Mathematics

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Rings and Fields

Definition 1.0.1: Rings

A ring $(R, +, \cdot)$ is set R equipped with two binary operations, addition $+: R \times R \to R$ and multiplication $\cdot: R \times R \to R$ satisfying the following conditions.

- (i) **Associative addition:** (x + y) + z = x + (y + z), for all $x, y, z \in R$.
- (ii) Commutative addition: x + y = y + x, for all $x, y \in R$.
- (iii) Additive identity: There exists an element, denoted $0 \in R$ such that x + 0 = x, for all $x \in R$.
- (iv) Additive inverse: For all $x \in R$, there exists an element $-x \in R$ such that x + (-x) = 0.
- (v) Associative multiplication: $(x \cdot y) \cdot z = x \cdot (y \cdot z)$, for all $x, y, z \in R$.
- (vi) Multiplicative identity: There exists an element, denoted $1 \in R$ such that $x \cdot 1 = 1 \cdot x = x$, for all $x \in R$.
- (vii) **Left distributivity:** $x \cdot (y+z) = (x \cdot y) + (x \cdot z)$, for all $x, y, z \in R$.
- (viii) **Right distributivity:** $(y+z) \cdot x = (y \cdot x) + (z \cdot x)$, for all $x, y, z \in R$.

Linear Algebra

Topology

Real Analysis

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