EXAMPLE (GE AND GJE)

EXAMPLE

Solve the following linear system using Gaussian elimination.

$$\left\{\begin{array}{rcrcrcrcrcl} & & 2w & + & x & + & 2y & - & z & = & 4 \\ & & w & & & + & y & - & z & = & 3 \\ 4v & + & 6w & + & x & + & 4y & - & 3z & = & 8 \\ 2v & + & 2w & & & + & y & - & z & = & 2 \end{array}\right.$$

We first write down the augmented matrix of the linear system.

$$\left(\begin{array}{ccccc|c} 0 & 2 & 1 & 2 & -1 & 4 \\ 0 & 1 & 0 & 1 & -1 & 3 \\ 4 & 6 & 1 & 4 & -3 & 8 \\ 2 & 2 & 0 & 1 & -1 & 2 \end{array}\right)$$

EXAMPLE

Identify leftmost column that is not all zero.

Since the topmost entry is 0, row swap is required.

$$\left(\begin{array}{ccccc|c} 0 & 2 & 1 & 2 & -1 & 4 \\ 0 & 1 & 0 & 1 & -1 & 3 \\ 4 & 6 & 1 & 4 & -3 & 8 \\ 2 & 2 & 0 & 1 & -1 & 2 \end{array}\right)$$

$$R_1 \leftrightarrow R_4$$

Pivot point

$$\left(\begin{array}{ccccc|c} 2 & 2 & 0 & 1 & -1 & 2 \\ 0 & 1 & 0 & 1 & -1 & 3 \\ 4 & 6 & 1 & 4 & -3 & 8 \\ 0 & 2 & 1 & 2 & -1 & 4 \end{array}\right) \xrightarrow{R_3 - 2R_1} \left(\begin{array}{ccccc|c} 2 & 2 & 0 & 1 & -1 & 2 \\ 0 & 1 & 0 & 1 & -1 & 3 \\ 0 & 2 & 1 & 2 & -1 & 4 \\ 0 & 2 & 1 & 2 & -1 & 4 \end{array}\right)$$

EXAMPLE

$$\left(\begin{array}{ccccc|c} 2 & 2 & 0 & 1 & -1 & 2 \\ 0 & 1 & 0 & 1 & -1 & 3 \\ 0 & 2 & 1 & 2 & -1 & 4 \\ 0 & 2 & 1 & 2 & -1 & 4 \end{array}\right)$$

Identify leftmost column (in the submatrix) that is not all zero.

No row swap required.

Pivot point

$$\xrightarrow{R_3 - 2R_2 \quad R_4 - 2R_2}$$

$$\left(\begin{array}{ccccc|c} 2 & 2 & 0 & 1 & -1 & 2 \\ 0 & 1 & 0 & 1 & -1 & 3 \\ 0 & 0 & 1 & 0 & 1 & -2 \\ 0 & 0 & 1 & 0 & 1 & -2 \end{array}\right)$$

Identify leftmost column (in the submatrix) that is not all zero.

No row swap required.

Pivot point

EXAMPLE

$$\left(\begin{array}{ccccc|c} 2 & 2 & 0 & 1 & -1 & 2 \\ 0 & 1 & 0 & 1 & -1 & 3 \\ 0 & 0 & 1 & 0 & 1 & -2 \\ 0 & 0 & 1 & 0 & 1 & -2 \end{array}\right) \xrightarrow{R_4 - R_3} \begin{array}{c} \begin{array}{cccccc} & & & y & z & \end{array} \\ \left(\begin{array}{ccccc|c} 2 & 2 & 0 & 1 & -1 & 2 \\ 0 & 1 & 0 & 1 & -1 & 3 \\ 0 & 0 & 1 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right) \end{array}$$

Row-echelon form

Let $y = s, z = t,\ s,t \in \mathbb{R}$

$x + t = -2 \Leftrightarrow x = -2 - t$

$w + s - t = 3 \Leftrightarrow w = -s + t + 3$

$2v + 2(-s + t + 3) + s - t = 2$

$\Leftrightarrow v = -2 + \frac{1}{2}s - \frac{1}{2}t$

$$\begin{cases} v &= -2 + \frac{1}{2}s - \frac{1}{2}t \\ w &= -s + t + 3 \\ x &= -2 - t \\ y &= s \\ z &= t, \qquad s,t \in \mathbb{R}. \end{cases}$$

EXAMPLE

Solve the following linear system using Gauss-Jordan elimination.

$$\begin{cases} \qquad\quad 2w + x + 2y - z = 4 \\ \qquad\quad w \qquad + y - z = 3 \\ 4v + 6w + x + 4y - 3z = 8 \\ 2v + 2w \qquad + y - z = 2 \end{cases}$$

$$\left(\begin{array}{ccccc|c} 2 & 2 & 0 & 1 & -1 & 2 \\ 0 & 1 & 0 & 1 & -1 & 3 \\ 0 & 0 & 1 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right) \xrightarrow{\frac{1}{2}R_1} \left(\begin{array}{ccccc|c} 1 & 1 & 0 & \frac{1}{2} & -\frac{1}{2} & 1 \\ 0 & 1 & 0 & 1 & -1 & 3 \\ 0 & 0 & 1 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

Row-echelon form

EXAMPLE

$$\left(\begin{array}{ccccc|c} 1 & 1 & 0 & \frac{1}{2} & -\frac{1}{2} & 1 \\ 0 & 1 & 0 & 1 & -1 & 3 \\ 0 & 0 & 1 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right) \xrightarrow{R_1 - R_2} \begin{array}{c} \begin{array}{cccccc} & & & y & z & \end{array} \\ \left(\begin{array}{ccccc|c} 1 & 0 & 0 & -\frac{1}{2} & \frac{1}{2} & -2 \\ 0 & 1 & 0 & 1 & -1 & 3 \\ 0 & 0 & 1 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right) \end{array}$$

Reduced row-echelon form

Let $y = s, z = t,\ s,t \in \mathbb{R}$

$x + t = -2 \Leftrightarrow x = -2 - t$

$w + s - t = 3 \Leftrightarrow w = -s + t + 3$

$v - \frac{1}{2}s + \frac{1}{2}t = -2 \Leftrightarrow v = -2 + \frac{1}{2}s - \frac{1}{2}t$

$$\begin{cases} v &= -2 + \frac{1}{2}s - \frac{1}{2}t \\ w &= -s + t + 3 \\ x &= -2 - t \\ y &= s \\ z &= t, \qquad s,t \in \mathbb{R}. \end{cases}$$