MA1512 TUTORIAL 3

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Link: http://tinyurl.com/MA1512Solutions

KEY CONCEPTS – CHAPTER 1 DIFFERENTIAL EQUATIONS

Consider the following 2nd order linear ordinary differential equation:

$$y'' + a(x)y' + b(x)y = r(x)$$

- If $r(x) \equiv 0$, this is known as a **homogeneous DE**. Then, the general solution is given by:

$$y = C_1y_1 + C_2y_2$$

  where y_1 and y_2 are linearly independent solutions to the DE.
- If $r(x) \not\equiv 0$, the DE becomes **non-homogeneous**. Thus, the general solution is given by:

$$y = y_h + y_p = (C_1y_1 + C_2y_2) + y_p$$

  where y_h is the general solution to the homogeneous DE and y_p is the particular solution satisfying the non-homogeneous DE.

What is the underlying principle for the characteristic equation?

- **Homogenous 2nd Order Linear ODEs with Constant Real Coefficients**:

$$ay'' + by' + cy = 0, \qquad a, b \in \mathbb{R}$$

Step 1 Find out the characteristic equation $a\lambda^2 + b\lambda + c = 0$. Then solve for λ.
Step 2 Choose case based on λ.

Case A: λ_1, λ_2 real and distinct G.S.: $y = C_1e^{\lambda_1 x} + C_2e^{\lambda_2 x}$	Case B: λ real, repeated G.S.: $y = C_1e^{\lambda x} + C_2xe^{\lambda x}$	Case C: $\lambda = \alpha \pm \beta i$ G.S.: $y = C_1e^{\alpha x}\cos\beta x + C_2e^{\alpha x}\sin\beta x$

To solve non-homogeneous problems:

- **Method of Variation of Parameters**: $y'' + p(x)y' + q(x)y = r(x), \;\; r(x) \not\equiv 0$

Step 1 Find the general solution to homogeneous DE $y'' + p(x)y' + q(x)y = 0$ given by

$$y_h = C_1y_1 + C_2y_2.$$

Step 2 Find Wronskian $W(y_1, y_2) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = y_1y_2' - y_1'y_2$.

Step 3 Use the formula to find u and v.

$$u = -\int \frac{y_2 r}{W(y_1, y_2)}dx \qquad v = \int \frac{y_1 r}{W(y_1, y_2)}dx \qquad y_p = uy_1 + vy_2$$

Step 4 The general solution is given by $y = (C_1y_1 + C_2y_2) + (uy_1 + vy_2) = y_h + y_p$.

- **Method of Undetermined Coefficients**: $y'' + p(x)y' + q(x)y = r(x), \ \ r(x) \not\equiv 0$

  Step 1 Find the general solution y_h to homogeneous DE $y'' + p(x)y' + q(x)y = 0$.

  Step 2 Choose case based on $r(x)$.

<table>
<tr><th>Case A: $r(x)$ is a polynomial</th><th>Case B: $r(x) = P(x)e^{kx}$</th><th>Case C: $r(x) \equiv P(x)e^{\alpha x}\sin\beta x$ or $r(x) \equiv P(x)e^{\alpha x}\cos\beta x$</th></tr>
<tr><td rowspan="2">Guess y_p to be a polynomial with unknown constant coefficients with the same highest power as the highest order in the DE.
• Otherwise, choose higher powers.</td><td>Guess $y_p = ue^{kx}$, where u is a polynomial.</td><td>Guess $y_p = ue^{(\alpha+i\beta)x}$.
If $r(x)$ has $\sin\beta x$, then $y_p = \operatorname{Im} ue^{(\alpha+i\beta)x}$.
If $r(x)$ has $\cos\beta x$, then $y_p = \operatorname{Re} ue^{(\alpha+i\beta)x}$.</td></tr>
<tr><td colspan="2">A. If the value of k is neither λ_1 or λ_2, then guess u to be a polynomial with the highest power same as $P(x)$.
B. If the value of k tallies with either simple root λ_1 or λ_2, then guess u to be a polynomial with highest power + 1 of highest power of $P(x)$.
C. If the value of k tallies tally with repeated root λ, then guess u to be a polynomial with highest power + 2 of highest power of $P(x)$.</td></tr>
</table>

  Step 3 The general solution is given by $y = y_h + y_p$.

