Week 01

MA1508E LINEAR ALGEBRA FOR ENGINEERING

The MA1508E teaching team

Module coordinator:

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- S17-07-20; DID 65161306

Teaching assistants:

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- Ms Adriana MARCIUK (matmae@nus.edu.sg, S17-06-12)

Lecture groups

Lecture group	Day	Time	Venue	Lecturer
LLE1	Tuesday	10am – 12noon	E1-06-09	Ms Adriana
LLE2	Monday	2pm – 4pm	E5-03-20	Mr Christian
LLE3	Monday	2pm – 4pm	E3-06-09	Dr Ng
LLE4	Monday	2pm – 4pm	E3-06-01	Ms Adriana
LLE5	Tuesday	4pm – 6pm	E1-06-03	Mr Christian
L6 (ISE+ESP)	Tuesday	8-10am	LT4	Dr Ng

Tutorial groups

Tutorial group	Day	Time	Venue	Tutor
TTE1	Thursday	10am – 12noon	E1-06-09	Ms Adriana
TTE2	Thursday	12noon – 2pm	E5-03-20	Mr Christian
TTE3	Wednesday	10am– 12noon	E5-03-20	Dr Ng
TTE4	Wednesday	2pm – 4pm	E3-06-01	Ms Adriana
TTE5	Friday	2pm – 4pm	E3-06-09	Mr Christian
T1 (ESP)	Friday	10am– 12noon	E3-06-01	Mr Christian
T2 (ISE)	Thursday	4pm – 6pm	E3-06-03	Mr Christian
T3 (ISE)	Friday	12noon – 2pm	E3-06-03	Mr Christian

Mode of teaching (flipped classroom)

Online lecture video clips

- Every week (week 1 to 12), there will be about 5-6 lecture video clips that will be released for viewing in IVLE.
- Each video will be around 10 minutes.
- No lecture notes will be provided for online videos.
- Instead, the script for each video clip will be provided in PDF format.
- Students are **strongly encouraged** to make their own notes upon viewing the videos.
- For most of the videos, viewing once is **usually insufficient**.
- Slides used in the videos will be uploaded in PDF format.

Mode of teaching (flipped classroom)

Face-2-Face (F2F) lectures

- Every week (week 1 to 13), there will be a 1.5 hour Face-2-Face lecture.
- During each F2F lecture, your lecturer will do the following
  - Quick summary of key points in last week's online videos
  - (Maybe) Discuss additional examples related to last week's online video materials
  - Discuss last week's online quiz questions/answers
  - Provide overview of this week's online videos content
  - Additional examples related to this week's content
  - Question and Answer session
  - F2F in-class quiz
- Slides used during F2F lectures will be uploaded in PDF format.

Mode of teaching (flipped classroom)

Tutorials

- Every week (week 3 to 13), there will be a 1.5 hour tutorial.
- The activities in each tutorial session includes
  - Discuss the solutions to the week's tutorial questions
  - Question and Answer session
  - In-class "practice session" – will be handed up for grading before students leave the class.

Face-2-Face in-class quiz

- A few simple computation-type of questions pertaining to what the F2F lecturer has just discussed in class.
- To be done independently (no discussion/collaboration). Students are allowed to refer to anything.
- Can be conducted anytime during the 1.5 hours
- Students who enter the class after the lecturer announces the quiz will not be allowed to take the quiz.
- Answers will be collected, marked and returned during the next F2F class.
- Duration of the quiz – no more than 15 minutes.

Online IVLE quiz

- 5 MCQ-type questions to be conducted on IVLE platform.
- Each week, the quiz session opens at 0000 hours, Monday and closes at 2200 hours, Sunday.
- There will be no extension of the quiz duration (please submit your quiz answers on time – WITH BUFFER TIME)
- You can resume a previously incomplete session on another day.
- You can submit multiple attempts but note that
  - Only the latest attempt will be graded
  - Do not give partial submissions (e.g. answer Q1, Q2 during first submission and Q3, Q4, Q5 during second submission. If you do this, IVLE will only mark Q3, Q4, Q5 and treat it that you did not answer Q1, Q2.). Make sure you click on “Submit”
- Your score for each week’s IVLE quiz can be checked in IVLE the following week.

