W04-01

Slide 01: In this unit, we will give a brief introduction to euclidean vectors.
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Slide 02: You should be familiar with vectors represented geometrically by a directed line segment. Recall that a vector is described by its direction as well as its magnitude.
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Two vectors are said to be equal if they have the same direction and magnitude. For example, among the vectors shown here, $\boldsymbol{u}$ and $\boldsymbol{v}$ are equal.

Slide 03: When we want to represent the addition of two vectors geometrically, say $\boldsymbol{u}+\boldsymbol{v}$,
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we first draw the vector $\boldsymbol{u}$
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and then place the initial point of $\boldsymbol{v}$ at the end point of $\boldsymbol{u}$.
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Then $\boldsymbol{u}+\boldsymbol{v}$ would be the vector drawn from the initial point of $\boldsymbol{u}$ to the end point of $\boldsymbol{v}$.
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Note that we can do the same by drawing $\boldsymbol{v}$ first then followed by $\boldsymbol{u}$,
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the resulting vector $\boldsymbol{v}+\boldsymbol{u}$, as can be seen here has the same direction and magnitude as $\boldsymbol{u}+\boldsymbol{v}$.
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Thus we see that $\boldsymbol{u}+\boldsymbol{v}$ is equal to $\boldsymbol{v}+\boldsymbol{u}$.
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The negative of a vector $\boldsymbol{u}$ is simply a change in the direction of $\boldsymbol{u}$ without changing the magnitude.

Slide 04: Once we have defined the negative of a vector, we can have the difference $\boldsymbol{u}-\boldsymbol{v}$ between $\boldsymbol{u}$ and $\boldsymbol{v}$. More precisely, $\boldsymbol{u}-\boldsymbol{v}$ is simply $\boldsymbol{u}$ plus the negative of $\boldsymbol{v}$.
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Using the same method discussed previously, we put the initial point of $-\boldsymbol{v}$ at the end point of $\boldsymbol{u}$,
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and the vector $\boldsymbol{u}-\boldsymbol{v}$ can be drawn easily.
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For any scalar k, the scalar multiple $k\boldsymbol{u}$ is a change in the magnitude of the vector $\boldsymbol{u}$. If k is negative, the vector $k\boldsymbol{u}$ points in the opposite direction as $\boldsymbol{u}$.

Slide 05: Let us consider a vector in the xy-plane that we are familiar with.
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Position the vector $\boldsymbol{u}$ with its initial point at the origin $(0,0)$.
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The end point of the vector can be described using two numbers, namely u_1 and u_2, where u_1 measures the distance from the origin in the x-direction while u_2 measures the distance from the origin in the y-direction.

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u_1 and u_2 are called the components of the vector $\boldsymbol{u}$.

Slide 06: Similarly, we can consider a vector $\boldsymbol{u}$ in the xyz-space.

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Position the initial point of $\boldsymbol{u}$ at the origin.

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The end point of the vector can be described by three numbers u_1, u_2 and u_3, each measuring the distance from the origin, in the x, y and z directions respectively.

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u_1, u_2 and u_3 are called the components of the vector $\boldsymbol{u}$.

Slide 07: Let us move on to discuss vector operations algebraically instead of geometrically. The addition of two vectors is done simply by adding the vectors component-wise.

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This is the same regardless of whether the vectors have two or three components.

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To multiply a scalar to a vector, we simply multiply the scalar to each component of the vector.

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So for example, if $\boldsymbol{u}$ has components u_1, u_2 and u_3, then the vector $k\boldsymbol{u}$ will have components ku_1, ku_2 and ku_3.

Slide 08: There is no reason why vectors must be restricted to having only two or three components. To generalise, we have a vector with n components, called an n-vector. Note that each of the component is a real number.

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u_1 is the first component, or first coordinate of the vector.

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u_i is the ith component of the vector.

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For two vectors $\boldsymbol{u}$ and $\boldsymbol{v}$, each having n components, they are said to be equal if and only if u_i is equal to v_i for all i.

Slide 09: Similar to the way we added vectors with two or three components, we can add vectors with n components by adding their corresponding components.

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Scalar multiplication is done in the same way.

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The negative of $\boldsymbol{u}$ is obtained by changing the sign of each component in the vector $\boldsymbol{u}$.

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The vector $\boldsymbol{u} - \boldsymbol{v}$ is obtained by subtracting the corresponding components.

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The zero vector is simply the vector with all components equal to zero.

Slide 10: You may have noticed by now that the way we deal with vector operations algebraically is very much similar to the way it was done for matrices. Thus, we often identify an n-vector with a $1 \times n$ row matrix. When we do this, the n-vector is written as a row vector.

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We could also identify the vector with a $n \times 1$ column matrix. In this case, the n-vector is written as a column vector.

Slide 11: This slide shows some simple results that are immediately carried over from our understanding on matrix operations. You may wish to pause for a while and look through these results before moving on.

Slide 12: Now that we have defined what is an n-vector, consider the collection of all such vectors with n components. This collection, in the form of a set, is known as the Euclidean n-space and we denote it by $\mathbb{R}^n$. Note that a vector $\boldsymbol{u}$ belongs to the set $\mathbb{R}^n$ if and only if $\boldsymbol{u}$ has exactly n components, each of which is a real number.

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A few important points to remember. First, for each poisitive integer n, the Euclidean n-space is a set.

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You may wonder how many elements are there in each $\mathbb{R}^n$. In other words, how many n-vectors are there?

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Since there are infinitely many different real numbers at each of the n-components, there are infinitely many different n-vectors. Thus each $\mathbb{R}^n$ contains an infinite number of vectors.

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It should also be clear that, for example, $\mathbb{R}^2$ and $\mathbb{R}^3$ contain entirely different type of vectors.

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Therefore, the Euclidean 2-space and the Euclidean 3-space have no vectors in common since all vectors in $\mathbb{R}^2$ have 2 components while those in $\mathbb{R}^3$ have three components.

Slide 13: Consider the following set S.

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It should be clear that S contains vectors with 3 components u_1, u_2 and u_3.

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The vertical line that separates the left from the right side is read as 'such that'.

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The conditions that must be satisfied by the components u_1, u_2 and u_3 are listed on the right side of the vertical line. So in this case, for a vector in $\mathbb{R}^3$ to be included in S, the first component must be 0 and the second and third components must be negatives of each other.

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Clearly, S is a sub-collection of $\mathbb{R}^3$. In other words, it is a subset of $\mathbb{R}^3$.

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Vectors like $(0, 1, -1)$ and $(0, 0, 0)$ are in S while $(1, 0, 0)$ does not belong to S.

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We can also write the set S explicitly as follows. Note that a here is any real number. It should also be clear now that S contains inifintely many vectors since a can take on inifinitely many different real numbers.

Slide 14: To summarise this unit,

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we introduced a vector both geometrically and also algebraically. Each vector can be described by its components.

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We discussed vector operations, like addition, subtraction and scalar multiplication.

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It is useful to remember that a vector is often identified with a matrix and this led to some vector operation laws.

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Lastly, for each positive integer n, we defined the set $\mathbb{R}^n$, otherwise known as the Euclidean n-space. We saw one example on how a subset of $\mathbb{R}^n$ can be described.