

Proofs problems

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Chapter 1

Intuitive Proofs

Fact 1.0.1: The pigeonhole principle

Simple form: If $n + 1$ objects are placed into n boxes, then at least one box has at least two objects in it.

General form: If $kn + 1$ objects are placed into n boxes, then at least one box has at least $k + 1$ objects in it.

Proposition

If one chooses $n + 1$ numbers from $\{1, 2, 3, \dots, 2n\}$, it is guaranteed that two of the numbers they chose are consecutive.

Proof. TODO

Quick maths

Proposition

If one selects any $n + 1$ numbers from the set $\{1, 2, \dots, 2n\}$, then two of the selected numbers will sum to $2n + 1$.

Proof. TODO

Quick maths

Proposition

If one chooses 31 numbers from the set $\{1, 2, 3, \dots, 60\}$, then two of the numbers must be relatively prime.

Proof. TODO

Quick maths

Problem

Determine whether or not the pigeonhole principle guarantees that two students at your school have the same 3-letter initials.

TODO

Chapter 2

Direct proofs