MATRICES

DEFINITION

A $\boxed{\text{matrix}}$ is a rectangular array of numbers.

The numbers in the array are called $\boxed{\text{entries}}$.

The $\boxed{\text{size}}$ of a matrix is $m \times n$ if it has m rows and n columns.

$$\boldsymbol{A} = \begin{pmatrix} 3 & 2.4 & 1 & -1 & 0 & 11 \\ -5 & 2 & 2 & 0 & 0 & 1 \\ 4.1 & \pi & 20 & 10 & -2 & 1 \end{pmatrix}$$

$\boldsymbol{A}$ is a 3×6 matrix and the $(1,4)$-entry of $\boldsymbol{A}$ is -1.

DEFINITION

A $\boxed{\text{column matrix}}$ is a matrix with only one column.

$$\begin{pmatrix} 2 \\ 3 \\ -1 \\ 0 \end{pmatrix}$$

A $\boxed{\text{row matrix}}$ is a matrix with only one row.

$$\begin{pmatrix} 1 & -1 & 0 \end{pmatrix}$$

NOTATION

$$\boldsymbol{A} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}$$ is a $m \times n$ matrix.

We can also write $\boldsymbol{A} = (a_{ij})_{m \times n}$ where a_{ij} is the (i, j)-entry of $\boldsymbol{A}$.

$$a_{ij}$$

i is the 'row index'

j is the 'column index'

We can also write $\boldsymbol{A} = (a_{ij})$.

DEFINITION

Square matrices are matrices with the same number of rows and columns.

$$(2) \quad \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

A square matrix with n rows and n columns is said to be of order n.

DEFINITION

Given a square matrix $\boldsymbol{A} = (a_{ij})$ of order n,

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \cdots & a_{nn} \end{pmatrix}$$

The diagonal of $\boldsymbol{A}$ is the sequence $a_{11}, a_{22}, a_{33}, \ldots, a_{nn}$.

$a_{11}, a_{22}, a_{33}, \ldots, a_{nn}$ are called the diagonal entries of $\boldsymbol{A}$.

a_{ij}, $i \neq j$, are called the non-diagonal entries of $\boldsymbol{A}$.

DEFINITION

$$\begin{pmatrix} a_{11} & 0 & 0 & \dots & 0 \\ 0 & a_{22} & 0 & \dots & 0 \\ 0 & 0 & a_{33} & \dots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \dots & a_{nn} \end{pmatrix}$$

A square matrix is a diagonal matrix if all its non-diagonal entries are zero.

$$\boldsymbol{A} = (a_{ij}) \text{ is diagonal } \Leftrightarrow \boxed{a_{ij} = 0 \text{ whenever } i \neq j.}$$

DEFINITION

$$\begin{pmatrix} c & 0 & 0 & \dots & 0 \\ 0 & c & 0 & \dots & 0 \\ 0 & 0 & c & \dots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \dots & c \end{pmatrix}$$

A diagonal matrix is a scalar matrix if all its diagonal entries are the same.

$\boldsymbol{A} = (a_{ij})$ is scalar $\Leftrightarrow a_{ij} = 0$ whenever $i \neq j$ and

$a_{ij} = c$ whenever $i = j$

(c is a constant).

DEFINITION

$$\begin{pmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \dots & 1 \end{pmatrix}$$

A diagonal matrix is an identity matrix if all its diagonal entries are equal to 1.

$\boldsymbol{I}_n$ is an identity matrix of order n.

If there is no danger of confusion, we simply write $\boldsymbol{I}$.

DEFINITION

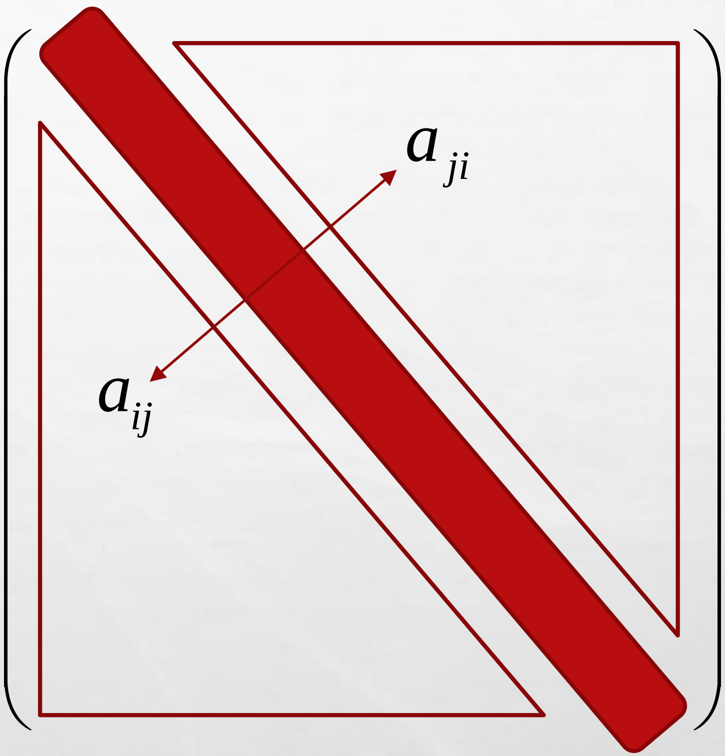
$$\begin{pmatrix} 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \dots & 0 \end{pmatrix}$$

