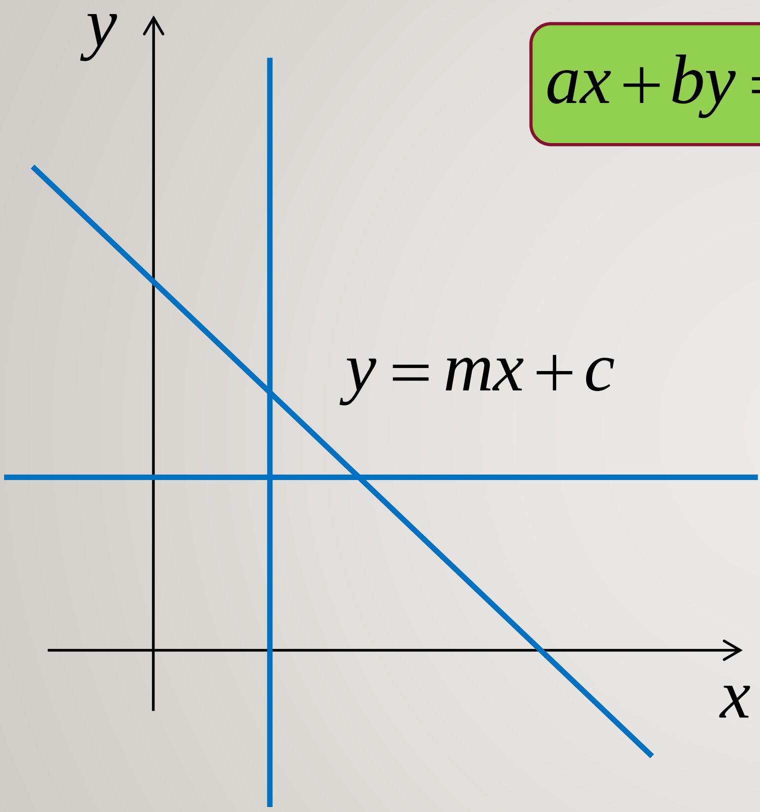
LINEAR SYSTEMS

LINEAR EQUATION IN TWO VARIABLES



$\boxed{ax + by = c}$ a, b not both zero

is a linear equation

in variables x and y.

$$y = -\frac{a}{b}x + \frac{c}{b} \quad \text{(if } b \neq 0\text{)}$$

$$x = \frac{c}{a} \quad \text{(if } b = 0, a \neq 0\text{)}$$

$$y = \frac{c}{a} \quad \text{(if } a = 0, b \neq 0\text{)}$$

DEFINITION (LINEAR EQUATIONS)

A linear equation in n variables $x_1, x_2, \ldots, x_n$ is

$$a_1 x_1 + a_2 x_2 + \ldots + a_n x_n = b$$

where $a_1, a_2, \ldots, a_n, b$ are real constants.

$x_1, x_2, \ldots, x_n$ are also called unknowns.

If $a_1, a_2, \ldots, a_n$ are all zero, we call it a zero equation.

DEFINITION (SOLUTIONS)

Linear equation: $a_1x_1 + a_2x_2 + \ldots + a_nx_n = b$ (*)

Given n real numbers $s_1, s_2, \ldots, s_n$, we say

$$x_1 = s_1, x_2 = s_2, \ldots, x_n = s_n$$

is a **solution** of the linear equation (*) if the equation is satisfied when we substitute $s_1, s_2, \ldots, s_n$ into (*).

$$a_1x_1 + a_2x_2 + \ldots + a_nx_n = b \quad (*)$$

DEFINITION (SOLUTION SET, GENERAL SOLUTION)

Put all solutions of an equation into a set

$\rightarrow$ Solution Set of the equation.

$$\{ \qquad\qquad\qquad \}$$

An expression that gives us all the solutions in the set

$\rightarrow$ General Solution of the equation.

$$\begin{cases} x = \ldots \\ y = \ldots \\ z = \ldots \end{cases}$$

EXAMPLE

$$\boxed{x+2y=2}$$

$x=1, y=\frac{1}{2}$ is a solution

$x=0, y=1$ is another solution

If $x=s$ is any real number, then

$$x=s, y=\frac{1}{2}(2-s)$$

is a solution to the equation.