CHAPTER 2 – APPLICATIONS OF ODES

The Harmonic Oscillator

<table>
<tr><td>Type 1 Simple Harmonic Oscillator:
$$m\ddot{x} + kx = 0, \text{ where } m, k > 0$$
m is the mass while k is the spring constant
General Solution:
$$\boxed{\begin{aligned} x(t) &= C\cos\omega t + D\sin\omega t \\ &= A\cos(\omega t - \delta) \end{aligned}}$$
Angular frequency: ω; Amplitude: A;
Phase Angle: δ
Period: $= \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$; Frequency, $f = \frac{1}{T}$;</td>
<td>Type 3 Forced Harmonic Oscillator without Damping:
$$m\ddot{x} + kx = F(t) = F_0\cos\alpha t$$
General Solution:
$$\boxed{x(t) = A\cos(\omega t - \delta) + \frac{F_0/m}{\omega^2-\alpha^2}\cos\alpha t}$$
Assume IC $x(0) = 0$ and $\dot{x}(0) = 0$, we get particular solution:
$$\begin{aligned} x(t) &= \frac{\left(\frac{F_0}{m}\right)}{\omega^2-\alpha^2}[\cos\alpha t - \cos\omega t] \\ &= \left[\frac{\left(\frac{2F_0}{m}\right)}{\alpha^2-\omega^2}\sin\left(\frac{\alpha-\omega}{2}t\right)\right]\sin\left(\frac{\alpha+\omega}{2}t\right) \\ &= A(t)\sin\left(\frac{\alpha+\omega}{2}t\right) \end{aligned}$$
Resonance: $\alpha = \omega$
$$\text{P.S.: } x(t) = \frac{F_0 t}{2m\omega}\sin(\omega t)$$</td></tr>
</table>

TUTORIAL PROBLEMS

Question 1

Solve the following differential equations:

Solutions

(a) $y'' + 6y' + 9y = 0 \qquad y(0) = 1 \qquad y'(0) = -1$

The characteristic equation is $\lambda^2 + 6\lambda + 9 = 0 \Longrightarrow \lambda = -3$ (repeated). The general solution is given by

$$y = C_1 e^{-3x} + C_2 x e^{-3x}$$
$$y' = -3C_1 e^{-3x} + C_2(-3xe^{-3x} + e^{-3x})$$

Substituting the two initial conditions into the above equations, we get $C_1 = 1$ and $C_2 = 2$. Thus, the particular solution is given by

$$y = e^{-3x} + 2xe^{-3x} = \boxed{e^{-3x}(1 + 2x)}$$

(b) $y'' - 2y' + (1 + 4\pi^2)y = 0 \qquad y(0) = -2 \qquad y'(0) = 2(3\pi - 1)$

The characteristic equation is $\lambda^2 - 2\lambda + (1 + 4\pi^2) = 0 \Longrightarrow \lambda = 1 \pm 2\pi i$. The G.S. is

$$y = C_1 e^x \cos 2\pi x + C_2 e^x \sin 2\pi x$$
$$y' = C_1 e^x \cos 2\pi x + C_2 e^x \sin 2\pi x + e^x(-2\pi C_1 \sin 2\pi x + 2\pi C_2 \cos 2\pi x)$$

Substituting the two initial conditions into the above equations, we get $C_1 = -2$ and $C_2 = 3$. Thus, the particular solution is given by

$$\boxed{y = -2e^x \cos 2\pi x + 3e^x \sin 2\pi x}$$

Question 2

Find **particular solutions** to the following:

Solutions

(a) $y'' + 2y' + 10y = 25x^2 + 3$. Assume $y_p = Ax^2 + Bx + C$.

$$\begin{aligned} y'' + 2y' + 10y &= 2A + 2(2Ax + B) + 10(Ax^2 + Bx + C) \\ &= 10Ax^2 + (4A + 10B)x + (2A + 2B + 10C) \\ &\equiv 25x^2 + 3 \end{aligned}$$

By comparison, $A = 2.5$, $B = -1$ and $C = 0$. Thus, the particular solution is given by

$$\boxed{y_p = \frac{5}{2}x^2 - x}$$

Try writing out its general solution.

