Matriculation Number: MA1512

A								

NATIONAL UNIVERSITY OF SINGAPORE

FACULTY OF SCIENCE

SEMESTER 1 EXAMINATION 2017-2018

MA1512

DIFFERENTIAL EQUATIONS FOR ENGINEERING

November/December 2017 Time allowed: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

1. **Write down your matriculation number neatly in the space provided above.** Do not write your name anywhere in this booklet. This booklet (and only this booklet) will be collected at the end of the examination. Do not insert any loose pages in the booklet.
2. This examination paper consists of **FOUR (4)** questions and comprises **SEVENTEEN (17)** printed pages.
3. Answer **ALL** questions. For each question, write your answer in the box and your working in the space provided inside the booklet following that question. The marks for each question are indicated at the beginning of the question. The maximum possible total score for this examination paper is 80 marks.
4. This is a **closed book (with authorized material)** examination. Students are only allowed to bring into the examination hall **ONE** piece A4 size help-sheet which can be used on both sides.
5. Candidates may use any calculators that satisfy MOE A-Level examination guidelines. However, they should lay out systematically the various steps in the calculations.

For official use only. Do not write below this line.

Question	1	2	3	4
(a)				
(b)				

Question 1 (a) [10 marks]

(i) Let $y(x)$ denote the solution of the differential equation

$$\frac{dy}{dx} = e^{x-y},$$

with $y\,(1) = 2$. Find the value of $y\,(4)$. Give your answer correct to two decimal places.

(ii) Let $y(x)$ denote the solution of the differential equation

$$y\frac{dy}{dx} - y^2 = x,$$

with $x > 0.6$, $y > 0$ and $y\,(1) = 1$. Find the value of $y\,(3)$. Give your answer correct to two decimal places.

Answer 1(a)(i)		**Answer 1(a)(ii)**	

(Show your working below and on the next page.)

(More working space for Question 1(a))

Question 1 (b) [10 marks]

(i) At time $t = 0$ a particle with mass 0.3 kg is projected vertically upwards at a velocity u metre per second towards the sky. It is observed that at time $t = 0.38$ second the particle reaches the highest point of its trajectory. If the gravitational constant is $g = 9.8$ metre per second square and the air resistance is equal to $0.3v^2$ when the velocity of the particle is v metre per second, find the value of u. Give your answer correct to two decimal places.

(ii) Let $y(x)$ be a solution of $y'' - 2y' + 10y = 0$, such that $y(0) = 1$ and $y'(0) = 7$. Find the value of $y\left(\frac{\pi}{4}\right)$. Give your answer correct to two decimal places. (Note: It is alright if you want to use Laplace Transform to solve this problem.)

Answer 1(b)(i)		**Answer 1(b)(ii)**	

(Show your working below and on the next page.)

(More working space for Question 1(b))

Question 2 (a) [10 marks]

(i) Let $y(x)$ be a solution of $y''-y'-2y = 3e^{2x}$, such that $y(0) = 2$ and $y'(0) = 2$. Find the value of $y(1)$. Give your answer correct to two decimal places. (Note: It is alright if you want to use Laplace Transform to solve this problem.)

(ii) Let $y(x)$ be the solution of the differential equation

$$y'' = y^2$$

such that

$$x < \sqrt{2}, \quad y > 0, \quad y' > 0, \quad y(0) = 3, \quad y'(0) = 3\sqrt{2}.$$

Find the value of $y(1)$. Give your answer correct to two decimal places.

Answer 2(a)(i)		**Answer 2(a)(ii)**	

(Show your working below and on the next page.)

(More working space for Question 2(a))

Question 2 (b) [10 marks]

(i) A particle moves along the x-axis in forced oscillation without friction such that the displacement x (measured in metre) of the particle from the origin at any time t (measured in second) satisfies the differential equation

$$\ddot{x} + 32x = 16\sqrt{2}\cos\alpha t.$$

Initially at time $t = 0$, the particle is at rest at the origin. It is known that α is the resonant frequency. Find the distance of the particle from the origin at time $t = 7$ second. Give your answer in metre correct to two decimal places.

