1 Question 1

1.1 Givens

Modes of transport $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $\mathbf{v}_2 = \begin{bmatrix} 6 \\ 3 \\ 8 \end{bmatrix}$ $\mathbf{v}_3 = \begin{bmatrix} 4 \\ 1 \\ 6 \end{bmatrix}$

1.2 Questions

a **Question:** Is there anywhere in $\mathbb{R}^3$ that old man Gauss can hide?

Answer: Maybe, let's do a quick RREF and have a look

$$\left[\begin{array}{ccc|c} 1 & 6 & 4 & x \\ 0 & 3 & 8 & y \\ 4 & 1 & 6 & z \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

As we're short a pivot we can be sure of a couple things. The third row is missing both a pivot, but, as importantly, it is missing a value in the agumented column. Given these things together, we can be sure that, one of the vectors is linearly dependent on the others, and that there are infinite solutions

2 Question 2

2.1 Givens

Suppose

$$\mathbf{v_1} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} \mathbf{v_2} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix} \mathbf{v_3} = \begin{bmatrix} 2 \\ 3 \\ h \end{bmatrix}$$

2.2 Question

Question: For what values of h are the vectors $\mathbf{v_1}$, $\mathbf{v_2}$, and $\mathbf{v_3}$ linearly dependent?

Answer: We can do this with the determinant this time around. The system should result in a determinant that is 0.

$$det(\begin{bmatrix} 1 & -1 & 2 \\ -1 & 2 & 3 \\ 2 & 3 & h \end{bmatrix}) = 0$$

$$1(2h-9) + 1(-h-6) + 2(-3-4) = 0$$

$$2h - 9 - h - 6 - 6 - 8 = 0$$

$$h - 29 = 0$$

$$h = 29$$

Check: So, given a value of $h = 29$ we can check our work by calculating

$$\mathbf{v_1} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} \mathbf{v_2} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix} \mathbf{v_3} = \begin{bmatrix} 2 \\ 3 \\ 29 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & x \\ -1 & 2 & 3 & y \\ 2 & 3 & 29 & z \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 7 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

As we have a row of all zeros, we can be sure that the vectors are linearly dependent.