

1 Intuition

The task we are given here is to take two modes of transportation, a hoverboard that moves 3 units east and 1 north, and a magic carpet that moves 2 units east and 1 north. From this we are to determine whether or not some combination of these two modes of transportation will allow us to arrive at old man Gauss's house at 107 units east and 64 miles north of our original position.

2 Further Details

We can represent this mathematically using column vectors and coefficients.

$$T_C = \begin{bmatrix} 3 \\ 1 \end{bmatrix}, T_M = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, T = \begin{bmatrix} 107 \\ 64 \end{bmatrix}$$

$$T = c_1 T_C + c_2 T_M$$

From here we simply want to find two corresponding values for the coefficients c_1 and c_2

$$c_1 \begin{bmatrix} 3 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 107 \\ 64 \end{bmatrix}$$

$$c_1 3 + c_2 1 = 107$$

$$c_1 2 + c_2 1 = 64$$

So, we can deduce with a little algebra that $c_1 = 30$ and $c_2 = 17$ and that

$$30 \begin{bmatrix} 3 \\ 1 \end{bmatrix} + 17 \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 107 \\ 64 \end{bmatrix}$$