(b) $y'' - 6y' + 8y = x^2 e^{3x}$

The characteristic equation to homogeneous DE is $\lambda^2 - 6\lambda + 8 = 0 \Longrightarrow \lambda_1 = 2$ and $\lambda_2 = 4$. The coefficient of the exponential power 3 does not tally with either λ_1 or λ_2. Thus, we try $y_p = ue^{3x} = (Ax^2 + Bx + C)e^{3x}$.

$$\begin{aligned} y_p &= (Ax^2 + Bx + C)e^{3x} \\ y_p' &= 3e^{3x}(Ax^2 + Bx + C) + e^{3x}(2Ax + B) \\ y_p'' &= 9e^{3x}(Ax^2 + Bx + C) + 6e^{3x}(2Ax + B) + e^{3x}(2A) \end{aligned}$$

Substituting the expressions into the DE:

$$y'' - 6y' + 8y = x^2 e^{3x}$$

$$e^{3x}\{[9Ax^2 + (12A + 9B)x + (2A + 6B + 9C)] - 6[3Ax^2 + (2A + 3B)x + (B + 3C)] + 8(Ax^2 + Bx + C)\} \equiv e^{3x}x^2$$

$$e^{3x}[-Ax^2 + 8Bx + (2A - C)] \equiv e^{3x}x^2$$

By comparison, $A = -1, B = 0$ and $C = -2$. Thus, the particular solution is given by

$$\boxed{y_p = -e^{3x}(x^2 + 2)}$$

(c) $y'' - y = 2x \sin x$

Let us consider the new differential equation $y'' - y = 2xe^{ix}$. The characteristic equation of the homogeneous case is given by $\lambda^2 - 1 = 0 \Longrightarrow \lambda = \pm 1$. The values of λ does not tally with the k value in the power of the exponential term (which is i). Let us try $y_{p1} = (Ax + B)e^{ix}$.

$$\begin{aligned} y'' - y &= \left[2Aie^{ix} - (Ax + B)e^{ix}\right] - (Ax + B)e^{ix} \\ &= e^{ix}[-2Ax + (2Ai - 2B)] \\ &\equiv 2xe^{ix} \end{aligned}$$

By comparison, $A = -1$ and $B = -i$. Thus, $y_{p1} = -e^{ix}(x + i)$. Since the original DE has the term $\sin x$, the imaginary part of y_{p1} will give us the required y_p.

$$\begin{aligned} y_p &= \operatorname{Im} y_{p1} \\ &= \operatorname{Im} [(\cos x + i \sin x)(-x - i)] \end{aligned}$$

$$\boxed{y_p = -x \sin x - \cos x}$$

(d) $y'' + 4y = \sin^2 x$

By trigonometric identity $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$, the new DE is now $y'' + 4y = \frac{1}{2}(1 - \cos 2x)$. There are trigonometric terms in the DE, thus we will consider the following DE:

$$y'' + 4y = \frac{1}{2}\left(1 - e^{2ix}\right)$$

Since the characteristic equation is $\lambda^2 + 4 = 0 \Longrightarrow \lambda = \pm 2i$, our polynomial in our guess must be +1 higher than the power of polynomial in the DE. We will guess $y_{p1} = A + (Bx + C)e^{2ix}$.