A zero matrix is a matrix with all entries equal to zero.

$\mathbf{0}_{m\times n}$ is a $m \times n$ zero matrix.

If there is no danger of confusion, we simply write $\mathbf{0}$.

DEFINITION



A symmetric matrix (a_{ij}) is a square matrix where

$$\boxed{a_{ij} = a_{ji} \text{ for all } i, j.}$$

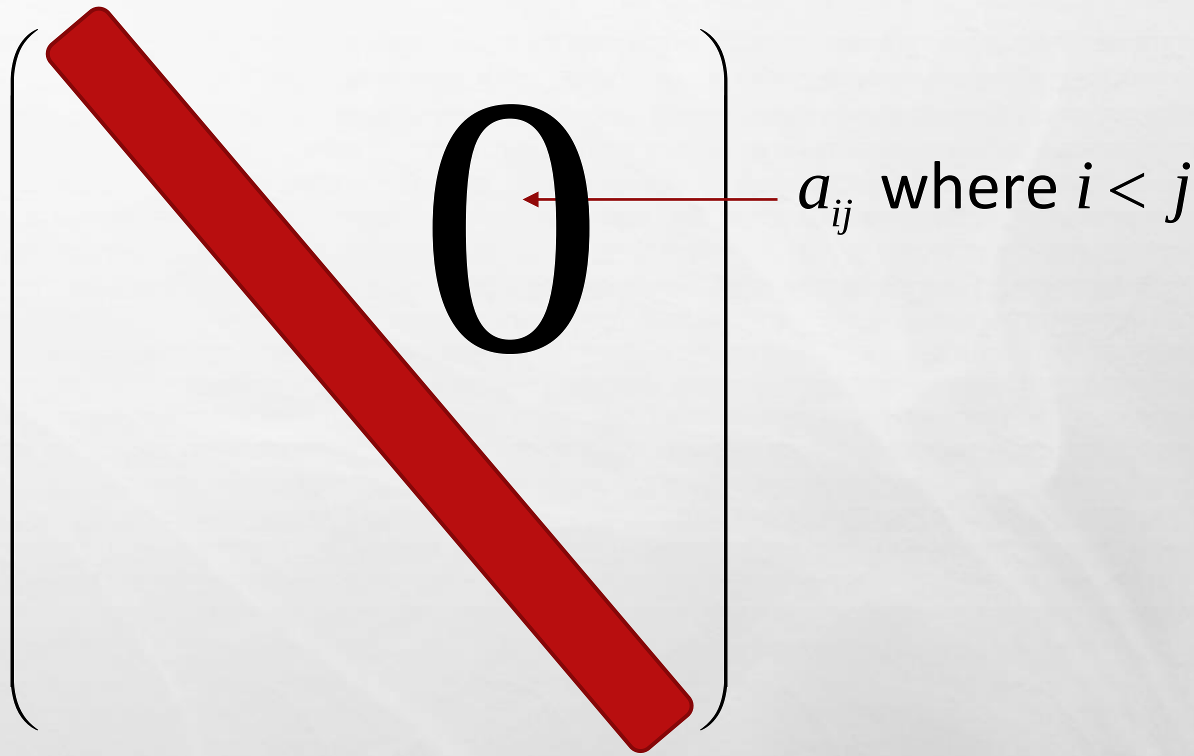
DEFINITION

a_{ij} where $i > j$

A square matrix (a_{ij}) is an upper triangular matrix if

$$\boxed{a_{ij} = 0 \text{ for all } i > j.}$$

DEFINITION



A square matrix (a_{ij}) is an lower triangular matrix if

$$\boxed{a_{ij} = 0 \text{ for all } i < j.}$$

SUMMARY

1) Matrices, entries, size, diagonal entries.

2) Diagonal matrix, scalar matrix, identity matrix, zero matrix.

3) Symmetric matrix, upper triangular matrix, lower triangular matrix.