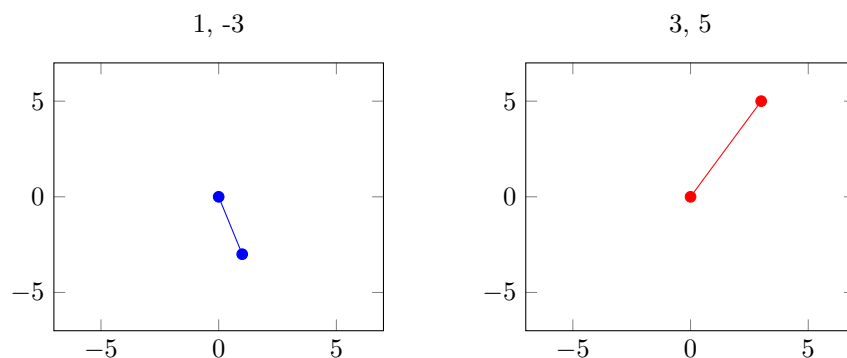


## 1 Question 1

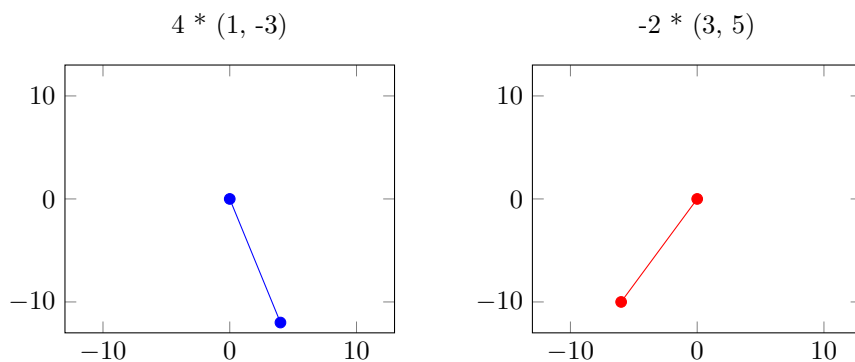
### 1.1 Consider

$$4 \begin{bmatrix} 1 \\ -3 \end{bmatrix} \text{ and } -2 \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

### 1.2 Graphical representation unscaled



### 1.3 Graphical representation scaled



### 1.4 Changes

As can be seen in the graphs above. The coefficient before each of the column matrices can be seen to scale any offset. For example, were we to take negative one times a matrix, then the offset would extend as far in the negative directions as a matrix extended in the positives.

## 2 Question 2

### 2.1 Quick reiteration

Having found that we were unable to reach old man Gauss's house using only one form of transport, justify this conclusion with two approaches.

### 2.2 First approach

**Induction** For this to be true we'd, much like before, need a coefficient with which we'd be able to multiply by our column vector to get to  $\begin{bmatrix} 107 \\ 64 \end{bmatrix}$

$$c_1 \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 107 \\ 64 \end{bmatrix}$$

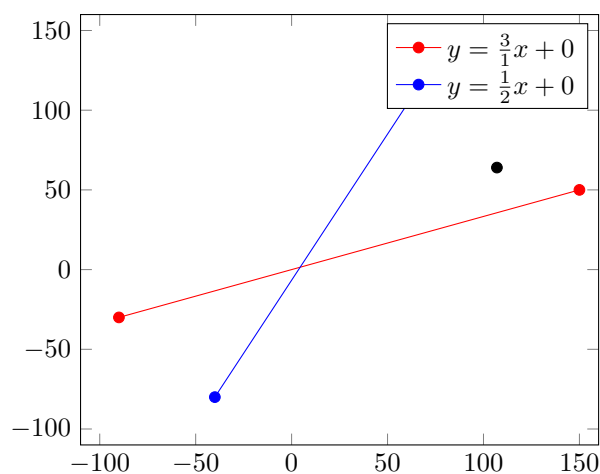
$$c_2 \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 107 \\ 64 \end{bmatrix}$$

However, there simply are not  $c_1$  or  $c_2 \in \mathbb{R}$  that satisfy either equation. It is therefor impossible to reach his home.

### 2.3 Second approach

**Visual** We can illustrate this as a form of argument. As  $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$  forms a line  $y = \frac{3}{1}x + 0$  That is, by scaling, we can point the end of this vector to any point on the line defined. However  $(107, 64)$  is not on that line. No possible scaling could result in a point that is not on the line.

3, 1 as a line



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**Analysis** As can be seen as depicted, either vector, when scaled, is insufficient to reach the point. It is only when we combine the two that we can reach both old man Gauss's house and the entirety of  $\mathbb{R}^2$