Week 05 IVLE Quiz

1. Let A and B be two square matrices of order 3 such that

$$E_2 E_1 A = E_4 E_3 B,$$

where

$$E_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad E_2 = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad E_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad E_4 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{pmatrix}.$$

   You may assume that $\det(A) \neq 0$. How of the following statements is/are definitely correct?

   (I) $Ax = 0$ and $Bx = 0$ have the same solution set.
   (II) $2\det(A) = 3\det(B)$.
   (III) $-3\det(A) = 2\det(B)$.
   (IV) $E_4^{-1} E_3^{-1} E_2 E_1 A = B$.

   (A) None
   (B) One
   (C) Two
   (D) Three or more.

   Answer: (C)

2. Which of the following statements regarding determinants is/are definitely correct?

   (I) If $\det(A) = 0$, then A is not a square matrix.
   (II) If A is a square matrix, then $\det(A) = -\det(A)$.
   (III) If A is singular, then $\det(A)$ is strictly less than zero.
   (IV) If A is invertible, then $\det(A^2)$ is always positive.

   (A) (I) and (IV) only
   (B) (I), (II) and (IV) only
   (C) (II) and (III) only
   (D) None of the given combinations is correct.

   Answer: (D)

3. How many statements below is/are corerct?

   (I) For any positive interger n, $\mathbb{R}^n$ is a set with inifinitely many elements.

   (II) $\mathbb{R}^2$ is a subset of $\mathbb{R}^4$.

   (III) $\{(1,1),(0,2),(1,-1)\}$ is a subset of $\mathbb{R}^2$.

   (IV) If $S = \{(x,y,z) \mid x = y = 2z$ and x,y,z are $integers\}$, then S is a subset of $\mathbb{R}^3$.

   (A) Exactly one.

   (B) Exactly two.

   (C) Exactly three.

   (D) All four.

   Answer: (C)

4. Suppose S is an orthogonal subset of $\mathbb{R}^4$ that **does not** contain the zero vector. Which of the following statements is/are definitely correct?

   (I) S is an orthonormal set of non zero vectors.

   (II) For any two distinct vectors $\boldsymbol{u}$ and $\boldsymbol{v}$ in S, the distance between $\boldsymbol{u}$ and $\boldsymbol{v}$ is 1.

   (III) If $\boldsymbol{u}$ is a vector in S, then the length of $-2\boldsymbol{u}$ is twice the length of $\boldsymbol{u}$.

   (IV) If $\boldsymbol{x}$ and $\boldsymbol{y}$ belong to S, then the dot product between $2\boldsymbol{x}$ and $3\boldsymbol{y}$ is zero.

   (A) (II) and (IV) only.

   (B) (III) only.

   (C) (I), (II) and (III) only.

   (D) (III) and (IV) only.

   Answer: (D) or (B)

5. Let $\boldsymbol{A}$ be a 4×4 matrix such that $\det(\boldsymbol{A}) = \frac{1}{2}$. How many of the equations below is/are correct?

   (I) $\det(2\boldsymbol{A}) = \det(\boldsymbol{A}^{-3})$

   (II) $\det((2\boldsymbol{A})^{-1}) = \det(3\boldsymbol{A})$

   (III) $\det(2\boldsymbol{A}^{-1}) = 2^3\det(\boldsymbol{A}^{-2})$

   (A) None of the equations is correct.

   (B) Exactly one of the equations is correct.

   (C) Exactly two of the equations are correct.

   (D) All of the equations are correct.

   Answer: (C)