Week 07 IVLE Quiz

1. Let $\boldsymbol{u}, \boldsymbol{v}, \boldsymbol{w}$ be vectors in $\mathbb{R}^3$. Which of the following statements below is/are always true?

   (I) $\{\boldsymbol{u}+\boldsymbol{v}, \boldsymbol{u}-\boldsymbol{v}, \boldsymbol{v}-\boldsymbol{w}, 2\boldsymbol{w}\}$ is a linearly dependent set.

   (II) $\{\boldsymbol{u}, \boldsymbol{v}, \boldsymbol{v}-\boldsymbol{w}\}$ is a linearly independent set.

   (III) If $a\boldsymbol{u}+b\boldsymbol{v}+c\boldsymbol{w}=\boldsymbol{0}$ has non trivial solutions, then $\boldsymbol{u}$ is a linear combination of $\boldsymbol{v}$ and $\boldsymbol{w}$.

   (A) (I) only.

   (B) (II) and (III) only.

   (C) (III) only.

   (D) None of the given combinations is correct.

   Answer: (A). (I) is correct because any set of 4 or more vectors in $\mathbb{R}^3$ is always linearly dependent. (II) may not be correct, because, for example one of the vectors, say $\boldsymbol{u}$ is the zero vector. (III) may not be correct, for example, when $\boldsymbol{u}=(1,0,0)$, $\boldsymbol{v}=(0,1,0)$, $\boldsymbol{w}=(0,2,0)$. Then $0\boldsymbol{u}-2\boldsymbol{v}+\boldsymbol{w}=\boldsymbol{0}$ but yet $\boldsymbol{u}$ is not a linear combination of $\boldsymbol{v}$ and $\boldsymbol{w}$.

2. If it is known that $\text{span}\{\boldsymbol{u_1}, \boldsymbol{u_2}, \boldsymbol{u_3}\}$ represents a plane in $\mathbb{R}^3$, how many of the statements below is/are deinitely **false**?

   (I) $\{\boldsymbol{u_2}, \boldsymbol{u_3}\}$ is a linearly dependent set.

   (II) The plane passes through the origin.

   (III) $\boldsymbol{u_1}$ is a scalar multiple of $\boldsymbol{u_2}$.

   (IV) $c_1\boldsymbol{u_1}+c_2\boldsymbol{u_2}+c_3\boldsymbol{u_3}=\boldsymbol{0}$ has only the trivial solution.

   (A) Exactly one.

   (B) Exactly two.

   (C) Exactly three.

   (D) All four.

   Answer: (A). Since we are told that $\text{span}\{\boldsymbol{u_1}, \boldsymbol{u_2}, \boldsymbol{u_3}\}$ is a plane, it means that there is exactly one redundant vector among the 3, but we do not know which one. (I) may not be false because it is possible that $\boldsymbol{u_2}$ is a multiple of $\boldsymbol{u_3}$. (II) is definitely true since the plane is a linear span (which always contain the origin). (III) may not be false because $\boldsymbol{u_1}$ may indeed be a multiple of $\boldsymbol{u_2}$ (which means that $\boldsymbol{u_2}$ and $\boldsymbol{u_3}$ are independent of each other). (IV) is definitely false because $\boldsymbol{u_1}, \boldsymbol{u_2}, \boldsymbol{u_3}$ are linearly dependent.

3. How many of the following sets is/are a basis for $\mathbb{R}^4$?

$$S_1 = \{(1,0,0,0),(0,0,0,1),(0,1,0,0),(0,0,1,0)\};$$

$$S_2 = \{(1,1,1,1),(0,1,1,1),(0,0,1,1)\};$$

$$S_3 = \{(1,1,1,1),(1,1,0,0),(0,0,1,1),(2,2,1,1)\}.$$

(A) None

(B) Exactly one

(C) Excatly two

(D) All three

Answer: (B). S_3 cannot be a basis for $\mathbb{R}^4$ since it has only 3 vectors (which cannot span $\mathbb{R}^4$. S_2 is a linearly dependent set since $(1,1,1,1) = (1,1,0,0) + (0,0,1,1)$. S_1 is a basis for $\mathbb{R}^4$, in fact this is the standard basis for $\mathbb{R}^4$.

4. Suppose $S = \{\boldsymbol{v_1}, \boldsymbol{v_2}, \boldsymbol{v_3}\}$ is a basis for a subspace V of $\mathbb{R}^4$. Let $\boldsymbol{w}$ be a non zero vector in V. Which of the following **cannot** be the coordinate vector of $\boldsymbol{w}$ relative to S?

(I) $(0,0,0)$

(II) $(-1,1,3)$

(III) $(2,-1,3,1)$

(A) (I) only.

(B) (II) only

(C) (I) and (III) only

(D) None of the given combinations is correct.

Answer: (C). (I) cannot be the coordinate vector of $\boldsymbol{w}$ relative to S because $\boldsymbol{w}$ is not the zero vector. (III) cannot be the coordinate vector of $\boldsymbol{w}$ relative to S because the basis S has 3 vectors so any coordinate vector relative to S has 3 components. (II) is the only possible coordinate vector.

5. Consider the matrix $\boldsymbol{A}$ and its row-echelon form.

$$\boldsymbol{A} = \begin{pmatrix} 1 & -2 & -1 & -1 \\ -1 & 3 & 3 & 0 \\ 2 & 1 & 8 & 3 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & -2 & -1 & -1 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 0 & 10 \end{pmatrix}.$$

Suppose the first 3 columns of $\boldsymbol{A}$ are vectors $\boldsymbol{u_1}, \boldsymbol{u_2}, \boldsymbol{u_3}$ respectively and the last column of $\boldsymbol{A}$ is $\boldsymbol{v}$. Which of the following statements is/are definitely true?

(I) $S = \{\boldsymbol{u_1}, \boldsymbol{u_2}, \boldsymbol{u_3}\}$ is a basis for span(S).

(II) $\boldsymbol{v}$ belongs to span(S).

(III) The reduced row-echelon form of $\boldsymbol{A}$ will allow us to find the coordinate vector of $\boldsymbol{v}$ relative to S.

(A) Only (II) is true.

(B) Only (I) and (III) are true.

(C) Only (II) and (III) are true.

(D) None of the given combination given is correct.

Answer: (D). From the row-echelon form, we see that $\boldsymbol{v}$ is not a linear combination of $\boldsymbol{u_1}$, $\boldsymbol{u_2}$ and $\boldsymbol{u_3}$. Thus (II) is incorrect. It is also clear from the row-echelon form that $\boldsymbol{u_1}, \boldsymbol{u_2}$ and $\boldsymbol{u_3}$ are linearly dependent, so S cannot be a basis, meaning (I) is incorrect. (III) is also incorrect since S is not even a basis in the first place.