A general solution to the equation is

$$\begin{cases} x &= s \\ y &= \frac{1}{2}(2-s) \end{cases}$$ where s is an arbitrary parameter

$$\begin{cases} x &= s \\ y &= \frac{1}{2}(2-s),\ s\in\mathbb{R} \end{cases}$$

EXAMPLE

$\boxed{x+2y=2}$

If $y=t$ is any real number, then

$x=2-2t, y=t$

is a solution to the equation.

A(nother) general solution to the equation is

$$\begin{cases} x &= 2-2t \\ y &= t \end{cases}$$ where t is an arbitrary parameter

$$\begin{cases} x &= 2-2t \\ y &= t,\ t\in\mathbb{R} \end{cases}$$

General solutions are not unique!

EXAMPLE

$$\boxed{x+2y=2}$$

$$\begin{cases} x &= s \\ y &= \frac{1}{2}(2-s),\ s\in\mathbb{R} \end{cases}$$

$$\begin{cases} x &= 2-2t \\ y &= t,\ t\in\mathbb{R} \end{cases}$$

How many solutions are there (in the solution set)?

Infinitely many!

EXAMPLE

$\boxed{x - 2y + 3z = 1}$

A general solution is: $\begin{cases} x &= 1 + 2s - 3t \\ y &= s \\ z &= t \qquad s, t \in \mathbb{R} \end{cases}$

$\boxed{x + 2y + 0z = 2}$

A general solution is: $\begin{cases} x &= 2 - 2s \\ y &= s \\ z &= t \qquad s, t \in \mathbb{R} \end{cases}$

DEFINITION (LINEAR SYSTEM)

A finite set of linear equations in the variables $x_1, x_2, \ldots, x_n$ is called a system of linear equations (or linear system).

$$\begin{cases} a_{11}x_1 & + & a_{12}x_2 & + & \ldots & + & a_{1n}x_n & = & b_1 \\ a_{21}x_1 & + & a_{22}x_2 & + & \ldots & + & a_{2n}x_n & = & b_2 \\ & \vdots & & \vdots & & \vdots & & \vdots & \\ a_{m1}x_1 & + & a_{m2}x_2 & + & \ldots & + & a_{mn}x_n & = & b_m \end{cases}$$

$a_{11}, a_{12}, \ldots, a_{mn}, b_1, b_2, \ldots, b_m$ are real constants.

DEFINITION (SOLUTIONS)

Linear system:

$$\begin{cases} a_{11}x_1 & + & a_{12}x_2 & + & \dots & + & a_{1n}x_n & = & b_1 \\ a_{21}x_1 & + & a_{22}x_2 & + & \dots & + & a_{2n}x_n & = & b_2 \\ & \vdots & & \vdots & & \vdots & & \vdots & \\ a_{m1}x_1 & + & a_{m2}x_2 & + & \dots & + & a_{mn}x_n & = & b_m \end{cases}$$

$$x_1 = s_1, x_2 = s_2, \dots, x_n = s_n$$

is a **solution** if it satisfies every equation in the linear system.

DEFINITION (SOLUTION SET, GENERAL SOLUTION)

Put all solutions of the linear system into a set

$\to$ Solution Set of the linear system.

$$\{ \qquad\qquad\qquad \}$$

An expression that gives us all the solutions in the set

$\to$ General Solution of the linear system.

$$\begin{cases} x = \ldots \\ y = \ldots \\ z = \ldots \end{cases}$$

SUMMARY

1) Linear equation (systems) in 2 (or more) variables.

2) Solution and solution set of a linear system.

3) General solution of a linear system.