Week 05 F2F Example Solutions

1. **Example 4.1**

   (a) $\det(\boldsymbol{C}) = 0$.

   (b) $\det(\boldsymbol{AC}) = 0$, so $(\boldsymbol{AC})\boldsymbol{x} = \boldsymbol{0}$ has infinitely many solutions.

2. **Example 4.2** $\begin{pmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{pmatrix} \begin{matrix} R_2 - R_1 \\ \longrightarrow \\ R_3 - R_1 \end{matrix} \begin{pmatrix} 1 & a & a^2 \\ 0 & b-a & b^2-a^2 \\ 0 & c-a & c^2-a^2 \end{pmatrix}$

   So $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = \begin{vmatrix} 1 & a & a^2 \\ 0 & b-a & b^2-a^2 \\ 0 & c-a & c^2-a^2 \end{vmatrix} = (b-a)(c^2-a^2) - (c-a)(b^2-a^2)$

   $= (b-a)(c-a)(c-b)$.

3. **Example 4.3**

   (a) $\det(3\boldsymbol{A}) = 3^4 \cdot \det(\boldsymbol{A}) = 729$.

   (b) $\frac{1}{9}$.

   (c) $\det(3\boldsymbol{A}^{-1}) = 3^4 \cdot \det(\boldsymbol{A}^{-1}) = 9$.

   (d) $\frac{1}{729}$.

4. **Example 4.4**

   (a) $\boldsymbol{B} \xrightarrow{R_4 + R_2} \xrightarrow{R_2 \leftrightarrow R_3} \xrightarrow{R_1 - R_2} \xrightarrow{3R_2} \xrightarrow{R_3 + 2R_1} \boldsymbol{A}$

   (b) $\det(\boldsymbol{A}) = 1 \cdot 2 \cdot 3 \cdot (-1) = -6$ and hence $\det(\boldsymbol{B}) = (-1) \cdot \frac{1}{3} \cdot \det(\boldsymbol{A}) = 2$.