Unit 008 Example (GE and GJE)

Slide 01: In this unit, we will go through an example of performing Gaussian and Gauss-Jordan elimination on an augmented matrix.

Slide 02: Suppose we wish to solve the following linear system using Gaussian elimination. Note that this is a linear system with 4 equations and 5 unknowns.

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To get started, we write down the augmented matrix of the linear system as shown.

Slide 03: Gaussian elimination begins by identifying the leftmost column that is not entirely zeros.

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The column identified in this case, is this highlighted column.

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The next step requires us to check if the topmost entry in the column identified is zero. In this case, it is zero and therefore, we need to perform a row swap.

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We have chosen to swap rows 1 and 4, to bring a non zero entry to the top of the identified column. Note that we could also have swapped rows 1 and 3.

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The topmost entry in the identified column is 2 and this will be our pivot point.

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We now proceed to eliminate the entries in the same column and below the pivot point. First, we add -2 times of row 1 to row 3. This is denoted by $R_3 - 2R_1$.

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All entries in the same column and below the pivot point is now zero and we are done with the first round of Gaussian elimination.

Slide 04: We now cover up the row containing the previously used pivot point and start again by identifying the leftmost column in the submatrix that is not all zero.

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The column identified is highlighted as shown. Since the topmost entry is 1, which is non zero, no row swap is required in this case.

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The topmost entry is now our next pivot point.

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We now proceed to eliminate the entries in the same column and below the pivot point. This is done via two elementary row operations. First we add -2 times of row 2 to row 3. This is followed by adding -2 times of row 2 to row 4. These two elementary row operations are represented by $R_3 - 2R_2$ and $R_4 - 2R_2$ respectively.

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The resulting matrix is shown here. Note that all entries in the same column and below the pivot point is now zero.

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We once again cover up the row containing the previously used pivot point and start by identifying the leftmost column in the remaining submatrix that is not all zero.

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The column identified is highlighted as shown and once again, no row swap is required.

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The topmost entry in the identified column is our next pivot point.

Slide 05: There is just one entry below our new pivot point and it can be eliminated by adding -1 times row 3 to row 4.

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The resulting matrix is shown here and you should recognise this as a matrix in row-echelon form. We are now ready to investigate this row echelon form further. As the last column of the matrix is not a pivot column, we know that the linear system is consistent. Furthermore, out of the five columns on the left side of the vertical line, three are pivot columns and two are non pivot. This means that this liner system is consistent and has infinitely many solutions. We are now ready to find a general solution for the linear system.

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As discussed in a previous unit, we identify the variables that corresponds to the non pivot columns. In this case, the variables are y and z, so we let $y = s$ and $z = t$ where s and t are arbitrary parameters.

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Starting with the last row of the matrix that is not all zero, we first observe that $x + t = -2$, in other words, $x = -2 - t$.

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Moving up to the next row of the matrix, we have $w + s - t = 3$, so $w = -s + t + 3$.

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Lastly, the first row of the matrix indicates that $2v + 2(-s + t + 3) + s - t = 2$. This simplifies to $v = -2 + \frac{1}{2}s - \frac{1}{2}t$.

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We can now write down a general solution to this linear system as shown.

Slide 06: Suppose we wish to solve the same linear system, now using Gauu-Jordan elimination.

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Notice that we have already obtained a row-echelon form of the augmented matrix in the earlier discussion. This matrix is shown here, with the 3 pivot points or leading entries.

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The next step in Gauss-Jordan eliminaiton requires multiplying the pivot rows by a suitably chosen constant.

Slide 07: It is easy to see that the leading entry in the lowest row already has all the entries in the same column above it equals to zero. We then move to the next lowest row. There is a 1 above the second lowest leading entry. We will eliminate it by adding -1 times of row 2 to row 1.

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The resulting matrix looks like this and you observe that this augmented matrix is now in reduced row-echelon form.

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Once again, we identify the columns that a non pivot. These variables corresponds to variables y and z. We let $y = s$ and $z = t$ to be aribitrary parameters.

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Once again, starting from the lowest row, we observe that $x + t = -2$, or equivalently $x = -2 - t$.

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The next lowest row gives us $w = -s + t + 3$.

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Lastly, the top row gives us $v = -2 + \frac{1}{2}s - \frac{1}{2}t$.

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This gives us a general solution to the linear system. You may now compare this general solution with the one obtained earlier by performing Gaussian elimination. Are they the same?