Week 01 F2F Example Solutions

1. **Example 1.1**

   (a) $x = 1 + 2y \Leftrightarrow x - 2y = 1$.

   (b) Substituting $x = t, y = \frac{1}{2}t - \frac{1}{2}$ into $x - 2y$, we have

   $$t - 2(\frac{1}{2}t - \frac{1}{2}) = t - t + 1 = 1.$$

2. **Example 1.2**

   (a) $x = 3 - 4y + z \Leftrightarrow x + 4y - z = 3$.

   (b)

   $$\begin{cases} x &= s \\ y &= \frac{1}{4}(3 - s + t) \\ z &= t, \quad s, t \in \mathbb{R} \end{cases} \qquad \begin{cases} x &= s \\ y &= t \\ z &= s + 4t - 3, \quad s, t \in \mathbb{R} \end{cases}$$

   (c)

   $$\begin{cases} x + 4y - z = 3 \\ 2x + 8y - 2z = 6 \end{cases}$$

3. **Example 1.3**

   (a) A plane in the xyz-space.

   (b) (i) A line in the xy-plane; (ii) A plane in the xyz-space.

   (c) The line of intersection between the two planes $x + y + z = 1$ and $x - y = 0$ in the xyz-space.

4. **Example 1.4**

   $$\boldsymbol{C} \xleftarrow{(1)} \boldsymbol{B} \xrightarrow{(2)} \boldsymbol{A} \xrightarrow{(3)} \xrightarrow{(4)} \boldsymbol{D}.$$

   (1): Multiply first row by 5

   (2): Add -1 times row 1 to row 2

   (3): Swap rows 1 and 2

   (4): Multiply second row by 2

5. **Example 1.5**

   (i) Matrix is in row-echelon form. Linear system is

   $$\begin{cases} -2x_1 - x_3 - 7x_4 = 8 \\ 3x_2 + 3x_4 = 2 \\ x_4 = -1 \end{cases}$$

General solution is

$$\begin{cases} x_1 &= -\frac{1}{2}(1+s) \\ x_2 &= \frac{5}{3} \\ x_3 &= s \\ x_4 &= -1, \quad s \in \mathbb{R} \end{cases}$$

(ii) Matrix is not in row-echelon form. Linear system is

$$\begin{cases} x_1 & & & + & 2x_3 & - & 2x_4 & + & 3x_5 & = & -2 \\ & & & & x_3 & + & x_4 & + & 3x_5 & = & 2 \\ 0x_1 & + & 0x_2 & + & 0x_3 & + & 0x_4 & + & 0x_5 & = & 0 \\ & & & & & & x_4 & + & 5x_5 & = & 5 \end{cases}$$

General solution is

$$\begin{cases} x_1 &= 14 - 17t \\ x_2 &= s \\ x_3 &= -3 + 2t \\ x_4 &= 5 - 5t \\ x_5 &= t, \quad s, t \in \mathbb{R}. \end{cases}$$

(iii) Matrix is in reduced row-echelon form. Linear system is

$$\begin{cases} x_1 & & - & 2x_3 & & + & 2x_5 & & = & -2 \\ & x_2 & & & & + & 2x_5 & & = & 4 \\ & & & & x_4 & - & x_5 & & = & 1 \\ & & & & & & & x_6 & = & 1 \end{cases}$$

General solution is

$$\begin{cases} x_1 &= -2 + 2s - 2t \\ x_2 &= 4 - 2t \\ x_3 &= s \\ x_4 &= 1 + t \\ x_5 &= t \\ x_6 &= 1, \quad s, t \in \mathbb{R} \end{cases}$$

6. **Example 1.6**

(a) Yes, for example

$$\begin{cases} x_1 & + & x_2 & + & x_3 & = & 3 \\ x_1 & + & x_2 & + & x_3 & = & 4 \end{cases}$$

(b) Yes, for example

$$\begin{cases} x & + & y & = & 1 \\ x & - & y & = & 0 \\ 2x & + & 2y & = & 2 \end{cases}$$

(c) No, it is impossible for the row-echelon form of the augmented matrix to have all pivot columns on the left (variable columns).

(d) Yes, for example

$$\begin{cases} x & + & y & = & 1 \\ 2x & + & 2y & = & 2 \\ 3x & + & 3y & = & 3 \end{cases}$$