in $\mathbb{R}^2$ or $\mathbb{R}^3$.

span{$\boldsymbol{u}$} is the set of all linear combinations (or scalar multiples) of $\boldsymbol{u}$.

Geometrically, span{$\boldsymbol{u}$} is a straight line passing through the origin.



Linear span of one vector

Let $\boldsymbol{u}$ be a nonzero vector in $\mathbb{R}^2$ or $\mathbb{R}^3$.

(In $\mathbb{R}^2$) $\boldsymbol{u} = (u_1, u_2)$, span$\{\boldsymbol{u}\} = \{(cu_1, cu_2) \mid c \in \mathbb{R}\}$

(can we find the equation of the line?)

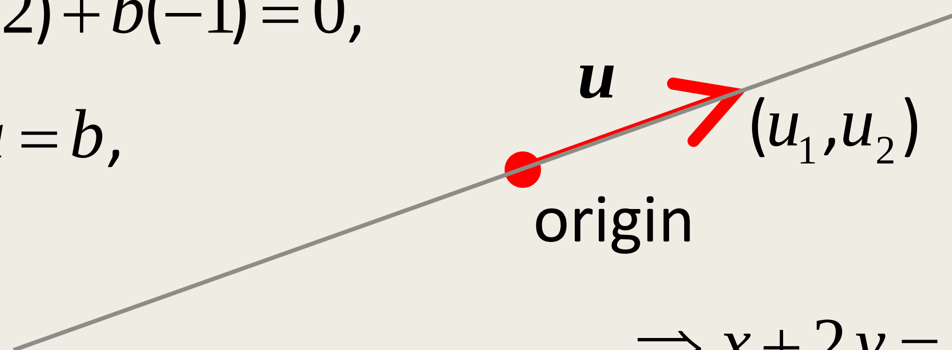
For example, if $(u_1, u_2) = (2, -1)$,

$\Rightarrow a(2) + b(-1) = 0,$

$ax + by = 0$

(u_1, u_2)

$\Rightarrow 2a = b,$



$\Rightarrow x + 2y = 0$ is the equation of the line spanned by $(2, -1)$

$\Rightarrow a = 1,\ b = 2$ is a solution

Linear span of one vector

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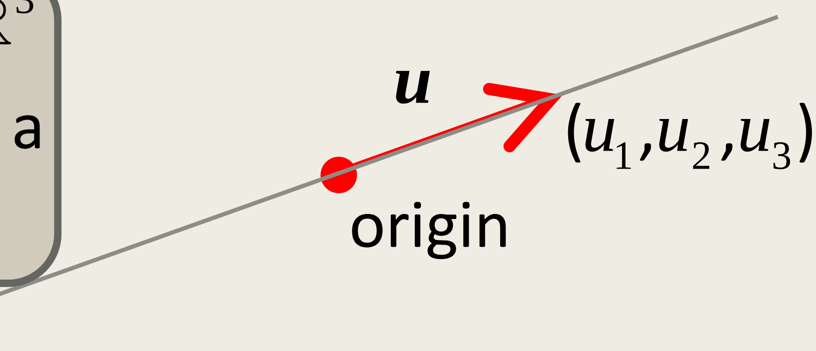
(In $\mathbb{R}^3$) $\boldsymbol{u} = (u_1, u_2, u_3)$, span$\{\boldsymbol{u}\} = \{(cu_1, cu_2, cu_3) \mid c \in \mathbb{R}\}$

Remember that a line in $\mathbb{R}^3$ cannot be represented by a single linear equation.



Linear span of two vectors

Let $\boldsymbol{u},\boldsymbol{v}$ be two nonzero vectors in $\mathbb{R}^2$ or $\mathbb{R}^3$.

span$\{\boldsymbol{u},\boldsymbol{v}\}$ is the set of all linear combinations of $\boldsymbol{u}$ and $\boldsymbol{v}$.

$$=\{s\boldsymbol{u}+t\boldsymbol{v} \mid s,t\in\mathbb{R}\}$$

What if $\boldsymbol{u}$ and $\boldsymbol{v}$ are parallel?

