ROW-ECHELON FORM

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An augmented matrix is said to be in row-echelon form if it has the following two properties.

1) If there are any rows consisting entirely of zeros, then they are grouped at the bottom of the matrix.

$$\begin{array}{l} \left.\begin{matrix} * & * & * & \cdots & * \\ \vdots & \vdots & \vdots & & \vdots \\ * & * & * & \cdots & * \end{matrix}\right\} \text{non zero rows} \\ \hline \left.\begin{matrix} 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \end{matrix}\right\} \text{zero rows (if any)} \end{array}$$

ROW-ECHELON FORM

2) In any two successive rows that are not entirely zeros, the first non zero number in the lower row occurs further to the right than the first non zero number in the higher row.

$$\text{non zero rows}\left\{\begin{pmatrix} 0 & 0 & \otimes & * & \dots & \dots & * \\ 0 & 0 & 0 & 0 & \otimes & * & * \end{pmatrix}\right\}\text{two successive rows}$$

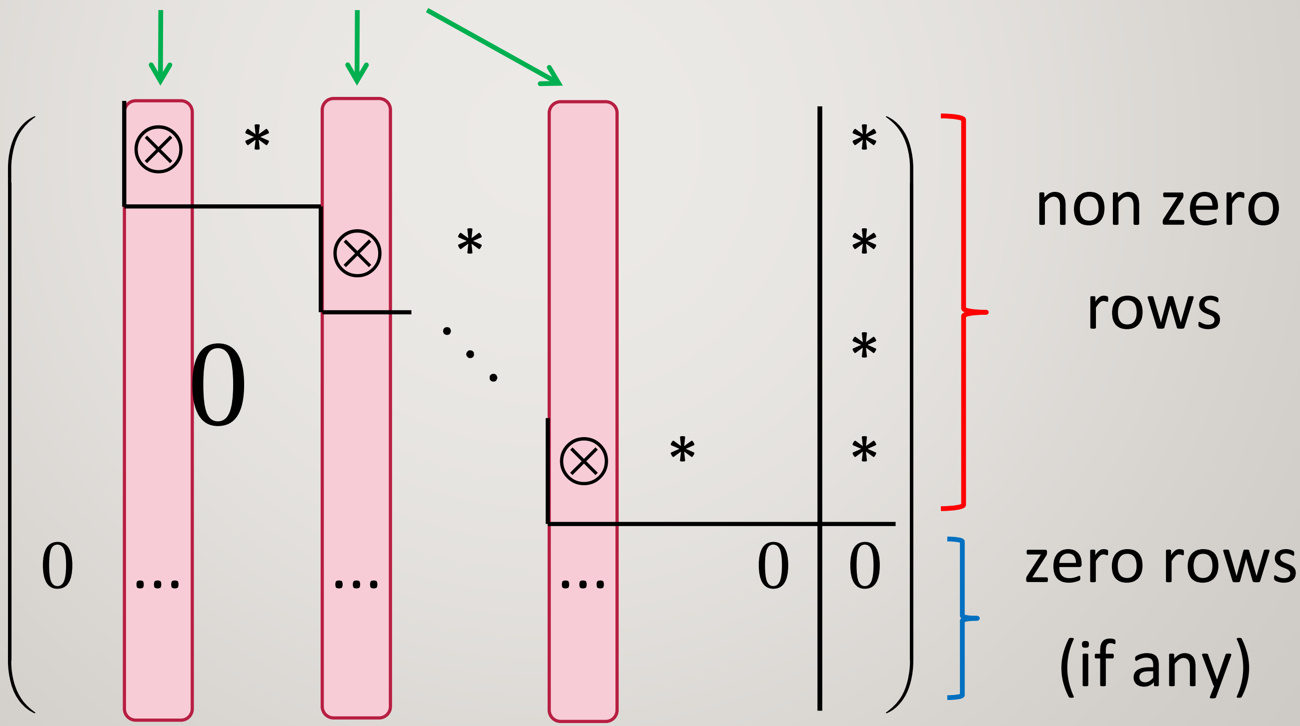
The first non zero number in every row is called the leading entry of that row.

$\otimes$: leading entry

PIVOT POINT, PIVOT COLUMN

A leading entry is also called a **pivot point.**

A column containing a leading entry / pivot point is called a **pivot column.**



REMARK

The concept of row-echelon forms can be applied to matrices in general, not just for augmented matrices.

