Chapter 1

The Reals

Definition 1.0.1: Fields

A *field* $(\mathbb{F}, +, \cdot)$ is a nonempty set $\mathbb{F}$, along with two binary operations, addition $+ : \mathbb{F} \times \mathbb{F} \to \mathbb{F}$ and multiplication $\cdot : \mathbb{F} \times \mathbb{F} \to \mathbb{F}$, satisfying the following,

1. **Associativity:** For all $x, y, z \in \mathbb{F}, (x + y) + z = x + (y + z)$ and $(x \cdot y) \cdot z = x \cdot (y \cdot z)$.
2. **Commutativity:** For all $x, y \in \mathbb{F}$, $x + y = y + x$ and $x \cdot y = y \cdot x$.
3. **Identities:** There exists elements $0, 1 \in \mathbb{F}$ such that for all $x \in \mathbb{F}, x + 0 = x$ and $x \cdot 1 = x$.
4. **Inverses:** For all $x \in \mathbb{F}$, there exists an element $-x \in \mathbb{F}$ such that $x + (-x) = 0$, and if $x \neq 0$, there exists an element x^{-1} such that $x \cdot x^{-1} = 1$.
5. **Distributive Property:** For all $x, y, z \in \mathbb{F}$, $x \cdot (y + z) = (x \cdot y) + (x \cdot z)$.