NATIONAL UNIVERSITY OF SINGAPORE
Department of Mathematics

Module: MA1508E Linear Algebra for Engineering
Year/Semester: 2018-2019 (Semester 2)
Practice Problem Set: 1

Name:

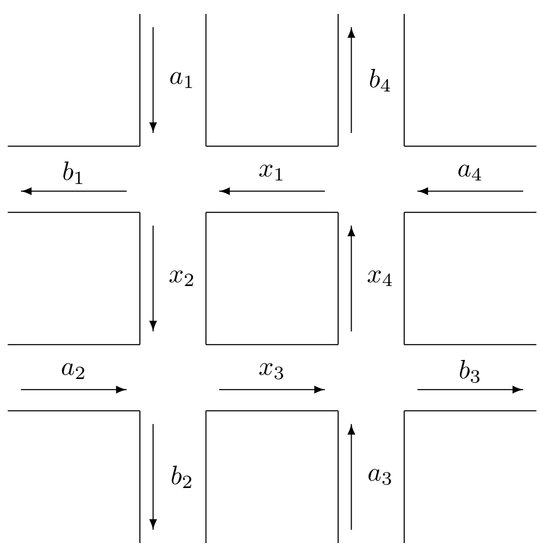
Matriculation Number:

Tutorial Group:

Write down the solutions to the problems below, showing all working involved (imagine it is an examination question). Hand in your answers **together with this question paper** before you leave the classroom. You may refer to any materials while answering the questions. You may also discuss with your friends but do not **copy blindly**.

1. Similar to Question 5 of Tutorial 1, the diagram below shows the downtown section of another city, where $a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4$ are fixed positive integers. Set up a linear system in the unknowns x_1, x_2, x_3, x_4 and show that the linear system will be consistent if and only if

$$a_1 + a_2 + a_3 + a_4 = b_1 + b_2 + b_3 + b_4.$$



Please turn over...

Solution: We set up a linear system as before:

$$\begin{cases} x_1 - x_2 = b_1 - a_1 \\ x_2 - x_3 = b_2 - a_2 \\ x_3 - x_4 = b_3 - a_3 \\ -x_1 + x_4 = b_4 - a_4 \end{cases}$$

Apply Gaussian elimination as follows:

$$\left(\begin{array}{cccc|c} 1 & -1 & 0 & 0 & b_1 - a_1 \\ 0 & 1 & -1 & 0 & b_2 - a_2 \\ 0 & 0 & 1 & -1 & b_3 - a_3 \\ -1 & 0 & 0 & 1 & b_4 - a_4 \end{array}\right) \xrightarrow{R_4 + R_1} \xrightarrow{R_4 + R_2} \xrightarrow{R_4 + R_3}$$

$$\left(\begin{array}{cccc|c} 1 & -1 & 0 & 0 & b_1 - a_1 \\ 0 & 1 & -1 & 0 & b_2 - a_2 \\ 0 & 0 & 1 & -1 & b_3 - a_3 \\ 0 & 0 & 0 & 0 & \sum_{i=1}^4 b_i - a_i \end{array}\right)$$

Thus the linear system is consistent if and only if $\sum_{i=1}^4 b_i - a_4 = 0$ or equivalently, $a_1 + a_2 + a_3 + a_4 = b_1 + b_2 + b_3 + b_4$.

2. Find all values of a for which the linear system has

   (a) No solution;

   (b) A unique solution;

   (c) Infinitely many solutions.

$$\begin{cases} x + y + z = 2 \\ 2x + 3y + 2z = 5 \\ 2x + 3y + (a^2 - 1)z = a + 1 \end{cases}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 2 & 3 & 2 & 5 \\ 2 & 3 & a^2 - 1 & a + 1 \end{array}\right) \underset{R_3 - 2R_1}{\overset{R_2 - 2R_1}{\longrightarrow}} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & a^2 - 3 & a - 3 \end{array}\right) \xrightarrow{R_3 - R_2} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & a^2 - 3 & a - 4 \end{array}\right)$$

   (a) The linear system has no solution when $a = \pm\sqrt{3}$.

   (b) The linear system has a unique solution when $a \neq \pm\sqrt{3}$.

   (c) The linear system cannot have infinitely many solutions.

THE END