Practice Problem Sessions

- Done during tutorials (on some weeks).
- Will contain some questions related to that week's tutorial problems (that has just been discussed).
- Students are allowed to discuss and collaborate.
- Students are allowed to consult their tutor.
- Answers are to be submitted and will be graded. They will be returned during next week's tutorial (or two weeks later).
- Duration of each session – no more than 30 minutes.

Assessment components

- Mid Term Test (week 7, date/time TBC) (20%)
- Face-2-Face in-class quizzes (10%)
- Online IVLE quizzes (10%)
- In-class practice problem sessions (10%)
- Final Examination (30th April, Tuesday, Evening) (50%)

Syllabus

- Chapter 1 – Linear systems and matrices
  - Linear systems and its solutions
  - Gaussian elimination
  - Matrix operations
  - Invertibility of a matrix
  - Determinants and computing inverse using determinants
  - Application of linear systems to Engineering

Syllabus

- Chapter 2 – Euclidean vectors and vector spaces
  - Euclidean vectors; norm, dot product, distance and angle.
  - Linear span, subspaces, linear combination and linear independence
  - Coordinates, basis and dimension
  - Vector spaces associated with a matrix
  - Rank and nullity theorem for matrices
  - Vector spaces in Engineering

Syllabus

- Chapter 3 – Orthogonality, least squares and linear approximation
  - Orthogonal basis and orthogonal projection
  - Gram-Schmidt process
  - Best approximation
  - Inconsistent linear systems and least squares solution

Syllabus

- Chapter 4 – Eigenvalues, eigenvectors and applications
  - Eigenvalues, eigenvectors and eigenspaces
  - Diagonalization
  - Applications of eigenvectors in real life problems.
  - Complex numbers (revision)
  - Solving systems of linear differential equations

Use of python

- This is not a programming course. Python exercises serve to (1) use python as a calculator and (2) manipulate existing programs.
- The purpose is to allow students to consistently utilise python as a tool to visualize engineering problems.
- Students will not be expected to write codes.
- You will provided with a guide on how Python can be used to investigate some of the linear algebra problems in this course.
- The proficiency and the use of python will not be part of any assessment component.

Final reminders/advice

- Linear algebra is VERY relevant to Engineering. You will see its usefulness in subsequent engineering courses that you take….so it augurs well that you take this module seriously.
- View (and review) each week's lecture videos diligently. Make notes that will help you remember/understand the concepts taught each week.
- Do not let the lecture content snow-ball. All concepts are inter-linked so you need to understand this week's content before appreciating next week's.
- Ask your lecturer/tutor questions and clarify your doubts as soon as you have them.
- Take the continuous assessments (online quizzes, F2F quizzes and practice sessions) seriously and you will benefit from them.

Finally…

ANY QUESTIONS?

Week 01 content (motivation)

- Many relationships between physical quantities are linear in nature.
- Sometimes a set of linear relations need to be satisfied simultaneously.
- We would like to learn about how (and what it means) the solutions of such linear relations can be obtained.
- Lower level systems can be visualised geometrically.
- For higher level (more complicated) systems, we would need to develop a systematic manner of finding solutions.

Week 01 (units 001-006) overview

001 Linear systems

- Linear equations (systems) in 2 or more variables
- Solution and solution set of a linear system
- General solution of a linear system

002 Geometrical interpretation

- Solutions to a linear equation in 2 variables (forms a line)
- Solutions to a linear equation in 3 variables (forms a plane)
- Solutions to a linear system in 2 variables (None? Exactly one? Infinitely many?)
- Solutions to a linear system in 3 variables (None? Exactly one? Infinitely many?)

