Week 05 F2F Example Solutions

1. **Example 5.1**

$$\left(\begin{array}{rrrr|c|}2 & 3 & -1 & 2 & 1 \\ 1 & -1 & 0 & 3 & 1 \\ 0 & 5 & 2 & -7 & \\ 3 & 2 & 1 & 3 & 1\end{array}\right) \longrightarrow \left(\begin{array}{cccr|c|}1 & 0 & 0 & 2 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1\end{array}\right)$$

   (a) Yes. $(2,3,-7,3) = 2\boldsymbol{u_1} - \boldsymbol{u_2} - \boldsymbol{u_3}$.

   (b) No.

2. **Example 5.2**

   (a) No, two vectors cannot span $\mathbb{R}^3$.

   (b) $\begin{pmatrix}1 & -2 & 1 \\ 1 & 2 & 5 \\ -1 & 1 & -2\end{pmatrix} \longrightarrow \begin{pmatrix}1 & 0 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 0\end{pmatrix}$. So the 3 vectors do not span $\mathbb{R}^3$.

   (c) $\begin{pmatrix}1 & -2 & 4 \\ 1 & 2 & 0 \\ -1 & 1 & 3\end{pmatrix} \longrightarrow \begin{pmatrix}1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1\end{pmatrix}$. The 3 vectors span $\mathbb{R}^3$.

   (d) $\begin{pmatrix}1 & -2 & -1 & 0 \\ 1 & 2 & 7 & 8 \\ -1 & 1 & -1 & -2\end{pmatrix} \longrightarrow \begin{pmatrix}1 & 0 & 3 & 4 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 0\end{pmatrix}$. So the 4 vectors do not span $\mathbb{R}^3$.

3. **Example 5.3**

$$\left(\begin{array}{rrc|r|c}2 & -1 & 0 & 1 & 0 \\ -2 & 1 & 0 & -1 & 1 \\ 0 & -1 & 9 & -5 & 1\end{array}\right) \longrightarrow \left(\begin{array}{ccc|c|c}1 & 0 & -\frac{9}{2} & 3 & 0 \\ 0 & 1 & -9 & 5 & 0 \\ 0 & 0 & 0 & 0 & 1\end{array}\right)$$

   So $\boldsymbol{v_2} \notin \operatorname{span}\{\boldsymbol{u_1}, \boldsymbol{u_2}, \boldsymbol{u_3}\}$ and $\operatorname{span}\{(\boldsymbol{u_1}, \boldsymbol{u_2}, \boldsymbol{u_3}\} \neq \operatorname{span}\{\boldsymbol{v_1}, \boldsymbol{v_2}\}$.

4. **Example 5.4**

   (a) Yes, the set is the solution set of the homogeneous linear system $w+x-y-z = 0$.

   (b) No, since $(1,0,1,0)$ and $(0,2,0,1)$ belongs to the set but $(1,0,1,0)+(0,2,0,1) = (1,2,1,1)$ does not.

   (c) No, since $(2,1,1,2)$ belongs to the set but $-(2,1,1,2) = (-2,-1,-1,-2)$ does not.

   (d) Yes, the set is $\operatorname{span}\{(0,1,0,0),(0,0,1,0)\}$.

   (e) No, since $(0,1,1,1)$ and $(1,1,1,0)$ belongs to the set but $(0,1,1,1)+(1,1,1,0) = (1,2,2,1)$ does not.

   (f) No, since $(0,0,0,0)$ does not belong to the set.

   (g) Yes, the set is the solution set of the homogeneous linear system

$$\left\{\begin{array}{ccccccccc} w & & & & & + & z & = & 0 \\ & & x & + & 4y & - & 4z & = & 0 \end{array}\right.$$