1 Question 1

1.1 Givens

Modes of transport $\mathbf{v}_1 = \begin{bmatrix}1\\1\\1\end{bmatrix}$ $\mathbf{v}_2 = \begin{bmatrix}6\\3\\8\end{bmatrix}$ $\mathbf{v}_3 = \begin{bmatrix}4\\1\\6\end{bmatrix}$

1.2 Questions

a **Question:** Is there anywhere in $\mathbb{R}^3$ that old man Gauss can hide?

Answer: Maybe, let's do a quick RREF and have a look

$$\left[\begin{array}{ccc|c} 1 & 6 & 4 & x \\ 0 & 3 & 8 & y \\ 4 & 1 & 6 & z \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

We're short a pivot in the bottom row but we do have an entry in the last column. This means that the system is inconsistent and there is no solution. So, there is somewhere in $\mathbb{R}^3$ that old man Gauss can hide. We can also see that the vectors are linearly dependent.

$$2\begin{bmatrix}1\\1\\1\end{bmatrix} + \begin{bmatrix}4\\1\\6\end{bmatrix} = \begin{bmatrix}6\\3\\8\end{bmatrix} = \mathbf{v_2}$$

2 Question 2

2.1 Givens

Suppose

$$\mathbf{v_1} = \begin{bmatrix}1\\-1\\2\end{bmatrix} \mathbf{v_2} = \begin{bmatrix}-1\\2\\3\end{bmatrix} \mathbf{v_3} = \begin{bmatrix}2\\3\\h\end{bmatrix}$$

2.2 Question

Question: For what values of h are the vectors $\mathbf{v_1}$, $\mathbf{v_2}$, and $\mathbf{v_3}$ linearly dependent?

Answer: We can do this with the determinant this time around. The system should result in a determinant that is 0.

$$det(\begin{bmatrix}1 & -1 & 2\\-1 & 2 & 3\\2 & 3 & h\end{bmatrix}) = 0$$

$$1(2h-9) + 1(-h-6) + 2(-3-4) = 0$$

$$2h - 9 - h - 6 - 6 - 8 = 0$$
$$h - 29 = 0$$
$$h = 29$$

Check: So, given a value of $h = 29$ we can check our work by calculating

$$\mathbf{v_1} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} \mathbf{v_2} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix} \mathbf{v_3} = \begin{bmatrix} 2 \\ 3 \\ 29 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & x \\ -1 & 2 & 3 & y \\ 2 & 3 & 29 & z \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & 7 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

As we have a row of all zeros, we can be sure that the vectors are linearly dependent.

3 Question 3

3.1 Question

We're asked whether given $\{\mathbf{x}, \mathbf{y}\}$ as linearly independent. And given $\{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$ as linearly dependent. Is $\mathbf{z}$ in the span of $\{\mathbf{x}, \mathbf{y}\}$? If false, give a counterexample.

3.2 Answer

Yes. For $\{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$ to be linearly dependent, one of the vectors must be a linear combination of the others. That is, we could write $\mathbf{z}$ as $\mathbf{z} = a\mathbf{x} + b\mathbf{y}$ for some a, $b \in \mathbb{R}$. This is another way of saying that $\mathbf{z}$ is in the span of $\{\mathbf{x}, \mathbf{y}\}$. So, if $\{\mathbf{x}, \mathbf{y}\}$ is linearly independent and $\{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$ is linearly dependent, then $\mathbf{z}$ is in the span of $\{\mathbf{x}, \mathbf{y}\}$.