$\Rightarrow \boldsymbol{v}$ is a linear combination (scalar multiple) of $\boldsymbol{u}$

span$\{\boldsymbol{u},\boldsymbol{v}\}=$ span$\{\boldsymbol{u}\}$

$=$ straight line passing through the origin.

Linear span of two vectors

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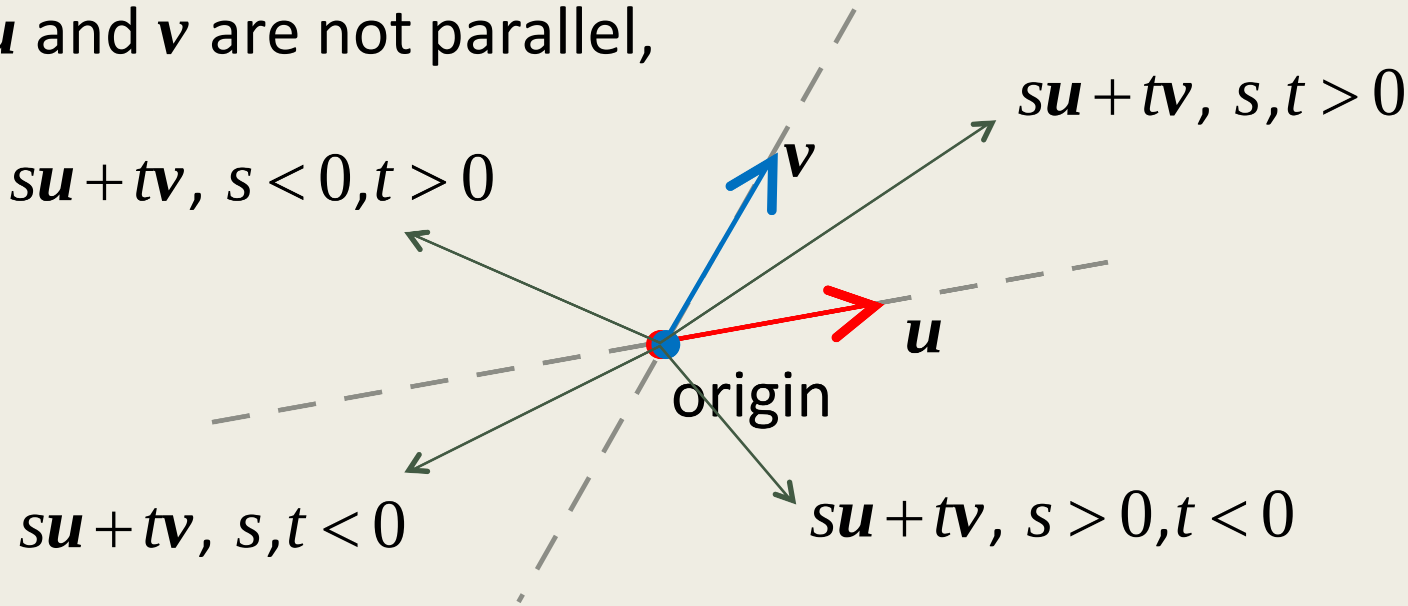
If $\boldsymbol{u}$ and $\boldsymbol{v}$ are not parallel,



