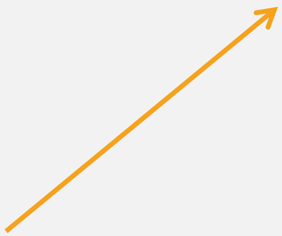
EUCLIDEAN VECTORS

GEOMETRIC VECTORS

Geometrically, a vector is represented by a directed line segment (or arrow).



direction of the arrow

length of the arrow (magnitude)

Two vectors u and v are said to be equal if they have the same length and direction.

GEOMETRIC VECTORS

1) Addition: $\boldsymbol{u}+\boldsymbol{v}$

$$\boldsymbol{u}+\boldsymbol{v}=\boldsymbol{v}+\boldsymbol{u}$$

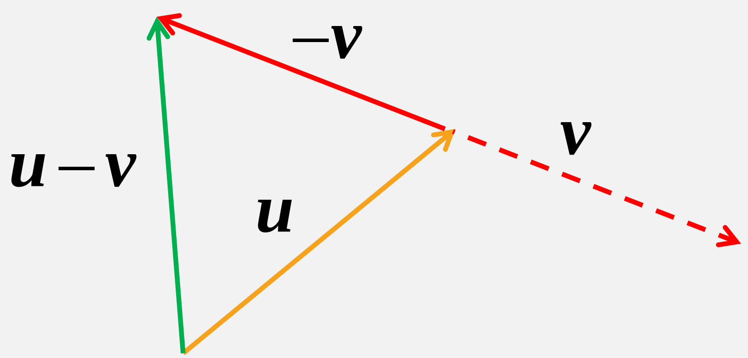
2) Negative of $\boldsymbol{u}$: $-\boldsymbol{u}$

change in direction,

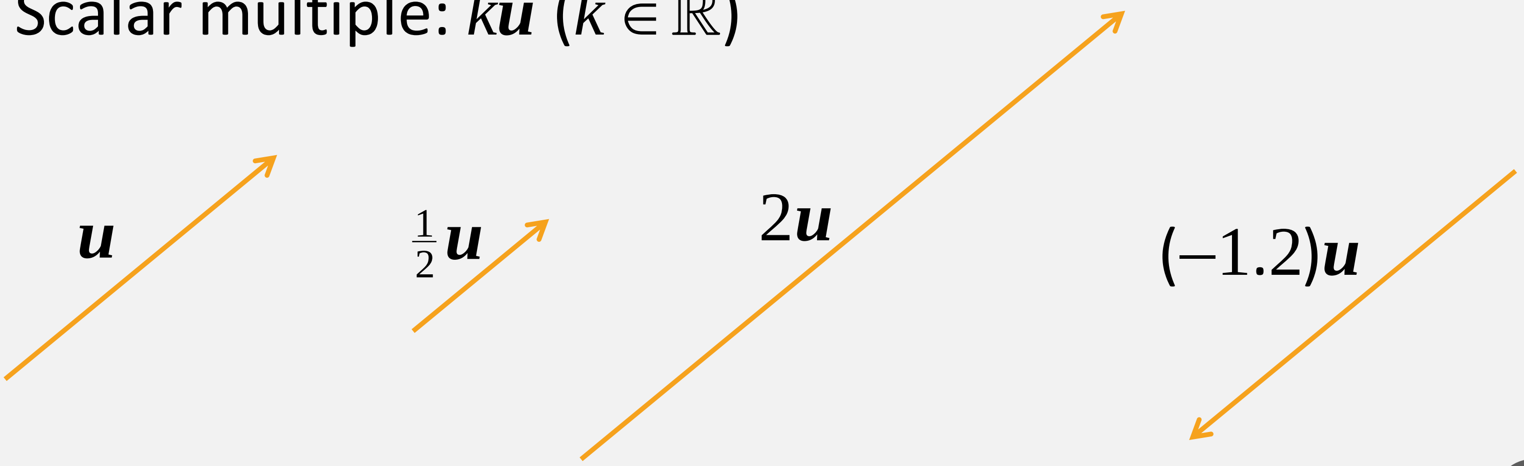
no change in magnitude

GEOMETRIC VECTORS

3) Difference: $\boldsymbol{u}-\boldsymbol{v}$ (same as $\boldsymbol{u}+(-\boldsymbol{v})$)



4) Scalar multiple: $k\boldsymbol{u}$ ($k \in \mathbb{R}$)

