

1 Intuition

Time for a little bit of further digging. We ascertained that we could reach old man Guass's house at the given offset, but, is there anywhere where we would be unable to reach him? Put another way. What is the span of the two column vectors representing the magic carpet and hoverboard?

2 Details

Examine

$$\text{span}\left(\begin{bmatrix} 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \end{bmatrix}\right) = c_1 \begin{bmatrix} 3 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 2 \end{bmatrix} \text{ where } c_1, c_2 \in \mathbb{R}$$

We can see that both are linearly independent as

$$\det\left(\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}\right) = 5 \neq 0$$

And from this deduce that our two methods of transport do indeed span \mathbb{R}^2

3 Conclusion

As the two column vectors representing the hoverboard and magic carpet do indeed span all of \mathbb{R}^2 , there is nowhere on the plane that old man Guass can hide. We'll always be able to find him with the help of our hoverboard and magic carpet.