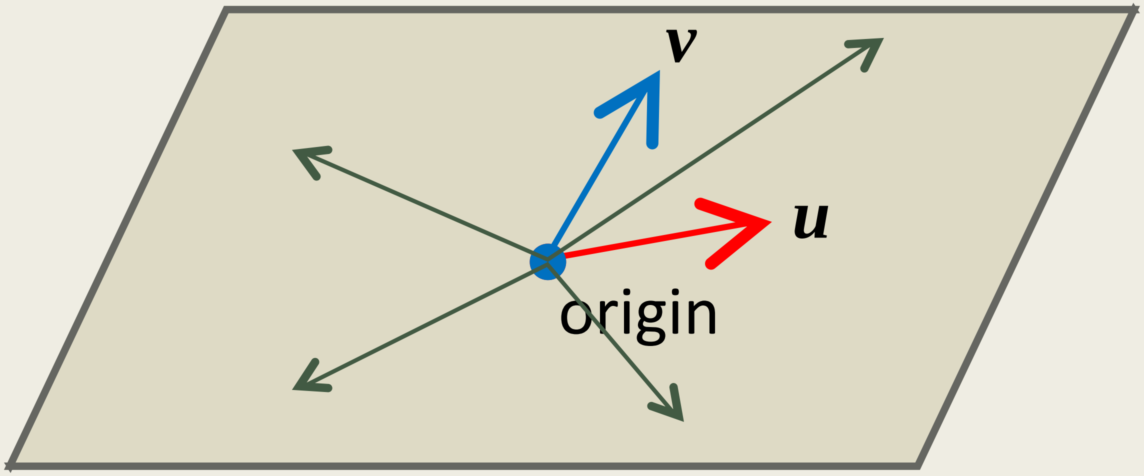
Linear span of two vectors

Let $\boldsymbol{u},\boldsymbol{v}$ be two nonzero vectors in $\mathbb{R}^2$ or $\mathbb{R}^3$.

span$\{\boldsymbol{u},\boldsymbol{v}\}$ is the set of all linear combinations of $\boldsymbol{u}$ and $\boldsymbol{v}$.

$$=\{s\mathbf{u}+t\mathbf{v}\mid s,t\in\mathbb{R}\}$$

If $\boldsymbol{u}$ and $\boldsymbol{v}$ are not parallel, span$\{\boldsymbol{u},\boldsymbol{v}\}$ is a plane containing the origin.



Linear span of two vectors

If $\boldsymbol{u}$ and $\boldsymbol{v}$ are not parallel,

$$\boldsymbol{u} = (u_1, u_2, u_3),\ \boldsymbol{v} = (v_1, v_2, v_3)$$

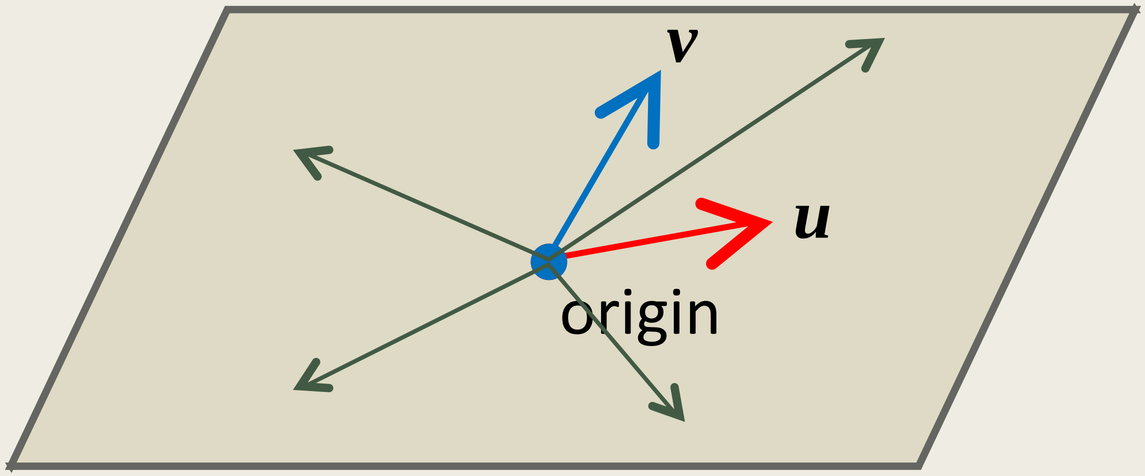
(In $\mathbb{R}^2$) $\text{span}\{\boldsymbol{u}, \boldsymbol{v}\} = \mathbb{R}^2$.

(In $\mathbb{R}^3$) $\text{span}\{\boldsymbol{u}, \boldsymbol{v}\} = \{s\boldsymbol{u} + t\boldsymbol{v} \mid s, t \in \mathbb{R}\}$

$$= \{s(u_1, u_2, u_3) + t(v_1, v_2, v_3) \mid s, t \in \mathbb{R}\}$$

(can we find the equation of the plane?)

$$ax + by + cz = 0$$



Linear span of two vectors

If $\boldsymbol{u}$ and $\boldsymbol{v}$ are not parallel, $\boxed{\boldsymbol{u}=(1,0,-2)}$, $\boxed{\boldsymbol{v}=(-1,1,0)}$