$$\begin{aligned} y_{p1} &= A + (Bx + C)e^{2ix} \\ y_{p1}' &= 2ie^{2ix}(Bx + C) + Be^{2ix} \\ y_{p1}'' &= 4Bie^{2ix} - 4(Bx + C)e^{2ix} \end{aligned}$$

$$\begin{aligned} y'' + 4y &= \left[4Bie^{2ix} - 4(Bx + C)e^{2ix}\right] + 4\left[A + (Bx + C)e^{2ix}\right] \\ &= 4A + 4Bie^{2ix} \\ &\equiv \frac{1}{2} - \frac{1}{2}e^{2ix} \end{aligned}$$

By comparison, $A = \frac{1}{8}$ and $B = \frac{i}{8}$ .Thus, $y_{p1} = \frac{1}{8} + \frac{i}{8}xe^{2ix}$. Since the original DE has the term $\cos 2x$, the real part of y_{p1} will give us the required y_p.

$$
\begin{aligned}
y_p &= \operatorname{Re} y_{p1} \\
&= \frac{1}{8} + \operatorname{Re} \frac{i}{8} x(\cos 2x + i \sin 2x)
\end{aligned}
$$

$$\boxed{y_p = \frac{1}{8} - \frac{1}{8} x \sin 2x}$$

For more practice, you can attempt Question 2 using the method of variation of parameters to see if you obtain the same answer.

Question 3

Use the method of variation of parameters to find **particular solutions** of

<u>Solutions</u>

(a) $y'' + 4y = \sin^2 x$

Since the characteristic equation is $\lambda^2 + 4 = 0 \Longrightarrow \lambda = \pm 2i$, the general solution of the homogeneous part is given by

$$y = C_1 \cos 2x + C_2 \sin 2x$$

Taking $y_1 = \cos 2x$ and $y_2 = \sin 2x$, we now calculate the Wronskian.

$$W(y_1, y_2) = \begin{vmatrix} \cos 2x & \sin 2x \\ -2\sin 2x & 2\cos 2x \end{vmatrix} = 2\cos^2 2x + 2\sin^2 2x = 2$$

Thus, using the formula,

$$
\begin{aligned}
u &= -\int \frac{y_2 r}{W(y_1, y_2)} dx = -\frac{1}{2} \int \sin 2x \cdot \sin^2 x \, dx = -\frac{1}{4} \int \sin 2x \cdot (1 - \cos 2x) \, dx \\
&= -\frac{1}{4}\left[\int \sin 2x \, dx - \frac{1}{2} \int \sin 4x \, dx\right] = -\frac{1}{4}\left[-\frac{1}{2}\cos 2x + \frac{1}{8}\cos 4x\right] \\
u &= \frac{1}{8}\cos 2x - \frac{1}{32}\cos 4x
\end{aligned}
$$

$$
\begin{aligned}
v &= \int \frac{y_1 r}{W(y_1, y_2)} dx = \frac{1}{2} \int \cos 2x \cdot \sin^2 x \, dx = \frac{1}{4} \int \cos 2x \cdot (1 - \cos 2x) \, dx \\
&= \frac{1}{4}\left[\int \cos 2x \, dx - \int \cos^2 2x \, dx\right] = \frac{1}{4}\left[\frac{1}{2}\sin 2x - \frac{1}{2}\int 1 + \cos 4x \, dx\right] \\
&= \frac{1}{4}\left[\frac{1}{2}\sin 2x - \frac{1}{2}\left(x + \frac{1}{4}\sin 4x\right)\right] = \frac{1}{8}\sin 2x - \frac{1}{8}x - \frac{1}{32}\sin 4x
\end{aligned}
$$

NB The constants of integration are not required in the integrals to give u and v because they will eventually be absorbed into y_h of the general solution.

