# 1 Question 1

## 1.1 Givens

**Modes of transport** $\mathbf{v}_1 = \begin{bmatrix}1\\1\\1\end{bmatrix}$ $\mathbf{v}_2 = \begin{bmatrix}6\\3\\8\end{bmatrix}$ $\mathbf{v}_3 = \begin{bmatrix}4\\1\\6\end{bmatrix}$

## 1.2 Questions

a **Question:** Is there anywhere in $\mathbb{R}^3$ that old man Gauss can hide?

**Answer:** Maybe, let's do a quick RREF and have a look

$$\left[\begin{array}{ccc|c} 1 & 6 & 4 & x \\ 0 & 3 & 8 & y \\ 4 & 1 & 6 & z \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

We're short a pivot in the bottom row but we do have an entry in the last column. This means that the system is inconsistent and there is no solution. So, there is somewhere in $\mathbb{R}^3$ that old man Gauss can hide. We can also see that the vectors are linearly dependent.

$$2\begin{bmatrix}1\\1\\1\end{bmatrix} + \begin{bmatrix}4\\1\\6\end{bmatrix} = \begin{bmatrix}6\\3\\8\end{bmatrix} = \mathbf{v_2}$$

# 2 Question 2

## 2.1 Givens

**Suppose**

$$\mathbf{v_1} = \begin{bmatrix}1\\-1\\2\end{bmatrix} \mathbf{v_2} = \begin{bmatrix}-1\\2\\3\end{bmatrix} \mathbf{v_3} = \begin{bmatrix}2\\3\\h\end{bmatrix}$$

## 2.2 Question

**Question:** For what values of $h$ are the vectors $\mathbf{v_1}$, $\mathbf{v_2}$, and $\mathbf{v_3}$ linearly dependent?

**Answer:** We can do this with the determinant this time around. The system should result in a determinant that is 0.

$$det(\begin{bmatrix}1 & -1 & 2\\-1 & 2 & 3\\2 & 3 & h\end{bmatrix}) = 0$$

$$1(2h-9)+1(-h-6)+2(-3-4) = 0$$

$$2h - 9 - h - 6 - 6 - 8 = 0$$
$$h - 29 = 0$$
$$h = 29$$

**Check:** So, given a value of $h = 29$ we can check our work by calculating

$$\mathbf{v_1} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} \mathbf{v_2} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix} \mathbf{v_3} = \begin{bmatrix} 2 \\ 3 \\ 29 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & x \\ -1 & 2 & 3 & y \\ 2 & 3 & 29 & z \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & 7 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

As we have a row of all zeros, we can be sure that the vectors are linearly dependent.

# 3 Question 3

## 3.1 Question

We're asked whether given $\{\mathbf{x}, \mathbf{y}\}$ as linearly independent. And given $\{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$ as linearly dependent. Is $\mathbf{z}$ in the span of $\{\mathbf{x}, \mathbf{y}\}$? If false, give a counterexample.

## 3.2 Answer

Yes. For $\{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$ to be linearly dependent, one of the vectors must be a linear combination of the others. That is, we could write $\mathbf{z}$ as $\mathbf{z} = a\mathbf{x} + b\mathbf{y}$ for some $a$, $b \in \mathbb{R}$. This is another way of saying that $\mathbf{z}$ is in the span of $\{\mathbf{x}, \mathbf{y}\}$. So, if $\{\mathbf{x}, \mathbf{y}\}$ is linearly independent and $\{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$ is linearly dependent, then $\mathbf{z}$ is in the span of $\{\mathbf{x}, \mathbf{y}\}$.

# 4 Question 4

## 4.1 Remember from class

a If a set contains two vectors that are scalar mulitples of one another, then the set is linearly dependent.

b If a set contains atleast two vectors that are scalar multiples of one another, then the set is linearly dependent.

c If a set contains $p$ vectors in $\mathbb{R}^n$ and $p > n$ then the set is linearly dependent.

d If a set contains the zero vector then it is linearly dependent.

e If a set contains exactly one vector, then the set is linearly independent if and only if the vector is non-zero.

f A set of vectors is linearly dependent if and only if one of the vectors can be expressed as a linear combination of the others.

