GER1000 QUANTITATIVE REASONING
TUTORIAL 1
Please work on the problems before coming to class. In class, you will engage in group work.

Question 1

On July 30 2018, the Straits Times article “Number of suicides among seniors hits record high” reported:

“The number of seniors taking their own lives hit a record high last year even as the total number of suicides dipped - raising concerns among counsellors that seniors may not be getting access to the support they need.”

“Seniors” means people of age 60 years or above.

Suicide Cases

Year	Age							Total
	60 and above	50-59	40-49	30-39	20-29	10-19	0-9	
2017	129	56	48	55	61	12	0	361
2016	123	74	66	67	77	22	0	429

(a) Which numbers from the table illustrate the quote from the ST?

There were 129 suicides among seniors in 2017, which is higher than 123, the number in 2016. But the total number in 2017 is less than that in 2016: 361 vs 429.

(b) Did the suicide rate among seniors increase from 2016 to 2017? Show your calculations. You might want to use the attached document extracted from Singstat (https://www.singstat.gov.sg/-/media/files/publications/population/population2017.pdf).

To calculate the rates, we need the number of seniors in 2016 and 2017. From the document, the number of seniors in 2016 is 251,853 + 198,020 + 103,796 + 87,955 + 53,556 + 44,243 = 739,423. So the percentage in 2016 is 123/739,423 x 100% = 0.01663%, or 123/739,423 x 100,000 = 16.6 suicides per 100,000 seniors. By the same method, the rate in 2017 is 129/779,343 x 100% = 0.01655%. In fact, the 2017 rate is slightly less.

Number of suicides per 100,000 people in each age group

Year	Age						
	60 and above	50-59	40-49	30-39	20-29	10-19	0-9
2017	16.6	9.1	7.8	9.5	11.1	2.7	0
2016	16.6	12.0	10.7	11.4	14.2	4.9	0

The table of rates show that for all other age groups above 9 years old, suicide rates have decreased from 2016 to 2017. This raises the question of why their trends are different from the seniors.

Question 2

Imagine that you are an intern at a tuition centre. Your employer wants to know if it is worthwhile to invest in iPads to improve students' proficiency in English. He asks you to research more about the matter, and gives you resources and authority to run experiments in the centre or to purchase data from other centres.

(a) Being prudent in the use of resources, you purchase data on classroom iPad usage from other tuition centres, and scores from a standardised English test given to the students at the end of the school year. Your colleague suggests categorising students as either Satisfactory (S) or Unsatisfactory (U) according to a threshold score of 50 points. Below are the results.

	Used iPad			No iPad		
	Number	S	Rate (%)	Number	S	Rate (%)
Males	220	50	23%	120	15	13%
Females	60	50	83%	400	250	63%
Overall	280	100	36%	520	265	51%

Fill in the blanks of the remaining missing data, and calculate the satisfactory rates within each group of students.

(b) Did the study find an overall association between iPad usage and test results? When your employer hears of this finding, he concludes and says to you, "Therefore, iPad usage causes poor English test results." How could you reply?

Rate(S | use iPad) = 100/280 x 100% = 36%.
Rate(S | no iPad) = 265/520 x 100% = 51%.
Yes, there is an overall association between S and iPad usage, and the association is negative.

This is not a randomised controlled experiment, so confounding (eg: sex) may be a serious issue. Even if the known confounders were controlled adequately, there could be other unknown confounders, such as test result before the study. Association is not causation. More investigation is needed to arrive at this conclusion.

If assignment to control and treatment were done at random, we expect similar percentage of males. However, rate(male | use iPad) = 220/280 x 100% = 79% and rate(male | no iPad) = 120/520 x 100% = 23%. This is the clue to concluding the study is most likely not a randomised experiment.

(c) Is there an association between iPad usage and test results among the males, and among the females? Are these findings surprising compared with the overall association in (b)?

Males:
Rate(S | use iPad) = 50/220 x 100% = 23%.
Rate(S | no iPad) = 15/120 x 100% = 13%.
Among males, iPad usage is positively associated with S.

Females:
Rate(S | use iPad) = 50/60 x 100% = 83%.
Rate(S | no iPad) = 250/400 x 100% = 63%.
Among females, iPad usage is positively associated with S.

iPad usage seems to improve test results, for both male and female students. Since the association is positive in both groups, yet the overall association is negative, a Simpson's paradox has occurred with sex as a confounder.

We may apply the Basic Rule on Rates. Since the iPad group has many more males than females, the overall rate of S, which must be between 23% and 83%, is close to 23%. Since the no iPad group has many more females than males, the overall rate of S, which must be between 13% and 63%, is close to 63%. Their combination accounts for the paradox.

(d) Your employer compares the findings in (b) and (c) and asks if they imply that iPad usage is associated to sex. Substantiate your answer with calculations based on a suitable 2×2 contingency table.

The relevant table is this or several rearrangements.

	Used iPad	No iPad	Row Sum
Males	220	120	340
Females	60	400	460
Column Sum	280	520	800

First method: compute rates of iPad usage among males and females.
Rate(used iPad | male) = 220/340 x 100% = 65%
Rate(used iPad | female) = 60/460 x 100% = 13%.
Hence iPad usage is associated with sex of the students.

Second method: compute rates of males among iPad users and non-iPad users.
Rate(male | used iPad) = 220/280 x 100% = 79%
Rate(male | no iPad) = 120/520 x 100% = 23%.

In general, if a Simpson's paradox occurs within sex subgroups, then sex is a confounder. In particular, sex is also associated to test results.

(e) How would you conduct an experiment to answer your employer's question? Provide brief reasons for your recommendations. Will you measure English proficiency twice (before and after the intervention), or will it be sufficient to just measure once (after the intervention)?

Relevant concepts: treatment and control groups, placebo, randomised assignment to treatment and control, blinding, confounding. With many subjects in a randomised controlled experiment, any given covariate is unlikely to be a confounder. It may not be feasible to blind subjects, but the reviewers that grade the English tests can and should be blinded. Ethical issues may also be a concern (although the observational study in (a)-(e) does not support the effectiveness of iPad usage).

By randomly assigning many students to treatment and control, a given covariate (age, sex, etc) is not likely to be associated to treatment. In particular, the pre-intervention English proficiency among those treated is likely to be similar to that in the control group. Hence a positive association observed between iPad usage and English proficiency after the intervention is sufficient evidence for the effectiveness of the treatment.

Note that with a randomized experiment, it is sufficient to measure proficiency just once, after intervention, in order to assess if it works.

If by some means, we are unable to randomly assign students, then we should control for potential confounders, by slicing or more sophisticated methods.