OBSERVATIONS

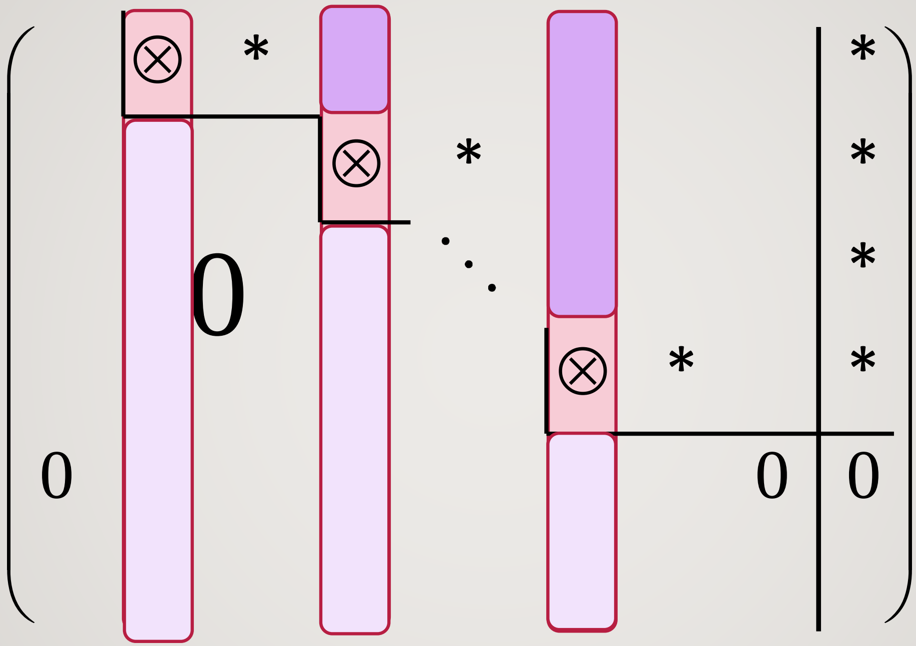
If an augmented matrix is in row-echelon form,

$$\left(\begin{array}{cccc|c} \otimes & * & & & * \\ & \otimes & * & & * \\ & \mathbf{0} & \ddots & & * \\ & & & \otimes \quad * & * \\ 0 & & & 0 & 0 \end{array}\right)$$

1) all entries (in the same column) below each leading entry must be 0.

OBSERVATIONS

If an augmented matrix is in row-echelon form,

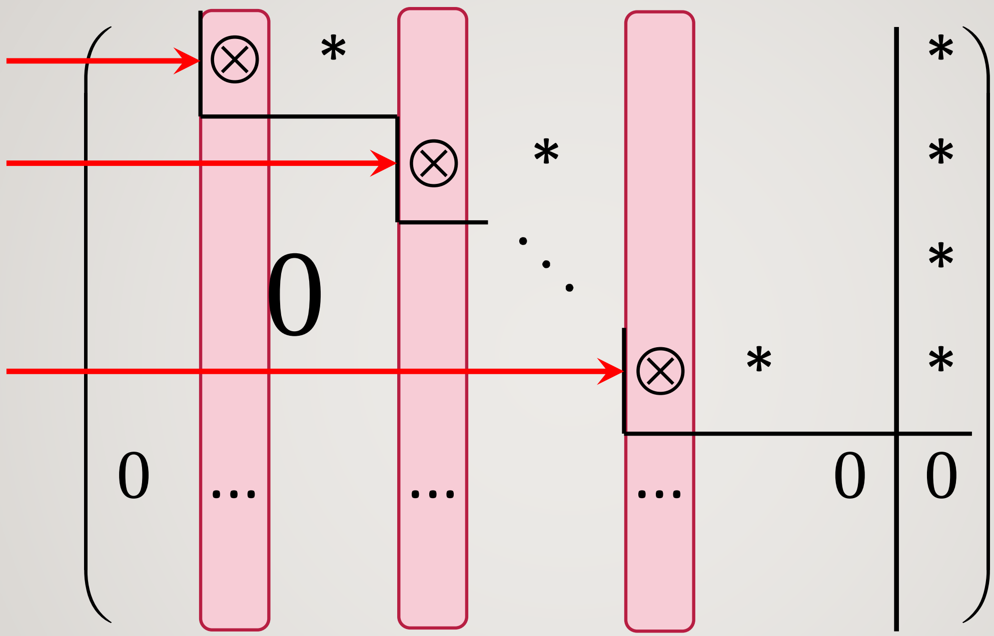


2) entries (in the same column) above each leading entry do not have to be 0.