Week 01 (units 001-006) overview

003 Elementary row operations

- Consistent and inconsistent systems. What is a unique solution?
- Using an augmented matrix to represent a linear system
- Three types of elementary row operations that can be performed on an augmented matrix

004 Row equivalent matrices

- Definition of row equivalent matrices
- The ‘reverse’ of an elementary row operation is also an elementary row operation
- If augmented matrices of two linear systems are row equivalent, then the two linear systems have the same solution set

Week 01 (units 001-006) overview

005 Row-echelon forms

- Definition of row-echelon form
- Definition of pivot point, pivot column, non-pivot column
- Definition of reduced row-echelon form
- Why are we interested in row-echelon forms?

006 Writing solutions from row-echelon forms

- What a row-echelon form can tell us
- Last column is a pivot column – inconsistent
- Last column is non-pivot – consistent
- Unique solution vs infinitely many solutions
- How to write down a general solution based on a (reduced) row-echelon form

Example 1.1

(a) Find a linear equation in the variables x and y that has a general solution $x = 1 + 2t$ and $y = t$ where t is an arbitrary parameter.

(b) Show that $x = t, y = \frac{1}{2}t - \frac{1}{2}$, where t is an arbitraray parameter, is also a general solution for the equation in Part (a).

Example 1.2

(a) Find a linear equation in the variables x, y and z that has a general solution

$$\begin{cases} x &= 3-4s+t \\ y &= s \\ z &= t \end{cases}$$

where s, t are arbitrary parameters.

(b) Express a general solution for the equation in Part (a) in two other different ways.

(c) Write down a linear system of two different non-zero linear equations such that the linear system has the same general solution as in Part (a).

Example 1.3

(a) Give a geometrical interpretation for the linear equation $x + y + z = 1$

(b) Give a geometrical interpretation for the linear equation $x - y = 0$ in

(i) the xy-plane; and (ii) the xyz-space.

(c) Give a geometrical interpretation for the solutions to the system of linear equations

$$\begin{cases} x + y + z = 1 \\ x - y = 0 \end{cases}$$

Example 1.4

Show that the following augmented matrices are row equivalent to each other.

$$A = \left(\begin{array}{cc|c} 1 & 3 & 2 \\ 0 & 0 & 0 \end{array}\right)$$

$$B = \left(\begin{array}{cc|c} 1 & 3 & 2 \\ 1 & 3 & 2 \end{array}\right)$$

$$C = \left(\begin{array}{cc|c} 5 & 15 & 10 \\ 1 & 3 & 2 \end{array}\right)$$

$$D = \left(\begin{array}{cc|c} 0 & 0 & 0 \\ 2 & 6 & 4 \end{array}\right)$$

Example 1.5

For each of the following augmented matrices, (i) determine whether the matrix is in row-echelon form, reduced row-echelon form, both, or neither; and (ii) find a system of linear equations corresponding to the augmented matrix and then solve the system (if possible). You may assume that the variables are x_1, x_2, etc.

$$\left(\begin{array}{cccc|c} -2 & 0 & -1 & -7 & 8 \\ 0 & 3 & 0 & 3 & 2 \\ 0 & 0 & 0 & 1 & -1 \end{array}\right)$$

$$\left(\begin{array}{ccccc|c} 1 & 0 & 2 & -2 & 3 & -2 \\ 0 & 0 & 1 & 1 & 3 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 5 & 5 \end{array}\right)$$

$$\left(\begin{array}{cccccc|c} 1 & 0 & -2 & 0 & 2 & 0 & -2 \\ 0 & 1 & 0 & 0 & 2 & 0 & 4 \\ 0 & 0 & 0 & 1 & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{array}\right)$$

Example 1.6

(a) Does an inconsistent linear system with more unknowns than equations exist?

(b) Does a linear system which has only one solution, but more equations than unknowns, exist?

(c) Does a linear system which has only one solution, but more unknowns than equations, exist?

(d) Does a linear system which has infinitely many solutions, but more equations than unknowns, exist?

Finally…

THE END