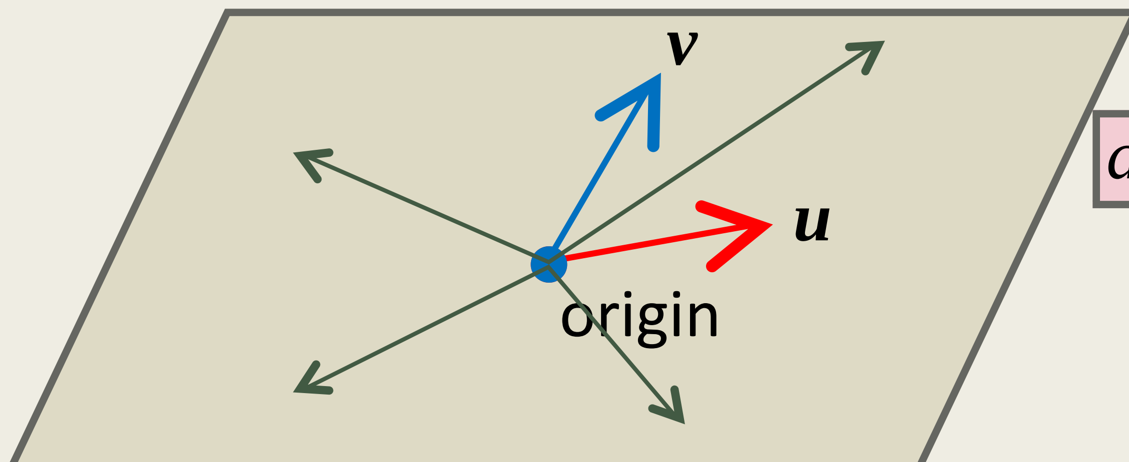
$$ax+by+cz=0$$

$$\boxed{2x+2y+z=0}$$

$$\begin{cases} a \qquad\qquad - 2c = 0 \\ -a + b \qquad\quad = 0 \end{cases}$$

$$\left(\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ -1 & 1 & 0 & 0 \end{array}\right) \longrightarrow \left(\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 1 & -2 & 0 \end{array}\right)$$

$$\begin{cases} a = 2s \\ b = 2s \\ c = s, \quad s \in \mathbb{R} \end{cases}$$



$$\boxed{a=2,b=2,c=1}$$

Subspaces of R^2

The following are all the subspaces of $\mathbb{R}^2$:

(1) span$\{\mathbf{0}\}$ (the zero subspace)

$=$ $\boxed{\text{the origin}}$

(2) span$\{\boldsymbol{u}\}$ ($\boldsymbol{u} \neq \mathbf{0}$)

$=$ $\boxed{\text{straight line through origin}}$

(3) span$\{\boldsymbol{u},\boldsymbol{v}\}$ ($\boldsymbol{u},\boldsymbol{v}$ not multiples of each other)

$= \boxed{\mathbb{R}^2}$

Subspaces of R^3

The following are all the subspaces of $\mathbb{R}^3$:

(1) span$\{\mathbf{0}\}$ (the zero subspace)

$= \boxed{\text{the origin}}$

(2) span$\{\boldsymbol{u}\}$ ($\boldsymbol{u} \neq \mathbf{0}$)

$= \boxed{\text{straight line through origin}}$

(3) span$\{\boldsymbol{u},\boldsymbol{v}\}$ ($\boldsymbol{u},\boldsymbol{v}$ not multiples of each other)

$= \boxed{\text{plane containing the origin}}$

(4) span$\{\boldsymbol{u},\boldsymbol{v},\boldsymbol{w}\}$

($\boldsymbol{u}$ is not a linear combination of $\boldsymbol{v},\boldsymbol{w}$)

($\boldsymbol{v}$ is not a linear combination of $\boldsymbol{u},\boldsymbol{w}$)

($\boldsymbol{w}$ is not a linear combination of $\boldsymbol{u},\boldsymbol{v}$)

$= \boxed{\mathbb{R}^3}$

Summary

1) Linear span of one vector (geometrical)

2) Linear span of two vectors (geometrical)

3) Characterisation of all subspaces of $\mathbb{R}^2$ and $\mathbb{R}^3$.