OBSERVATIONS

If an augmented matrix is in row-echelon form,

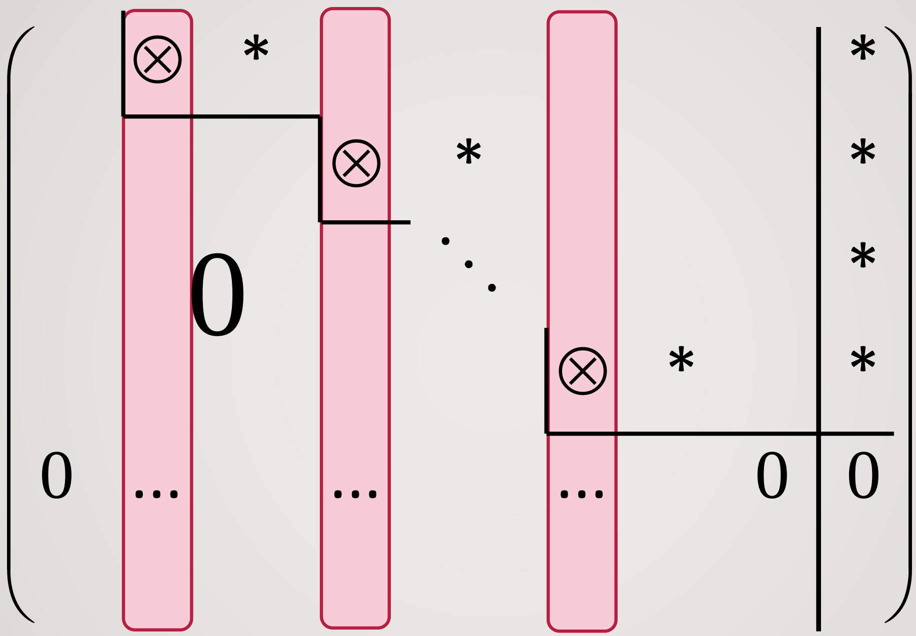
$\otimes$: leading entries



3) every non zero row has one and exactly one leading entry.

OBSERVATIONS

If an augmented matrix is in row-echelon form,



4) every pivot column has one and exactly one pivot point;

a column without a pivot point is a non-pivot column.

REDUCED ROW-ECHELON FORM

An augmented matrix is said to be in reduced row-echelon form if it is in row-echelon form and has the following two additional properties.

3) all leading entries must be 1.

$$\begin{array}{l} \left(\begin{array}{ccccccc|c} 1 & * & & & & & & * \\ & & 1 & * & & & & * \\ & 0 & & & \ddots & & & * \\ & & & & & 1 & * & * \\ \hline 0 \;\; \cdots & & \cdots & & & \cdots & 0 & 0 \end{array}\right) \end{array} \begin{array}{l} \left.\begin{array}{l} \\ \\ \\ \\ \end{array}\right\} \text{non zero rows} \\ \left.\begin{array}{l} \\ \end{array}\right\} \text{zero rows (if any)} \end{array}$$

REDUCED ROW-ECHELON FORM

4) in each pivot column, other than the pivot point, all other entries are zero.

$$\left(\begin{array}{cccccc|c} & 1 & * & 0 & & 0 & * \\ & & & 1 & * & 0 & * \\ & & \mathbf{0} & & \ddots & 0 & * \\ & & & & & 1 \quad * & * \\ 0 & & & & & 0 & 0 \end{array}\right)$$

WHY ROW-ECHELON FORM?

row equivalent?

$$\left(\begin{array}{ccc|c} 1 & 1 & 3 & 0 \\ 2 & -2 & 2 & 4 \\ 3 & 9 & 0 & 3 \end{array}\right) \qquad \begin{cases} x + y + 3z = 0 \\ 2x - 2y + 2z = 4 \\ 3x + 9y = 3 \end{cases}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 3 & 0 \\ 0 & -4 & -4 & 4 \\ 0 & 0 & -15 & 9 \end{array}\right)$$

row-echelon form

$$\begin{cases} x + y + 3z = 0 \\ \quad - 4y - 4z = 4 \\ \quad\quad - 15z = 9 \end{cases}$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & \frac{11}{5} \\ 0 & 1 & 0 & -\frac{2}{5} \\ 0 & 0 & 1 & -\frac{3}{5} \end{array}\right)$$

reduced row-echelon form

$$\begin{cases} x = \frac{11}{5} \\ y = -\frac{2}{5} \\ z = -\frac{3}{5} \end{cases}$$

SUMMARY

1) Definition of row-echelon form.

2) Definition of pivot point, pivot column,
   non pivot column.

3) Definition of reduced row-echelon form.

4) Why are we interested in row-echelon forms?