# 1 Question 1

**Given** We're given a set of points that have different offsets based on two different basis vectors. First, let's grab these points and where they map to in the new basis.

## 1.1 Part A

| $\mathbf{B}_x$ | $\mathbf{B}_y$ | $\mathbf{B}'_x$ | $\mathbf{B}'_y$ |
|---|---|---|---|
| 1 | 4 | 0 | 1 |
| 4 | 7 | 1 | 2 |
| -8 | -5 | -3 | -2 |
| 7 | 1 | 1 | 0 |
| $5\frac{1}{2}$ | -5 | 3 | $-\frac{1}{2}$ |
| -6 | $-1\frac{1}{2}$ | $-2\frac{1}{2}$ | -1 |

Where I've denoted $\mathbf{B}$ as the black basis, and $\mathbf{B}'$ as the red. These taken together give us plenty of information to find a change of basis.

## 1.2 Part B

The points above are of the form $[\mathbf{v}_n]_\mathbf{B} \to [\mathbf{v}_n]_{\mathbf{B}'}\ \forall n$. So we can form a system mapping two columns of the first to that of the second.

**The system** Let $\mathbf{P}$ be the change of basis we're attempting to find.

$$\mathbf{P}\begin{bmatrix}1 & 4\\4 & 7\end{bmatrix} = \begin{bmatrix}0 & 1\\1 & 2\end{bmatrix}$$

We need to clear out the left hand side, so take the matrix besides $\mathbf{P}$ as $\mathbf{A}$, and find the inverse. As $\mathbf{A}\mathbf{A}^{-1} = \mathbf{I}_2$

$$\mathbf{P} = \begin{bmatrix}0 & 1\\1 & 2\end{bmatrix}\begin{bmatrix}1 & 4\\4 & 7\end{bmatrix}^{-1} = \begin{bmatrix}0 & 1\\1 & 2\end{bmatrix}\frac{1}{9}\begin{bmatrix}7 & -4\\-4 & 1\end{bmatrix} = \frac{1}{9}\begin{bmatrix}4 & -1\\1 & 2\end{bmatrix}$$

**Checking solution**

Let's check this solution by multiplying $\mathbf{P}$ by another point in the original basis, and see if it maps to the correct point in the new basis.

$$\mathbf{P}\begin{bmatrix}-8\\-5\end{bmatrix} = \frac{1}{9}\begin{bmatrix}4 & -1\\1 & 2\end{bmatrix}\begin{bmatrix}-8\\-5\end{bmatrix} = \frac{1}{9}\begin{bmatrix}-32+5\\-8-10\end{bmatrix} = \frac{1}{9}\begin{bmatrix}-27\\-18\end{bmatrix} = \begin{bmatrix}-3\\-2\end{bmatrix}$$

This matches the expected output, so our change of basis is correct.

## 1.3 Part C

Now we're after a way to go in reverse. We could be super diligent and do the same process, but we can also just take the inverse of $\mathbf{P}$.

$$\mathbf{P}^{-1} = \begin{bmatrix} \frac{4}{9} & -\frac{1}{9} \\ \frac{1}{9} & \frac{2}{9} \end{bmatrix}$$

**Checking solution**

Let's check this solution by multiplying $\mathbf{P}^{-1}$ by another point in the red basis, and see if it maps to the correct point in the black basis.

$$\mathbf{P}^{-1}[\mathbf{v}_2]_{\mathbf{B}'} = \begin{bmatrix} 2 & 1 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 4 \\ 7 \end{bmatrix} = [\mathbf{v}_2]_{\mathbf{B}}$$

Again, seems the intuition is correct. For this point atleast, it seems our change of basis is mapping back from black to red.

## 1.4 Part D

Ok. I'm reading stretch so, eigenvalues, let's give things a name before we get going

#### Given

**First line** We are given a stretch factor of 2 corresponding to the line $y = -\frac{1}{2}x$ So that gives us

$$T(\begin{bmatrix} 2 \\ -1 \end{bmatrix}) = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$$

Where we take $\lambda_1 = 2$ as the eigenvalue

**Second line** For the second we're given a stretch factor of $-1$ and a line $y = 4x$. From this

$$T(\begin{bmatrix} 1 \\ 4 \end{bmatrix}) = \begin{bmatrix} -1 \\ -4 \end{bmatrix}$$

Where we take $\lambda_2 = -1$ as the eigenvalue