(ii) The monkey population at the Bukit Timah Nature Reserve follows a logistic model with a birth rate per capita of 10% per year. Initially at time $t = 0$ there were 2000 monkeys at the Reserve. After a very long time, the population settled down to an equilibrium value of M monkeys. If there were 1200 monkeys when time $t = 10$ year, find the value of M. Give your answer correct to the nearest integer.

Answer 2(b)(i)		**Answer 2(b)(ii)**	

(Show your working below and on the next page.)

(More working space for Question 2(b))

Question 3 (a) [10 marks]

(i) The growth of a type of bacteria follows a Malthus model with a birth rate per capita of 1.23 per bacteria per hour and a death rate per capita of D per bacteria per hour. If the number of bacteria doubles every two hours, find the value of D. Give your answer correct to two decimal places.

(ii) Let B, s and E denote three positive constants with $E < \frac{B^2}{4s}$. It is known that the differential equation $\frac{dx}{dt} = Bx - sx^2 - E$ has a stable equilibrium solution $x = \lambda$ and an unstable equilibrium solution $x = \alpha$. If $\frac{B^2}{sE} = \frac{17}{4}$, find the value of $\frac{\lambda}{\alpha}$. Give your answer correct to two decimal places.

Answer 3(a)(i)		**Answer 3(a)(ii)**	

(Show your working below and on the next page.)

(More working space for Question 3(a))

Question 3 (b) [10 marks]

(i) Let $F(s) = L((te^t)\,u(t-1))$, where L denotes the Laplace transform and u denotes the unit step function. Find the value of $F(1.8)$. Give your answer correct to two decimal places.
(ii) At time $t = 0$ a doctor injected 150 mg of morphine into a patient. At time $t = 2$ day the doctor injected 100 mg of morphine into the same patient. If the half-life of morphine in the patient's body is 0.5 day, find the amount of morphine in the patient's body at time $t = 3$ days. Give your answer in mg correct to two decimal places.

Answer 3(b)(i)		**Answer 3(b)(ii)**	

(Show your working below and on the next page.)

(More working space for Question 3(b))

Question 4 (a) [10 marks]

(i) Let $f(t) = L^{-1}\left(\frac{1}{(s-1)^2(s-2)^2}\right)$, where L^{-1} denotes the inverse Laplace transform. Find the value of $f(1.5)$. Give your answer correct to two decimal places.

(ii) Let $y(t)$ be the solution of the differential equation

$$y'' + 3y' + 2y = 2\{u(t-2) - u(t-4)\}$$

such that

$$y(0) = 0 \text{ and } y'(0) = 0,$$

where u denotes the unit step function. Find the value of $y(4.1)$. Give your answer correct to two decimal places.

Answer 4(a)(i)		**Answer 4(a)(ii)**	

(Show your working below and on the next page.)

(More working space for Question 4(a))

Question 4 (b) [10 marks]

(i) Let $w = w(x, y)$ denote a function of two variables x and y. If $w(x, y)$ is the answer that you get by applying the method of separation of variables to solve the partial differential equation $x^2(\frac{\partial w}{\partial x}) = w + y\frac{\partial w}{\partial y}$, with $x > 0$, $y > 0$ and $w(1, 1) = \frac{3}{e^2}$, find the value of $w(3, 3)$. Give your answer correct to two decimal places.

(ii) Let $y(t, x)$ be the solution of the wave equation

$$y_{tt} = y_{xx} \quad , \quad 0 \leq t, \ 0 \leq x \leq \pi,$$

with $y(t, 0) = y(t, \pi) = 0$, $y(0, x) = \sin^3 x$, $y_t(0, x) = 0$.
Find the value of $y\left(\frac{\pi}{6}, \frac{\pi}{3}\right)$. Give your answer correct to two decimal places.
(Suggestion: You may want to use d'Alembert's solution to the wave equation.)

Answer 4(b)(i)		**Answer 4(b)(ii)**	

(Show your working below and on the next page.)

(More working space for Question 4(b))

END OF PAPER