Unit 011 Matrix operations

Slide 01: We will introduce some basic matrix operations in this unit.

Slide 02: We have done real numbers operations before, starting with adding, subtracting, multiplying and also dividing. Can we do the same for matrices?

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Before we discuss matrix operations, we first need to define what is meant by saying that one matrix is equal to another.

Slide 03: Two matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ are said to be equal, written as $\boldsymbol{A} = \boldsymbol{B}$, if they have the same size and the entries in both matrices are equal at every corresponding position. In other words, we write $a_{ij} = b_{ij}$ for all i, j.

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So for example, in the matrices shown here, $\boldsymbol{A}$ will be equal to $\boldsymbol{B}$ if and only if $x = 0, y = 1$ and $z = -1$. $\boldsymbol{A}$ will not be equal to $\boldsymbol{C}$ because there is at least one entry in the two matrices that is not equal. Matrix $\boldsymbol{D}$ does not have the same size as $\boldsymbol{A}$, $\boldsymbol{B}$ or $\boldsymbol{C}$ so $\boldsymbol{D}$ can not be equal to any of them.

Slide 04: We will now define a few simple matrix operations. Let $\boldsymbol{A}$ and $\boldsymbol{B}$ be two $m \times n$ matrices. The matrix $\boldsymbol{A} + \boldsymbol{B}$ is also a $m \times n$ matrix whose (i, j)-entry is simply the sum of the (i, j)-entries in $\boldsymbol{A}$ and $\boldsymbol{B}$. In other words, to obtain the matrix $\boldsymbol{A} + \boldsymbol{B}$, we simply add the two matrices entry by entry.

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Similarly for subtraction, the matrix $\boldsymbol{A} - \boldsymbol{B}$ is again a $m \times n$ matrix whose (i, j)-entry is $a_{ij} - b_{ij}$.

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For a real number c, if we multiply c to a matrix $\boldsymbol{A}$, we simply multiply c to every matrix in $\boldsymbol{A}$. Thus, if the (i, j)-entry in $\boldsymbol{A}$ is a_{ij}, then the (i, j)-entry in $c\boldsymbol{A}$ will be ca_{ij}.

Slide 05: Let us go through a few remarks. Firstly, it is obvious that we cannot add or subtract matrices that are not the same size.

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The negative of a matrix $\boldsymbol{A}$ is obtained by multiplying -1 to $\boldsymbol{A}$.

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$\boldsymbol{A} - \boldsymbol{B}$ is just adding the negative of $\boldsymbol{B}$ to $\boldsymbol{A}$.

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In what follows we will go through some simple matrix operation laws, that are similar to what we have for real numbers operation.

Slide 06: The first law is the commutative law for matrix addition. Essentially, this means that $\boldsymbol{A}+\boldsymbol{B}$ is the same as $\boldsymbol{B}+\boldsymbol{A}$. It should be obvious why this is so and actually it follows directly from the fact that for real numbers addition, $x+y$ is the same as $y+x$.

Slide 07: The second law is the associative law for matrix addition. This law states that if $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{C}$ are matrices of the same size, we can add them up in any order and the result would be the same. Once again, this follows naturally from real numbers addition.

Slide 08: The next few results involves multiplying scalars to matrices. Following natrually from how a scalar is multiplied to a matrix, these results should not surprise you.

Slide 09: Let us conclude this unit with a few remarks. Because of the associative law for matrix addition, if we wish to add up a collection of matrices $\boldsymbol{A}_1$ to $\boldsymbol{A}_k$, we can write down this sum without including any parantheses since there will be no ambiguity.

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Next, it is clear that adding the zero matrix to $\boldsymbol{A}$ does not change the matrix $\boldsymbol{A}$.

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Subtracting a matrix from itself results in the zero matrix.

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And lastly, multiplying the scalar 0 to a matrix results in the zero matrix.

Slide 10: In this short unit,

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we defined what is meant by two matrices being equal to each other.

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This is followed by introducing matrix operations like addition, subtraction and scalar multiplication, together with some laws. It should be noted that the operations we have discussed in this unit are identical to real numbers operation.