Thus, a particular solution is given by

$$
\begin{aligned}
y_p &= u y_1 + v y_2 \\
&= \cos 2x \left(\frac{1}{8}\cos 2x - \frac{1}{32}\cos 4x\right) + \sin 2x \left(\frac{1}{8}\sin 2x - \frac{1}{8}x - \frac{1}{32}\sin 4x\right) \\
&= \frac{1}{8}\cos^2 2x - \frac{1}{32}\cos 2x \cos 4x + \frac{1}{8}\sin^2 2x - \frac{1}{8}x \sin 2x - \frac{1}{32}\sin 2x \sin 4x \\
&= \frac{1}{8} - \frac{1}{8}x \sin 2x - \frac{1}{64}(\cos 6x + \cos 2x) - \frac{1}{64}(\cos 2x - \cos 6x) \\
&= \frac{1}{8} - \frac{1}{8}x \sin 2x - \frac{1}{32}\cos 2x
\end{aligned}
$$

$$\boxed{y_p = \frac{1}{8} - \frac{1}{8}x \sin 2x}$$

Why can the '$-\frac{1}{32}\cos 2x$' in the second last line be dropped?

The answer obtained in this question is exactly the same as the answer obtained in Q2d.

NB Note the importance of trigonometric identities in this question (double-angle formula, product-to-sum formula, basic identities etc.)

(b) $y'' + y = \sec x$

The characteristic equation is given by $\lambda^2 + 1 = 0 \Longrightarrow \lambda = \pm i$. Thus, the general solution is:

$$y = C_1 \cos x + C_2 \sin x$$

Taking $y_1 = \cos x$ and $y_2 = \sin x$, we calculate Wronskian to be $W(y_1, y_2) = 1$. Using the formula,

$$u = -\int \frac{y_2 r}{W(y_1, y_2)} dx = -\int \sin x \sec x \, dx = -\ln|\sec x| = \ln|\cos x|$$

$$v = \int \frac{y_1 r}{W(y_1, y_2)} dx = \int \cos x \sec x \, dx = x$$

Write down the general solution for all ODEs in Question 2 and 3.

A particular solution is given as

$$y_p = u y_1 + v y_2$$

$$\boxed{y_p = \cos x \ln|\cos x| + x \sin x}$$

Question 4

A simple pendulum of length 1 metre is at rest at its stable equilibrium position. At time $t = 0$, it is given an initial angular velocity of 1 radian per second. Neglecting friction, determine the angular displacement of the pendulum at time $t = 0.8$ s. Take the value of the gravitational constant $g = 9.8$ m/s^2. Give your answer correct to two decimal places.

<u>Solutions</u>

From lecture notes, the 2$^{\text{nd}}$-order ODE governing the pendulum motion is given by

$$\frac{d^2\theta}{dt^2} + \frac{g}{L}\theta = 0$$

$$\Rightarrow \theta = A \cos\sqrt{9.8}t + B \sin\sqrt{9.8}t$$

Using initial conditions $\theta(0) = 0$ and $\theta'(0) = 1$, we get $A = 0$ and $B = \frac{1}{\sqrt{9.8}}$. Thus, the particular solution is

$$\theta = \frac{1}{\sqrt{9.8}} \sin\sqrt{9.8}t.$$

To find angular displacement at $t = 0.8$, substitute $t = 0.8$ into θ to get $\theta = \mathbf{0.19\ m}$.

Question 5

A fully loaded large oil tanker can be modelled as a solid object with perfectly vertical sides and a perfectly horizontal bottom, so all horizontal cross-sections have the same area, equal to A. Archimedes' principle [http://en.wikipedia.org/wiki/Buoyancy] states that the upward force exerted on a ship by the sea is equal to the weight of the water pushed aside by the ship. Let ρ be the mass density of seawater, and let M be the mass of the ship, so that its weight is Mg, where g is 9.8 m/s^2. When the ship is at rest, find the distance d from sea level to the bottom of the ship. This is called the **draught** of the ship.