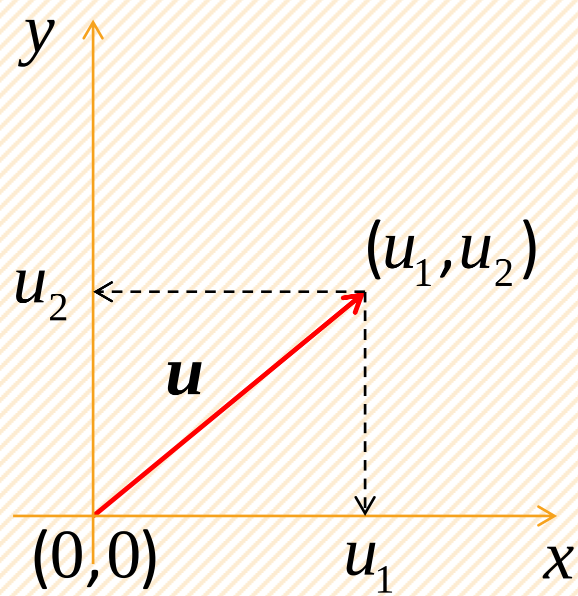


COMPONENTS OF A VECTOR

Position $\boldsymbol{u}$ with its initial point at $(0,0)$

$$\boldsymbol{u} = (u_1, u_2)$$

components of $\boldsymbol{u}$



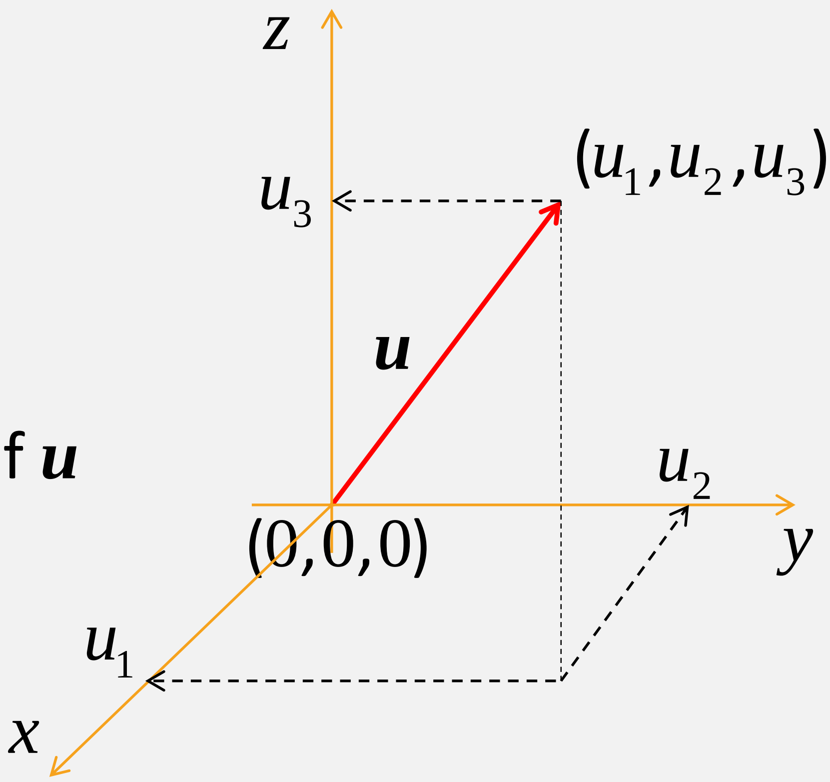
$xy-$plane

COMPONENTS OF A VECTOR

Position $\boldsymbol{u}$ with its initial point at $(0,0,0)$

$\boldsymbol{u} = (u_1, u_2, u_3)$

components of $\boldsymbol{u}$



$xyz-$space

VECTOR OPERATIONS

1) Addition: add component-wise.

$$(u_1, u_2) + (v_1, v_2) = (u_1 + v_1, u_2 + v_2)$$

$$(u_1, u_2, u_3) + (v_1, v_2, v_3) = (u_1 + v_1, u_2 + v_2, u_3 + v_3)$$

2) Scalar multiplication: multiply to each component.

$$\boldsymbol{u} = (u_1, u_2), k \in \mathbb{R} \qquad k\boldsymbol{u} = (ku_1, ku_2)$$

$$\boldsymbol{u} = (u_1, u_2, u_3), k \in \mathbb{R} \qquad k\boldsymbol{u} = (ku_1, ku_2, ku_3)$$

MORE THAN 3 COMPONENTS

n-vector: $(u_1, u_2, \ldots, u_i, \ldots, u_n)$

$u_1, u_2, \ldots, u_n$ are real numbers.

u_1 : 1st component (or 1st coordinate) of the vector

u_i : ith component (or ith coordinate) of the vector

$\boldsymbol{u} = (u_1, u_2, \ldots, u_n) \quad \boldsymbol{v} = (v_1, v_2, \ldots, v_n)$

Equality: $\boldsymbol{u} = \boldsymbol{v}$ if and only if

$u_i = v_i$ for all $i = 1, 2, \ldots, n$.

