

1 Question 1

Given

$$\text{Given } \vec{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \text{ is } \vec{v}_3 \in \text{span}\{\vec{v}_1, \vec{v}_2\}?$$

$$\text{where } \vec{v}_1 = \begin{bmatrix} 1 \\ 1 \\ -1 \\ 1 \end{bmatrix} \text{ and } \vec{v}_2 = \begin{bmatrix} 1 \\ -1 \\ 1 \\ 0 \end{bmatrix}.$$

Solution We're given that we can consider these vectors as

$$A = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}, C = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

We're still asking the same question, so let's see how that breaks down

$$c_1 \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} + c_2 \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

This gives us the following system of equations

$$c_1 + c_2 = 1$$

$$c_1 - c_2 = 2$$

$$c_1 = 3$$

$$c_1 = 4$$

Seen as we can't reconcile the last two equations, we can see that this system is inconsistent.

2 Question 2

$$\text{Restate Is } \mathbf{v}_4 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \in \text{span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}?$$

Values

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} -2 \\ -3 \\ 1 \end{bmatrix}$$

Form $\text{span}\{p(x), q(x), r(x)\}$ is of the form

$$\begin{aligned} p(x) &= 1 - 2x + 0x^2 \\ q(x) &= 0 + 1x - 1x^2 \\ r(x) &= -2 - 3x + 1x^2 \end{aligned}$$

Investigation

$$\begin{aligned} 1 &= 1c_1 + 0c_2 - 2c_3 \\ 1 &= -2c_1 + 1c_2 - 3c_3 \\ 1 &= 0c_1 - 1c_2 + 1c_3 \end{aligned}$$

Augment We can express this as an augmented matrix

$$\left[\begin{array}{ccc|c} 1 & 0 & -2 & 1 \\ -2 & 1 & -3 & 1 \\ 0 & -1 & 1 & 1 \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -2 & 1 \\ 0 & 1 & 0 & -1/2 \\ 0 & 0 & 1 & 5/2 \end{array} \right]$$

Analysis Not only consistent, but unique as well! So. Yes. We can use these coefficients to express \mathbf{v}_4 as a linear combination of $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$.

3 Question 3

3.1 Question

Using RREF. How would you determine whether a system of linear equations is consistent, inconsistent, or has infinite solutions?

3.2 Answer

Been looking forwards to this one. Was one of the first things I had to spend a moment clarifying for myself.

Inconsistent This one's the easiest. If the system that I produce in anyway leads to a contradiction, then the system is inconsistent. Too. If there is no solution to a row in the RREF form, then the system is inconsistent.

Example

$$\left[\begin{array}{ccc|c} 1 & 0 & -2 & 1 \\ 0 & 1 & 0 & -1/2 \\ 0 & 0 & 0 & 5 \end{array} \right]$$

Here we're lead to the conclusion that $0 = 5$, which is clearly a contradiction.

Heuristic If you have a row of all zeros on the left side and a non-zero value on the right side, then the system is inconsistent.

Consistent We can just be lazy and say that consistency is the absence of a contradiction. We can subtype this into two categories, unique solutions and infinite solutions.

Infinite

$$\left[\begin{array}{ccc|c} 1 & 0 & -2 & 1 \\ 0 & 1 & 0 & -1/2 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

Here we can see that there is a free variable, which means that there are infinite solutions. We could express the solution set as

$$\begin{bmatrix} 1 + 2t \\ -1/2 \\ t \end{bmatrix} \text{ for any } t \in \mathbb{R}$$

Hence, infinite solutions.

Unique

$$\left[\begin{array}{ccc|c} 1 & 0 & -2 & 1 \\ 0 & 1 & 0 & -1/2 \\ 0 & 0 & 1 & 5/2 \end{array} \right]$$

Here we can see that there is a unique solution. Way more boring. We can express the solution set as

$$\begin{bmatrix} 1 \\ -1/2 \\ 5/2 \end{bmatrix}$$

Hence, a unique solution.