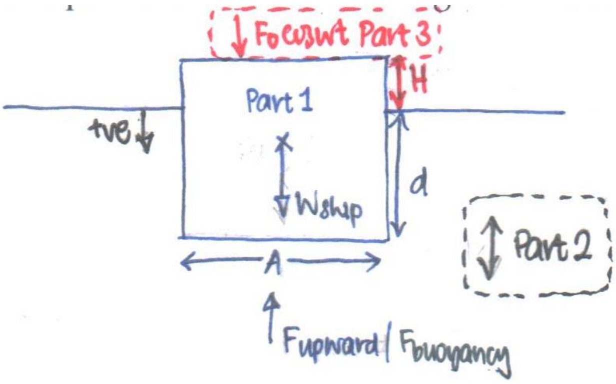
Suppose now that the ship is **not** at rest; instead it is moving in the vertical direction. Let $d + x(t)$ be the distance from sea level to the bottom of the ship, where d is the draught as above. Show that, if gravity and buoyancy are the only forces acting on the ship, it will bob up and down with an angular frequency given by $\omega = \sqrt{\rho A g/M}$.

Next, suppose that waves from a storm strike the ship [which is initially at rest with $x(0) = 0$] and exert a vertical force $F_0 \cos \omega t$ on the ship, where F_0 is the amplitude of the wave force. Let H be the height of the deck of the ship above sea level when the ship is at rest. [We assume that the ship is heavily loaded, so H is much less than d.] Write down a formula which allows you to compute when the ship sinks. [That is, find an equation satisfied by t_{sink}, the time at which the ship's deck first goes under water. You do not need to solve this equation — though of course that can be done by a computer.]

Solutions

Part 1 Determine the **draught** of the ship.

Firstly, let us consider the **Archimedes' Principle**: It states that the upward force exerted on a ship by the sea is equal to the weight of the water pushed aside by the ship. We will need to consider the balance of upward force and the weight of the water.



To consider upward force, we will consider the volume of the ship V submerged under water (and eventually linking it through density). Since the distance from sea level to the bottom of the ship is given by d while the horizontal cross-sectional area is given by A, the volume of ship submerged is given by $V = Ad$. Since the volume of the water displaced by the ship is the same as the volume of ship submerged under water, the upward force exerted on the ship by the sea-water is given by:

$$F_{upward} = mg = \rho V g = \rho A d g$$

The relationship above is achieved by Archimedes' Principle. Since the ship is at rest, there should be no net force exerting on the ship (i.e. $F_{upward} = W_{ship}$). Next, we consider the weight of the ship W_{ship}, which is a downward acting force. This is given by:

$$W_{ship} = Mg$$

By balancing the forces to ensure that the ship remains stationary, $F_{upward} = W_{ship}$:

$$d = \frac{M}{\rho A}. \tag{1}$$

Part 2 Show that motion of ship in vertical direction will cause a wave-like movement.

We will now need to use Newton's Second Law of Motion ($F_{resultant} = M\ddot{x}$) to help us.

Since the ship is now moving vertically, the distance from the sea level to the bottom of the ship will change accordingly with time, and is hence expressed as $d + x(t)$. Note that $x(t)$ is a displacement term from the original position. Setting the downward direction as positive, the buoyancy force (upward acting, allowing the ship to float) is now given as:

$$F_{upward} = -mg = -\rho V g = -\rho A\big(d + x(t)\big)g$$

Thus, by Newton's Second Law of Motion and equation (1),

$$\begin{aligned} F_{resultant} = M\ddot{x} &= W_{ship} + F_{upward} \\ &= Mg - \rho A\big(d + x(t)\big)g \\ &= Mg - \rho A\left(\frac{M}{\rho A} + x\right)g \end{aligned}$$

$$\boxed{\ddot{x} = -\frac{\rho A g}{M}x}$$

This differential equation represents **Type 1** simple harmonic motion with angular frequency $\omega = \sqrt{\frac{\rho A g}{M}}$. Thus, by the theory of SHM, the ship will bob up and down with that frequency.

Part 3 Find out how long it takes for the ship to sink if additional force (waves) was introduced.