MORE THAN 3 COMPONENTS

$$\boldsymbol{u}=(u_1,u_2,\ldots,u_n) \qquad \boldsymbol{v}=(v_1,v_2,\ldots,v_n)$$

Addition: $\boldsymbol{u}+\boldsymbol{v}=(u_1+v_1,u_2+v_2,\ldots,u_n+v_n)$

Scalar multiplication: $c\in\mathbb{R},\ c\boldsymbol{u}=(cu_1,cu_2,\ldots,cu_n)$

Negative: $-\boldsymbol{u}=(-u_1,-u_2,\ldots,-u_n)$

Subtraction: $\boldsymbol{u}-\boldsymbol{v}=(u_1-v_1,u_2-v_2,\ldots,u_n-v_n)$

Zero vector: $\mathbf{0}=(0,0,\ldots,0)$

VECTORS AND MATRICES

Identifying an n-vector $(u_1, u_2, \ldots, u_n)$ with:

$1 \times n$ matrix $\begin{pmatrix} u_1 & u_2 & \ldots & u_n \end{pmatrix}$ (row vector)

or

$n \times 1$ matrix $\begin{pmatrix} u_1 \\ u_2 \\ : \\ u_n \end{pmatrix}$ (column vector)

STANDARD RESULTS

Let $\boldsymbol{u}, \boldsymbol{v}, \boldsymbol{w}$ be n-vectors and a, b be real numbers.

1) $\boldsymbol{u}+\boldsymbol{v}=\boldsymbol{v}+\boldsymbol{u}$

2) $\boldsymbol{u}+(\boldsymbol{v}+\boldsymbol{w})=(\boldsymbol{u}+\boldsymbol{v})+\boldsymbol{w}$

3) $\boldsymbol{u}+\mathbf{0}=\boldsymbol{u}=\mathbf{0}+\boldsymbol{u}$

4) $\boldsymbol{u}+(-\boldsymbol{u})=\mathbf{0}$

5) $a(b\mathbf{u})=(ab)\boldsymbol{u}$

6) $a(\mathbf{u}+\mathbf{v})=a\mathbf{u}+a\mathbf{v}$

7) $(a+b)\mathbf{u}=a\mathbf{u}+b\boldsymbol{u}$

8) $1\boldsymbol{u}=\boldsymbol{u}$

EUCLIDEAN N-SPACE

Euclidean n-space, denoted by $\mathbb{R}^n$, is the set of all n-vectors $(u_1, u_2, \ldots, u_n)$ where u_i, $i = 1, \ldots, n$, is a real number.

$$\boldsymbol{u} = (u_1, u_2, \ldots, u_n) \in \mathbb{R}^n \text{ if and only if } u_1, \ldots, u_n \in \mathbb{R}.$$

Note:

1) For any positive integer n, $\mathbb{R}^n$ is a set.

2) How many vectors does $\mathbb{R}^n$ contain?

Infinitely many!

3) Do $\mathbb{R}^2$ and $\mathbb{R}^3$ have any vector in common?

No!

$(2, -3)$ $(-1.5, 3\pi, 0)$

EXAMPLE

$$S = \{\boxed{(u_1, u_2, u_3)} \mid \boxed{u_1 = 0 \text{ and } u_2 = -u_3}\}$$

S contains vectors $\boxed{(u_1, u_2, u_3) \text{ from } \mathbb{R}^3}$ $\boxed{\text{such that}}$

$$\boxed{u_1 = 0 \text{ and } u_2 = -u_3}$$

S is a subset of $\mathbb{R}^3$ $\quad (0,1,-1) \in S \quad (0,0,0) \in S \quad (1,0,0) \notin S$

We can also write S as

$$S = \{(0, a, -a) \mid a \in \mathbb{R}\}$$

SUMMARY

1) A vector and the components of a vector.

2) Addition and subtraction of vectors; and multiplying a scalar to a vector.

3) Identifying a vector with a matrix. Some vector operation laws.

4) Definition of the Euclidean $n-$space, $\mathbb{R}^n$; subsets of $\mathbb{R}^n$.