1 Question 1

Given We are given three different vectors to try to map through a given function. We are then to plot everything.

1. $\mathbf{A} = \begin{bmatrix} 3 & 0 \\ 0 & -2 \end{bmatrix}$
2. $T : \mathbb{R}^2 \to \mathbb{R}^2$
3. $T(\mathbf{x}) = \mathbf{Ax}$

Find

1. The image of u under T where $\mathbf{u} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$
2. The image of c under T where $\mathbf{v} = \begin{bmatrix} 0 \\ 1.5 \end{bmatrix}$
3. The image of $\mathbf{u} + \mathbf{v}$

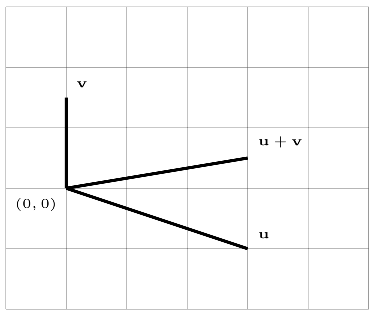
1.1 Work

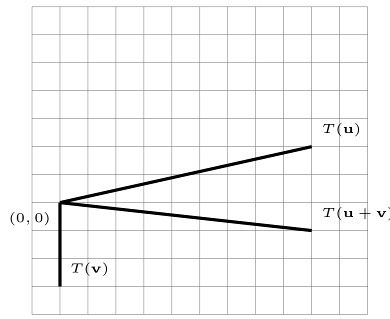
Images The image under is just going to be the result of having the function applied. We can do as such like so, for each of the given terms

1. $T(\mathbf{u}) = A\mathbf{u} = \begin{bmatrix} 3 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} 3 \\ -1 \end{bmatrix} = \begin{bmatrix} 9 \\ 2 \end{bmatrix}$
2. $T(\mathbf{v}) = A\mathbf{v} = \begin{bmatrix} 3 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} 0 \\ 1.5 \end{bmatrix} = \begin{bmatrix} 0 \\ -3 \end{bmatrix}$
3. $T(\mathbf{u} + \mathbf{v}) = A(\mathbf{u} + \mathbf{v}) = \begin{bmatrix} 3 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} 0 \\ 1.5 \end{bmatrix} = \begin{bmatrix} 9 \\ -1 \end{bmatrix}$

1.2 Illustration

Scaling Throughout we should expect to see a few things. The origin should remain constant, and relationships about perpendicularity and being parallel should contiue to hold.





2 Question 2

Given the matrix A:

$$A = \begin{bmatrix} 1 & -5 & -7 \\ -3 & 7 & 5 \end{bmatrix}$$

Define the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^2$ by $T(\mathbf{x}) = A\mathbf{x}$.

2.1 Prompts

(a) Find the image under T of $\mathbf{u} = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$ So. We have a 2x3 and we're multiplying by a 3x1. The result is going to be a 2x1

Checking solution

$$\begin{bmatrix} 1 & -5 & -7 \\ -3 & 7 & 5 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 4 \\ -4 \end{bmatrix}$$

(b) Find a vector $\mathbf{x}$ whose image under T is $\mathbf{b} = \begin{bmatrix} -12 \\ 12 \end{bmatrix}$

Thinking We can se up the system of equations so that we're solving for this. As long as the solution $y \in \text{span}\{\mathbf{u}, \mathbf{v}\}$

Solution

$$\begin{bmatrix} 1 & -5 & -7 \\ -3 & 7 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -12 \\ 12 \end{bmatrix}$$

$$\begin{aligned} x_1 - 5x_2 - 7x_3 &= -12 \\ -3x_1 + 7x_2 + 5x_3 &= 12 \end{aligned}$$

$$\begin{aligned} x_1 + 3x_3 &= 3 \\ x_2 + 2x_3 &= 3 \end{aligned}$$

Particular solution Let $x_3 = 0$ then $x_1 = 3$ and $x_2 = 3$.

Checking solution

$$\begin{bmatrix} 1 & -5 & -7 \\ -3 & 7 & 5 \end{bmatrix} \begin{bmatrix} 3 \\ 3 \\ 0 \end{bmatrix} = \begin{bmatrix} -12 \\ 12 \end{bmatrix}$$