# 1 Question 1

**Given**

$$\text{Given } \vec{v_3} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \text{ is } \vec{v_3} \in \operatorname{span}\{\vec{v_1}, \vec{v_2}\}?$$

$$\text{where } \vec{v_1} = \begin{bmatrix} 1 \\ 1 \\ -1 \\ 1 \end{bmatrix} \text{ and } \vec{v_2} = \begin{bmatrix} 1 \\ -1 \\ 1 \\ 0 \end{bmatrix}.$$

**Solution** We're given thast we can consider these vectors as

$$A = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}, C = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

We're still asking the same question, so let's see how that breaks down

$$c_1 \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} + c_2 \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

This gives us the following system of equations

$$\begin{aligned} c_1 + c_2 &= 1 \\ c_1 - c_2 &= 2 \\ c_1 &= 3 \\ c_1 &= 4 \end{aligned}$$

Seen as we can't reconcile the last two equations, we can see that this system is inconsistent.

# 2 Question 2

**Restate** Is $\mathbf{v}_4 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \in \operatorname{span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$?

**Values**

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} -2 \\ -3 \\ 1 \end{bmatrix}$$

**Form** $\text{span}\{p(x), q(x), r(x)\}$ is of the form

$$\begin{aligned} p(x) &= 1 - 2x + 0x^2 \\ q(x) &= 0 + 1x - 1x^2 \\ r(x) &= -2 - 3x + 1x^2 \end{aligned}$$

**Investigation**

$$\begin{aligned} 1 &= 1c_1 + 0c_2 - 2c_3 \\ 1 &= -2c_1 + 1c_2 - 3c_3 \\ 1 &= 0c_1 - 1c_2 + 1c_3 \end{aligned}$$

**Augment** We can express this as an augmented matrix

$$\left[\begin{array}{ccc|c} 1 & 0 & -2 & 1 \\ -2 & 1 & -3 & 1 \\ 0 & -1 & 1 & 1 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -2 & 1 \\ 0 & 1 & 0 & -1/2 \\ 0 & 0 & 1 & 5/2 \end{array}\right]$$

**Analysis** Not only consistent, but unique as well! So. Yes. We can use these coefficients to express $\mathbf{v}_4$ as a linear combination of $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$.