Week 02 IVLE Quiz

1. Suppose an augmented matrix $\boldsymbol{A}$ has 4 rows and 6 columns. If a row-echelon form of $\boldsymbol{A}$ has 3 leading entries, which of the following statements is definitely true?

   (I) The linear system that is represented by $\boldsymbol{A}$ is inconsistent.

   (II) The linear system that is represented by $\boldsymbol{A}$ is either inconsistent or has infinitely many solutions.

   (III) The linear system that is represented by $\boldsymbol{A}$ is either inconsistent or has a unique solution.

   (A) (I) and (II)

   (B) (II) only

   (C) (III) only

   (D) None of the combinations provided is correct.

   Answer: (B)

2. Consider the following augmented matrix $\boldsymbol{A}$ that represents a linear system. How many of the statements below is/are correct?

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 2 & 0 & 0 \\ 0 & 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

   (I) The augmented matrix is in reduced row-echelon form.

   (II) The linear system has the trivial solution.

   (III) The linear system is homogeneous.

   (IV) The linear system is inconsistent.

   (A) None.

   (B) Exactly one.

   (C) Exactly two.

   (D) More than two.

   Answer: (A)

3. Let $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{C}$ be $1 \times n$, $n \times 1$ and $p \times n$ matrices respectively, where $n \geq 2$. Note that $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{C}$ are non-zero matrices. Which of the following statements is/are definitely true?

(I) The matrix $\boldsymbol{AB}$ has only one entry.

(II) The homogeneous linear system $(\boldsymbol{BA})\boldsymbol{x} = \boldsymbol{0}$ has infinitely many solutions.

(III) It is possible that the reduced row-echelon form of $\boldsymbol{CB}$ has 2 leading entries.

(A) (I) and (III) only

(B) (I) and (II) only

(C) (II) and (III) only

(D) All three statements are true.

Answer: (B)

4. Suppose $\boldsymbol{A}$ and $\boldsymbol{B}$ are square matrices of the same size. How many of the statements below is/are true?

(I) If $\boldsymbol{A}$ and $\boldsymbol{B}$ are diagonal, then $\boldsymbol{AB}$ is diagonal.

(II) If $\boldsymbol{A}$ and $\boldsymbol{B}$ are upper triangular, then $\boldsymbol{AB}$ is upper triangular.

(III) If $\boldsymbol{A}$ is upper triangular and $\boldsymbol{B}$ is lower triangular, then $\boldsymbol{AB}$ is diagonal.

(A) None.

(B) One.

(C) Two.

(D) All three.

Answer: (C)

5. A consistent linear system's augmented matrix has the following reduced row-echelon form. We know that $(x_1, x_2, x_3, x_4) = (-1, -2, 2, 1)$ is a solution to the linear system. Which of the following are possible values for the constants a, b and c?

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 2 & 0 \\ 0 & 0 & 1 & -1 & b \\ 0 & 0 & 0 & 0 & a+b+c \end{array}\right).$$

(A) $a = 1, b = 1, c = 0$.

(B) $a = 0, b = 1, c = -1$.

(C) $a = 0, b = 0, c = 0$.

(D) $a = -1, b = 2, c = 0$.

Answer: (B)