Unit 001 Linear systems

Slide 01: This is a unit on linear systems.

Slide 02: Most of you would be familiar with the equation of a straight line from your high school days. We would often represent such a line using the equation $y = mx + c$.

(#)

For our purpose, we will use something equivalent to $y = mx+c$, namely $ax+by = c$ to represent a straight line. This is a linear equation in two variables x and y.

(#)

It is easy to rearrange the equation $ax + by = c$ into the following form, as long as b is not 0.

(#)

Note that in the special case where b is zero and a is not zero, the equation reduces to $x = \frac{c}{a}$ which is essentially a vertical line.

(#)

On the other hand, if a is zero but b is not zero, then the equation becomes $y = \frac{c}{a}$ which is a horizontal line.

Slide 03: To generalise a linear equation with 2 variables, we can now define a linear equation in n variables involving variables x_1, x_2 to x_n. Note that the as and b are just some real numbers.

(#)

Sometimes the variables x_1 to x_n are also called unknowns.

(#)

In the event that the as are zll zero, the equation reduces into what is called a zero equation. For our purpose, zero equations are not really interesting.

Slide 04: Consider the following linear equation with n variables.

(#)

What is a solution to the equation?

(#)

It is basically a set of real number s_1, s_2 to s_n such that when substituted into the linear equation, we have equality between both sides of the equation. In other words, we say that the equation is satisfied when we subsitute the values s_1, s_2 to s_n into it.

Slide 05: When all solutions to an equation are put into a set, that set is known as the solution set of the equation.

(#)

If we can obtain an expression that gives us all the solutions in the solution set, then the expression is known as a general solution of the equation. It would be clear how general solutions can be written after we have seen some examples.

Slide 06: Consider the linear equation $x + 2y = 2$. It is clear that $x = 1$, $y = \frac{1}{2}$ is a solution to the equation. Similarly, $x = 0$, $y = 1$ is also a solution to the equation. In fact, if x takes on any real number s, then as long as y takes on the value $\frac{1}{2}(2 - s)$, then $x = s$, $y = \frac{1}{2}(2 - s)$ will always be a solution to the equation.

(#)

Thus we can now write down a general solution to the equation in the following way. Note that s is said to be an arbitrary parameter, meaning that it can take on all possible real number values.

(#)

We can also write s belongs to the set of all real numbers $\mathbb{R}$ in our general solution.

Slide 07: It is not necessarily the case where we let x to take on any real number, like we did previously. We could alternatively let y be any real number t and as along as x takes on the value $2 - 2t$, then $x = 2 - 2t$, $y = t$ will again be a solution to the equation.

(#)

Thus we have yet another way to write down a general solution to the equation.

(#)

This simple example shows us that general solutions are not unique and can be written in many different ways.

Slide 08: How many solutions are there in the solution set? Since there are infinitely many diffferent real numbers that an arbitary parameter can take on, and different choices of real numbers results in different solutions in the solution set,

(#)

The linear equation in this case would have infinitely many solutions. In other words, there are infinitely many different pairs of values (x, y) that will satisfy the equation $x + 2y = 2$.

Slide 09: Let us look at another example, this time a linear equation with three variables x, y and z. It is easy to see that a general solution for this equation can be obtained by letting y and z to be arbitrary parameters s and t respectively. Now as long as x is equal to $1 + 2s - 3t$, then the equation will always be satisfied.

(#)

This next linear equation is also one with three variables x, y and z. However, note that in this case, the coefficient of z is 0.

(#)

A general solution for the equation also involves two arbitrary parameters s and t. However, note that while z is equal to t, x is only dependent on s and not on t. Thus, z can take on any arbitrary value without affecting the values of the other two variables.

Slide 10: A linear system is essentially a generalisation of a single linear equation. In a linear system, we have finite collection of linear equations involving the variables x_1 to x_n. Here, like in the case of a single linear equation, the as and the bs are just some real constants.

Slide 11: For a set of numbers s_1, s_2 to s_n to be a solution to the linear system, they would have to satisfy not just one but every equation in the system. This means that we need to substitute s_1, s_2 to s_n into every equation in the system and make sure that each equation holds.

Slide 12: The definition of the solution set of a linear system, as well as a general solution of a linear system is the same as what we have defined for a single linear equation.

Slide 13: Let us recap the main points in this unit. We defined what is a linear equation, or more generally, a linear system of equations that involves 2 or more variables or unknowns. We also defined what is a solution to a linear equation or a system of linear equations. And finally, an expression that gives us all the solutions in the solution set is called a general solution.