## 4.2 Question

Choose two and write a reasonable explanation/proof.

## 4.3 Answers

**Is a set containing two scalar multiples dependent?** Yes. If we have two vectors $\mathbf{v_1}$ and $\mathbf{v_2}$ such that $\mathbf{v_2} = k\mathbf{v_1}$ for some $k \in \mathbb{R} \setminus \{0\}$. Then we can write

$$k\mathbf{v_1} - \mathbf{v_2} = \mathbf{0}$$

As we've expressed that atleast one vector is a scalar multiple of another, the set is linearly dependent.

**A set containing the zero vector is dependent?** Yes. If we have a set of vectors $\{\mathbf{v_1}, \mathbf{v_2}, \ldots, \mathbf{v_n}\}$ and one of those vectors is the zero vector, say $\mathbf{v_1} = \mathbf{0}$. Then we can write

$$1 \cdot \mathbf{0} + 0 \cdot \mathbf{v_2} + \ldots + 0 \cdot \mathbf{v_n} = \mathbf{0}$$

As we've expressed that one of the vectors is a linear combination of the others, the set is linearly dependent.

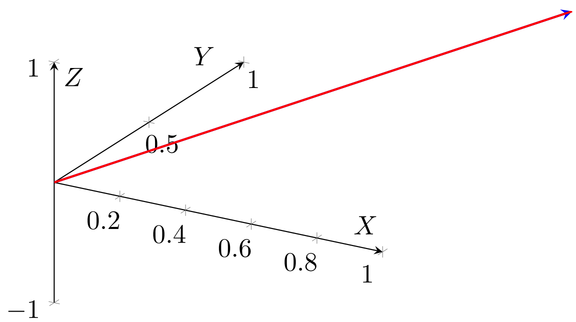
## 4.4 Question 5

Take a row from task 4 and describe the span of the vectors.

**A set of 2 vectors in $\mathbb{R}^3$**

**Linearly Independent**

$$\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} \}$$



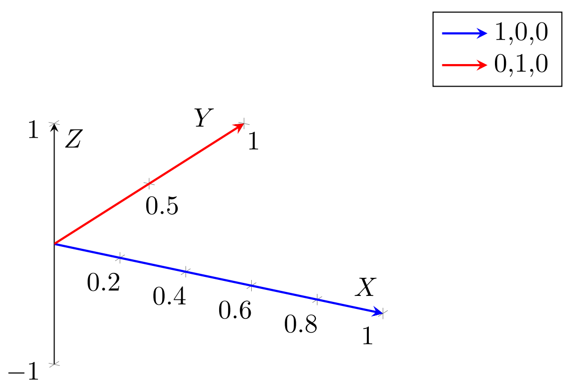

We can see from abovec that the two columns are coincident. This means that the span of these two vectors is a line in $\mathbb{R}^3$.

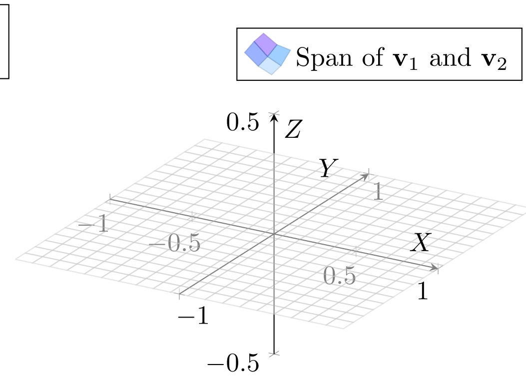
$$\mathbf{v}_2 = k\mathbf{v}_1 = 2\mathbf{v}_1$$

Taking either from the pairing would give us the same line.

**Linearly Dependent**

$$\begin{bmatrix}1\\0\\0\end{bmatrix} \begin{bmatrix}0\\1\\0\end{bmatrix}$$




So, as we are able to provide coefficients to each of either vector to reach any point for $x, y \in \mathbb{R}^2$ and $z = 0$, We have no control over $z$.