Definition 0.1: Topological Space

A *topology* $\mathcal{T} \subseteq \mathcal{P}(X)$ on a set X forms a *topological space* $(X, \mathcal{T})$ if it satisfies the following,

(i) $\emptyset, X \in \mathcal{T}$.

(ii) If $S \subseteq \mathcal{T}$ then $\bigcup S \in \mathcal{T}$.

(iii) If $S_1, S_2 \in \mathcal{T}$ then $S_1 \cap S_2 \in \mathcal{T}$.

Definition 0.2: Basis for a Topology

A subset $\beta \subseteq \mathcal{T}$ for a topology $(X, \mathcal{T})$ is a basis of the topology if every open set can be represented as a union of the elements of β. That is, for all open sets $\mathcal{O} \in \mathcal{T}$ there exists a subset $E \subseteq \beta$ such that $\bigcup E = \mathcal{O}$. Elements of β are called *basic open sets*.