**Week 02 F2F Example Solutions**

1. **Example 2.1**

   (a)

$$\begin{cases} a &= \frac{1}{2} + \frac{5s}{2} - 4t \\ b &= 1 + 3s - 4t \\ c &= s \\ d &= t, \quad s, t \in \mathbb{R} \end{cases}$$

   (b) $x = 1, y = 2, z = 3$.

2. **Example 2.2**

   (a) (i) No solution when $a = 5, b \neq 4$. (ii) Exactly one solution when $a \neq 5$. (iii) Infinitely many solutions when $a = 5, b = 4$.

   (b) (i) No solution when $a = 0$ or $2$ and $b \neq 0$. (ii) Exactly one solution when $a \neq 0, 2$. (iii) Infinitely many solutions when $a = 0$ or $2$ and $b = 0$.

3. **Example 2.3** Since the solution set is a line that contains the origin, the linear system is homogeneous, which implies $d = g = k = 0$ and $a = e = 1, h = b = 0$ (since reduced row-echelon form). Since the line passes through $(1, 1, 1)$, a general solution for the system can be

$$\begin{cases} x &= s \\ y &= s \\ z &= s, \quad s \in \mathbb{R}. \end{cases}$$

   So $c = -1, f = -1$.

4. **Example 2.4**

   (a) True.

   (b) False. A non-homogeneous system has at least one equation where the left hand side is non-zero. In this case, $x_1 = x_2 = ... = x_n = 0$ does not satisfy this equation and thus the system cannot have the trivial solution.

   (c) False. Homogeneous systems can have both trivial and non-trivial solutions.

   (d) False. Homogeneous systems always have the trivial solution.

   (e) True.

   (f) False. The homogeneous system can have non-trivial solutions too.

   (g) True.

5. **Example 2.5**

   (a) $(3, 4)$-entry of $\boldsymbol{AB}$.

   (b) $(3, 2)$-entry of $\boldsymbol{BA}$.

(a) $\sum_{k=1}^{p} c_{ik} b_{kj}$

(b) $\sum_{r=1}^{p} \left( \sum_{k=1}^{n} b_{ik} a_{kr} \right) c_{rj}$