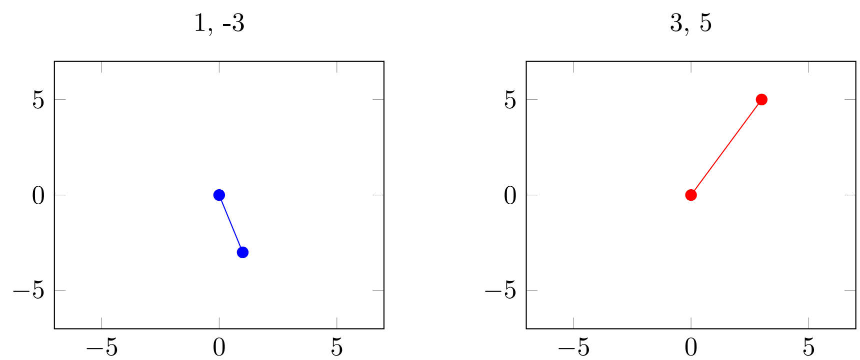
1 Question 1

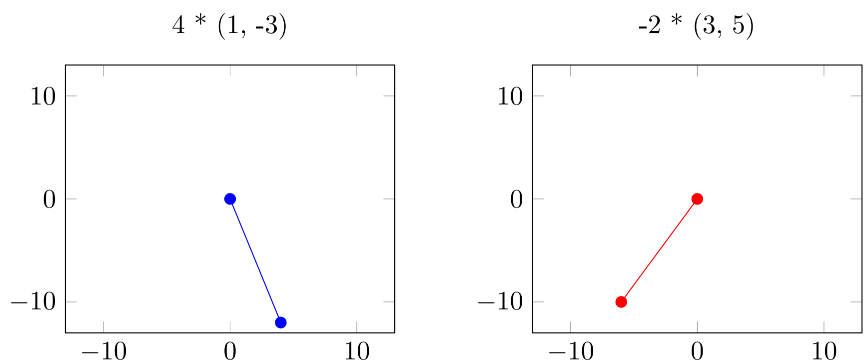
1.1 Consider

$$4\begin{bmatrix}1\\-3\end{bmatrix} \text{ and } -2\begin{bmatrix}3\\5\end{bmatrix}$$

1.2 Graphical representation unscaled

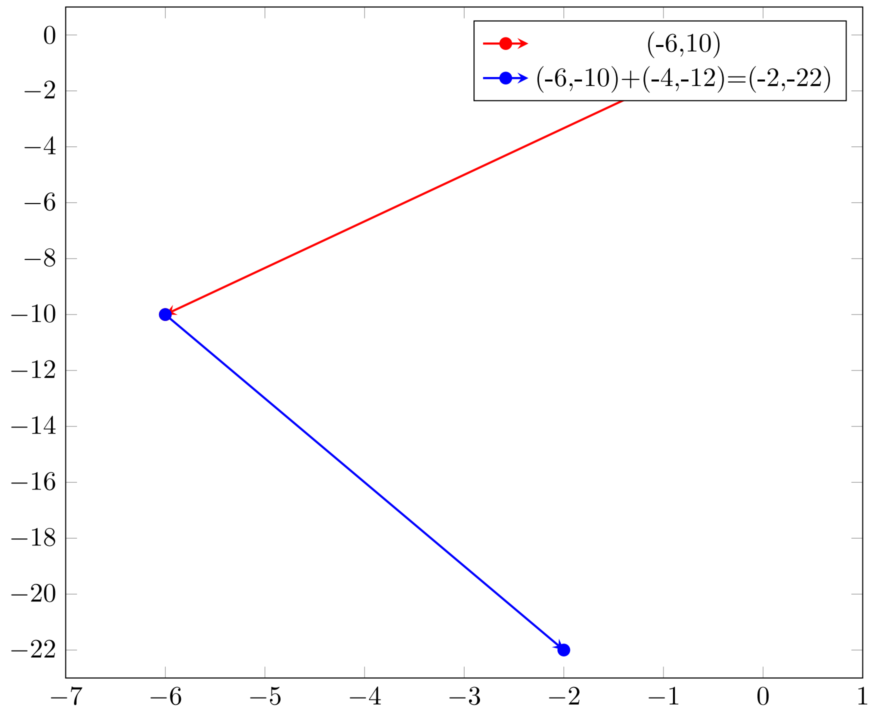


1.3 Graphical representation scaled



1.4 Added together

Combined vectors



1.5 Changes

As can be seen in the graphs above. The coefficient before each of the column matrices can be seen to scale any offset. For example, were we to take negative one times a matrix, then the offset would extend as far in the negative directions as a matrix extended in the positives. When we scale them by a multiplier and then add (subtract) from one another.

2 Question 2

2.1 Quick reiteration

Having found that we were unable to reach old man Gauss's house using only one form of transport, justify this conclusion with two approaches.

2.2 First approach

Induction For this to be true we'd, much like before, need a coefficient with which we'd be able to multiply by our column vector to get to $\begin{bmatrix}107\\64\end{bmatrix}$

$$c_1 \begin{bmatrix}3\\1\end{bmatrix} = \begin{bmatrix}107\\64\end{bmatrix}$$

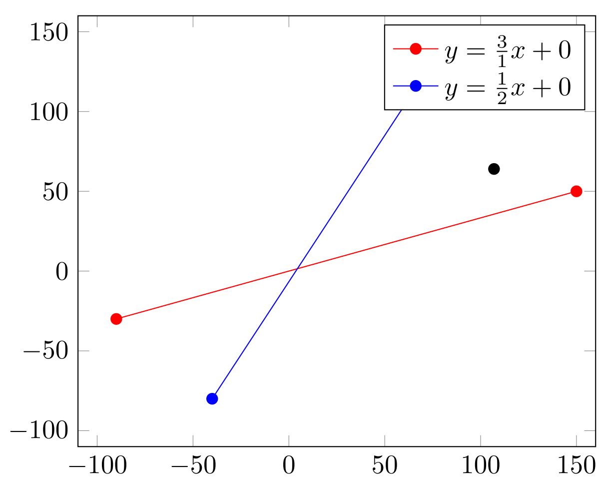
$$c_2 \begin{bmatrix}1\\2\end{bmatrix} = \begin{bmatrix}107\\64\end{bmatrix}$$

However, there simply are not c_1 or $c_2 \in \mathbb{R}$ that satisfy either equation. It is therefor impossible to reach his home.

2.3 Second approach

Visual We can illustrate this as a form of argument. As $\begin{bmatrix}3\\1\end{bmatrix}$ forms a line $y = \frac{3}{1}x + 0$ That is, by scaling, we can point the end of this vector to any point on the line defined. However $(107, 64)$ is not on that line. No possible scaling could result in a point that is not on the line.

3, 1 as a line



Analysis As can be seen as depicted, either vector, when scaled, is insufficient to reach the point. It is only when we combine the two that we can reach both old man Gauss's house and the entirety of $\mathbb{R}^2$