By performing force balance again, we get

$$\begin{aligned} M\ddot{x} &= Mg - \rho A\big(d + x(t)\big)g + F_0 \cos \omega t \\ M\ddot{x} + \rho A g x &= F_0 \cos \omega t \end{aligned}$$

This is exactly **Type 2 Forced** Harmonic Oscillator without Damping, where $k = \rho A g$. Also, it was given that the external frequency, $\alpha = \omega = \sqrt{\frac{\rho A g}{M}}$. As such, **resonance** is present. Together with initial conditions $x(0) = 0$ and $\dot{x}(0) = 0$, we get (from lecture notes)

$$x(t) = \frac{F_0 t}{2M\omega}\sin\left(\frac{\alpha + \omega}{2}t\right) = \frac{F_0 t}{2m\omega}\sin(\omega t)$$

As $x(t)$ is the displacement of the instantaneous sea level from the equilibrium position of the ship, the ship will sink when the amplitude of the wave exceeds the top of the ship, which is a distance H from the original sea level. Thus, t_{sink} will be the smallest positive value (why?) satisfying the following equation:

$$H = \frac{F_0 t_{sink}}{2M\omega}\sin(\omega t_{sink}).$$

Question Will the ship sink?

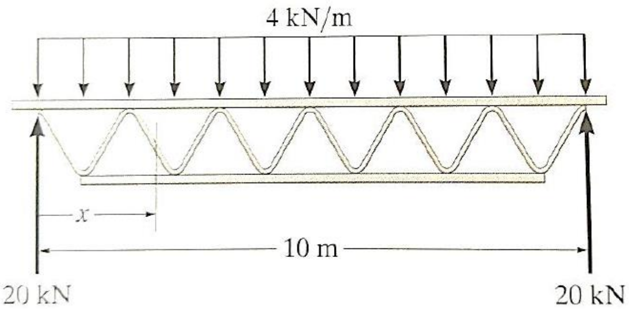
Question Can we solve this equation by hand if all the values H, F_0, M, ω are given? Why?

Question 6

In a given beam set-up, the amount of deflection can be mathematically related to the load acting on the beam through an equation called the *Moment Equation*. Given some simplification, the moment equation is expressed as

$$EI\frac{d^2v}{dx^2} = M,$$

where v is the deflection, M is the moment function for a given beam set-up, E is the Young's modulus, I is the beam's moment of inertia about the neutral axis. EI taken together represents the beam flexural rigidity or resistance to bending and is assumed a constant for a length of the beam, while x is the distance from the origin (selected arbitrarily). The first derivative of v is known as the slope of the bend at any point along the beam – used as a boundary condition.



Each simply-supported floor joist is shown in the photo is subjected to a uniform design loading of 4 kN/m. Given that the moment function is as follows,

$$M = 20x - 4x\left(\frac{x}{2}\right) = 20x - 2x^2,$$

determine the maximum deflection of the joist, taking EI as a constant.

<u>Solutions</u>

Solve the 2nd-order differential equation

$$EI\frac{d^2v}{dx^2} = 20x - 2x^2$$

$$EI\frac{dv}{dx} = 10x^2 - \frac{2}{3}x^3 + C$$

$$EIv = \frac{10}{3}x^3 - \frac{1}{6}x^4 + Cx + D$$

The two ends of the joist are supported, let us take the left end to be the origin. Since there are no deflection at the ends of the joist, we have the boundary conditions $v(0) = v(10) = 0$. Substituting in the conditions,

$$v(0) = 0 \Rightarrow D = 0$$

$$v(10) = 0 \Rightarrow 0 = \tfrac{10}{3}(10)^3 - \tfrac{1}{6}(10)^4 + 10C \Rightarrow C = -\tfrac{500}{3}$$

Thus,

$$EIv = \frac{10}{3}x^3 - \frac{1}{6}x^4 - \frac{500}{3}x$$

Maximum deflection should occur in the middle of the floor joist, i.e. when $x = 5$. Thus, substituting $x = 5$,

$$v = \frac{1}{EI}\left[\frac{10}{3}(5)^3 - \frac{1}{6}(5)^4 - \frac{500}{3}(5)\right] = -\boldsymbol{\frac{3125}{6EI}}$$